Problems for Discussion 1, 09/11/13

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1 Alternate Expressions for Sequences

Let $x[n] = \begin{cases} \left(\frac{1}{2}\right)^n, & n \text{ nonnegative multiple of } 4 \\ -\left(\frac{1}{2}\right)^n, & n \text{ nonnegative multiple of } 2, \text{ but not a nonnegative multiple of } 4 \\ 0, & \text{otherwise} \end{cases}$

Express $x[n]$ mathematically in three different ways

2 Nonlinear Systems

Give an example of a system that is nonlinear but satisfies $\mathcal{T}\{\alpha x[n]\} = \alpha\mathcal{T}\{x[n]\}$ for all sequences $x[n]$ and for all scalars $\alpha \in \mathbb{R}$.

3 Length of Convolution

Let $x[n]$ be non-zero only over $N_1 \leq n \leq N_2$ and $h[n]$ be non-zero only over $M_1 \leq n \leq M_2$. Let $y[n] = x[n] * h[n]$. Then $y[n]$ is only non-zero over $L_1 \leq n \leq L_2$. Define L_1 and L_2 in terms of N_1, N_2, M_1, and M_2.

4 Distributivity of Convolution

Prove the distributive property of convolution.

5 Computing Discrete Convolution

Let $y[n] = x[n] * h[n]$. Find an expression for $y[n]$.

(a)

$x[n] = \begin{cases} 1, & n = -2, 0, 1 \\ 2, & n = -1 \\ 0, & \textit{otherwise} \end{cases}$

$h[n] = \delta[n] - \delta[n-1] + \delta[n-4] + \delta[n-5]$

(b)

$x[n] = u[n+1] - u[n-4] - \delta[n-5]$
$h[n] = (u[n+2] - u[n-3])\,(3 - |n|)$

6 Convolution and Signal Energy

Let $y[n] = x[n] * h[n]$. Prove that $\left(\sum_{n=-\infty}^{\infty} y[n]\right) = \left(\sum_{n=-\infty}^{\infty} x[n]\right)\left(\sum_{n=-\infty}^{\infty} h[n]\right)$.

7 Impulse Response of a BIBO Stable System

Let $h[n]$ be the impulse response of a BIBO stable system. (Remember that impulse responses are only defined for LSI systems, so this system is also LSI.) What must hold true for $h[n]$?

8 Impulse Response of a Causal System

Let $h[n]$ be the impulse response of a causal system. What must hold true for $h[n]$?

9 Impulse Response of an Invertible System

A system $\mathcal{T}_1$ is invertible if there exists a system $\mathcal{T}_2$ such that $\mathcal{T}_2\{\mathcal{T}_1\{x[n]\}\} = x[n]$ for all $n \in \mathbb{Z}$. Let $h[n]$ be the impulse response of an invertible system. What must hold true for $h[n]$?

10 Eigensequences

A sequence $x[n]$ is an eigensequence of a system $\mathcal{T}$ if $\mathcal{T}\{x[n]\} = \lambda x[n]$ for some scalar $\lambda \in \mathbb{C}$. What are the eigensequences for the following systems?

10.1 $\mathcal{T}\{x[n]\} = 3x[n]$

10.2 $\mathcal{T}\{x[n]\} = x[n]u[n]$

10.3 **causal moving average:** $\mathcal{T}\{x[n]\} = \frac{1}{M}\sum_{k=0}^{M-1} x[n-k]$

10.4 **general LSI system:** $\mathcal{T}\{x[n]\} = \sum_{k=-\infty}^{\infty} h[k]x[n-k]$

11 Geometric Basis for Sequences

Consider the signal $\gamma[n] = a^n u[n]$ for $0 < a < 1$.

(a)

Show that any sequence $x[n]$ can be decomposed as $x[n] = \sum_{n=-\infty}^{\infty} c_k \gamma[n-k]$ and express c_k in terms of $x[n]$.

(b)

Use the properties of linearity and time invariance to express the output $y[n] = \mathcal{T}\{x[n]\}$ in terms of the input $x[n]$ and the signal $g[n] = \mathcal{T}\{y[n]\}$, where $\mathcal{T}$ is an LTI system.

(c)

Express the impulse response $h[n] = \mathcal{T}\{\delta[n]\}$ in terms of $g[n]$.

12 Steady State of Stable Systems

Let $\mathcal{T}$ be an LTI, BIBO stable system. Show if x[n] is bounded and tends to a constant, the corresponding output, $y[n]$, will also tend to a constant.