(5.17) $s(n) = s_1(n) + j s_2(n)$, $s_1(n)$ & $s_2(n)$ ARE REAL

a) BECAUSE THE DFT IS LINEAR

$$S(k) = S_1(k) + j S_2(k)$$

b) $\mathrm{Re}[S(k)] = \mathrm{Re}[S_1(k)] - \mathrm{Im}[S_2(k)]$

$\mathrm{Im}[S(k)] = \mathrm{Im}[S_1(k)] + \mathrm{Re}[S_2(k)]$

BECAUSE $S_1(k)$ & $S_2(k)$ ARE COMPLEX-VALUED, EVALUATING $\mathrm{Re}[S(k)]$ & $\mathrm{Im}[S(k)]$ IS NOT SIMPLE.

c) <u>NOTE</u>: $\mathrm{Re}[S_1(k)]$ & $\mathrm{Re}[S_2(k)]$ ARE EVEN: $\mathrm{Re}[S_2(k)] = \mathrm{Re}[S_2(N-k)]$
$\mathrm{Im}[S_1(k)]$ & $\mathrm{Im}[S_2(k)]$ ARE ODD: $\mathrm{Im}[S_2(k)] = -\mathrm{Im}[S_2(N-k)]$

EVALUATE $\dfrac{\mathrm{Re}[S(k)] + \mathrm{Re}[S(N-k)]}{2}$ [THE EVEN PART]

$$= \frac{\mathrm{Re}[S_1(k)] - \mathrm{Im}[S_2(k)] + \mathrm{Re}[S_1(N-k)] - \mathrm{Im}[S_2(N-k)]}{2}$$

$$= \frac{1}{2}\left[\mathrm{Re}[S_1(k)] - \mathrm{Im}[S_2(k)] + \mathrm{Re}[S_1(k)] + \mathrm{Im}[S_2(k)]\right]$$

$\frac{1}{2}[\mathrm{Re}[S(k)] + \mathrm{Re}[S(N-k)]] = \mathrm{Re}[S_1(k)]$

$$\therefore \quad \frac{1}{2}\left[\mathrm{Im}[S(k)] - \mathrm{Im}[S(N-k)]\right] = \mathrm{Im}[S_1(k)]$$

$$\frac{1}{2}\left[\mathrm{Re}[S(k)] - \mathrm{Re}[S(N-k)]\right] = -\mathrm{Im}[S_2(k)]$$

$$\frac{1}{2}\left[\mathrm{Im}[S(k)] + \mathrm{Im}[S(N-k)]\right] = \mathrm{Re}[S_2(k)]$$

$\therefore$ BY EVALUATING THE EVEN & ODD PARTS OF THE REAL & IMAGINARY PARTS, WE CAN FIND <u>ALL</u> THE PIECES.

d) INSTEAD OF TWO FFTs, WE CAN USE ONE AND SAVE A FACTOR OF TWO IN COMPUTATIONAL TIME.