EECS351 Discussion 1, 09/8/16

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1 What is a signal? What is a system?

focus on discrete signals and discrete-domain systems

2 Properties of Discrete Signals

2.1 Periodicity

As signal $x[n]$ is periodic if there exists a number n_0 such that $x[n] = x[n-n_0]$ for all n. The period of the signal is n_0.

For sinusoidal functions (i.e. $cos\,(\omega n + \phi)$), the function is periodic iff ω is a rational multiple of π, i.e. $\omega = \frac{M}{N}\pi$ for integers M and N.

2.2 Boundedness

A signal is bounded if there exists a positive, finite number B such that $|x[n]| \leq B$ for all n.

2.3 Causality

A signal $x[n]$ is causal if $x[n] = 0$ for $n < 0$.

2.4 Symmetry

A signal $x[n]$ is symmetric if $x[n] = x[-n]$. Signals with symmetry are also called "even".

If instead a signal has the property that $-x[n] = x[-n]$, then it is called "odd".

3 Properties of Discrete-Domain Systems

3.1 Causality

A system $\mathcal{T}$ is causal if $\mathcal{T}\{x[n]\}$ depends only on $x[n]$ for $n < 0$. In other words, it depends only on the present and past values of $x[n]$ and not the future values.

3.2 Linearity

A system $\mathcal{T}$ is linear iff for any inputs $x_1[n]$ and $x_2[n]$ and any scalars a_1 and a_2, $\mathcal{T}\{a_1x_1[n] + a_2x_2[n]\} = a_1\mathcal{T}\{x_1[n]\} + a_2\mathcal{T}\{x_2[n]\}$.

3.3 Shift-Invariance (a.k.a. Time-Invariance)

Let $y[n] = \mathcal{T}\{x[n]\}$. Then the system $\mathcal{T}$ is shift-invariant if $\mathcal{T}\{x[n-n_0]\} = y[n-n_0]$. In other words, the output of the delayed signal is the same as the delay of the output signal.

3.4 Bounded Input Bounded Output (BIBO) Stability

A system $\mathcal{T}$ is called BIBO stable if any bounded input signal $x[n]$ results in a bounded output signal $y[n]$. If a bound B_x exists for bounded $x[n]$, then there exists a bound B_y such that $|y[n]| = |\mathcal{T}\{x[n]\}| \leq B_y$ for all n.

4 Kronecker Delta (a.k.a. Delta Sequence, Unit Sample Sequence)

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

4.1 Decomposition of Discrete Signals with Shifted Deltas

Any sequence $x[n]$ can be written as $x[n] = \sum\limits_{k=-\infty}^{\infty} x[k]\delta[n-k]$

4.2 Sampling Property

$\delta[n-n_0]x[n] = \delta[n-n_0]x[n_0]$

4.3 Sifting Property

$$\sum_{n=-\infty}^{\infty} \delta[n-n_0]x[n] = x[n_0]$$

4.4 Scaling Property

$\delta[2n] = \delta[n]$ Note that there is no scaling factor!

5 Complex Numbers

6 Convolution

7 Miscellany

In class, she mentioned carrier frequency modulation as a motivation for ideal filters. Want to discuss IQ demodulation?