$$X_{2\pi}(\omega) = X_{2\pi}(e^{j\omega}) = \sum_{n=\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(\omega)e^{j\omega n}\,d\omega$$

2.) Periodicity:

Show $X(\omega)$ is periodic with 2π = period

$$X(\omega+2\pi) = \sum x[n]e^{-j(\omega+2\pi)n}$$

$$= \sum x[n]e^{-j\omega n}e^{-j2\pi n}$$

$$= \sum x[n]e^{-j\omega n}$$

$$= X(\omega) \quad \checkmark$$

3.) Linearity: Show $\alpha x_1[n] + \beta x_2[n] \overset{DTFT}{\longleftrightarrow} \alpha X_1(\omega)$ $\beta X_2(\omega)$

$$\sum_{n=-\infty}^{\infty}\left(\alpha x_1[n] + \beta x_2[n]\right)e^{-j\omega n}$$

$$\sum_{n=-\infty}^{\infty}\alpha x[n]e^{-j\omega n} + \sum_{n=-\infty}^{\infty}\beta x(n)e^{-j\omega n}$$

$$\alpha X(\omega) + \beta X_2(\omega) \quad \checkmark$$

4.) Shift in time

$x[n-a] \longleftrightarrow X(\omega)e^{-j\omega a}$

$\sum_{n=-\infty}^{\infty} x[n-a]e^{-j\omega n}$ Let $n' = n-a$

$= \sum_{n'=-\infty}^{\infty} x[n']e^{-j\omega(n'+a)}$

$= \sum_{n'=-\infty}^{\infty} x[n']e^{-j\omega n'}e^{-j\omega a}$

$= (e^{-j\omega a}) \sum_{n'=-\infty}^{\infty} x[n']e^{-j\omega n'} = e^{-j\omega a}X(\omega)$ ✓

5.) Modulation

$x[n]e^{jan} \longleftrightarrow X(e^{j(\omega-a)})$

$\sum x[n]e^{jan}e^{-j\omega n}$

$\sum x[n]e^{-jn(\omega-a)}$

Let $\theta = \omega - a$

$= \sum x[n]e^{-jn\theta}$

$= X(\theta) = X(e^{j\theta})$

$= X(e^{j(\omega-a)})$

8.) Zero Padding

Let $x_1[n] = [x_0 \ldots x_{N-1}]$ Length N

Let $x_2[n] = x_1[n]$ followed by N zeros

Find $X_2(\omega)$ in terms of $X_1(\omega)$

Hint expand the sum $\sum x[n]e^{-j\omega n}$ one term at a time

$$X_1(\omega) = x[0]e^{0} + x[1]e^{-j\omega} + x[2]e^{-j2\omega} + \ldots x[n-1]e^{-j\omega(n-1)}$$

$$X_2(\omega) = \sum_{n=0}^{2N-1} x_2[n]e^{-j\omega n} = \text{"} \quad \text{"} + 0e^{-j\omega N} + 0e^{-j\omega N+1} + \ldots$$

$$= X_1(\omega)$$

9.) Let $x_1[n] = [x_0 \ldots x_{n-1}]$

$x_3[n] = [x_0 \ 0 \ x_1 \ 0 \ x_2 \ 0 \ x_3 \ 0 \ \ldots \ x_{n-1} \ 0]$

Find $X_3(\omega)$ in terms of $X_1(\omega)$

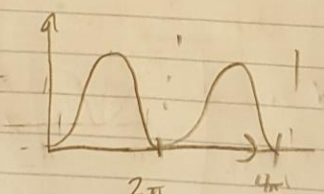
$$X_3(\omega) = x_0 e^{0} + x_1 e^{-j2\omega} + x_2 e^{-j4\omega} + x_3 e^{-j6\omega}$$

$$= \sum_{n=-\infty}^{\infty} x_1[n] e^{-j2\omega n}$$ Let $\hat{\omega} = 2\omega$

$$= \sum_{n=-\infty}^{\infty} x_1[n] e^{-j\hat{\omega} n}$$

$$= X_{1_{2\pi}}(\hat{\omega}) = X_{1_{2\pi}}(2\omega) = X_3(\omega)$$

Suppose $X_1(\omega)$ -



$X_3(\omega)$