

Exercise 9

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Exercise 3

MEAN

```
##      1/2   1/3   2/3
##      0.04 0.06 1.45
```

Order is 1 < 3 < 2

MEDIAN

```
##      1/2   1/3   2/3
##      0.8  0.72 0.91
```

Order is 1 < 2 < 3

HUBER

```
##      1/2   1/3   2/3
##      0.52 0.51 0.98
```

Order is 1 < 2 < 3

Exercise 4

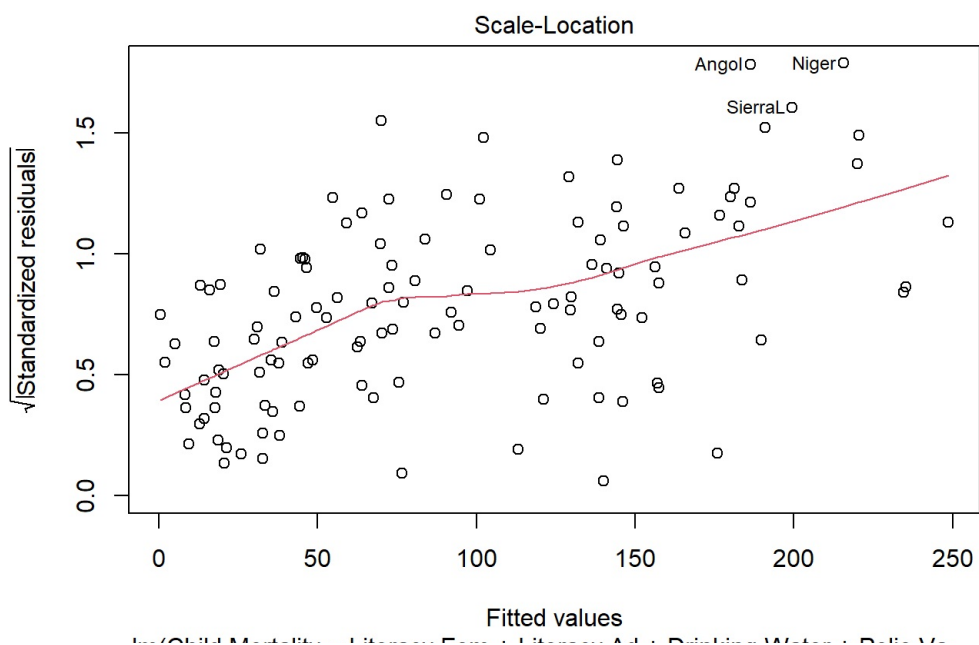
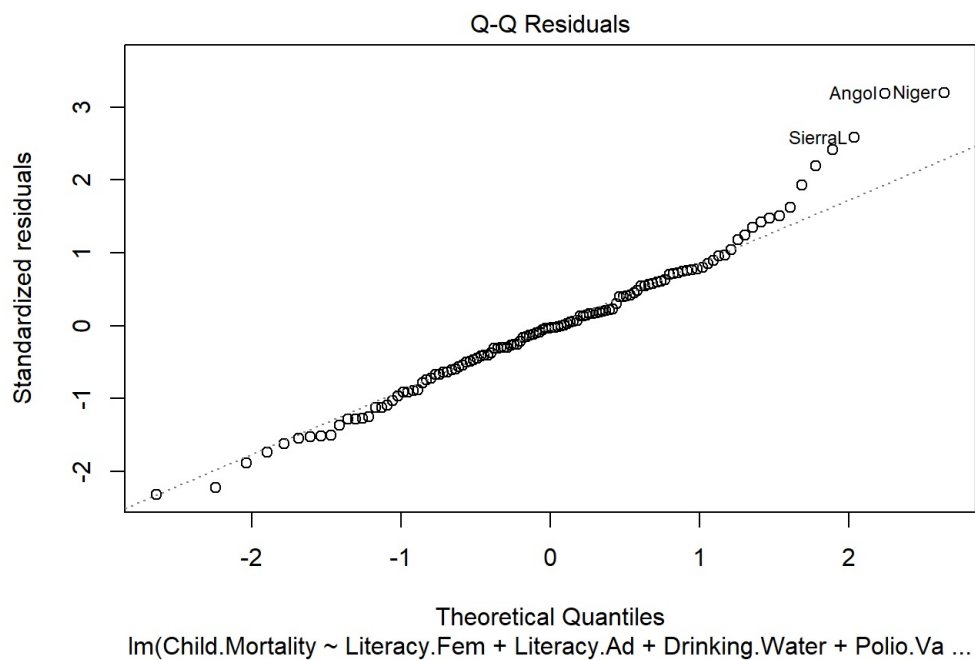
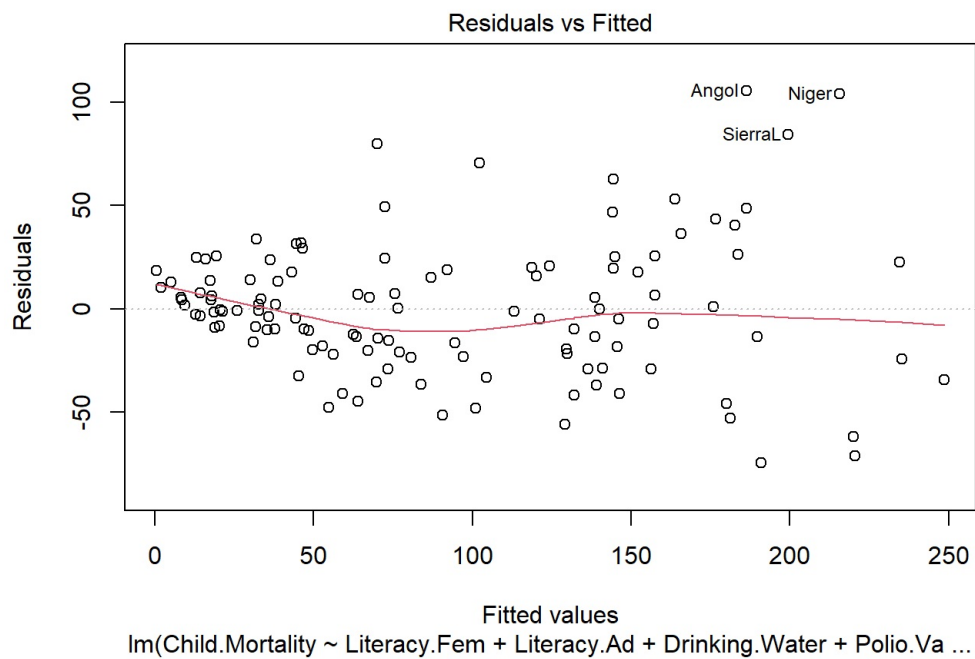
Summaries of models with 121 observations

```
##
## Call:
## lm(formula = Child.Mortality ~ Literacy.Fem + Literacy.Ad + Drinking.Water +
##      Polio.Vacc + Tetanus.Vacc.Preg + Urban.Pop + Foreign.Aid,
##      data = unicef)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -84.802 -19.570  -3.072  16.142 100.297
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    333.4750     16.7638   19.893 < 2e-16 ***
## Literacy.Fem     -1.1577      0.4432   -2.612  0.01021 *
## Literacy.Ad      -0.2405      0.4167   -0.577  0.56497
## Drinking.Water  -0.8695      0.2004   -4.339 3.13e-05 ***
## Polio.Vacc       -0.7159      0.2362   -3.031  0.00302 **
## Tetanus.Vacc.Preg -0.0985      0.1593   -0.618  0.53750
## Urban.Pop        -0.4112      0.1952   -2.107  0.03736 *
## Foreign.Aid       0.2878      0.1759    1.636  0.10459
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 36.27 on 113 degrees of freedom
## Multiple R-squared:  0.7587, Adjusted R-squared:  0.7437
## F-statistic: 50.75 on 7 and 113 DF,  p-value: < 2.2e-16
```

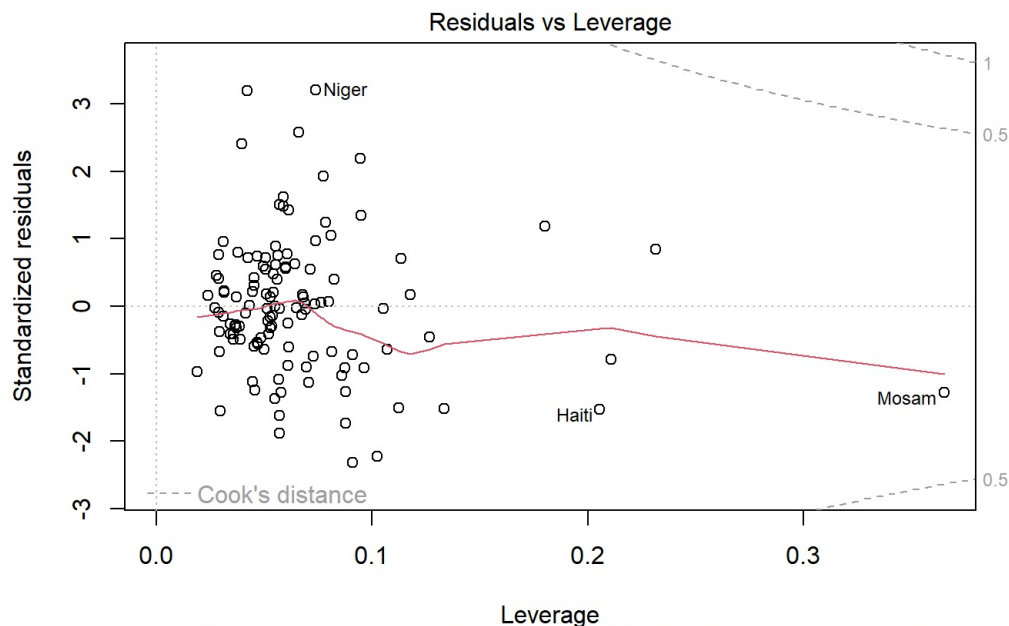
```
##
## Call:
## lmrob(formula = Child.Mortality ~ Literacy.Fem + Literacy.Ad + Drinking.Water +
##       Polio.Vacc + Tetanus.Vacc.Preg + Urban.Pop + Foreign.Aid, data = unicef)
## \--> method = "MM"
## Residuals:
##      Min       1Q   Median       3Q      Max
## -238.8820  -14.2924   -0.4143   21.3896  123.7362
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    277.88469    34.15661     8.136 5.91e-13 ***
## Literacy.Fem     -1.14738     0.55415    -2.071 0.040683 *
## Literacy.Ad       0.01122     0.43620     0.026 0.979529
## Drinking.Water   -0.61264     0.19972    -3.067 0.002702 **
## Polio.Vacc       -0.63284     0.36036    -1.756 0.081775 .
## Tetanus.Vacc.Preg -0.15987     0.13705    -1.166 0.245872
## Urban.Pop        -0.32653     0.16752    -1.949 0.053752 .
## Foreign.Aid      1.25256     0.31866     3.931 0.000146 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Robust residual standard error: 24.46
## Multiple R-squared:  0.8142, Adjusted R-squared:  0.8027
## Convergence in 24 IRWLS iterations
##
## Robustness weights:
## 3 observations c(4,80,91) are outliers with |weight| = 0 ( < 0.00083);
## 9 weights are ~1. The remaining 109 ones are summarized as
##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.
## 0.04766 0.85490 0.94130 0.87120 0.98690 0.99900
## Algorithmic parameters:
##      tuning.chi          bb      tuning.psi      refine.tol
##      1.548e+00          5.000e-01      4.685e+00      1.000e-07
##      rel.tol          scale.tol      solve.tol      zero.tol
##      1.000e-07          1.000e-10      1.000e-07      1.000e-10
##      eps.outlier          eps.x warn.limit.reject warn.limit.meanrw
##      8.264e-04          3.165e-10      5.000e-01      5.000e-01
##      nResample          max.it      best.r.s      k.fast.s      k.max
##      500              50          2          1          200
##      maxit.scale      trace.lev          mts      compute.rd fast.s.large.n
##      200              0          1000          0          2000
##      psi          subsampling          cov
##      "bisquare"          "nonsingular"          ".vcov.avar1"
## compute.outlier.stats
##      "SM"
## seed : int(0)
```

Linear model's checks

```
##
## Call:
## lm(formula = Child.Mortality ~ Literacy.Fem + Literacy.Ad + Drinking.Water +
##     Polio.Vacc + Tetanus.Vacc.Preg + Urban.Pop + Foreign.Aid,
##     data = unicef120)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -74.948  -19.955   -1.076   18.563  105.683
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    313.6320    16.3503   19.182 < 2e-16 ***
## Literacy.Fem     -1.4658     0.4202   -3.488 0.000696 ***
## Literacy.Ad       0.1100     0.3979    0.277 0.782621
## Drinking.Water   -0.6912     0.1919   -3.602 0.000472 ***
## Polio.Vacc       -0.8500     0.2228   -3.815 0.000224 ***
## Tetanus.Vacc.Preg -0.0499     0.1492   -0.335 0.738591
## Urban.Pop        -0.2603     0.1858   -1.401 0.163884
## Foreign.Aid      1.1833     0.2692    4.395 2.53e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 33.87 on 112 degrees of freedom
## Multiple R-squared:  0.7914, Adjusted R-squared:  0.7784
## F-statistic: 60.71 on 7 and 112 DF, p-value: < 2.2e-16
```

lm(Child.Mortality ~ Literacy.Fem + Literacy.Ad + Drinking.vvater + Polio.va ...

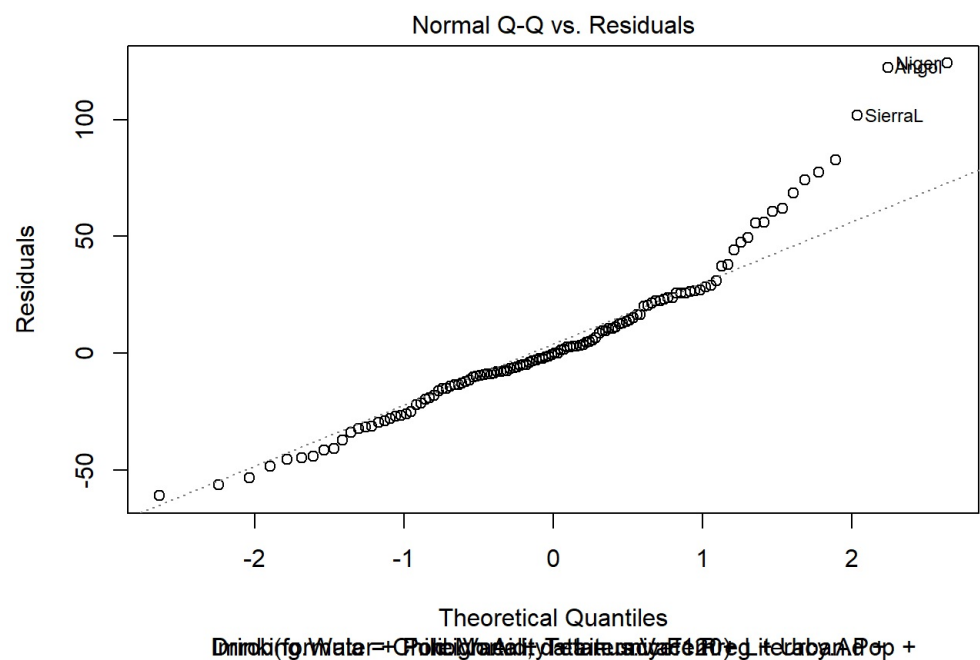
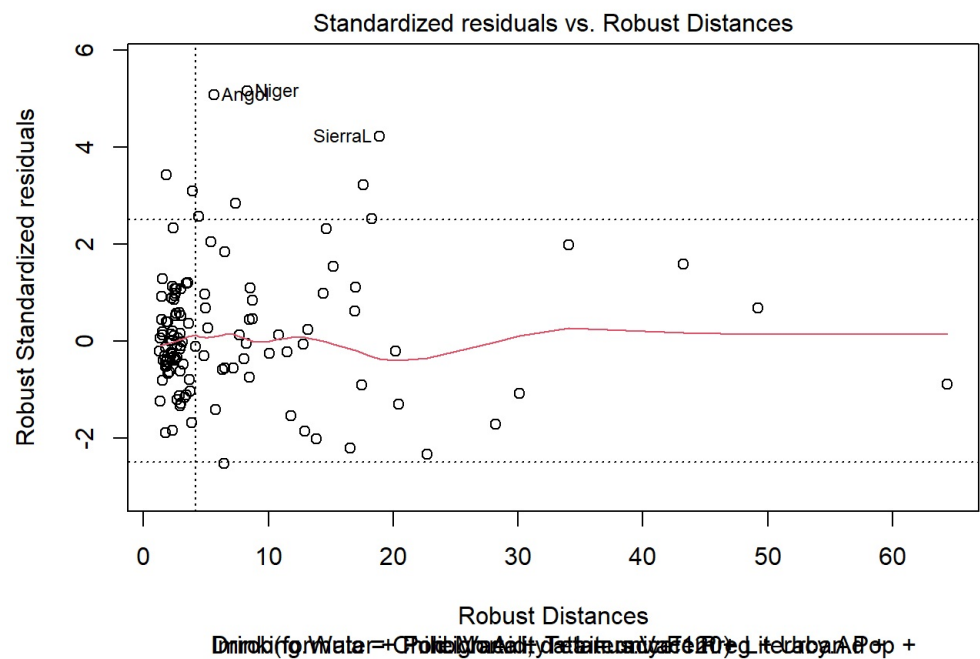


Robust linear model's checks

```
##
## Call:
## lmrob(formula = Child.Mortality ~ Literacy.Fem + Literacy.Ad + Drinking.Water +
##       Polio.Vacc + Tetanus.Vacc.Preg + Urban.Pop + Foreign.Aid, data = unicef120)
## \--> method = "MM"
## Residuals:
##      Min       1Q   Median       3Q      Max
## -61.0825 -13.7633  -0.3464  21.5446 124.1333
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    2.770e+02  3.522e+01   7.863 2.54e-12 ***
## Literacy.Fem   -1.131e+00  5.629e-01  -2.010 0.046850 *
## Literacy.Ad     -9.009e-04  4.415e-01  -0.002 0.998375
## Drinking.Water  -6.113e-01  2.009e-01  -3.043 0.002922 **
## Polio.Vacc      -6.245e-01  3.695e-01  -1.690 0.093790 .
## Tetanus.Vacc.Preg -1.638e-01  1.368e-01  -1.197 0.233744
## Urban.Pop       -3.287e-01  1.674e-01  -1.964 0.052015 .
## Foreign.Aid      1.252e+00  3.199e-01   3.914 0.000156 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Robust residual standard error: 24.06
## Multiple R-squared:  0.8158, Adjusted R-squared:  0.8043
## Convergence in 24 IRWLS iterations
##
## Robustness weights:
## 2 observations c(4,80) are outliers with |weight| = 0 ( < 0.00083);
## 9 weights are ~1. The remaining 109 ones are summarized as
##      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.03456 0.85200 0.94010 0.86760 0.98690 0.99900
## Algorithmic parameters:
##      tuning.chi          bb      tuning.psi      refine.tol
##      1.548e+00        5.000e-01    4.685e+00    1.000e-07
##      rel.tol          scale.tol    solve.tol      zero.tol
##      1.000e-07        1.000e-10    1.000e-07    1.000e-10
##      eps.outlier          eps.x warn.limit.reject warn.limit.meanrw
##      8.333e-04        2.001e-10    5.000e-01    5.000e-01
##      nResample      max.it      best.r.s      k.fast.s      k.max
##      500            50          2            1            200
##      maxit.scale    trace.lev      mts      compute.rd fast.s.large.n
##      200            0            1000        0            2000
##      psi            subsampling      cov
##      "bisquare"      "nonsingular"    ".vcov.avar1"
## compute.outlier.stats
##      "SM"
## seed : int(0)
```

```
## recomputing robust Mahalanobis distances
```

```
## saving the robust distances 'MD' as part of 'unicef120mm'
```



Comparing t-test results

Baseline model

```
##      (Intercept) Literacy.Fem Literacy.Ad Drinking.Water Polio.Vacc
## [1,] 19.89255    -2.612494   -0.5771748    -4.338649   -3.030994
## [2,] 19.18199    -3.488185    0.2765692    -3.602400   -3.814594
##      Tetanus.Vacc.Preg Urban.Pop Foreign.Aid
## [1,]  -0.6184877  -2.106637    1.636137
## [2,]  -0.3345491  -1.401319    4.395282
```

Robust model

```
##      (Intercept) Literacy.Fem  Literacy.Ad Drinking.Water Polio.Vacc
## [1,]    8.135604    -2.070507  0.025716048      -3.067454  -1.756135
## [2,]    7.862837    -2.009862 -0.002040581      -3.042612  -1.690101
##      Tetanus.Vacc.Preg Urban.Pop Foreign.Aid
## [1,]      -1.166491 -1.949182    3.930774
## [2,]      -1.197233 -1.963937    3.914437
```

Comparing vectors of estimators of the regression coefficients

Baseline model

```
##      (Intercept) Literacy.Fem Literacy.Ad Drinking.Water Polio.Vacc
## [1,]    333.475    -1.157742  -0.2405001    -0.8695181  -0.7159093
## [2,]    313.632    -1.465829   0.1100508    -0.6912333  -0.8500264
##      Tetanus.Vacc.Preg Urban.Pop Foreign.Aid
## [1,]      -0.09850199 -0.4112021    0.2877856
## [2,]      -0.04989845 -0.2603098    1.1833540
```

Robust model

```
##      (Intercept) Literacy.Fem  Literacy.Ad Drinking.Water Polio.Vacc
## [1,]    277.8847    -1.147380  0.0112172780    -0.6126395  -0.6328382
## [2,]    276.9589    -1.131354 -0.0009008566    -0.6112957  -0.6245238
##      Tetanus.Vacc.Preg Urban.Pop Foreign.Aid
## [1,]      -0.1598670 -0.3265270    1.252564
## [2,]      -0.1637969 -0.3286928    1.252097
```

The robust regression using the MM-estimator appears to be less sensitive to outliers compared to the standard least squares linear regression because:

- The robust model shows smaller residuals and more stable coefficient estimates, indicating reduced sensitivity to outliers, it reports a lower robust residual standard error, suggesting a better fit to the majority of data points and a decreased impact of outliers.
- T-test results for coefficients in the robust model exhibit smaller t-values, indicating less influence from extreme observations.
- The vectors of estimators in the robust model are more stable, with smaller changes in coefficients when compared to the baseline model.