M3 Reserving Claim Counts

Topics in Insurance, Risk, and Finance [1]

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[1] References: Chapter 2 of Taylor (2000) | → latest slides

1. Exposure
   - General idea

General idea

Assume that we there exists $e(i)$ so that we can write

$$E[N(i,j)] \equiv e(i)\mu(j),$$

where (this is important), $\mu(j)$ does **not depend on** i.

- $e(i)$ will be called the **exposure** in period of occurrence i (for instance, "car-years")
- $\mu(j)$ may be interpreted as **relative claim frequency** in period j (per unit of exposure)
- The cumulative version

$$\frac{E[N(i,\cdot)]}{e(i)} = \mu(\cdot)$$

is the **relative frequency of claim occurence per period**.

In practice, this is not always achievable, that is,

$$\frac{E[N(i,j)]}{e(i)} = \mu(i,j),$$

with only *weak dependency* of $\mu(i,j)$ on i.

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2 IBNR claims

- Exposure based methods
- Normalised methods

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Allowing for an i effect in μ

Assume

$$\mu(i,j) = f(i)v(j),$$

where $f(i)$ is some known function (otherwise determined).

We have then

$$E[N(i,j)] = e(i)f(i)v(j).$$

We will discuss how to work with this.

Examples of $f(i)$

- A simple example could be:

$$f(i) = \alpha + \beta i,$$

which leads to a linear adjustment across rows (periods of origins i) to development "base frequency" $v(j)$ (for given development period j)
- It is unlikely to hold across all columns without modification, so one could extend this to

$$\mu(i,j) = f_j(i)v(j)$$

so that α and β will (potentially) depend on j as well
- This could lead to a highly over-parametrised model

- In practice, changes in "development speed" often occur in the first two development periods mostly, and in opposite direction (justifying a separate α and β).
- We could then use

$$\begin{aligned}
\mu(i,0) &= f_0(i) = \alpha_0 + \beta_0 i, \\
\mu(i,1) &= f_1(i) = \alpha_1 + \beta_1 i, \\
\mu(i,j) &= v(j), j = 2, 3, \ldots .
\end{aligned}$$

Estimating $N(i,j)$

Now, assume

$$N(i,j) \sim \text{Poisson}(e(i)f(i)v(j))$$

so that (assuming independence across periods of origin)

$$N(\cdot,j) \sim \text{Poisson}\left(v(j)\sum_i e(i)f(i)\right)$$

Then we can show that

$$\hat{v}(j) = \frac{N(\cdot,j)}{\sum_i e(i)f(i)}$$

is a maximum likelihood, minimum variance, unbiased, consistent estimator (in short, a good one!).

In the end, our estimator for $E[N(i,j)]$ is

$$\widehat{E[N(i,j)]} = e(i)f(i)\left[\sum_{j=I-i+1}^{I} + \sum j = I + 1^{\infty}\right]\hat{v}(j).$$

- the first $\sum$ include $\hat{v}\hat{}$s estimated from available data
- the second $\sum$ cannot be estimated from data, and will be extrapolated from the former set

Example

Taylor (2000), Table 2.1 and Table 2.2 provide an example of such calculations, where $f(i) = 1$.

2 IBNR claims

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Motivation

- In the previous section, one hoped that claim notification rates (as proportions of exposure) would constant across periods of origin i, or at least approximatively or predictively so.
- There may not always be such an exposure available.
- For instance, consider Public Liability of a manufacturer of toys
  - would time a good measure of exposure? or revenue?
  - this would unlikely to be satisfactory if the mix of business (which toys are sold and at what level) changes of time

The idea

- We keep the idea
$$E[N(i,j)] = \alpha(i)\mu(j).$$
- We will “anchor” our prediction on a subset S of existing data, say:
$$\sum_{m\in S} N(i,m),$$
where S is typicall the first m development periods.
- Then we assume that the ratio
$$\frac{E[N(i,j)]}{E\left[\sum_{m\in S} N(i,m)\right]} = \frac{\alpha(i)\mu(j)}{\alpha(i)\sum_{m\in S}\mu(m)} = \frac{\mu(j)}{\sum_{m\in S}\mu(m)},$$
which is independent of i, can be estimated from data and used to “complete the rectangle”.

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References I

Selected references:

Taylor, Greg. 2000. *Loss Reserving: An Actuarial Perspective*. Huebner International Series on Risk, Insurance and Economic Security. Kluwer Academic Publishers.