

Name: _____

Net ID: _____

Quiz 4

Please write your name and net-id eg: Ojas Kanhere, ok671

Do NOT write your N# number

Closed book/closed notes. No electronics, no calculators.

Total 20 points

Time: 40 minutes

1. (10 points) Let $U(t)$ and $V(t)$ be independent, WSS random processes with zero means and the same autocorrelation function $R(\tau) = R_U(\tau) = R_V(\tau)$. Let $Z(t)$ be the random process defined by $Z(t) = U(t) \cos t + V(t) \sin t$.
 - (a) Find the mean of $Z(t)$, $E(Z(t))$.
 - (b) Find the autocorrelation function of $Z(t)$, $R_Z(t, t + \tau) = E(Z(t)Z(t + \tau))$.
 - (c) Is $Z(t)$ is a WSS random process? Explain.
 - (d) Suppose $U(t)$ and $V(t)$ are independent SSS random processes. Is $Z(t)$ a SSS random process? Explain.

Hint:

$$\sin(\theta \pm \phi) = \sin \theta \cos \phi \pm \cos \theta \sin \phi$$

$$\cos(\theta \pm \phi) = \cos \theta \cos \phi \mp \sin \theta \sin \phi$$

2. (10 points) Let $N(t)$ be a Poisson process with rate $\lambda > 0$. Hence

$$P(N(t) = n) = \frac{(\lambda t)^n e^{-\lambda t}}{n!}, n = 0, 1, \dots$$

Let X_1 be the time of the first arrival, X_2 be the time of the second arrival. Suppose you are told that there is exactly one arrival in the interval $[0, t_0]$.

- (a) Show that the two events “there is exactly one arrival in the interval $[0, t_0]$ ” and “ $N(t_0) = 1$ ” are the same.
- (b) Find the conditional pdf of X_1 given that there is exactly one arrival in the interval $[0, t_0]$.
- (c) Find the conditional pdf of X_2 given that there is exactly one arrival in the interval $[0, t_0]$.

