Set Theory

1 Equivalence

The law of equivalence

$$a = a$$

2 Definition of element $\{a\}$

$$\{a\} := a$$

3 Definition of Empty Set $\{\}$

$$\varnothing := \{\}$$

4 Definition of Cardinality of element a $|a|$

$$|\varnothing| := 0$$
$$|\{a\}| := 1$$

5 Definition of Union $\cup$

$$\varnothing \cup \{a\} := \{a\}$$
$$\{a_x\} \cup \{a_y\} := \{a_x; a_y\}$$

5.1 Notation

Union $\cup$ can be denoted as ,

$$\cup = ,$$

5.2 Equivalent Expressions

$$\varnothing, a = \varnothing \cup a = \{\} \cup \{a\} = \{\}, \{a\}$$
$$a_x, a_y = a_x \cup a_y = \{a_x\} \cup \{a_y\} = \{a_x\}, \{a_y\}$$

6 Definition of Intersection $\cap$

$$\varnothing \cap a := \varnothing$$

$$a_x \cap a_y := \varnothing$$

$$a_x \cap a_x := a_x$$

7 Definition of Deletion $\setminus$

$$a_x \ \setminus \ a_x := \varnothing$$

8 Definition of Set S

$$S := a_1 \cup a_2 \cup ... \cup a_N$$

$$= \{a_1\} \cup \{a_2\} \cup ... \cup \{a_N\}$$

$$= \{a_1;\ a_2;\ ... ; a_N\}$$

9 Definition of Magnitude of a Set $|S|$

$$S := a_1 \cup a_2 \cup ... \cup a_N = \{a_1;\ a_2;\ ... ; a_N\}$$

10 Definition of In $\in$

11 Definition of For All $\forall$

12 Definition of Vector

13 Null Property of Vector V

$$< \varnothing_i, \varnothing_j, ... >= \varnothing$$

13.1 Proof

$$< \varnothing_i, \varnothing_j, ... >:= \{\varnothing_i, \varnothing_j, ...\}$$

$$\{\varnothing_i \cup \varnothing_j \cup ...\}$$

$$\{\varnothing\}$$

14 Properties of Empty Set $\varnothing$

14.1 $\varnothing_c = \varnothing_d$

Prove $\varnothing_c = \varnothing_d$

$$\varnothing_c := \{\}$$
$$\varnothing_d := \{\}$$
$$\therefore \varnothing_c = \{\} = \varnothing_d$$

14.2 $\varnothing \cup \varnothing = \varnothing$

Prove

$$\varnothing \cup \varnothing = \varnothing$$

$$\varnothing := \{\}$$
$$\varnothing \cup \varnothing = \{\} \cup \{\} := \{\} = \varnothing$$
$$\therefore \varnothing \cup \varnothing = \varnothing$$

15 Definition of Universal Set

Define Universal Set

$$\Omega := s_i \in \Omega, \forall i$$

16 Set Union $\cup$

Define $\cup$ the union of two elements

$$\cup[l_i, l_j] = l_i \cup l_j = \{l_i\} \cup \{l_j\} :=$$
$$\begin{gathered} \cup[l_i, l_j] \rightarrow \{l_i\} \;\; i = j \\ \cup[l_i, l_j] \rightarrow \{l_i, l_j\} \;\; i \neq j \end{gathered}$$

16.1 Translation

$\cup$ is often read as "and"

16.2 Comma ,

In set notation the comma "," denotes union $\cup$

$$l_1 \cup l_2 = \{l_1\} \cup \{l_2\} = \{l_1, l_2\}$$

17 Set Intersection $\cap$

Define $\cap$, the intersection of two elements

$$l_1 \cap l_2 = \{l_1\} \cap \{l_2\}$$

$$\cap[l_i, l_j] \rightarrow \{l_i\} \;\; i = j$$
$$\cap[l_i, l_j] \rightarrow \varnothing \;\; i \neq j$$

18 Set Subtraction $\setminus$

19 Sets

19.1 Definition

Define set S as an ordered union of elements s_i

$$S := s_1 \cup s_2 \cup \;\ldots \cup s_{n-1} \cup \;\; s_n = \{s_1, s_2, ..s_N\}$$

19.2 Alternate Notation

$$S := s_i \in S : i = 1, 2, ..., N-1, N$$
$$S = \{s_1, s_2, ..., s_{N-1}, s_N\}$$

19.3 Magnitude of a Set

$$|S| = |\{x_1, ..., x_N\}| = N$$

19.4 Counting

$$1, 2, ..., N = 1 : N$$

19.5 Definition Unordered Set

Set S is unordered if

$$S = \{x_1, x_2, ..x_n\} :=$$
$$x_i, x_j \in S; \;\; x_i = x_j; \quad \forall i, j \neq i$$

19.6 Definition of Unique Set

$$a_i, a_j \in S$$
$$a_i \neq a_j \quad \forall i, j \neq i$$

19.7 Definition of Countable/Uncountable set

Potentially just a line?

19.8 Define line $\mathbb{L}$

Define line $\mathbb{L}$

$$\mathbb{L} := \{l_0, l_1, l_2, ..., l_{N-1}, l_N, l_{N+1}, ...$$
$$\iff \exists l_i \in \mathbb{L} \ \ \forall i$$

20 Hierarchy of Elements to Sets

Every element is a set, but not all sets are elements

21 Containment

21.1 Contains

21.2 Equals =

Define set equivalence =

$$S_1 \subseteq S_2; \quad S_2 \subseteq S_1 \iff S_1 = S_2$$

21.3 Subset

21.4 Proper Subset Citation

21.5 Definition of Complement

$$S = \{s_1, s_2, ..., s_N\}$$
$$S^C :=$$
$$s_j : \{s_j \in \Omega\} \cap \{s_j \notin S\}; \ \ \forall j$$

21.6 Alternate Notation

Wikipedia definition of complement

$$S^C = U - S = \{x \in \Omega : x \notin S\}$$

https://en.wikipedia.org/wiki/Complement_(set_theory)

Appendix

22 Criticism of Union $\cup$

23 Alternate Translations

23.1 Set

Collection

23.2 Magnitude

Count

23.3 Intersection

Mutual Elements

23.4 Union

And

24 Proofs and Properties

1. $\Omega \subset \varnothing$
2. $\Omega \cap \Omega = \Omega$
3. $\Omega \cup \Omega = \Omega$
4. $\Omega \cup \varnothing = \Omega$
5. $\Omega \cap \varnothing = \varnothing$
6. $\Omega \cap S = S$
7. $\Omega \cup S = \Omega$
8. $\varnothing \nsubseteq \Omega$
9. $\varnothing \cup \varnothing = \varnothing$
10. $\varnothing \cap \varnothing = \varnothing$

11. $\varnothing \cap S = \varnothing$

12. $\varnothing \cup S = S$

13. $\varnothing = \varnothing$

14. $\varnothing = \Omega^C$

15. $\Omega \subseteq S$