EECS 451 Exam 3, 2004-4-27

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1. (20 points)

Consider the following signal $x[n] = \begin{cases} (1/3)^n, & n = 2, 6, 10, 14, 18, \ldots \\ 0, & \text{otherwise.} \end{cases}$

- Determine the discrete-time Fourier transform of $x[n]$.
- Determine the 6-point discrete Fourier transform of $x[n]$.

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2. (10 points)

A nonzero, finite-length signal $x[n]$ is the input to a stable LTI system with frequency response $H(\omega) = \dfrac{3\, \mathrm{e}^{-\jmath 2\omega}}{3 + 2\, \mathrm{e}^{-\jmath\omega}}$.
Determine the transient response portion of the output signal as precisely as you can given only this information.

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3. (10 points)

Explain briefly why the following statement is true or false.

"The sum of any two causal, symmetric, FIR, linear-phase filters of the same length is a linear-phase filter."

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4. (10 points)

Determine the 8-point DFT of the signal $y[n]$ defined by $y[n] = \sum_{l=-\infty}^{\infty} x[n - 8l]$, where $x[n] = \left(\frac{1}{3}\right)^n u[n]$.

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5. (10 points)

The 50-point DFT of a signal $x[n]$ is given by $X[k] = \begin{cases} 10, & k = 0, \ldots, 20 \\ 0, & k = 21, \ldots, 29 \\ 10 & k = 30, \ldots, 49. \end{cases}$

The signal $y[n]$ is defined by up-sampling $x[n]$ by two: $y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd.} \end{cases}$

Sketch the 100-point DFT of $y[n]$.

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6. (10 points)

Determine the output of the following MATLAB command:

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fft(fft([3 4 5 6]))
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7. (10 points)

Determine the output of the following MATLAB command:

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ifft(fft([1 2 3 0 0 0]) .* fft([1 0 1 0 0 0]))
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8. (10 points)

A signal $x_\mathrm{a}(t)$ was sampled at rate $F_1 = 30$kHz and its samples $x_1[n]$ were stored. Later it is desired to recover this signal using an D/A converter that works at the sampling rate $F_2 = 20$kHz. The following system has been proposed for solving this sampling rate conversion problem.

$$\xrightarrow{x_1[n]} \boxed{H_2(\omega)} \xrightarrow{x_2[n]} \boxed{\uparrow 2} \xrightarrow{x_3[n]} \boxed{H_3(\omega)} \xrightarrow{x_4[n]} \boxed{\downarrow 3} \xrightarrow{x_5[n]} \boxed{H_4(\omega)} \xrightarrow{x_6[n]} \boxed{\begin{array}{c}\text{D/A} \\ F_2 = 20\text{kHz}\end{array}} \rightarrow \hat{x}_\mathrm{a}(t).$$

As usual, $\boxed{\uparrow 2}$ denotes upsampling by zero insertion and $\boxed{\downarrow 3}$ denotes down sampling by discarding.
Specify the magnitude responses of the three filters $H_2(\omega)$, $H_3(\omega)$, and $H_4(\omega)$ so that the final output signal will be as close to the original signal as possible.
Only the shape of the filter will be graded; not its gain. If a filter is not needed, say so.

9. (10 points)
The following alternative system for solving the preceding rate conversion problem has also been proposed.

$$\xrightarrow{x_1[n]} \boxed{H_2(\omega)} \xrightarrow{x_2[n]} \boxed{\downarrow 3} \xrightarrow{x_3[n]} \boxed{H_3(\omega)} \xrightarrow{x_4[n]} \boxed{\uparrow 2} \xrightarrow{x_5[n]} \boxed{H_4(\omega)} \xrightarrow{x_6[n]} \boxed{\begin{array}{c}\text{D/A} \\ F_2 = 20\text{kHz}\end{array}} \rightarrow \hat{x}_{\mathrm{a}}(t)$$

Explain why this system would be *preferable to* or *inferior to* or *equivalent to* the system proposed in the previous problem. (Pick one of these three and explain with a couple of sentences.)