Abstracts of ALH Talks

Adaricheva, Kira

Title. Well known and little known Nation

Speaker. Kira Adaricheva

Institution. Department of Mathematics, Hofstra University, Hempstead NY

Abstract. Starting from famous Malcev conference in Novosibirsk in 1989, the author describes the stride to approach the solution of Birkhoff-Malcev problem, where Ralph, JB and Bill all made their contribution. With JB Nation in particular, since R. Mckenzie's conference in 2002, it went into publishing uncountably many joint papers and finitely many chapters in the volume 2 of Lattice Theory: Special Topics and Applications (2016), edited by G.Grätzer and F. Wehrung. Surprisingly, the byproducts of this stride were as valuable as the the progress in Birkhoff-Malcev problem itself, bringing understanding to infinite convex geometries, and new applications in data science among others.

Aglianó, P.

Title. Splitting residuated (semi)lattices

Speaker. P. Aglianó

Institution. Via Roma 56 53100, Siena, Italy

Abstract. The concept of splitting is of lattice-theoretic origin; if $\mathbf{L}$ is any lattice a pair $(a, b) \in L^2$ of elements of L is a **splitting pair** if L is equal to the disjoint union of the ideal generated by a and the filter generated by b. If $\mathcal{V}$ is any variety, an algebra $\mathbf{A} \in \mathcal{V}$ is **splitting in** $\mathcal{V}$ if $\mathbb{V}(\mathbf{A})$ is the right member of a splitting pair in the lattice of subvarieties of $\mathcal{V}$. A more transparent definition is the following: $\mathbf{A}$ is splitting in $\mathcal{V}$ if there is a subvariety $\mathcal{W}_\mathbf{A} \subseteq \mathcal{V}$ (the {\bf conjugate variety of $\mathbf{A}$}) such that for any variety $\mathcal{U} \subseteq \mathcal{V}$ either $\mathcal{U} \subseteq \mathcal{W}_\mathbf{A}$ or $\mathbf{A} \in \mathcal{U}$.

Splitting lattices were introduced by P. Whitman (1943) in the early 40's; thirty years later R. McKenzie (1972) explored the concept with greater fortune. Note that if (a, b) is a splitting pair then, by a standard lattice theoretic argument, b must be completely meet prime and a completely join prime. If we apply this to the splitting pair $(\mathcal{W}_\mathbf{A}, \mathbb{V}(\mathbf{A}))$ then $\mathcal{W}_\mathbf{A}$ is axiomatized by a single equation and $\mathbf{A}$ is a finitely generated subdirectly irreducible algebra. If moreover $\mathcal{V}$ is generated by its finite algebras, then $\mathbf{A}$ must in fact be finite and if $\mathcal{V}$ is also congruence distributive then (by Jónsson Lemma) $\mathbf{A}$ is in fact unique. We summarize all this in a theorem:

Theorem 1. [McKenzie (1972)] Let $\mathcal{V}$ be a congruence distributive variety that is generated by its finite algebras; then any splitting algebra $\mathbf{A}$ in $\mathcal{V}$ is finite and subdirectly irreducible. Moreover the conjugate variety $\mathcal{W}_\mathbf{A}$ is axiomatizable by a single equation in the language of $\mathcal{W}$.

Being generated by its finite algebras is the *finite model property* for the equational theory of $\mathcal{V}$: if an equation fails in $\mathcal{V}$, then it fails in some finite algebra in $\mathcal{V}$. The equation whose existence is postulated by Theorem 1 is called the **splitting equation** of $\mathbf{A}$. Note that the theorem does not say that if the hypotheses hold, then there is a splitting algebra in $\mathcal{V}$; nor its proof produces an effective way of determining the splitting equation of $\mathbf{A}$, in case it is splitting. Both the existence and the splitting equation require *ad hoc* arguments; and in 1972 McKenzie did exactly that, characterizing the splitting algebras in the variety of all lattices and also giving an algorithm (still the only one available) for their splitting equations.

Almost in the same period of time in which R. McKenzie was working towards his results, in a different part of the world V. Jankov was studying intermediate logics, which amounts to studying subvarieties of the variety $\mathcal{HA}$ of Heyting algebras. In his investigations Jankov (1968) found a way to associate to any finite subdirectly irreducible Heyting algebra $\mathbf{A}$ a term $J_\mathbf{A}$ (called the *Jankov formula*) and was able to prove essentially that:

1. the largest variety of Heyting algebras not containing $\mathbb{V}(\mathbf{A})$ is axiomatized by $J_\mathbf{A} \approx 1$;

2. hence any finite subdirectly irreducible Heyting algebra is splitting in $\mathcal{HA}$ with splitting equation $J_{\mathbf{A}} \approx 1$.

Subdirectly irreducible Heyting algebras are easily described: they are exactly all Heyting algebras having a unique maximal element below the top element 1; a fancy way of saying that it that they can be represented as $\mathbf{H} \oplus \mathbf{2}$, where $\mathbf{2}$ is the two element Boolean algebra and $\oplus$ is the *ordinal sum*. So we can restate Jankov's result in this fashion:

Theorem 2. A finite Heyting algebra is splitting if and only if it is of the form $\mathbf{H} \oplus \mathbf{2}$ for some Heyting algebra $\mathbf{H}$.

We will explain how this result can be generalized to several classes of residuated (semi)lattices.

Aten, Charlotte

Title. Multiplayer Rock-Paper-Scissors

Speaker. Charlotte Aten

Institution. Department of Mathematics, University of Rochester, Rochester, NY

Abstract. We examine a class of algebras generalizing the game of Rock-Paper-Scissors. We give a numerical constraint on the arity and order of such algebras by using a result on binomial coefficients communicated by Erdos. This constraint is sharp in the sense that such algebras always exist when our condition is satisfied. Some elementary algebraic properties of the class are also discussed.

Bergman, Clifford

Title. Joins and Maltsev Products of Congruence Permutable Varieties

Speaker. Clifford Bergman

Institution. Department of Mathematics, Iowa State University, Ames, Iowa, 50011

Abstract. Let $\mathcal{A}$ and $\mathcal{B}$ be idempotent varieties and suppose that the variety $\mathcal{A} \vee \mathcal{B}$ is congruence permutable. Then the Maltsev product $\mathcal{A} \circ \mathcal{B}$ is also congruence permutable.

Clark, David M.

Title. Asymptotic Properties of Finite Groupoids

Speaker. David M. Clark

Institution. SUNY New Paltz, NY

Abstract. Recent work has shown that randomizing algorithms drawn from the methods of evolutionary computation can efficiently find a term representing a given term operation on an idemprimal groupoid provided the groupoid has two other properties: NSR and AC. This work motivates a deeper study of these two properties which prove to be worthy of investigation quite apart from their application to these algorithms.

Dang, V. V.

Title. Minimal SH-approximation of semigroups

Speaker. V. V. Dang

Institution. Department of Applied Sciences, Vietnam National University-HCMC, Hochiminhcity University of Technology, Saigon, Vietnam

Abstract. The problem of SH-approximation of a semigroup with respect to the predicate of a possible belonging of an element to a subsemigroup is considered. Several explicit conditions for SH-approximation with respect to this predicate are presented. We constructed a special semigroup acting the role of a minimal SH-approximation of semigroup for many predicates. This semigroup has neither identity nor additive identity. It contains an infinite number of idempotents, and the presence of each idempotent is mandatory.

Davey, Brian

Title. A Tale of Three Papers: a beautiful theorem, four open problems, a surprising connection. (My joint work with Bill, JB and Ralph.)

Speaker. Brian Davey

Institution. Department of Mathematics, Latrobe University, Australia

Abstract. I have published one paper with each of Bill, JB and Ralph.

- Davey, B. A.; Idziak, P. M.; Lampe, W. A.; McNulty, G. F.: Dualizability and graph algebras. Discrete Math. 214 (2000), 145–172.
- Davey, B. A.; Nation, J. B.; McKenzie, R. N.; Palfy, P. P.: Braids and their monotone clones. Algebra Universalis 32 (1994), 153–176.
- Clark, D. M.; Davey, B. A.; Freese, R. S.; Jackson, M.: Standard topological algebras: syntactic and principal congruences and profiniteness. Algebra Universalis 52 (2004), 343–376.

The talk will be based around the results in these papers and developments (mainly in the theory of natural dualities) related to them.

DeMeo, William

Title. Thank you Bill, JB and Ralph

Speaker. William DeMeo

Institution. Department of Mathematics, University of Colorado, Boulder.

Abstract. I will tell some stories about what it was like to be a student of Bill, JB and Ralph in Hawaii. My time at UH was the most formative of my mathematical training. The three honorees, through their brilliance and their generosity, taught me what it means to be a mathematician and a gentleman. They showed me that it is possible to devote one's scientific life to lattice theory and universal algebra, and I am grateful anyway.

Gjonbalaj, Gent

Title. Description of the closure operator for a convex geometry of convex dimension two

Speaker. Gent Gjonbalaj

Institution. Department of Mathematics, Hofstra University, Hempstead, NY

Abstract. Convex geometry is a closure space (G, ϕ) with the anti-exchange property. A classical result of Edelman and Jamison (1985) claims that every finite convex geometry is a join of several linear sub-geometries, and the smallest number of such sub-geometries necessary for representation is called a convex dimension. In our work we find necessary and sufficient conditions on a closure operator ϕ of convex geometry (G, ϕ) so that its convex dimension is 2,

equivalently, they are represented by segments on a line. These conditions can be checked in polynomial time in two parameters: the size of the base set $|G|$ and the size of the implicational basis of (G, ϕ).

Idziak, Paweł M.

Title. Solving Equations – kith and kin

Speaker. Paweł M. Idziak

Institution. Jagiellonian University, Faculty of Mathematics and Computer Science, Department of Theoretical Computer

Abstract. The talk is intended to present latest achievements in searching structural algebraic conditions a finite algebra A has to satisfy in order to have a polynomial time algorithm that decides if an equation $s(x_1, \ldots, x_n) = t(x_1, \ldots, x_n)$, where s and t are polynomials over A, has a solution in A.

Jipsen, Peter

Title. Greatest Hits of the Hawaiian Legends of Universal Algebra and Lattice Theory

Speaker. Peter Jipsen

Institution. Chapman University

Abstract. In this talk I will cover some of the highlights of the many deep and influential results that were discovered by Ralph Freese, Bill Lampe and JB Nation. Be prepared for a whirlwind tour of the classics spanning half a century of hits that reverberated around the globe and continue to delight and inspire mathematicians across space and time.

Kearnes, Keith

Title. Is supernilpotence super nilpotence?

Speaker. Keith Kearnes

Institution. Department of Mathematics, University of Colorado, Boulder

Abstract. An algebra is nilpotent of class at most 2 if it satisfies the commutator condition $[1, [1, 1]] = 0$, and it is supernilpotent of class at most 2 if it satisfies the higher commutator condition $[1, 1, 1] = 0$. There are higher-class nilpotence conditions for both commutators. How are they related? From the names, one expects that, for every k there is some ℓ such that

class-k supernilpotence = (class-ℓ nilpotence + ε)

holds. I will speak about this equation.

Kjos-Hanssen, Bjørn

Title. Automatic complexity of monotone Boolean functions

Speaker. Bjørn Kjos-Hanssen

Institution. Department of Mathematics, University of Hawai’i, HI

Abstract. Champarnaud and Pin (1989) found the maximum complexity, in terms of finite automata, of a Boolean function on n variables. We study the special case of monotone functions. It is inspired by an application to complexity of financial securities. Mathematically we relate it to a certain lattice homomorphism problem.

Kehayopulu, Niovi

Title. How we pass from ordered semigroups to ordered hypersemigroups

Author. Niovi Kehayopulu

Institution. University of Athens, Department of Mathematics, 15784 Panepistimiopolis, Greece

Dedicated to Professor J. B. Nation

Abstract. We have seen in [N. Kehayopulu, Lobachevskii J. Math. 39, no. 1 (2018), 121–128] the way we pass from semigroups to hypersemigroups. The aim of the present paper is to show the way we pass from ordered semigroups to ordered hypersemigroups.

An hypersemigroup is a nonempty set H with an "operation" $\circ : H \times H \to P*(H) \mid (a, b) \to a \circ b$ on H called hyperoperation (as it maps to each couple a, b of elements of S a nonempty subset $a \circ b$ of S) and an operation $* : P*(H) \times P*(H) \to P*(H) \mid (A, B) \to A * B$ on $P*(H)$ (induced by the operation of H) such that (1) $A * B = (a \circ b)$ for every $A, B \in P*(H)$ $(a,b) \in A \times B$ and (2) $\{a\} * (b \circ c) = (a \circ b) * \{c\}$ for every $a, b, c \in H$ ($P*(H)$ denotes the set of nonempty subsets of H). If H is an hypersemigroup and "≤" an order relation on H, we denote by (A] the subset of H defined by $(A] := \{t \in H \mid t \le a \text{ for some } a \in A\}$; and by "≼" the relation on $P*(H)$ defined by

$$\preccurlyeq := \{(A, B) | \forall a \in A \exists b \in B \text{ such that } (a, b) \in \le\}.$$

So, for $A, B \in P*(H)$, we write $A \preccurlyeq B$ if for every $a \in A$ there exists $b \in B$ such that $a \le b$. This is a reflexive and transitive relation on $P*(H)$, that is, a preorder on $P*(H)$. An hypersemigroup $(H, \circ)$ is called an ordered hypersemigroup if there exists an order relation "≤" on H such that $a \le b$ implies $a \circ c \preccurlyeq b \circ c$ and$c \circ a \preccurlyeq c \circ b$ for every $c \in H$; which means that if $a \le b$, then for every $u \in a \circ c$ there exists $v \in b \circ c$ such that $u \le v$ and for every $u \in c \circ a$ there exists $v \in c \circ b$ such that $u \le v$. If $(H, \circ, \le)$ is an ordered hypersemigroup, a nonempty subset A of H is called a left (resp. right) ideal of H if (1) $H * A \subseteq A$ (resp. $A * H \subseteq A$) and (2) if $a \in A$ and $H \ni b \le a$ implies $b \in A$; it is called a quasi-ideal of H if (1) $(A * H] \cap (H * A] \subseteq A$ and (2) if $a \in A$ and $H \ni b \le a$, then $b \in A$. Recall that $H * A \subseteq A$ is equivalent to $h \circ a \subseteq A$ for every $h \in H$ and every $a \in A$. A nonempty subset A of H is called idempotent if $(A * A] = A$.

We introduce the concept of regular ordered hypersemigroups, as follows: An ordered hypersemigroup H is called regular if for every $a \in H$ there exists $x \in H$ such that $\{a\} \preccurlyeq (a \circ x) * \{a\}$; which means that for every $a \in H$ there exist $x, t \in H$ such that $t \in (a \circ x) * \{a\}$ and $a \le t$. Some well known results on semigroups or on ordered semigroups due to Iséki, Lajos and Szász, Calais and Kehayopulu are examined in case of ordered hypersemigroups. Based on the result by Calais [Demi-groupes quasi-inversifs, C.R. Acad. Sci. Paris 252 (1961), 2357–2359], for example, we prove that an ordered hypersemigroup H is regular if and only if the right ideals and the left ideals of H are idempotent and for every right ideal A and every left ideal B of H, the set $(A * B]$ is a quasi-ideal of H. The corresponding results on hypersemigroups (without order), can be also obtained as application of the results of the present paper and this is because every hypersemigroup endowed with the equality relation "=" (that is, $(x, y) \in \le \Leftrightarrow x = y$) is an ordered hypersemigroup. We tried to use sets instead of elements in our investigation, to show the pointless character of the results (in the same spirit with the abstract formulation of general topology –the so-called topology without points– initiated by Koutský and Nöbeling); illustrative examples using a table of multiplication (: the hyperoperation) and an order relation are given at the end of the paper. The paper serves as an example to show the way we pass from ordered semigroups to ordered hypersemigroups and its aim is to give the right information concerning this structure.

Lampe, Bill

Title. Revisiting Grätzer-Schmidt

Speaker. Bill Lampe

Abstract. The theorems Ross Willard will talk/has talked about each require a special congruence representation. Our congruence representations will be built using variations on the techniques pioneered by Grätzer and Schmidt. So we now proceed to revisit those techniques and try to gain some further insight into how they work.

We will present their constructions in a slightly new way which makes many of the details easier and makes many of them disappear. Although we will present the construction in a slightly new way, the algebras at the end of either way are isomorphic.

We will also discuss some previously unnoticed features of their construction.

Kjos-Hanssen, Bjørn

Title. Automatic complexity of monotone Boolean functions

Speaker. Bjørn Kjos-Hanssen

Institution. Department of Mathematics, University of Hawai'i, HI

Abstract. Champarnaud and Pin (1989) found the maximum complexity, in terms of finite automata, of a Boolean function on n variables. We study the special case of monotone functions. It is inspired by an application to complexity of financial securities. Mathematically we relate it to a certain lattice homomorphism problem.

McKenzie, Ralph

Title. Lattice Theory in Hawaii: Bill, JB and Ralph

Speaker. Ralph McKenzie

Institution. Vanderbilt University, Nashville TN

Abstract. Theorems and People, Stories and Memories.

McNulty, George F.

Title. Hawai'ian Excursions into Equational Logic

Speaker. George F. McNulty

Institution. Department of Mathematics, University of South Carolina, Columbia, SC

Abstract. I will talk about the impact of the work of Bill Lampe, Ralph Freese, and JB Nation on equational logic. While this talk will set some well-known results in the context of equational logic and will pose some open problems, the talk will not explain how to lecture in barefeet, how to give your doctoral students the fatherly advice they need on the streets of Mexico City, where to go on your honeymoon, nor the relationship between fluid consumption and mathematical progress. But I will talk about natural undecidable equational theories, finite height (or depth, really), inherently nonfinitely based lattices, and zippers.

Nation, J. B.

Title. Extending partial projective planes

Speaker. J. B. Nation

Insitution. Department of Mathematics, University of Hawai'i, Honolulu

Abstract. This note discusses a computational method for constructing finite projective planes.

There are a number of interesting problems concerning finite non-desarguean projective planes. One would hope that these problems would admit an algebraic, or geometric, or combinatorial solution. But it may just be that the existence, or non-existence, of certain types of planes is an accident of nature. With that in mind, since 1999 the author has been trying various computer programs to construct non-desarguean projective planes. While all these attempts have failed, hope springs eternal, and this note describes a set of problems and some ideas for addressing them.

This note is based on a talk given to the Courant Institute geometry seminar in October 2017. The author appreciates the hospitality and encouragement from the participants of the seminar.

Nishida, Joy

Title. Primer of Quasivariety Lattices

Speaker. Joy N. Nishida

Abstract. The lattice of subquasivarieties of a quasivariety can be represented as the lattice $\mathrm{S}_p(\mathbf{L}, H)$ of H-closed algebraic subsets of an algebraic lattice $\mathbf{L}$ with a monoid H of operators. This representation is used to develop new restrictions on the equational closure operator. A method that enables us to represent some finite lattices as lattices of subquasivarieties is described.

Pálfy, Péter Pal

Title. The role of twisted wreath products in the finite congruence lattice problem

Speaker. Péter Pal Pálfy

Institution. Rényi Institute, Hungarian Academy of Sciences, Budapest

Abstract. On the poster of the conference we see Bill, JB and Ralph looking at a blackboard showing a particular 7-element lattice. This is the smallest lattice that is not known to be representable as the congruence lattice of a finite algebra (William DeMeo's Thesis, UH, 2012).

Although I am not able to solve the representation problem for this lattice, I will show how to reduce the problem to representing finite lattices as intervals in subgroup lattices of finite groups of two particular types: (1) almost simple groups; (2) twisted wreath products with peculiar properties. This is based on works of Baddeley, Lucchini, Börner, Aschbacher, and Shareshian. Twisted wreath products were introduced by B. H. Neumann in 1963. I will indicate how naturally this group theoretic construction arises and what relevance it has for the finite congruence lattice representation problem.

Seselja, Branimir

Title. Ω-algebras

Speaker. Branimir Seselja

Institution. Faculty of Sciences, University of Novi Sad, Serbia

Abstract. Starting with Ω-sets where Ω is a complete lattice, we introduce the notion of an Ω-algebra. This is a classical algebra equipped with an Ω-valued equality replacing the ordinary one. In these new structures identities hold as appropriate lattice-theoretic formulas. Our investigation is related to weak congruences of the basic algebra to which a

generalized equality is associated. Namely every Ω-algebra uniquely determines a closure system in the lattice of weak congruences of the basic algebra. By this correspondence we formulate a representation theorem for Ω-algebras.

Silberger, Sylvia

Title. Sums of finitely many distinct reciprocals

Speaker. Sylvia Silberger

Institution. Hofstra University, Hemstead, NY

Abstract. Let $\mathcal{F}$ denote the family of all finite nonempty $S \subseteq \mathbb{N} := \{1, 2, \ldots\}$, and let $\mathcal{F}(X) := \mathcal{F} \cap \{S : S \subseteq X\}$ when $X \subseteq \mathbb{N}$. In this talk we treat the function $\sigma : \mathcal{F} \to \mathbb{Q}^+$ given by $\sigma : S \mapsto \sigma S := \sum\{1/x : x \in S\}$, and the function $\delta : \mathcal{F} \to \mathbb{N}$ defined by $\sigma S = \nu S/\delta S$ where the integers νS and δS are coprime.

We then discuss the following results.

Theorem 1.1.
For each $r \in \mathbb{Q}^+$, there exists an infinite pairwise disjoint subfamily $\mathcal{H}_r \subseteq \mathcal{F}$ such that $r = \sigma S$ for all $S \in \mathcal{H}_r$.

Theorem 1.2.
Let X be a pairwise coprime set of positive integers. Then $\sigma \restriction \mathcal{F}(X)$ and $\delta \restriction \mathcal{F}(X)$ are injective. Also, $\sigma C \in \mathbb{N}$ for $C \in \mathcal{F}(X)$ only if $C = \{1\}$.

Valeriote, Matthew

Title. UACalc and testing for Maltsev conditions

Speaker. Matthew Valeriote

Institution. Department of Mathematics, McMaster University, Canada

Abstract. In the 1990's several researchers, including Ralph Freese, Emil Kiss, and the speaker, produced various versions of a software package that is now known as UACalc. In addition to being a very useful research tool for investigating finite algebras (I can't recall the last time that I had to compute the labelled congruence lattice of a finite algebra by hand), it also raised several interesting questions about the computational complexity of testing for certain properties of finite algebras and the varieties that they generate.

I plan to talk about the role of the UH group in the development of computational tools for studying algebras and lattices and then will focus on a set of questions and results about the complexity of testing for certain Maltsev conditions that were in part motivated by using UACalc and that have helped to expand the scope of the calculator.

Taylor, Walter

Title. Some two-dimensional lattices that fail distributivity or fail modularity and some other laws

Author. Walter Taylor

Institution. Mathematics Department, University of Colorado, Boulder

Abstract. For L a finite simple lattice, we can examine the class of topological spaces that can support a lattice (with continuous operations) that admits L as a 0,1-sublattice. We consider this problem for various modular L, such as M_n ($n \geq 3$) and possibly $M_{p,q}$. Also a brief look at the distributive case. So far our spaces are small finite geometric examples, very user-friendly.

Willard, Ross

Title. Independence of multi-term commutators and centralizers

Author. Ross Willard

Institution. Department of Mathematics, University of Waterloo, Canada

Abstract. In 1977, Bill Lampe co-discovered the “term condition” and used it to prove the existence of algebraic lattices which cannot be represented as the congruence lattice of an algebra with just one basic operation. Lampe’s key observation was that there exist algebraic lattices whose shape forces every representing algebra to satisfy the term condition.

In the 1980s, a relativized version of the term condition came to form the standard foundation for what we now call the “usual” commutator. In modern language, Lampe’s observation was that there exist algebraic lattices whose shape forces the commutator operation of any representing algebra to be constantly zero. Hence the commutator is not independent of the congruence lattice on which it sits.

There exist “unusual” commutators, all of which coincide with the usual commutator in congruence modular varieties but outside of that context can be strictly bigger than the usual commutator. Each of these commutators is defined by systems of implications involving two or more terms. In 1997 Lampe, Keith Kearnes and I asked whether there might exist an algebraic lattice whose shape forces these larger commutators to be constantly zero. Thus began a “disastrous” collaboration which is now, finally, reaching consummation. It turns out that not only are these larger commutators independent of the congruence lattice (modulo a few obvious necessary conditions such as subadditivity), they are independent of each other (again modulo some obvious necessary restrictions). Stronger still, the corresponding centralizer relations can be abstractly axiomatized, and modulo this axiomatization, they are essentially independent of the congruence lattice and each other.

In this lecture I will try to connect these results to greed, Blackstone growlers, and Bill’s personal hygiene.

List of Speakers