EECS351 Discussion 6 Problems, 10/13/16

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1 Convolution equations

$$x[n] * h[n] = \sum_{\tau=-\infty}^{\infty} x[\tau]h[n-\tau]$$

$$x[n] * h[n] \longleftrightarrow X(\omega)H(\omega)$$

Note:
(a) x[n] * h[n] is a function of n, not τ.
(b) For EVERY index of n, you need to compute a potentially infinite sum.

2 Graphical Intuition

Understanding the graphical intuition behind its definition is one way to compute convolution. Here is my step by step process for computing convolution by looking at the problem graphically:

$$x[n] * h[n] = y[n]$$

(1) Pick either x[n] or h[n] (doesn't matter which) and call this x_1[n]. Draw this in solid ink, because you won't be changing it.
(2) Pick the other signal and call it x_2[n]. Draw x_2[n] on separate axes directly underneath x_1[n]. You will be shifting x_2[n] frequently, so don't draw it too permanently.
(3) Flip x_2[n] about the y-axis, call this x_3.
(4) For EVERY value of n, compute the infinite sum associated with convolution for a fixed n. This infinite sum is computed by adding the element-wise product of x_1 and the shifted version of x_3 at each value of tau. This value of the infinite sum is equal to y[n] for a fixed n. A value of n = 3, corresponds to shifting x_3[n] to the right by 3. A value of n = -5, corresponds to shifting x_3[n] to the left by 5.

3 Convolution example

Let x[n] = [1, -1]
Let h[n] = [3, 5, 1]

As a class, use the steps for graphical convolution to verify:

x[n] * h[n] = [3, 2, -4, -1]

4 Basic Problem

Let x_1 have length N, let x_2 have length M.

What is the length of x_3[n] = x_1[n] * x_2[n]

5 Faster Method for Convolution

There is a faster method for computing convolution, but it requires less understanding. This is what we call the "stack and sum" method:

$$x[n] * h[n] = y[n]$$

$$y[n] = h[0]x[n]$$

$$+[0\ \ h[1]x[n]]$$

$$+[0\ \ 0\ \ h[2]x[n]] + ...$$

$$+[0...(Nzeros)\ \ \ h[N-1]x[n]]$$

6 Stack and Sum Method

Let x[n] = [3, 5, 1]
Let h[n] = [2, 1, 6]

(a) Find x[n] * h[n] using the stack and sum method
(b) Find h[n] * x[n] using the stack and sum method
(c) Is part (a) the same as part (b)? What is the length of x[n] * h[n], does your answer agree with Problem 4?

7 Challenge Problem: Convolution

In analog signal processing, it was often easier to think about multiplication in the frequency domain, rather than convolution in the time domain. Here's a problem where it is probably easier to think in terms of convolution.

Let:

$$x[n] = [0\ 1\ 1\ 1]$$

$$h[n] = [\ldots\ ,\ -1\ ,2,\ -1,\ 2,\ -1\ ,\ \ldots]\ \text{ with } h[0] = -1$$

(a) Find y[n] = x[n] * h[n]
(b) Express X(ω) as a sum of three complex exponentials. Express H(w) as a sum of infinity complex exponentials.
(c) Find Y(ω) and take the inverse DTFT to obtain y[n], you should get the same answer as part a

8 Challenge Problem: Proofs

(a) Show that x[n] * h[n] = h[n] * x[n]
(b) Show the convolution property of the DTFT:

$$x[n] * h[n] \longleftrightarrow X(\omega)H(\omega)$$