Lecture 16 - tensor flow - math

What is A “tensor” and how will it “flow”?

In the general definition “tensors” are arrays of numbers organized into an n-dimensional grid.

A scalar is a 1-ish number. This is the simplest kind of tensor:

```
import tensorflow as tf

x = tf.constant(-2.0, name="x", dtype=tf.float32)
a = tf.constant(5.0, name="a", dtype=tf.float32)
b = tf.constant(13.0, name="b", dtype=tf.float32)

y = tf.Variable(tf.add(tf.multiply(a, x), b))

print ( "result is:" )
tf.print ( y )
```

Elements are positionaly identifiable. So A at i,j,k is `Ai,j,k` .

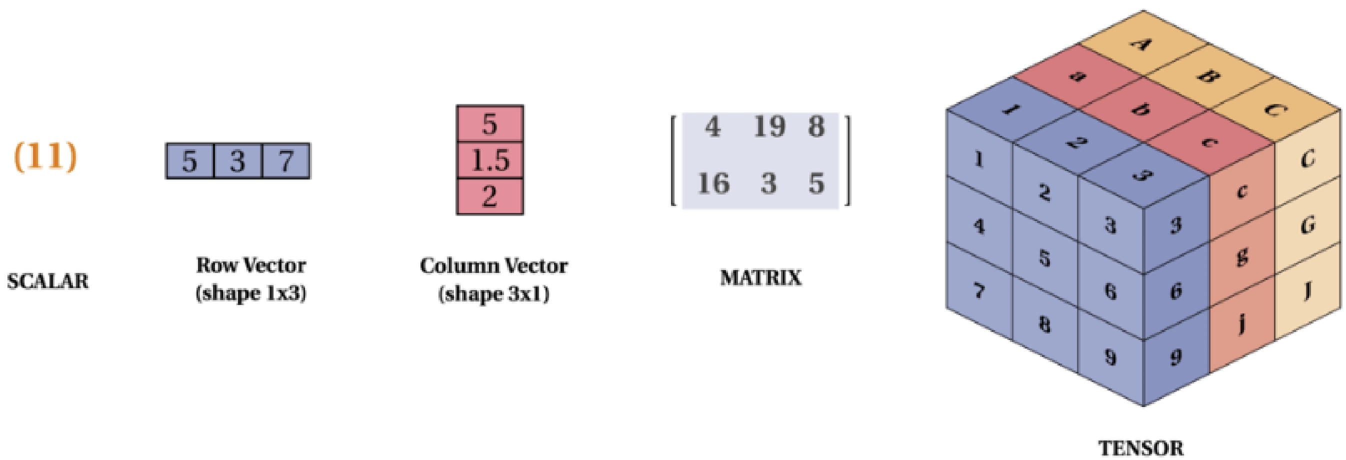
A vector is a 1x array of numbers. `[1,2,4]` that is the x,y,z distance from the origin.

The tensor is the 3d vector of each of these.

You are not limited to 3d data.

So...

Tensor:



Ranking:

rank 0 - scalar

rank 1 - vector [1,2,3]

rank 2 - matrix [[1,2] , [2, 3]]

rank 3 - 3 tensor

rank 4 - 4 tensor

Add of 2 matrix tensors

add1.py:

```
import tensorflow as tf

# let's create a ones 3x3 rank 2 tensor
rank_2_tensor_A = tf.ones([3, 3], name='MatrixA')
print("3x3 Rank 2 Tensor A: \n{}\n".format(rank_2_tensor_A))

# let's manually create a 3x3 rank two tensor and specify the data type as float
rank_2_tensor_B = tf.constant([[1, 2, 3], [4, 5, 6], [7, 8, 9]], name='MatrixB', dtype=tf.float32)
print("3x3 Rank 2 Tensor B: \n{}\n".format(rank_2_tensor_B))

# addition of the two tensors
rank_2_tensor_C = tf.add(rank_2_tensor_A, rank_2_tensor_B, name='MatrixC')
print("Rank 2 Tensor C with shape={} and elements: \n{}".format(rank_2_tensor_C.shape, rank_2_tensor_C))
```

Some matrix multiplication: $[\mathrm{A}]_{m\times n}[\mathrm{B}]_{n\times p} = [\mathrm{C}]_{m\times p}$

Definition of multiply

$$\mathrm{c}_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

Let's multiply using TF:

File matmul1.py:

```
import tensorflow as tf

# Matrix A and B with shapes (2, 3) and (3, 4)
mmv_matrix_A = tf.ones([2, 3], name="matrix_A")
mmv_matrix_B = tf.constant([[1, 2, 3, 4], [1, 2, 3, 4], [1, 2, 3, 4]], \
    name="matrix_B", dtype=tf.float32)

# Matrix Multiplication: C = AB with C shape (2, 4)
matrix_multiply_C = tf.matmul(mmv_matrix_A, mmv_matrix_B, \
    name="matrix_multiply_C")

print("""Matrix A: shape {0} \nelements: \n{1} \n\n
Matrix B: shape {2} \nelements: \n{3}\n
Matrix C: shape {4} \nelements: \n{5}""". \
    format(mmv_matrix_A.shape, mmv_matrix_A, mmv_matrix_B.shape, \
    mmv_matrix_B, matrix_multiply_C.shape, matrix_multiply_C))
```

output matmul1.out:

```
Matrix A: shape (2, 3)
elements:
[[1. 1. 1.]
 [1. 1. 1.]]


Matrix B: shape (3, 4)
elements:
[[1. 2. 3. 4.]
 [1. 2. 3. 4.]
 [1. 2. 3. 4.]]

Matrix C: shape (2, 4)
elements:
[[ 3.  6.  9. 12.]
 [ 3.  6.  9. 12.]]
```

Compare to a matrix multiply directly in code.

File matmul3.py:

```
# Example of multipying 2 matricies in Python

# 3x3 matrix
X = [[12,7,3],
     [4 ,5,6],
     [7 ,8,9]]
# 3x4 matrix
Y = [[5,8,1,2],
     [6,7,3,0]]
# result is 3x4
result = [[0,0,0,0],
          [0,0,0,0],
          [0,0,0,0]]

# iterate through rows of X
for i in range(len(X)):
   # iterate through columns of Y
   for j in range(len(Y[0])):
       # iterate through rows of Y
       for k in range(len(Y)):
           result[i][j] += X[i][k] * Y[k][j]

a = "["
com = ","
for i in range(len(result)):
   if i+1 == len(result):
       com = "]"
   print("{}{}{}".format(a,result[i],com))
   a = " "
```

output:

```
[[102, 145, 33, 24],
 [50, 67, 19, 8],
 [83, 112, 31, 14]]
```

Inner Dimentions must be the same.

A by hand example:

$$c_{11}=a_{11}b_{11}+a_{12}b_{21}+a_{13}b_{31}+a_{14}b_{41}$$

$$\begin{bmatrix} \boxed{a_{11} \quad a_{12} \quad a_{13} \quad a_{14}} \\ a_{21} \quad a_{22} \quad a_{23} \quad a_{24} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

2 x 4 4 x 3 2 x 3

$$c_{22}=a_{21}b_{12}+a_{22}b_{22}+a_{23}b_{32}+a_{24}b_{42}$$

$$\begin{bmatrix} a_{11} \quad a_{12} \quad a_{13} \quad a_{14} \\ \boxed{a_{21} \quad a_{22} \quad a_{23} \quad a_{24}} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \\ b_{41} & b_{42} & b_{43} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \end{bmatrix}$$

With Some Data

First a 1x example:

$$\begin{bmatrix} 5 \\ 3 \\ 7 \\ 1 \end{bmatrix} \begin{bmatrix} 6 & 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 30 & 10 & 15 & 20 \\ 18 & 6 & 9 & 12 \\ 42 & 14 & 21 & 28 \\ 6 & 2 & 3 & 4 \end{bmatrix}$$

Matrices are useful

Calculate Inverse of a Matrix:

File: inv.py:

```
import tensorflow as tf

jim_matrix_A = tf.constant([[2, 3], [2, 2]], name='MatrixA', dtype=tf.float32)
```

```

try:
    # Tensorflow function to take the inverse

    inverse_matrix_A = tf.linalg.inv(iim_matrix_A)

    # Creating a identity matrix using tf.eye
    identity_matrix = tf.eye(2, 2, dtype=tf.float32, name="identity")

    iim_RHS = identity_matrix
    iim_LHS = tf.matmul(inverse_matrix_A, iim_matrix_A, name="LHS")

    predictor = tf.reduce_all(tf.equal(iim_RHS, iim_LHS))
    def true_print(): print("""A^-1 times A equals the Identity Matrix
Matrix A: \n{0} \n\nInverse of Matrix A: \n{1} \n\nRHS: I: \n{2} \n
LHS: A^(-1) A: \n{3}""".format(iim_matrix_A, inverse_matrix_A,
    iim_RHS, iim_LHS))
    def false_print(): print("Condition Failed")
    tf.cond(predictor, true_print, false_print)

except:
    print("""A^-1 doesnt exist
    Matrix A: \n{} \n\nInverse of Matrix A: \n{} \n\nRHS: I: \n{}
    \nLHS: (A^(-1) A): \n{}""".format(iim_matrix_A, inverse_matrix_A, iim_RHS, iim_LHS))
```

And the output:

```
A^-1 times A equals the Identity Matrix
Matrix A:
[[2. 3.]
 [2. 2.]]

Inverse of Matrix A:
[[-1.   1.5]
 [ 1.  -1. ]]

RHS: I:
[[1. 0.]
 [0. 1.]]

LHS: A^(-1) A:
[[1. 0.]
 [0. 1.]]
```