Problems for Discussion 6, 10/30/13

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1 CTFT of a Dirac Comb

Derive the following CTFT pair:

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) \leftrightarrow S(\Omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\Omega - k\Omega_s),\ \Omega_s = \frac{2\pi}{T}$$

2 Noble Identities

Downsampling by M followed by a filter $H(\omega)$ is equivalent to what, followed by downsampling?

$$(\downarrow M)H(\omega) = \_\_\_\_\_(\downarrow M)$$

Filtering, then upsampling by L is equivalent to upsampling by L and then what?

$$H(\omega)(\uparrow L) = (\uparrow L)\_\_\_\_\_$$

3 Circular Convolution

Let $x_1[n] = \begin{cases} 1, & 0 \le n \le 19 \\ 0, & otherwise \end{cases}$ and $x_2[n] = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & otherwise \end{cases}$.

Determine and sketch (a) the linear convolution $x_1[n] * x_2[n]$, (b) the 20-point circular convolution, $x_1[n] \circledast x_2[n]$ and (c) the 25-point circular convolution $x_1[n] \circledast x_2[n]$.

4 DFS and Fundamental Period

O&S 8.2

Suppose $\tilde{x}[n]$ is a periodic sequence with period N. Then $\tilde{x}[n]$ is also periodic with period $3N$. Let $\tilde{X}[k]$ denote the DFS coefficients of $\tilde{x}[n]$ considered as a periodic sequence with period N and let $\tilde{X}_3[k]$ denote the DFS coefficients of $\tilde{x}[n]$ as considered as a periodic sequence with period $3N$. Express $\tilde{X}_3[k]$ in terms of $\tilde{X}[k]$.

Verify your result for $x[n] = \{\underline{1}, 2, 1, 2, ...\}$, with $N = 2$.