

Diagnostic Quiz

1. If $A \cap B = \emptyset$, $P(A) \neq 0$, $P(B) \neq 0$, $P(C) \neq 0$, then state whether the following are true or false. Explain.
 - (a) $P(A|B) = P(A)$.
 - (b) $P(A \cap B^c) = P(A)$.
 - (c) $P(A \cup B) < P(A) + P(B)$.
 - (d) $P(A^c \cap B^c) = P(A^c)P(B^c)$.
 - (e) $P(A \cup B|C) = P(A|C) + P(B|C)$.

2. Consider a communication channel that flips each transmitted bit with probability α . In order to protect transmitted information, we repeat the same bit multiple times. For example, to communicate a 0, we send 000; to communicate a 1, we send 111. At the receiver if we receive more 0's than 1's, we decide the original bit was a 0; otherwise we decide the original bit was a 1.
- (a) Suppose we would like to communicate 0. Find probability that we receive 101.
 - (b) If probability of communicating a 0 and 1 are both 0.5, find the probability that the receiver makes an error.

3. (a) Suppose two random variables X and Y are Gaussian and uncorrelated. Are X and Y independent? Explain.
- (b) Consider two independent random variables X and Y . Are $\cos(X)$ and X^3 independent? Explain.
- (c) Suppose we pick a number uniformly in the unit interval $(0, 1)$. What is the probability that we pick a rational number? Explain.