Homework 8

Solution

Contents

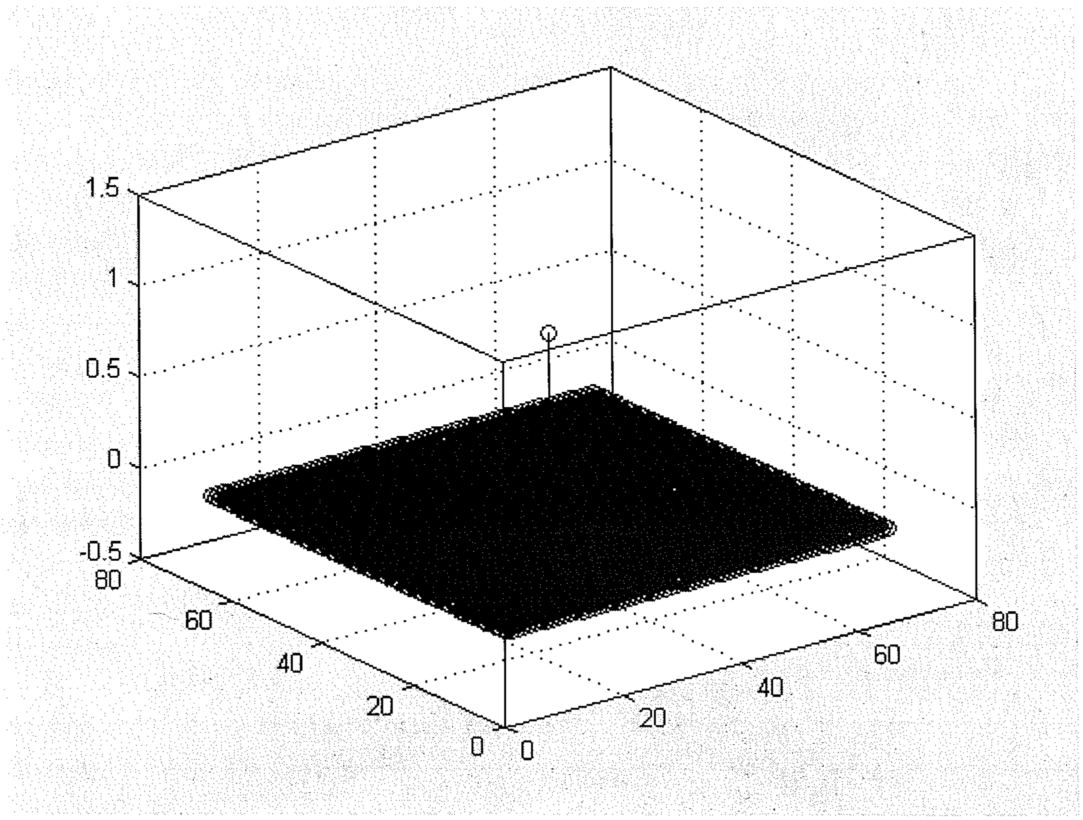
Homework 8 Problem 1

```
clear
close all
clc

load Hmwk8.mat
```

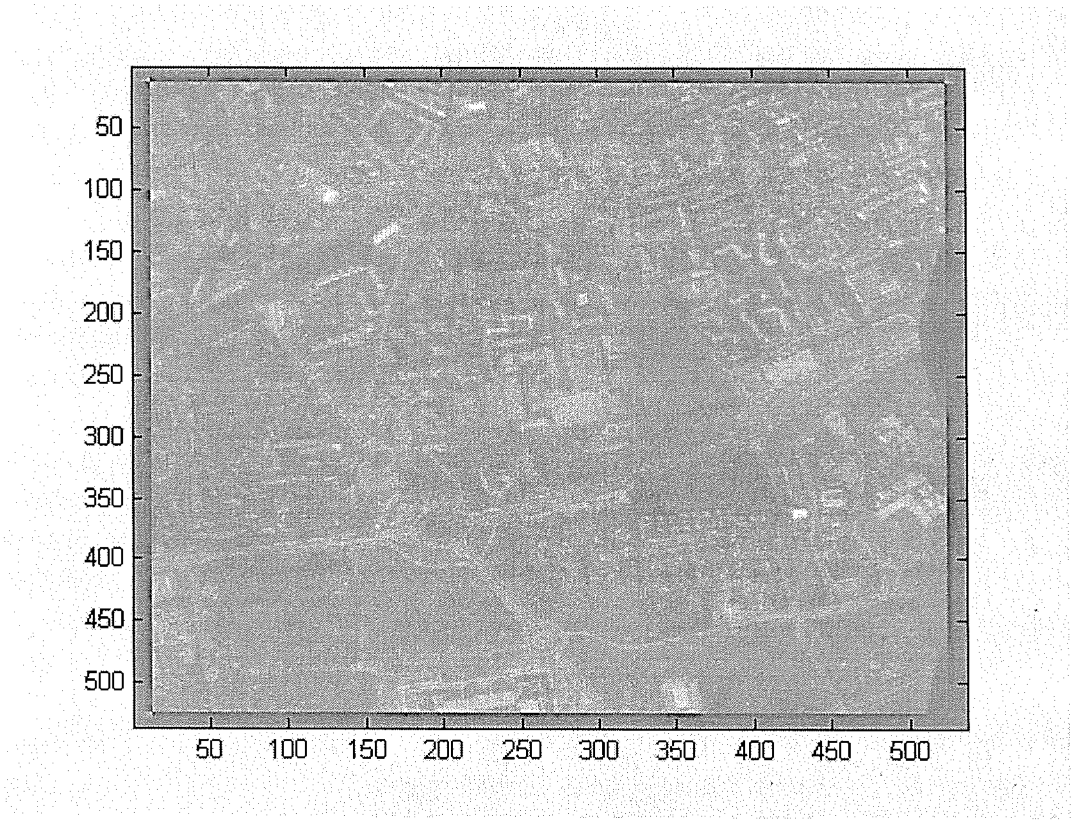
Problem 1a)

```
stem3(conv2(invblur,blur))
%check to see that this is an impulse by looking at stem plot, or any other
%method
```



Problem 1b)

```
i_nimes_b = conv2(invblur,nimes_b);
figure
imagesc(i_nimes_b)
colormap(gray)
```



Problem 1c)

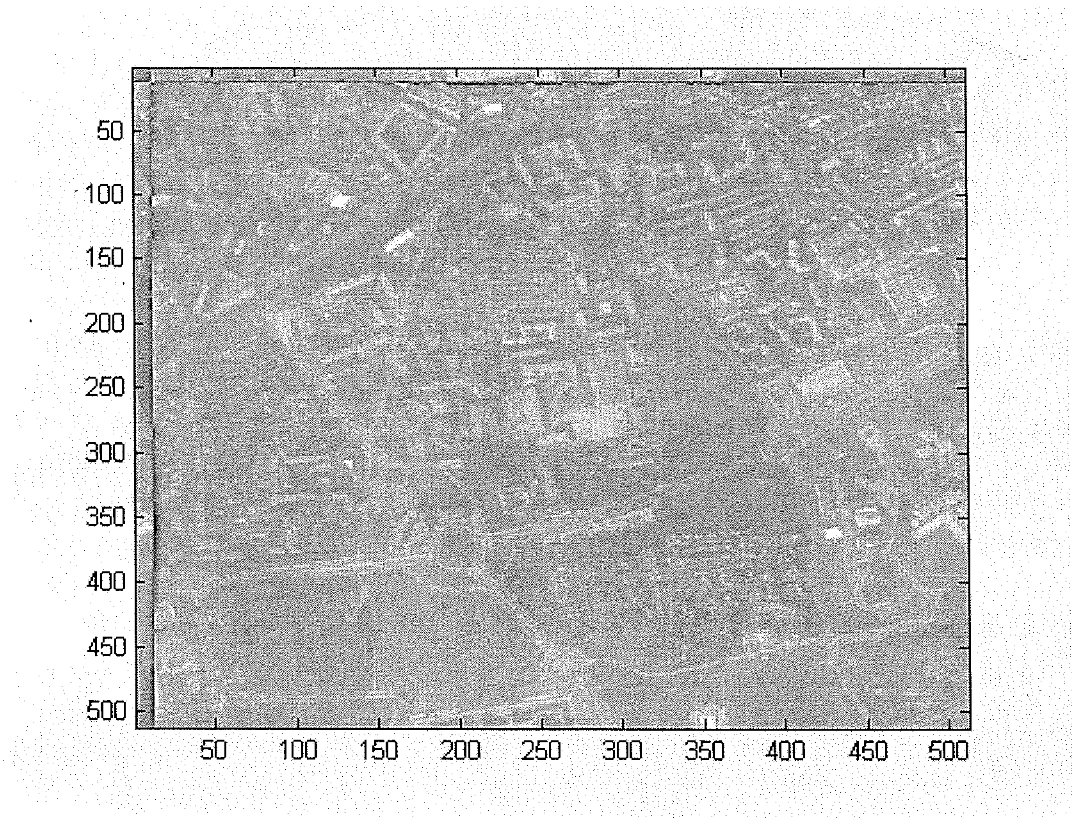
```
[n m] = size(nimes_b)
%zero pad invblur to make it same size as nimes
i_nimes_b_dft = ifft2(fft2(nimes_b).*fft2(invblur,n,m));
figure
imagesc(real(i_nimes_b_dft))
colormap(gray)


n =

   512


m =

   512
```

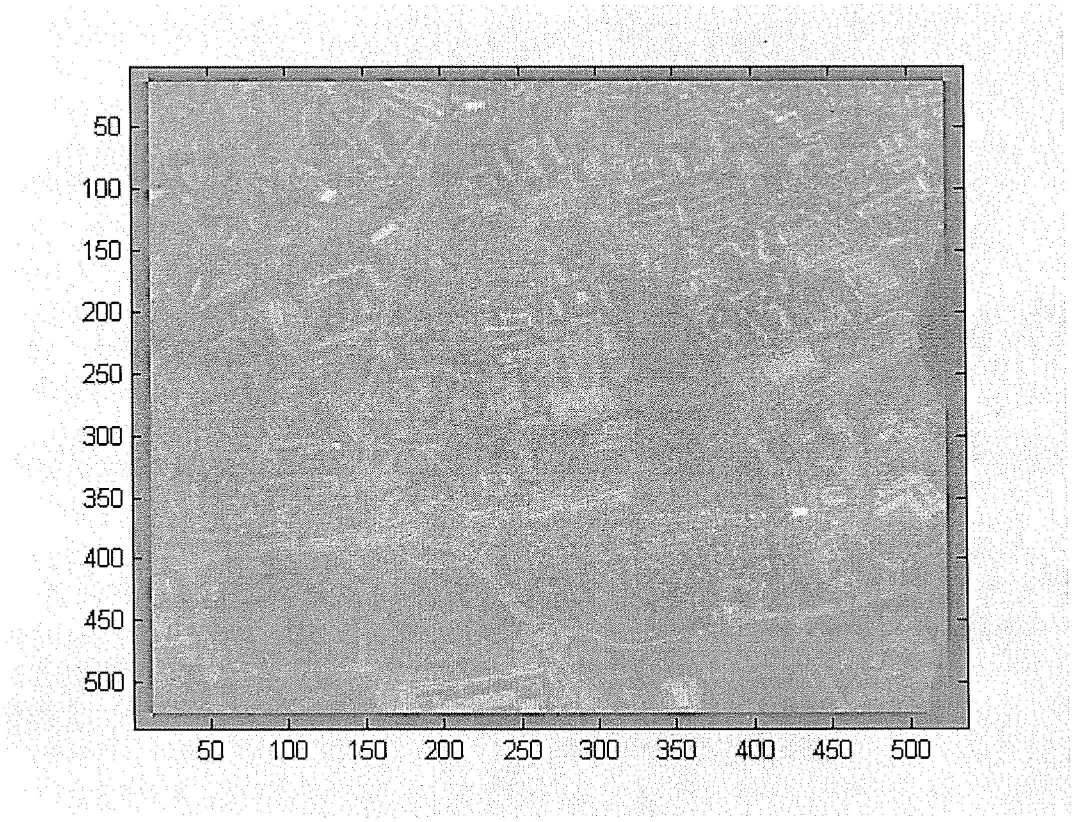


Problem 1d)

We have to zero pad so that each dimension is N+L-1 (where N is size of nimes, L is size of filter.

```
[m1,n1] = size(nimes_b);
[m2,n2] = size(invblur);

i_nimes_b_dft = ifft2(fft2(nimes_b,m1+m2-1,n1+n2-1).*fft2(invblur,m1+m2-1,n1+n2-1));
figure
imagesc(real(i_nimes_b_dft))
colormap(gray)
```



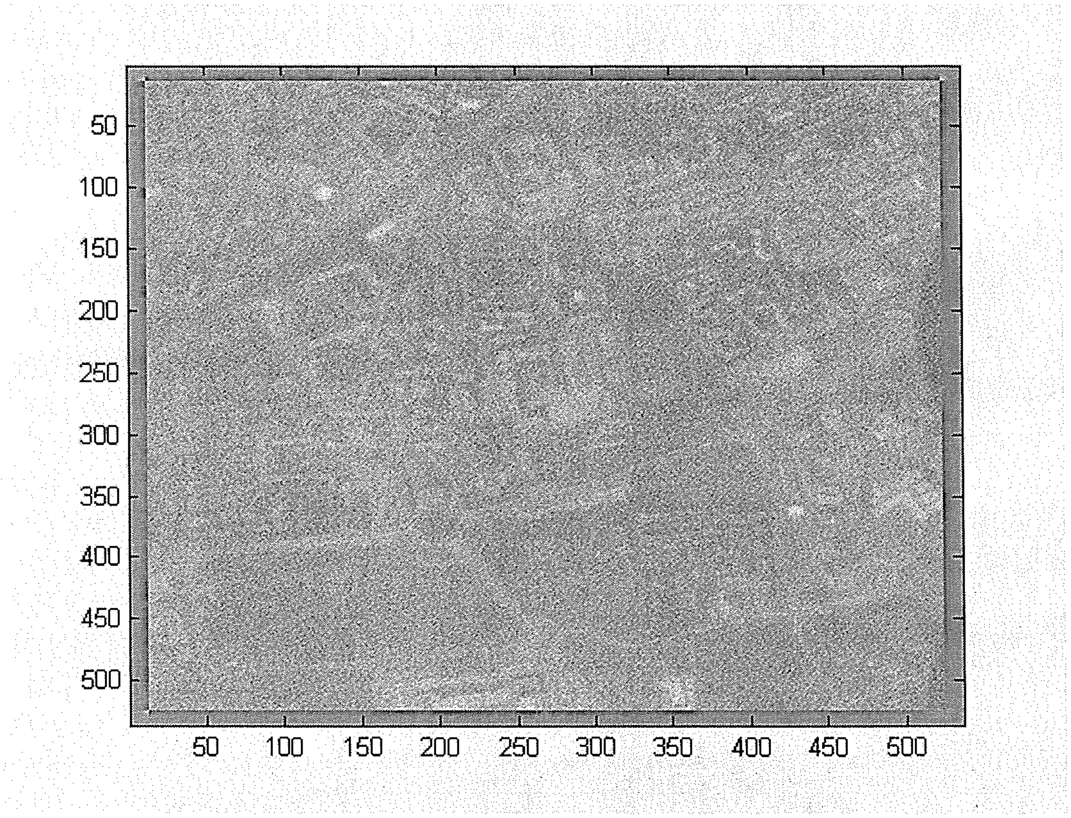
Problem 1e)

```
[m1,n1] = size(nimes_bn);
[m2,n2] = size(invblur);

i_nimes_bn_dft = ifft2(fft2(nimes_bn,m1+m2-1,n1+n2-1).*fft2(invblur,m1+m2-1,n1+n2-1));
figure
imagesc(real(i_nimes_bn_dft))
colormap(gray)

%Do not grade second question of 1e)
%- you could do any number of things - one
%be use the deconvwnr (deconvolve using weiner filter) built-in matlab
%command

J = deconvwnr(nimes_bn, blur);
figure
imagesc(J)
colormap(gray)
figure
imagesc(nimes_bn)
colormap(gray)
```


50
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250
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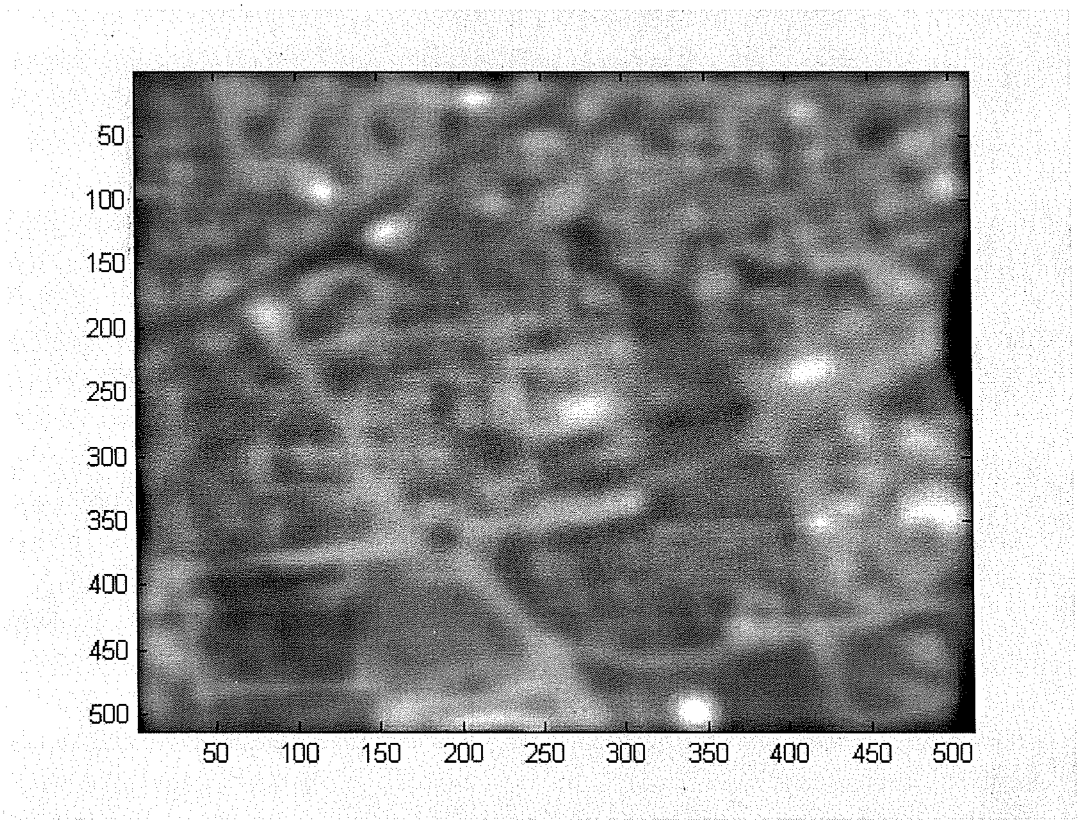
400
450
500
50 100 150 200 250 300 350 400 450 500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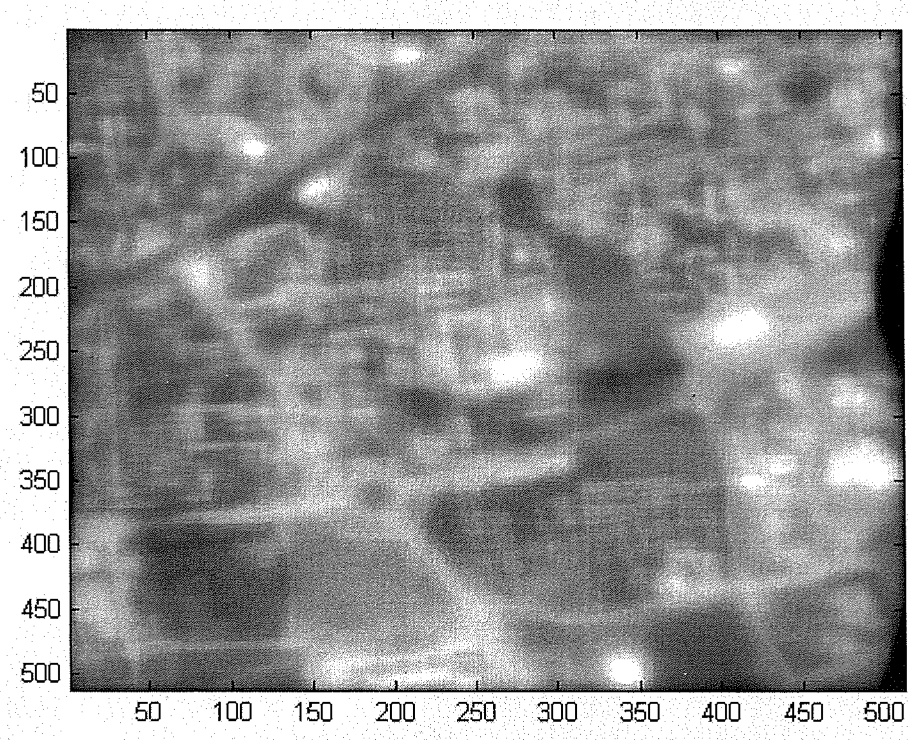
50 100 150 200 250 300 350 400 450 500



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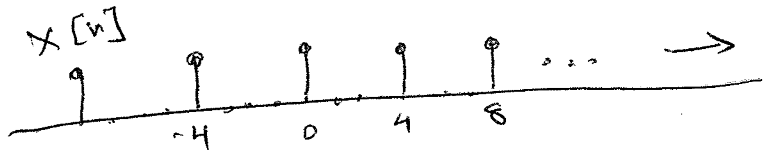
3.25

a) $\displaystyle\sum_{k=-\infty}^{\infty} \delta[n-4k]$



$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta[n-4k] z^{-n}$$

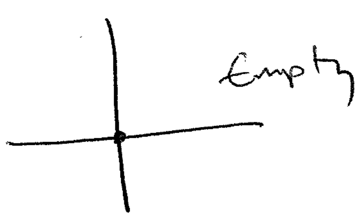
$$= \sum_{n=-\infty}^{\infty} z^{-n} \underbrace{\sum_{k=-\infty}^{\infty} \delta[n-4k]}$$

0 unless n is multiple of 4.

$$= \sum_{n=-\infty}^{\infty} z^{-4n}$$

<u>Does NOT converge</u> for $z \neq 0$.

ROC



3.25 b

$$\frac{1}{2}\left[e^{j\pi n} + \cos\left(\frac{\pi}{2}n\right) + \underbrace{\sin\left(\frac{\pi}{2} + 2\pi n\right)}_{= \cos(-2\pi n) = 1 \text{ all } n}\right] u[n]$$

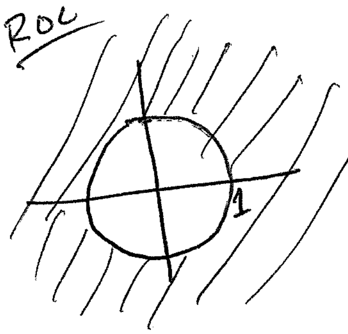
Use tables

$$X(z) = \frac{1}{2}\left[\frac{1}{1-e^{j\pi}z^{-1}} + \frac{1-\cos\left(\frac{\pi}{2}\right)z^{-1}}{1-2\cos(\pi/2)z^{-1}+z^{-2}} + \frac{1}{1-z^{-1}}\right]$$

$|z|>1$ $\quad$ $|z|>1$ $\quad$ $|z|>1$

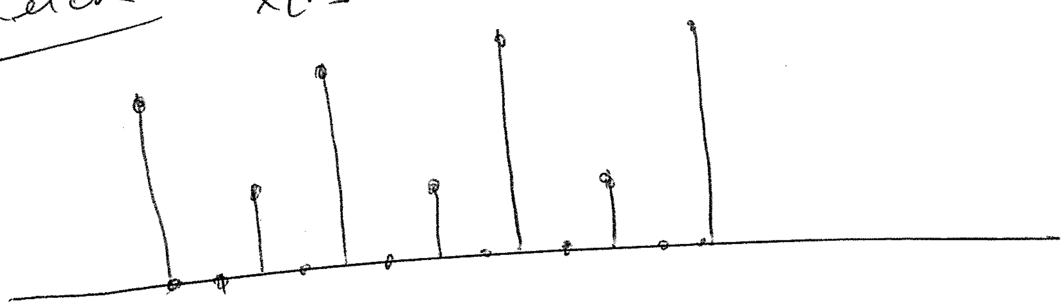
$$= \frac{1}{2}\left[\frac{1}{1+z^{-1}} + \frac{1}{1+z^{-2}} + \frac{1}{1-z^{-1}}\right]$$

ROC



$|z|>1$

Sketch $x[n]$



3.31

a) Long division:

$$X(z) = \frac{1 - \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}}, \quad x[n] \text{ right sided}$$

$$\begin{array}{r l}
 & \quad -1 \\
-\frac{1}{3}z^{-1} + 1 & \overline{\big)\ \frac{1}{3}z^{-1} + 1} \\
 & \ \ -\left(\frac{1}{3}z^{-1} - 1\right) \\
\cline{2-2}
 & \qquad\quad 2
\end{array}$$

$$X(z) = -1 + \frac{2}{1 + \frac{1}{3}z^{-1}}$$

Check:

$$\frac{2}{1 + \frac{1}{3}z^{-1}} - \frac{1 + \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}} = \frac{1 - \frac{1}{3}z^{-1}}{1 + \frac{1}{3}z^{-1}}$$

$$x[n] = -\delta[n] + 2\left(-\frac{1}{3}\right)^n u[n]$$

3.31 b

Partial Fraction:

$$X(z) = \frac{3}{z - 1/4 - 1/8 z^{-1}} \qquad x[n] \text{ stable}$$

$$= \frac{3z^{-1}}{1 - 1/4 z^{-1} - 1/8 z^{-2}}$$

$$= \frac{3z^{-1}}{(1 - \frac{1}{2}z^{-1})(1 + 1/4 z^{-1})}$$

$$= \frac{A}{(1 - 1/2 z^{-1})} + \frac{B}{(1 + 1/4 z^{-1})}$$

$$\Rightarrow \quad A(1 + 1/4 z^{-1}) + B(1 - 1/2 z^{-1}) = 3z^{-1}$$

$$A + B + \frac{1}{4}Az^{-1} - \frac{1}{2}Bz^{-1} = 3z^{-1}$$

$$A + B = 0$$

$$1/4 A - 1/2 B = 3$$

$$A = 4$$

$$B = -4$$

$$\Rightarrow X(z) = \frac{4}{(1-1/2\, z^{-1})} - \frac{4}{(1+1/4\, z^{-1})}$$

$$x[n] = 4 \cdot \frac{1}{2}^{n} u[n] - 4\,(-1/4)^{n}\, u[n]$$

Since $x[n]$ is stable, must be right-sided

3.31 c

$$X(z) = \ln(1-4z) \qquad |z| < 1/4$$

Power Series

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n} \qquad \text{for } |x| < 1$$

So

$$X(z) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{(-4z)^n}{n} \qquad |z| < 1/4$$

$$\Rightarrow |-4z| < 1$$

$$\checkmark \; |z| < 1/4$$

Let $k = -n$

$$\sum_{k=-\infty}^{-1} (-1)^{-k+1} \frac{(-4)^{-k} z^{-k}}{-k}$$

$$\rightarrow \quad x[n] = \frac{(-1)^{-n+1} (-4)^{-n}}{-n} \cdot u[-n-1]$$

3.31 d

$$X(z) = \frac{1}{1 - \frac{1}{3} z^{-3}} \qquad |z| > (3)^{-1/3}$$

Geometric Series

$$\sum_{k=0}^{\infty} a^k = \frac{1}{1-a} \qquad |a| < 1$$

$$\frac{1}{1 - \frac{1}{3} z^{-3}} = \sum_{k=0}^{\infty} \left(\frac{1}{3} z^{-3}\right)^k \quad \text{for} \quad \left|\tfrac{1}{3} z^{-3}\right| < 1$$

$$|z^{-3}| < 3$$

$$z^3 > 3$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k z^{-3k} \qquad |z| > 3^{1/3}$$

$$= \left(\frac{1}{3}\right)^0 z^{-3\cdot 0} + \left(\frac{1}{3}\right)^1 z^{-3} + \left(\frac{1}{3}\right)^2 z^{-6} + \dots$$

So, substitute $n = 3k$ and change index

$$= \sum_{n=0,3,6\dots}^{\infty} \left(\frac{1}{3}\right)^{n/3} z^{-n}$$

by inspection

$$x[n] = \begin{cases} \left(\frac{1}{3}\right)^{n/3} & n = 0, 3, 6, 9 \dots \\ 0 & \text{else} \end{cases}$$