Assignment 1
Due at 2:00 PM on Friday, September 30th

1. (25 marks) Draw a DFA, NFA or ε-NFA for each of the following languages, and show informally why your automaton accepts exactly the language described. That is, show that your automaton accepts all of the words in L, and no other words.

   (a) (5 marks) $L = \{x \in \{0,1\}^* \mid$ all 0s in x precede all 1s.$\}$

   (b) (5 marks) $L = \{x \in \{0,1\}^* \mid n_0(x) \geq 2$ and $n_1(x) \leq 1\}$

   (c) (5 marks) $L = \{x \in \{0,1\}^* \mid$ there are three consecutive 1s in x.$\}$

   (d) (10 marks) L is the the set of all binary string beginning with 1 that, when interpreted as binary integers, is a multiple of 5.

2. (20 marks) Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA with $F = \{q_0\}$. Let $w \in L(M)$ be any string in the language of M. Prove by induction that for all $k \geq 0$, the string $w^k \in L(M)$.

3. (10 marks + up to 20 bonus points) Find efficient algorithms for the following problems, you may assume that $\Sigma = \{0,1\}$. Aim for solutions that are clear and concise, some marks will be allocated to clarity of exposition.

   (a) Given an NFA $M = (Q, \Sigma, \delta, q_0, F)$ and an integer $n \geq 0$, find the lexicographically minimal word of length n accepted by M. What is the running time of your algorithm in terms of n?

   (b) Given an NFA $M = (Q, \Sigma, \delta, q_0, F)$ and a number $n \geq 0$, output *all words of length* n that are accepted by M in lexicographical order. What is the running time of your algorithm in terms of n?

   (c) Given an NFA $M = (Q, \Sigma, \delta, q_0, F)$ and a number $m \geq 1$, output the m first lexicographically minimal words accepted by M, listing these words in lexicographical order. If M accepts less than m words, then output all the words that it accepts. Analyse the running time of your algorithm (you may choose the parameters in the analysis).

   To earn all bonus points, you must find linear time, $O(n)$, solutions to parts (a) and (b), and have an efficient solution to part (c).