Probability and Stochastic Processes (EL6303), Section I — September 5, 2018
NYU Tandon School of Engineering, Fall 2018
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Diagnostic Quiz

1. If $A \subset B$, then state whether the following are true or false. Explain.

   (a) $P(B|A^c) = 0$.

   (b) $P(A|B^c) = 0$.

   (c) $P(B|A) = 1$.

   (d) $P(A) > P(A \cap B)$.

   (e) $P(A) = P(B)$.

Solution 1

(a) **False**
$$P(B|A^c) = \frac{P(B \cap A^c)}{P(A^c)}$$
$A \subset B$. Since A is a proper subset of B,
$P(B \cap A^c) = P(B - A) \neq 0$
Thus, $P(B|A^c) \neq 0$

(b) **True**
$$P(A|B^c) = \frac{P(A \cap B^c)}{P(B^c)}$$
$A \subset B$. Since A is a proper subset of B,
$P(A \cap B^c) = P(A - B) = 0$.
Thus, $P(A|B^c) = 0$

(c) **True**
$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$
$A \subset B$. Since A is a proper subset of B,
$P(B \cap A) = P(A)$
Thus, $P(B|A) = 1$

(d) **False** $A \subset B$. Since A is a proper subset of B,
$P(A \cap B) = P(A)$
which is not less than P(A)

(e) **False** $A \subset B$. Since A is a proper subset of B, at least one element in B $\notin$ A.
Thus, $P(A) \neq P(B)$

2. In the lab you have phones coming from two vendors S and N. Probability that a phone coming from vendor S is faulty is 0.1, from vendor N is 0.2.

   (a) If you have an equal number of phones from vendors S and N, what is the probability that a randomly chosen phone is faulty?

   (b) If you would like to have the probability of a randomly chosen phone being faulty no more than 0.11, what is the smallest proportion of phones you need to buy from vendor S?

Solution 2

(a)

$$\begin{aligned} P(\text{faulty}) &= P(\text{faulty}|S) \cdot P(S) + P(\text{faulty}|N) \cdot P(N) \\ &= 0.1 * 0.5 + 0.2 * 0.5 \\ &= 0.15 \end{aligned}$$

(b) Let the proportion of phones you buy from vendor S be λ.
Thus, $P(S) = \lambda$ and $P(N) = 1 - \lambda$

$$\begin{aligned} P(\text{faulty}) &\leq 0.11 \\ P(\text{faulty}|S) \cdot P(S) + P(\text{faulty}|N) \cdot P(N) &\leq 0.11 \\ 0.1 * \lambda + 0.2 * (1 - \lambda) &\leq 0.11 \\ 0.09 &\leq 0.1\lambda \\ \lambda &\geq 0.9 \end{aligned}$$

3. (a) What does it mean for two random variables X and Y to be independent? Explain.
   (b) What does it mean for two random variables X and Y to be uncorrelated? Explain.
   (c) If X, Y are uncorrelated, are they independent? Explain.
   (d) If X, Y are independent, are they uncorrelated? Explain.

Solution 3

(a) X and Y are independent means their joint distribution satisfies $p(x, y) = p(x)p(y)$.

(b) X and Y are uncorrelated means $E(XY) = E(X)E(Y)$.

(c) Not necessarily. Suppose X is uniformly distributed in the interval $(-1, 1)$, and $Y = X^2$. Then obviously X and Y are not independent, but they are uncorrelated.

(d) Yes. Below we prove for the discrete random variables, the continuous case is similar.

$$E(XY) = \sum_x \sum_y p(x, y) = \sum_x \sum_y p(x)p(y)$$
$$= \sum_x p(x) \sum_y p(y) = E(X)E(Y).$$