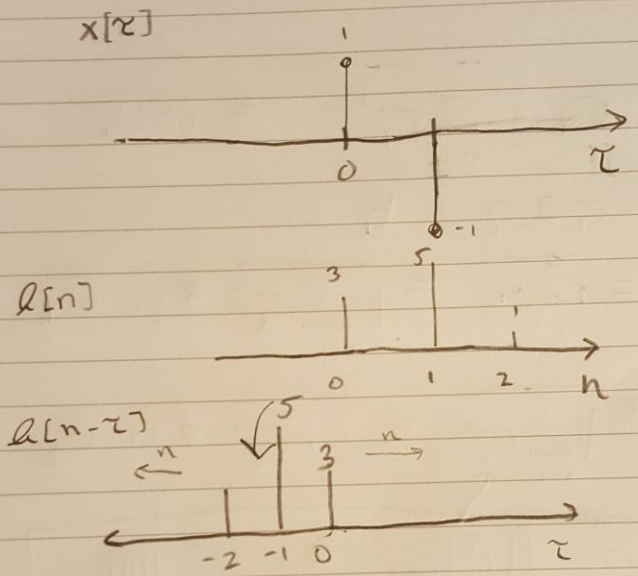
# Problem 3

$x[n] = [\underline{1}, -1]$

$h[n] = [\underline{3}, 5, 1]$




$y[0] = 3 \cdot 1 = 3$

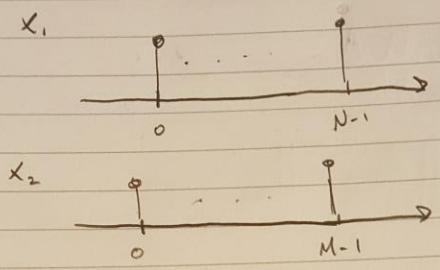
$y[1] = 5 \cdot 1 + 3 \cdot (-1) = 2$

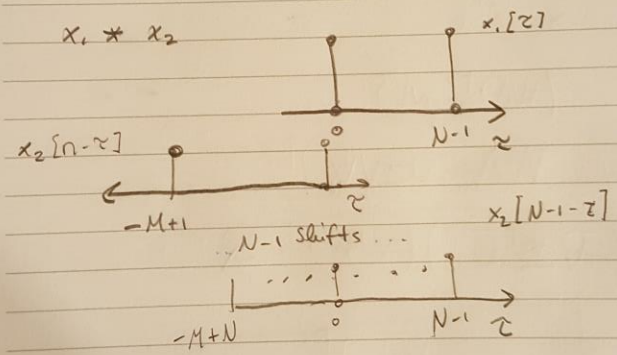
$y[2] = 1 \cdot 1 - 5 \cdot 1 = -4$

$y[3] = -1 \cdot 1 = -1$

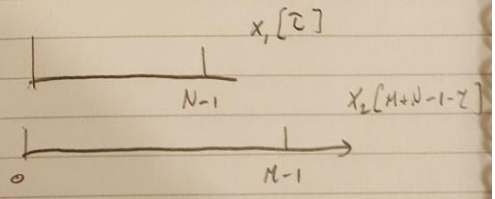
Problem 4

Let $x_1$ have length $N$, $x_2$ length $M$




M more shifts left




$= N + M - 1$ shifts

# Problem 6

(a)

$$
\begin{array}{lll}
^{3}(2\ 1\ 6)\ 0\ 0 & & 6\ 3\ 18\ 0\ 0 \\
0\ ^{5}(2\ 1\ 6)\ 0 & = & 0\ 10\ 5\ 30\ 0 \\
0\ 0\ ^{1}(2\ 1\ 6) & & 0\ 0\ 2\ 1\ 6 \\
\hline
 & & 6\ 13\ 25\ 31\ 6
\end{array}
$$

(b.)

$$
\begin{array}{lll}
^{2}(3\ 5\ 1)\ 0\ 0 & & 6\ 10\ 2\ 0\ 0 \\
0\ ^{1}(3\ 5\ 1)\ 0 & = & 0\ 3\ 5\ 1\ 6 \\
0\ 0\ ^{6}(3\ 5\ 1) & & 0\ 0\ 18\ 30\ 6 \\
 & & 6\ 13\ 25\ 31\ 6
\end{array}
$$

c.) Length $= M+N-1 = 3+3-1 = 5$

(a) & (b) agree

Problem 7.

(a) Using Stack & Sum

$$\{\cdots, 2, \underline{-1}, 2, \cdots\}$$

$$2\{\cdots, -1, \underline{2}, -1\cdots\}$$

$$= \{\cdots 0, 3, 0, \underline{3}, 0, 3, 0\cdots\} = x[n] * h[n]$$

b.)

$$X(\omega) = \sum_{i=0}^{N-1} x[n] e^{-j\omega n}$$

$$X(\omega) = x[0]e^{0} + x[1]e^{-j\omega} + x[2]e^{-2j\omega} + x[3]e^{-3j\omega}$$

→ substitute $x(n)$

$$X(\omega) = e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega}$$

c.) $Y(\omega) = X(\omega) H(\omega)$

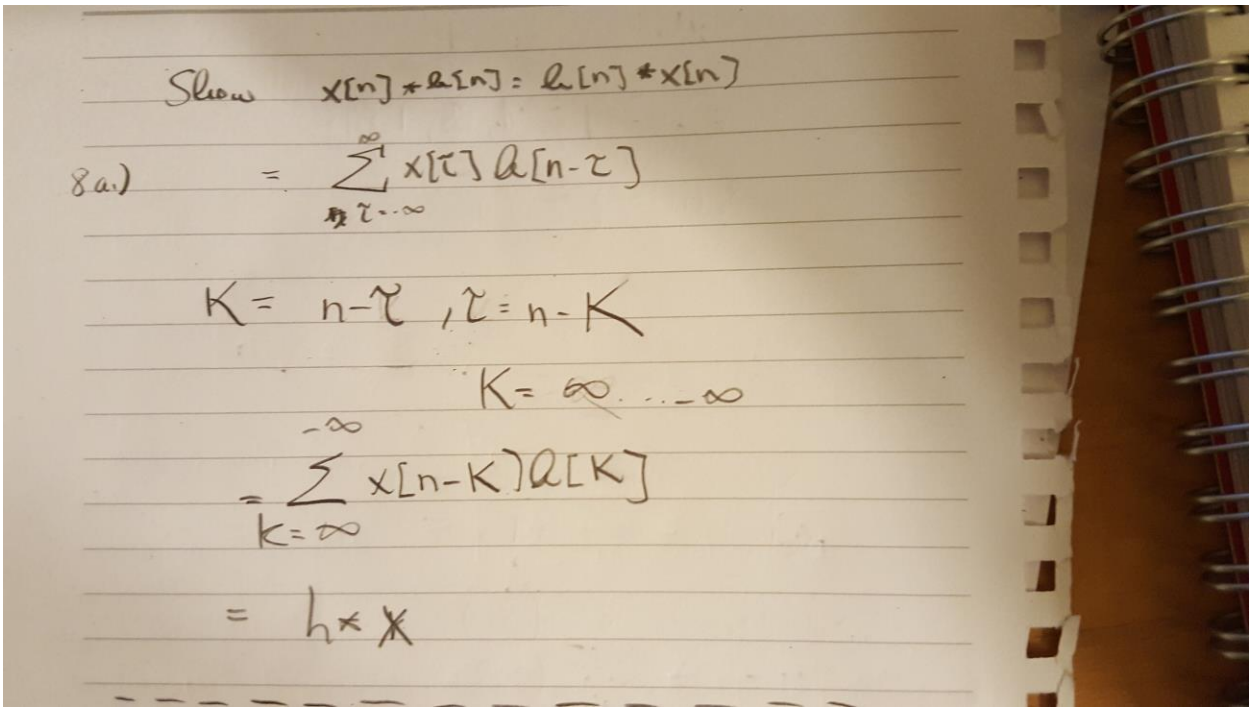
$$= (e^{-j\omega} + e^{-2j\omega} + e^{-3j\omega}) H(\omega)$$

$$= H(\omega)e^{-j\omega} + H(\omega)e^{-2j\omega} + e^{-3j\omega} H(\omega)$$

$\Downarrow$ IDFT $\quad H(\omega)e^{-j\omega\alpha} \Rightarrow h[n-\alpha]$

$$y[n] = h[n-1] + h[n-2] + h[n-3]$$

$$= \{\cdots 0, 3, 0, \underline{3}, 0, 3, 0\cdots\}.$$



Show $x[n] * h[n] = h[n] * x[n]$

8a.) $= \sum_{\tau=-\infty}^{\infty} x[\tau]\, h[n-\tau]$

$K = n - \tau \quad , \tau = n - K$

$K = \infty \ldots -\infty$

$= \sum_{K=\infty}^{-\infty} x[n-K]\, h[K]$

$= h * x$

Show the convolution prop of DTFT

↯ $x[n] * h[n] \hookrightarrow X(\omega)\,H(\omega)$

$$\sum_{n=-\infty}^{\infty} \left( \sum_{\tau=-\infty}^{\infty} x[\tau]\,h[n-\tau] \right) e^{-j\omega n}$$

$$= \sum_{\tau=-\infty}^{\infty} x[\tau] \sum_{n=-\infty}^{\infty} h[n-\tau]\,e^{-j\omega n}$$

Let $n' = n-\tau$, $(n' = -\inf \ldots \inf)$

$$n = n' + \tau$$

$$= \sum_{\tau=-\infty}^{\infty} x[\tau] \sum_{n'=-\infty}^{\infty} h[n']\,e^{-j\omega n'}\,e^{-j\omega\tau}$$

$$= \sum_{\tau=-\infty}^{\infty} x[\tau]\,e^{-j\omega\tau} \sum_{n'=-\infty}^{\infty} h[n']\,e^{-j\omega n}$$

$$= X(\omega)\,H(\omega)$$