Problems for Discussion 4, 10/02/13

Compiled by Mai Le, some problems from Prof. Fessler, Prof. Yagle, and O&W

1 Inverse Z-Transform

$X(z) = \frac{2z^2-1.5z}{z^2-1.5z+0.5}$

Determine the three possible ROCs of this z-transform and the corresponding time-domain sequences (inverse z-transforms).

2 All zeros and all poles

If $X(z)$ has the form $X(z) = \sum_{k=0}^{M} b_k z^{-k} = \frac{1}{z^M} \sum_{k=0}^{M} b_k z^{M-k} = \frac{\prod_{k=1}^{M}(z-z_k)}{z^M}$, then what can we say about the poles and zeros? What can we say about the duration of $x[n]$?

If $X(z)$ has the form $X(z) = \frac{b_0}{1+\sum_{k=1}^{N} a_k z^{-k}} = \frac{b_0 z^N}{\sum_{k=0}^{N} a_k z^{N-k}} = b_0 \frac{z^N}{\prod_{k=1}^{N}(z-p_k)}$, then what can we say about the poles and zeros? What can we say about the duration of $x[n]$?

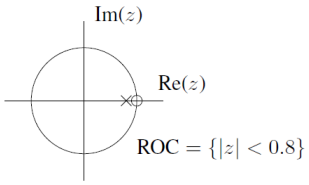
This problem was intended to be a fun theoretical exploration of the effect of poles and zeros. However, I'm cutting it from the list because it generated more confusion than understanding. I also want to emphasize that this type of analysis is not expected in the exam. If you are curious about this topic, talk to me in office hours, or wait until we discuss system response functions formally in lecture. Sorry for the mess!

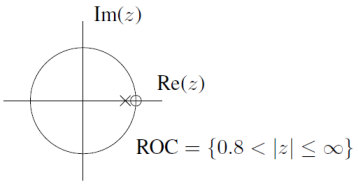
3 ROCs and Convolution

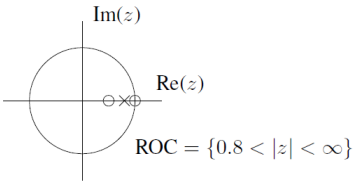
Let $h[n] = \delta[n]-\delta[n-1]$ (discrete-time differentiator), $x[n] = u[n-2]$, and $y[n] = x[n]*h[n]$. What is the ROC of $Y(z)$?

4 Causality and Pole-Zero Plots

One of the following pole-zero plots corresponds to a causal system. Which one?







5 Causality and the Z-Transform

Which of the following could be the z-transform of a causal sequence? Do not evaluate the inverse transform. You should be able to give the answer by inspection.

(a) $\frac{(1-z^{-1})^2}{(1-\frac{1}{2}z^{-1})}$

(b) $\frac{(z-1)^2}{(z-\frac{1}{2})}$

(c) $\frac{(z-\frac{1}{4})^5}{(z-\frac{1}{2})^6}$

(d) $\frac{(z-\frac{1}{4})^6}{(z-\frac{1}{2})^5}$

6 Z-Transform Deduction

We are given the following facts about a discrete-time signal $x[n]$ with z-transform $X(z)$:

1. $X(z)$ can be written as a ratio of polynomials in z.
2. $x[n]$ is real and right-sided
3. $X(z)$ has exactly 2 poles.
4. $X(z)$ has two zeros at the origin and no other zeros.
5. $X(z)$ has a pole at $z = \frac{1}{2}e^{j\pi/3}$
6. $X(1) = \frac{8}{3}$

Determine $X(z)$ and its region of convergence.

7 Inverse Z-Transform with Multiple Poles at Origin

Consider $X(z) = \frac{z^3+2z^2+3z+4}{z^2(z-1)}$. Use the "convolution in time is multiplication in z" theorem to find a right-sided $x[n]$.