

Discussion Sol's

P1

$$(a) \quad H(z) = 8 + \frac{-7 + 8z^{-1}}{1 - .75z^{-1} + .125z^{-2}}$$

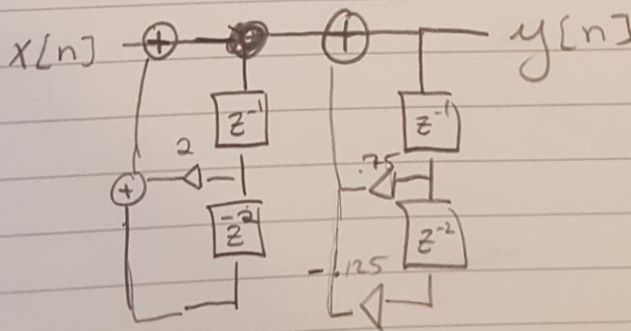
$$H(z) = \frac{8 - 6z^{-1} + z^{-2}}{1 - .75z^{-1} + .125z^{-2}} + \frac{-7 + 8z^{-1}}{1 - .75z^{-1} + .125z^{-2}}$$

$$H(z) = \frac{z^{-2} + 2z^{-1} + 1}{(1 - .75z^{-1} + .125z^{-2})}$$

$$Y(z)(1 - .75z^{-1} + .125z^{-2}) = X(z)(z^{-2} + 2z^{-1} + 1)$$

$$y[n] = x[n] + 2x[n-1] + x[n-2] + .75y[n-1] - .125y[n-2]$$

(b.)



B P2

$$\frac{2z(z - \frac{1}{2})}{(z - \frac{1}{2})(z + \frac{1}{3})} = X(z)$$

get z^{-1} on top / bottom

(multiply by $(\frac{z}{z})$)

$$\frac{2z(z^{-1}) z^{-1}(z - \frac{1}{2})}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{3}z^{-1})}$$

$$\frac{2(1 - \frac{1}{2}z^{-1})}{(1 - \frac{1}{2}z^{-1})(1 + \frac{1}{3}z^{-1})} = \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 + \frac{1}{3}z^{-1}}$$

$$A, B = 1$$

$$X[n] = \left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{3}\right)^n u[n]$$

$$a^n u[n] \leftrightarrow \frac{1}{1 - az^{-1}}$$

$$\text{ROC } |z| > |a|$$

P3

$$\log(1 + \alpha z^{-1}) \quad |z| > |\alpha|$$

take deriv.

$$\frac{dX(z)}{dz} = \frac{-\alpha z^{-2}}{1 + \alpha z^{-1}}$$

$$-z \frac{dX(z)}{dz} = \frac{-\alpha z^{-1}}{1 + \alpha z^{-1}}$$

$$= \frac{1}{1 + \alpha z^{-1}} \text{ (shift) (scale)}$$

$$\alpha^n u[n]$$

$$\alpha^{n-1} u[n-1]$$

$$(-\alpha) \alpha^{n-1} u[n-1] = nx[n]$$

thus

$$x[n] = \frac{1}{n} (-\alpha) (\alpha)^{n-1} u[n-1]$$

Hint USE : $nx[n] \longleftrightarrow -z \frac{dx}{dz}$

p4

(a)

$$\alpha_1 \begin{bmatrix} -1 & -2 \\ 0 & 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \alpha_3 \begin{bmatrix} 0 & 0 \\ -3 & -3 \end{bmatrix} + \alpha_4 \begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix} = 0$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$$

So not a valid basis

$$(b) \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \alpha_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 0 + 0 + 0$$

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \frac{1}{2} \left(\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \right) \quad \alpha_2 = \alpha_3 = \frac{1}{2}$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \rightarrow \alpha_1 = -1 \quad \alpha_3 = 1 \quad \alpha_4 = -1$$

$$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \alpha_4 = 1$$

So these are a valid basis

c.) Since the matrices in (b) is a basis for $\mathbb{R}^{2 \times 2}$

yes! any matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

can be formed
as a linear combination
of a valid basis.