# Joining up to minority

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I will show how minority is the join of two varieties defined by linear equations.

We take two varieties: $V_1$ is given by the equations

$$\begin{aligned}
w(x,x,y) &= w(x,y,x) = w(y,x,x) = m(x,y,x)\\
m(x,x,y) &= y\\
m(y,x,x) &= y.
\end{aligned}$$

and $V_2$ by the three cube-term style equations

$$\begin{aligned}
t(y,x,x,z,y,y,z) &= y\\
t(x,y,x,y,z,y,z) &= y\\
t(x,x,y,y,y,z,z) &= y.
\end{aligned}$$

One possible member of $V_1$ is $\mathbb{Z}_4$ with $w(x,y,z) = -x-y-z$ and $m(x,y,z) = x-y-z$, so $V_1$ does not have a minority. For a member of $V_2$, consider $\mathbb{Z}_6$ with $t(x_1,\dots,x_7) = 3x_1 + 3x_2 + 3x_3 + 2x_4 + 2x_5 + 2x_6 + 4x_7$. Again, this shows that $V_2$ does not have minority.

Taking the join of $V_1$ and $V_2$ we will find the minority operation: Consider the term $p(x,y,z) = t(x,y,z,m(z,x,y),m(x,y,z),m(y,z,x),w(x,y,z))$. We get

$$\begin{aligned}
p(x,x,y) &= t(x,x,y,m(y,x,x),m(x,x,y),m(x,y,x),w(x,x,y))\\
&= t(x,x,y,y,y,w(x,x,y),w(x,x,y)) = y
\end{aligned}$$

etc.

However the variety $V_1$ and $V_2$ is term-equivalent to just the variety with a minority operation: If $n$ is a minority operation then we can let $w(x,y,z) =$

$m(x,y,z) = n(x,y,z)$ and $t(x_1,\ldots,x_7) = n(n(x_1,x_2,x_3),n(x_4,x_5,x_6),x_7)$ and get the equations above.

As to testing for $V_1$ and $V_2$, Dima's example will have the $V_2$ terms locally (but not globally), and $V_1$ terms globally: We can take $w(x,y,z) = t_i(y,t_i(x,y,z),y)$ and $m(x,y,z) = t_i(x,y,z)$ for any $i$.

Thus $V_2$ invalidates my conjecture that local to global will work whenever dropping any single equation results in a trivial Maltsev condition.