1) $|\psi\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$

P.T. :- $|\psi\rangle$ can't be decomposed as $|\psi_1\rangle \otimes |\psi_2\rangle$

Let $\alpha_1, \alpha_2, \beta_1, \beta_2 \in \mathbb{C}$ & $|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$;

$\boxed{|\alpha_1|^2 + |\beta_1|^2 = |\alpha_2|^2 + |\beta_2|^2 = 1}$ $\quad |\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$

& $|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$

$= \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle$

equating with $\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$;

$\alpha_1\alpha_2 = \frac{1}{\sqrt{2}}$; $\beta_1\beta_2 = \frac{1}{\sqrt{2}}$; $\alpha_1\beta_2 = \beta_1\alpha_2 = 0$

$\cancel{\Rightarrow |\alpha_1\alpha_2|^2 + |\beta_1\beta_2|^2 = \frac{1}{}}$

$|\alpha_1\alpha_2|^2 + |\alpha_1\beta_2|^2 = \frac{1}{2}$.

$\Rightarrow |\alpha_1|^2\left(|\alpha_2|^2 + |\beta_2|^2\right) = \frac{1}{2}$

$\Rightarrow |\alpha_1|^2 = \frac{1}{2}$ $\quad \left[|\alpha_2|^2 + |\beta_2|^2 = 1\right.$ as $|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle\left.\right]$

Similarly $|\beta_1\beta_2|^2 + |\beta_1\alpha_2|^2 = \frac{1}{2}$

$\Rightarrow |\beta_1|^2\left(|\alpha_1|^2 + |\beta_1|^2\right) = \frac{1}{2}$ .. (i)

as $|\psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$

hence, $|\alpha_1|^2 + |\beta_1|^2 = 1$

Now, $\cancel{\Rightarrow |\beta_1|^2 =}$ from (i); $|\beta_1|^2 = \frac{1}{2}$

$|\alpha_1\beta_2|^2 = 0$ but $|\alpha_1|^2 = \frac{1}{2} \Rightarrow |\beta_2|^2 = 0$

$|\beta_1\alpha_2|^2 = 0$ but $|\beta_1|^2 = \frac{1}{2} \Rightarrow |\alpha_2|^2 = 0$

$\Rightarrow |\alpha_2|^2 + |\beta_2|^2 = 0 \Rightarrow$ can't be possible as $|\alpha_2|^2 + |\beta_2|^2$ must be equal to 1

$\Rightarrow$ contradiction $\quad [|\psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle]$

$\Rightarrow |\psi\rangle$ can't be written as

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$

2) CCNOT gate

1st & 2nd bit 1 $\longrightarrow$ 3rd bit gets flipped

$$\left.\begin{array}{l}
|000\rangle \longrightarrow |000\rangle \\
|001\rangle \longrightarrow |001\rangle \\
|010\rangle \longrightarrow |010\rangle \\
|011\rangle \longrightarrow |011\rangle \\
|100\rangle \longrightarrow |100\rangle \\
|101\rangle \longrightarrow |101\rangle
\end{array}\right\} \text{stays the same}$$

$$\left.\begin{array}{l}
|110\rangle \longrightarrow |111\rangle \\
|111\rangle \longrightarrow |110\rangle
\end{array}\right\} \text{flips in these}$$

$$M_1 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{pmatrix}$$

CSWAP

$|101\rangle \longrightarrow |110\rangle$ & $|110\rangle \rightarrow |101\rangle$

all others stay same

$$M_2 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$