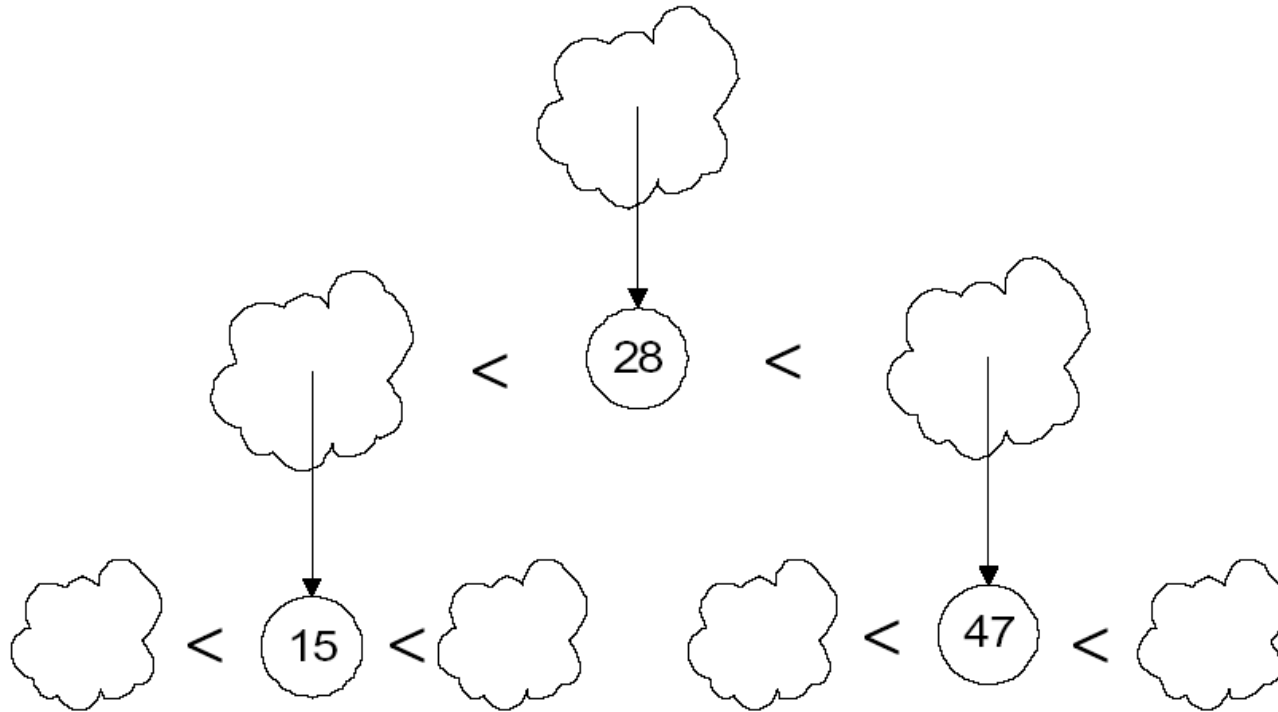


Hızlı Sıralama (Quick-Sort)

- Bu algoritma Böl-Yönet yaklaşımına dayanmaktadır.

Hızlı Sıralama



Bir pivot eleman seçilir. Pivottan küçük olanlar ve büyük olanlar olarak dizi ikiye bölünür.
Herbir taraf özyineli olarak sıralanır.

Hızlı sıralama (Quicksort) Algoritması

Tamsayılardan oluşan n elemanlı bir dizi verilmiş olsun.

- If {dizi sadece bir elemandan oluşuyorsa} , return
- Else
 - *pivot* olarak kullanmak için bir eleman seç
 - Diziyi aşağıdaki gibi iki alt dizi halinde böl:
 - *Pivot*'a eşit yada pivottan küçük olanlar
 - *Pivottan* büyük olanlar
 - İki alt diziyi Hızlı sırala
 - Return sonuç dizi

Örnek

Verilen diziyi algoritmaya göre sıralayalım

40	20	10	80	60	50	7	30	100
----	----	----	----	----	----	---	----	-----

Pivot Eleman seç

- Pivot eleman değişik şekillerde seçilebilir. Bu örnekte ilk eleman pivot olarak seçilecektir.

40	20	10	80	60	50	7	30	100
----	----	----	----	----	----	---	----	-----

Diziyi bölme

- Pivottan küçük yada eşit olanlardan oluşan bir altdizi(\leq pivot)
- Pivottan büyük olan elemanlardan oluşan bir dizi($>$ pivot)
- **Altdiziler orijinal dizide tutulur.**
- **Bölme işlemini diziler parçalanamayacak hale gelene kadar rekürsif olarak tekrarla**

pivot_index = 0

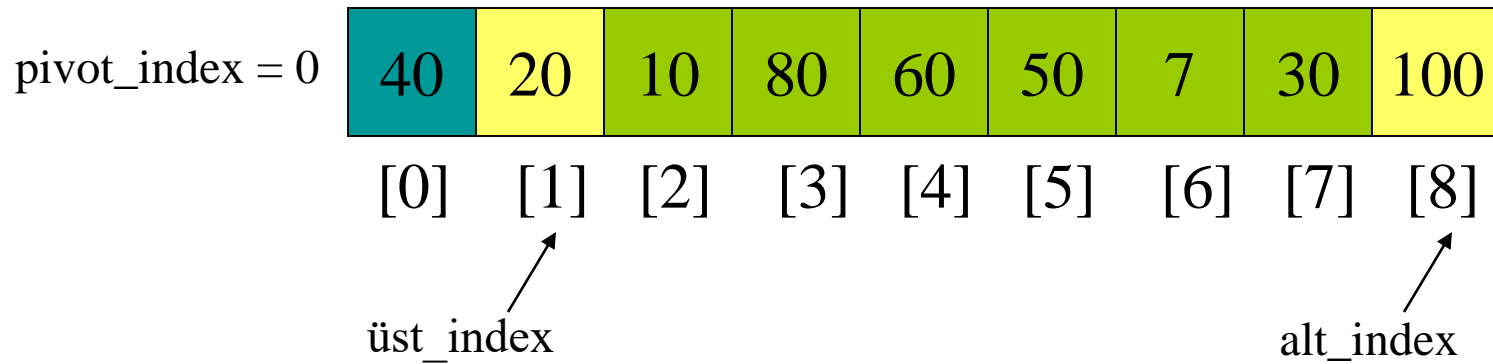
40	20	10	80	60	50	7	30	100
----	----	----	----	----	----	---	----	-----

[0] [1] [2] [3] [4] [5] [6] [7] [8]

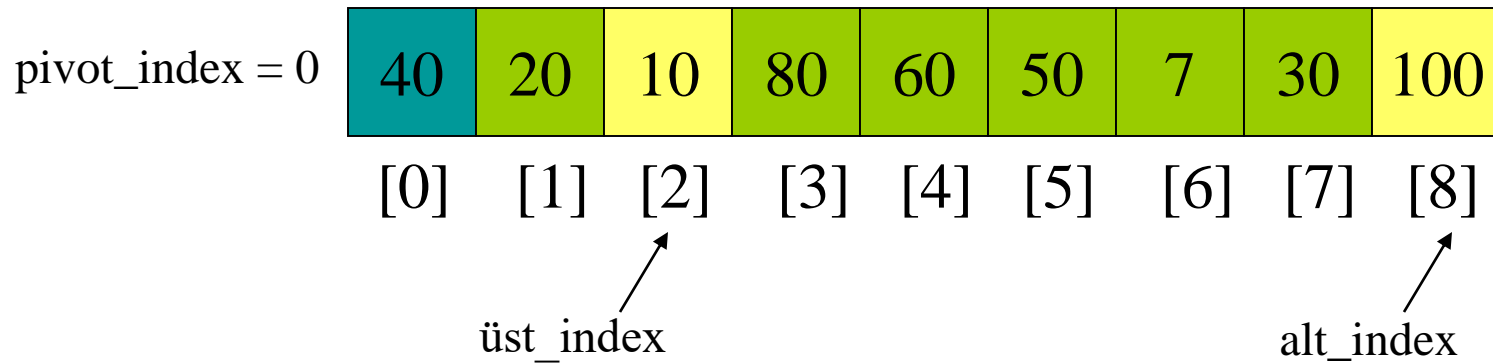
üst_index

alt_index

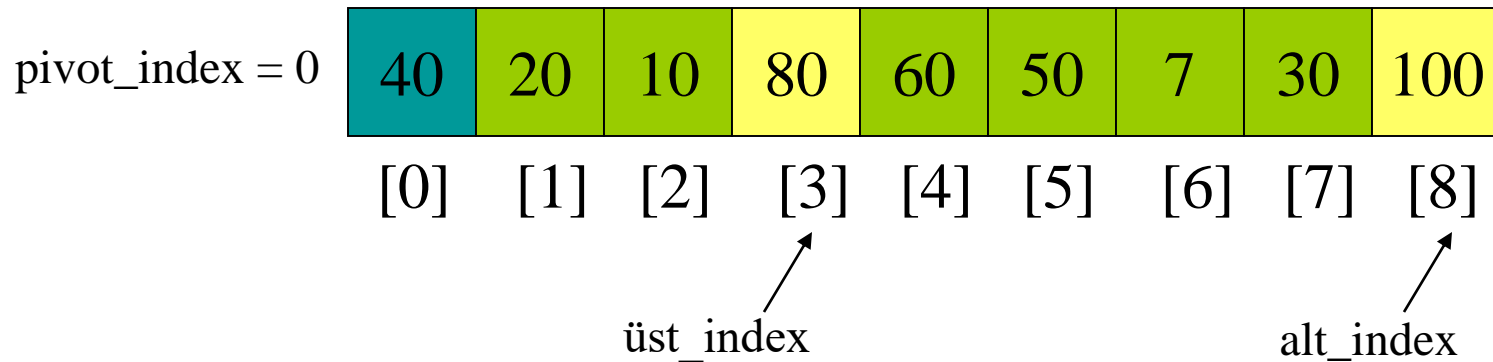
1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
 $++\text{üst_index}$



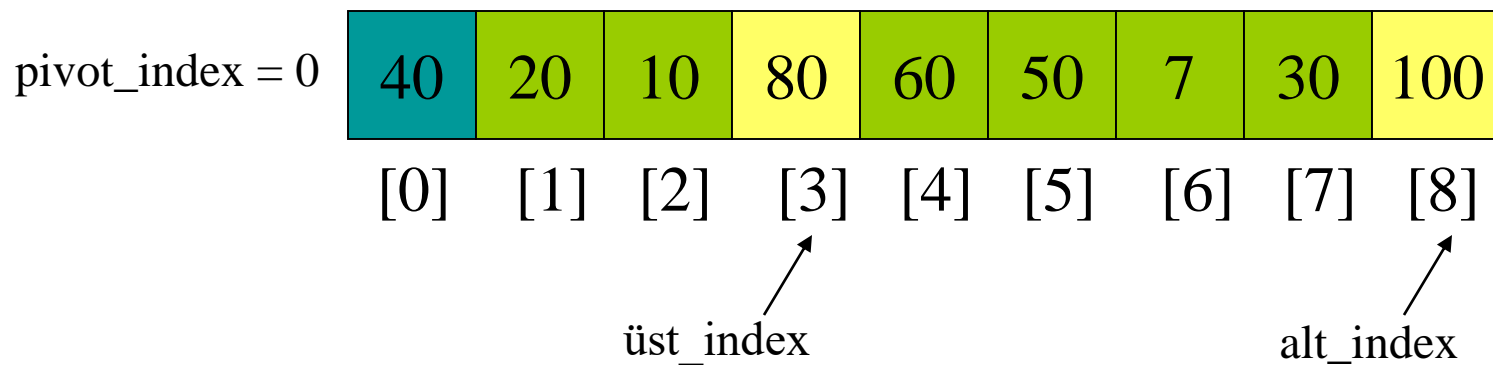
1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
 $++\text{üst_index}$



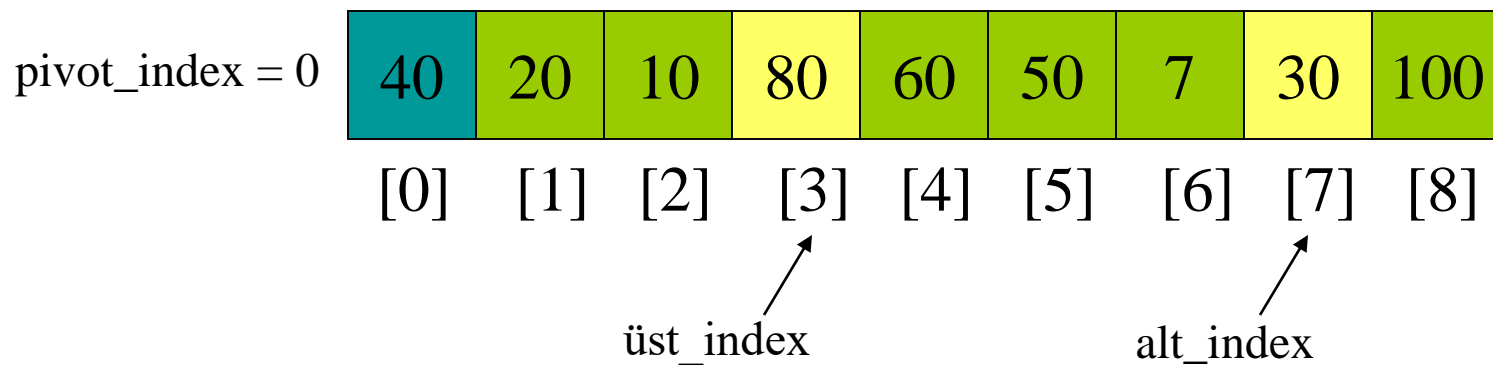
1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
 $++\text{üst_index}$



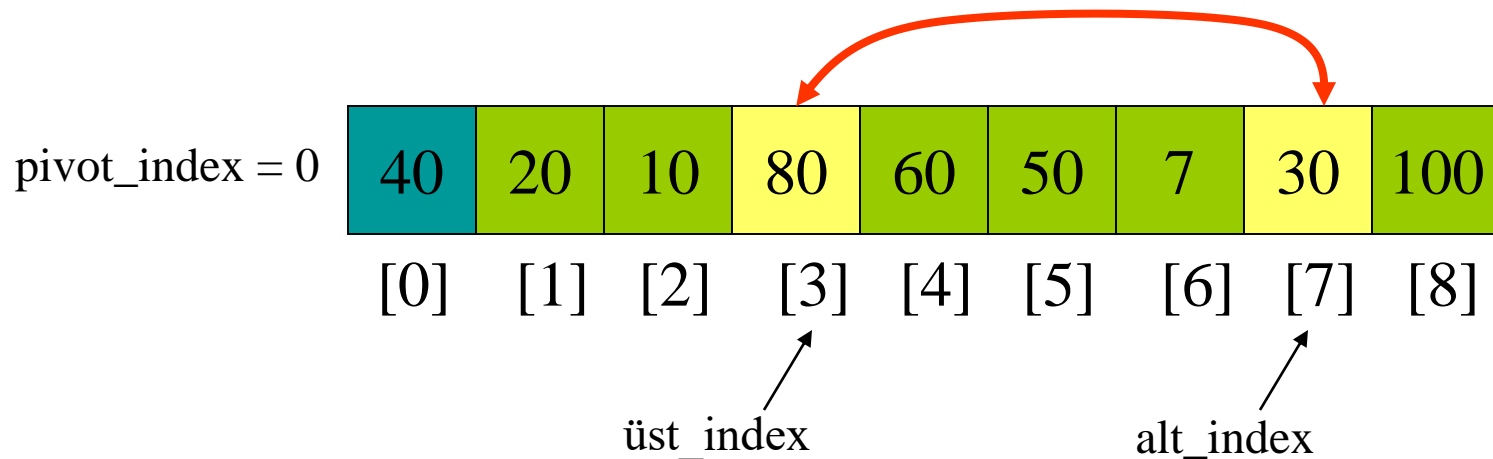
1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
 ++üst_index
2. While $\text{data}[\text{alt_index}] > \text{data}[\text{pivot}]$
 --alt_index



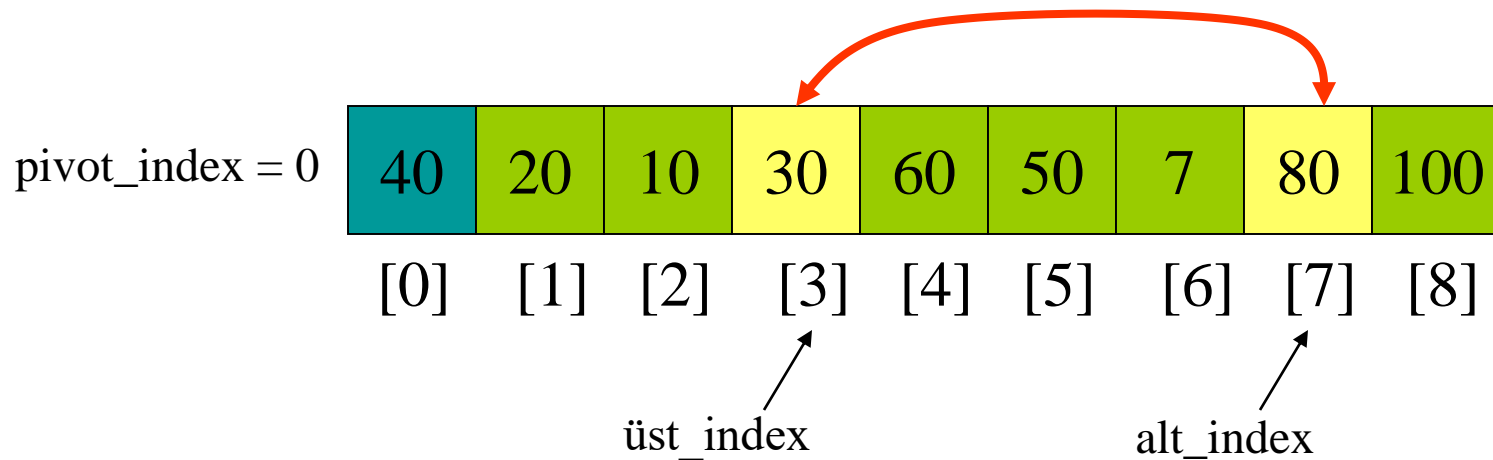
1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
 $++\text{üst_index}$
2. While $\text{data}[\text{alt_index}] > \text{data}[\text{pivot}]$
 $--\text{alt_index}$



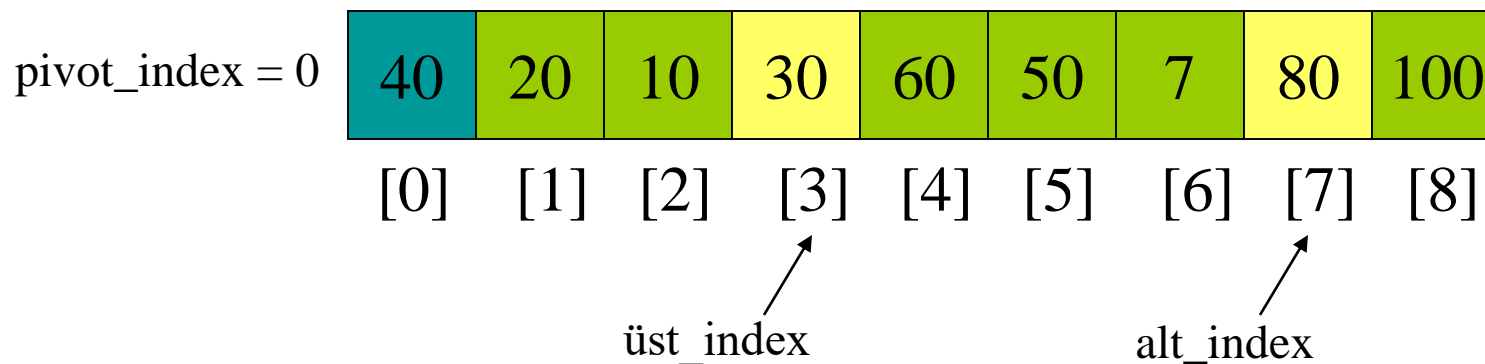
1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
 ++üst_index
2. While $\text{data}[\text{alt_index}] > \text{data}[\text{pivot}]$
 --alt_index
3. If $\text{üst_index} < \text{alt_index}$
 swap $\text{data}[\text{üst_index}]$ and $\text{data}[\text{alt_index}]$



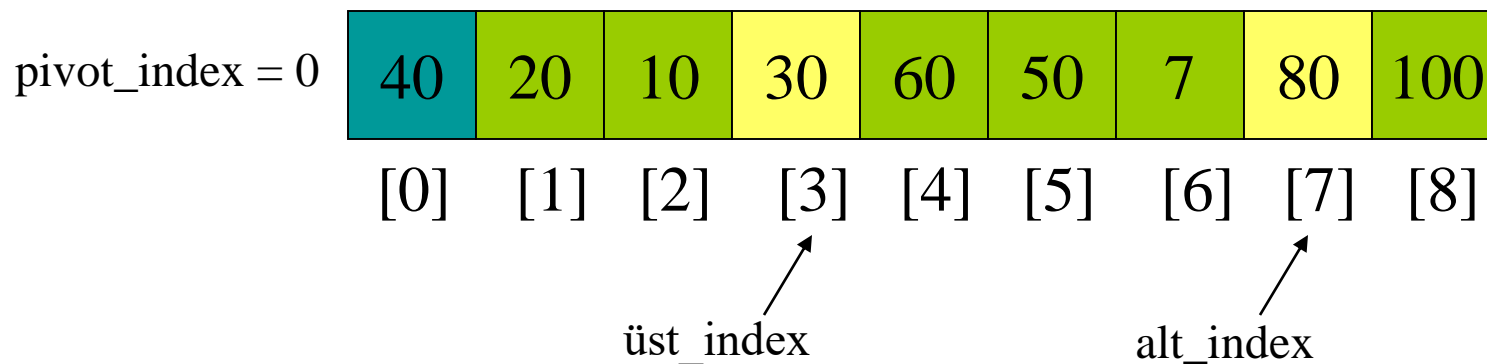
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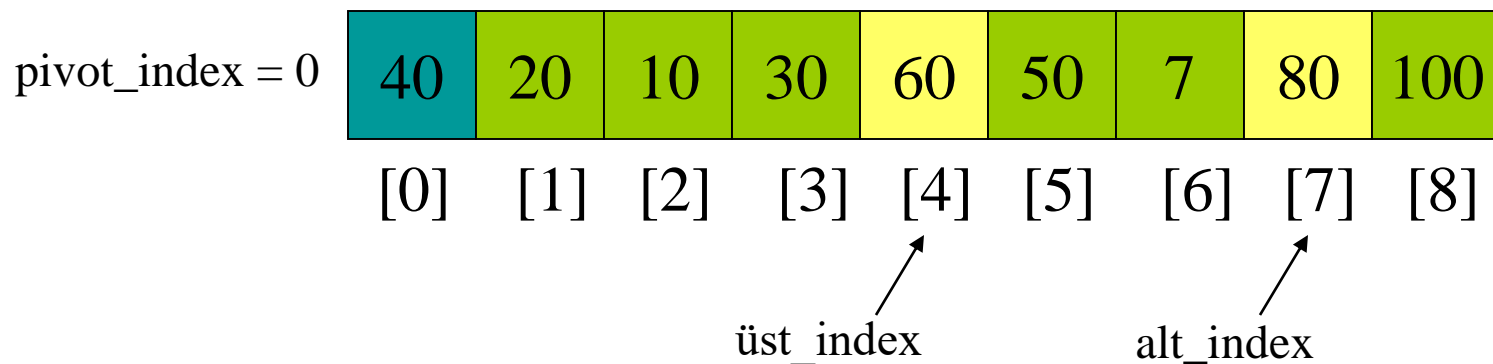
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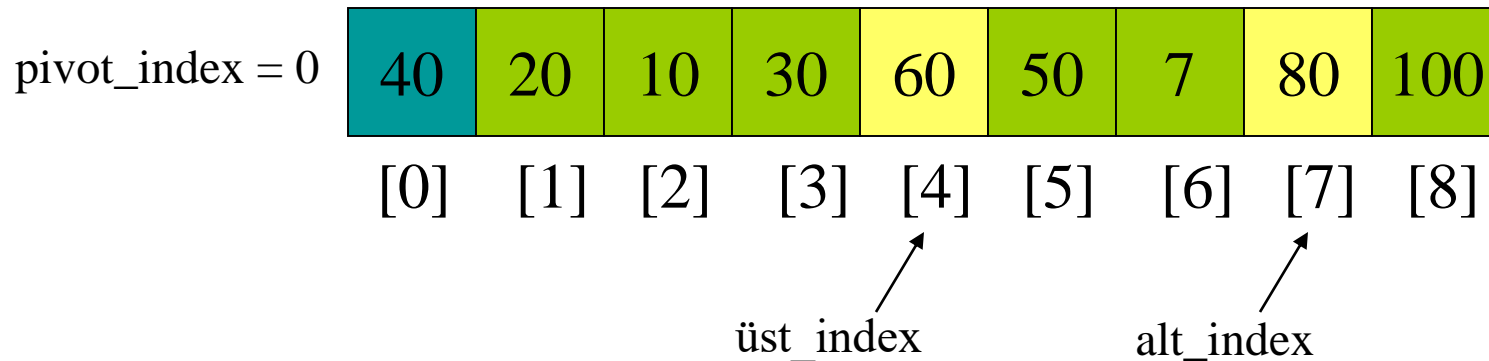
- 1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
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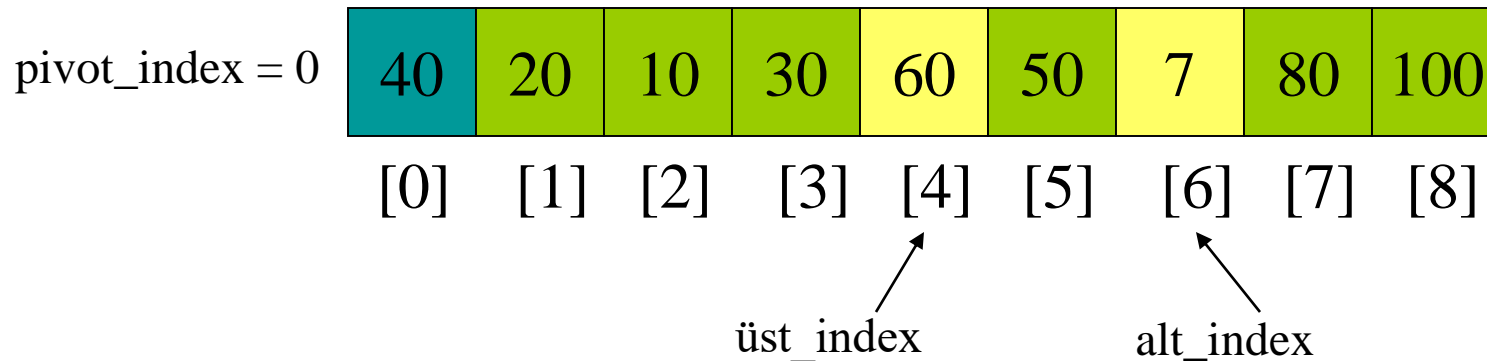
- 1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
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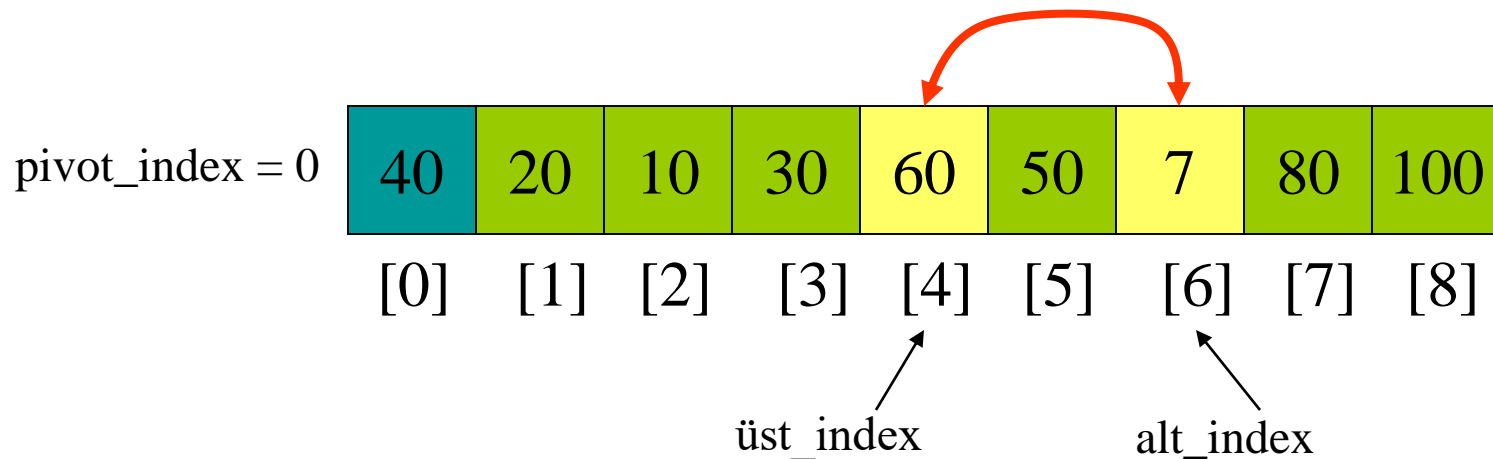
1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
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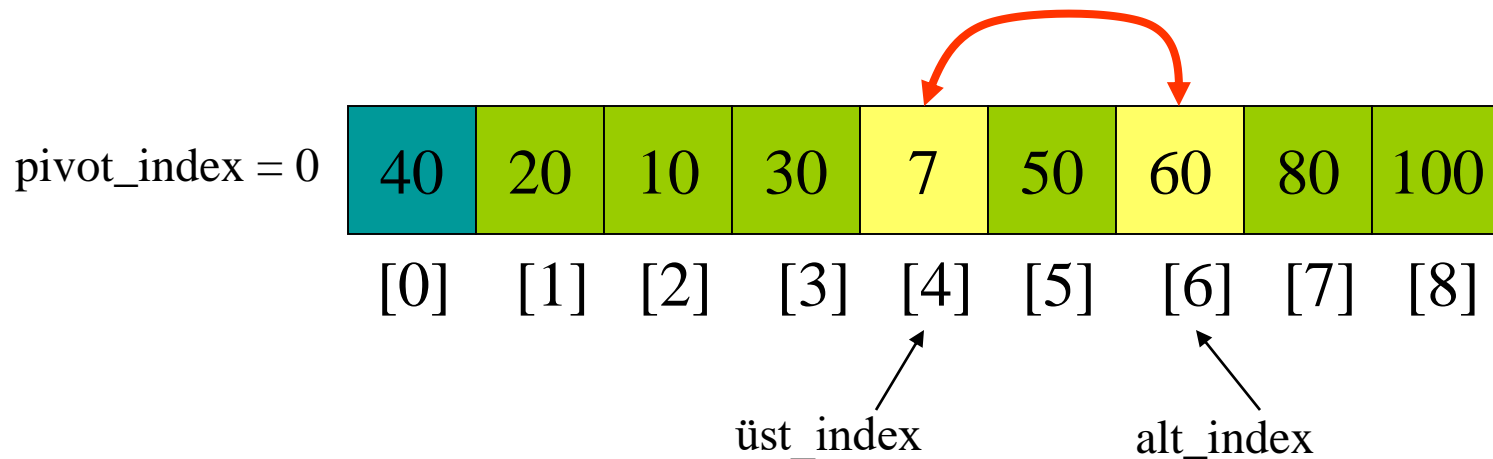
1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
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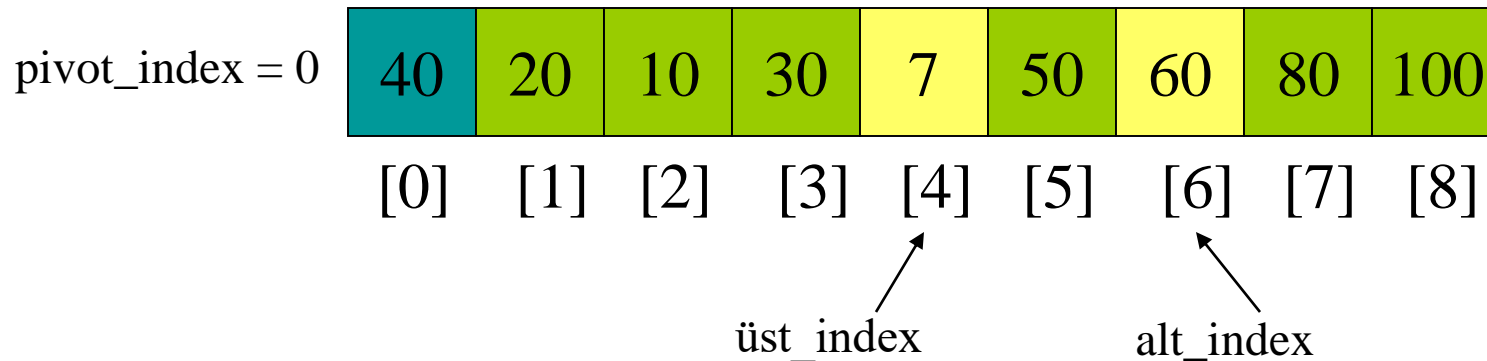
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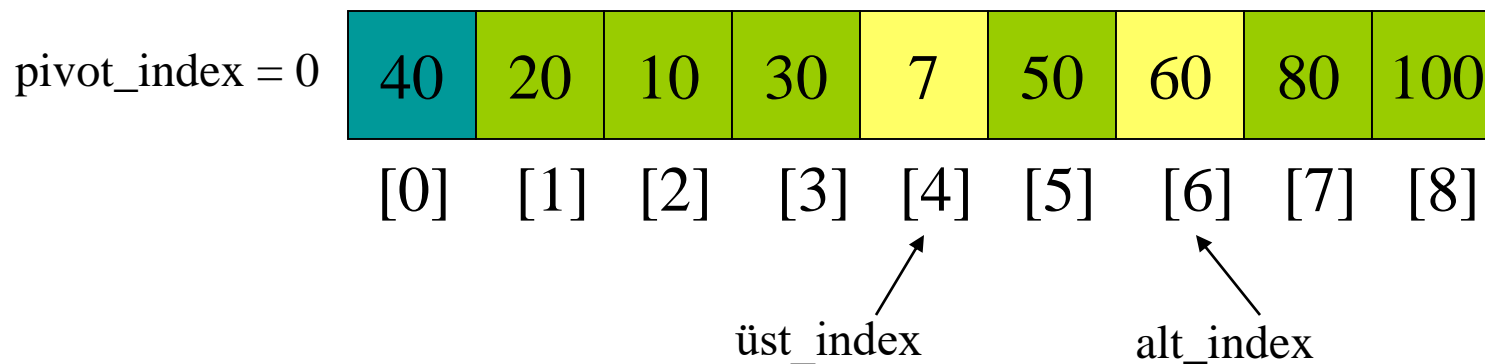
1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
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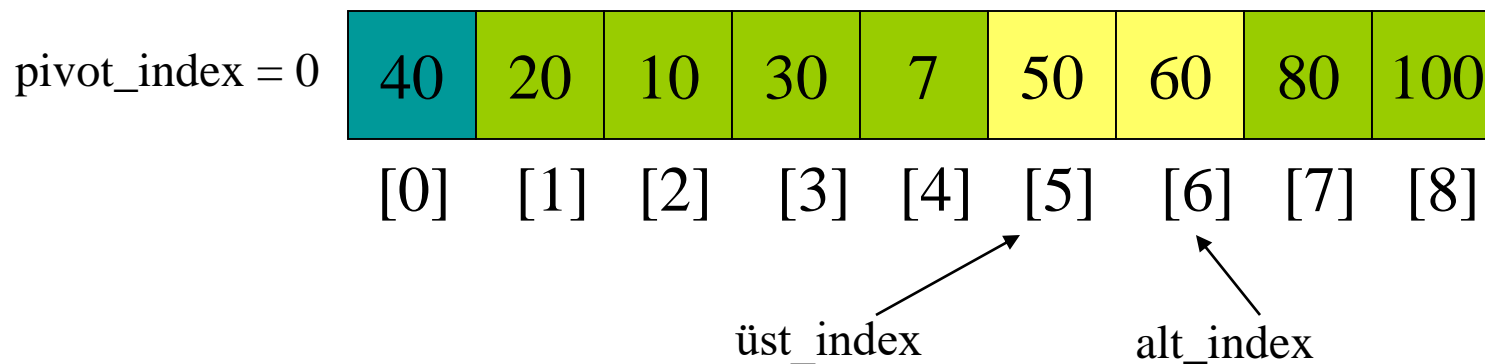
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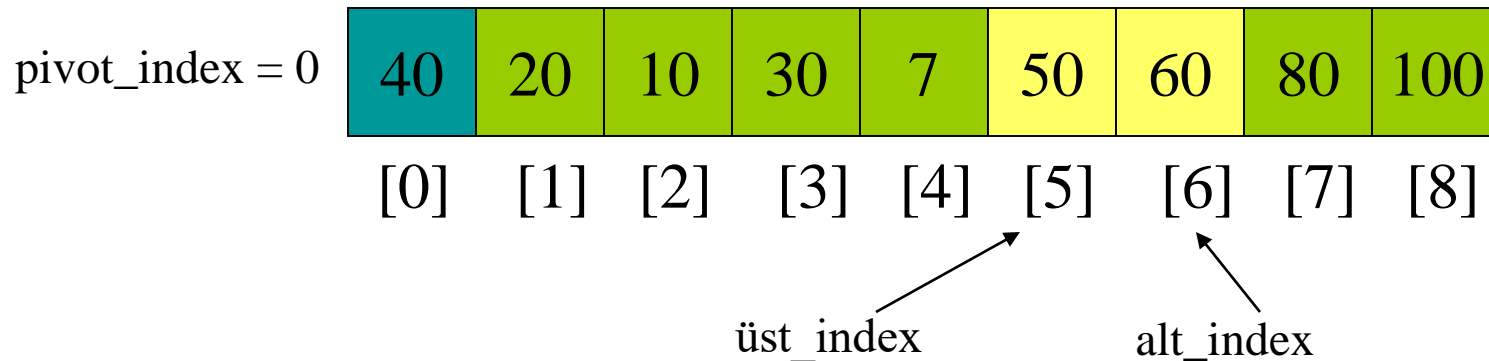
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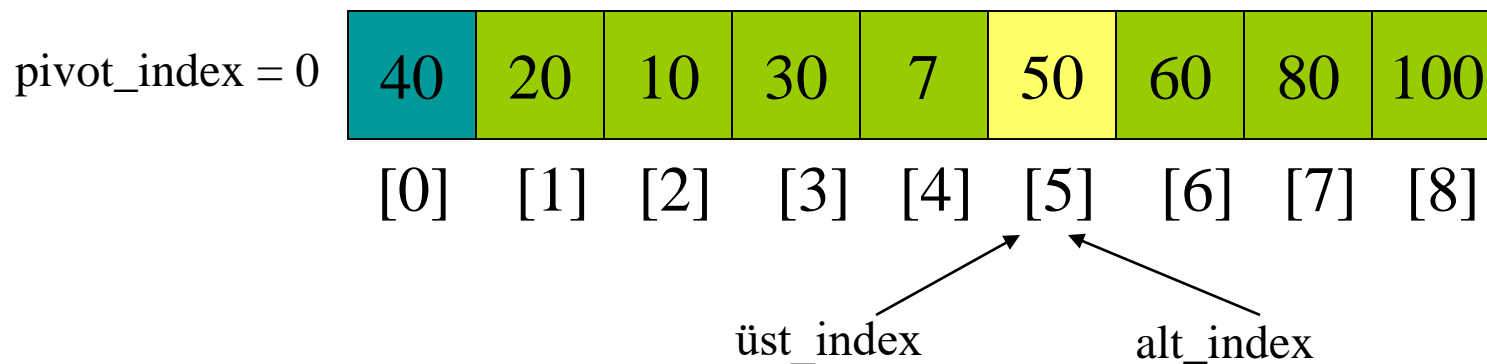
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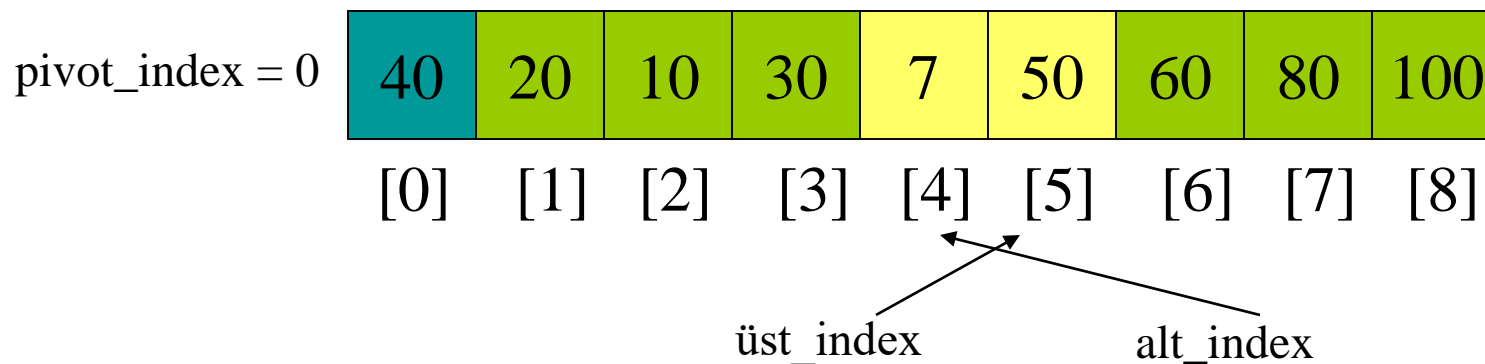
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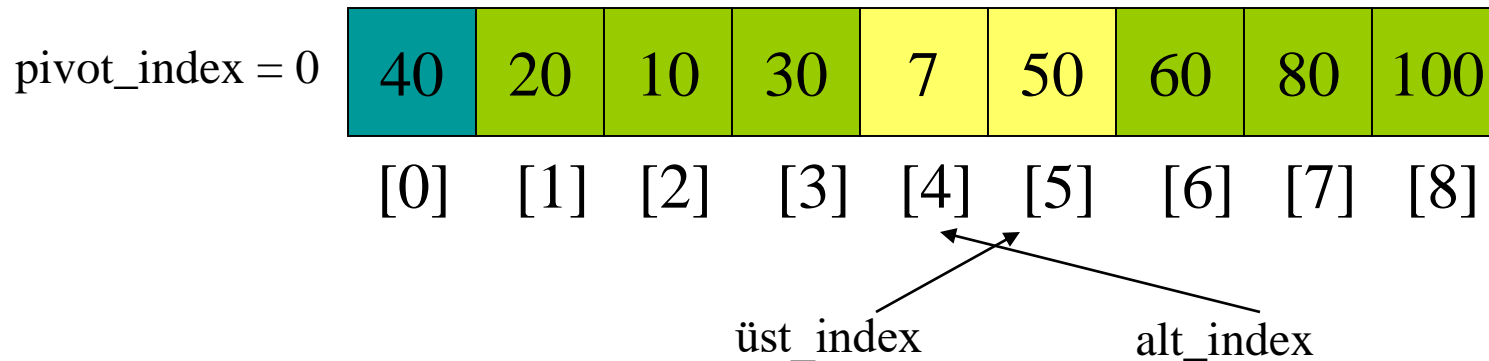
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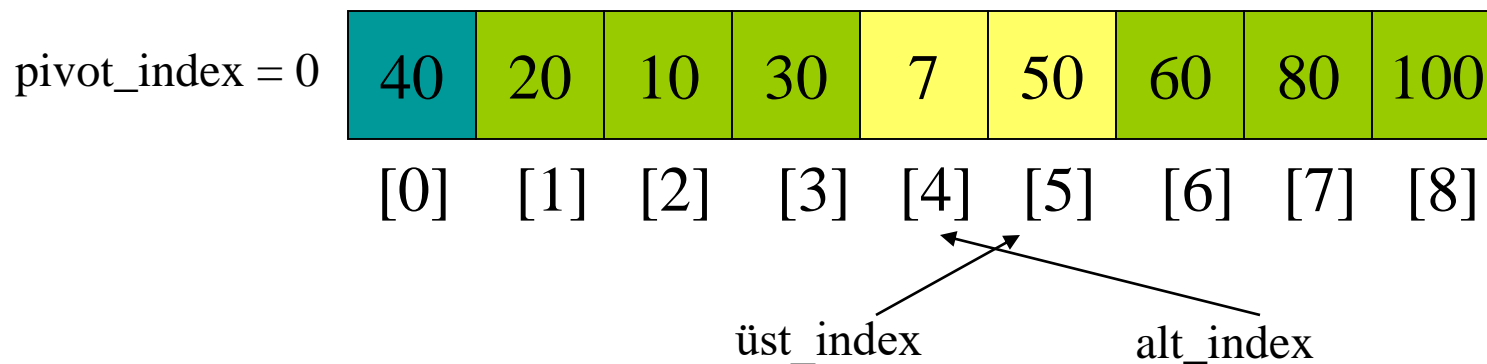
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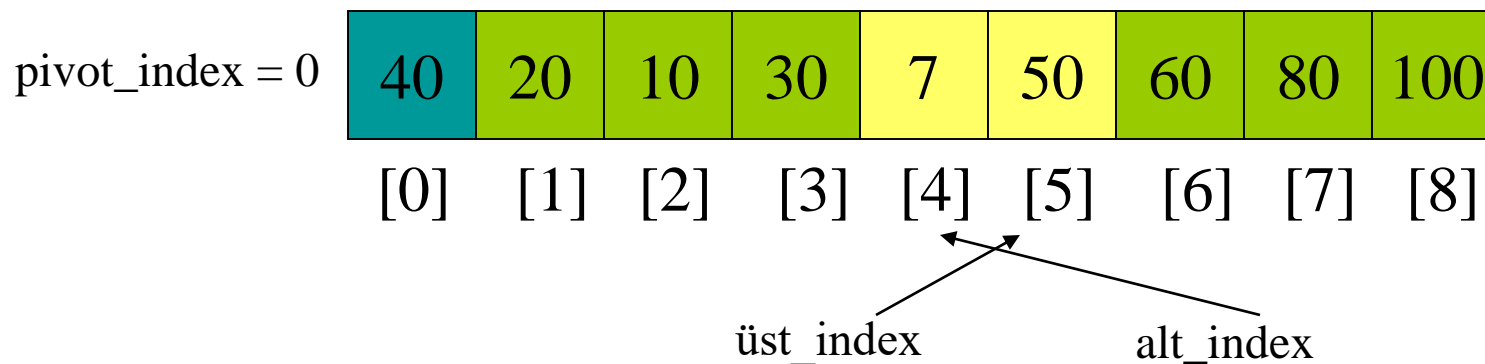
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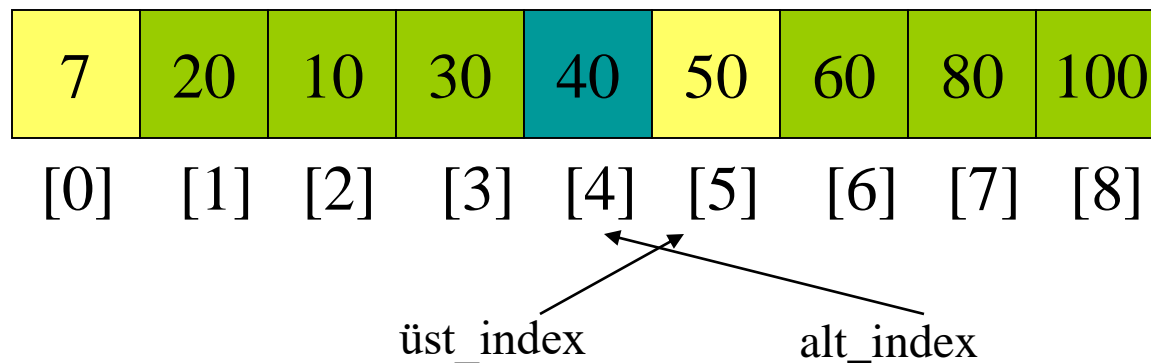


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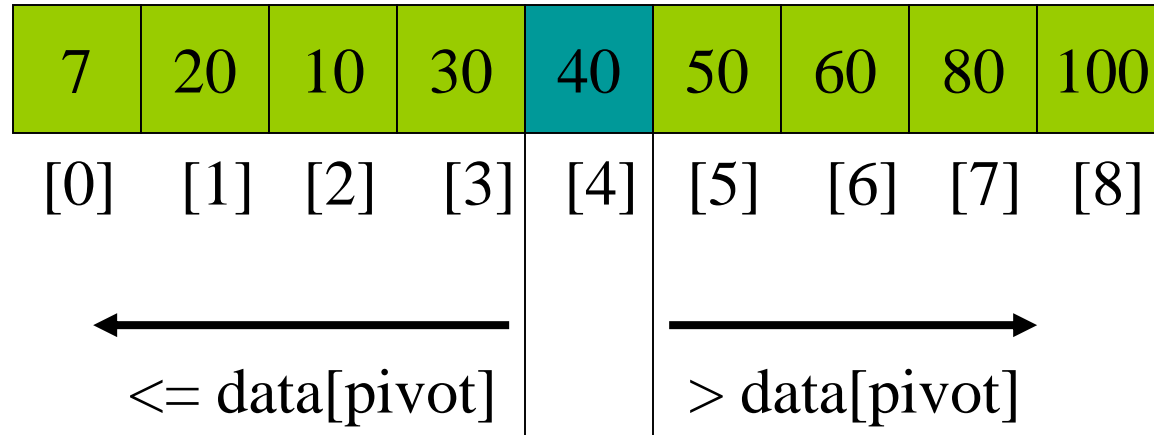


1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
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- 5. Swap $\text{data}[\text{alt_index}]$ and $\text{data}[\text{pivot_index}]$

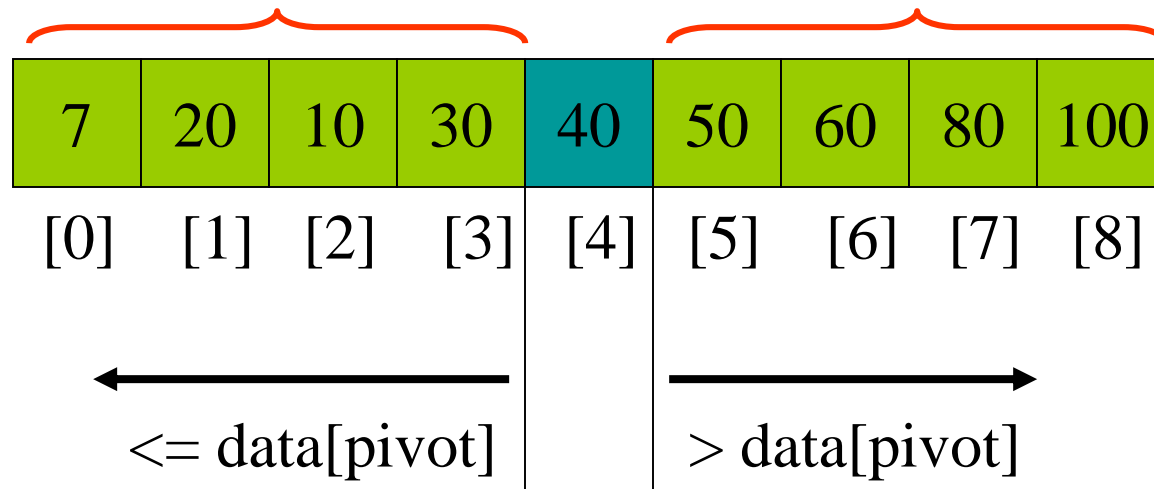
pivot_index = 4



Oluşturulan iki alt dizi

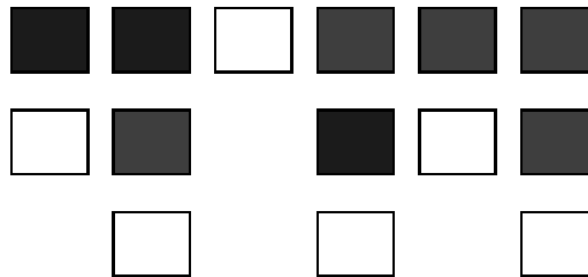


Recursion: Altdizilere Quicksort



Quicksort Algoritmasının Analizi

- Elemanlar düzensiz dağılmışsa,
- **“Best case” çalışma zamanı: $O(n \log_2 n)$**
 - Pivot eleman her zaman ortada
 - Her rekürsif çağrıda dizi iki eş parçaya ayrılmakta.



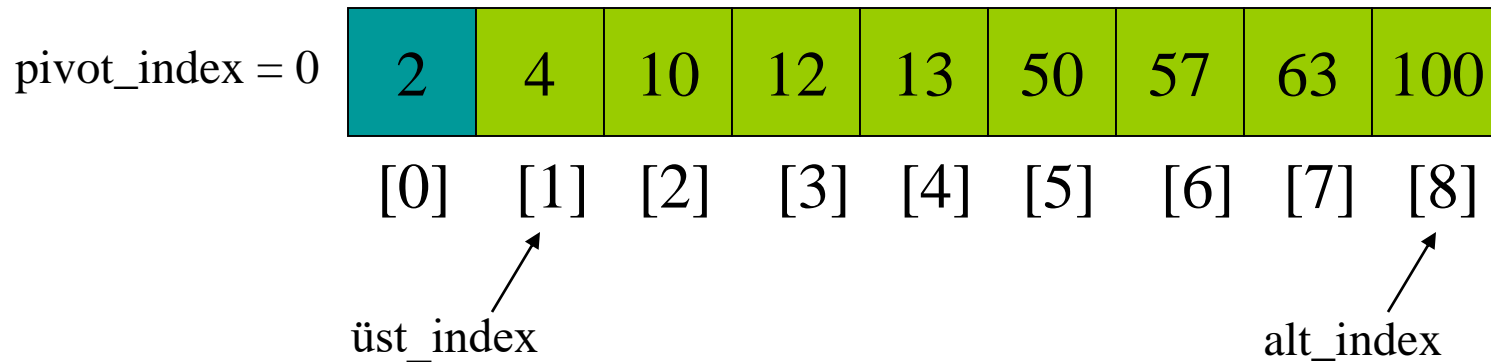
Quicksort Analysis

Worst case: $O(N^2)$

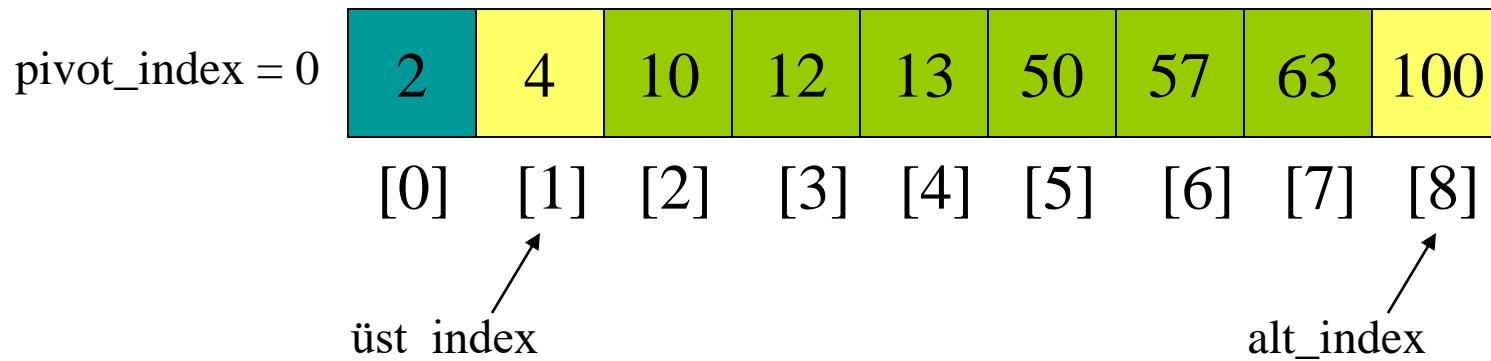
- Her rekürsif çağrıda pivot en küçük elemandır.
- Dizi zaten sıralı olması halidir.

Quicksort: Worst Case

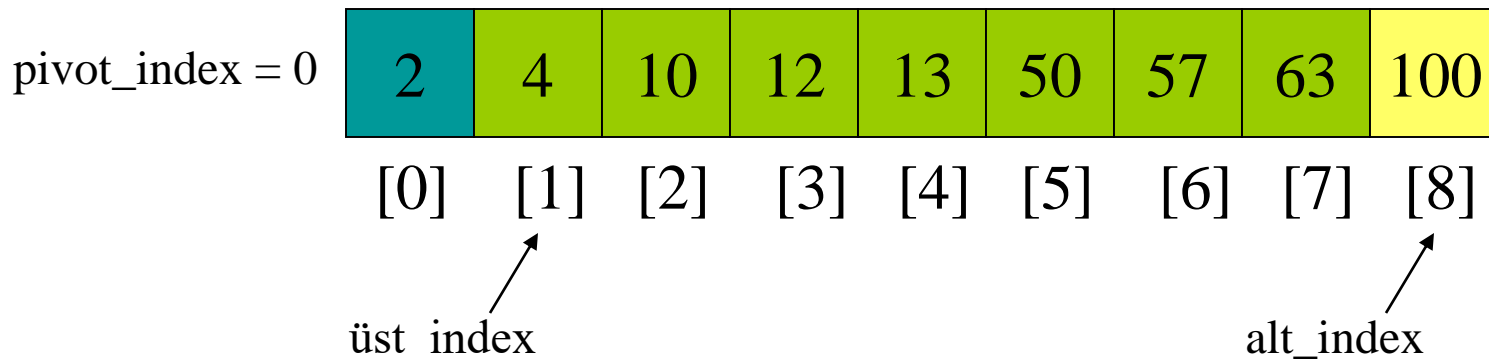
- İlk eleman pivot olarak seçilmiş olsun.
- Dizi zaten sıralı olsun



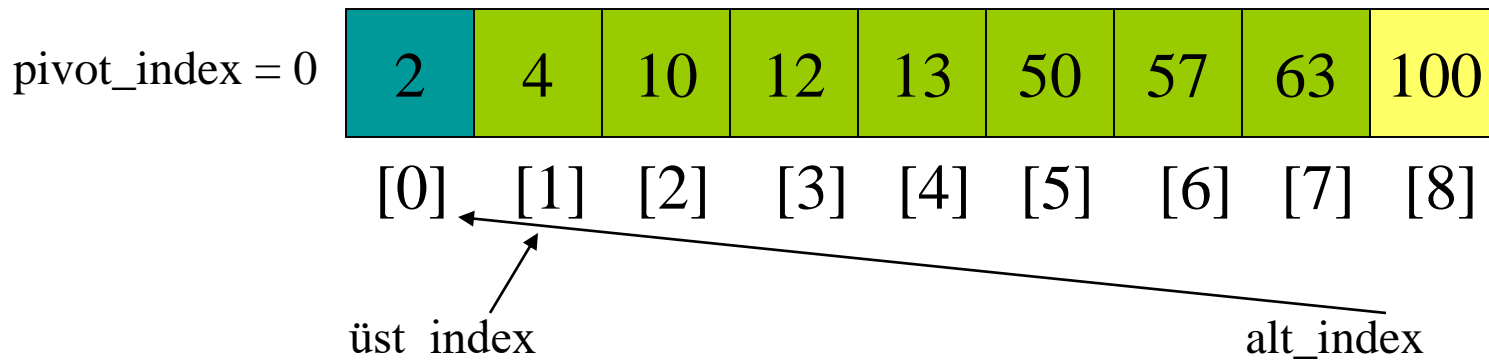
- 1. While $\text{data}[\text{üst_index}] \leq \text{data}[\text{pivot}]$
 ++üst_index
2. While $\text{data}[\text{alt_index}] > \text{data}[\text{pivot}]$
 --alt_index
3. If $\text{üst_index} < \text{alt_index}$
 swap $\text{data}[\text{üst_index}]$ and $\text{data}[\text{alt_index}]$
4. While $\text{alt_index} > \text{üst_index}$, go to 1.
5. Swap $\text{data}[\text{alt_index}]$ and $\text{data}[\text{pivot_index}]$



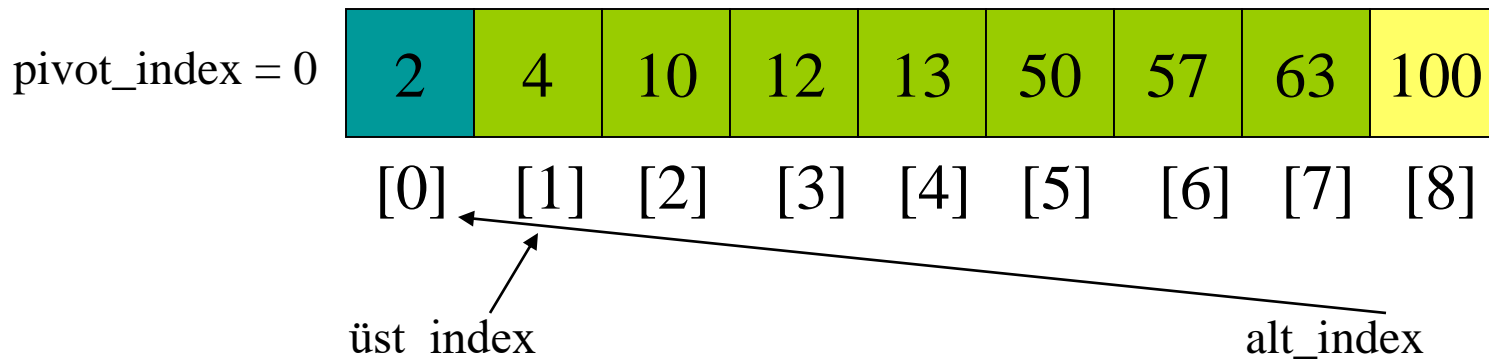
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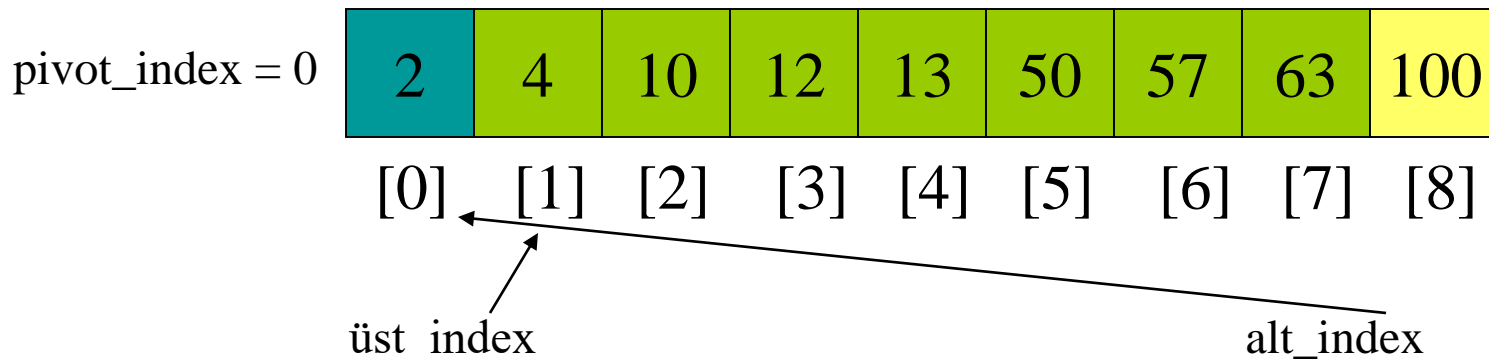
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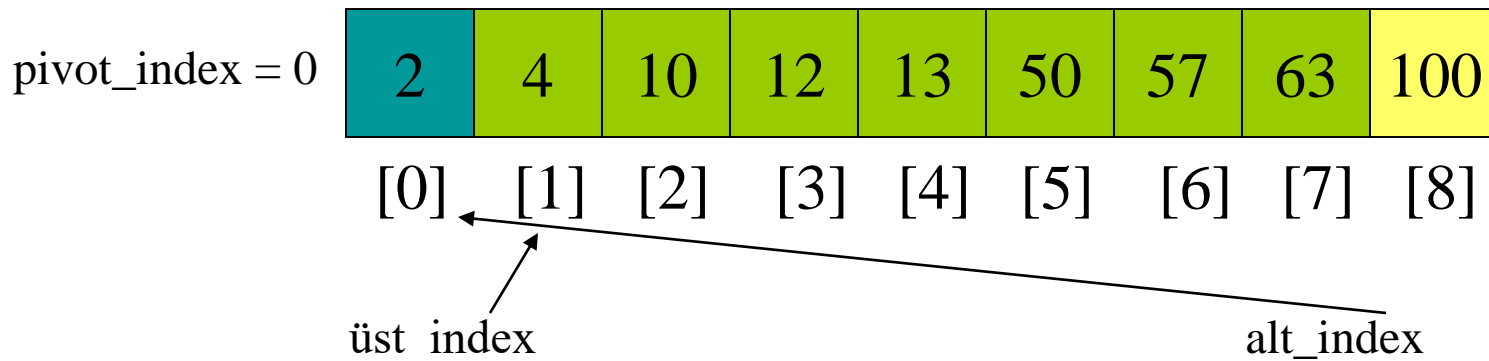
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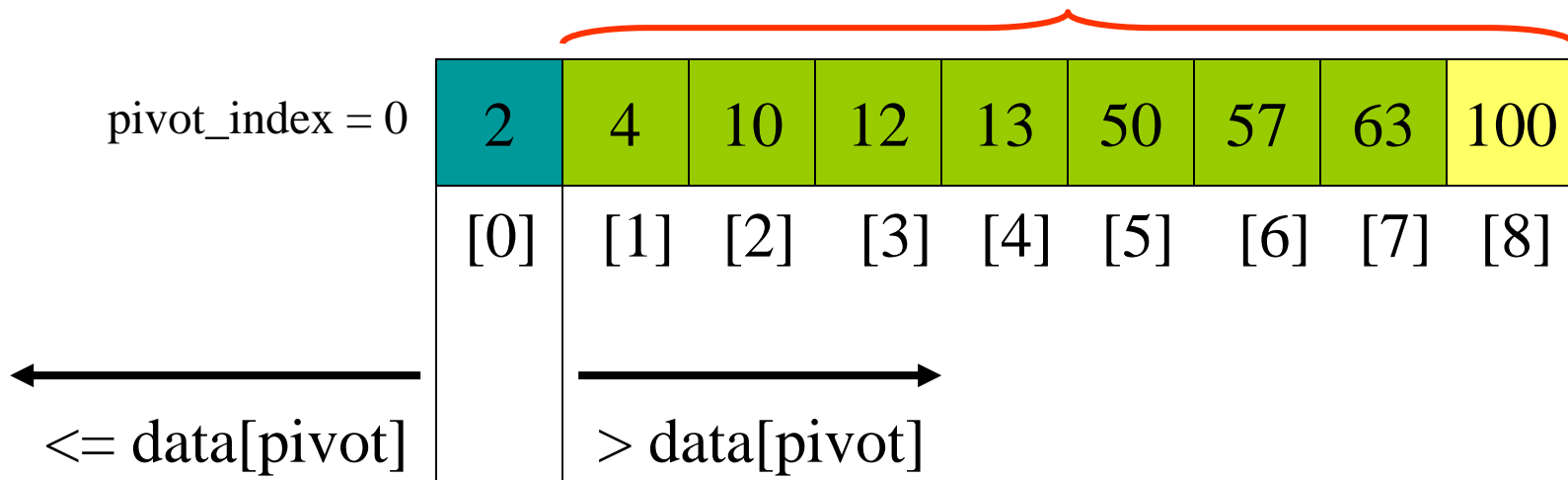
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$$T(n) = T(k) + T(n - k) + \alpha n$$

En kötü durumda dizinin bölünmesi 1 ve n-1 elemanlı biçiminde olacaktır.

$$T(n) = T(1) + T(n - 1) + \alpha n$$

$$T(n) = T(n - 1) + T(1) + \alpha n \Rightarrow [T(n - 2) + T(1) + \alpha(n - 1)] + T(1) + \alpha n$$

$$= T(n - 2) + 2T(1) + \alpha(n - 1 + n)$$

$$= [T(n - 3) + T(1) + \alpha(n - 2)] + 2T(1) + \alpha(n - 1 + n)$$

$$= T(n - 3) + 3T(1) + \alpha(n - 2 + n - 1 + n)$$

$$= [T(n - 4) + T(1) + \alpha(n - 3)] + 3T(1) + \alpha(n - 2 + n - 1 + n)$$

$$= T(n - 4) + 4T(1) + \alpha(n - 3 + n - 2 + n - 1 + n)$$

$$= T(n - i) + iT(1) + \alpha(n - i + 1 + \dots + n - 2 + n - 1 + n)$$

$$= T(n - i) + i.T(1) + \alpha(\sum_{j=0}^{i-1} (n - j))$$

$$= T(1) + (n-1)T(1) + \alpha(\sum_{j=0}^{n-2} (n - j))$$

$$\begin{matrix} \underline{n-i=1} \\ \underline{i=n-1} \end{matrix}$$

$$\sum_{j=0}^{n-2} j = \sum_{j=1}^{n-2} j = (n-2)(n-1)/2$$

Karesel
çalışma
zamanı

Eniyi Durum Analizi

En iyi durumda dizi her defasında tam olarak ikiye bölünecektir.

$$T(n) = 2T(n/2) + \alpha n = 2(2T(n/4) + \alpha n/2) + \alpha n$$

$$T(n/2) = 2T(n/4) + \alpha n/2$$

$$T(n) = 2T(n/2) + \alpha n$$

$$= 2^2T(n/4) + 2\alpha n$$

$$= 2^2(2T(n/8) + \alpha n/4) + 2\alpha n$$

$$= 2^3T(n/8) + 3\alpha n$$

$$= 2^kT(n/2^k) + k\alpha n$$

$$n = 2^k$$

$$n/2^k < 1$$

$$k = \log n.$$

$$T(n) = nT(1) + \alpha n \log n, \text{ which is } O(n \log n).$$

Quicksort Algoritmasının Çalışma Zamanı Analizi

- **Best case** : $O(n \log_2 n)$
- **Worst case** : $O(n^2)$
- **Average case** : $O(n \log_2 n)$

Quicksort

QUICKSORT(A, p, r)

if $p < r$

 then $q \leftarrow \text{PARTITION}(A, p, r)$

 QUICKSORT(A, p, $q-1$)

 QUICKSORT(A, $q+1$, r)

Algoritma rekürsif yapıda olduğundan bilgisayarın yığıtı yoğun bir biçimde kullanılmaktadır.