# EECS351 Discussion 4 Problems, 09/26/16

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## 1 Discrete-domain Basis Sequences

Let $x[n] = \{\underline{1}, 1, 2, 3, 5\}$. Represent $x[n]$ in each of the following bases.

Note: You won't be asked to do anything like this on the homework or exams, but hopefully it will help you solidify your understanding of bases from lecture.

### Unit Step Sequences

Recall $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$. Let $\mathcal{S}_u = \{u[n-n_0] | n_0 \in \mathbb{Z}\}$ (unit steps shifted by any integer $n_0$) be your basis sequences. Represent $x[n]$ in $\mathcal{S}_u$.

In other words, show you can write $x[n] = \sum_{k=-\infty}^{\infty} c_k u[n-k]$ by finding the values for $c_k$.

### Three-Tap Rectangles

Let $r[n] = \{\underline{1}, 1, 1\}$ and $\mathcal{S}_r = \{r[n-n_0] | n_0 \in \mathbb{Z}\}$. Represent $x[n]$ in $\mathcal{S}_r$.

### Three-Tap Triangle

Let $t[n] = \{\underline{1}, 2, 1\}$ and $\mathcal{S}_t = \{t[n-n_0] | n_0 \in \mathbb{Z}\}$. Represent $x[n]$ in $\mathcal{S}_t$.

## 2 Problems

### Problem 1

Show that B is orthonormal.

$$B = \begin{bmatrix} 3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & -3/\sqrt{10} \end{bmatrix}$$

*Solution:* Lets show that v1 and v2 can express the standard/canonical basis vectors using a weighted sum:

That is, find a particular $\alpha$ and $\beta$ such that

$$\alpha v_1 + \beta v_2 = [1\ 0]^T$$

And find a particular $c$ and $d$ such that

$$c v_1 + d v_2 = [0\ 1]^T$$

$$B = \begin{bmatrix} v_1 & v_2 \end{bmatrix}$$

Consider $(\sqrt{10}v_1$ - $3\sqrt{10}v_2)/10 =$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Consider $(3\sqrt{10}v_1 + \sqrt{10}v_2)/10 =$

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

## 2.2 Useful Property

Suppose B is a valid orthogonal basis matrix

Then

$$(\mathrm{B}^{-1}) \text{ exists}$$

and

$$(\mathrm{B}^{-1}) = \mathrm{B}^T$$

### 2.3

Last week we spent a lot of time discussing what constitutes a basis. Suppose B is any valid basis, then any signal $\mathbf{x} \in \mathbb{R}^2$ (length 2) can be expressed:

$$\mathrm{B}\mathbf{c} = \mathbf{x}$$

Where B is a valid basis for $\mathbb{R}^2$

$\mathbf{c} = [c_1 \; c_2]^T$ is called *the coordinates*

$$B = \begin{bmatrix} b_1 & b_3 \\ b_2 & b_4 \end{bmatrix}$$

Let $v_1 = [b_1 \; b_2]^T$, $v_2 = [b_3 \; b_4]^T$

$$\mathrm{B} = [v_1 \; v_2]$$

Graphically, $v_1$ and $v_2$ represent coordinate axes. The coordinates tell you how many $v_1$'s and $v_2$'s are needed to express $\mathbf{x}$. So if $c_1 = 2$ and $c_2 = 3$, then you need $2v_1$ and $3v_2$ to create $\mathbf{x}$. Symbolically:

$$2v_1 + 3v_2 = \mathbf{x}$$

## 2.4 Change of Basis

Represent the signal $\mathbf{x} = [2\ 1]^T$ the following ways:
(a) using the standard/canonical basis
(b) using the basis B from Problem 1, find the coordinates with respect to the new basis which represents the signal $\mathbf{x} = [2\ 1]^T$
(c) Draw the graphical representation of (a) and (b) in the same $\mathbb{R}^2$ plane. Verify that even though the coordinates for (a) and (b) are different, they still represent the same vector.

*Solution:*
(a)

$$\mathrm{B}\mathbf{c} = \mathbf{x}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Thus $c_1 = 2$, and $c_2 = 1$. We just found the coordinates with respect to the canonical basis.
(b)

$$\mathrm{B}\mathbf{c} = \mathbf{x}$$

$$= \begin{bmatrix} 3/\sqrt{10} & 1/\sqrt{10} \\ 1/\sqrt{10} & -3/\sqrt{10} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Using the Property from 2:

$$\mathrm{B}\mathbf{c} = \mathbf{x}$$

$$\mathbf{c} = B^{-1}\ \mathbf{x}$$

$$\mathbf{c} = B^T\ \mathbf{x}$$

$$\mathbf{c} = [2.21\ \text{-}.32]^T$$

This $c_1 = 2.21$, and $c_2 = \text{-}.32$. We just found the coordinates with respect to the basis B.

### (c)

## 3 Inner Product

$$||z||^2$$

$$= ||x + y||^2$$

$$= (x+y)(x+y)$$

$$= x^2 + 2xy + y^2$$

$$xy = 0 \text{ since } x \perp y$$

$$= x^2 + y^2 = ||x||^2 + ||y||^2$$