Logic

1 Statement s

$$"Statement" := s :$$
$$\mathbb{T} \doteq s \;\oplus\; \mathbb{F} \doteq s$$

2 Definition of Positive Implication

$$"Positive\ Implication" := s_{in} \cup s_{result} :$$
$$\mathbb{T} \doteq s_{in} \Rightarrow \mathbb{T} \doteq s_{result}$$

3 Definition of Negative Implication

$$"Negative\ Implication" := s_{in} \cup s_{result} :$$
$$\mathbb{F} \doteq s_{in} \Rightarrow \mathbb{F} \doteq s_{result}$$

4 Definition of Preventative Implication

$$"Preventative\ Implication" := s_{in} \cup s_{result} :$$
$$\mathbb{T} \doteq s_{in} \Rightarrow \mathbb{F} \doteq s_{result}$$

5 Definition of Consequential Implication

$$"Consesquential\ Implication" := s_{in} \cup s_{result} :$$
$$\mathbb{F} \doteq s_{in} \Rightarrow \mathbb{T} \doteq s_{result}$$

6 Fact t

6.1 Definition of "Fact" t

$$"Fact" := t :$$
$$\mathbb{T} \doteq t$$

6.2 Prove a fact is a statement

6.3 English Examples

Let $t =$ Five plus five is equal to ten

$$\text{Five plus five is equal to ten is a true statement}$$
$$\text{Flive plus five is equal to ten is a fact}$$

7 Lie l

7.1 Definition of "lie" l

$$"lie" := l :$$
$$\mathbb{F} \doteq l$$

7.2 Prove a "lie" is a statement

7.3 English Example

Consider the statement five plus five is equal to nine

$$\text{Five plus five is equal to nine is a false statement}$$
$$\text{Five plus five is equal to nine is a lie}$$

8 Not s $\neg$ s

8.1 Definition of "Not s" $\neg$ s

$$\neg s :=$$

$\mathbb{T} \doteq s$

$$\mathbb{F} \doteq \neg s$$

$\mathbb{F} \doteq s$

$$\mathbb{T} \doteq \neg s$$

8.2 English Example

Let

$$s = \text{Humans can travel faster than the speed of light.}$$
$$\neg s = \text{Humans can not travel faster than the speed of light.}$$

$$\mathbb{F} \doteq s$$
$$\text{s is false}$$

$$\mathbb{T} \doteq \neg s$$
$$\neg s \text{ is a fact}$$

9 Definition of Or $\vee$

The law of excluded middle

9.1 The Law of Total Equivalence

10 Definition of And $\wedge$

The law of non-contradiction

$$s \wedge \neg s = \mathbb{F}$$

10.1 Definition of Contradiction

$$s \wedge \neg s = \mathbb{T}$$

11 Remaining 2 Variable Logical Definitions

Express explicitly; Express in terms of the above definitions

11.1 XOR

11.2 NOR

11.3 XNOR

11.4 NAND

12 Universality of Logical Expressions

12.1 Universality of Not $\neg$; Logical Or $\vee$; Logical $\wedge$

Appendix

13 "-ness"

Happy is an adjective
Happyiness is a noun
The dog is happy
The dog has happiness
Happiness is a (current or permanent) quality of the dog

14 is vs is a

The cat is a feline
The cat is a member of the set of felines
vs
The cat is hairy
The cat has hair. The cat has the quality of hairyness
vs
The cat is a hairy cat
The cat is a member of the set of cats having the quality of hairyness

15 Overloaded "is"

Object c is a cat
c = cat

That cat is a feline
cat = feline?
cat $\Rightarrow$ feline
cat inherits felineness

That cat is hairy
cat = hairy?
The cat has hairyness; the quality of having hair

16 Criticism of "Is True" and "Is False"

Consider true statement t
"Statement t is true" is equivalent to saying "Statement t equals True"
Statement t is not equivalent to True. Statement has the property of truth.

Consider false statement f
"Statement f is false" is equivalent to saying "Statement f equals False"
False statement f is not equivalent to False. Statement f has the property of falsehood.

17 English Translation of Logical Or $\vee$

b_1 Logical Or b_2 is spoken in English as "at least one of the following is true. b_1. b_2."

17.1 English Example

At least one of the following is true.
Most dogs have four legs.
Two plus three is equal to 5.

17.2 Criticism of "Logical Or" In Computer Science

In Computer Science, b_1 Logical Or b_2 is often spoken as "b_1 or b_2". "b_1 or b_2" can lead to inconsistent statements.

$$b_1 = (\text{int } 3 \subset [1, 2, 3])$$
$$b_2 = ((2 + 2) == 5)$$

The following is a valid expression in Computer Science

$$b_1 \vee b_2 = \mathbb{T}$$

The expression is read in English as "3 is in the list 1 2 3 or 2 plus 2 is equal to 5". The expression is True by definition but b_1 or b_2 do not necessarily imply Truth.

$$b_1 = (\text{int } 3 \subset [1, 2, 3])$$
$$b_2 = (\text{int } 3 \not\subset [1, 2, 3])$$

The following is a valid expression in Computer Science

$$b_1 \vee b_2 = b_1 \vee \neg b_1 = \mathbb{T}$$

The expression is read in English as "3 is in the list 1 2 3 or 3 is not in the list 1 2 3 is True". The expression is necessarily true.

Now consider

$$b_1 \wedge b_2 = b_1 \wedge \neg b_1 = \mathbb{F}$$

The expression is read in English as "3 is in the list 1 2 3 and 3 is not in the list 1 2 3 is True". The expression in computer science evaluates to false

18 English Translation of "Exclusive Or" XOR

"b_1 Exclusive Or b_2" is read in English as "Either b_1 Or b_2"

18.1 Example

Either
Three plus four is equal to seven
or
Three plus four is equal to eight

18.2 Criticism of English Expression of "Exclusive Or"

"Exclusive Or" is often expressed as "Or" in English.
I can order the salad for lunch
or
I can order tofu for lunch

19 Commentary on "Logical And"

In English, "b_1 Logical And b_2" is read as "And"

19.1 Example

Most dogs have four legs
and
Most cats have four legs

20 Criticism logical union, set union, logical and, set and

- logical or is a function logical and is a function
- language muks up our understanding

Logical or $\vee$ is different from $\cup$ Logical and $\wedge$ is different from $\cap$
Logical or, only one has to be true
Logical and, both have to be true –> I'll take the intersection

Set and, I'll take bag 1 and bag 2 i'll take both –> I'll take the union
set or, I'll take bag 1 or bag 2 I'll take just one

Do we ever confuse set union, set and with logical or, and?
(Don't we describe set union $\cup$ as "or")