Problems for Discussion 3, 09/25/13

Compiled by Mai Le, some problems from Prof. Yagle, Prof. Fessler

1 Using DTFT Properties

Let $x[n] = \{1, 4, 3, 2, 5, 7, \underline{-45}, 7, 5, 2, 3, 4, 1\}$. Let $X(\omega) = DTFT(x[n])$. Compute the following without actually computing $X(\omega)$. Use DTFT properties!

1.1 **(a)** $X(\pi)$

1.2 **(b)** $\mathbf{arg}X(\omega)$

1.3 **(c)** $\int_{-\pi}^{\pi} X(\omega)d\omega$

1.4 **(d)** $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

2 Computing the Z-Transform

Compute the z-transform and the corresponding ROCs of the following sequences.

(a) $x[n] = \left(\frac{1}{5}\right)^n u[n]$

(b) $x[n] = 2^n u[-n] + \left(\frac{1}{3}\right)^n u[n]$

(c) $x[n] = \{-1, \underline{0}, 1\}$

(d) $x[n] = \sum_{k=-\infty}^{n} 3^k u[n]$

3 Z-Transform Manipulation

Express the z-transform of $y[n] = \sum\limits_{k=-\infty}^{n} x[k]$ in terms of $X(z)$.

4 Inverse z-transform strategies

Consider the z-transform $X(z) = \frac{1+2z^{-1}+z^{-2}}{1-\frac{3}{2}z^{-1}+\frac{1}{2}z^{-2}}$. We can rewrite $X(z)$ into a friendlier form $X(z) = B(z) + \frac{A_1}{D_1} + \frac{A_2}{D_2}$, where $B(z)$, $A_1(z)$, $A_2(z)$, $D_1(z)$, and $D_2(z)$ are polynomials of z (or rather z^{-1}).

Use long division to find $B(z)$ and partial fraction expansion to find the remaining $A_1(z)$, $A_2(z)$, $D_1(z)$, and $D_2(z)$.