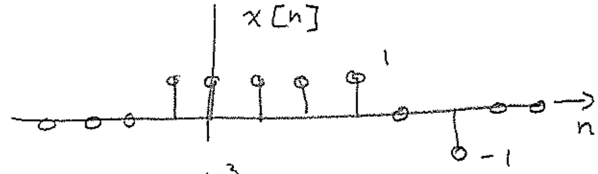
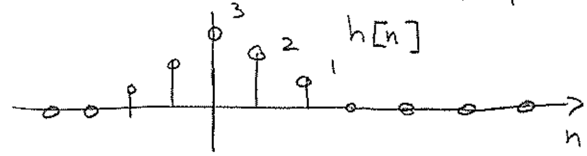
5 b) $y[n] = x[n] * h[n]$

$x[n] = u[n+1] - u[n-4] - \delta[n-5]$

$h[n] = \left(u[n+2] - u[n-3]\right)\left(3 - |n|\right)$

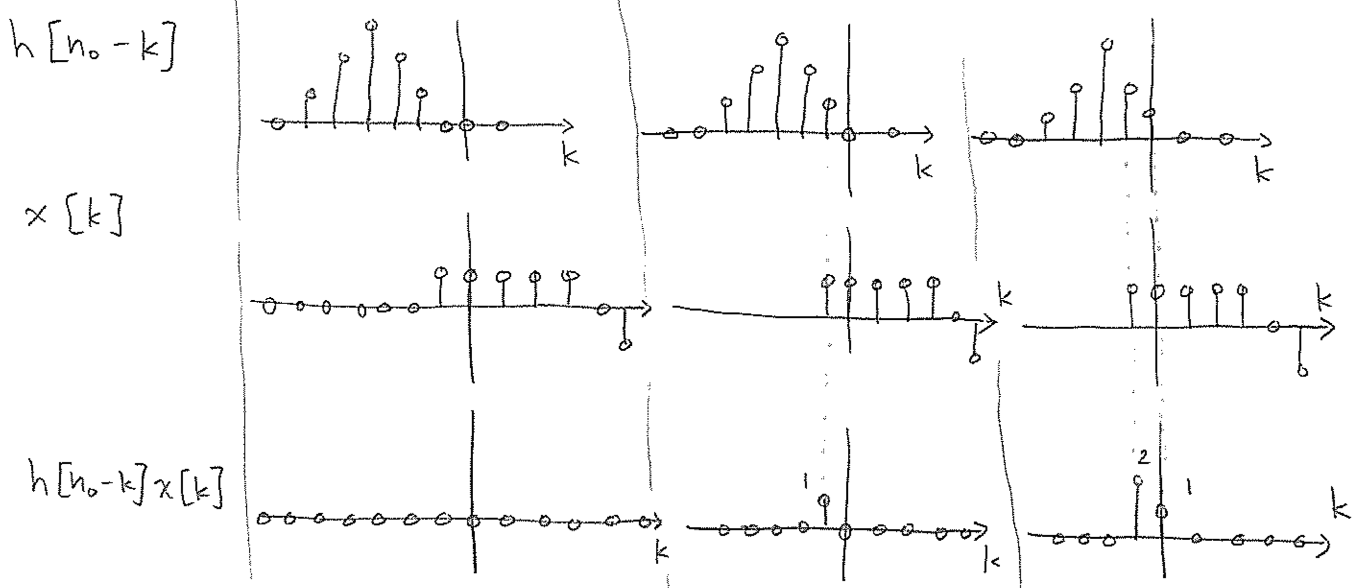




$\boxed{1}$ FLIP-AND-SLIDE

1. flip $h[k]$ to get $h[-k]$. Since $h[k]$ is symmetric, we get $h[k]$ $(= h[-k])$.
2. take flipped $h[n]$ and pull it all the way to the left
   this represents $h[n-k]$ for $n = -\infty$.
3. to evaluate $y[n_0]$ for all values of n_0, we slide flipped $h[k]$ from left to right, representing $h[n_0-$

	① $n_0 = -4$	② $n_0 = -3$	③ $n_0 = -2$



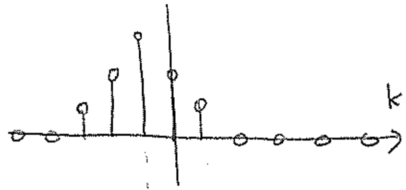
	① $n_0 = -4$	② $n_0 = -3$	③ $n_0 = -2$
$\sum_{k=-\infty}^{\infty} h[n_0-k]\,x[k] = y[n_0]$	0	1	$2+1 = 3$

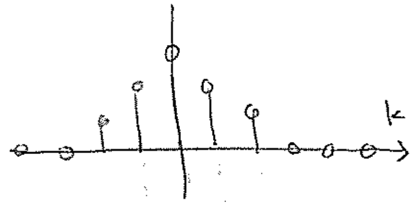
Note: $h[n_0-k]$ (the top sequence) is the one sliding left to right

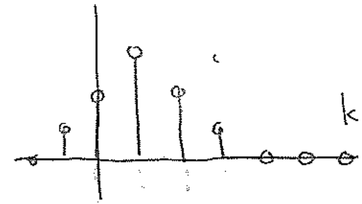
continued →

$n_0 = -1$ | $n_0 = 0$ | $n_0 = 1$

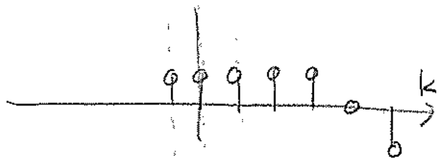
$h[n_0 - k]$

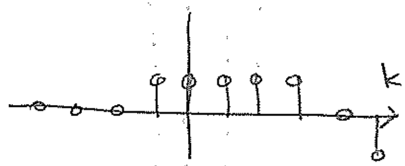


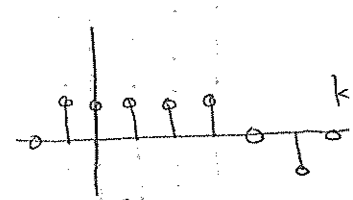




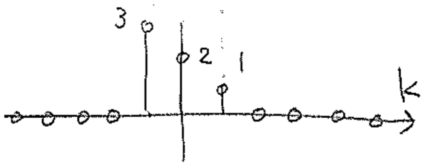
$x[k]$

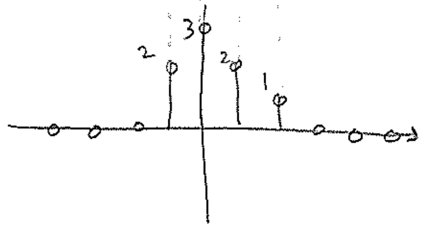


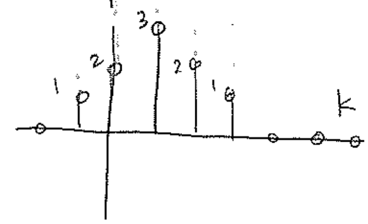




$h[n_0 - k]\,x[k]$





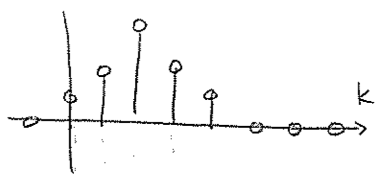


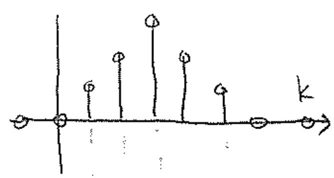
$\sum_{k=-\infty}^{\infty} h[n_0-k]\,x[k] = y[n_0]$

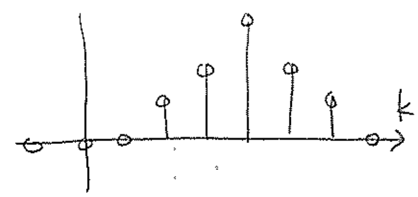
$3+2+1 = 6$ | $2+3+2+1 = 8$ | $1+2+3+2+1 = 9$

$n_0 = 2$ | $n_0 = 3$ | $n_0 = 4$

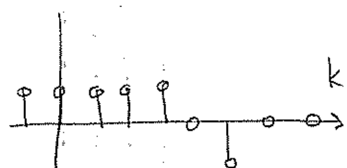
$h[n_0 - k]$

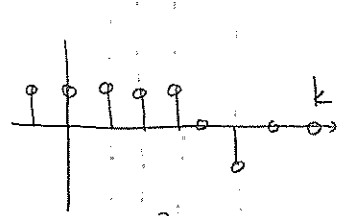


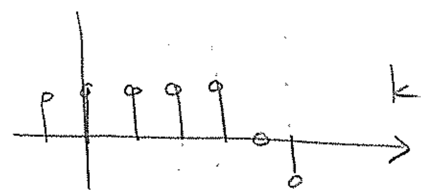




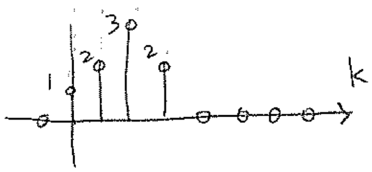
$x[k]$

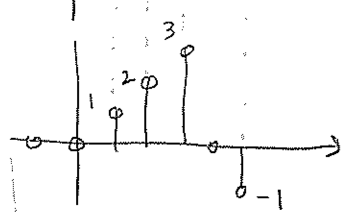


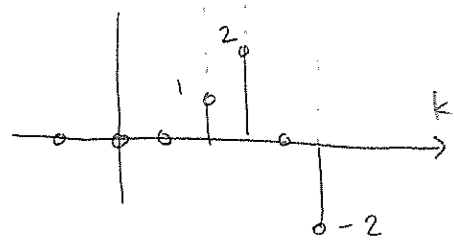




$h[n_0 - k]\,x[k]$



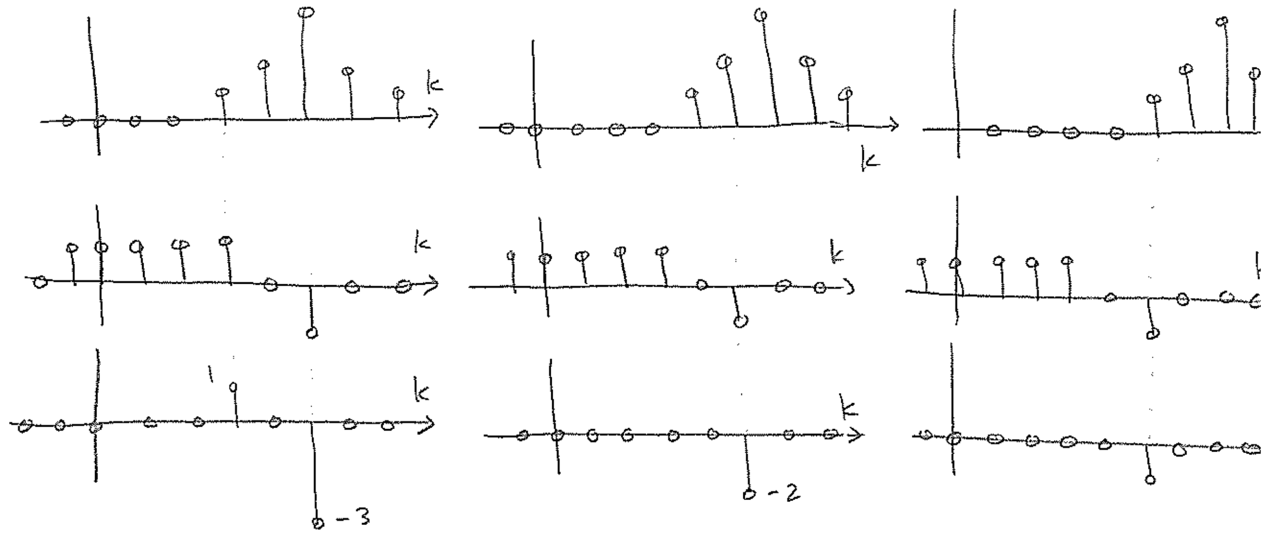




$\sum_{k=-\infty}^{\infty} h[n_0-k]\,x[k] = y[n_0]$

$1+2+3+2 = 8$ | $1+2+3-1 = 6$ | $1+2-2 = 1$

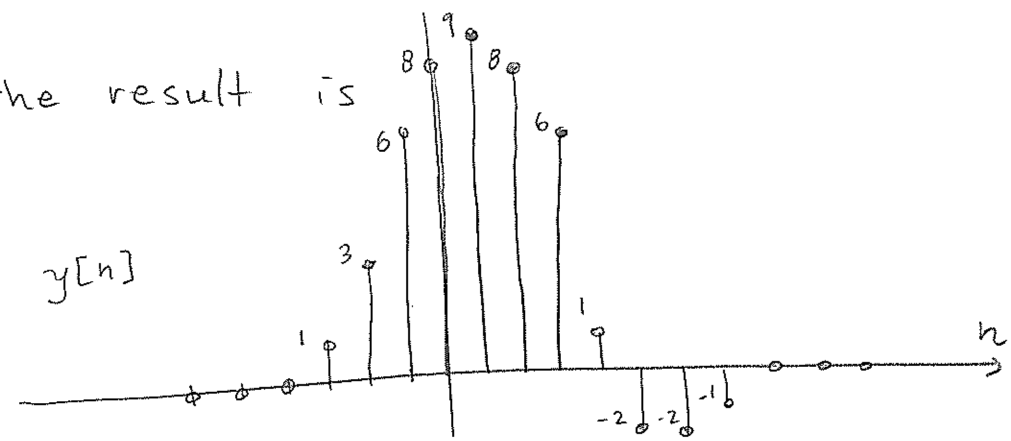
	$n_0 = 5$	$n_0 = 6$	$n_0 = 1$
$h[n_0-k]$			
$x[k]$			
$h[n_0-k]x[k]$			



$$\sum_{k=-\infty}^{\infty} h[n_0-k]x[k] = y[n_0]$$

$n_0 = 5$: $1 - 3 = -2$ $n_0 = 6$: -2 $n_0 = 1$: -1

So the result is



$$y[n] = \{1, 3, 6, \underline{8}, 9, 8, 6, 1, -2, -2, -1\}$$

$\boxed{2}$ STACKING SEQUENCES

Using shift-invariance of convolution, we can see

$$h[n] * \delta[n-\ell] = h[n-\ell]$$

Convolution of signal with delayed delta is the input signal delayed.

Recall also we can decompose $x[n]$ into a sum of shifted deltas.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] = \delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] - \delta[n-5]$$

$\longrightarrow$

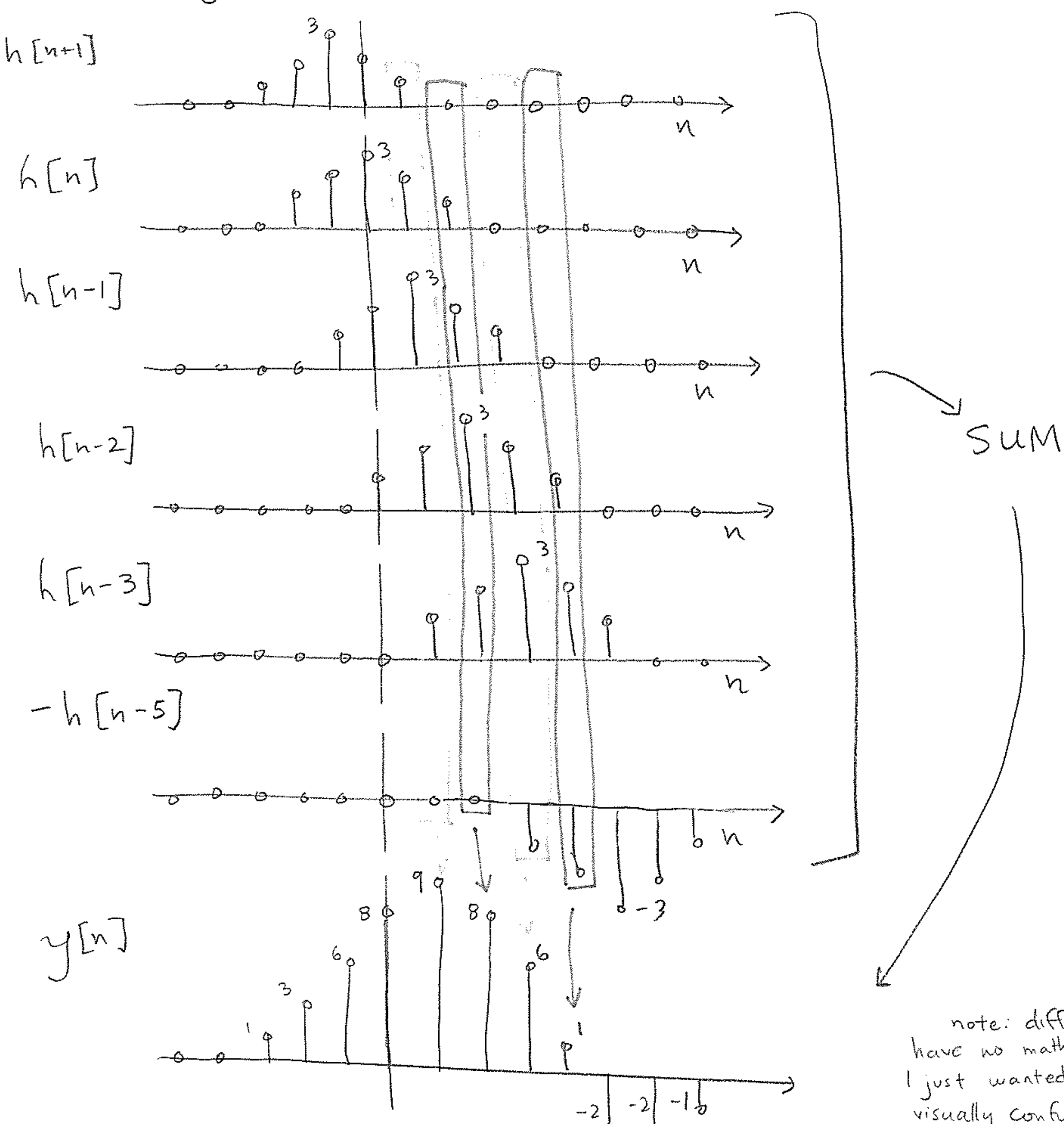
so $y[n] = h[n] * x[n] = h[n] * (\delta[n+1] + \delta[n] + \delta[n-1] + \delta[n-2] + \delta[n-3] - \delta[n-5])$

using the distributive property of convolution,

$$y[n] = h[n] * \delta[n+1] + h[n] * \delta[n] + h[n] * \delta[n-1] + h[n] * \delta[n-2] + h[n] * \delta[n-3] - h[n] * \delta[n-5]$$

$$y[n] = h[n+1] + h[n] + h[n-1] + h[n-2] + h[n-3] - h[n-5]$$

so, $y[n]$ is just a sum of shifted $h[n]$'s!



note: different color boxes have no mathematical significance I just wanted it to be less visually confusing :)