Logic

1 Definition of Bool b

Define Bool b

$$b :=$$

$b \Rightarrow \mathbb{T}$

$$b = \mathbb{T}$$

$b \Rightarrow \mathbb{F}$

$$b = \mathbb{F}$$

2 Definition of Not $\neg$

Define not b; $\neg$ b

$$\neg b :=$$

$b \Rightarrow \mathbb{T}$

$$\neg b = \mathbb{F}$$

$b \Rightarrow \mathbb{F}$

$$\neg b = \mathbb{T}$$

2.1 Alternate Notation

$$\neg b = !b$$

3 Definition of Contradiction

4 Definition of Logical Or

4.1 Definition of Logical Or $\vee$

Define Logical Or $\vee$

$$a \vee b :=$$

$a = \mathbb{F};\ b = \mathbb{F}$

$$a \vee b = \mathbb{F}$$

$a = \mathbb{F};\ b = \mathbb{T}$

$$a \vee b = \mathbb{T}$$

$a = \mathbb{T};\ b = \mathbb{F}$

$$a \vee b = \mathbb{T}$$

$a = \mathbb{T};\ b = \mathbb{T}$

$$a \vee b = \mathbb{T}$$

5 Definition of Logical And

5.1 Definition of Logical And $\wedge$

Define Logical Or $\wedge$

$$a \wedge b :=$$

$a = \mathbb{F};\ b = \mathbb{F}$

$$a \wedge b = \mathbb{F}$$

$a = \mathbb{F};\ b = \mathbb{T}$

$$a \wedge b = \mathbb{F}$$

$a = \mathbb{T};\ b = \mathbb{F}$

$$a \wedge b = \mathbb{F}$$

$a = \mathbb{T};\ b = \mathbb{T}$

$$a \wedge b = \mathbb{T}$$

6 Remaining 2 Bool Logical Definitions

Express explicitly; Express in terms of the above definitions

6.1 XOR

6.2 NOR

6.3 XNOR

6.4 NAND

7 Universality of Logical Expressions

7.1 Universality of Not $\neg$; Logical Or $\vee$; Logical $\wedge$

Appendix

8 Criticism logical union, set union, logical and, set and

- logical or is a function logical and is a function
- language muks up our understanding

Logical or $\vee$ is different from $\cup$ Logical and $\wedge$ is different from $\cap$
Logical or, only one has to be true
Logical and, both have to be true –> I'll take the intersection

Set and, I'll take bag 1 and bag 2 i'll take both –> I'll take the union
set or, I'll take bag 1 or bag 2 I'll take just one

Do we ever confuse set union, set and with logical or, and?
(Don't we describe set union $\cup$ as "or")