LU3IN005
Stats

TD5

29/10/2020

Exercice 28 : Salaires

$E(X_i) = 2000$

28.1) $P(X > a) \leq \frac{E(X)}{a}$ inégalité de Markov

$P(X > 2200) \leq \frac{2000}{2200} \simeq 0,90$

28.2) $P(\forall i,\ X_i > 2200)$

$= P(X_1 > 2200,\ X_2 > 2200, \ldots)$

$= \prod P(X_i > 2200) \leq (0,9)^n \mapsto 0$

28.3) $P(A) = 1 - P(B) \geqslant 1 - 0,9^n \geqslant 0,95$

$\Leftrightarrow 0,05 \geqslant 0,9^n$

$\Leftrightarrow \log(0,05) \geqslant n \log(0,9)$

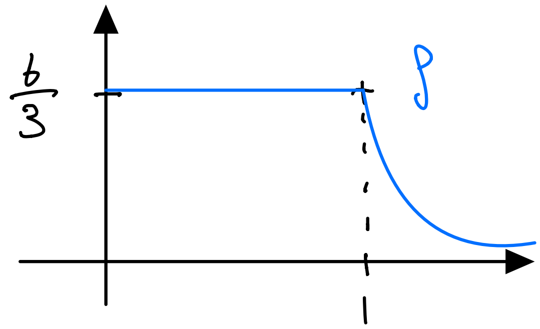
$\Leftrightarrow n \geqslant \frac{\log 0,05}{\log 0,9} \geqslant 31,43$

Exercice 29 : Loi à densité

$$f(x) = \begin{cases} 0 & \text{si } x < 0 \\ b/3 & 0 \leq x \leq 1 \quad \text{où } c \text{ cte} \\ c/x^4 & x \geqslant 1 \end{cases}$$

$(x^a)' = a x^{a-1}$

29.1) $x = 1$, $\frac{b}{3} = c \iff b = 3c$



$$\int_{-\infty}^{+\infty} f(x)\,dx = 1$$

$$\int_0^1 \frac{b}{3}\,dx + \int_1^{+\infty} \frac{c}{x^4}\,dx = 1$$

$$\frac{b}{3} + c\left[\frac{-x^{-3}}{3}\right]_1^{+\infty} = 1 \Rightarrow \frac{b}{3} + \frac{c}{3} = 1 \iff b + c = 3$$

or $b = 3c$ donc $c = \frac{3}{4}$ et $b = \frac{9}{4}$

29.2) $F(t) = \int_{-\infty}^{t} f(x)\,dx$

$t \leq 0$, $F(t) = 0$

$t \in [0;1]$, $F(t) = \frac{3}{4}t$

$t \geq 1$, $F(t) = F(1) + \int_1^t \frac{c}{x^4}\,dx$

$$= \frac{3}{4} + \left(\frac{t^{-3}}{3} - \frac{1}{3}\right)c$$

29.3) $P(X=2) = 0$

$P(0,5 < X < 2) = F(2) - F(0,5)$

$$P(X > 3 \mid X > 1) = \frac{P(X>3, X>1)}{P(X>1)}$$

$$= \frac{P(X>3)\,P(X>1)}{P(X>1)}$$ car indep

$$= P(X>3)$$

Exercice 30 : Marketing

30.1) $P(X=0, Y=0) = P(X=0)\,P(Y=0)$
$= e^{-2}e^{-1}$

Loi de Poisson : $P(X=k) = \dfrac{\lambda^k e^{-\lambda}}{k!}$ $\quad E(X)=\lambda$

$P(Y=0, X>0) = P(Y=0)\,P(X>0)$
$= e^{-1}(1-e^{-2})$

$P(X>0, Y>0) = (1-e^{-1})(1-e^{-2})$

30.2) $Z = 8X + 12Y$

$E(Z) = 8E(X) + 12E(Y)$
$= 16 + 12 = 28$

$E(X) = 2$
$E(Y) = 1$

$V(Z) = V(8X + 12Y)$
$= V(8X) + V(12Y)$ car X et Y indep
$= 8^2 \times 2 + 12^2$
$V(Z) = 272$

30.3.1) $W = \sum_{i=1}^{350} Z_i = 28$

$E(W) = 350\, Z_i = 9800$
$V(W) = 350^2\, Z_i = 3\,430\,000$

30.3.2) $P(9000 > W > 10600)$

$P(9000 - E(X) > W - E(X) > 10600 - E(X))$

$= P(9000 - 9800 > W - E(X) > 10600 - 9800)$

$= P(-800 > W - E(X) > 800)$

d'après l'inégalité de Tchebychev, on a :

$$P(|W - E(W)| \geqslant \lambda) \leqslant \frac{V(W)}{\lambda^2}$$

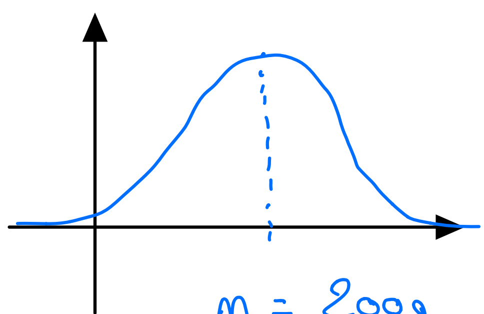
ici, $\lambda = 800$, on a donc :

$$P(W - E(W) > 800) \leqslant \frac{3\,430\,000}{800^2}$$

$$P(W - E(W) > 800) \leqslant \frac{3\,430\,000}{640\,000}$$

$$P(W - E(W) > 800) \leqslant 5,3$$

Exercice 27 : Loi Normale



27.1) $X \sim \mathcal{N}(200, 70)$

27.1.1) $P(X > 250)$

$$Z = \frac{X - 200}{70} \sim \mathcal{N}(0,1)$$

$$P\left(Z > \frac{250 - 200}{70}\right) = P\left(Z > \frac{5}{7}\right)$$

$= P(Z > 0,71)$

$= 0,24$

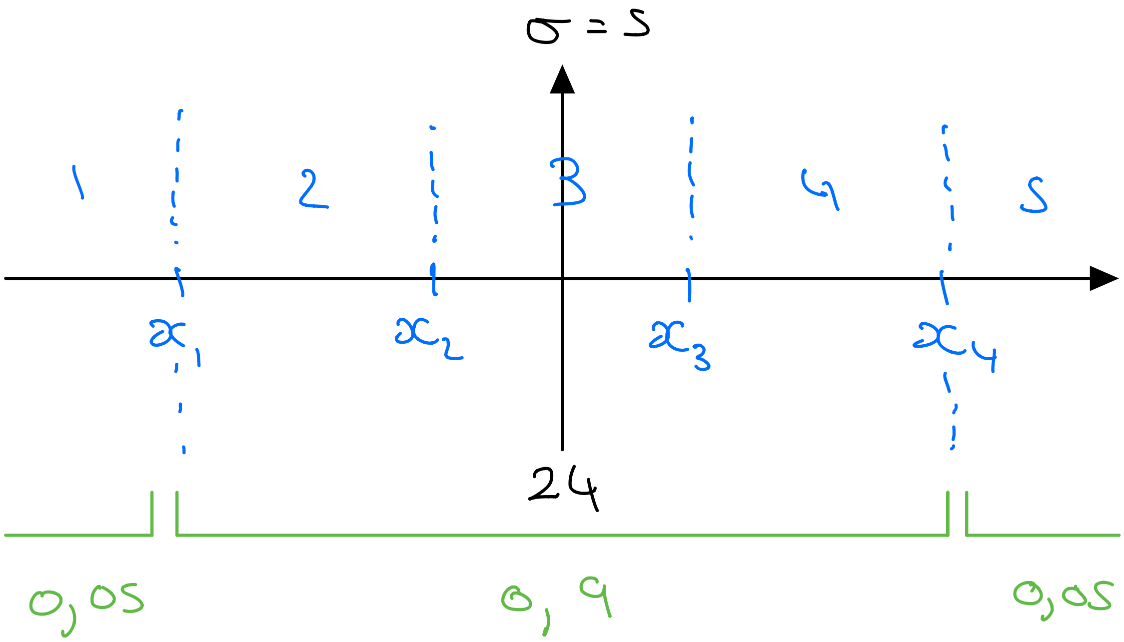
$$P(X \leqslant 180) = P\left(Z < \frac{180-250}{70}\right)$$

$$= P\left(Z \leqslant \frac{-2}{7}\right) = 1 - P\left(Z > \frac{2}{7}\right)$$

$$= 1 - P(Z > 0{,}26)$$

$$= 1 - 0{,}39 = 0{,}61$$

27.2)



27.2.1) $\dfrac{x_1 + x_4}{2} = \dfrac{x_2 + x_3}{2} = 24$

$$x_2 - x_1 = x_3 - x_2 = x_4 - x_3 = \ell$$

$$P(x_1 \leqslant X \leqslant x_4) = 0{,}9$$

$$\Rightarrow P(X > x_4) = 0{,}05$$

$$\Rightarrow P\left(\frac{X-24}{3} > \frac{x_4 - 24}{3}\right) = 0{,}05$$

d'après la table de la loi Normale, on a :

$$\frac{x_4 - 24}{3} = 1{,}64$$

$$\Leftrightarrow x_4 = 3 \times 1{,}64 + 24$$

donc $x_4 = 28{,}92$

comme $\frac{x_1 + x_4}{2} = 24$ alors $x_1 = 2 \times 24 - x_4$

donc $x_1 = 48 - 28,92 \Rightarrow x_1 = 19,08$

on a $\ell = \frac{x_4 - x_1}{3} = \frac{28,92 - 19,08}{3}$

donc $\ell = 3,28$

on sait que $x_2 - x_1 = \ell$, donc $x_2 = \ell + x_1$

donc $x_2 = 3,28 + 19,08$

$\Rightarrow x_2 = 22,36$

de m̂, $x_3 = x_1 + 2\ell$

$= 19,08 + 2 \times 3,28$

$x_3 = 25,64$

FIN