

Logic

1 Definition of Bool b

Define Bool b

$$b := \mathbb{T}$$

2 Definition of Not $\neg b$

Define Bool b

$$\neg b := \mathbb{F}$$

3 Definition of Logical Or

3.1 Definition of Logical Or \vee

$$\vee :=$$

4 Definition Logical And

The law of non-contradiction

$$b_1 \wedge \neg b_1 = \mathbb{F}$$

5 Contradiction

6 Remaining 2 Variable Logical Definitions

Express explicitly; Express in terms of the above definitions

6.1 XOR

6.2 NOR

6.3 XNOR

6.4 NAND

7 Universality of Logical Expressions

7.1 Universality of Not \neg ; Logical Or \vee ; Logical \wedge

8 Definition of Not \neg b

Define not b

$$\neg b := \mathbb{F}$$

9 Definition of Contradiction

Appendix

10 English Translation of Logical Or \vee

b_1 Logical Or b_2 is spoken in English as "at least one of the following is true.
 b_1 . b_2 ."

10.1 English Example

At least one of the following is true.
Most dogs have four legs.
Two plus three is equal to 5.

10.2 Criticism of "Logical Or" In Computer Science

In Computer Science, b_1 Logical Or b_2 is often spoken as " b_1 or b_2 ". " b_1 or b_2 " can lead to inconsistent statements.

$$b_1 = (\text{int } 3 \in [1, 2, 3])$$

$$b_2 = ((2 + 2) == 5)$$

The following is a valid expression in Computer Science

$$b_1 \vee b_2 = \mathbb{T}$$

The expression is read in English as "3 is in the list 1 2 3 or 2 plus 2 is equal to 5". The expression is True by definition but b_1 or b_2 do not necessarily imply Truth.

$$b_1 = (\text{int } 3 \in [1, 2, 3])$$

$$b_2 = (\text{int } 3 \notin [1, 2, 3])$$

The following is a valid expression in Computer Science

$$b_1 \vee b_2 = b_1 \vee \neg b_1 = \mathbb{T}$$

The expression is read in English as "3 is in the list 1 2 3 or 3 is not in the list 1 2 3 is True". The expression is necessarily true.

Now consider

$$b_1 \wedge b_2 = b_1 \wedge \neg b_1 = \mathbb{F}$$

The expression is read in English as "3 is in the list 1 2 3 and 3 is not in the list 1 2 3 is True". The expression in computer science evaluates to false

11 English Translation of "Exclusive Or" XOR

" b_1 Exclusive Or b_2 " is read in English as "Either b_1 Or b_2 "

11.1 Example

Either
Three plus four is equal to seven
or
Three plus four is equal to eight

11.2 Criticism of English Expression of "Exclusive Or"

"Exclusive Or" is often expressed as "Or" in English.
I can order the salad for lunch
or
I can order tofu for lunch

12 Commentary on "Logical And"

In English, " b_1 Logical And b_2 " is read as "And"

12.1 Example

Most dogs have four legs
and
Most cats have four legs

13 Criticism logical union, set union, logical and, set and

- logical or is a function logical and is a function
- language mucks up our understanding

Logical or \vee is different from \cup Logical and \wedge is different from \cap

Logical or, only one has to be true
Logical and, both have to be true \rightarrow I'll take the intersection

Set and, I'll take bag 1 and bag 2 i'll take both \rightarrow I'll take the union
set or, I'll take bag 1 or bag 2 I'll take just one

Do we ever confuse set union, set and with logical or, and?
(Don't we describe set union \cup as "or")