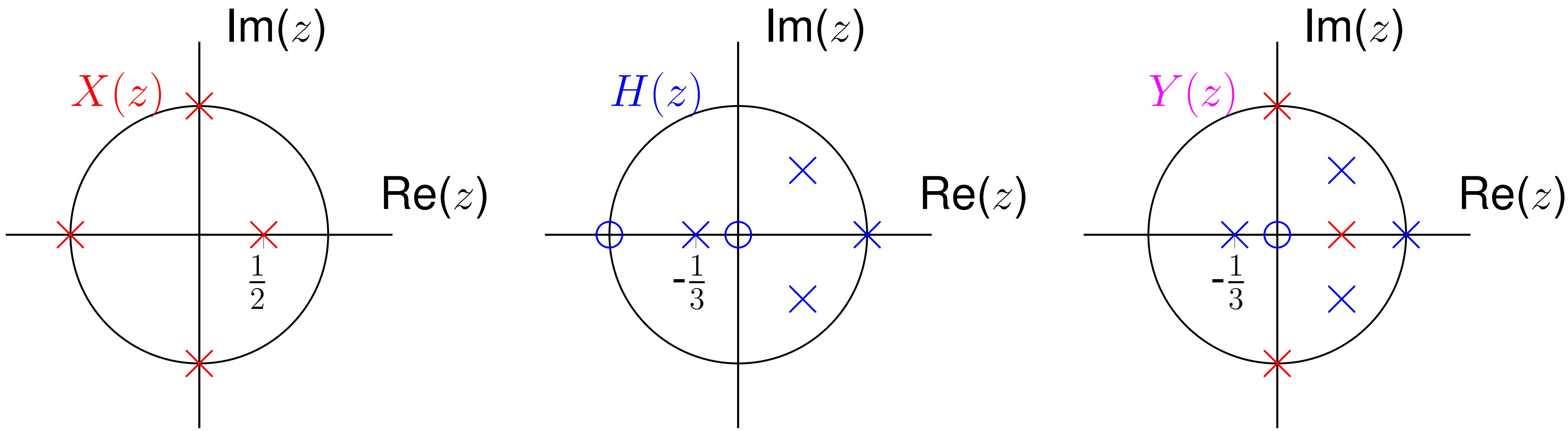
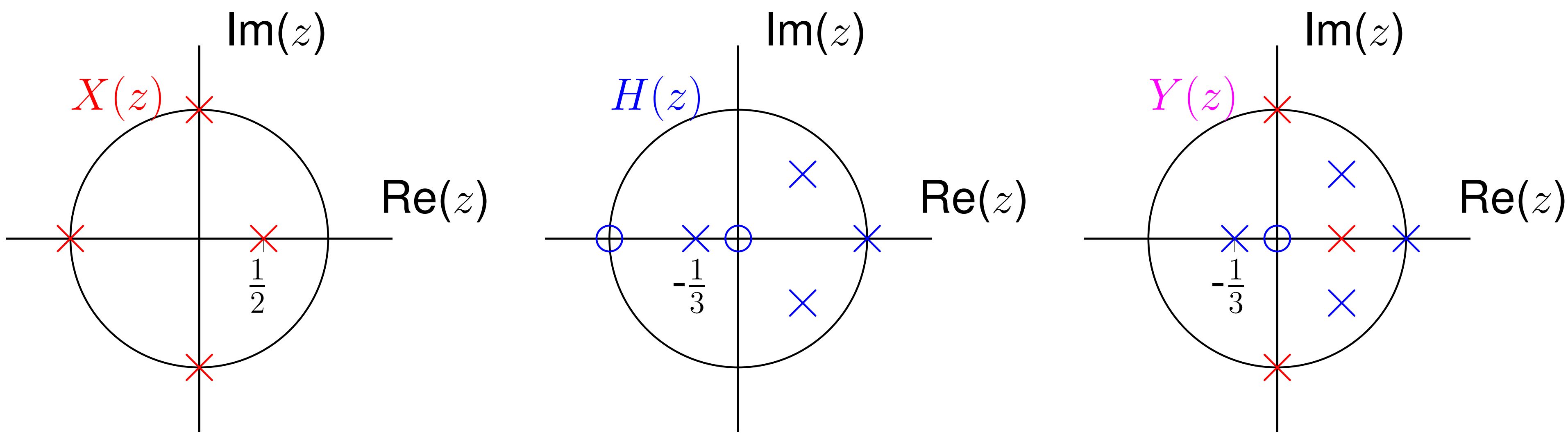
System Response



$$
\begin{aligned}
Y(z) &= X(z)H(z) \\
&= g \frac{z}{\underbrace{(z-\jmath)(z+\jmath)\left(z-\frac{1}{2}\right)}_{\text{input}}\;\underbrace{\left(z+\frac{1}{3}\right)\left(z-\frac{1}{\sqrt{2}}e^{\jmath\pi/4}\right)\left(z-\frac{1}{\sqrt{2}}e^{-\jmath\pi/4}\right)(z-1)}_{\text{system}}} \\
&= \underbrace{\frac{r_1}{z-\jmath}+\frac{r_1^*}{z+\jmath}+\frac{r_2}{z-\frac{1}{2}}}_{\text{input}}+\underbrace{\frac{r_3}{z+\frac{1}{3}}+\frac{r_4}{z-\frac{1}{\sqrt{2}}e^{\jmath\pi/4}}+\frac{r_4^*}{z-\frac{1}{\sqrt{2}}e^{-\jmath\pi/4}}+\frac{r_5}{z-1}}_{\text{system}}
\end{aligned}
$$

One input component is nulled due to pole-zero cancellation.
Since $Y(z)$ is proper, there are no $k\,\delta[n-\cdot]$ terms.

Response



$$Y(z) = \underbrace{\frac{r_1}{z-\jmath} + \frac{r_1^*}{z+\jmath} + \frac{r_2}{z-\frac{1}{2}}}_{\text{input}} + \underbrace{\frac{r_3}{z+\frac{1}{3}} + \frac{r_4}{z-\frac{1}{\sqrt{2}}\,\mathrm{e}^{\jmath\pi/4}} + \frac{r_4^*}{z-\frac{1}{\sqrt{2}}\,\mathrm{e}^{-\jmath\pi/4}} + \frac{r_5}{z-1}}_{\text{system}}$$

$$y[n] = \underbrace{\mathcal{H}\left(\frac{\pi}{2}\right)\alpha_1 \cos\left(\frac{\pi}{2}n + \angle\mathcal{H}\left(\frac{\pi}{2}\right)\right)u[n] + r_2\left(\frac{1}{2}\right)^n u[n]}_{\text{forced (due to input)}}$$

$$+\underbrace{r_3\left(\frac{-1}{3}\right)^n u[n] + |r_4|\left(\frac{1}{\sqrt{2}}\right)^n \cos\left(\frac{\pi}{4}n + \angle r_4\right)u[n] + r_5 1\,u[n]}_{\text{natural (due to system)}}$$

Note that $r_1 = \mathcal{H}(\omega)\big|_{\omega=\frac{\pi}{2}}$, where $\mathcal{H}(\omega) = H(\mathrm{e}^{\jmath\omega})$. This is particularly important!

Response

$$x[n] = \alpha_1 \cos\left(\frac{\pi}{2}n\right) + \alpha_2 \left(\frac{1}{2}\right)^n u[n] + \alpha_3 \left(-1\right)^n u[n]$$

$$y[n] = \underbrace{\mathcal{H}\left(\frac{\pi}{2}\right)\alpha_1 \cos\left(\frac{\pi}{2}n + \angle\mathcal{H}\left(\frac{\pi}{2}\right)\right) u[n] + r_2 \left(\frac{1}{2}\right)^n u[n]}_{\text{forced (due to input)}}$$

$$+ \underbrace{r_3 \left(\frac{-1}{3}\right)^n u[n] + |r_4| \left(\frac{1}{\sqrt{2}}\right)^n \cos\left(\frac{\pi}{4}n + \angle r_4\right) u[n] + r_5 1\, u[n]}_{\text{natural (due to system)}}$$

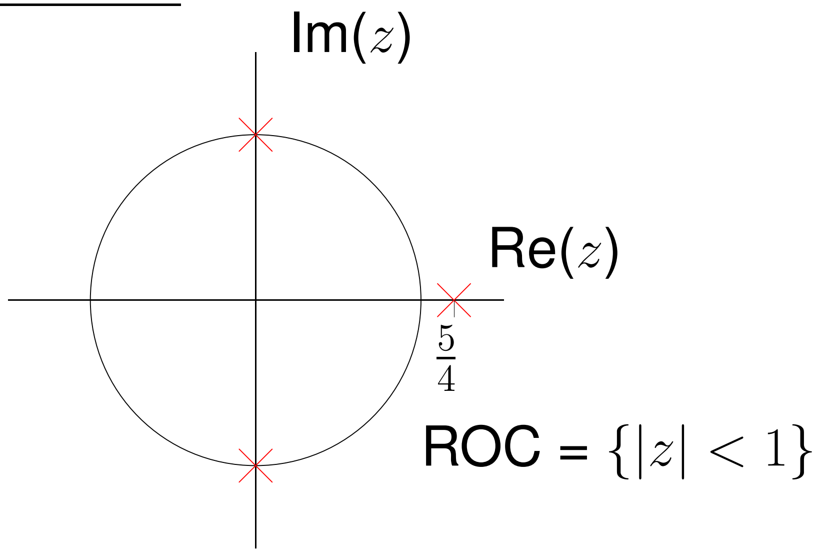
$$= \underbrace{\mathcal{H}\left(\frac{\pi}{2}\right)\alpha_1 \cos\left(\frac{\pi}{2}n + \angle\mathcal{H}\left(\frac{\pi}{2}\right)\right) + 0(-1)^n u[n]}_{\text{steady-state}} + \underbrace{r_2 \left(\frac{1}{2}\right)^n u[n]}_{\text{transient?}}$$

$$+ \underbrace{r_3 \left(\frac{-1}{3}\right)^n u[n] + |r_4| \left(\frac{1}{\sqrt{2}}\right)^n \cos\left(\frac{\pi}{4}n + \angle r_4\right) u[n]}_{\text{transient}} + \underbrace{r_5 1\, u[n]}_{\text{steady-state?}}$$

Response

- What if $x[n]$ is finite duration? ($X(z)$ is all zeros except poles at origin.)
- What if $x[n]$ is left-sided?

  Example.



$x[n] = \alpha_1 \cos\left(\frac{\pi}{2}n\right) u[-n-1] + \alpha_2 \left(\frac{5}{4}\right)^n u[-n-1]$, which is 0 for $n \geq 0$.

For $n \geq 0$, the output signal $y[n]$ consists only of the transient reponse, *i.e.*,

$$y_{\mathrm{tr}}[n] = r_3 \left(\frac{-1}{3}\right)^n + |r_4| \left(\frac{1}{\sqrt{2}}\right)^n \cos\left(\frac{\pi}{4}n + \angle r_4\right),$$

where only the residues r_3 and r_4 depend on the input signal.