

Name: \_\_\_\_\_

Net ID: \_\_\_\_\_

**Quiz 2**

**Please write your name and net-id eg: Ojas Kanhere, ok671**

**Do NOT write your N# number**

*Closed book/closed notes. No electronics, no calculators.*

*Total 20 + 3 points*

*Time: 40 minutes*

1. (15 points)

Consider,  $X \sim \text{Exponential}(\lambda)$  random variable. Hence  $f_X(x) = \lambda e^{-\lambda x}, x \geq 0$ .

- (a) Find  $P(X \geq a)$ .
- (b) Find  $E(X)$  and  $\text{Var}(X)$ .
- (c) Use Markov's inequality to find an upper bound for  $P(X \geq a)$ .
- (d) Use Chebyshev's Inequality to find an upper bound for  $P(X \geq a)$ .
- (e) For what values of  $a$  is the bound in part (c) tighter than part (d)?
- (f) Recall that the moment generating function is  $\Phi_X(s) = E(e^{sx})$ .  
Find  $\Phi_X(s)$  for  $X \sim \text{Exponential}(\lambda)$ .
- (g) Show that for any random variable  $X$ ,

$$P(X \geq a) \leq \min_{s \geq 0} [e^{-sa} \Phi_X(s)].$$

Hint: Note that  $I(X \geq a) \leq e^{s(X-a)}, s \geq 0$ ,

$$\text{where } I(X \geq a) = \begin{cases} 1 & x \geq a \\ 0 & x < a \end{cases}$$

- (h) Using part (f) and (g), find an upper bound for  $P(X \geq a)$  for  $X \sim \text{Exponential}(\lambda)$ .
- (i) For  $a = 1$  and  $\lambda = 1$ , compare bounds in part (c), (d) and (h).







2. (8 points)

Consider a Laplace random variable  $X$  with pdf:

$$f_X(x) = \frac{1}{2a} e^{-\frac{|x|}{a}}$$

where  $a > 0$ .

The “self-information” of outcome  $X$  is defined as:

$$i(x) = -\ln(f_X(x)).$$

(a) Find the pdf of  $i(X)$ .

(b) The “differential entropy” of  $X$  is

$$h(X) \triangleq E(i(X)).$$

Compute  $h(X)$  for the Laplace random variable above.

(c) Is the differential entropy of a Laplace random variable always non-negative?

**Explain.**





