351 Discussion 4 Solutions

1.

$$\left|\left|z\right|\right|^2$$

$$= \left|\left|x + y\right|\right|^2$$

$$= (x + y)(x + y)$$

$$= x^2 + 2xy + y^2$$

$$=> xy = 0 \; since \; x \; and \; y \; are \; orthogonal \; (given \; in \; the \; directions)$$

$$= x^2 + y^2$$

$$= \left|\left|x\right|\right|^2 + \left|\left|y\right|\right|^2$$

2.

a)

$$< x, x >$$

$$= x_1^3 x_1^3 + x_2 x_2$$

$$= x_1^6 + x_2^2$$

$$since \; x_1^6 \geq 0 \; and \; x_2^2 \geq 0, \qquad equal \; to \; 0 \; when \; x_1, x_2 = 0$$

Thus $< x, x >$ satisfies the positivity requirement

b)

$$< \alpha x, y >$$

$$= (\alpha x_1)^3 y_1^3 + \alpha x_2 y_2$$

$$\neq$$

$$= \alpha(x_1^3 y_1^3 + x_2 y_2)$$

$$= \alpha < x, y >$$

Scaling Property is not satisfied

3.

a)

$$\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\e^{\frac{j2\pi}{3}}\\e^{\frac{j4\pi}{3}}\end{bmatrix}, \begin{bmatrix}1\\e^{\frac{j4\pi}{3}}\\e^{\frac{j8\pi}{3}}\end{bmatrix}$$

b)

$$< w_i, w_j > = \sum w_i^* w_j$$

$$< w_0, w_1 > = \ 1 + \left(-.5 + \frac{\sqrt{3}j}{2}\right) + \left(-.5 - \frac{\sqrt{3}j}{2}\right) = 0$$

$$< w_0, w_2 > = \ 1 + \left(-.5 - \frac{\sqrt{3}j}{2}\right) + \left(-.5 + \frac{\sqrt{3}j}{2}\right) = 0$$

$$< w_1, w_2 > = \ 1 + \left(-.5 + \frac{\sqrt{3}j}{2}\right) + \left(-.5 - \frac{\sqrt{3}j}{2}\right) = 0$$

c)

$$X[k] = \sum_n x[n] e^{-\frac{j2\pi nk}{N}}$$

$$X[0] = \sum_n x[n] e^{-\frac{j2\pi n0}{N}}$$

$$X[0] = \sum_n x[n] = 3 + 5 + 1 = 9$$

$$X[1] = \sum_n x[n] e^{-\frac{j2\pi n1}{N}}$$

$$X[1] = \sum_n x[n] = 3 + 5e^{-\frac{j2\pi}{3}} + 1e^{-\frac{j4\pi}{3}} = -2\sqrt{3}j$$

$$X[2] = \sum_n x[n] e^{-\frac{j2\pi 2n}{N}}$$

$$X[2] = \sum_n x[n] = 3 + 5e^{-\frac{j4\pi}{3}} + 1e^{-\frac{j8\pi}{3}} = 2\sqrt{3}j$$

d)

The inverse DFT will simply be x[n] = [3 5 1]

e)

$$c_0 = < w_0, x > = \frac{1}{\sqrt{3}}(3+5+1) = 3\sqrt{3}$$

$$c_1 = < w_1, x > = \frac{1}{\sqrt{3}}\left(3+5e^{-\frac{j2\pi}{3}}+e^{-\frac{j4\pi}{3}}\right) = -2j$$

$$c_2 = < w_2, x > = \frac{1}{\sqrt{3}}\left(3+5e^{-\frac{j4\pi}{3}}+e^{-\frac{j8\pi}{3}}\right) = 2j$$

f)

$$\gamma = \frac{1}{\sqrt{3}}$$

$$coordinates\ from\ e) = \gamma X[k]$$

Since X[k] are the coordinates for the un-normalized basis in part a), our result makes sense from two viewpoints: DFT or change of basis.

4.

This is not a valid basis. At least three vectors are required (and they must be linearly independent)

5.

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = b_1 + b_2 - b_3$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = b_3$$

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = (b_3 - b_2)\left(\frac{1}{8}\right) = -\frac{1}{8}b_2 + \frac{1}{8}b_3$$

6. The three normalized basis vectors are:

$$\frac{1}{\sqrt{2}}\left(\begin{bmatrix}1\\1\end{bmatrix},\begin{bmatrix}1\\-1\end{bmatrix}\right)$$

$$x = c_0 w_0 + c_1 w_1$$

$$x = \begin{bmatrix}8\\1\end{bmatrix}$$

7.

The weights for the sum of $\frac{1}{N} W_N^{-nk}$ are simply the DFT of x

$$x[n] = \frac{1}{N}\sum_k X[k]e^{2\pi nk/N} = \frac{1}{N}\sum_k X[k]W_N^{-nk}$$

So we just need to calculate the DFT of [1 2 1 4]

$$\text{fft(x)} = \begin{bmatrix}8\\2j\\-4\\-2j\end{bmatrix}$$

It can be verified that

$$\text{x[n]} = \frac{1}{4}8W_N^{-n0} + \frac{1}{4}2jW_N^{-n1} + \frac{1}{4}(-4)W_N^{-n2} + \frac{1}{4}(-2j)W_N^{-n3}$$