MATHEMATICS

Duration: Three Hours *Maximum Marks*:100

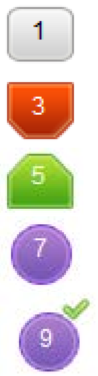
Please read the following instructions carefully:

General Instructions:

1. Total duration of examination is 180 minutes (3 hours).

2. The clock will be set at the server. The countdown timer in the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You will not be required to end or submit your examination.

3. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:



The Marked for Review status for a question simply indicates that you would like to look at that question again. ***If a question is answered and Marked for Review, your answer for that question will be considered in the evaluation.***

Navigating to a Question

4. To answer a question, do the following:
   a. Click on the question number in the Question Palette to go to that question directly.
   b. Select an answer for a multiple choice type question. Use the virtual numeric keypad to enter a number as answer for a numerical type question.
   c. Click on **Save and Next** to save your answer for the current question and then go to the next question.
   d. Click on **Mark for Review and Next** to save your answer for the current question, mark it for review, and then go to the next question.
   e. Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on its question number.

5. You can view all the questions by clicking on the **Question Paper** button. Note that the options for multiple choice type questions will not be shown.

Answering a Question

6. Procedure for answering a multiple choice type question:
   a. To select your answer, click on the button of one of the options
   b. To deselect your chosen answer, click on the button of the chosen option again or click on the **Clear Response** button
   c. To change your chosen answer, click on the button of another option
   d. To save your answer, you MUST click on the **Save and Next** button
   e. To mark the question for review, click on the **Mark for Review and Next** button**.** ***If an answer is selected for a question that is Marked for Review, that answer will be considered in the evaluation.***

7. Procedure for answering a numerical answer type question:
   a. To enter a number as your answer, use the virtual numerical keypad
   b. A fraction (eg.,-0.3 or -.3) can be entered as an answer with or without '0' before the decimal point
   c. To clear your answer, click on the **Clear Response** button
   d. To save your answer, you MUST click on the **Save and Next** button
   e. To mark the question for review, click on the **Mark for Review and Next** button**.** ***If an answer is entered for a question that is Marked for Review, that answer will be considered in the evaluation.***

8. To change your answer to a question that has already been answered, first select that question for answering and then follow the procedure for answering that type of question.

9. Note that ONLY Questions for which answers are saved or marked for review after answering will be considered for evaluation.

Paper specific instructions:

1. There are a total of 65 questions carrying 100 marks. Questions are of multiple choice type or numerical answer type. A multiple choice type question will have four choices for the answer with only **one** correct choice. For numerical answer type questions, the answer is a number and no choices will be given**. A number as the answer should be entered** using the virtual keyboard on the monitor.

2. Questions Q.1 – Q.25 carry 1mark each. Questions Q.26 – Q.55 carry 2marks each. The 2marks questions include two pairs of common data questions and two pairs of linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is not attempted, then the answer to the second question in the pair will not be evaluated.

3. Questions Q.56 – Q.65 belong to General Aptitude (GA) section and carry a total of 15 marks. Questions Q.56 – Q.60 carry 1mark each, and questions Q.61 – Q.65 carry 2marks each.

4. Questions not attempted will result in zero mark. Wrong answers for multiple choice type questions will result in **NEGATIVE** marks. For all 1 mark questions, ⅓ mark will be deducted for each wrong answer. For all 2 marks questions, ⅔ mark will be deducted for each wrong answer. However, in the case of the linked answer question pair, there will be negative marks only for wrong answer to the first question and no negative marks for wrong answer to the second question. There is no negative marking for questions of numerical answer type.

5. Calculator is allowed. Charts, graph sheets or tables are **NOT** allowed in the examination hall.

6. Do the rough work in the Scribble Pad provided.

USEFUL DATA FOR MA: MATHEMATICS

Notations and Symbols used

$\mathbb{R}$: The set of all real numbers.

$\mathbb{Z}$: The set of all integers.

$\mathbb{C}$: The set of all complex numbers.

$\mathbb{N}$: The set of all positive integers.

$\mathbb{Q}$: The set of all rational numbers.

$\mathbb{Z}_n$: The cyclic group of order n.

S_n: The group of permutations of the set $\{1, 2, \ldots, n\}$.

X^t: Transpose of the matrix X.

$\mathrm{Re}(z)$: Real part of the complex number z.

$\mathrm{Im}(z)$: Imaginary part of the complex number z.

$\overline{A}$: Closure of the set A.

A°: Interior of the set A.

$\langle a \rangle$: Ideal generated by an element a

$P(E)$: Probability of the event E.

$E[X]$: Expectation of the random variable X.

$\mathrm{Var}(X)$: Variance of the random variable X.

$\log x$: Natural logarithm of the positive real number x.

Q. 1 – Q. 25 carry one mark each.

Q.1 The possible set of eigen values of a 4×4 skew-symmetric orthogonal real matrix is

(A) $\{\pm i\}$ (B) $\{\pm i, \ \pm 1\}$ (C) $\{\pm 1\}$ (D) $\{0, \pm i\}$

Q.2 The coefficient of $(z-\pi)^2$ in the Taylor series expansion of

$$f(z) = \begin{cases} \dfrac{\sin z}{z-\pi} & \text{if } z \neq \pi \\ -1 & \text{if } z = \pi \end{cases}$$

around π is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{1}{6}$ (D) $-\frac{1}{6}$

Q.3 Consider $\mathbb{R}^2$ with the usual topology. Which of the following statements are **TRUE** for all $A, B \subseteq \mathbb{R}^2$?

P:$\overline{A \cup B} = \overline{A} \cup \overline{B}$.
Q:$\overline{A \cap B} = \overline{A} \cap \overline{B}$.
R:$(A \cup B)^\circ = A^\circ \cup B^\circ$.
S:$(A \cap B)^\circ = A^\circ \cap B^\circ$.

(A) P and R only (B) P and S only (C) Q and R only (D) Q and S only

Q.4 Let $f: \mathbb{R} \to \mathbb{R}$ be a continuous function with $f(1) = 5$ and $f(3) = 11$. If $g(x) = \int_1^3 f(x+t)dt$ then $g'(0)$ is equal to ______

Q.5 Let P be a 2×2 complex matrix such that trace$(P) = 1$ anddet$(P) = -6$. Then, trace$(P^4 - P^3)$ is ______

Q.6 Suppose that R is a unique factorization domain and that $a, b \in R$ are distinct irreducible elements. Which of the following statements is **TRUE**?

(A) The ideal $\langle 1 + a \rangle$ is a prime ideal
(B) The ideal $\langle a + b \rangle$ is a prime ideal
(C) The ideal $\langle 1 + ab \rangle$ is a prime ideal
(D) The ideal $\langle a \rangle$ is not necessarily a maximal ideal

Q.7 Let X be a compact Hausdorff topological space and let Y be a topological space. Let $f: X \to Y$ be a bijective continuous mapping. Which of the following is **TRUE**?

(A) f is a closed mapbut not necessarily an open map
(B) f is an open map but not necessarily a closed map
(C) f is both an open map and a closed map
(D) f need not be an open map or a closed map

Q.8 Consider the linear programming problem:

$$\begin{aligned} \text{Maximize} \quad & x + \frac{3}{2}y \\ \text{subject to} \quad & 2x + 3y \leq 16, \\ & x + 4y \leq 18, \\ & x \geq 0, y \geq 0. \end{aligned}$$

If S denotes the set of all solutions of the above problem, then

(A) S is empty (B) S is a singleton
(C) S is a line segment (D) S has positive area

Q.9 Which of the following groups has a proper subgroup that is **NOT** cyclic?

(A) $\mathbb{Z}_{15} \times \mathbb{Z}_{77}$
(B) S_3
(C) $(\mathbb{Z}, +)$
(D) $(\mathbb{Q}, +)$

Q.10 The value of the integral

$$\int_0^\infty \int_x^\infty \left(\frac{1}{y}\right) e^{-y/2}\, dy dx$$

is ______

Q.11 Suppose the random variable U has uniform distribution on $[0,1]$ and $X = -2\log U$. The density of X is

(A) $f(x) = \begin{cases} e^{-x} \text{if } x > 0 \\ 0 \quad\quad \text{otherwise} \end{cases}$

(B) $f(x) = \begin{cases} 2e^{-2x} \text{if } x > 0 \\ 0 \quad\quad \text{otherwise} \end{cases}$

(C) $f(x) = \begin{cases} \frac{1}{2} e^{-\frac{x}{2}} \text{if } x > 0 \\ 0 \quad\quad \text{otherwise} \end{cases}$

(D) $f(x) = \begin{cases} 1/2 & \text{if } x \in [0,2] \\ 0 & \text{otherwise} \end{cases}$

Q.12 Let f be an entire function on $\mathbb{C}$ such that $|f(z)| \le 100 \log|z|$ for each z with $|z| \ge 2$. If $f(i) = 2i$, then $f(1)$

(A) must be 2
(B) must be $2i$
(C) must be i
(D) cannot be determined from the given data

Q.13 The number of group homomorphisms from $\mathbb{Z}_3$ to $\mathbb{Z}_9$ is ______

Q.14 Let $u(x,t)$ be the solution to the wave equation

$$\frac{\partial^2 u}{\partial x^2}(x,t) = \frac{\partial^2 u}{\partial t^2}(x,t), \qquad u(x,0) = \cos(5\pi x), \qquad \frac{\partial u}{\partial t}(x,0) = 0.$$

Then, the value of $u(1,1)$ is ______

Q.15 Let $f(x) = \sum_{n=1}^{\infty} \frac{\sin(nx)}{n^2}$. Then

(A) $\lim_{x\to 0} f(x) = 0$
(B) $\lim_{x\to 0} f(x) = 1$
(C) $\lim_{x\to 0} f(x) = \pi^2/6$
(D) $\lim_{x\to 0} f(x)$ does not exist

Q.16 Suppose X is a random variable with $P(X = k) = (1-p)^k p$ for $k \in \{0,1,2,...\}$ and some $p \in (0,1)$. For the hypothesis testing problem

$$H_0: p = \frac{1}{2} H_1: p \neq \frac{1}{2}$$

consider the test "Reject H_0 if $X \leq A$ or if $X \geq B$", where$A < B$ are given positive integers. The type-I error of this test is

(A) $1 + 2^{-B} - 2^{-A}$
(B) $1 - 2^{-B} + 2^{-A}$
(C) $1 + 2^{-B} - 2^{-A-1}$
(D) $1 - 2^{-B} + 2^{-A-1}$

Q.17 Let G be a group of order 231. The number of elements of order 11 in G is ______

Q.18 Let $f: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by $f(x,y) = (e^{x+y}, e^{x-y})$. The area of the image of the region $\{(x,y) \in \mathbb{R}^2 : 0 < x, y < 1\}$ under the mapping f is

(A) 1 (B) $e - 1$ (C) e^2 (D) $e^2 - 1$

Q.19 Which of the following is a field?

(A) $\mathbb{C}[x] / \langle x^2 + 2 \rangle$

(B) $\mathbb{Z}[x] / \langle x^2 + 2 \rangle$

(C) $\mathbb{Q}[x] / \langle x^2 - 2 \rangle$

(D) $\mathbb{R}[x] / \langle x^2 - 2 \rangle$

Q.20 Let$x_0 = 0$. Define $x_{n+1} = \cos x_n$ for every $n \geq 0$. Then

(A) $\{x_n\}$ is increasing and convergent
(B) $\{x_n\}$ is decreasing and convergent
(C) $\{x_n\}$ is convergent and $x_{2n} < \lim_{m\to\infty} x_m < x_{2n+1}$ for every $n \in \mathbb{N}$
(D) $\{x_n\}$ is not convergent

Q.21 Let C be the contour $|z| = 2$ oriented in the anti-clockwise direction. The value of the integral $\oint_C z e^{3/z}\, dz$ is

(A) $3\pi i$ (B) $5\pi i$ (C) $7\pi i$ (D) $9\pi i$

Q.22 For each $\lambda > 0$, let X_λ be a random variable with exponential density $\lambda e^{-\lambda x}$ on$(0, \infty)$. Then, $\mathrm{Var}(\log X_\lambda)$

(A) is strictly increasing in λ
(B) is strictly decreasing in λ
(C) does not depend on λ
(D) first increases and then decreases in λ

Q.23 Let $\{a_n\}$ be the sequence of consecutive positive solutions of the equation $\tan x = x$ and let $\{b_n\}$ be the sequence of consecutive positive solutions of the equation $\tan\sqrt{x} = x$. Then

(A) $\sum_{n=1}^{\infty}\frac{1}{a_n}$ converges but $\sum_{n=1}^{\infty}\frac{1}{b_n}$ diverges (B) $\sum_{n=1}^{\infty}\frac{1}{a_n}$ diverges but $\sum_{n=1}^{\infty}\frac{1}{b_n}$ converges

(C) Both $\sum_{n=1}^{\infty}\frac{1}{a_n}$ and $\sum_{n=1}^{\infty}\frac{1}{b_n}$ converge (D) Both $\sum_{n=1}^{\infty}\frac{1}{a_n}$ and $\sum_{n=1}^{\infty}\frac{1}{b_n}$ diverge

Q.24 Let f be an analytic function on $\overline{D} = \{z \in \mathbb{C}: |z| \le 1\}$. Assume that $|f(z)| \le 1$ for each $z \in \overline{D}$. Then, which of the following is **NOT** a possible value of $\left(e^f\right)''(0)$?

(A) 2 (B) 6 (C) $\frac{7}{9}e^{1/9}$ (D) $\sqrt{2} + i\sqrt{2}$

Q.25 The number of non-isomorphic abelian groups of order 24 is ______

Q. 26 to Q. 55 carry two marks each.

Q.26 Let V be the real vector space of all polynomials in one variable with real coefficients and having degree at most 20. Define the subspaces

$$W_1 = \left\{p \in V : p(1) = 0, \quad p\left(\frac{1}{2}\right) = 0, \quad p(5) = 0, \quad p(7) = 0\right\},$$
$$W_2 = \left\{p \in V : p\left(\frac{1}{2}\right) = 0, \quad p(3) = 0, \quad p(4) = 0, \quad p(7) = 0\right\}.$$

Then the dimension of $W_1 \cap W_2$ is ______

Q.27 Let $f, g : [0,1] \to \mathbb{R}$ be defined by

$$f(x) = \begin{cases} x & \text{if } x = \frac{1}{n} \text{ for } n \in \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$

and

$$g(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \cap [0,1] \\ 0 & \text{otherwise.} \end{cases}$$

Then

(A) Both f and g are Riemann integrable
(B) f is Riemann integrable and g is Lebesgue integrable
(C) g is Riemann integrable and f is Lebesgue integrable
(D) Neither f nor g is Riemann integrable

Q.28 Consider the following linear programming problem:

$$\begin{array}{ll} \text{Maximize} & x + 3y + 6z - w \\ \text{subject to} & 5x + y + 6z + 7w \le 20, \\ & 6x + 2y + 2z + 9w \le 40, \\ & x \ge 0,\ y \ge 0,\ z \ge 0,\ w \ge 0. \end{array}$$

Then the optimal value is ______

Q.29 Suppose X is a real-valued random variable. Which of the following values **CANNOT** be attained by $E[X]$ and $E[X^2]$, respectively?

(A) 0 and 1 (B) 2 and 3 (C) $\frac{1}{2}$ and $\frac{1}{3}$ (D) 2 and 5

Q.30 Which of the following subsets of $\mathbb{R}^2$ is **NOT** compact?

(A) $\{(x,y) \in \mathbb{R}^2 : -1 \le x \le 1,\ y = \sin x\}$

(B) $\{(x,y) \in \mathbb{R}^2 : -1 \le y \le 1,\ y = x^8 - x^3 - 1\}$

(C) $\{(x,y) \in \mathbb{R}^2 : \ y = 0,\ \sin(e^{-x}) = 0\}$

(D) $\left\{(x,y) \in \mathbb{R}^2 : x > 0,\ y = \sin\left(\frac{1}{x}\right)\right\} \cap \left\{(x,y) \in \mathbb{R}^2 : x > 0,\ y = \frac{1}{x}\right\}$

Q.31 Let M be the real vector space of 2×3 matrices with real entries. Let $T: M \to M$ be defined by

$$T\left(\begin{bmatrix} x_1 & x_2 & x_3 \\ x_4 & x_5 & x_6 \end{bmatrix}\right) = \begin{bmatrix} -x_6 & x_4 & x_1 \\ x_3 & x_5 & x_2 \end{bmatrix}.$$

The determinant of T is ______

Q.32 Let $\mathcal{H}$ be a Hilbert space and let $\{e_n : n \ge 1\}$ be an orthonormal basis of $\mathcal{H}$. Suppose $T: \mathcal{H} \to \mathcal{H}$ is a bounded linear operator. Which of the following **CANNOT** be true?

(A) $T(e_n) = e_1$ for all $n \ge 1$

(B) $T(e_n) = e_{n+1}$ for all $n \ge 1$

(C) $T(e_n) = \sqrt{\frac{n+1}{n}}\, e_n$ for all $n \ge 1$

(D) $T(e_n) = e_{n-1}$ for all $n \ge 2$ and $T(e_1) = 0$

Q.33 The value of the limit

$$\lim_{n \to \infty} \frac{2^{-n^2}}{\sum_{k=n+1}^{\infty} 2^{-k^2}}$$

is

(A) 0 (B) some $c \in (0,1)$ (C) 1 (D) ∞

Q.34 Let $f: \mathbb{C} \setminus \{3i\} \to \mathbb{C}$ be defined by $f(z) = \frac{z-i}{iz+3}$. Which of the following statements about f is **FALSE**?

(A) f is conformal on $\mathbb{C} \setminus \{3i\}$
(B) f maps circles in $\mathbb{C} \setminus \{3i\}$ onto circles in $\mathbb{C}$
(C) All the fixed points of f are in the region $\{z \in \mathbb{C} : \operatorname{Im}(z) > 0\}$
(D) There is no straight line in $\mathbb{C} \setminus \{3i\}$ which is mapped onto a straight line in $\mathbb{C}$ by f

Q.35 The matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 3 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ can be decomposed uniquely into the product $A = LU$, where $L = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ and $U = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$. The solution of the system $LX = [1 \quad 2 \quad 2]^t$ is

(A) $[1 \quad 1 \quad 1]^t$ (B) $[1 \quad 1 \quad 0]^t$ (C) $[0 \quad 1 \quad 1]^t$ (D) $[1 \quad 0 \quad 1]^t$

Q.36 Let $S = \left\{x \in \mathbb{R} : x \geq 0,\ \sum_{n=1}^{\infty} x^{\sqrt{n}} < \infty\right\}$. Then the supremum of S is

(A) 1 (B) $\frac{1}{e}$ (C) 0 (D) ∞

Q.37 The image of the region $\{z \in \mathbb{C} : \mathrm{Re}(z) > \mathrm{Im}(z) > 0\}$ under the mapping $z \mapsto e^{z^2}$ is

(A) $\{w \in \mathbb{C} : \mathrm{Re}(w) > 0,\ \mathrm{Im}(w) > 0\}$ (B) $\{w \in \mathbb{C} : \mathrm{Re}(w) > 0, \mathrm{Im}(w) > 0, |w| > 1\}$

(C) $\{w \in \mathbb{C} : |w| > 1\}$ (D) $\{w \in \mathbb{C} : \mathrm{Im}(w) > 0, |w| > 1\}$

Q.38 Which of the following groups contains a unique normal subgroup of order four?

(A) $\mathbb{Z}_2 \oplus \mathbb{Z}_4$ (B) The dihedral group, D_4, of order eight

(C) The quaternion group, Q_8 (D) $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$

Q.39 Let B be a real symmetric positive-definite $n \times n$ matrix. Consider the inner product on $\mathbb{R}^n$ defined by $\langle X, Y\rangle = Y^t BX$. Let A be an $n \times n$ real matrix and let $T: \mathbb{R}^n \to \mathbb{R}^n$ be the linear operator defined by $T(X) = AX$ for all $X \in \mathbb{R}^n$. If S is the adjoint of T,then $S(X) = CX$ for all $X \in \mathbb{R}^n$,where C is the matrix

(A) $B^{-1}A^tB$ (B) BA^tB^{-1} (C) $B^{-1}AB$ (D) A^t

Q.40 Let X be an arbitrary random variable that takes values in$\{0, 1, \dots, 10\}$. The minimum and maximum possible values of the variance of X are

(A) 0 and 30 (B) 1 and 30 (C) 0 and 25 (D) 1 and 25

Q.41 Let M be the space of all4×3 matrices with entries in the finite field of three elements. Then the number of matrices of rank three in M is

(A) $(3^4 - 3)(3^4 - 3^2)(3^4 - 3^3)$
(B) $(3^4 - 1)(3^4 - 2)(3^4 - 3)$
(C) $(3^4 - 1)(3^4 - 3)(3^4 - 3^2)$
(D) $3^4(3^4 - 1)(3^4 - 2)$

Q.42 Let V be a vector space of dimension $m \geq 2$. Let $T: V \to V$ be a linear transformation such that $T^{n+1} = 0$ and $T^n \neq 0$ for some $n \geq 1$. Then which of the following is necessarily **TRUE**?

(A) Rank $(T^n) \leq$ Nullity (T^n) (B) trace $(T) \neq 0$
(C) T is diagonalizable (D) $n = m$

Q.43 Let X be a convex region in the plane bounded by straight lines. Let X have 7 vertices. Suppose$f(x, y) = ax + by + c$ has maximum value$M$ and minimum value$N$ on X and $N < M$. Let $S = \{P : P \text{ is a vertex of} X \text{ and } N < f(P) < M\}$. If S has n elements, then which of the following statements is **TRUE**?

(A) n cannot be 5 (B) n can be 2
(C) n cannot be 3 (D) n can be 4

Q.44 Which of the following statements are **TRUE**?

P: If $f \in L^1(\mathbb{R})$, then f is continuous.
Q: If $f \in L^1(\mathbb{R})$ and $\lim_{|x|\to\infty} f(x)$ exists, then the limit is zero.
R: If $f \in L^1(\mathbb{R})$, then f is bounded.
S: If $f \in L^1(\mathbb{R})$ is uniformly continuous, then $\lim_{|x|\to\infty} f(x)$ exists and equals zero.

(A) Q and S only (B) P and R only (C) P and Q only (D) R and S only

Q.45 Let u be a real valued harmonic function on $\mathbb{C}$. Let $g: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$g(x, y) = \int_0^{2\pi} u\big(e^{i\theta}(x + iy)\big) \sin\theta \, d\theta.$$

Which of the following statements is **TRUE**?

(A) g is a harmonic polynomial
(B) g is a polynomial but not harmonic
(C) g is harmonic but not a polynomial
(D) g is neither harmonic nor a polynomial

Q.46 Let $S = \{z \in \mathbb{C} : |z| = 1\}$ with the induced topology from $\mathbb{C}$ and let $f: [0,2] \to S$ be defined as $f(t) = e^{2\pi i t}$. Then, which of the following is **TRUE**?

(A) K is closed in $[0,2] \Longrightarrow f(K)$ is closed in S
(B) U is open in $[0,2] \Longrightarrow f(U)$ is open in S
(C) $f(X)$ is closed in $S \Longrightarrow X$ is closed in $[0,2]$
(D) $f(Y)$ is open in $S \Longrightarrow Y$ is open in $[0,2]$

Q.47 Assume that all the zeros of the polynomial $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ have negative real parts. If $u(t)$ is any solution to the ordinary differential equation

$$a_n \frac{d^n u}{dt^n} + a_{n-1} \frac{d^{n-1} u}{dt^{n-1}} + \cdots + a_1 \frac{du}{dt} + a_0 u = 0,$$

then $\lim_{t\to\infty} u(t)$ is equal to

(A) 0 (B) 1 (C) $n - 1$ (D) ∞

Common Data Questions

Common Data for Questions 48 and 49:

Let c_{00} be the vector space of all complex sequences having finitely many non-zero terms. Equip c_{00} with the inner product $\langle x, y\rangle = \sum_{n=1}^{\infty} x_n \overline{y_n}$ for all $x = (x_n)$ and $y = (y_n)$ in c_{00}. Define $f: c_{00} \to \mathbb{C}$ by $f(x) = \sum_{n=1}^{\infty} \frac{x_n}{n}$. Let N be the kernel of f.

Q.48 Which of the following is **FALSE**?

(A) f is a continuous linear functional
(B) $\|f\| \le \frac{\pi}{\sqrt{6}}$
(C) There does not exist any $y \in c_{00}$ such that $f(x) = \langle x, y\rangle$ for all $x \in c_{00}$
(D) $N^{\perp} \neq \{0\}$

Q.49 Which of the following is **FALSE**?

(A) $c_{00} \neq N$
(B) N is closed
(C) c_{00} is not a complete inner product space
(D) $c_{00} = N \oplus N^{\perp}$

Common Data for Questions 50 and 51:

Let $X_1,\ X_2,\ \ldots, X_n$ be an i.i.d. random sample from exponential distribution with mean μ. In other words, they have density

$$f(x) = \begin{cases} \frac{1}{\mu} e^{-x/\mu} \text{if } x > 0 \\ 0 \qquad \text{otherwise.} \end{cases}$$

Q.50 Which of the following is **NOT** an unbiased estimate of μ?

(A) X_1
(B) $\frac{1}{n-1}(X_2 + X_3 + \cdots + X_n)$
(C) $n \cdot (\min\{X_1, X_2, \ldots, X_n\})$
(D) $\frac{1}{n}\max\{X_1, X_2, \ldots, X_n\}$

Q.51 Consider the problem of estimating μ. The m.s.e (mean square error) of the estimate

$$T(X) = \frac{X_1 + X_2 + \cdots + X_n}{n+1}$$

is

(A) μ^2 (B) $\frac{1}{n+1}\mu^2$ (C) $\frac{1}{(n+1)^2}\mu^2$ (D) $\frac{n^2}{(n+1)^2}\mu^2$

Linked Answer Questions

Statement for Linked Answer Questions 52 and 53:

Let $X = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\} \cup ([-1,1] \times \{0\}) \cup (\{0\} \times [-1,1])$.
Let $n_0 = \max\{k : k < \infty$, there are k distinct points $p_1, \ldots, p_k \in X$ such that $X \setminus \{p_1, \ldots, p_k\}$ is connected$\}$

Q.52 The value of n_0 is ______

Q.53 Let $q_1, \ldots, q_{n_0+1}$ be $n_0 + 1$ distinct points and $Y = X \setminus \{q_1, \ldots, q_{n_0+1}\}$. Let m be the number of connected components of Y. The maximum possible value of m is ______

Statement for Linked Answer Questions 54 and 55:

Let $W(y_1, y_2)$ be the Wrönskian of two linearly independent solutionsy_1 and y_2 of the equation $y'' + P(x)y' + Q(x)y = 0$.

Q.54 The product $W(y_1, y_2)P(x)$ equals

(A) $y_2y_1'' - y_1y_2''$ (B) $y_1y_2' - y_2y_1'$
(C) $y_1'y_2'' - y_2'y_1''$ (D) $y_2'y_1' - y_1''y_2''$

Q.55 If $y_1 = e^{2x}$ and $y_2 = xe^{2x}$, then the value of $P(0)$ is

(A) 4 (B) -4 (C) 2 (D) -2

General Aptitude (GA) Questions

Q. 56 – Q. 60 carry one mark each.

Q.56 A number is as much greater than 75 as it is smaller than 117. The number is:

(A) 91 (B) 93 (C) 89 (D) 96

Q.57 The professor ordered to the students to go out of the class.
I: The professor; II: ordered to; III: the students to go; IV: out of the class

Which of the above underlined parts of the sentence is grammatically incorrect?

(A) I (B) II (C) III (D) IV

Q.58 Which of the following options is the closest in meaning to the word given below:

Primeval

(A) Modern (B) Historic
(C) Primitive (D) Antique

Q.59 Friendship, no matter how __________it is, has its limitations.

(A) cordial

(B) intimate

(C) secret

(D) pleasant

Q.60 Select the pair that best expresses a relationship similar to that expressed in the pair:
Medicine: Health

(A) Science: Experiment (B) Wealth: Peace
(C) Education: Knowledge (D) Money: Happiness

Q. 61 to Q. 65 carry two marks each.

Q.61 X and Y are two positive real numbers such that $2X + Y \leq 6$ and $X + 2Y \leq 8$. For which of the following values of (X, Y) the function $f(X, Y) = 3X + 6Y$ will give maximum value?

(A) (4/3, 10/3)

(B) (8/3, 20/3)

(C) (8/3, 10/3)

(D) (4/3, 20/3)

Q.62 If $|4X - 7| = 5$ then the values of $2\,|X| - |-X|$ is:

(A) 2, 1/3 (B) 1/2, 3 (C) 3/2, 9 (D) 2/3, 9

Q.63 Following table provides figures (in rupees) on annual expenditure of a firm for two years - 2010 and 2011.

Category	**2010**	**2011**
Raw material	5200	6240
Power & fuel	7000	9450
Salary & wages	9000	12600
Plant & machinery	20000	25000
Advertising	15000	19500
Research & Development	22000	26400

In 2011, which of the following two categories have registered increase by same percentage?

(A) Raw material and Salary & wages

(B) Salary & wages and Advertising

(C) Power & fuel and Advertising

(D) Raw material and Research & Development

Q.64 A firm is selling its product at Rs. 60 per unit. The total cost of production is Rs. 100 and firm is earning total profit of Rs. 500. Later, the total cost increased by 30%. By what percentage the price should be increased to maintained the same profit level.

(A) 5 (B) 10 (C) 15 (D) 30

Q.65 Abhishek is elder to Savar.
Savar is younger to Anshul.

Which of the given conclusions is logically valid and is inferred from the above statements?

(A) Abhishek is elder to Anshul

(B) Anshul is elder to Abhishek

(C) Abhishek and Anshul are of the same age

(D) No conclusion follows

END OF THE QUESTION PAPER