Solutions to EECS 451 Exam 3, 2004-4-27

Regrade requests must be submitted to Prof. Fessler *in writing*, within 1 week of when the exam is returned in class. All problems will be re-examined, and scores may increase or decrease.
Discussing the exam with a professor or GSI nullifies the opportunity to submit a regrade request.
For elaboration on these solutions, please come to office hours.

p/xn,ft1

1.

- [10] $x[n] = \sum_{k=0}^{\infty} (1/3)^{4k+2}\,\delta[n-(4k+2)] \overset{Z}{\leftrightarrow} X(z) = \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^{4k+2} z^{-(4k+2)} = z^{-2}\frac{1}{9}\sum_{k=0}^{\infty}\left(3^{-4}z^{-4}\right)^k$
  $= z^{-2}\frac{1}{9}\frac{1}{1-3^{-4}z^{-4}} = \frac{1}{9}\frac{z^2}{z^4-1/3^4}$ so $\boxed{X(\omega) = \frac{1}{9}\frac{e^{\jmath 2\omega}}{e^{\jmath 4\omega}-1/3^4}}.$
  [20% correct. many attempted down-sampling instead of upsampling.] *(HW 5-8)*
- [10] $X[k] = \sum_{n=0}^{5} x[n]\,e^{-\jmath\frac{2\pi}{6}kn} = \frac{1}{3^2}\,e^{-\jmath\frac{2\pi}{6}k2} = \boxed{\frac{1}{9}\,e^{-\jmath\frac{2\pi}{3}k}}$ for $k = 0,\ldots,5$. *[85% correct]* *(HW 9-2)*

p/transient2

2. (10)
$H(z) = \frac{3z^{-2}}{3+2z^{-1}} = \frac{z^{-1}}{z+\frac{2}{3}}$. Since there is a pole at $\frac{2}{3}$, the transient response is $\boxed{y_{tr}[n] = r_1\left(\frac{-2}{3}\right)^n}$ for some constant r_1 that depends on the input signal. (There are also Kronecker impulse term(s) that one could consider part of the transient response.) *[mostly correct. many forgot the residue r_1 (-2).]* *(HW 7-8)*

p/linphase1

3. (10)
True. Since $h_1[n]$ and $h_2[n]$ are both symmetric, $h[n] = h_1[n] + h_2[n]$ is also symmetric, since
$h[N-1-n] = h_1[N-1-n] + h_2[N-1-n] = h_1[n] + h_2[n] = h[n]$.
[Many students argued that it is false using two "symmetric" shifted impulse responses of the same length, but different "midpoints." This is not consistent with the book's definition of symmetry for FIR filters, but was also accepted since the term "length" is not necessarily the same as the "M."
50% correct. many tried to claim it was false using non-*symmetric filters. (0 pts).]* *(HW 10-4)*

p/dft,persup1

4. (10)
$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$, so the DTFT of $x[n]$ is $X(\omega) = \frac{1}{1-\frac{1}{3}e^{-\jmath\omega}}$. Since $y[n]$ is the 8-point periodic superposition of $x[n]$, the 8-point DFT of $y[n]$ is the usual samples of $X(\omega)$: $\boxed{Y[k] = \frac{1}{1-\frac{1}{3}\,e^{-\jmath\frac{2\pi}{8}k}}, \quad k = 0,\ldots,7.}$
[70% correct. Some tried to "find $y[n]$" first, and then take its DFT, usually without success.] *(HW 10-3c)*

p/dft,odd0

5. (10)
$Y[k] = X[k \bmod 50]$ which is 10 for $k \in \{0,\ldots,20,\ 30,\ldots 49,\ 50,\ldots,70,\ 80,\ldots,99\}$ and zero otherwise.
[80% correct.] *(HW 10-5)*

p/fft,fft1

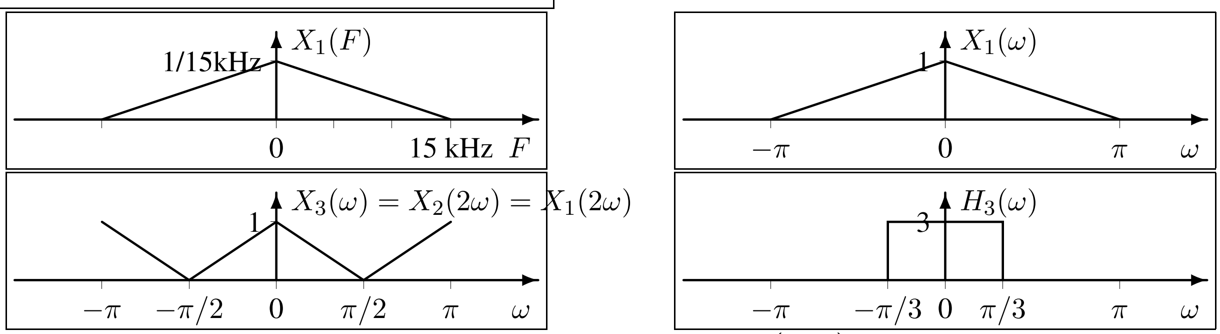
6. (10)
The output is the product of $N = 4$ times the N-point circular time reversal of the input signal, *i.e.*, $\boxed{\texttt{[12 24 20 16]}}$ *[Some used two DFTs, rather than using the property, usually with errors.]* *(HW 11-3)*

p/fft,conv1

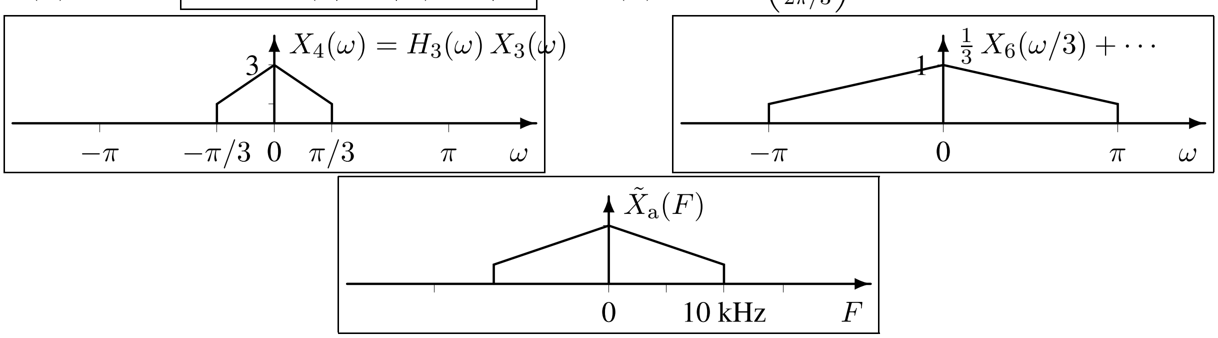
7. (10)
In general this would be the 6-point circular convolution of the two signals. In this case, however, there is adequate zero padding, so the circular convolution results in the same thing as linear convolution of $x[n] = \{\underline{1}, 2, 3\}$ with $h[n] = \delta[n] + \delta[n-2]$. So the output is $x[n] * h[n] = x[n] + x[n-2] = \{\underline{1}, 2, 4, 2, 3, 0\}$.
[mostly correct. many students performed circular convolution by hand, apparently not recognizing that the zero "padding" results in (effectively) linear convolution. a few tried to do this by taking two DFTs, multiplying, and then taking the inverse DFT; very painful and error prone.] *(HW 10-2)*

p/rate1a

8. (10)
We can either assume that $x_a(t)$ is band-limited to 15kHz, or that there is an anti-alias filter.
$\boxed{\text{For this design, we do not need } H_2(\omega) \text{ or } H_4(\omega).}$ To illustrate, we sketch spectra at each stage.



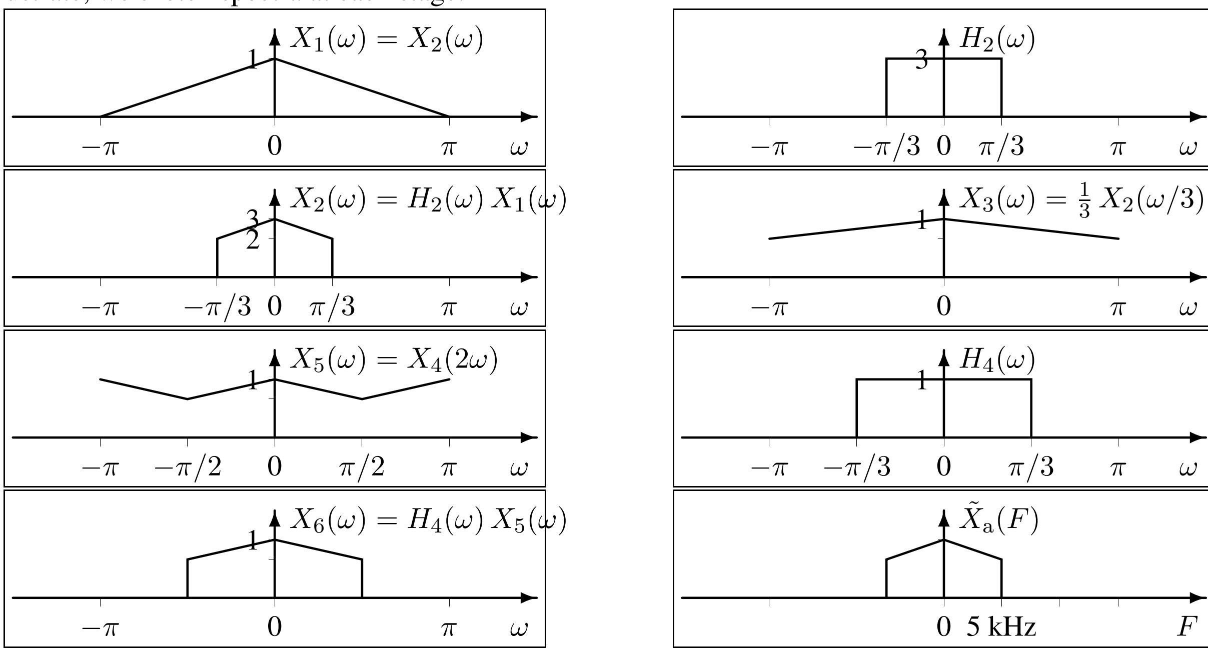
For $H_3(\omega)$ we need $\boxed{\omega_c = \min(\pi/2, \pi/3) = \pi/3,}$ so $H_3(\omega) = 3\,\text{rect}\left(\frac{\omega}{2\pi/3}\right)$ (anti-alias before downsampling).



Of course we have lost the frequency components above $F_2/2 = 10$kHz.
[30% got both #8 and #9 correct. many did not seem to realize that $H_4(\omega) = \text{rect}\left(\frac{\omega}{2\pi}\right)$ does nothing. (-1) Several used $H_3(\omega)$ and $H_4(\omega)$ with cutoff of $\pi/2$. This preserves up to 7.5kHz only (-4).] *(HW 11-7)*

p/rate1b

9. (10)
This system would be *inferior*, because downsampling by 3 first would reduce the effective sampling rate to only 10 kHz, so, to prevent aliasing, all frequency components above 5kHz would have to be eliminated by the filter $H_2(\omega)$ (with a cutoff of $\pi/3$). Furthermore, although $H_3(\omega)$ is not needed, we would still need $H_4(\omega)$, so this system requires more components.
To illustrate, we sketch spectra at each stage.



[A few students used the same filters here as were designed in #8. Since it was perhaps not clear that the filters could (and should!) be re-designed, correct arguments based on #8 filters were also accepted.] *(HW 11-7)*

Exam scores with APPROXIMATE grades.
52 undergraduate students: mean=65.7, median=65, std=19.5
a+ 99 98 a 94 93 92 90 90 90 90 90 89 88 a- 83 81
b+ 78 78 77 b 74 74 72 70 69 69 69 68 67 b- 63 63 62 62 61 61 61
c+ 58 c 56 54 52 52 51 50 50 c- 47 46
d 43 42 41 40 40 40 38 e 32 21

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19 graduate students: mean=80.5, median=85, std=17.4, std=11.4 (without the lowest score);
a 100 100 98 95 92 90 89 87 85 85 b 82 79 76 74 73 ? 70 68 62 f 25