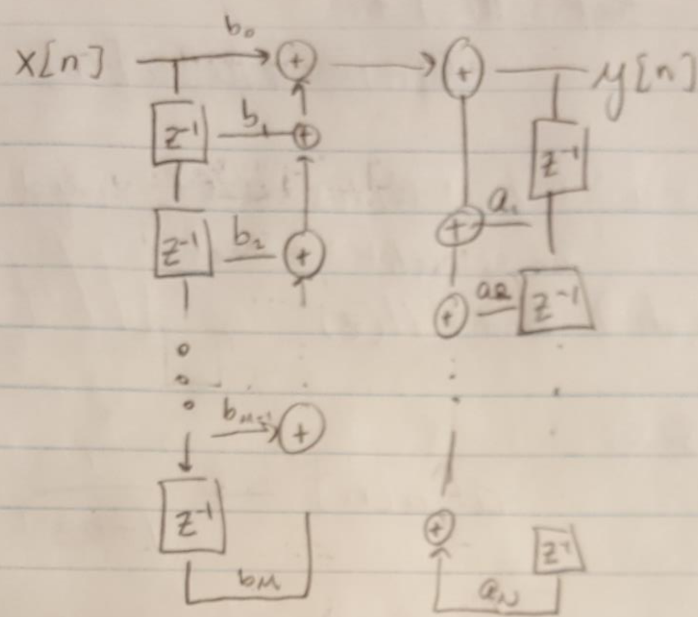
Direct form 1:

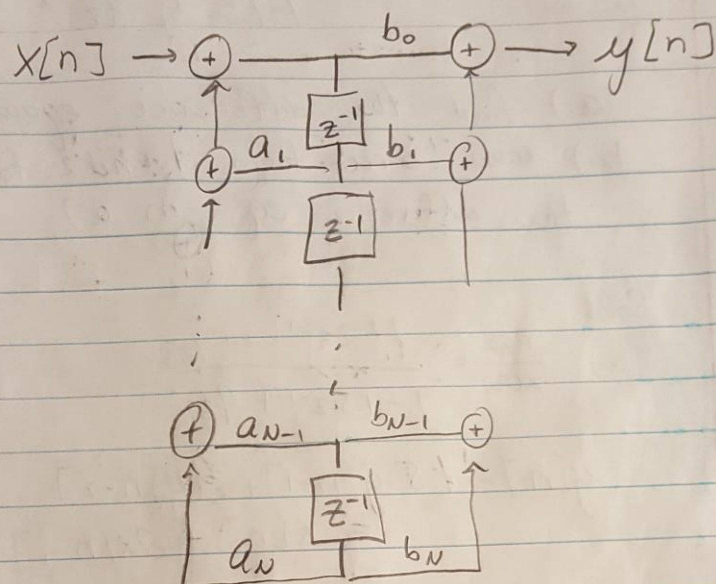


$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{\ell=0}^{M} b_\ell x[n-\ell]$$

Direct Form 1:

- Delays for x[n] and y[n] separate
- Adders oriented inwards from delays

Direct form 2



$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{l=0}^{N} b_l x[n-l]$$

Direct form 2:

- Delays for x[n] and y[n] share delays
- adders oriented outwards from delays

Continuous Sampling

$x(t)$: continuous time signal

$x[n]$: sampled version of $x(t)$ @ f_s

$X(\omega)$: $\omega = [-\pi, \pi]$

$X[K]$: $K = [0, N-1]$

$X(\omega)$: digital frequency, how do we get the real frequency "f"?

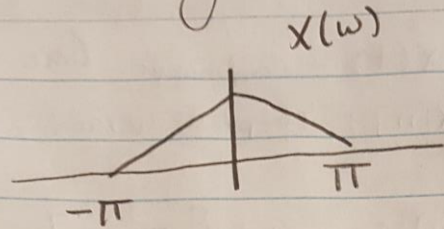
$$f_{max} = f_s/2 \qquad f = [-f_s/2, f_s/2]$$

Resolution: $\frac{\text{Bandwidth}}{\text{Number Samples}} = \frac{F_s}{N}$

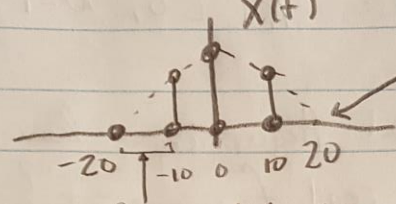
$$f_{true} = \frac{1}{2\pi}\frac{\omega_0}{T_s} = \frac{f_s\,\omega_0}{2\pi}$$

where ω_0 is digital freq

Frequency Resolution : $f_s = 40$



4-pt DFT



X(ω) periodic
So we don't obtain
this point

Resolution : $f_s/N = 40/4 = 10$

$$\text{Resolution} = \frac{\left(\frac{f_s}{2} - \frac{f_s}{N}\right) - \left(-f_s/2\right)}{N-1}$$

$$= f_s/N$$

1.) $x[n] = \left(-\frac{1}{3}\right)^n u[n] - \left(\frac{1}{2}\right)^n u[-n-1]$

a.) Find $X(z)$

b.) draw the ROC

c.) Does this sequence have a DTFT

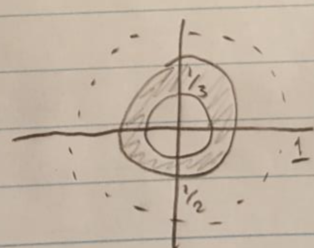
Sol:

a.) $\left(-\frac{1}{3}\right)^n u[n] \rightarrow \frac{1}{1+\frac{1}{3}z^{-1}} \qquad |z| > \frac{1}{3}$

$-\left(\frac{1}{2}\right)^n u[-n-1] \rightarrow \frac{1}{1-\frac{1}{2}z^{-1}} \qquad |z| < \frac{1}{2}$

$\therefore \quad X(z) = \frac{1}{1+\frac{1}{3}z^{-1}} + \frac{1}{1-\frac{1}{2}z^{-1}} \qquad |z| > \frac{1}{3} \cap |z| < \frac{1}{2}$

b.)



c.) ~~Does~~ ROC does not contain unit circle so DTFT does not exist

2. Show $\text{DTFT}\{x[n-n_0]\} = X(\omega)e^{-j\omega n_0}$

$$\sum_{n=-\infty}^{\infty} x[n-n_0]e^{-j\omega n}$$

Let $m = n-n_0$

$m+n_0 = n$

$$= \sum_{m=-\infty}^{\infty} x[m]e^{-j\omega(m+n_0)}$$

$$= e^{-j\omega n_0} \sum_{m=-\infty}^{\infty} x[m]e^{-j\omega n}$$

$$= e^{-j\omega n_0} X(\omega)$$

3.) Use z-transform properties to find $H(z)$ for

$$h[n] = n a^n u[n]$$

a.) is $h[n]$ left-sided or right sided

b.) find $H(z)$

Sol:

$$a^n u[n] \longrightarrow \frac{1}{1-az^{-1}} \qquad |z| > |a|$$

for right sided

$$n(a^n u[n]) \rightarrow -z \frac{d}{\partial z}\left(\frac{1}{1-az^{-1}}\right)$$

$$= -z \frac{1}{(1-az^{-1})^2}(az^{-2})$$

$$\boxed{H(z) = \frac{-az^{-1}}{(1-az^{-1})^2} \qquad |z| > |a|}$$

1.

$$H(z) = \frac{1+2z^{-1}}{1-1.5z^{-1}+.9z^{-2}}$$

a.) find the difference equation

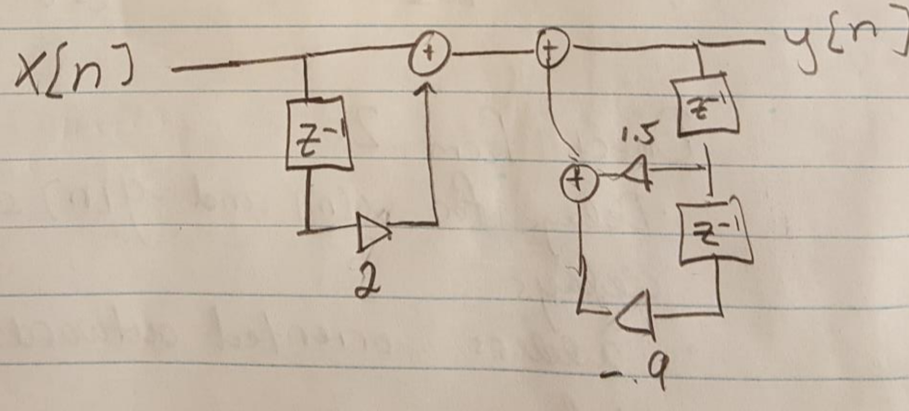
b.) draw direct form 1 and 2 for the difference eq. in a)

Sol:

a) $$\frac{Y(z)}{X(z)} = \frac{1+2z^{-1}}{1-1.5z^{-1}+.9z^{-2}}$$

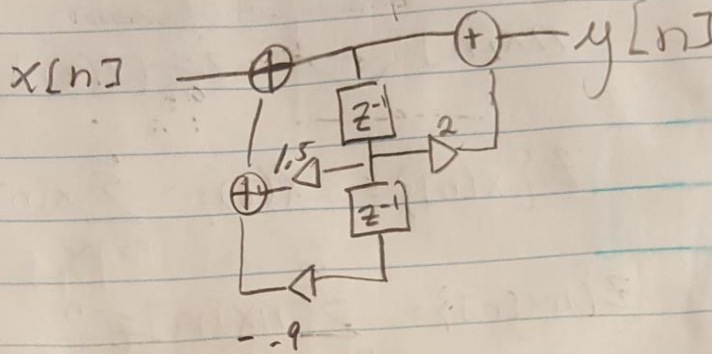
$$y[n] - 1.5y[n-1] + .9y[n-2] = x[n] + 2x[n-1]$$

b.) direct form 1 - Separate delays for x and y



4 b.) direct form 2

- Shared delays for x and y



5.) Show derivative property of z-transform

$$n x[n] \longrightarrow -z\frac{\partial}{\partial z}(X(z))$$

$$Z\{x[n]\} = X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$Z\{n x[n]\} = \sum_{n=-\infty}^{\infty} n x[n] z^{-n}$$

$$= -\sum_{n=-\infty}^{\infty} (-n) x[n]\, z\, z^{-n-1}$$

$$= -z \sum_{n=-\infty}^{\infty} -n x[n] z^{-n-1}$$

$$=$$

$$= -z \frac{\partial}{\partial z} \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$\boxed{Z\{n x[n]\} = -z\frac{\partial}{\partial z} X(z)}$$

Note $\frac{\partial}{\partial z} \sum_{n=-\infty}^{\infty} x[n]z^{-n} = \sum_{n=-\infty}^{\infty} -n x[n] z^{-n-1}$