Problem 3

(a) $y[n] = x[n-1]$

$\Downarrow$

$$Y(\omega) = X(\omega)e^{-j\omega}$$

$$\frac{Y(\omega)}{X(\omega)} = \frac{e^{-j\omega}}{1}$$

(b.) $2y[n] + 4y[n-4] = x[n] + 3x[n-1]$

$$Y(\omega)\left(2 + 4e^{-4j\omega}\right) = X(\omega)\left(1 + 3e^{-j\omega}\right)$$

$$Y(\omega)/X(\omega) = \frac{1 + 3e^{-j\omega}}{2 + 4e^{-4j\omega}}$$

c.)

$$= \frac{\sum_{k=0}^{M} b_k e^{-j\omega k}}{\sum_{z=0}^{M} a_z e^{-j\omega z}} = H(\omega)$$

Problem 4

(a) $Y(z) = z^{-1} X(z)$

$$H(z) = \cancel{z}\, z^{-1}$$

(b)

$$H(z) = \frac{1 + 3z^{-1}}{2 + 4z^{-4}}$$

(substitute) $z = e^{j\omega}$ from problem 3

(c.)

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{i=0}^{N} a_i z^{-i}}$$

Problem 6

$$H(z) = \frac{1}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{2}z^{-1})} \qquad |z| > \frac{1}{2}$$

$$= \frac{A_1}{(1-\frac{1}{4}z^{-1})} + \frac{A_2}{(1-\frac{1}{2}z^{-1})}$$

$$1 = (1-\tfrac{1}{2}z^{-1})A_1 + (1-\tfrac{1}{4}z^{-1})A_2$$

Let $z = \frac{1}{4}$ and $z = \frac{1}{2}$

$z = \frac{1}{4}$

$$1 = -A_1$$

$z = \frac{1}{2}$

$$1 = 0 + (1-\tfrac{1}{2})A_2$$

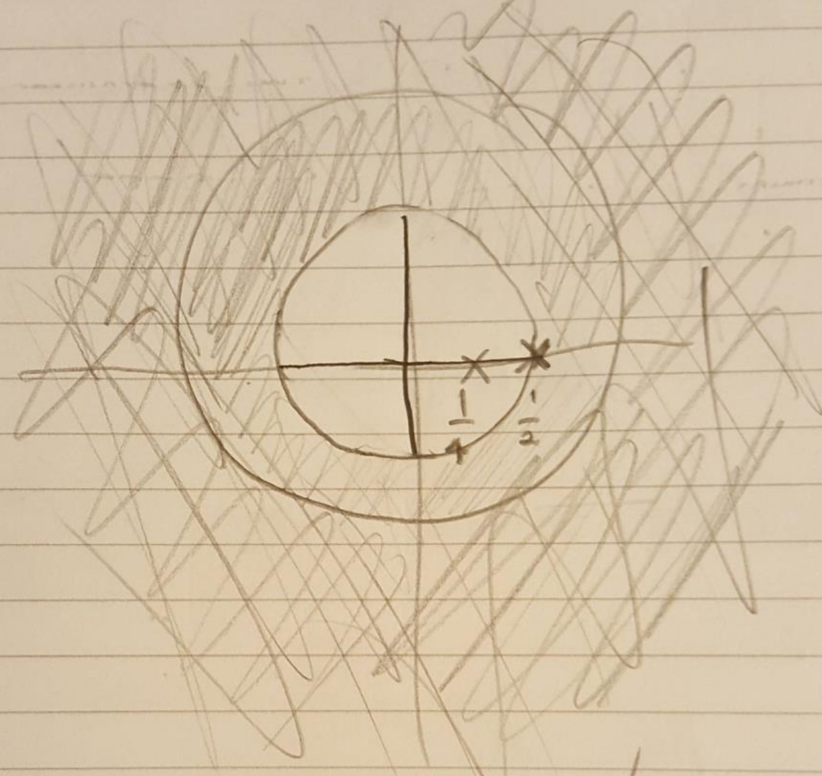
$$A_2 = 2$$

$$X(z) = \frac{-1}{(1-\frac{1}{4}z^{-1})} + \frac{2}{(1-\frac{1}{2}z^{-1})} \qquad |z| > \frac{1}{2}$$

I-Z-Transform

$$\Rightarrow x[n] = -\left(\frac{1}{4}\right)^n u[n] + 2\left(\frac{1}{2}\right)^n u[n]$$

Pole / Zero / ROC



Right - Sided!

8 Extra Problems

(a) When $z = e^{j\omega}$, the z-transform becomes the DTFT. So if ROC contains ~~unit~~ unit circle, the DTFT exists and the system must be stable by Part 7 of this discussion.

(b) $$\frac{s+3}{s(s+2)^2(s+5)}$$

$$= \frac{A_1}{s} + \frac{A_2}{s+2} + \frac{A_3}{(s+2)^2} + \frac{A_4}{(s+5)}$$

$$s+3 = A_1(s+2)^2(s+5) + A_2(s)(s+2)(s+5) + A_3 s(s+5) + A_4(s)(s+2)^2$$

$s=-2$:

$$1 = 3A_1 - 6A_2 - 6A_3 - 2A_4 \rightarrow \boxed{A_3 = -.1\overline{6} \quad A_2 = -.19\overline{4}}$$

$s=0$

$$3 = 20A_1 \qquad \boxed{A_1 = 3/20 = .15}$$

$s=-5$

$$-2 = -45A_4 \qquad \boxed{A_4 = 2/45 = .044\ldots}$$

$s=1$ $$4 = 54A_1 + 18A_2 + 6A_3 + 9A_4$$

(c)

$$\delta[n] - h_{lp}[n] = h_{hp}[n]$$

$\delta[n] \Rightarrow 1$