Unit-6 Curve Fitting:

Let $(x_1,y_1)(x_2,y_2) \ldots (x_n,y_n)$ are the given data points to fit a curve $y=f(x)$ by using method of least squares which states that we can fit a best curve when the sum of the squares of the residues is the most minimum

i.e $d_i = y_i - f(x_i)$ is the general term of the residue and $\sum_{i=1}^{n} d_i^2 \simeq 0$

Fitting of straight line: $(y = a + bx)$

$$\sum d_i^2 = \sum [y_i - f(x_i)]^2 = 0$$

$$\sum [y_i - f(x_i)]^2 = 0$$

$$\Rightarrow \sum [y_i - (a+bx_i)]^2 = 0 \quad \text{—— (1)}$$

differentiate eq (1) w.r.to 'a'.

$$2\left[\sum [y_i - (a+bx_i)\right](-1) = 0$$

$$\sum y_i - \sum (a+bx_i) = 0$$

$$\sum y_i = \sum a + \sum bx_i \Rightarrow \boxed{\sum y = na + b\sum x} \text{ —— (2)}$$

differentiate 1, w.r.t 'b'

$$\Rightarrow 2\left[\sum y_i - (a+bx_i)\right](-x_i) = 0$$

$$\sum y_i x_i - \sum (a+bx_i)(x_i) = 0$$

$$\boxed{\sum yx = a\sum x + b\sum x^2} \text{ —— (3)}$$

the normal equation for fitting the straight line are

$$\boxed{\Sigma y = na + b\Sigma x}$$

$$\boxed{\Sigma yx = a\Sigma x + b\Sigma x^2}$$

fit a straight line by using method of least squares using the following data.

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3
xy	0	1.8	6.6	13.5	25.2

$\Sigma x = 10$
$\Sigma y = 16.9$
$\Sigma xy = 47.1$

Normal equations are.

$$\boxed{\Sigma y = na + \Sigma x}$$

$$\Sigma xy = a\Sigma x + b\Sigma x^2$$

$\Sigma x^2 = 0+1+4+9+16 = 30.$

$\therefore$ Normal equations are

$$16.9 = 5a + 10b$$

$$47.1 = 10a + 30b.$$

$$\Rightarrow \begin{aligned} 33.8 &= 10a + 20b \\ 47.1 &= 10a + 30b \\ \hline -13.3 &= -10b \end{aligned}$$

$$\boxed{b = 1.33}$$

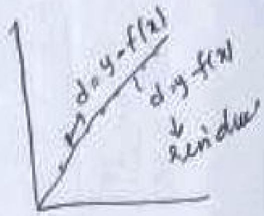
$$5a = 16.9 - 13.3$$

$$a = \frac{3.6}{5}$$

$$\boxed{a = 0.72}$$

$$\boxed{\therefore\ y = 0.72 + (1.33)x}$$

* Fit a straight line by taking x as dependent Variable for the following data.



x	y	y^2	xy
1	1	1	1
3	2	4	6
4	4	16	16
6	4	16	24
8	5	25	40
9	7	49	63
11	8	64	88
14	9	81	126
$\Sigma x = 56$	40	256	364

$$\Sigma x = na + b\Sigma y$$

$$\Sigma xy = a\Sigma y + b\Sigma y^2$$

$$56 = 8a + 40b$$

$$7 = a + 5b \leftarrow (1)$$

$$364 = 40a + 256b$$
$$280 = 40a + 200b$$

$$84 = 56b$$

$$b = \frac{84}{56} = 1.5$$

$$a = 7 - 7.5$$

$$\boxed{a = -0.5}$$

$$\boxed{x = -0.5 + 1.5y}$$

Fitting of second degree polynomial:

$y = f(x) = a_0 + a_1 x + a_2 x^2$

$d_i = y_i - f(x_i)$

$= y_i - (a_0 + a_1 x + a_2 x^2)$

since $\sum d_i^2 = 0$

$\sum [y_i - (a_0 + a_1 x + a_2 x^2)]^2 = 0$ —— (1)

differentiate w.r.to 'a_0'

$2 \sum [y_i - (a_0 + a_1 x_i + a_2 x_i^2)] (-1) = 0$.

$\sum y_i - \sum (a_0 + a_1 x_i + a_2 x_i^2) = 0$

$\sum y_i = \sum a_0 + \sum a_1 x_i + \sum a_2 x_i^2$.

$\boxed{\sum y_i = n a_0 + a_1 \sum x_i + a_2 \sum x_i^2}$ — (2)

differentiate eq (1) w.r.t a_1

$2 [\sum y_i - (a_0 + a_1 x_i + a_2 x_i^2)] (-x_i) = 0$

$\boxed{\sum y_i x_i = a_0 \sum x_i + a_1 \sum x_i^2 + a_2 \sum x_i^3}$ — (3)

differentiate w.r.t a_2

$\boxed{\sum y_i x_i^2 = a_0 \sum x_i^2 + a_1 \sum x_i^3 + a_2 \sum x_i^4}$ — (4)

The equations 2, 3, 4 are the normal equations for fitting a second degree polynomial curve.

Fit a quadratic curve to the following data

x_i	1	2	3	4
y_i	1.7	1.8	2.3	3.2

Also estimate y at $x = 4$.

x	y	x^2	x^3	x^4	xy	x^2y
1	1.7	1	1	1	1.7	1.7
2	1.8	4	8	16	3.6	7.2
3	2.3	9	27	81	6.9	20.7
4	3.2	16	64	256	12.8	51.2
$\Sigma x = 10$	$\Sigma y = 9$	$30 = \Sigma x^2$	$100 = \Sigma x^3$	$354 = \Sigma x^4$	$25 = \Sigma xy$	$80.8 = \Sigma x^2 y$

$\boxed{n = 4}$

Let the curve be $y = a_0 + a_1 x + a_2 x^2$

Normal equations are

$$\Sigma y = n a_0 + a_1 \Sigma x + a_2 \Sigma x^2$$

$$\Sigma xy = a_0 \Sigma x + a_1 \Sigma x^2 + a_2 \Sigma x^3$$

$$\Sigma x^2 y = a_0 \Sigma x^2 + a_1 \Sigma x^3 + a_2 \Sigma x^4$$

$$9 = 4a_0 + a_1(10) + a_2(30)$$

$$4a_0 + 10a_1 + 30a_2 = 9 \quad \text{---} (1)$$

$$25 = 10a_0 + 30a_1 + 100a_2$$

$$10a_0 + 30a_1 + 100a_2 = 25 \quad \text{---} (2)$$

$$80.8 = 30a_0 + 100a_1 + 354a_3$$

$$30a_0 + 100a_1 + 354a_3 = 80.8 \quad \text{---} (3)$$

$$a_0 = 2, \quad a_1 = -0.5, \quad a_2 = 0.2$$

The curve is $y = 2 + (-0.5)x + 0.2x^2$

$$y = 2 - \frac{x}{2} + \frac{x^2}{5}$$

$$y = 2 - \frac{2.4}{2} + \frac{\overset{0.48}{2.4 \times 2.4}}{8}$$

$$y = 2 - 1.2 + 1.152$$

$$y = 1.952 \text{ (at } x = 2.4).$$

Fitting of exponential curve:

$$y = ae^{bx}$$

taking logarithm on both sides.

$$\log_e y = \log_e (ae^{bx})$$

$$\ln y = \ln a + \ln e^{bx}$$

$$\ln y = \ln a + bx.\log_e^e$$

$$\boxed{Y = A + bx.}$$ where $Y = \ln y$, $A = \ln a$.

The normal equations are.

$$\sum Y = nA + b\sum x$$

$$\sum xY = A\sum x + b\sum x^2$$

* Fit an exponential curve to the following data. estimate y value at 3.5.

x	1	2	3	4
y	7	11	17	27

x	y	$\ln y = Y$	x^2	xY
1	7	1.94	1	1.94
2	11	2.39	4	4.78
3	17	2.83	9	8.49
4	27	3.29	16	13.16
$\Sigma x = 10$	$\Sigma y = 62$	10.45	30	28.37

Normal equations are.

$$10.45 = 4A + 10b$$

$$28.37 = 10A + 30b.$$

$$A = 1.49$$

$$b = 0.449.$$

$A = \ln a \Rightarrow a = e^{A}$

$a = e^{1.49} = 4.44$

$b = 0.449.$

$x_n \quad y$

$\therefore$ The curve is $y = ae^{bx}$

$y = 4.44 \cdot e^{0.449x}$

$y = 4.44 \cdot e^{0.449 \times 3.5}$

$= 4.44 \cdot e^{1.57}$

$\boxed{y_{(3.5)} = 21.34}$

Fitting of a curve $y = ax^{b}$:

$y = ax^{b}$

taking log on both sides.

$\log y = \log(ax^{b})$

$\log y = \log a + b \cdot \log x$

$\Rightarrow Y = \log_{10} y \quad , \log_{10} a = A, \quad \log_{10} x = X$

$\boxed{Y = A + bX}$

Normal equations are:

$\Sigma Y = nA + b\Sigma X$

$\Sigma YX = A\Sigma X + b\Sigma X^{2}$

x	y	$Y = \log y$	$X = \log x$	$XY = (\log x)(\log y)$

Fitting of a Curve $x^a y = b$.

taking logarithm.

$\log x^a y = \log b$

$a \cdot \log x + \log y = \log b$

$\log y = \log b - a \cdot \log x$

$\boxed{Y = B - aX}$

$Y = \log y$, $B = \log b$, $\log x = X$

$\boxed{Y = B + (-a)X}$

Normal equations are

$\Sigma Y = nB + (-a)\Sigma X$

$\Sigma XY = B\Sigma X + (-a)\Sigma X^2$

Fit a curve $y = ax^b$ to the following data and estimate y at $x = 12$.

x	20	16	10	11	14
y	22	41	120	89	56

x	y	$X = \log x$	$Y = \log y$	XY	X^2
20	22	1.3	1.34	1.74	1.69
16	41	1.2	1.61	1.93	1.44
10	120	1	2.08	2.08	1
11	89	1.04	1.95	2.03	1.08
14	56	1.15	1.75	2.01	1.32
		5.7	8.73	9.79	6.53

$\Sigma y = A(n) + b\,\Sigma x$

$8.73 = A(5) + b(5.69)$

$8.73 = 5A + 5.69b$ — (1)

$\Sigma xy = A\,\Sigma x + b\,\Sigma x^2$

$9.79 = A(5.69) + b(6.53)$

$9.79 = 5.69A + 6.53b$ — (2)

$A = 4.75,\; b = -2.64$

$\log a = 4.75$

$a = 10^{4.75} \Rightarrow a = 5.6 \times 10^{4}$

$\therefore$ The curve is $y = a \cdot x^{b}$

$y = 5.6 \times 10^{4} \cdot x^{-2.64}$

at $x = 12$

$y = 5.6 \times 10^{4} \cdot (12)^{-2.64}$

$\boxed{y(12) = 78.4}$

$x^a y = b$ (fit)

Normal equations are

$$\sum y = B + (-a)\sum x$$

$$\sum xy = B\sum x + (-a)\sum x^2$$

$$8.73 = B \cdot 6 - a(5.69)$$

$$8.73 = B - 5.69a \quad \text{①}$$

$$9.79 = B(5.69) - a(6.53)$$

$$9.79 = 5.69B - 6.53a \quad \text{②}$$

$$a = -2.8516, \quad B = 4.99$$

$$b = 10^B = 10^{4.99} = 97723.722$$

$$x^a y = b$$

$\therefore$ The curve is $x^{-2.8516} \times y = 97723.722$

$$y \text{ at } 12 = y = \frac{97723.722}{(12)^{-2.8516}}$$

$$= 116785974.6$$

Fitting of $y = ab^x$

taking logarithms

$$\log y = \log ab^x$$

$$\log y = \log a + x \cdot \log b$$

put $\log y = Y$, $\log a = A$, $\log b = B$

$$\boxed{Y = A + Bx}$$

x	y	$Y = \log y$	x^2	$xY = x\log y$

Normal equations are

$$\boxed{\begin{aligned} \sum Y &= nA + B\sum x \\ \sum xY &= A\sum x + B\sum x^2 \end{aligned}}$$

x	y	$Y = \log y$	x^2	$xY = x\cdot\log y$
20	22	1.34	400	26.8
16	41	1.61	256	25.76
10	120	2.08	100	20.8
11	89	1.95	121	21.45
14	56	1.75	196	24.5
71		8.73	1093	119.31

$$8.73 = 5(A) + B(\overset{71}{\cancel{119.31}})$$

$$8.73 = 5A + \overset{71}{\cancel{119.31}}B \quad \text{—} (1)$$