CHAPTER 1

INTRODUCTION
NUMBER SYSTEMS AND CONVERSION

Contents

Objectives

Topics introduced in this chapter:

- Difference between Analog and Digital System
- Difference between Combinational and Sequential Circuits
- Binary number and digital systems
- Number systems and Conversion
- Add, Subtract, Multiply, Divide Positive Binary Numbers
- 1's Complement, 2's Complement for Negative binary number
- BCD code, 6-3-1-1 code, excess-3 code

1.1 Digital Systems and Switching Circuits

- Digital systems: computation, data processing, control,
  communication, measurement
  - Reliable, Integration

- Analog – Continuous
  - Natural Phenomena
    (Pressure, Temperature, Speed…)
  - Difficulty in realizing, processing using electronics
- Digital – Discrete
  - Binary Digit ➔ Signal Processing as Bit unit
  - Easy in realizing, processing using electronics
  - High performance due to Integrated Circuit Technology

Binary Digit?

- Binary:- Two values(0, 1)
  - Each digit is called as a "bit"

Good things in Binary Number

- Number representation with only two values (0,1)
- Can be implemented with simple electronics devices

  (ex: Voltage High(1), Low(0)

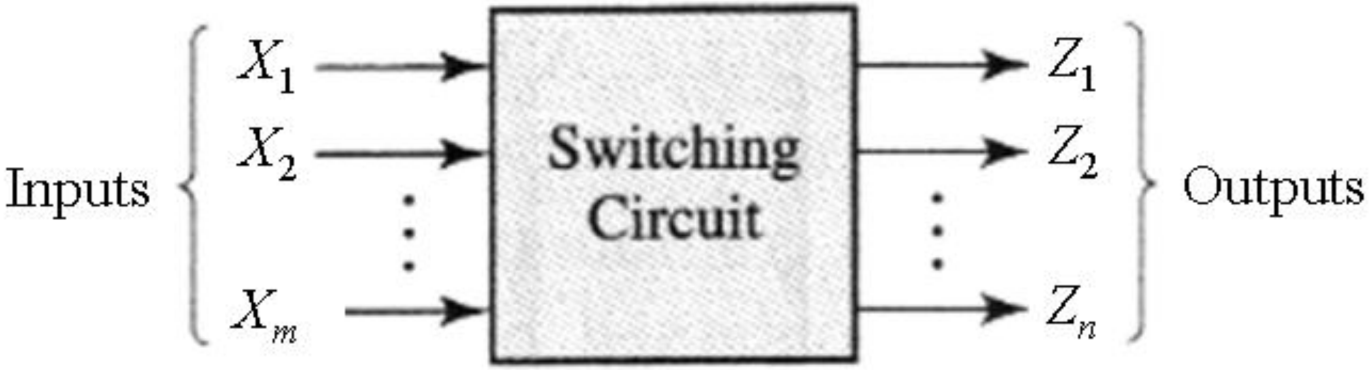
  Switch On (1) Off(0)...)

Switching Circuit

- Combinational Circuit :

  outputs depend on only present inputs, not on past inputs

- Sequential Circuit:
  - outputs depend on both present inputs and past inputs
  - have “memory” function



1.2 Number Systems and Conversion

Decimal:

$$953.78_{10} = 9\times10^{2} + 5\times10^{1} + 3\times10^{0} + 7\times10^{-1} + 8\times10^{-2}$$

Binary:

$$1011.11_{2} = 1\times2^{3} + 0\times2^{2} + 1\times2^{1} + 1\times2^{0} + 1\times2^{-1} + 1\times2^{-2}$$

$$= 8 + 0 + 2 + 1 + \frac{1}{2} + \frac{1}{4} = 11\frac{3}{4} = 11.75_{10}$$

Radix(Base):

$$N = (a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3})_R$$

$$= a_4\times R^4 + a_3\times R^3 + a_2\times R^2 + a_1\times R^1 + a_0\times R^0$$

$$+ a_{-1}\times R^{-1} + a_{-2}\times R^{-2} + a_{-3}\times R^{-3}$$

Example:

$$147.3_{8} = 1\times8^{2} + 4\times8^{1} + 7\times8^{0} + 3\times8^{-1} = 64 + 32 + 7 + \frac{3}{8}$$

$$= 103.375_{10}$$

Hexa-Decimal:

$$A2F_{16} = 10\times16^{2} + 2\times16^{1} + 15\times16^{0} = 2560 + 32 + 15 = 2607_{10}$$

1.2 Number Systems and Conversion

Conversion of Decimal to Base-R

$$N = (a_n a_{n-1} \cdots a_2 a_1 a_0)_R = a_n R^n + a_{n-1} R^{n-1} + \cdots + a_2 R^2 + a_1 R^1 + a_0$$

$$\frac{N}{R} = a_n R^{n-1} + a_{n-1} R^{n-2} + \cdots + a_2 R^1 + a_1 = Q_1, \text{ remainder } a_0$$

$$\frac{Q_1}{R} = a_n R^{n-2} + a_{n-1} R^{n-3} + \cdots + a_3 R^1 + a_2 = Q_2, \text{ remainder } a_1$$

$$\frac{Q_2}{R} = a_n R^{n-3} + a_{n-1} R^{n-4} + \cdots + a_3 = Q_3, \text{ remainder } a_2$$

1.2 Number Systems and Conversion

Example: Decimal to Binary Conversion

2	53	
2	26	rem. = 1 = a_0
2	13	rem. = 0 = a_1
2	6	rem. = 1 = a_2
2	3	rem. = 0 = a_3
2	1	rem. = 1 = a_4
	0	rem. = 1 = a_5

$$53_{10} = 110101_2$$

1.2 Number Systems and Conversion

Conversion of a decimal fraction to Base-R

$$F = (.a_{-1}a_{-2}a_{-3}\cdots a_{-m})_R = a_{-1}R^{-1} + a_{-2}R^{-2} + a_{-3}R^{-3} + \cdots + a_{-m}R^{-m}$$

$$FR = a_{-1} + a_{-2}R^{-1} + a_{-3}R^{-2} + \cdots + a_{-m}R^{-m+1} = a_{-1} + F_1$$

$$F_1R = a_{-2} + a_{-3}R^{-1} + \cdots + a_{-m}R^{-m+2} = a_{-2} + F_2$$

$$F_2R = a_{-3} + \cdots + a_{-m}R^{-m+3} = a_{-3} + F_3$$

Example:

$$\begin{array}{rr} F = & .625 \\ & \times \quad 2 \\ \hline & 1.250 \\ & (a_{-1} = 1) \end{array} \qquad \begin{array}{rr} F_1 = & .250 \\ & \times \quad 2 \\ \hline & 0.500 \\ & (a_{-2} = 0) \end{array} \qquad \begin{array}{rr} F_2 = & .500 \\ & \times \quad 2 \\ \hline & 1.000 \\ & (a_{-3} = 1) \end{array}$$

$$.625_{10} = .101_2$$

1.2 Number Systems and Conversion

Example: Convert 0.7 to binary

$$
\begin{array}{r}
.7 \\
\underline{\quad 2} \\
(1).4 \\
\underline{\quad 2} \\
(0).8 \\
\underline{\quad 2} \\
(1).6 \\
\underline{\quad 2} \\
(1).2 \\
\underline{\quad 2} \\
(0).4 \\
\underline{\quad 2} \\
(0).8
\end{array}
$$

← **Process starts repeating here because .4 was previously obtained**

$$0.7_{10} - 0.1\,\underline{0110}\,\underline{0110}\,\underline{0110}\cdots_2$$

1.2 Number Systems and Conversion

Example: Convert 231.3 to base-7

$$231.3_4 = 2\times 16 + 3\times 4 + 1 + \frac{3}{4} = 45.75_{10}$$

$$\begin{array}{rrl}
7 & \underline{\big/\,45} & \\
7 & \underline{\big/\ \ 6} & \text{rem.}3 \\
 & 0 & \text{rem.}6
\end{array}
\qquad
\begin{array}{r}
.75 \\
\underline{7} \\
(5).25 \\
\underline{7} \\
(1).75 \\
\underline{7} \\
(5).25 \\
\underline{7} \\
(1).75
\end{array}$$

$$45.75_{10} = 63.5151\cdots_7$$

$$1001101.010111_2 = \underbrace{0100}_{4}\ \underbrace{1101}_{D}\ \underbrace{0101}_{5}\ \underbrace{1100}_{C} = 4D.5C_{16}$$

1.2 Number Systems and Conversion

Conversion of Binary to Octal, Hexa-decimal

- $(101011010111)_2$
  $= (\qquad)_8$, octal

- $(10111011)_2$
  $= (\qquad)_8$, octal

- $(1010111100100101)_2$
  $= (\qquad)_{16}$, Hexadecimal

- $(110110100)_2$
  $= (\qquad)_{16}$, Hexadecimal

1.3 Binary Arithmetic

Addition

$$0+0=0$$

$$0+1=1$$

$$1+0=1$$

$$1+1=0$$ **and carry 1 to the next column**

Example:

$$
\begin{array}{rrr}
 & 1111 & \leftarrow \textbf{carries} \\
13_{10} = & 1101 & \\
11_{10} = & \underline{1011} & \\
 & 11000 = 24_{10} &
\end{array}
$$

1.3 Binary Arithmetic

Subtraction

$$0-0=0$$

$$0-1=1$$ **and borrow 1 from the next column**

$$1-0=1$$

$$1-1=0$$

Example:

$$\begin{array}{r} 1 \\ 11101 \\ -10011 \\ \hline 1010 \end{array}$$

$\leftarrow$ (indicates a borrow From the 3rd column)

$$\begin{array}{r} 1111 \\ 10000 \\ -\quad 11 \\ \hline 1101 \end{array}$$

$\leftarrow$ borrows

$$\begin{array}{r} 111 \\ 111001 \\ -\quad 1011 \\ \hline 101110 \end{array}$$

$\leftarrow$ borrows

1.3 Binary Arithmetic

Subtraction Example with Decimal

column 2 column 1

$$\begin{array}{r} 205 \\ -\ \ 18 \\ \hline 187 \end{array}$$

$$\begin{aligned}
205-18 &= [2\times10^2+0\times10^1+5\times10^0] \\
&\quad -[\qquad\qquad 1\times10^1+8\times10^0]
\end{aligned}$$

note borrow from column 1

$$\begin{aligned}
&= [2\times10^2+(0-1)\times10^1+(10+5)\times10^0] \\
&\quad -[\qquad\qquad 1\times10^1+\qquad 8\times10^0]
\end{aligned}$$

note borrow from column 2

$$\begin{aligned}
&= [(2-1)\times10^2+(10+0-1)\times10^1+15\times10^0] \\
&\quad -[\qquad\qquad\qquad\qquad 1)\times10^1+8\times10^0] \\
&= [1\times10^2\quad +\quad 8\times10^1\qquad +\ 7\times10^0]=187
\end{aligned}$$

1.3 Binary Arithmetic

Multiplication

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

Multiply: 13 x11(10)

$$\begin{array}{r} 1101 \\ \underline{1011} \\ 1101 \\ 1101 \\ 0000 \\ \underline{1101} \\ 10001111 = 143_{10} \end{array}$$

1111	multiplicand
1101	multiplier
1111	first partial product
0000	second partial product
(01111)	sum of first two partial products
1111	third partial product
(1001011)	sum after adding third partial product
1111	fourth partial product
11000011	final product (sum after adding fourth partial prodoct)

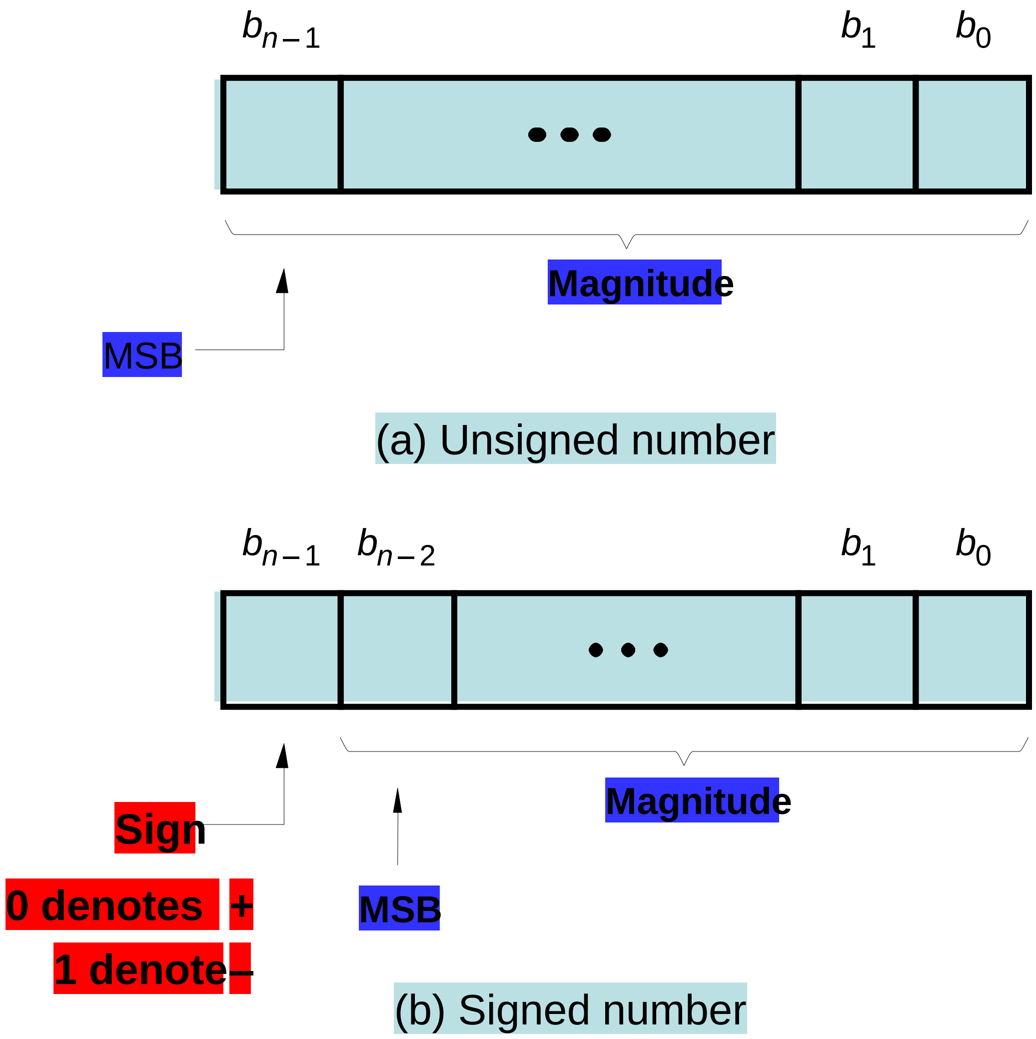
1.3 Binary Arithmetic

Division

$$
\begin{array}{r@{\,}l}
 & 1101 \\
\cline{2-2}
1011 & \big)\,10010001 \\
 & \underline{1011} \\
 & 1110 \\
 & \underline{1011} \\
 & 1101 \\
 & \underline{1011} \\
 & 10
\end{array}
$$

The quotient is 1101 with a remainder of 10.

1.4 Representation of Negative Numbers



1.4 Representation of Negative Numbers

2's complement representation for Negative Numbers

$$N^* = 2^n - N$$

+N	Positive integers (all systems)	-N	Negative integers: Sign and magnitude	Negative integers: 2's complement N^*	Negative integers: 1's complement $\overline{N}$
+0	0000	-0	1000	-	1111
+1	0001	-1	1001	1111	1110
+2	0010	-2	1010	1110	1101
+3	0100	-3	1011	1101	1100
+4	0101	-4	1100	1100	1011
+5	0110	-5	1101	1011	1010
+6	0111	-6	1110	1010	1001
+7		-7	1111	1001	1000
		-8	-	1000	-

1.4 Representation of Negative Numbers

1's complement representation for Negative Numbers

$$\overline{N} = (2^n - 1) - N$$

Example:

$$2^n - 1 = 111111$$

$$N = 010101$$

$$\overline{N} = 101010$$

$$N^* = 2^n - N = (2^n - 1 - N) + 1 = \overline{N} + 1$$

==➔ 2's complement: 1's complement + '1'

$$N = 2^n - N^* \text{ and } N = (2^n - 1) - \overline{N}$$

$$2^n - 2^{n-1} = 2^{n-1}$$

1.4 Representation of Negative Number

Addition of 2's complement Numbers

Case 1

+3	0011	
+4	0100	
+7	0111	(correct answer)

Case 2

+5	0101	
+6	0110	
	1011	← wrong answer because of **overflow** (+11 requires 5 bits including sign)

Case 3

+5	0101	
−6	1010	
	1111	(correct answer)

Case 4

−5	1011	
+6	0110	
	(1)0001	← correct answer when the carry from the sign bit is ignored (this is *not* an overflow)

1.4 Representation of Negative Numbers

Addition of 2's complement Numbers

Case 5

$$\begin{array}{rr} -3 & 1101 \\ \underline{-4} & \underline{1100} \\ -7 & (1)1001 \end{array}$$

$\leftarrow$ correct answer when the last carry is ignored (this is *not* an overflow)

Case 6

$$\begin{array}{rr} -5 & 1011 \\ \underline{-6} & \underline{1010} \\ & (1)0101 \end{array}$$

$\leftarrow$ wrong answer because of overflow (-11 requires 5 bits including sign)

1.4 Representation of Negative Numbers

Addition of 1's complement Numbers

Case 3

$$\begin{array}{rrl} +5 & 0101 & \\ \underline{-6} & \underline{1001} & \\ -1 & 1110 & \text{(correct answer)} \end{array}$$

Case 4

$$\begin{array}{rrrl} & & 1010 & \\ -5 & & \underline{0110} & \\ \underline{+6} & (1) & 0000 & \\ & & \underline{1} & \text{(end-around carry)} \\ & & 0001 & \text{(correct answer, } \textit{no} \text{ overflow)} \end{array}$$

Case 5

$$\begin{array}{rrrl} & & 1100 & \\ -5 & & \underline{1011} & \\ \underline{+6} & (1) & 0111 & \\ & & \underline{1} & \text{(end-around carry)} \\ & & 1000 & \text{(correct answer, } \textit{no} \text{ overflow)} \end{array}$$

1.4 Representation of Negative Numbers

Addition of 1's complement Numbers

Case 6

$$
\begin{array}{rrr}
 & & 1010 \\
-5 & & \underline{1001} \\
\underline{-6} & (1) & 0011 \\
 & & \underline{1} \\
 & & 0100
\end{array}
$$

(end-around carry)

(wrong answer because of overflow)

$$Case\,4: \quad -A+B \;\; (\text{where } B>A)$$

$$\overline{A}+B=(2^n-1-A)+B=2^n+(B-A)-1$$

$$Case\,5: \quad -A-B \;\; (A+B<2^{n-1})$$

$$\overline{A}+\overline{B}=(2^n-1-A)+(2^n-1-B)=2^n+[2^n-1-(A+B)]-1$$

1.4 Representation of Negative Numbers

Addition of 1's complement Numbers

$$
\begin{array}{rrl}
 & 11110100 & \quad (-11) \\
 & \underline{11101011} & \quad \underline{+(-20)} \\
(1) & 11011111 & \\
 & \underline{1} & \quad \text{(end-around carry)} \\
 & 11100000 = 30 &
\end{array}
$$

Addition of 2's complement Numbers

$$
\begin{array}{rl}
11111000 & \quad (-8) \\
\underline{00010011} & \quad \underline{+19} \\
\cancel{(1)}00001011 & \quad = +11
\end{array}
$$

(end-around carry)

1.5 Binary Codes

$$\overbrace{1001}^{9}\ \overbrace{0011}^{3}\ \overbrace{0111}^{7}\ .\ \overbrace{0010}^{2}\ \overbrace{0101}^{5}$$

Decimal Digit	8-4-2-1 Code (BCD)	6-3-1-1 Code	Excees-3 Code	2-out-of-5 Code	Gray Code
0	0000	0000	0011	00011	0000
1	0001	0001	0100	00101	0001
2	0010	0011	0101	00110	0011
3	0011	0100	0110	01001	0010
4	0100	0101	0111	01010	0110
5	0101	0111	1000	01100	1110
6	0110	1000	1001	10001	1010
7	0111	1001	1010	10010	1011
8	1000	1011	1011	10100	1001
9	1001	1100	1100	11000	1000

1.5 Binary Codes

6-3-1-1 Code:

$$N = w_3 a_3 + w_2 a_2 + w_1 a_1 + w_0 a_0$$

$$N = 6 \cdot 1 + 3 \cdot 0 + 1 \cdot 1 + 1 \cdot 1 = 8$$

ASCII Code

1010011	1110100	1100001	1110010	1110100
S	t	a	r	t