EECS351 Discussion 5 Problems, 10/13/16

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1 Digital Frequency vs. Real Frequency

$$f_{real} = \frac{\omega_{digital}}{2\pi T_s}$$

The highest digital frequency represented is π thus

$$f_{max} = F_s/2$$

The maximum frequency represented by a 44,100 sampling rate is 22,050. This is easy to remember because humans hear a frequency spectrum of approximately 20-20,000 Hz.

```
»N = length(x)
»frequencies = -Fs/2:Fs/N:Fs/2-Fs/N      // final -Fs/N term ensures length N
»plot(frequencies,fftshift(abs(fft(x))))
```

2 Discrete-Time Fourier Transform DTFT equations

$$X(\omega) = X_{2\pi}(w) = X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

$$x[n] = (1/2\pi)\int_{2\pi} X(\omega)e^{j\omega n}d\omega$$

3 Periodicity of DTFT

The DTFT is often written the following way, to denote that any DTFT is periodic with period 2π:

$$X_{2\pi}(w) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

Show that the DTFT is periodic with period 2π.

4 Linearity of the DTFT

The following property is called the Linearity Property of the DTFT:

$$\alpha x[n] + \beta y[n] \longleftrightarrow \alpha X(\omega) + \beta Y(\omega)$$

Proof: (always start with the original equation !!!!!!)

$$Z(e^{j\omega}) = \sum_{n=-\infty}^{\infty} z[n] e^{-j\omega n}$$

Let z[n] = $\alpha x[n] + \beta y[n]$

$$Z(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (\alpha x[n] + \beta y[n]) e^{-j\omega n}$$

$$Z(e^{j\omega}) = \sum_{n=-\infty}^{\infty}(\alpha x[n])e^{-j\omega n} + \sum_{n=-\infty}^{\infty}(\beta y[n])e^{-j\omega n}$$

$$Z(e^{j\omega}) = \alpha \sum_{n=-\infty}^{\infty}(x[n])e^{-j\omega n} + \beta \sum_{n=-\infty}^{\infty}(y[n])e^{-j\omega n}$$

$$Z(e^{j\omega}) = \alpha X(\omega) + \beta Y(\omega)$$

5 Shift in Time Property of DTFT

The following property is used to find the DTFT of a signal that's been shifted:

$$x[n-\alpha] \longleftrightarrow X(\omega)e^{-j\alpha\omega}$$

Using the definition of DTFT, show that the Shift in Time Property is true.

6 Modulation Property of DTFT (shift in frequency)

The following property is used to find the DTFT of a signal that's been shifted:

$$e^{j\alpha n}x[n] \longleftrightarrow X(\omega - \alpha)$$

Using the definition of DTFT, show that the Modulation Property is true.

7 Summary of (Some) DTFT Properties

$$\alpha x[n] + \beta y[n] \longleftrightarrow \alpha X(\omega) + \beta Y(\omega)$$
$$x[n-\alpha] \longleftrightarrow X(\omega)e^{-j\alpha\omega}$$
$$e^{j\alpha n}x[n] \longleftrightarrow X(\omega - \alpha)$$
$$x[kn] \longleftrightarrow X(\omega/k)$$
$$x[-n] \longleftrightarrow X(-\omega)$$
$$x[n]^* \longleftrightarrow X(-\omega)^*$$
$$x[-n]^* \longleftrightarrow X(\omega)^*$$
$$-(jn)x[n] \longleftrightarrow (d/d\omega)X(\omega)$$
$$x[n] * y[n] \longleftrightarrow X(\omega)Y(\omega)$$

8 Extra Problems: Cross Correlation

Recall the definition of convoluion:

$$a[n] * b[n] = z[n] = \sum_{\tau=-\infty}^{\infty} a[\tau]b[n-\tau]$$

There is an expression called the cross-correlation property of the DTFT:

$$a^*[-n] * b[n] \longleftrightarrow ?$$

(a) Find the DTFT of $a^*[-n] * b[n]$ using DTFT properties from part 6.
(b) Prove your answer from part (a) without using DTFT properties.

9 Extra Problems: Zero Padding 1

Let x_1[n] = $[x_0\ x_1\ x_2\ ...\ x_{N-1}]^T$, a signal of length N.

Now consider x_2, which is x_1 followed by N zeros:
Let x_2[n] = $[x_0\ x_1\ x_2\ ...\ x_{N-1}\ 0\ 0\ 0\ ...\ 0]^T$, a signal of length 2N.

You might be tempted to think we can gain some kind of information about the DTFT by adding zeros to the end of this signal. Find $X_2(\omega)$ in terms of $X_1(\omega)$ by expanding the DTFT equation one term at a time.

10 Extra Problems: Zero Padding 2

Let x_1[n] = $[x_0\ x_1\ x_2\ ...\ x_{N-1}]^T$, a signal of length N.

Now consider x_3, which is x_1 with a zero inserted between each index:
Let x_3[n] = $[x_0\ 0\ x_1\ 0\ x_2\ ...\ 0\ x_{N-1}\ 0\ ]^T$, a signal of length 2N.

You might be tempted to think we can gain some kind of information about the DTFT by inserting zeros into this signal. Find $X_3(\omega)$ in terms of $X_1(\omega)$ by expanding the DTFT equation one term at a time.