EECS351 Discussion 4 Problems, 09/26/16

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1 Discrete-domain Basis Sequences

Let $x[n] = \{\underline{1}, 1, 2, 3, 5\}$. Represent $x[n]$ in each of the following bases.

Note: You won't be asked to do anything like this on the homework or exams, but hopefully it will help you solidify your understanding of bases from lecture.

Unit Step Sequences

Recall $u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$. Let $\mathcal{S}_u = \{u[n - n_0] | n_0 \in \mathbb{Z}\}$ (unit steps shifted by any integer n_0) be your basis sequences. Represent $x[n]$ in $\mathcal{S}_u$.

In other words, show you can write $x[n] = \sum\limits_{k=-\infty}^{\infty} c_k u[n-k]$ by finding the values for c_k.

Three-Tap Rectangles

Let $r[n] = \{\underline{1}, 1, 1\}$ and $\mathcal{S}_r = \{r[n - n_0] | n_0 \in \mathbb{Z}\}$. Represent $x[n]$ in $\mathcal{S}_r$.

Three-Tap Triangle

Let $t[n] = \{\underline{1}, 2, 1\}$ and $\mathcal{S}_t = \{t[n - n_0] | n_0 \in \mathbb{Z}\}$. Represent $x[n]$ in $\mathcal{S}_t$.

2 Graphical Depiction of Change of Basis

(a) Draw an x and y axis in $\mathbb{R}^2$ representing the canonical basis. Represent the signal x = $[2\ 2]^T$ graphically in this basis. Show that B is orthonormal.
(b) Draw an two axes in $\mathbb{R}^2$ corresponding the following basis:

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, v_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

(c) Find the coordinates for x = $[2\ 2]^T$ with respect to the basis from part (b)
(d) Draw a graphical depiction of part (c) on the same axes you used for part (a). Verify that your signal vector is the same for both bases, even though your coordinates/coefficients will be different.

3 Inner Product

Show that the Euclidean dot product in $\mathbb{R}^n$ satisfies the following conditions of an inner product:

$$<\mathrm{x,y}> = \sum_{i=1}^{n} x_i y_i$$

(a) $<\mathrm{u} + \mathrm{v}, \mathrm{w}> = <\mathrm{u,w}> + <\mathrm{v,w}>$
(b) $<\mathrm{v,v}> \geq 0$

4 $\mathbb{R}^3$

Express the following signal of length three using any basis *other than the canonical basis* (be sure to use convenient axes):

$$x = \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix}$$

(b) Graphically depict your answer from part (a)