CHAPTER 5

KARNAUGH MAPS

Contents

5.1 Minimum Forms of Switching Functions
5.2 Two- and Three-Variable Karnaugh Maps
5.3 Four-Variable Karnaugh Maps
5.4 Determination of Minimum Expressions
5.5 Five-Variable Karnaugh Maps
5.6 Other Uses of Karnaugh Maps
5.7 Other Forms of Karnaugh Maps

Objectives

1. Given a function (completely or in completely specified) of three to five variable, plot it on a Karnaugh map. The function may be given in minterm, maxterm, or algebraic form.

2. Determine the essential prime implicants of a function from a map.

3. Obtain the minimum sum-of-products or minimum product-of-sums form of a function from the map.

4. Determine all of the prime implicants of a function from a map.

5. Understand the relation between operations performed using the map and the corresponding algebraic operation.

5.1 Minimum Forms of Switching Functions

1. **Combine terms by using** $XY' + XY = X$

Do this repeatedly to eliminates as many literals as possible.

A given term may be used more than once because $X + X = X$

2. **Eliminate redundant terms by using the consensus theorems.**

5.1 Minimum Forms of Switching Functions

Example: Find a minimum sum-of-products

$$F(a,b,c) = \sum m(0,1,2,5,6,7)$$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= \quad a'b' \quad + \quad b'c \quad + \quad bc' \quad + \quad ab$$

$$F = a'b'c' + a'b'c + a'bc' + ab'c + abc' + abc$$

$$= \quad a'b' \quad + \quad bc' \quad + \quad ac$$

5.1 Minimum Forms of Switching Functions

Example: Find a minimum product-of-sums

$$(A+B'+C+D')(A+B'+C'+D')(A+B'+C'+D)(A'+B'+C'+D)(A+B+C'+D)(A'+B+C'+D)$$

$$= (A+B'+D') \quad (A+B'+C') \quad (B'+C'+D) \quad (B+C'+D)$$

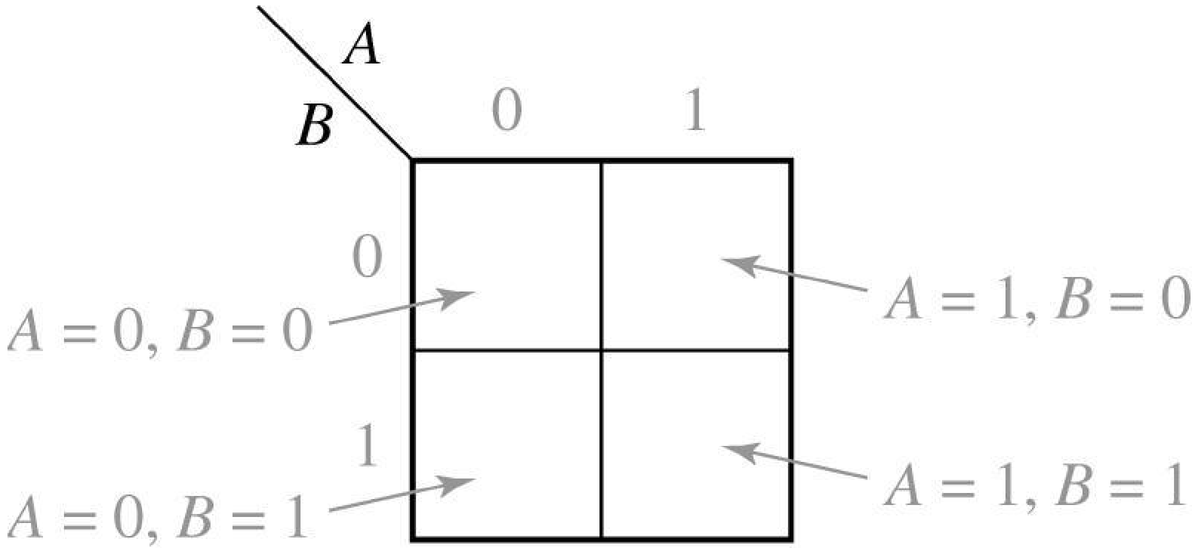
$$= (A+B'+D') \quad \underbrace{(A+B'+C')} \quad (C'+D)$$

$$= (A+B'+D')(C'+D)$$

Eliminate by consensus

5.2 Two- and Three-Variable Karnaugh Maps

A 2-variable Karnaugh Map

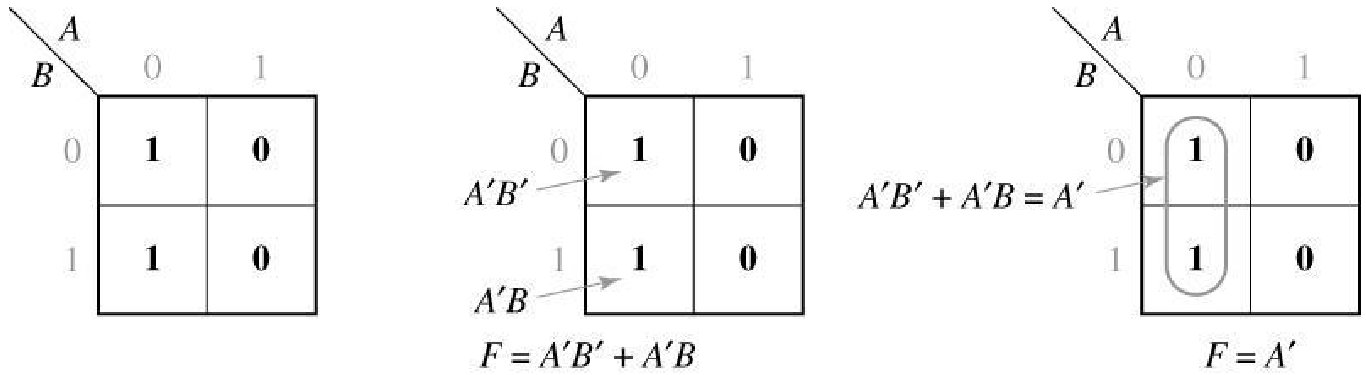


5.2 Two- and Three-Variable Karnaugh Maps

Truth Table for a function F

A B	F
0 0	1
0 1	1
1 0	0
1 1	0

(a)



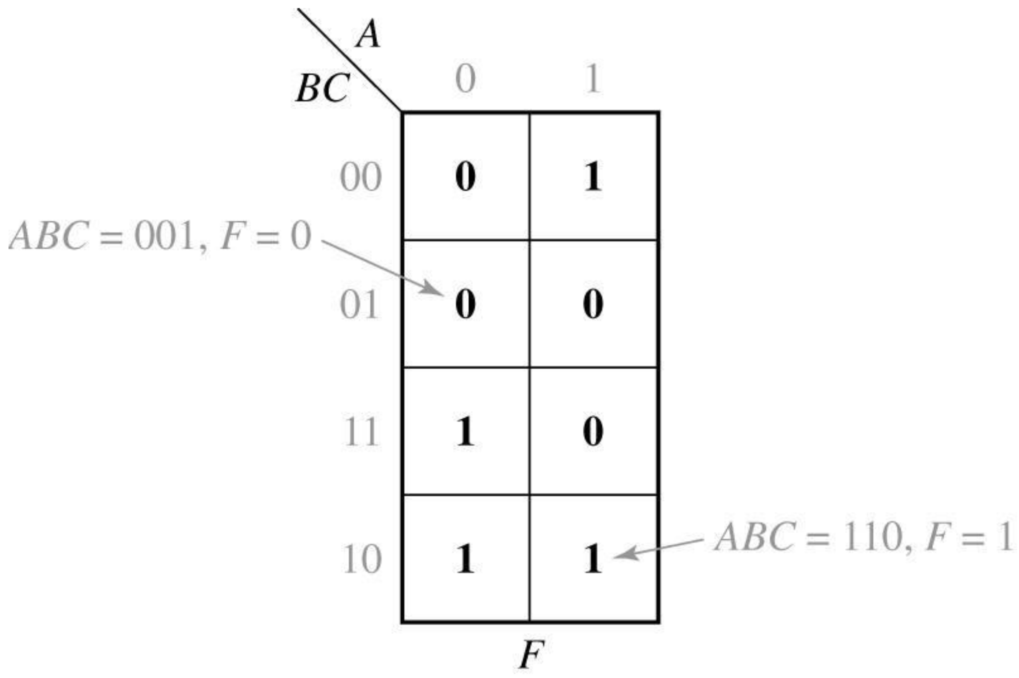
(b) (c) (d)

5.2 Two- and Three-Variable Karnaugh Maps

Truth Table and Karnaugh Map for Three-Variable Function

A B C	F
0 0 0	0
0 0 1	1
0 1 0	0
0 1 1	1
1 0 0	0
1 0 1	1
1 1 0	0
1 1 1	1

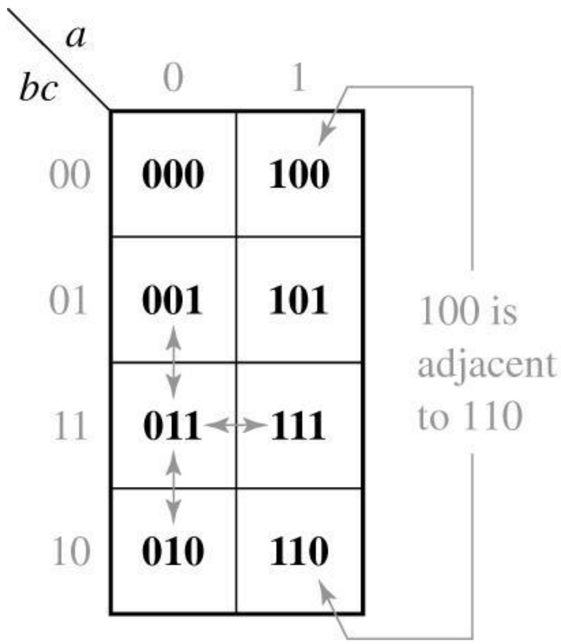
(a)



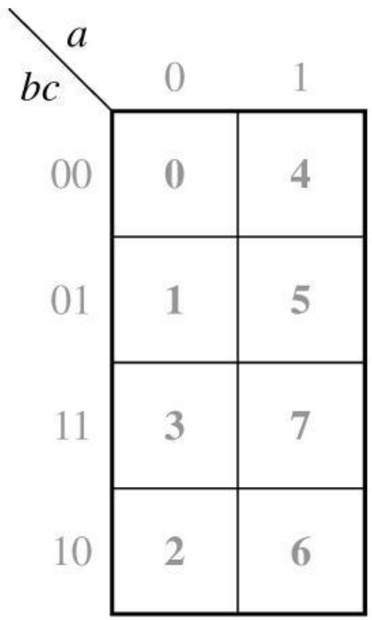
(b)

5.2 Two- and Three-Variable Karnaugh Maps

Location of Minterms on a Three-Variable Karnaugh Map



(a) Binary notation



(b) Decimal notation

5.2 Two- and Three-Variable Karnaugh Maps

Karnaugh Map of F(*a*, *b*, *c*) = ∑ m(1, 3, 5) = ∏(0, 2, 4, 6, 7)

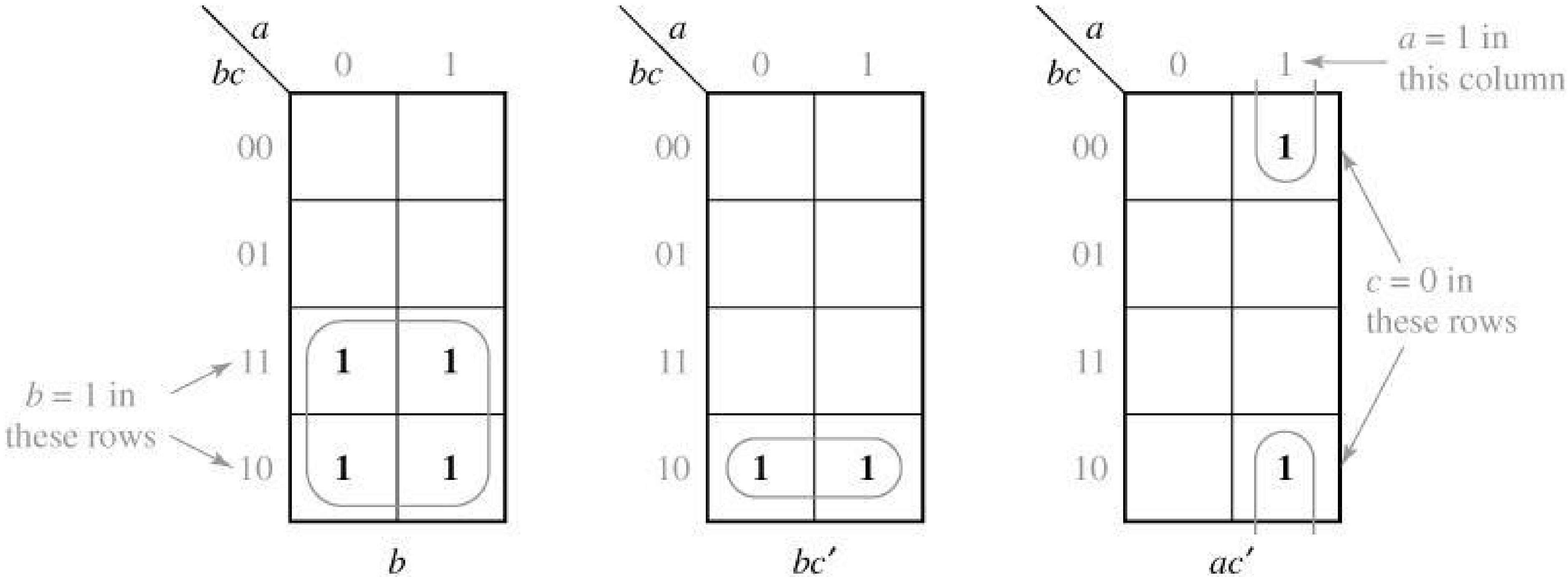
bc \ *a*	0	1
00	**0** $_{0}$	**0** $_{4}$
01	**1** $_{1}$	**1** $_{5}$
11	**1** $_{3}$	**0** $_{7}$
10	**0** $_{2}$	**0** $_{6}$

$$F(a,b,c) = m_1 + m_3 + m_5$$

$$= M_0 + M_2 + M_4 + M_6 + M_7$$

5.2 Two- and Three-Variable Karnaugh Maps

Karnaugh Maps for Product Terms

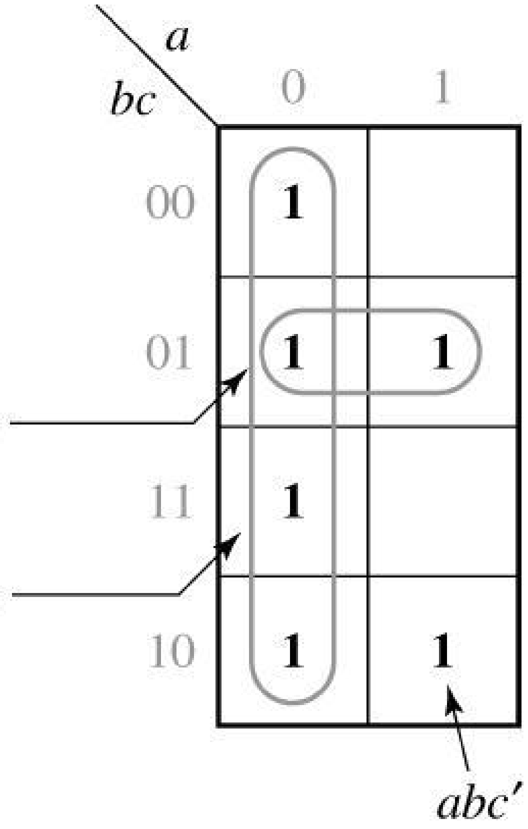


5.2 Two- and Three-Variable Karnaugh Maps

Given Function

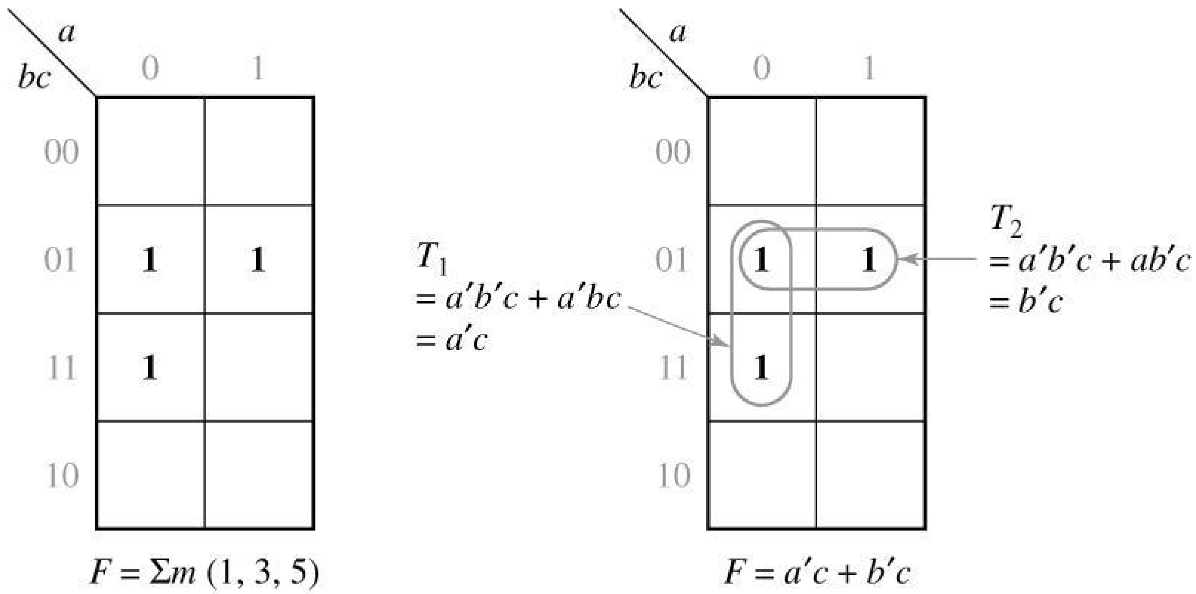
$$f(a,b,c) = abc' + b'c + a'$$

1. The term abc' is 1 when $a = 1$ and $bc = 10$, so we place a 1 in the square which corresponds to the $a = 1$ column and the $bc = 10$ row of the map.
2. The term $b'c$ is 1 when $bc = 01$, so we place 1's in both squares of the $bc = 01$ row of the map.
3. The term a' is 1 when $a = 0$, so we place 1's in all the squares of the $a = 0$ column of the map. (Note: Since there already is a 1 in the abc = 001 square, we do not have to place a second 1 there because $x + x = x$.)



5.2 Two- and Three-Variable Karnaugh Maps

Simplification of a Three-Variable Function



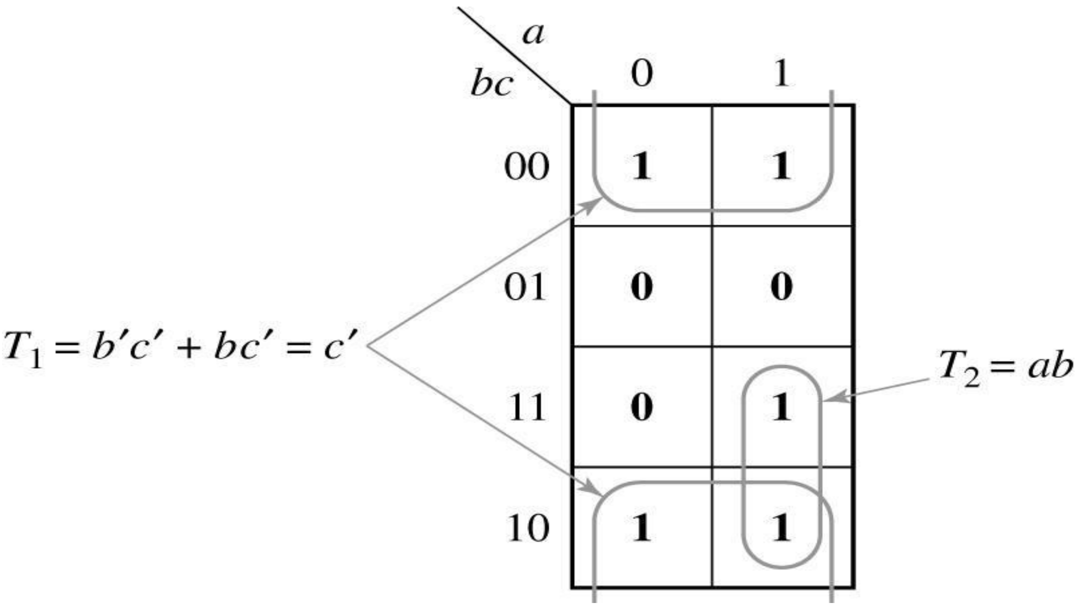
(a) Plot of minterms

(b) Simplified form of F

$$F = T_1 + T_2 = a'c + b'c$$

5.2 Two- and Three-Variable Karnaugh Maps

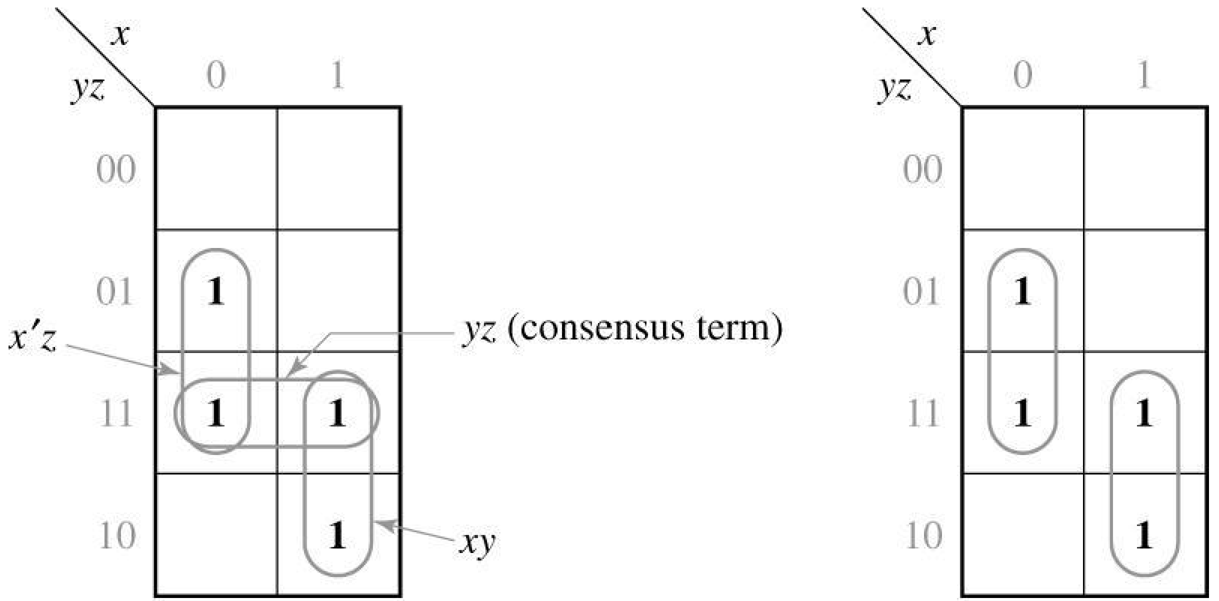
Complement of Map in Figure 5-6(a)



$$F = T_1 + T_2 = c' + ab$$

5.2 Two- and Three-Variable Karnaugh Maps

Karnaugh Maps Which Illustrate the Consensus Theorem



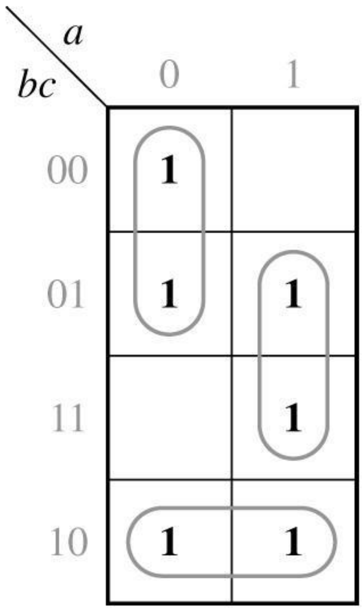
$$xy + x'z + yz = xy + x'z$$

Consensus term is redundant

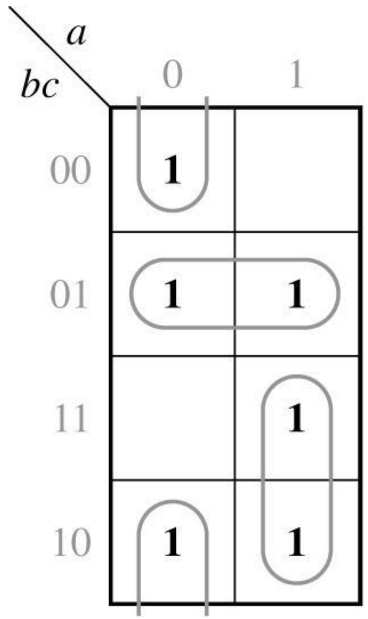
5.2 Two- and Three-Variable Karnaugh Maps

Function with Two Minimal Forms

$$F = \sum m(0,1,2,5,6,7)$$



$F = a'b' + bc' + ac$



$F = a'c' + b'c + ab$

Same Expression

5.3 Four-Variable Karnaugh Maps

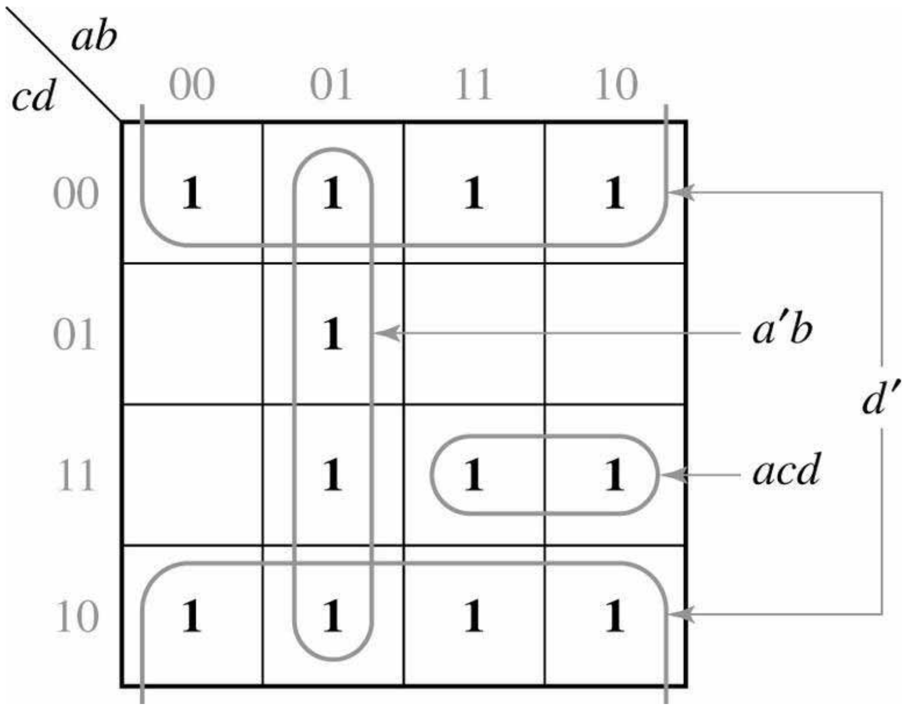
Location of Minterms on Four-Variable Karnaugh Map

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

5.3 Four-Variable Karnaugh Maps

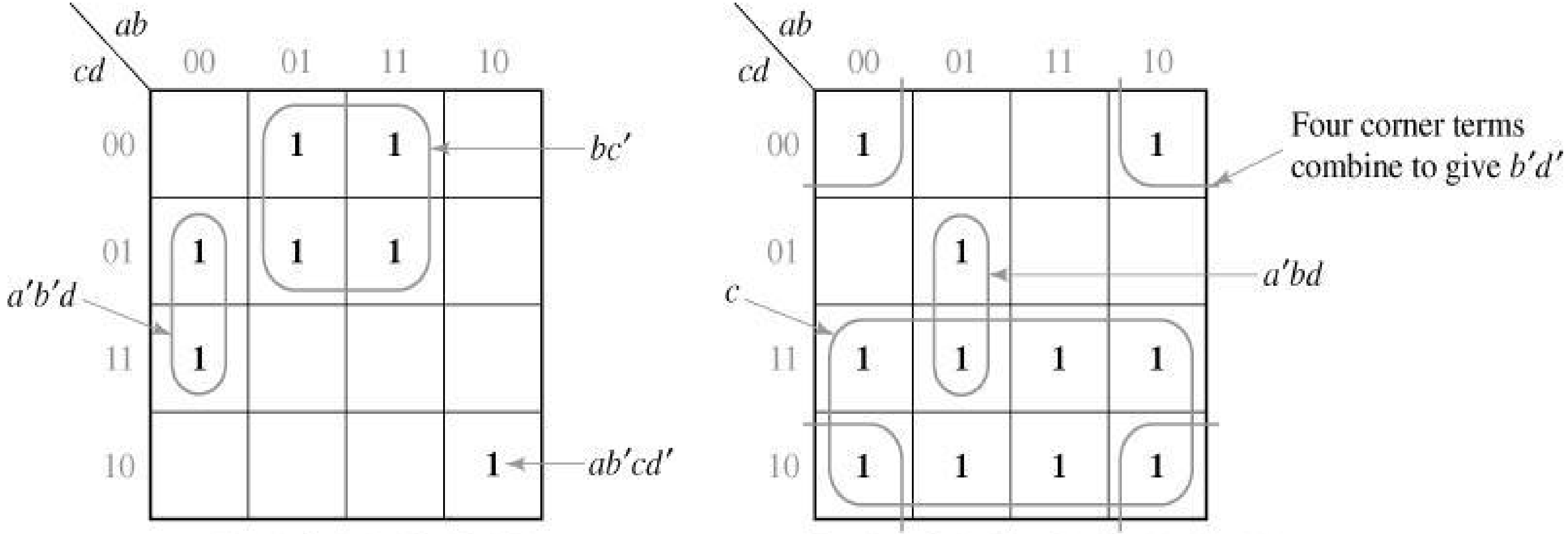
Plot of ***acd* + *a'b* + *d'***

$$f(a,b,c,d) = acd + a'b + d'$$



5.3 Four-Variable Karnaugh Maps

Simplification of Four-Variable Functions



$f_1 = \Sigma m(1, 3, 4, 5, 10, 12, 13)$
$= bc' + a'b'd + ab'cd'$

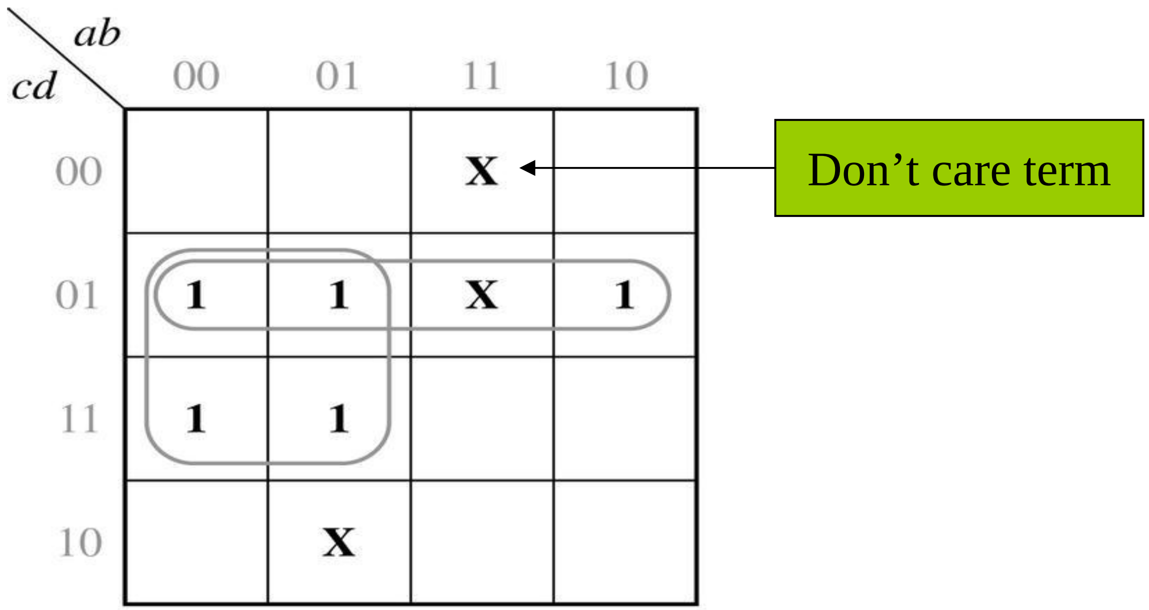
(a)

$f_2 = \Sigma m(0, 2, 3, 5, 6, 7, 8, 10, 11, 14, 15)$
$= c + b'd' + a'bd$

(b)

5.3 Four-Variable Karnaugh Maps

Simplification of an Incompletely Specified Function



$$f = \Sigma m(1, 3, 5, 7, 9) + \Sigma d(6, 12, 13)$$
$$= a'd + c'd$$

5.3 Four-Variable Karnaugh Maps

Figure 5-14

1's of f

$$f = x'z' + wyz + w'y'z' + x'y$$

0's of f

$$f' = y'z + wxz' + w'xy$$

$$f = (y + z')(w' + x' + z)(w + x' + y')$$

minimum product of sum for f

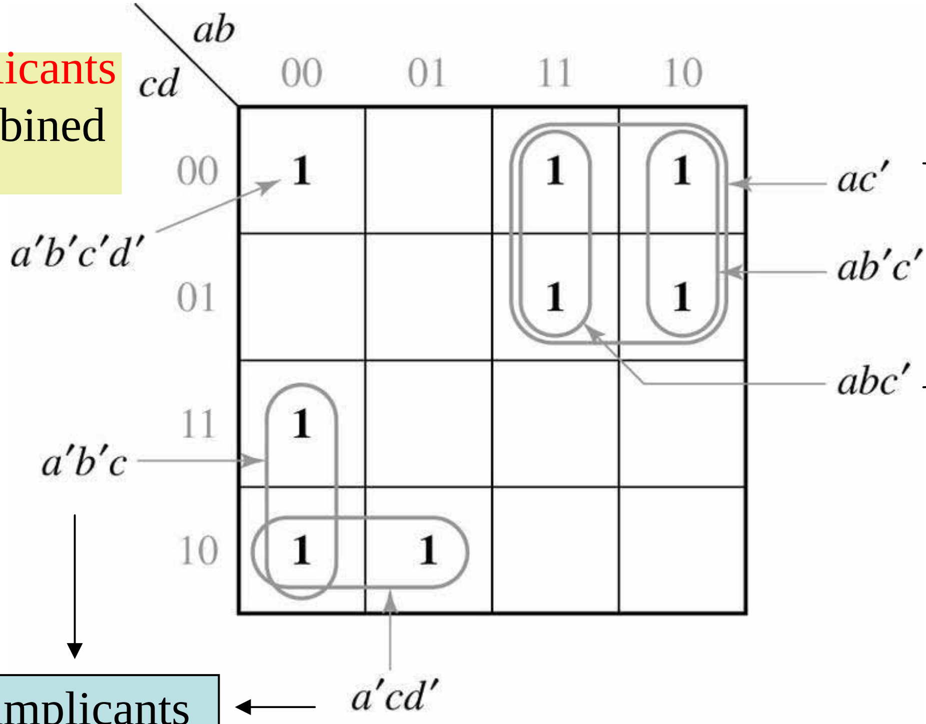
$yz \backslash wx$	00	01	11	10
00	**1**	**1**	**0**	**1**
01	**0**	**0**	**0**	**0**
11	**1**	**0**	**1**	**1**
10	**1**	**0**	**0**	**1**

5.4 Determination of Minimum Expressions Using Essential Prime Implicants

- ***Implicants of F*** : Any single ‘1’ or any group of “1’s which can be combined together on a Map

- ***prime Implicants of F*** : A product term if it can not be combined with other terms to eliminate variable

It is not Prime implicants since it can be combined with other terms

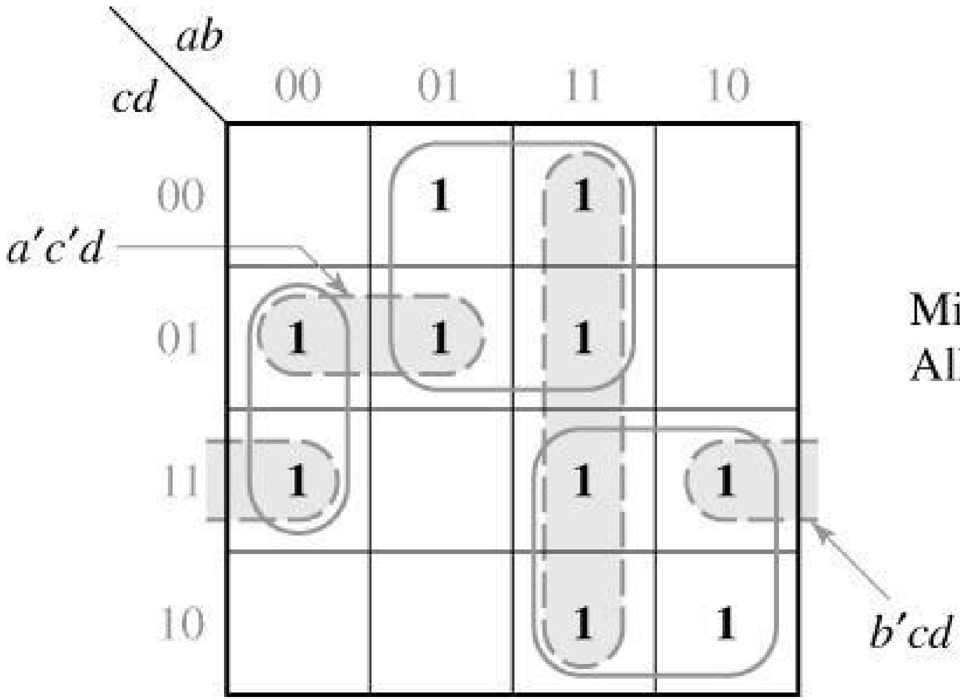


Prime implicants

Prime implicants

5.4 Determination of Minimum Expressions Using Essential Prime Implicants

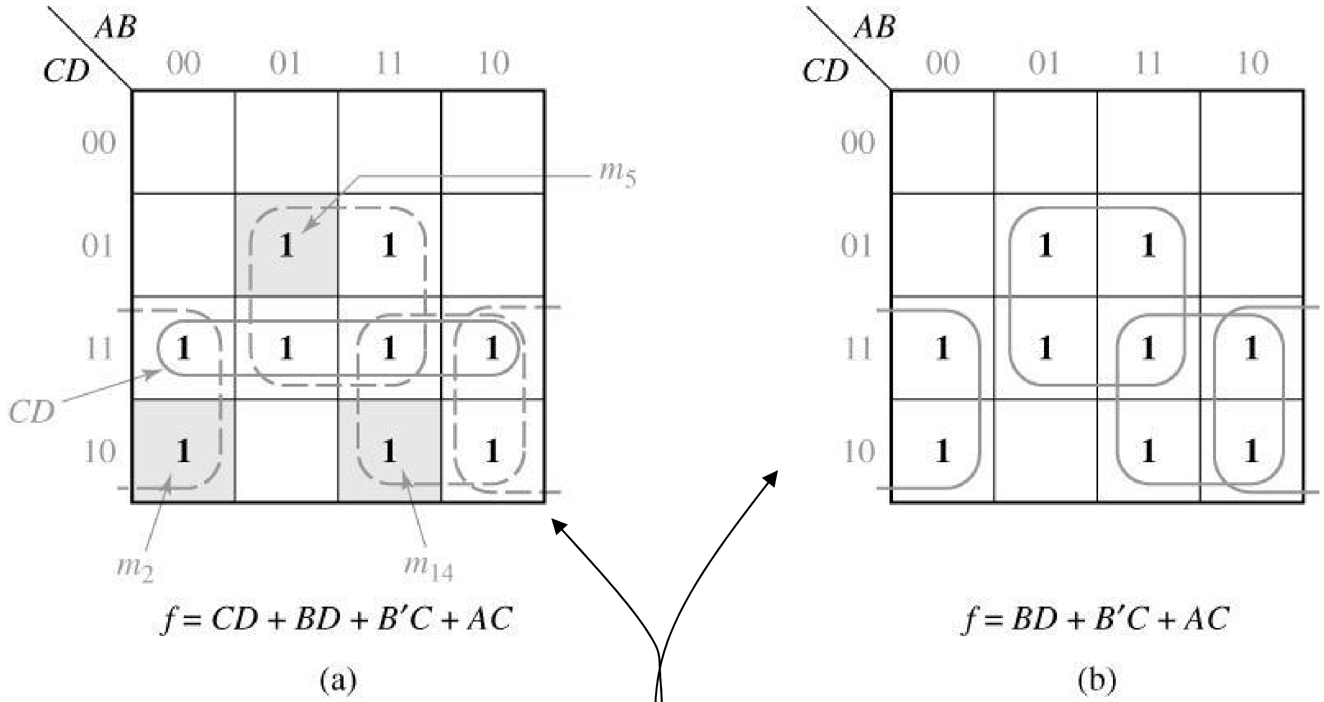
Determination of All Prime Implicants



Minimum solution: $F = a'b'd + bc' + ac$
All prime implicants: $a'b'd$, bc', ac, $a'c'd$, ab, $b'cd$

5.4 Determination of Minimum Expressions Using Essential Prime Implicants

Because all of the prime implicants of a function are generally not needed in forming the minimum sum of products, selecting prime implicants is needed.



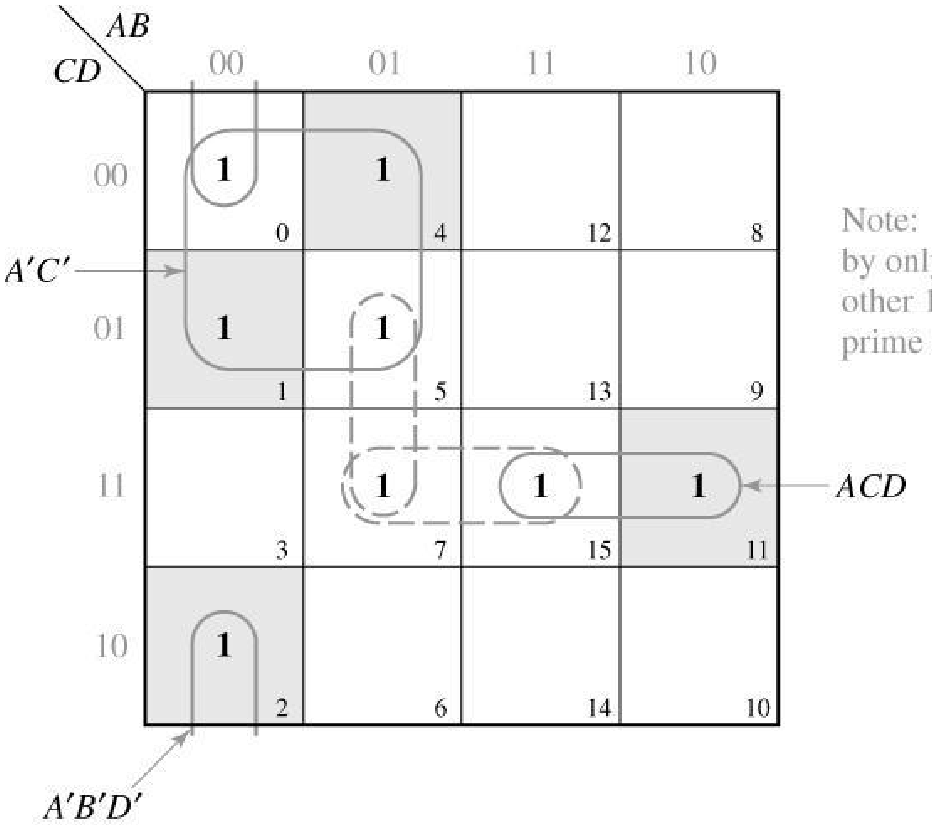
- ***CD*** **is not needed to cover for minimum expression**

-*B'C* , *AC* , *BD* are "essential" prime implicants

- ***CD*** **is not an "essential " prime implicants**

5.4 Determination of Minimum Expressions Using Essential Prime Implicants

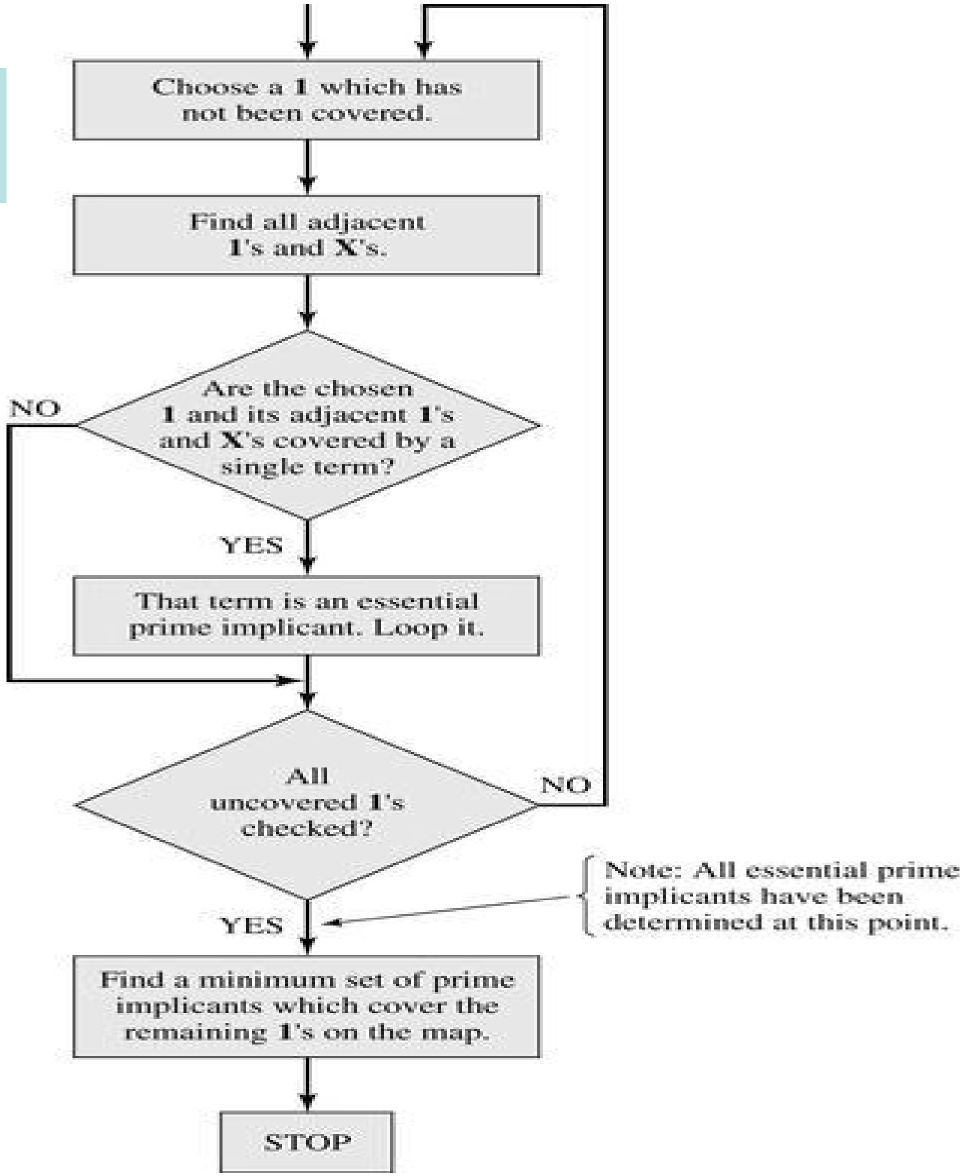
1. **First, find essential prime implicants**
2. **If minterms are not covered by essential prime implicants only, more prime implicants must be added to form minimum expression.**



$$A'C + A'B'D' + ACD + \begin{cases} A'BD \\ \text{or} \\ BCD \end{cases}$$

5.4 Determination of Minimum Expressions Using Essential Prime Implicants

Flowchart for Determining a Minimum Sum of Products Using a Karnaugh Map

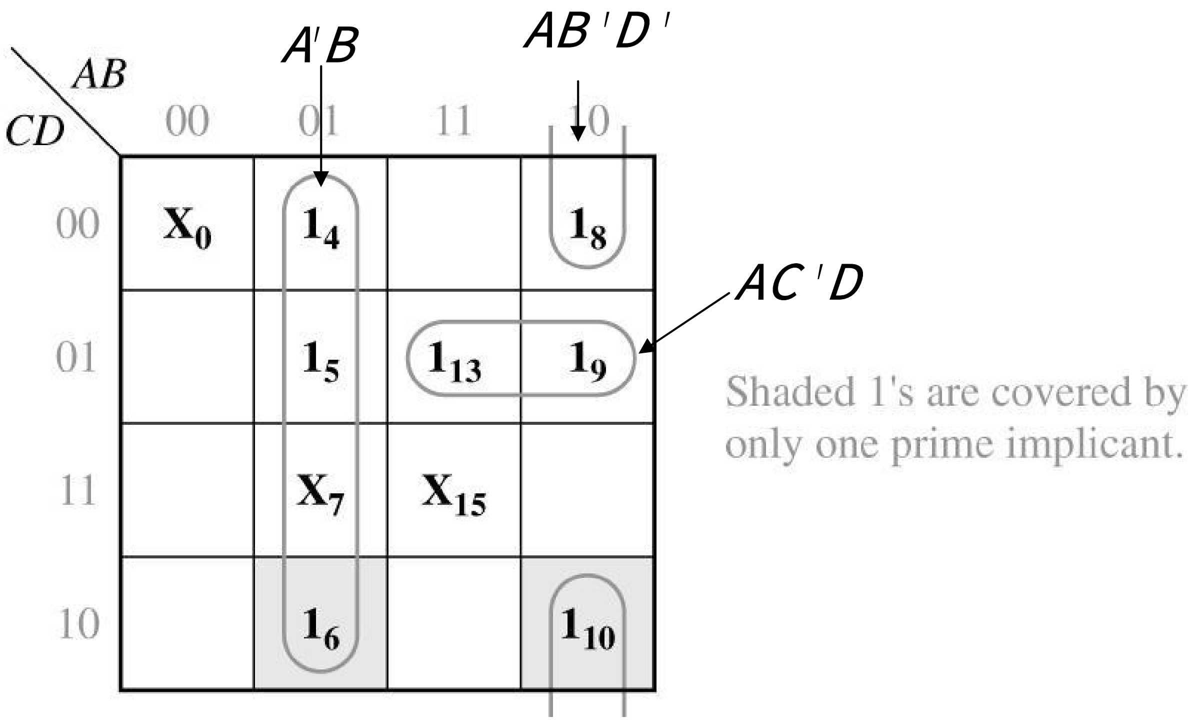


5.4 Determination of Minimum Expressions Using Essential Prime Implicants

1) *A'B* covers I_6 and its adjacent → essential PI

2) *AB'D'* covers I_{10} and its adjacent → essential PI

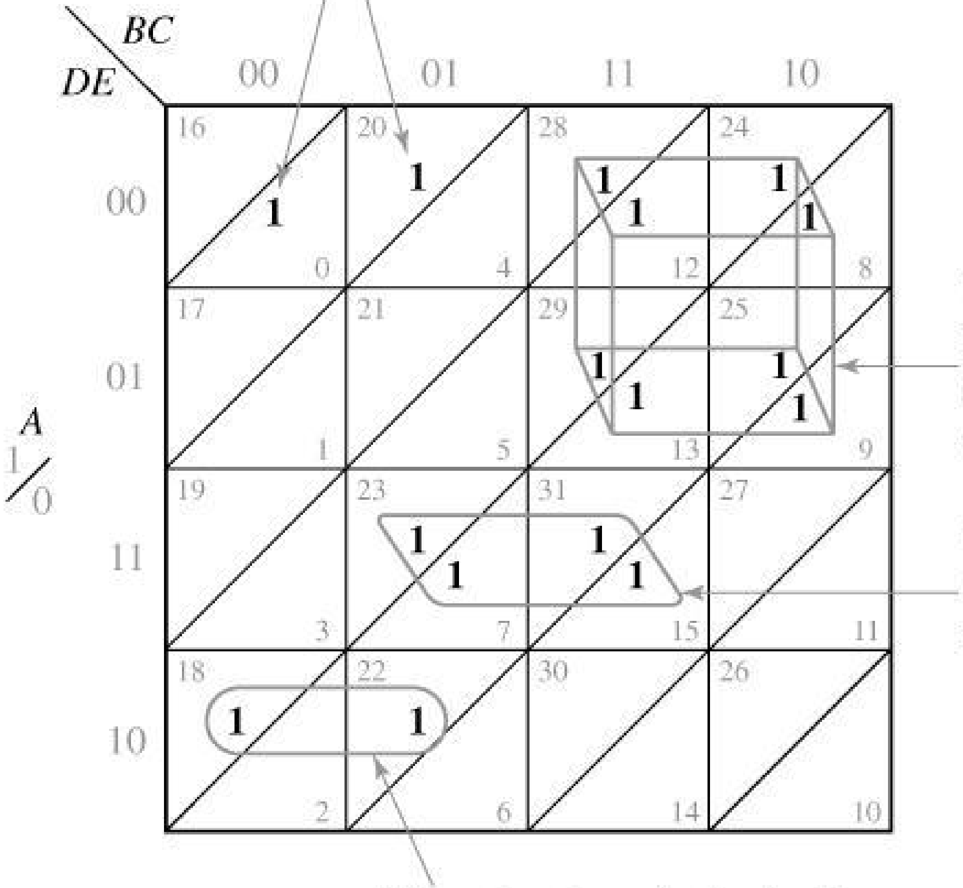
3) *AC'D* is chosen for minimal cover → *AC'D* is not an essential PI



5.5 Five-Variable Karnaugh Maps

Five-Variable Karnaugh Map

These terms do not combine because they are in different layers and different columns (they differ in two variables).



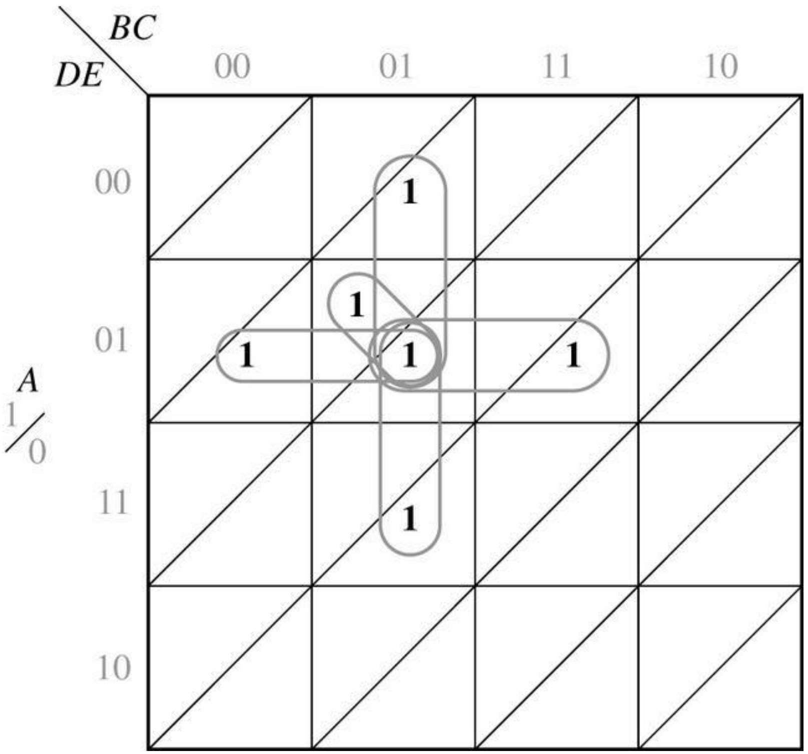
These eight terms combine to give BD' (B from last two columns and D' from top two rows; A is eliminated because four terms are in the top layer and four in the bottom).

These four terms (two from top layer and two from bottom) combine to yield CDE (C from the middle two columns and DE from the row).

These two terms in the top layer combine to give $AB'DE'$.

5.5 Five-Variable Karnaugh Maps

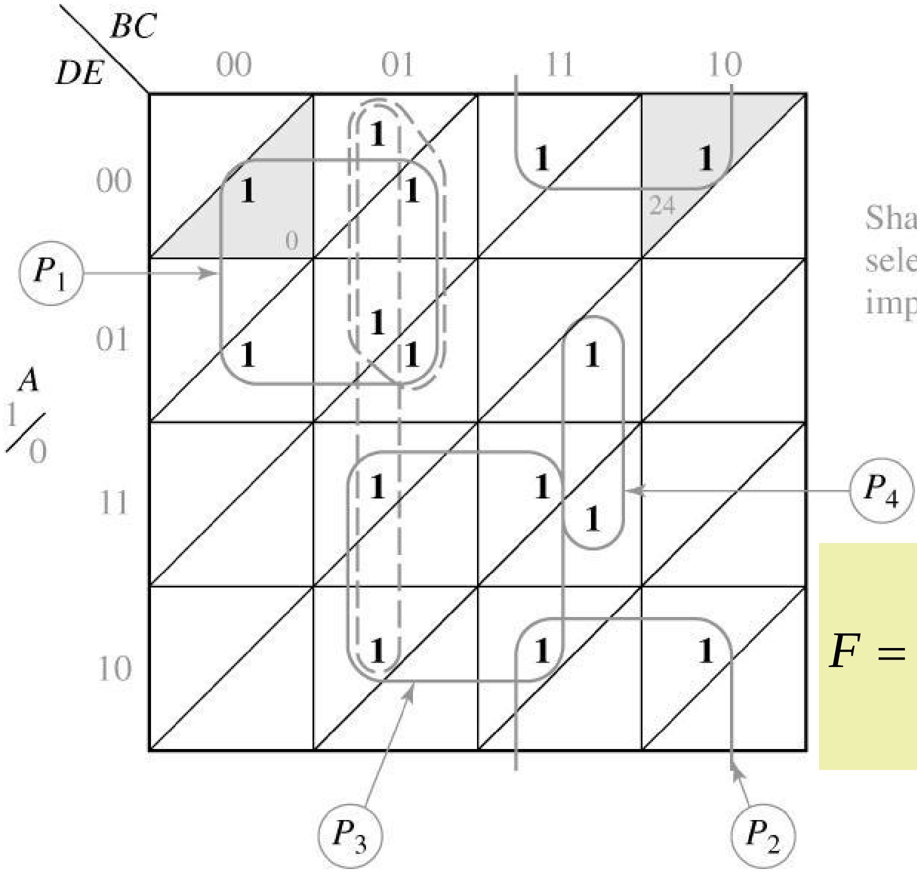
Figure 5-22



5.5 Five-Variable Karnaugh Maps

Figure 5-23

$F(A,B,C,D,E)=\sum m(0,1,4,5,13,15,20,21,22,23,24,26,28,30,31)$



Shaded 1's are used to select essential prime implicants.

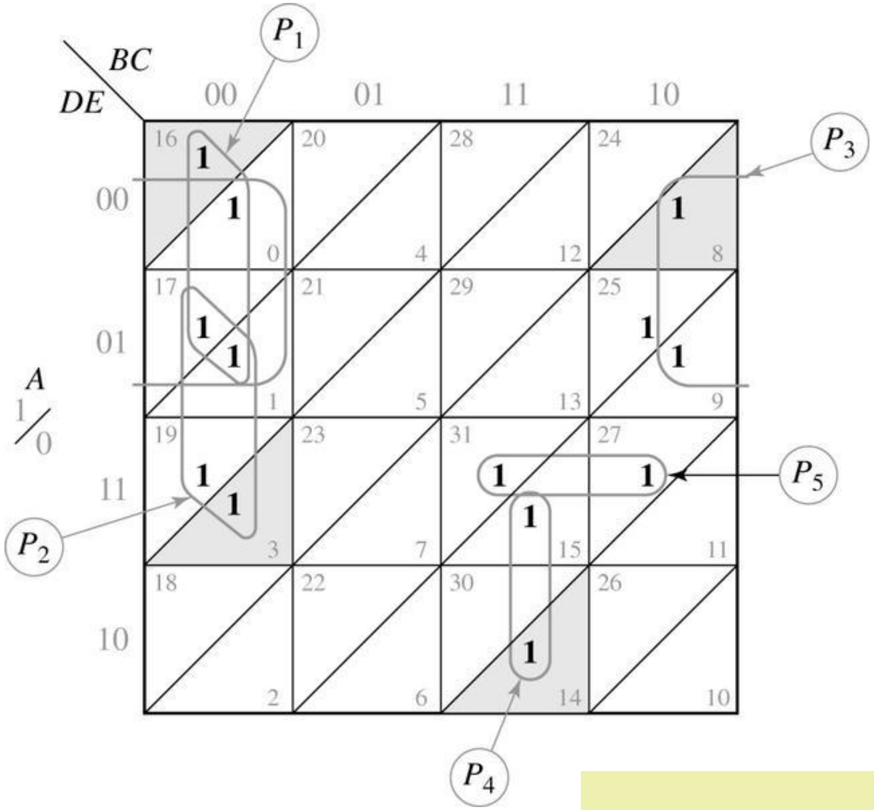
Resulting minimum solution

$$F = \underbrace{A'B'D'}_{P_1} + \underbrace{ABE'}_{P_2} + \underbrace{ACD}_{P_3} + \underbrace{A'BCE}_{P_4} + \left\{ \begin{array}{c} AB'C \\ \text{or} \\ B'CD' \end{array} \right\}$$

5.5 Five-Variable Karnaugh Maps

Figure 5-24

$F(A,B,C,D,E) = \sum m(0,1,3,8,9,14,15,16,17,19,25,27,31)$



Final solution

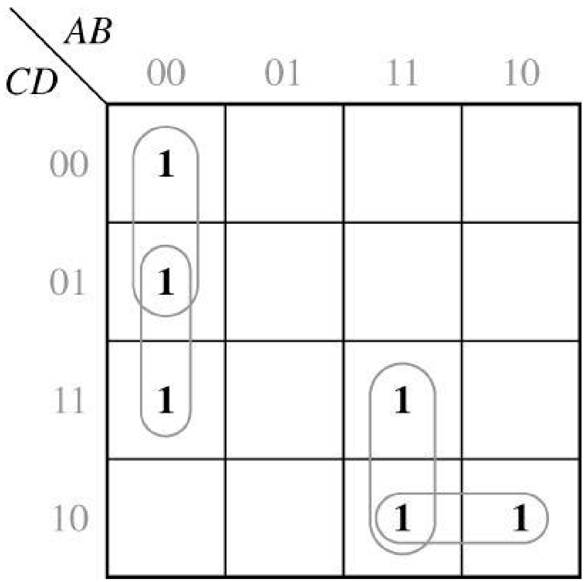
$$F = \underbrace{B'C'D'}_{P_1} + \underbrace{B'C'E}_{P_2} + \underbrace{A'C'D'}_{P_3} + \underbrace{A'BCD}_{P_4} + \underbrace{ABDE}_{P_5} + \begin{array}{c} C'D'E \\ \text{or} \\ AC'E \end{array}$$

5.6 Other Uses of Karnaugh Maps

minturm expansion of f is $f = \sum m(0,2,3,4,8,10,11,15)$
maxterm expansion of f is $f = \prod M(1,5,6,7,9,12,13,14)$

same

Figure 5-25



$F = A'B'(C' + D) + AC(B + D')$

5.6 Other Uses of Karnaugh Maps

Figure 5-26

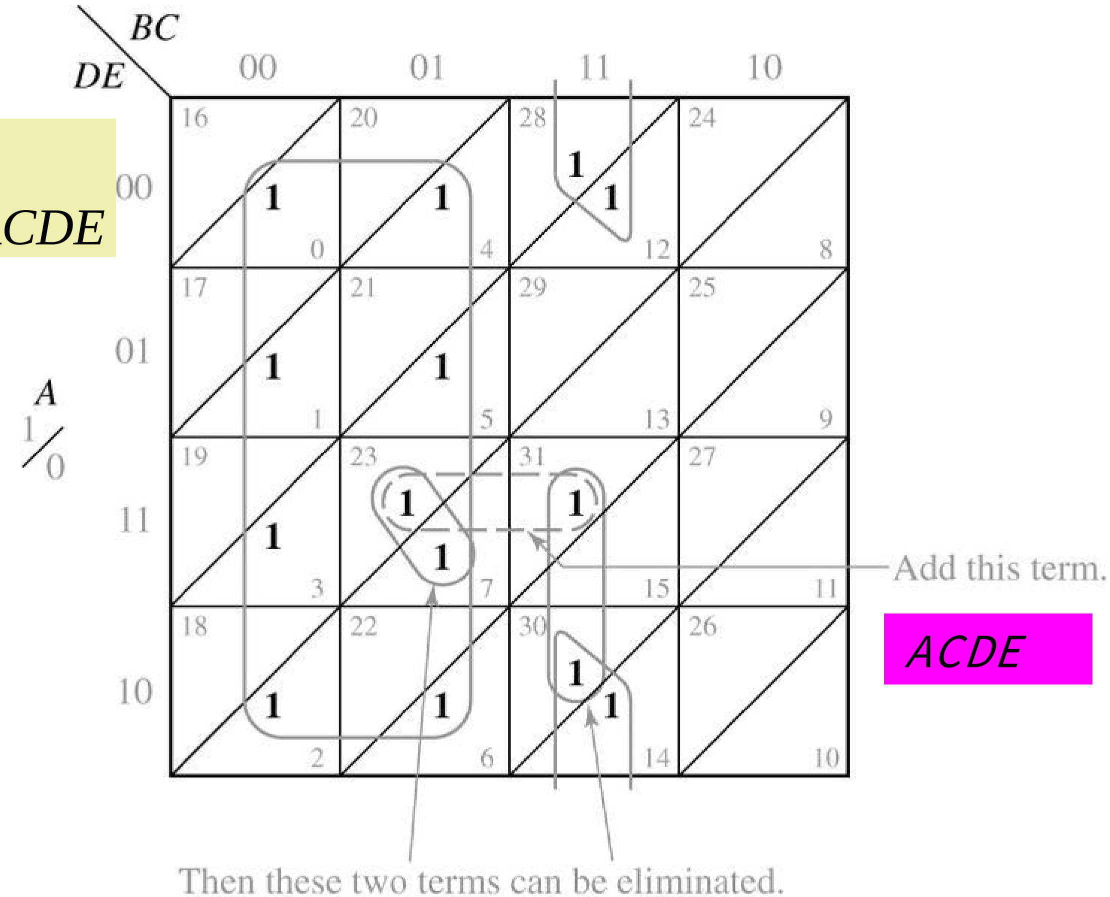
$$F = ABCD + B'CDE + A'B' + BCE'$$

Using the consensus theorem:

$$F = ABCD + B'CDE + A'B' + BCE' + ACDE$$

minimum solution:

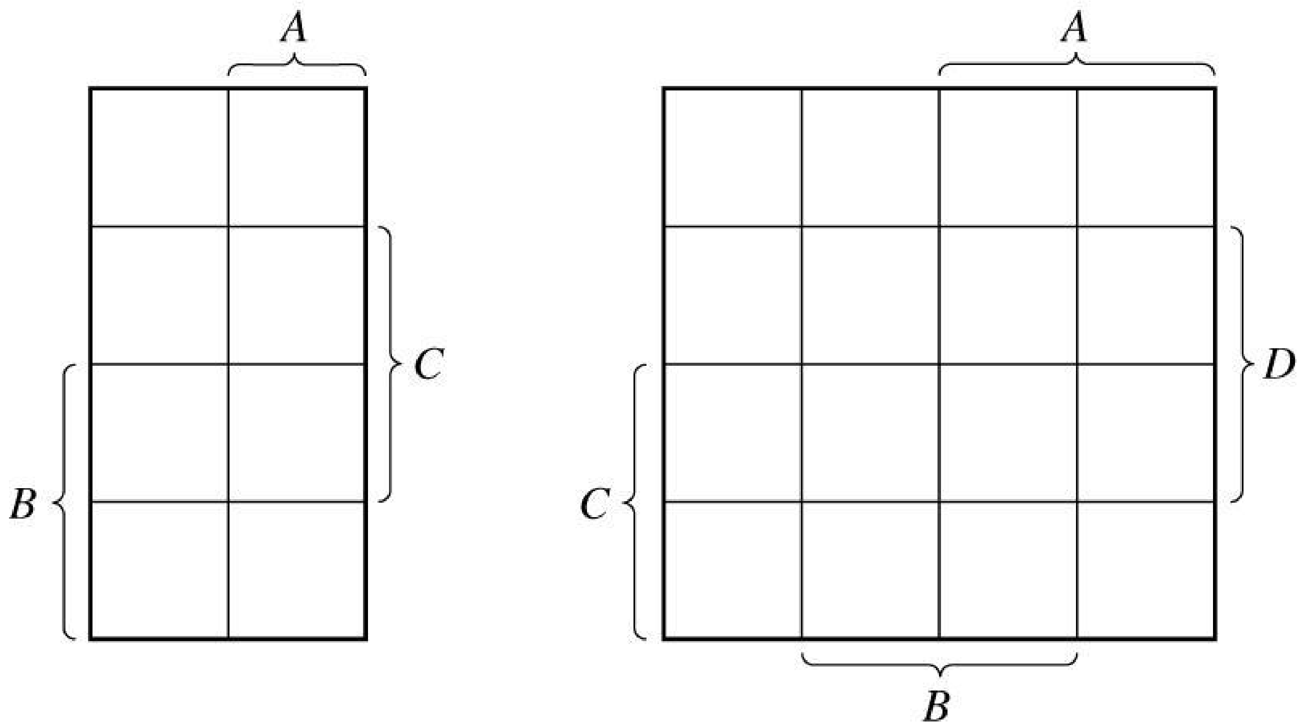
$$F = A'B' + BCE' + ACDE$$



ACDE

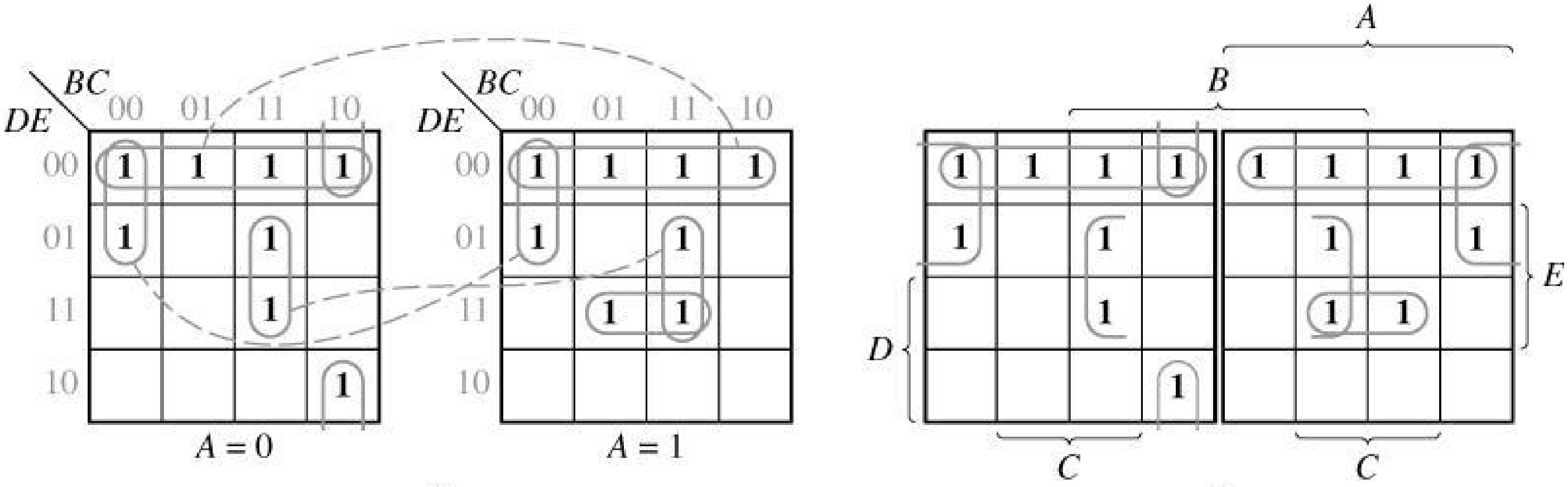
5.7 Other Forms of Karnaugh Maps

Figure 5-27. Veitch Diagrams



5.7 Other Forms of Karnaugh Maps

Figure 5-28. Other Forms of Five-Variable Karnaugh Maps



(a) (b)

$$F = D'E' + B'C'D' + BCE + A'BC'E' + ACDE$$