Homework #9, EECS 451, W04. Due **Fri. Apr. 2**, in class

Skill Problems

1. [B 30] Text 8.19ab. Concept(s): ***FIR min-max filter design.*** 20pts for (a), 10 for (b).
   For (a), plot $|H(\omega)|$ on a dB scale and report the actual passband ripple and stopband attenuation.

2. [B 25] Concept(s): ***DFT***
   Consider the signal $x[n] = \{\underline{3}, 0, 1, 0, 0, 0, 1, 0\}$.
   (a) [0] Find the DTFT of $x[n]$.
   (b) [0] Find the 8-point DFT of $x[n]$.
   (c) [0] Find the 16-point DFT of $x[n]$.
   (d) [0] Find the 16-point inverse DFT from your $X[k]$'s in the previous part.
   (e) [5] Find the 4-point DFT of $x[n]$.
   (f) [5] Compute the 4-point inverse DFT from your $X[k]$'s in the previous part.
   (g) [0] Find $Y[k] = X(\omega_k)$ for $\omega_k = \frac{2\pi}{4}k$ for $k = 0, \ldots, 3$.
   (h) [10] Compute the 4-point inverse DFT from your $Y[k]$'s in the previous part. Call it $y[n]$.
   (i) [5] Relate your answers in (f) and (h) to $x[n]$.

3. [B 0] Text 5.1. Concept(s): ***DFT symmetry properties***

4. [B 10] Concept(s): ***DFT modulation property.***
   Let $x[n] \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} X[k]$. Determine the N-point DFTs of the following signals in terms of $X[k]$.
   (a) [5] $x_c[n] = x[n] \cos\left(\frac{2\pi}{N}k_0 n\right)$
   (b) [5] $x_s[n] = x[n] \sin\left(\frac{2\pi}{N}k_0 n\right)$

5. [B 30] Concept(s): ***DFT implementation***
   This problem explores a simple "brute force" implementation of the DFT. (Later we will use the more efficient FFT.)
   (a) [5] Create a MATLAB function named `dft.m` that computes and returns the N-point DFT of any input sequence $x[0], \ldots, x[N-1]$ passed as N-element vector `xv`. Print your m-file and circle the part(s) you changed from `dtfs1.m` (to make grading easier). Hint: modify of your `dtfs1.m` slightly.
   (b) [10] Use your `dft` function to compute the 8-point DFT of the ramp signal $x[n] = \{\underline{6}, 5, \ldots, 1\}$.
   Plot the magnitude and phase of the DFT values $\{X[k]\}_{k=0}^{N-1}$.
   Hint: your `dft` routine and MATLAB's `fft` routine should produce essentially identical results.
   (c) [5] Create a MATLAB function named `idft.m` that computes and returns the N-point inverse DFT of any input sequence $\{X[k]\}_{k=0}^{N-1}$ passed as N-element vector `Xv`. As always, print out your m-file.
   Hint: this is a minor modification of your `dft.m` function. Caution: $X[k]$ is `Xv(k+1)` in MATLAB.
   Hint: compare your results on any test input vector to the output of MATLAB's `ifft` routine.
   (d) [0] Take the DFT vector of values `Xv` you computed in part (b) and execute the following MATLAB command: `Yv = ([Xv(1) Xv(8:-1:2)] + Xv) / 2`. Now use your `idft` routine to compute the 8-point inverse DFT $y[n]$ from `Yv`. Plot the real and imaginary parts of $y[n]$.
   (e) [10] Explain the relationship between $y[n]$ and $x[n]$ mathematically.
   (f) [0] Modify your `dft.m` function to accept a second optional argument that allows the user to specify a $N \geq L$, where $L =$ `length(xv)`. (The `fft` routine accepts such a useful argument.)

Mastery Problems

6. [B 10] Concept(s): ***Odd zeroing (precursor to downsampling)***
Suppose you compute the N-point DFT of a signal $x[n]$ and get $\{X[k]\}_{k=0}^{N-1}$. Now define $Y[k] = X[k]$ for even k and $Y[k] = 0$ for odd k. Then you compute the N-point inverse DFT to get a signal $y[n]$.
Relate $y[n]$ to $x[n]$. Assume N is even.

7. [B 10] Concept(s): ***Effect of*** N

Let $x[n]$ be a N-periodic signal. Consider the following DFTs: $x[n] \overset{\text{DFT}}{\underset{N}{\longleftrightarrow}} X_1[k]$, and $x[n] \overset{\text{DFT}}{\underset{3N}{\longleftrightarrow}} X_3[k]$.

(a) [10] Determine the relationship between $X_1[k]$ and $X_3[k]$.
Hints. Write the analysis formula for $X_3[k]$ and split the sum into three N-point terms. Or do (b) first.
You should arrive at an expression involving $1 + \mathrm{e}^{-\jmath \frac{2\pi}{3} k} + ?$, and this simplifies a lot.

(b) [0] Find the 2-point and 6-point DFTs of the signal $x[n] = \frac{3}{2} + \frac{1}{2}\cos(\pi n) = \{\underline{2}, 1\}_2$.

(c) [0] Verify that your relation in part (a) holds for the signal in part (b).