

Algorithms

Module Code: ELEE1147

Module Name: Programming for Engineers

Credits: 15

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O Notation

- **Big-O Notation (O -notation):**

- Represents the upper bound of the running time of an algorithm.
- Shows the worst-case complexity of an algorithm.

- **Omega Notation (Ω -notation):**

- Represents the lower bound of the running time of an algorithm.
- Provides the best case complexity of an algorithm.

- **Theta Notation (Θ -notation):**

- Theta notation encloses the function from above and below.
- Used for analysing the average-case complexity of an algorithm.

Why Big O?

Importance:

- Efficient algorithms are crucial in computer science and programming.
- Big O helps in quantifying and comparing algorithm efficiency.
- Allows for better decision-making in algorithm selection.

Analysing Algorithm Complexity

Factors Affecting Complexity:

- **Time Complexity:**
 - How the runtime of an algorithm increases with the input size.
- **Space Complexity:**
 - How the memory requirements of an algorithm scale with the input size.

Time Complexity

Time complexity represents the amount of time an algorithm takes to complete as a function of the input size.

- Constant Time $\implies O(1)$
- Logarithmic Time $\implies O(\log n)$
- Linear Time $\implies O(n)$
- Log-linear Time $\implies O(n \log n)$
- Quadratic Time $\implies O(n^2)$
- ...

Time Complexity Metrics

Big O Notation	n	$n \log n$	n^2	n^3	2^n	$n!$
n = 10	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	4 sec
n = 30	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 18 min	10^{25} years
n = 100	< 1 sec	< 1 sec	< 1 sec	1s	10^{17} years	Very Long Time
n = 1000	< 1 sec	< 1 sec	1 sec	18 min	Very Long Time	Very Long Time
n = 10,000	< 1 sec	< 1 sec	2 min	12 days	Very Long Time	Very Long Time
n = 100,000	< 1 sec	2 sec	3 hours	32 years	Very Long Time	Very Long Time
n = 1,000,000	1 sec	20 sec	12 days	31,710 years	Very Long Time	Very Long Time

Space Complexity

Space complexity represents the amount of memory space an algorithm requires as a function of the input size.

- Constant Space $\implies O(1)$
- Linear Space $\implies O(n)$
- Log-linear Space $\implies O(n \log n)$
- Quadratic Space $\implies O(n^2)$
- ...

Recognising Algorithms Complexity

- Constant runtime is represented by $O(1)$
- linear growth is $O(n)$
- logarithmic growth is $O(\log n)$
- log-linear growth is $O(n \log n)$
- quadratic growth is $O(n^2)$
- exponential growth is $O(2^n)$
- factorial growth is $O(n!)$



Table of Big O

Big O Notation	Relationship with 'n'	Description	Assumption
$O(1)$	Constant	The algorithm's runtime is constant regardless of the input size.	The algorithm performs a single operation.
$O(\log n)$	Logarithmic	The algorithm's runtime grows logarithmically as the input size increases.	The algorithm divides the input in half at each step (e.g., binary search).
$O(n)$	Linear	The algorithm's runtime grows linearly with the input size.	The algorithm iterates through the input once.
$O(n \log n)$	Linearithmic	The algorithm's runtime grows in between linear and logarithmic as the input size increases.	Typically seen in efficient sorting algorithms like merge sort or quicksort.
$O(n^2)$	Quadratic	The algorithm's runtime grows quadratically with the input size.	The algorithm has nested iterations over the input (e.g., nested loops).
$O(n^3)$	Cubic	The algorithm's runtime grows cubically with the input size.	The algorithm has triple nested iterations over the input.
$O(2^n)$	Exponential	The algorithm's runtime grows exponentially with the input size.	The algorithm performs exhaustive search or generates all subsets of the input.

Real-world Applications

- Choosing the right data structures and algorithms for software development.
- Optimizing database queries.

	Seach in a table	Seach in an index
Seach Algorithm	Linear Scan	Binary Scan
Complexity	$O(N)$	$O(\log N)$

- Designing efficient algorithms for large-scale data processing.

Examples of Big O Notation

- Linear Search $\implies O(n)$
- Binary Search $\implies O(\log n)$
- Bubble Sort $\implies O(n^2)$
- Merge Sort $\implies O(n \log n)$
- ...

Linear Search Example, $O(n)$:

- Searching for a value and its index
- Unordered List, Small Data Sets, Linked Lists.

```
#include <stdio.h>

int linearSearch(int arr[], int size, int target) {
    for (int i = 0; i < size; i++) {
        if (arr[i] == target) {
            return i; // Target found
        }
    }
    return -1; // Target not found
}
```

```
int main() {
    int arr[] = {3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5};
    int size = sizeof(arr) / sizeof(arr[0]);
    int target = 4;

    int result = linearSearch(arr, size, target);

    if (result != -1) {
        printf("Target %d found at index %d\n", target, result);
    } else {
        printf("Target %d not found\n", target);
    }

    return 0;
}
```

Second example of $O(n)$, finding Max:

```
// Linear complexity: O(n)
int FindMaxElement(int[] array)
{
    int max = int.MinValue;
    for (int i = 1; i < array.Length; i++)
    {
        if (array[i] > max)
        {
            max = array[i];
        }
    }
    return max;
}
```

- The algorithm's time complexity is linearly dependent on the size of the input (each additional element in the array results in one more iteration through the loop)
- it is denoted as $O(n)$, where n is the length of the array. This makes it an efficient linear time algorithm for finding the maximum element in an array.

Binary search $O(\log n)$ code example:

```
#include <stdio.h>

int binarySearch(int arr[], int size, int target) {
    int low = 0, high = size - 1;

    while (low <= high) {
        int mid = (low + high) / 2;
        if (arr[mid] == target) {
            return mid; // Target found
        } else if (arr[mid] < target) {
            low = mid + 1;
        } else {
            high = mid - 1;
        }
    }

    return -1; // Target not found
}
```

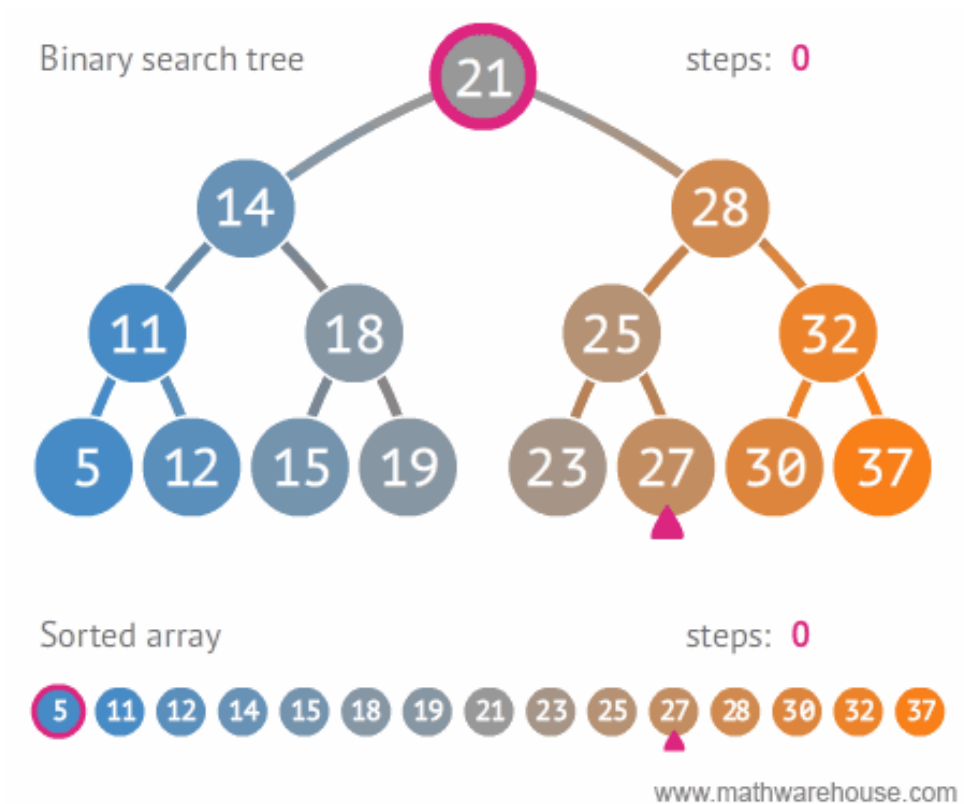
```
int main() {
    int arr[] = {5, 11, 12, 14, 15, 18, 19, 21, 23,
                 27, 25, 28, 30, 32, 37};
    int size = sizeof(arr) / sizeof(arr[0]);
    int target = 27;

    int result = binarySearch(arr, size, target);

    if (result != -1) {
        printf("Target %d found at index %d\n",
               target, result);
    } else {
        printf("Target %d not found\n", target);
    }

    return 0;
}
```

Binary search $O(\log n)$ and Linear(Seq) Search $O(n)$



Binary search

steps: 0



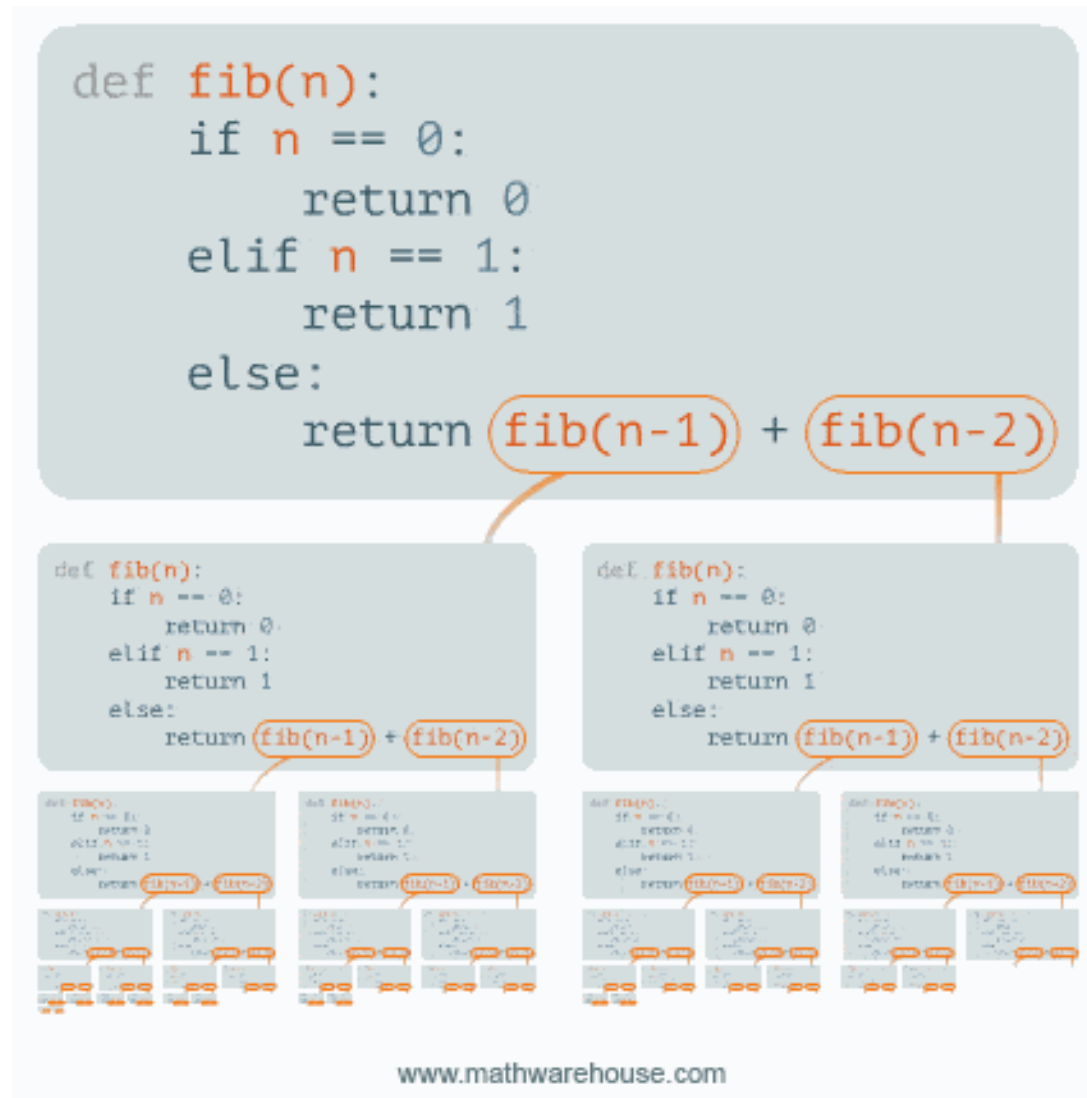
Sequential search

steps: 0



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Exponential growth is $O(2^n)$, Fibonacci:



Exponential growth is $O(2^n)$, Fibonacci:

```
// Exponential complexity:  $O(2^n)$ 
long Fibonacci(int n)
{
    if (n == 0)
    {
        return 1;
    }
    else if (n == 1)
    {
        return 1;
    }
    else
    {
        return Fibonacci(n - 1) + Fibonacci(n - 2);
    }
}
```

- For each Fibonacci number, the algorithm makes two recursive calls: `Fibonacci(n - 1)` and `Fibonacci(n - 2)`. This leads to an exponential growth in the number of recursive calls as the input `n` increases.
- Similar to binary as this is base 2 too.
- An algorithm's performance can degrade rapidly as the input size increases.

Greatest Common Demonator $O(\log n)$

```
// logarithmic growth is  $O(\log n)$ 
int GCD(int a, int b)
{
    if (b > a) // Ensure that a is greater than b
    {
        int temp = a;
        a = b;
        b = temp;
    }

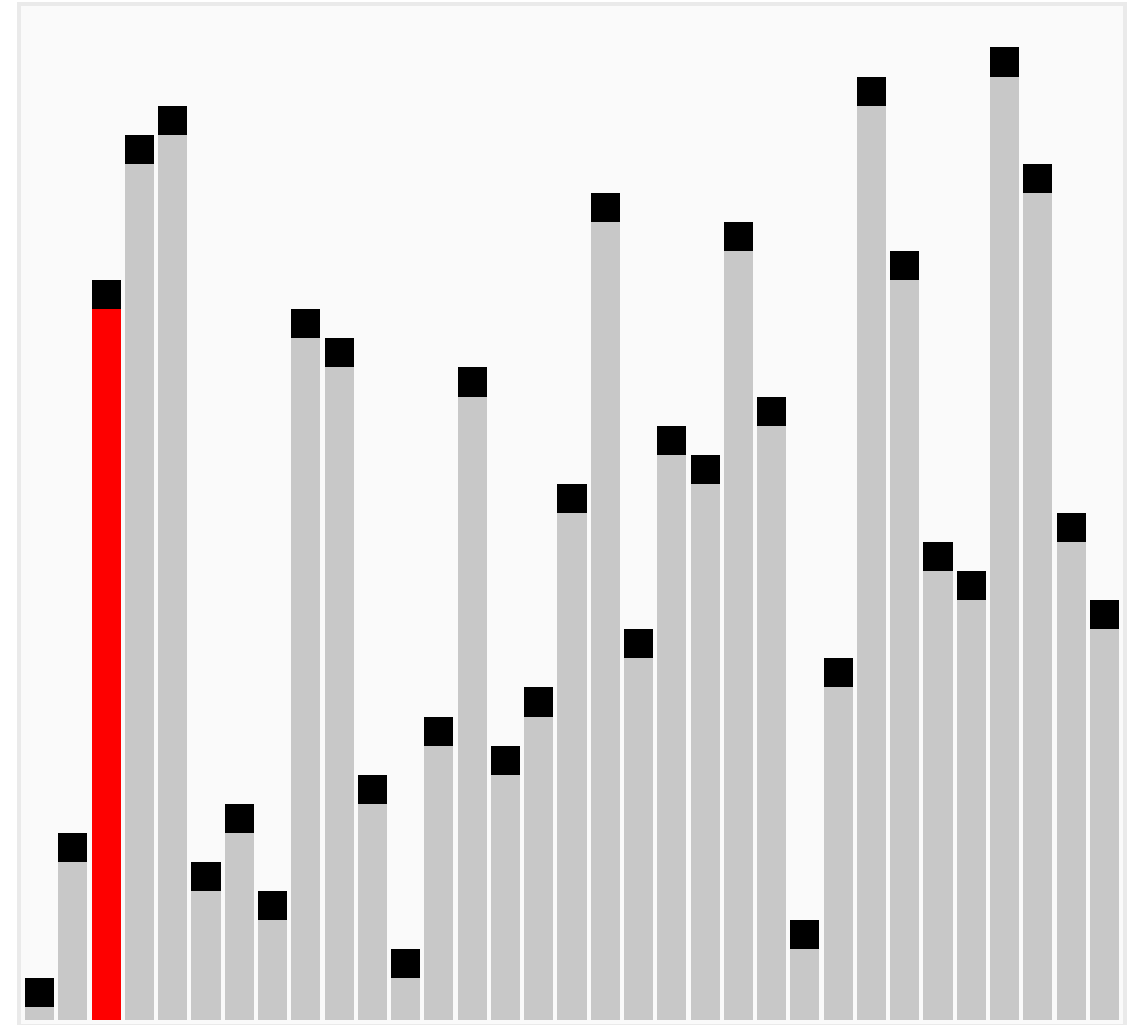
    // Compute GCD using the Euclidean algorithm
    while (b > 0)
    {
        int r = a % b;
        a = b;
        b = r;
    }

    return a;
}
```

- The Euclidean algorithm has a time complexity of $O(\log N)$, where N is the larger of the two input integers.
- This is because, in each iteration of the algorithm, the size of the inputs is reduced by a factor of at least 2,
- which means that the number of iterations required is proportional to $\log N$.

Bubble Sort $\implies O(n^2)$:

```
for (c = 0 ; c < n - 1; c++)  
{  
    for (d = 0 ; d < n - c - 1; d++)  
    {  
        if (array[d] > array[d+1])  
        {  
            swap      = array[d];  
            array[d]   = array[d+1];  
            array[d+1] = swap;  
        }  
    }  
}
```



Quick Sort $\implies O(n \log n)$:

```
void quicksortMiddle(int arr[], int low, int high) {  
    if (low < high) {  
        // Selecting the middle element as the pivot  
        int pivot = arr[(low + high) / 2];  
        int i = low, j = high, temp;  
  
        while (i <= j) {  
            // Moving elements smaller than pivot to the left  
            while (arr[i] < pivot) i++;  
            // Moving elements greater than pivot to the right  
            while (arr[j] > pivot) j--;  
  
            if (i <= j) {  
                temp = arr[i]; // Swapping elements  
                arr[i] = arr[j];  
                arr[j] = temp;  
                i++;  
                j--;  
            }  
        }  
        // Recursively sort the two partitions  
        if (low < j) quicksortMiddle(arr, low, j);  
        if (i < high) quicksortMiddle(arr, i, high);  
    }  
}
```

