2010 – CA

QUESTION BOOKLET CODE **A**

Test Paper Code : CA

Time : 3 Hours

Max. Marks : 300

INSTRUCTIONS

A. General :

1. This Question Booklet is your Question Paper.
2. This Question Booklet contains **24** pages and has **100** questions.
3. The Question Booklet **Code** is printed on the right-hand top corner of this page.
4. The Question Booklet contains blank spaces for your rough work. No additional sheets will be provided for rough work.
5. **Clip board, log tables, slide rule, calculator, cellular phone or electronic gadgets in any form are NOT allowed.**
6. Write your **Name** and **Registration Number** in the space provided at the bottom.
7. All answers are to be marked only on the machine gradable Optical Response Sheet **(ORS)** provided along with this booklet, as per the instructions therein.
8. The Question Booklet along with the Optical Response Sheet **(ORS)** must be handed over to the Invigilator before leaving the examination hall.
9. Refer to **Special Instructions/Useful Data** on reverse of this sheet.

B. Filling-in the ORS :

10. Write your Registration Number in the boxes provided on the upper left-hand-side of the **ORS** and darken the appropriate bubble under each digit of your Registration Number using a **HB pencil**.
11. Ensure that the **code** on the **Question Booklet** and the **code** on the **ORS** are the same. If the codes do not match, report to the Invigilator immediately.
12. On the lower-left-hand-side of the **ORS**, write your Name, Registration Number, and Name of the Test Centre and put your signature in the appropriate box with ball-point pen. Do not write these anywhere else.

C. Marking of Answers on the ORS :

13. Each question has **4 choices** for its answer : (A), (B), (C) and (D). Only **ONE** of them is the correct answer.
14. On the right-hand-side of **ORS**, for each question number, darken with a **HB Pencil** ONLY one bubble corresponding to what you consider to be the most appropriate answer, from among the four choices.
15. There will be **negative marking** for wrong answers.

MARKING SCHEME :

(a) For each correct answer, you will be awarded **3 (Three)** marks.

(b) For each wrong answer, you will be awarded **–1 (Negative one)** mark.

(c) Multiple answers to a question will be treated as a wrong answer.

(d) For each un-attempted question, you will be awarded **0 (Zero)** mark.

Name							
Registration Number							

A

Special Instructions / Useful Data

$\mathbb{N}$ denotes the set of natural numbers.

$\mathbb{Z}$ denotes the set of integers.

$\mathbb{Q}$ denotes the set of rational numbers.

$\mathbb{R}$ denotes the set of real numbers.

$\bar{x}$ denotes complement of a Boolean variable x.

LPP denotes Linear Programming Problem.

Max f denotes Maximum of f.

Min f denotes Minimum of f.

f' denotes derivative of f.

$E(X)$ denotes the expected value of a random variable X.

$Var(X)$ denotes the variance of a random variable X.

$P(A \mid B)$ denotes the conditional probability of A given B.

For all **C programs**, assume that all standard library functions are accessible.

DO NOT WRITE ON THIS PAGE

A

Q.1 If (2, 1) is a critical point of $f(x, y)$ and $f_{xx}(2,1)\, f_{yy}(2,1) - [f_{xy}(2,1)]^2 < 0$, then

(A) (2, 1) is a saddle point
(B) (2, 1) is a point of local maximum
(C) (2, 1) is a point of local minimum
(D) further investigation is required to determine the nature of the point

Q.2 If $f_x(x, y) = 2xy + y^2$, $f_y(x, y) = x^2 + 2xy$ and $f(-1, -1) = 5$, then

(A) $f(x, y) = x^2y + xy^2 + 5$ (B) $f(x, y) = x^2y + xy^2 - 5$
(C) $f(x, y) = x^2y + xy^2 + 7$ (D) $f(x, y) = x^2y + xy^2 - 7$

Q.3 If $f(x, y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2} & \text{if} \quad (x, y) \neq (0, 0), \\ 0 & \text{if} \quad (x, y) = (0, 0), \end{cases}$

then

(A) $f_x(0, 0) = 0$, $f_y(0, 0) = 1$ (B) $f_x(0, 0) = 1$, $f_y(0, 0) = 0$
(C) $f_x(0, 0) = 1$, $f_y(0, 0) = -1$ (D) $f_x(0, 0) = -1$, $f_y(0, 0) = 1$

Q.4 If $f(x, y) = x^3 \sin^{-1}\left(\dfrac{y}{x}\right) - y^3 \cos^{-1}\left(\dfrac{x}{y}\right)$, $x > 0$, $y > 0$, then $x^2 \dfrac{\partial^2 f}{\partial x^2} + 2xy \dfrac{\partial^2 f}{\partial x\, \partial y} + y^2 \dfrac{\partial^2 f}{\partial y^2}$ is equal to

(A) $2f$ (B) $3f$ (C) $5f$ (D) $6f$

Q.5 The value of the integral $\displaystyle\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\sin y}{y}\, dy\, dx$ is

(A) 0 (B) $\dfrac{1}{2}$ (C) 1 (D) 2

Q.6 The volume of the solid bounded by the planes $x + 2y + z = 2$, $x = 2y$, $x = 0$ and $z = 0$ is

(A) $\displaystyle\int_0^1 \int_0^{2-2y} \int_0^{2-x-2y} dz\, dx\, dy$ (B) $\displaystyle\int_0^1 \int_{\frac{x}{2}}^{1-\frac{x}{2}} \int_0^{2-x-2y} dz\, dy\, dx$

(C) $\displaystyle\int_0^1 \int_0^{2y} \int_0^{2-x-2y} dz\, dx\, dy$ (D) $\displaystyle\int_0^1 \int_0^{\frac{1}{2}} \int_0^{2-x-2y} dz\, dy\, dx$

Q.7 The area of the region bounded by the curves $r = 1$ and $r^2 = \cos 2\theta$, $0 \le \theta \le \frac{\pi}{2}$, is

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{8}$

Q.8 If $f(x) = \int_a^{x^2} t(t-1)\,dt$, then

(A) f has a local maximum at $x = 0$ and a local minimum at $x = 1$
(B) f has local minima at $x = 0$ and $x = 1$
(C) f has a local maximum at $x = 1$ and a local minimum at $x = 0$
(D) f has local maxima at $x = 1$ and $x = 0$

Q.9 If $f(x) = ax^3 + bx^2 + x + 1$ has a local maximum value 3 at $x = -2$, then

(A) $a = \frac{3}{4}, b = \frac{5}{2}$ (B) $a = \frac{3}{2}, b = \frac{5}{4}$ (C) $a = \frac{3}{4}, b = \frac{5}{4}$ (D) $a = \frac{3}{2}, b = \frac{5}{2}$

Q.10 If a real valued function f is given by $\int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x} + b$, $x > 0$, where $a > 0$ and b are real constants, then $f(4)$ is equal to

(A) 4 (B) 6 (C) 8 (D) 10

Q.11 The integral $\int_1^{\infty} \frac{\log_e x}{x}\,dx$

(A) converges to e
(B) converges to $\frac{1}{e}$
(C) converges to 1
(D) diverges

Q.12 Let $I = \int_0^2 \int_{\sqrt{4-y^2}}^{\sqrt{9-y^2}} 2xy\,dx\,dy + \int_2^3 \int_0^{\sqrt{9-y^2}} 2xy\,dx\,dy$. Then using the transformation $x = r\cos\theta$, $y = r\sin\theta$, integral I is equal to

(A) $\int_0^{\frac{\pi}{2}} \int_0^3 r^3 \sin 2\theta\,dr\,d\theta$ (B) $\int_0^{\frac{\pi}{2}} \int_0^2 r^3 \sin 2\theta\,dr\,d\theta$

(C) $\int_0^{\frac{\pi}{2}} \int_2^3 r^3 \sin 2\theta\,dr\,d\theta$ (D) $\int_0^{\frac{\pi}{2}} \int_2^3 r^2 \sin 2\theta\,dr\,d\theta$

Q.13 Using Taylor's polynomial of the first degree, the approximate value of $\sqrt{(3.01)^2 + (3.98)^2}$ is equal to

(A) 4.99 (B) 4.95 (C) 5.01 (D) 4.92

Q.14 If $z = y + f(x^2 - y^2)$, where f is differentiable, then $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y}$ is equal to

(A) x^2y (B) xy^2 (C) $2xy$ (D) x

Q.15 If $z = f(x, y)$, where $x = g(t)$, $y = h(t)$, $g(3) = 2$, $g'(3) = 5$, $h(3) = 7$, $h'(3) = -4$, $f_x(2, 7) = 6$, $f_y(2, 7) = -8$, then the value of $\frac{dz}{dt}$ at $t = 3$ is

(A) 62 (B) 60 (C) 72 (D) 52

Q.16 Consider the following C Program

```
#include<stdio.h>
void main()
{  int a[] = {10,20,30,40};
    int j;
    int *p = a+3;
    for (j=3; j>=0; j--)
      {
          printf("%d   ", --*p);
              p=p-1;
       }
}
```

The output of the above program is

(A) 9 19 29 39 (B) 39 29 19 9 (C) 40 30 20 10 (D) 38 28 18 8

Q.17 Consider the following C program fragment:

```
int a=100, b=20;
int c,d,e;
c = a&b;
d = a|b;
e = c&d;
printf("%d", e);
```

The output of the above fragment is

(A) 116 (B) 100 (C) 20 (D) 4

Q.18 Consider the following C program fragment:

```
void get(int n)
 {
    if (n<1)
      return;
     get(n-1);
      printf("%d", n);
     get(n-2);
 }
```

The value returned by the function call `get(4)` is

(A) 3421211 (B) 3421121 (C) 1231412 (D) 1234121

Q.19 Binary equivalent of the hexadecimal number B81F is

(A) 1011100000011111 (B) 1011101000011011
(C) 1011110110011111 (D) 1011111000011001

Q.20 Consider the following C program fragment:

```
int i=10, m, n;
    m=++i;
printf("%d,%d,", m, i);
    n=i++;
printf("%d,%d", n, i);
```

The output of the above fragment is

(A) 11, 11, 11, 12 (B) 11, 11, 12, 12 (C) 11, 12, 12, 11 (D) 11, 11, 12, 11

Q.21 Consider the following C program fragment:

```
int i;
int a[5]={1000,800,600,400,200};
for (i=0; i<5; i++)
  printf("%d ", i[a]);
```

Which of the following is true about this fragment?

(A) Error in the definition of array i
(B) Execution results in an infinite loop
(C) The output is 1000 800 600 400 200
(D) The output is 1001 801 601 401 201

Q.22 Decimal value of $(122)_{16} \div (22)_8$ lies in the interval

(A) (15.5, 16) (B) (16, 16.5) (C) (16.5, 17) (D) (17, 17.5)

Q.23 Which one of the following units **CANNOT** be used to measure the speed of computer?

(A) MIPS (B) MFLOPS (C) FLOPS (D) BAUD

Q.24 Two's complement of the binary number 1011.01 is

(A) 0100.10 (B) 0100.11 (C) 1011.10 (D) 0100.01

Q.25 Flip-Flop circuits can be used for

(A) scaling (B) rectification (C) modulation (D) counting

Q.26 What will the following C statement print?

```
printf("%d", ++8);
```

(A) 8 (B) 9
(C) 7 (D) an error message

Q.27 The following C program segment

```
int a=4, b=6;
printf("%d", a==b);
```

(A) prints 1 (B) prints 0
(C) gives run time error (D) gives compile time error

Q.28 Consider the following C program fragment:

```
int i
char str[6]="sachin";
for (i=0; i<5; i+=2)
printf("%c", *(1+str+i));
```

The output from the above program will be

(A) sci (B) aci (C) ahn (D) ahi

Q.29 Windows operating system released in 2009 has been named as

(A) Windows Vista (B) Windows 7
(C) Windows 8 (D) Windows XP++

Q.30 Consider the following C program fragment:

```
void main()
{ int a[5]={50,60};
   printf("\n%d   %d   %d", a[2],a[3],a[4]);
}
```

The output of the above program will be

(A) Garbage values
(B) 50 60 60
(C) 50 60 0
(D) 0 0 0

Q.31 Consider the following C program fragment:

```
void main()
 {
   int x=20, y=50;
   if (x<y) return (x=x+y);
   else printf("Z1");
   printf("Z2");
 }
```

Then,

(A) the output will be Z1Z2
(B) the output will be Z1
(C) the output will be Z2
(D) no output will be displayed

Q.32 The optimal solution of the LPP

Maximize $f = 3x_1 - 2x_2$

subject to $-2x_1 + 3x_2 \le 6,$
$3x_1 + 2x_2 \le 6,$
$2x_1 - x_2 = 2,$
$x_1, x_2 \ge 0,$

is

(A) $x_1 = \frac{10}{7}, x_2 = \frac{6}{7}, Max\ f = \frac{18}{7}$
(B) $x_1 = 2, x_2 = 0, Max\ f = 6$
(C) $x_1 = 1, x_2 = 0, Max\ f = 3$
(D) $x_1 = 2, x_2 = 1, Max\ f = 4$

Q.33 The optimal solution(s) of the LPP

Maximize $f = x_1 - x_2$

subject to $x_1 - x_2 \le 1,$

$2x_1 + 3x_2 \ge 6,$

$x_1, x_2 \ge 0,$

(A) is unbounded

(B) are infinitely many along $x_1 - x_2 = 1$ with $x_1 \ge \frac{9}{5}$, $Max\ f = 1$

(C) are infinitely many along $x_1 - x_2 = 2$ with $x_1 \ge \frac{9}{5}$, $Max\ f = 2$

(D) $x_1 = 1,\ x_2 = 0,\ Max\ f = 1$

Q.34 The optimal solution(s) of the LPP

Minimize $f = 3x_1 + 2x_2$

subject to $2x_1 + x_2 \ge 2,$

$3x_1 + 2x_2 \le 6,$

$x_1, x_2 \ge 0,$

(A) is unique (B) is bounded

(C) are infinitely many along $3x_1 + 2x_2 \le 6$ (D) does not exist

Q.35 The LPP formulation of the unconstrained optimization problem

Maximize $y = \min \{|3x_1 + 7x_2|, |3x_1 - 7x_2|\};\ x_1, x_2 \ge 0,$

is

(A) $Max\ y$ such that $3x_1 + 7x_2 - y \ge 0, 3x_1 - 7x_2 - y \ge 0, 3x_1 - 7x_2 + y \le 0; x_1, x_2, y \ge 0$

(B) $Max\ y$ such that $3x_1 + 7x_2 + y \le 0, 3x_1 - 7x_2 - y \ge 0, 3x_1 - 7x_2 + y \le 0; x_1, x_2, y \ge 0$

(C) $Max\ y$ such that $3x_1 + 7x_2 - y \ge 0, 3x_1 - 7x_2 + y \ge 0, 3x_1 - 7x_2 + y \le 0; x_1, x_2, y \ge 0$

(D) $Max\ y$ such that $3x_1 + 7x_2 - y \ge 0, 3x_1 - 7x_2 - y \ge 0, 3x_1 - 7x_2 - y \le 0; x_1, x_2, y \ge 0$

Q.36 Let

$S_1 = \{(x_1, x_2) : 2x_1 + x_2 = 2, x_1 \ge 0, x_2 \in \mathrm{R}\}$, $S_2 = \{(x_1, x_2) : 2x_1 + x_2 = 2, x_1 \in \mathrm{R}, x_2 \in \mathrm{R}\}$.

Then the set $S_1 \cup S_2$ is

(A) convex and unbounded (B) not convex but bounded

(C) both convex and bounded (D) neither convex nor bounded

Q.37 With the conversion rate 1 U.S.D. = 48 INR, ten million U.S.D. is equivalent to

(A) 0.48 crores INR
(B) 4.80 crores INR
(C) 48 crores INR
(D) 48,000 lacs INR

Q.38 Which one of the following cricketers did **NOT** play in the final match of 1983 Prudential World Cup won by India?

(A) Sunil Gavaskar
(B) Mohinder Amarnath
(C) Krishnamachari Srikkanth
(D) Dilip Vengsarkar

Q.39 Consider the following lists:

List I	List II
1. Ustad Alla Rakha	K. Saxophone
2. Ustad Bismillah Khan	L. Tabla
3. Kunnakudi Vaidyanathan	M. Shehnai
4. Kadri Gopalanath	N. Violin

Then the correct match is

(A) (1, L), (2, M), (3, K), (4, N)
(B) (1, N), (2, M), (3, L), (4, K)
(C) (1, N), (2, L), (3, M), (4, K)
(D) (1, L), (2, M), (3, N), (4, K)

Q.40 Who among the following is **NOT** related to space exploration?

(A) Kalpana Chawla
(B) Bachendri Pal
(C) Rakesh Sharma
(D) Sunita Williams

Q.41 A tap fills a water tank in 20 minutes. Another tap fills the same tank in 30 minutes. If both taps are opened simultaneously, then the time taken (in minutes) to fill the tank is

(A) 10 (B) 12 (C) 16 (D) 20

Q.42 Who gave the slogan "Jai Jawan, Jai Kisan"?

(A) Netaji Subhash Chandra Bose
(B) Charan Singh
(C) Lal Bahadur Shastri
(D) Morarji Desai

Q.43 Which one of the following is **NOT** a permanent storage device?

(A) Pen drive (B) Hard disk (C) Compact disk (D) RAM

Q.44 National Anthem of India was written by

(A) Mohammed Iqbal
(B) C. Rajagopalachari
(C) Rabindranath Tagore
(D) Bankim Chandra Chatterjee

Q.45 Who among the following has **NOT** been the Secretary General of United Nations Organization?

(A) Kofi Annan
(B) Ban Ki Moon
(C) U. Thant
(D) F. Mitterand

Q.46 The first Indian to win the World Junior Badminton Championship is

(A) Saina Nehwal
(B) Sania Mirza
(C) Pullela Gopichand
(D) Prakash Padukone

Q.47 The average marks of a class of 25 students in a class test is 80. On rechecking of records, it is found that marks of 2 students which were actually 85 and 90 have been wrongly entered as 65 and 55 respectively. The correct average will be

(A) 81.40 (B) 81.70 (C) 83.50 (D) 82.20

Q.48 If the following table

p		*r*	
*		*s*	
	s		*q*
	**		*r*

is to be filled with the alphabets from the set {*p*, *q*, *r*, *s*} such that each row and each column has distinct alphabets, then * and ** are respectively

(A) *r* and *p* (B) *q* and *q* (C) *r* and *q* (D) *q* and *p*

Q.49 In the sequence 1, 3, 5, 4, 8, 9, 12, 17, *x*, ..., ..., the value of *x* is

(A) 19 (B) 20 (C) 21 (D) 29

Q.50 Consider the following C program fragment:

```
void main()
{
int p,b,c,d,e;
int a=2;
p=10;
c=p*p;

switch(p<<a)
  {
   case 20: {b=p*a; printf("%d", b); break;}
   case 40: {b=p*a*a; printf("%d", b); break;}
   case 80: {b=p*a*a*a; printf("%d", b); break;}
    default: {b=p*a*a*a*a; printf("%d", b); break;}
  }

}
```

The output of the above program fragment yields

(A) 20 (B) 40 (C) 80 (D) 160

Q.51 Consider the following lists:

List I	List II
1. FIFO	P. Linear Programming
2. LIFO	Q. Queue
3. OS	R. Stack
4. LINDO	S. Unix

The correct match is

(A) (1, R), (2, P), (3, S), (4, Q) (B) (1, R), (2, Q), (3, S), (4, P)
(C) (1, Q), (2, R), (3, S), (4, P) (D) (1, Q), (2, P), (3, S), (4, R)

Q.52 **http** stands for

(A) hyper text transfer protocol (B) hyper text transmission protocol
(C) high transfer transport protocol (D) hyper transfer text protocol

Q.53 Consider the following lists:.

List I	List II
1. Procedural Oriented Language	P. COBOL
2. Object Oriented Programming	Q. HTML
3. Business Oriented Language	R. C++
4. Web Page	S. Pascal

The correct match is

(A) (1, S), (2, Q), (3, P),(4, R) (B) (1, S), (2, R), (3, P),(4, Q)
(C) (1, R), (2, S), (3, P),(4, Q) (D) (1, S), (2, P), (3, Q),(4, R)

Q.54 Let a_1, a_2 and a_3 be chosen randomly in Boolean algebra. The probability that $a_3 = 0$, given that $(a_1 \bullet a_2) \oplus (\overline{a}_1 \bullet a_3) = 0$, is

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Q.55 Let $T : \mathbb{N} \cup \{0\} \to \mathbb{N} \cup \{0\}$ be defined as follows.

$$T(0) = 0,\ T(1) = 1 \text{ and for } n \geq 2,\ T(n) = \begin{cases} 2T\left(\frac{n}{2}\right) + T(n-2), & \text{if } n \text{ is even} \\ 2T\left(\frac{n-1}{2}\right) + T(n-2), & \text{if } n \text{ is odd} \\ 0, & \text{otherwise} \end{cases}$$

Then $T(7)$ is

(A) 13 (B) 12 (C) 7 (D) 6

Q.56 Which one of the following is **NOT** a search engine?

(A) Zing (B) Google (C) Yahoo (D) Bing

Q.57 Which one of the following matrices **CANNOT** be obtained by elementary row operations on the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{pmatrix}$?

(A) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ (B) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{pmatrix}$ (C) $\begin{pmatrix} 3 & 0 & 1 \\ 2 & 2 & 2 \\ 1 & 3 & 1 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 3 & 1 \\ 2 & 2 & 2 \\ 3 & 1 & 3 \end{pmatrix}$

Q.58 Consider the non-homogeneous system $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{A}$ is a square matrix of order n. If the matrix $\mathbf{A}$ and the augmented matrix $(\mathbf{A} \mid \mathbf{b})$ have the same rank r where $r < n$, then the system has

(A) no solution
(B) a unique solution
(C) exactly two solutions
(D) infinite number of solutions

Q.59 Let W be the subspace of $\mathbb{R}^4$ given by $W = \{(x, y, z, w) : x + z + w = 0, y + z + w = 0\}$. Then the dimension of W is

(A) 4 (B) 3 (C) 2 (D) 1

Q.60 Eigenvectors of the matrix $\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ corresponding to the eigenvalues $(1+i)$ and $(1-i)$ are respectively (where $i = \sqrt{-1}$)

(A) $\begin{pmatrix} -1 \\ i \end{pmatrix}$ and $\begin{pmatrix} i \\ 1 \end{pmatrix}$
(B) $\begin{pmatrix} 1 \\ -i \end{pmatrix}$ and $\begin{pmatrix} i \\ -1 \end{pmatrix}$
(C) $\begin{pmatrix} i \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -i \\ 1 \end{pmatrix}$
(D) $\begin{pmatrix} -i \\ 1 \end{pmatrix}$ and $\begin{pmatrix} i \\ 1 \end{pmatrix}$

Q.61 Let N be a nilpotent matrix of order 4 with real entries. Then which one of the following statements is true about eigenvalues of N?

(A) All eigenvalues are non-zero real numbers
(B) All eigenvalues are purely imaginary
(C) Zero is the only eigenvalue
(D) At least one eigenvalue is real and at least one eigenvalue has non-zero imaginary part

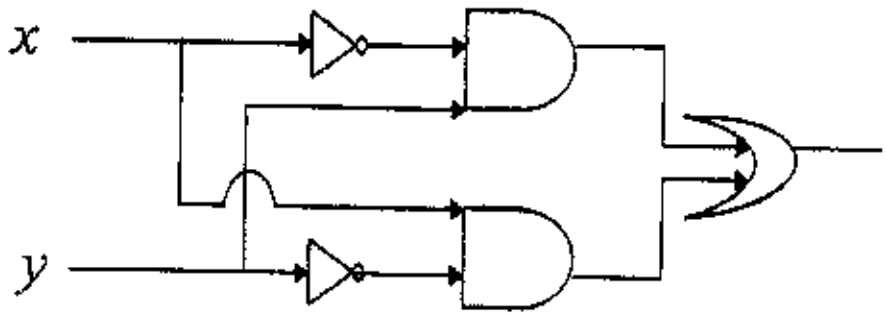
Q.62 The nullity of the matrix

$$\begin{pmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{pmatrix}$$

is

(A) 1 (B) 2 (C) 3 (D) 4

Q.63 The Boolean expression for the logic circuit



is

(A) $x \cdot y$ (B) $x \oplus y$

(C) $\overline{x} \oplus y$ (D) $x \oplus y \oplus (x \cdot y)$

Q.64 The Boolean expression $\overline{(\overline{x} + \overline{y})} \oplus \overline{(x \oplus y)}$ is equivalent to

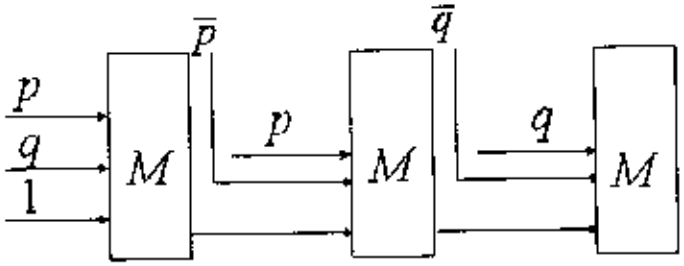
(A) OR gate (B) NAND gate

(C) NOR gate (D) XOR gate

Q.65 Let S be the set of all four variable Boolean functions f having the property $f(x_1, x_2, x_3, x_4) = f(x_2, x_3, x_4, x_1) = f(x_3, x_4, x_1, x_2) = f(x_4, x_1, x_2, x_3)$. Then the cardinality of S is

(A) 2^8 (B) 2^6 (C) 2^5 (D) 2^4

Q.66 In Boolean algebra, the majority function $M(x, y, z)$ is equal to 1 if at least two of x, y and z are equal to 1. Then the output of the following circuit



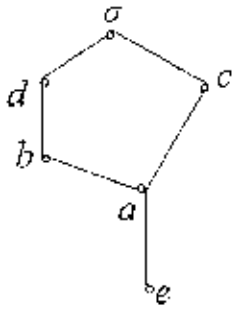
is

(A) $p+q+p\bullet q$ (B) $p+q+\overline{p}\bullet\overline{q}$ (C) $p\bullet q+\overline{p}$ (D) $p\bullet q+\overline{p}\bullet\overline{q}$

Q.67 In Boolean algebra, if $p\bullet\overline{q}+\overline{p}\bullet q=r$, then $p\bullet\overline{r}+\overline{p}\bullet r$ is equal to

(A) p (B) $\overline{q}$ (C) r (D) q

Q.68 Consider the lattice given by the following Hasse diagram:



This lattice is

(A) distributive (B) non-distributive
(C) complemented (D) a Boolean algebra

Q.69 Let $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$. Then $\nabla(\vec{r}\bullet\nabla(\vec{r}\bullet\vec{r}))$ is equal to

(A) $2\vec{r}$ (B) $4\vec{r}$ (C) $\frac{\vec{r}}{2}$ (D) $\frac{\vec{r}}{4}$

Q.70 For any two unit vectors $\vec{a}$ and $\vec{b}$, $|\vec{a}\times\vec{b}|^2$ is equal to

(A) $1+(\vec{a}\bullet\vec{b})^2$ (B) $(\vec{a}\bullet\vec{b})^2-1$ (C) $1-(\vec{a}\bullet\vec{b})^2$ (D) $(\vec{a}\bullet\vec{b})^2$

Q.71 Let P be the point (3, 4, 1). Let L be the line through P parallel to the vector $\vec{i}+\vec{j}+\vec{k}$. If Q is a point on L in the first octant such that $|\overrightarrow{PQ}|^2 = 48$, then Q is

(A) (7, 8, 5) (B) (8, 7, 5) (C) (5, 8, 7) (D) (8, 5, 7)

Q.72 The unit tangent vectors to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at the point (1, 0) are

(A) $\pm\left(\frac{3\vec{i}+4\vec{j}}{5}\right)$ (B) $\pm\left(\frac{3\vec{i}-4\vec{j}}{5}\right)$ (C) $\pm\left(\frac{4\vec{i}+3\vec{j}}{5}\right)$ (D) $\pm\left(\frac{4\vec{i}-3\vec{j}}{5}\right)$

Q.73 Consider the differential equation $\frac{dy}{dx} - y = -y^2$. Then $\lim_{x\to\infty} y(x)$ is equal to

(A) -1 (B) 0 (C) 1 (D) ∞

Q.74 The solution of the differential equation $\frac{d^2y}{dx^2} - y = e^x$ satisfying $y(0) = 0$ and $\frac{dy}{dx}(0) = \frac{3}{2}$ is

(A) $y(x) = \sinh x + \frac{x}{2}e^x$ (B) $y(x) = x\cosh x + \frac{x}{2}e^x$

(C) $y(x) = \sinh x - \frac{x}{2}e^x$ (D) $y(x) = 2x\cosh x - \frac{x}{2}e^x$

Q.75 The general solution of the differential equation $\frac{d^3y}{dx^3} - 9\frac{dy}{dx} = \cos x$ is

(A) $y(x) = C_1e^{3x} + C_2e^{-3x} + C_3 + \frac{1}{10}\sin x$

(B) $y(x) = C_1e^{3x} + C_2e^{-3x} + C_3 - \frac{1}{10}\sin x$

(C) $y(x) = C_1e^{3x} + C_2e^{-3x} + C_3 + \frac{1}{10}\cos x$

(D) $y(x) = C_1e^{3x} + C_2e^{-3x} + C_3 - \frac{1}{10}\cos x$

Q.76 An integrating factor of the differential equation $2xy\,dx + (y^2 - x^2)\,dy = 0$ is

(A) y (B) $\frac{1}{y}$ (C) $\frac{1}{y^2}$ (D) $\frac{1}{y^3}$

Q.77 The differential equation representing all circles centred at (1, 0) is

(A) $x + y\frac{dy}{dx} = 1$ (B) $x - y\frac{dy}{dx} = 1$ (C) $y + x\frac{dy}{dx} = 1$ (D) $y - x\frac{dy}{dx} = 1$

Q.78 Consider the following data:

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	0.45	0.47	0.52	0.58	0.63

Using the central difference formulae for numerical differentiation, the values of $f'(0.3)$ and $f''(0.3)$ are respectively

(A) 0.85, 0.99 (B) 0.86, 0.99 (C) 0.86, 1.00 (D) 0.85, 1.00

Q.79 Consider the following forward difference table:

x	f	Δf	$\Delta^2 f$	$\Delta^3 f$
–2	–15			
		11		
–1	–4		y	
		x		z
0	0		16	
		20		
1	20			

The values of x, y and z are respectively

(A) 4, –7, 9 (B) –11, 22, 6 (C) 4, –3, 1 (D) 4, –7, 23

Q.80 Consider the non-linear equation $x^3 - 2x - 3 = 0$. If $x_0 = 2$ is the initial approximation of the root, then the value of the root at first iteration using the Newton-Raphson method yields

(A) 2.20 (B) 2.10 (C) 1.90 (D) 1.80

Q.81 For evaluating the integral $\int_2^{11} y\,dx$ by Simpson's one-third rule, the error term for $2 < w < 11$ is

(A) $-\frac{1}{20}h^4 y^{(iv)}(w)$ (B) $-\frac{3}{4}h^4 y^{(iv)}(w)$

(C) $-\frac{1}{12}h^4 y^{(iv)}(w)$ (D) $-\frac{1}{6}h^4 y^{(iv)}(w)$

Q.82 Consider the following table:

x	1	2	3
y	–5	0	7

Then by Lagrange's interpolation $y(1.5)$ is

(A) –2.50 (B) –2.75 (C) –2.25 (D) –3.25

Q.83 If A and B are two independent events and $P(A \mid B) = \frac{2}{5}$, $P(B) = \frac{1}{3}$, then $P(A \cup B)$ is

(A) $\frac{11}{15}$ (B) $\frac{3}{5}$ (C) $\frac{13}{15}$ (D) $\frac{2}{15}$

Q.84 Let $P(A) = \frac{3}{5}$, $P(B) = \frac{4}{5}$ and $P(A \cap B) = \frac{13}{25}$. Consider the following lists :

List I	**List II**
1. $P(A \mid B)$	P. $\frac{7}{20}$
2. $P(A \mid B^c)$	Q. $\frac{13}{20}$
3. $P(A^c \mid B)$	R. $\frac{3}{5}$
4. $P(A^c \mid B^c)$	S. $\frac{2}{5}$

Then the correct match is

(A) (1, Q), (2, S), (3, R), (4, P) (B) (1, S), (2, R), (3, P), (4, Q)
(C) (1, Q), (2, S), (3, P), (4, R) (D) (1, S), (2, R), (3, Q), (4, P)

Q.85 Eight couples are participating in a game. Four persons are chosen randomly. The probability that at least one couple will be among the chosen persons is

(A) $\frac{5}{13}$ (B) $\frac{1}{26}$ (C) $\frac{25}{26}$ (D) $\frac{2}{5}$

Q.86 Let X and Y be binomial random variables with the same number of trials such that $E(X) = 2E(Y)$ and $2Var(X) = Var(Y)$. Then the respective probabilities of successes are

(A) $\frac{5}{7}$ and $\frac{5}{14}$ (B) $\frac{2}{7}$ and $\frac{1}{7}$ (C) $\frac{6}{7}$ and $\frac{3}{7}$ (D) $\frac{3}{14}$ and $\frac{3}{28}$

Q.87 The random variable X follows the Poisson distribution with variance 3. Then $P(X = 1|X > 0)$ is

(A) $\frac{1-e^{-3}}{e^{-3}}$ (B) $\frac{1-e^{-\sqrt{3}}}{e^{-\sqrt{3}}}$ (C) $\frac{\sqrt{3}e^{-\sqrt{3}}}{1-e^{-\sqrt{3}}}$ (D) $\frac{3e^{-3}}{1-e^{-3}}$

Q.88 Let $S = \left\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{0, 1\}\right\}$. An element of S is chosen randomly. Then the probability that the chosen matrix is an invertible matrix is

(A) $\frac{3}{8}$ (B) $\frac{1}{2}$ (C) $\frac{5}{8}$ (D) $\frac{3}{4}$

Q.89 Let X denote the set of all real-valued functions defined on $\mathbb{Z}$. Define a relation ~ in X by $f \sim g$ if $f(0) \neq g(0)$. Then the relation ~ is

(A) both reflexive and symmetric
(B) neither reflexive nor symmetric
(C) transitive but not reflexive
(D) neither transitive nor reflexive

Q.90 Let $X = \{1, 2, 3, 4\}$. Then the total number of partitions of the set X is

(A) 5 (B) 12 (C) 15 (D) 16

Q.91 The total number of non-trivial proper subgroups of the group $\mathbb{Z}_{12}$ under addition modulo 12 is

(A) 4 (B) 5 (C) 6 (D) 7

Q.92 The total number of generators of the cyclic group $\mathbb{Z}_{21}$ under addition modulo 21 is

(A) 18 (B) 19 (C) 20 (D) 21

Q.93 Let S_5 denote the group of all permutations on the finite set $\{1, 2, 3, 4, 5\}$ under the operation of permutation multiplication. Then the order of the subgroup of S_5 generated by $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 4 & 1 & 2 \end{pmatrix}$ is

(A) 3 (B) 6 (C) 12 (D) 24

Q.94 The number of distinct group homomorphisms from $(\mathbb{Z}, +)$ onto $(\mathbb{Z}, +)$ is

(A) 1 (B) 2 (C) 3 (D) infinite

Q.95 Let $< (0, 2) >$ denote the subgroup generated by $(0, 2)$ in $\mathbb{Z}_4 \times \mathbb{Z}_8$. Then the order of $(3, 1) + < (0, 2) >$ in the quotient group $\mathbb{Z}_4 \times \mathbb{Z}_8 / < (0, 2) >$ is

(A) 4 (B) 8 (C) 16 (D) 32

Q.96 Let $\mathbf{R}$ be a commutative ring with unity of characteristic 3. For $a, b \in \mathbf{R}$, $(a+b)^6$ is equal to

(A) $\mathbf{0}$, the additive identity in the ring $\mathbf{R}$ (B) $a^6 + b^6$

(C) $a^6 - a^3b^3 + b^6$ (D) $a^6 + a^3b^3 + b^6$

Q.97 The remainder of 8^{31} when it is divided by 13 is

(A) 1 (B) 5 (C) 8 (D) 12

Q.98 The total number of linear maps from the vector space $R^3(R)$ to the vector space $R(R)$ which are **NOT** onto is

(A) 0 (B) 1 (C) 3 (D) infinite

Q.99 Let $M_5(\mathbb{R})$ denote the vector space consisting of all 5×5 matrices with real entries over the real number field. Let $W \subset M_5(\mathbb{R})$ be the subspace of all skew-symmetric matrices. Then the dimension of W is

(A) 10 (B) 15 (C) 20 (D) 25

Q.100 Consider the following subsets of R^2

$W_1 = \{ (x, y) : 2x + y = 0 \}$,
$W_2 = \{ (x, y) : xy = 0 \}$,
$W_3 = \{ (x, y) : \sin^2 x + \sin^2 y = 0 \}$,
$W_4 = \{ (x, y) : \sin^2 x + \cos^2 x = 1 \}$.

Which one of the following pairs has the property that they are **NOT** subspaces of R^2?

(A) W_1, W_2 (B) W_2, W_3 (C) W_3, W_4 (D) W_2, W_4

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