EECS351 Discussion 8 Problems, 11/10/16

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1 System Diagrams

Consider the following Transfer Function:

$$H(z) = 8 + \frac{-7+8z^{-1}}{1-.75z^{-1}+.125z^{-2}}$$

Note:
(a) Find the LCCDE (difference equation) y[n] in terms of shifted y[n] and shifted x[n].
(b) Draw the system diagram for the difference equation you found in part (a)

2 Properties of ROC

Consider a signal x[n], which has the following z-transform:

$$X(z) = \frac{2z(z-\frac{1}{12})}{(z-\frac{1}{2})(z+\frac{1}{3})}$$

$|z| > 1/2$

(a) Determine the partial fraction expansion of X(z)
(b) Sketch the ROC for X(z). Based on your sketch, does the DTFT for x[n] exist? Is x[n] right sided, left sided or neither? Is x[n] causal?
(c) Find x[n] using the inverse z-transform. Based on your answer for x[n]: Is x[n] right sided, left sided or neither? Is x[n] causal?

3 Z-transform

Determine the inverse z-transform for the following:

$$X(z) = log(1 + \alpha z^{-1})$$

$|z| > |\alpha|$

$$\text{HINT: nx[n]} \longleftrightarrow -z\frac{dX(z)}{dz}$$

4 2D Basis

Recall in 1D, the standard (canonical) basis for $\mathbb{R}^2$:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

In 2D, the standard basis for $\mathbb{R}^{2x2}$:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Similarly to $\mathbb{R}^2$, we can show the standard basis is valid by showing:

$$\alpha_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + \alpha_3 \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} + \alpha_4 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Implies $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0$
We could also show that a linear combination of basis matrices form the standard basis.

5 2D Basis Question

(a) Show

$$\begin{bmatrix} -1 & -2 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ -3 & -3 \end{bmatrix}, \begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix}$$

is not a valid basis for $\mathbb{R}^{2x2}$

2D Basis Question continued

(b) Consider the following for matrices:

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Show the canonical (standard) basis can be formed as a linear combination of the above matrices.

(c) Consider the following matrix:

$$\begin{bmatrix} 1296 & 4899 \\ 3516 & 0135 \end{bmatrix}$$

Can this matrix be expressed as a linear combination of the matrices from part (b)?