# NOTES ON VARIETIES WITH A MINORITY TERM

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A *minority term* (for an algebra or variety) is a 3-variable term $q(x, y, z)$ such if two of the variables are equal, its value is the other one; that is,

$$q(x, x, y) \approx q(x, y, x) \approx q(y, x, x) \approx y.$$

We are interested in an algebraic description of varieties having a minority term. One possible conjecture is:

**Conjecture 1.** *$\mathcal{V}$ has a minority term if and only if it is CP and its ring has characteristic 2.*

**Fact 2.** *If $\mathcal{V}$ has a minority term, then it is CP.*

*Proof.* Clearly a minority term is a Maltsev term. □

**Fact 3.** *A CD variety $\mathcal{V}$ has a minority term if and only if it is CP.*

*Proof.* A variety is CD and CP if and only if it has a Pixley term. If $p(x, y, z)$ is a Pixley term then

$$q(x, y, z) = p(p(x, y, z), x, p(x, z, y))$$

is a minority term. The fact can be derived from these observations. □

**Fact 4.** *A variety of groups has a minority term if and only the variety has exponent 2 (well, or 1).*

*Proof.* If a variety has exponent 2 then it is abelian and so, using additive notation, $x + y + z$ is a minority term.

If the exponent is not 2 then the variety contains a group which contains an element of order $n$, where $n > 2$ (or infinite). This element generates a cyclic group. In any abelian algebra the Maltsev term operation is unique and so must be $x - y + z$. But one easily checks that this is not a minority term when $n > 2$. □

**Lemma 5.** *Let $\mathcal{V}$ be a CP variety with a Maltsev term $p(x, y.z)$. The the following are equivalent:*

(1) *The ring $R(\mathcal{V})$ of $\mathcal{V}$ has characteristic 2.*

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(2) *On every block of every abelian congruence, $p$ restricted to the block is a minority term; that is, satisfies $p(a,b,a) = b$.*
(3) *If $\theta = \mathrm{Cg}^{\mathbf{F}_{\mathcal{V}}(x,y)}(x,y)$, then, on each block of $\theta/[\theta,\theta]$, $p$ is a minority term.*

*If $\mathcal{V}$ has a minority term, then these conditions hold.*

*Proof.* By commutator theory (give some specific refs, also that the ring is det by the strucure of $\theta/[\theta,\theta]$) the Maltsev term operation on an abelian algebra, or even a block of an abelian congruence, is unique and it is $p(x,y,z) = x - y + z$ for some abelian group. It is easy that $x - y + z$ is a minority if and only if the abelian group has exponent 2. $\square$

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