Homework #7, EECS 451, W04. Due Fri. Mar. 12, in class

Review Problems

R1. [B 0] Text 4.49. Concept(s): ***Frequency response from pole-zero plot.*** Make the sketches by hand.

Skill Problems

1. [B 20] Concept(s): ***Minimum phase systems.***
   (a) [5] Determine if the system with impulse response $h[n] = \{10, 9, 7, \underline{-8}, 0, 5, 3\}$ is minimum phase. Hint: use MATLAB or a calculator.
   (b) [5] Explain why or give a contradiction to this claim: "the series cascade of two minimum phase systems is always minimum phase."
   (c) [10] Explain why or give a contradiction to this claim: "the parallel sum of two minimum phase systems is always minimum phase."

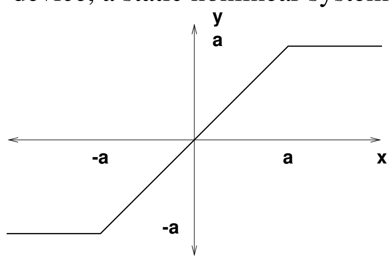
2. [B 25] Concept(s): ***Mellowing a square wave.***
   The MATLAB file `mellow1_show.m`, available on web site, generates a signal that is one second of a sinusoid, followed by one second of a square wave, followed by one second of a sinusoid.
   (a) [5] Determine the fundamental frequency (in Hz) of the sinusoid part of the signal when played back through the D/A converter using the `sound` command. Repeat for the square wave portion.
   (b) [0] Use MATLAB's `filter` command to create a signal $y[n]$ from $x[n]$ by filtering $x[n]$ with a LTI system whose impulse response is $h[n] = p^n\, u[n] - 0.5p^{n-1}\, u[n-1]$ where $p = 0.8$.
   (c) [5] Use the `sound` command to listen to both $x[n]$ and $y[n]$. Describe briefly the effects of the filter.
   (d) [0] Repeat but use $p = 0.4$ instead of $p = 0.8$.
   (e) [5] Describe briefly the difference qualitatively between the output when $p = 0.8$ and the output when $p = 0.4$.
   (f) [10] Explain the difference by considering the frequency response of the two filters and the frequency content of the signal segments.
   (g) [0] Experiment with other values of p.

3. [B 15] Concept(s): ***Filtering out a squeaky violin signal.***
   Design a causal, stable, FIR filter that will eliminate (in steady state) the sawtooth violin signal of HW#5. (Your system must pass some signals, *i.e.*, $h[n] = 0$ is unacceptable.)
   (a) [5] Specify the pole-zero plot of your filter.
   (b) [5] Find the frequency response of your system. Hint. Your answer may have a $\prod_{k=0}^{9}$ in it.
   (c) [5] Find the steady-state response to a unit step function input signal. Hint. The answer is very simple.

Mastery Problems

4. [B 10] Concept(s): ***Linear phase systems.***
The lecture notes show how to generate a causal all-zero (FIR) system that has linear phase. Give an example of a (possibly noncausal) *all-pole*, stable, *linear phase*, *low-pass*, IIR filter. Sketch the pole-zero plot (including ROC) of your design, and determine its impulse response.

5. [B 30] Concept(s): ***Distortions due to nonlinearities.***
A common way to create the distorted electric guitar sound that is ubiquitous in rock music is to use a "clipping" device, a static nonlinear system with an input-output relationship like the following:



$$y[n] = \begin{cases} x[n]\,, & |x[n]| < a \\ -a, & x[n] < -a \\ a, & x[n] > a. \end{cases}$$

Suppose the clipper is used in the following system:

$$\text{audio input}\, x_{\rm a}(t) \to \boxed{\text{Sampler}} \xrightarrow{x[n]} \boxed{\text{Clipper}} \xrightarrow{y[n]} \boxed{\text{Filter } h[n]} \to z[n]$$

Assume the sampling rate is 20kHz, the clipper limit is $a = 2$, and the filter impulse response is $h[n] = \delta[n] + \delta[n-1]$.

(a) [5] Find the steady-state output of the filter when the input signal is $x_{\rm a}(t) = \sin(2\pi F_0 t)\, u(t)$ with F_0 =2.5kHz.

(b) [15] Repeat when $x_{\rm a}(t) = 2\sqrt{2}\sin(2\pi F_0 t)\, u(t)$ with $F_0 = 2.5$kHz.
Hint: your answers will be quite different.

(c) [10] Plot (by hand is ok) the power density spectrum of the steady-state output signals for the two cases. Label the horizontal axis in terms of CT frequencies (Hz), rather than in radians.

(d) [0] Considering (c), describe the effect of the clipper qualitatively.

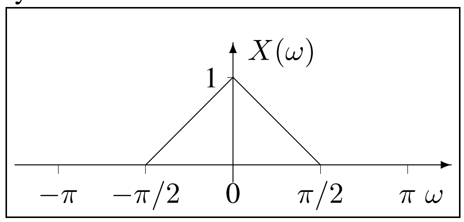
6. [B 15] Concept(s): ***DSP compensation for imperfect analog audio component.***
You record an audio signal using an imperfect microphone whose frequency response is

$$H_{\rm a}(F) = \begin{cases} 1 - |F|/F_{\rm c}, & |F| < F_{\rm c} \\ 0, & \text{otherwise,} \end{cases}$$

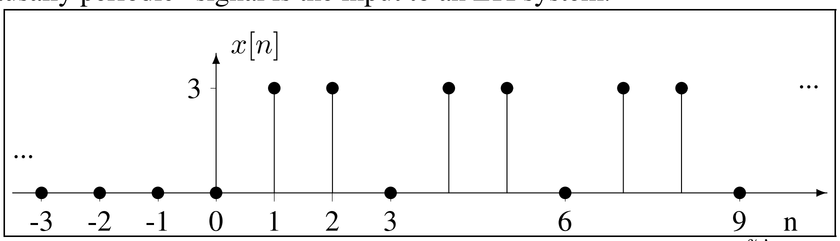
where $F_{\rm c} = 30$kHz. To compensate for the signal attenuation of this microphone, you decide to sample the recorded signal at 44kHz (using an ideal anti-alias filter) and then process the signal using a DSP system to restore the audio signal (over the audio range) to its original spectral content.
Specify the frequency response of the ideal compensating DT filter for this problem.
Hint. The system looks like this, where $y_{\rm a}(t)$ is the acoustic wave entering the microphone, $v_{\rm a}(t)$ is the analog signal output of the microphone, and $x_{\rm a}(t)$ is the output of the (analog) anti-alias filter. Your task is to determine what $H(\omega)$ should be so that $y[n]$ would be the same as what $x[n]$ would have been if the microphone were perfect, *i.e.*, if $H_{\rm a}(F) = 1$.

$$y_{\rm a}(t) \to \boxed{H_{\rm a}(F)} \xrightarrow{v_{\rm a}(t)} \boxed{\text{Anti Alias}} \xrightarrow{x_{\rm a}(t)} \boxed{\text{Sampler}} \xrightarrow{x[n]} \boxed{H(\omega)} \to y[n]$$

7. [B 10] Concept(s): ***Ideal interpolation.***
A discrete-time signal $x[n]$ with the following spectrum is the input to an ideal D/A converter. Determine the analog output signal $x_{\mathrm{a}}(t)$.
Hint. Your answer should be simple and will involve sinc^2; there should not be any infinite summations in it. Start by working in the frequency domain.



8. [B 20] The following "causally periodic" signal is the input to an LTI system.



Determine the output signal $y[n]$ if the system frequency response is $H(\omega) = \dfrac{e^{\jmath\omega}}{e^{\jmath\omega} - \frac{3}{4}}$.
In your expression for the output signal, identify the transient components and the steady-state components.
Hints. You will need to use several tools, including DTFS and PFE. The inverse DTFT is not very useful.
Use MATLAB as much as you like.