

Lecture 16 - tensor flow - math

What is A "tensor" and how will it "flow"?

In the general definition "tensors" are arrays of numbers organized into an n-dimensional grid.

A scalar is a 1-ish number. This is the simplest kind of tensor:

```
1: import tensorflow as tf
2:
3: x = tf.constant(-2.0, name="x", dtype=tf.float32)
4: a = tf.constant(5.0, name="a", dtype=tf.float32)
5: b = tf.constant(13.0, name="b", dtype=tf.float32)
6:
7: y = tf.Variable(tf.add(tf.multiply(a, x), b))
8:
9: print ( "result is:" )
10: tf.print ( y )
```

Elements are positionally identifiable. So A at i,j,k is $A_{i,j,k}$.

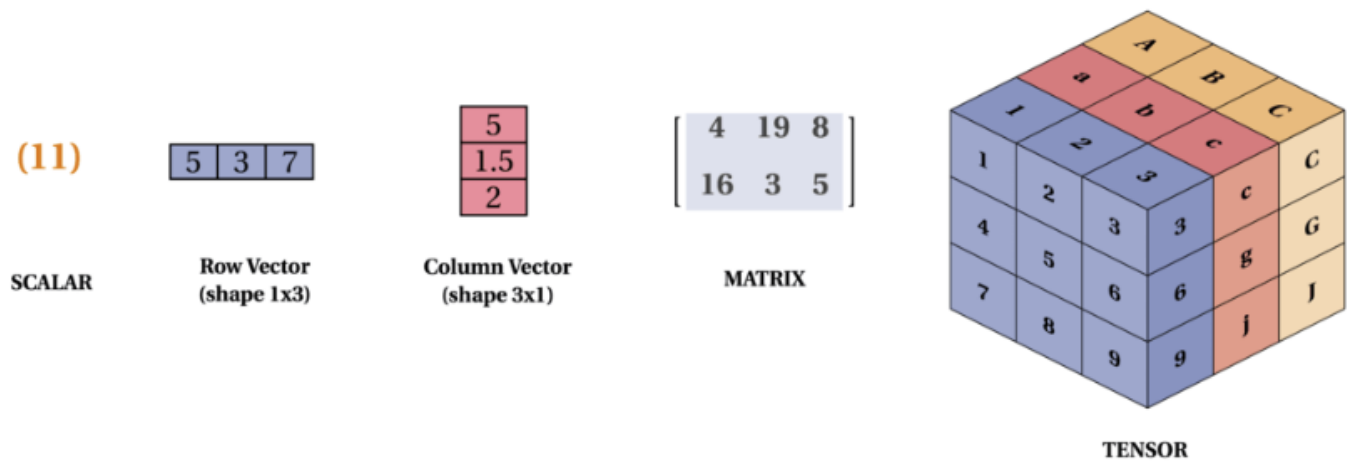
A vector is a 1x array of numbers. $[1,2,4]$ that is the x,y,z distance from the origin.

The tensor is the 3d vector of each of these.

You are not limited to 3d data.

So...

Tensor:



Ranking:

rank 0 - scalar

rank 1 - vector [1,2,3]

rank 2 - matrix [[1,2] , [2, 3]]

rank 3 - 3 tensor

rank 4 - 4 tensor

Add of 2 matrix tensors

add1.py:

```

1: import tensorflow as tf
2:
3: # let's create a ones 3x3 rank 2 tensor
4: rank_2_tensor_A = tf.ones([3, 3], name='MatrixA')
5: print("3x3 Rank 2 Tensor A: \n{}\n".format(rank_2_tensor_A))
6:
7: # let's manually create a 3x3 rank two tensor and specify the data type as float
8: rank_2_tensor_B = tf.constant([[1, 2, 3], [4, 5, 6], [7, 8, 9]], name='MatrixB', dtype=tf.float32)
9: print("3x3 Rank 2 Tensor B: \n{}\n".format(rank_2_tensor_B))
10:
11: # addition of the two tensors
12: rank_2_tensor_C = tf.add(rank_2_tensor_A, rank_2_tensor_B, name='MatrixC')
13: print("Rank 2 Tensor C with shape={} and elements: \n{}\n".format(rank_2_tensor_C.shape, rank_2_tensor_C))

```

$$[A]_{m \times n} [B]_{n \times p} = [C]_{m \times p}$$

Some matrix multiplication:

Definition of multiply

$$c_{ij} = \sum_{k=1}^n a_{ik} b_{kj}$$

Let's multiply using TF:

File matmul1.py:

```
1: import tensorflow as tf
2:
3: # Matrix A and B with shapes (2, 3) and (3, 4)
4: mmv_matrix_A = tf.ones([2, 3], name="matrix_A")
5: mmv_matrix_B = tf.constant([[1, 2, 3, 4], [1, 2, 3, 4], [1, 2, 3, 4]], \
6:     name="matrix_B", dtype=tf.float32)
7:
8: # Matrix Multiplication: C = AB with C shape (2, 4)
9: matrix_multiply_C = tf.matmul(mmv_matrix_A, mmv_matrix_B, \
10:     name="matrix_multiply_C")
11:
12: print("""Matrix A: shape {0} \nelements: \n{1} \n\n
13: Matrix B: shape {2} \nelements: \n{3}\n
14: Matrix C: shape {4} \nelements: \n{5}"""). \
15:     format(mmv_matrix_A.shape, mmv_matrix_A, mmv_matrix_B.shape, \
16:     mmv_matrix_B, matrix_multiply_C.shape, matrix_multiply_C))
```

output matmul1.out:

```
Matrix A: shape (2, 3)
elements:
[[1. 1. 1.]
 [1. 1. 1.]]
```

```
Matrix B: shape (3, 4)
elements:
[[1. 2. 3. 4.]
 [1. 2. 3. 4.]
 [1. 2. 3. 4.]]
```

```
Matrix C: shape (2, 4)
elements:
[[ 3.  6.  9. 12.]
 [ 3.  6.  9. 12.]]
```

Compare to a matrix multiply directly in code.

File matmul3.py:

```

1: # Example of multiplying 2 matrices in Python
2:
3: # 3x3 matrix
4: X = [[12,7,3],
5:      [4 ,5,6],
6:      [7 ,8,9]]
7: # 3x4 matrix
8: Y = [[5,8,1,2],
9:      [6,7,3,0]]
10: # result is 3x4
11: result = [[0,0,0,0],
12:           [0,0,0,0],
13:           [0,0,0,0]]
14:
15: # iterate through rows of X
16: for i in range(len(X)):
17:     # iterate through columns of Y
18:     for j in range(len(Y[0])):
19:         # iterate through rows of Y
20:         for k in range(len(Y)):
21:             result[i][j] += X[i][k] * Y[k][j]
22:
23: a = "["
24: com = ","
25: for i in range(len(result)):
26:     if i+1 == len(result):
27:         com = "]"
28:     print("{}{}{}".format(a,result[i],com))
29:     a = " "

```

output:

```

[[102, 145, 33, 24],
 [50, 67, 19, 8],
 [83, 112, 31, 14]]

```

Inner Dimensions must be the same.

A by hand example:

$$c_{11} = a_{11}b_{11} + a_{12}b_{21} + a_{13}b_{31} + a_{14}b_{41}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

$2 \times 4 \qquad \qquad 4 \times 3 \qquad \qquad 2 \times 3$

$$c_{22} = a_{21}b_{12} + a_{22}b_{22} + a_{23}b_{32} + a_{24}b_{42}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

With Some Data

First a 1x example:

$$\begin{bmatrix} 5 \\ 3 \\ 7 \\ 1 \end{bmatrix} \begin{bmatrix} 6 & 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 30 & 10 & 15 & 20 \\ 18 & 6 & 9 & 12 \\ 42 & 14 & 21 & 28 \\ 6 & 2 & 3 & 4 \end{bmatrix}$$

Matrices are useful

Calculate Inverse of a Matrix:

File: inv.py:

```

1: import tensorflow as tf
2:
3: iim_matrix_A = tf.constant([[2, 3], [2, 2]], name='MatrixA', dtype=tf.float32)
4:
5: try:
6:     # Tensorflow function to take the inverse
7:     inverse_matrix_A = tf.linalg.inv(iim_matrix_A)
8:
9:     # Creating a identity matrix using tf.eye
10:    identity_matrix = tf.eye(2, 2, dtype=tf.float32, name="identity")
11:
12:    iim_RHS = identity_matrix
13:    iim_LHS = tf.matmul(inverse_matrix_A, iim_matrix_A, name="LHS")
14:
15:    predictor = tf.reduce_all(tf.equal(iim_RHS, iim_LHS))
16:    def true_print(): print("""A^-1 times A equals the Identity Matrix
17: Matrix A: \n{0} \n\nInverse of Matrix A: \n{1} \n\nRHS: I: \n{2} \n
18: LHS: A^(-1) A: \n{3}""").format(iim_matrix_A, inverse_matrix_A,
19:    iim_RHS, iim_LHS))
20:    def false_print(): print("Condition Failed")
21:    tf.cond(predictor, true_print, false_print)
22:
23: except:
24:    print("""A^-1 doesnt exist
25: Matrix A: \n{0} \n\nInverse of Matrix A: \n{1} \n\nRHS: I: \n{2}
26: \nLHS: (A^(-1) A): \n{3}""").format(iim_matrix_A, inverse_matrix_A, iim_RHS, iim_LHS))

```

And the output:

```

A^-1 times A equals the Identity Matrix
Matrix A:
[[2. 3.]
 [2. 2.]]

Inverse of Matrix A:
[[-1.  1.5]
 [ 1. -1. ]]

RHS: I:
[[1. 0.]
 [0. 1.]]

LHS: A^(-1) A:
[[1. 0.]
 [0. 1.]]

```