E 25/01/2023

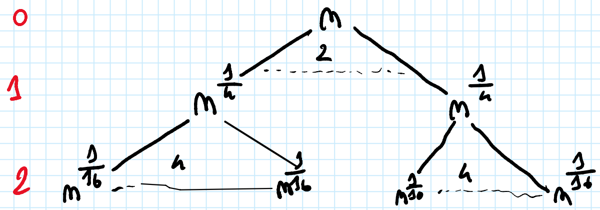
martedì 7 marzo 2023 16:36

$$T(n) = \begin{cases} 1, & \text{se } n \le 2; \\ 2 \cdot T(\sqrt[4]{n}) + \log(2n), & \text{altrimenti.} \end{cases}$$

LIVELLO		CONTRIBUTI SINGOLI	NUM RAMI	CONTRIBUTO TOTALE
0	$T(n) = 2 \cdot T(n^{\frac{1}{4}}) + \log_2(2n)$	$\log(2n)$ $= 1 + \log(n)$	1	$\log(2n) =$ $= 1 + \log(n)$
1	$T(n^{\frac{1}{4}}) = 2 \cdot T(n^{\frac{1}{16}}) + \log(2\, n^{\frac{1}{4}})$	$\log(2\, n^{\frac{1}{4}})$ $= 2 + \frac{1}{4} \log(n)$	2	$2 \cdot \log(2\, n^{\frac{1}{4}})$ $= 2 \cdot (\log(2) +$ $\log(n^{\frac{1}{4}}))$ $=$ $= 2 + \frac{1}{2} \log(n)$
2	$T(n^{\frac{1}{16}}) = 2 \cdot T(n^{\frac{1}{64}}) + \log(2\, n^{\frac{1}{16}})$	$\log(2\, n^{\frac{1}{16}})$ $= 1 + \frac{1}{16} \log(n)$	$2 \cdot 2 = 4$	$4 \cdot (1 + \frac{1}{16} \log(n))$ $= 4 + \frac{1}{4} \log(n)$

POTREI ANDARE AVANTI, MA NON CREDO SIA NECESSARIO, IL CONTRIBUTO PER LIVELLO È:

$$2^i + \frac{1}{2^i} \log(n)$$



MENTRE L'ALTEZZA DELL'ALBERO SEGUE LA REGOLA

$$n^{\left(\frac{1}{2^2}\right)^i}$$

RICAVIAMOCI L'ALTEZZA DELL'ALBERO $\boxed{H}$ SEGUENDO IL CASO PEGGIORE

$$n^{\frac{1}{2^{2h}}} = 2 \Rightarrow \log_2\left(n^{\left(\frac{1}{2^2}\right)^h}\right) = \log_2 2 \Rightarrow$$

$$\left(\frac{1}{2^2}\right)^h \log_2(n) = 1 \Rightarrow \frac{1}{2^{2h}} \log_2(n) = 1 \Rightarrow \log_2(n) = 2^{2h}$$

$$\log_2(\log_2(n)) = \log_2(2^{2h}) \Rightarrow \log_2(\log_2(n)) = 2h \Rightarrow h = \frac{\log_2(\log_2(n))}{2}$$

CALCOLO DELLA SOMMATORIA

$$\sum_{i=0}^{h} 2^i + \sum_{i=0}^{h} \frac{1}{2^i} \log(n) =$$

$$\sum_{i=0}^{h} 2^i = 2 > 1$$ QUINDI SERIE GEOMETRICA COMPLETA $$\Rightarrow \frac{2^{h+1} - 1}{2 - 1} = 2^{h+1} - 1$$

$$\Rightarrow 2^{\frac{\log_2(\log_2(n))}{2} + 1} - 1$$

$\log(n) \sum_{i=0}^{h} \frac{1}{2^n} = \frac{1}{2^h}$ E' COMPRESO TRA 0 E 1 $\Rightarrow \frac{1}{1 - \frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2 \log_2(n)$

$$\boxed{\begin{array}{c}\text{SFRUTTO LA PROPRIETA'} \\ n = a^{\log_a(n)}\end{array}}$$

$$2^{\frac{\log_2(\log_2(n))}{2} + 1} - 1 + 2 \log_2(n)$$

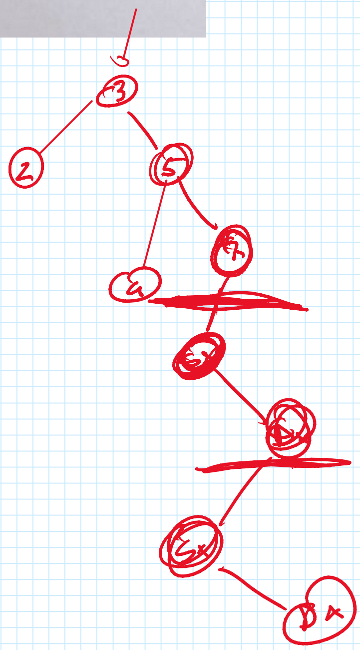
$$\sqrt{2^{\log_2(\log_2(n))}} \cdot 2 - 1 + 2 \log_2(n) \Rightarrow \sqrt{\log_2(n)} \cdot 2 - 1 + 2 \log_2(n)$$

$$\Theta(\log_2(n))$$

2

2. Si scriva un **algoritmo iterativo** che **simuli precisamente** l'algoritmo ricorsivo di seguito riportato, dove Z è una funzione esterna non meglio specificata.

```
function Algoritmo(T, h)
  if T = Nil then
    return Z(0, h)
  else
    a ← 0
    if T→key ≡ 0 mod 2 then
      a ← a + Algoritmo(T→dx, 2 · h)
    if T→key ≡ 1 mod 3 then
      a ← a − Algoritmo(T→sx, 3 · h)
    return Z(T→key, a)
```




```
ALGO_IT(T,h)

    st_t = st_h = st_a = NULL
    cH = h
    cT = T
    start = true
    last = NULL


    while(start = true OR st_t != NULL)

        if(start = true)
            if(cT = NIL)
                last = cT
                start = false
                ret = Z(0,cH)

            else

                st_t = push(st_t, cT)
                st_h = push(st_h, cH)

                if(cT->key = 0 mod 2)
                    cT = cT->dx
                    cH = 2 * cH
                else
                    start = false
                    ret = 0


        else
            cT = top(st_t)
            cH = top(st_h)

            if(cT->sx != last) // TORNO DA PRIMA CHIAMATA
                a = ret

                if(cT->key = 1 mod 3)
                    cT = cT->sx
                    cH = 3 * cH
                    start = true
                    st_a = push(st_a, a)
                else
                    st_t = pop(st_t)
                    st_h = pop(st_h)
                    start = false // RIPETITIVO
                    ret = Z(cT->key, a)
                    last = cT

            else // TORNO DA SECONDA CHIAMATA

                a = top(st_a)
                a <- a - ret
                start = false // RIPETITIVO
```

```
ret = Z(cT->key, a)
st_t = pop(st_t)
st_h = pop(st_h)
st_a = pop(st_a)
last = cT

return ret
```

if $j - i >= 1$ **then**

$y = Rand()\%2$

if $(x = 1)$ **then**

IF RET% 2-1 =0

$ret =$ ALGORITMO$(A, \frac{(i+j)}{2} + 1, j, y)$

if $ret\%2 = 0$ **then**

$ret =$ ALGORITMO$(A, i, \frac{(i+j)}{2}, 1 - y)$

else

$ret =$ ALGORITMO$(A, i, \frac{(i+j)}{2}, y)$

if $ret\%2 = 1$ **then**

$ret = ret +$ ALGORITMO$(A, \frac{(i+j)}{2} + 1, j, 1 - y)$

else

$ret = A[i]$

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$$T(n) = \begin{cases} 1, & \text{se } n \le 27; \\ 3n^2 \cdot T(\sqrt[3]{n}) + 2n^3, & \text{altrimenti.} \end{cases}$$

LIVELLO		CONTRIBUTI SINGOLI	NUM RAMI	CONTRIBUTO TOTALE
0	$T(n) =$ $3n^2 \cdot T(\sqrt[3]{n}) + 2n^3,$ $T(n^{\frac{1}{3}})$	$2n^3$	1	$2n^3$
1	$T(n^{\frac{1}{3}}) = 3 \cdot (n^{\frac{1}{3}})^2 \cdot T((n^{\frac{1}{3}})^{\frac{1}{3}}) + 2 \cdot (n^{\frac{1}{3}})^3 =$ $= 3n^{\frac{2}{3}} \cdot T(n^{\frac{1}{9}}) + 2n$	$2n$	$3n^2$	$2n \cdot 3n^2$ $6n^3$
2	$T(n^{\frac{1}{9}}) = 3 \cdot n^{\frac{2}{9}} \cdot T((n^{\frac{1}{9}})^{\frac{1}{3}}) + 2 \cdot (n^{\frac{1}{9}})^3 =$ $3n^{\frac{2}{9}} \cdot T(n^{\frac{1}{27}}) + 2 \cdot (n^{\frac{1}{3}})$	$2n^{\frac{1}{3}}$	$3n^{\frac{2}{3}} \cdot 3n^2$ $= 9n^{\frac{8}{3}}$	$2n^{\frac{1}{3}} \cdot 9n^{\frac{8}{3}} =$ $18n^3$

CASO PEGGIORE

CONTRIBUTO GENERALE SOMMATORIA

$$2n^{\frac{3}{3^h}}$$

$$n^{\frac{1}{3^i}} \le 27 \qquad n^{\frac{1}{3^i}} = 27 \Rightarrow$$

$$\log_3 n^{\frac{1}{3^h}} = \log_3 27 \Rightarrow \frac{1}{3^h} \cdot \log_3(n) = 3 \Rightarrow$$

$$\log_3(n) = 3 \cdot 3^h \Rightarrow \log_3(n) = 3^{h+1} \Rightarrow$$

$$\Rightarrow h+1 = \log_3 \log_3(n) \Rightarrow h = \log_3(\log_3(n)) - 1$$

SOLUZIONE PERFETTA X MOGAVERO

$$n^{1/3^i} = 27$$

$$\log_{27} n^{1/3^i} = 1$$

$$\frac{1}{3^i} \cdot \log_{27}(n) = 1$$

$$\log_{27}(n) = 3^i \Rightarrow i = \log_3 \log_{27}(n)$$

$$T(n) = \sum_{i=0}^{h} 3^i \cdot 2n^3 = 2n^3 \sum_{i=0}^{h} 3^i = 2n^3 \cdot \left(\frac{3^{h+1} - 1}{3-1}\right)$$

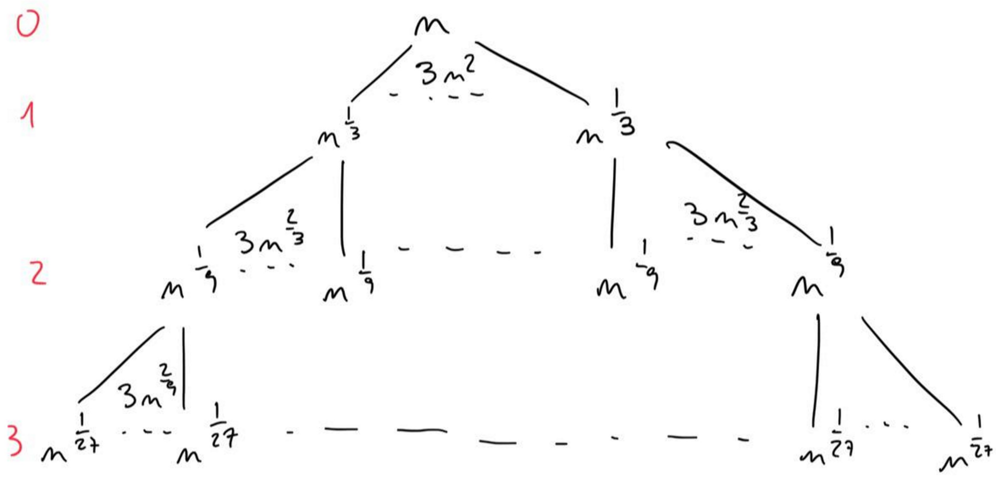
$$= \cancel{2} n^3 \cdot \frac{3^{h+1} - 1}{\cancel{2}} = n^3 \cdot \left(3^{\left(\log_3(\log_3(n)) \cancel{-1} \cancel{+1}\right)} - 1\right) =$$

$$n^3 \, 3^{\log_3(\log_3(n))} - n^3$$

$$\Theta\left(n^3 \cdot \log_3(n)\right)$$

SOLUZIONE LUCA

$$T(n) = \begin{cases} 1 & n \le 27 \\ 3n^2 \cdot T(\sqrt[3]{n}) + 2n^3 & \text{altrimenti} \end{cases}$$



NUMERO NODI	LIVELLO
1	0
$3n^2$	1
$3n^2 \cdot 3n^{\frac{2}{3}}$	2
$3n^2 \cdot 3n^{\frac{2}{3}} \cdot 3n^{\frac{2}{9}}$	3

LIV.		CONTRIBUTO SINGOLO	CONTRIBUTO TOTALE
0	$3n^2 \cdot T(n^{\frac{1}{3}}) + 2n^3$	$2n^3$	$2n^3$
1	$3n^2\left(T(n^{\frac{1}{3}})\right) =$ $= 3n^2\left(3n^{\frac{2}{3}} \cdot T(n^{\frac{1}{9}}) + 2n^{\frac{3}{3}}\right)$	$2n^{\cancel{\frac{3}{3}}}$	$3n^2 \cdot 2n =$ $= 6n^3$
2	$3n^{\frac{2}{3}} \cdot 3n^2 \cdot \left(T(n^{\frac{1}{9}})\right) =$ $= \left(3n^{\frac{2}{9}} \cdot T(n^{\frac{1}{27}}) + 2n^{\cancel{\frac{3}{9}}\frac{1}{3}}\right)$ $\cdot 3n^{\frac{2}{3}} \cdot 3n^2$	$2n^{\cancel{\frac{3}{9}}\frac{1}{3}}$	$3n^2 \cdot 3n^{\frac{2}{3}} \cdot$ $2n^{\frac{1}{3}} =$ $= 18n^3$

POTREI ANDARE AVANTI, MA NON CREDO SIA NECESSARIO, IL CONTRIBUTO PER LIVELLO È:

$$3^i \cdot 2 \cdot n^3$$

MENTRE L'ALTEZZA DELL'ALBERO SEGUE LA REGOLA

$$n^{\left(\frac{1}{3}\right)^i}$$

E POSSO RICAVARE LA i COSì:

$$n^{\left(\frac{1}{3}\right)^i} = 27 \Rightarrow \log_{27} n^{\left(\frac{1}{3}\right)^i} = \cancel{\log_{27} 27}^{\,1}$$

$$\left(\frac{1}{3}\right)^i \log_{27} n = 1$$

$$\log_{27} n = 3^i$$

$$\log_3 \log_{27} n = i$$

OPPURE

$$n^{\left(\frac{1}{3}\right)^i} = 27 \Rightarrow \log_3 n^{\left(\frac{1}{3}\right)^i} = \cancel{\log_3 27}^{\,3}$$

$$\left(\frac{1}{3}\right)^i \log_3 n = 3 \Rightarrow \log_3 n = 3 \cdot 3^i$$

$$\log_3 n = 3^{i+1} \Rightarrow \log_3(\log_3 n) = \log_3\left(3^{i+1}\right)$$

$$\log_3(\log_3 n) = i+1 \left(\cancel{\log_3 3}^{\,1}\right)$$

$$i = \log_3(\log_3 n) - 1$$

CALCOLO DELLA SOMMATORIA

$$\sum_{i=0}^{h} 2 \cdot 3^i \cdot n^3 \Rightarrow 2n^3 \sum_{i=0}^{h} 3^i$$

$3>1$, USO LA FORMULA COMPLETA

$$2n^3 \left(\frac{3^{h+1} - 1}{3-1}\right) = \cancel{2} n^3 \cdot \frac{1}{\cancel{2}} \cdot \left(3^{h+1} - 1\right) =$$

$$= n^3 \cdot \left(3^{h+1} - 1\right)$$

SOSTITUISCO h COL RICAVATO DI PRIMA

$$= n^3 \cdot \left(3^{\log_3(\log_3 n) \cancel{-1+1}} - 1\right) =$$

$$= n^3 \cdot 3^{\log_3(\log_3 n)} - n^3 =$$

$$= n^3 \cdot 3^{\log_3(\log_3 n)} - n^3 =$$

$$= n^3 \cdot (\log_3 n)^{\cancel{\log_3 3}} - n^3 \qquad \boxed{a^{(\log_a (x))} = x}$$

$$\Theta\left(n^3 \log_3(n)\right)$$

2

2. Si scriva un **algoritmo iterativo** che **simuli precisamente** l'algoritmo ricorsivo di seguito riportato, dove Z_l e Z_r sono due funzioni esterne non meglio specificate che soddisfano la seguente proprietà: $p < Z_l(A, p, s) < Z_r(A, p, s) \leq s$, quando $p + 1 < s$.

```
function Algoritmo(A, p, s)
  if s ≤ p + 1 then
    return 0
  else
    q ← Z_l(A, p, s)
    r ← Z_r(A, p, s)
    a ← Algoritmo(A, p, q)
    a ← a − Algoritmo(A, q, r)
    a ← a + Algoritmo(A, r, s)
    return a + (r − q)
```

IF
C1
ELSE IF
C2
ELSE
C3

```
ALGO_IT(A,p,s)

    Cs = s
    Cp = p
    Start = true
    Last = NIL
    Ret = 0
    St_p = st_s = st_a = NIL


    While(start OR st_p != NIL) DO

        If (start) then

            If(cs <= cp + 1) THEN
                Ret = 0
                Start = false
                Last = cp

            Else
                // NUOVA CHIAMATA
                /* SIMULO FINO ALLA 1 CHIAMATA */

                q <- Zl(A,cp,cs)
                r <- Zl(A,cp,cs)
                St_p = push(st_p, cp)
                St_s = push(st_s, cs)



        else   // VECCHIA CHIAMATA
            /* RECUPERO CONTESTO */
```

```
                    Cp = top(st_p)
                    Cs = top(st_s)
                    q <- Zl(A,cp,cs)
                    r <- Zl(A,cp,cs)


                    If(last = cp) THEN //torno 1 CHIAMATA DA CHIAMARE LA 2

                        a <- ret
                        push(st_a, a)
                        Cp = q
                        Cs = r
                        Start = true

                    else // 2 CHIAMATA

                        If(last = q) THEN   // torno dalla 2 CHIAMATA da CHIAMARE LA 3
                            a = top(st_a)
                            A <- a - ret
                            St_a = pop(st_a)
                            Push(st_a, a)
                            Cp = r
                            Cs = cs
                            Start = true



                        else    // TORNO DALLA 3 CHIAMATA ///// LAST = R

                            a = top(st_a)
                            a <- a + ret
                            St_a = pop(st_a)
                            Push(st_a, a)

                            Ret = a + (r - q)
                            St_p = pop(st_p)
                            St_s = pop(st_s)
                            Last = cp

                            Start = false



        Return ret

    }
```

```
AlgoritmoIter(A, p, s)
    cp = p
    cs = s
    st_p = st_s = st_a = NIL
    last = NIL
    start = true
    WHILE start || st_p != NIL DO
        IF start THEN
            IF cp + 1 > cs THEN
                q <- Z_l(A, cp, cs)
                r <- Z_r(A, cp, cs)
                st_p = push(st_p, cp)
                st_s = push(st_s, cs)
                cs = q
                /*start = true*/
            ELSE /*caso base cs <= cp + 1*/
                ret = 0
                last = cp
                start = false
        ELSE /*ripresa del contesto*/
            cp = top(st_p)
            cs = top(st_s)
            q <- Z_l(A, cp, cs)
            r <- Z_r(A, cp, cs)
            IF last = cp THEN
                a <- ret
                cp = q
                cs = r
                start = true
            ELSE IF last = q THEN
                a <- a - ret
                cp = r
                /*cs = cs*/
                start = true
            ELSE /* last = r*/
                a <- a + ret
                ret = a + (r - q)
                st_p = pop(st_p)
                st_s = pop(st_s)
                last = cp
                /*start = false*/
    RETURN ret
```

a <- ret
push(st_a, a)

a = top(st_a)
a <- a - ret
st_a = pop(st_a)
push(st_a, a)

a = top(st_a)
a <- a + ret
St_a = pop(st_a)

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1. Si risolva la seguente equazione di ricorrenza, calcolandone l'**andamento asintotico**:

$$T(n) = \begin{cases} 1, & \text{se } n \le 27; \\ 3n^2 \cdot T(\sqrt[3]{n}) + 2n^3, & \text{altrimenti.} \end{cases}$$

2. Si scriva un **algoritmo iterativo** che **simuli precisamente** l'algoritmo ricorsivo di seguito riportato, dove Z_l e Z_r sono due funzioni esterne non meglio specificate che soddisfano la seguente proprietà: $p < Z_l(A,p,s) < Z_r(A,p,s) \le s$ quando $p+1 < s$.

function Algoritmo(A, p, s)
- **if** $s \le p + 1$ **then**
  - **return** 0
- **else**
  - $q \leftarrow Z_l(A, p, s)$
  - $r \leftarrow Z_r(A, p, s)$
  - $a \leftarrow$ Algoritmo(A, p, q)
  - $a \leftarrow a -$ Algoritmo(A, q, r)
  - $a \leftarrow a +$ Algoritmo(A, r, s)
  - **return** $a + (r - q)$

ALGO_IT(A,p,s)

While(){

If(){ // NUOVA CHIAMATA
/* SIMULO FINO ALLA 1 CHIAMATA */

}else{ // VECCHIA CHIAMATA
/* RECUPERO CONTESTO */

If() { //torno 1 CHIAMATA con 2 CHIAMATA != da NULL

}else{ // 2 CHIAMATA

If(){ // torno dalla 1 CHIAMATA con 2 CHIAMATA = NULL

}else{ //TORNO DALLA 2 CHIAMATA
If(){ //torno dalla 2 CHIAMATA con 3 CHIAMATA != NULL

}else{ //TORNO dalla 3 CHIAMATA

}

}

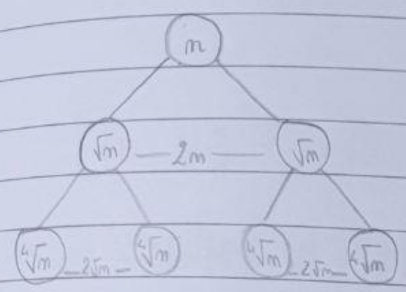
```
    }

    Return

}
```

EQUAZIONE 1

$$T(n) = \begin{cases} 1 & \text{se } n \leq 2 \\ 2n\,T(\sqrt{n}) + n^2 & \text{se } n > 2 \end{cases}$$



LIV.	ISTANZA	N° NODI	CONTR. NODO	CONTR. LIV.
0	n	1	n^2	n^2
1	$\sqrt{n}$	$2n$	n	$2n^2$
2	$\sqrt[4]{n}$	$2n \cdot 2\sqrt{n}$	$\sqrt{n}$	$2n \cdot 2\sqrt{n} \cdot \sqrt{n} = 4n^2$
i	$\sqrt[2^i]{n}$	$2n \cdot 2\sqrt{n} \cdot 2\sqrt[2^i]{n}$	$\left(\sqrt[2^i]{n}\right)^2$	$2^i n^2$

Calcolo $h := \sqrt[2^i]{n} = 2 \iff n^{\frac{1}{2^i}} = 2 \Rightarrow \frac{1}{2^i} = \log_n 2 \iff 2^i = \frac{1}{\log_n 2} \Rightarrow i = \log_2(\log_2 n)$

Calcoliamo la sommatoria

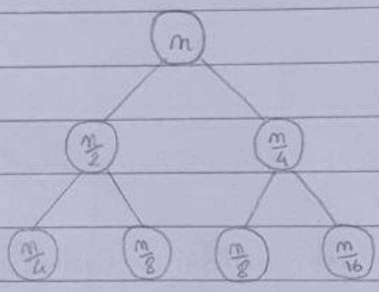
$T(n) = \sum_{i=0}^{h} 2^i \cdot n^2 = \sum_{i=0}^{\log_2(\log_2 n)} 2^i \cdot n^2 \Rightarrow n^2 \cdot \sum_{i=0}^{\log_2(\log_2 n)} 2^i$ serie geometrica con ragione > 1

$= n^2 \cdot \left(\frac{2^{\log_2 \log_2 n + 1} - 1}{2 - 1} \right) = n^2 \left(2^{\log_2 \log_2 n} \cdot 2 - 1 \right) = n^2 \cdot (2 \log_2 n - 1) = 2n^2 \cdot \log_2 n - n^2$

$= \Theta(n^2 \log_2 n)$

EQUAZIONE 2

$$T(n) = \begin{cases} 1 & \text{se } n \leq 1 \\ T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + n^2 & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO
0	n^2
1	$\left(\frac{n}{2}\right)^2 + \left(\frac{n}{4}\right)^2 = \frac{5}{16} n^2$
2	$\left(\frac{n}{4}\right)^2 + \left(\frac{n}{8}\right)^2 + \left(\frac{n}{8}\right)^2 + \left(\frac{n}{16}\right)^2 = \frac{25}{256} n^2$
i	$\left(\frac{5}{16}\right)^i n^2$

Calcolo $h_1 := \frac{n}{2^i} = 1 \iff 2^i = n \Rightarrow i = \log_2 n$

Calcolo $h_2 := \frac{n}{4^i} = 1 \iff 4^i = n \Rightarrow i = \log_4 n$

Calcolo la sommatoria:

$$T_{h_1}(n) = \sum_{i=0}^{h_1} \left(\frac{5}{16}\right)^i \cdot n^2 = \sum_{i=0}^{\log_2 n} \left(\frac{5}{16}\right)^i \cdot n^2 = n^2 \sum_{i=0}^{\log_2 n} \left(\frac{5}{16}\right)^i$$ serie geometrica con ragione < 1

$$= n^2 \cdot \frac{1}{1 - \frac{5}{16}} = n^2 \cdot \frac{1}{\frac{11}{16}} = 4n^2 = \Theta(n^2)$$

Calcolo la sommatoria:

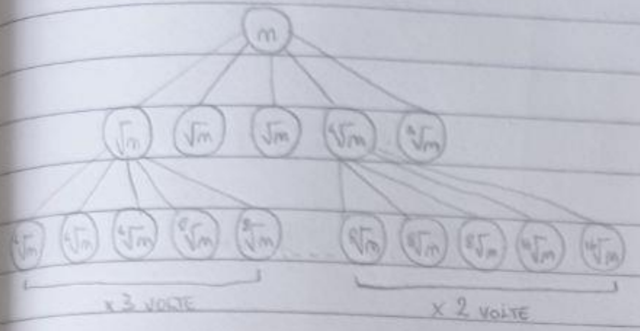
$$T_{h_2}(n) = \sum_{i=0}^{h_2} \left(\frac{5}{16}\right)^i \cdot n^2 = \sum_{i=0}^{\log_4 n} \left(\frac{5}{16}\right)^i \cdot n^2 = n^2 \cdot \sum_{i=0}^{\log_4 n} \left(\frac{5}{16}\right)^i$$ serie geometrica con ragione < 1

$$= n^2 \cdot \frac{1}{1 - \frac{5}{16}} = n^2 \cdot \frac{1}{\frac{11}{16}} = 4n^2 = \Theta(n^2)$$

Sapendo che $T_{h_1}(n) \leq T(n) \leq T_{h_2}(n) \Rightarrow \Theta(n^2) \leq T(n) \leq \Theta(n^2) \Rightarrow T(n) = \Theta(n^2)$

EQUAZIONE 3

$$T(m) = \begin{cases} 1 & \text{se } m \leq 2 \\ 3T(\sqrt{m}) + 2T(\sqrt[4]{m}) + \log m & \text{se } m > 2 \end{cases}$$



LIVELLO	CONTRIBUTO
0	$\log m$
1	$3\log m^{\frac{1}{2}} + 2\log m^{\frac{1}{4}} = \left(\frac{3}{2} + \frac{1}{2}\right)\log m = 2\log m$
2	$3\left(3\log m^{\frac{1}{4}} + 2\log m^{\frac{1}{8}}\right) + 2\left(3\log m^{\frac{1}{8}} + 2\log m^{\frac{1}{16}}\right) = \left(\frac{9}{4} + \frac{3}{4}\right)\log m + \left(\frac{3}{4} + \frac{1}{4}\right)\log m = 3\log m + \log m = 4\log m$
i	$2^i \log m$

Calcolo $h_1 := \sqrt[2^i]{m} = 2 \iff m^{\frac{1}{2^i}} = 2 \iff \frac{1}{2^i} = \log_m 2 \iff 2^i = \log_2 m \Rightarrow i = \log_2(\log_2 m)$

Calcolo $h_2 := \sqrt[4^i]{m} = 2 \iff m^{\frac{1}{4^i}} = 2 \iff \frac{1}{4^i} = \log_m 2 \iff i = \log_4(\log_2 m) = \frac{\log_2(\log_2 m)}{2}$

Calcolo la sommatoria

$$T_{h_1}(m) = \sum_{i=0}^{h_1} 2^i \cdot \log m = \sum_{i=0}^{\log_2(\log_2 m)} 2^i \cdot \log m = \log m \cdot \sum_{i=0}^{\log_2 \log_2 m} 2^i$$ serie geometrica con ragione > 1

$$= \log m \cdot \left(\frac{2^{\log_2(\log_2 m)+1} - 1}{2 - 1}\right) = \log m \left(2^{\log_2(\log_2 m)} \cdot 2 - 1\right) = \left(2\log_2 m - 1\right) \cdot \log m = 2\log^2 m - \log m = \Theta(\log^2 m)$$

Calcolo la sommatoria:

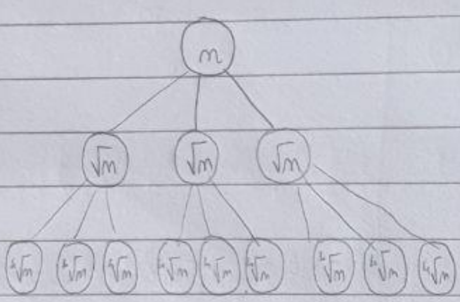
$$T_{h_2}(m) = \sum_{i=0}^{h_2} 2^i \cdot \log m = \sum_{i=0}^{\log_4(\log_2 m)} 2^i \cdot \log m = \log m \cdot \sum_{i=0}^{\log_4(\log_2 m)} 2^i$$ serie geometrica con ragione > 1

$$= \log m \cdot \left(\frac{2^{\log_4(\log_2 m)+1} - 1}{2 - 1}\right) = \log m \cdot \left(2^{\frac{\log_2(\log_2 m)}{2}} \cdot 2 - 1\right) =$$

$$= \log m \left[2(\log_2 m)^{\frac{1}{2}} - 1\right] = 2(\log_2 m)^{\frac{3}{2}} - \log_2 m = \Theta\left(\log^{\frac{3}{2}}(m)\right) \Rightarrow \Theta(\log^2 m) \leq T(m) \leq \Theta\left(\log^{\frac{3}{2}}(m)\right)$$

EQUAZIONE 4

$$T(n) = \begin{cases} 1 & \text{se } n \leq 2 \\ 3T(\sqrt{n}) + \log n & \text{se } n > 2 \end{cases}$$



LIVELLO	CONTRIBUTO
0	$\log n$
1	$3 \log n^{\frac{1}{2}} = \frac{3}{2} \log n$
2	$9 \log n^{\frac{1}{4}} = \frac{9}{4} \log n$
i	$\left(\frac{3}{2}\right)^i \cdot \log n$

Calcolo h: $\sqrt[2^i]{n} = 2 \iff n^{\frac{1}{2^i}} = 2 \iff \frac{1}{2^i} = \log_n 2 \Rightarrow i = \log_2(\log_2 n)$

Calcolo la sommatoria:

$$T(n) = \sum_{i=0}^{h} \left(\frac{3}{2}\right)^i \cdot \log n = \sum_{i=0}^{\log_2 \log_2 n} \left(\frac{3}{2}\right)^i \cdot \log n = \log n \cdot \sum_{i=0}^{\log_2 \log_2 n} \left(\frac{3}{2}\right)^i$$ serie geometrica con ragione > 1

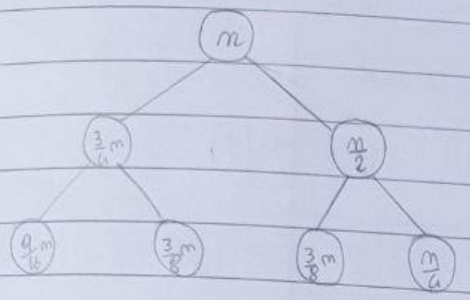
$$= \log n \cdot \frac{\left(\frac{3}{2}\right)^{\log_2(\log_2 n)+1} - 1}{\frac{3}{2} - 1} = \log n \cdot 2\left(\left(\frac{3}{2}\right)^{\log_2 \log_2 n} \cdot \frac{3}{2} - 1\right) = 3 \log n \cdot \left(\frac{3}{2}\right)^{\log_2 \log_2 n} - 2 \log n$$

$$= 3 \log n \cdot (\log_2 n)^{\log_2\left(\frac{3}{2}\right)} - 2 \log n = 3 \log n \cdot (\log_2 n)^{\log_2(3) - 1} - 2 \log n$$

$$= 3 \cancel{\log n} \cdot \frac{(\log_2 n)^{\log_2(3)}}{\cancel{\log_2 n}} - 2 \log n = 3(\log_2 n)^{\log_2(3)} - 2 \log n \Rightarrow \Theta\left((\log_2 n)^{\log_2(3)}\right)$$

EQUAZIONE 5

$$T(m) = \begin{cases} 1 & \text{se } m \le 1 \\ T\left(\frac{3}{4}m\right) + T\left(\frac{m}{2}\right) + m & \text{se } m > 1 \end{cases}$$



LIVELLO	CONTRIBUTO
0	m
1	$\frac{3}{4}m + \frac{1}{2}m = \frac{5}{4}m$
2	$\frac{9}{16}m + \frac{3}{8}m + \frac{3}{8}m + \frac{1}{4}m = \frac{25}{16}m = \left(\frac{5}{4}\right)^2 m$
i	$\left(\frac{5}{4}\right)^i \cdot m$

Calcolo $h_1 := \left(\frac{3}{4}\right)^i m = 1 \Rightarrow \left(\frac{4}{3}\right)^i = m \Rightarrow i = \log_{\frac{4}{3}} m$

Calcolo $h_2 := \frac{m}{2^i} = 1 \Rightarrow 2^i = m \Rightarrow i = \log_2 m$

Calcolo la sommatoria:

$T_{h_1}(m) = \sum_{i=0}^{h_1} \left(\frac{5}{4}\right)^i \cdot m = \sum_{i=0}^{\log_{\frac{4}{3}} m} \left(\frac{5}{4}\right)^i \cdot m = m \cdot \sum_{i=0}^{\log_{\frac{4}{3}} m} \left(\frac{5}{4}\right)^i$ serie geometrica con ragione > 1

$= m \frac{\left(\frac{5}{4}\right)^{h+1} - 1}{\frac{5}{4} - 1} = m \cdot 4\left(\left(\frac{5}{4}\right)^h \cdot \frac{5}{4} - 1\right) = 5m \cdot \left(\frac{5}{4}\right)^{\log_{\frac{4}{3}} m} - 4m = 5m \cdot m^{\log_{\frac{4}{3}} \frac{5}{4}} - 4m$

$= 5 \cdot m^{\log_{\frac{4}{3}}\left(\frac{5}{4}\right) + 1} - 4m \Rightarrow \Theta\left(m^{\log_{\frac{4}{3}}\left(\frac{5}{4}\right) + 1}\right)$

Calcolo la sommatoria:

$T_{h_2}(m) = \sum_{i=0}^{h_2} \left(\frac{5}{4}\right)^i \cdot m = \sum_{i=0}^{\log_2 m} \left(\frac{5}{4}\right)^i \cdot m = m \cdot \sum_{i=0}^{\log_2 m} \left(\frac{5}{4}\right)^i$ serie geometrica con ragione > 1

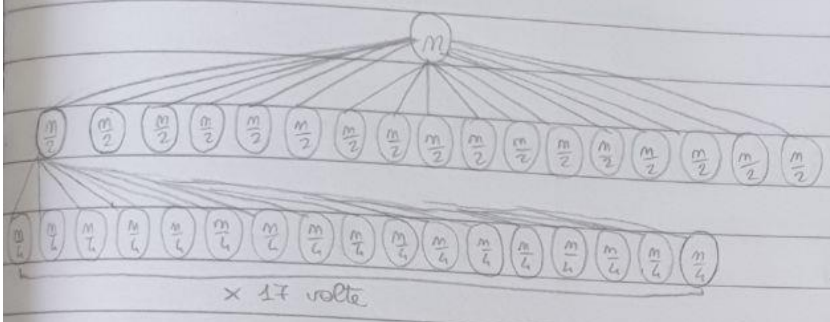
$= 5m \left(\frac{5}{4}\right)^{\log_2 m} - 4m = 5m \cdot m^{\log_2\left(\frac{5}{4}\right)} - 4m = 5m^{\log_2\left(\frac{5}{4}\right) + 1} - 4m \Rightarrow \Theta\left(m^{\log_2\left(\frac{5}{4}\right) + 1}\right)$

Essendo $T_{h_2}(m) \le T(m) \le T_{h_1}(m)$ avremo $\Theta\left(m^{\log_2\left(\frac{5}{4}\right) + 1}\right) \le T(m) \le \Theta\left(m^{\log_{\frac{4}{3}}\left(\frac{5}{4}\right) + 1}\right)$

N.B. Abbiamo scritto che $T_{h_2}(m) \le T(m) \le T_{h_1}(m)$ poiché $T_{h_2}(m)$ decresce più velocemente e quindi termina prima di $T_{h_1}(m)$

EQUAZIONE 7

$$T(n) = \begin{cases} 1 & \text{se } n \le 1 \\ 17T\left(\frac{n}{2}\right) + n^4 & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO
0	n^4
1	$17 \cdot \left(\frac{n}{2}\right)^4 = 17 \cdot \frac{n^4}{16}$
2	$17^2 \left(\frac{n}{4}\right)^4 = 17^2 \cdot \frac{n^4}{256} = 17^2 \cdot \frac{n^4}{16^2}$
i	$17^i \cdot \frac{n^4}{16^i} = \left(\frac{17}{16}\right)^i \cdot n^4$

Calcolo h: $\frac{n}{2^i} = 1 \Rightarrow 2^i = n \Rightarrow i = \log_2 n$

Calcolo la sommatoria:

$$T(n) = \sum_{i=0}^{h} \left(\frac{17}{16}\right)^i \cdot n^4 = \sum_{i=0}^{\log_2 n} \left(\frac{17}{16}\right)^i \cdot n^4 = n^4 \cdot \sum_{i=0}^{\log_2 n} \left(\frac{17}{16}\right)^i$$ serie geometrica con ragione > 1

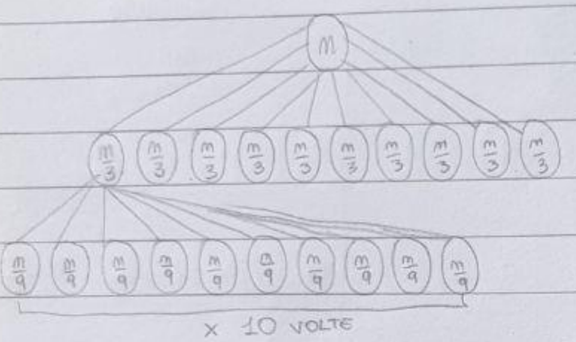
$$= n^4 \cdot \frac{\left(\frac{17}{16}\right)^{h+1} - 1}{\frac{17}{16} - 1} = n^4 \cdot \frac{\left(\frac{17}{16}\right)^{\log_2 n + 1} - 1}{\frac{1}{16}} = n^4 \left[16 \left(\left(\frac{17}{16}\right)^{\log_2 n} \cdot \frac{17}{16} - 1\right)\right]$$

$$= 17\, n^4 \cdot \left(\frac{17}{16}\right)^{\log_2 n} - 16\, n^4 = 17\, n^4 \cdot n^{\log_2\left(\frac{17}{16}\right)} - 16\, n^4 = 17\, n^{\log_2\left(\frac{17}{16}\right) + 4} - 16\, n^4$$

$$\Rightarrow T(n) = \Theta\left(n^{\log_2\left(\frac{17}{16}\right) + 4}\right)$$

EQUAZIONE 8

$$T(m) = \begin{cases} 1 & \text{se } m \leq 2 \\ 10T\left(\frac{m}{3}\right) + m^2 & \text{se } m > 2 \end{cases}$$



LIVELLO	CONTRIBUTO
0	m^2
1	$10\left(\frac{m}{3}\right)^2 = 10 \cdot \frac{m^2}{9}$
2	$10^2\left(\frac{m}{9}\right)^2 = 10^2 \cdot \frac{m^2}{81}$
i	$10^i \cdot \left(\frac{m}{3^i}\right)^2 = 10^i \cdot \frac{m^2}{9^i} = \left(\frac{10}{9}\right)^i m^2$

Calcolo $h :=$ $\frac{m}{3^i} = 2 \Rightarrow 3^i = \frac{m}{2} \Rightarrow i = \log_3\left(\frac{m}{2}\right)$

Calcolo la sommatoria:

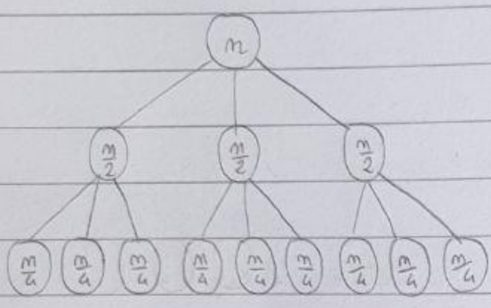
$$T(m) = \sum_{i=0}^{h} \left(\frac{10}{9}\right)^i \cdot m^2 = \sum_{i=0}^{\log_3\left(\frac{m}{2}\right)} \left(\frac{10}{9}\right)^i \cdot m^2 = m^2 \cdot \sum_{i=0}^{\log_3\left(\frac{m}{2}\right)} \left(\frac{10}{9}\right)^i$$ serie geometrica con ragione > 1

$$= m^2 \cdot \frac{\left(\frac{10}{9}\right)^{h+1} - 1}{\frac{10}{9} - 1} = m^2 \cdot \frac{\left(\frac{10}{9}\right)^{\log_3\left(\frac{m}{2}\right)+1} - 1}{\frac{1}{9}} = m^2\left[9\left(\left(\frac{10}{9}\right)^{\log_3\left(\frac{m}{2}\right)} \cdot \frac{10}{9} - 1\right)\right]$$

$$= m^2\left[\left(\frac{10}{9}\right)^{\log_3\left(\frac{m}{2}\right)} \cdot 10 - 9\right] = 10m^2 \cdot \left(\frac{10}{9}\right)^{\log_3\left(\frac{m}{2}\right)} - 9m^2 = 10m^2 \cdot \left(\frac{m}{2}\right)^{\log_3\left(\frac{10}{9}\right)} - 9m^2$$

EQUAZIONE 10

$$T(m) = \begin{cases} 1 & \text{se } m \leq 1 \\ 3T\left(\frac{m}{2}\right) + m & \text{se } m > 1 \end{cases}$$



LIVELLO	CONTRIBUTO
0	m
1	$\frac{m}{2} + \frac{m}{2} + \frac{m}{2} = \frac{3}{2}m$
2	$9\left(\frac{n}{4}\right) = \frac{9}{4}m = \left(\frac{3}{2}\right)^2 m$
i	$\left(\frac{3}{2}\right)^i \cdot m$

Calcolo h: $\frac{n}{2^i} = 1 \Rightarrow 2^i = m \Rightarrow i = \log_2 m$

Calcolo la sommatoria:

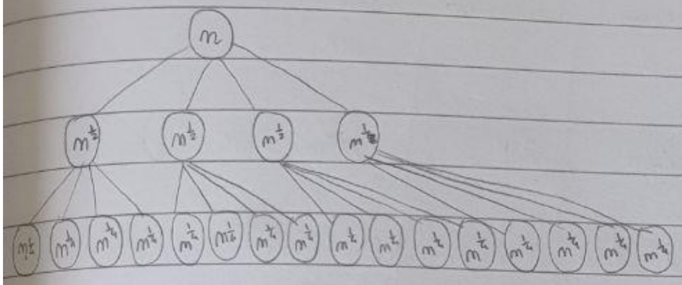
$$T(m) = \sum_{i=0}^{h} \left(\frac{3}{2}\right)^i \cdot m = \sum_{i=0}^{\log_2 m} \left(\frac{3}{2}\right)^i \cdot m = n \cdot \sum_{i=0}^{\log_2 m} \left(\frac{3}{2}\right)^i$$ serie geometrica con ragione > 1

$$= n \cdot \frac{\left(\frac{3}{2}\right)^{h+1} - 1}{\frac{3}{2} - 1} = m\left[2\left(\frac{3}{2} \cdot \left(\frac{3}{2}\right)^{\log_2 m} - 1\right)\right] = 3m\left(\frac{3}{2}\right)^{\log_2 m} - 2m$$

$$= 3m \cdot m^{\log_2\left(\frac{3}{2}\right)} - 2m = 3m^{\log_2\left(\frac{3}{2}\right)+1} - 2m \Rightarrow T(m) = \Theta\left(m^{\log_2\left(\frac{3}{2}\right)+1}\right)$$

EQUAZIONE 11

$$T(m) = \begin{cases} 1 & \text{se } m \le 2 \\ 4T(\sqrt{m}) + \log m & \text{se } m > 2 \end{cases}$$



LIVELLO	CONTRIBUTO
0	$\log m$
1	$4 \log \sqrt{m} = 4 \log m^{\frac{1}{2}}$
2	$16 \log \sqrt[4]{m} = 4^2 \log m^{\frac{1}{4}}$
i	$4^i \log \sqrt[2^i]{m} = 4^i \log m^{\frac{1}{2^i}} = 2^i \cdot \log m$

Calcolo h:: $m^{\frac{1}{2^i}} = 2 \Rightarrow \frac{1}{2^i} = \log_m 2 \Rightarrow 2^i = \log_2 m \Rightarrow i = \log_2 \log_2 m$

Calcolo la sommatoria:

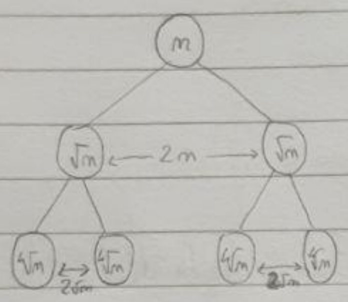
$$T(m) = \sum_{i=0}^{h} 2^i \cdot \log m = \sum_{i=0}^{\log_2 \log_2 m} 2^i \cdot \log m = \log m \cdot \sum_{i=0}^{\log_2 \log_2 m} 2^i$$ serie geometrica con ragione > 1

$$= \log m \cdot \frac{2^{h+1} - 1}{2 - 1} = \log m \cdot \left(2^{\log_2 \log_2 m + 1} - 1\right) = \log m \left(2^{\log_2 \log_2 m} \cdot 2 - 1\right)$$

$$= 2 \cdot \log m \cdot \log_2 m - \log m \Rightarrow T(m) = \Theta(\log^2 m)$$

EQUAZIONE 12

$$T(m) = \begin{cases} 1 & \text{se } m \le 2 \\ 2m\,T(\sqrt{m}) + m^2 & \text{se } m > 2 \end{cases}$$



LIVELLO	ISTANZA	N° NODI	CONTR. NODO	CONTR. LOCALE LIVELLO
0	m	1	m^2	m^2
1	$\sqrt{m}$	$2m$	$(\sqrt{m})^2 = m$	$2m \cdot m = 2m^2$
2	$\sqrt[4]{m}$	$2m \cdot 2\sqrt{m}$	$(\sqrt[4]{m}) = \sqrt{m}$	$2m \cdot 2\sqrt{m} \cdot \sqrt{m} = 4m^2 = 2^2 m^2$
i	$\sqrt[2^i]{m}$	$2m \cdot 2\sqrt{m} \cdot \ldots \cdot 2\sqrt[2^i]{m}$	$(\sqrt[2^i]{m})^2$	$2^i \cdot m^2$

Calcolo h: $\sqrt[2^i]{m} = 2 \Rightarrow m^{\frac{1}{2^i}} = 2 \Rightarrow \frac{1}{2^i} = \log_m 2 \Rightarrow i = \log_2(\log_2 m)$

Calcolo la sommatoria:

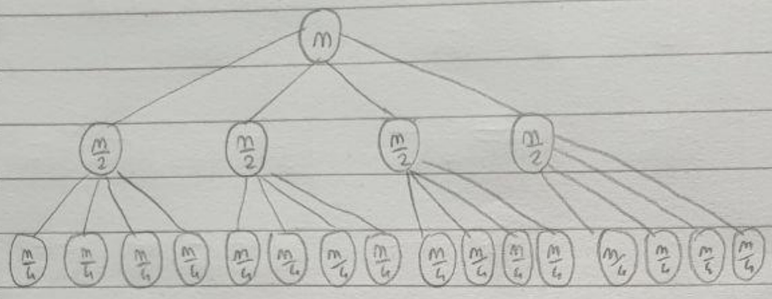
$$T(m) = \sum_{i=0}^{h} 2^i \cdot m^2 = \sum_{i=0}^{\log_2 \log_2 m} 2^i \cdot m^2 = m^2 \cdot \sum_{i=0}^{\log_2 \log_2 m} 2^i$$ serie geometrica con ragione > 1

$$= m^2 \cdot \frac{2^{h+1} - 1}{2 - 1} = m^2 \cdot \left(2^{\log_2 \log_2 m + 1} - 1\right) = m^2 \cdot \left(2^{\log_2 \log_2 m} \cdot 2 - 1\right)$$

$$= 2m^2 \log_2 m - m^2 \Rightarrow T(m) = \Theta(m^2 \log_2 m)$$

EQUAZIONE 14

$$T(m) = \begin{cases} 1 & \text{se } m \leq 1 \\ 4T\left(\frac{m}{2}\right) + m & \text{se } m > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	m
1	$\frac{4}{2}m = 2m$
2	$\frac{16}{4}m = 4m = 2^2 \cdot m$
i	$2^i \cdot m$

Calcolo h: $\frac{m}{2^i} = 1 \Rightarrow 2^i = m \Rightarrow i = \log_2 m$

Calcolo la sommatoria:

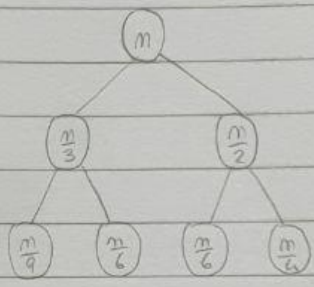
$$T(m) = \sum_{i=0}^{h} 2^i \cdot m = \sum_{i=0}^{\log_2 m} 2^i \cdot m = m \cdot \sum_{i=0}^{\log_2 m} 2^i$$ serie geometrica con ragione > 1

$$= m \cdot \frac{2^{h+1} - 1}{2 - 1} = m\left(2^{\log_2 m + 1} - 1\right) = m\left(2^{\log_2 m} \cdot 2 - 1\right) = m(m \cdot 2 - 1)$$

$$= 2m^2 - m \Rightarrow T(m) = \Theta(m^2)$$

EQUAZIONE 18

$$T(m) = \begin{cases} 1 & \text{se } m \leq 1 \\ T\left(\frac{m}{3}\right) + T\left(\frac{m}{2}\right) + m & \text{se } m > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	m
1	$\frac{m}{3} + \frac{m}{2} = \frac{2m+3m}{6} = \frac{5}{6}m$
2	$\frac{m}{9} + \frac{m}{6} + \frac{m}{6} + \frac{m}{4} = \frac{4m+6m+6m+9m}{36} = \frac{25}{36}m = \left(\frac{5}{6}\right)^2 m$
i	$\left(\frac{5}{6}\right)^i \cdot m$

Calcolo h_1: $= \frac{m}{3^i} = 1 \Rightarrow 3^i = m \Rightarrow i = \log_3 m$

Calcolo h_2: $= \frac{m}{2^i} = 1 \Rightarrow 2^i = m \Rightarrow i = \log_2 m$

Calcolo la sommatoria:

$$T_{h_1}(m) = \sum_{i=0}^{h_1} \left(\frac{5}{6}\right)^i \cdot m = \sum_{i=0}^{\log_3 m} \left(\frac{5}{6}\right)^i \cdot m = m \cdot \sum_{i=0}^{\log_3 m} \left(\frac{5}{6}\right)^i$$ serie geometrica con ragione < 1

$$= m \cdot \frac{1}{1-\frac{5}{6}} = m \cdot \frac{1}{\frac{1}{6}} = 6m \Rightarrow T_{h_1}(m) = \Theta(m)$$

Calcolo la sommatoria:

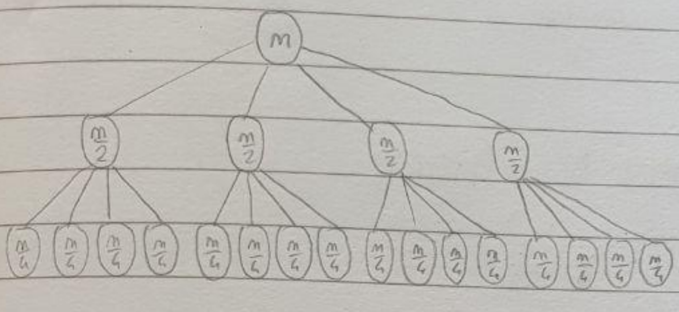
$$T_{h_2}(m) = \sum_{i=0}^{h_2} \left(\frac{5}{6}\right)^i \cdot m = \sum_{i=0}^{\log_2 m} \left(\frac{5}{6}\right)^i \cdot m = m \cdot \sum_{i=0}^{\log_2 m} \left(\frac{5}{6}\right)^i$$ serie geometrica con ragione < 1

$$= m \cdot \frac{1}{1-\frac{5}{6}} = m \cdot \frac{1}{\frac{1}{6}} = 6m \Rightarrow T_{h_2}(m) = \Theta(m)$$

Essendo $T_{h_1}(m) \leq T(m) \leq T_{h_2}(m)$, avremo che $T(m) = \Theta(m)$

EQUAZIONE 19

$$T(n) = \begin{cases} 1 & \text{se } n \leq 1 \\ 4T\left(\frac{n}{2}\right) + n^2 & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	n^2
1	$4\left(\frac{n}{2}\right)^2 = 4 \cdot \frac{n^2}{4} = n^2$
2	$16\left(\frac{n}{4}\right)^2 = 16 \cdot \frac{n^2}{16} = n^2$
i	$4^i\left(\frac{n}{2^i}\right)^2 = n^2$

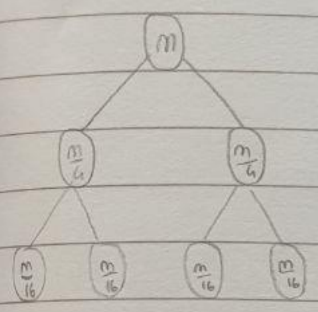
Calcolo h: $\frac{n}{2^i} = 1 \Rightarrow 2^i = n \Rightarrow i = \log_2 n$

Calcolo la sommatoria:

$$T(n) = \sum_{i=0}^{h} n^2 = \sum_{i=0}^{\log_2 n} n^2 = n^2 \cdot \sum_{i=0}^{\log_2 n} (1) \Rightarrow T(n) = \Theta(n^2)$$

EQUAZIONE 21

$$T(n) = \begin{cases} 1 & \text{se } n \leq 1 \\ 2T\left(\frac{n}{4}\right) + \sqrt{n} & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	$\sqrt{n}$
1	$\sqrt{\frac{n}{4}} + \sqrt{\frac{n}{4}} = \frac{\sqrt{n}}{2} + \frac{\sqrt{n}}{2} = \sqrt{n}$
2	$4\sqrt{\frac{n}{16}} = \cancel{4} \cdot \frac{\sqrt{n}}{\cancel{4}} = \sqrt{n}$
i	$2^i \cdot \sqrt{\frac{n}{4^i}} = \cancel{2^i} \cdot \frac{\sqrt{n}}{\cancel{2^i}} = \sqrt{n}$

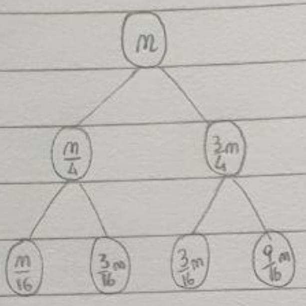
Calcolo $h := \frac{n}{4^i} = 1 \Rightarrow 4^i = n \Rightarrow \log_4 n$

Calcolo la sommatoria

$$T(n) = \sum_{i=0}^{h} \sqrt{n} = \sum_{i=0}^{\log_4 n} \sqrt{n} = \sqrt{n} \cdot \sum_{i=0}^{\log_4 n} 1 \Rightarrow T(n) = \Theta(\sqrt{n})$$

EQUAZIONE 22

$$T(n) = \begin{cases} 1 & \text{se } n \leq 1 \\ T\left(\frac{n}{4}\right) + T\left(\frac{3}{4}n\right) + n & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	n
1	$\frac{n}{4} + \frac{3}{4}n = n$
2	$\frac{n}{16} + \frac{3}{16}n + \frac{3}{16}n + \frac{9}{16}n = \cancel{\frac{16}{16}}n$
i	n

Calcolo h_1: $\frac{n}{4^i} = 1 \Rightarrow 4^i = n \Rightarrow i = \log_4 n$

Calcolo h_2: $\left(\frac{3}{4}\right)^i n = 1 \Rightarrow n = \left(\frac{4}{3}\right)^i \Rightarrow i = \log_{\frac{4}{3}} n$

Calcolo la sommatoria

$$T_{h_1}(n) = \sum_{i=0}^{h_1} n = \sum_{i=0}^{\log_4 n} n = n \cdot \sum_{i=0} 1 \Rightarrow T_{h_1}(n) = \Theta(n)$$

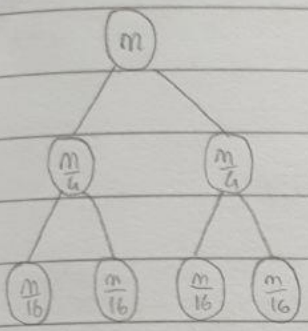
Calcolo la sommatoria

$$T_{h_2}(n) = \sum_{i=0}^{h_2} n = \sum_{i=0}^{\log_{\frac{4}{3}} n} n = n \cdot \sum_{i=0}^{\log_{\frac{4}{3}} n} 1 \Rightarrow T_{h_2}(n) = \Theta(n^2)$$

Essendo $T_{h_1}(n) \leq T(n) \leq T_{h_2}(n)$, avremo che $T(n) = \Theta(n)$

EQUAZIONE 23

$$T(n) = \begin{cases} 1 & \text{se } n \leq 1 \\ 2T\left(\frac{n}{4}\right) + n^3 & \text{se } n > 1 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	n^3
1	$2\left(\frac{n}{4}\right)^3 = 2 \cdot \frac{n^3}{\cancel{64}_{32}} = \frac{n^3}{32}$
2	$4\left(\frac{n}{16}\right)^3 = \cancel{4} \cdot \frac{n^3}{\cancel{4096}_{1024}} = \frac{n^3}{1024}$
i	$\frac{n^3}{32^i} = \left(\frac{1}{32}\right)^i \cdot n^3$

Calcolo h: $\frac{n}{4^i} = 1 \Rightarrow 4^i = n \Rightarrow i = \log_4 n$

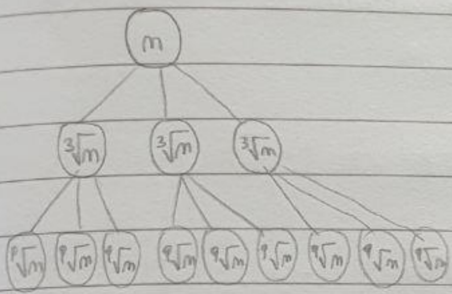
Calcolo la sommatoria:

$$T(n) = \sum_{i=0}^{h} \left(\frac{1}{32}\right)^i \cdot n^3 = \sum_{i=0}^{\log_4 n} \left(\frac{1}{32}\right)^i \cdot n^3 = n^3 \cdot \sum_{i=0}^{\log_4 n} \left(\frac{1}{32}\right)^i$$ serie geometrica con ragione < 1

$$= n^3 \cdot \frac{1}{1 - \frac{1}{32}} = n^3 \cdot \frac{1}{\frac{31}{32}} = \frac{32}{31} n^3 \Rightarrow T(n) = \Theta(n^3)$$

EQUAZIONE 27

$$T(m) = \begin{cases} 1 & \text{se } m \leq 3 \\ 3T(\sqrt[3]{m}) + \log_3 m & \text{se } m > 3 \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	$\log_3 m$
1	$3 \log_3 \sqrt[3]{m} = 3 \log_3 m^{\frac{1}{3}} = 3 \cdot \frac{1}{3} \log_3 m$
2	$3^2 \cdot \log_3 \sqrt[9]{m} = 3^2 \cdot \log_3 m^{\frac{1}{9}} = 3^2 \cdot \frac{1}{9} \log_3 m$
i	$\log_3 m$

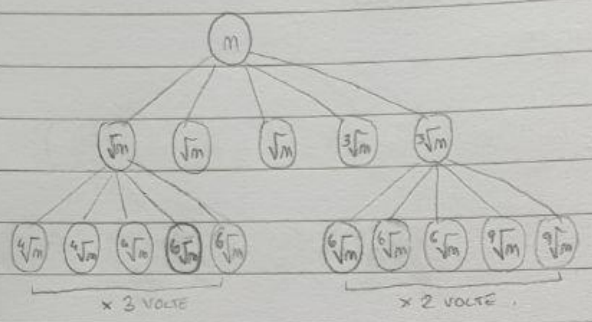
Calcolo h: $\sqrt[3^i]{m} = 3 \Rightarrow m^{\frac{1}{3^i}} = 3 \Rightarrow \frac{1}{3^i} = \log_m 3 \Rightarrow 3^i = \frac{1}{\log_m 3} \Rightarrow 3^i = \log_3 m \Rightarrow i = \log_3(\log_3 m)$

Calcolo la sommatoria:

$$T(m) = \sum_{i=0}^{h} \log_3 m = \sum_{i=0}^{\log_3 \log_3 m} \log_3 m = \log_3 m \cdot \sum_{i=0}^{\log_3 \log_3 m} 1 \Rightarrow T(m) = \Theta(\log_3 m)$$

EQUAZIONE 28

$$T(m) = \begin{cases} \text{costante} & \text{se } m \leq x \\ 3T(\sqrt{m}) + 2T(\sqrt[3]{m}) + \log_x m & \text{se } m > x \end{cases}$$



LIVELLO	CONTRIBUTO LIVELLO
0	$\log_x m$
1	$3\log_x m^{\frac{1}{2}} + 2\log_x m^{\frac{1}{3}} = \left(\frac{3}{2}+\frac{2}{3}\right)\log_x m = \frac{13}{6}\log_x m$
2	$3\left(3m^{\frac{1}{4}}+2m^{\frac{1}{6}}\right)+2\left(3m^{\frac{1}{6}}+2m^{\frac{1}{9}}\right) = \left(\frac{169}{36}\right)\log_x m = \left(\frac{13}{16}\right)^2 \log_x m$
i	$\left(\frac{13}{16}\right)^i \cdot \log_x m$

Calcolo $h_1 := \sqrt[2^i]{m} = x \Rightarrow m^{\frac{1}{2^i}} = x \Rightarrow \frac{1}{2^i} = \log_m x \Rightarrow 2^i = \log_x m \Rightarrow i = \log_2(\log_x m)$

Calcolo $h_2 := \sqrt[3^i]{m} = x \Rightarrow m^{\frac{1}{3^i}} = x \Rightarrow \frac{1}{3^i} = \log_m x \Rightarrow 3^i = \log_x m \Rightarrow i = \log_3(\log_x m)$

Calcolo la sommatoria:

$$T_{h_1}(m) = \sum_{i=0}^{h_1}\left(\frac{13}{16}\right)^i \cdot \log_x m = \sum_{i=0}^{\log_2(\log_x m)}\left(\frac{13}{16}\right)^i \cdot \log_x m = \log_x m \cdot \sum_{i=0}^{\log_2(\log_x m)}\left(\frac{13}{16}\right)^i$$ serie geometrica con ragione < 1

$$= \log_x m \cdot \frac{1}{1-\frac{13}{16}} = \log_x m \cdot \left(\frac{16}{3}\right) = \frac{16}{3}\log_x m \Rightarrow T_{h_1}(m) = \Theta(\log_x m)$$

Calcolo la sommatoria:

$$T_{h_2}(m) = \sum_{i=0}^{h_2}\left(\frac{13}{16}\right)^i = \sum_{i=0}^{\log_3(\log_x m)}\left(\frac{13}{16}\right)^i \cdot \log_x m = \log_x m \cdot \sum_{i=0}^{\log_3(\log_x m)}\left(\frac{13}{16}\right)^i$$ serie geometrica con ragione < 1

$$= \log_x m \cdot \frac{1}{1-\frac{13}{16}} = \frac{16}{3}\log_x m \Rightarrow T_{h_2}(m) = \Theta(\log_x m)$$

Essendo $T_{h_1}(m) \leq T(m) \leq T_{h_2}$, avremo che $T(m) = \Theta(\log_x m)$