ANALISI I

RICEVIMENTO ONLINE I

27/9/2022

DISEQUAZIONI

1. $(x-1)(x^3-x^2-2x)>0$;

2. $\dfrac{x^2-4}{x-1}\leq 0$;

3. $2^{2x+1}<4^{x^2}$;

4. $\left(\dfrac{1}{2}\right)^{2x+1}<\left(\dfrac{1}{2}\right)^{\frac{x^2}{x\ \ 1}}$;

5. $\log_{\frac{1}{2}}\left(\dfrac{x^2-1}{x}\right)>0$;

6. $2\log(3x)<\log(3x+2)$;

7. $\sqrt{x^2+4\ \ 4x}>x\ \ 3$;

8. $\sqrt{x\ \ 1}+\sqrt{x+1}>\sqrt{3x}$;

9. $\sqrt[3]{x^2+x}<x$;

10. $\sqrt{x+4}<x+3$;

11. $3^{-2x}\ \ 4\cdot 3^{-x}+5\leq 0$;

12. $|x^2+5x+3|>3$;

(1) $(x-1)(x^3 - x^2 - 2x) > 0 \rightarrow +$

STRETTAMENTE MAGGIORE DI ZERO

$(x-1)\,x\,(x^2 - x - 2)$

$P_1 \quad P_2 \quad P_3$

$P_1: \quad x - 1 \geq 0 \longrightarrow x \geq 1$

$P_2: \quad x > 0$

$P_3: \quad x^2 - x - 2 \geq 0$

$x^2 - x - 2 = 0$

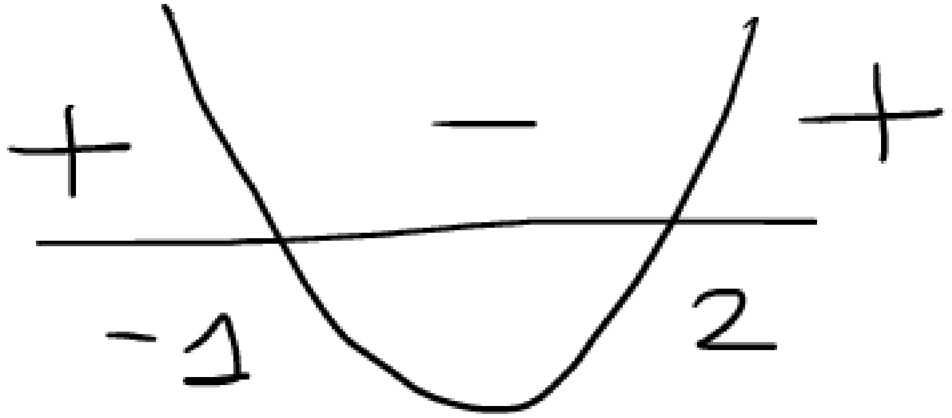
$a = 1 \quad b = -1 \quad c = -2$

$x^2 - x - 2 = 0$

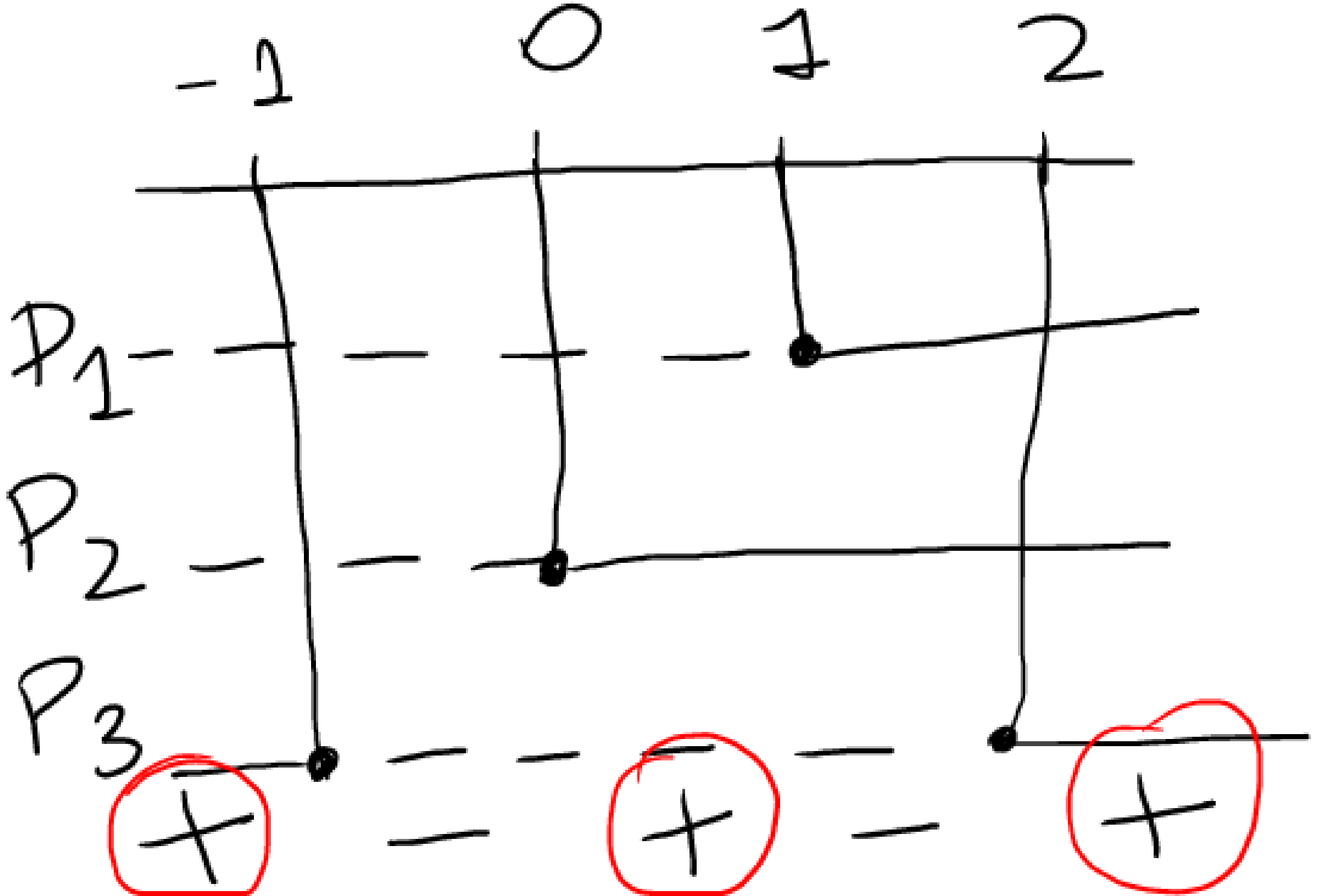
$1+8$

$\Delta = 1 - 4(1)(-2) = 9 > 0$

$x_{1,2} = \dfrac{1 \pm 3}{2} = \begin{cases} 2 \\ -1 \end{cases}$



METTIAMO INSIEME :



SOLUZIONE :

$x < -1 \vee 0 < x < 1 \vee$
$x > 2$

$x < -1 \quad \vee \quad 0 < x < 1 \quad \vee \quad x > 2$

NOTAZIONE PER INTERVALLI:

$\boxed{(-\infty, -1) \cup (0, 1) \cup (2, +\infty)}$

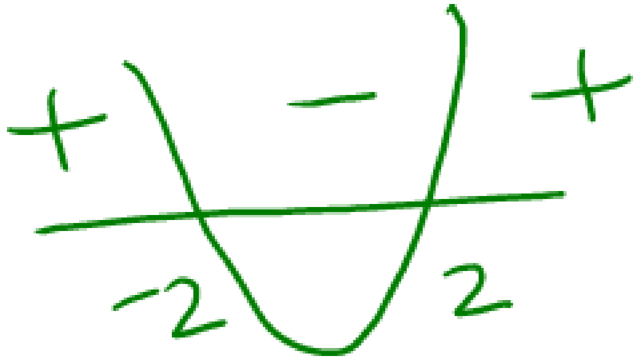
② $\begin{array}{l} N \rightarrow \\ D \rightarrow \end{array} \dfrac{x^2-4}{x-1} \le 0$ → NEGATIVO O ZERO

(C.E. $x-1 \neq 0 \Rightarrow x \neq 1$)

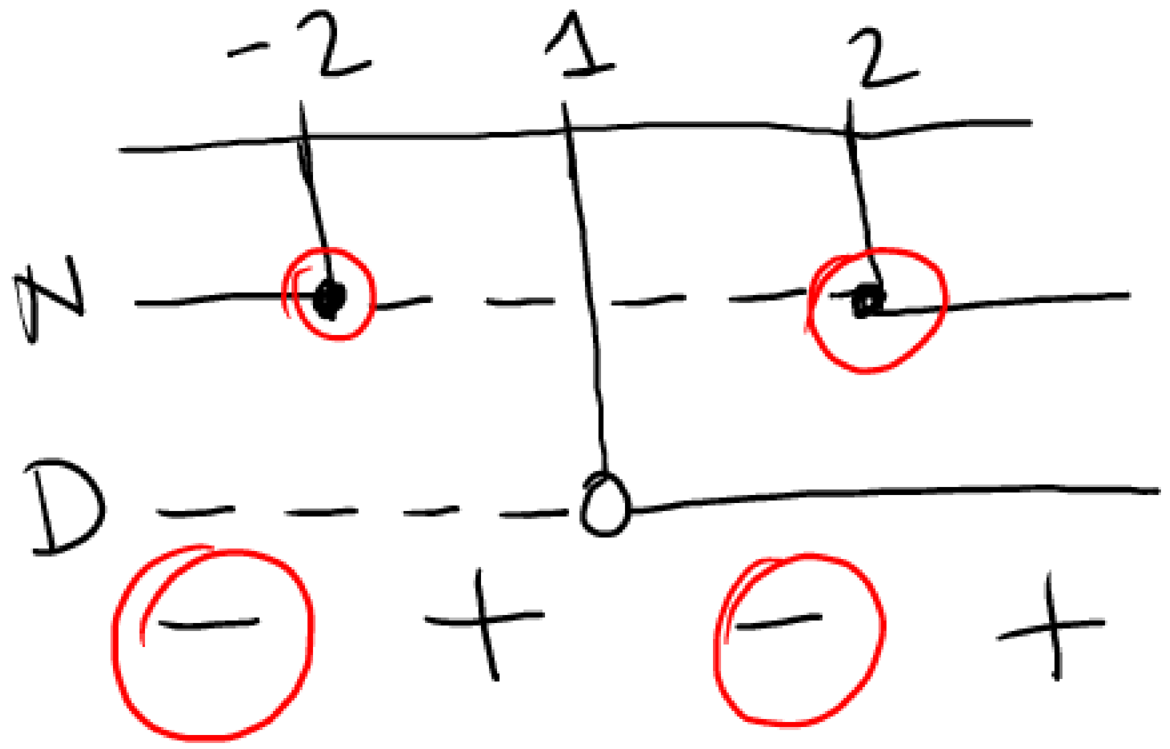
N: $x^2-4 \geq 0$

$x^2-4=0$

$x = \pm 2$



D: $x-1>0 \longrightarrow x>1$



SOLUZIONE:

$$x \le -2 \ \vee \ 1 < x \le 2$$

$$(-\infty, -2] \cup (1, 2]$$

3

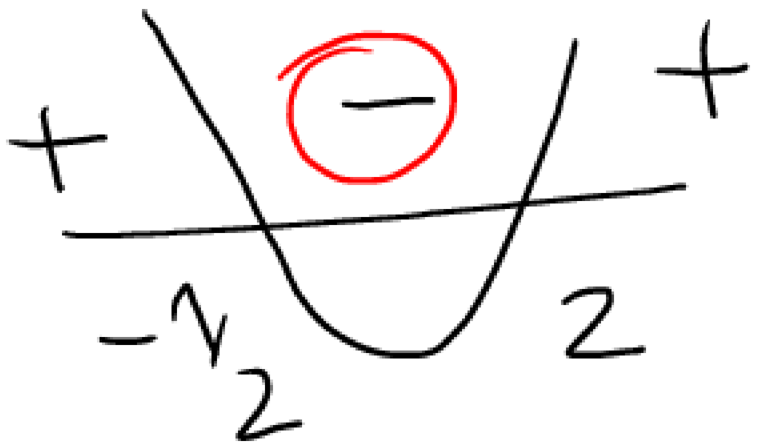
$$\begin{cases} 2x^2 - 3x - 2 < 0 & (1) \\ 5x - 3 \ge 0 & (2) \end{cases}$$

(1) $2x^2 - 3x - 2 < 0 \longrightarrow 2x^2 - 3x - 2 = 0$

$$2x^2 - 3x - 2 = 0$$

$$\Delta = 9 + 16 = 25 > 0$$

$$x_{1,2} = \frac{3 \pm 5}{4} = \begin{cases} \nearrow 2 \\ \searrow -\frac{1}{2} \end{cases}$$



SOLUZIONE ①:

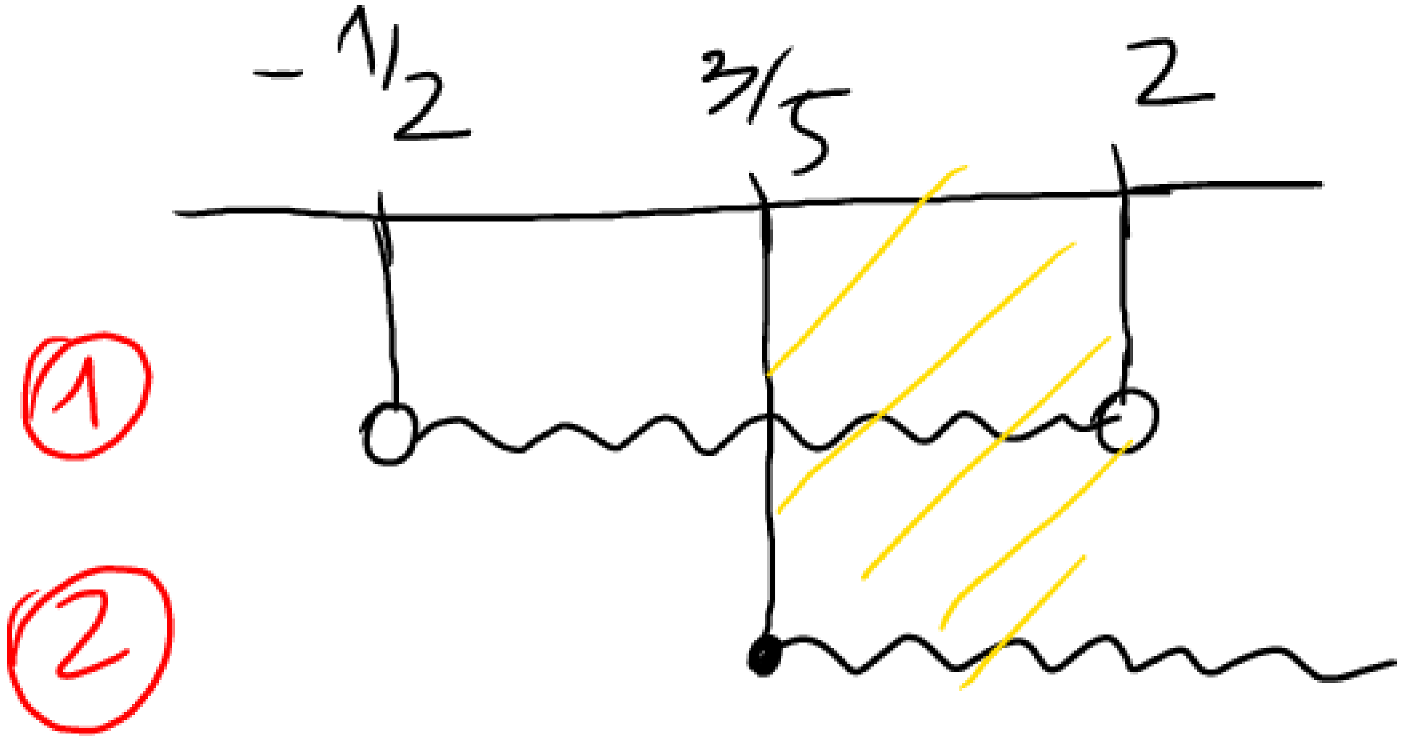
$-\frac{1}{2} < x < 2$ cioè $\left(-\frac{1}{2}, 2\right)$

② $5x - 3 \geq 0$

$\to 5x \geq 3 \Rightarrow x \geq \frac{3}{5}$

SOLUZIONE ②: $x \geq \frac{3}{5}$ cioè $\left[\frac{3}{5}, +\infty\right)$

METTO INSIEME :



SOLUZIONE : $\frac{3}{5} \leq x < 2$ OVVERO

$[3/5, 2)$.

④ $\sqrt{x^2-4x+4} > x-3$ ← RISOLVO

DISUGUAGLIANZA IRRAZIONALE

$\sqrt{A(x)} > B(x) \longrightarrow \begin{cases} A(x) \geq 0 \\ B(x) \geq 0 \\ A(x) > B^2(x) \end{cases} \cup \begin{cases} A(x) \geq 0 \\ B(x) < 0 \end{cases}$

POSITIVO SEMPRE

EQUIVALENTE A

$$\sqrt{A(x)} < B(x) \longrightarrow \begin{cases} A(x) \geq 0 \\ B(x) > 0 \\ A(x) < B^2(x) \end{cases}$$

EQUIVALENTE

$$\sqrt{x^2 - 4x + 4} > x - 3$$

$$(1) \begin{cases} x^2 - 4x + 4 \geq 0 \\ x - 3 > 0 \\ x^2 - 4x + 4 > (x-3)^2 \end{cases} \cup \quad (2) \begin{cases} x^2 - 4x + 4 \geq 0 \\ x - 3 \leq 0 \end{cases}$$

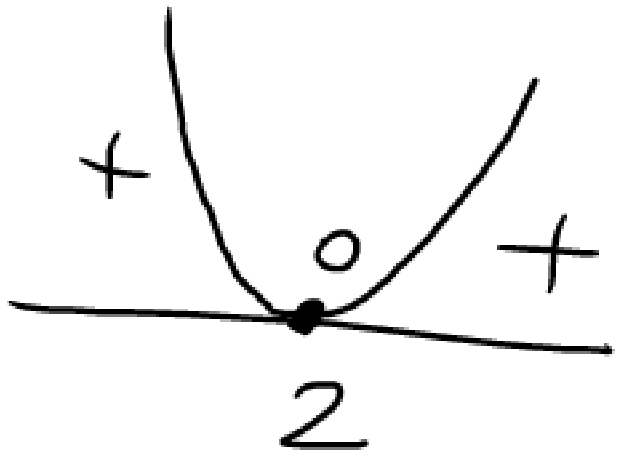
1

a) $x^2-4x+4 \geq 0$

$(x-2)^2 \geq 0$

$\Delta = 0$

$a > 0$

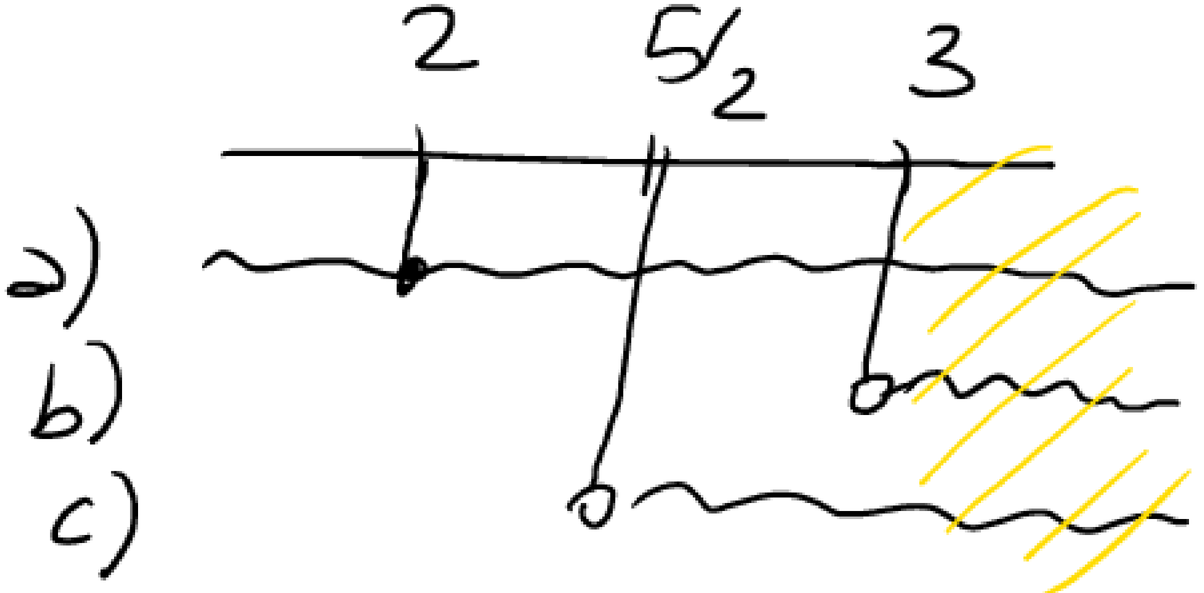


SEMPRE VERA!

b) $x-3>0 \rightarrow x>3$

c) $\cancel{x^2}-4x+4 > (x-3)^2 = \cancel{x^2}-6x+9$

$2x > 5 \longrightarrow x > 5/2$



SOLUZIONE: $x>3$ OVVERO $(3,+\infty)$

② a): $x^2 - 4x + 4 \geq 0 \rightarrow$ SEMPRE VERA

b) $x - 3 \leq 0 \rightarrow x \leq 3$

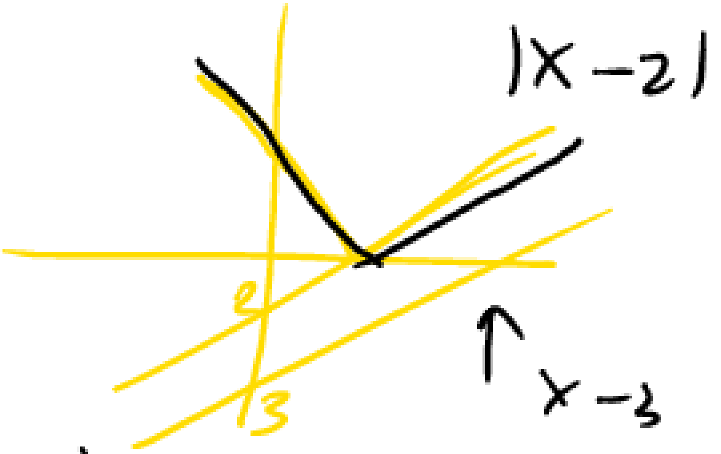
SOLUZIONE ②:

$x \leq 3$ ovvero $(-\infty, 3]$

UNIAMO ① E ②: $(-\infty, 3] \cup (3, +\infty) = \overset{\mathbb{R}}{(-\infty, +\infty)}$

SEMPRE VERA!

OSSERVAZIONE:



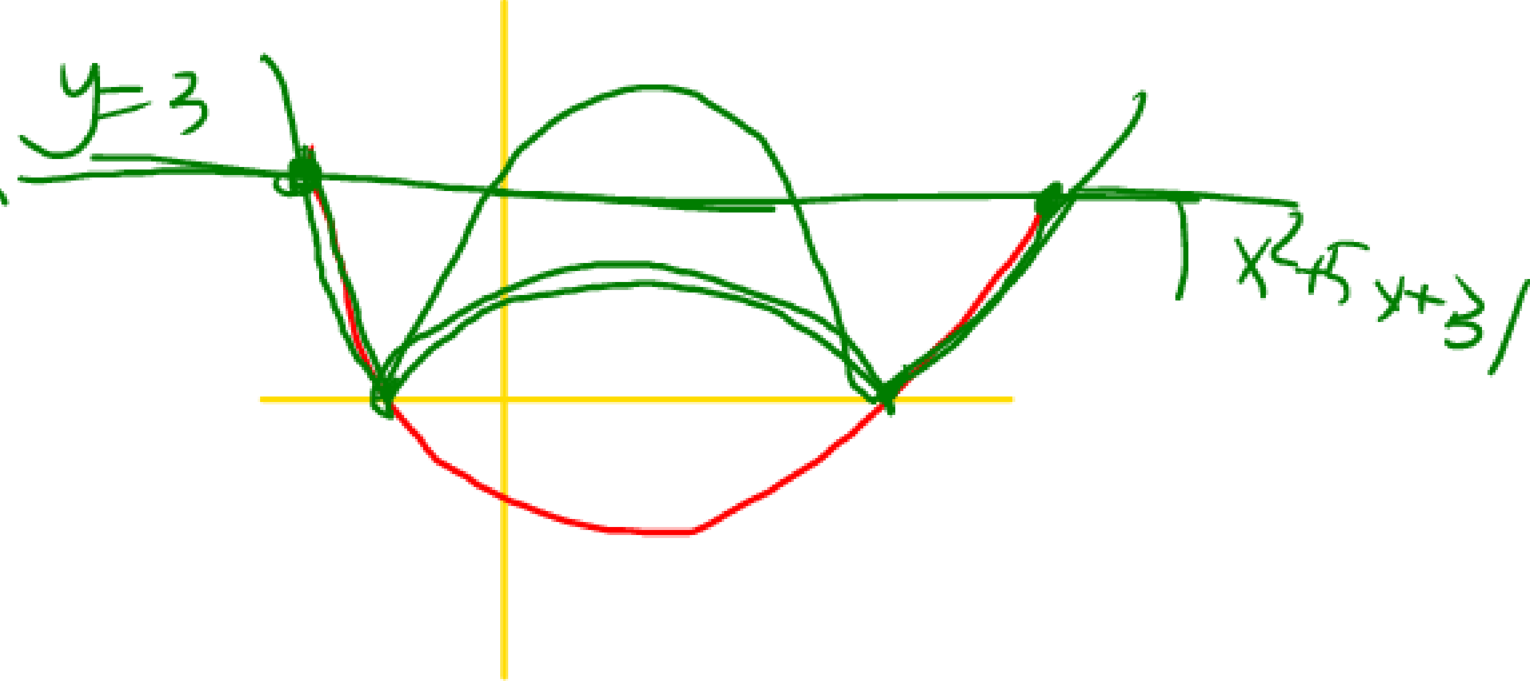
$$\sqrt{x^2-4x+4} = \sqrt{(x-2)^2} = |x-2| > x-3$$

$$\sqrt{x^2} = |x| \; !!!!$$

$$(\sqrt{x})^2 = x \qquad x \geq 0$$

$$x=-2 \longrightarrow x^2 = 4 \longrightarrow \sqrt{x^2} = \sqrt{4} = 2 = |-2|$$

⑤ $|x^2+5x+3| > 3$



VALORE ASSOLUTO (MODULO)

$$|x| = \begin{cases} x ; & \text{SE } x \geq 0 \\ -x ; & \text{SE } x \leq 0 \end{cases} = \text{MAX}(x, -x)$$

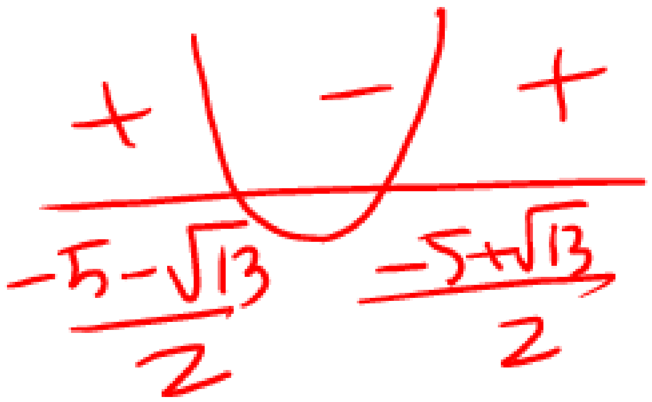
$$|x^2+5x+3| = \begin{cases} x^2+5x+3 ; & \text{SE } x^2+5x+3 \geq 0 \\ -x^2-5x-3 ; & \text{SE } x^2+5x+3 \leq 0 \end{cases}$$

$$|x^2+5x+3| > 3$$

È EQUIVALENTE A

$$\left\{\begin{array}{l} x^2+5x+3 \geq 0 \\ x^2+5x+\cancel{3} > \cancel{3} \end{array}\right. \quad 1 \qquad \cup \qquad \left\{\begin{array}{l} x^2+5x+3 \leq 0 \\ -x^2-5x-3 > 3 \end{array}\right. \quad 2$$

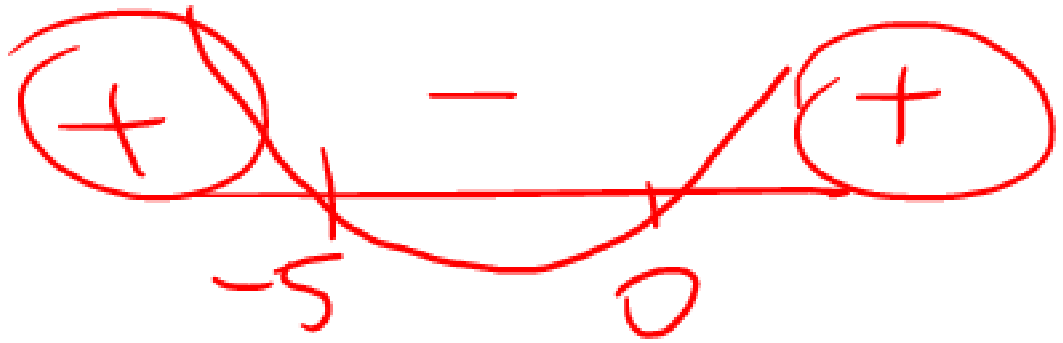
$$x^2+5x+3=0 \qquad \Delta = 25-12 = 13 > 0$$

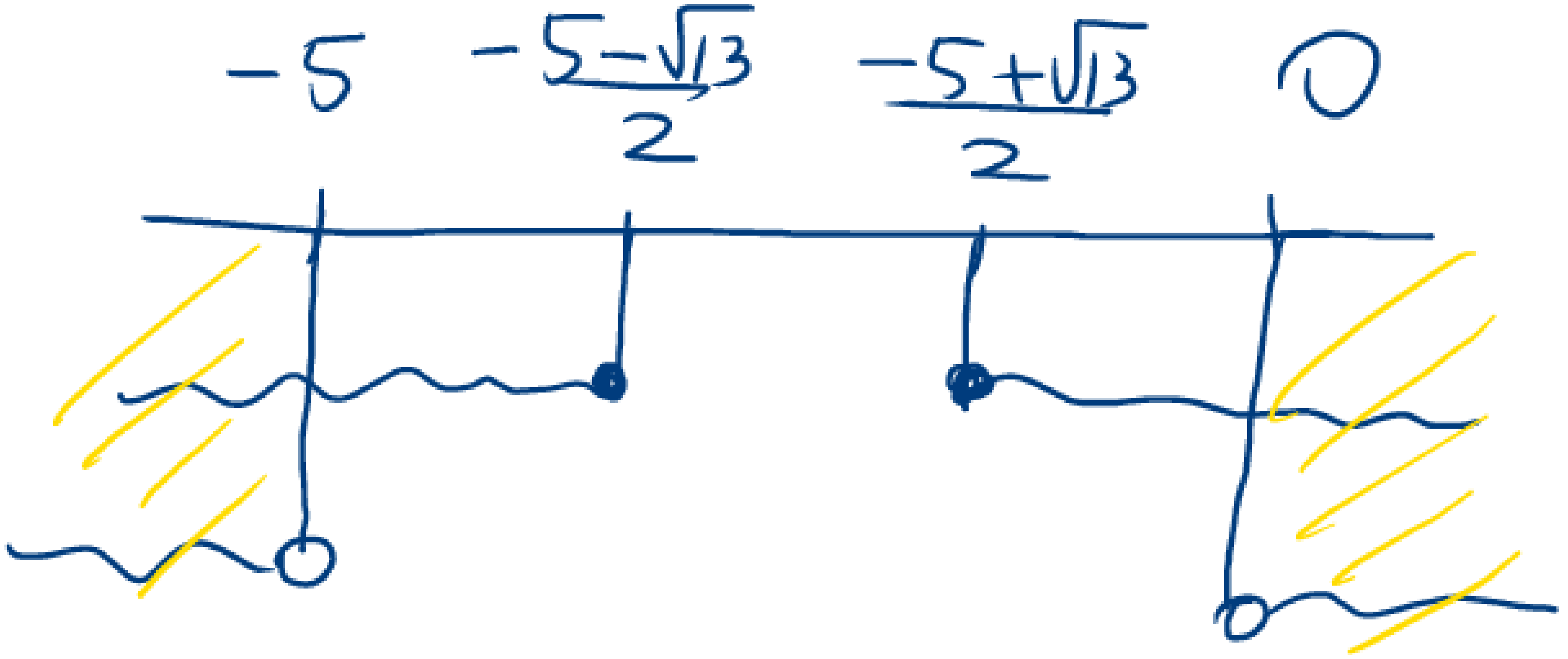


$$x_{1,2} = \frac{-5 \pm \sqrt{13}}{2} = \begin{cases} \dfrac{-5-\overset{3{,}6}{\sqrt{13}}}{2} \cong -4{,}3 \\ \dfrac{-5+\sqrt{13}}{2} = -0{,}7 \end{cases}$$

1

$$\begin{cases} x \le \dfrac{-5-\sqrt{13}}{2} \ \vee \ x \ge \dfrac{-5+\sqrt{13}}{2} \\ x^2+5x>0 \longrightarrow x<-5 \ \vee \ x>0 \end{cases}$$





$x<-5 \ \vee \ x>0 \Rightarrow (-\infty,-5) \cup (0,+\infty)$

②

$$\begin{cases} \dfrac{-5-\sqrt{13}}{2} < x < \dfrac{-5+\sqrt{13}}{2} \\ -x^2-5x-3 > 3 \end{cases}$$

$\longrightarrow -3 < x < -2$

SOLUZIONE

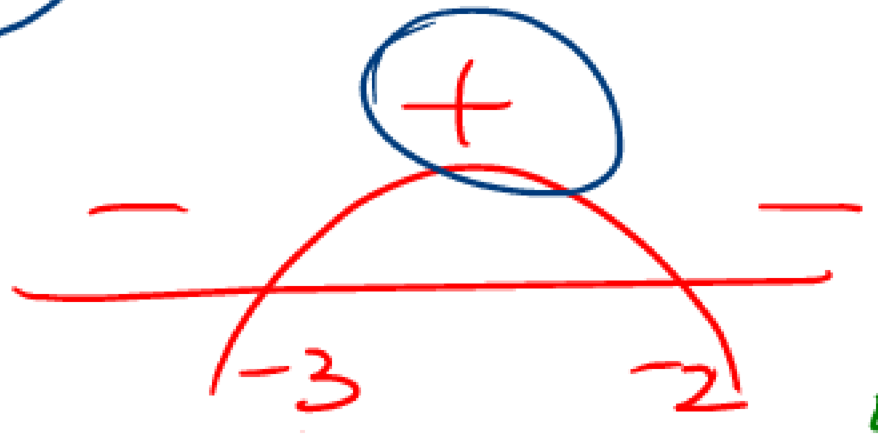
$(-3, -2)$

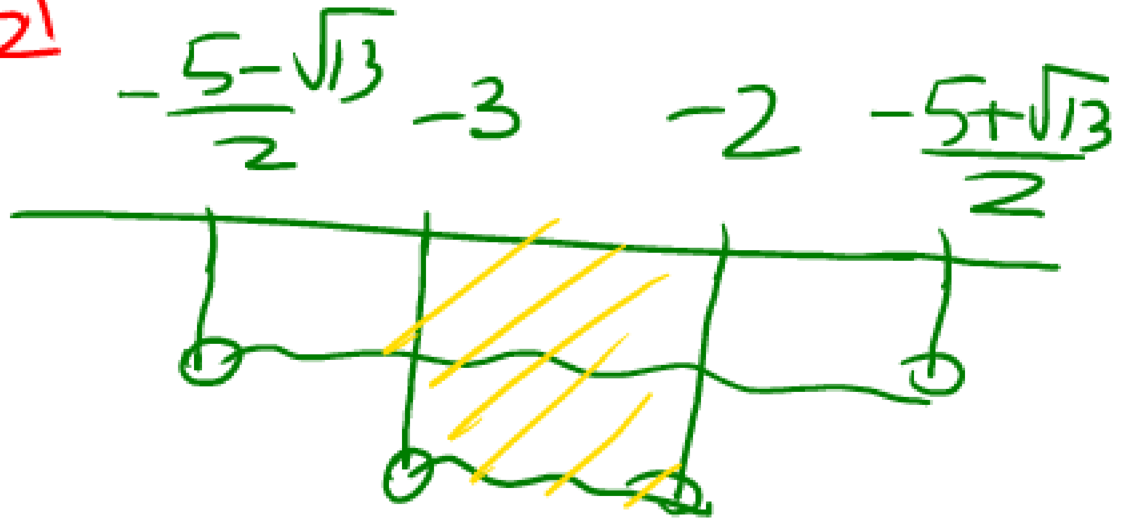
$$-x^2-5x-6 > 0$$

$a = -1$

$\Delta = 25 - 24 = 1 > 0$

$$x_{1,2} = \frac{5 \pm 1}{-2} = \begin{matrix} \nearrow -3 \\ \searrow -2 \end{matrix}$$

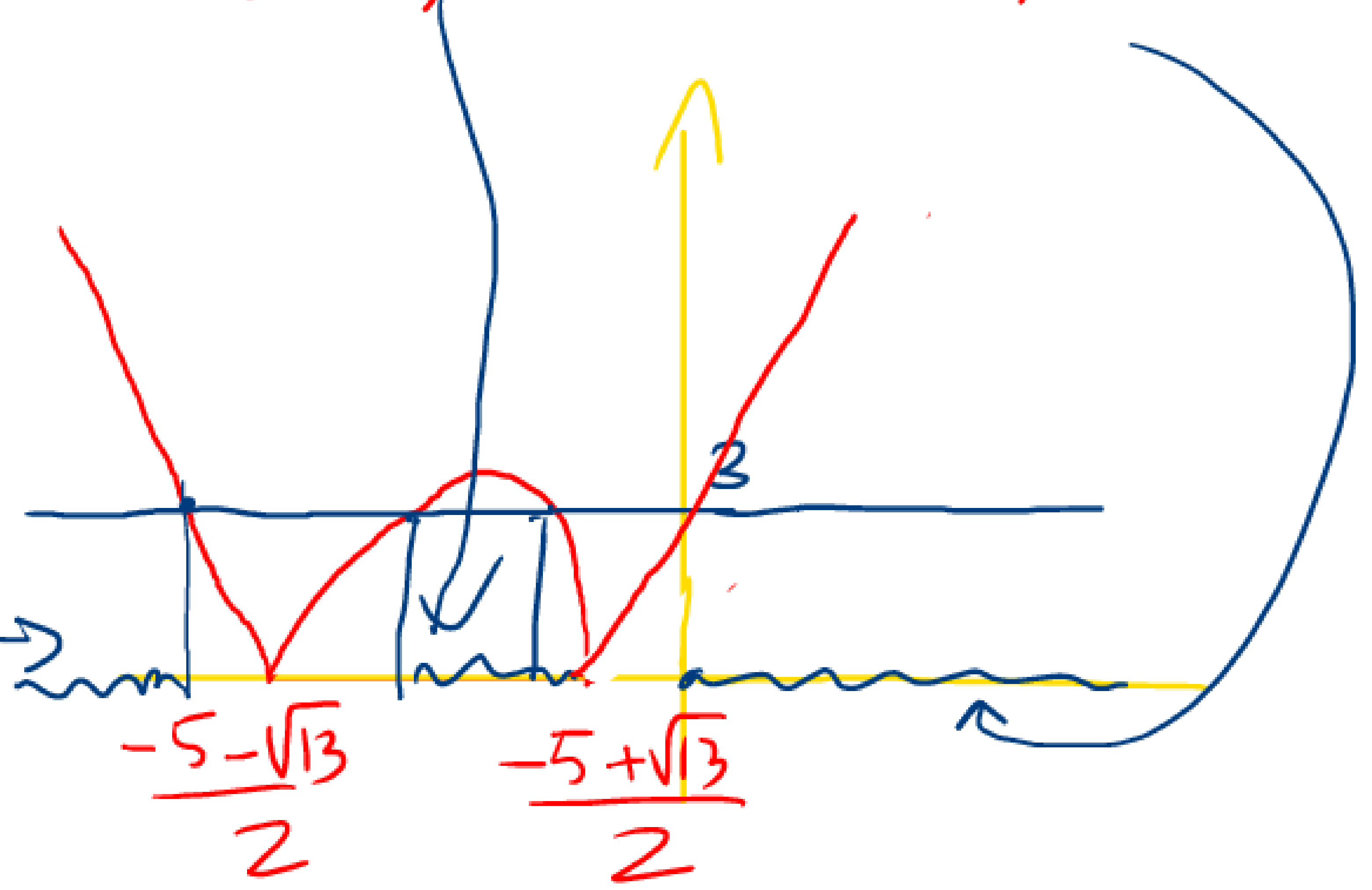




UNISCO LE SOLUZIONI ① E ②:

$$(-\infty, -5) \cup (0, +\infty) \cup (-3, -2) =$$

$$(-\infty, -5) \cup (-3, -2) \cup (0, +\infty)$$



RICEVIMENTO ONLINE 2

18/10/2022

DOMINI DI FUNZIONI

1. Determinare l'insieme di definizione della seguente funzione:

$$f(x) = |x| \arcsin(x-4) + \log_{\frac{1}{2}}(\sqrt{x}-1).$$

$$\begin{cases} -1 \leq x-4 \leq 1 & A \\ x \geq 0 & B \\ \sqrt{x}-1 > 0 & C \end{cases}$$

SOLUZIONE:

$3 \leq x \leq 5$

OVVERO $[3,5]$

A $\quad -1 \leq x-4 \Rightarrow x \geq 3$

$\quad x-4 \leq 1 \Rightarrow x \leq 5$

$\longrightarrow \quad 3 \leq x \leq 5$

B $\quad x \geq 0$

C $\quad \sqrt{x} > 1 \longrightarrow x > 1$

1. Determinare l'insieme di definizione della seguente funzione:

$$f(x)=\frac{\sqrt[4]{1+\log_{1/3}(3x^2-12x)}}{|3x-2|}$$

$$\begin{cases} 1+\log_{1/3}(3x^2-12x)\geq 0 & \text{(A)} \\ 3x^2-12x>0 & \text{(B)} \\ |3x-2|\neq 0 & \text{(C)} \end{cases}$$

(A) $\log_{1/3}(3x^2-12x)\geq -1$

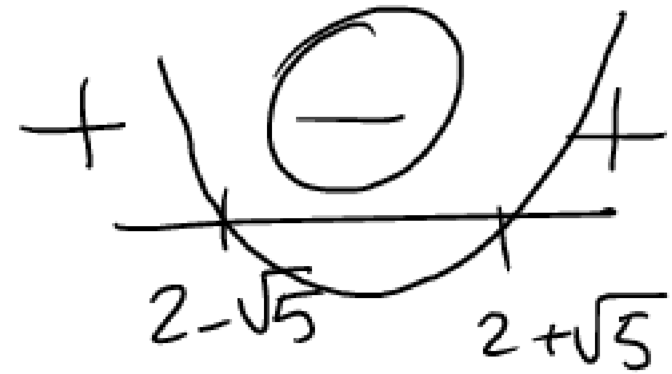
$$\Rightarrow \left(\tfrac{1}{3}\right)^{\log_{1/3}(3x^2-12x)} \leq \left(\tfrac{1}{3}\right)^{-1}$$

$$\underbrace{\left(\tfrac{1}{3}\right)^{\log_{1/3}(3x^2-12x)}}_{=\,3x^2-12x} \quad\Rightarrow\quad 3x^2-12x \leq 3$$

APPLICO LA F. INVERSA DI $\log_{1/3}$ CIOÈ EXP DI BASE $\frac{1}{3}$

A $\cancel{3}x^2 - \overset{4}{\cancel{12}}x - \cancel{3} \le 0$

$x^2 - 4x - 1 \le 0$



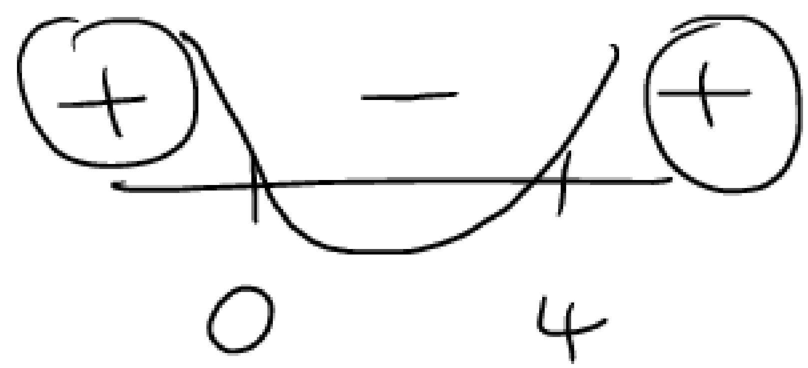
$\Delta = 16 + 4 = 20 > 0$

$$x_{1,2} = \frac{4 \pm \sqrt{20}}{2} = \frac{\overset{2}{\cancel{4}} \pm \cancel{2}\sqrt{5}}{\cancel{2}} = 2 \pm \sqrt{5}$$

$2 - \sqrt{5} \le x \le 2 + \sqrt{5}$

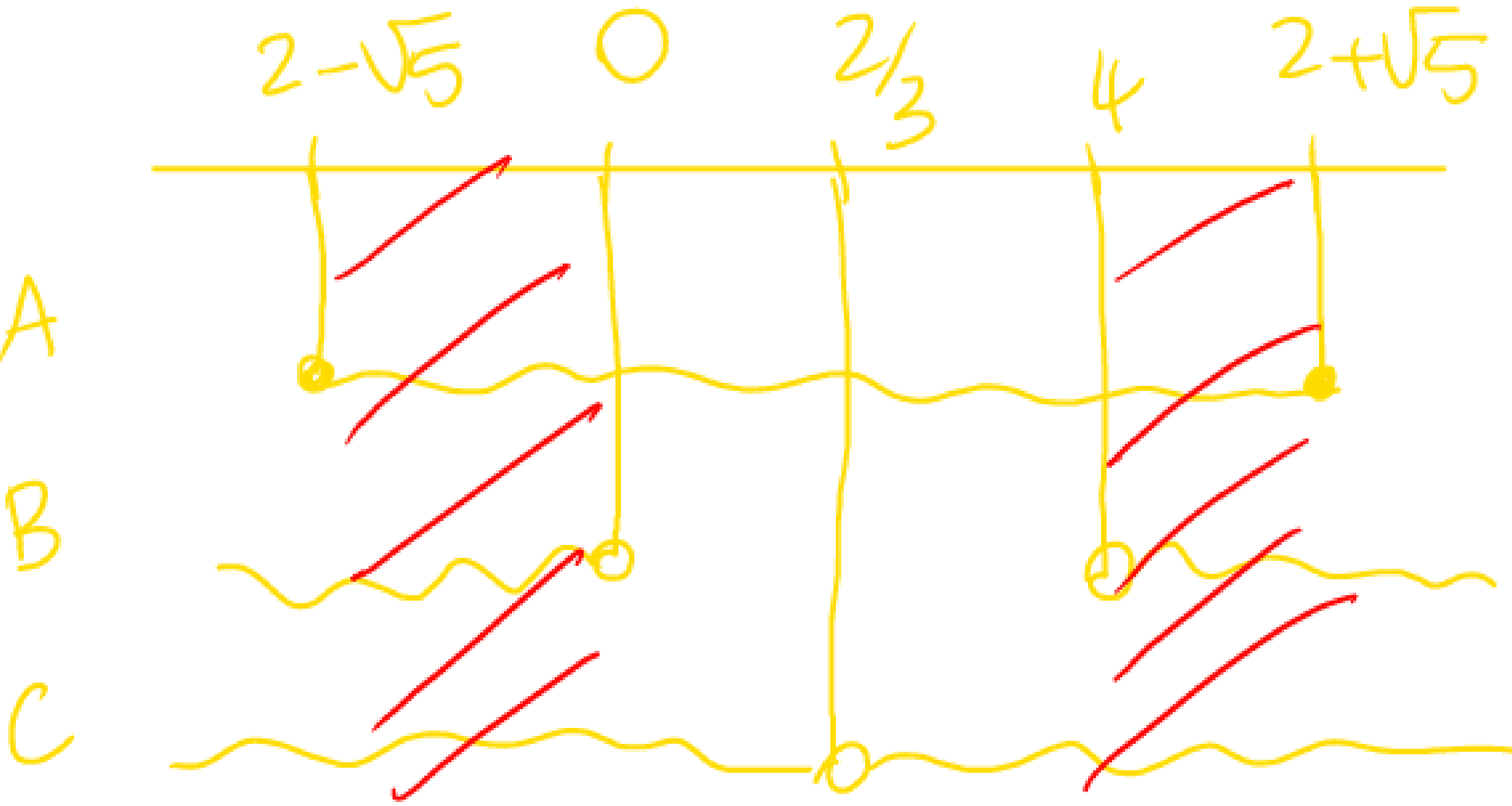
B $3x^2 - 12x > 0$

$3x(x-4) > 0$



$x < 0 \ \vee\ x > 4$

C $|3x-2| \neq 0 \Rightarrow x \neq 2/3$



SOLUZIONE: $2-\sqrt{5} \leq x < 0 \quad \vee$

$4 < x \leq 2+\sqrt{5}$.

OVVERO $[2-\sqrt{5}, 0) \cup (4, 2+\sqrt{5}]$.

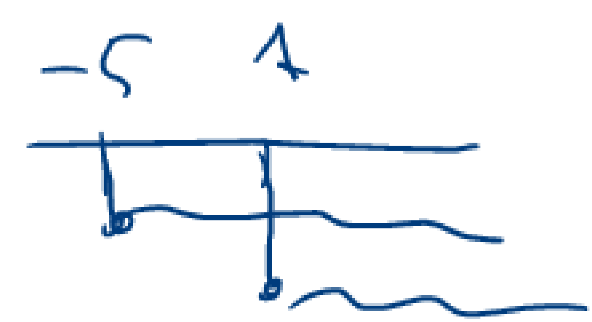
1. Determinare l'insieme di definizione della seguente funzione

$$f(x) = \log\left(x - 1 + \sqrt{x+5}\right).$$

$$\begin{cases} x+5 \geq 0 \\ x-1+\sqrt{x+5} > 0 \end{cases} \qquad x \geq -5$$

$$\sqrt{x+5} > 1-x$$

$$\begin{cases} x+5 \geq 0 \\ 1-x > 0 \\ x+5 > (1-x)^2 \end{cases} \cup \begin{cases} x+5 \geq 0 \\ 1-x \leq 0 \end{cases} \qquad \begin{matrix} x \geq -5 \\ x \geq 1 \end{matrix}$$



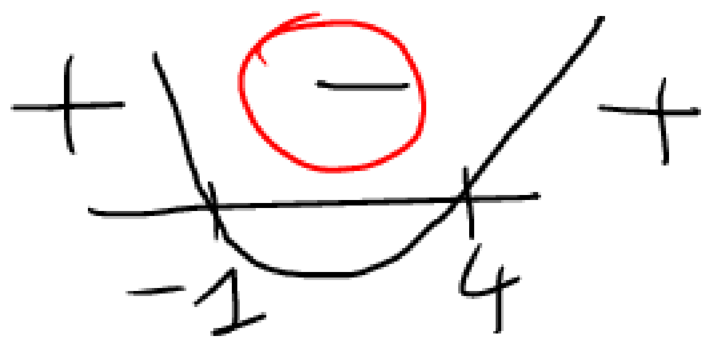
$[1,+\infty)$ SOLZ: $x \geq 1$

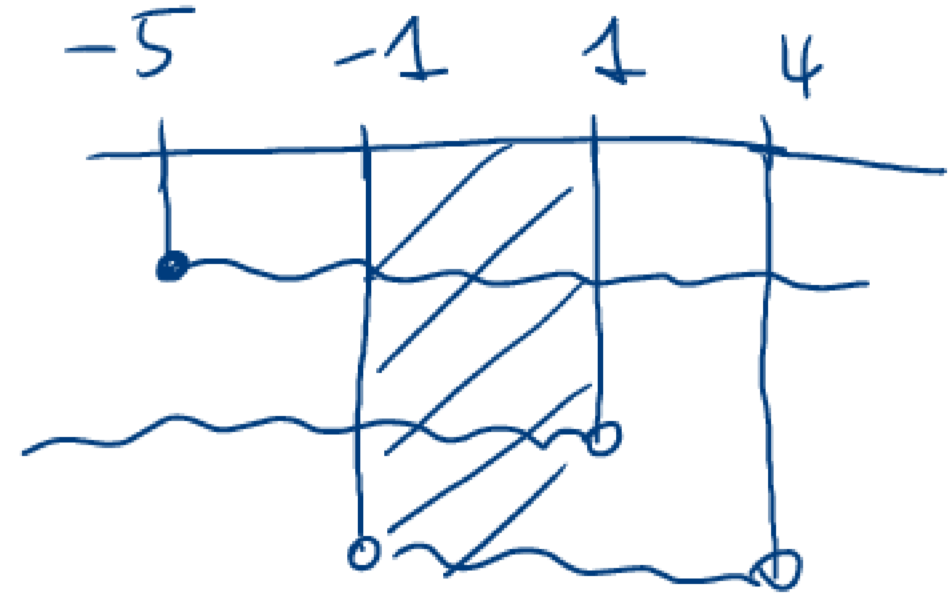
$$\begin{cases} x \geq -5 \\ x < 1 \\ x+5 > x^2+1-2x \end{cases}$$

$$x^2 - 3x - 4 < 0$$

$$\Delta = 9 + 16 = 25 > 0$$

$$x_{1,2} = \frac{3 \pm 5}{2} = \begin{cases} 4 \\ -1 \end{cases}$$





Sol1 : $-1 < x < 1$

ovvero $(-1, 1)$

METTO INSIEME SOL 1 E SOL 2:

$$(-1,1) \cup [1,+\infty) = (-1,+\infty)$$

SOLUZIONE È $x > -1$.

RICEVIMENTO

$\cos^2 x \neq 0 \to x \neq \pi/2 + k\pi$

1. $$\sin^2 x - (1+\sqrt{3})\sin x \cos x + \sqrt{3}\cos^2 x = 0$$

$$\tan^2 x - (1+\sqrt{3})\tan x + \sqrt{3} = 0$$

$$y^2 - (1+\sqrt{3})y + \sqrt{3} = 0$$

$$y_{1,2} = \frac{(1+\sqrt{3}) \pm \sqrt{4+2\sqrt{3}-4\sqrt{3}}}{2}$$

$$= \frac{1+\sqrt{3}}{2} \pm \frac{\sqrt{4-2\sqrt{3}}}{2}$$

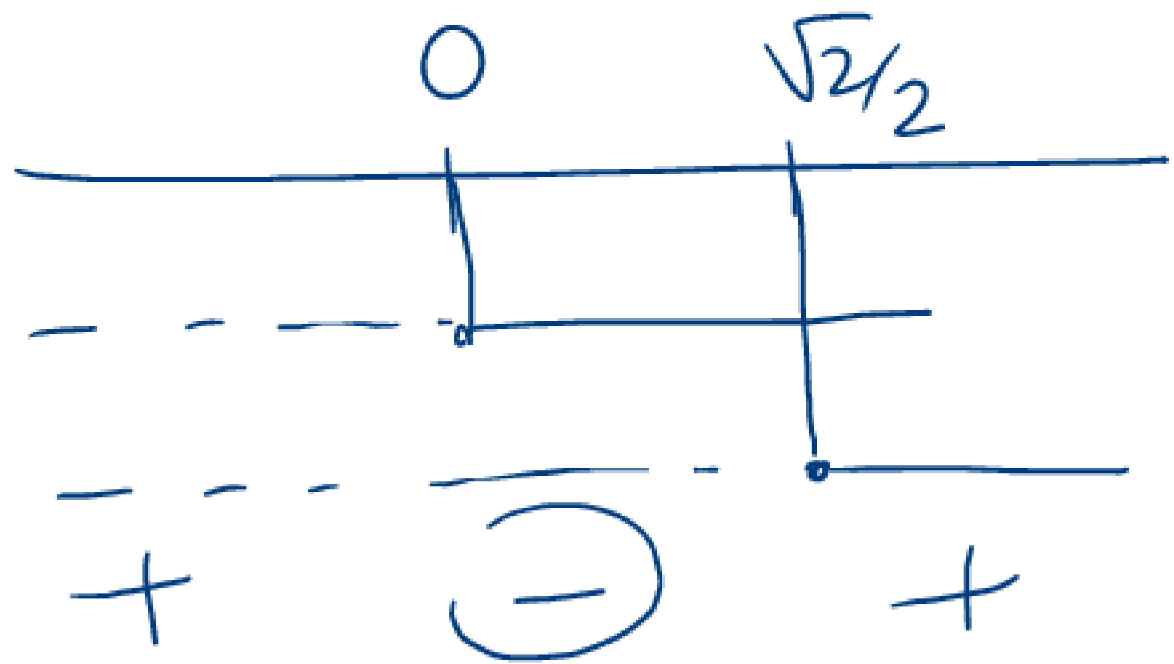
② $$\operatorname{sen}^2 x - \frac{\sqrt{2}}{2}\operatorname{sen} x \le 0$$

$y = \operatorname{sen} x$

$$y^2 - \frac{\sqrt{2}}{2} y \le 0$$

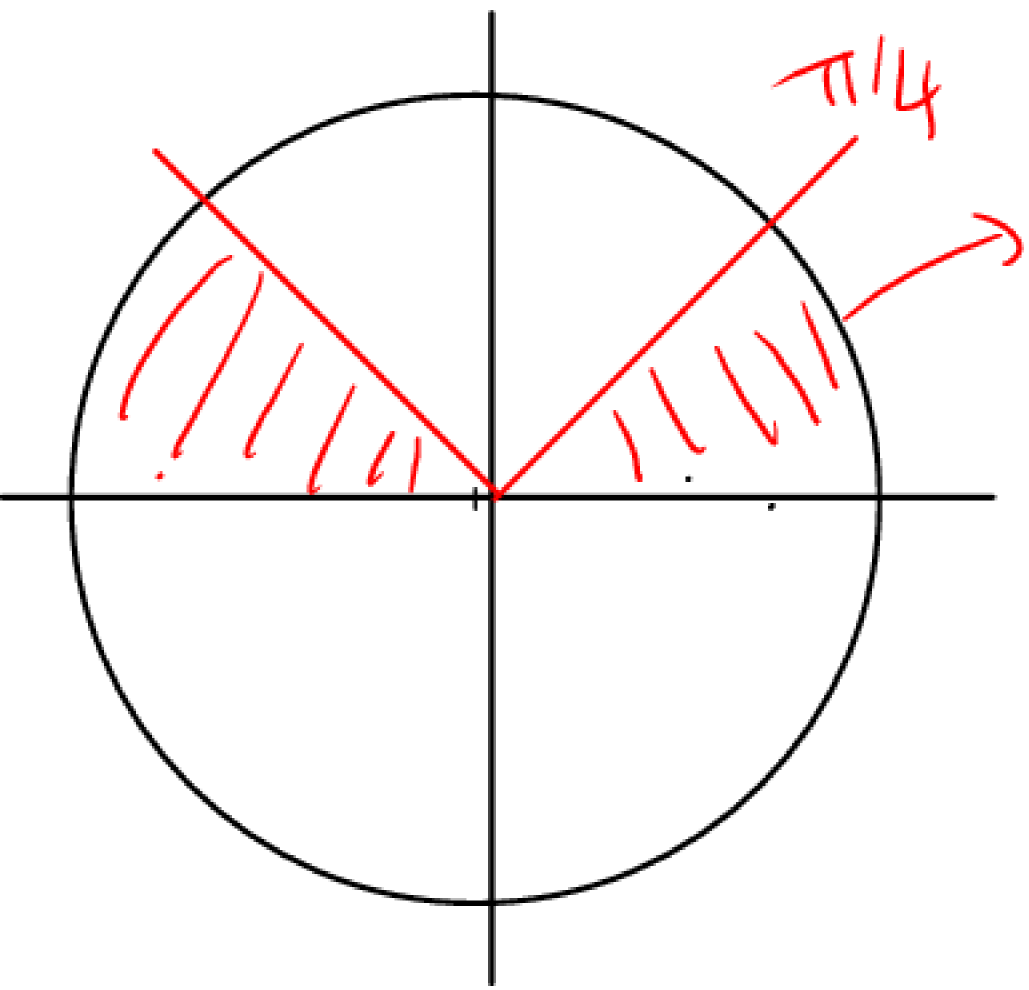
$$y\left(y - \frac{\sqrt{2}}{2}\right) \le 0$$

$$0 \le y \le \frac{\sqrt{2}}{2}$$



Se $0 \le y \le \frac{\sqrt{2}}{2} \to$

$$0 \le \operatorname{sen} x \le \frac{\sqrt{2}}{2}$$



$$0 \le x \le \pi/4 \;\cup$$

$$\tfrac{3}{4}\pi \le x \le \pi$$

$$(+2k\pi)$$

ANALISI I

15/11/2022

MASSIMI E MINIMI

VINCOLATI

MASSIMI E MINIMI ASSOLUTI

TEO DI WEIERSTRASS

SIA $f \in C([a,b])$. ALLORA f AMMETTE MASSIMO E MINIMO.

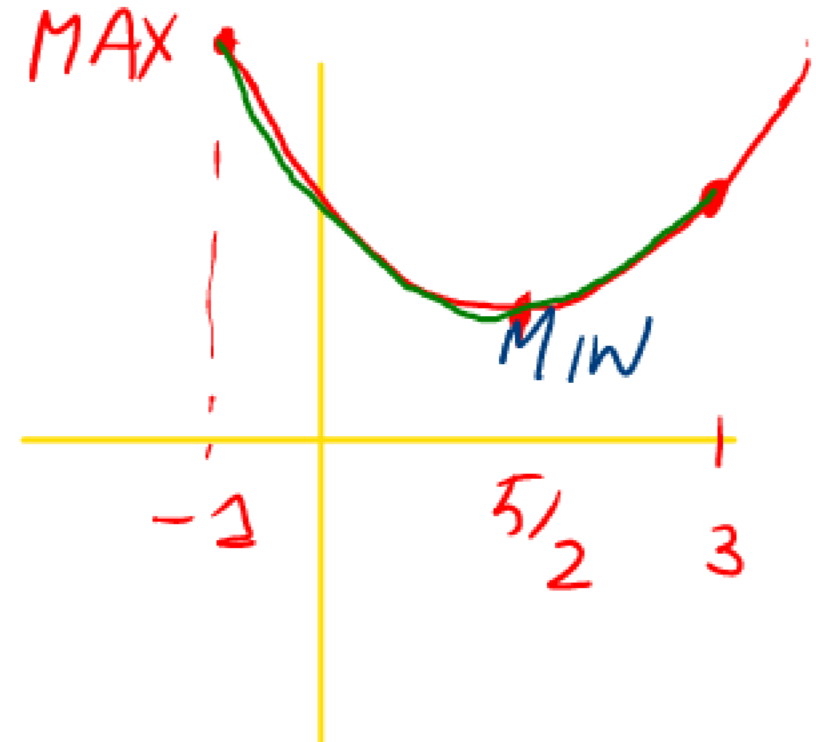
COME SI TROVANO?? SE f È DERIVABILE E MASSIMO O IL MINIMO SONO INTERNI ALLORA DAL TEOREMA DI FERMAT $f'(x_0) = 0$.

STRATEGIA: COSTRUIRE UNA LISTA DI CANDIDATI AD ESSERE MAX / MIN.

1. PUNTI CRITICI ($f'(x) = 0$)
2. " " (DI NON DERIVABILITÀ)
3. ESTREMI DELL'INTERVALLO.

VALUTO $f(x)$ IN TUTTI QUESTI PUNTI:
IL VALORE PIÙ GRANDE È IL MASSIMO,
IL VALORE PIÙ PICCOLO È IL MINIMO.

$f(x) = x^2 - 5x + 7$ nell'itervallo $[-1, 3]$



$f \in C^1([-1, 3])$

$\Rightarrow$ NON CI SONO PUNTI DI NON DERIVABILITÀ

$$f'(x) = 2x - 5 = 0 \Rightarrow x = \frac{5}{2}$$

LISTA: $-1, \ 5/2, \ 3$

$f(-1) = 3$ MAX

$f(5/2) = 7 - \frac{25}{4} = 3/4$ MIN

$f(3) = 1$

$f(x) = x^3 - 6x^2 + 9x + 2$ nell'itervallo $[0, 4]$

$f \in C^1([0,4]) \Rightarrow$ NO PUNTI DI NON DERIVABILITÀ

$f'(x) = 3x^2 - 12x + 9 = 0$

$$x_{1,2} = \frac{6 \pm \sqrt{36-27}}{3} = \frac{6 \pm 3}{3} = \begin{cases} 3 \\ 1 \end{cases}$$

LISTA: 0, 1, 3, 4

MIN $f(0) = 2$

MAX $f(1) = 6$

MAX $f(4) = 6$

$f(3) = \cancel{27} - \cancel{54} + \cancel{27} + 2 = 2$ MIN

$f(x) = x^2 e^x$ nell'itervallo $[-5, 1]$

$$f \in C^1([-5,1]).$$

$$f'(x) = 2xe^x + x^2 e^x = \cancel{e^x}(x^2+2x) = 0$$

$$\iff x(x+2) = 0 \iff x = 0 \ \vee \ x = -2$$

LISTA:

x	$f(x)$		
-5	$25e^{-5}$	$\simeq \dfrac{25}{3^5} = \dfrac{25}{273}$	
-2	$4e^{-2}$	$\simeq \dfrac{4}{9}$	
0	0		MIN
1	e		MAX

$f(x) = \ln(x^2 + 2x + 4)$ nell'itervallo $[-4, 3]$

$\mathrm{Dom}\, f: \quad x^2+2x+4>0 \Rightarrow \Delta = 4-16 = -12 < 0$

SEMPRE $\Rightarrow \mathrm{Dom}\, f = \mathbb{R}$

$f \in C^1([-4,3]) \Rightarrow$ NO PTI NON DERIVABILITA'

$$f'(x) = \frac{2x+2}{\cancel{x^2+2x+4}} = 0 \Rightarrow x = -1$$

x	$f(x)$	
-4	$\ln(12)$	
-1	$\ln(3)$	MIN
3	$\ln(19)$	MAX

$f(x) = \sqrt[3]{(x+1)^2}$ nell'itervallo $[-3, 4]$

$$f'(x) = \left((x+1)^{2/3} \right)' = \frac{2}{3}(x+1)^{-1/3} =$$

$$= \frac{2}{3\sqrt[3]{(x+1)}} = 0 \quad \text{MAI!}$$

NON È DEFINITA IN $\sqrt[3]{(x+1)} = 0 \Rightarrow x = -1$

CIOÈ $x = -1$ È UN PUNTO DI NON DERIVABILITÀ.

x	$f(x)$
-3	$\sqrt[3]{4}$
-1	0 MIN
4	$\sqrt[3]{25}$ MAX

ESERCIZI PER CASA:

1. $f(x) = x\,\mathrm{e}^{-x},\ x \in [0,2];$
2. $f(x) = x \log x,\ x \in [\frac{1}{\mathrm{e}^2}, 1];$
3. $f(x) = (2x+1)\,\mathrm{e}^{-x},\ x \in [0,1];$
4. $f(x) = x + 2\cos x,\ x \in [0, \pi/2];$
5. $f(x) = \dfrac{\mathrm{e}^{x/2}}{x+1},\ x \in [0,2].$

ANALISI I

6/12/2022

ESERCIZI SUGLI INTEGRALI

4. Calcolare il seguente integrale indefinito

$$\int \frac{e^{4x}}{e^{2x} - 3e^{x} + 2}\, dx$$

$$\int \frac{e^{4x}}{e^{2x}-3e^{x}+2}\, dx =$$

$$\left[\begin{array}{l} y = e^{x} \\ dy = e^{x}\, dx \end{array}\right] \rightarrow \left[\begin{array}{l} x = \log y \\ dx = \frac{1}{y}\, dy \end{array}\right]$$

$$= \int \frac{y^{\cancel{4}\,3}}{y^2-3y+2} \frac{1}{\cancel{y}}\, dy = \int \frac{y^3}{y^2-3y+2}\, dy$$

$$\overset{(*)}{=} \int (y+3)\, dx + \int \frac{7y-6}{y^2-3y+2}\, dy =$$

(*) DIVISIONE LUNGA:

$$
\begin{array}{rrrr|l}
y^3 & 0 & 0 & 0 & y^2-3y+2 \\
\cline{5-5}
-y^3 & +3y^2 & -2y & & y+3 \\
\cline{1-4}
// & 3y^2 & -2y & & \\
 & -3y^2 & +9y & -6 & \\
\cline{2-4}
 & // & 7y & -6 &
\end{array}
$$

$$= \frac{y^2}{2} + 3y + \int \frac{7y-6}{(y-1)(y-2)}\,dy =$$

FRATTI SEMPLICI :

$$\frac{7y-6}{(y-1)(y-2)} = \frac{A}{y-1} + \frac{B}{y-2}$$

$$\frac{7y-6}{y-2} = A + \frac{B\cancel{(y-1)}}{\cancel{y-2}} \xrightarrow{y=1} A = -1$$

$$\frac{7y-6}{y-1} = \frac{A\cancel{(y-2)}}{\cancel{y-1}} + B \xrightarrow{y=2} B = 8$$

$$= -\int \frac{1}{y-1}\,dy + 8\int \frac{1}{y-2}\,dy =$$

$$= -\log|y-1| + 8\log|y-2| + c$$

METTENDO INSIEME:

$$\int \frac{e^{4x}}{e^{2x}-3e^{x}+2}\,dx = \frac{y^2}{2} + 3y - \log|y-1| + 8\log|y-2| + c$$

$$= \frac{e^{2x}}{2} + 3e^{x} - \log|e^{x}-1| + 8\log|e^{x}-2| + c$$

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4. Calcolare il seguente integrale indefinito

$$\int \frac{(\sqrt{\cos(x)}+1)\sqrt{\cos(x)}\tan(x)}{\cos(x)+2}\,dx$$

$$\int \frac{(\sqrt{\cos x}+1)\sqrt{\cos x}}{\cos(x)+2}\,\frac{\sin x}{\cos x}\,dx =$$

$-dy$

$$\left[\begin{aligned} y &= \cos x \\ dy &= -\sin x\,dx \end{aligned}\right]$$

$$= -\int \frac{(\sqrt{y}+1)\cancel{\sqrt{y}}}{(y+2)\,\cancel{\sqrt{y}}\,y}\,dy =$$

$$= -\int \frac{(\sqrt{y}+1)}{(y+2)\sqrt{y}}\,dy =$$

$$\left[\begin{aligned} z &= \sqrt{y} \\ dz &= \frac{1}{2\sqrt{y}}\,dy \end{aligned}\right]$$

$$= -2\int \frac{z+1}{z^2+2}\,dz =$$

$\longrightarrow$

$$= -2\int \frac{z+1}{z^2+2}\,dz \quad =$$

$$= -\int \frac{2z}{z^2+2}\,dz - 2\int \frac{1}{z^2+2}\,dz$$

$$\left(w = \frac{z}{\sqrt{2}} \quad dw = \frac{1}{\sqrt{2}}\,dz \right)$$

$$= -\log(z^2+2) - \frac{\cancel{2}}{\cancel{2}}\int \frac{1}{\left(\frac{z}{\sqrt{2}}\right)^2+1}\,dz$$

$$= -\log(z^2+2) - \sqrt{2}\arctan\left(\frac{z}{\sqrt{2}}\right) + c$$

$$= -\log(\cos x + 2) - \sqrt{2}\arctan\left(\sqrt{\frac{\cos x}{2}}\right) + c\,.$$

$$\left(\arctan\left(\frac{z}{\sqrt{2}}\right)\right)' = \frac{1}{\left(\frac{z}{\sqrt{2}}\right)^2+1}\,\frac{1}{\sqrt{2}}$$

4. Calcolare il seguente integrale indefinito

$$\int x \log\left(\frac{x^2}{x^2-2}\right) dx$$

$$\int x \log\left(\frac{x^2}{x^2-2}\right) dx =$$

$$\left[\begin{aligned} y &= x^2 \\ dy &= 2x\,dx \end{aligned}\right]$$

$$= \frac{1}{2} \int \log\left(\frac{y}{y-1}\right) dy = \quad [\text{PER PARTI}]$$

$$= \frac{1}{2}\left[y \log\left(\frac{y}{y-1}\right) - \int \cancel{y}\, \frac{\cancel{y-1}}{\cancel{y}} \cdot \frac{(\cancel{y}-1)-\cancel{y}}{(y-1)^{\cancel{2}}}\, dy \right]$$

$$= \frac{1}{2}\left[y \log\left(\frac{y}{y-1}\right) + \int \frac{1}{y-1}\, dy \right] =$$

$$= \frac{1}{2} y \log\left(\frac{y}{y-1}\right) + \frac{1}{2} \log|y-1| + c \quad =$$

$$= \frac{1}{2} x^2 \lg\left(\frac{x^2}{x^2-2}\right) + \frac{1}{2} \lg |x^2 - 1| + C .$$

4. Calcolare il seguente integrale indefinito

$$\int e^{2x} \cosh(e^x + 1)\, dx$$

$$\int e^{2x} \cosh(e^x+1)\, dx =$$

$e^x \cdot e^x$

$$\left[\begin{array}{l} y = e^x + 1 \to e^x = y-1 \\ dy = e^x\, dx \end{array}\right]$$

$$= \int (y-1) \cosh(y)\, dy =$$

[PER PARTI]

$$= (y-1)\,\mathrm{senh}(y)$$

$$- \int 1 \cdot \mathrm{senh}(y)\, dy = (y-1)\,\mathrm{senh}(y) - \cosh(y)$$

$$= e^x\, \mathrm{senh}(e^x+1) - \cosh(e^x+1) + C \,.$$

4. Calcolare il seguente integrale indefinito

$$\int \frac{\sqrt{x}+2}{x+4}\,dx$$

$$\int \frac{\sqrt{x}+2}{x+4}\,dx =$$

$y = \sqrt{x} \longrightarrow x = y^2$

$dx = 2y\,dy$

$$= \int \frac{y+2}{y^2+4} \cdot 2y\,dy =$$

$$= 2\int \frac{y^2+2y}{y^2+4}\,dy =$$

$$= 2\left[\int \frac{y^2+4-4}{y^2+4}\,dy + \int \frac{2y}{y^2+4}\,dy\right] =$$

$$= 2\left[\int 1\, dy - 4\int \frac{1}{y^2+4}\, dy + \int \frac{2y}{y^2+4}\, dy\right] =$$

$$= 2\left[y - \frac{\cancel{4}}{\cancel{4}}\int \frac{1}{\left(\frac{y}{2}\right)^2+1}\, dy + \log(y^2+4)\right]$$

$$= 2y - 4\arctan(y/2) + 2\log(y^2+4) + c$$

$$= 2\sqrt{x} - 4\arctan\left(\frac{\sqrt{x}}{2}\right) + 2\log(x+4) + c$$

$$\left[\begin{array}{l} z = \frac{y}{2} \Rightarrow \\ dz = \frac{1}{2}dy \rightarrow dy = 2dz \end{array}\right. \qquad \int \frac{1}{(y/2)^2+1}\, dy = \int \frac{2}{z^2+1}\, dz$$

$$= 2\arctan(z) = 2\arctan\left(\frac{y}{2}\right)\Big]$$

4. Calcolare il seguente integrale indefinito

$$\int \frac{1}{x^4}\cos\left(\frac{1}{x}\right)\,dx$$

$$\int \frac{1}{x^4}\cos\left(\frac{1}{x}\right)dx = \quad \left[\begin{array}{l} z = \dfrac{1}{x} \\ x = \dfrac{1}{z} \rightarrow dx = -\dfrac{1}{z^2}dz \end{array}\right]$$

$$= \int z^{\cancel{4}\,2}\cos(z)\left(-\frac{1}{\cancel{z^2}}dz\right) =$$

$$= -\int z^2\cos(z)\,dz = \quad [\text{PER PARTI}]$$

$$= -\left[z^2 \operatorname{sen}(z) - \int 2z\operatorname{sen}(z)\,dz\right] =$$

$$-\left[2z(-\cos(z)) + \int 2\cos z\,dz\right]$$

$$= -\left[z^2 \operatorname{sen}(z) + 2z\cos(z) - \int 2\cos(z)\, dz \right]$$

$$= -\left[z^2 \operatorname{sen}(z) + 2z\cos(z) - 2\operatorname{sen}(z)\, dz \right]$$

$$= -\frac{1}{x^2}\operatorname{sen}\left(\frac{1}{x}\right) - \frac{2}{x}\cos\left(\frac{1}{x}\right) + 2\operatorname{sen}\left(\frac{1}{x}\right) + C \, .$$