Solutions to EECS 451 Exam 3, 12/11/1998

1.

- $x[n] = y[n] - \delta[n]$ where $y[n] = a^n\, u[n]$ so $Y_k = \sum_{n=0}^{N-1} a^n e^{-j2\pi kn/N} = \sum_{n=0}^{N-1} [ae^{-j2\pi k/N}]^n = \frac{1-a^N}{1-ae^{-j2\pi k/N}}$. So $\boxed{X_k = \dfrac{1-a^N}{1-ae^{-j2\pi k/N}} - 1.}$ *[1U/16G correct. Many sampled z-Transform, despite $L = \infty$.]*

- Since $x[n]$ is time-limited to $L = 4$, $X_k = X(\omega)|_{\omega=2\pi k/N}$ for $N \geq L$. Thus the 8 values given are samples of $X(\omega)$ every $\pi/4$ around unit circle. So the 4-point DFT is $\boxed{\{5, 3, 1, 3\}_4.}$ *[12U/14G correct.]*

2.

- $x[n] = 3\,\delta[n+2] + 7\,\delta[n] + 2\,\delta[n-11] + 4\,\delta[n-13]$ and the 11-point IDFT yields $\boxed{x_p[n] = \{9, 0, 4, 0, 0,\ 0, 0, 0, 0, 3,\ 0\}_{11}.}$ *[5U/13G correct. confusion between $x((n))_N$ and $x_p[n]$]*

- $x((n))_N = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{j2\pi kn/N}$ so $x((n))_4 = 2e^{j2\pi n/4} + 2e^{j2\pi 3n/4} = 4\cos(\pi/2n)$. In time-limited case, $x[n] = x((n))_4$ for $n = 0, 1, 2, 3$ and is 0 otherwise, so $x[n] = \{4, 0, -4, 0\}$. Thus $\boxed{X(\omega) = 4 - 4e^{-j2\omega}.}$ *[10U/19G correct. A few used (5.1.39) correctly.]*

3.

- $x[n] = \sin(\pi/2n) = \{0, 1, 0, -1\}_4$ so $\boxed{X_k = e^{-j2\pi k/4} - e^{-j2\pi 3k/4} = -2j\sin((\pi/2)\,k) = \{0, -2j, 0, 2j\}_4.}$ *[10U/20G correct.]*

- $y[n] = H(\pi/2)\sin((\pi/2)\,n + \angle H(\pi/2))$, where $H(\omega) = 1 - e^{-j2\omega}$ so $H(\pi/2) = 1 - e^{-j\pi} = 2$. Thus $y[n] = 2\,x[n]$ and $\boxed{Y_k = 2X_k = \{0, -4j, 0, 4j\}.}$ *[13U/23G correct.]*

- If $\boxed{T = 2\text{msec or } F_s = 500\text{Hz},}$ then $x[n] = \sin(\pi n) = 0$ so $X_k = 0$. *[17U/23G correct.]*

4.

- $\boxed{y[n] = \dfrac{1}{2}x((n))_N.}$ *[5U/10G correct.]*

- $x((n))_4 = \{0, 8, 16, 24\}$ so $y[n]$ is $\boxed{8,\ 4,\ 12,\ 8,\ 0}$ for the values of n given. *[6U/17G correct.]*

5.

- From z-Transform table: $X_2(z) = 1/4z^{-1}/(1 - 1/4z^{-1})^2 = 1/4z^{-1}/(1 - 1/2z^{-1} + 1/16z^{-2})$.

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n = 0:40
x1 = cos(0.3*n) .* (n < 20)
y = filter([0 1/4], [1 -1/2 1/16], x1);
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[4U correct. -3 for wrong n range. -3 for $b = [1/4]$]

- $q(n) = X((-n))_{10}/10$ so $\boxed{\texttt{q = [0.7 0.4 0 0 0.5 0 0 0.2 0.1 0].}}$ *[15G correct.]*

Exam scores, with *approximate* grades:
28 Graduate Students: Mean=79.9, Median=81.5, Std=16.3
a 100 100 100 99 98 98 96 94 91 89 89 86 86, b 83 80 78 78 77 75 73, ? 67 65 60 59 59 57 54, c 46
25 Undergraduate Students: Mean=62.1, Median=63, Std=19.3
a 98 92 87 81 79 78 77, b 72 71 71 71 63 63 63 58 57 55 ? 50 50, c 44 42 ? 37 35 33, d 25