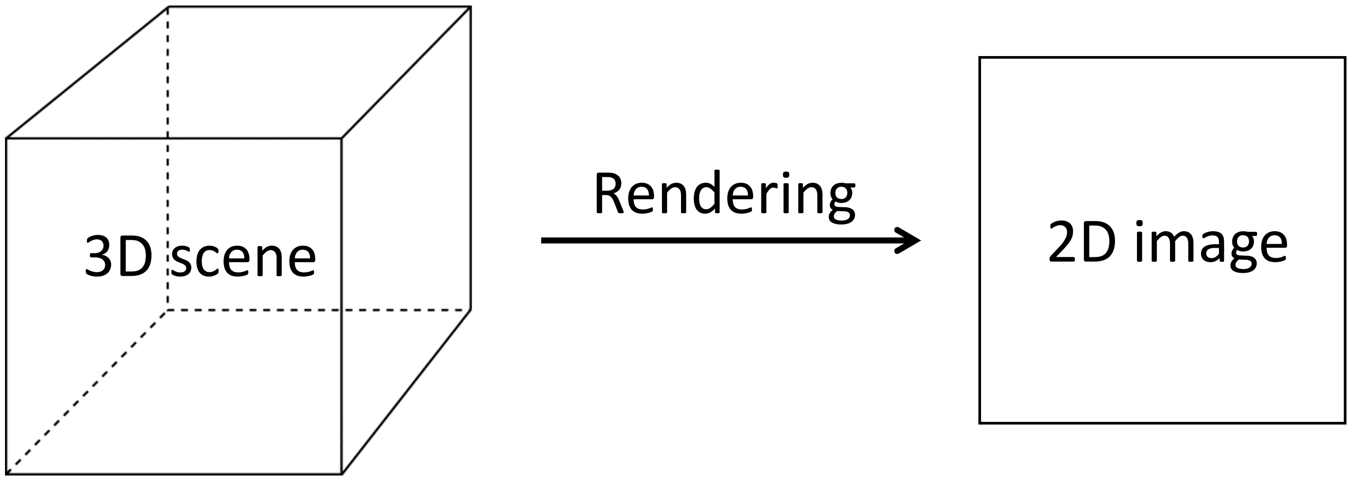
Computer Graphics and Visualization

Dr Wojciech Pałubicki

Computer graphics in a nutshell

- Images generated with computers
- In this course we focus on 3D rendering



Films, special effects

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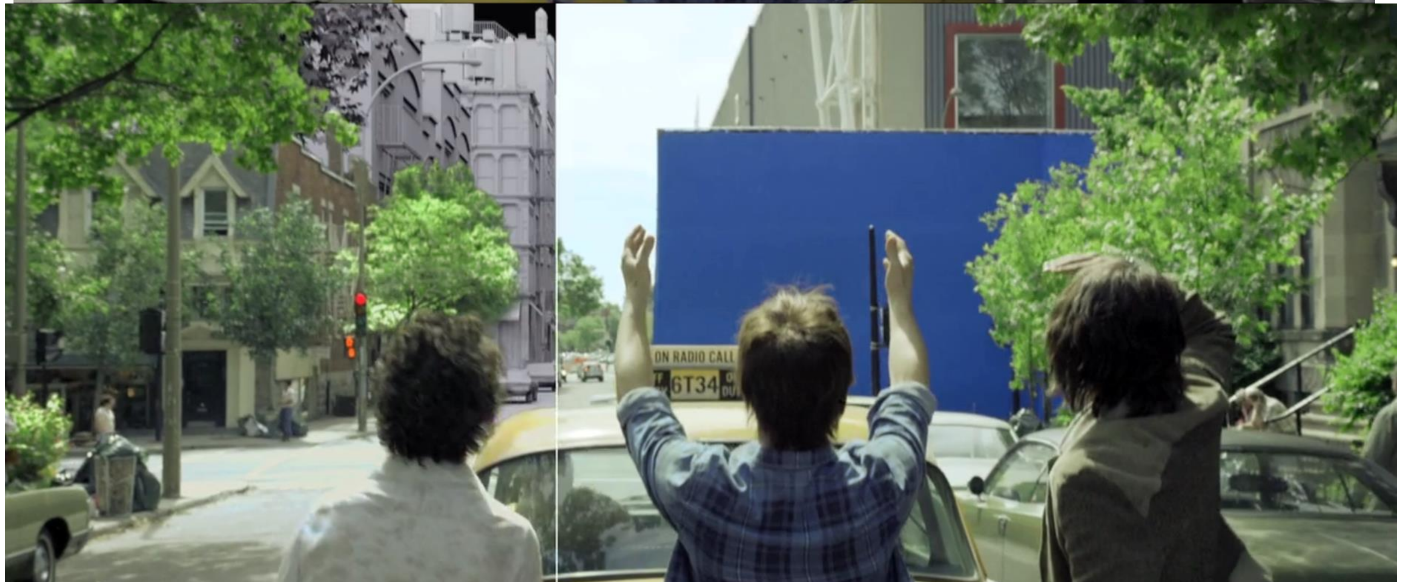
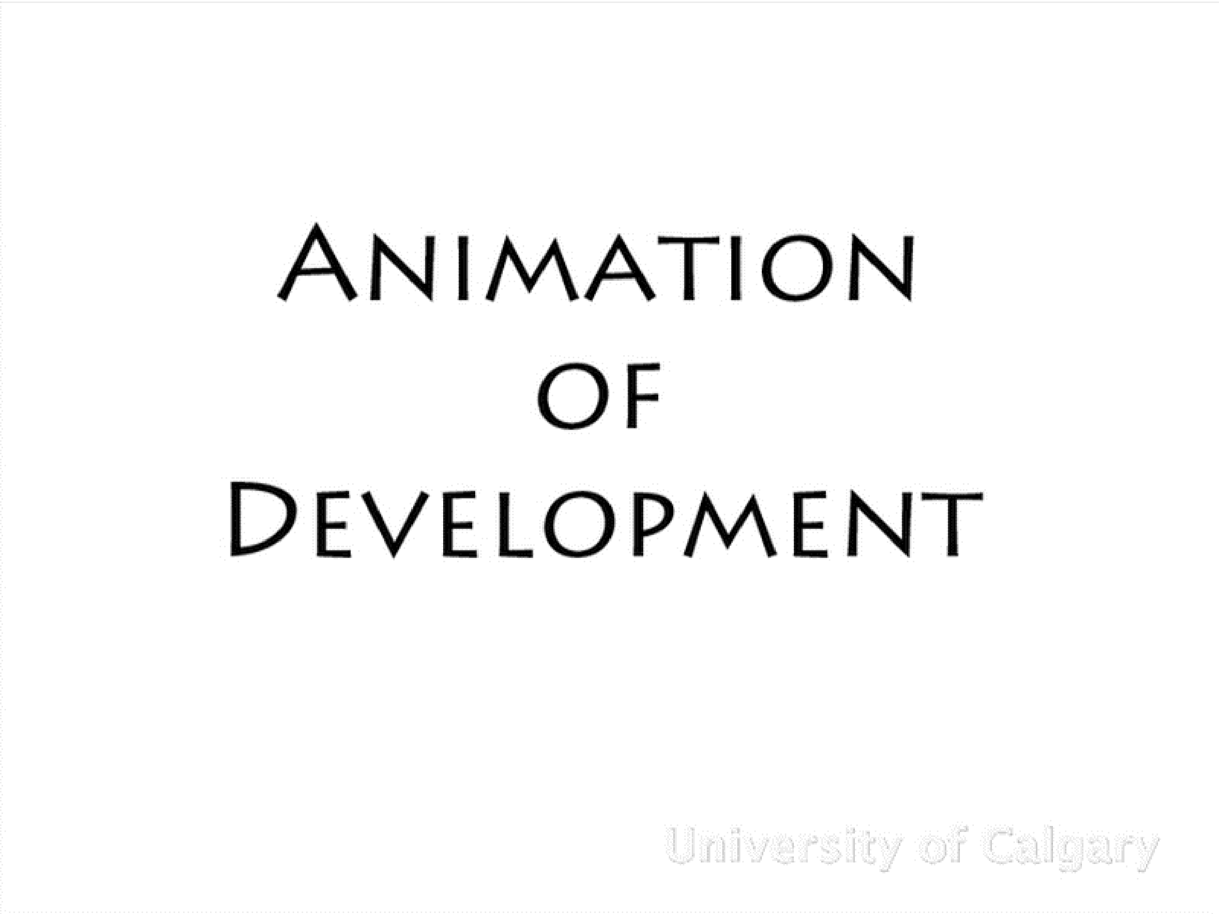
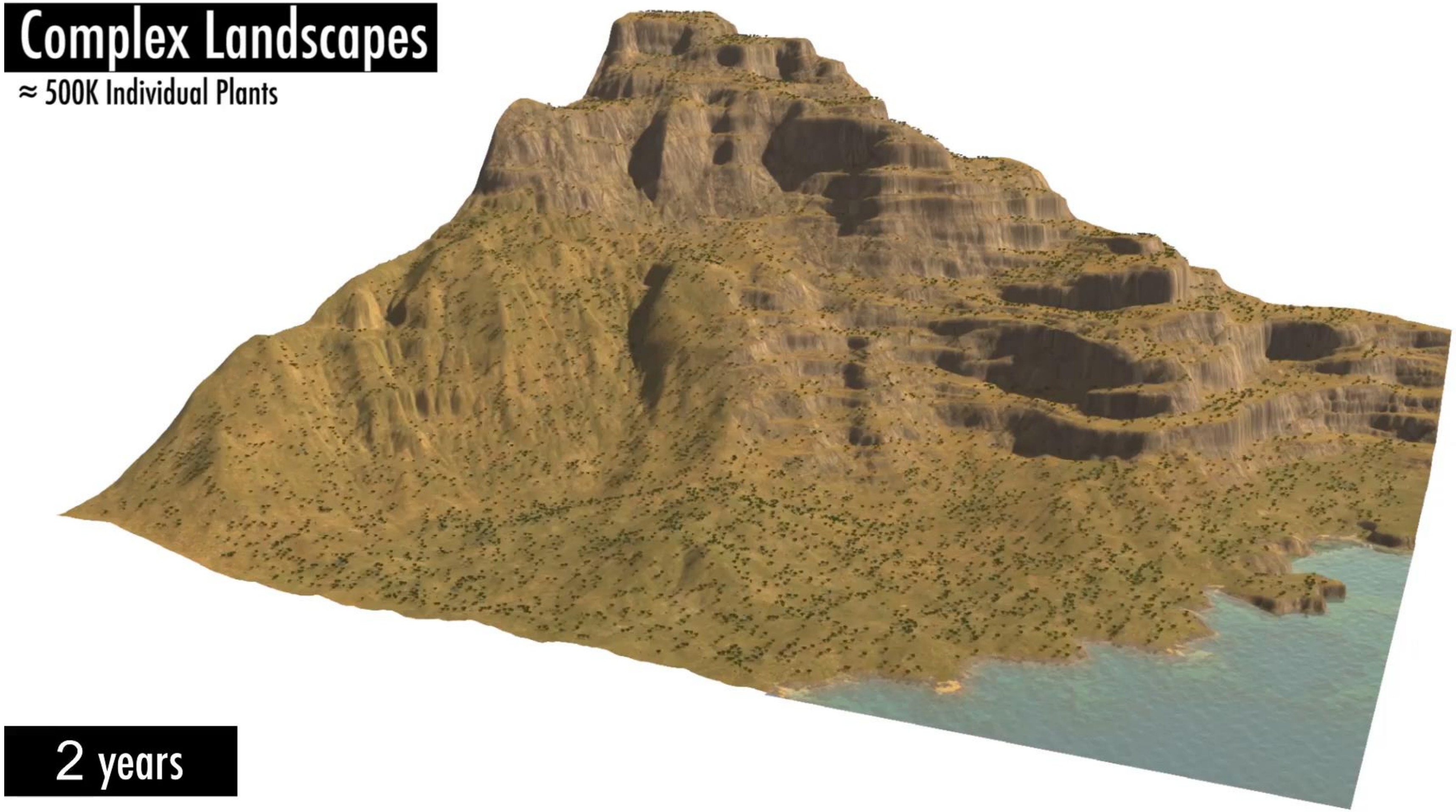


Computer Games

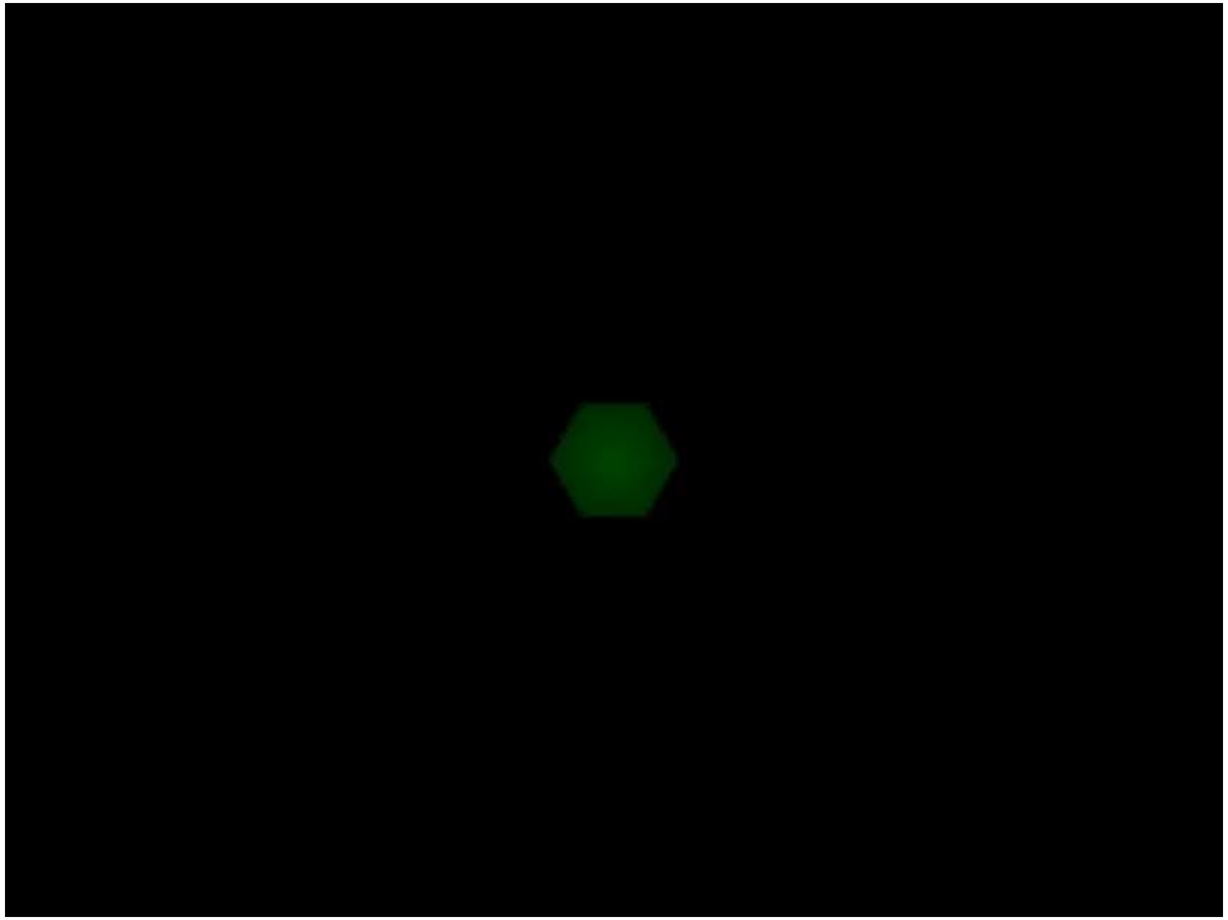
Simulations



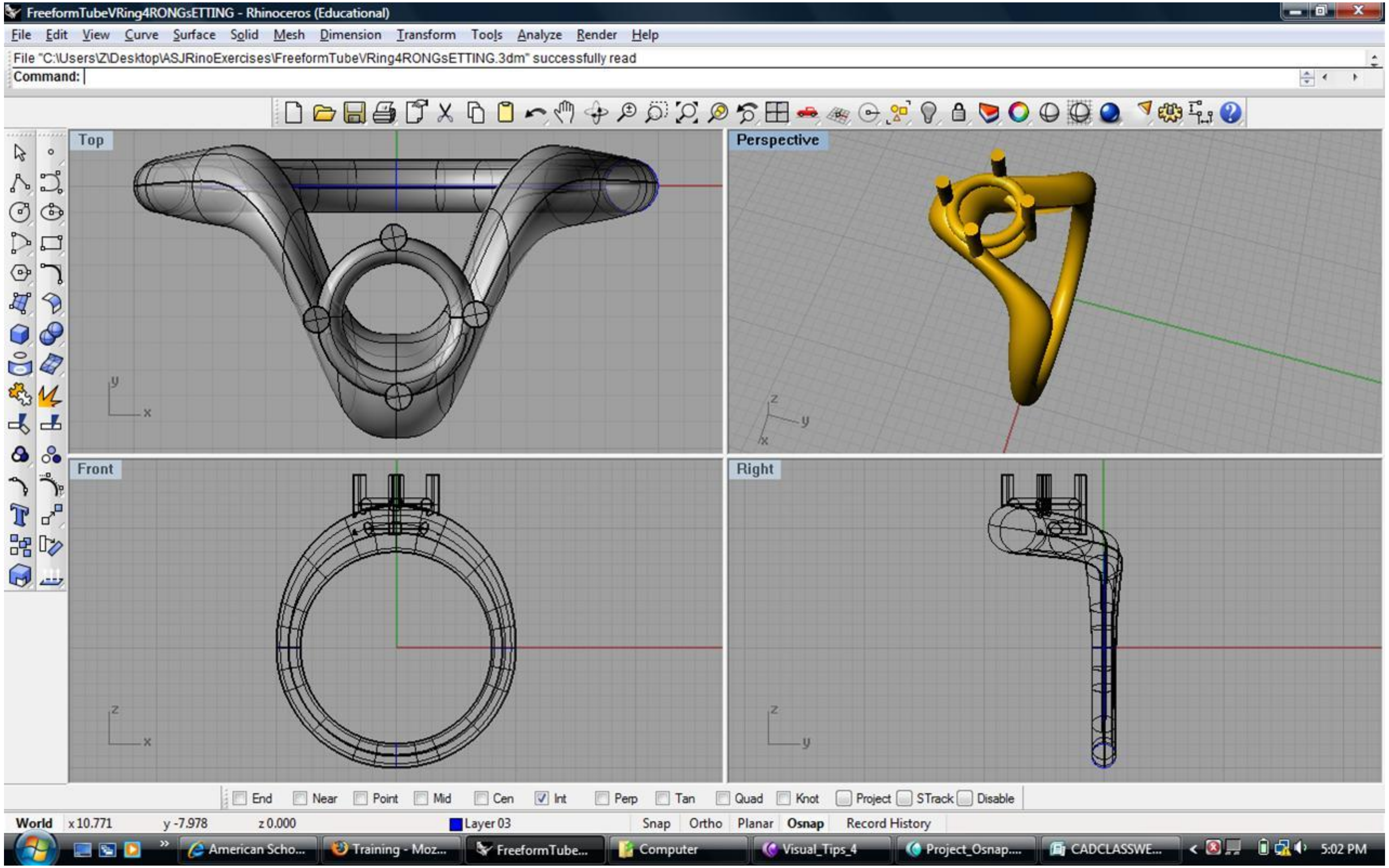
Simulations



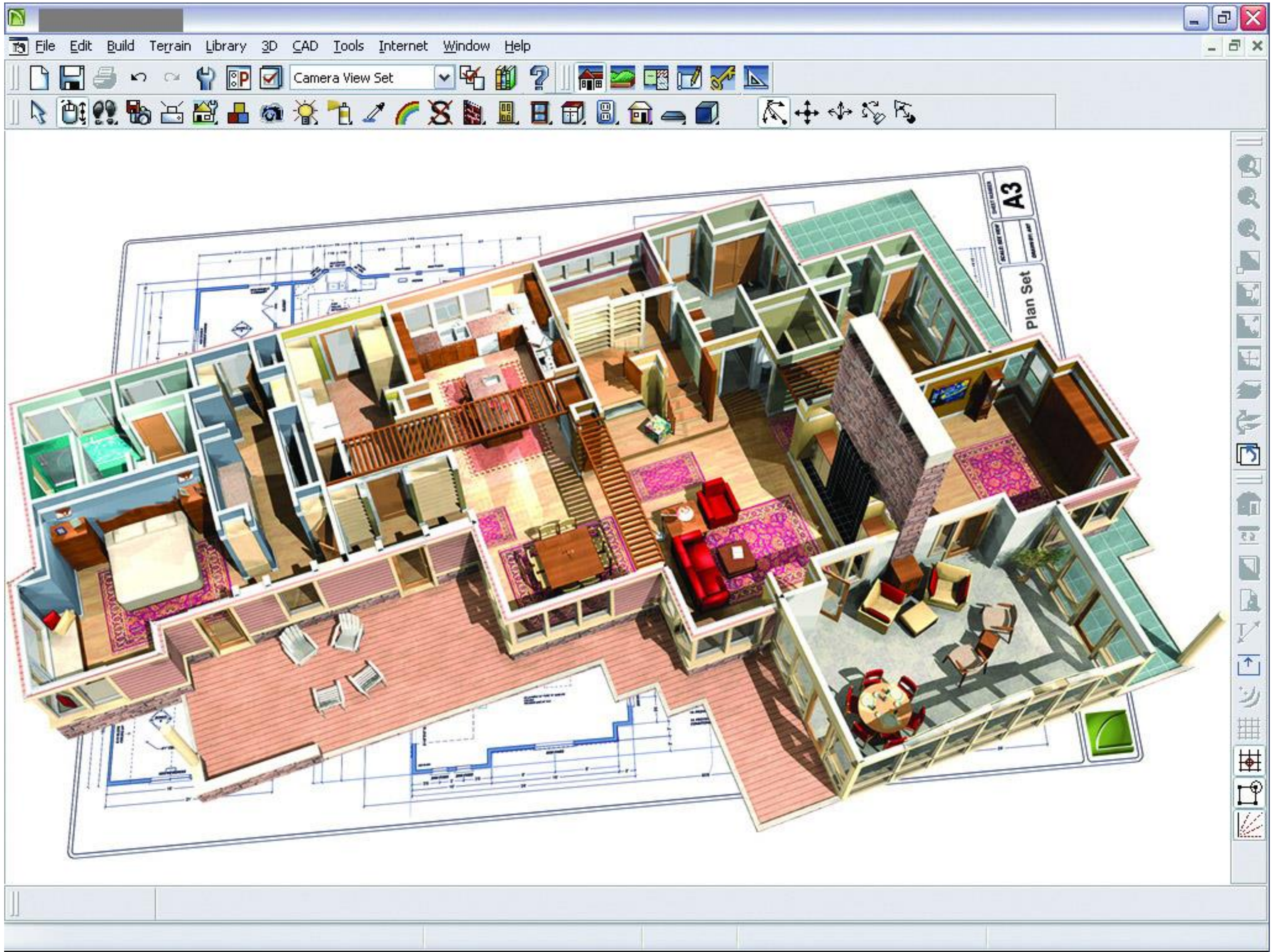
Simulations



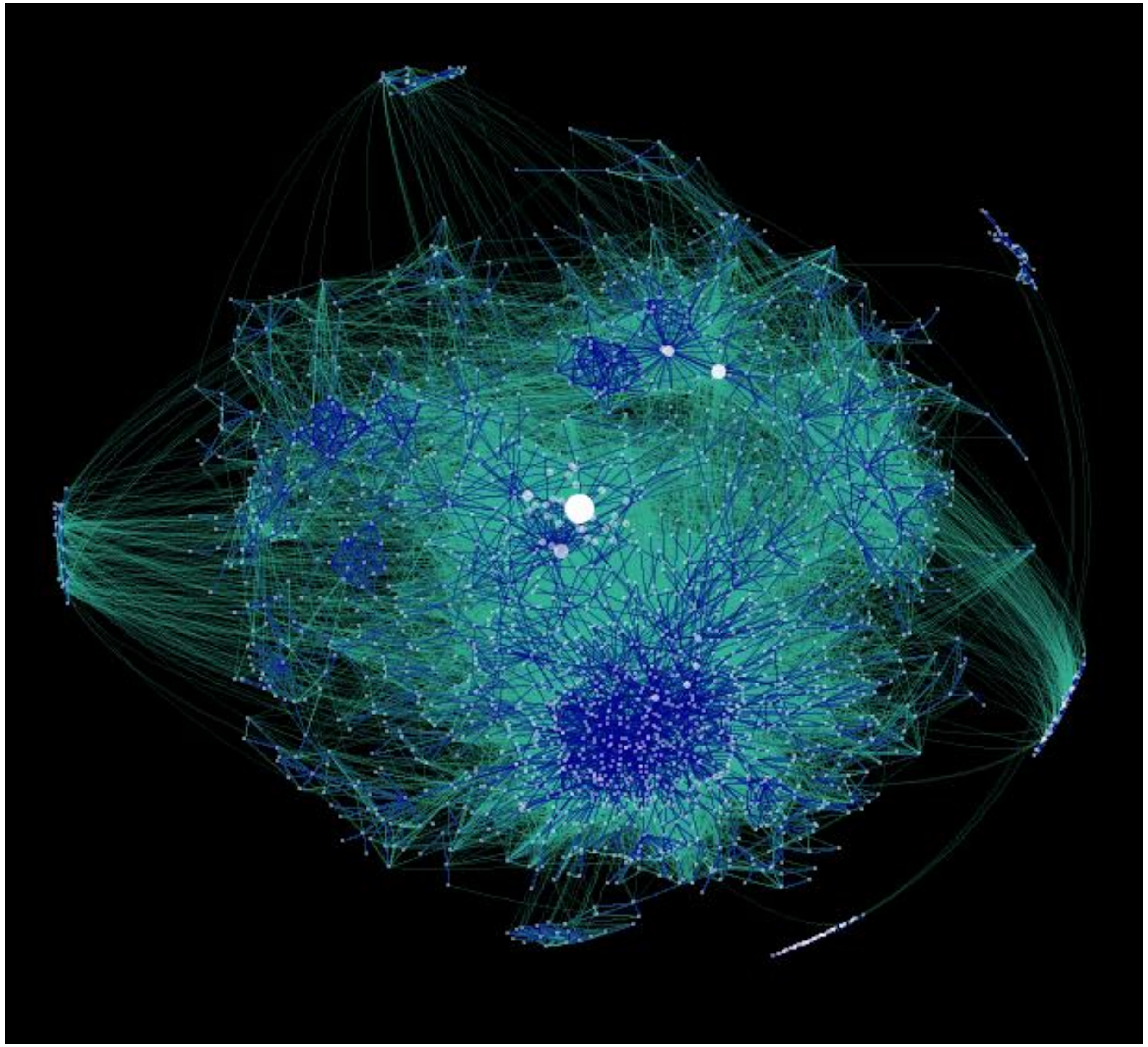
CAD & CAM Design



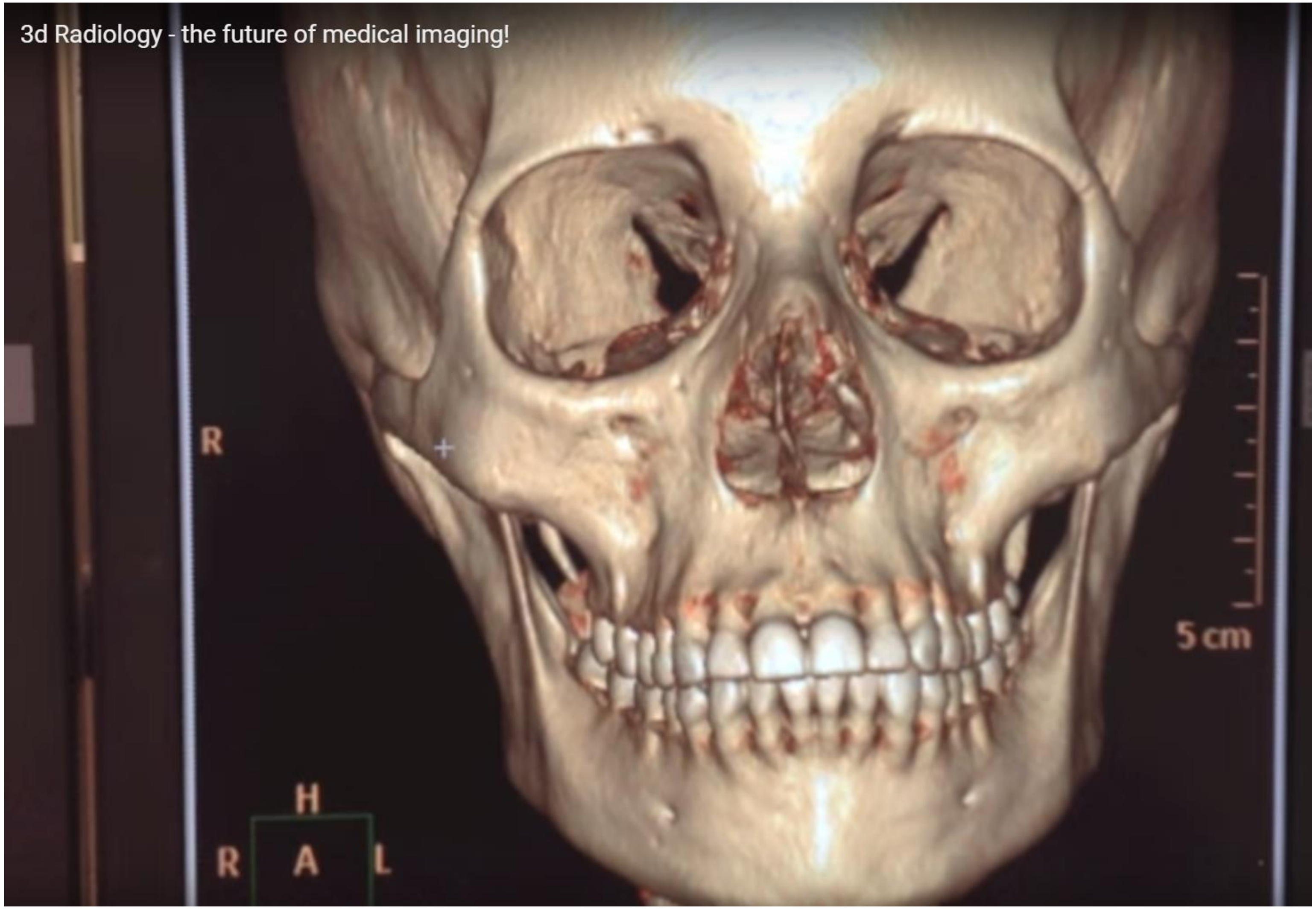
Architecture



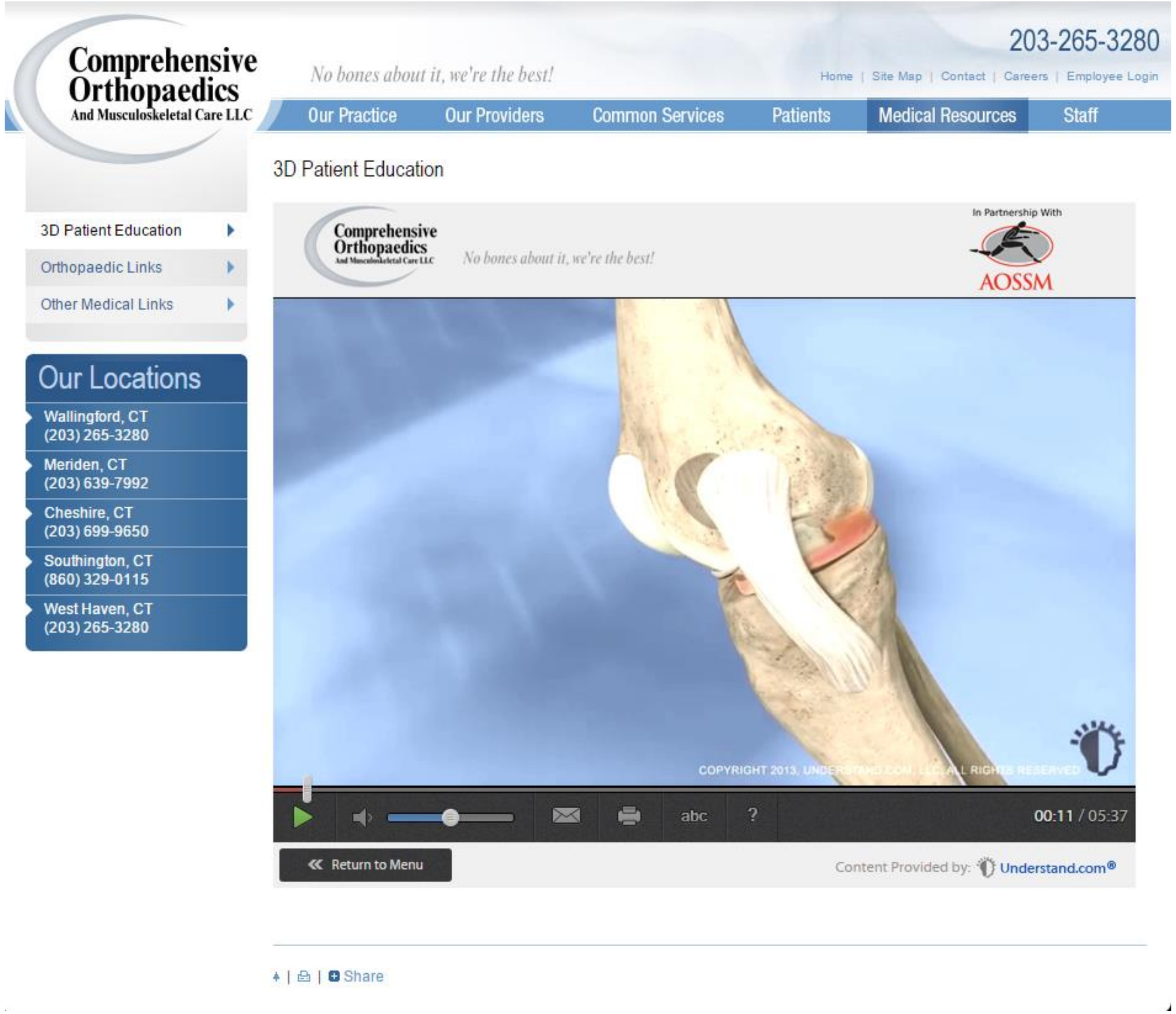
Data Visualization



Medical Imaging



Education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Applications

Rendering specifications

- Fundamental specifications for rendering are **OpenGL** and **Direct3D**

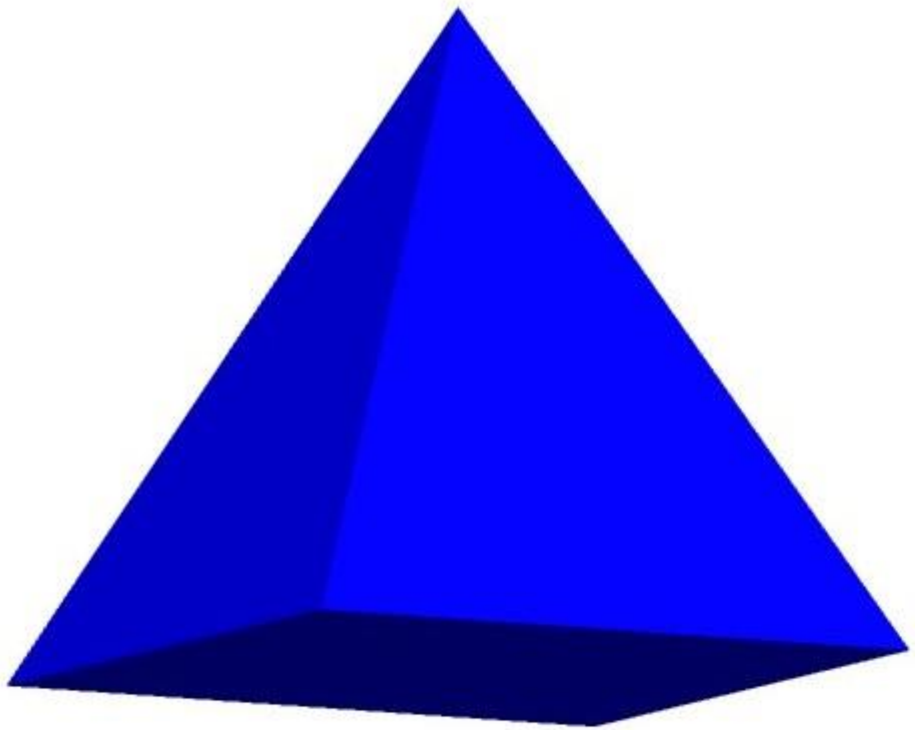
What will you learn

- Fundamental theory of computer graphics
- Rendering pipeline (how to generate 2D images from 3D scenes)
- **OpenGL**
- Experience with **C++**
- Fundamental elements of **GLSL**, a programming language executed on the graphics card

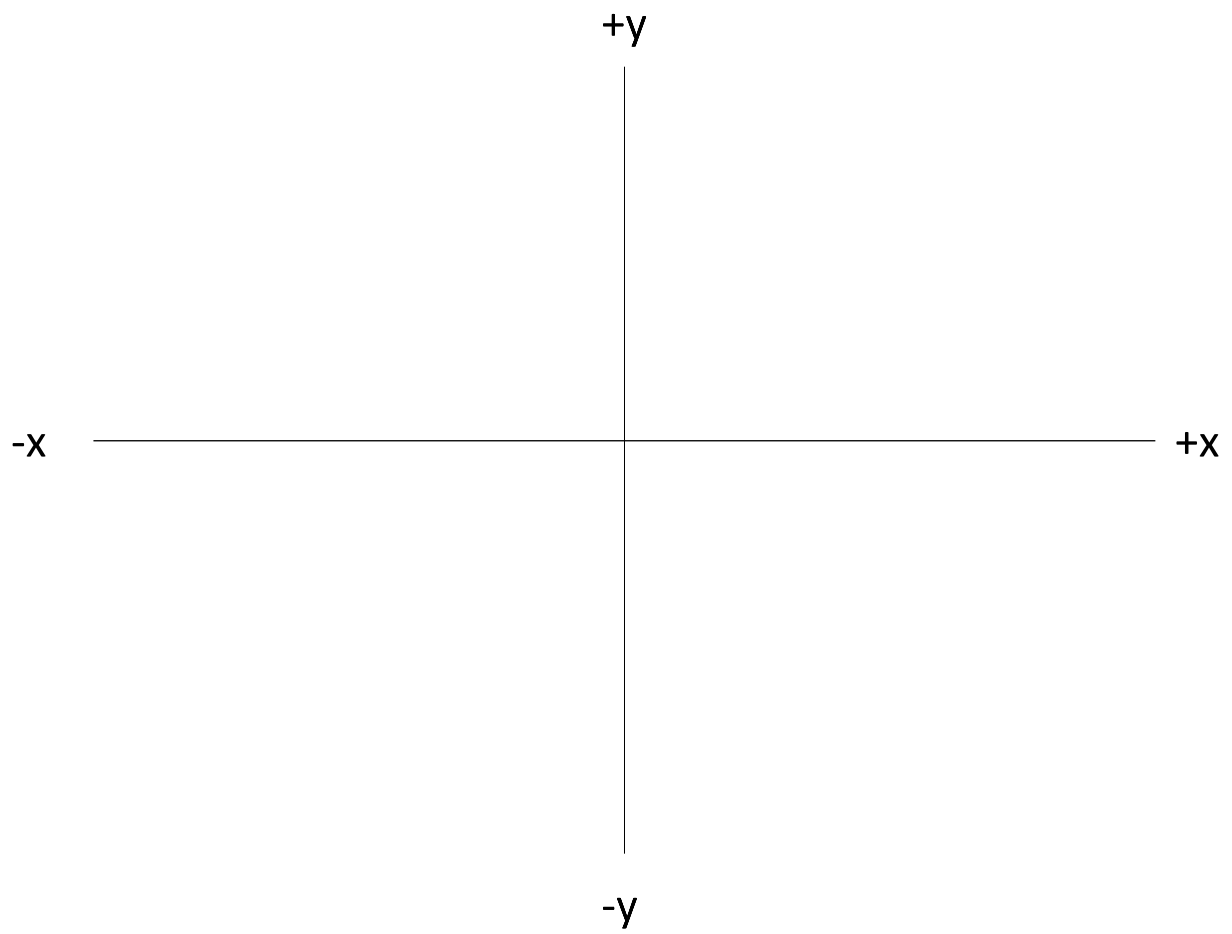
Passing Computer graphics

- Multiple choice tests (50%) - plus max. 1-2 bonus points for lab exercises (n-2 best tests are taken into account)
- Semester project (50%) – 1/4 research presentation + 3/4 project (minimum 15%)
- More details in the labs (e.g. dates)
- Information will be available: MS Teams and https://wp.faculty.wmi.amu.edu.pl/GRK.html

How to express 3D objects mathematically?



2D coordinate system



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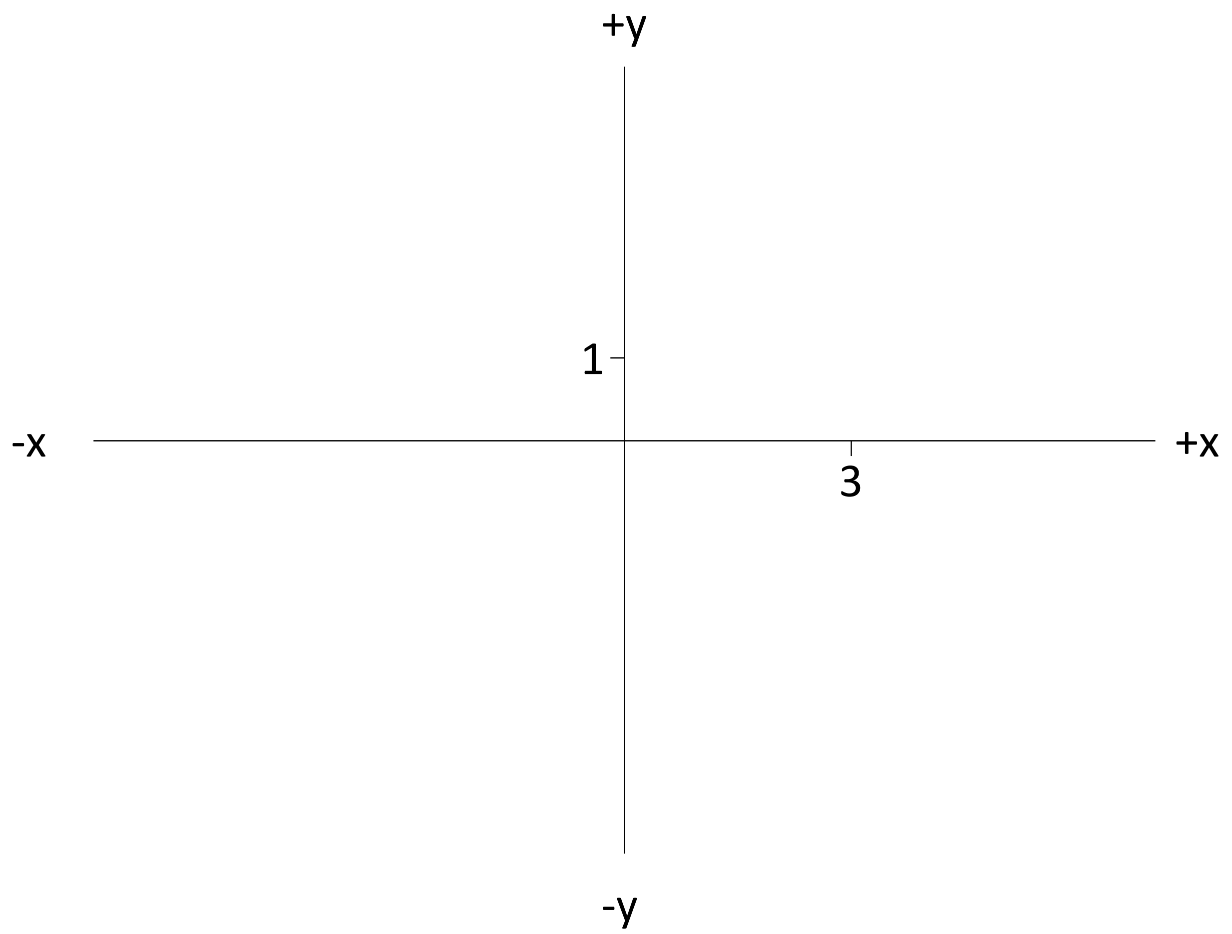

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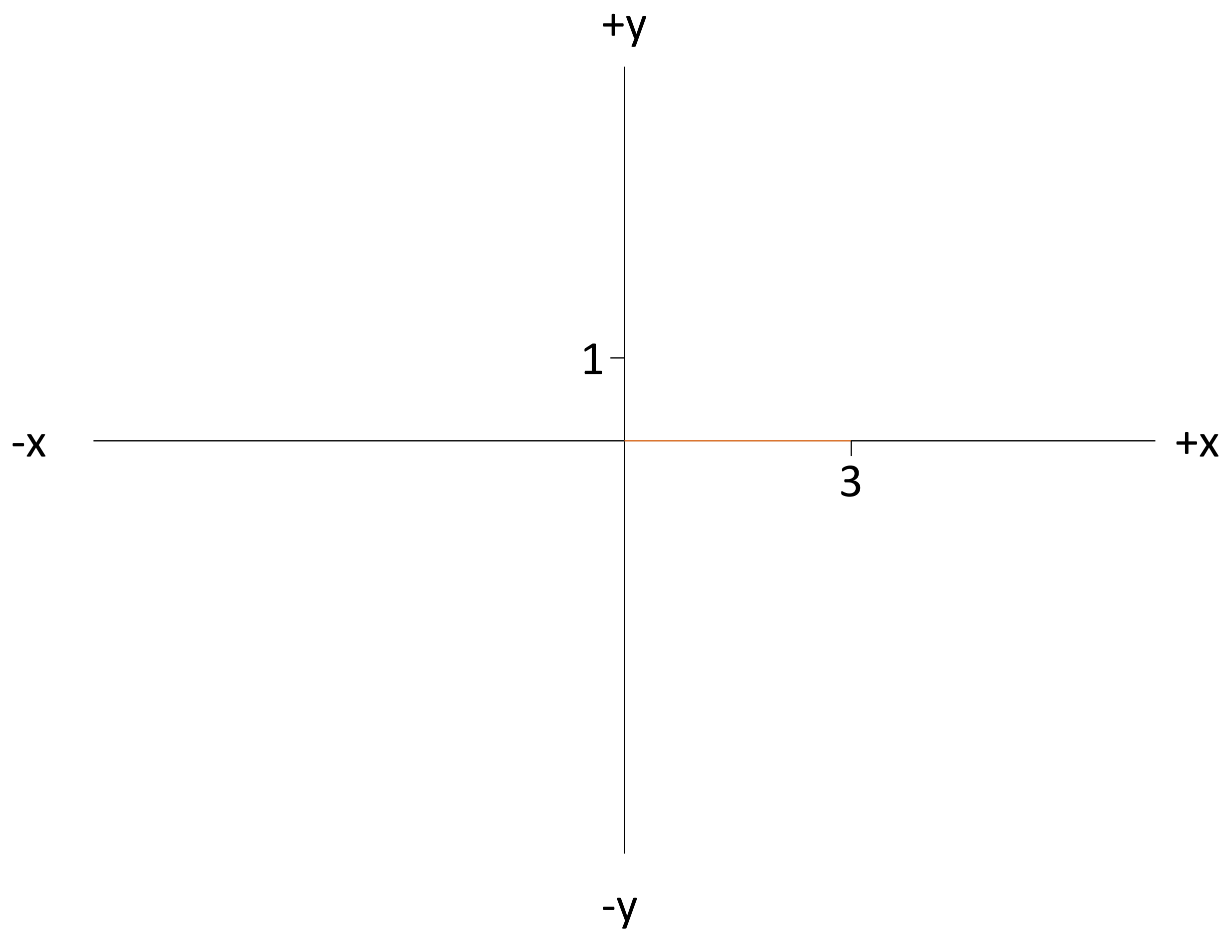

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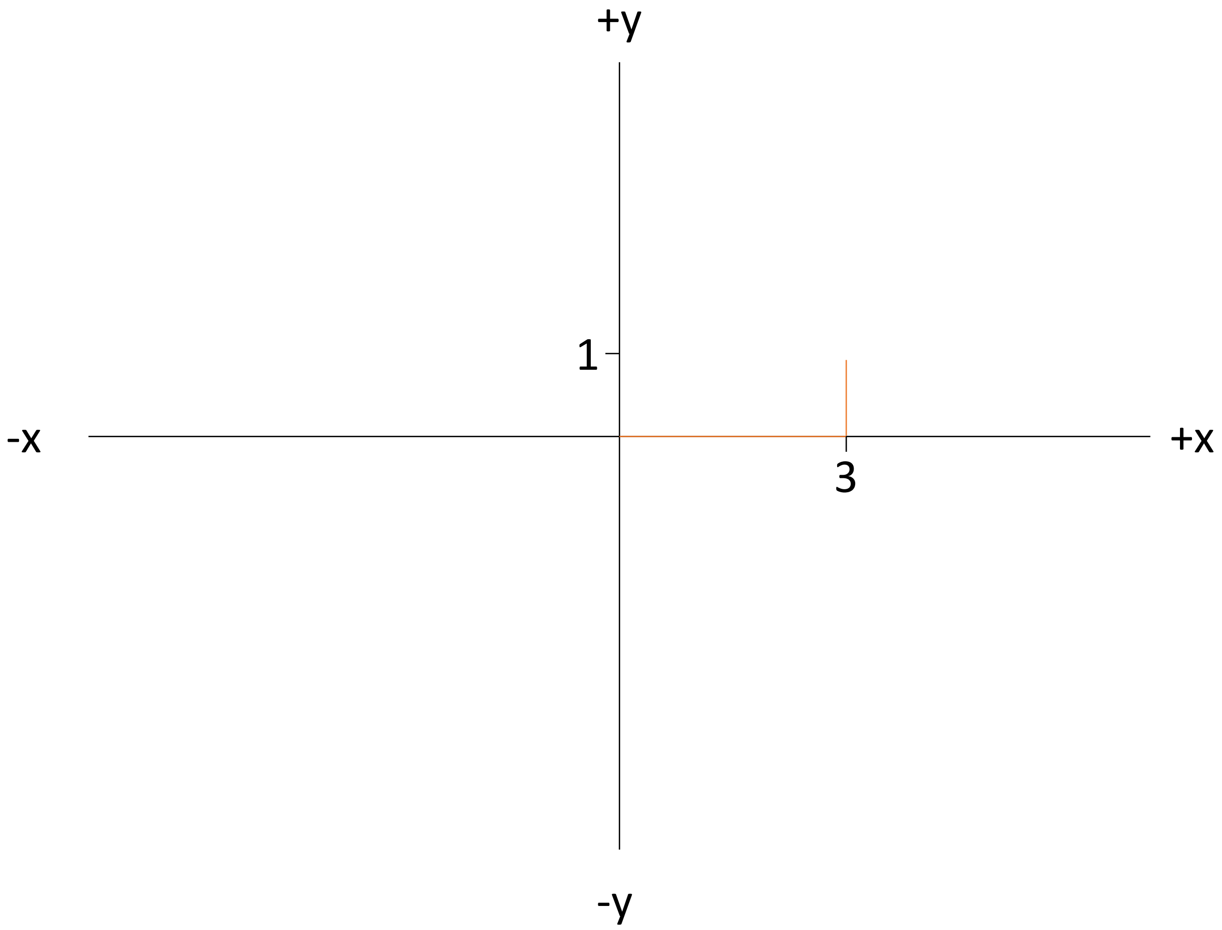
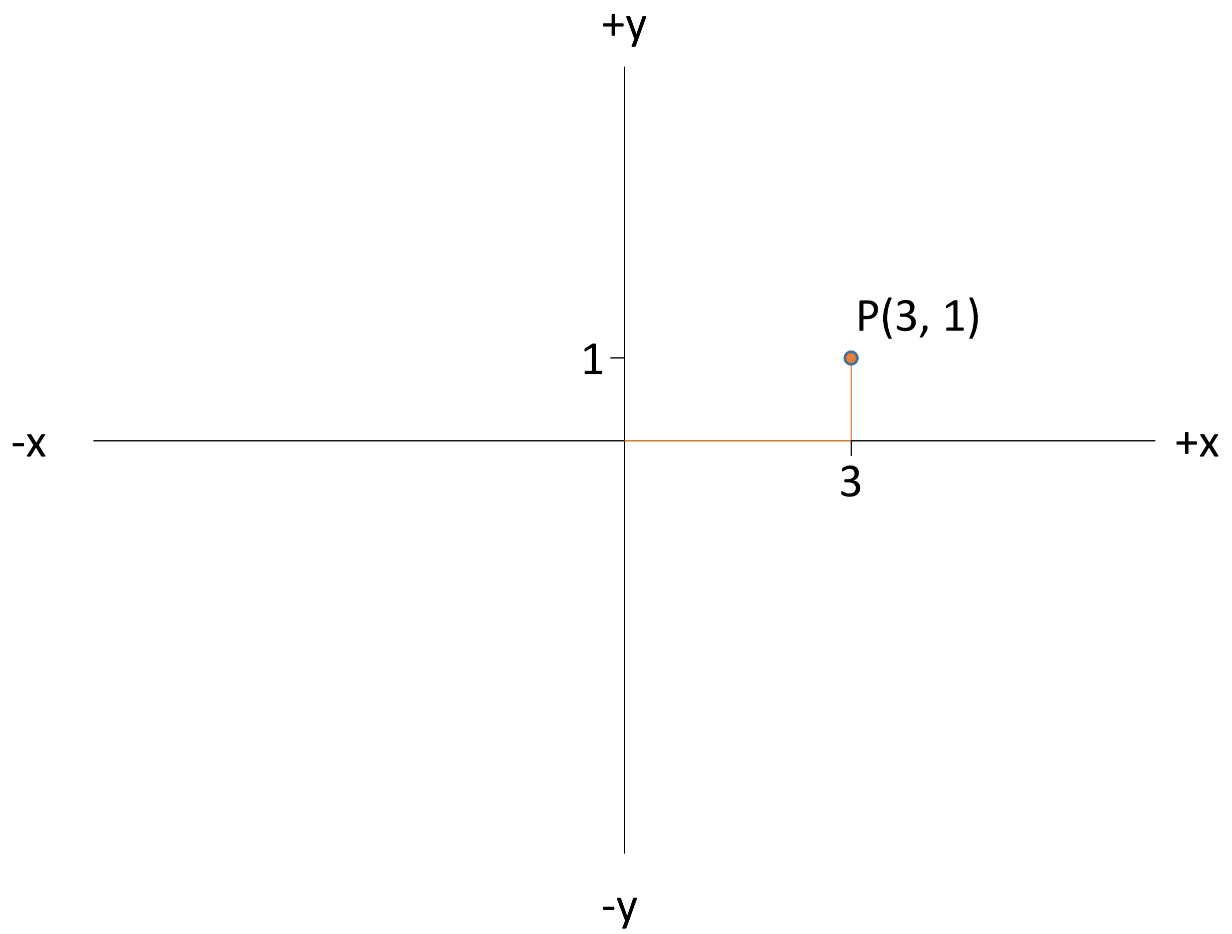
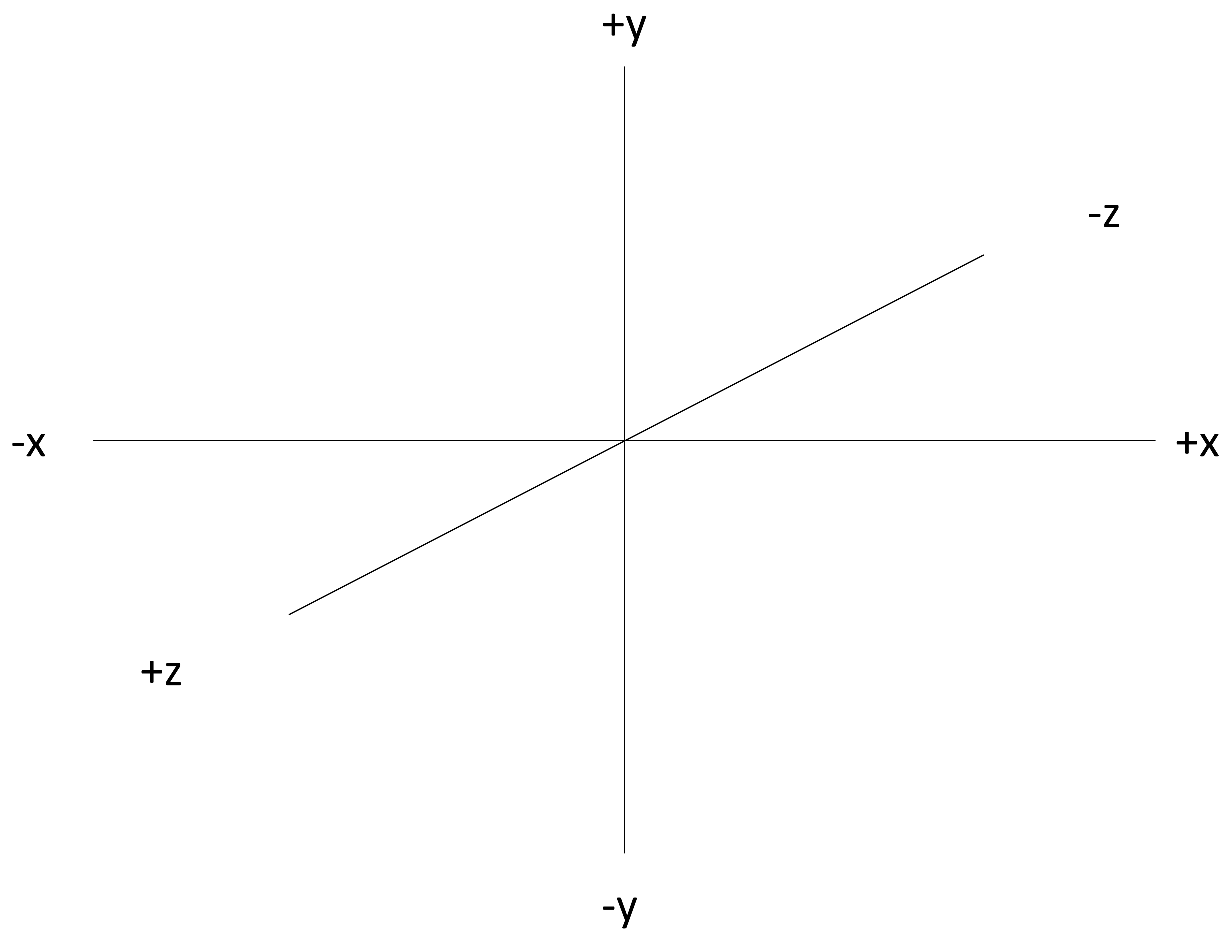


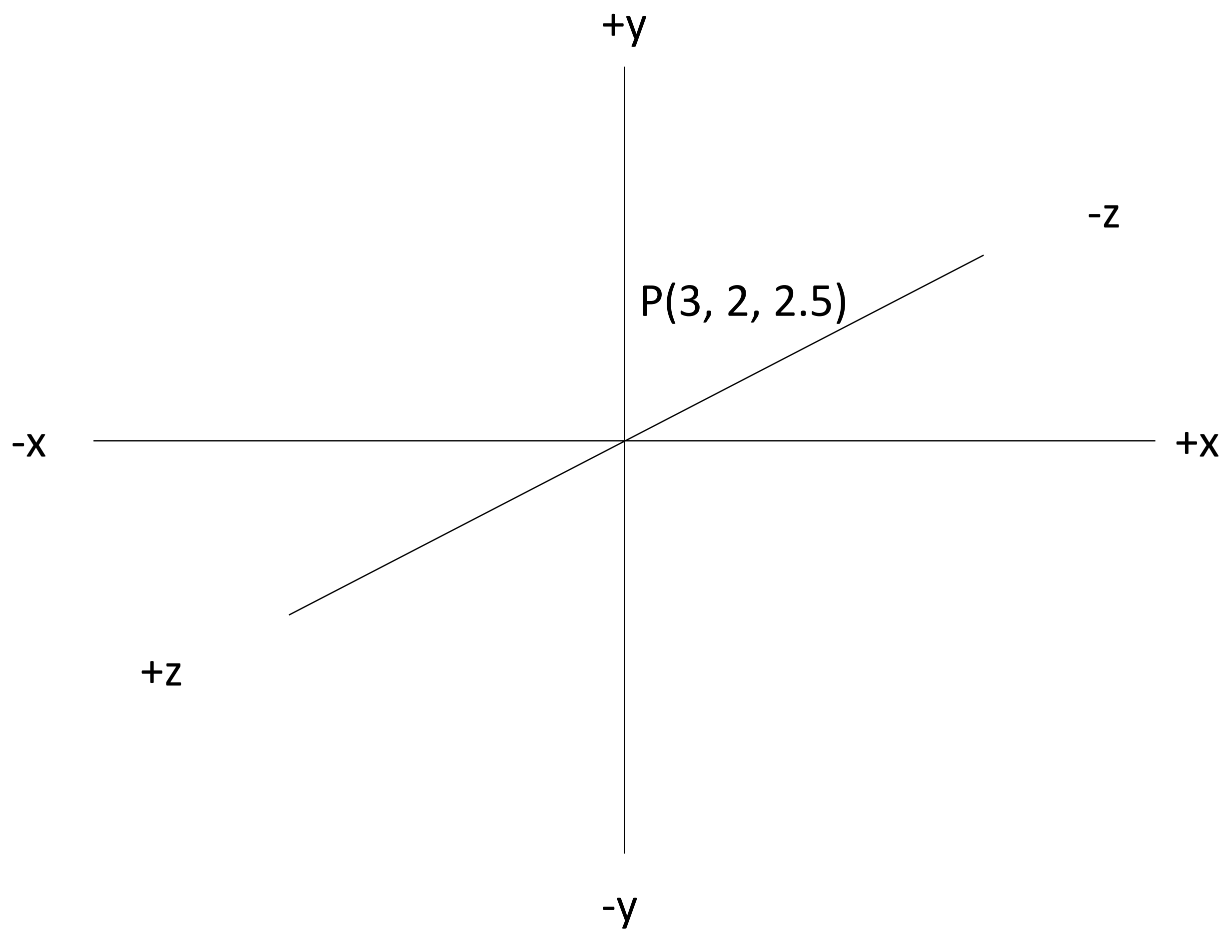
2D coordinate system



3D coordinate system



3D coordinate system



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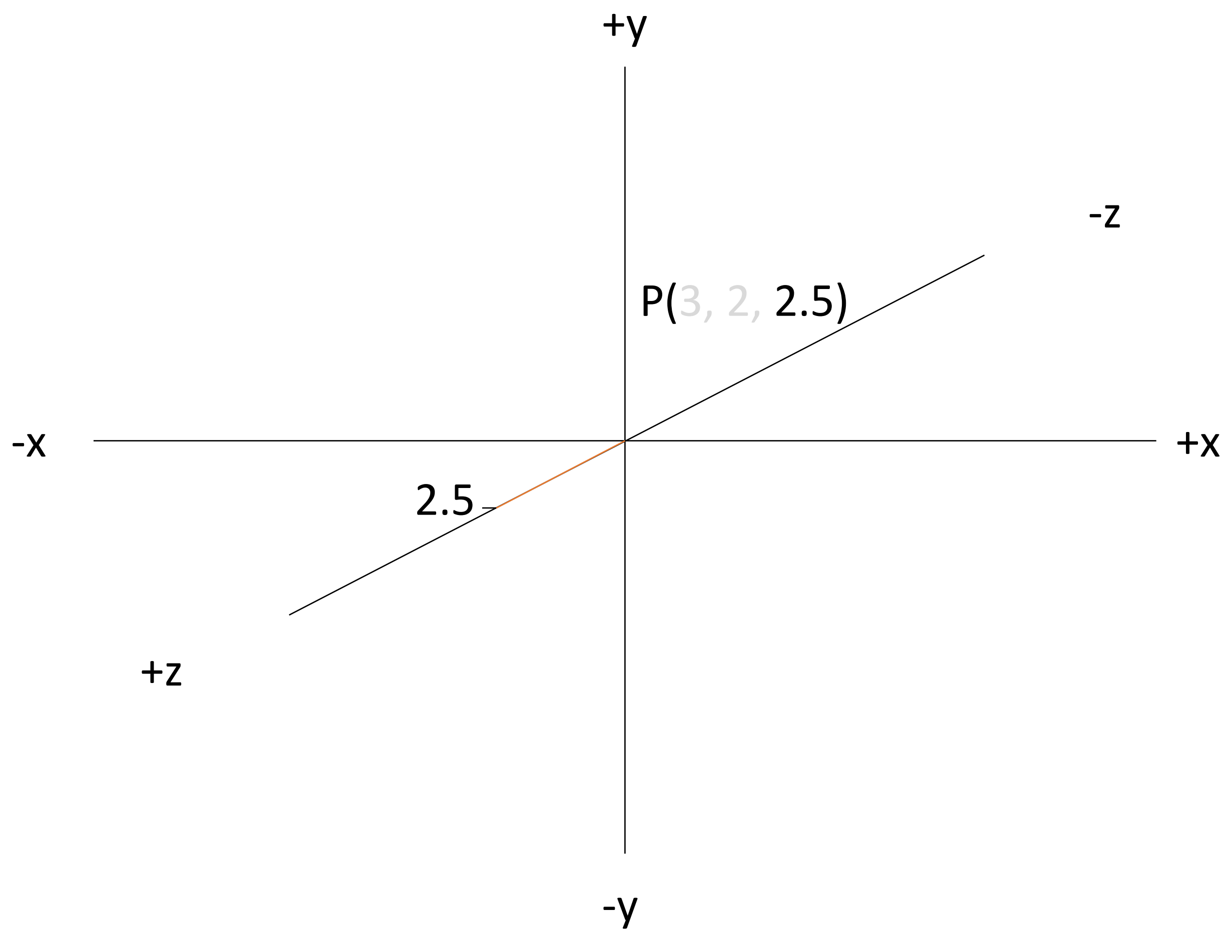
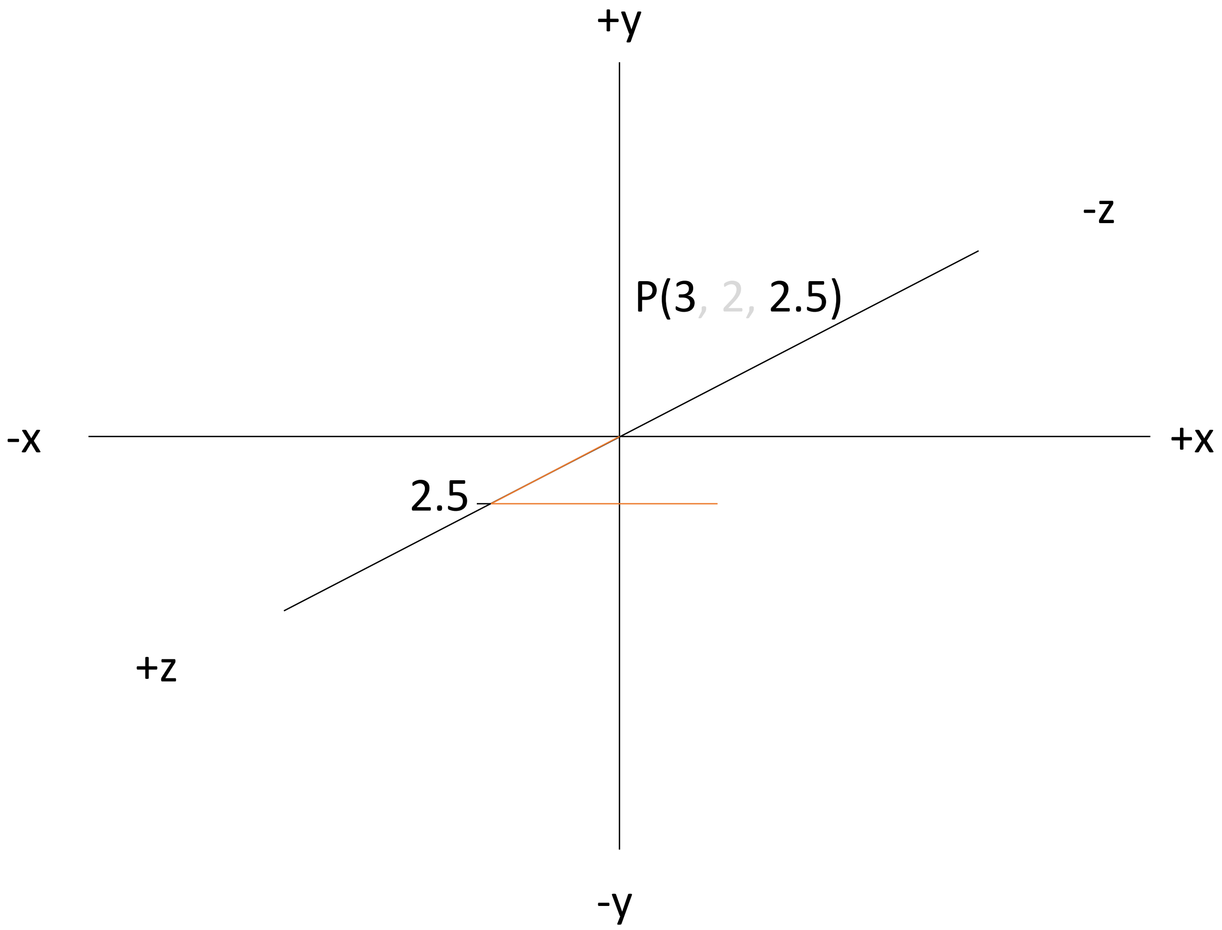
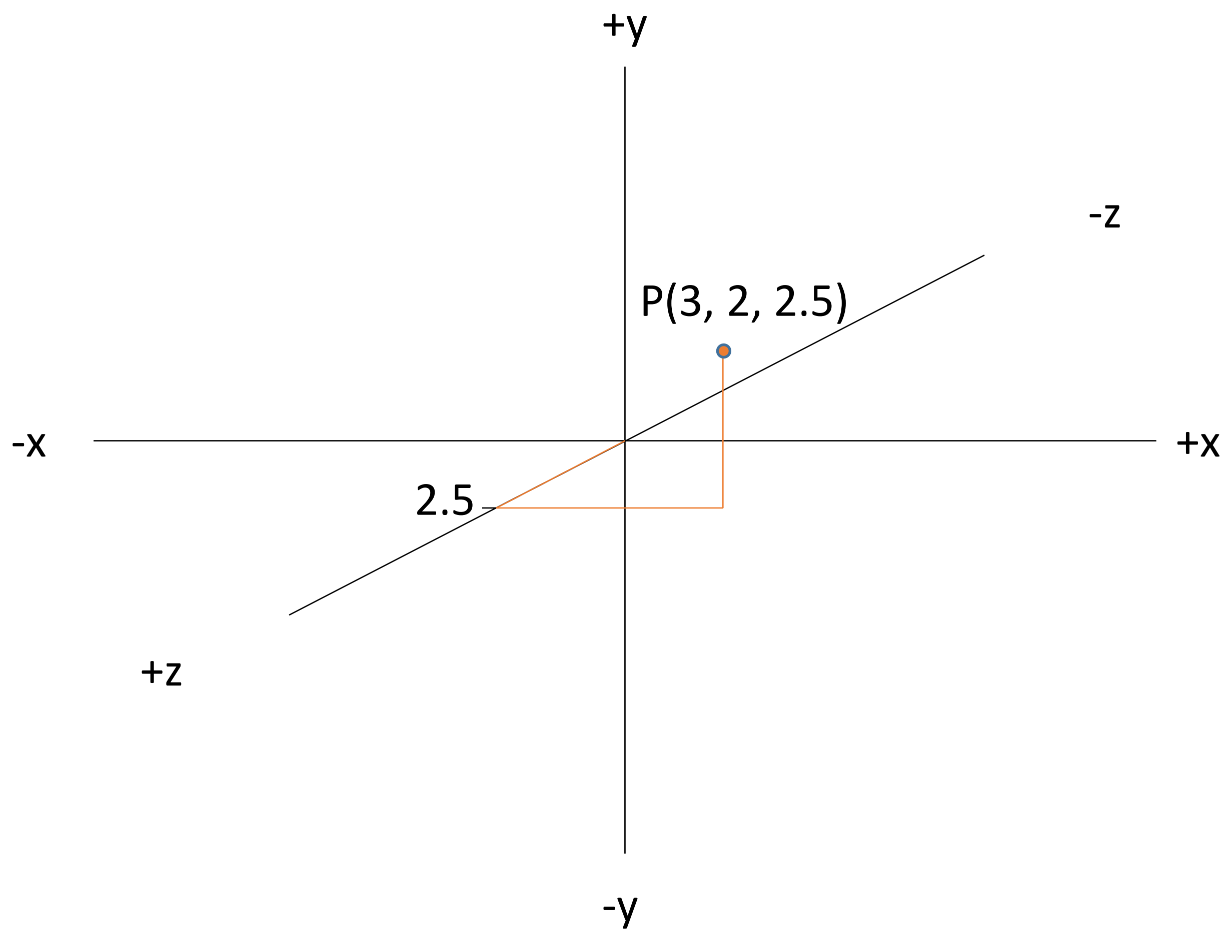


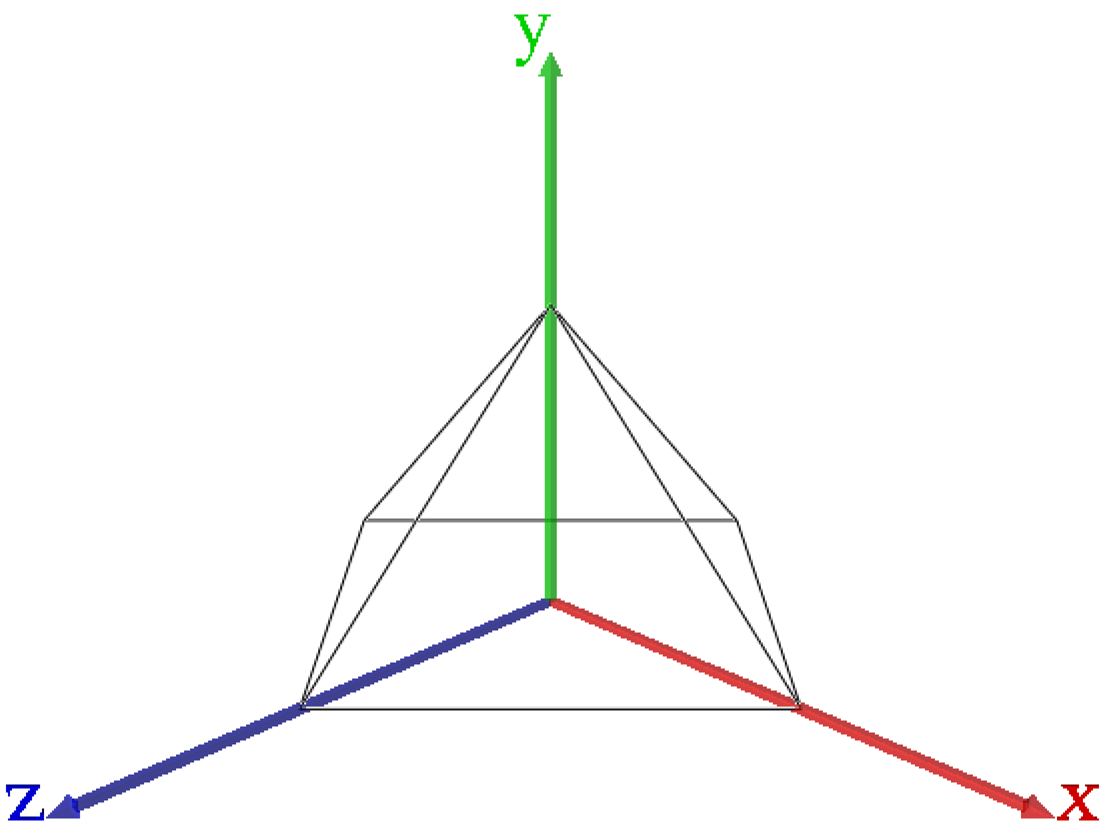
3D coordinate system



3D coordinate system



Example: Pyramid



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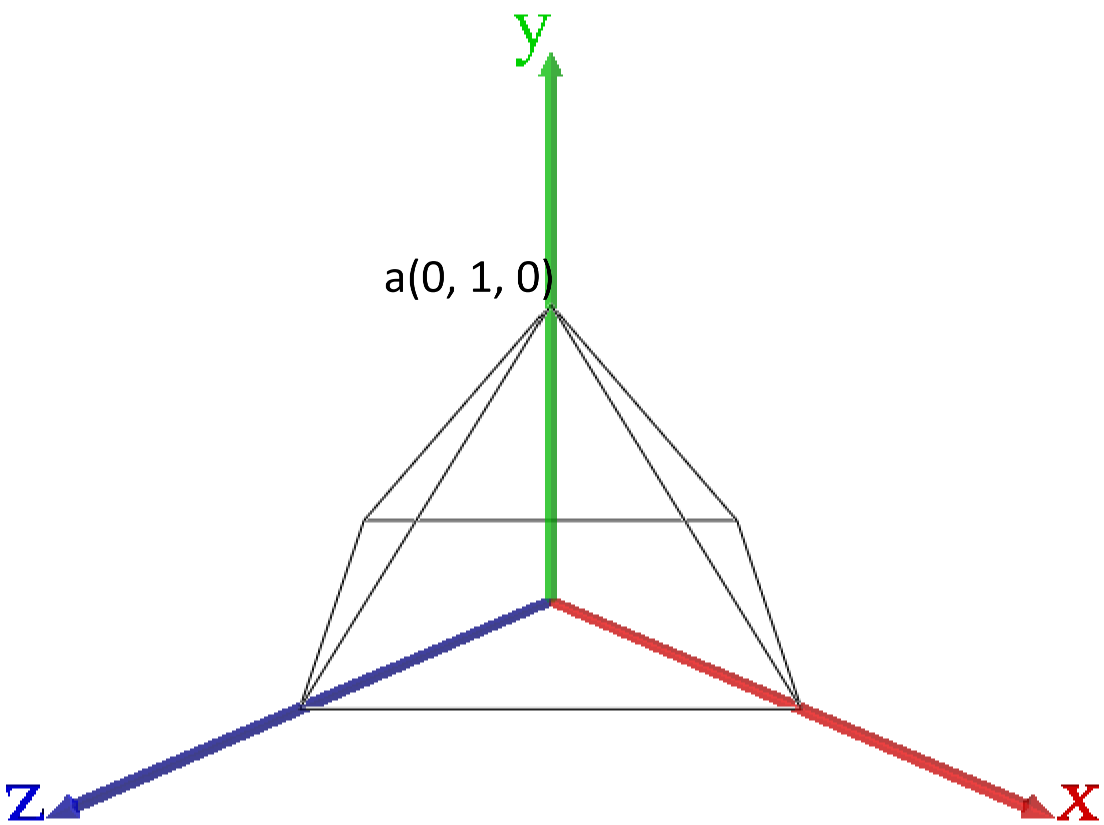

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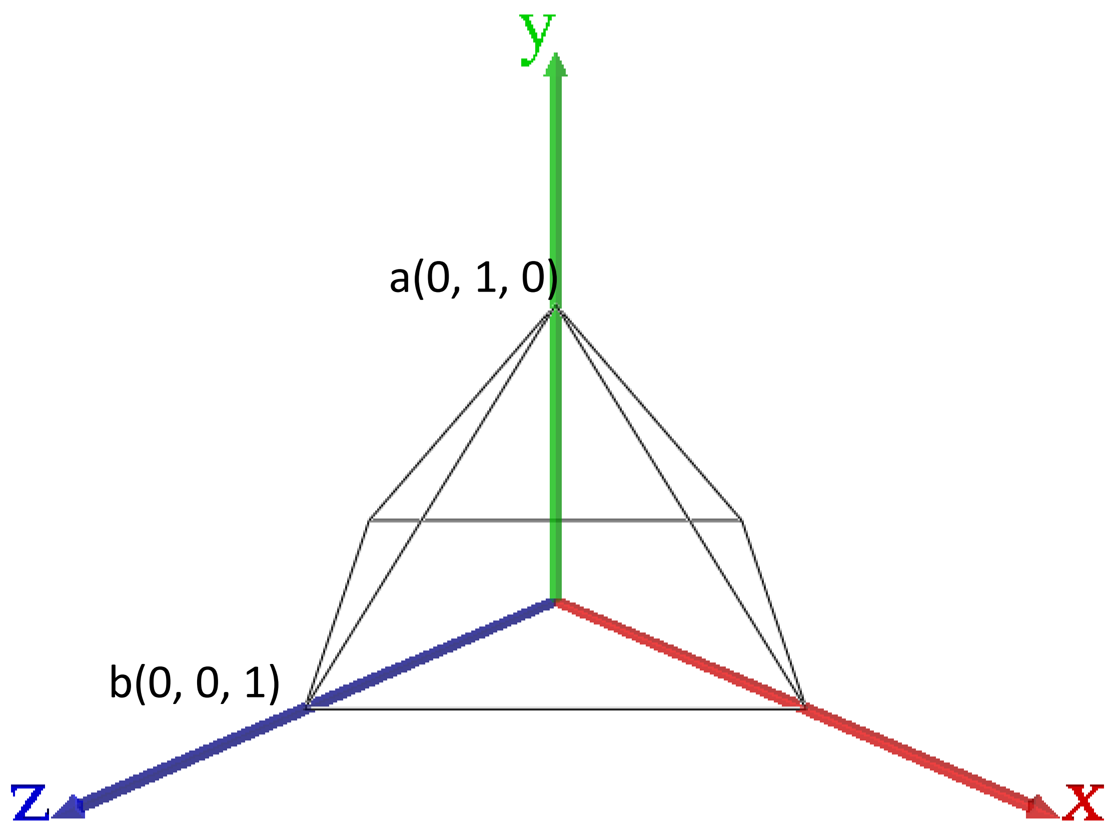

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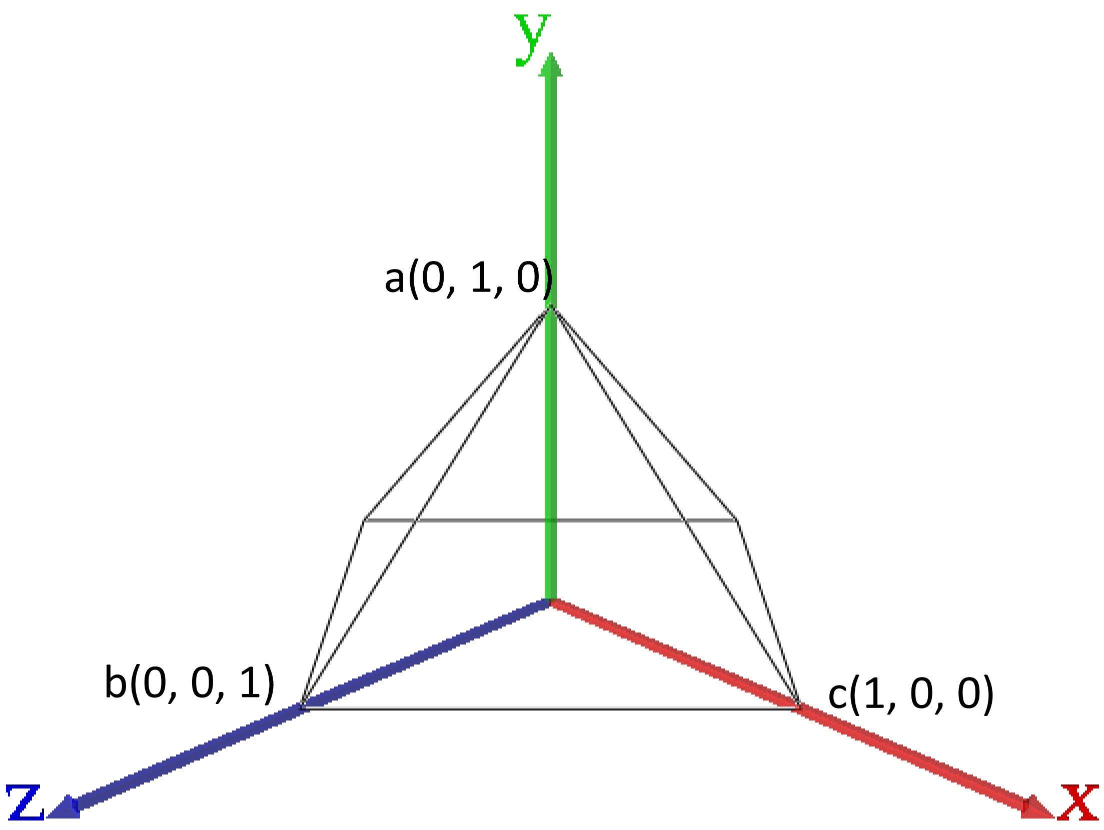

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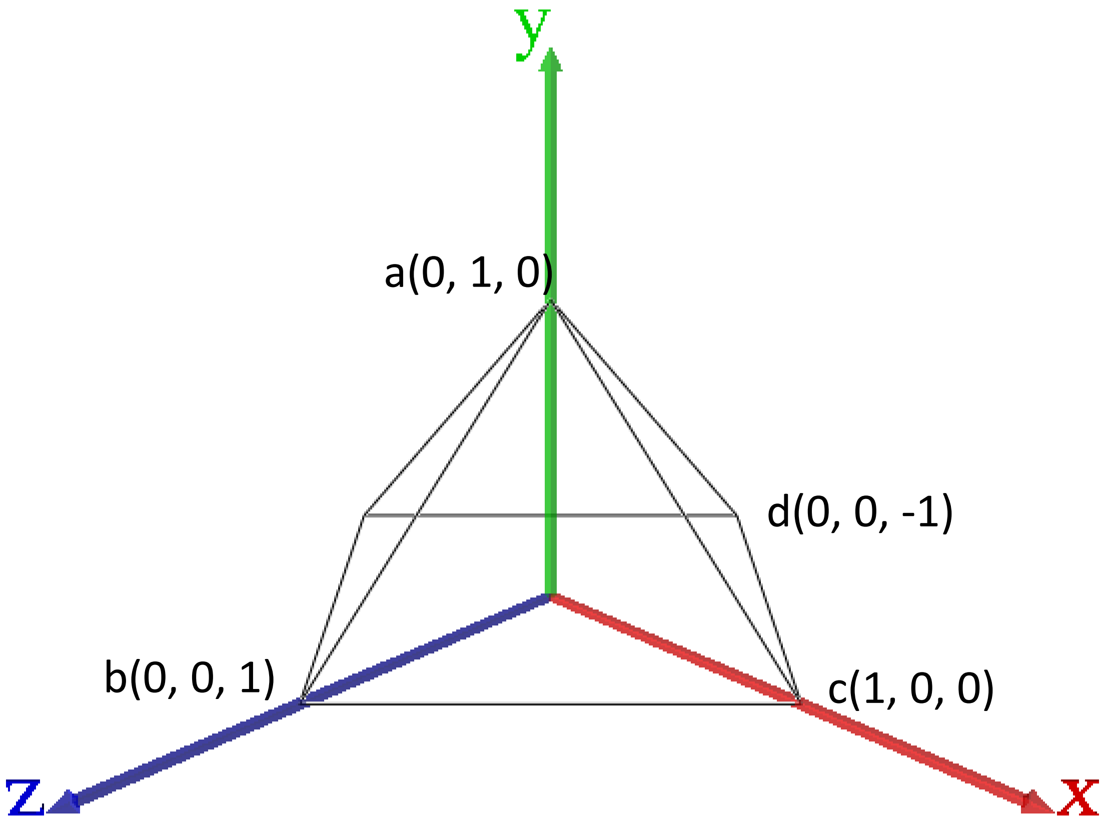

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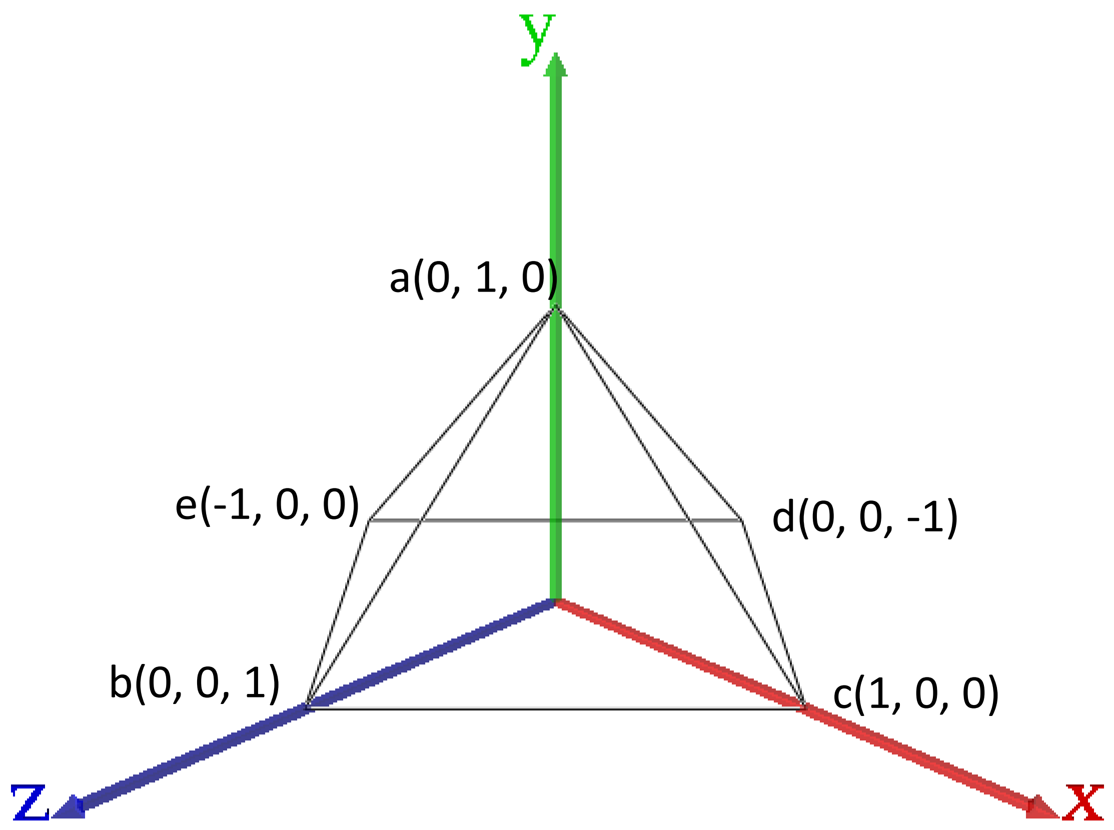
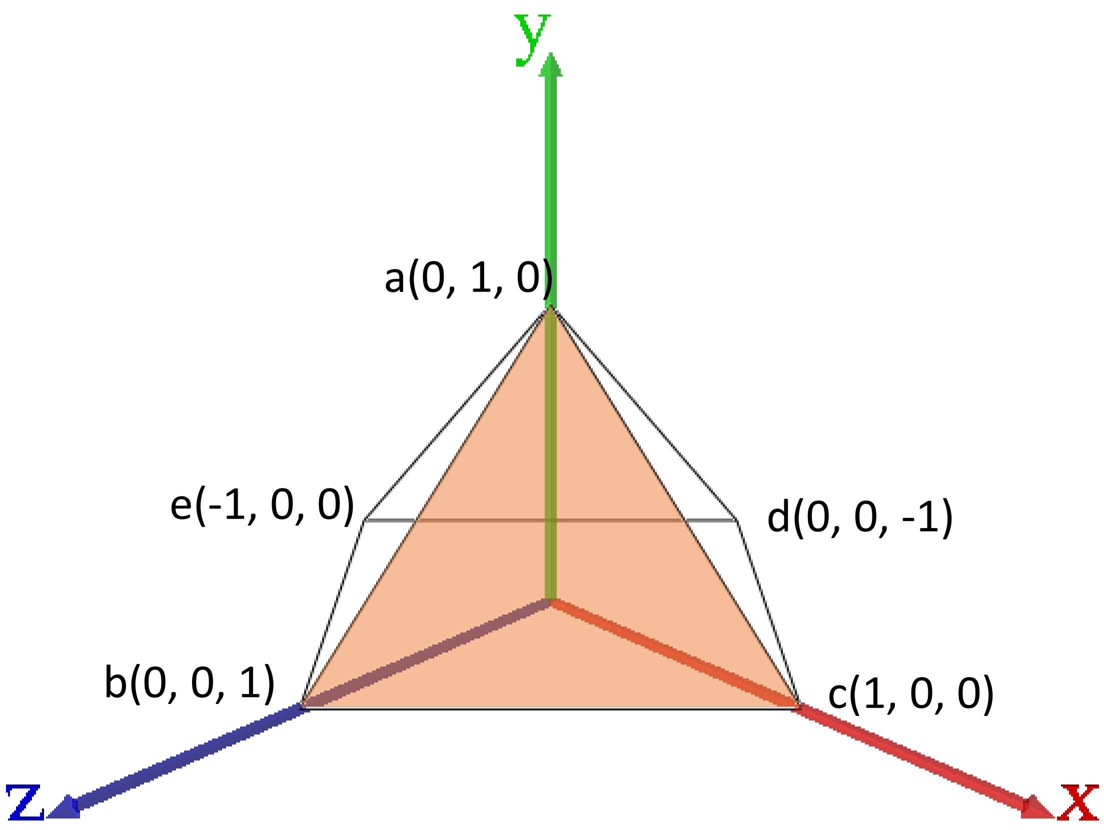
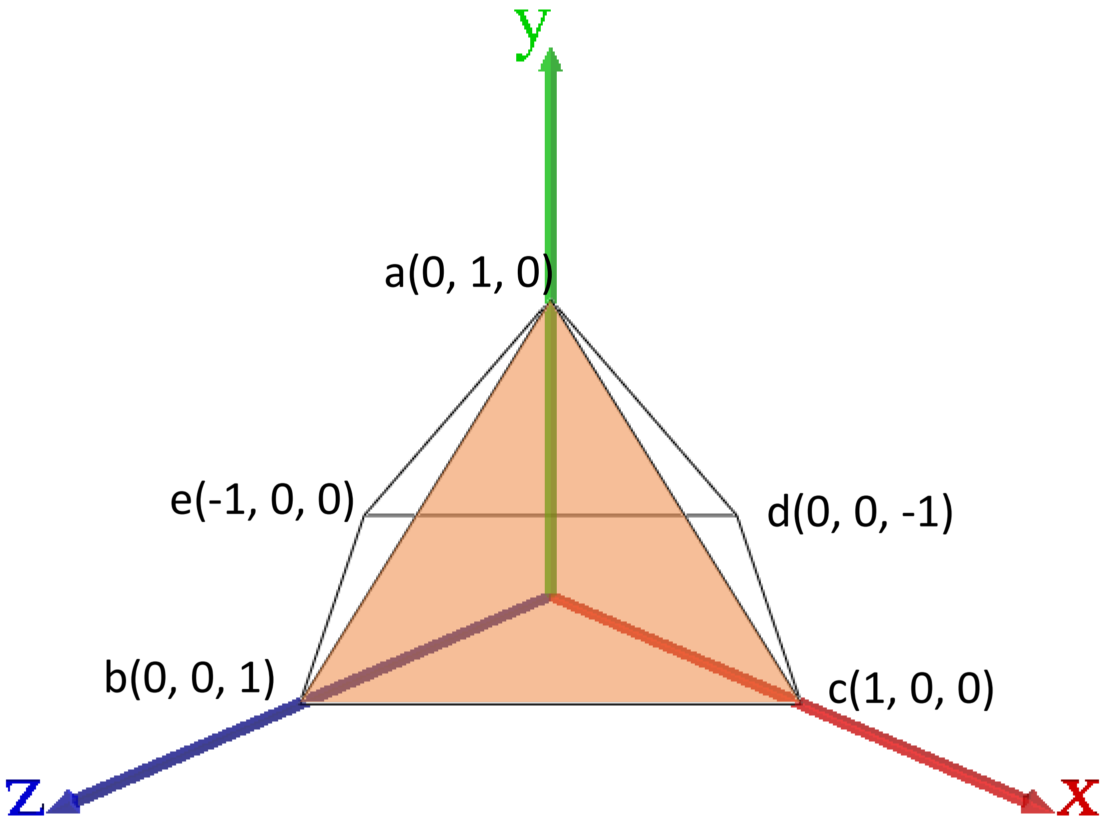


Face

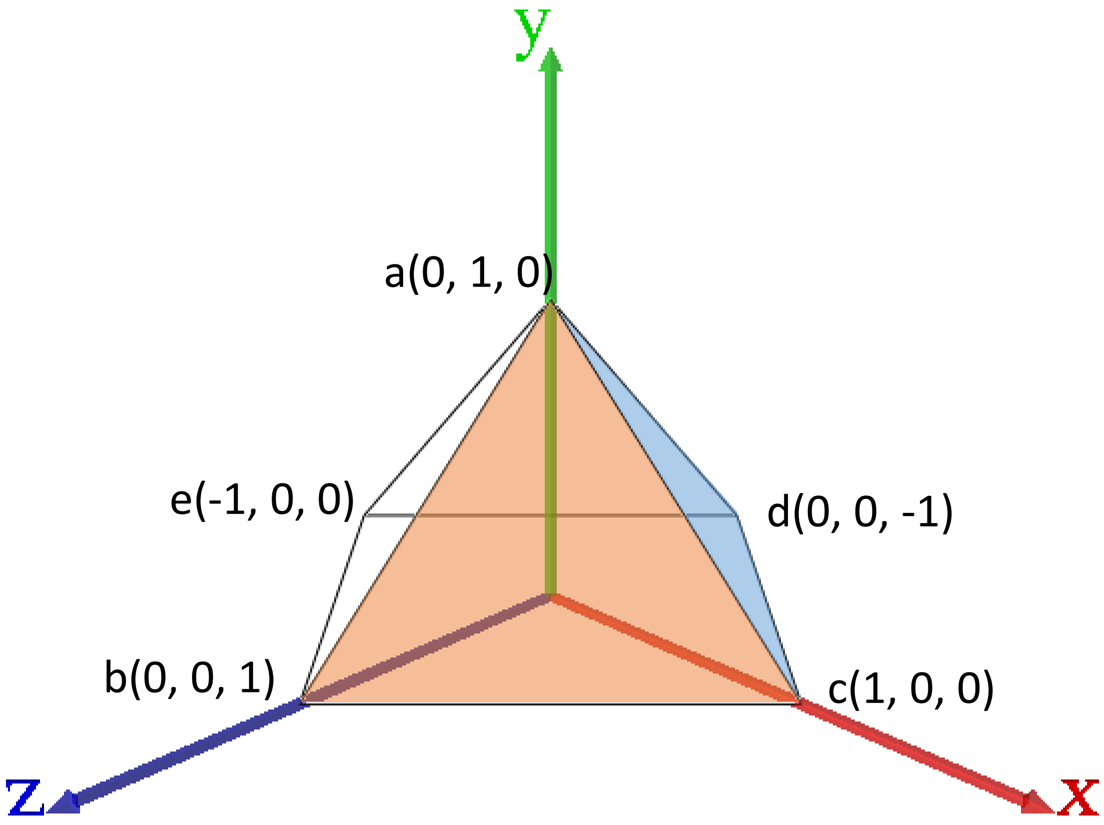


Face



For example: (0, 1, 0) (0, 0, 1) (1, 0, 0)

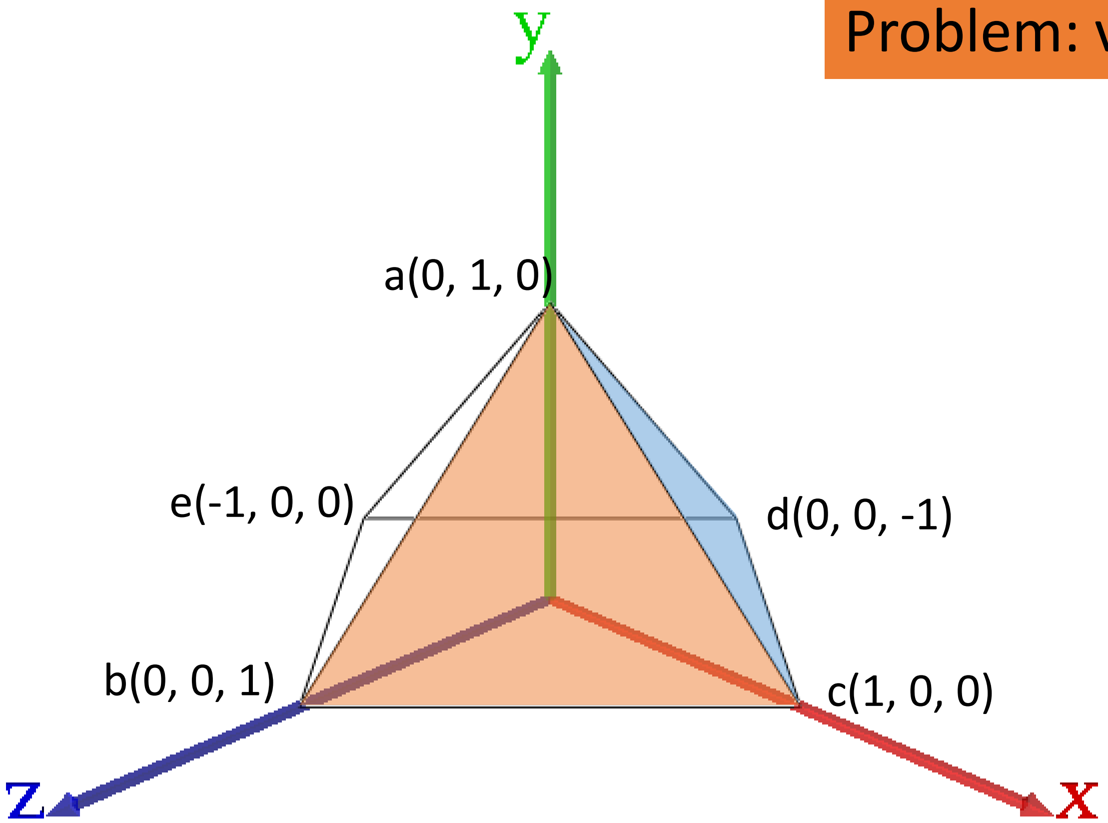
Face



For example: (0, 1, 0) (0, 0, 1) (1, 0, 0) or (0, 1, 0) (1, 0, 0) (0, 0, -1)

Face

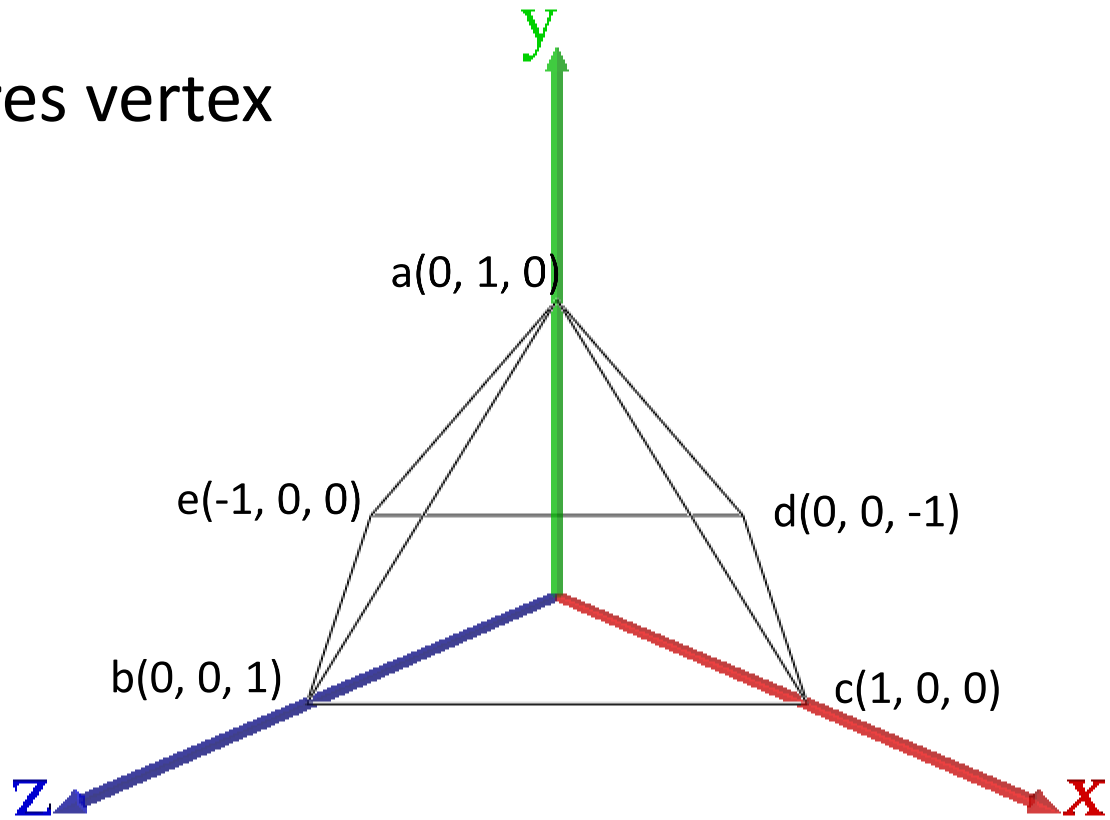
Problem: vertex duplication!



For example: (0, 1, 0) (0, 0, 1) (1, 0, 0) or (0, 1, 0) (1, 0, 0) (0, 0, -1)

Solution: new data structure

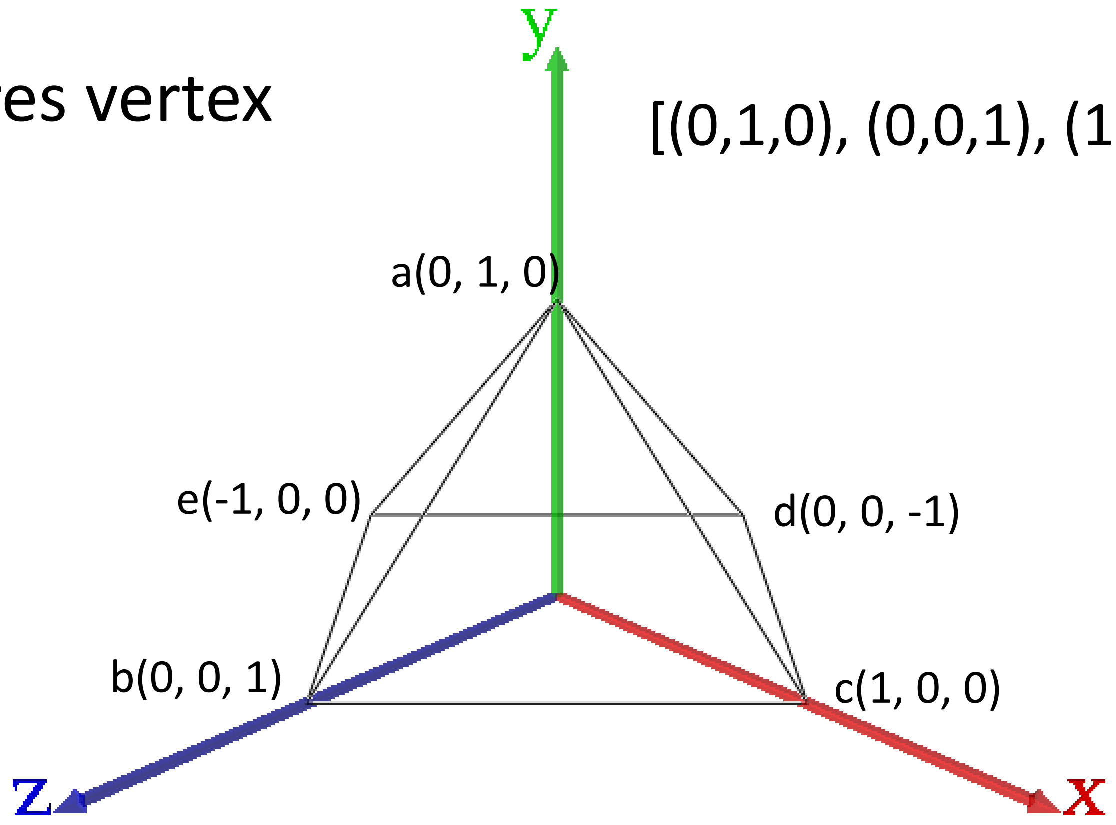
Vertex buffer stores vertex information



Solution: new data structure

Vertex buffer stores vertex information

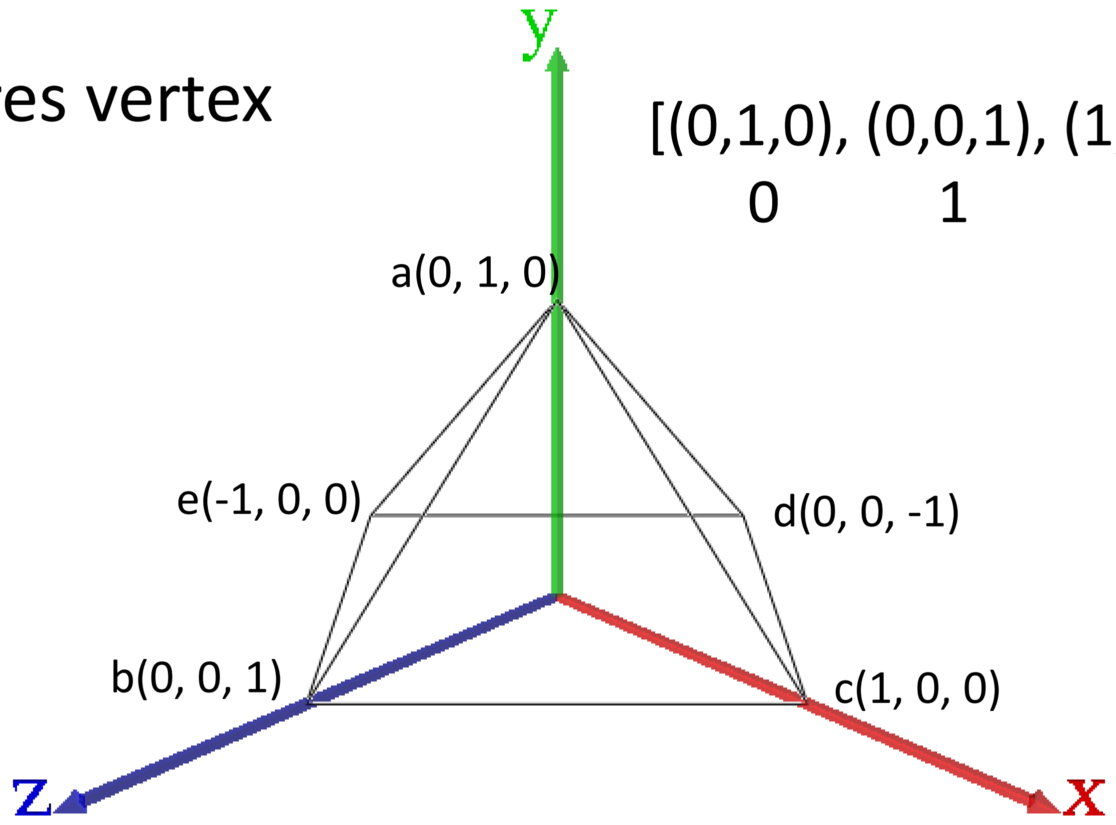
[(0,1,0), (0,0,1), (1,0,0), (0,0,-1), (-1,0,0)]



Solution: new data structure

Vertex buffer stores vertex information

[(0,1,0), (0,0,1), (1,0,0), (0,0,-1), (-1,0,0)]
0 1 2 3 4



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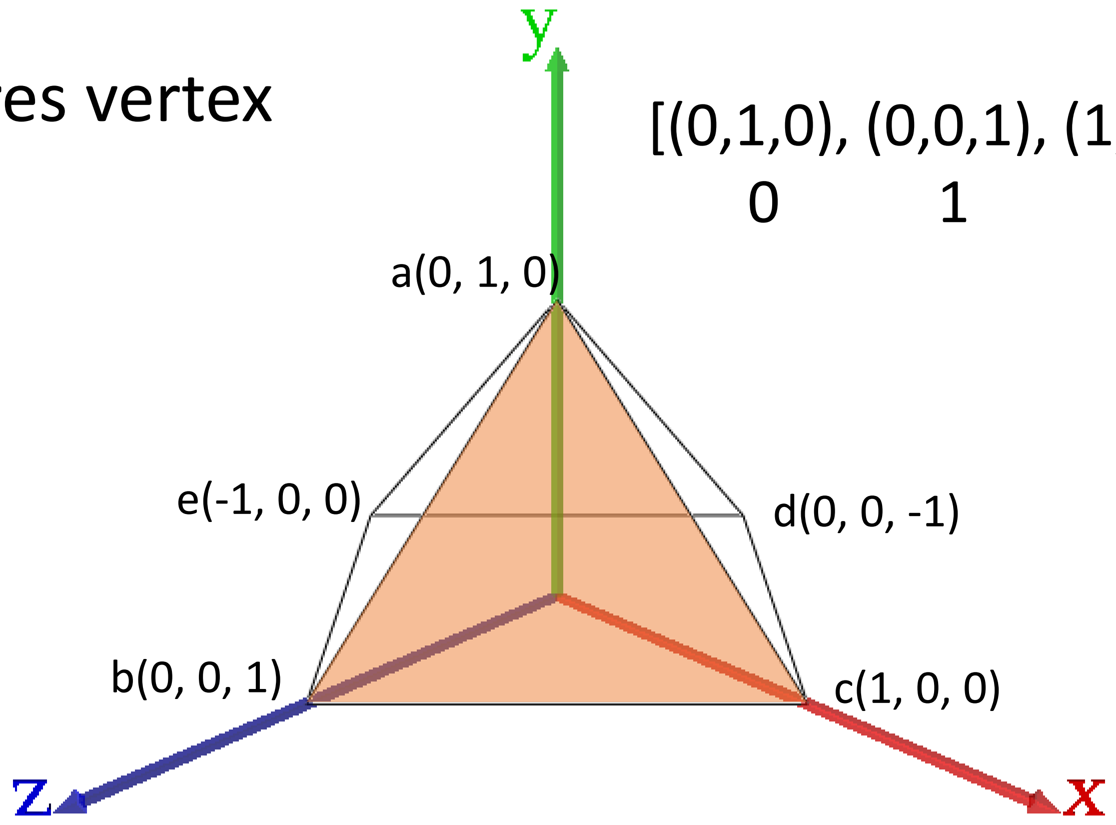
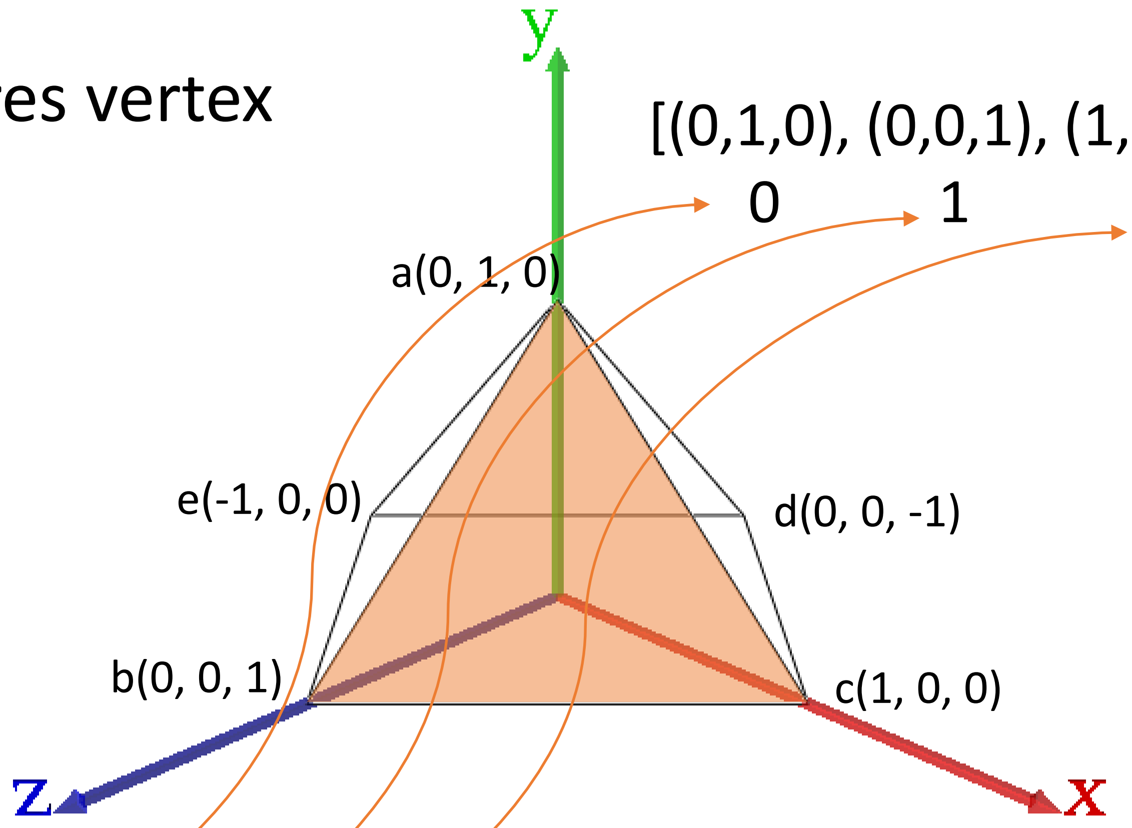


For example: (0, 1, 2)

Solution: new data structure

Vertex buffer stores vertex information

[(0,1,0), (0,0,1), (1,0,0), (0,0,-1), (-1,0,0)]
0 1 2 3 4



For example: (0, 1, 2)

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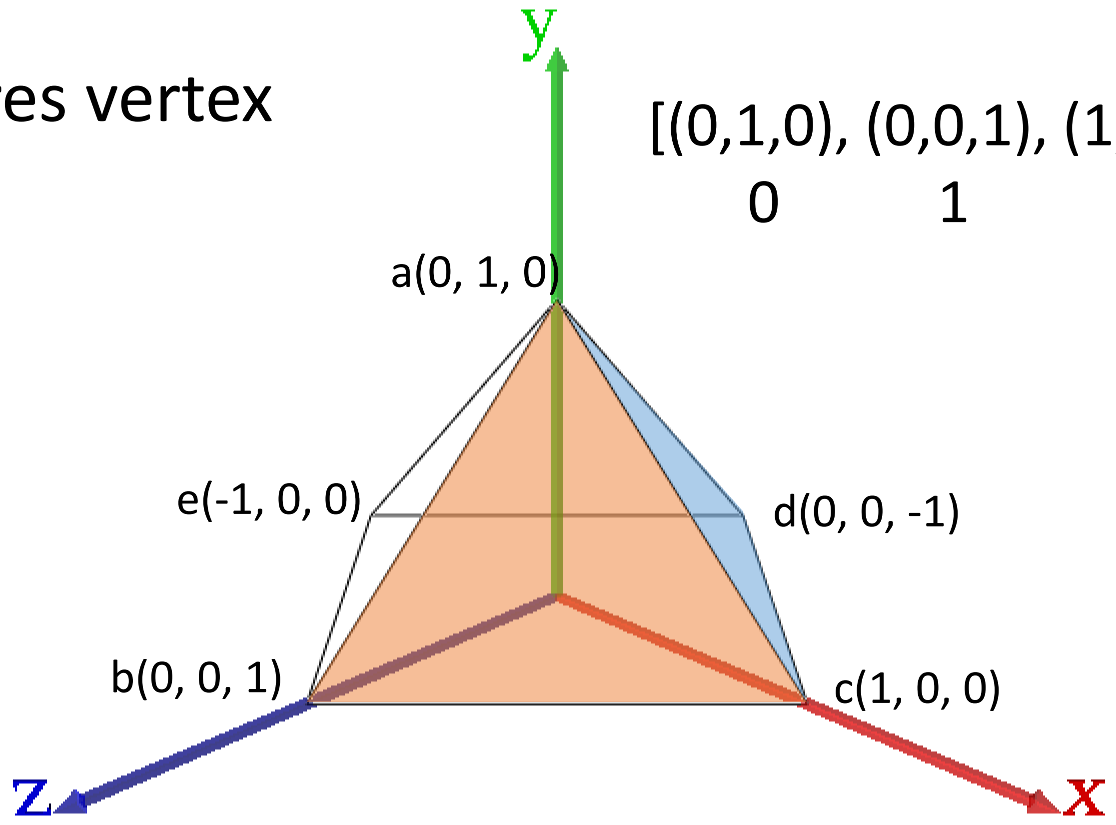
[(0,1,0), (0,0,1), (1,0,0), (0,0,-1), (-1,0,0)]
0 1 2 3 4



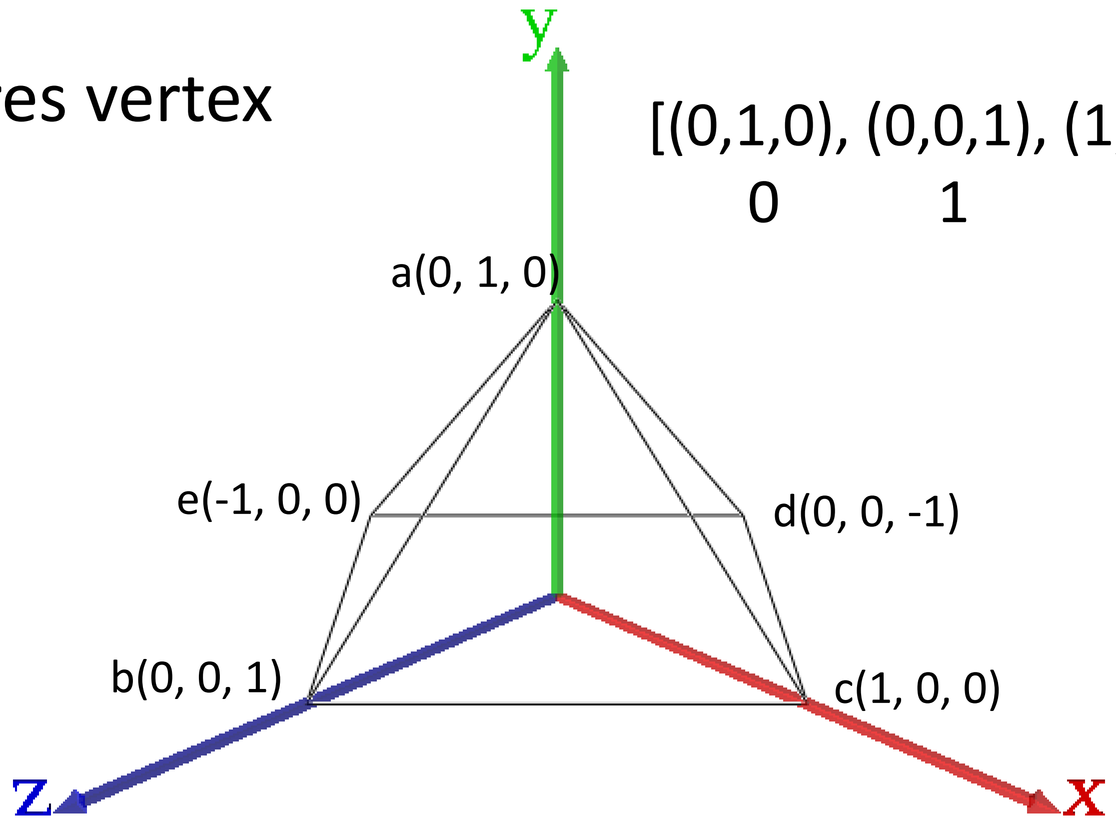
For example: (0, 1, 2) or(0, 2, 3)

Solution: new data structure

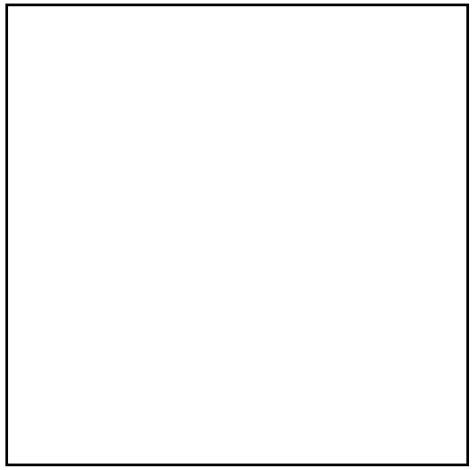
Vertex buffer stores vertex information

[(0,1,0), (0,0,1), (1,0,0), (0,0,-1), (-1,0,0)]

0 1 2 3 4

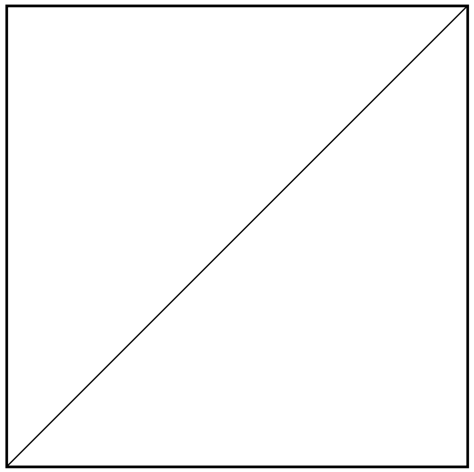


Triangles



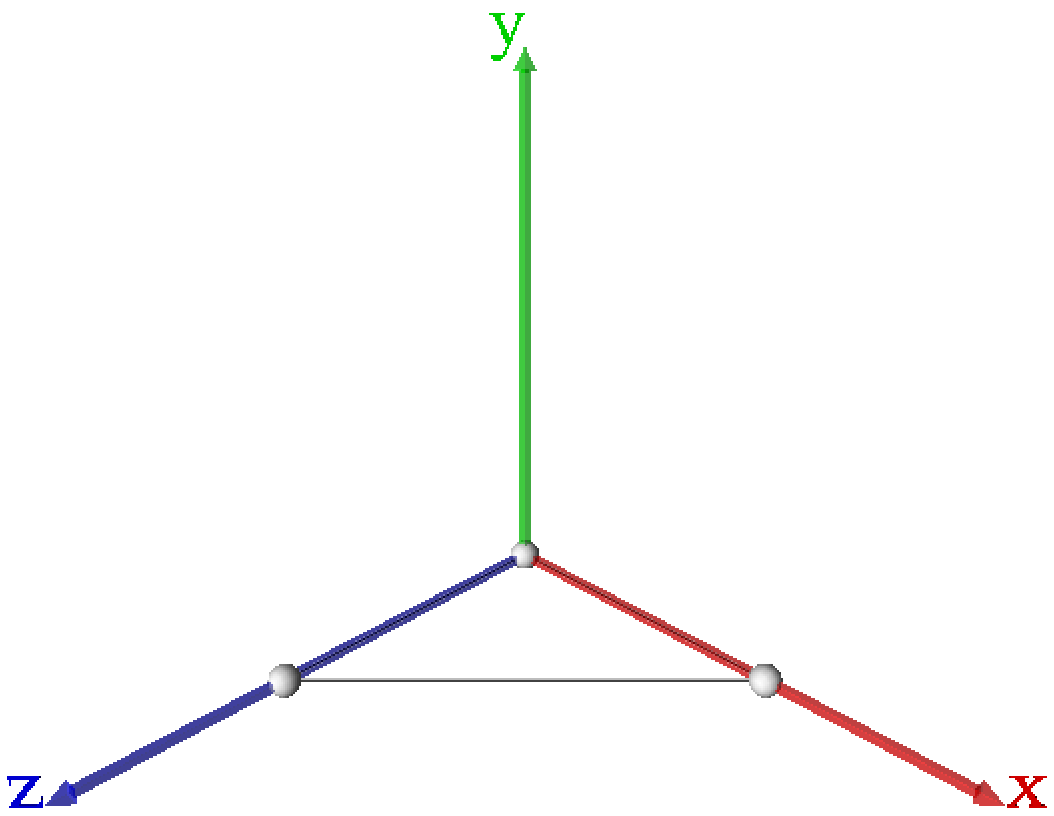
bottom of the pyramid

Triangles



bottom of the pyramid

Triangles - coplanarity



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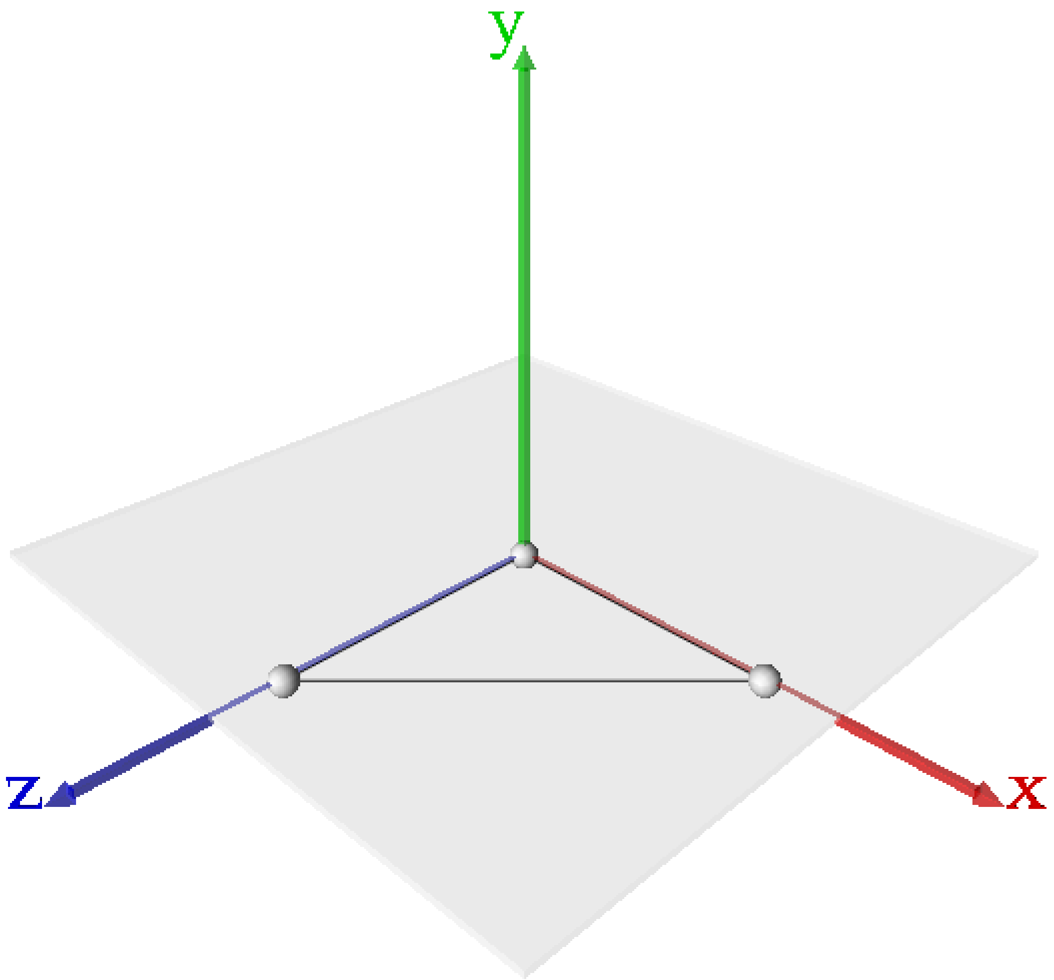
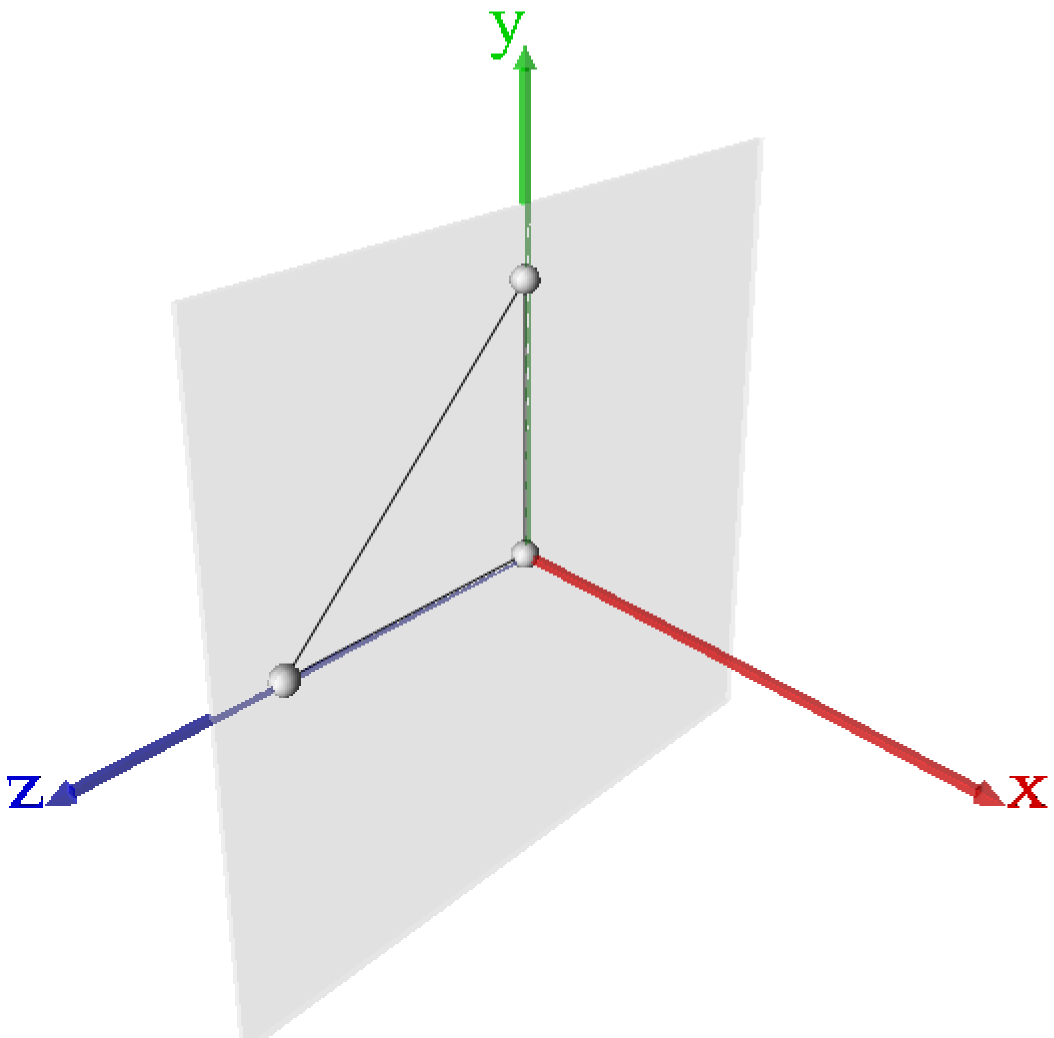
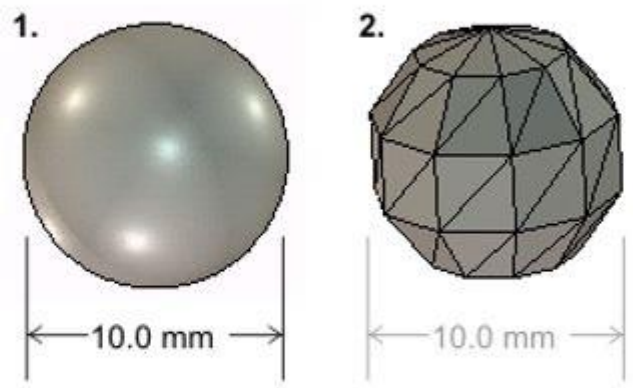


Triangles - coplanarity

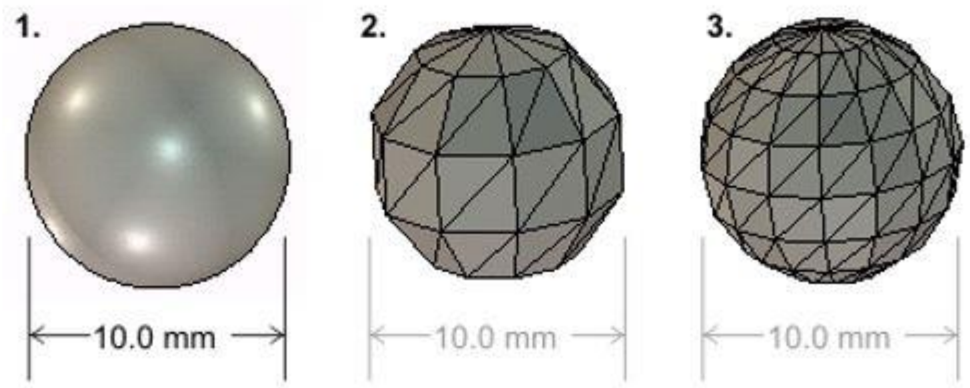


Shape approximation with triangles

Shape approximation with triangles



Shape approximation with triangles



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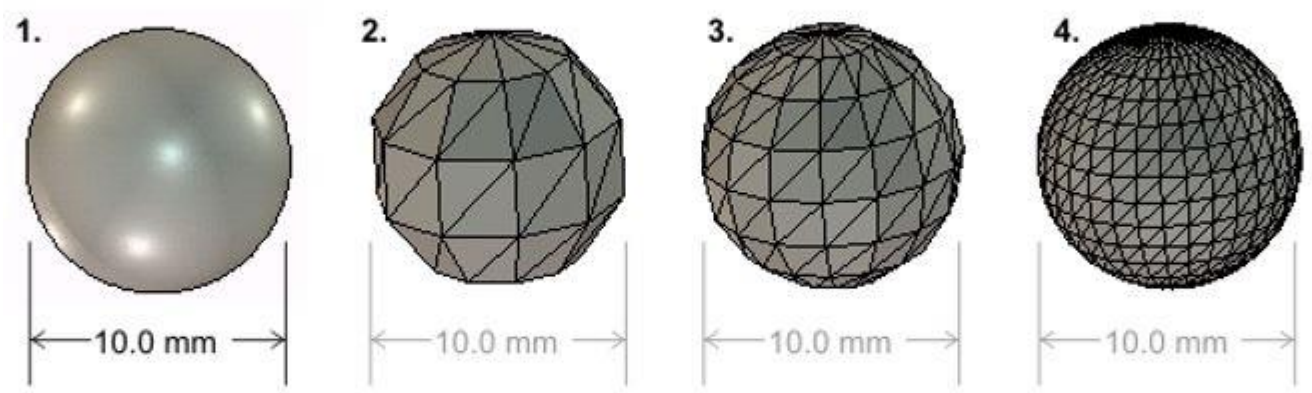
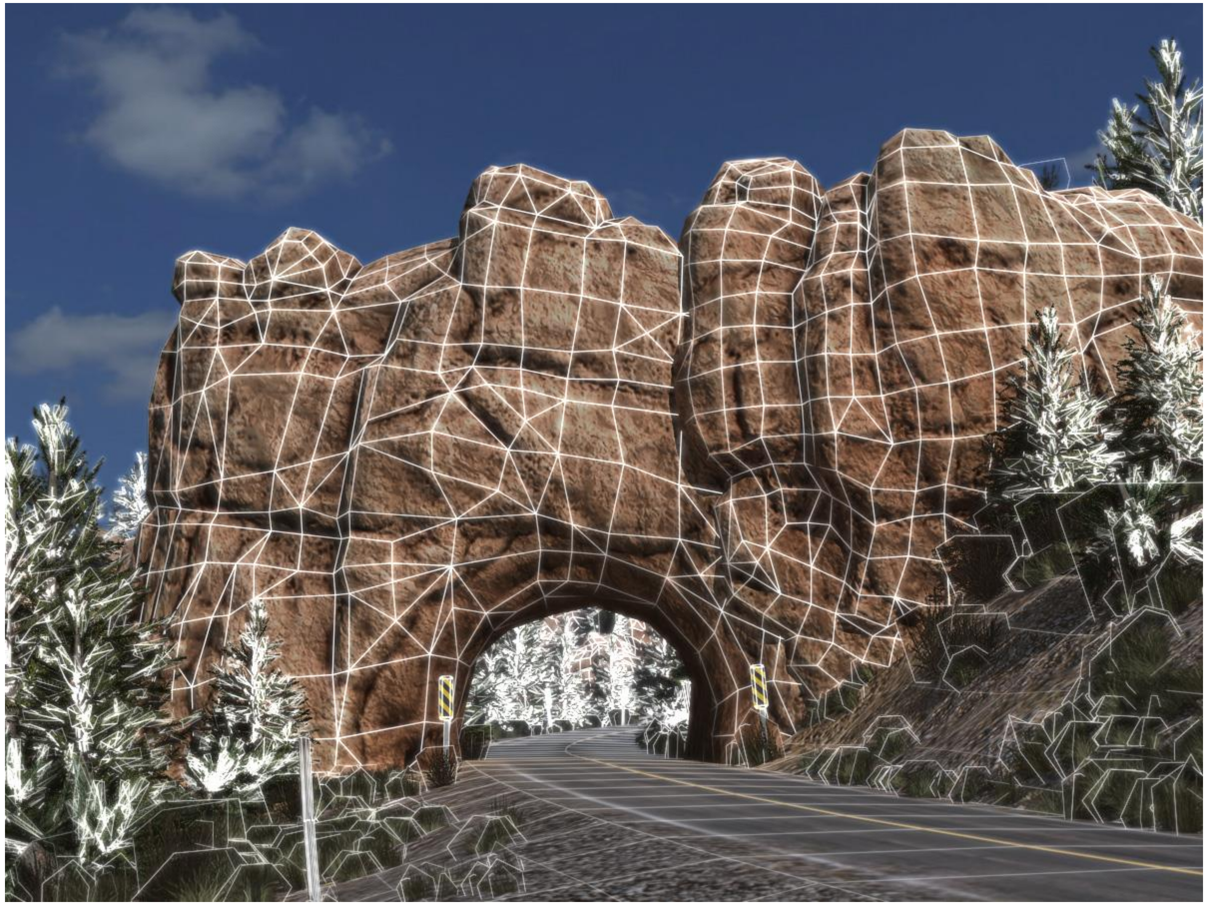


World of polygons (triangles)

http://www.jeroenbackx.com/

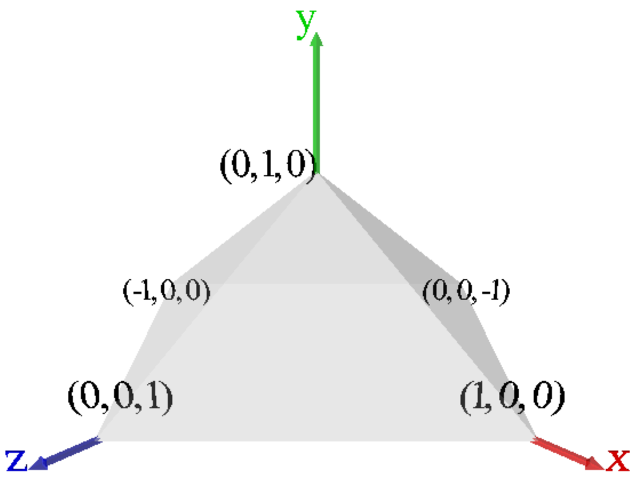
World of polygons (triangles)



http://www.jeroenbackx.com/

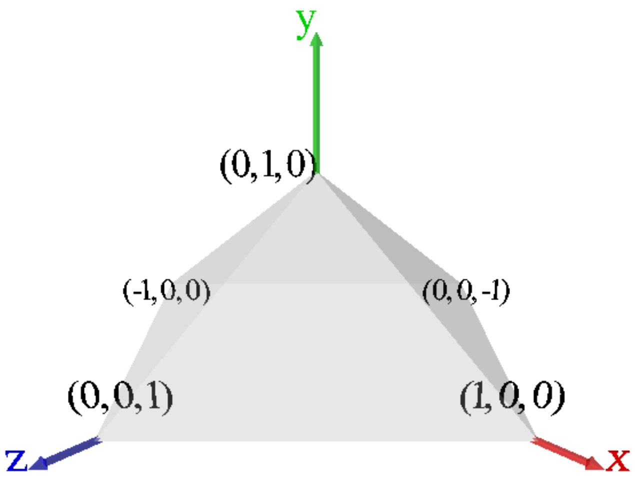
Translation of vertices?

Vertex transformations



Scaling

Vertex transformations



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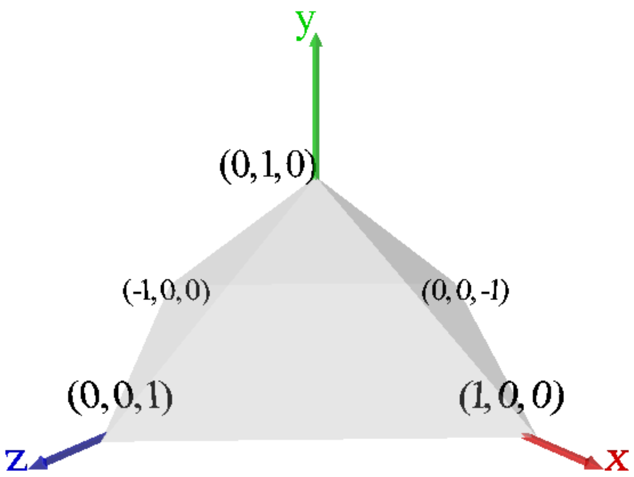
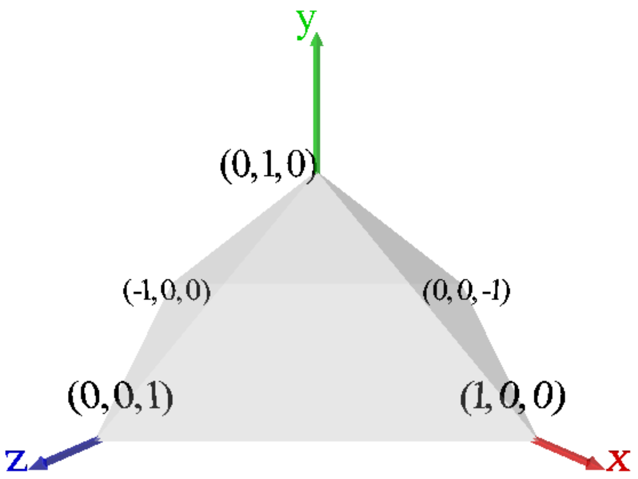
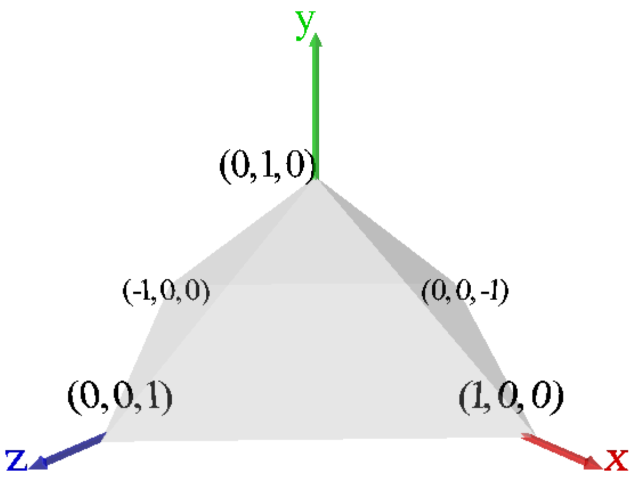


Rotation

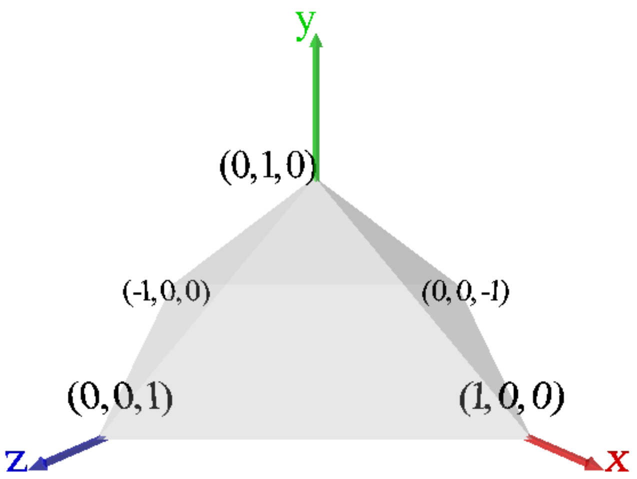
Vertex transformations



Scaling



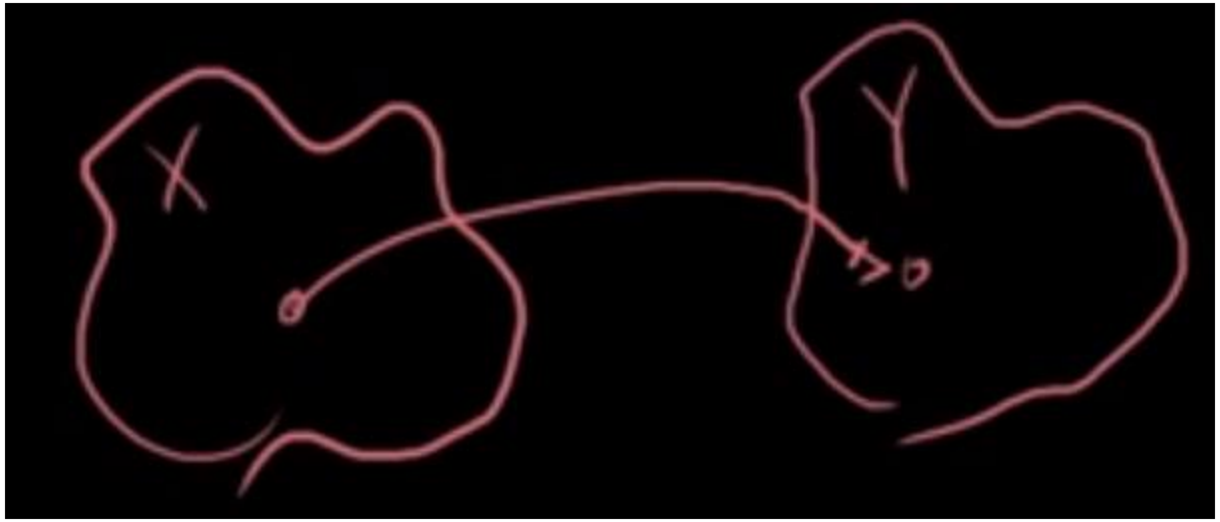
Rotation



Translation

Transformation

- $f: X \rightarrow Y$

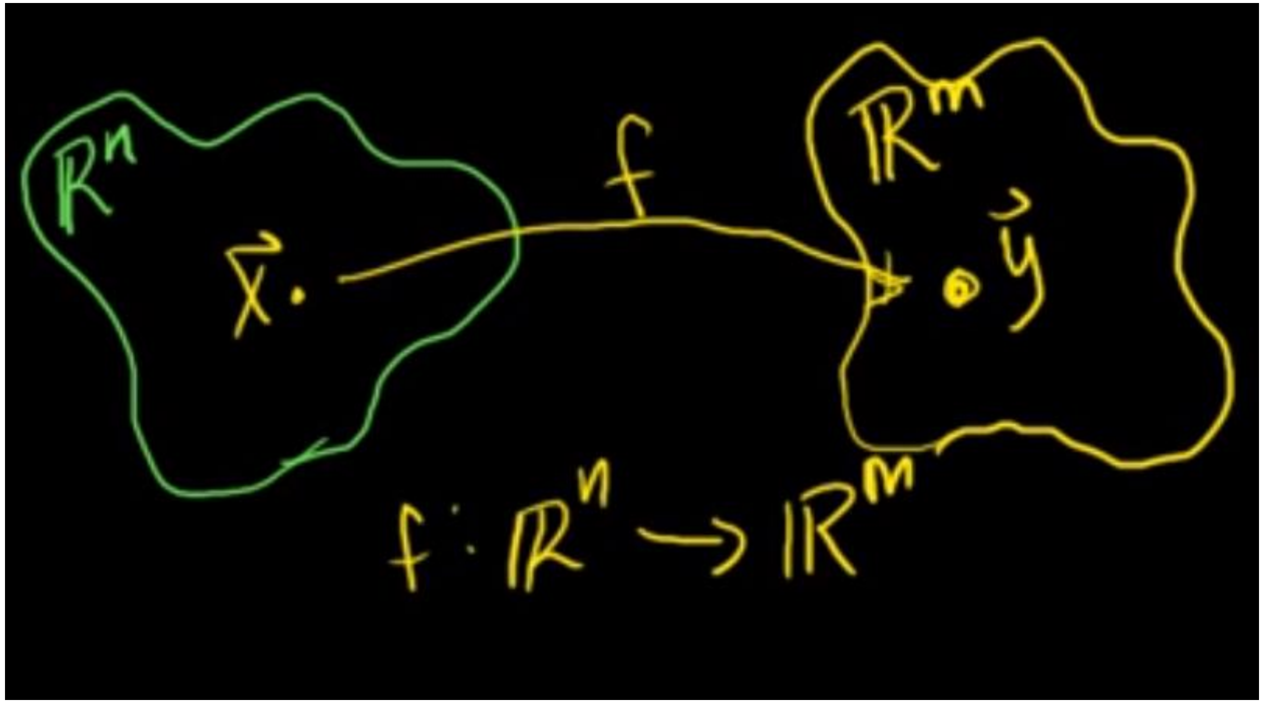


Transformation

- $f: X \rightarrow Y$

Vector-valued functions

$\mathbb{R}^n = \{(x_1, \dots, x_n): x_1, \dots, x_n \in \mathbb{R}\}$



Transformation

$f: \mathbb{R}^3 \to \mathbb{R}^2$

Transformation

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$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ 3x_3 \end{bmatrix}$$

Transformation

$f: \mathbb{R}^3 \to \mathbb{R}^2$

$$f\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ 3x_3 \end{bmatrix}$$

$$f\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right)$$

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$$f\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 + 2 \cdot 1 \\ 3 \cdot 1 \end{bmatrix}$$

Transformation

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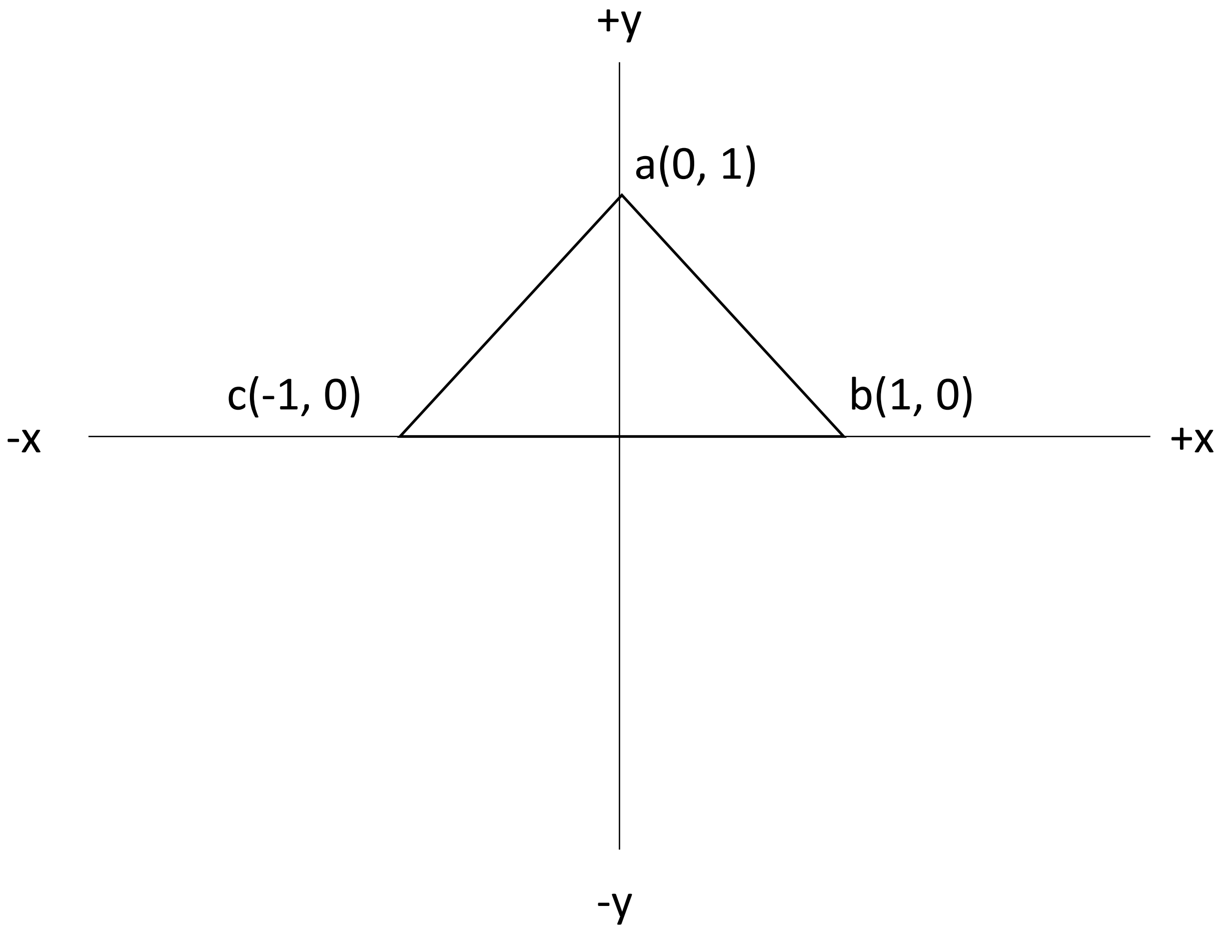
$$f\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 + 2 \cdot 1 \\ 3 \cdot 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

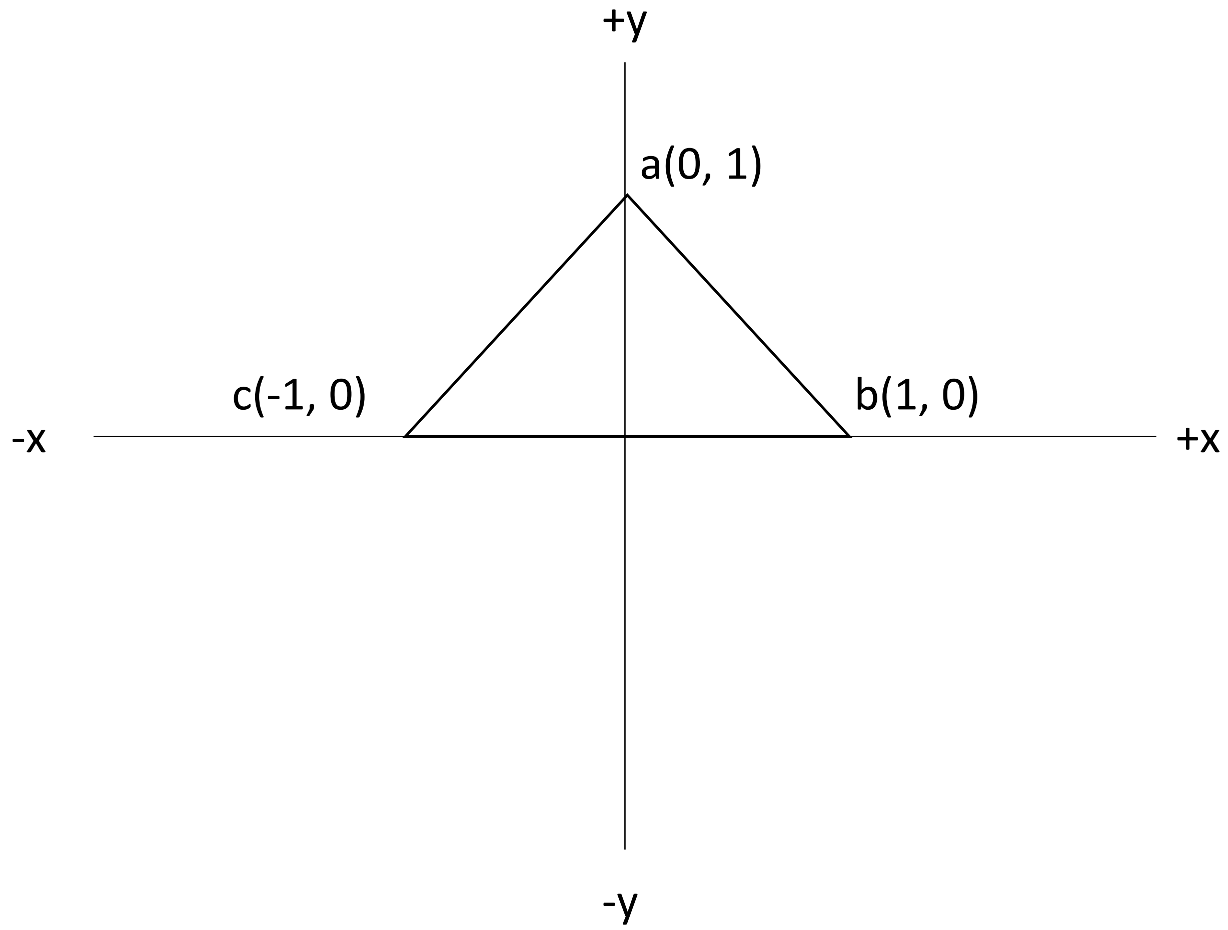
Scaling

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \cdot s_x \\ y \cdot s_y \end{bmatrix}$$



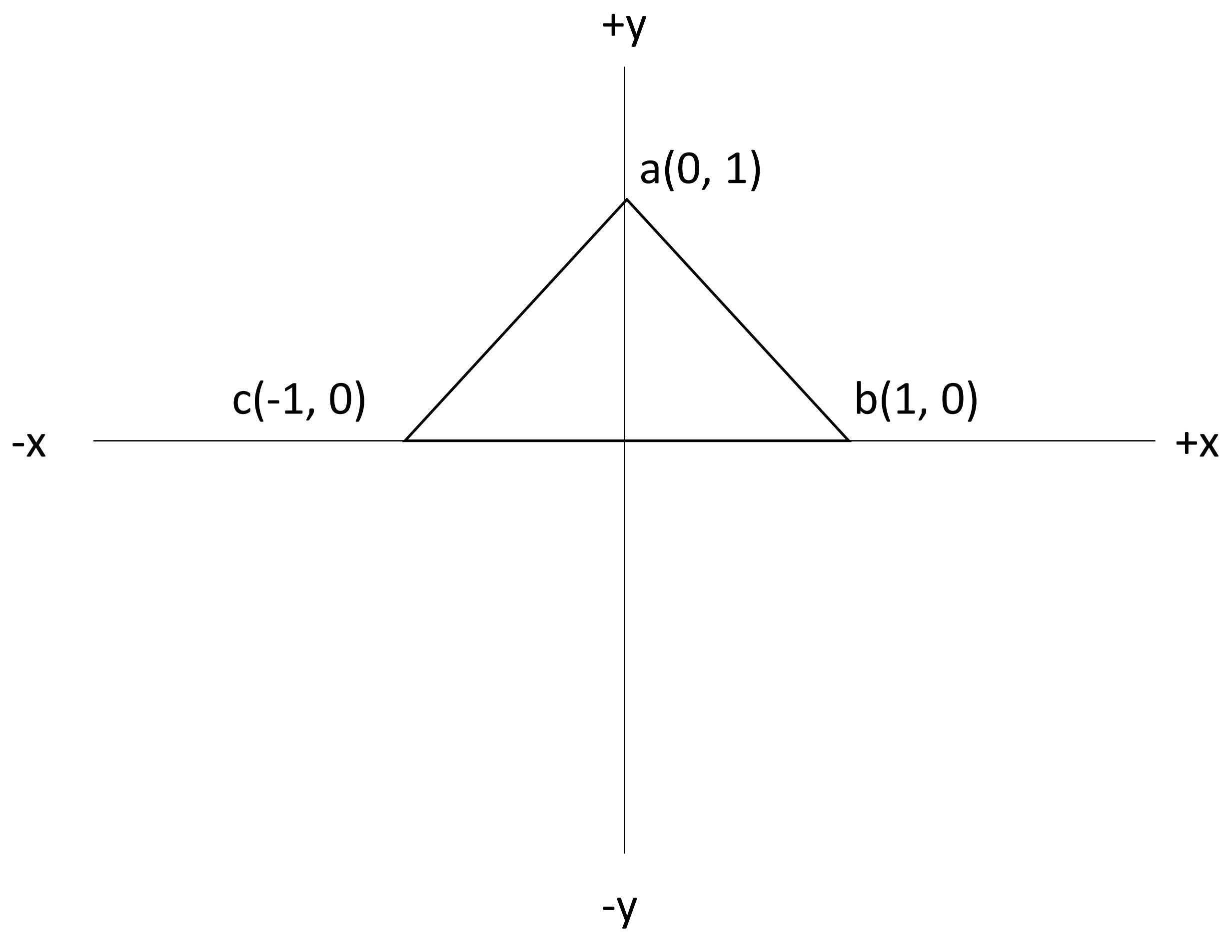
Scaling

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \cdot 2 \\ y \cdot 1 \end{bmatrix}$$



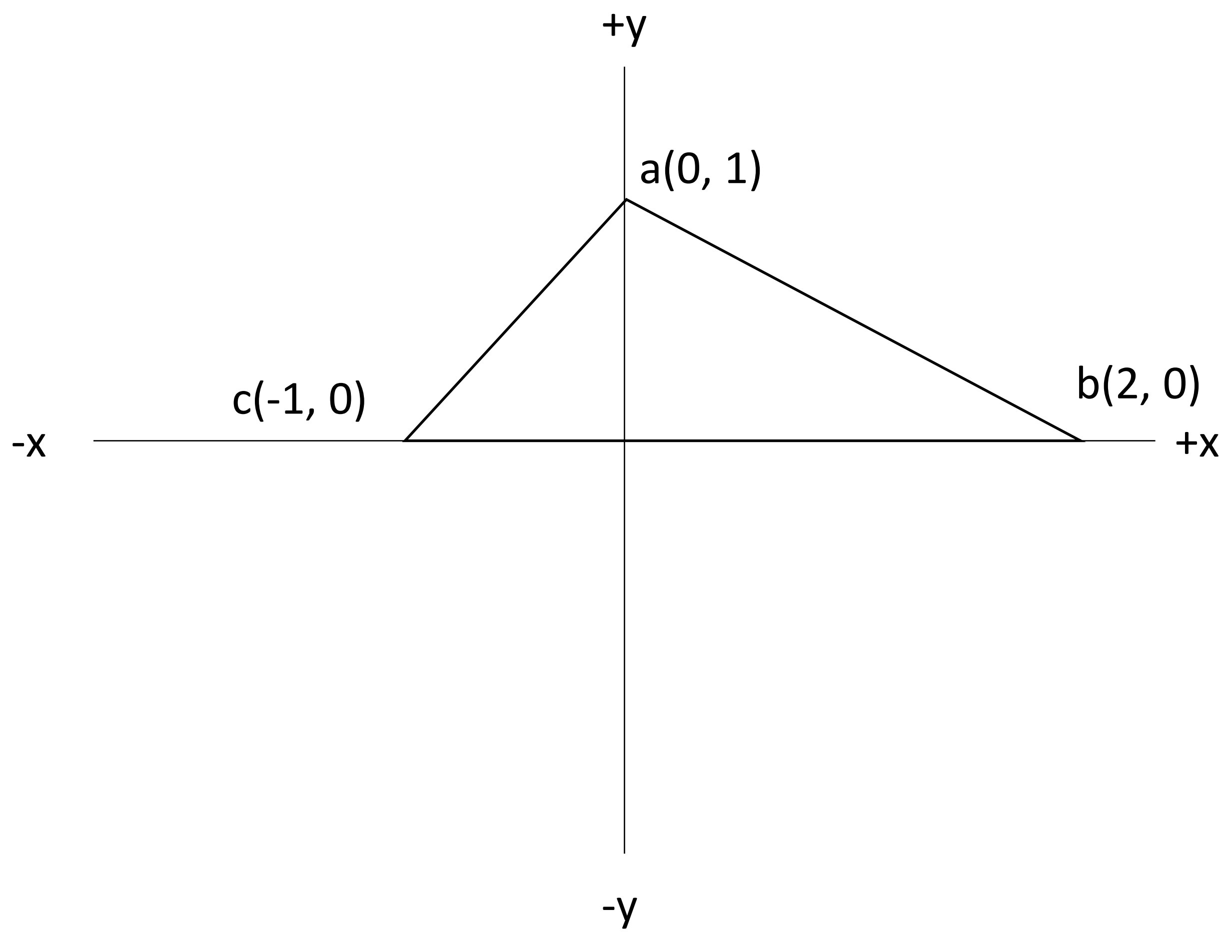
Scaling

$$f(\vec{b}) = \begin{bmatrix} 1 \cdot 2 \\ 0 \cdot 1 \end{bmatrix} = \begin{vmatrix} 2 \\ 0 \end{vmatrix}$$



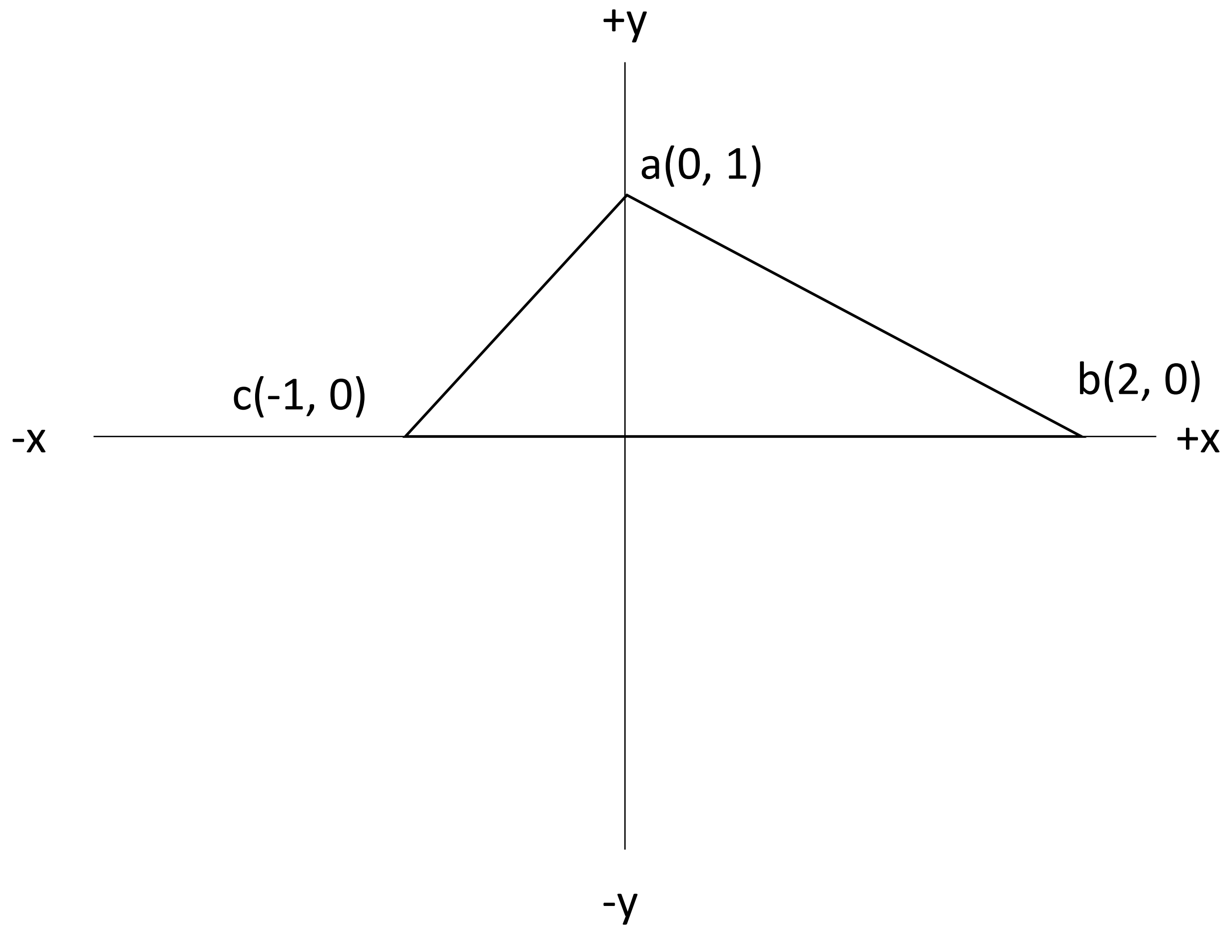
Scaling

$$f(\vec{b}) = \begin{bmatrix} 1 \cdot 2 \\ 0 \cdot 1 \end{bmatrix} = \begin{vmatrix} 2 \\ 0 \end{vmatrix}$$



Scaling

$$f(\vec{c}) = \begin{bmatrix} -1 \cdot 2 \\ 0 \cdot 1 \end{bmatrix} = \begin{vmatrix} -2 \\ 0 \end{vmatrix}$$



Scaling

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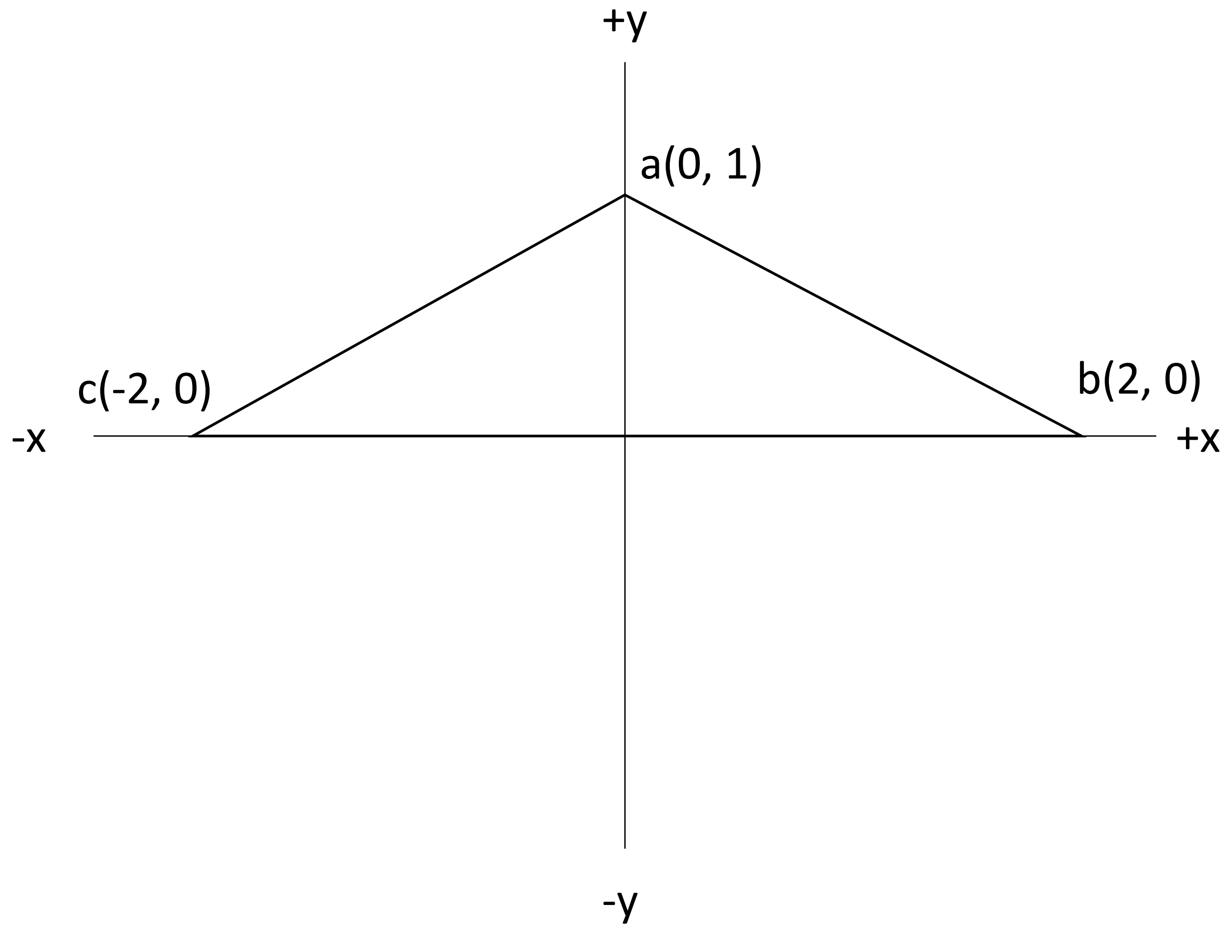
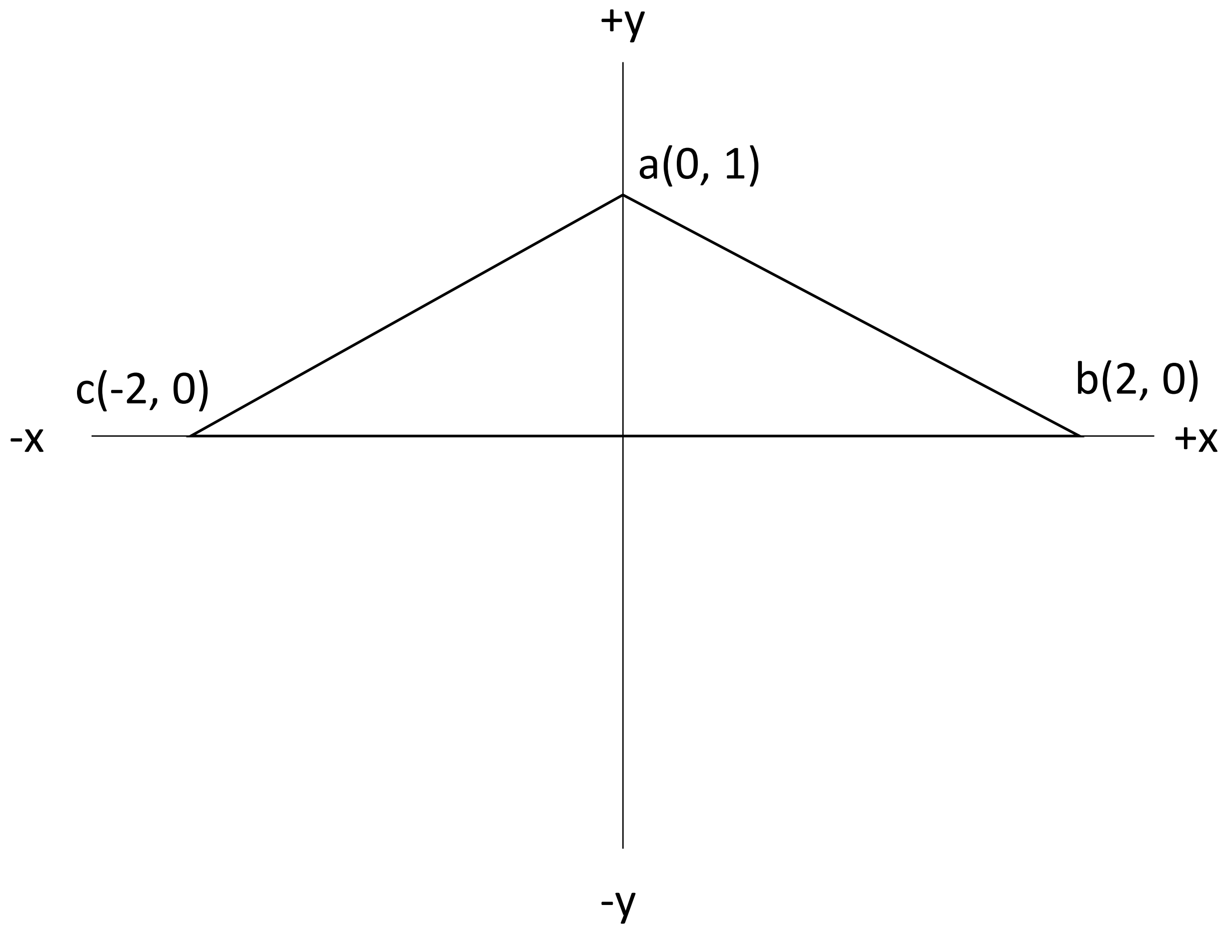


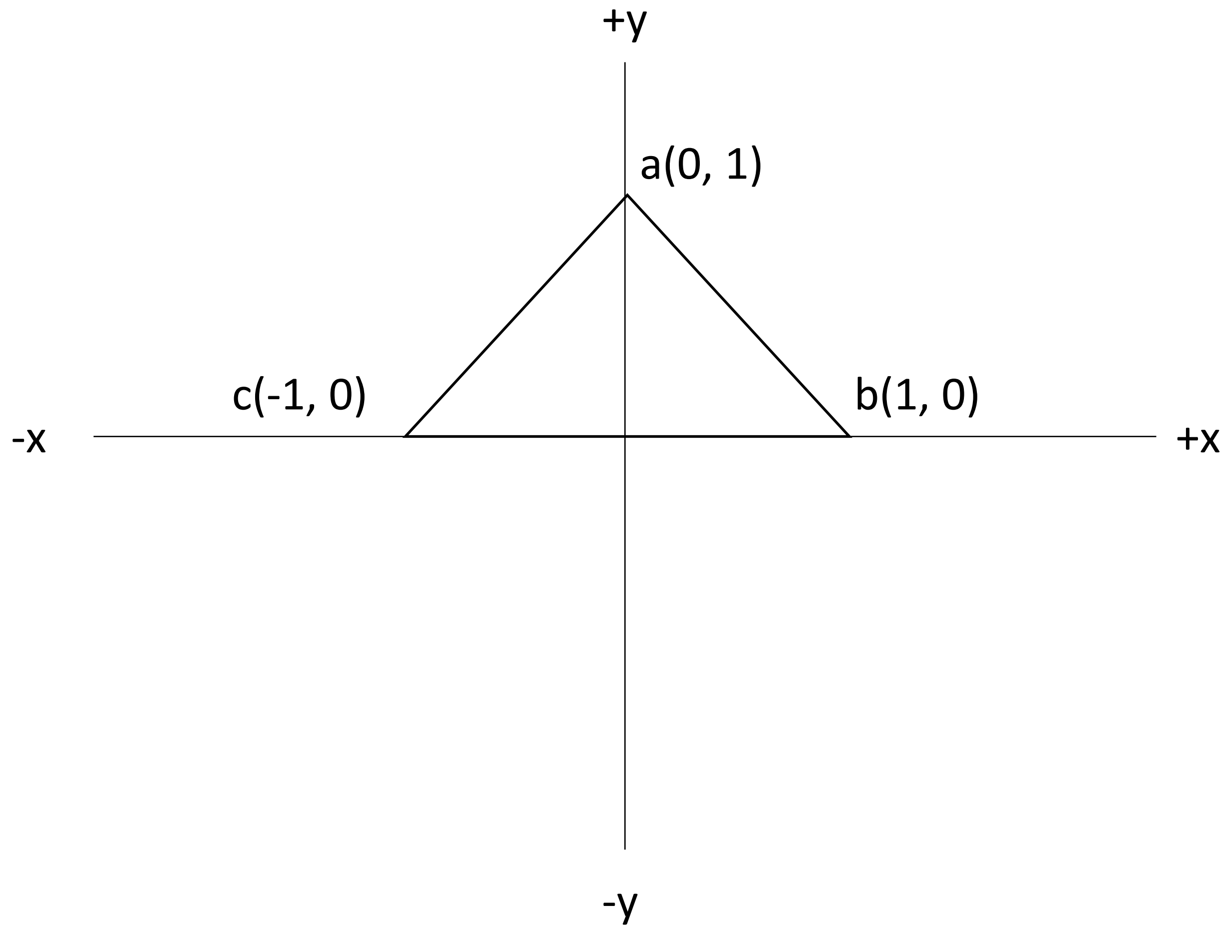
Scaling

$$f(\vec{a}) = \begin{bmatrix} 0 \cdot 2 \\ 1 \cdot 1 \end{bmatrix} = \begin{vmatrix} 0 \\ 1 \end{vmatrix}$$



Scaling - XY

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \cdot 0.5 \\ y \cdot 0.5 \end{bmatrix}$$



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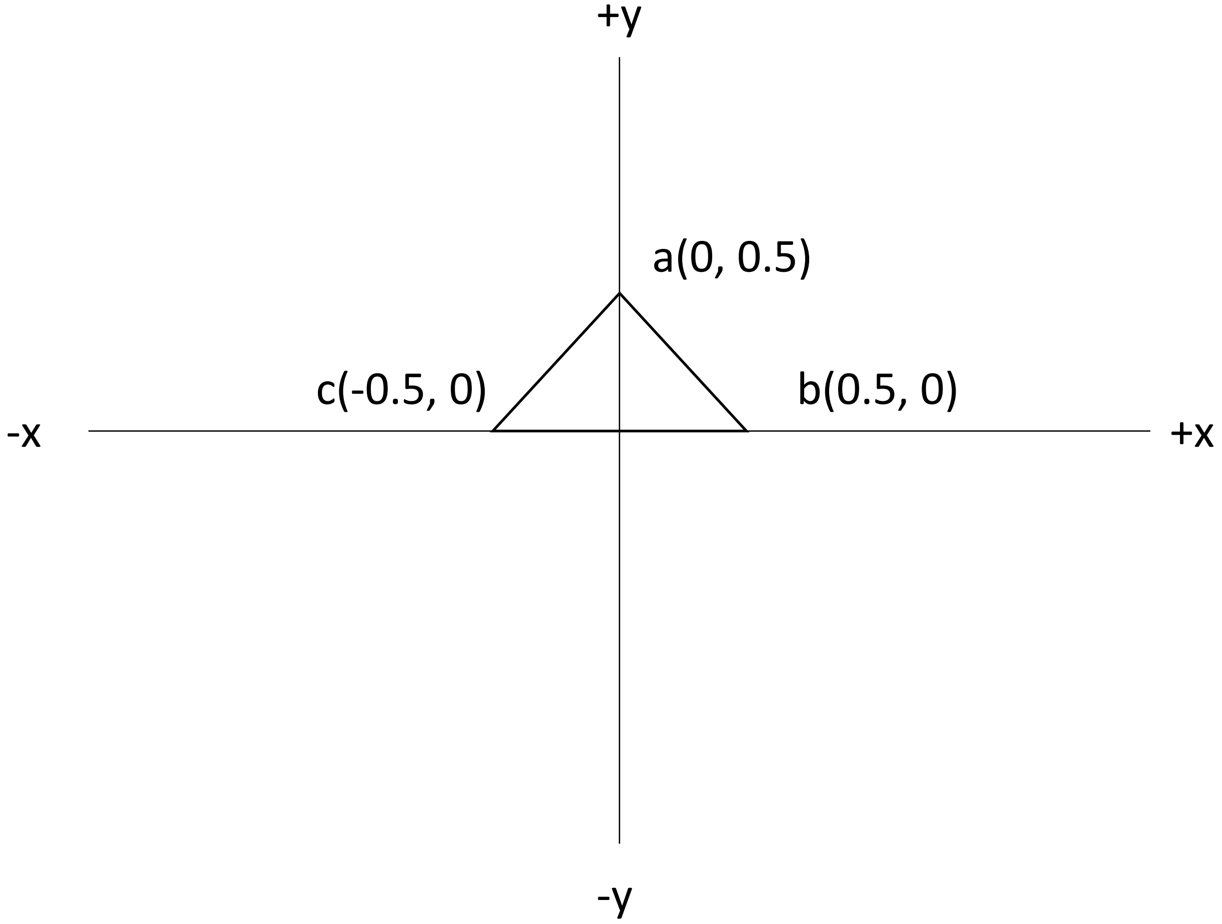
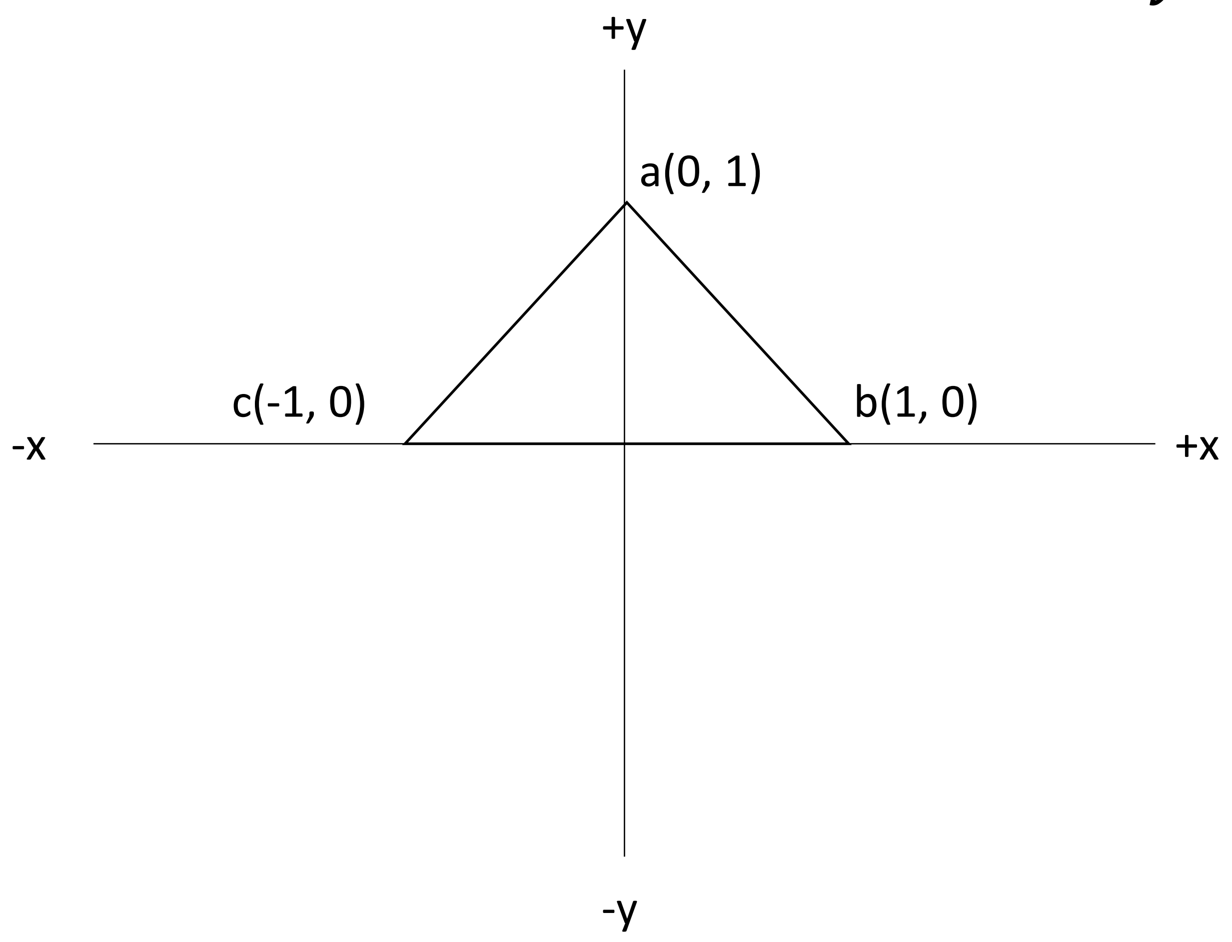


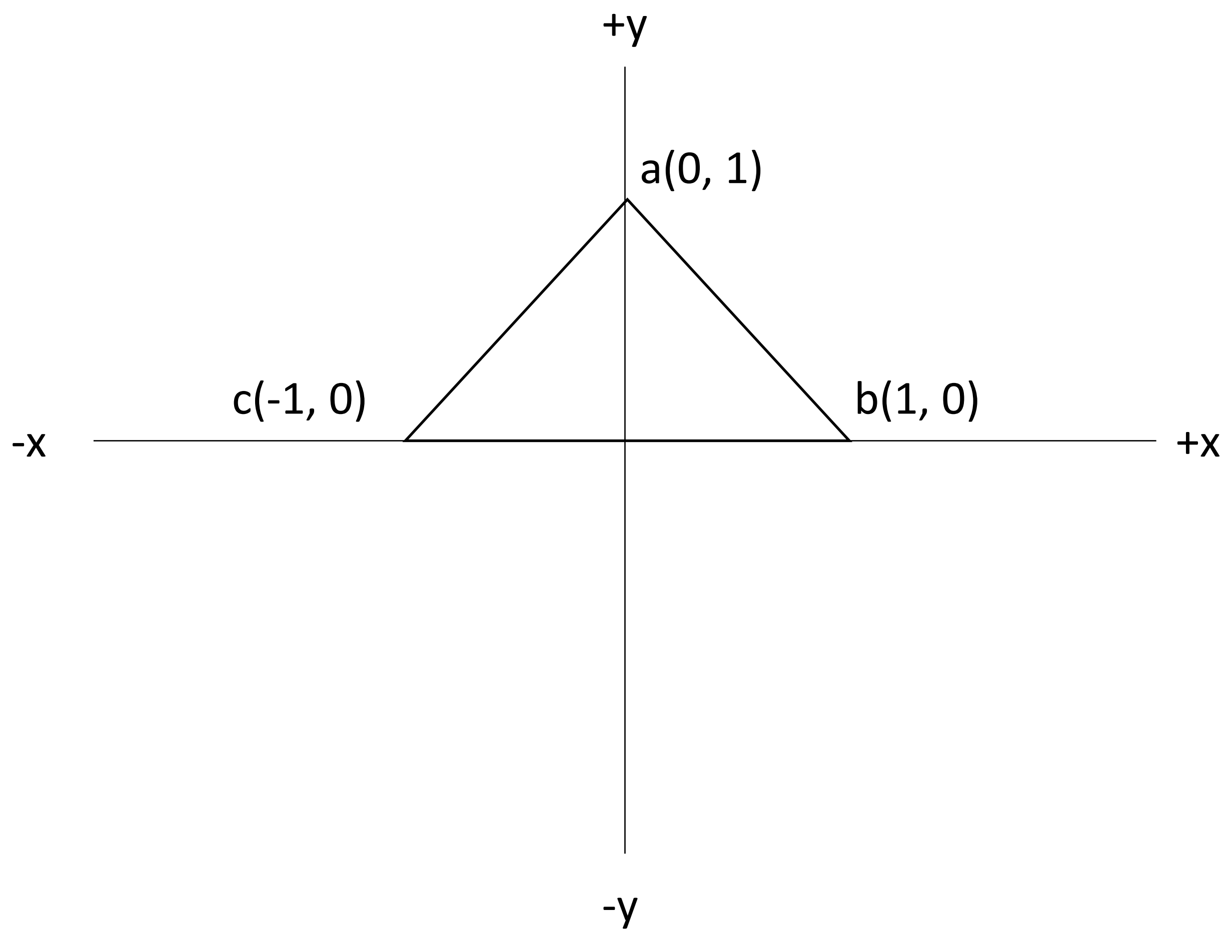
Translation

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + T_x \\ y + T_y \end{bmatrix}$$



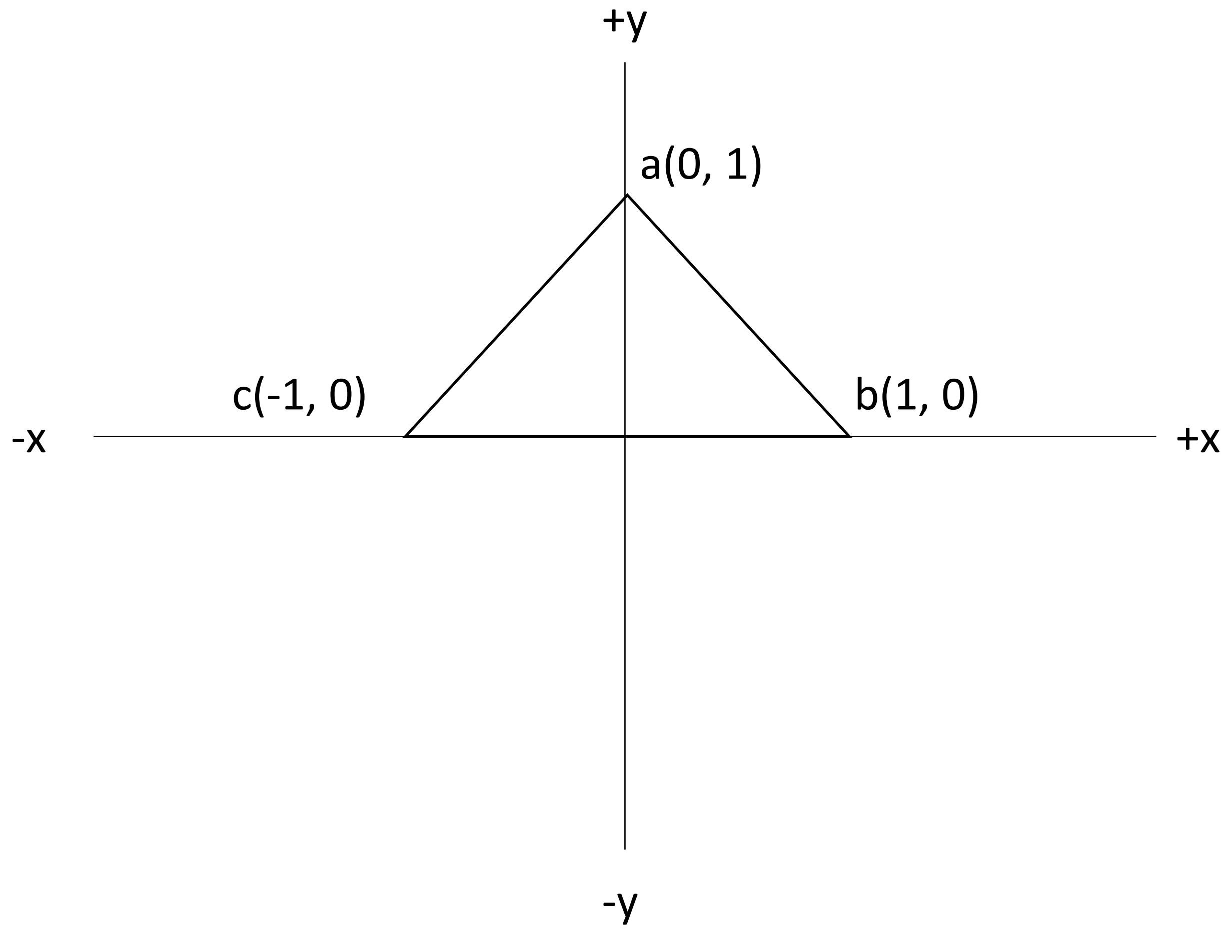
Translation

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + 1 \\ y \end{bmatrix}$$



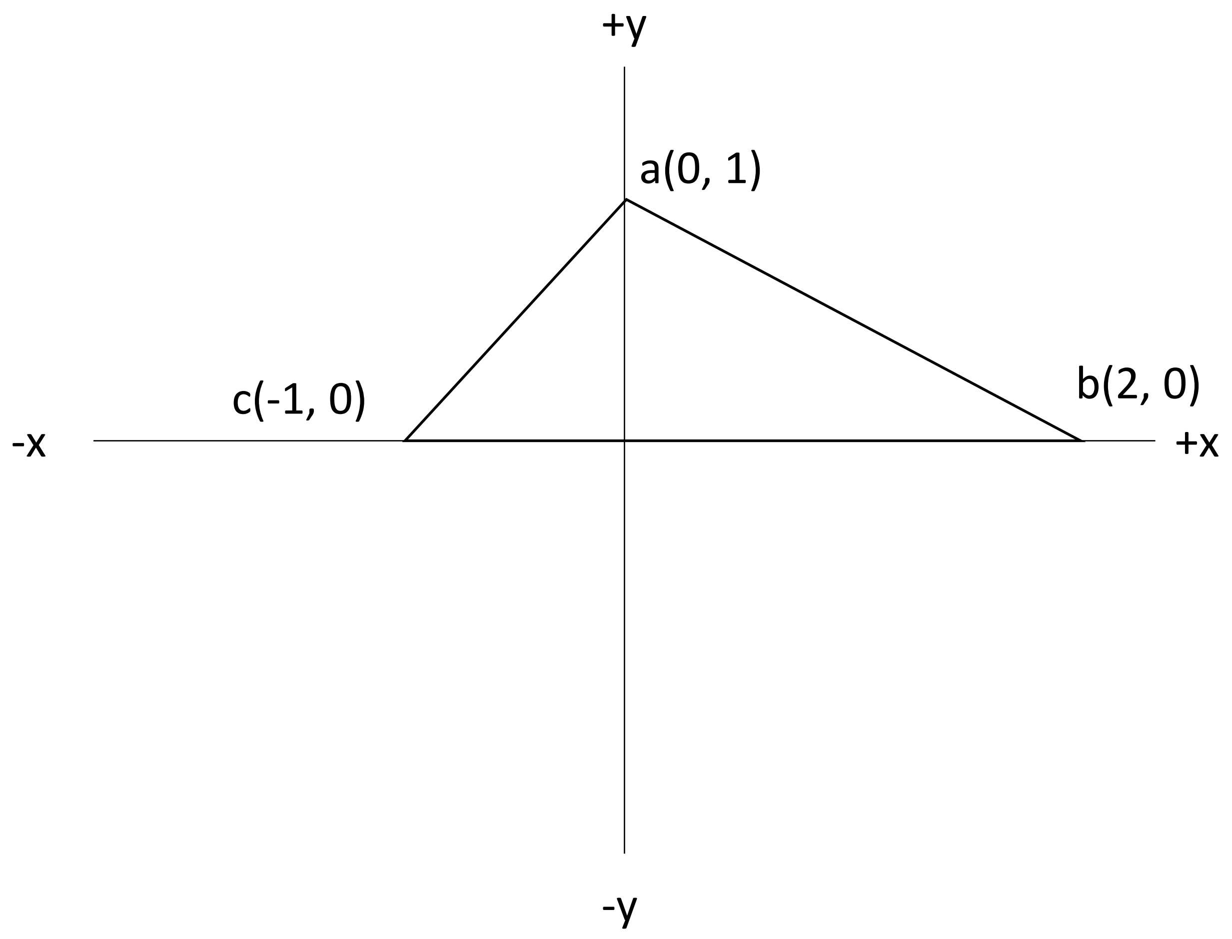
Translation

$$f(\vec{b}) = \begin{bmatrix} x + 1 \\ y \end{bmatrix} = \begin{vmatrix} 2 \\ 0 \end{vmatrix}$$



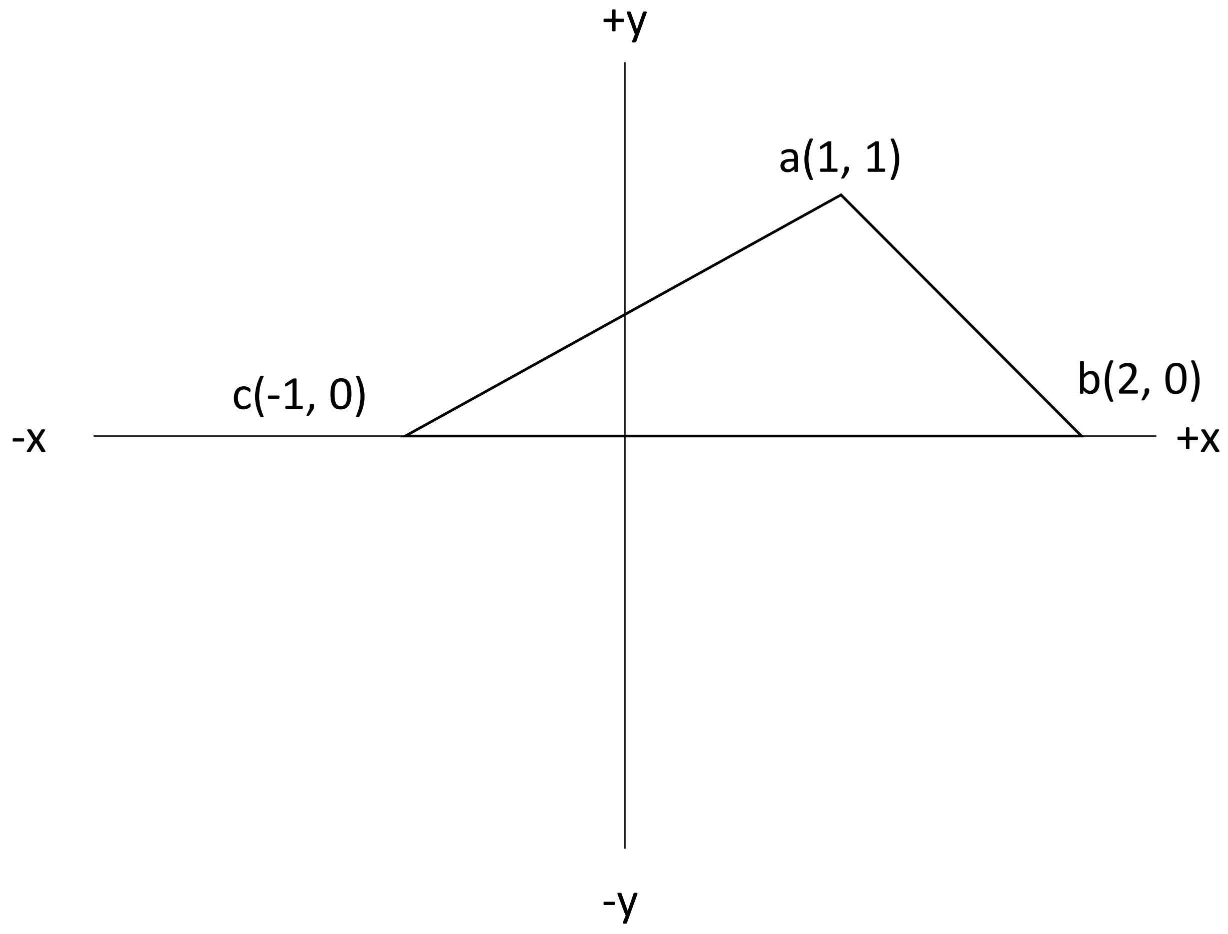
Translation

$$f(\vec{b}) = \begin{bmatrix} 1+1 \\ 0 \end{bmatrix} = \begin{vmatrix} 2 \\ 0 \end{vmatrix}$$



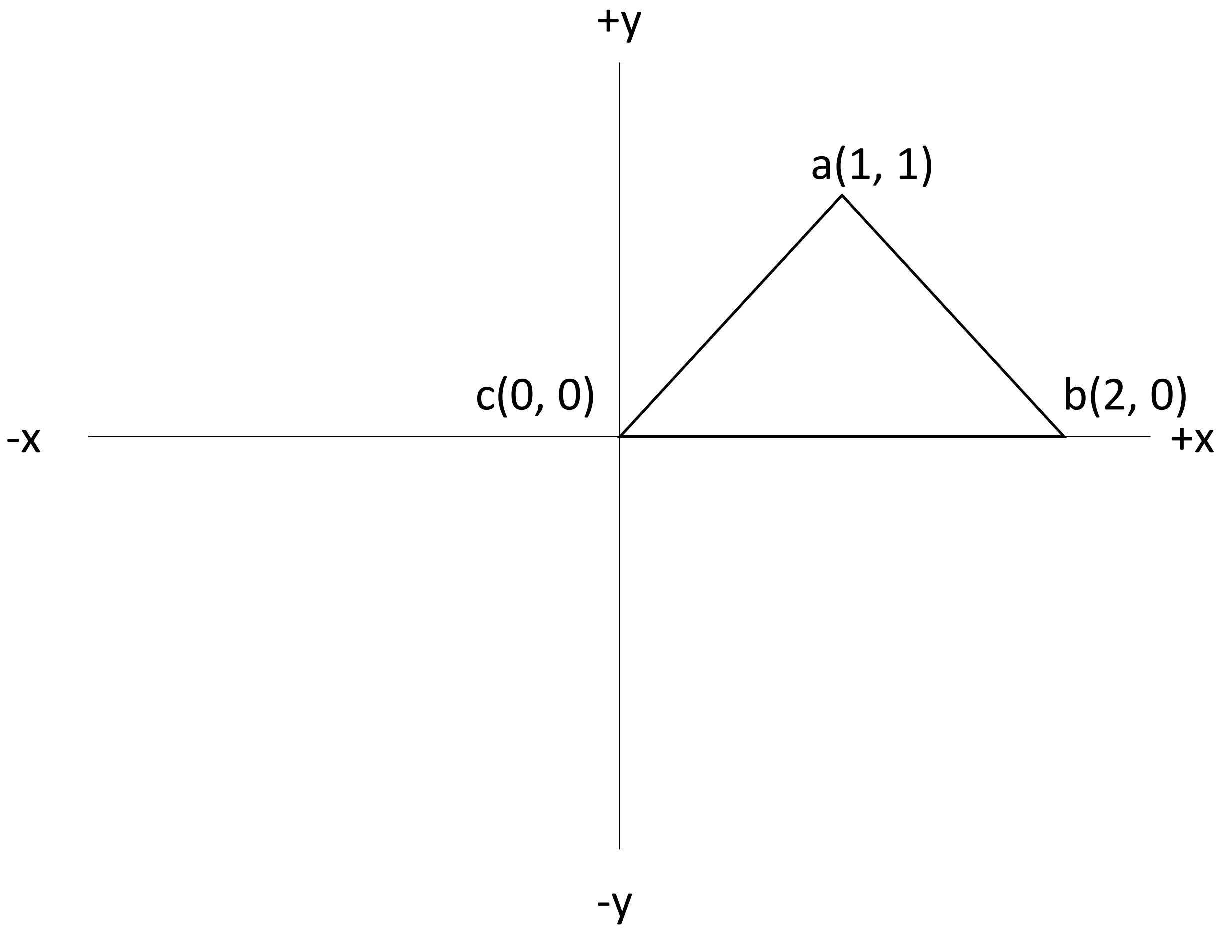
Translation

$$f(\vec{a}) = \begin{bmatrix} 0+1 \\ 1 \end{bmatrix} = \begin{vmatrix} 1 \\ 1 \end{vmatrix}$$



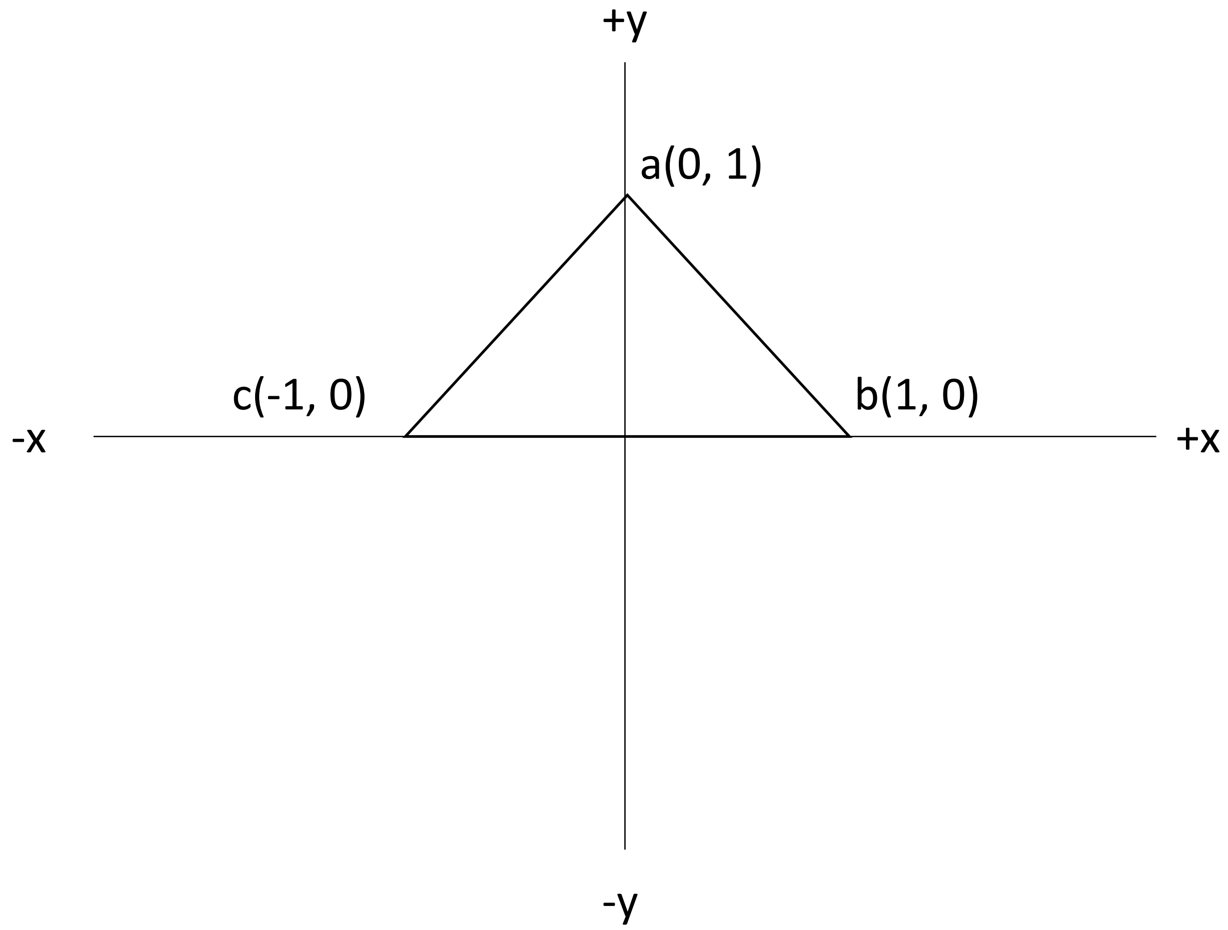
Translation

$$f(\vec{c}) = \begin{bmatrix} -1+1 \\ 0 \end{bmatrix} = \begin{vmatrix} 0 \\ 0 \end{vmatrix}$$



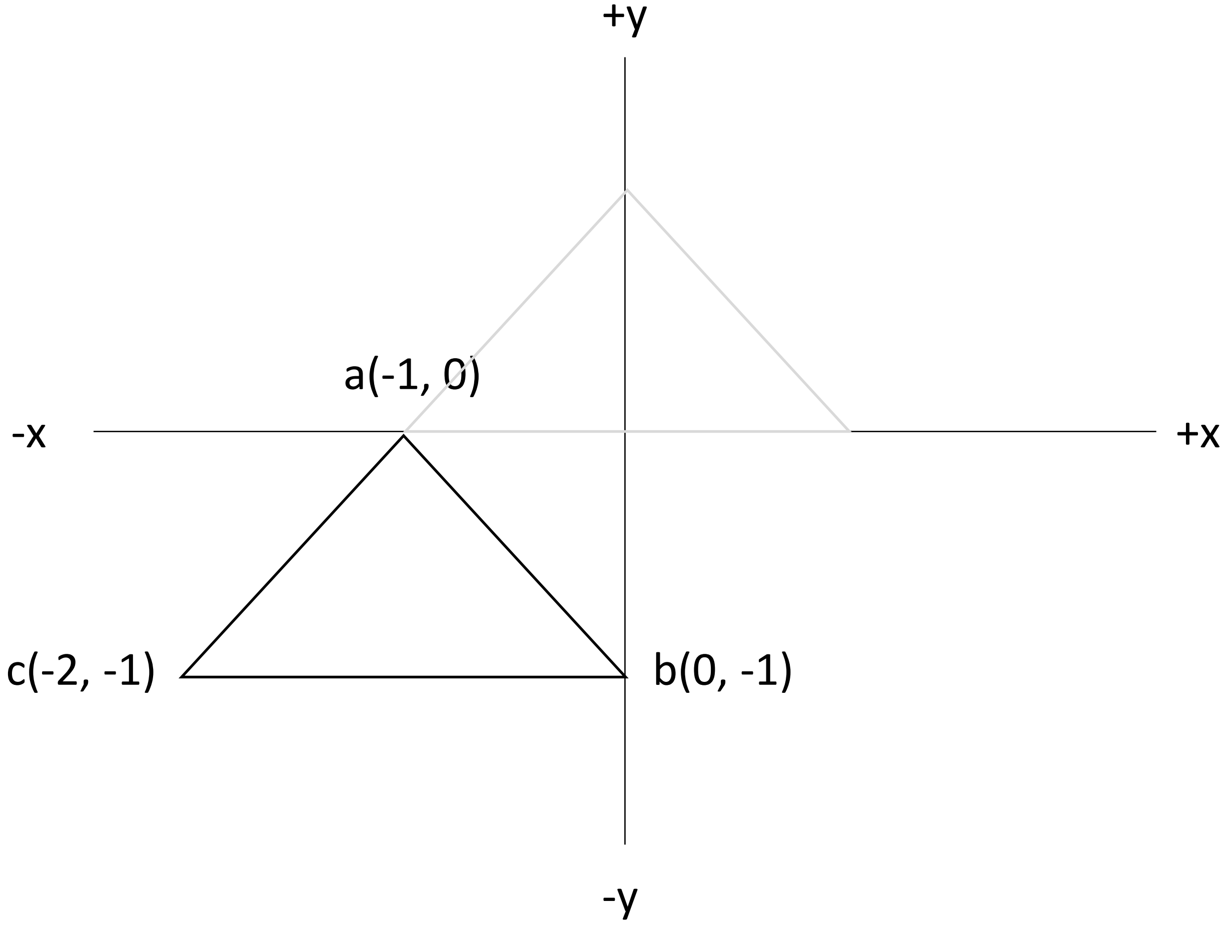
Translation - XY

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - 1 \\ y - 1 \end{bmatrix}$$

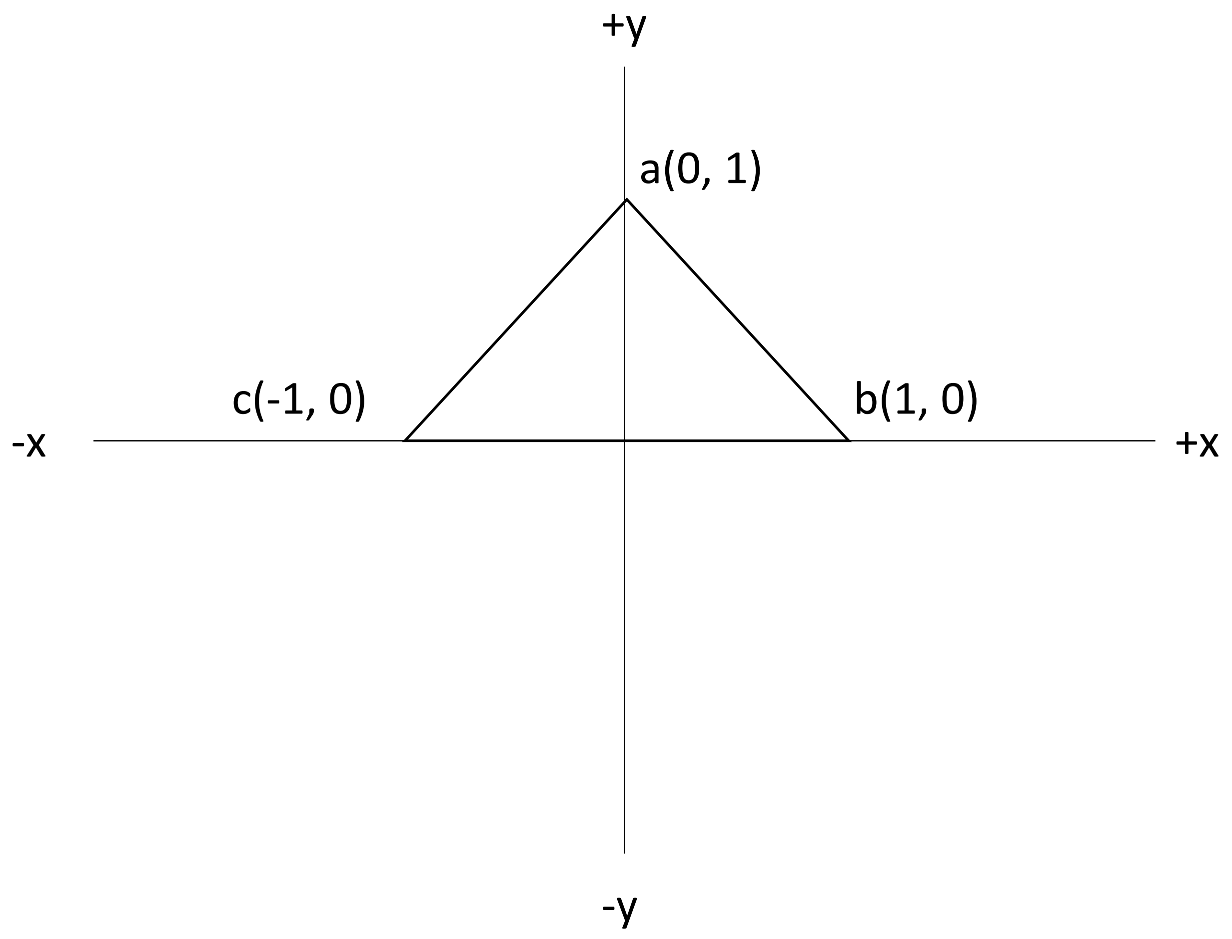


Translation - XY

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - 1 \\ y - 1 \end{bmatrix}$$

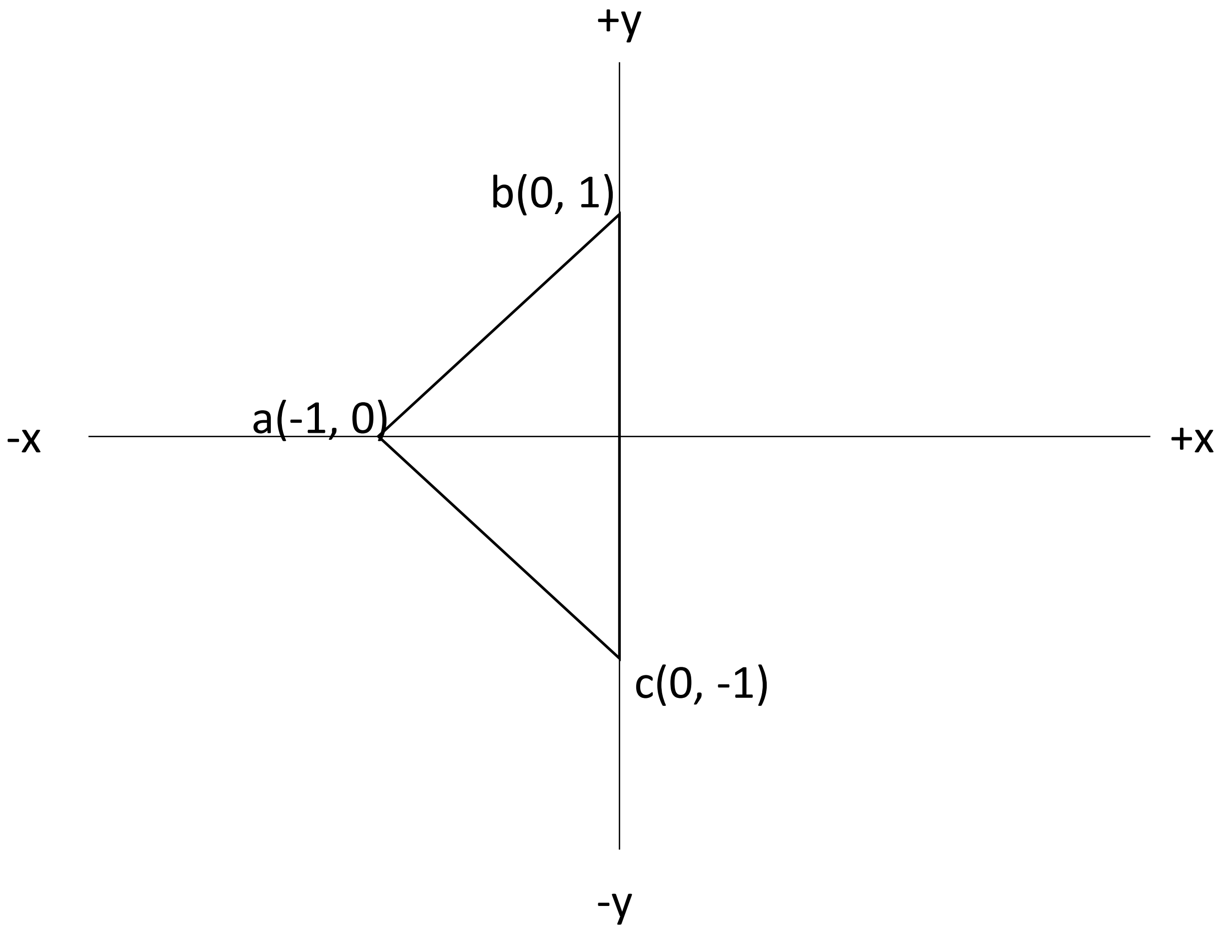


Rotation



Rotation

$$\delta = 90°$$

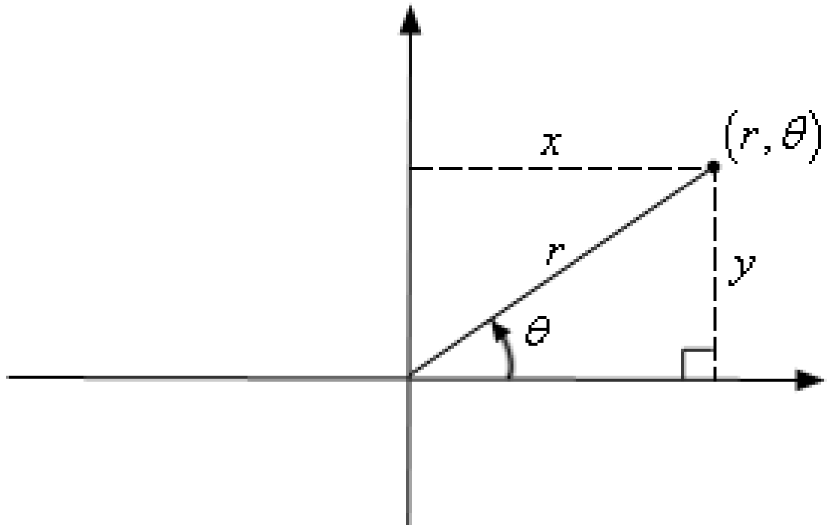


Rotation

Polar coordinates :

$$x \quad = r \cdot \cos(\theta)$$

$$y \quad = r \cdot \sin(\theta)$$

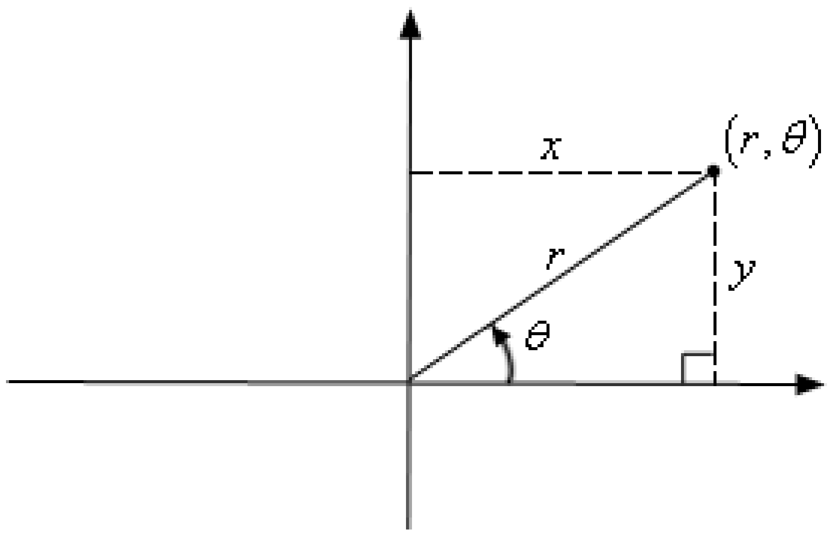


Rotation

Polar coordinates :

$$x \quad = r \cdot \cos(\theta)$$
$$y \quad = r \cdot \sin(\theta)$$

$$x' = r \cdot \cos(\theta + \ \delta)$$
$$y' = r \cdot \sin(\theta + \ \delta)$$

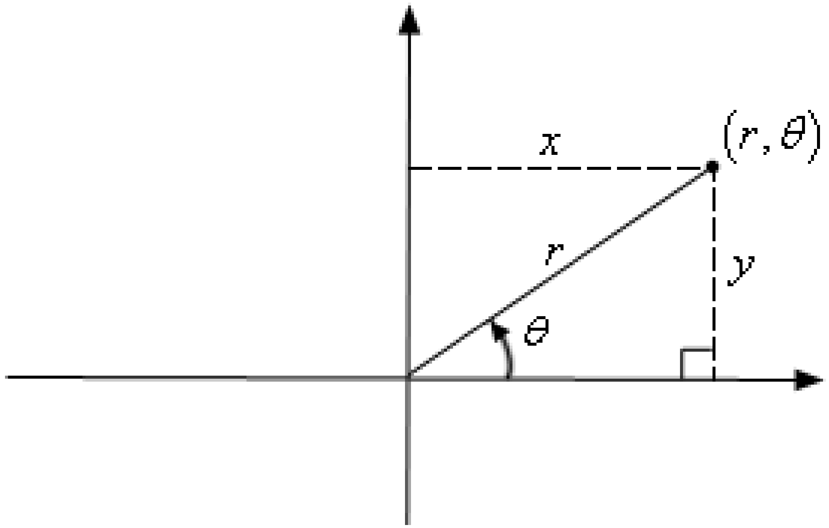


Rotation

Polar coordinates :

$x \quad = r \cdot \cos(\theta)$

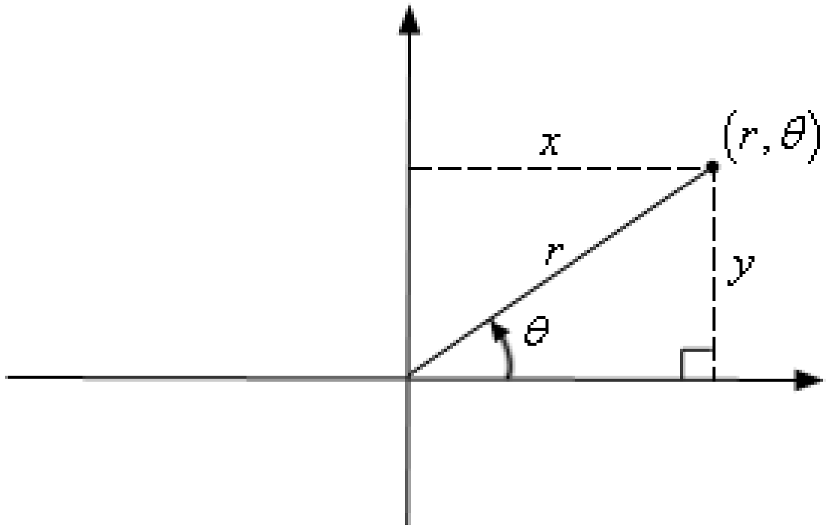
$y \quad = r \cdot \sin(\theta)$



$x' = r \cdot \cos(\theta + \ \delta) = r\cos(\theta)\cos(\delta) - r\sin(\theta)\sin(\delta)$

$y' = r \cdot \sin(\theta + \ \delta) = r\sin(\theta)\cos(\delta) + r\cos(\theta)\sin(\delta)$

Rotation



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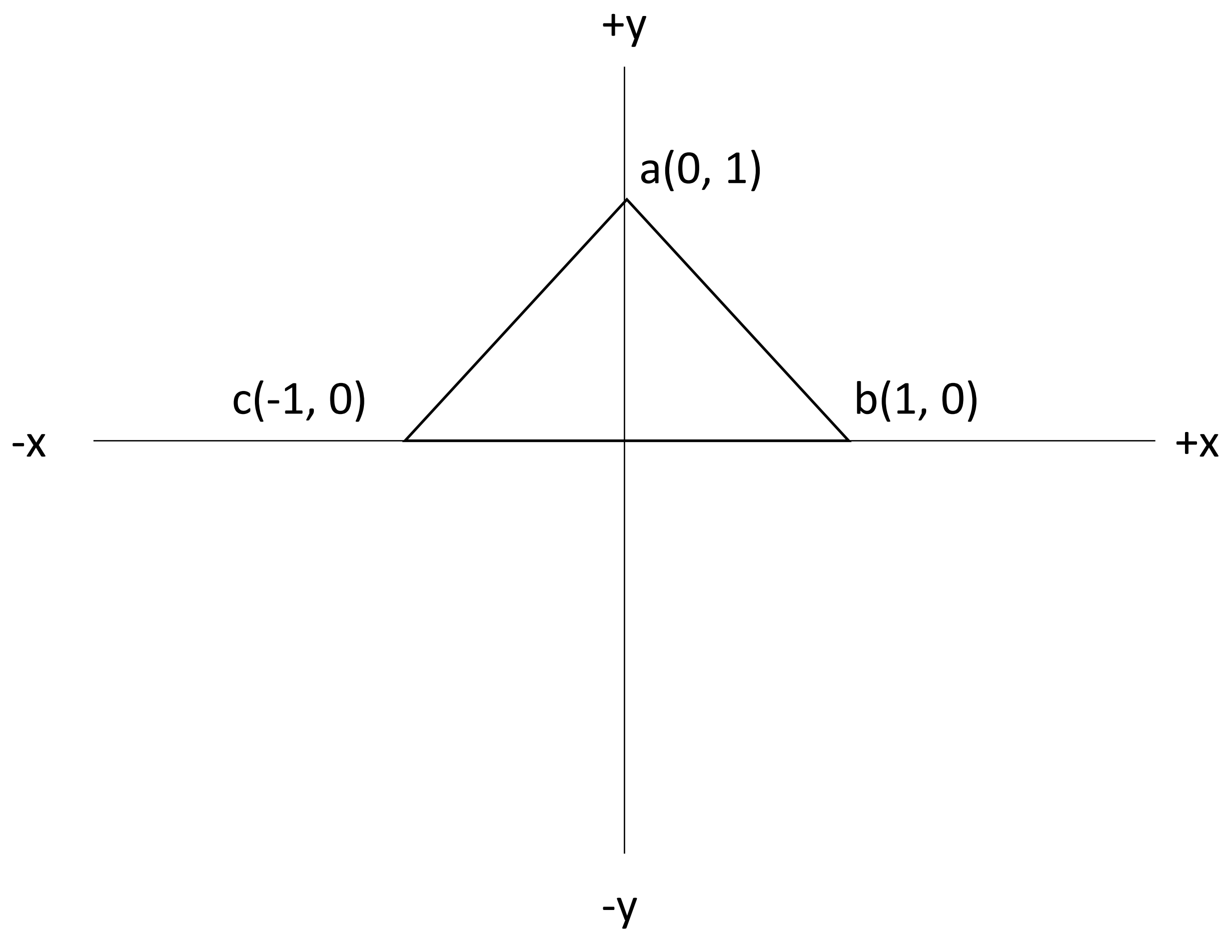
$$x' = x \cdot \cos(\delta) - y \cdot \sin(\delta)$$
$$y' = x \cdot \sin(\delta) + y \cdot \cos(\delta)$$

Rotation

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \cdot \cos(\theta) - y \cdot \ \sin(\theta) \\ x \cdot \sin(\theta) + y \cdot \cos(\theta) \end{bmatrix}$$

$\Theta = 90°$

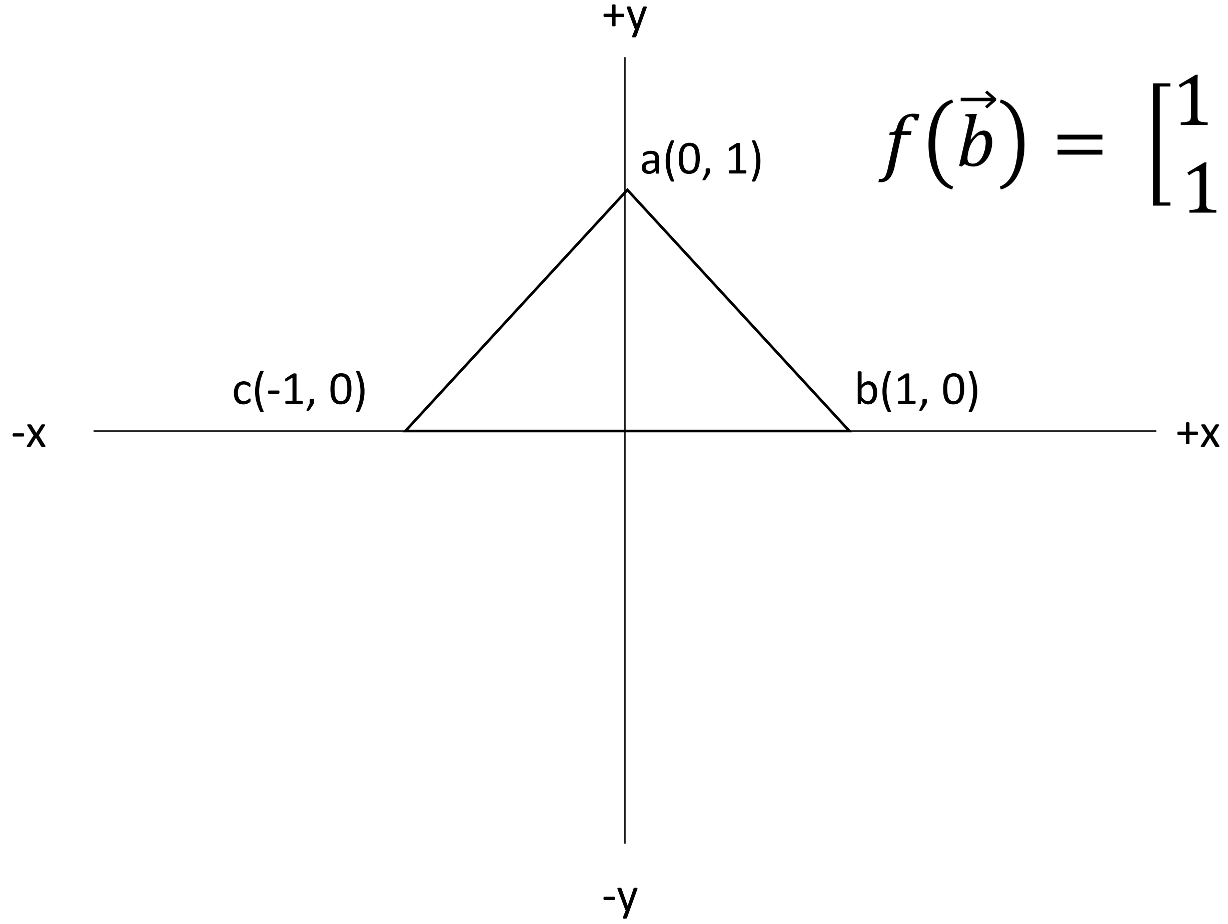


Rotation

$$\Theta = 90°$$

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \cdot \cos(\theta) - y \cdot \ \sin(\theta) \\ x \cdot \sin(\theta) + y \cdot \cos(\theta) \end{bmatrix}$$

$$f(\vec{b}) = \begin{bmatrix} 1 \cdot 0 - 0 \cdot \ 1 \\ 1 \cdot 1 + 0 \cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

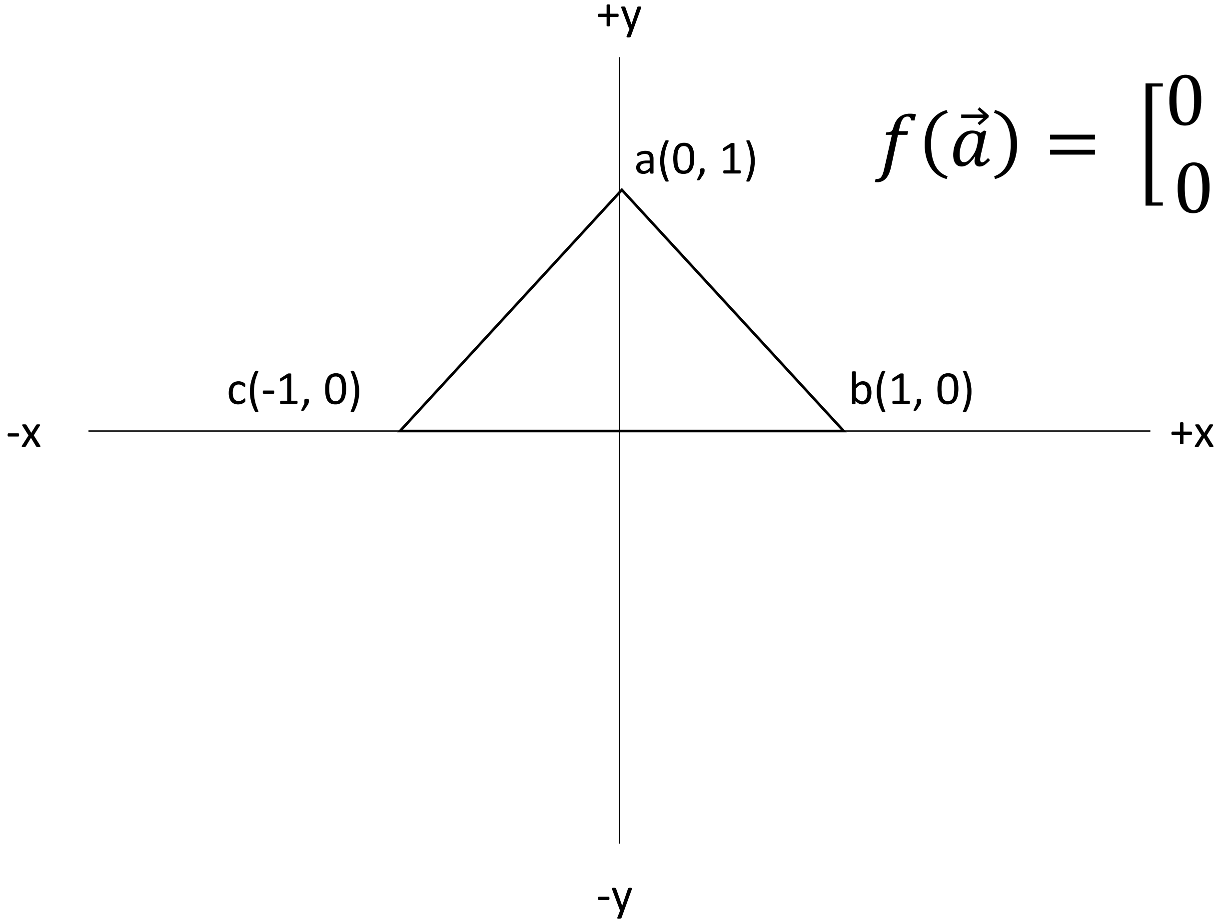


Rotation

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \cdot \cos(\theta) - y \cdot \ \sin(\theta) \\ x \cdot \sin(\theta) + y \cdot \cos(\theta) \end{bmatrix}$$

$$\Theta = 90°$$

$$f(\vec{a}) = \begin{bmatrix} 0 \cdot 0 - 1 \cdot \ 1 \\ 0 \cdot 1 + 1 \cdot 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

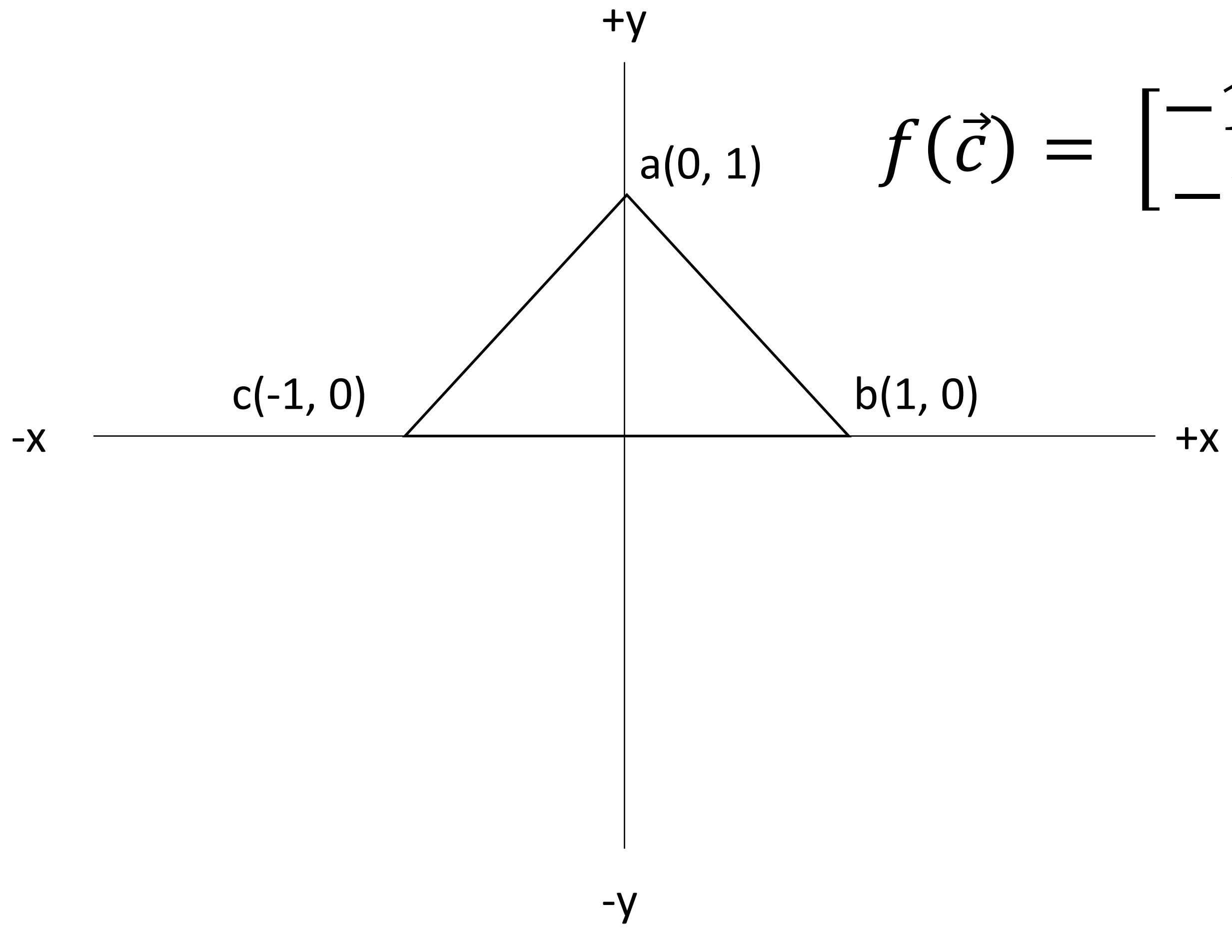


Rotation

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \cdot \cos(\theta) - y \cdot \sin(\theta) \\ x \cdot \sin(\theta) + y \cdot \cos(\theta) \end{bmatrix}$$

$$\Theta = 90°$$

$$f(\vec{c}) = \begin{bmatrix} -1 \cdot 0 - 0 \cdot 1 \\ -1 \cdot 1 + 0 \cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$



Rotation

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x \cdot \cos(\theta) - y \cdot \sin(\theta) \\ x \cdot \sin(\theta) + y \cdot \cos(\theta) \end{bmatrix}$$

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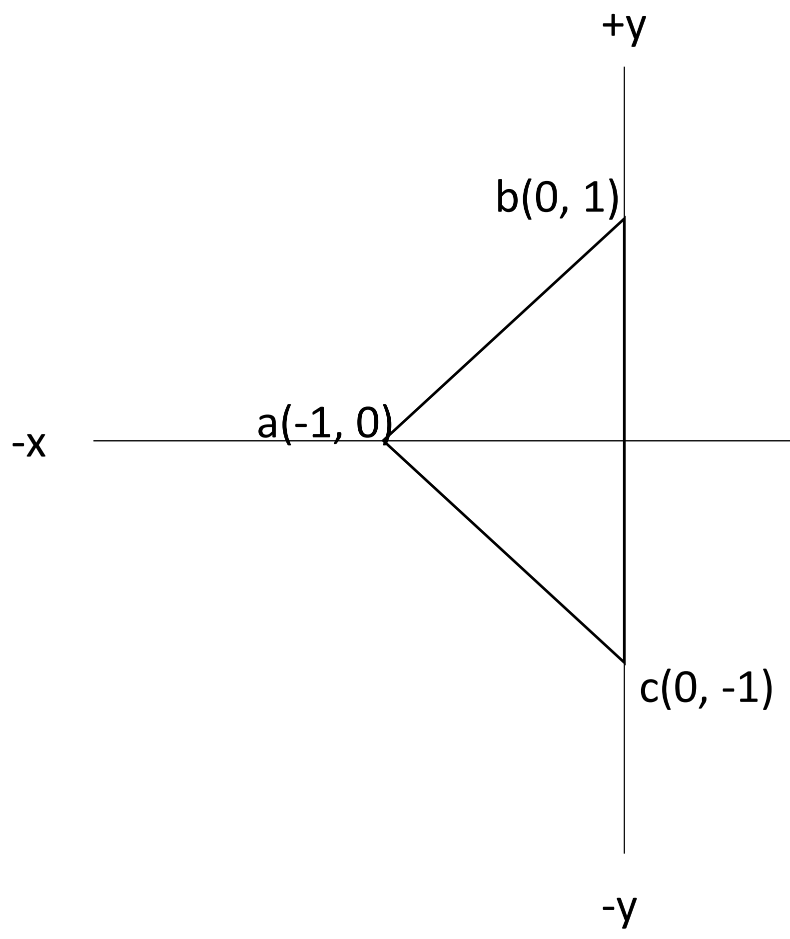
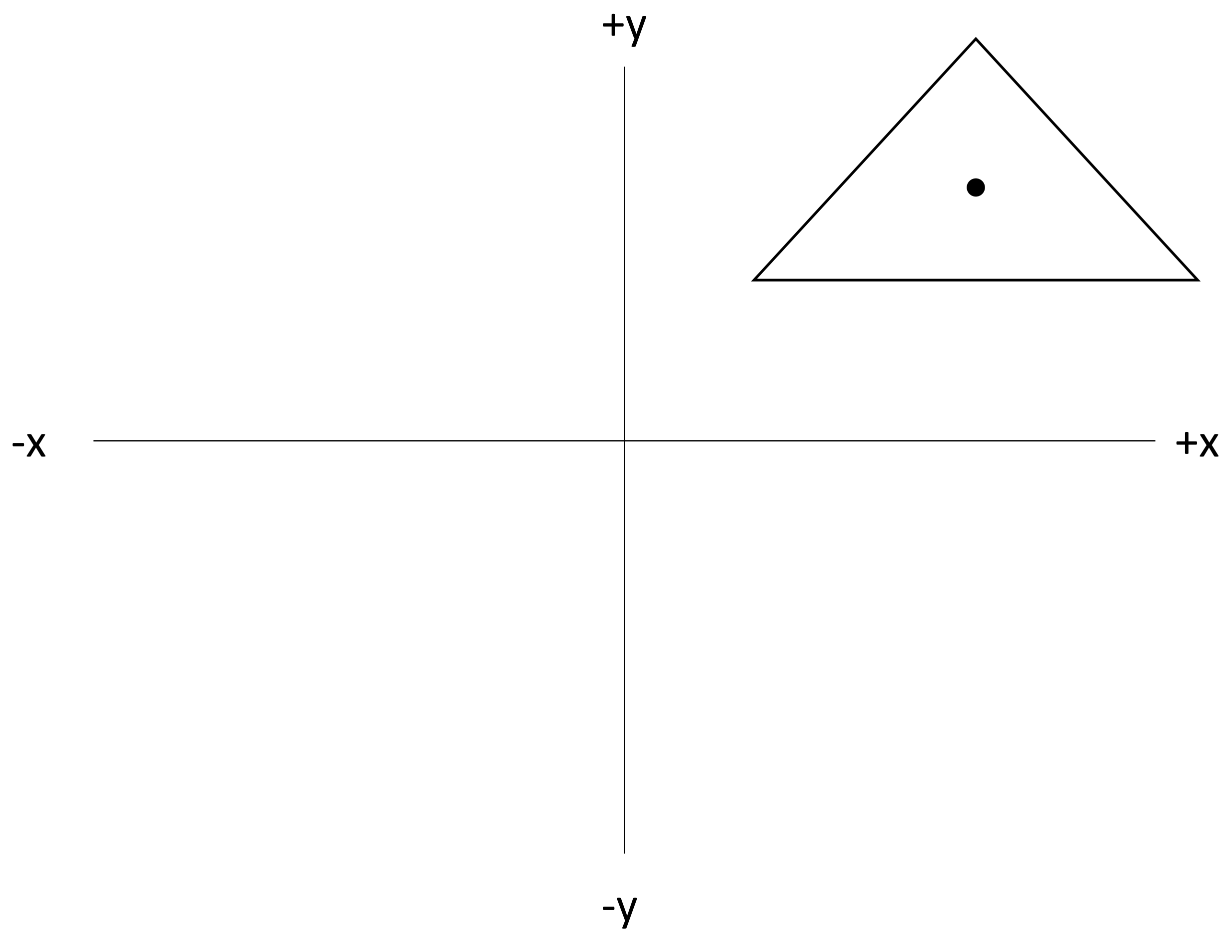
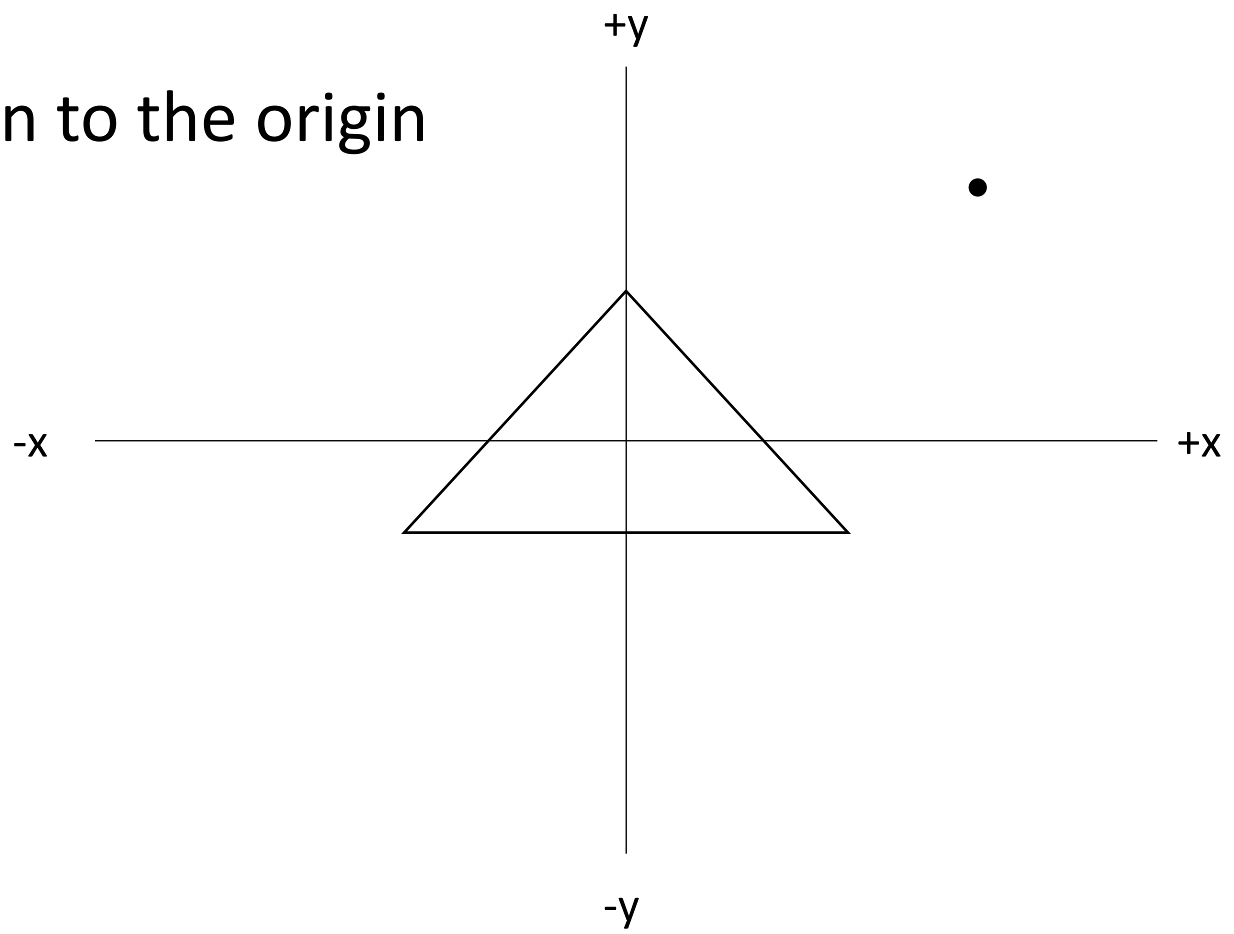


Rotation around the geometric center



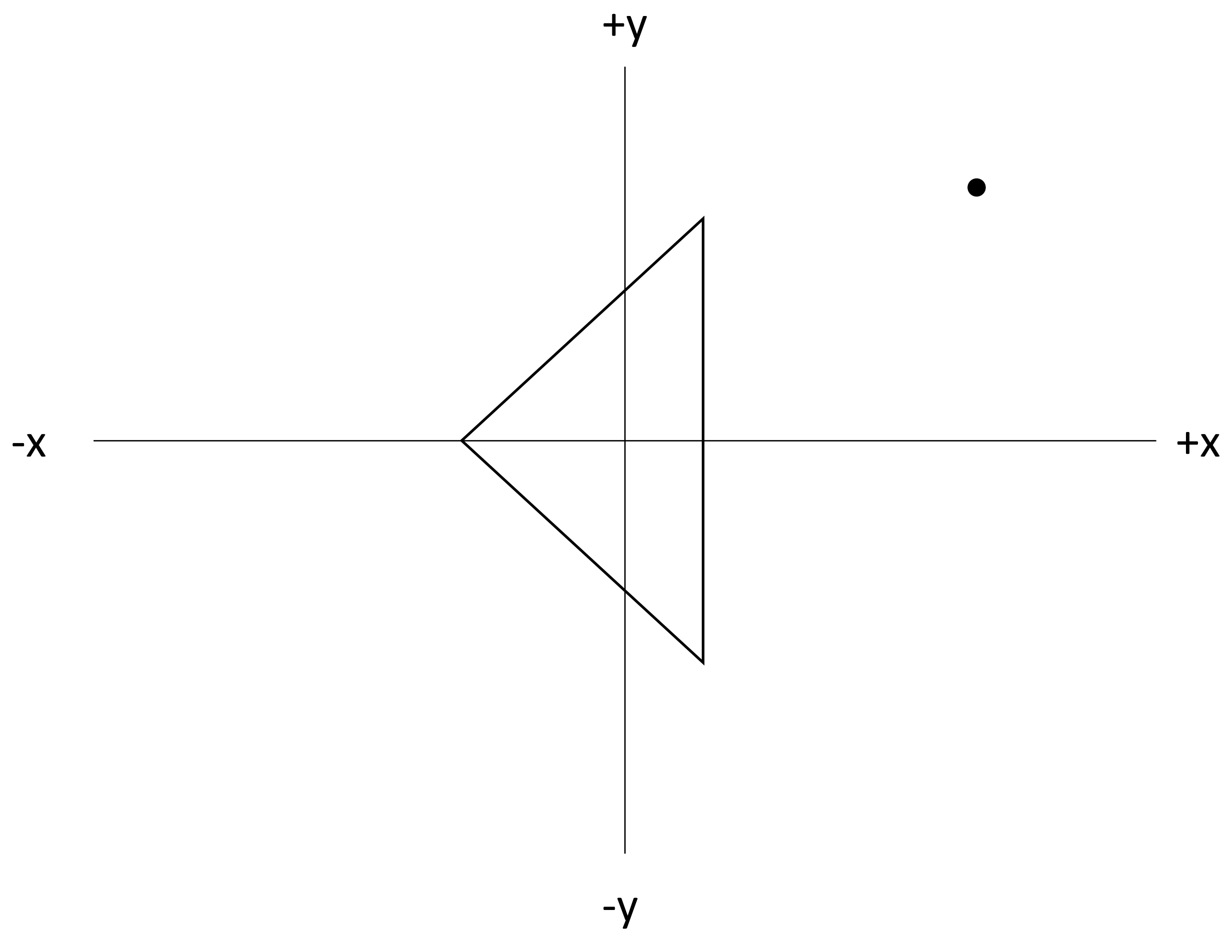
Rotation around the geometric center

Translation to the origin



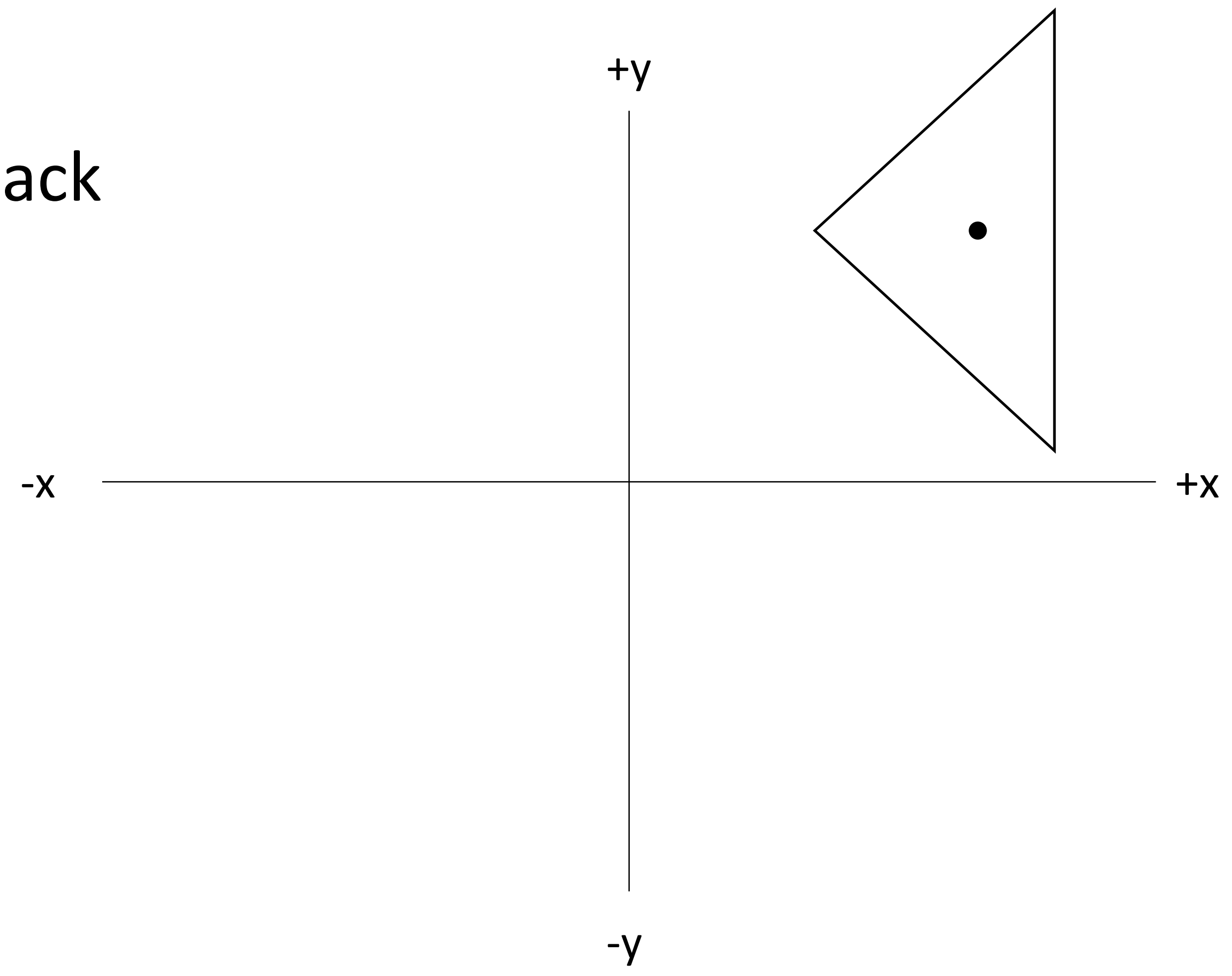
Rotation around the geometric center

Rotation

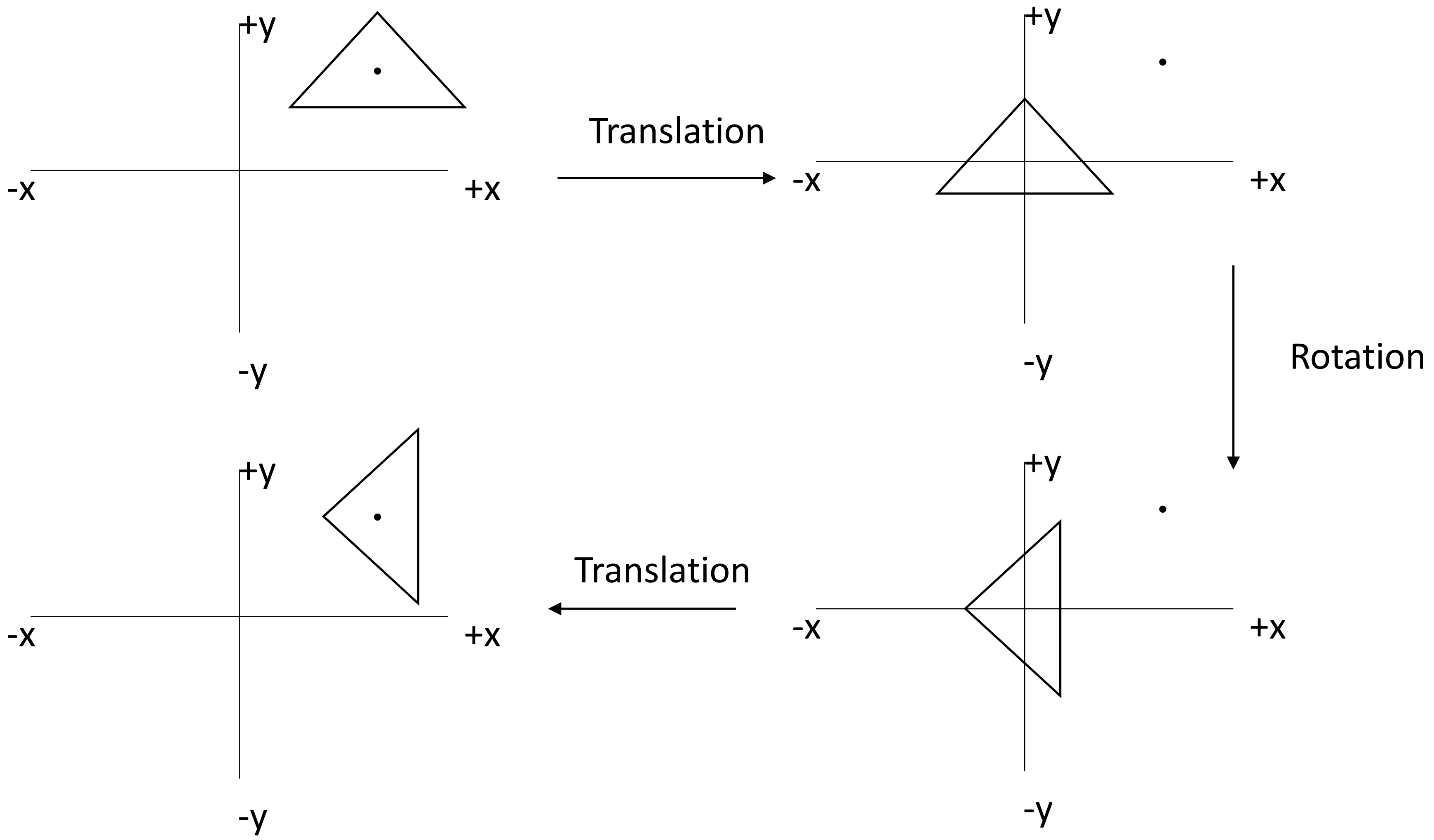


Rotation around the geometric center

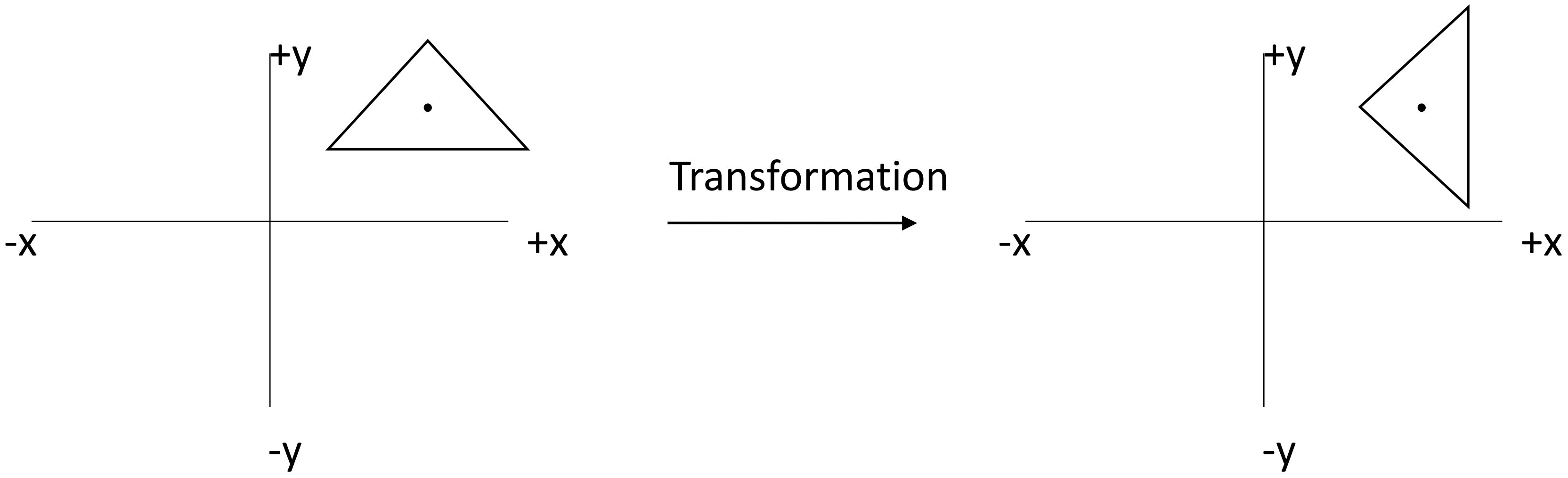
Translation back



How to apply transformations instantaneously



How to apply transformations instantaneously?



Transformation Matrices: Associative

- Let x be a vertex
- A and B transformation matrices and C the product of A and B
- Then $A(Bx) = (AB)x = Cx$

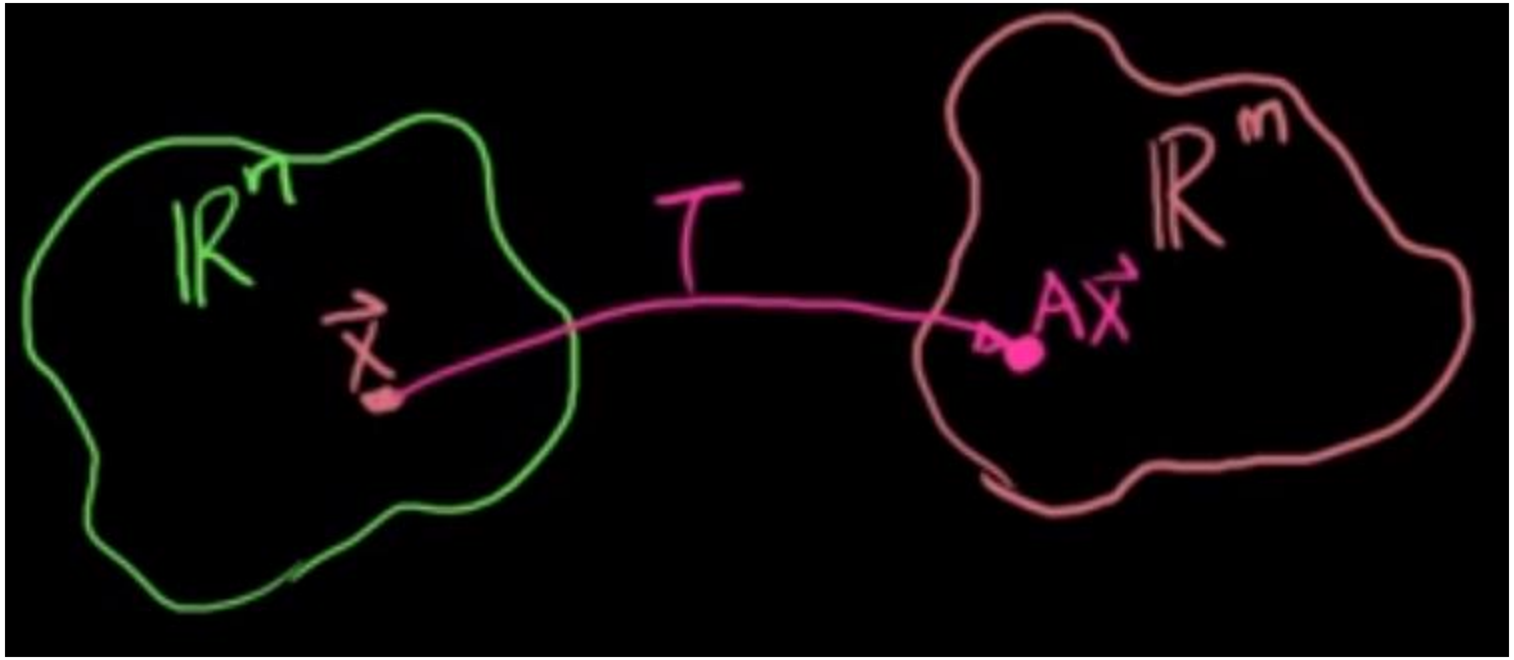
Transformation Matrices: Associative

- Let x be a vertex
- A and B transformation matrices and C the product of A and B
- Then A(Bx) = (AB)x = Cx
  - → applies transformations in a single Matrix-vector multiplication
- If we have a scene composed of millions of vertices this is a significant optimization

Matrix-Vector Product as a Transformation

$T: \mathbb{R}^n \to \mathbb{R}^m$

$T(\vec{x}) = A\vec{x}$



Matrix-Vector Product as a Transformation

$T: \mathbb{R}^2 \to \mathbb{R}^2$

Matrix-Vector Product as a Transformation

$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$

$$B = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

Matrix-Vector Product as a Transformation

$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$

$$B = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

$T(\vec{x}) = B\vec{x}$

Matrix-Vector Product as a Transformation

$T: \mathbb{R}^2 \to \mathbb{R}^2$

$$B = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

$$T(\vec{x}) = B\vec{x} = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Matrix-Vector Product as a Transformation

$T: \mathbb{R}^2 \to \mathbb{R}^2$

$$B = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

$$T(\vec{x}) = B\vec{x} = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_2 \\ 3x_1 + 4x_2 \end{bmatrix}$$

Matrix-Vector Product as a Transformation

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$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 2x_1 - x_2 \\ 3x_1 + 4x_2 \end{bmatrix}$$

Scaling 2D

Scaling matrix: $\begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$

$$P = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x \cdot x + 0 \cdot y \\ 0 \cdot x + s_y \cdot y \end{bmatrix} = \begin{bmatrix} s_x \cdot x \\ s_y \cdot y \end{bmatrix}$$

Rotation 2D

Rotation matrix: $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$

$$P = \begin{bmatrix} \cos(\Pi/4) & -\sin(\Pi/4) \\ \sin(\Pi/4) & \cos(\Pi/4) \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \cdot 0 \ - 0 \cdot 1 \\ 1 \cdot 1 + 0 \cdot 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Concatenation of transformation matrices

$$P' = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Concatenation of transformation matrices

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Concatenation of transformation matrices

$$P' = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Concatenation of transformation matrices

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$$P'' = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} x' \\ y' \end{bmatrix}$$

Concatenation of transformation matrices

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Concatenation of transformation matrices

$$P' = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

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This is identical to (associative multiplication of matrices)

$$P'' = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \cdot \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Concatenation of transformation matrices

$$P' = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} s_x x \\ s_y y \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

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Translation

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + T_x \\ y + T_y \end{bmatrix}$$

Translation

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + T_x \\ y + T_y \end{bmatrix} \rightarrow$$

it is impossible to express such a transformation with 2D matrix multiplications

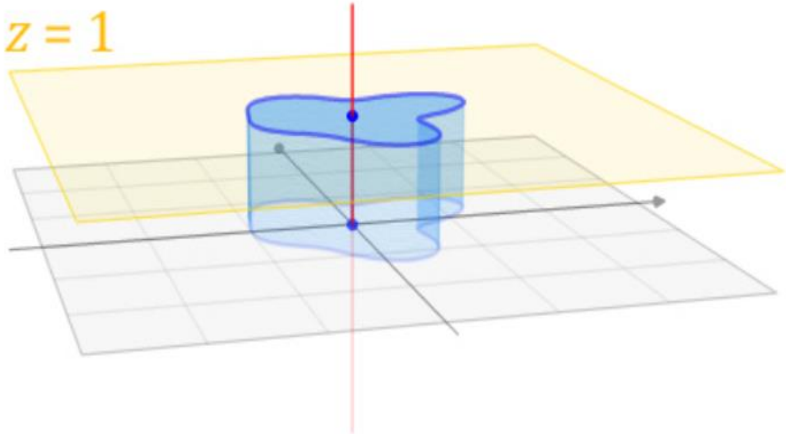
Translation

$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x + T_x \\ y + T_y \end{bmatrix}$ → it is impossible to express such a transformation with 2D matrix multiplications

Hence, we embed 2D space in 3D where the third coordinate will be equal to 1. Our 2D space resides in the z = 1 plane.

$$\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Geometric interpretation of 2D translation



Translation T(T_x, T_y)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Translation T(T_x, T_y)

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Resulting in:

$x' = x + 1 \cdot T_x$
$y' = y + 1 \cdot T_y$
$1 = 1$

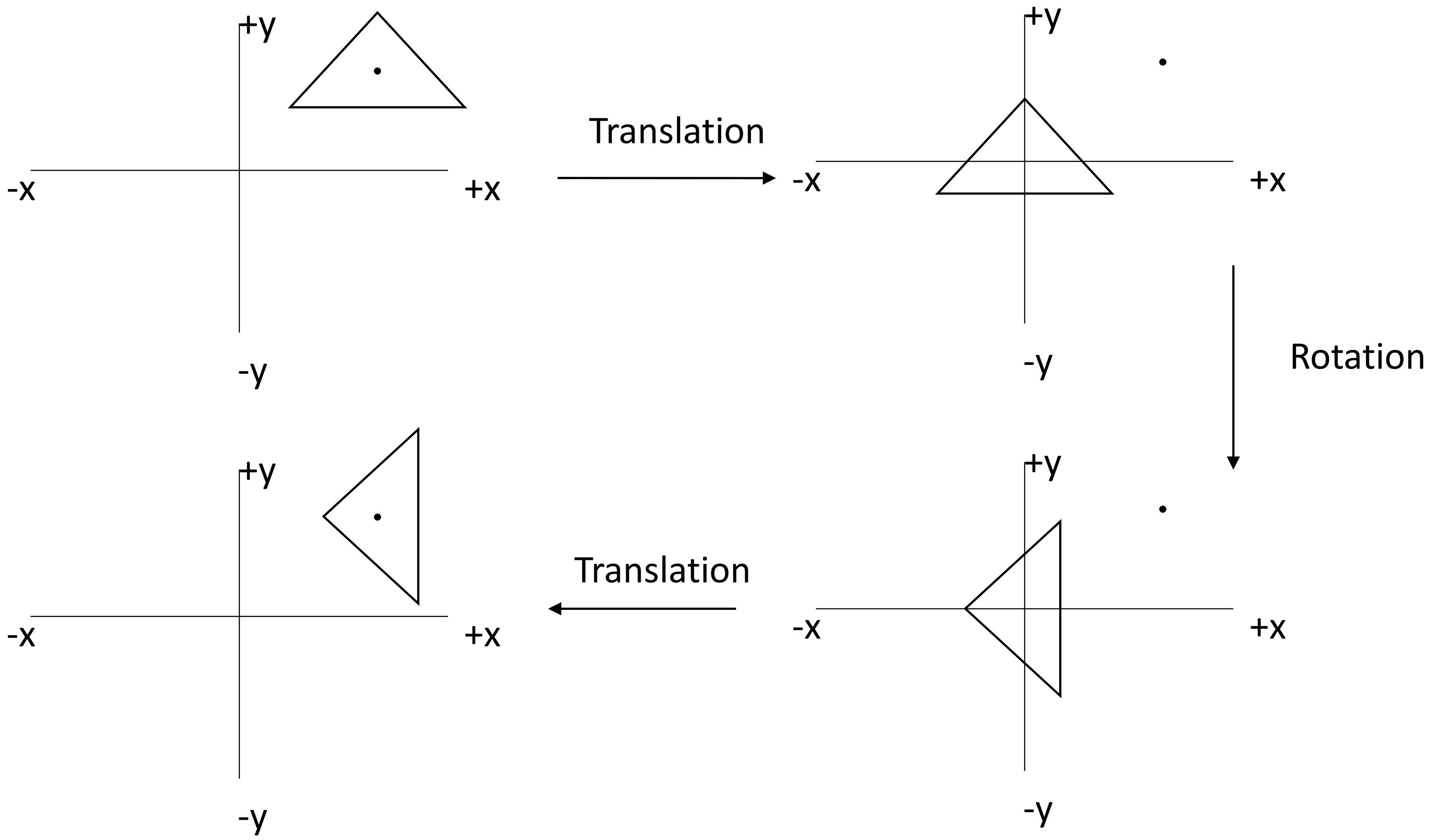
Scaling, Rotation and Translation in 2D

$$S(s_x, s_y) = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(T_x, T_y) = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

How to apply transformations instantaneously



Rotation matrix M around $P_0(x_0, y_0)$

Rotation matrix M around $P_0(x_0, y_0)$

1. Translation to origin: $T(-x_0, -y_0)$

Rotation matrix M around $P_0(x_0, y_0)$

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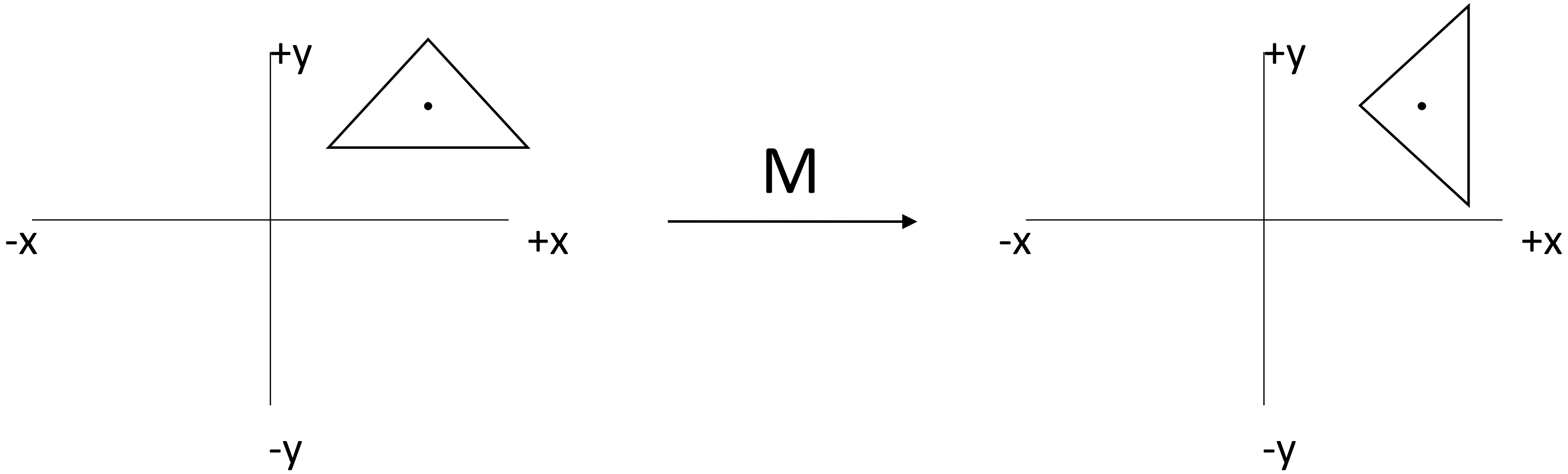

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- → 4 operations per vertex

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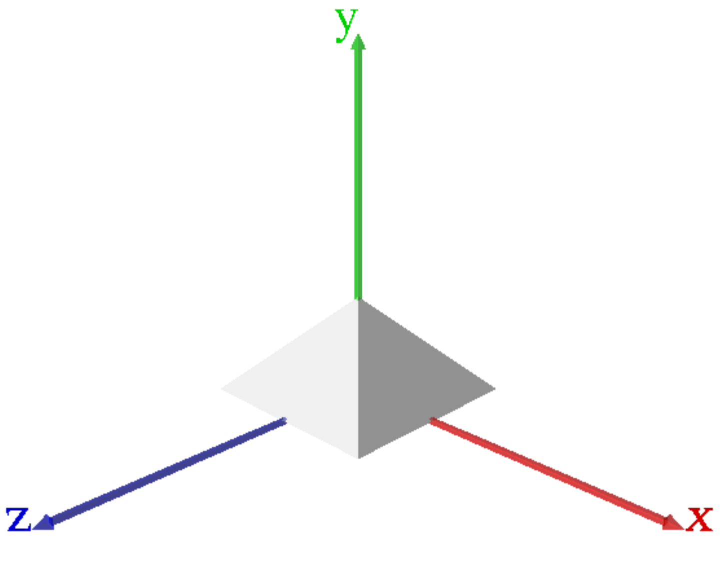
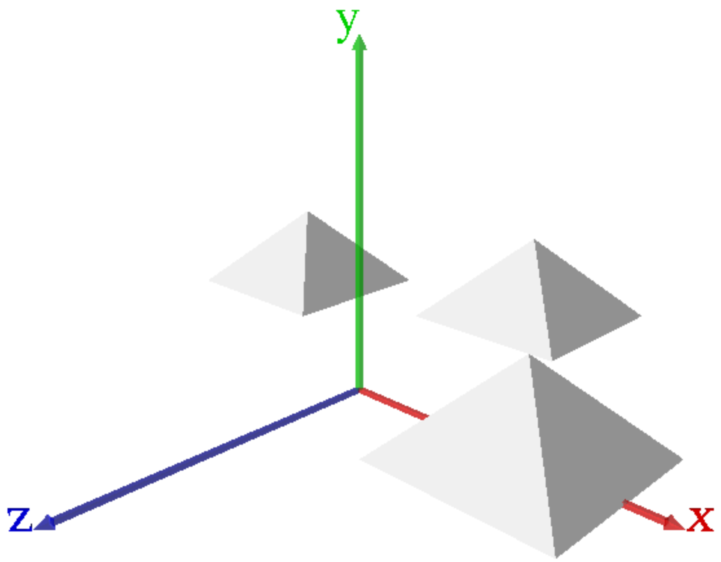
- Compare to the naïve approach: N*k*2

World space transformation



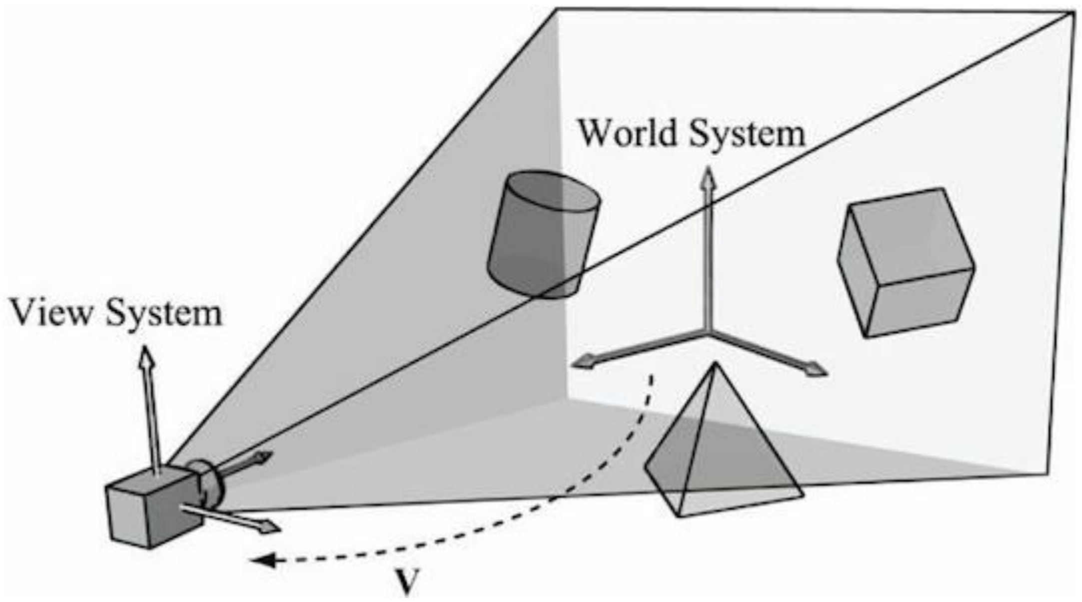
Object Space

Model Matrix



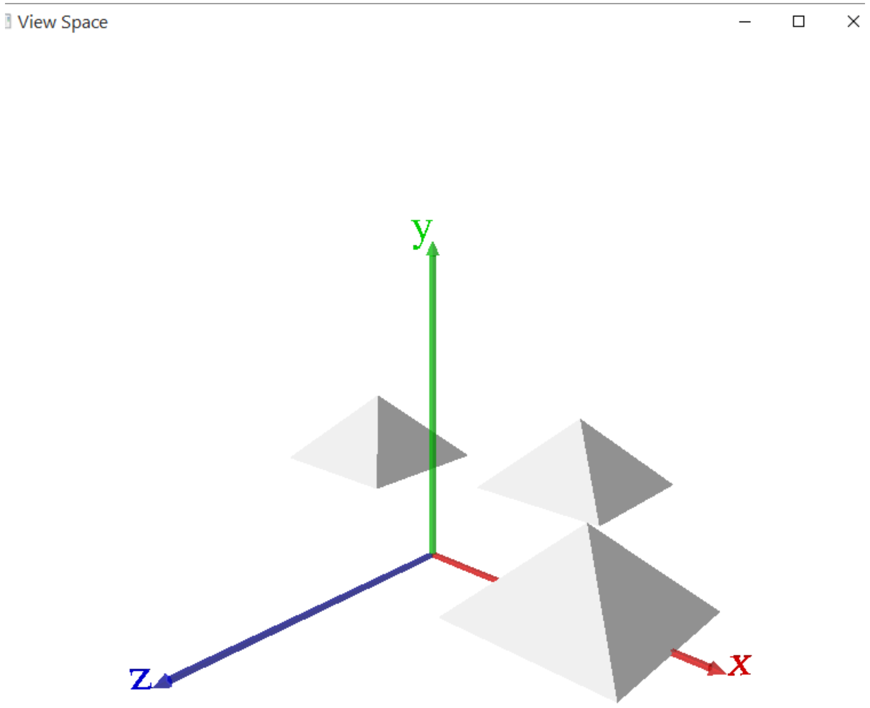
World Space

View (eye) space transformation

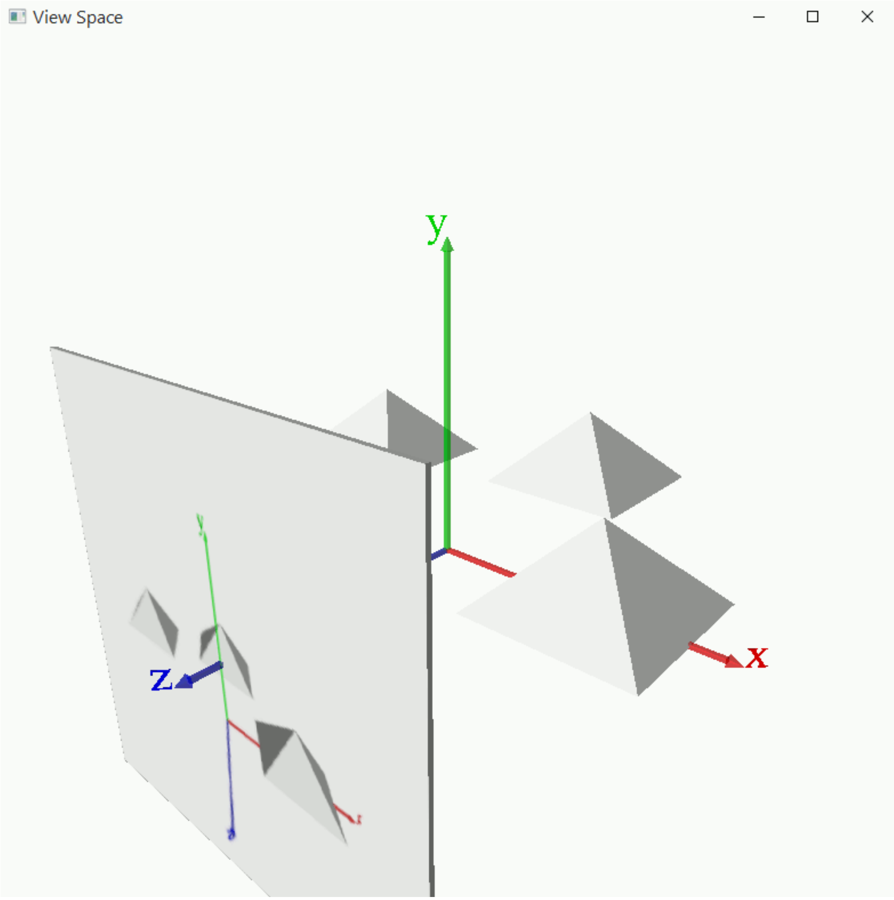


Define a "**view frustum**" that contains all visible objects

View (eye) space transformation

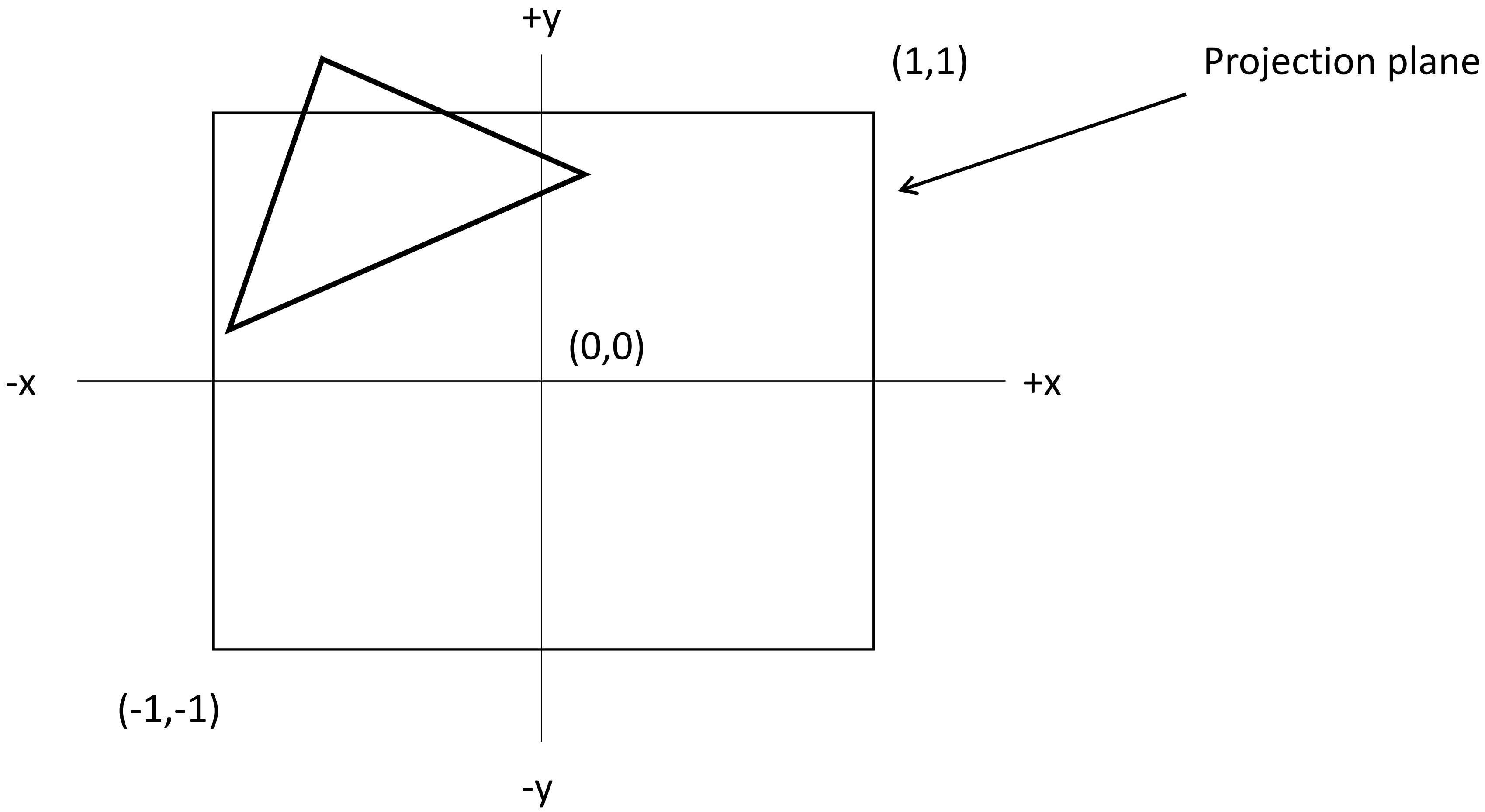


Projection transformation

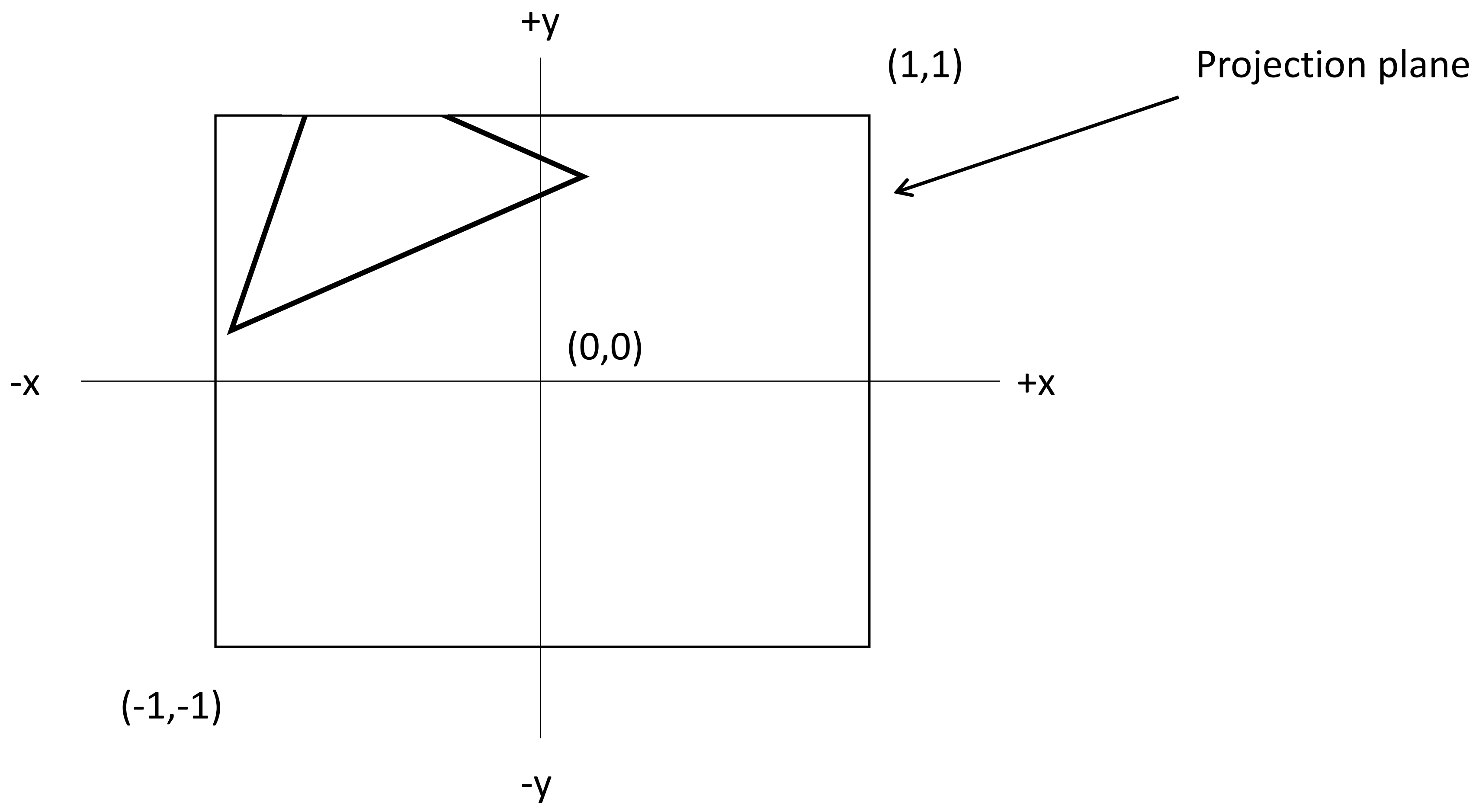


Project scene inside the view frustum onto a "**projection plane**"

Clipping



Clipping



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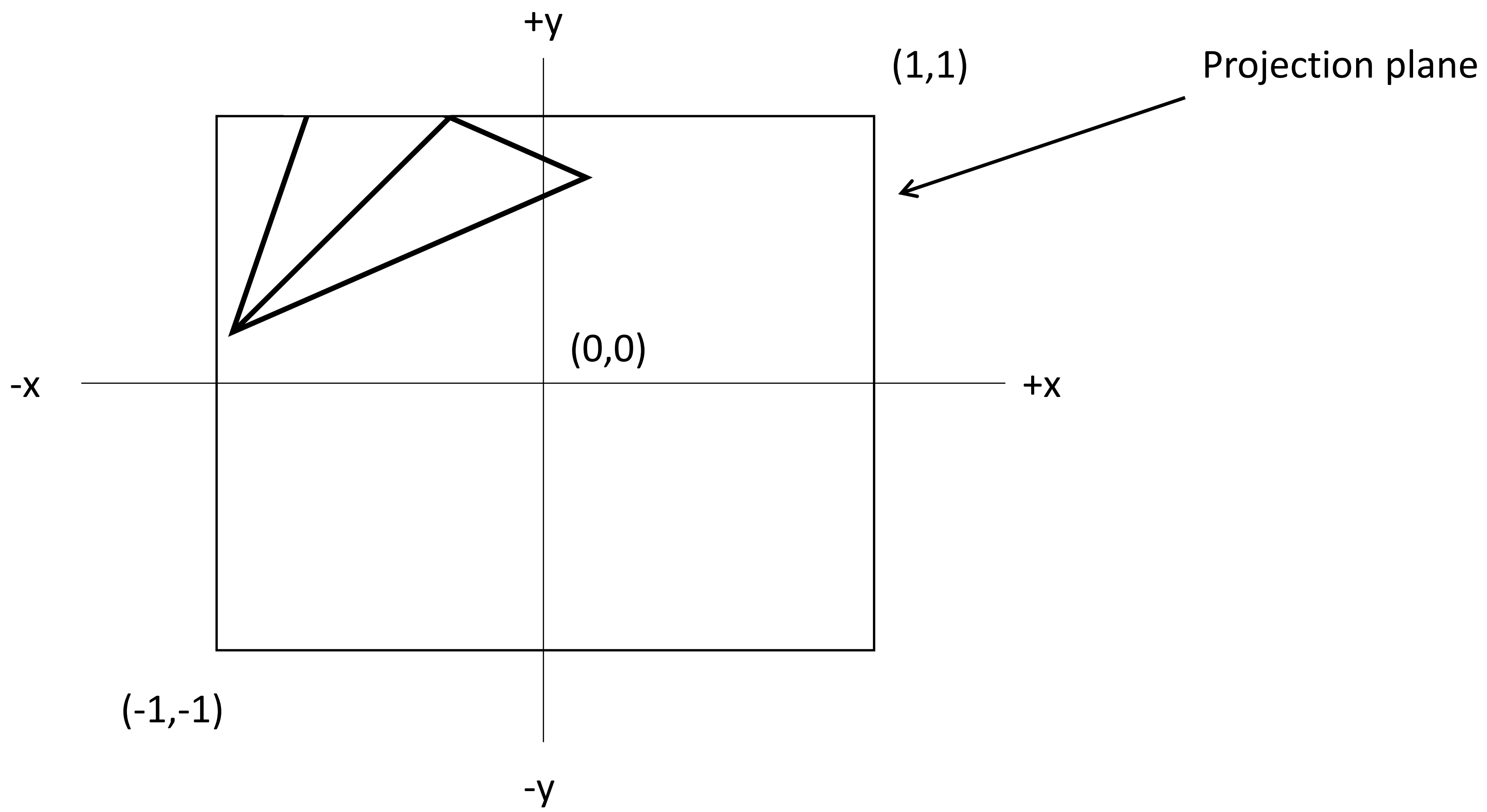
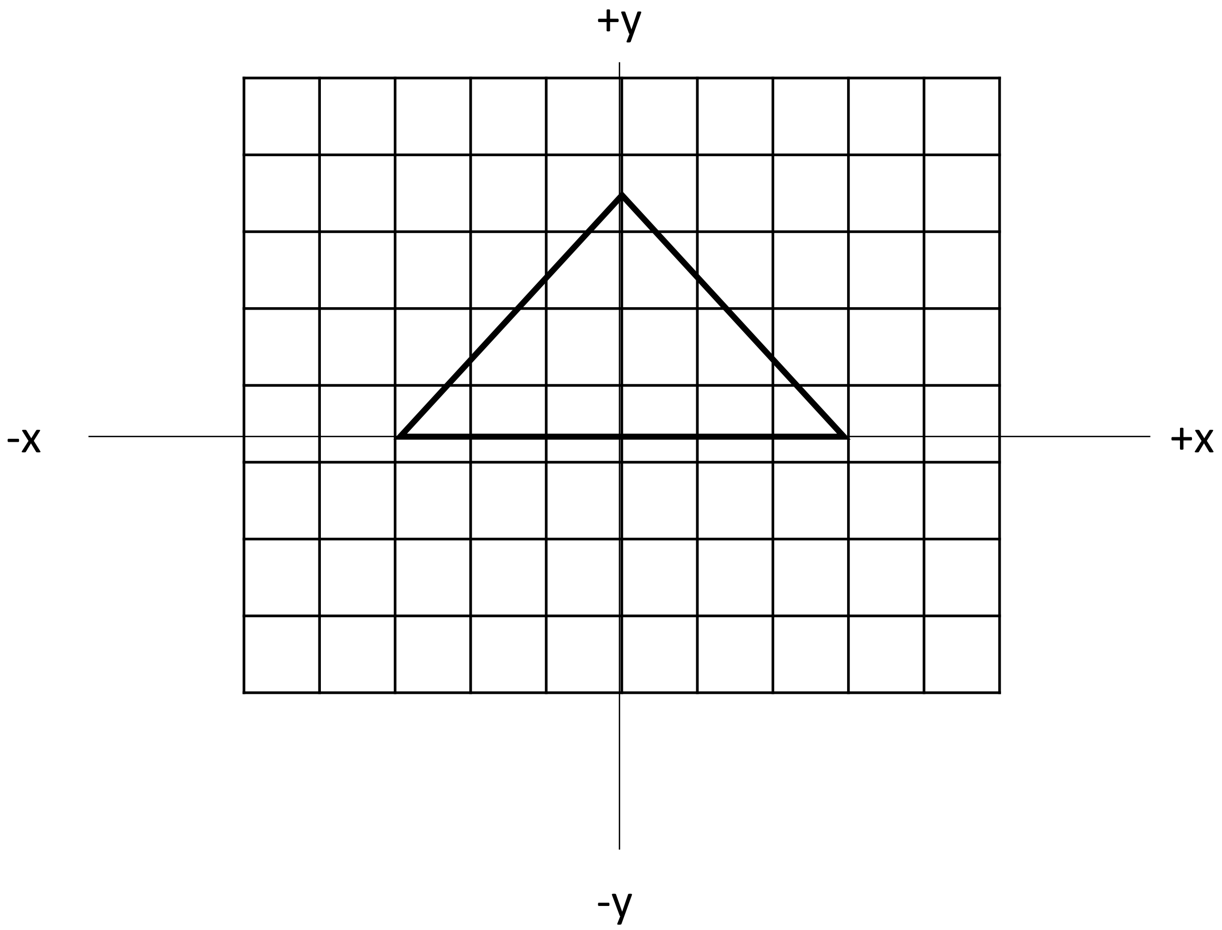


Scan conversion or rasterization



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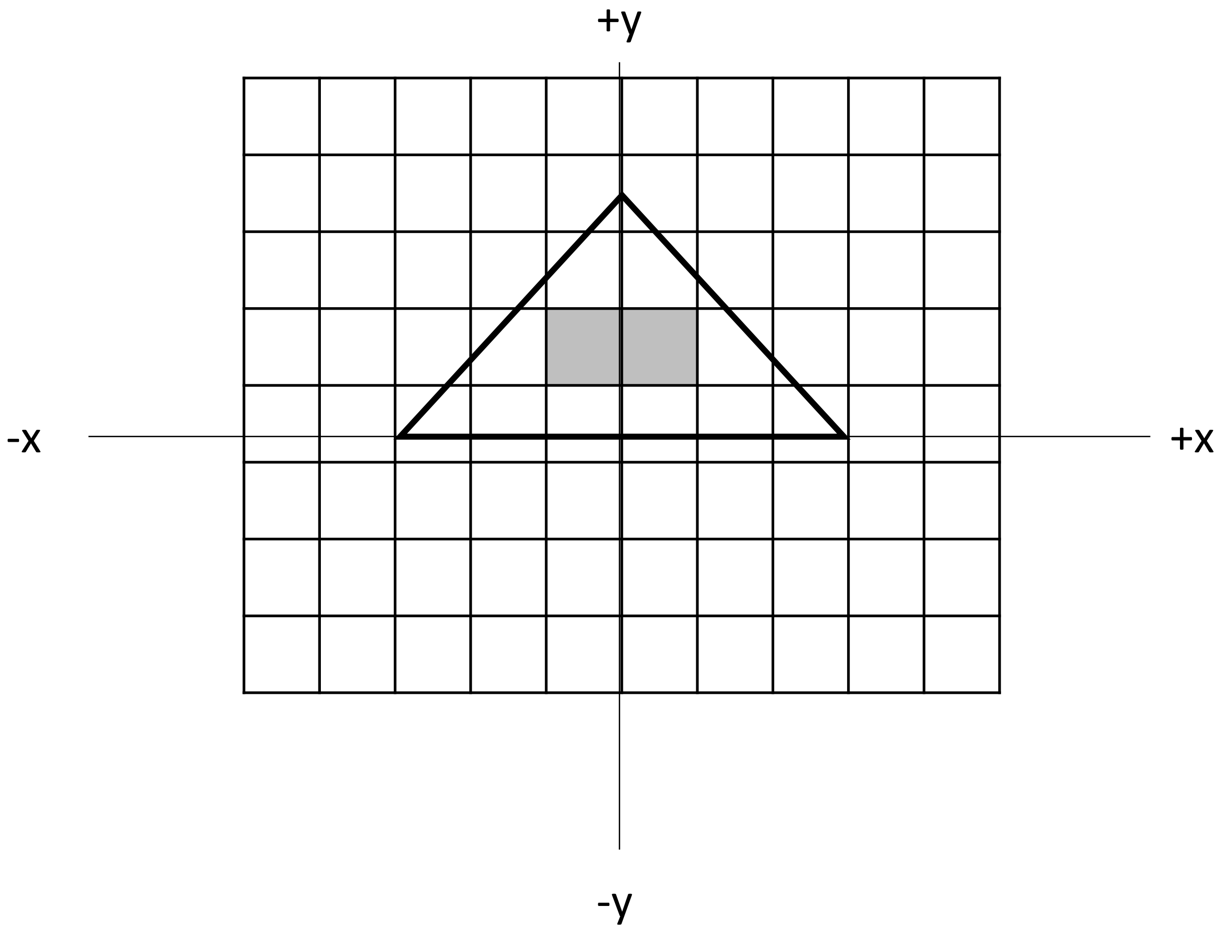

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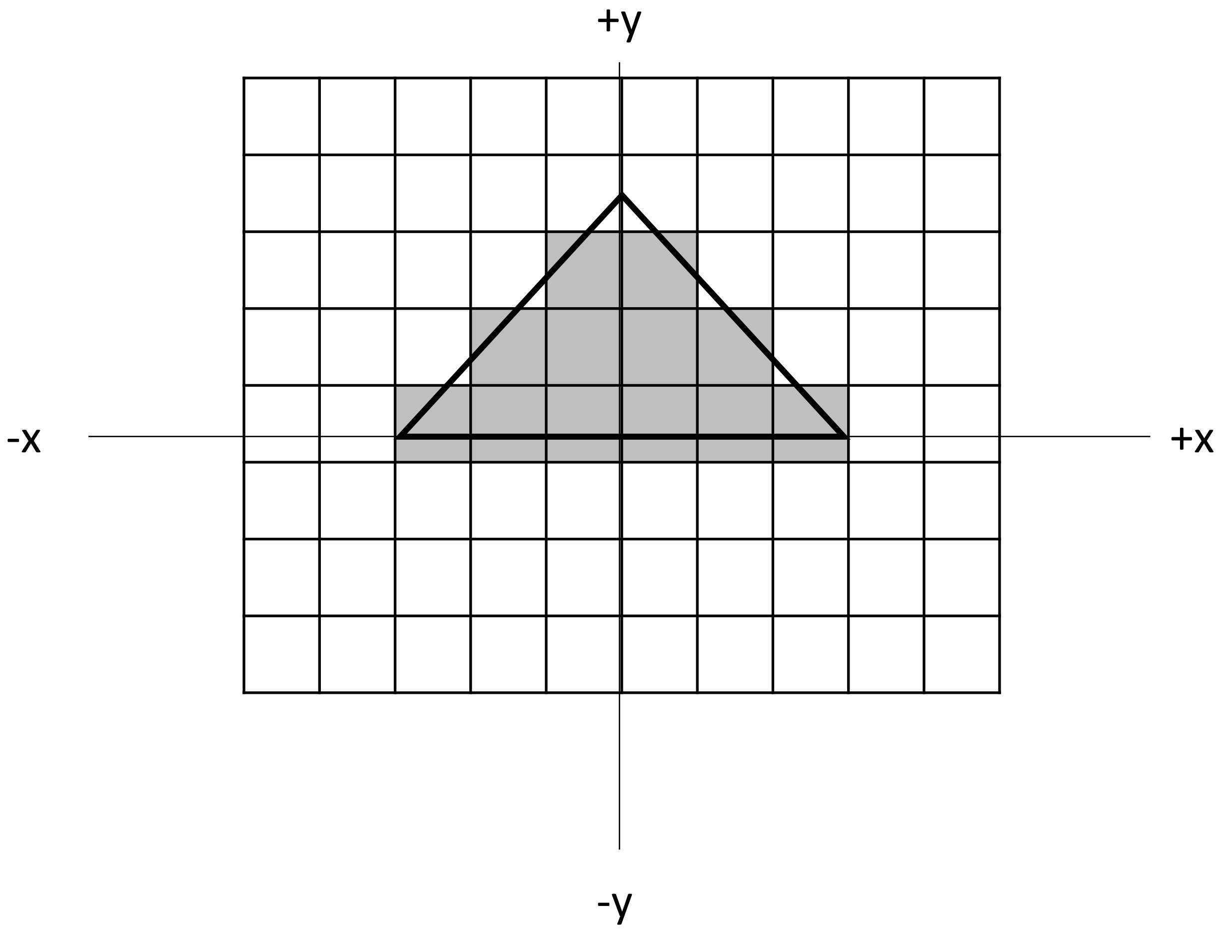

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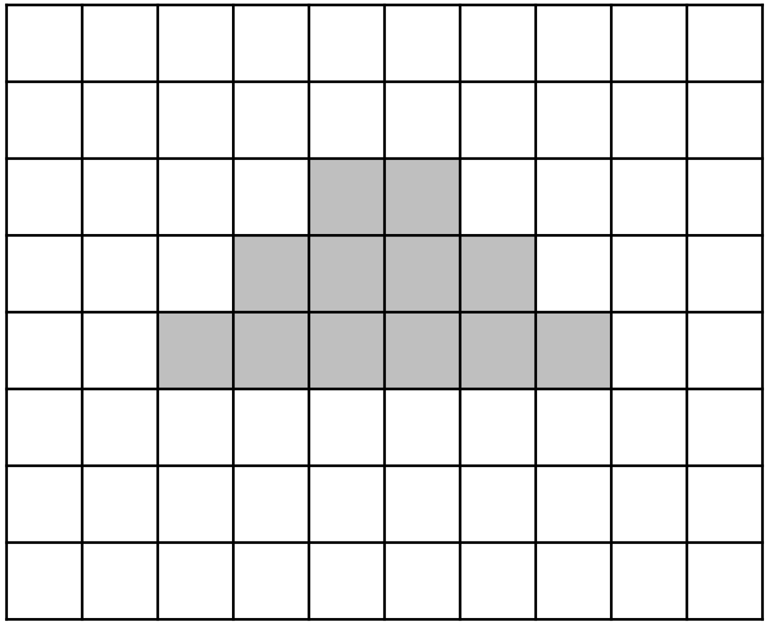
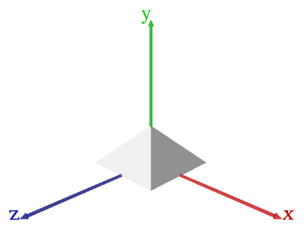


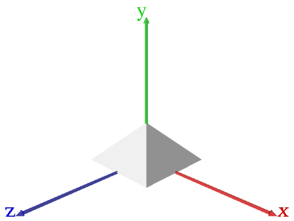
Simplified Rendering Pipeline



Object Space

Simplified Rendering Pipeline

Model Matrix



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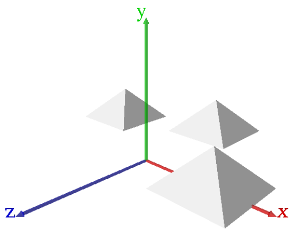
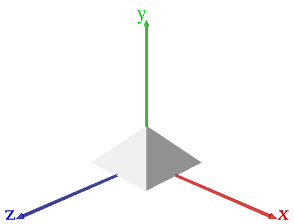
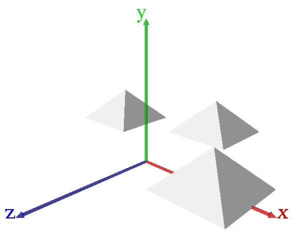


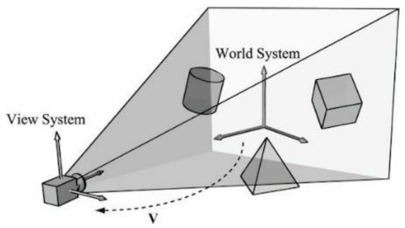
World Space

Simplified Rendering Pipeline

Model Matrix → View/Camera Matrix →







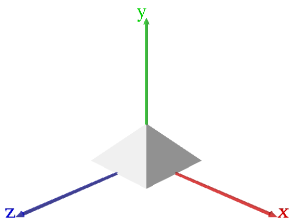
Object Space

World Space

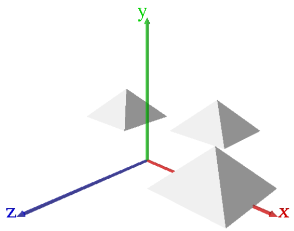
View Space

Simplified Rendering Pipeline

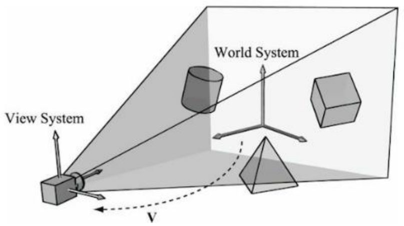
Model Matrix → View/Camera Matrix → Projection Matrix →



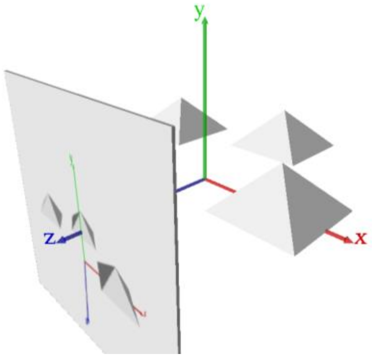
Object Space



World Space



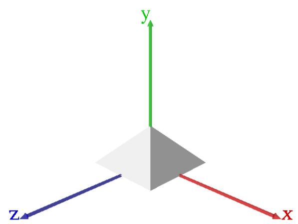
View Space

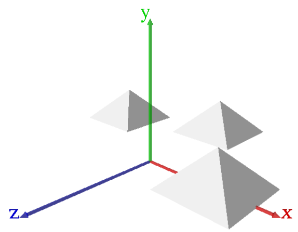


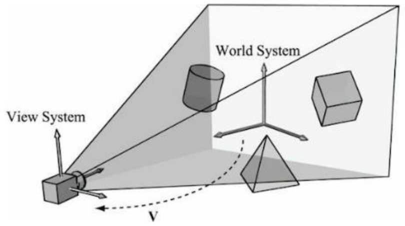
Clip Space

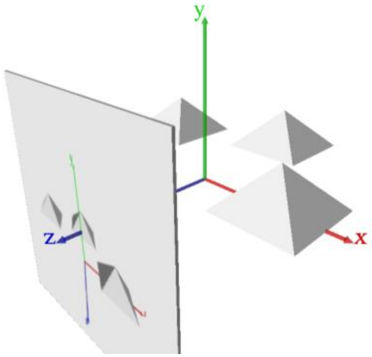
Simplified Rendering Pipeline

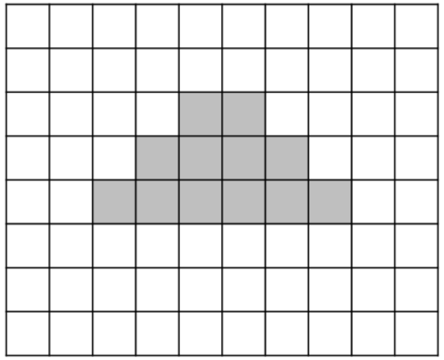
Model Matrix → View/Camera Matrix → Projection Matrix → Viewport Transform →











Object Space | World Space | View Space | Clip Space | Screen/Window Space