EECS351 Discussion 3, 1/23/2017

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1 Inner Product

Prove the Pythagorean theorem for norms defined by inner products. That is, if z = x + y and x $\perp$ y, then $||x||^2 + ||y||^2 = ||z||^2$

2 Valid or Invalid Inner Product

Consider signals in $\mathbb{R}^2$. Your friend tells you he has discovered a new inner product. He writes the expression for the new inner product below (length 2 signals).

$$<x,y> = {x_1}^3{y_1}^3 + x_2y_2$$

a)
Does the proposed computation satisfy the positivity requirement of an inner product?
($<x,x> \geq 0$ and $<x,x> = 0$ iff x = 0)

b)
Recall that for an inner product space, the following property must be true. $<\alpha x, y> = \alpha^* <x,y>$.
For a real vector space, only containing real entries, the following property must be satisfied.

$$<\alpha x, y> = \alpha <x,y>$$

Has your friend satisfied this requirement?

3 DFT Basis

Recall the following valid un-normalized basis function.

$$w_k[n] = W_N^{-nk} = e^{\frac{j2\pi nk}{N}}$$

a) Generate the Fourier Basis Vectors for $\mathbb{R}^3$

b) Verify that the Vectors you found in a) are orthogonal

c) Recall the definition of the DFT:

$$X[k]= \sum_n x[n]e^{-j2\pi nk/N}$$

Compute X[k] for x[n] = [3 5 1]

d) Compute the inverse DFT of X[k]

e) Compute the coordinates of x[n] with respect to the normalized Basis from part a)

f) Let γ represent the normalizing constant you used in e) to create the normalized basis from a). Using γ explain the relationship between X[k] and the coordinates you found in e)

4 Valid Basis?

Are the following set of vectors a valid basis for $\mathbb{R}^3$

$$b_1 = \begin{bmatrix} 1 \\ 0 \\ 8 \end{bmatrix}$$

$$b_2 = \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$$

5 Valid Basis?

Are the following set of vectors a valid basis for $\mathbb{R}^3$

$$b_1 = \begin{bmatrix} 1 \\ 0 \\ 8 \end{bmatrix}$$

$$b_2 = \begin{bmatrix} 0 \\ 1 \\ -8 \end{bmatrix}$$

$$b_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$B = \begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix}$$

6 A different way of thinking

Your friend has a favorite two digit number. But your friend is cheeky and he doesn't want to give you this important information without a bit of trickery.
The secret number is encoded as a length two signal. He gives you the **coordinates with respect to the Normalized Basis Function from part 3** of the length two signal.

$$c_0 = \frac{9}{\sqrt{2}}$$

$$c_1 = \frac{7}{\sqrt{2}}$$

What is your friend's favorite number?

7 Free Response

Express the length 4 signal $x = [1\ 2\ 1\ 4]^T$ as a weighted sum of $\frac{1}{N}W_N^{-nk}$ terms (4 terms total)