CHAPTER – 11

Probability Distributions

Exercise 11.1

1. Suppose X is the number of tails occurred when three fair coins are tossed once simultaneously. Find the values of the random variable X and number of points in its inverse images.

Solution:

$$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$$

$n(S) = 8$

Let X be the random variable denotes the number of tails.

X : 0, 1, 2, 3 (tails)

$X(0) = 1 \qquad \{HHH\}$

$X(1) = 3 \qquad \{HHT, HTH, THH\}$

$X(2) = 3 \qquad \{HTT, THT, TTH\}$

$X(3) = 1 \qquad \{TTT\}$

Values of random variable	0	1	2	3	Total
No. of points in inverse image	1	3	3	1	8

2. An urn contains 5 mangoes and 4 apples. Three fruits are taken at random. If the number of apples taken is random variable, then find the values of random variable and number of points in its inverse images.

Solution:

Total fruits = 9

$$n(S) = 9C_3 = \frac{9\times8\times7}{3\times2\times1} = 84$$

Let X be the random variable denotes no. of apples taken

X : 0, 1, 2, 3 (no. of apples)

$$X(0) = 4C_0 \times 5C_3 = 1 \times \frac{5\times4\times3}{3\times2\times1} = 10$$

$$X(1) = 4C_1 \times 5C_2 = 4 \times \frac{5\times4}{2\times1} = 40$$

$$X(2) = 4C_2 \times 5C_1 = \frac{4\times3}{2\times1} \times 5 = 30$$

$$X(3) = 4C_3 \times 5C_0 = \frac{4\times3\times2}{3\times2\times1} \times 1 = 4$$

Values of random variable	0	1	2	3	Total
No. of points in inverse image	10	40	30	4	84

3. Two balls are chosen randomly from an urn containing 6 red and 8 black balls. Suppose that we win ₹15 for each red ball selected and we lose ₹10 for each black ball selected. If X denotes the winning amount, find the values of X number of points in its inverse image.

Solution:

Total balls = 14

$$n(S) = 14C_2 = \frac{14\times13}{2\times1} = 91$$

Let X be the random variable denotes the winning amount

X : −20, 5, 30 (winning amount)

$X(-20) = X(2\ black)$

$$= 6C_0 \times 8C_2 = 1 \times \frac{8\times7}{2\times1}$$

$X(-20) = 28$

$X(5) = X(1\ black, 1\ red)$

$$= 6C_1 \times 8C_1 = 6 \times 8$$

$X(5) = 48$

$X(30) = X(2\ red)$

$$= 6C_2 \times 8C_0 = \frac{6\times5}{2\times1} \times 1$$

$X(30) = 15$

Values of random variable	−20	5	30	Total
No. of points in inverse image	28	48	15	91

4. A six sided die is marked '2' on one face, '3' on two of its faces, and '4' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find the values of the random variable and number of points in its inverse image.

Solution:

$$S = \{2,3,3,4,4,4\} \times \{2,3,3,4,4,4\}$$

$n(S) = 36$

•	2	3	3	4	4	4
2	4	5	5	6	6	6
3	5	6	6	7	7	7
3	5	6	6	7	7	7
4	6	7	7	8	8	8
4	6	7	7	8	8	8
4	6	7	7	8	8	8

Let X be the random variable denotes total score of die

X : 4, 5, 6, 7, 8 (total score)

$X(4) = 1, X(5) = 4, X(6) = 10, X(7) = 12, X(8) = 9$

Values of random variable	4	5	6	7	8	Total
No. of points in inverse image	1	4	10	12	9	36

Exercise 11.2

1. Three fair coins are tossed simultaneously. Find the probability mass function of heads occurred.

Solution:

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$

Let X be the random variable denotes the no. of heads

$X : 0, 1, 2, 3$ (heads)

$P(X = 0) = \frac{1}{8}$ $\{TTT\}$

$P(X = 1) = \frac{3}{8}$ $\{HTT, THT, TTH\}$

$P(X = 2) = \frac{3}{8}$ $\{HHT, HTH, THH\}$

$P(X = 3) = \frac{1}{8}$ $\{HHH\}$

The probability mass function is,

X	0	1	2	3	Total
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	1

2. A six sided die is marked '1' on one face, '3' on two of its faces, and '5' on remaining three faces. The die is thrown twice. If X denotes the total score in two throws, find

(i) the probability mass function

(ii) the cumulative distribution function

(iii) $P(4 \le X < 10)$ (iv) $P(X \ge 6)$

Solution:

$S = \{1,3,3,5,5,5\} \times \{1,3,3,5,5,5\}$

$n(S) = 36$

•	1	3	3	5	5	5
1	2	4	4	6	6	6
3	4	6	6	8	8	8
3	4	6	6	8	8	8
5	6	8	8	10	10	10
5	6	8	8	10	10	10
5	6	8	8	10	10	10

Let X be the random variable denotes total score of die

$X : 2, 4, 6, 8, 10$ (total score)

$P(X = 2) = \frac{1}{36}, P(X = 4) = \frac{4}{36}, P(X = 6) = \frac{10}{36}$

$P(X = 8) = \frac{12}{36}, P(X = 10) = \frac{9}{36}$

(i) The probability mass function is,

X	2	4	6	8	10	Total
$P(X = x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$	1

(ii) The cumulative distribution function is,

X	2	4	6	8	10
$F(x)$	$\frac{1}{36}$	$\frac{5}{36}$	$\frac{15}{36}$	$\frac{27}{36}$	$\frac{36}{36}$

(iii) $P(4 \le X < 10) = P(X = 4) + P(X = 6) + P(X = 8)$

$= \frac{4}{36} + \frac{10}{36} + \frac{12}{36} = \frac{26}{36}$

$= \frac{13}{18}$

(iv) $P(X \ge 6) = P(X = 6) + P(X = 8) + P(X = 10)$

$= \frac{10}{36} + \frac{12}{36} + \frac{9}{36}$

$= \frac{31}{36}$

3. Find the probability mass function and cumulative distribution function of number of girl child in families with 4 children, assuming equal probabilities for boys and girls.

Solution:

$n(S) = 2^4 = 16$

Let X be the random variable denotes the no. of girl child

$X : 0, 1, 2, 3, 4$ (girl)

$X(0) = 1$ $\{BBBB\}$

$X(1) = 4$ $\{GBBB, BGBB, BBGB, BBBG\}$

$X(2) = 6$ $\{GGBB, BBGG, GBGB, BGBG, BGGB, GBBG\}$

$X(3) = 4$ $\{BGGG, GGGB, GBGG, GGBG\}$

$X(4) = 1$ $\{GGGG\}$

The probability mass function is,

X	0	1	2	3	4	Total
$P(X = x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$	1

The cumulative distribution function is,

X	0	1	2	3	4
$F(x)$	$\frac{1}{16}$	$\frac{5}{16}$	$\frac{11}{16}$	$\frac{15}{16}$	$\frac{16}{16}$

---

4. Suppose a discrete random variable can only take the values 0,1, and 2. The probability mass function is defined by

$$f(x) = \begin{cases} \frac{x^2+1}{k} & for\ x = 0,1,2 \\ 0 & otherwise \end{cases}$$

Find (i) the value of k

(ii) cumulative distribution function (iii) $P(X \geq 1)$

Solution:

(i) we know that, $\sum f(x) = 1$

$$f(0) + f(1) + f(2) = 1$$

$$\frac{0+1}{k} + \frac{1^2+1}{k} + \frac{2^2+1}{k} = 1$$

$$\frac{1}{k} + \frac{2}{k} + \frac{5}{k} = 1$$

$$\frac{8}{k} = 1$$

$$k = 8$$

The probability mass function is,

X	0	1	2	Total
$P(X = x)$	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{5}{8}$	1

(ii) The cumulative distribution function is,

X	0	1	2
$F(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{8}{8}$

(iii) $P(X \geq 1) = P(X = 1) + P(X = 2)$

$$= \frac{2}{8} + \frac{5}{8}$$

$$= \frac{7}{8}$$

---

5. The cumulative distribution function of a discrete random variable is given by

$$f(x) = \begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \leq x < 0 \\ 0.35 & 0 \leq x < 1 \\ 0.60 & 1 \leq x < 2 \\ 0.85 & 2 \leq x < 3 \\ 1 & 3 \leq x < \infty \end{cases}$$

Find (i) The probability mass function

(ii) $P(X < 1)$ (iii) $P(X \geq 2)$.

Solution:

(i) The probability mass function is,

X	-1	0	1	2	3
$F(x)$	0.15	0.35	0.60	0.85	1
$f(x) = P(X = x)$	0.15	0.20	0.25	0.25	0.15

(ii) $P(X < 1) = P(X = 0) + P(X = -1)$

$$= 0.15 + 0.20$$

$$= 0.35$$

(iii) $P(X \geq 2) = P(X = 2) + P(X = 3)$

$$= 0.25 + 0.15$$

$$= 0.40$$

---

6. A random variable X has the following probability mass function:

X	1	2	3	4	5
$P(X = x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2 \leq X < 5)$ (iii) $P(3 < X)$

Solution:

(i) we know that, $\sum f(x) = 1$

$$k^2 + 2k^2 + 3k^2 + 2k + 3k = 1$$

$$6k^2 + 5k - 1 = 0$$

$$(6k - 1)(k + 1) = 0$$

$$\begin{array}{c|c} & -6 \\ -1 & 6 \\ \hline 6 & 6 \\ & 5 \end{array}$$

$$k = \frac{1}{6} \text{ or } k = -1$$

$$k = \frac{1}{6} \qquad (\because 0 \leq k \leq 1)$$

(ii) $P(2 \leq X < 5) = P(X = 2) + P(x = 3) + P(X = 4)$

$$= \frac{2}{36} + \frac{3}{36} + \frac{2}{6}$$

$$= \frac{5}{36} + \frac{2}{6}$$

$$= \frac{17}{36}$$

(iii) $P(3 < X) = P(X = 4) + P(X = 5)$

$$= \frac{2}{6} + \frac{3}{6}$$

$$= \frac{5}{6}$$

---

7. The cumulative distribution function of a discrete random variable is given by

$$f(x) = \begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{2} & 0 \le x < 1 \\ \frac{3}{5} & 1 \le x < 2 \\ \frac{4}{5} & 2 \le x < 3 \\ \frac{9}{10} & 3 \le x < 4 \\ 1 & 4 \le x < \infty \end{cases}$$

Find (i) the probability mass function (ii) $P(X < 3)$ (iii) $P(X \ge 2)$.

Solution:

(i) The probability mass function is,

X	0	1	2	3	4
$F(x)$	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{9}{10}$	1
$f(x) = P(X = x)$	$\frac{1}{2}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

(ii) $P(X < 3) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= \frac{1}{2} + \frac{1}{10} + \frac{1}{5}$$

$$= \frac{5+1+2}{10} = \frac{8}{10}$$

$$= \frac{4}{5}$$

(iii) $P(X \ge 2) = P(X = 2) + P(X = 3) + P(X = 4)$

$$= \frac{1}{5} + \frac{1}{10} + \frac{1}{10}$$

$$= \frac{4}{10}$$

$$= \frac{2}{5}$$

Exercise 11.3

Probability density function, $\int_{-\infty}^{\infty} f(x)\,dx = 1$

P.d.f., $f(x) = F'(x)$

Cumulative distribution function, $F(x) = \int_{-\infty}^{x} f(t)\,dt$

Properties:

(i) $0 \le F(x) \le 1$

(ii) $F(-\infty) = 0$ and $F(\infty) = 1$

(iii) $P(X > x) = 1 - P(X \le x) = 1 - F(x)$

(iii) $P(a < X < b) = F(b) - F(a)$

Hint: $\int_0^{\infty} x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$

1. The probability density function of X is given by

$$f(x) = \begin{cases} k\,x\,e^{-2x} & for\ x > 0 \\ 0 & for\ x \le 0 \end{cases}$$ **find the value of k.**

Solution:

Since $f(x)$ is a probability density function,

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{0} 0\,dx + \int_0^{\infty} kxe^{-2x}\,dx = 1$$

$$k\int_0^{\infty} xe^{-2x}\,dx = 1$$

$$k\left[\frac{1!}{2^2}\right] = 1 \qquad \text{(by hint)}$$

$$k = 4$$

2. The probability density function of X is

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \le x < 2 \\ 0 & otherwise \end{cases}$$

Find (i) $P(0.2 \le X < 0.6)$ (ii) $P(1.2 \le X < 1.8)$ (iii) $P(0.5 \le X < 1.5)$

Solution:

(i) $P(0.2 \le X < 0.6) = \int_{0.2}^{0.6} f(x)\,dx$

$$= \int_{0.2}^{0.6} x\,dx$$

$$= \left[\frac{x^2}{2}\right]_{0.2}^{0.6} = \frac{(0.6)^2}{2} - \frac{(0.2)^2}{2}$$

$$= \frac{0.36}{2} - \frac{0.04}{2}$$

$$= \frac{0.36}{2} = 0.16$$

(ii) $P(1.2 \le X < 1.8) = \int_{1.2}^{1.8} f(x)\,dx$

$$= \int_{1.2}^{1.8} (2 - x)\,dx$$

$$= \left[2x - \frac{x^2}{2}\right]_{1.2}^{1.8}$$

$$= \left[3.6 - \frac{3.24}{2}\right] - \left[2.4 - \frac{1.44}{2}\right]$$

$$= 3.6 - 1.62 - 2.4 + 0.72$$

$$= 0.3$$

(iii) $P(0.5 < X < 1.5) = \int_{0.5}^{1.0} f(x)\,dx + \int_{1.0}^{1.5} f(x)\,dx$

$$= \int_{0.5}^{1.0} x\,dx + \int_{1.0}^{1.5} (2 - x)\,dx$$

$$= \left[\frac{x^2}{2}\right]_{0.5}^{1.0} + \left[2x - \frac{x^2}{2}\right]_{1.0}^{1.5}$$

$$= \left[\frac{1}{2} - \frac{0.25}{2}\right] + \left[\left(3 - \frac{2.25}{2}\right) - \left(2 - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2} - \frac{0.25}{2} + 3 - \frac{2.25}{2} - \frac{3}{2}$$

$$= \frac{1 - 0.25 + 6 - 2.25 - 3}{2} = \frac{1.5}{2}$$

$$= 0.75$$

3. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function

$$f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & otherwise \end{cases}$$

Find (i) the value of k (ii) the distribution function

(iii) the probability that daily sales will fall between 300 litres and 500 litres?

Solution:

(i) Since $f(x)$ is a probability density function

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{200}^{600} k\, dx = 1$$

$$k(600-200) = 1$$

$$k = \frac{1}{400}$$

(ii) Distribution function, $F(x) = \int_{-\infty}^{x} f(t)\, dt$

When $x \in (-\infty, 200)$

$$F(x) = \int_{-\infty}^{x} f(t)\, dt = 0 \qquad \text{----------(1)}$$

When $x \in [200,600]$

$$F(x) = \int_{-\infty}^{200} f(t)\, dt + \int_{200}^{x} f(t)\, dt$$

$$= 0 + \int_{200}^{x} k\, dt$$

$$= k\,[t]_{200}^{x}$$

$$= \frac{1}{400}(x-200)$$

$$F(x) = \frac{x}{400} - \frac{1}{2} \qquad \text{----------(2)}$$

When $x \in (600, \infty)$

$$F(x) = \int_{-\infty}^{200} f(t)\, dt + \int_{200}^{600} f(t)\, dt + \int_{600}^{x} f(t)\, dt$$

$$= 0 + \int_{200}^{600} k\, dt + 0$$

$$= k\,[t]_{200}^{600}$$

$$= \frac{1}{400}(600-200)$$

$$F(x) = 1 \qquad \text{----------(3)}$$

The distribution function is,

$$F(x) = \begin{cases} 0 & -\infty < x < 200 \\ \frac{x}{400} - \frac{1}{2} & 200 \le x \le 600 \\ 1 & 600 < x < \infty \end{cases}$$

(iii) $P(300 < x < 500) = F(500) - F(300)$

$$= \frac{500}{400} - \frac{1}{2} - \left(\frac{300}{400} - \frac{1}{2}\right)$$

$$= \frac{500-300}{400} - \frac{1}{2} + \frac{1}{2}$$

$$= \frac{200}{400} = \frac{1}{2}$$

---

4. The probability density function of X is given by

$$f(x) = \begin{cases} k\, e^{-\frac{x}{3}} & for\ x > 0 \\ 0 & for\ x \le 0 \end{cases}$$

Find (i) the value of k (ii) the distribution function

(iii) $P(X < 3)$ (iv) $P(5 \le X)$ (v) $P(X \le 4)$

Solution:

(i) Since $f(x)$ is probability density function,

$$\int_{-\infty}^{\infty} f(x)\, dx = 1$$

$$\int_{0}^{\infty} k\, e^{-\frac{x}{3}}\, dx = 1$$

$$k\left[-3e^{-\frac{x}{3}}\right]_0^{\infty} = 1$$

$$3k = 1$$

$$k = \frac{1}{3}$$

(ii) Distribution function, $F(x) = \int_{-\infty}^{x} f(t)\, dt$

When $x \in (-\infty, 0)$

$$F(x) = \int_{-\infty}^{x} f(t)\, dt$$

$$F(x) = 0 \qquad \text{----------(1)}$$

When $x \in (0, \infty)$

$$F(x) = \int_{-\infty}^{0} f(t) dt + \int_{0}^{x} f(t)\, dt$$

$$= 0 + \int_{0}^{x} k\, e^{-\frac{t}{3}}\, dt$$

$$= -3k\left[e^{-\frac{t}{3}}\right]_0^{x}$$

$$= -3\left(\frac{1}{3}\right)\left(e^{-\frac{x}{3}} - 1\right)$$

$$F(x) = 1 - e^{-\frac{x}{3}} \qquad \text{----------(2)}$$

The distribution function is,

$$F(x) = \begin{cases} 1 - e^{-\frac{x}{3}} & for\ x > 0 \\ 0 & for\ x \le 0 \end{cases}$$

(iii) $P(X < 3) = F(3) - F(-\infty)$

$$= 1 - e^{-\frac{3}{3}} - 0$$

$$= 1 - e^{-1}$$

(iv) $P(5 \le X) = P(X \ge 5)$

$$= F(\infty) - F(5)$$

$$= 1 - \left(1 - e^{-\frac{5}{3}}\right)$$

$$= e^{-\frac{5}{3}}$$

(v) $P(X \le 4) = F(4) - F(-\infty)$

$$= 1 - e^{-\frac{4}{3}}$$

5. If X is the random variable with probability density function $f(x)$ given by,

$$f(x) = \begin{cases} x+1 & -1 \le x < 0 \\ -x+1 & 0 \le x < 1 \\ 0 & otherwise \end{cases} \text{ then find,}$$

(i) the distribution function $F(x)$ (ii) $P(-0.5 \le X \le 0.5)$

Solution:

(i) Distribution function, $F(x) = \int_{-\infty}^{x} f(t)\,dt$

When $x \in (-\infty, -1)$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt = 0 \qquad \text{---------(1)}$$

When $x \in (-1, 0)$

$$F(x) = \int_{-\infty}^{-1} f(t)\,dt + \int_{-1}^{x} f(t)\,dt$$

$$= 0 + \int_{-1}^{x} (t+1)\,dt$$

$$= \left[\frac{t^2}{2} + t\right]_{-1}^{x}$$

$$= \frac{x^2}{2} + x - \left(\frac{1}{2} - 1\right)$$

$$F(x) = \frac{x^2}{2} + x + \frac{1}{2} \qquad \text{----------(2)}$$

When $x \in (0, 1)$

$$F(x) = \int_{-\infty}^{-1} f(t)\,dt + \int_{-1}^{0} f(t)\,dt + \int_{0}^{x} f(t)\,dt$$

$$= 0 + \int_{-1}^{0} (t+1)\,dt + \int_{0}^{x} (-t+1)\,dt$$

$$= \left[\frac{t^2}{2} + t\right]_{-1}^{0} + \left[-\frac{t^2}{2} + t\right]_{0}^{x}$$

$$= \left[0 - \left(\frac{1}{2} - 1\right)\right] + \left[\left(-\frac{x^2}{2} + x\right) - 0\right]$$

$$F(x) = \frac{1}{2} - \frac{x^2}{2} + x \qquad \text{----------(3)}$$

When $x \in (1, \infty)$

$$F(x) = \int_{-1}^{0} f(t)\,dt + \int_{0}^{1} f(t)\,dt + \int_{1}^{x} f(t)\,dt$$

$$= \frac{1}{2} + \int_{0}^{1} (-t+1)\,dt + 0$$

$$= \frac{1}{2} + \left[-\frac{t^2}{2} + t\right]_{0}^{1}$$

$$= \frac{1}{2} + \left[\left(-\frac{1}{2} + 1\right) - 0\right]$$

$$= \frac{1}{2} + \frac{1}{2}$$

$$F(x) = 1 \qquad \text{----------(4)}$$

The distribution function is,

$$F(x) = \begin{cases} 0 & x \le -1 \\ \frac{x^2}{2} + x + \frac{1}{2} & -1 \le x < 0 \\ \frac{1}{2} - \frac{x^2}{2} + x & 0 \le x < 1 \\ 1 & 1 \le x \end{cases}$$

(ii) $P(-0.5 \le X \le 0.5) = F(0.5) - F(-0.5)$

$$= \left(\frac{1}{2} - \frac{0.25}{2} + 0.5\right) - \left(\frac{0.25}{2} - 0.5 + \frac{1}{2}\right)$$

$$= -0.25 + 1$$

$$= 0.75$$

6. If X is the random variable with distribution function $F(x)$ given by,

$$F(x) = \begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{2}(x^2 + x) & 0 \le x < 1 \\ 1 & otherwise \end{cases} \text{ then}$$

Find (i) the probability density function $f(x)$

(ii) $P(0.3 \le X \le 0.6)$

Solution:

(i) Probability density function, $f(x) = F'(x)$

$$f(x) = \begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{2}(2x+1) & 0 \le x < 1 \\ 0 & otherwise \end{cases}$$

$$f(x) = \begin{cases} 0 & otherwise \\ \frac{1}{2}(2x+1) & 0 \le x < 1 \end{cases}$$

(ii) $P(0.3 \le X \le 0.6) = F(0.6) - F(0.3)$

$$= \frac{1}{2}[(0.6)^2 + 0.6] - \frac{1}{2}[(0.3)^2 + 0.3]$$

$$= \frac{1}{2}[0.96] - \frac{1}{2}[0.39]$$

$$= \frac{1}{2}(0.96 - 0.39)$$

$$= \frac{1}{2}(0.57)$$

$$= 0.285$$

Exercise 11.4

Mean: $E(X), \mu$

$$\Rightarrow \quad E(X) = \begin{cases} \sum x\, f(x) & X \text{ is discrete} \\ \int_{-\infty}^{\infty} x\, f(x)\,dx & X \text{ is continuous} \end{cases}$$

Variance: $V(X), Var(X), \sigma^2$

$$\Rightarrow \quad V(X) = E(X^2) - \left(E(X)\right)^2$$

1. For the random variable X with the given probability mass function as below, find the mean and variance.

(i) $f(x) = \begin{cases} \frac{1}{10} & x = 2, 5 \\ \frac{1}{5} & x = 0, 1, 3, 4 \end{cases}$

Solution:

The probability mass function is,

x	0	1	2	3	4	5
$f(x)$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$
$x\,f(x)$	0	$\frac{1}{5}$	$\frac{2}{10}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{5}{10}$
$x^2\,f(x)$	0	$\frac{1}{5}$	$\frac{4}{10}$	$\frac{9}{5}$	$\frac{16}{5}$	$\frac{25}{10}$

Mean:

$$E(X) = \sum x\,f(x)$$

$$= 0 + \frac{1}{5} + \frac{2}{10} + \frac{3}{5} + \frac{4}{5} + \frac{5}{10}$$

$$= \frac{2+2+6+8+5}{10}$$

$$E(X) = \frac{23}{10} = 2.3$$

Variance:

$$V(X) = E(X^2) - \left(E(X)\right)^2$$

$$E(X^2) = 0 + \frac{1}{5} + \frac{4}{10} + \frac{9}{5} + \frac{16}{5} + \frac{25}{10}$$

$$= \frac{2+4+18+32+25}{10}$$

$$E(X^2) = \frac{81}{10} = 8.1$$

$$V(X) = 8.1 - (2.3)^2$$

$$V(X) = 8.1 - 5.29 = 2.81$$

(ii) $f(x) = \begin{cases} \frac{4-x}{6} & x = 1, 2, 3 \end{cases}$

Solution:

The probability mass function is,

x	1	2	3
$f(x)$	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$
$x\,f(x)$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{3}{6}$
$x^2 f(x)$	$\frac{3}{6}$	$\frac{8}{6}$	$\frac{9}{6}$

Mean:

$$E(X) = \sum x\,f(x)$$

$$= \frac{3}{6} + \frac{4}{6} + \frac{3}{6}$$

$$E(X) = \frac{10}{6} = 1.67$$

Variance:

$$V(X) = E(X^2) - \left(E(X)\right)^2$$

$$E(X^2) = \frac{3}{6} + \frac{8}{6} + \frac{9}{6}$$

$$E(X^2) = \frac{20}{6}$$

$$V(X) = \frac{20}{6} - \left(\frac{10}{6}\right)^2$$

$$= \frac{20}{6} - \frac{100}{36}$$

$$= \frac{120-100}{36} = \frac{20}{36}$$

$$V(X) = 0.56$$

(iii) $f(x) = \begin{cases} 2(x-1) & 1 < x < 2 \\ 0 & otherwise \end{cases}$

Solution:

Mean:

We know that, $E(X) = \int_{-\infty}^{\infty} x\,f(x)\,dx$

$$E(X) = 2\int_1^2 x\,(x-1)\,dx$$

$$= 2\int_1^2 (x^2 - x)\,dx$$

$$= 2\left[\frac{x^3}{3} - \frac{x^2}{2}\right]_1^2$$

$$= 2\left[\left(\frac{8}{3} - \frac{4}{2}\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right]$$

$$= 2\left[\frac{7}{3} - \frac{3}{2}\right]$$

$$= 2\left[\frac{14-9}{6}\right]$$

$$E(X) = \frac{5}{3}$$

Variance: $V(X) = E(X^2) - \left(E(X)\right)^2$

$$E(X^2) = 2\int_1^2 x^2(x-1)\,dx$$

$$= 2\int_1^2 (x^3 - x^2)\,dx$$

$$= 2\left[\frac{x^4}{4} - \frac{x^3}{3}\right]_1^2$$

$$= 2\left[\left(\frac{16}{4} - \frac{8}{3}\right) - \left(\frac{1}{4} - \frac{1}{3}\right)\right]$$

$$= 2\left[\frac{15}{4} - \frac{7}{3}\right]$$

$$= 2\left[\frac{45-28}{12}\right]$$

$$E(X^2) = \frac{17}{6}$$

$$V(X) = \frac{17}{6} - \left(\frac{5}{3}\right)^2$$

$$= \frac{17}{6} - \frac{25}{9}$$

$$= \frac{153-150}{54} = \frac{3}{54}$$

$$V(X) = \frac{1}{18}$$

Hint: $\int_0^\infty x^n e^{-ax} dx = \frac{n!}{a^{n+1}}$

(iv) $f(x) = \begin{cases} \frac{1}{2} e^{-\frac{x}{2}} & for\ x > 0 \\ 0 & otherwise \end{cases}$

Solution:

Mean: $E(X) = \int_{-\infty}^{\infty} x\, f(x)\, dx$

$$E(X) = \frac{1}{2}\int_0^\infty x e^{\frac{-x}{2}}\, dx$$

$$= \frac{1}{2}\left(\frac{1!}{\left(\frac{1}{2}\right)^2}\right) \qquad \text{(by hint)}$$

$$= \frac{1}{2}\left(\frac{4}{1}\right)$$

$$E(X) = 2$$

Variance: $V(X) = E(X^2) - \left(E(X)\right)^2$

$$E(X^2) = \frac{1}{2}\int_0^\infty x^2 e^{\frac{-x}{2}}\, dx$$

$$= \frac{1}{2}\left(\frac{2!}{\left(\frac{1}{2}\right)^3}\right) \qquad \text{(by hint)}$$

$$= \frac{1}{2}\left(\frac{16}{1}\right)$$

$$E(X^2) = 8$$

$$V(X) = 8 - 2^2 = 4$$

---

2. Two balls are drawn in succession without replacement from an urn containing four red balls and three black balls. Let X be the possible outcomes drawing red balls. Find the probability mass function and mean for X.

Solution:

Total balls $= 7$

$$n(S) = 7C_2 = \frac{7\times 6}{2\times 1} = 21$$

Let X be the random variable denotes the no. of red balls.

X : 0, 1, 2 (red balls)

$$x(0) = 4C_0 \times 3C_2 = \frac{3\times 2}{2\times 1} = 3$$

$$x(1) = 4C_1 \times 3C_1 = 4 \times 3 = 12$$

$$x(2) = 4C_2 \times 3C_0 = \frac{4\times 3}{2\times 1} = 6$$

The probability mass function is,

x	0	1	2
$f(x)$	$\frac{1}{7}$	$\frac{4}{7}$	$\frac{2}{7}$
$x\,f(x)$	0	$\frac{4}{7}$	$\frac{4}{7}$

Mean:

$$E(X) = \sum x\, f(x)$$

$$= 0 + \frac{4}{7} + \frac{4}{7}$$

$$E(X) = \frac{8}{7}$$

Hint: $E(aX + b) = aE(X) + b$

3. If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X+3) = 10$ and $E(X+3)^2 = 116$, find μ and σ^2.

Solution:

Mean:

$$E(X+3) = 10$$

$$E(X) + 3 = 10$$

$$E(X) = 7$$

Variance:

$$E(X+3)^2 = 116$$

$$E(X^2 + 6X + 9) = 116$$

$$E(X^2) + 6E(X) + 9 = 116$$

$$E(X^2) = 116 - 9 - 42$$

$$E(X^2) = 65$$

$$V(X) = 65 - 7^2 = 16$$

Hence, $\mu = 7$ and $\sigma^2 = 16$

---

4. Four fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred.

Solution:

$$n(S) = 2^4 = 16$$

Let x be the random variable denotes the number of heads

X : 0, 1, 2, 3, 4 (head)

$$P(X=0) = \frac{4C_0}{16} = \frac{1}{16}$$

$$P(X=1) = \frac{4C_1}{16} = \frac{4}{16}$$

$$P(X=2) = \frac{4C_2}{16} = \frac{6}{16} \qquad \left(4C_2 = \frac{4\times 3}{2} = 6\right)$$

$$P(X=3)=\frac{4C_3}{16}=\frac{4}{16}$$

$$P(X=4)=\frac{4C_4}{16}=\frac{1}{16}$$

The probability mass function is,

x	0	1	2	3	4
$f(x)$	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$
$x\,f(x)$	0	$\frac{4}{16}$	$\frac{12}{16}$	$\frac{12}{16}$	$\frac{4}{16}$
$x^2 f(x)$	0	$\frac{4}{16}$	$\frac{24}{16}$	$\frac{36}{16}$	$\frac{16}{16}$

Mean:

$$E(X)=\sum x\,f(x)$$

$$=0+\frac{4}{16}+\frac{12}{16}+\frac{12}{16}+\frac{4}{16}$$

$$=\frac{32}{16}$$

$$E(X)=2$$

Variance:

$$E(X^2)=\sum x^2\,f(x)$$

$$=0+\frac{4}{16}+\frac{24}{16}+\frac{36}{16}+\frac{16}{16}$$

$$=\frac{80}{16}$$

$$E(X^2)=5$$

$$V(X)=E(X^2)-\left(E(X)\right)^2$$

$$V(X)=5-2^2=1$$

5. A commuter train arrives punctually at a station every half hour. Each morning, a student leaves his house to the train station. Let X denote the amount of time, in minutes, that the student waits for the train from the time he reaches the train station. It is known that the pdf of X is $f(x)=\begin{cases}\frac{1}{30} & 0<x<30\\ 0 & elsewhere\end{cases}$ obtain and interpret the expected value of the random variable X.

Solution:

Given X be the random variable denotes the waiting time.

$$E(X)=\int_{-\infty}^{\infty} x\,f(x)\,dx$$

$$E(X)=\int_0^{30} x\left(\frac{1}{30}\right)dx$$

$$=\frac{1}{30}\left[\frac{x^2}{2}\right]_0^{30}$$

$$=\frac{1}{30}\left[\frac{900}{2}\right]=\frac{30}{2}$$

$$E(X)=15$$

∴ The expected value of waiting time is 15 minutes.

6. The time to failure in thousands of hours of an electronic equipment used in a manufactured computer has the density function $f(x)=\begin{cases}3e^{-3x} & x>0\\ 0 & elsewhere\end{cases}$ find the expected life of this electronic equipment.

Solution:

$$E(X)=\int_{-\infty}^{\infty} x\,f(x)\,dx$$

$$=\int_0^{\infty} x(3e^{-3x})\,dx$$

$$=3\int_0^{\infty} x\,e^{-3x}\,dx$$

$$=3\left(\frac{1!}{(-3)^2}\right)\qquad\left(\because \int_0^{\infty} x^n e^{-ax}dx=\frac{n!}{a^{n+1}}\right)$$

$$E(X)=\frac{1}{3}$$

7. The probability density function of the random variable X is given by $f(x)=\begin{cases}16xe^{-4x} & for\ x>0\\ 0 & for\ x\le 0\end{cases}$ find the mean and variance of X.

Solution:

Mean:

$$E(X)=\int_0^{\infty} x\,f(x)\,dx$$

$$=\int_0^{\infty} x(16xe^{-4x})\,dx$$

$$=16\int_0^{\infty} x^2e^{-4x}\,dx$$

$$=16\left(\frac{2!}{(-4)^3}\right)=\frac{32}{64}\qquad\left(\because \int_0^{\infty} x^n e^{-ax}dx=\frac{n!}{a^{n+1}}\right)$$

$$E(X)=\frac{1}{2}$$

Variance:

$$E(X^2)=\int_0^{\infty} x^2(16xe^{-4x})\,dx$$

$$=16\int_0^{\infty} x^3e^{-4x}\,dx$$

$$=16\left(\frac{3!}{(-4)^4}\right)\qquad\left(\because \int_0^{\infty} x^n e^{-ax}dx=\frac{n!}{a^{n+1}}\right)$$

$$=16\left(\frac{6}{16\times16}\right)$$

$$E(X^2)=\frac{3}{8}$$

$$V(X)=E(X^2)-\left(E(X)\right)^2$$

$$=\frac{3}{8}-\left(\frac{1}{2}\right)^2$$

$$=\frac{3}{8}-\frac{1}{4}$$

$$V(X)=\frac{1}{8}$$

8. A lottery with 600 tickets gives one prize of ₹200, four prize of ₹100, and six prizes of ₹50. If the ticket cost is ₹2, find the expected profit amount of a ticket.

Solution:

$$n(S) = 600$$

Let X be the random variable denotes the profit amount.

$$X : 200, 100, 50, 0$$

$$P(X = 200) = \frac{1}{600}$$

$$P(X = 100) = \frac{4}{600}$$

$$P(X = 50) = \frac{6}{600}$$

$$P(X = 0) = \frac{589}{600}$$

x	200	100	50	0
$f(x)$	$\frac{1}{600}$	$\frac{4}{600}$	$\frac{6}{600}$	$\frac{589}{600}$
$x\,f(x)$	$\frac{200}{600}$	$\frac{400}{100}$	$\frac{300}{100}$	0

Mean:

$$E(X) = \sum x\,f(x)$$

$$= \frac{200}{600} + \frac{400}{600} + \frac{300}{600} + 0$$

$$= \frac{900}{600} = \frac{3}{2}$$

$$E(X) = 1.50$$

∴ Expected winning amount is ₹ 1.50

Profit $= 1.50 - 2 = -0.50$

∴ Expected loss is ₹ 0.50

Exercise 11.5

Binomial distribution: $\boldsymbol{P(X = x) = f(x)}$

$$\Rightarrow \quad f(x) = \binom{n}{x} p^x q^{n-x}, x = 0, 1 \dots n$$

Mean $\boldsymbol{\mu = np}$ **, Variance** $\boldsymbol{\sigma^2 = npq}$**, where** $\boldsymbol{q = 1 - p}$

1. Compute $P(X = k)$ for the binomial distribution, $B(n, p)$ where

(i) $n = 6, p = \frac{1}{3}, k = 3$

Solution:

Given $n = 6, p = \frac{1}{3}$ and $k = 3$

$$q = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(X = k) = \binom{n}{k} p^k q^{n-k}, k = 0,1,2 \dots n$$

$$P(X = 3) = \binom{6}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3$$

$$= \frac{6\times5\times4}{3\times2\times1}\left(\frac{2^3}{3^6}\right)$$

$$= 20\left(\frac{8}{729}\right)$$

$$P(X = 3) = \frac{160}{729}$$

(ii) $n = 10, p = \frac{1}{5}, k = 4$

Solution:

$$q = 1 - \frac{1}{5} = \frac{4}{5}$$

$$P(X = k) = \binom{n}{k} p^k q^{n-k}, k = 0,1,2 \dots n$$

$$P(X = 4) = \binom{10}{4}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^6$$

$$= \frac{10\times9\times8\times7}{4\times3\times2\times1}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^6$$

$$P(X = 4) = 210\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^6$$

(iii) $n = 9, p = \frac{1}{2}, k = 7$

Solution:

$$q = 1 - \frac{1}{2} = \frac{1}{2}$$

$$P(X = k) = \binom{n}{k} p^k q^{n-k}, k = 0,1,2 \dots n$$

$$P(X = 7) = \binom{9}{7}\left(\frac{1}{2}\right)^7\left(\frac{1}{2}\right)^2$$

2. The probability that Mr. Q hits a target at any trial is $\frac{1}{4}$. Suppose he tries at the target 10 times. Find the probability that he hits the target (i) exactly 4 times (ii) at least one time.

Solution:

Here $n = 10, p = \frac{1}{4}$

$$q = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(X = x) = \binom{n}{x} p^x q^{n-x}, x = 0,1,2 \dots n$$

$$P(X = x) = \binom{10}{x}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^{10-x}, x = 0,1,2 \dots 10$$

(i) exactly 4 times

$$P(X = 4) = \binom{10}{4}\left(\frac{1}{4}\right)^4\left(\frac{3}{4}\right)^6$$

(ii) at least one time

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - \binom{10}{0}\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{10}$$

$$P(X \geq 1) = 1 - \left(\frac{3}{4}\right)^{10}$$

3. Using binomial distribution find the mean and variance of X for the following experiments

(i) A fair coin is tossed 100 times, and X denote the number of heads

Solution:

Here $n = 100$

$$p = p(\text{probability of getting head})$$

$$p = \frac{1}{2}, q = 1 - \frac{1}{2} = \frac{1}{2}$$

Mean: $np = 100\left(\frac{1}{2}\right) = 50$

Variance: $npq = 100\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 25$

(ii) A fair die is rolled 240 times, and X denote the numbr of times that four appeared.

Solution:

Here $n = 240$

$$p = p(\text{probability of getting 4})$$

$$p = \frac{1}{6}, q = 1 - \frac{1}{6} = \frac{5}{6}$$

Mean: $np = 240\left(\frac{1}{6}\right) = 40$

Variance: $npq = 240\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) = \frac{200}{6} = \frac{100}{3}$

4. The probability that a certain kind of component will survive a electrical test is $\frac{3}{4}$. Find the probability that exactly 3 of 5 components tested survive.

Solution:

Here $n = 5, p = \frac{3}{4}$

$$q = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P(X = x) = \binom{n}{x} p^x q^{n-x}, x = 0,1,2 \dots n$$

$$P(X = x) = \binom{5}{x}\left(\frac{3}{4}\right)^x\left(\frac{1}{4}\right)^{5-x}, x = 0,1,2,\dots 5$$

$$P(X = 3) = \binom{5}{3}\left(\frac{3}{4}\right)^3\left(\frac{1}{4}\right)^2$$

$$= \frac{5\times4\times3}{3\times2\times1}\left(\frac{3^3}{4^5}\right) = 10\left(\frac{27}{1024}\right)$$

$$P(X = 3) = \frac{270}{1024}$$

5. A retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 5%. The inspector of the retailer randomly pics 10 items from a shipment. What is the probability that there will be (i) at least one defective item (ii) exactly two defective items?

Solution:

Here $n = 10, p = \frac{5}{100} = 0.05$

$$q = 1 - 0.05 = 0.95$$

$$P(X = x) = \binom{n}{x} p^x q^{n-x}, x = 0,1,2 \dots n$$

$$P(X = x) = \binom{10}{x}(0.05)^x(0.95)^{10-x}, x = 0,1,2 \dots 10$$

(i) at least one defective item

$$P(X \geq 1) = 1 - P(X < 1)$$

$$= 1 - P(X = 0)$$

$$= 1 - \binom{10}{0}(0.05)^0(0.95)^{10}$$

$$P(X \geq 1) = 1 - (0.95)^{10}$$

(ii) exactly two items

$$P(X = 2) = \binom{10}{2}(0.05)^2(0.95)^8$$

6. If the probability that a fluorescent light has a useful life of at least 600 hours is 0.9, find the probabilities that among 12 such lights

(i) exactly 10 will have a useful life of at least 600 hours;

(ii) at least 11 will have a useful life of at least 600 hours;

(iii) at least 2 will not have a useful life of at least 600 hours.

Solution:

Here $n = 12, p = 0.9$

$$q = 1 - 0.9 = 0.1$$

$$P(X = x) = \binom{n}{x} p^x q^{n-x}, x = 0,1,2 \dots n$$

$$P(X = x) = \binom{12}{x}(0.9)^x(0.1)^{12-x}, x = 0,1,2 \dots 12$$

(i) exactly 10 will have a useful life of at least 600 hours

$$P(X = 10) = \binom{12}{10}(0.9)^{10}(0.1)^2$$

(ii) at least 11 will have a useful life of at least 600 hours

$$P(X \geq 11) = P(X = 11) + P(X = 12)$$

$$= \binom{12}{11}(0.9)^{11}(0.1)^1 + \binom{12}{12}(0.9)^{12}(0.1)^0$$

$= 12(0.9)^{11}(0.1) + (0.9)^{12}$

$= (0.9)^{11}[12 \times 0.1 + 0.9]$

$= (0.9)^{11}[2.1]$

$P(X \geq 11) = 2.1(0.9)^{11}$ ---------(1)

(iii) at least 2 will not have useful life of at least 600 hour

$= P(\text{at most 10 will have a useful life of 600 hours})$

$= P(X \leq 10)$

$= 1 - P(X > 10)$

$= 1 - [P(X = 11) + P(X = 12)]$

$= 1 - [2.1(0.9)^{11}]$ From (1)

Hint: Standard deviation $\sigma = \sqrt{V(x)}$

7. The mean and the standard deviation of a binomial variate X are respectively 6 and 2. Find

(i) the probability mass function

(ii) $P(X = 3)$ (iii) $P(X \geq 2)$

Solution:

Mean $np = 6$ ----------(1)

Standard deviation $\sigma = 2$

Variance $npq = 4$ ----------(2)

Divide (2) by (1) we get,

$\frac{npq}{np} = \frac{4}{6}$

$q = \frac{2}{3}$

$p = 1 - \frac{2}{3} = \frac{1}{3}$

From (1), $\frac{n}{3} = 6$

$n = 18$ ----------(3)

(i) Probability mass function

$$P(X = x) = \binom{n}{x} p^x q^{n-x}, x = 0,1,2 \ldots n$$

$$P(X = x) = \binom{18}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{18-x}, x = 0,1, \ldots 18$$

(ii) $P(X = 3) = \binom{18}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^{15}$

(iii) $P(X \geq 2) = 1 - P(X < 2)$

$= 1 - [P(X = 0) + P(X = 1)]$

$= 1 - \left[\binom{18}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{18} + \binom{18}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{17}\right]$

$= 1 - \left[\left(\frac{2}{3}\right)^{18} + 18\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^{17}\right]$

$= 1 - \left(\frac{2}{3}\right)^{17}\left[\frac{2}{3} + \frac{18}{3}\right]$

$P(X \geq 2) = 1 - \left(\frac{2}{3}\right)^{17}\left(\frac{20}{3}\right)$

8. If $X \sim B(n, p)$ such that $4P(X = 4) = P(X = 2)$ and $n = 6$. Find the distribution, mean and standard deviation of X.

Solution:

Here $n = 6$

$$P(X = x) = \binom{n}{x} p^x q^{n-x}, x = 0,1,2 \ldots n$$

$4P(X = 4) = P(X = 2)$

$4\left[\binom{6}{4} p^4 q^2\right] = \binom{6}{2} p^2 q^4$

$4\binom{6}{2} p^2 = \binom{6}{2} q^2$ $\left[\because \binom{6}{4} = \binom{6}{2}\right]$

$4p^2 = q^2$

$2p = q$ ----------(1)

$p = 1 - 2p$ $(\because p = 1 - q)$

$3p = 1$

$p = \frac{1}{3}, q = 1 - \frac{1}{3} = \frac{2}{3}$

The distribution is,

$$P(X = x) = \binom{6}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{6-x}, x = 0,1,2, \ldots 6$$

Mean $np = 6\left(\frac{1}{3}\right) = 2$

Variance $npq = 6\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{4}{3}$

Standard deviation $= \frac{2}{\sqrt{3}}$

9. In a binomial distribution consisting of 5 independents trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the mean and variance of the random variable.

Solution:

Here $n = 5$

$P(X = 1) = 0.4096$

$\binom{5}{1} p^1 q^4 = 0.4096$

$5pq^4 = 0.4096$ ----------(1)

$P(X = 2) = 0.2048$

$\binom{5}{2} p^2 q^3 = 0.2048$

$10p^2q^3 = 0.2048$ ----------(2)

(2) ÷ (1) we get,

$$\frac{10p^2q^3}{5pq^4} = \frac{0.2048}{0.4096}$$

$$\frac{2p}{q} = \frac{1}{2}$$

$$q = 4p$$

$$p = 1 - 4p$$

$$5p = 1$$

$$p = \frac{1}{5}, q = 1 - \frac{1}{5} = \frac{4}{5}$$

Mean $np = 5\left(\frac{1}{5}\right) = 1$

Variance $npq = 5\left(\frac{1}{5}\right)\left(\frac{4}{5}\right) = \frac{4}{5}$