AAE 537 Homework #1

Fall, 2019

1. Using the standard atmosphere model provided, compute conditions along constant q trajectories of q=800, 1000, and 1200 psf. Produce the following plots:
   a. The flight corridor relating Mach number (x-axis) to altitude (y-axis) for each q (on the same plot)
   b. Stagnation temperature vs altitude for the three q values noted
   c. Recovery temperature vs. altitude for the three q values noted
   d. Stagnation pressure vs. altitude for the three q values noted
   e. Static pressure vs. altitude for the three q values noted

Solution:

a)
$$q=\frac{1}{2}\rho u^2=\frac{1}{2}\frac{P}{RT}\gamma RTM^2=\frac{1}{2}\gamma PM^2 \Rightarrow M=\sqrt{\frac{2q}{\gamma P}} \tag{0.1}$$

b)
$$T_0=T\left(1+\frac{\gamma-1}{2}M^2\right) \tag{0.2}$$

c) Recovery temperature requires Prandtl number(Pr), which can be assumed to be function of temperature alone. Since the atmospheric temperature variation with altitude is minimal, Pr can be considered constant, between 0.71 and 0.74. There are several relations for recovery factor, some require Reynolds' number, so we will ignore those and use the simplest:

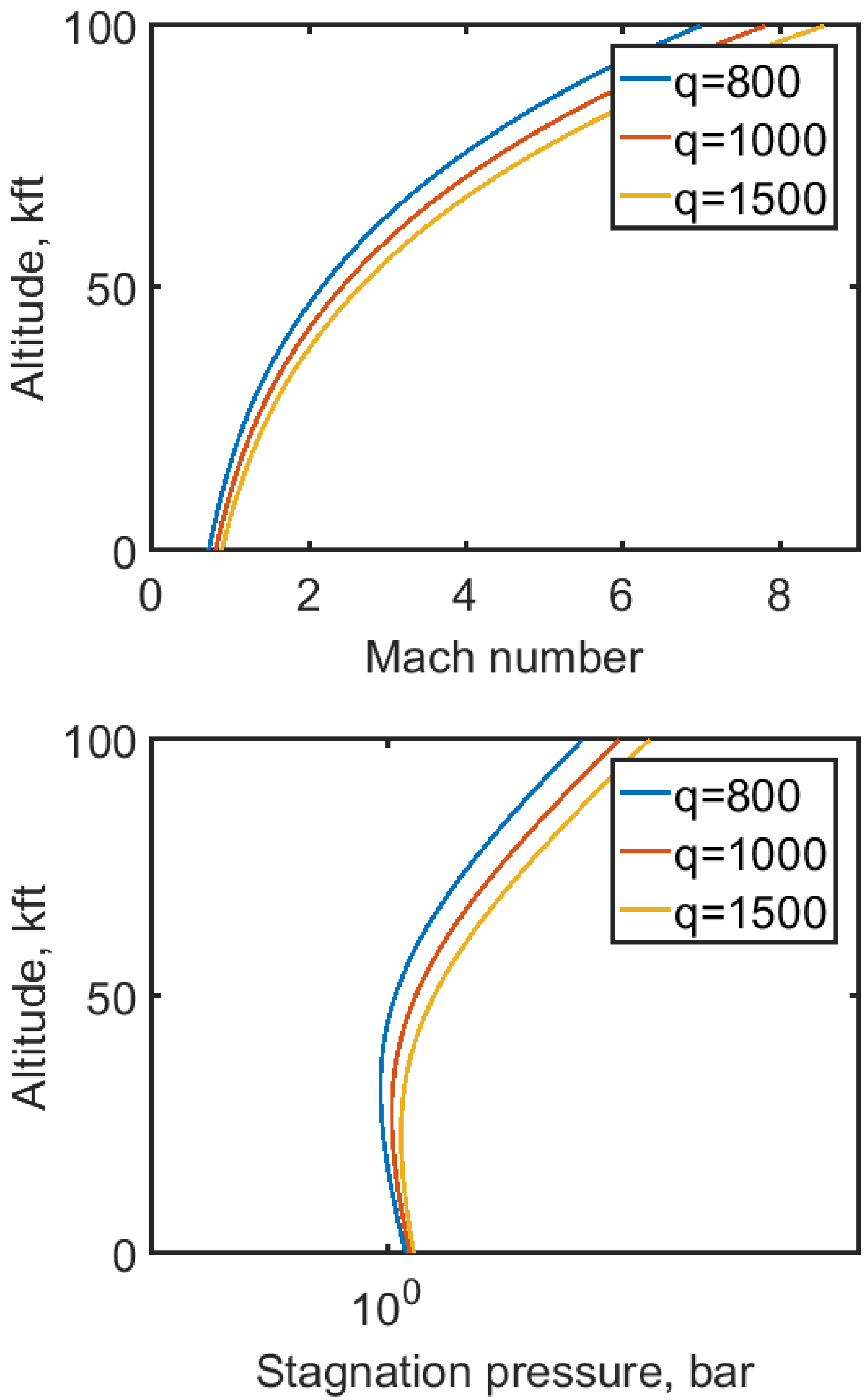
$$r=\mathrm{Pr}^{\frac{1}{3}} \tag{0.3}$$

$$r=\frac{T_{ad}-T}{T_0-T}\Rightarrow T_{ad}=T+r\left(T_0-T\right) \tag{0.4}$$

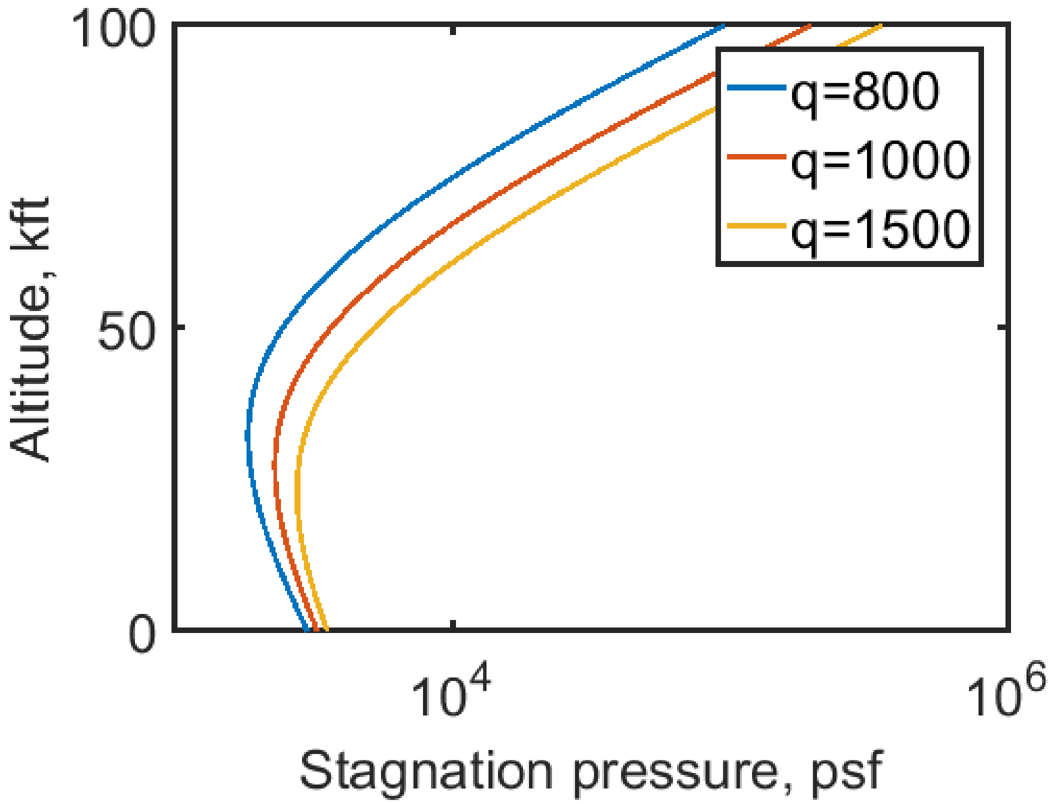
d)
$$P_0=P\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} \tag{0.5}$$

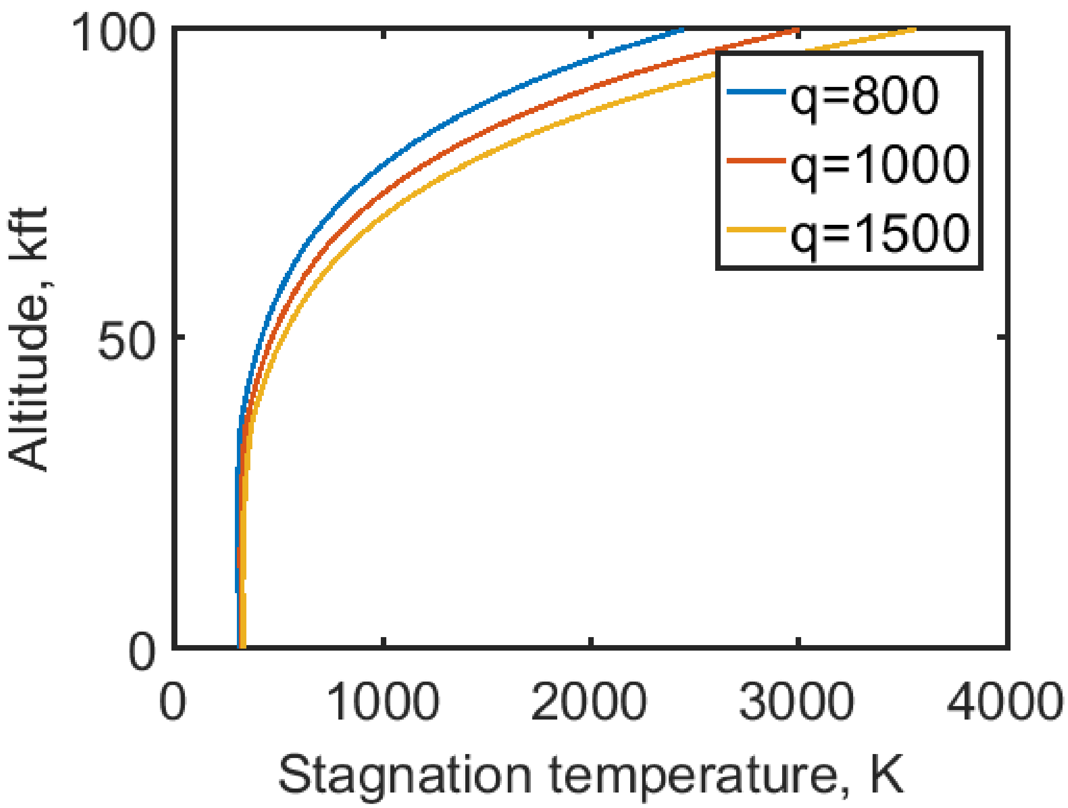
e) This is the same as the atmospheric model output.

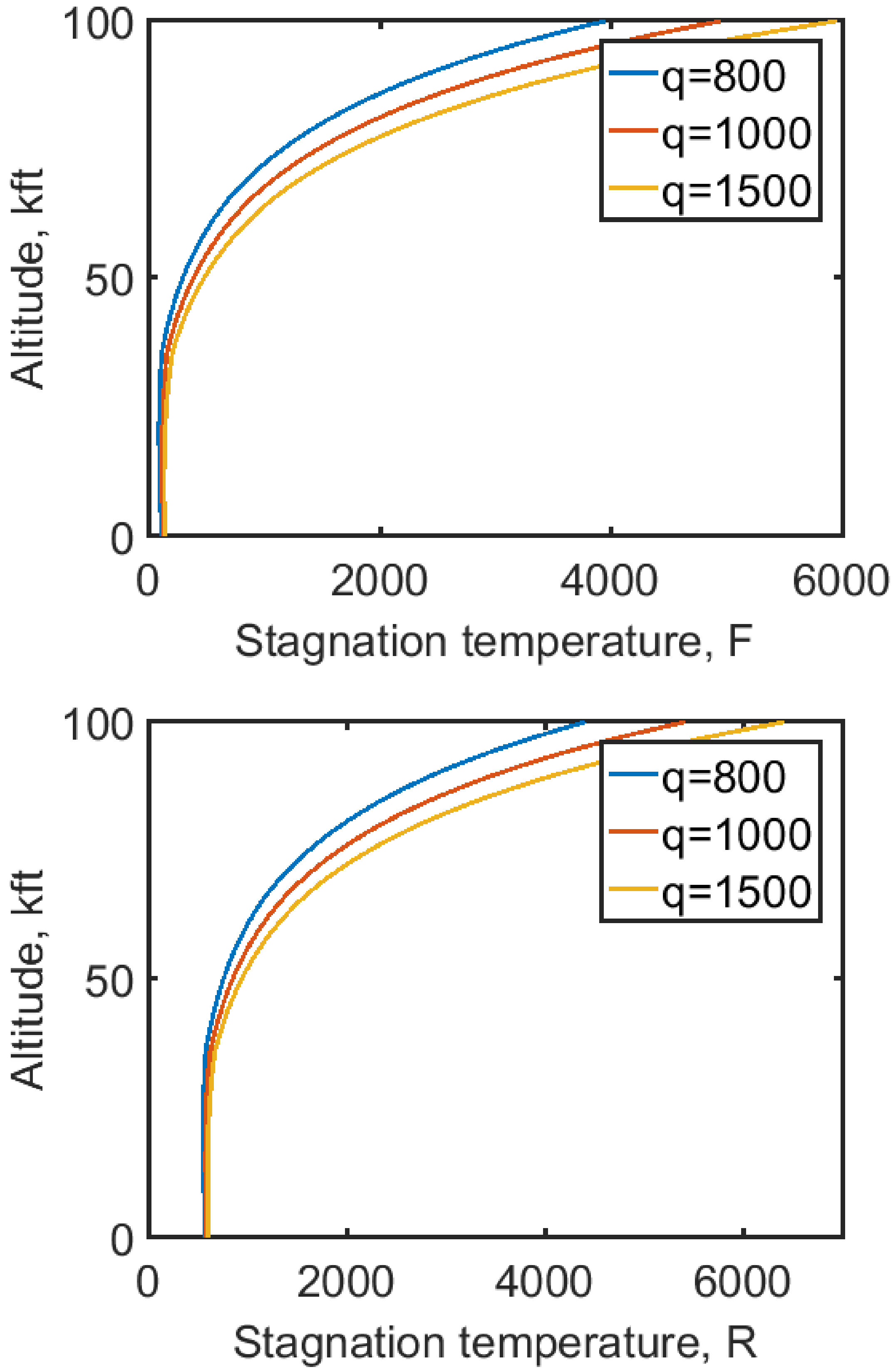
The required plots are attached.



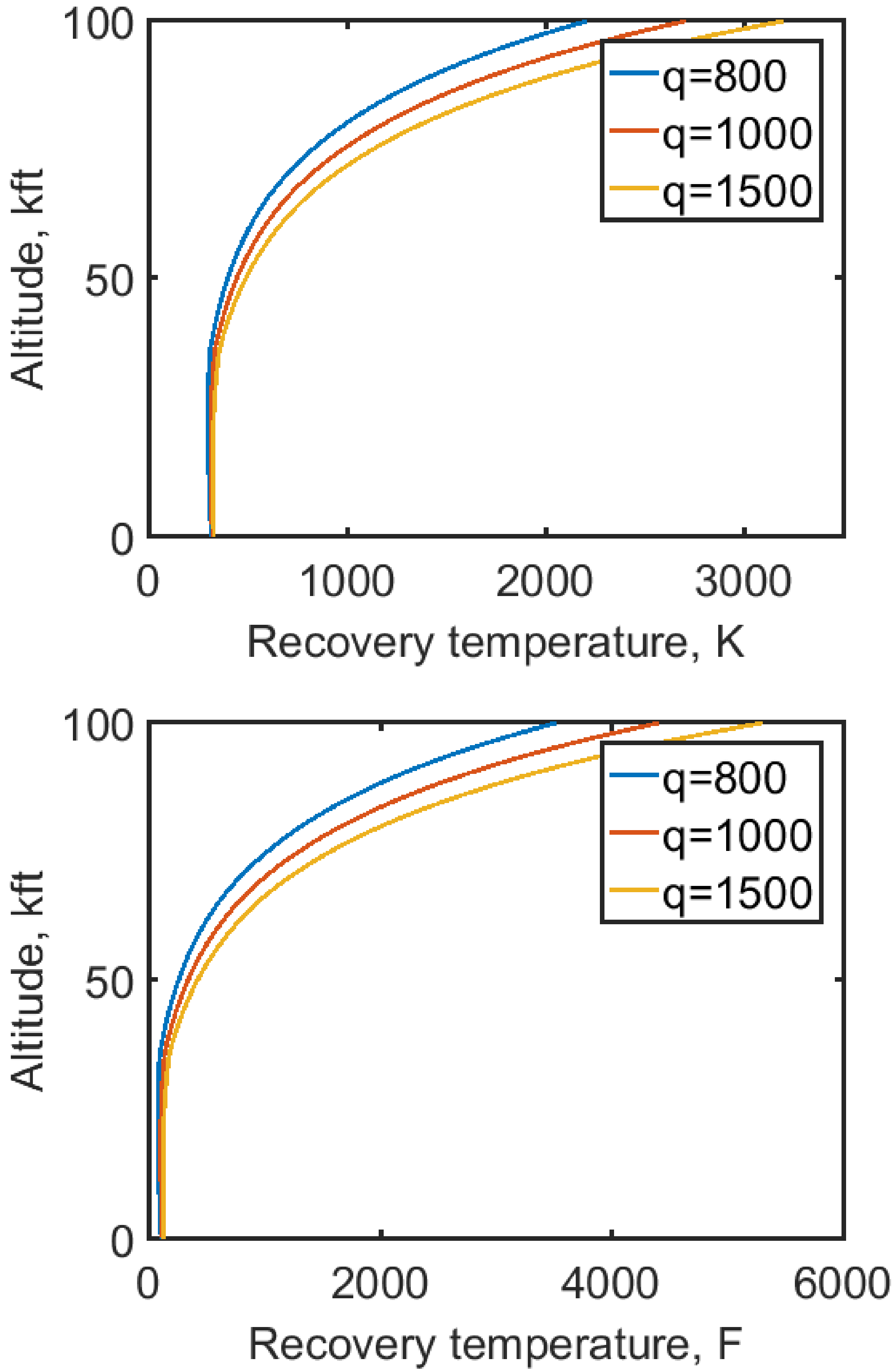
100
50
0
Altitude, kft
q=800
q=1000
q=1500
0
2
4
6
8
Mach number
100
50
0
Altitude, kft
q=800
q=1000
q=1500
10^0
Stagnation pressure, bar


100
50
0
Altitude, kft
10^4
10^6
Stagnation pressure, psf
q=800
q=1000
q=1500

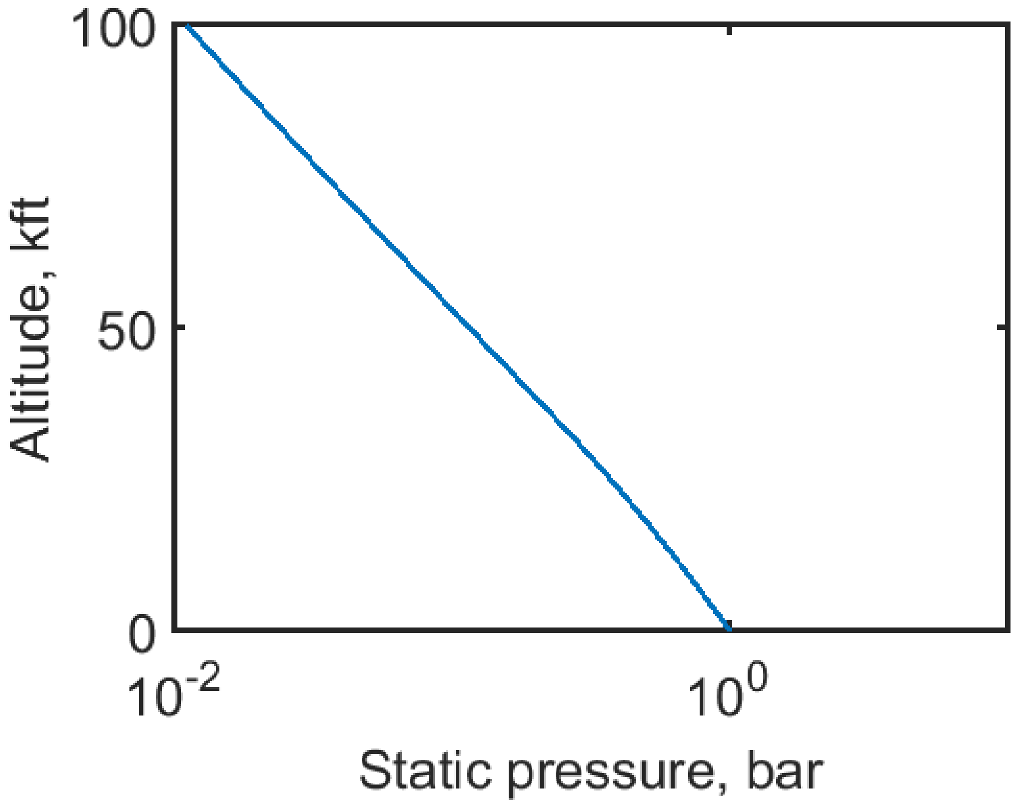

100
50
0
Altitude, kft
0
1000
2000
3000
4000
Stagnation temperature, K
q=800
q=1000
q=1500



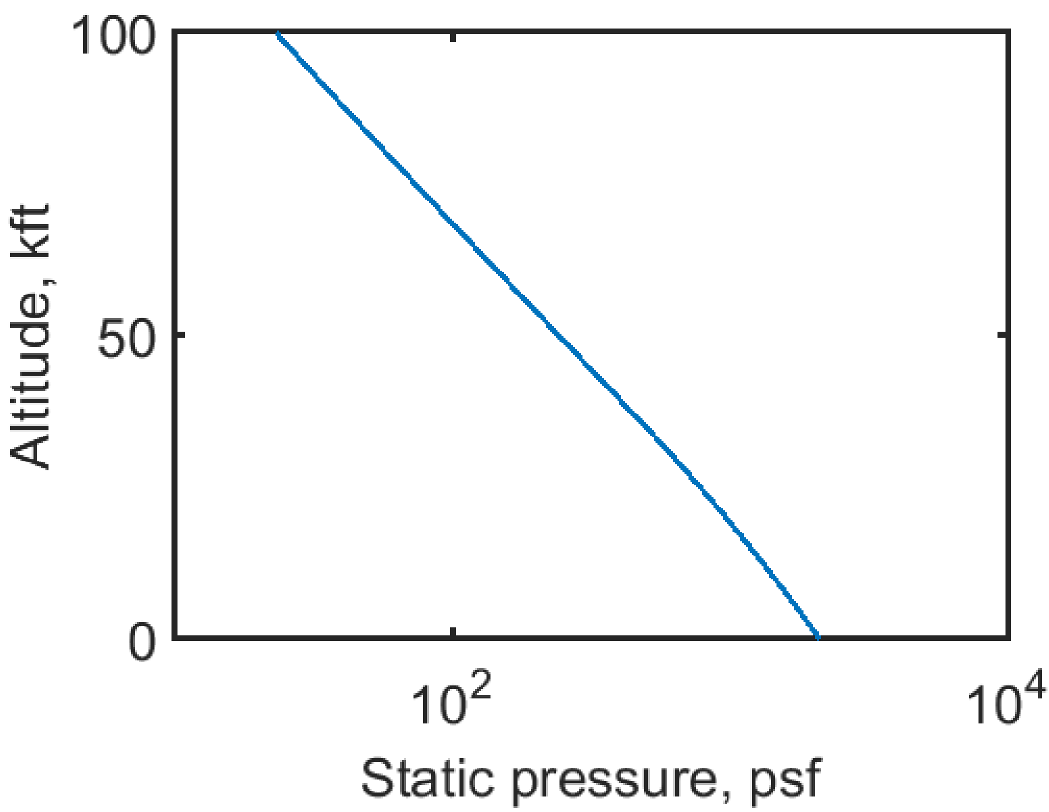
100
50
0
0
2000
4000
6000
Altitude, kft
Stagnation temperature, F
q=800
q=1000
q=1500
100
50
0
0
2000
4000
6000
Altitude, kft
Stagnation temperature, R
q=800
q=1000
q=1500



100
50
0
0
1000
2000
3000
Altitude, kft
Recovery temperature, K
q=800
q=1000
q=1500
100
50
0
0
2000
4000
6000
Altitude, kft
Recovery temperature, F
q=800
q=1000
q=1500

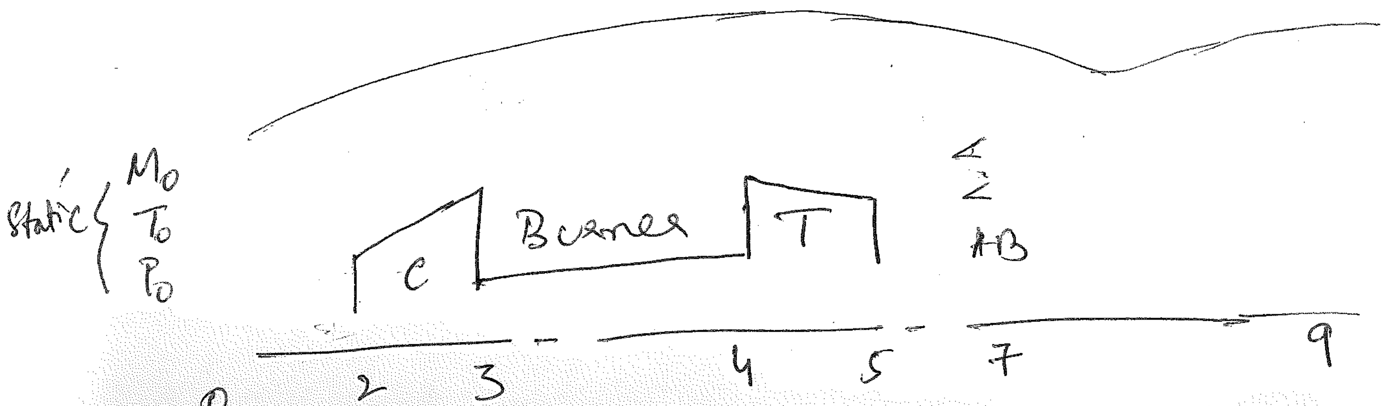

100
50
0
Altitude, kft
10^{-2}
10^{0}
Static pressure, bar

a


100
50
0
Altitude, kft
10^{2}
10^{4}
Static pressure, psf

1(a)

For Turbojet, Turbojet with AB, Ramjet and Scramjet the following set of equations can be incorporated and altered based on components for each system



Dynamic pressure can be written as

$$q = \frac{1}{2}\rho V^2 \quad \text{where} \quad q = 1500\,\text{psf (constant)}$$

→ For every Altitude step h, we can get P, T and ρ from atmosphere code.

→ $$q = \frac{1}{2}\rho(h)\,V^2(h) \;\Rightarrow\; V(h) = \sqrt{\frac{2q}{\rho(h)}}$$

and $$M_0 = \frac{V(h)}{\sqrt{\gamma R T(h)}}$$

→ From Mach number, static conditions at particular altitude and Isentropic relations, we can get stagnation conditions. P_{t0} and T_{t0}

AAE 537 Homework #1
Fall, 2017

1. In this problem, develop Isp-Mach relationships for the following ideal systems: **(60 points)**

   i) Turbojet engine with CPR=10
   ii) Afterburning Turbojet with CPR=10
   ii) Turbofan engine with CPR=30, FPR=2, BPR=6
   iii) Ramjet engine with start condition at M=1.7
   iv) Scramjet engine with start condition at M=3.5
   v) Bipropellant rocket engine with liquid oxygen oxidizer at Pc=3000 psi

   Assume that all systems are operating within an airbreathing corridor at a dynamic pressure of 1500 psf. For all airbreathing systems:
   - Consider kerosene fuel with a heating value (heat of combustion) of 18,500 BTU/lbm.
   - Assume ideal components throughout with the exception of the inlet. For this component assume inlet recovery per Mil Std 5008B.
   - Assume an average Cp of 0.3 BTU/lbm-deg F.

   Turbine inlet temperatures are limited to 2600 F, ramjet burner or afterburner exit temperature is limited to 3500 F and scramjet burner exit is limited to 4000 F. You may assume that all nozzles are perfectly expanded at all Mach conditions. The rocket engine is operating at optimal O/F. Take the following steps in your study:

   a) Derive relationships for specific thrust and Isp for the turbofan engine. Here, FPR and BPR are fan pressure ratio and bypass ratio respectively. **(15)**
   b) Run CEA at a number of altitudes to determine rocket performance as a function of Mach number **(15)**
   c) Generate Isp-Mach relationships for all systems and plot all results on the same graph. **(15)**
   d) Discuss the implications of your results, i.e. which system is preferred over which Mach range? Why? **(15)**

2. Using the standard atmosphere model provided, make a plot of the altitude-Mach relationship for constant q trajectories of q=1000 psf and q=2000 psf. **(40)**

→ At Station (2)

$$T_{t2} = T_{t0} = T_0 \left(1 + \frac{\gamma-1}{2} M_0^2\right)$$

$$P_{t2} = P_{t0} \left(1 - 0.0076 (M_0 - 1)^{1.35}\right) \rightarrow \text{MIL Std 5008 B}$$

→ Station (3)

$$P_{t3} = CPR \times P_{t2} \quad \left(\text{For Turbofan } P_{t3} = CPR \times FPR \times P_{t2}\right)$$

$$T_{t3} = T_{t2} (CPR)^{\frac{\gamma-1}{\gamma}}$$

→ Station (4)

From energy balance

$$f = \frac{(T_{t4} - T_{t3})\, C_p}{\Delta H_B} \Rightarrow \text{Gives } f$$

where T_{t4} is turbine inlet temperature in the HW statement

C_p is given

$$P_{t4} = P_{t3}$$ assuming constant pressure combustion

→ Station (5)

Powerbalance between compressor and Turbine

$$\dot{m}\, C_p (T_{t3} - T_{t2}) = (\dot{M} + \dot{m}_f)\, C_p (T_{t4} - T_{t5})$$

* Add fan power in case of Turbofans

This equation gives $\boxed{T_{t5}}$

$$P_{t5} = P_{t4}\left(\frac{T_{t5}}{T_{t4}}\right)^{\gamma/\gamma-1}$$

→ Afterburner

Energy balance gives

$$(1+f_{AB})T_{t7} = T_{t5} + f_{AB}\frac{\Delta H_B}{c_p}$$

where T_{t7} is limiting temperature at AB exit

f_{AB} can now be calculated

→ Perfectly expanded nozzle means

$$P_9 = P_0 \;,\quad P_{t9} = P_{t7} = P_{t5}$$

Exit Velocity is $V_{exit} = V_9 = \sqrt{2 c_p T_{t7}\left(1-\left(\frac{P_9}{P_{t7}}\right)^{\frac{\gamma-1}{\gamma}}\right)}$

Specific thrust $\frac{F}{\dot{m}} = (1+f)V_9 - V_0$

$$ISP = \frac{F/\dot{m}}{f + f_{AB}}$$

→ For Turbofan, include thrust from Bypass flow

β = Bypass Ratio (BPR)

$$\frac{F}{\dot{m}} = \frac{(1+f)V_9 + \beta V_{fan\,exit} - (\beta+1)V_0}{\beta+1}$$

$$ISP = \frac{1}{fg}\,\frac{[(1+f)V_9 + \beta V_{fan\,exit} - (\beta+1)V_0]}{\beta+1}$$

→ For Scramjet, from Class notes

$M_2 = \frac{M_1}{3}$, $T_{t3} = T_{t2} = T_{t0}$ and T_{t4} is given

use these conditions to calculate f

* Extrapolate pressure, temperature and densities for Mach numbers below $M = 1$. Because $q = 1500$ psf does not give rise to subsonic domain which is important for turbomachinery plots.

1. (b) Rocket Analysis using CEA → Run CEA for LOx / Kerosene using $\left(\frac{P_c}{P_a}\right)$ where P_a comes from atm. Code

→ ISP or C^* can be chosen to optimize O/F, But ISP is the Natural choice

→ Tabulate M vs ISP and plot results

→ Choose Optimal O/F of 2.56

1. (c) Check figure

1(d) → Different propulsion systems perform better in different Mach regimes. At low Mach numbers Turbofans and Turbojets do well.

→ But they quickly drop in ISP at slightly higher (Supersonic) Mach Numbers.

→ Material limitations and Turbine Inlet temp. limit the performance

→ Scramjet and Ramjet perform well at high Mach Numbers

(2) following similar procedure as Q1

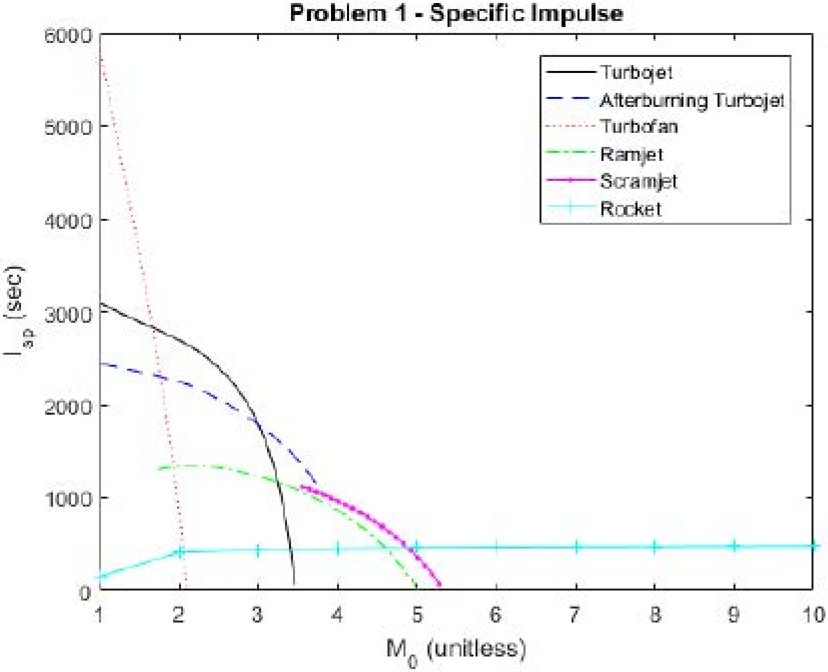
dynamic pressure $q = \frac{1}{2}\rho V^2$ or $V = \sqrt{\frac{2q}{\rho(h)}}$

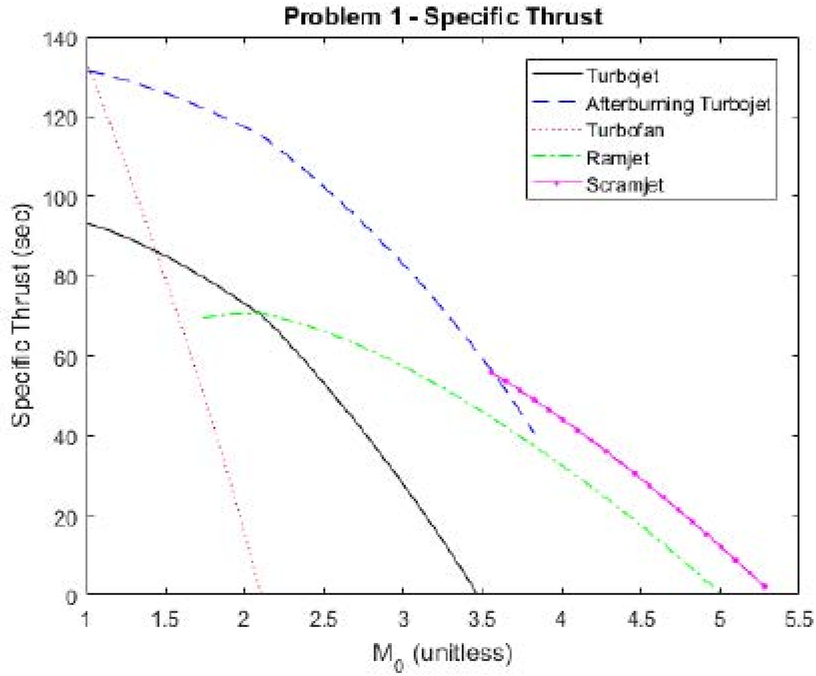
$$M(h) = \frac{V(h)}{\sqrt{\gamma R T(h)}} \Bigg\} \text{Plot } M \text{ vs } h \text{ for different } q\text{'s}$$

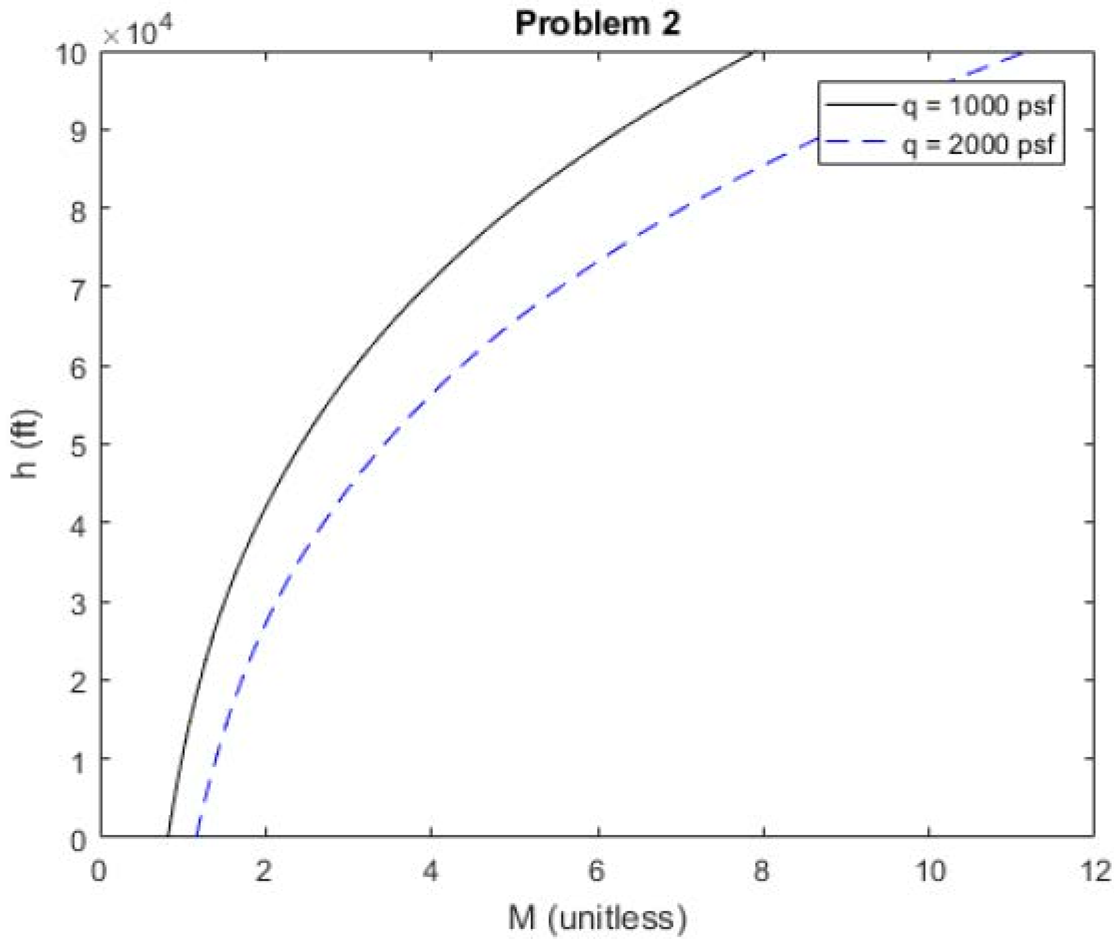
Similarly you can plot T_t and P_t as a function of altitude using atmosphere code and local $M(h)$.

* See Plots

1)


Problem 1 - Specific Impulse
I_{sp} (sec)
M_0 (unitless)
Turbojet
Afterburning Turbojet
Turbofan
Ramjet
Scramjet
Rocket
0
1000
2000
3000
4000
5000
6000
1
2
3
4
5
6
7
8
9
10


Problem 1 - Specific Thrust
Specific Thrust (sec)
M_0 (unitless)
Turbojet
Afterburning Turbojet
Turbofan
Ramjet
Scramjet
0
20
40
60
80
100
120
140
1
1.5
2
2.5
3
3.5
4
4.5
5
5.5



Problem 2
×10^4
q = 1000 psf
q = 2000 psf
h (ft)
M (unitless)
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10
12

```
%AAE 537 HW1 code

clear all;
close all;
clc;

workdir = 'C:\Users\ssardesh\Google Drive\dropbox\Classes\AAE537\HW1';
Altitude = [0:1:100];   %kft
Altft = Altitude.*1e3;
q = [800, 1000, 1200]*47.77;

[T,P,Rho,dummy]=atmosphere4(Altft,1);

T = T'*5/9;   %K
P = P'.*47.77;   %Pa
Rho = Rho'*515.379;  %kg/m3
R = 287;
gamma = 1.4;
Pr = 0.71;
r = Pr^(1/3);
for i = 1:length(q)
    a(:,i) = sqrt(gamma*R*T);
    U(:,i) = (sqrt(2*q(i)./Rho));
    M(:,i) = U(:,i)./a(:,i);
    P0(:,i) = P.*(1+(gamma-1)/2.*M(:,i).^2).^(gamma/(gamma-1));
    P0psf(:,i) = P0(:,i)./47.77;
    T0(:,i) = T.*(1+(gamma-1)/2.*M(:,i).^2);
    T0R(:,i) = 9/5.*T0(:,i);
    T0F(:,i) = T0R(:,i)-459.67;
    Tad(:,i) = T+r.*(T0(:,i)-T);
    TadR(:,i) = Tad(:,i).*9/5;
    TadF(:,i) = TadR(:,i) - 459.67;
end

figure
plot(M(:,1),Altitude,M(:,2),Altitude,M(:,3),Altitude);
xlabel('Mach number');
ylabel('Altitude, kft');
legend('q=800','q=1000','q=1500');
s = hgexport('readstyle','537');
fname = [workdir '\Mach.png'];
hgexport(gcf, fname, s,'applystyle',true);
print('-dpng','-r100',fname);

figure
plot(T0(:,1),Altitude,T0(:,2),Altitude,T0(:,3),Altitude);
xlabel('Stagnation temperature, K');
ylabel('Altitude, kft');
legend('q=800','q=1000','q=1500');
s = hgexport('readstyle','537');
fname = [workdir '\T0.png'];
hgexport(gcf, fname, s,'applystyle',true);
print('-dpng','-r100',fname);

figure
plot(T0R(:,1),Altitude,T0R(:,2),Altitude,T0R(:,3),Altitude);
xlabel('Stagnation temperature, R');
ylabel('Altitude, kft');
legend('q=800','q=1000','q=1500');
s = hgexport('readstyle','537');
fname = [workdir '\T0R.png'];
hgexport(gcf, fname, s,'applystyle',true);
print('-dpng','-r100',fname);
```

```
figure
plot(T0F(:,1),Altitude,T0F(:,2),Altitude,T0F(:,3),Altitude);
xlabel('Stagnation temperature, F');
ylabel('Altitude, kft');
legend('q=800','q=1000','q=1500');
s = hgexport('readstyle','537');
fname = [workdir '\T0F.png'];
hgexport(gcf, fname, s,'applystyle',true);
print('-dpng','-r100',fname);

figure
plot(Tad(:,1),Altitude,Tad(:,2),Altitude,Tad(:,3),Altitude);
xlabel('Recovery temperature, K');
ylabel('Altitude, kft');
legend('q=800','q=1000','q=1500');
s = hgexport('readstyle','537');
fname = [workdir '\Tad.png'];
hgexport(gcf, fname, s,'applystyle',true);
print('-dpng','-r100',fname);

figure
plot(TadR(:,1),Altitude,TadR(:,2),Altitude,TadR(:,3),Altitude);
xlabel('Recovery temperature, R');
ylabel('Altitude, kft');
legend('q=800','q=1000','q=1500');
s = hgexport('readstyle','537');
fname = [workdir '\TadR.png'];
hgexport(gcf, fname, s,'applystyle',true);
print('-dpng','-r100',fname);

figure
plot(TadF(:,1),Altitude,TadF(:,2),Altitude,TadF(:,3),Altitude);
xlabel('Recovery temperature, F');
ylabel('Altitude, kft');
legend('q=800','q=1000','q=1500');
s = hgexport('readstyle','537');
fname = [workdir '\TadF.png'];
hgexport(gcf, fname, s,'applystyle',true);
print('-dpng','-r100',fname);

figure
plot(P0(:,1)/1e5,Altitude,P0(:,2)/1e5,Altitude,P0(:,3)/1e5,Altitude);
xlabel('Stagnation pressure, bar');
ylabel('Altitude, kft');
legend('q=800','q=1000','q=1500');
ax = gca;
ax.XScale = 'log';
s = hgexport('readstyle','537');
fname = [workdir '\P0.png'];
hgexport(gcf, fname, s,'applystyle',true);
print('-dpng','-r100',fname);

figure
plot(P0psf(:,1),Altitude,P0psf(:,2),Altitude,P0psf(:,3),Altitude);
xlabel('Stagnation pressure, psf');
ylabel('Altitude, kft');
legend('q=800','q=1000','q=1500');
ax = gca;
ax.XScale = 'log';
s = hgexport('readstyle','537');
fname = [workdir '\P0psf.png'];
hgexport(gcf, fname, s,'applystyle',true);
print('-dpng','-r100',fname);
```

```
figure
plot(P(:,1)/1e5,Altitude);
xlabel('Static pressure, bar');
ylabel('Altitude, kft');
%legend('q=800','q=1000','q=1500');
ax = gca;
ax.XScale = 'log';
s = hgexport('readstyle','537');
fname = [workdir '\P.png'];
hgexport(gcf, fname, s,'applystyle',true);
print('-dpng','-r100',fname);

figure
plot(P(:,1)/47.77,Altitude);
xlabel('Static pressure, psf');
ylabel('Altitude, kft');
%legend('q=800','q=1000','q=1500');
ax = gca;
ax.XScale = 'log';
s = hgexport('readstyle','537');
fname = [workdir '\Ppsf.png'];
hgexport(gcf, fname, s,'applystyle',true);
print('-dpng','-r100',fname);
```