UNIT-3 Test of hypothesis

The principal objective of statistical inference is to draw inference (or generalize) about the population on the basis of data collected by sampling from the population.

Statistical inference consists of two major areas.

(1) Estimation
(2) Test of hypothesis.

In test of hypothesis a postulate or a statement about a parameter of the population is tested for its validity or hypothesis.

* Statistical hypothesis is an assumption or guess about the parameter of population distribution.

Null hypothesis: This is denoted by H_0. This statement is tested for possible acceptance or rejection under the assumption that it is true.

Example(1): Suppose the average height of Indian solider is 164cm then we set up null hypothesis as

H_0: The avg height of the soldiers is 164

$H_0: \mu = \mu_0 = 164cm$.

Ex(2): Suppose we want to test the average mark between two groups then we setup null hypothesis as

H_0: The avg marks b/w 2 groups are equal

$H_0: \mu_1 = \mu_2$.

Alternate Hypothesis: Any hypothesis which is complementary to null hypothesis is called an alternate hypothesis & is denoted by H_1.

Example (1): Suppose the average height of Indian solider is 164cm then we set up

$\mu \neq \mu_0 \rightarrow (1)$

Alternate hypothesis is further divided into two statements

H_1: The average height of a Indian solider is greater than 164cm.

$H_1: \mu > \mu_0 \longrightarrow (2)$

H_1: The average height of Indian soilder is less than 164cm

$H_1: \mu < \mu_0 \longrightarrow (3)$

Similarly for the Example(2) we have

H_1: The average marks b/w two groups are not equal.

$H_1: \mu_1 \neq \mu_2$

H_1: The average marks of group one is more than group2

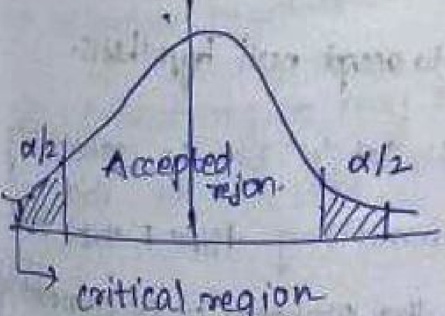
$H_1: \mu_1 > \mu_2$

H_1: The average marks of group one is ~~more~~ less than group2

$H_1: \mu_1 < \mu_2$.

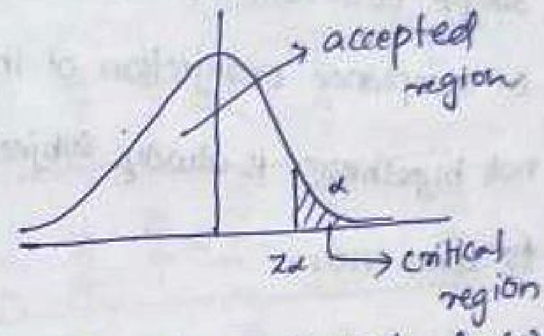
* To test the hypothesis the test conducted for (1) is known as 'Two tailed Test' (TTT)

for two tailed test the critical region lies on both ends



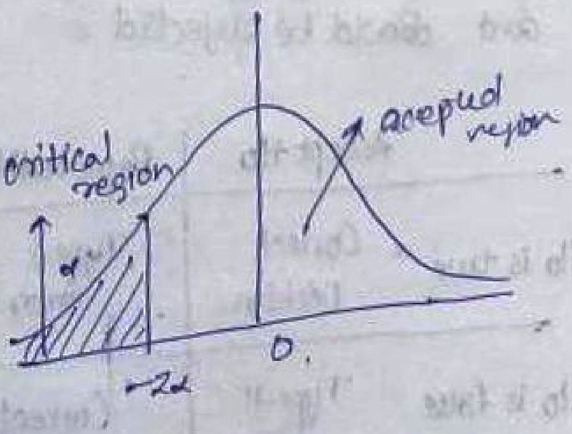
* The test conducted for (2) is known as 'Right Tailed Test' (RTT)

In this test the critical region lies entirely on the right hand side of the distribution.



* The test conducted for (3) is known as 'Left Tailed Test' (LTT)

In this test critical region lies entirely towards left side.



$Z_\alpha \Rightarrow$ critical value.

* Test of hypothesis decides whether a statement concerning a parameter is true or false instead of estimating the value of the parameter.

* Since the test is based on sample observations, the decision of acceptance or rejection of the null hypothesis is always subjected to some error.

Types of errors in test of hypothesis:

Type-I Error: Involves rejection of null hypothesis when it should: should be accepted.

Type-II Error. Involves acceptance of null hypothesis when it is false and should be rejected

	Accept Ho	Rejected Ho
Ho is true	Correct Decision	Type-I Error
Ho is false	Type-II Error	Correct Decision

Level of Significance (LOS):

level of significance of a test is denoted by 'α' is the probability of comitting error. Thus LOS measures the amount of risk or error associated in taking decisions.

* LOS is also known as the 'size of the test'.

Examples: level of significance $\alpha = 5\%$ means there are 5 chances in 100 that null hypothesis is rejected.

i.e; 95% chances are there for to accept null hypothesis.

If level of significance $\alpha = 1\%$ means there is 1 chance in 100 that null hypothesis is rejected.

i.e; 99% chances are there for to accept null hypothesis.

Note: let 'α' be the probability of committing type I-error and β be the probability of committing type-II error then.

'α' and 'β' are known as producers risk and consumers risk respectively

* When the size of the sample is increased the probability of committing both types of errors can be reduced simultaneously

* When both α, β are small, the test procedure is good for taking correct decisions.

critical region: Critical region is the region of rejection of Null hypothesis.

* The area of the critical region is equals to the level of significance 'α'

* The critical region always lies on the tail or tails of the distribution depending on the nature of the alternate hypothesis.

ie: Critical region may lie one side or both sides of the tails.

Critical value: The value where the null hypothesis H_0 is rejected are known as critical values and the region is called critical region

Table of critical values:

α%	15%	10%	5%	4%	1%	0.5%	0.2%
α	0.15	0.1	0.05	0.04	0.01	0.05	0.02
$-z_{\alpha/2}$ & $z_{\alpha/2}$	−1.44 & 1.44	−1.645 & 1.645	−1.96 & 1.96	−2.06 & 2.06	−2.58 & 2.58	−2.81 & 2.81	−3.08 and 3.08
$-z_{\alpha/2}$	−1.04	−1.28	−1.645	−2.6	−2.33	−2.58	−2.88
$z_{\alpha/2}$	1.04	1.28	1.645	2.6	2.33	2.58	2.88

Test of hypothesis concerning to single main:

To test whether the population mean 'μ' equals to a specified constant 'μ_0' or not, formulate test of hypothesis as follows.

1. Null Hypothesis (H_0) : $\mu = \mu_0$
2. Alternate Hypothesis (H_1) : $\mu \neq \mu_0$
3. level of significance : α
4. critical region: Since A.H is not equal to type a two tailed test is considered. for a given 'α', critical values $-z_{\alpha/2}$ and $z_{\alpha/2}$ are determined from table.
5. Compute the test static 'z' denoted by z_{cal} and is given by formula

$$z_{cal} = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

6. Conclusion : Null hypothesis is rejected z_{cal} falls in the critical region.

1. The length of life of a certain computer is approximately normally distributed with mean 800 hrs and standard deviation of 40 hrs. If a random sample of 30 computers has an average life 788 hours test the null hypothesis at 5% level of significance.

$\mu = 800$ hrs

$\sigma = 40$ hrs

$\alpha = 5\%$

$n = 30$

$\bar{x} = 788$ hrs

N H (H_0) : $\mu = 800$ hrs

A.H (H_1) : $\mu \neq 800$ (TTT)

LOS (α) = 5% = $\alpha = 0.05$

$$\frac{\alpha}{2} = \frac{0.05}{2} = 0.025$$

$$z_{cal} = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

$n = 30$

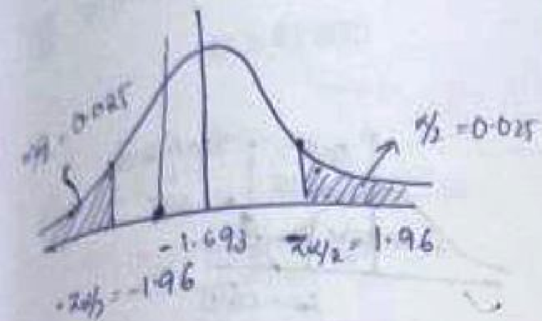
$\bar{x} = 788$

$\sigma = 40$ hours.

$$z_{cal} = \frac{788 - 800}{40/\sqrt{30}} = \frac{-12}{7.3029}$$

$= -1.643$

since $\frac{\alpha}{2} = 0.025$



$z = \pm 1.96$

critical values

$z_{cal} > z_{tab}$, z_{cal} fall in the critical region

so Ho is accepted

is mean ~~life~~time of computer is 500 hours.

A company claims that the mean thermal efficiency of digi disel engine produced by them is 32.3%. To test this claim, a random sample of 40 engines were examined which showed the main thermal efficiency of 31.4% and standard deviation of 1.6% can the claim be accepted or not at 1% level of significance

① Null hypothesis (Ho)

$\mu = 32.3\%$

② Alternate hypothesis

$\mu \neq 32.3\%$

③ LOS $\alpha = 1\%$

$\alpha = 0.01$

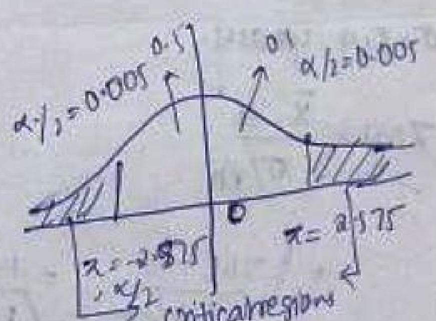
$\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

$z_{cal} = \frac{\bar{x} - \mu}{[\sigma/\sqrt{n}]}$

$\frac{-0.9}{2.5298} = \frac{31.4 - 32.3}{(1.6/\sqrt{40})}$

$= -9/4 = -2.25$

-0.3557



critical values

$= z_{tab} = \pm 2.575$

$z_{cal} < z_{tab}$

∴ Ho is rejected

* can it be concluded that the average life span of indian is more than 70 years. If a random sample of 100 indians has an avg lifespan of 71.8 years with s.d of 8.9 years. L.O.S $\alpha = 5\%$.

$\mu = 70$

① Null hypothesis

$\mu = 70$.

② Alternate hypothesis

$\mu > 70$

③ $\alpha = 5\%$
$= 0.05$

~~$\alpha/2 = 0.05/2$~~

$n = 100$
$\bar{x} = 71.8$ years.
$\sigma = 8.9$ years.

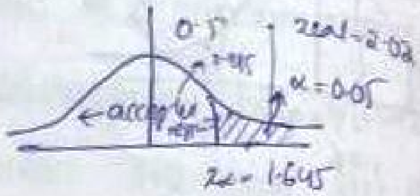
$$Z_{cal} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{71.8 - 70}{8.9/\sqrt{100}} = \frac{1.8}{(8.9/10)}$$

$$= 2.022$$

Since $\alpha = 0.05$, 4

RTT



$Z_{tab} = Z_{\alpha} = 0.05 = 1.645$

$Z_{cal} = 2.02 > Z_{tab} = 1.645$

H_0 is rejected

It can be concluded that the avg lifespan of Indian cannot be more than 70 years.

*A manufacturer of tyres guarantees that the avg life time of its tyres is more than 28,000 miles if 40 tyres of this company were tested yeilds a mean life time 27463 miles with a s.d of 1348 miles. can the gaurantee be accepted at 1% level of significant

① Null hypothes

$\mu = 2800$ miles

② Alternate hypothes

$\mu > 28000$

③ $\alpha = 1\% = \underline{0.01}$

$\boxed{\alpha = 0.01}$

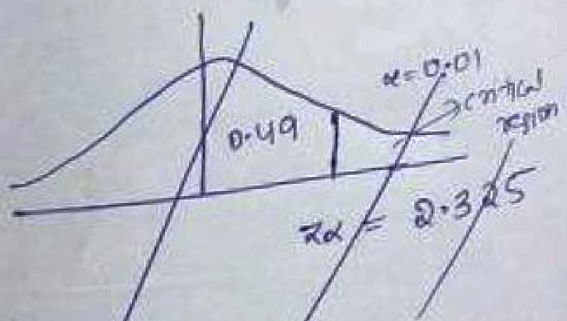
$$z_{cal} = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$= \frac{27463 - 280000}{1348/\sqrt{40}}$$

$$= -2.519$$

since $\alpha = 0.01$

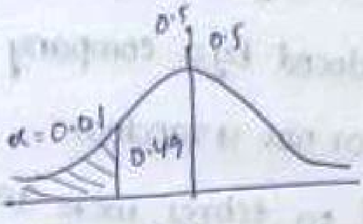
RTT



$z_\alpha = 2.325$

$z_{tab} = 2.325 = z_\alpha$

$z_{cal} = -2.519$



$\therefore -z_\alpha = -3.8$

$z_{tab} = -3.8 < z_{cal} = -2.519$

H_0 is $\underline{\text{accepted}}$

To determine wether the mean breaking strength of synthetic fibre produced by a company is 8kgs or not a random sample of 50 fibres were tested yeilding a mean breaking streaming of 7·8kg and S.D of 0.5kg L.O.S = 1%.

(1) Null hypothesis

$\mu = 8kgs$

(2) Alternate hypothesis

$\mu \neq 8kg$

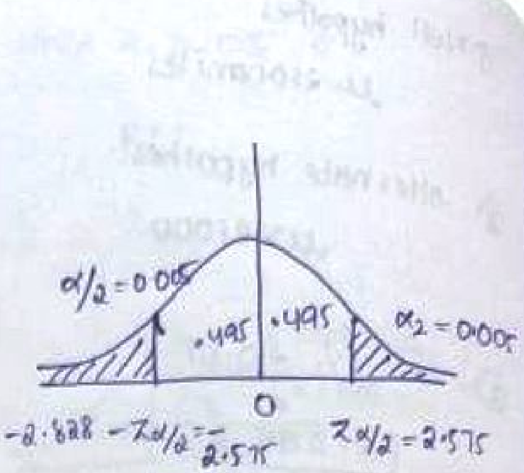
(3) $\alpha = 0.01$

$\frac{\alpha}{2} = 0.005$

(4) $Zcal = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{7.8 - 8}{0.5/\sqrt{50}}$

$= \frac{-0.2}{0.0707}$

$Zcal = -2.828$



Z_{tab} = critical values

$= \pm 2.575$

$Zcal = -2.828 < Z_{tab} = 2.575$

H_0 is rejected.

It can be concluded that the mean breaking strength of synthetic fibre produced is not 8 kgs

Test of hypothesis concerning to one proportion:

The test statistic formula for single proportion of large samples is

$$\boxed{Z_{cal} = \frac{p - P}{\sqrt{nPQ}}}$$

n → total no. of obs

p = sample proportion value

P = population proportion value

$\boxed{Q = 1 - P}$

* If a random sample of 600 cars making a right turn at certain traffic junction 157 drives made a mistake test wether 30% of drivers make this mistake or not at 0.05 & 0.01 levels of significance.

$n = 600$

$p = 157$

$H_0 : P = 30\%$

$H_a : P \neq 30\%$. (TTT)

$P = \frac{30}{100} = 0.3$

$Q = 1 - P = 0.7$

$$Z_{cal} = \frac{157 - (0.3 \times 600)}{\sqrt{600 \times 0.3 \times 0.7}}$$

$$= \frac{157 - 180}{\sqrt{600 \times 0.3 \times 0.7}}$$

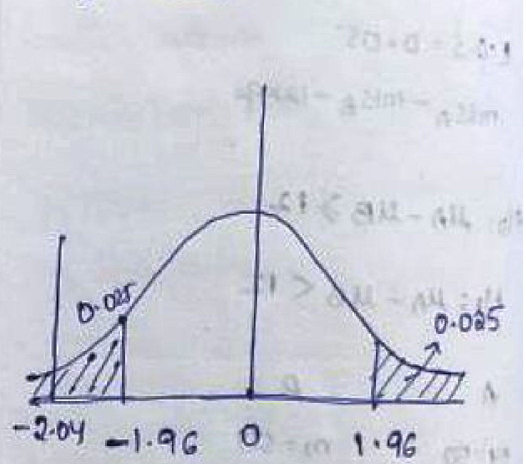
$$= \frac{-23}{3\sqrt{14}}$$

$$= -2.049$$

(i) LOC 0.05

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

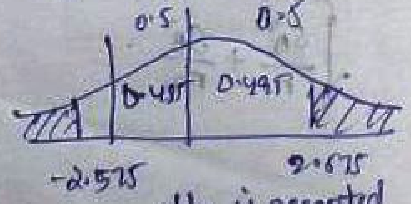


H_0 is rejected

H_a is accepted

(ii) 0.01

$\frac{\alpha}{2} = 0.005$



H_0 is accepted

Test the claim of manufacturer that 95% of his stabilizers confirm to ISI specifications if out of a random sample of 200 stabilizers are considered shows 18 were faulty test the at 0.05 & 1% on LOS

$n = 200$

$p = 18$

$H_0: P = 95\%$

$H_a = P \neq 95\%$

$P = \frac{95}{100} = 0.95$

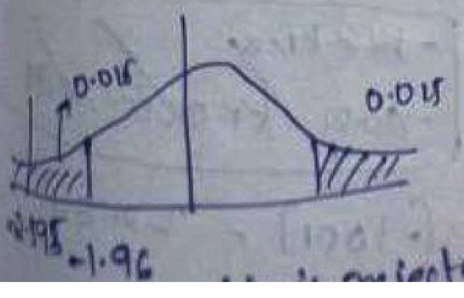
$Q = 1 - 0.95 = 0.05$

$$Z_{cal} = \frac{182 - 190}{\sqrt{200 \times 9.5}}$$

$$\frac{-8}{\sqrt{9.5}} = -2.595$$

(ii) LOS 0.05

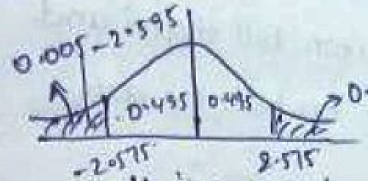
$\frac{\alpha}{2} = 0.025$



... is rejected

(b) 0.01

$\frac{\alpha}{2} = 0.005$



H_0 is rejected

H_a is accepted

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