Math

1 Definition of a line $\mathbb{L}$

1.1 Definition

$$\mathbb{L} := \{l_i, ...$$
$$\exists l_i \in \mathbb{L} \quad \forall i$$

2 Whole Numbers $\mathbb{W}$

2.1 Definition

Define the positive number line $\mathbb{W}$, the set of whole numbers

$$\mathbb{W} :=$$
$$|S_i| \in \mathbb{W} \quad \forall i$$

3 Injection (one-to-one)

3.1 Definition[1]

$$S_1 = \{a_1, a_2, ..., a_n\}; \;\; S_2 = \{b_1, b_2, ..., b_m\} \;\; m \geqslant n$$
$$injective[S_1, S_2] \rightarrow (\forall a_i \in S_1 \;\; \exists b_j \in S_2 : f[a_i] \rightarrow b_j)$$
$$\forall a_i, a_j \in S_1; f[a_i] = f[a_j] \Longleftrightarrow a_i = a_j \;\; [2]$$

4 Surjection (onto)

4.1 Definition [2]

$$S_1 = \{a_1, a_2, ..., a_n\}; \;\; S_2 = \{b_1, b_2, ..., b_m\} \;\; m \geqslant n$$
$$surjective[S_1, S_2] \rightarrow (\forall a_i \in S_1 \;\; \exists b_j \in S_2 : f[a_i] \rightarrow b_j)$$

5 Bijection (One-to-one and onto)

Also known as invertible

5.1 Definition [3]

$$S_1 = \{a_1, a_2, ..., a_n\}; \;\; S_2 = \{b_1, b_2, ..., b_n\}$$
$$Invertible[S_1, S_2] \rightarrow$$
$$(\forall a_i \in S_1 \;\; \exists b_j \in S_2 : f[a_i] \rightarrow b_j) \wedge (\forall b_j \in S_2 \;\; \exists a_i \in S_1 : g[b_j] \rightarrow a_i)$$

6 Hierarchy of Bijections Surjections and Injections

7 Vector

Define vector as a set of more than one cardinalities

7.1 Definition

$$\vec{v} :=$$
$$|S_1| = x_1; |S_2| = x_2; ...; |S_N| = x_N$$
$$\vec{v} = \{|S_1|, |S_2|, ..., |S_N|\} =< x_1, x_2, ..., x_N >$$

7.2 Dimensionality

$$dim[\vec{v}] = |\vec{v}| :=$$
$$\vec{v} = \{|S_1|, |S_2|, ..., |S_N|\} =< x_1, x_2, ..., x_N >$$
$$|\vec{v}| = N$$

8 Vector Space

Define Vector Space $\mathbf{V}$ as a set of more than one lines

8.1 Definition

$$\mathbf{V} := \{\mathbb{L}_1, \mathbb{L}_2, ..., \mathbb{L}_N\}$$

9 Spans

A function/system mapping to a vector space

9.1 Bijective Span

9.2 Injective Span

9.3 Surjective Span

10 Summation Notation

Define summation $\sum$; the notation for successive additions

$$\sum_{i=1}^{N} a :=$$
$$\sum_{i=1}^{N} a = a + \sum_{i=2}^{N} a =$$
$$a + a + \sum_{i=3}^{N} a = ... = a + a + ... + a = a * N$$

11 Definition of Series

Define series

$$series :=$$
$$\sum_{i=1}^{N} x_i = x_1 + x_2 + ... + x_N$$

12 Convergent Series

Defines the bound of a series; also known as a limit

12.1 Definition

$$f[n] = \sum_{i=1}^{n} x_i$$
$$convergent[f[n]] \rightarrow$$
$$\exists C :$$
$$(C > f[n] \quad \forall n) \wedge (\nexists K : C - \sum_{i=1}^{n} x_i > K \quad \forall n; K > 0)$$

12.2 Convergence

$$f[n] = \sum_{i=1}^{n} x_i$$
$$\forall f[n] : convergent[f[n]] \rightarrow \mathbb{T}$$
$$\lim_{n \rightarrow \infty} f[n] = C :=$$

$$C > f[n] \quad \forall n$$

$$\nexists K : C - \sum_{i=1}^{n} x_i > K \quad \forall n; K > 0$$

Appendix

13 Properties of Sums $\sum$

14 For all convergent series, there exists only one bound

$$\forall f[n] : convergent[f[n]] \rightarrow \mathbb{T}$$
$$\lim_{n\rightarrow\infty} f[n] = C_1; \lim_{n\rightarrow\infty} f[n] = C_2$$
$$\Longleftrightarrow C_1 = C_2$$

14.1 Proof

Proof by contradiction

Citations

[1] https://en.wikipedia.org/wiki/Bijection,_injection_and_surjection#Injection

[2] https://en.wikipedia.org/wiki/Bijection,_injection_and_surjection#Surjection

[3] https://en.wikipedia.org/wiki/Bijection,_injection_and_surjection#Bijection