Canonical Comparators

1 Comparators

1.1 Domain

Let statement s_j

$$s_j = s : (\mathbb{T} \doteq s_j) \oplus (\mathbb{F} \doteq s_j)$$

$$"The\ set\ of\ all\ arguments" =$$

$$\mathbb{L} := \{X_i, ...\} :$$

$$(\exists \doteq X_i) \wedge (X_i = s_1 \cup s_2 \cup ...)$$

1.2 Definition of a Comparator

Let

$$X_n^{comparator} = \{x_1, x_2, ..., x_n\} :$$
$$(\mathbb{T} \doteq x_1 \oplus \mathbb{F} \doteq x_1) \wedge (\mathbb{T} \doteq x_2 \oplus \mathbb{F} \doteq x_2) \wedge ... \wedge (\mathbb{T} \doteq x_n \oplus \mathbb{F} \doteq x_n)$$

$$"Comparator" = f[x_1, x_2, ...] \rightarrow y_o :=$$
$$(\mathbb{T} \doteq y_o) \oplus (\mathbb{T} \doteq y_o)\ \ \forall$$

2 ==

2.1 Definition

$$(a == b) = (== [a, b]) :=$$

a = b

$$a == b \rightarrow \mathbb{T}$$

a ≠ b

$$a == b \rightarrow \mathbb{F}$$

2.2 Domain

$$\{a \mid a \in \{\mathbb{T}, \mathbb{F}\}\}$$
$$\{b \mid b \in \{\mathbb{T}, \mathbb{F}\}\}$$

3 $\vee$

3.1 Definition

$$a \vee b = \vee[a, b] :=$$

a = b = $\mathbb{T}$

$$a \vee b = \mathbb{T}$$

a = b = $\mathbb{F}$

$$a \vee b = \mathbb{F}$$

$\mathbb{T} = a \neq b = \mathbb{F}$

$$a \vee b = \mathbb{T}$$

$\mathbb{F} = a \neq b = \mathbb{T}$

$$a \vee b = \mathbb{T}$$

3.2 Domain

$$\{a \mid a \in \{\mathbb{T}, \mathbb{F}\}\}$$
$$\{b \mid b \in \{\mathbb{T}, \mathbb{F}\}\}$$

4 $\wedge$

4.1 Definition

$$a \wedge b = \wedge[a, b] :=$$

a = b = $\mathbb{T}$

$$a \wedge b = \mathbb{T}$$

a = b = $\mathbb{F}$

$$a \wedge b = \mathbb{F}$$

$\mathbb{T} = a \neq b = \mathbb{F}$

$$a \wedge b = \mathbb{F}$$

$\mathbb{F} = a \neq b = \mathbb{T}$

$$a \wedge b = \mathbb{F}$$

4.2 Domain

$$\{a \mid a \in \{\mathbb{T}, \mathbb{F}\}\}$$
$$\{b \mid b \in \{\mathbb{T}, \mathbb{F}\}\}$$

5 $>$

5.1 Definition

$$\begin{aligned} a > b \ &= \ > [a, b] := \\ 0 > 1 \ &= \ > [0, 1] \to \mathbb{F} \\ 1 > 0 \ &= \ > [1, 0] \to \mathbb{T} \end{aligned}$$

5.2 Domain

$$\{a \mid a \in \{0, 1\}\}$$
$$\{b \mid b \in \{0, 1\}\}$$

6 $<$

6.1 Definition

$$
\begin{aligned}
a < b &= < [a, b] := \\
0 < 1 &= < [0, 1] \to \mathbb{T} \\
1 < 0 &= < [1, 0] \to \mathbb{F}
\end{aligned}
$$

6.2 Domain

$$
\{a \mid a \in \{0, 1\}\}
$$
$$
\{b \mid b \in \{0, 1\}\}
$$

7 Prove all comparators can be expressed with $==$,$<$,$>$

8 $\neg$

8.1 Definition

$$\neg = \neg[b] :=$$

$$\neg[\mathbb{T}] \to \mathbb{F}$$

$$\neg[\mathbb{F}] \to \mathbb{T}$$

8.2 Domain

$$\{b \mid b \in \{\mathbb{T}, \mathbb{F}\}\}$$

9 $\cup$

9.1 Definition

$$a \cup b = \cup[a, b] :=$$

$a = b \neq \varnothing$

$$a \cup a = b \cup b \to \{a\} = \{b\}$$

$\varnothing \neq a \neq b \neq \varnothing$

$$a \cup b \to \{a, b\}$$

$\varnothing = a \neq b$

$$\varnothing \cup b \to \{b\}$$

$a \neq b = \varnothing$

$$a \cup \varnothing \to \{a\}$$

$a = b = \varnothing$

$$\varnothing \cup \varnothing \to \varnothing$$

9.2 Domain

$$\{a \mid a \in \Omega\}$$
$$\{b \mid b \in \Omega\}$$

10 $\setminus$

10.1 Definition

$$a \setminus b = \setminus[a, b] :=$$

$a \neq b = \varnothing$

$$a \setminus b \to a$$

$a = b$

$$a \setminus b \to \varnothing$$

$\varnothing = a \neq b$

$$\text{Undefined}$$

10.2 Domain

$$\{a \mid a \in \Omega\}$$
$$\{b \mid b \subseteq a\}$$

11 $\cap$

11.1 Definition

$$a \cap b = \cap[a, b] :=$$

a = b

$$a \cap a = b \cap b \rightarrow \{a\} = \{b\}$$

$a \neq b$

$$a \cap b \rightarrow \varnothing$$

11.2 Domain

$$\{a \mid a \in \Omega\}$$

$$\{b \mid b \in \Omega\}$$

12 Cardinality | |

12.1 Definition

$$|S| = |\ |[\mathrm{S}]:=$$

$S = \varnothing$

$$|S| \to 0$$

$S = \{s_1\}$

$$|S| = |\{s_1\}| \to 1$$

$S = \{s_1, s_2, ..., s_N\}$

$$|S| = |\{s_1, s_2, ..., s_N\}| \to N$$

12.2 Domain

$$\{S \mid S \subset \Omega\}$$

13 Definition of Get

$$\text{"}get\text{"} = get[a] :=$$

$\exists \doteq a$

$$get[a] \to a$$

13.1 Domain

$$\{a \mid a \in \Omega\}$$

14 Definition of Assign

$$\text{"}set\text{"} = set[a] :=$$

$\exists \doteq a$

$$set[a]$$

14.1 Domain

$$\{a \mid a \in \Omega\}$$