Number_Systems

Module Code: GEEN1064

Module Name: Engineering Design and Implementation

Lecturer: Seb Blair BEng(H) PGCAP MIET MIHEEM FHEA

Outline

- Information Theory
- Binary
- Hexadecimal
- Octal

Information Theory

- A Mathematical Theory of Communication, (Shannon,1948)
  - Term *bit* was introduced as something that can have the representation of **1** or **0** most commonly an electrical signal
  - Foundations of the electrical computer was built off of this knowledge.
- Sequences of bits have no intrinsic meaning except for the representation that we assign to them, both by convention and by building particular operations into the hardware.

Binary

By choosing an appropriate representation, you can use bits to represent any value you can imagine:

- Characters are represented using numeric character codes.
- Floating-point representation supports real numbers.
- Two-dimensional arrays of bits represent images.
- Sequences of images represent video.
- and so on...

> > *can be read right to left or left to right (an important distinction)*

Binary

Decimal	Binary	Decimal	Binary
0	000000	10	001010
1	000001	11	001011
2	000010	12	001100
3	000011	13	001101
4	000100	14	001110
5	000101	15	001111
6	000110	16	001000
7	000111	17	010001
8	001000	...	...
9	001001	63	111111

Binary (2^n)

The number of combinations can be found by doing the following:

$$N_{combinations} = 2^n$$

You can calculate the maximum number you can have:

$$N_{maxValue} = 2^n - 1$$

	MSB									**LSB**
	2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
2^n	1024	512	256	128	64	32	16	8	4	2	1
$2^n - 1$	1023	511	255	127	63	31	15	7	3	1	0

Binary to Decimal

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1
0	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**1**	**1**	**0**

Therefore the decimal equivalent value is:

$$\begin{aligned} 1 \cdot 2^7 &= 128_{10} \\ 1 \cdot 2^4 &= 16_{10} \\ 1 \cdot 2^2 &= 4_{10} \\ 1 \cdot 2^1 &= 2_{10} \; + \\ \hline [0]000\ 1001\ 0110_2 &\equiv 150_{10} \end{aligned}$$

Your turn

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1

- 1111
  - ▶ Answer
- 11111101
  - ▶ Answer

Little or Big Endian

- The order in which binary is read is very important.
- consider:
  - $10000000_2 = 256_{10} \,||\, 1_{10}$
- or:
  - $10011001_2 = 153_{10} \,||\, 153_{10}$
- Intel processors use little endian
- ISO standard for network address is big endian

Decimal to Binary

We can convert decimal to binary in a similar way eg: 51

- $64 < 51 = 0$
- $32 < 51 = 1$
- $(32 + 16) \rightarrow 48 <= 51 = 1$
- $(48 + 8) \rightarrow 56 <= 51 = 0$
- $(48 + 4) \rightarrow 52 <= 51 = 0$
- $(48 + 2) \rightarrow 50 <= 51 = 1$
- $(50 + 1) \rightarrow 51 <= 51 = 1$

64	32	16	8	4	2	1
0	1	1	0	0	1	1

Your Turn

2^{10}	2^9	2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1024	512	256	128	64	32	16	8	4	2	1
0	**0**	**0**	**1**	**0**	**0**	**1**	**0**	**1**	**1**	**0**

- 75_{10}
  - ▶ Answer
- 126_{10}
  - ▶ Answer
- 487_{10}
  - ▶ Answer

Hexadecimal (16^n)

Hexadecimal is base 16:

$$N_{combinations} = 16^n$$

You can calculate the maximum number you can have:

$$N_{maxValue} = 16^n - 1$$

16^6	16^5	16^4	16^3	16^2	16^1	16^0
16777216	1048576	65536	4096	256	16	1

Hexadecimal, Binary, Decimal

Binary	1111	1110	1101	1100	1011	1010	1001	1000
Unsigned Int	15	14	13	12	11	10	9	8
Hexadecimal	F	E	D	C	B	A	9	8
Binary	0111	0110	0101	0100	0011	0010	0001	0000
Unsigned Int	7	6	5	4	3	2	1	0
Hexadecimal	7	6	5	4	3	2	1	0

Hexadecimal to Binary

Remember that binary and Hex have a relationship defined as 4 bits per Hexadecimcal symbol.

$$\begin{array}{cccccccc} & & & & 1111 & 1111 & F & F \\ 1100 & 1010 & 0101 & 0111 & C & A & 5 & 7 \end{array}$$

Hexadecimal to Decimal

In hex each position's weight should be 16 times the previous.

16^3	16^2	16^1	16^0
4096	256	16	1

$$
\begin{aligned}
? &= E5_{16} \\
&\rightarrow (E \cdot 16) + (5 \cdot 1) \\
&\rightarrow (14 \cdot 16) + (5 \cdot 1) \\
&\rightarrow 224 + 5 \\
229_{10} &=
\end{aligned}
$$

Hexadecimal to Decimal

The conversion can be done between 1 and 4 hexadecimal positions, much like Hexadecimal $\longleftrightarrow$ Binary.

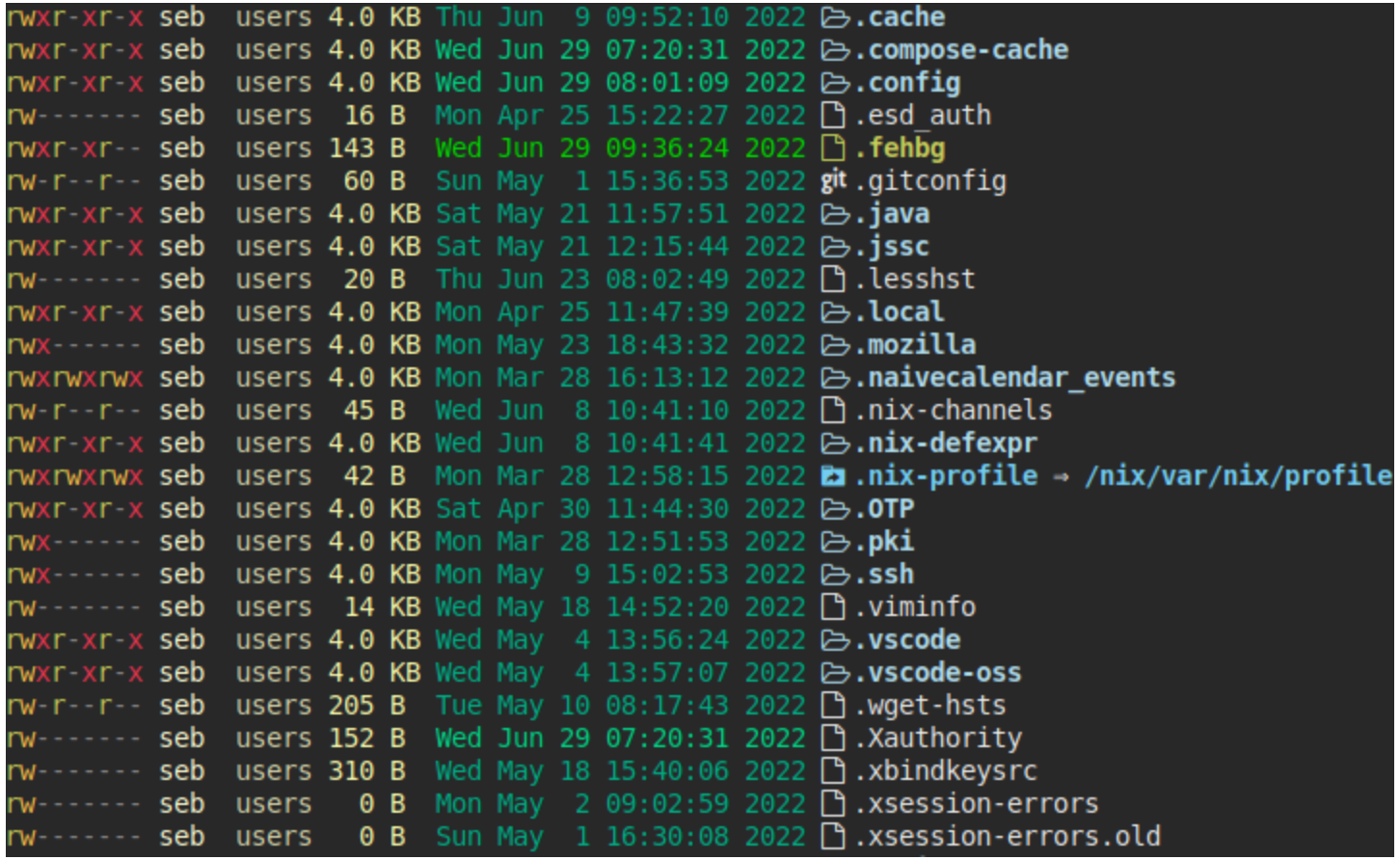
16^3	16^2	16^1	16^0
4096	256	16	1

$$
\begin{aligned}
? &= B2F8_{16} \\
&\rightarrow (B \cdot 4096) + (2 \cdot 256) + (F \cdot 16) + (8 \cdot 1) \\
&\rightarrow (11 \cdot 4096) + (2 \cdot 256) + (15 \cdot 16) + (8 \cdot 1) \\
&\rightarrow 45056 + 512 + 240 + 8 \\
45816_{10} &=
\end{aligned}
$$

Octal (8^n)

Octal you'll see more commonly in Linux systems for file system permissions

Binary	Octal	Permission
111	7	`rwx`
110	6	`rw-`
101	5	`r-x`
100	4	`r--`
011	3	`-wx`
010	2	`-w-`
001	1	`--x`
000	0	`---`



Octal $\longleftrightarrow$ Binary

All you need to remember is the octal only uses three bits to represent is value per symbol.

Eg, 7301_8

$$
\begin{aligned}
001_2 &= 1_8 \\
000_2 &= 0_8 \\
011_2 &= 3_8 \\
111_2 &= 7_8 \\
\hline
111\ 011\ 000\ 001_2 &\equiv 7301_8
\end{aligned}
$$

Representing Characters

- Computers use numeric encodings to represent character data inside the memory of the machine, in which each character is assigned an integral value.
- Character codes, however, are not very useful unless they are standardised. When different computer manufacturers use different coding sequence (as was indeed the case in the early years), it is harder to share such data across machines.
- The first widely adopted character encoding was ASCII (American Standard Code for Information Interchange).
- With only 256 possible characters, the ASCII system proved inadequate to represent the many alphabets in use throughout the world. It has therefore been superseded by Unicode, which allows for a much larger number of characters.

American Standard Code for Information Interchange (ASCII)

Dec	Char	Dec	Char	Dec	Char	Dec	Char	Dec	Char	Dec	Char	Dec	Char	Dec	Char
32	SPACE	45	-	58	:	71	G	84	T	97	a	110	n	123	{
33	!	46	.	59	;	72	H	85	U	98	b	111	o	124	
34	"	47	/	60	<	73	I	86	V	99	c	112	p	125	}
35	#	48	0	61	=	74	J	87	W	100	d	113	q	126	~
36	$	49	1	62	>	75	K	88	X	101	e	114	r	127	DEL
37	%	50	2	63	?	76	L	89	Y	102	f	115	s		
38	&	51	3	64	@	77	M	90	Z	103	g	116	t		
39	'	52	4	65	A	78	N	91	[	104	h	117	u		
40	(	53	5	66	B	79	O	92	\	105	i	118	v		
41	)	54	6	67	C	80	P	93	]	106	j	119	w		
42	*	55	7	68	D	81	Q	94	^	107	k	120	x		
43	+	56	8	69	E	82	R	95	_	108	l	121	y		
44	,	57	9	70	F	83	S	96	`	109	m	122	z		