Computation Continued

1 Discrete Derivatives

1.1 First Order Derivative

1.2 K Order Derivative

2 Convergent Complexity

2.1 Definition

Define Convergent Complexity; the set of solutions with complexity satisfying

$$limit_{n\to\infty}\frac{O[n+1]}{O[n]} = c$$

where c is a constant

2.2 Derivative Property of Convergent Solutions

There exists an nth derivative equal to zero

$$limit_{n\to\infty}\frac{O[n+1]}{O[n]} = c$$

2.3 Theorem of Polynomial Subfunctions

Consider solution s^+ with polynomial total complexity O[n] containing z subfunctions $Sub_k[X_i]$ k = 1..z

$$X_i = \{x_1, ..., x_n, C\};\ \ \hat{X}_i = \{x_1, ..., x_{n+1}, C\}$$
$$s^+ = s^+[X_i] := P :$$
$$(P[X_i] \to y_o == a_o \quad \forall X_i) \quad \cap \quad (P[\hat{X}_i] \supseteq P[X_i] \quad \forall X_i, \hat{X}_i)$$

$$s^+ = \{s_1, s_2, ..., s_N | b_1, b_2, ..., b_M, y_o\} = \{s_1, s_2, ..., s_{O_T[n]}, b_1, b_2, ..., b_{O_S[n]}, y_o\}$$
$$= \{\mathcal{L}, \mathcal{M}, y_o\}$$

$$Sub_h[X_i] := S_h = \{s_j, ... | b_k, ..., y_o\} :$$
$$s_j, b_k \in s^+ \quad \forall s_j, b_k \in S_h$$

$$s^+ = Sub_1[X_i] \cup Sub_2[X_i] \cup ... \cup Sub_z[X_i]$$

$$O[n] = (\lambda_K n)^K + (\lambda_{K-1} n)^{K-1} ... + \lambda_1 n + C \quad \forall n$$

$$O[n] = O_{T_1}[n] + O_{T_2}[n] + ... + O_{T_z}[n] + |O_{S_1}[n] \cup O_{S_2}[n] \cup ... \cup O_{S_z}[n]|$$
$$= O_{T_1}[n] + O_{T_2}[n] + ... + O_{T_z}[n] + O_S[n]$$

By property of polynomial complexity

$$limit_{n\to\infty} \frac{O[n+1]}{O[n]} = 1$$
$$limit_{n\to\infty} \frac{O_{T_1}[n+1]+O_{T_2}[n+1]+...+O_{T_z}[n+1]+O_S[n+1]}{O_{T_1}[n]+O_{T_2}[n]+...+O_{T_z}[n]+O_S[n]]} = 1$$
$$limit_{n\to\infty} \frac{O_h[n+1]}{O[n]} \leqslant 1 \;\; \forall h$$

2.4 The Union of Two Converging Subfunctions is Convergent

Let

$$Sub_1[X] \text{ with total complexity } O_1[n]$$
$$O_1[n] :$$
$$limit_{n\to\infty} \frac{O_1[n+1]}{O_1[n]} = c_1$$

$$Sub_2[X] \text{ with total complexity } O_2[n]$$
$$O_2[n] :$$
$$limit_{n\to\infty} \frac{O_2[n+1]}{O_2[n]} = c_2$$

$$Sub_{1;2}[X] = Sub_1[X] \cup Sub_2[X] \text{ with complexity } O[n]$$
$$limit_{n\to\infty} \frac{O[n+1]}{O[n]} = c$$

2.4.1 Proof

Show

$$limit_{n\to\infty} \frac{O[n+1]}{O[n]} = c$$

$$Sub_{1;2}[X] = Sub_1[X] \cup Sub_2[X] \text{ with complexity } O[n]$$
$$O_1[n] = O_{T_1}[n] + O_S[n]$$

$$O_2[n] = O_{T_2}[n] + O_S[n]$$
$$O[n] = O_{T_1}[n] + O_{T_2}[n] + O_S[n]$$

$$\frac{O[n+1]}{O[n]} = \frac{O_{T_1}[n+1]+O_{T_2}[n+1]+O_S[n+1]}{O_{T_1}[n]+O_{T_2}[n]+O_S[n]}$$
$$\frac{O[n+1]}{O[n]} = \frac{O_{T_1}[n+1]+O_S[n+1]}{O_{T_1}[n]+O_{T_2}[n]+O_S[n]} + \frac{O_{T_2}[n+1]}{O_{T_1}[n]+O_{T_2}[n]+O_S[n]}$$

For all non-decreasing functions f[n], g[n]

$$f_{n+1} \text{ goes to 0 faster}$$

2.5 The Union of Two Divergent Subfunctions is Divergent

Let

$$Sub_1[X] \text{ with total complexity } O_1[n]$$
$$O_1[n]:$$
$$limit_{n\to\infty}\frac{O_1[n+1]}{O_1[n]}diverges$$

$$Sub_2[X] \text{ with total complexity } O_2[n]$$
$$O_2[n]:$$
$$limit_{n\to\infty}\frac{O_2[n+1]}{O_2[n]}diverges$$

$$Sub_{1;2}[X] = Sub_1[X] \cup Sub_2[X] \text{ with complexity } O[n]$$
$$limit_{n\to\infty}\frac{O[n+1]}{O[n]}diverges$$

2.5.1 Proof

Show

$$limit_{n\to\infty}\frac{O[n+1]}{O[n]}diverges$$

2.6 The Union of a convergent and divergent subfunction is Divergent

Let

$$Sub_1[X] \text{ with total complexity } O_1[n]$$
$$O_1[n]:$$
$$limit_{n\to\infty}\frac{O_1[n+1]}{O_1[n]} = c_1$$

$$Sub_2[X] \text{ with total complexity } O_2[n]$$
$$O_2[n]:$$
$$limit_{n\to\infty}\frac{O_2[n+1]}{O_2[n]} = diverges$$

$$Sub_{1;2}[X] = Sub_1[X] \cup Sub_2[X] \text{ with complexity } O[n]$$

Show

$$limit_{n\to\infty}\frac{O[n+1]}{O[n]} diverges$$

2.6.1 Proof

2.7 Theorem of Divergent Subfunctions

2.7.1 $limit_{n\to\infty}\frac{O[n+1]}{O[n]}$ diverges $\Rightarrow$ $\exists Sub_h[X_i] : limit_{n\to\infty}\frac{O_h[n+1]}{O_h[n]}$ diverges

If any subfunction of s^+ diverges, then O[n+1]/O[n] diverges, f_{n+1}/O[n] diverges Consider solution s^+ with polynomial total complexity O[n] containing z subfunctions $Sub_k[X_i]$ k = 1..z
FIX!!! concerns about OS memory complexity; $c_h = c_{T_h} + c_{S_h}$; c_{S_h} is the same for all subfuncrions

$$X_i = \{x_1, ..., x_n, C\};\ \ \hat{X}_i = \{x_1, ..., x_{n+1}, C\}$$
$$s^+ = s^+[X_i] := P:$$
$$(P[X_i] \to y_o == a_o \quad \forall X_i) \quad \cap \quad (P[\hat{X}_i] \supseteq P[X_i] \quad \forall X_i, \hat{X}_i)$$

$$s^+ = \{s_1, s_2, ..., s_N | b_1, b_2, ..., b_M, y_o\} = \{s_1, s_2, ..., s_{O_T[n]}, b_1, b_2, ..., b_{O_S[n]}, y_o\}$$
$$= \{\mathcal{L}, \mathcal{M}, y_o\}$$

$$Sub_h[X_i] := S_h = \{s_j, ...|b_k, ..., y_o\} :$$
$$s_j, b_k \in s^+ \quad \forall s_j, b_k \in S_h$$

$$s^+ = Sub_1[X_i] \cup Sub_2[X_i] \cup ... \cup Sub_z[X_i]$$

$$O[n] = O_{T_1}[n] + O_{T_2}[n] + ... + O_{T_z}[n] + |O_{S_1}[n] \cup O_{S_2}[n] \cup ... \cup O_{S_z}[n]|$$
$$= O_{T_1}[n] + O_{T_2}[n] + ... + O_{T_z}[n] + O_S[n]$$

By defintion of divergent complexity

$$limit_{n\to\infty}\frac{O[n+1]}{O[n]} \text{ diverges}$$

Suppose there does not exist a diverging subfunction $Sub_h[X_i]$ for all h

$$\nexists Sub_h[X_i] :$$
$$limit_{n\to\infty}\frac{O_h[n+1]}{O_h[n]} \text{ diverges} \quad \forall h$$
$$\Rightarrow limit_{n\to\infty}\frac{O_h[n+1]}{O_h[n]} = c_h \;\; \forall h$$
$$limit_{n\to\infty}\frac{O_1[n+1]+O_2[n+1]+...+O_z[n+1]}{O_1[n]+O_2[n]+...+O_z[n]}$$

Let

$$g_h[n] = \textstyle\sum_{i\neq h} O_i[n] \geqslant 0^*$$
$$\Rightarrow 0 \leqslant limit_{n\to\infty}\frac{O_h[n+1]}{O_h[n]+g_h[n]} \leqslant c_h$$
$$limit_{n\to\infty}\frac{O_1[n+1]}{O_1[n]+g_1[n]} + \frac{O_2[n+1]}{O_2[n]+g_2[n]} + ... + \frac{O_z[n+1]}{O_1[n]+g_z[n]}$$
$$0 \leqslant limit_{n\to\infty}\frac{O_1[n+1]}{O_1[n]+g_1[n]} + \frac{O_2[n+1]}{O_2[n]+g_2[n]} + ... + \frac{O_z[n+1]}{O_1[n]+g_z[n]} \leqslant \textstyle\sum_{i=1}^{z} c_i$$
$$\Rightarrow limit_{n\to\infty}\frac{O_1[n+1]}{O_1[n]+g_1[n]} + \frac{O_2[n+1]}{O_2[n]+g_2[n]} + ... + \frac{O_z[n+1]}{O_1[n]+g_z[n]} = \tilde{C}$$
$$0 \leqslant \tilde{C} \leqslant \textstyle\sum_{i=1}^{z} c_i$$

*O$_i[n] \geqslant 0$ is a non-decreasing function

Assuming

$$\nexists Sub_h[X_i]:$$

$$limit_{n\to\infty}\frac{O_h[n+1]}{O_h[n]} \text{ diverges } \quad \forall h$$

$$\Rightarrow limit_{n\to\infty}\frac{O_[n+1]}{O_1[n]} = \tilde{C}$$

Contradicting the definition of divergent solution

$$\therefore \exists Sub_h[X_i]:$$

$$limit_{n\to\infty}\frac{O_h[n+1]}{O_h[n]} \text{ diverges}$$

2.7.2 $\exists Sub_h[X_i] : limit_{n\to\infty}\frac{O_h[n+1]}{O_h[n]}$ diverges $\Rightarrow$ $limit_{n\to\infty}\frac{O[n+1]}{O[n]}$ diverges

FIX!!! SPACE OS portion

$$X_i = \{x_1, ..., x_n, C\};\ \ \hat{X}_i = \{x_1, ..., x_{n+1}, C\}$$
$$s^+ = s^+[X_i] := P :$$
$$(P[X_i] \to y_o == a_o \quad \forall X_i) \quad \cap \quad (P[\hat{X}_i] \supseteq P[X_i] \quad \forall X_i, \hat{X}_i)$$

$$s^+ = \{s_1, s_2, ..., s_N | b_1, b_2, ..., b_M, y_o\} = \{s_1, s_2, ..., s_{O_T[n]}, b_1, b_2, ..., b_{O_S[n]}, y_o\}$$
$$= \{\mathcal{L}, \mathcal{M}, y_o\}$$

$$Sub_h[X_i] := S_h = \{s_j, ...| b_k, ..., y_o\} :$$
$$s_j, b_k \in s^+ \quad \forall s_j, b_k \in S_h$$

$$s^+ = Sub_1[X_i] \cup Sub_2[X_i] \cup ... \cup Sub_z[X_i]$$
$$O[n] = O_{T_1}[n] + O_{T_2}[n] + ... + O_{T_z}[n] + |O_{S_1}[n] \cup O_{S_2}[n] \cup ... \cup O_{S_z}[n]|$$
$$= O_{T_1}[n] + O_{T_2}[n] + ... + O_{T_z}[n] + O_S[n]$$

Suppose

$$\exists Sub_h[X_i] : limit_{n\to\infty}\frac{O_h[n+1]}{O_h[n]} \text{ diverges}$$

$$\frac{O_h[n+1]}{O_h[n]} \geqslant 1^* \quad \forall h$$

$^*O_h[n]$ is a positive non-decreasing function

$$limit_{n\to\infty}\frac{O[n+1]}{O[n]}$$
$$= limit_{n\to\infty}\frac{O_1[n+1]+O_2[n+1]+...+O_z[n+1]}{O_1[n]+O_2[n]+...+O_z[n]}$$
$$limit_{n\to\infty}\frac{O_1[n+1]}{O[n]} + ... + \frac{O_h[n+1]}{O[n]} + ... + \frac{O_z[n+1]}{O[n]}$$

$$limit_{n\to\infty}\frac{O_h[n+1]}{O[n]} = limit_{n\to\infty}\frac{O_h[n+1]}{O_h[n]+g_h[n]}$$

$$= limit_{n\to\infty}(\frac{O_h[n+1]}{O_h[n]]} - \frac{g_h[n]O_h[n+1]}{O_h[n](O_h[n]+g_h[n])})$$

$$= limit_{n\to\infty}(\frac{O_h[n+1]}{O_h[n]]} - \frac{(O[n]-O_h[n])(O_h[n]+f_{n+1}[n])}{O_h[n]O[n]})$$

$$= limit_{n\to\infty}(\frac{O_h[n+1]}{O_h[n]]} + \frac{-O_h[n]O[n]-f_{n+1}[n]O[n]+O_h^2[n]+f_{n+1}O_h[n]}{O_h[n]O[n]})$$

$$= limit_{n\to\infty}(\frac{O_h[n+1]}{O_h[n]]} - 1 - \frac{f_{n+1}^h[n]}{O_h[n]} + \frac{O_h[n]}{O[n]} + \frac{f_{n+1}^h[n]}{O[n]})$$

2.8 Sum of convergent, divergent, and constant subfunctions

Let

$$s^+ = \cup_{i=1}^z Sub_i[X]$$

3 Optimal Complexity

Conjecture is that you start as the optimal solution and then as you add new inputs the complexity remains optimal
There's an easier proof that the optimal inductive function converges to optimal as n approaches infty

3.1 Definition

Define Optimal Complexity; the minimum total complexity required to solve a decision problem

$$O_{opt}[n] := O[n] :$$
$$\nexists O_i[n] < O[n] \quad \forall s_i \in S^+$$

3.2 Proof of Existence

Prove the existence of at least one $O_{min}[n]$ by induction/contradiction
Induction, let n = 1
There must exists an O_{min}
subfunction property all the way up

4 Optimal solution

Define an optimal solution s_{opt}^+

4.1 Definition

$$X_i = \{x_1, ..., x_n\}$$

$$D_j := f[X_i] \rightarrow a_o \in \{\mathbb{T}, \mathbb{F}\} \quad \forall X_i$$

$$s^+ := P[X_i] \rightarrow y_o : y_o = a_o \quad \forall X_i$$

$$s^+_{opt} := s^+ :$$

$$\nexists \hat{O}[n] < O_{opt}[n] \quad \forall n,\ s^+ \in S^+_j$$

4.2 Optimal Time Complexity Solution

$$X_i = \{x_1, ..., x_n\}$$

$$D_j := f[X_i] \rightarrow a_o \in \{\mathbb{T}, \mathbb{F}\} \quad \forall X_i$$

$$s^+ := P[X_i] \rightarrow y_o : y_o = a_o \quad \forall X_i =$$

$$\{s_1, s_2, ..., s_{O_T[n]}, b_1, b_2, ..., b_{O_S[n]}, X_i, y_o\} = \{\mathcal{L}, \mathcal{M}, X_i, y_o\}$$

$$O_T[n] := |\mathcal{L}| = N$$

$$s^+_T := s^+ :$$

$$\nexists \hat{O_T}[n] < O_T[n] \quad \forall n,\ s^+ \in S^+_j$$

4.3 Optimal Space Complexity Solution

$$X_i = \{x_1, ..., x_n\}$$

$$D_j := f[X_i] \rightarrow a_o \in \{\mathbb{T}, \mathbb{F}\} \quad \forall X_i$$

$$s^+ := P[X_i] \rightarrow y_o : y_o = a_o \quad \forall X_i =$$

$$\{s_1, s_2, ..., s_{O_T[n]}, b_1, b_2, ..., b_{O_S[n]}, X_i, y_o\} = \{\mathcal{L}, \mathcal{M}, X_i, y_o\}$$

$$O_S[n] := |\mathcal{M}| = M$$

$$s^+_S := s^+ :$$

$$\nexists \hat{O_S}[n] < O_S[n] \quad \forall n,\ s^+ \in S^+_j$$

4.4 Conjecture of Optimal Solutions

$O_{T_{min}}$ subject to $O_{S_{opt}} = 1$

$$\exists s^+_{opt} :$$

$$O_{opt}[n] = 1 + O_T[n] = O_{S_{opt}} + O_T[n] \quad \forall s^+ \in S^+$$

4.4.1 Proof

4.5 Theorem of Optimal Solutions

Alternate way of expressing $O_{opt}[n]$ possibly with efficiency function and equivalence functions?
$f_{S \to T}[n, O_S[n]]$ as a function of space complexity order K?
Efficiency function ($f_{S \to T}[n, O_S[n]]$ as a function of space complexity order K) is strictly decreasing for Polynomial Functions
Or there's an inflection point
Efficiency function might have a general pattern for all problems in D

5 Duality of $O_T[n]$, $O_S[n]$?

5.0.1 O_T to O_S

Define equivalence function $f_{T \to S}$; a function converting logical operations into memory elements

$$f_{T \to S} := f :$$
$$O_S[n] = f[n, O_T[n]] \quad \forall n, s^+ \in S^+$$

5.0.2 O_S to O_T

Define equivalence function $f_{S \to T}$; a function converting memeory elements into logical operations

$$f_{S \to T} := f :$$
$$O_T[n] = f[n, O_S[n]] \quad \forall n, s^+ \in S^+$$

5.0.3 Invertibility?

5.0.4 Polynomial Bounded?

5.1 Efficiency Function?

Function relating the decrease in O[n] as $O_S[n]$ increases in order
$f_{S \to T}[n, O_S[n]]$ as a function of space complexity order K?

6 Theorem of Computational Duality?

For all Problems in P there exists a duality function
Formally define dynamic programming, Optimal polynomial complexity minimizes the difference between time and space complexity order

$$D \in \mathbb{P}$$
$$O[n] := O_T[n] + O_S[n]$$
$$limit_{n \to \infty} \frac{O[n+1]}{O[n]} = 1 \quad \forall s^+ \in S^+_{\mathbb{P}}$$

$$O_T[n] = f_{S \to T}[n, O_S[n]]$$
$$O_S[n] = f_{T \to S}[n, O_T[n]]$$

$$limit_{n\to\infty}\frac{O_T[n+1]+O_S[n+1]}{O_T[n]+O_S[n]} = 1 \quad \forall s^+ \in S^+_{\mathbb{P}}$$

$$limit_{n\to\infty}\frac{f_{S\to T}[n+1,O_S[n+1]]+O_S[n+1]}{f_{S\to T}[n,O_S[n]]+O_S[n]} = 1 \quad \forall s^+ \in S^+_{\mathbb{P}}$$

$$limit_{n\to\infty}\frac{O_T[n+1]+f_{T\to S}[n+1,O_T[n+1]]}{O_T[n]+f_{T\to S}[n,O_T[n]]} = 1 \quad \forall s^+ \in S^+_{\mathbb{P}}$$

7 Imaginary Problems

Problems with a contradictory subproblem
O[n] = n^n
Show no general finite solution exists only
General approach is obtained by negative recursive set span

8 Efficient Approximations

9 Universal Bound of Computation?

maybe

$$O[n] < n^n \quad \forall s^+$$

9.1 Convergent Solutions

9.2 Show Convergent Union Divergent Solutions represent the universe of solutions

O[n+1]/O[n] converges or diverges represents the universe of outcomes, does converging to 1 imply all converging solutions?

9.3 Show Polynomial Solutions are bounded by n^n

9.4 Show Divergent Solutions are bounded by n^n?

10 Theorem of Prime Numbers "Riemann Hypothesis"

Riemann Zeta Function

$$\zeta(s) \equiv \sum_{n=1}^{\infty} \frac{1}{n^s} \quad [2]$$

"The prime number theorem determines the average distribution of the primes. The Riemann hypothesis tells us about the deviation from the average. Formulated in Riemann's 1859 paper, it asserts that all the 'non-obvious' zeros of the zeta function are complex numbers with real part 1/2." [3]

Prove the problem is divergent
There fore it can only be proven to a certain degree
The limit as n approaches infinity implies a real part of one half
Connection with the real and imaginary part of O $[n]$

10.1 Determine a duality function for the Riemann Hypothesis

10.2 Determine an expression for O[n+1] as a function of O[n]

10.3 Prove O_{opt} is performing O_{opt} recursively for the ints less than square root of n

Testing the primes less than sqrt(n)? double check
1. Optimal solution for n=1,2,3, everything else is a recursive optimal proof by induction
Time Complexity seems to be on the order of n log n... implies divergence or lack of bound? Add in the complexity of division.. probably approaches n^n

10.4 Since divergent, no s^+ exists.. only rules

Express as a limit

10.5 Show that the limit as n $\rightarrow \infty$ implies the real part is 1/2

1/2 $\pm$ 14.134725 i 1/2 $\pm$ 21.022040 i 1/2 $\pm$ 25.010858 i 1/2 $\pm$ 30.424876 i 1/2 $\pm$ 32.935062 i 1/2 $\pm$ 37.586178 i

Z $= \zeta(1/2 + it)$

10.6 Notation, real imaginary parts of the problem

Even numbers and numbers ending in 5 are automatically convergent
Testing numbers ending in 1,3,7,9 results in divergent expression
we can continue to add rules to a certain degree

Computation Continued

Permitting contradiction?

11 Beyond Simple Computation

Recall the definition of complexity and simple computational complexity

12 Relationship to Simple Computation

Simply an abstraction above Logical operations and Memory; typically empowered by hardware

13 Fuzzy Logic

Allowing partial truth and false values

14 Simultaneous Computation

Seemingly a contradiction, perhaps a new dimension of complexity is how close to simultaneous

15 Paraconsistent Computation

Might not be possible