Logic

1 Law of Non-Contradiction

Let a denote a boolean statement

$$a = a$$

1.1 Commentary

In digital logic, the Law of Non-Contradiction is often referred to as a buffer

2 Not $\neg$

2.1 Definition

$$\neg a = \neg[a] :=$$
$$\neg[\mathbb{T}] \to \mathbb{F}$$
$$\neg[\mathbb{F}] \to \mathbb{T}$$

3 Contradiction

$$a = \neg a \Rightarrow \text{contradiction}$$

3.1 Proof by Contradiction

4 Logical Function

4.1 Definition

Define logical function with boolean input(s) $a_1, ..., a_n$ and a boolean output b

$$f_{logical} :=$$
$$f[a_1, ..., a_n] \to b$$

5 Non-Trivial Logical Functions

5.1 Definition

5.2 Proof All Logical Functions Can Be Built from Not with Any Non-Trivial Logical Function

6 Logical And $\wedge$

6.1 Definition

$$\begin{gathered} a \wedge b = \wedge[a, b] := \\ \neg(\exists \mathbb{F} \in \{a, b\}) = \\ \nexists \mathbb{F} \in \{a, b\} \end{gathered}$$

6.2 Alternate Definition

$$\begin{gathered} \wedge[\mathbb{F}, \mathbb{F}] = \mathbb{F} \\ \wedge[\mathbb{F}, \mathbb{T}] = \mathbb{F} \\ \wedge[\mathbb{T}, \mathbb{F}] = \mathbb{F} \\ \wedge[\mathbb{T}, \mathbb{T}] = \mathbb{T} \end{gathered}$$

7 Logical Or $\vee$

7.1 Definition

$$\begin{gathered} a \wedge b = \wedge[a, b] := \\ \exists \mathbb{T} \in \{a, b\}) \end{gathered}$$

7.2 Alternate Definition

$$\begin{gathered} \wedge[\mathbb{F}, \mathbb{F}] = \mathbb{F} \\ \wedge[\mathbb{F}, \mathbb{T}] = \mathbb{T} \\ \wedge[\mathbb{T}, \mathbb{F}] = \mathbb{T} \\ \wedge[\mathbb{T}, \mathbb{T}] = \mathbb{T} \end{gathered}$$

8 Exclusive Or (Xor)

8.1 Definition

8.2 Alternate Definition

9 Not Or (Nor)

9.1 Definition

9.2 Alternate Definition

10 Exclusive Nor (Xnor)

10.1 Definition

10.2 Alternate Definition

11 Not And (Nand)

11.1 Definition

11.2 Alternate Definition

Appendix

12 Criticism logical union, set union, logical and, set and

- logical or is a function logical and is a function
- language muks up our understanding

Logical or $\vee$ is different from $\cup$ Logical and $\wedge$ is different from $\cap$
Logical or, only one has to be true
Logical and, both have to be true –> I'll take the intersection

Set and, I'll take bag 1 and bag 2 i'll take both –> I'll take the union
set or, I'll take bag 1 or bag 2 I'll take just one

Do we ever confuse set union, set and with logical or, and?
(Don't we describe set union $\cup$ as "or")