Ch. 4 Computation

1 Definition of a Program

Program P is defined as an ordered set of logical operations s_i

$$P \equiv \{s_1, s_2, ..., s_N\} \quad (Definition 3)$$

2 Definition of a Decision Problem

A decision problem is a program whos output is True/False

$$d_i \equiv \{s_1, s_2, ..., return \quad 0/1, ..., s_N - 1, return \quad 0/1\} \quad (Definition 4)$$

2.1 Time Complexity of a Decision Problem

The Time Complexity $O_T[n]$ of Decision Problem d

$$O_T[n] <= n(d) (Theorem 1)$$

3 Definition Proper Decision Problem

$$D \equiv \{s_1, s_2, , ..., s_N - 1, return \quad 0/1\}$$

3.1 Time Complexity of a Proper Decision Problem

The Time Complexity $O_T[n]$ of Proper Decision Problem D

$$O_T[n] = n(d) (Theorem 2)$$

4 Definition of Complexity

Define Complexity $O[n]$ as a Tensor of dimension N

$$\mathbf{O}[n] \equiv \quad < O_T[n], O_S[n], ..., O_N[n] > \quad (Definition 1)$$

4.1 Time Complexity

Define Time Complexity O_T as the maximum number of logical operations in a Program P

$$O_T[n] \equiv n(P) \quad (Definition 2)$$

4.2 Space Complexity

Define Time Complexity O_T as the maximum number of bits required to complete Program P

5 Definition of Polynomial Time Complexity

A proper decision problem D with Time Complexity $O_T[n]$ can be solved with Polynomial Time Complexity if

$$\exists K, C \quad \ni \quad O_T[n] < n^K + C, \quad \forall n$$

5.1 Definition of Polynomial Problems

Define P, the set of Proper Decision Problems that can be solved with Polynomial Time Complexity

$$P \equiv \{D_1, D_2, .., D_N\}$$
$$\exists K, C \quad \ni \quad O_T[n] < n^K + C, \quad \forall D_i \in P$$

5.2 Proof of the existence of P

Trivial

5.3 Definition of Non-Polynomial Problems

Define $\mathcal{N}$, the set of Proper Decision Problems that cannot be solved with Polynomial Time Complexity

$$\mathcal{N} \equiv \{D_1, D_2, .., D_N\}$$
$$\nexists K, C \quad \ni \quad O_T[n] < n^K + C, \quad \forall D_i \in P$$

5.4 Proof of the existence of $\mathcal{N}$

Non-trivial

5.5 Definition of Divergent Programs

A program is a function that solves

6 Fundamental Theorem of Computation - "Theorem of Divergent Programs"

$$O_\perp[n] \equiv (O[n])^n$$

7 Proof of "P $\neq$ NP"