Equivalence

1 Equivalence =

Let = denote equivalence

$$x_i = x_i \ \ \forall i$$

2 Is equal to ==

$$\begin{gathered} == [x_i, x_j] := \\ == [x_i, x_j] \to \mathbb{T} \ \ i = j \\ == [x_i, x_j] \to \mathbb{F} \ \ i \neq j \end{gathered}$$

2.1 Alternate Notation

$$x_i == x_j$$

3 Is not equal to !=

$$\begin{gathered} ! = [x_i, x_j] := \\ ! = [x_i, x_j] \to \mathbb{T} \ \ i \neq j \\ ! = [x_i, x_j] \to \mathbb{F} \ \ i = j \end{gathered}$$

3.1 Alternate Notation

$$x_i \, ! = x_j$$

Set Theory

4 Element

Define element l

$$l = \{l\}$$

5 Identity

Define identity of element l

$$|l| := 1$$

6 Empty Set

Define Empty Set, the set containing no elements

$$\varnothing := \{\}$$

7 Null Identity

Define the null identity

$$|\varnothing| := 0$$

8 Union $\cup$

Define $\cup$ the union of two elements

$$\cup[l_i, l_j] = l_i \cup l_j = \{l_i\} \cup \{l_j\} :=$$
$$\cup[l_i, l_j] \to \{l_i\} \;\; i = j$$
$$\cup[l_i, l_j] \to \{l_i, l_j\} \;\; i \neq j$$

8.1 Translation

$\cup$ is often read as "and"

8.2 Comma ,

In set notation the comma "," denotes union $\cup$

$$l_1 \cup l_2 = \{l_1\} \cup \{l_2\} = \{l_1, l_2\}$$

9 Intersection $\cap$

Define $\cap$, the intersection of two elements

$$l_1 \cap l_2 = \{l_1\} \cap \{l_2\}$$

$$\cap[l_i, l_j] \rightarrow \{l_i\} \;\; i = j$$
$$\cap[l_i, l_j] \rightarrow \varnothing \;\; i \neq j$$

10 Sets

10.1 Definition

Define set S as an ordered union of elements s_i

$$S := s_1 \cup s_2 \cup \;\; ... \cup s_{n-1} \cup \;\; s_n = \{s_1, s_2, ..s_N\}$$

10.2 Alternate Notation

$$S := s_i \in S : i = 1, 2, ..., N-1, N$$
$$S = \{s_1, s_2, ..., s_{N-1}, s_N\}$$

10.3 Magnitude of a Set

$$|S| = |\{x_1, ..., x_N\}| = N$$

10.4 Counting

$$1, 2, ..., N = 1 : N$$

10.5 Definition Unordered Set

Set S is unordered if

$$S = \{x_1, x_2, ..x_n\} :=$$
$$x_i, x_j \in S; \;\; x_i = x_j; \quad \forall i, j \neq i$$

10.6 Definition of Unique Set

$$a_i, a_j \in S$$
$$a_i \neq a_j \quad \forall i, j \neq i$$

10.7 Definition of Countable/Uncountable set

Potentially just a line?

10.8 Define line $\mathbb{L}$

Define line $\mathbb{L}$

$$\mathbb{L} := \{l_0, l_1, l_2, ..., l_{N-1}, l_N, l_{N+1}, ...$$
$$\iff \exists l_i \in \mathbb{L} \ \forall i$$

11 Hierarchy of Elements to Sets

Every element is a set, but not all sets are elements

12 Universal Set

12.1 Definition

Define Universe, the "Universal Set" containing all elements

$$\Omega := s_i \in \Omega, \forall i$$

13 Containment

13.1 Contains

13.2 Equals $=$

Define set equivalence $=$

$$S_1 \subseteq S_2; \quad S_2 \subseteq S_1 \iff S_1 = S_2$$

13.3 Subset

13.4 Proper Subset Citation

13.5 Definition of Complement

$$S = \{s_1, s_2, ..., s_N\}$$
$$S^C :=$$
$$s_j : \{s_j \in \Omega\} \cap \{s_j \notin S\}; \quad \forall j$$

13.6 Alternate Notation

Wikipedia definition of complement

$$S^C = U - S = \{x \in \Omega : x \notin S\}$$
https://en.wikipedia.org/wiki/Complement_(set_theory)

Appendix

14 Proofs and Properties

1. $\Omega \subset \emptyset$
2. $\Omega \cap \Omega = \Omega$
3. $\Omega \cup \Omega = \Omega$
4. $\Omega \cup \emptyset = \Omega$
5. $\Omega \cap \emptyset = \emptyset$
6. $\Omega \cap S = S$
7. $\Omega \cup S = \Omega$
8. $\emptyset \nsubseteq \Omega$
9. $\emptyset \cup \emptyset = \emptyset$
10. $\emptyset \cap \emptyset = \emptyset$
11. $\emptyset \cap S = \emptyset$
12. $\emptyset \cup S = S$
13. $\emptyset = \emptyset$
14. $\emptyset = \Omega^C$
15. $\Omega \subseteq S$