Problems for Discussion 2, 09/18/13

Compiled by Mai Le, some problems from Prof. Fessler

1 Discrete-domain Basis Sequences

Let $x[n] = \{\underline{1}, 1, 2, 3, 5\}$. Represent $x[n]$ in each of the following bases.

Note: You won't be asked to do anything like this on the homework or exams, but hopefully it will help you solidify your understanding of bases from lecture.

Unit Step Sequences

Recall $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$. Let $\mathcal{S}_u = \{u[n - n_0] | n_0 \in \mathbb{Z}\}$ (unit steps shifted by any integer n_0) be your basis sequences. Represent $x[n]$ in $\mathcal{S}_u$.

In other words, show you can write $x[n] = \sum_{k=-\infty}^{\infty} c_k u[n-k]$ by finding the values for c_k.

Three-Tap Rectangles

Let $r[n] = \{\underline{1}, 1, 1\}$ and $\mathcal{S}_r = \{r[n - n_0] | n_0 \in \mathbb{Z}\}$. Represent $x[n]$ in $\mathcal{S}_r$.

Three-Tap Triangle

Let $t[n] = \{\underline{1}, 2, 1\}$ and $\mathcal{S}_t = \{t[n - n_0] | n_0 \in \mathbb{Z}\}$. Represent $x[n]$ in $\mathcal{S}_t$.

2 Properties of Even More Systems

Are the following linear? time-invariant? causal? stable?

(a)

$y[n] = x^2[n + 1]$

(b)

$$y[n] = \begin{cases} x[-n], & n < 0 \\ x[n], & n \geq 0 \end{cases}$$

(c)

$y[n] = \sum_{k=-\infty}^{\infty} x[n-k]p[k]$ where $p[n] = \{-10, \ldots, -1, \underline{0}, 1, \ldots, 10\}$.

3 Proof of Parseval's Theorem

Prove that $\sum_{n=-\infty}^{\infty} |x[n]|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$.

4 Orthogonality of Complex Exponentials

(a)

Prove $\frac{1}{N}\sum\limits_{n=0}^{N-1} e^{j\frac{2\pi}{N}kn} = \begin{cases} 1, & k = 0, \pm N, \pm 2N, ... \\ 0, & otherwise \end{cases}$.

(b)

Show that harmonically related complex exponential signals $s_k[n] = e^{j\frac{2\pi}{N}kn}$ are orthogonal over any interval of length N. I.e. $\sum\limits_{n=n_0}^{n_0+N-1} s_k[n]s_l^*[n] = 0$ if $k \neq l$.

5 Symmetry and the DTFT

(a)

Let $y[n] = x^*[-n]$. What is $Y(\omega)$ in terms of $X(\omega)$?

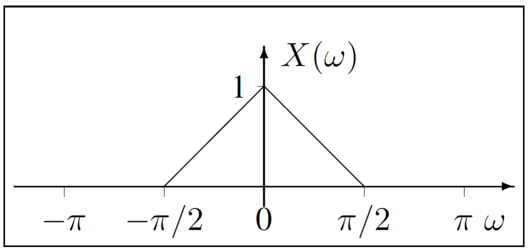
(b)

If $x[n]$ is real, what can we say about the symmetry of $X(\omega)$?

6 Inverse DTFT

(a) Inverse DTFT of a Triangle

Find the signal $x[n]$ that has the following spectrum. Hint: integration by parts!



(b)

Given: $h[n] = \delta[n-1] + \delta[n+1]$ has spectrum $H(\omega) = 1e^{-j\omega} + 1e^{j\omega} = 2cos(\omega)$. (You should be able to verify this.)

Find the signal $y[n]$ that has the spectrum $Y(\omega) = cos^2(\omega)$ using information you learned in part a. (Don't do the integration!)

7 Inverse DTFT

Find the signal having the following spectrum.