Logic

1 Not $\neg$

1.1 Definition

$$\neg a = \neg[a] :=$$
$$\neg[\mathbb{T}] \to \mathbb{F}$$
$$\neg[\mathbb{F}] \to \mathbb{T}$$

2 Logical And $\wedge$

2.1 Definition

$$a \wedge b = \wedge[a, b] :=$$
$$\neg(\ \exists\ \mathbb{F} \in \{a, b\}) =$$
$$\nexists\ \mathbb{F} \in \{a, b\}$$

2.2 Alternate Definition

$$\wedge[\mathbb{F}, \mathbb{F}] = \mathbb{F}$$
$$\wedge[\mathbb{F}, \mathbb{T}] = \mathbb{F}$$
$$\wedge[\mathbb{T}, \mathbb{F}] = \mathbb{F}$$
$$\wedge[\mathbb{T}, \mathbb{T}] = \mathbb{T}$$

3 Law of Non-Contradiction

Introduce the symbol for contradiction, proof by contradiction

4 All Logical Functions Can Be Built from Not And

4.1 Proof

5 Criticism logical union, set union, logical and, set and

Verify on Wiki
- logical or is a function logical and is a function
- language muks up our understanding

Logical or $\vee$ is different from $\cup$ Logical and $\wedge$ is different from $\cap$

Logical or, only one has to be true
Logical and, both have to be true –> I'll take the intersection

Set and, I'll take bag 1 and bag 2 i'll take both –> I'll take the union
set or, I'll take bag 1 or bag 2 I'll take just one

Do we ever confuse set union, set and with logical or, and?
(Don't we describe set union $\cup$ as "or")

Appendix

6 Logical Or $\vee$

6.1 Definition

$$a \vee b = \vee[a, b] :=$$
$$\exists\, \mathbb{T} \in \{a, b\}$$

6.2 Alternate Definition

$$\vee[\mathbb{F}, \mathbb{F}] = \mathbb{F}$$
$$\vee[\mathbb{F}, \mathbb{T}] = \mathbb{T}$$
$$\vee[\mathbb{T}, \mathbb{F}] = \mathbb{T}$$
$$\vee[\mathbb{T}, \mathbb{T}] = \mathbb{T}$$

7 Exclusive Or (Xor)

7.1 Definition

7.2 Alternate Definition

8 Not Or (Nor)

8.1 Definition

8.2 Alternate Definition

9 Exclusive Nor (Xnor)

9.1 Definition

9.2 Alternate Definition

10 Not And (Nand)

10.1 Definition

10.2 Alternate Definition