Discussion Sols

P1

(a) $$H(z) = 8 + \frac{-7 + 8z^{-1}}{1 - .75z^{-1} + .125z^{-2}}$$

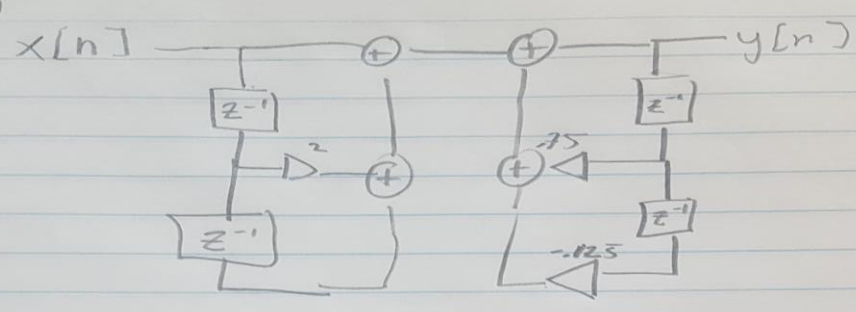
$$H(z) = \frac{8 - 6z^{-1} + z^{-2}}{1 - .75z^{-1} + .125z^{-2}} + \frac{-7 + 8z^{-1}}{1 - .75z^{-1} + .125z^{-2}}$$

$$H(z) = \frac{z^{-2} + 2z^{-1} + 1}{(1 - .75z^{-1} + .125z^{-2})}$$

$$Y(z)(1 - .75z^{-1} + .125z^{-2}) = X(z)(\cancel{2}\,1 + 2z^{-1} + z^{-2})$$

$$\boxed{y[n] = x[n] + 2x[n-1] + x[n-2] + .75y[n-1] - .125y[n-2]}$$

16.)



B P2

$$\frac{2z(z-\frac{1}{12})}{(z-\frac{1}{2})(z+\frac{1}{3})} = X(z)$$

get z^{-1} on top/bottom

(multiply by $\left(\frac{z^{-2}}{z^{-2}}\right)$)

$$\frac{2z(z^{-1})\,z^{-1}(z-\frac{1}{12})}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})}$$

$$\frac{2(1-\frac{1}{12}z^{-1})}{(1-\frac{1}{2}z^{-1})(1+\frac{1}{3}z^{-1})} = \frac{A}{1-\frac{1}{2}z^{-1}} + \frac{B}{1+\frac{1}{3}z^{-1}}$$

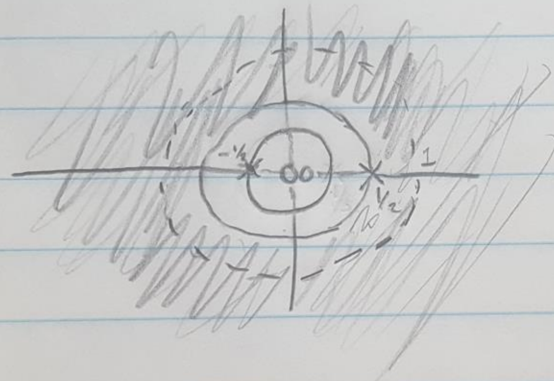
$$A, B = 1$$

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + \left(-\frac{1}{3}\right)^n u[n]$$

$$a^n u[n] \longleftrightarrow \frac{1}{1-az^{-1}}$$

ROC $|z| > |a|$

2.b.)



x[n] rightsided, causal, DTFT exists

2c.) right-sided, causal

P3

$\log(1+\alpha z^{-1})$ $|z|>|\alpha|$

take deriv.

$$\frac{\partial X(z)}{\partial z} = \frac{-\alpha z^{-2}}{1+\alpha z^{-1}}$$

$$-z\frac{\partial X(z)}{\partial z} = \frac{-\alpha z^{-1}}{1+\alpha z^{-1}}$$

$$= \frac{1}{1+\alpha z^{-1}} \text{ (shift) (scale)}$$

$a^n u[n]$

$a^{n-1} u[n-1]$

$(-\alpha)\,\alpha^{n-1} u[n-1] = n\,x[n]$

thus

$$x[n] = \frac{1}{n}(-\alpha)(\alpha)^{n-1} u[n-1]$$

Hint USE : $n x[n] \longleftrightarrow -z\frac{dx}{dz}$

P4

(a)

$$\alpha_1 \begin{bmatrix} -1 & -2 \\ 0 & 0 \end{bmatrix} + \alpha_2 \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} + \alpha_3 \begin{bmatrix} 0 & 0 \\ -3 & -3 \end{bmatrix} + \alpha_4 \begin{bmatrix} -1 & -1 \\ -1 & -2 \end{bmatrix} = 0$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 1$$

So not a valid basis

(b) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} = \alpha_1 \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 0 + 0 + 0$

$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \frac{1}{2}\left(\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\right)$ $\alpha_2 = -\alpha_3 = \frac{1}{2}$

$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \rightarrow \alpha_1 = -1 \quad \alpha_3 = 1 \quad \alpha_4 = -1$

$\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow \alpha_4 = 1$

So these are a valid basis

c.) Since the matrices in (b) is a basis for $\mathbb{R}^{2\times 2}$

yes! any matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ can be formed as a linear combination of a valid basis.