

Functions

1 + Addition

1.1 Definition

$\forall S_i, S_j$

$$+[S_i, S_j] := |S_i \cup S_j| + |S_i \cap S_j| \quad [1]$$

1.2 Alternative Definition

Alternatively $+$ can be defined as the magnitude of a disjoint union

$$+[S_i, S_j] := |S_i \cup S_j|$$

$$\forall i, j : S_i \cap S_j = \emptyset$$

2 \ Set Difference

2.1 Definition

Define \setminus , the operator for (subset) subtraction.

$$\begin{aligned} S_1 \subseteq S_2 \\ \setminus[S_1, S_2] = S_1 \setminus S_2 = \{s_i, \dots\} : s_i \notin S_1, s_i \notin S_2 \quad \forall i \end{aligned}$$

2.2 Notation

3 Assignment \leftarrow

3.1 Elemental Definition

3.2 Set Definition

$$\begin{aligned} \leftarrow [S, \{x_1, \dots, x_N\}] = S \leftarrow \{x_1, \dots, x_N\} := \\ S = \emptyset \cup \{x_1, \dots, x_N\} = \{x_1, \dots, x_N\} \end{aligned}$$

4 Delete

4.1 Elemental Definition

4.2 Set Definition

$$delete [S] :=$$

$$\text{delete } [S] \rightarrow S = \emptyset$$

5 Append

5.1 Elemental Definition

5.2 Set Definition

$$\begin{aligned} \text{append } [S_1, S_2] &:= \\ S_1 &= \{a_1, a_2, \dots, a_N\}; \quad S_2 = \{b_1, b_2, \dots, b_M\} \\ \text{append } [S_1, S_2] \rightarrow S_1 &= \{a_1, a_2, \dots, a_N, b_1, b_2, \dots, b_M\} = \\ S_1 &= \{a_1, a_2, \dots, a_N, b_{N+1}, b_{N+2}, \dots, b_{N+M}\} \end{aligned}$$

6 Insertion

6.1 Elemental Definition

6.2 Set Definition

$$\begin{aligned} \text{insert } [S_1, S_2, K] &:= \\ S_1 &= \{a_1, a_2, \dots, a_N\}; \quad S_2 = \{b_1, b_2, \dots, b_M\}; \quad 0 \leq K \leq N \\ \text{insert } [S_1, S_2, K] \rightarrow S_1 &= \{a_1, a_2, \dots, a_K, b_1, b_2, \dots, b_M, a_{K+1}, \dots, a_N\} = \\ S_1 &= \{a_1, a_2, \dots, a_K, b_{K+1}, b_{K+2}, \dots, b_{K+M}, a_{K+M+1}, \dots, a_{M+N}\} \end{aligned}$$

7 Remove

7.1 Indexed Definition

$$\begin{aligned} \text{remove } [S, K] &:= \\ S &= \{a_1, a_2, \dots, a_N\}; \quad 1 \leq K \leq N \\ \text{remove } [S, K] \rightarrow &\{a_1, a_2, \dots, a_{K-1}, a_{K+1}, \dots, a_N\} \end{aligned}$$

7.2 Elemental Definition

$$\begin{aligned} \text{remove } [S, a_M] &:= \\ S &= \{a_1, a_2, \dots, a_N\}; \quad 1 \leq M \leq N \\ \text{remove } [S, a_M] \rightarrow &\{a_1, a_2, \dots, a_{M-1}, a_{M+1}, \dots, a_N\} \end{aligned}$$

7.3 Set Definition

8 Iteration \mathbb{C}

8.1 Definition

Define iteration \mathbb{C}

$$\begin{aligned}\mathbb{C}[S, k, n, l] &:= \\ S &= \{s_1, s_2, \dots, s_n\} \\ \mathbb{C}[S, k, n] &\rightarrow\end{aligned}$$

8.2 Define $*$ = \mathbb{C}

Define multiplication $x * n$

$$\begin{aligned}*[x, n] &= x * n := \\ S &= \{s_1, s_2, \dots, s_x\} \\ x * n &\rightarrow |\mathbb{C}[S, 0, x, n]|\end{aligned}$$

9 Nested Iteration \mathbb{C}^n

9.1 Definition

Define nested iteration \mathbb{C}^n

9.2 Definition Exponentiation

Define exponentiation x^n

$$\begin{aligned}exponentiate[x, N] &= x^N := \\ S &= \{s_1, s_2, \dots, s_x\} \\ x^N &\rightarrow |\mathbb{C}^n[S, 0, x, x, N]|\end{aligned}$$

10 Division

10.1 Definition

Define division, iterative subtraction with remainder

Citations

[1] <https://en.wikipedia.org/wiki/Cardinality>