EECS 451 FALL 2013 HW 4 DUE OCTOBER 1

After a genuine attempt to solve the homework problems by yourself, you are free to collaborate with your fellow students to find solutions to the homework problems. Regardless of whether you collaborate with other 451 students, you are required to write your own solutions to hand in. Copying homework solutions from another student or from existing solutions will be considered a violation of the honor code. Finally, if you choose to collaborate, you must include the names of your collaborators on your submitted homework.

Please take advantage of the Piazza discussion forum on CTools and the professor's and GSI's office hours. We are all in this together!!!

Also please keep a copy of these solutions after you hand it in; it will not be graded before the exam.

1. (20 points) In HW 3 we considered a noise-removal filter in the textbook problem 2.88. It was described as a filter for removing noise from images. For the n^{th} pixel value ($x[n]$) we essentially averaged out its value with the neighboring two pixel values.

   However, since images are two-dimensional, a pixel in reality has more than two neighboring values. In this problem we will use Matlab's built-in function `conv2` to perform 2-d convolution on an image.

   (a) Download the files
   `http://web.eecs.umich.edu/~girasole/teaching/451/earl.mat`
   `http://web.eecs.umich.edu/~girasole/teaching/451/hw4.m`

   First run the file. It will apply a filter to the image and then stop at the "pause" command. Qualitatively describe what the filter code is doing to the image. Is the filter linear? Why or why not?

   (b) Now edit the second half of the code to convolve the following 5 signals with the output image from part (a). You can simply use `conv2`. This Matlab function will perform a 2-d convolution. Please use the default settings, i.e. call `conv2(`x, h_i`)`. If you choose to use a 'shape' parameter, please say why you chose the one you did. Turn in plots of the resulting images (i.e. the new images), and qualitatively describe what each convolution did.

$$h_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} /4$$

$$h_2 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} /25$$

$$h_3 = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} /16$$

$$h_4 = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$$

$$h_5 = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

(c) We can consider the image as a sequence on two variables, the horizontal pixel number and the vertical pixel number, $x[n, m]$. Then 2-d convolution is defined as

$$x[n, m] * h[n, m] = \sum_{k=-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} x[k, \ell] h[n-k, m-\ell] \ .$$

Given this, what can you say about how is h_3 related to h_1?

(d) If we performed these convolutions on the original image (before part (a)), and then applied the filter from part (a), would we get the same output? Why or why not?

2. (10 points) Textbook 3.24
3. (10 points) Textbook 3.26
4. (10 points) Find the Z-transforms and ROC of the following sequences.
   (a) $x[n] = n \cos^2(\omega_0 n) u[n]$ for a fixed ω_0.
   (b) $\sin\left(\frac{\pi}{3}n - \frac{\pi}{6}\right) u[n-3]$
   (c) $x[n] = u[n] \sum_{k=-\infty}^{n} 5^k$
   (d) $x[n] = 5^n u[-n]$
   (e) $x[n] = 4^n u[n] * u[n] * (3^n u[n] - 3^{n-1} u[n-1])$
5. (10 points) **Challenge problem:** In class it was given that the inverse Z-transform is gotten by a contour integral:

$$x[n] = \frac{1}{2\pi j} \oint_C X(z) z^{n-1} dz \ , \tag{0.1}$$

for a C which is a counterclockwise closed path that (1) encircles the origin and (2) is entirely contained within the region of convergence. Show that we can also get Equation 0.1 from the inverse DTFT.

Hint: We have seen a motivation for why the book uses the notation $X(e^{j\omega})$, that if you let $z = e^{j\omega}$ you can get the forward DTFT from the forward Z-transform. Consider instead letting $z = e^{\theta + j\omega}$. Start by writing down the inverse DTFT of $x[n]e^{-\theta n}$ and multiply both sides by $e^{\theta n}$.