Taming the Zoo of Logical Systems by formalizing it

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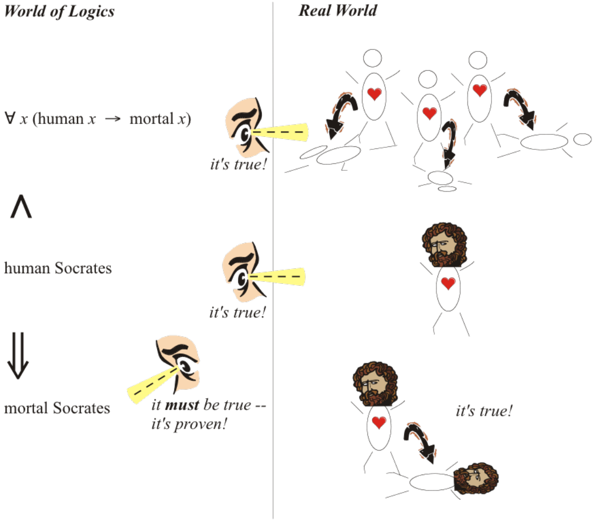
What is Logic?

- Logic $\widehat{=}$ formal languages, inference and their relation with the world
  - Formal language $\mathcal{FL}$: set of formulae ($2 + 3/7$, $\forall x.x + y = y + x$)
  - Formula: sequence/tree of symbols ($x, y, f, g, p, 1, \pi, \in, \neg, \wedge \forall, \exists$)
  - Model: things we understand (e.g. number theory)
  - Interpretation: maps formulae into models ($[\![\text{three plus five}]\!] = 8$)
  - Validity: $\mathcal{M} \models \mathbf{A}$, iff $[\![\mathbf{A}]\!]^{\mathcal{M}} = \mathsf{T}$ (five greater three is valid)
  - Entailment: $\mathbf{A} \models \mathbf{B}$, iff $\mathcal{M} \models \mathbf{B}$ for all $\mathcal{M} \models \mathbf{A}$. (generalize to $\mathcal{H} \models \mathbf{A}$)
  - Inference: rules to transform (sets of) formulae ($\mathbf{A}, \mathbf{A} \Rightarrow \mathbf{B} \vdash \mathbf{B}$)
  - Syntax: formulae, inference (just a bunch of symbols)
  - Semantics: models, interpr., validity, entailment (math. structures)
- Important Question: relation between syntax and semantics?

The miracle of logics

- Purely formal derivations are true in the real world!



Modeling Natural Language Semantics

- Problem: Find formal (logic) system for the meaning of natural language
- History of ideas
  - Propositional logic [ancient Greeks like Aristotle]
    **Every human is mortal*
  - First-Order Predicate logic [Frege $\leq$ 1900]
    **I believe, that my audience already knows this.*
  - Modal logic [Lewis18, Kripke65]
    **A man sleeps. He snores.* $((\exists X.\mathrm{man}(X) \wedge \mathrm{sleep}(X))) \wedge \mathrm{snore}(X)$
  - Various dynamic approaches (e.g. DRT, DPL)
    **Most men wear black*
  - Higher-order Logic, e.g. generalized quantifiers
  - ...

Natural Language Semantics?

$$
\begin{tikzcd}
\mathcal{M} = \langle \mathcal{D}, \mathcal{I} \rangle \arrow[r, dotted, "\text{induces}"] & \models \subseteq \mathcal{FL} \times \mathcal{FL} \arrow[d, leftrightarrow, "\models \equiv \vdash_{\mathcal{C}}?"'] \\
\mathcal{L} = \mathit{wff}(\Sigma) \arrow[u, "\mathcal{I}_{\varphi}"] \arrow[r, "\text{formulae}"] & \vdash_{\mathcal{C}} \subseteq \mathcal{FL} \times \mathcal{FL} \arrow[d, leftrightarrow, "\models_{\mathcal{NL}} \equiv \vdash_{\mathcal{C}}?"'] \\
\mathcal{NL} \arrow[u, "\text{Analysis}"] \arrow[r, dotted, "\text{induces}"] & \models_{\mathcal{NL}} \subseteq \mathcal{NL} \times \mathcal{NL}
\end{tikzcd}
$$

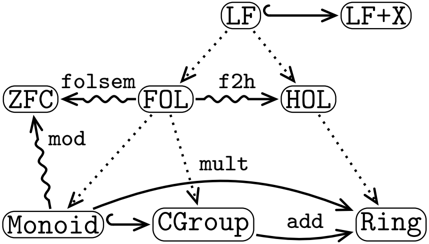
Comp Ling

Representation language (MMT)

- MMT = module system for mathematical theories
- Formal syntax and semantics
  - needed for mathematical interface language
  - but how to avoid foundational commitment?
- Foundation-independence
  - identify aspects of underlying language that are necessary for large scale processing
  - formalize exactly those, be parametric in the rest
  - observation: most large scale operations need the same aspects
- Module system
  - preserve mathematical structure wherever possible
  - formal semantics for modularity
- Web-scalable
  - build on XML, *OpenMath*, *OMDoc*
  - URI-based logical identifiers for all declarations
- Implemented in the MMT API system.

Representing Logics and Foundations as Theories

- **Example 0.1.** Logics and foundations represented as MMT theories



- **Definition 0.2.** Meta-relation between theories – special case of inclusion
- Uniform Meaning Space: morphisms between formalizations in different logics become possible via meta-morphisms.
- **Remark 0.3.** Semantics of logics as views into foundations, e.g., `folsem`.
- **Remark 0.4.** Models represented as views into foundations (e.g. ZFC)
- **Example 0.5.** $\mathtt{mod} := \{G \mapsto \mathbb{Z}, \circ \mapsto +, e \mapsto 0\}$ interprets Monoid in ZFC.