University of Michigan Winter 2008

EECS451: Solution to Problem Set 4

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1. (a) $x[n] = n\cos^2(w_0 n)u[n] = n\frac{1+\cos(2w_0 n)}{2}u[n] = \frac{1}{2}\{nu[n] + n\cos(2w_0 n)u[n]\}$. Now note that the z-transform of $nu[n]$ is $\frac{z^{-1}}{(1-z^{-1})^2}$, $|z| > 1$. Further, the z-transform of $n\cos(2w_0 n)u[n]$ is equal to $\frac{1}{2}\left\{\frac{e^{j2w_0}z^{-1}}{(1-e^{j2w_0}z^{-1})^2} + \frac{e^{-j2w_0}z^{-1}}{(1-e^{-j2w_0}z^{-1})^2}\right\}$ with $|z| > 1$. Hence the z-transform of $x[n]$ is given by

$$\frac{1}{4}\left\{\frac{2z^{-1}}{(1-z^{-1})^2} + \frac{e^{j2w_0}z^{-1}}{(1-e^{j2w_0}z^{-1})^2} + \frac{e^{-j2w_0}z^{-1}}{(1-e^{-j2w_0}z^{-1})^2}\right\}, |z| > 1$$

   (b) Define $w_0 = \pi/3$ and $\phi = 5\pi/6$. We have

$$x[n] = \sin\left(w_0[n-3] + \phi\right)u[n-3].$$

   Using the z-transform of $\sin(w_0 n + \phi)u[n]$, we get the z-transform of $x[n]$ as

$$z^{-3}\left(\frac{e^{j\phi}}{2j(1-e^{jw_0}z^{-1})} - \frac{e^{-j\phi}}{2j(1-e^{-jw_0}z^{-1})}\right).$$

   ROC=$\{z : |z| > 1\}$.

   (c) Now let us do a change of variables: $l = n - k$. Hence when $k = n$, we get $l = 0$ and when $k = -\infty$, we get $l = \infty$. Thus we can rewrite $x[n]$ as

$$x[n] = u[n]\sum_{l=0}^{\infty} 5^{n-l} = 5^n u[n]\sum_{l=0}^{\infty}\frac{1}{5^l} = 5^n u[n]\frac{1}{1-\frac{1}{5}} = \frac{5}{4}5^n u[n].$$

   Hence its z-transform is given by

$$\frac{5}{4}\frac{1}{(1-5z^{-1})},$$

   and ROC=$\{z : |z| > 5\}$.

   (d) The z-transform is given by

$$-z\frac{d}{dz}\frac{z^{-1}}{(1-z^{-1})^2} = -z\frac{(1-z^{-1})^2(-z^{-2}) - z^{-1}2(1-z^{-1})(-1)(-z^{-2})}{(1-z^{-1})^4}.$$

   Hence the z-transform is given by

$$\frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3},$$

   ROC=$\{z : |z| > 1\}$.

(e) $x[n] = v[-n]$, where $v[n] = \frac{1}{5^n}u[n]$. We know that

$$V(z) = \frac{1}{1 - 0.2z^{-1}}$$

with ROC=$\{z : |z| > 0.2\}$. Hence the z-transform of $x[n]$ is given by

$$X(z) = V(z^{-1}) = \frac{1}{1 - 0.2z},$$

with ROC=$\{z : |z| < 5\}$.

(f) Using convolution property of z-transforms, we can write the z-transform of $x[n]$ as follows:

$$X(z) = \frac{1}{(1 - 4z^{-1})(1 - z^{-1})}\left(\frac{1}{(1 - 3z^{-1})} - \frac{z^{-1}}{1 - 3z^{-1}}\right) = \frac{1}{(1 - 4z^{-1})(1 - z^{-1})}\frac{(1 - z^{-1})}{(1 - 3z^{-1})}$$

$$= \frac{1}{(1 - 4z^{-1})(1 - 3z^{-1})}$$

with ROC=$\{z : |z| > 3\}$.

2. (a) Looking at the expression for $y_1[n]$ we can infer that $y_1[n] = y[n] * h[n]$ where $h[n] = \delta[n] + \delta[n-1]$, and

$$y[n] = \left\{ \begin{array}{cc} x[n/2] & \text{if } n \text{ even} \\ 0 & \text{if } n \text{ odd} \end{array} \right\}$$

Hence

$$Y_1(z) = Y(z)(1 + z^{-1}) = X(z^2)(1 + z^{-1}).$$

(b) By noting that $y_2[n] = 0.5x[n](1 + (-1)^n)$, we get

$$Y_2(z) = \frac{1}{2}\left(X(z) + X(-z)\right).$$