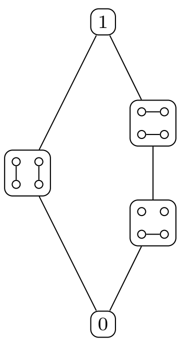
SMALL UNARY ALGEBRAS FOR CONGRUENCE LATTICES OF SIZE ≤ 7

PETER JIPSEN

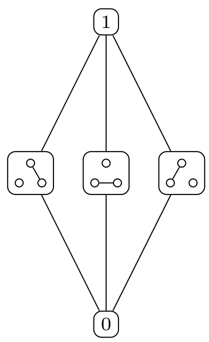
Distributive lattices and lattices that are ordinal sums of smaller lattices are omitted. The base set of each algebra is $\{0, 1, \ldots, n-1\}$, and each unary operation is specified by a vector of values of these elements. Algebras of size less than 11 are known to be minimal-size algebras that produce the corresponding congruence lattice. The algebra for 33 (M_5) is also known to be minimal in size. Currently only one of the lattices (10) is not known to be the congruence lattice of a finite algebra. Thirteen of the 35 lattices below are subdirectly reducible (specifically: 6, 7, 12, 13, 26, 27, 28, 29, 30, 31, 32, 34, 35).

(1,0,3,2)
(1,0,1,0)



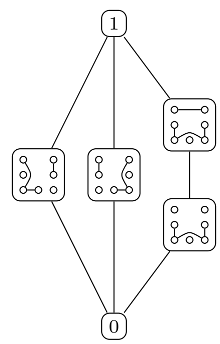
1 (N_5)

(1,0,3,2)
(2,3,0,1)



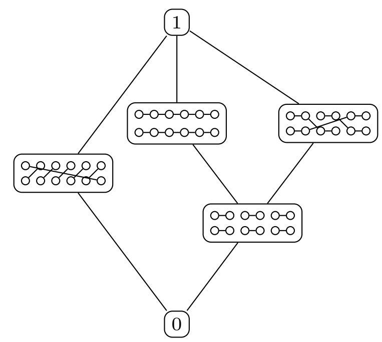
2 (M_3)

p_0 = (0,1,2,1,2,1,0)
p_1 = (0,3,4,3,4,3,0)
p_2 = (6,5,2,5,2,5,6)
p_3 = (0,1,2,0,0,2,2)



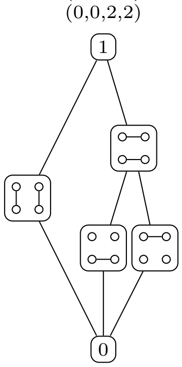
3 (V_1)

(1,2,3,4,5,0,7,8,9,10,11,6)
(6,11,10,9,8,7,0,5,4,3,2,1)
(0, 0, 0,6,0,0,0,0,6,0,0,0)



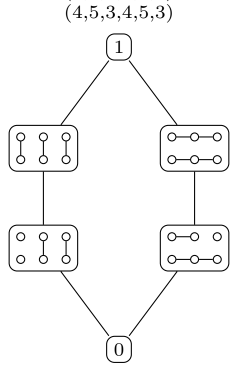
4 (L_5)

(1,0,3,2)
(0,0,2,2)



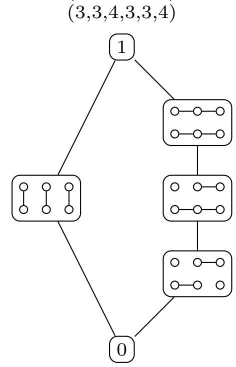
5 (L_4)

(2,2,1,5,5,4)
(3,4,4,0,1,1)
(4,5,3,4,5,3)



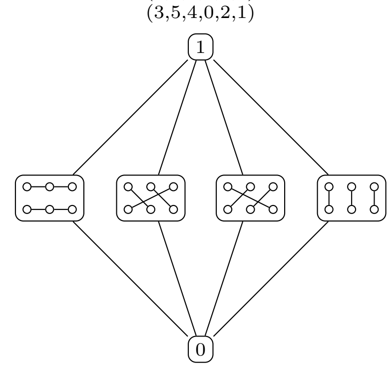
6

(1,0,0,4,3,3)
(4,5,5,1,2,2)
(3,3,4,3,3,4)



7

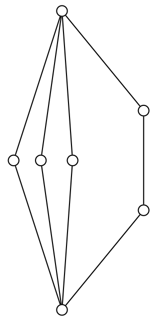
(1,2,0,4,5,3)
(3,5,4,0,2,1)



8 (M_4)

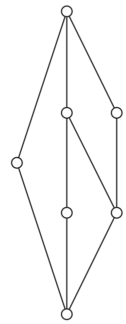
Date: May 31, 2011.

(0,0,0,0,0,0,2,1,2,1,3,4,5,3,4,5)
(0,0,0,0,0,0,6,7,6,7,10,11,12,10,11,12)
(13,14,15,1,9,8,15,14,13,15,1,9,8,8,1,9)
(R. Freese "rabbit ears")



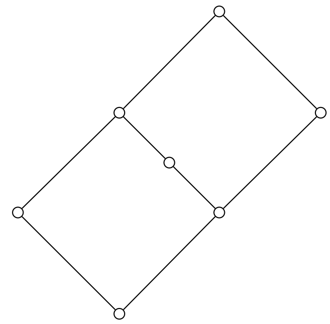
9

No finite algebra
known with this
congruence lattice



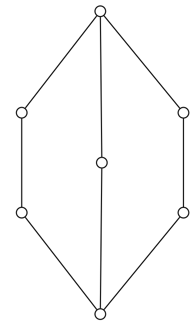
10

Finite algebra with
108 elements known (W. DeMeo)



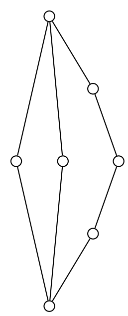
11

(0,0,3,3,3,6,6,6,0)
(0,0,8,8,8,1,1,1,0)
(0,5,5,4,0,0,5,4,4)
(4,2,2,3,4,4,2,3,3)
(5,5,7,7,7,6,6,6,5)
(found by W. DeMeo)



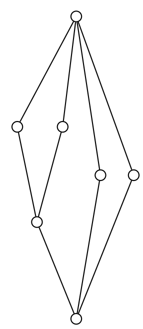
12

(0,1,2,1,2,1,0,0,1,2,2,1,0,0,1,2,1,2,0)
(0,1,2,0,0,2,2,0,3,4,0,4,4,6,5,2,6,6,2)
(0,1,2,3,4,5,6,0,1,2,4,5,6,0,1,2,3,4,6)
(7,8,9,3,10,11,12,3,3,3,3,3,3,11,11,11,
11,11,11),(13,14,15,16,17,5,18,13,16,17,
17,16,13,5,5,5,5,5,5)("rabbit ears")



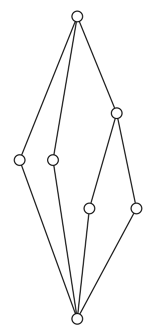
13

Upper interval in Sub(A_6)
algebra of size 90 (W. DeMeo)



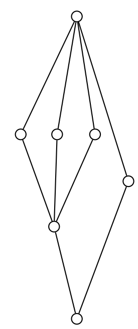
14

(1,0,3,2)



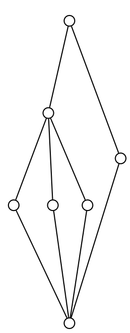
15

Upper interval in Sub($C_2.A_6$)
algebra of size 180 (W. DeMeo)



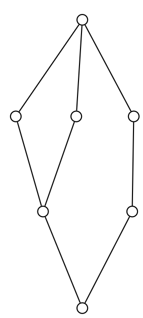
16

(1,0,3,2,5,4,7,6,9,8,11,10)
(4,7,5,6,8,11,9,10,0,3,1,2)
(0,0,0,0,5,5,5,5,10,10,10,10)
(W. Demeo, filter-ideal in Sub(A_4))



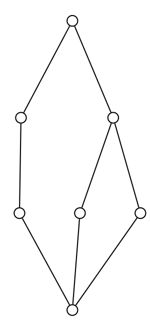
17

Dual of 19, no explicit
small representation known?



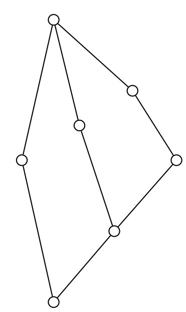
18

(0,1,1,0,4,5,5,4)
(0,2,3,1,0,2,3,1)
(7,6,6,7,3,2,2,3)
(P. Jipsen, search in Equ(8))



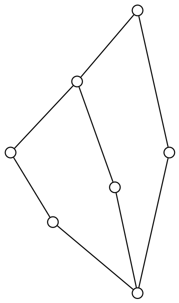
19

(W. DeMeo in GAP
SmallGroup(216,153))



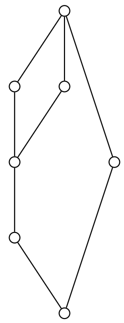
20

(3,3,4,8,8,2,2,3,4)
(0,0,6,1,1,0,0,5,6)
(4,5,5,7,8,8,7,4,4)
(R. Freese, in Equ(9))



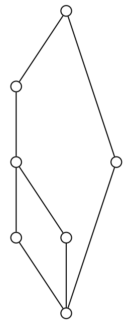
21

Dual of 23, no explicit small representation known?



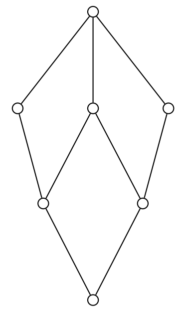
22

(0,1,0,1,4,4)
(1,1,3,3,4,5)
(3,2,3,2,5,5)
(4,1,5,3,4,5)



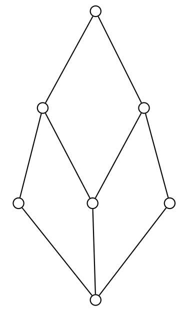
23

(1,1,2,2)
(2,3,3,2)



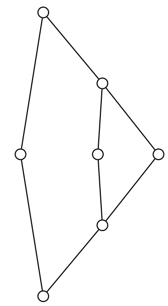
24 (L_2)

(0,0,2,2,2)
(0,1,0,1,1)
(1,1,4,4,4)
(2,3,2,3,3)



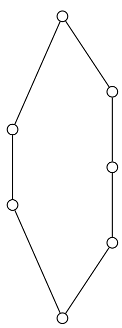
25 (L_1)

(1,0,3,2,0,2)
(4,4,5,5,1,3)
(0,0,0,0,1,1)
(3,5,3,5,3,3)



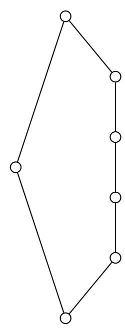
26

(0,1,2,3,4,5,0,0,0,0,0,2,2,2,2,2)
(4,5,3,4,5,3,5,3,4,5,3,4,5,4,5,3)
(2,2,1,5,5,4,2,1,5,5,4,2,2,5,5,4)
(3,4,4,0,1,1,4,4,0,1,1,3,4,0,1,1)
(0,6,7,8,9,10,6,7,8,9,10,0,6,8,9,10)
(11,12,2,13,14,15,12,2,13,14,15,
11,12,13,14,15) (“rabbit ears”)



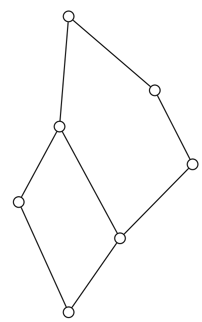
27

(0,1,2,3,4,5,0,0,0,0,0,2,2,2,2,2)
(3,3,4,3,3,4,3,4,3,3,4,3,3,3,3,4)
(1,0,0,4,3,3,0,0,4,3,3,1,0,4,3,3)
(4,5,5,1,2,2,5,5,1,2,2,4,5,1,2,2)
(0,6,7,8,9,10,6,7,8,9,10,0,6,8,9,10)
(11,12,2,13,14,15,12,2,13,14,15,
11,12,13,14,15) (“rabbit ears”)



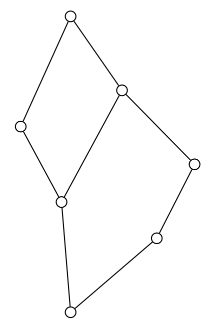
28

(1,0,3,2,2)
(2,4,2,4,3)



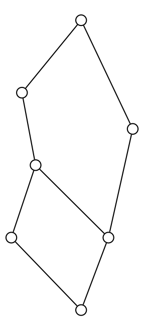
29

(0,3,4,3,4)
(2,2,1,4,3)



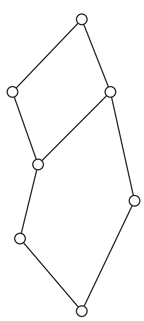
30

(0,1,1,0,0)
(1,1,2,2,2)
(3,2,2,4,4)



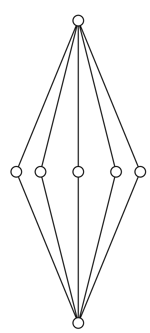
31

(0,1,1,3,3)
(1,2,2,4,4)
(3,3,4,3,4)



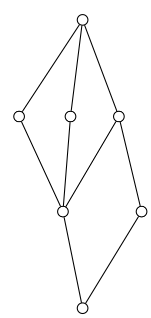
32

(1,3,2,0,9,11,10,8,13,15,14,12,5,7,6,4)
(11,8,10,9,7,4,6,5,15,12,14,13,3,0,2,1)
(14,15,12,13,10,11,8,9,6,7,4,5,2,3,0,1)



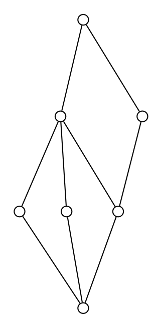
33 (M_5)

(0,1,3,2)



34

(1,1,2,3)
(2,3,3,3)



35