ECE9343 Homework 6

All problem/exercise numbers are for the third edition of CLRS text book

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1. Exercise 24.1-1, reverse the order of relaxing edges in the figure, i.e., in each pass, start with edge (s, y), finish with edge (t, x);

2. Exercise 24.2-1, using vertex t as the source;

3. Exercise 24.3-1, only use t as the source

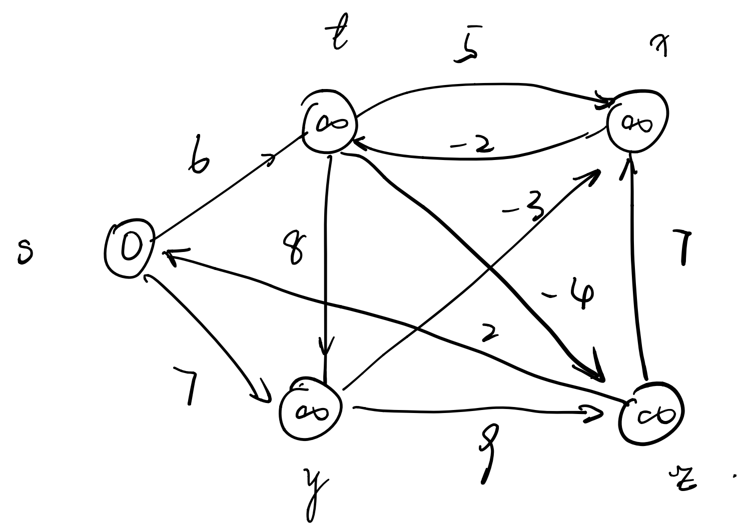
4. Exercise 24.3-6

5. Exercise 24.3-8

6. Exercise 25.2-1

7. Exercise 25.2-7

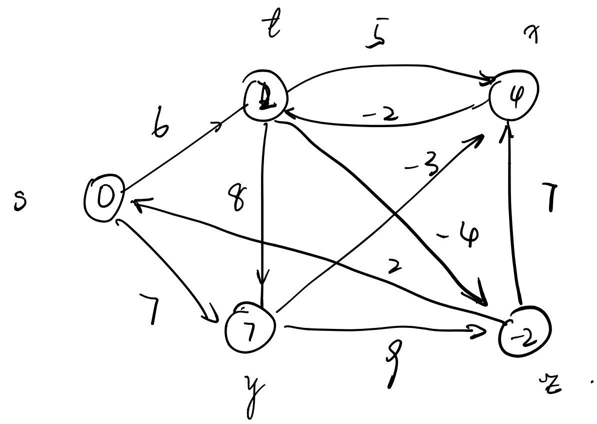
Sol:



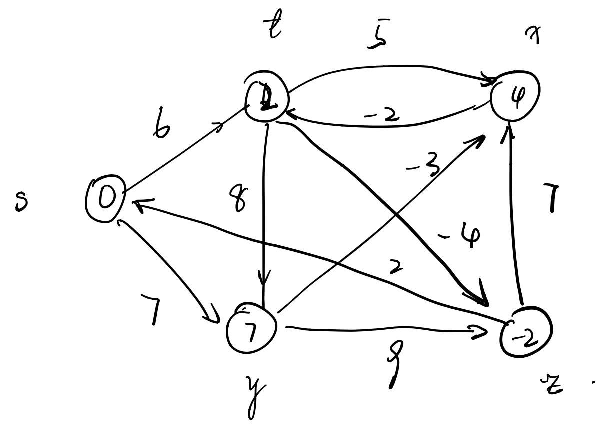
Order : (s, y), (s, t), (z, s) (z, x) (y, z) (y, x)

(x, t), (t, z), (t, y), (t, x)

Update :

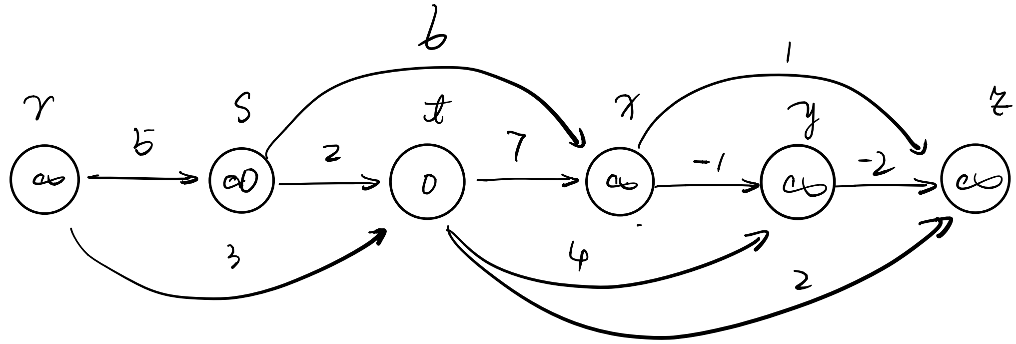


Round 1



Converge.

2.



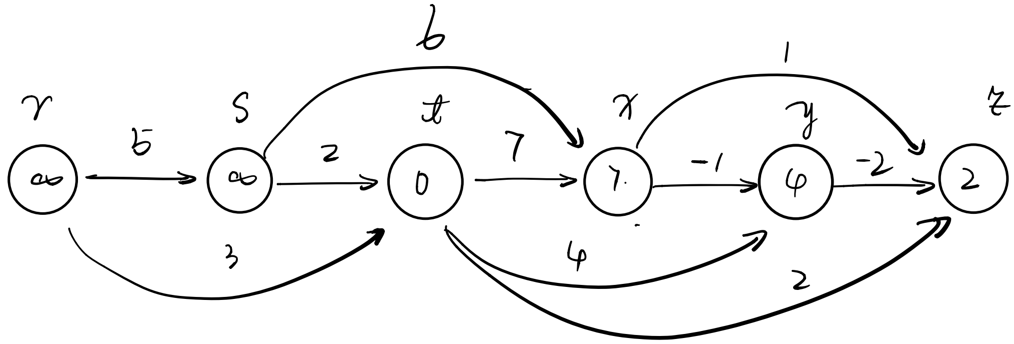
DAG - Topological Sort as shown above:

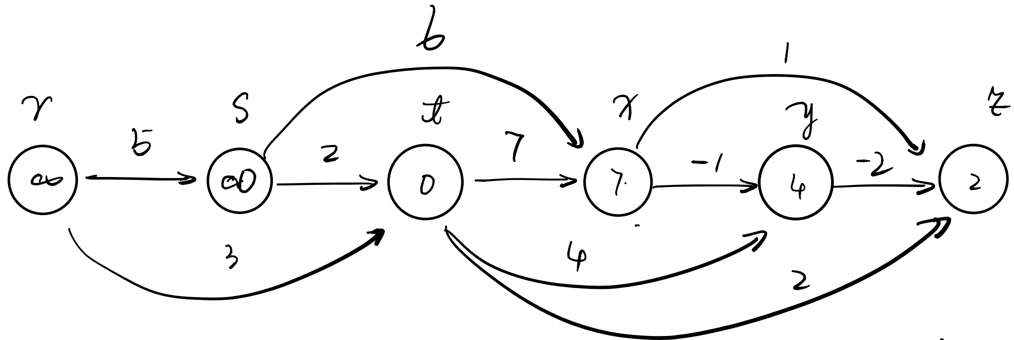
initialize t as the source as above:

Order from r -> z

for each v in adj [u].

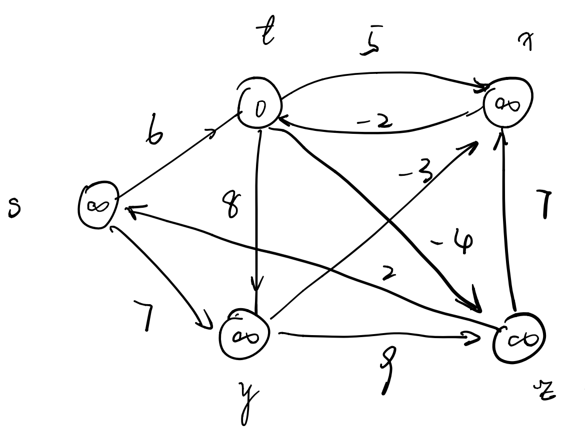
relax (u, v, w).





Converge.

3.



Dijkstra:

initialize t as the source.

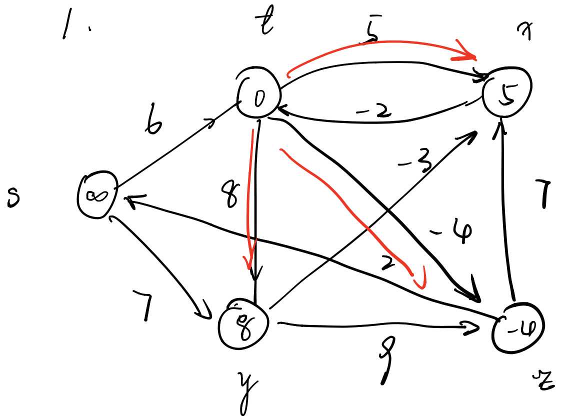
$S = \phi$

Round one. $Q = \{ t, s, y, x, z \}$.

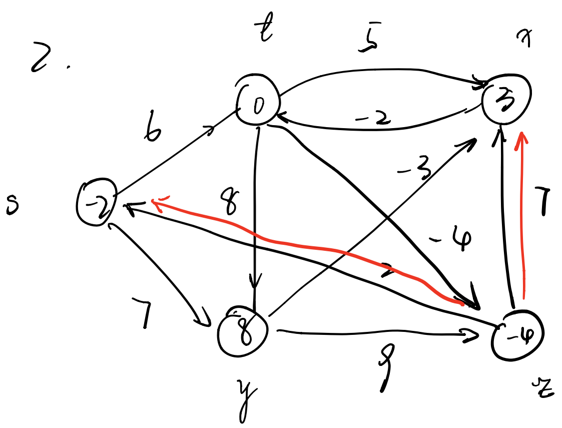
$u = \text{extract-min}(Q)$ $S = \{t\}$.

Q	t	s	y	x	z
	$\boxed{0}$	∞	∞	∞	∞
		∞	8	5	$\boxed{-4}$
		$\boxed{-2}$	8	3	
			5	$\boxed{3}$	
			$\boxed{5}$		

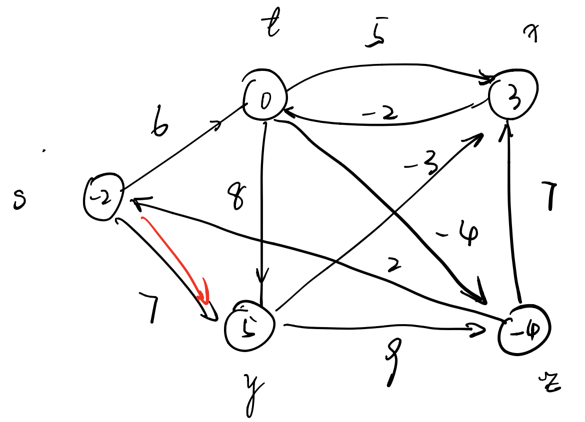
1.



2.



3.



4. unchanged

5. converge.

4. Most reliable path between two vertices. find a path $(v_s, \dots v_t)$

s.t. $\max \prod_{i=s}^{t-1} r(v_i, v_{i+1})$.

$\iff \max_{(v_s \dots v_t)} \sum_{i=s}^{t-1} \log r(v_i, v_{i+1})$

So the Algorithm would be:

for $(u,v) \in E$:

$r(u,v) = \log(r(u,v))$

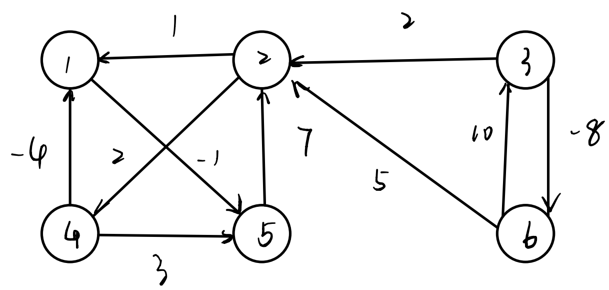
Run Bellman-Ford (G, r, s)

5. $G = (V, E)$ weighted, directed graph $w: E \to \{0, 1, \cdots W\}$
Modify Dijkstra's Algorithm to compute the shortest paths given s in $O(WV + E)$ time.

Modified Dijkstra (G, w, s): Discrete. w.

```
//Initialize:
for each v ∈ G.V
    : v.d = VW+1   v.π = NULL   s.d = 0.
Initial new array A[VW+2]
A[0].insert(s).
A[1 : VW+2] are linked lists of vertices.
k = 0
for i = 1 : |V|
        while A[k] = NULL :
                k = k+1
        u = A[k].head
        A[k].delete(u).
        for v ∈ G.Adj[u] :
            if v.d > u.d + w(u,v) then
                A[v.d].delete(v.list)
                v.d = u.d + w(u,v)
                v.π = u
                A[v.d].insert(v)
                v.list = A[v.d].head.
```

6. Run Floyd-Warshall algorithm on



$d_{ij}^{(k)}$

$= \min\left(d_{ij}^{(k-1)},\ d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\right)$

$$D^{(0)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & \infty & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & 3 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{bmatrix} \quad \Pi^{(0)} = \begin{bmatrix} NIL & NIL & NIL & NIL & 1 & NIL \\ 2 & NIL & NIL & 2 & NIL & NIL \\ NIL & 3 & NIL & NIL & NIL & 3 \\ 4 & NIL & NIL & NIL & 4 & NIL \\ NIL & 5 & NIL & NIL & NIL & NIL \\ NIL & 6 & 6 & NIL & NIL & NIL \end{bmatrix}$$

$$d_{ij}^{(1)} = \min\left(d_{ij}^{(0)},\ d_{i1}^{(0)} + d_{1j}^{(0)}\right)$$

$$D^{(1)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ \infty & 2 & 0 & \infty & \infty & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ \infty & 7 & \infty & \infty & 0 & \infty \\ \infty & 5 & 10 & \infty & \infty & 0 \end{bmatrix} \quad \Pi^{(1)} = \begin{bmatrix} NIL & NIL & NIL & NIL & 1 & NIL \\ 2 & NIL & NIL & 2 & 1 & NIL \\ NIL & 3 & NIL & NIL & 1 & 3 \\ 4 & 1 & 1 & NIL & 1 & 1 \\ NIL & 5 & NIL & NIL & NIL & NIL \\ NIL & 6 & 6 & NIL & 1 & NIL \end{bmatrix}$$

$$d_{ij}^{(2)} = \min\left(d_{ij}^{(1)},\ d_{i2}^{(1)} + d_{2j}^{(1)}\right)$$

$$D^{(2)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix} \quad \Pi^{(2)} = \begin{bmatrix} NIL & NIL & NIL & NIL & 1 & NIL \\ 2 & NIL & NIL & 2 & 1 & NIL \\ 2 & 3 & NIL & 2 & 2 & 3 \\ 4 & 1 & 1 & NIL & 1 & 1 \\ 2 & 5 & NIL & 2 & NIL & NIL \\ 2 & 6 & 6 & 2 & 2 & NIL \end{bmatrix}$$

$$d_{ij}^{(3)} = \min(d_{ij}^{(2)}, d_{i3}^{(2)} + d_{3j}^{(2)})$$

$$D^{(3)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ 1 & 0 & \infty & 2 & 0 & \infty \\ 3 & 2 & 0 & 4 & 2 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 8 & 7 & \infty & 9 & 0 & \infty \\ 6 & 5 & 10 & 7 & 5 & 0 \end{bmatrix} \quad \Pi^{(3)} = \begin{bmatrix} NIL & NIL & NIL & NIL & 1 & 3 \\ 2 & NIL & NIL & 2 & 1 & 3 \\ 2 & 3 & NIL & 2 & 2 & 3 \\ 4 & 1 & 1 & NIL & 1 & 3 \\ 2 & 5 & NIL & 2 & NIL & 3 \\ 2 & 6 & 6 & 2 & 2 & NIL \end{bmatrix}$$

$$d_{ij}^{(4)} = \min(d_{ij}^{(3)}, d_{i4}^{(3)} + d_{4j}^{(3)})$$

$$D^{(4)} = \begin{bmatrix} 0 & \infty & \infty & \infty & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & \infty & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix} \quad \Pi^{(4)} = \begin{bmatrix} NIL & NIL & NIL & NIL & 1 & 3 \\ 4 & NIL & 4 & 2 & 4 & 4 \\ 4 & 3 & NIL & 2 & 4 & 3 \\ 4 & 1 & 1 & NIL & 1 & 3 \\ 4 & 5 & NIL & 2 & NIL & 3 \\ 4 & 6 & 6 & 2 & 4 & NIL \end{bmatrix}$$

$$d_{ij}^{(5)} = \min(d_{ij}^{(4)}, d_{i5}^{(4)} + d_{5j}^{(4)})$$

$$D^{(5)} = \begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ 0 & 2 & 0 & 4 & -1 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix} \quad \Pi^{(5)} = \begin{bmatrix} \text{NIL} & 5 & 5 & 5 & 1 & 5 \\ 4 & \text{NIL} & 5 & 2 & 4 & 5 \\ 4 & 3 & \text{NIL} & 2 & 4 & 3 \\ 4 & 5 & 5 & \text{NIL} & 1 & 5 \\ 4 & 5 & \text{NIL} & 2 & \text{NIL} & 3 \\ 4 & 5 & 6 & 2 & 4 & \text{NIL} \end{bmatrix}$$

$$d_{ij}^{(6)} = \min\left(d_{ij}^{(5)},\ d_{i6}^{(5)} + d_{6j}^{(5)}\right)$$

$$D^{(6)} = \begin{bmatrix} 0 & 6 & \infty & 8 & -1 & \infty \\ -2 & 0 & \infty & 2 & -3 & \infty \\ -5 & -3 & 0 & -1 & -6 & -8 \\ -4 & 2 & \infty & 0 & -5 & \infty \\ 5 & 7 & \infty & 9 & 0 & \infty \\ 3 & 5 & 10 & 7 & 2 & 0 \end{bmatrix} \quad \Pi^{(6)} = \begin{bmatrix} \text{NIL} & 5 & 5 & 5 & 1 & 5 \\ 4 & \text{NIL} & 5 & 2 & 4 & 5 \\ 6 & 6 & \text{NIL} & 6 & 6 & 3 \\ 4 & 5 & 5 & \text{NIL} & 1 & 5 \\ 4 & 5 & \text{NIL} & 2 & \text{NIL} & 3 \\ 4 & 6 & 6 & 2 & 4 & \text{NIL} \end{bmatrix}$$

7. Using $\phi_{ij}^{(k)}$ for $i, j, k = 1, 2, \ldots, n$ where $\phi_{ij}^{(k)}$ is the highest-numbered intermediate vertex of a shortest path from i to j in which all intermediate vertices are in the set $\{1, \ldots, k\}$

Recursive Formulation given:

Initialize $\Phi = \Pi^{(0)}$ $\left[D^{(0)}\right]_{ij} = \begin{cases} 0 & i \neq j \\ w_{ij} & \text{if } (i,j) \text{ in edges.} \\ \infty & \text{otherwise.} \end{cases}$

$n = \text{len}(\Pi^{(0)}.\text{row})$.

$k = 0$.

for $k = 1 \ldots n$.

for $i = 1 \ldots n$:

for $j = 1 \dots n$:

$$d_{ij}^{(k)} = \min\left(d_{ij}^{(k-1)},\ d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \right)$$

$$\Pi_{ij}^{(k)} = \Pi_{ij}^{(k-1)} \text{ if } d_{ij}^{(k-1)} < d_{ik}^{(k-1)} + d_{kj}^{(k-1)} \text{ else } k$$

if $d_{ik}^{(k)} + d_{kn}^{(k)} \geq d_{ij}^{(k-1)}$:

$$\phi_{ij}^{(k)} = \phi_{ij}^{(k-1)}$$

else:

$$\phi_{ij}^{(k)} = k$$

Reconstructing the path is as for example. $\Pi_{ij}^{(n)} = k_1$. then find out what's in $\Pi_{ik}^{(n)}$

But from $\phi_{ij}^{(k)}$, we first find out $\phi_{ij}^{(n)} = k_1$. then $\phi_{ik_1}^{(n-1)} = k_2$. then $\phi_{ik_2}^{(n-2)} \dots$