EL9343 Homework 1
(Due September 17th, 2019)

All problem/exercise numbers are for the third edition of CLRS text book

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1. Prove the *Symmetry* property of $\Theta(\cdot)$, i.e. $f(n) = \Theta(g(n))$ if and only if $g(n) = \Theta(f(n))$.

2. Problem 3-2 in CLRS Text book;

3. Problem 3-3 (a) in CLRS Text book;

4. First use the iteration method to solve the recurrence:

$$T(n) = 2T\left(\frac{2n}{3}\right) + n$$

then use the substitution method to verify your solution.

5. First use the iteration method to solve the recurrence

$$T(n) = T\left(\frac{n}{4}\right) + T\left(\frac{n}{2}\right) + n^2$$

then use the substitution method to verify your solution.

6. Solving the recurrence:

$$T(n) = 9T(n^{\frac{1}{6}}) + \log^2(n).$$

(*Hint: Making change of variable*)

7. Give asymptotic upper and lower bounds for $T(n)$ in each of the following recurrences. Make your bounds as tight as possible, and justify your answers.

(a) $T(n) = 2T(n/3) + n^{\frac{1}{2}} \log n$

(b) $T(n) = 25T(n/5) + n^2$

(c) $T(n) = 4T(n/2) + n^2\sqrt{n}$

8. Determine the time complexity of following code pieces (Figure 1), using big-O notation (every single statement has O(1) time complexity):

9. Analyze the time complexity of the following piece of code (Figure 2), using big-O notation (assume 'A' in the following code is an integer array):

Sequence of statements

```
statement 1;
statement 2;
  ...
statement k;
```

If-Then-Else

```
if (cond) then
   block 1 (sequence of statements)
else
   block 2 (sequence of statements)
end if;
```

Loops

```
for I in 1 .. N loop
   sequence of statements
end loop;
```

Nested loops

```
for I in 1 .. N loop
   for J in 1 .. M loop
      sequence of statements
   end loop;
end loop;
```

Figure 1: Code Pieces

```
low = 0
high = N - 1
while (low <= high) {
    // invariants: value > A[i] for all i < low
                   value < A[i] for all i > high
    mid = (low + high) / 2
    if (A[mid] > value)
        high = mid - 1
    else if (A[mid] < value)
        low = mid + 1
    else
        return mid
```

Figure 2: Code