Solutions to EECS 451 Exam 2, 2004-3-26

Regrade requests must be submitted to Prof. Fessler *in writing*, within 1 week of when the exam is returned in class. All problems will be re-examined, and scores may increase or decrease.
Discussing the exam with a professor or GSI nullifies the opportunity to submit a regrade request.
For elaboration on these solutions, please come to office hours.

p/transient1

1.

- [15] $H_I(z) = 1/H(z) = \frac{(z+1/2)(z-2/3)}{z} = z - \frac{1}{6} - \frac{1}{3}z^{-1} \Longrightarrow \boxed{h_I[n] = \delta[n+1] - \frac{1}{6}\,\delta[n] - \frac{1}{3}\,\delta[n-1]\,.}$

  5 for $H(z)$, 5 for $H_I(z)$, 5 for $h_I[n]$. *(HW 6-4)*

- [10] For large n, the output signal is just the transient response which has the form $\boxed{r_1(2/3)^n + r_2(-1/2)^n,}$ where the residue values depend on the input signal. *(HW 4-9, 7-8) [5 correct.]*

- [15] The system function is $H(z) = \frac{z}{(z+1/2)(z-2/3)} \Longrightarrow H(\omega) = \frac{e^{\jmath\omega}}{(e^{\jmath\omega}+1/2)(e^{\jmath\omega}-2/3)}$.

  So the frequency responses at $\omega = 0$ and $\omega = \pi$ are $H(0) = 2$ and $H(\pi) = -\frac{6}{5}$.

  By DTFS, $x[n] = 5\,u[n] + 3\cos(\pi n)\,u[n]$. The steady-state response to a right-sided sinusoid equals the response to an eternal sinusoid, so $\boxed{y_{ss}[n] = 5\,H(0) + 3\,|H(\pi)|\cos(\pi n + \angle\, H(\pi)) = 10 - \frac{18}{5}\cos(\pi n)\,.}$ *(HW 7-5)*

p/design1

2. (15)

$|H_d(\omega)| = 2\,\mathrm{rect}\left(\frac{\omega}{2(4\pi/5)}\right) - \mathrm{rect}\left(\frac{\omega}{2(\pi/3)}\right) \Longrightarrow h_d[n] = 2\frac{4}{5}\,\mathrm{sinc}\left(\frac{4}{5}(n - \frac{M-1}{2})\right) - \frac{1}{3}\,\mathrm{sinc}\left(\frac{1}{3}(n - \frac{M-1}{2})\right)\,.$

So the correct line is

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hd = 2 * 4/5 * sinc(4/5*(n-15)) - 1/3 * sinc(1/3*(n-15));
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(HW 5-9, 8-3)

p/sample,down1

3. (20)

An ideal anti-alias has frequency response $H_a(F) = \mathrm{rect}(F/F_s)$, so $X_2(F) = X_a(F)\,H_a(F) = |F|\,\mathrm{rect}(F/F_s)$, which has peak value $F_s/2$.

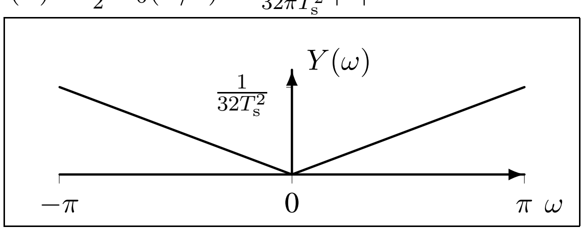
After sampling, $X_3(\omega) = \frac{1}{T_s}\,X_2\left(\frac{\omega}{2\pi T_s}\right) = \frac{1}{T_s}\left|\frac{\omega}{2\pi T_s}\right|$ for $|\omega| \le \pi$, which has peak value $F_s^2/2$.

$X_4(\omega)$ is the same except $|\omega| \le \pi/2$, and the peak value is $F_s^2/4$.

In general, $X_5(\omega) = \frac{1}{2}\,X_4(\omega/2) + \frac{1}{2}\,X_4(\omega/2 \pm \pi)$. Since in this case $X_4(\omega)$ is bandlimited to $|\omega| \le \pi/2$, so only the first term matters for $|\omega| \le \pi$, for which $X_5(\omega) = \frac{1}{2}\,X_4(\omega/2) = \frac{1}{8\pi T_s^2}\,|\omega|$, which has peak value $F_s^2/8$.

$X_6(\omega)$ is the same except $|\omega| \le \pi/2$, with peak value $F_s^2/16$.

For $|\omega| \le \pi$, $Y(\omega) = \frac{1}{2}\,X_6(\omega/2) = \frac{1}{32\pi T_s^2}\,|\omega|$.



(HW 6-7, 6-8, 8-6)

p/updown2

4.

- [15] $V(\omega) = X(2\omega)$ (4 pts). $W(\omega) = H_1(\omega)\, V(\omega) = H_1(\omega)\, X(2\omega)$ (3 pts).
  $Y(\omega) = \frac{1}{2}\, W(\omega/2) + \frac{1}{2}\, W(\omega/2 + \pi)$ (6 pts) $= \frac{1}{2}\, H_1(\omega/2)\, X(\omega) + \frac{1}{2}\, H_1(\omega/2 + \pi)\, X(\omega)$ (2 pts).
  $\boxed{Y(\omega) = \frac{1}{2}\left[H_1\left(\frac{\omega}{2}\right) + H_1\left(\frac{\omega}{2} + \pi\right)\right] X(\omega)\,.}$ (Similar problem in z-domain on Exam1.) *(HW 6-7, 8-6)*

- [10] From the above relation between $Y(\omega)$ and $X(\omega)$, we have $H(\omega) = \frac{1}{2}\left[H_1\left(\frac{\omega}{2}\right) + H_1\left(\frac{\omega}{2} - \pi\right)\right]$.
  In this case we have $H_1(\omega) = \mathrm{rect}\left(\frac{\omega}{\pi}\right)$ for $|\omega| \leq \pi$ (and otherwise periodic).
  So $H(\omega) = \frac{1}{2}\left[\mathrm{rect}\left(\frac{\omega}{2\pi}\right) + \mathrm{rect}\left(\frac{\frac{\omega}{2} - \pi}{\pi}\right)\right] = \frac{1}{2}\left[\mathrm{rect}\left(\frac{\omega}{2\pi}\right) + \mathrm{rect}\left(\frac{\omega - 2\pi}{\pi}\right)\right] = \frac{1}{2},\ \forall\omega.$
  Hence by inverse DTFT $\boxed{h[n] = \frac{1}{2}\,\delta[n]\,.}$ *(HW 5-9, 6-1)*

Exam scores with APPROXIMATE grades.
52 undergraduate students: mean=53.7, median=54.5, std=16.66
a+ 93 a 81 79 78 77 73 73 72 a- 70 69 67 66 65 65
? 63 62 62 62 61 60 59 59 59 59
b+ 57 55 54 54 53 52 b 50 50 47 47 47 45 45 45 44 b- 42
c 40 37 37 36 35 ? 33 32 32 29 26 20 15

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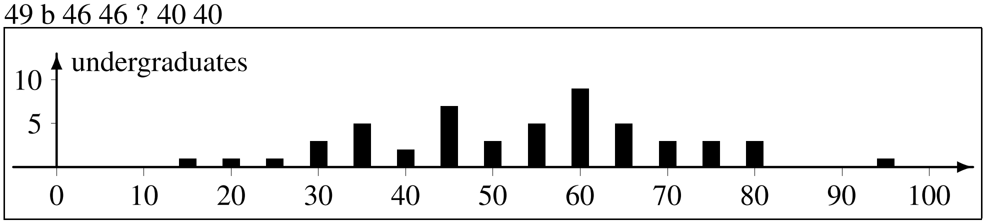
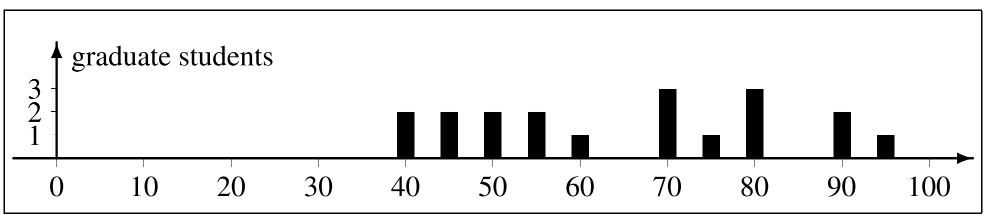
19 graduate students: mean=65.2, median=68, std=17.6;
a 95 88 88 82 80 80 76 a- 72 70 68
b+ 59 56 55 49 49 b 46 46 ? 40 40





When computing final grades, most likely I will add 20 points to undergrad scores and 10 points to grad student scores for this exam, to equalize the means of Exam1 and Exam2.