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1 What is a digital signal? Discrete Time vs. Continuous Time

A digital signal is simply a sequence of discrete values. A digital signal x[n] is only valid for integer n. A continuous signal x(t) is valid for all t.

Digital signals can be denoted using brackets, braces, or parenthesis.

Example: x[n] =[$\underline{1}$, 2, 3, 3, 5, 1]

NOTE: We underline a value to denote the 0th index of a signal (n = 0)

2 Kronecker Delta (a.k.a. Delta Sequence, Unit Sample Sequence)

$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

2.1 Decomposition of Discrete Signals with Shifted Deltas

Any sequence $x[n]$ can be written as $x[n] = \sum\limits_{k=-\infty}^{\infty} x[k]\delta[n-k]$

2.2 Dirac Delta (a.k.a. Unit Impulse Function)

A loose definition:

$$\delta(t) = \begin{cases} +\infty, & t = 0 \\ 0, & t \neq 0 \end{cases} \quad \text{s.t. } \int_{-\infty}^{\infty} \delta(t)dt = 1.$$

A more precise definition:

$$\delta(t) = \lim_{\Delta \to 0} \frac{1}{\Delta} rect(\frac{t}{\Delta})$$

2.3 Decomposition of Continuous Signals with Shifted Deltas

Any continuous-domain function $x(t)$ can be written as $x(t) = \int\limits_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$

2.4 Kronecker vs. Dirac

What's the deal with these different delta functions?

The Kronecker delta, denoted $\delta[n]$, is a sequence indexed over a discrete domain, in other words, $n \in \mathbb{Z}$. The Dirac delta, denoted $\delta(t)$ or drawn as an upward pointing arrow, is a function over a continuous domain, in other words $t \in \mathbb{R}$. In this class, we're dealing exclusively with discrete-time signals, so in the time domain, we'll have Kronecker deltas. However, when we take the DTFT of a discrete-time signal, we get a result in the continuous frequency domain (ω), so if we have an impulse in the continuous frequency domain, it will be a Dirac delta, $\delta(\omega)$.

Spiritually, these functions are the same though.

3 Properties of Discrete Signals

3.1 Periodicity

As signal $x[n]$ is periodic if there exists a number n_0 such that $x[n] = x[n - n_0]$ for all n. The period of the signal is n_0.

For sinusoidal functions (i.e. $cos\,(\omega n + \phi)$), the function is periodic iff ω is a rational multiple of π, i.e. $\omega = \frac{M}{N}\pi$ for integers M and N.

3.2 Boundedness

A signal is bounded if there exists a positive, finite number B such that $|x[n]| \leq B$ for all n.

3.3 Causality

A signal $x[n]$ is causal if $x[n] = 0$ for $n < 0$.

3.4 Symmetry

A signal $x[n]$ is symmetric if $x[n] = x[-n]$. Signals with symmetry are also called "even".

If instead a signal has the property that $-x[n] = x[-n]$, then it is called "odd".

4 Properties of Discrete-Domain Systems

4.1 Causality

A system $\mathcal{T}$ is causal if $\mathcal{T}\{x[n]\}$ depends only on $x[n]$ for $n < 0$. In other words, it depends only on the present and past values of $x[n]$ and not the future values.

4.2 Linearity

A system $\mathcal{T}$ is linear iff for any inputs $x_1[n]$ and $x_2[n]$ and any scalars a_1 and a_2, $\mathcal{T}\{a_1 x_1[n] + a_2 x_2[n]\} = a_1 \mathcal{T}\{x_1[n]\} + a_2 \mathcal{T}\{x_2[n]\}$.

4.3 Shift-Invariance (a.k.a. Time-Invariance)

Let $y[n] = \mathcal{T}\{x[n]\}$. Then the system $\mathcal{T}$ is shift-invariant if $\mathcal{T}\{x[n-n_0]\} = y[n-n_0]$. In other words, the output of the delayed signal is the same as the delay of the output signal.

4.4 Bounded Input Bounded Output (BIBO) Stability

A system $\mathcal{T}$ is called BIBO stable if any bounded input signal $x[n]$ results in a bounded output signal $y[n]$. If a bound B_x exists for bounded $x[n]$, then there exists a bound B_y such that $|y[n]| = |\mathcal{T}\{x[n]\}| \leq B_y$ for all n.

5 Properties of the Delta Functions

5.1 Sampling Property of the Kronecker Delta

$\delta[n-n_0]x[n] = \delta[n-n_0]x[n_0]$

5.2 Sifting Property of the Kronecker Delta

$\sum\limits_{n=-\infty}^{\infty} \delta[n-n_0]x[n] = x[n_0]$

5.3 Scaling Property of the Dirac Delta

$\delta[2n] = \delta[n]$ Note that there is no scaling factor!

5.4 Sampling Property of the Dirac Delta

$\delta(t-t_0)x(t) = \delta(t-t_0)x(t_0)$

note: The result is a function!

5.5 Sifting Property of the Dirac Delta

$\int\limits_{-\infty}^{\infty} \delta(t - t_0)x(t) = x(t_0)$

note: The result is a scalar!

5.6 Scaling Property of the Dirac Delta

$\delta(\alpha t) = \frac{1}{|\alpha|}\delta(t)$ for any scalar α