Homework #4, EECS 451, W04. Due **Fri. Feb. 6**, in class

Notes

- Exam 1 is Wed. Feb. 11 in class (50 minutes). Last name A-L in 1001, M-Z in 1005.
- It will cover Chapters 1-3 and Homework 1-4.
- This HW will not be graded and returned before the exam. You may want to photocopy your answers for comparison with the posted solutions.
- Guaranteed exam topics include: system characteristics, convolution, and inverse z-transforms by PFE.
- You may bring a scientific calculator to the exam, and one 8.5x11" piece of paper with your own handwriting on both sides. Tables of common transforms will not be provided on the exam.

Review Problems

R1. [B0] Text 3.14ach. Concept(s): ***inverse z-transform by PFE.***

R2. [B0] Text 3.16bd. Concept(s): ***convolution via z-transform and PFE.***

Skill Problems

1. [U10] Text 3.25a. Concept(s): ***Z^{-1} for unspecified ROC.***

2. [B 40] Concept(s): ***use of MATLAB for system analysis.***
   (a) [20] Use MATLAB's `residuez` function to determine the impulse response of the discrete-time system described by the following difference equation:
   $$y[n] = -0.6\, y[n-1] + 0.11\, y[n-2] - 0.084\, y[n-3] - 0.09\, y[n-4] + x[n] - 2\, x[n-4]\,.$$
   (b) [5] Plot $h[n]$ using MATLAB.
   (c) [5] Make a pole-zero plot for this system using MATLAB. Sketch the ROC on the pole-zero plot (by hand or use MATLAB plotting commands). Hint: the MATLAB commands `tf2zp` and/or `zplane` may be useful here.
   (d) [5] Is this system BIBO stable?
   (e) [5] Is this system causal?

3. [B 20] Concept(s): ***pole-zero diagram from impulse response.***
   Consider an LTI system having impulse response $h[n] = \{\underline{0}, 0, 1, 1, 2, 2, 3, 3, \ldots\}$.
   (a) [10] Sketch the pole-zero diagram of this system. Hint: use upsampling ideas.
   (b) [5] Rewrite the impulse response in terms that indicate clearly the modes of the system, *i.e.*, not as a list or an infinite summation. Hint. Use PFE.
   (c) [5] Sketch the direct form II realization of this system.

4. [B 10] Concept(s): ***system response given $H(z)$ and $x[n]$.***
   An LTI system has system function $H(z) = \mathrm{e}^{z} + \mathrm{e}^{1/z}$.
   Determine the system response when the input signal is $x[n] = \delta[n-2] - 3\,\delta[n-4]$.
   Hint: think carefully about whether time domain or z-domain is easier.
   Hint: $\mathrm{e}^{t} = \sum_{k=0}^{\infty} t^k/k!$

5. [B 0] Concept(s): ***convolution via z-transform and series.***
   Convolve $x[n] = \dfrac{a^n}{n!}\, u[n]$ with $h[n] = \dfrac{(-b)^n}{n!}\, u[n]$ using z-transforms. Hint: consider power-series for e^{t}.

Mastery Problems

6. [B 0] Concept(s): ***z-transform properties.***
Assuming $x[n] \stackrel{Z}{\leftrightarrow} X(z)$, prove the following.
   (a) [0] conjugation: $x^*[n] \stackrel{Z}{\leftrightarrow} X^*(z^*)$
   (b) [0] real part: $\text{real}(x[n]) \stackrel{Z}{\leftrightarrow} \frac{1}{2}[X(z) + X^*(z^*)]$
   (c) [0] complex modulation: $e^{\jmath\omega_0 n} x[n] \stackrel{Z}{\leftrightarrow} X(z\,e^{-\jmath\omega_0})$

7. [B10] Text 3.50. Concept(s): ***zeros of even FIR filters.***
(This property is very important for practical FIR filter design.)

8. [G10] Text 3.36. Concept(s): ***nonuniqueness of autocorrelation function.***

9. [B 40] Concept(s): ***using z-transforms for financial systems.***
At the start of the first day of each month you deposit \$100 into a savings account at Bank2. At the end of the last day of each month, Bank2 pays 3/12% interest on the balance held for the month[a]. (So if your balance were \$1000 in a given month, Bank2 would add \$2.50 to your account at the end of that month.) At the end of the last day of each month, you withdraw \$25 to buy groceries.
   (a) [20] Determine how many years pass (since opening the account) until you have saved enough money to buy a \$8000 used car. Use LTI system methods to solve this problem.
   Hint: let $y[n]$ denote the balance at the end of month n, where you open the account on month 0. If there were no interest paid, and you deposited \$60 each month and made no withdrawals, then $y[n] = y[n-1] + 60$.
   (b) [10] Decompose the $y[n]$ expression you found above into a forced response component and a natural response component.
   (c) [5] Is there a transient response component? Explain.
   (d) [5] Is there a steady-state response component? Explain.

[a] This corresponds to an annual percentage rate (APR) of $[(1 + 0.03/12)^{12} - 1] \cdot 100 = 3.0416$ %.