EL-GY9343

Data Structure and Algorithm

Lecture 1: Syllabus, Introduction, and Asymptotic notation

Instructor: Yong Liu

Why taking this course

- You can learn
  - Classic algorithms for classic problems, mathematical insights
  - Techniques for analyzing performance of various algorithms
  - How to design good algorithms for solving real-world problems

- Improving programming skills
  - Then you can rock in classes, job interviews, etc.

- It's also fun!

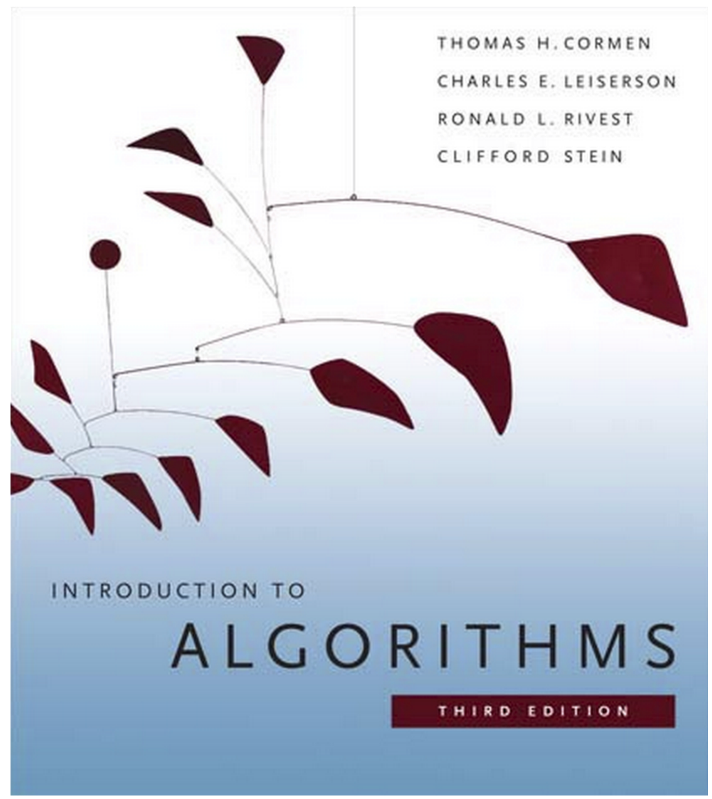
Prerequisites

- Basic knowledge of fundamental data structures
  - stacks, queues, heaps, ...
- Some programming experience
  - C, C++, Python, Java, etc.
- Discrete mathematics, probabilities, ...
- Should not take this course if you have taken a similar course e.g. CS6033 with a B or better grade.

Textbook

Introduction to Algorithms, 3rd Edition, by Thomas H. Cormen, Charles E. Leiserson, Rondald L. Rivest and Clifford Stein, MIT Press, 2009; ISBN-13: 9780262033848. It is known as CLRS.

Free access to CLRS on books24x7 (on the library web site *http:// library.poly.edu*, go to Databases A-Z, then letter B, then books24x7).



Grading policy

- Your final grade will be calculated as:

Homework	10%
Midterm	40%
Final	50%

- No extra work to improve grade!

Homework

- Key component to mastering the course material
  - Very good exercise and practice
  - Will not do well on exams if you have not done the hw
  - Will be assigned weekly or biweekly

Exams

- One mid-term
- One final exam
- Close-book and limited notes
- Attendance at exams is mandatory

Remember: If you miss an exam without a valid excuse (need documents to prove), you will receive a grade of zero.

What is an algorithm?

- An *algorithm* is any well-defined computational procedure that takes some values as *input* and produces some values as *output*.

- Provide a step-by-step method for solving a computational problem. (Like cooking recipes)

- Not dependent on a particular programming language, machine, system, or compiler. (Unlike programs)

Example on sorting

- Problem: Sorting
  - **Input:** sequence of n numbers $(a_1, a_2, \cdots, a_n)$
  - **Output:** a permutation $(a'_1, a'_2, \cdots, a'_n)$ of the input instance such that $a'_1 \leq a'_2 \leq \cdots \leq a'_n$

For example:

Input: 53, 12, 35, 21, 59, 15
Output: 12, 15, 21, 35, 53, 59

- Algorithms: Insertion sort, merge sort, quick sort, . . .

Issues in algorithm analysis & design

- Two fundamental issues: correctness and efficiency

- Steps to analyze and design an algorithm
  - Formally define a problem
  - Clearly describe an algorithm
  - Prove correctness of the algorithm
  - Analyze the efficiency of the algorithm

Efficiency of algorithms

- Goal
  - To compare algorithms mainly in terms of running time but also in terms of other factors (e.g., memory requirement, programmer's effort).

- Running time analysis
  - Determine how running time increases as the size of the problem increases.

How do we compare algorithms?

- Need to define a number of objective measures

  (1) Compare execution time?

  Not good: time is specific to a particular computer !!

  (2) Count the number of statements executed?

  Not good: number of statements vary with the programming language as well as the style of the individual programmer.

- Ideal solution
  - Express running time as a function of the input size n (i.e., $f(n)$).
  - Compare different functions corresponding to running time.
  - Such an analysis is independent of machine time, programming style, etc.

Random-Access Machine (RAM)

- A computational model
  - All memory equally expensive to access
  - No concurrent operations
  - All reasonable instructions take unit time
    - Except, of course, function calls

Example

- Associate a "cost" with each statement.
- Find the "total cost" by finding the total number of times each statement is executed.

Algorithm	***Cost***
sum = 0;	c_1
for(i=0; i<N; i++)	c_2
for(j=0; j<N; j++)	c_2
sum += arr[i][j];	c_3

$$c_1 + c_2 \times (N+1) + c_2 \times N \times (N+1) + c_3 \times N^2$$

Input size (number of elements in the input)

- How we characterize input size is problem-specific:
  - Sorting: number of input items
  - Multiplication: total number of bits
  - Graph algorithms: number of nodes & edges
  - ...

Common orders of magnitude

Table 2.1 The running times (rounded up) of different algorithms on inputs of increasing size, for a processor performing a million high-level instructions per second. In cases where the running time exceeds 10^{25} years, we simply record the algorithm as taking a very long time.

	n	$n \log_2 n$	n^2	n^3	1.5^n	2^n	$n!$
$n = 10$	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	4 sec
$n = 30$	< 1 sec	< 1 sec	< 1 sec	< 1 sec	< 1 sec	18 min	10^{25} years
$n = 50$	< 1 sec	< 1 sec	< 1 sec	< 1 sec	11 min	36 years	very long
$n = 100$	< 1 sec	< 1 sec	< 1 sec	1 sec	12,892 years	10^{17} years	very long
$n = 1{,}000$	< 1 sec	< 1 sec	1 sec	18 min	very long	very long	very long
$n = 10{,}000$	< 1 sec	< 1 sec	2 min	12 days	very long	very long	very long
$n = 100{,}000$	< 1 sec	2 sec	3 hours	32 years	very long	very long	very long
$n = 1{,}000{,}000$	1 sec	20 sec	12 days	31,710 years	very long	very long	very long

Types of analysis

- Worst case
  - Provides an upper bound on running time
  - An absolute guarantee that the algorithm would not run longer
- Best case
  - Provides a lower bound on running time
  - Input is the one for which the algorithm runs the fastest

$$\textit{Lower Bound} \le \textit{Running Time} \le \textit{Upper Bound}$$

- Average case
  - Provides a prediction about the running time
  - Very useful, but treat with care: what is “average”?
    - Random (equally likely) inputs
    - Real-life inputs

Asymptotic Analysis

- To compare two algorithms with running times *f(n)* and *g(n),* we need a rough measure that characterizes how fast each function grows.

- Simplifications
  - Ignore actual and abstract statement costs
  - Order of growth: highest-order term is what counts
    - Remember, we are doing asymptotic analysis
    - As the input size grows larger, the high order term dominates

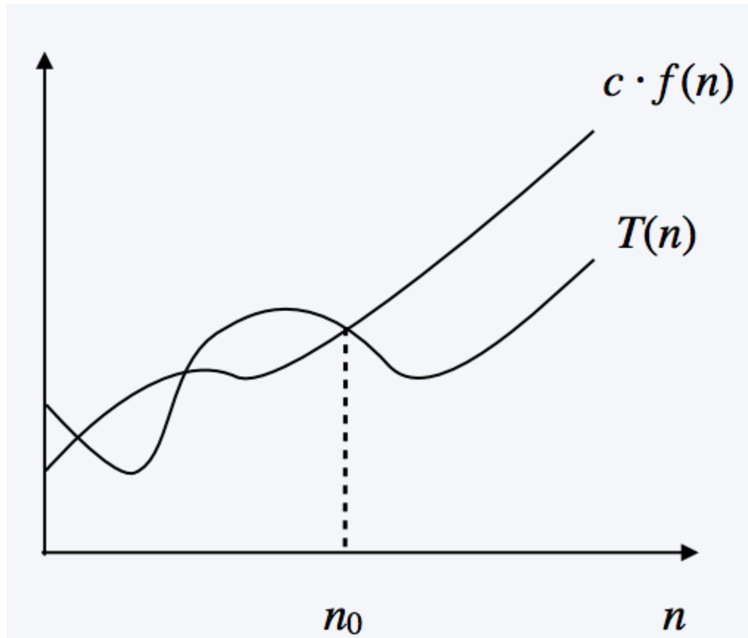
- For example: $5n^3 + 100n^2 + 10n + 50 \quad \sim \quad n^3$

Asymptotic notation: Big-Oh notation

- Upper bounds. $T(n)$ is $O(f(n))$ if there exist constants $c > 0$ and $n_0 \geq 0$ such that $T(n) \leq c \cdot f(n)$ for all $n \geq n_0$.

Ex. $T(n) = 32n^2 + 17n + 1$.

- $T(n)$ is $O(n^2)$. ← choose $c=50, n_0=1$
- $T(n)$ is also $O(n^3)$.
- $T(n)$ is neither $O(n)$ nor $O(n \log n)$.

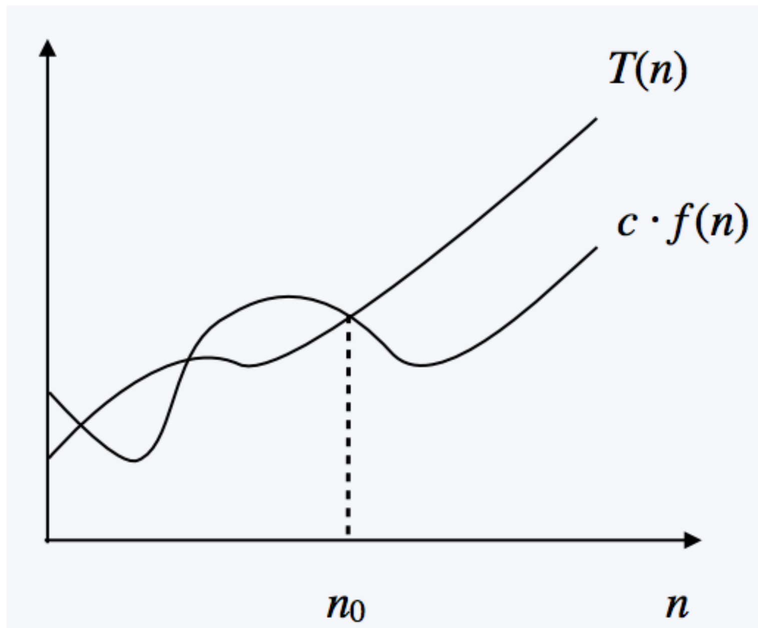


Asymptotic notation: Big-Omega notation

- Lower bounds. $T(n)$ is $\Omega(f(n))$ if there exist constants $c > 0$ and $n_0 \geq 0$ such that $T(n) \geq c \cdot f(n)$ for all $n \geq n_0$.

Ex. $T(n) = 32n^2 + 17n + 1$.

- $T(n)$ is $\Omega(n^2)$. ← choose $c=32, n_0=1$
- $T(n)$ is also $\Omega(n)$.
- $T(n)$ is neither $\Omega(n^3)$ nor $\Omega(n^3 \log n)$.

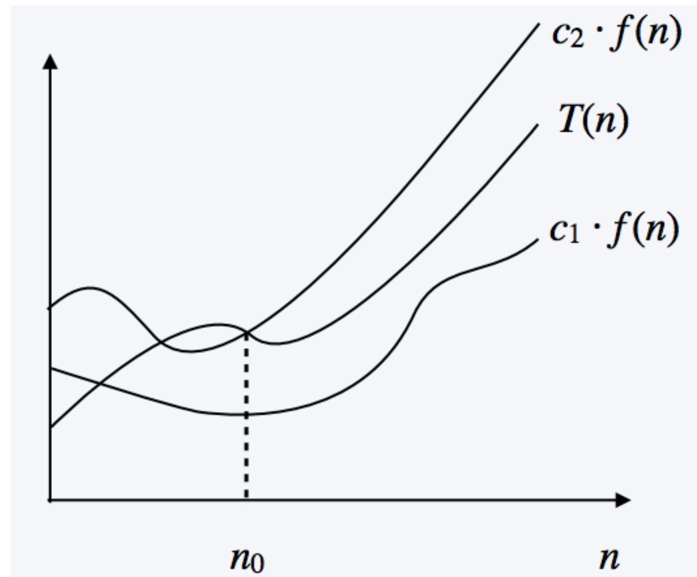


Asymptotic notation: Big-Theta notation

- Tight bounds. $T(n)$ is $\Theta(f(n))$ if there exist constants $c_1 > 0$, $c_2 > 0$ and $n_0 \geq 0$ such that $c_1 \cdot f(n) \leq T(n) \leq c_2 \cdot f(n)$ for all $n \geq n_0$.

Ex. $T(n) = 32n^2 + 17n + 1$.

- $T(n)$ is $\Theta(n^2)$. ← choose $c_1=32, c_2=50, n_0=1$
- $T(n)$ is neither $\Theta(n)$ nor $\Theta(n^3)$.



Asymptotic notation: little-o, and little-ω

- Little-o: $T(n)$ is $o(f(n))$ if for any constant $c > 0$, there exists $n_0 \geq 0$ such that $T(n) < c \cdot f(n)$ for all $n \geq n_0$

- Little-ω: $T(n)$ is $\omega(f(n))$ if for any constant $c > 0$, there exists $n_0 \geq 0$ such that $T(n) > c \cdot f(n)$ for all $n \geq n_0$.

- Intuitively
  - o() is like <   O() is like ≤
  - ω() is like >   Ω() is like ≥
  - Θ() is like =

Properties

- *Theorem:*

  $f(n) = \Theta(g(n)) \Leftrightarrow f = O(g(n))$ and $f = \Omega(g(n))$

- Transitivity:
  - $f(n) = \Theta(g(n))$ and $g(n) = \Theta(h(n)) \Rightarrow f(n) = \Theta(h(n))$
  - Same for O and Ω
- Reflexivity:
  - $f(n) = \Theta(f(n))$
  - Same for O and Ω
- Symmetry:
  - $f(n) = \Theta(g(n))$ if and only if $g(n) = \Theta(f(n))$
- Transpose symmetry:
  - $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$

What's next...

- Recurrences, Divide-and-Conquer
  - Substitution, Iteration, Master method
  - Read CLRS Chapter 4