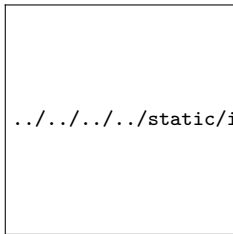


M5 Reserving Combination

Topics in Insurance, Risk, and Finance ¹

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- 1 Background
- 2 Combining the results of the different models
- 3 Allowance for prior expectations

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1 Background

- Summary
- Motivation
- Criteria for choice

Summary

We have seen covered a number of reserving methods:

- ① unadjusted chain ladder on paid losses
- ② inflation adjusted chain ladder on paid losses
- ③ unadjusted chain ladder on incurred losses
- ④ inflation adjusted chain ladder on incurred losses
- ⑤ separation
- ⑥ payments per claim incurred (PPCI)

The IBNR claims (counts) were estimated according to a number of methods, too:

- exposure
- normaliser
- chain ladder

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Motivation

- All those methods are producing outstanding liability \$ estimates in nominal terms.
- There are a number of differences:
 - whether inflation is accounted for explicitly (2, 4, 5, 6) or not (1, 3)
 - whether inflation is assumed for past data (2, 4), or a result of the method (1, 3, 5, 6)
 - whether paid (1, 2, 6) or incurred (3, 4, 5) losses are used
 - whether aggregate amounts (1, 2, 3, 4) or amounts per claim (5, 6) are used. The latter require an estimate of IBNR counts.
- All methods have strengths and weaknesses.

There is obviously a good mix of assumptions and approaches. Which one should we choose? Or should we combine them?

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Criteria for choice

Decision factors include:

- analytical properties of the various models
 - e.g., we know there has been a change in the past, that affected the development of claims. Can the method allow for that?
- average claim sizes for various periods of origin
 - e.g., do those average claims make sense, given our (also qualitative) knowledge of the claims development processes??
 - e.g., do trends in average claim size agree with our beliefs around claims inflation and superimposed inflation?
- relation of forecasts of liability to case estimates
 - e.g., they are not directly comparable, but the evolution of the ratio should be smooth enough.

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2 Combining the results of the different models

- Idea

Idea

- Let $\hat{P}_h^*(i, j)$ be the estimate of outstanding liability of model $h = 1, 2, \dots$, at end of development period j and for period of origin i .
- Assume the we want to combine such estimates (at the end of experience period k) as follows:

$$\overline{\hat{P}^*}(i, k) = \sum_h w_h(i) \overline{\hat{P}_h^*}(i, k),$$

where $\overline{\hat{P}_h^*}(i, k) = \hat{P}_h^*(i, k - i)$, $w_h(i)$ are weights allocated to to model h ,

$$\sum_h w_h(i) = 1.$$

- Note that in general those weights depend on i as well; different reserving models will typically perform better for different levels of maturity (in complex environments - understand “non chain ladder like”).

Those weights can be determined in different ways:

- Judgmentally, by considering the properties of the models available, and their respective strenghts and weaknesses for different i .
- With respect to some sort of objective criteria.
 - This is done to some extent in the book (Chapter 12).
 - This has been done a lot more rigorously only very recently by Avanzi et al. (2023) via ensembling, which was awarded the 2023 Hachemeister Prize by the American Casualty Actuarial Society (CAS).
 - It is still a relevant topic!

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3 Allowance for prior expectations

- Idea
- Choice of credibility weights
- Prior expectations

Idea

- Imagine one might to combine a “prior expectation” (or belief) with the estimate of liability provided by a method (or ensemble thereof).
- This can be done in a way which is routinely referred to as “credibility weighting” by actuaries:

$$\text{estimate} = [1 - z(i)]\overline{P}_0^*(i, k) + z(i)\overline{\hat{P}}^*(i, k),$$

where

- $\hat{P}^*(i, k)$ is the quantity defined earlier
- $\overline{P}_0^*(i, k)$ is the prior expectation (examples later), and
- $z(i)$ is the credibility assigned to the model estimate $\overline{\hat{P}}^*(i, k)$
- One probably should give more credibility to models in more mature years (small i).
- The formula above will yield equivalent results if $z(i)$ is applied on incurred amounts instead (see Taylor (2000)).

3 Allowance for prior expectations

- Idea
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- Prior expectations

Bornhuetter-Ferguson

Bornhuetter and Ferguson (1972) suggested the most simple approach, which is to use either

- $z(i) = 0$: outstanding liability is exclusively calculated on the basis of prior expectations; or
- $z(i) = 1$: outstanding liability is entirely based on models, ignoring prior expectations totally.

The “textbook”, “plain” vanilla Bornhuetter-Ferguson (BF):

- applies $z(i)$ on all or only some subset of the most immature years
- calculates the prior expectation based on premium and loss ratios

An example is provided below.

More generally

The following would make sense:

- $z(i) = 0$ when no information has been collected (start of development period 0);
- $z(i) = 1$ at the end of the running off period (when all claims and their costs are known and certain);
- some monotonic progression between those two extremes.

For instance, $1/\pi$ from chain ladder:

- It satisfies the criteria above:
- It is somewhat reflective of the amount of information gathered so far
- In particular, a highly leveraged line, which would benefit from averaging with prior expectation, will have a very low $z(I) = 1/\pi(0)$.

3 Allowance for prior expectations

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Prior expectations

In this section we review some possible choices for the “prior expectations”.

BF loss ratio

The loss method proceeds as follows. For each i

- Define $EP(i)$ as the gross aggregate premium earned for period of origin i .
- Define $C(i)$ as the aggregate sum of all payments made for period of origin i (the ultimate).
- The loss ratio is then defined as

$$LR(i) = \frac{C(i)}{EP(i)}.$$

The method projects $LR(i)$ from past values and infers $C(i)$ from observable $EP(i)$. We have then (assuming chain ladder development patterns)

$$\overline{P}_0^*(i, k) = LR(i)EP(i) \left(1 - \frac{1}{\pi(k)}\right)$$

which is typically applied with $z(i) = 1$ in immature (or all) years.

Example

Consider the following triangle (cumulative claims):

| Origin | EP | DY1 | DY2 | DY3 | DY4 |
|--------|-------|-----|-----|-----|-----|
| 2020 | 860 | 473 | 620 | 690 | 715 |
| 2021 | 940 | 512 | 660 | 750 | |
| 2022 | 980 | 611 | 700 | | |
| 2023 | 1,020 | 647 | | | |

It is assumed that the ultimate loss ratios for underwriting years 2021-2023 are expected to be in line with year 2020.

First calculate the development factors:

$$\hat{f}_1 = \frac{620 + 660 + 700}{473 + 512 + 611} = 1.2406$$

$$\hat{f}_2 = \frac{690 + 750}{620 + 660} = 1.125$$

$$\hat{f}_3 = \frac{715}{690} = 1.0362$$

Then calculate the loss ratio LR and the prior expectations of ultimate:

$$LR(1)EP(1) = LR \cdot 860 = 715 \implies LR = 0.8314$$

$$LR(2)EP(2) = LR \cdot 940 = 781.52$$

$$LR(3)EP(3) = LR \cdot 980 = 814.77$$

$$LR(4)EP(4) = LR \cdot 1020 = 848.02$$

Now, we have

$$\begin{aligned}\text{outstanding} &= \sum_{i=1}^4 \overline{P}_0^*(i, 4) \\ &= \sum_{i=1}^4 LR(i)EP(i) \left(1 - \frac{1}{\pi(i)}\right) \\ &= 0 + 27.30 + 115.82 + 261.64 \\ &= 404.76.\end{aligned}$$

Remember that π is normally defined as a $\pi(j)$, so that $\pi(1)$ applies to that year where we have only $j = 1$ cell available - the last row. Similarly, $\pi(3)$ is the one that applies to the amount in the diagonal that is at $j = 3$ - here 781.41.

Extensions

- The problem with the plain vanilla BF is that it does not specify objectively how $LR(i)$ is determined. A priori, this is too judgmental.
- There are a number of methods which try to make this choice more objective, such as “modified BF”, “Cape Cod”, ...

References I

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- Taylor, Greg. 2000. *Loss Reserving: An Actuarial Perspective*. Huebner International Series on Risk, Insurance and Economic Security. Kluwer Academic Publishers.