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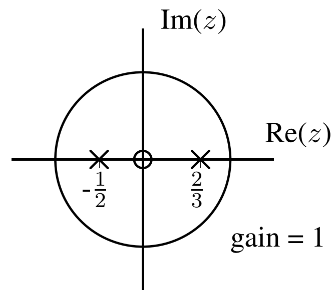
1. (40 points)

Consider the causal LTI system described by the following pole-zero plot.



- Determine the impulse response of the *inverse system* corresponding to this system.
- Suppose the input to the system described by the above pole-zero plot is a signal that is zero for $n \geq 20$. Determine the output signal for $n > 21$ as precisely as you can given only this information.
  Hint: there may be some undetermined constants in your answer.
- Determine the steady-state response of this system to the input signal $x[n] = \begin{cases} 8, & n \geq 0,\ n \text{ even} \\ 2, & n \geq 0,\ n \text{ odd} \\ 0, & n < 0. \end{cases}$

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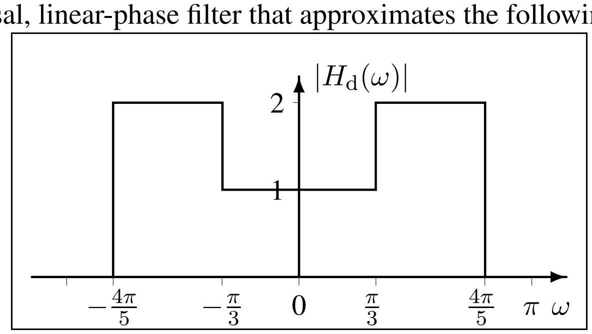
2. (15 points)
An engineer is designing a causal, linear-phase filter that approximates the following magnitude response.



She uses the following MATLAB commands.

```
M = 31; n = [0:(M-1)]';
hd = ??
h = hd .* hann(M);
```

Determine what the “??” line should be above, using as correct MATLAB syntax as you can.

3. (20 points)
An analog signal $x_{\mathrm{a}}(t)$ with spectrum $X_{\mathrm{a}}(F) = |F|$ is processed by the following system.

$$x_{\mathrm{a}}(t) \rightarrow \boxed{\begin{array}{c}\text{Ideal}\\ \text{anti-alias}\\ H_{\mathrm{a}}(F)\end{array}} \xrightarrow{x_2(t)} \boxed{\begin{array}{c}\text{Sample}\\ \text{at rate: } F_{\mathrm{s}}\end{array}} \xrightarrow{x_3[n]} \boxed{H(\omega)} \xrightarrow{x_4[n]} \boxed{\downarrow 2} \xrightarrow{x_5[n]} \boxed{H(\omega)} \xrightarrow{x_6[n]} \boxed{\downarrow 2} \rightarrow y[n]$$

The system component $\boxed{\downarrow 2}$ denotes down sampling by discarding the odd samples.
The filter $H(\omega)$ is an ideal lowpass filter with a cutoff frequency $\omega_{\mathrm{c}} = \pi/2$.
Carefully sketch the spectrum of the output signal $y[n]$, or find an expression for its spectrum.
Strategy: to maximize partial credit, show the spectra of the intermediate signals.

4. (25 points)
A signal $x[n]$ is processed by upsampling, filtering, and downsampling as follows:

$$x[n] \rightarrow \boxed{\text{upsample}} \rightarrow v[n] \rightarrow \boxed{H_1(\omega)} \rightarrow w[n] \rightarrow \boxed{\text{downsample}} \rightarrow y[n]\,.$$

The upsampler inserts a zero after each sample. The downsampler discards the odd samples.

- Find the relationship between the spectrum of the input signal and the spectrum of the output signal.
- The entire system above can be simplified into a single LTI filter, *i.e.*, $x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$. Suppose that $H_1(\omega)$ above denotes the ideal lowpass filter with cutoff frequency $\omega_{\mathrm{c}} = \pi/2$.
  Determine the impulse response $h[n]$ of the equivalent filter. Hint. First find the frequency response $H(\omega)$.