

1.

(10): Find the N -point DFT of $x[n] = a^n u[n-1]$ for $|a| \neq 1$.

(10): Suppose $x[n]$ is time-limited to $n = 0, 1, 2, 3$ and the 8-point DFT of $x[n]$ is given by

$$\{\underline{5}, 3 - j\sqrt{2}, 3, 3 - j\sqrt{2}, 1, 3 + j\sqrt{2}, 3, 3 + j\sqrt{2}\}.$$

Find the 4-point DFT of $x[n]$. (Be sure to explain your method.)

2.

(10): A signal $x[n]$ has DTFT $X(\omega) = 4e^{-j13\omega} + 3e^{j2\omega} + 2e^{-j11\omega} + 7$.

Find the 11-point inverse DFT of $\left\{ X(\omega)|_{\omega=2\pi k/11}, k = 0, \dots, 10 \right\}$.

(10): Suppose $X_0 = 0, X_1 = 8, X_2 = 0$, and $X_3 = 8$ is the 4-point DFT of a signal $x[n]$ that is time-limited to $n = 0, 1, 2, 3$. Find $X(\omega)$.

3.

(10): The CT signal $x_a(t) = \sin(2\pi F_0 t)$ with $F_0 = 250\text{Hz}$ is sampled according to $x[n] = x_a(nT)$ with $T = 1\text{msec}$. Find X_k , the 4-point DFT of $x[n]$.

(10): Suppose that the above DT signal $x[n]$ is filtered by a LTI system with impulse response $h[n] = \delta[n] - \delta[n-2]$ to form a DT signal $y[n]$.

Find Y_k , the 4-point DFT of $y[n]$.

(10): Find the fastest sampling rate for which $X_k = 0$.

4.

(10): Let X_k denote the N -point DFT of a signal $x[n]$. Suppose you upsample the DFT values as follows:

$$Y_k = \begin{cases} X_{k/2}, & k \text{ even} \\ 0, & k \text{ odd.} \end{cases}$$

Relate $y[n]$, the $2N$ -point inverse DFT of the Y_k 's, to the values $x[n]$ of the original signal.

(10): Continuing the previous problem, suppose $x[n] = 8n u[n]$ and $N = 4$.

Find $y[n]$ for $n = -2, 1, 3, 6$, and 8 .

5.

(10): **[Undergraduates only]**

Write a short MATLAB m-file that uses MATLAB's `filter` command to compute the *exact* convolution between the following two signals:

$$x_1[n] = \begin{cases} \cos(0.3n), & n = 0, \dots, 19 \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad x_2[n] = n(1/4)^n u[n].$$

Your MATLAB program should compute the result for $n = 0, 1, \dots, 40$.

(10): **[Graduate students only]**

As part of a MATLAB session, a user types:

```
X = [7 0 1 2 0 0 5 0 0 4];
```

```
x = ifft(ifft(X))
```

Determine what the MATLAB output is.