Probability and Stochastic Processes (EL6303) October 5, 2018
NYU Tandon School of Engineering, Fall 2018
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Quiz 1

Closed book/closed notes. No electronics, no calculators.
Time: 40 minutes

1. *(10 points)* Consider the following communication channel known as the erasure channel. The channel input is either a '0' (with probability α) or a '1' (with probability β). We have $\alpha + \beta = 1$ and $0 \le \alpha \le 1$. When a '0' is transmitted, the channel output is a '0' with probability $1 - p$ and is an erasure (e) with probability p. Similarly, when a '1' is transmitted, the channel output is a '1' with probability $1 - q$ and is an erasure (e) with probability q. Here $0 \le p, q \le 1$.

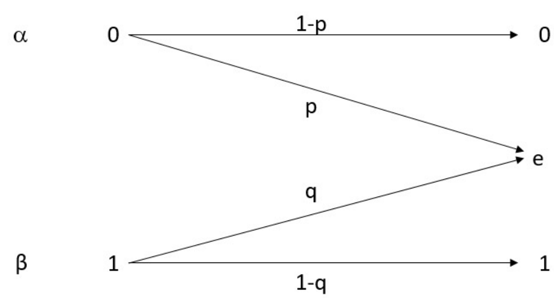

Figure 1: The erasure channel

(a) Find P(e)

(b) Find P('0' transmitted | e received)

(c) Find P('0' transmitted | '0' received)

(d) What is the probability that, by looking at the channel output, you will not be able to determine the channel input with certainty?

(e) Now you decide to send the same input three times. So, we either transmit '000' or '111'. We assume the channel acts independently each time. For this part, we assume $\alpha = \beta = 1/2$, $p = q$.

i. Find P('000' transmitted | 'ee0' received)

ii. Find P('000' transmitted | '*eee*' received)

iii. What is the probability that by looking at the channel output, you will not be able to determine the channel input with certainty?

2. *(10 points + 3 bonus)* You go to the basketball courts in West 4 street to shoot hoops. Assume you make a successful shot with probability $p \in [0, 1]$. You shoot several times with independent outcomes.

   (a) You decide to stop as soon as you make one successful shot. The total number of shots is a random variable X. Find the PMF of X.

   (b) Now you decide to stop as soon as you make k successful shots. The total number of shots is a random variable Y. Find the PMF of Y.

   (c) You realize that you have a 'hot hand'. That is

   $$\mathrm{P}(\text{shot } n+1 \text{ successful } | \text{shot } n \text{ successful}) = q$$

   and

   $$\mathrm{P}(\text{shot } n+1 \text{ successful } | \text{shot } n \text{ not successful}) = p,$$

   where $n = 1, 2, \cdots$ and $q > p$. We assume P(shot 1 successful) $= p$.

   i. Find the probability that you make 3 successful shots out of 4 total shots.

   ii. *(Bonus: 3 pts)* You decide to stop as soon as you make 2 successful shots. The total number of shots is a random variable Z. Find the PMF of Z.