Set Theory

1 Definition of Element a

$$"Element" := a :$$
$$\exists \doteq a \;\; \oplus \;\; \nexists \doteq a$$

2 Definition of Empty Set $\varnothing$

$$"Empty\ set" = \varnothing := a :$$
$$\nexists \doteq a$$

3 Definition of Cardinality of Element a

$\exists \doteq a$

$$|a| := 1$$

$\nexists \doteq a$

$$|\varnothing| := 0$$

4 Definition of Union $\cup$

Define Union $\cup$

5 Definition of Intersection $\cap$

6 Definition of Deletion $\setminus$

7 Definition of Set

8 Definition of Universal Set

Define Universal Set

$$\Omega := s_i \in \Omega, \forall i$$

9 Set Union $\cup$

Define $\cup$ the union of two elements

$$\cup[l_i, l_j] = l_i \cup l_j = \{l_i\} \cup \{l_j\} :=$$

$$\cup[l_i, l_j] \to \{l_i\} \;\; i = j$$
$$\cup[l_i, l_j] \to \{l_i, l_j\} \;\; i \neq j$$

9.1 Translation

$\cup$ is often read as "and"

9.2 Comma ,

In set notation the comma "," denotes union $\cup$

$$l_1 \cup l_2 = \{l_1\} \cup \{l_2\} = \{l_1, l_2\}$$

10 Set Intersection $\cap$

Define $\cap$, the intersection of two elements

$$l_1 \cap l_2 = \{l_1\} \cap \{l_2\}$$

$$\cap[l_i, l_j] \to \{l_i\} \;\; i = j$$
$$\cap[l_i, l_j] \to \varnothing \;\; i \neq j$$

11 Set Subtraction $\setminus$

12 Sets

12.1 Definition

Define set S as an ordered union of elements s_i

$$S := s_1 \cup s_2 \cup \;\; ... \cup s_{n-1} \cup \;\; s_n = \{s_1, s_2, ..s_N\}$$

12.2 Alternate Notation

$$S := s_i \in S : i = 1, 2, ..., N-1, N$$
$$S = \{s_1, s_2, ..., s_{N-1}, s_N\}$$

12.3 Magnitude of a Set

$$|S| = |\{x_1, ..., x_N\}| = N$$

12.4 Counting

$$1, 2, ..., N = 1 : N$$

12.5 Definition Unordered Set

Set S is unordered if

$$S = \{x_1, x_2, ..x_n\} :=$$
$$x_i, x_j \in S; \;\; x_i = x_j; \quad \forall i, j \neq i$$

12.6 Definition of Unique Set

$$a_i, a_j \in S$$
$$a_i \neq a_j \quad \forall i, j \neq i$$

12.7 Definition of Countable/Uncountable set

Potentially just a line?

12.8 Define line $\mathbb{L}$

Define line $\mathbb{L}$

$$\mathbb{L} := \{l_0, l_1, l_2, ..., l_{N-1}, l_N, l_{N+1}, ...$$
$$\iff \exists l_i \in \mathbb{L} \;\; \forall i$$

13 Hierarchy of Elements to Sets

Every element is a set, but not all sets are elements

14 Containment

14.1 Contains

14.2 Equals =

Define set equivalence =

$$S_1 \subseteq S_2; \quad S_2 \subseteq S_1 \iff S_1 = S_2$$

14.3 Subset

14.4 Proper Subset Citation

14.5 Definition of Complement

$$S = \{s_1, s_2, ..., s_N\}$$
$$S^C :=$$
$$s_j : \{s_j \in \Omega\} \cap \{s_j \notin S\}; \quad \forall j$$

14.6 Alternate Notation

Wikipedia definition of complement

$$S^C = U - S = \{x \in \Omega : x \notin S\}$$

https://en.wikipedia.org/wiki/Complement_(set_theory)

Appendix

15 Proofs and Properties

1. $\Omega \subset \varnothing$
2. $\Omega \cap \Omega = \Omega$
3. $\Omega \cup \Omega = \Omega$
4. $\Omega \cup \varnothing = \Omega$
5. $\Omega \cap \varnothing = \varnothing$
6. $\Omega \cap S = S$
7. $\Omega \cup S = \Omega$
8. $\varnothing \nsubseteq \Omega$
9. $\varnothing \cup \varnothing = \varnothing$
10. $\varnothing \cap \varnothing = \varnothing$
11. $\varnothing \cap S = \varnothing$
12. $\varnothing \cup S = S$
13. $\varnothing = \varnothing$
14. $\varnothing = \Omega^C$
15. $\Omega \subseteq S$