PLAKSHA UNIVERSITY

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U20220090

COMPUTATIONAL METHOD AND OPTIMIZATION

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ASSIGNMENT ONE

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Question 1 Assignment 1

Taylor Series

Question 1

$f(x) = \frac{1}{1-x^2}$

Taylor series expansion Note the expansion is centered at 0

$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$ Note $a = 0$ (center)

$f(x) = \sum_{0}^{\infty} \frac{f^{(n)}(0)(x)^n}{n!}$

First expansion

$f(a) = \frac{1}{1-a^2} = \frac{1}{1-0^2} = 1$

Second expansion

$f'(a) = \frac{2a}{(1-a^2)^2}$ at $a=0$

$= 0$

$\frac{f'(a)x^1}{1!} = 0$

Third expansion

$f''(a) = \frac{(1-a^2)^2(2) - 2a(1-a^2)(2)(-2a)}{(1-a^2)^2}$

$= \frac{8a^2}{(1-a^2)^3} + \frac{2}{(1-a^2)^2}$

at $a=0$ $f''(a) = 2$

for fourth expansion

$$f'''(a) = [(8a^2)(-3(1-a^2)^{-4}(-2a)) + (1-a^2)^{-3}(16a)] + [2x(-2(1-a^2)^{-3}(-2a))]$$

$$= [(48a^3)(1-a^2)^{-4} + (1-a^2)^{-3}(16a)] + [8a(1-a^2)^{-3}]$$

$$= \frac{48a^3}{(1-a^2)^4} + \frac{24a}{(1-a^2)^3}$$

$f'''(a) = 0$

$$\frac{f'''(a)(x)^3}{3!} = 0$$

from the above the odd term just go to zero

So the expansion

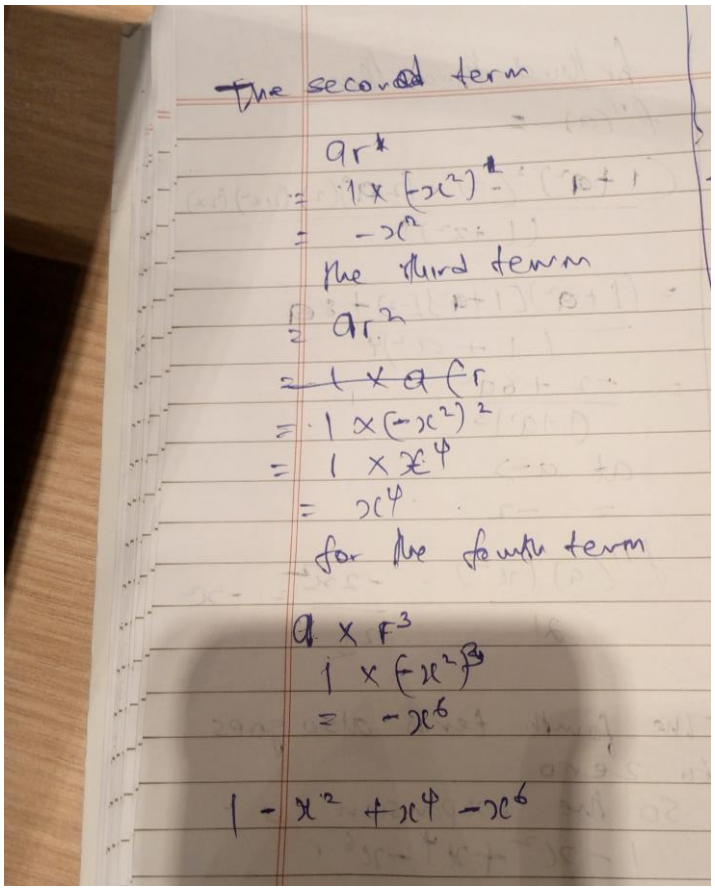
$1 + x^2 + x^4 + x^6 \ldots$

For the Geometric expansion / Series.

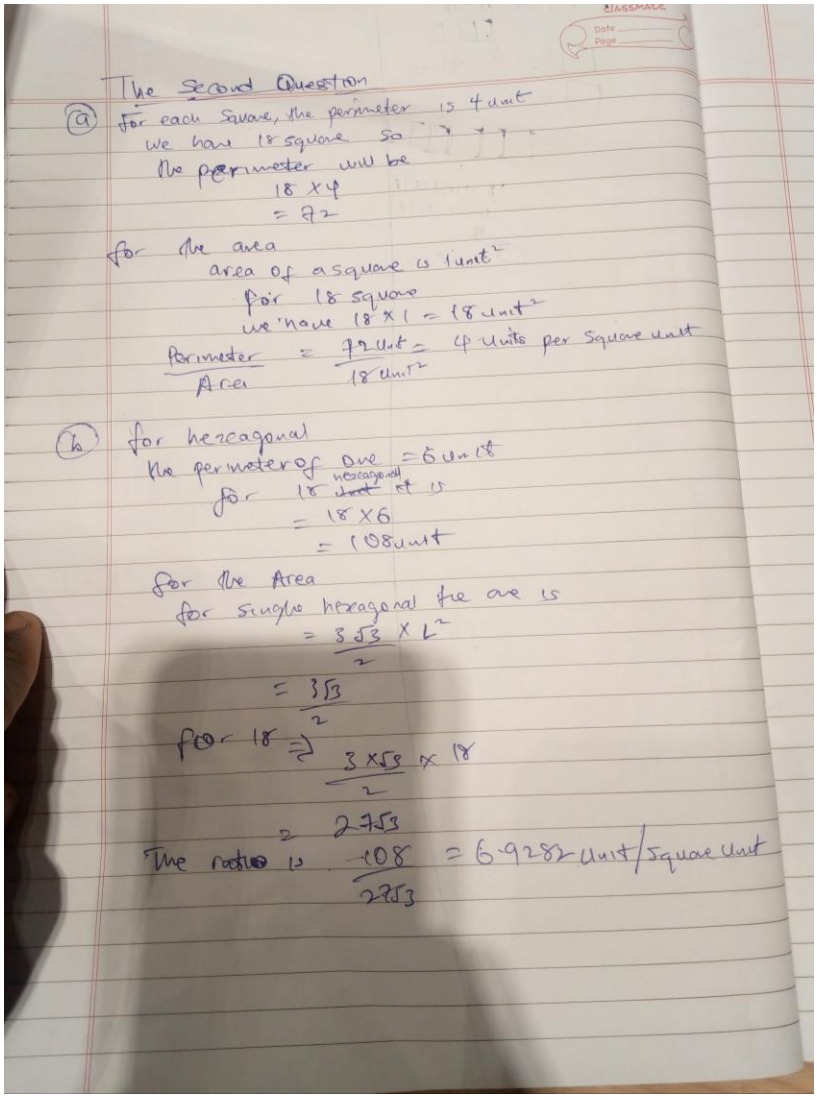
$$\frac{1}{1-x^2}$$

$$f(x) = \sum_0^\infty ar^n = \frac{a}{1-r}$$

$$\frac{1}{1-x^2} \quad \therefore \quad a = 1 \quad r = x^2.$$



The two agree both the Taylor series and the Geometric, because the Taylor series is centered at point zero which is around the radius of convergence for the geometric series.



The hexagonal is preferred for efficient space-filling because it optimizes the ratio of perimeter to area, using fewer resources (wax) to enclose a given area compared to squares. Bees use hexagons for their hives to minimize the wax they need to produce, as it's a more efficient use of resources.

NOTE:- OTHER COMMENT IS IN THE .ipnyb file