

1. (20 points)

Consider the following signal $x[n] = \begin{cases} (1/3)^n, & n = 2, 6, 10, 14, 18, \dots \\ 0, & \text{otherwise.} \end{cases}$

- Determine the discrete-time Fourier transform of $x[n]$.
- Determine the 6-point discrete Fourier transform of $x[n]$.

2. (10 points)

A nonzero, finite-length signal $x[n]$ is the input to a stable LTI system with frequency response $H(\omega) = \frac{3e^{-j2\omega}}{3 + 2e^{-j\omega}}$. Determine the transient response portion of the output signal as precisely as you can given only this information.

3. (10 points)

Explain briefly why the following statement is true or false.

“The sum of any two causal, symmetric, FIR, linear-phase filters of the same length is a linear-phase filter.”

4. (10 points)

Determine the 8-point DFT of the signal $y[n]$ defined by $y[n] = \sum_{l=-\infty}^{\infty} x[n - 8l]$, where $x[n] = (\frac{1}{3})^n u[n]$.

5. (10 points)

The 50-point DFT of a signal $x[n]$ is given by $X[k] = \begin{cases} 10, & k = 0, \dots, 20 \\ 0, & k = 21, \dots, 29 \\ 10 & k = 30, \dots, 49. \end{cases}$

The signal $y[n]$ is defined by up-sampling $x[n]$ by two: $y[n] = \begin{cases} x[n/2], & n \text{ even} \\ 0, & n \text{ odd.} \end{cases}$

Sketch the 100-point DFT of $y[n]$.

6. (10 points)

Determine the output of the following MATLAB command:

```
fft(fft([3 4 5 6]))
```

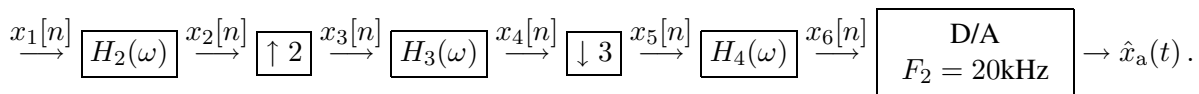
7. (10 points)

Determine the output of the following MATLAB command:

```
ifft(fft([1 2 3 0 0 0]) .* fft([1 0 1 0 0 0]))
```

8. (10 points)

A signal $x_a(t)$ was sampled at rate $F_1 = 30\text{kHz}$ and its samples $x_1[n]$ were stored. Later it is desired to recover this signal using an D/A converter that works at the sampling rate $F_2 = 20\text{kHz}$. The following system has been proposed for solving this sampling rate conversion problem.



As usual, $\boxed{\uparrow 2}$ denotes upsampling by zero insertion and $\boxed{\downarrow 3}$ denotes down sampling by discarding.

Specify the magnitude responses of the three filters $H_2(\omega)$, $H_3(\omega)$, and $H_4(\omega)$ so that the final output signal will be as close to the original signal as possible.

Only the shape of the filter will be graded; not its gain. If a filter is not needed, say so.

9. (10 points)

The following alternative system for solving the preceding rate conversion problem has also been proposed.

$$x_1[n] \xrightarrow{\quad} \boxed{H_2(\omega)} \xrightarrow{\quad} x_2[n] \xrightarrow{\quad} \boxed{\downarrow 3} \xrightarrow{\quad} x_3[n] \xrightarrow{\quad} \boxed{H_3(\omega)} \xrightarrow{\quad} x_4[n] \xrightarrow{\quad} \boxed{\uparrow 2} \xrightarrow{\quad} x_5[n] \xrightarrow{\quad} \boxed{H_4(\omega)} \xrightarrow{\quad} x_6[n] \xrightarrow{\quad} \boxed{\begin{array}{c} \text{D/A} \\ F_2 = 20\text{kHz} \end{array}} \rightarrow \hat{x}_a(t)$$

Explain why this system would be *preferable to* or *inferior to* or *equivalent to* the system proposed in the previous problem. (Pick one of these three and explain with a couple of sentences.)