Blind (Uninformed) Search

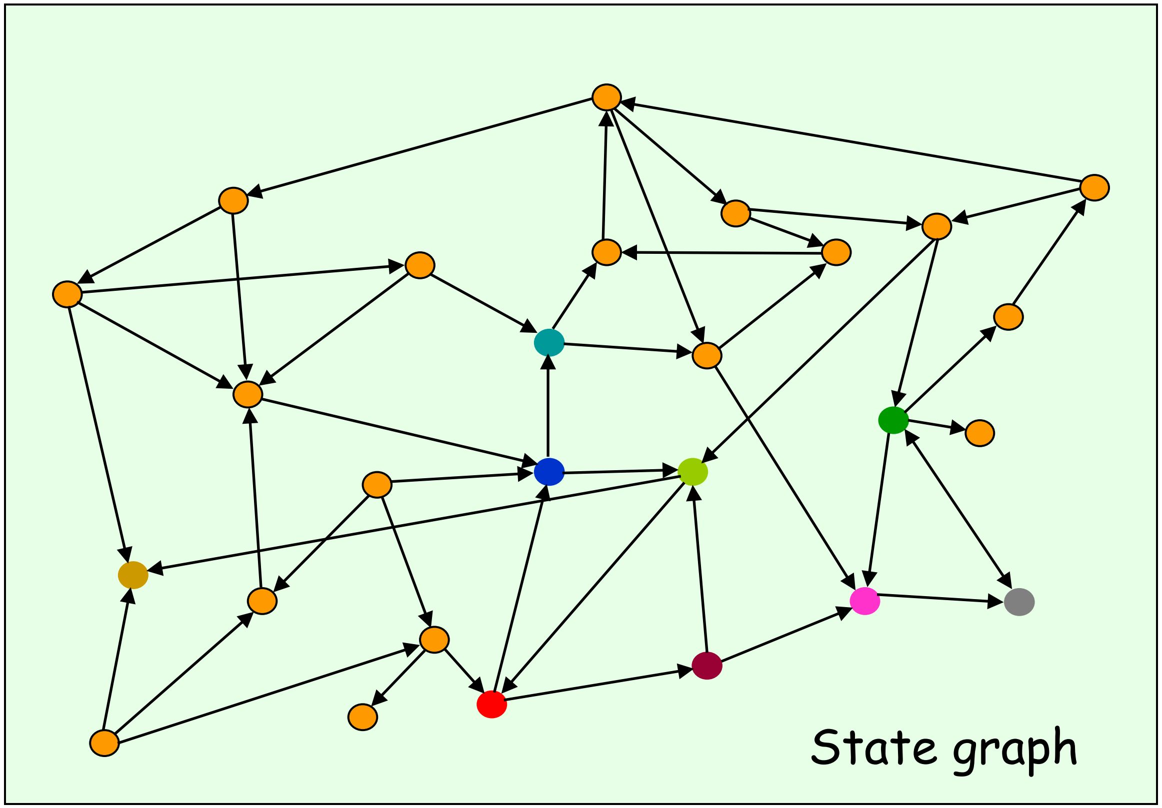
(Where we systematically explore alternatives)

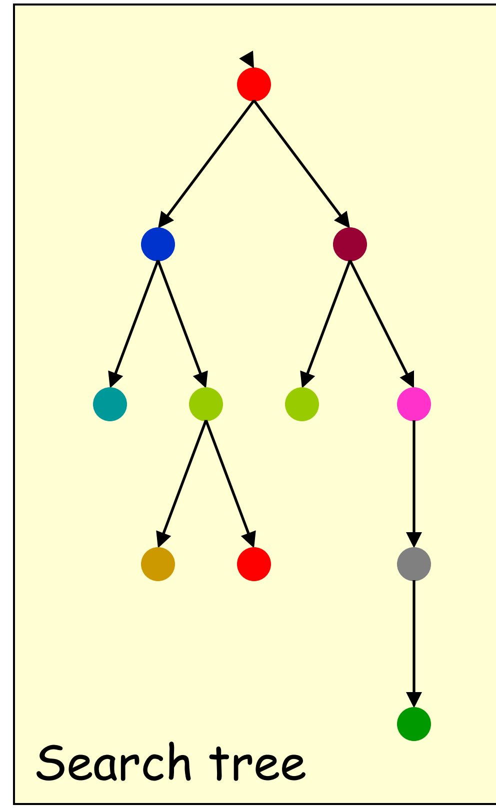
R&N: Chap. 3, Sect. 3.3-5

Simple Problem-Solving-Agent Agent Algorithm

1. $s_0 \leftarrow$ sense/read initial state
2. GOAL? $\leftarrow$ select/read goal test
3. Succ $\leftarrow$ read successor function
4. solution $\leftarrow$ **search**(s_0, GOAL?, Succ)
5. perform(solution)

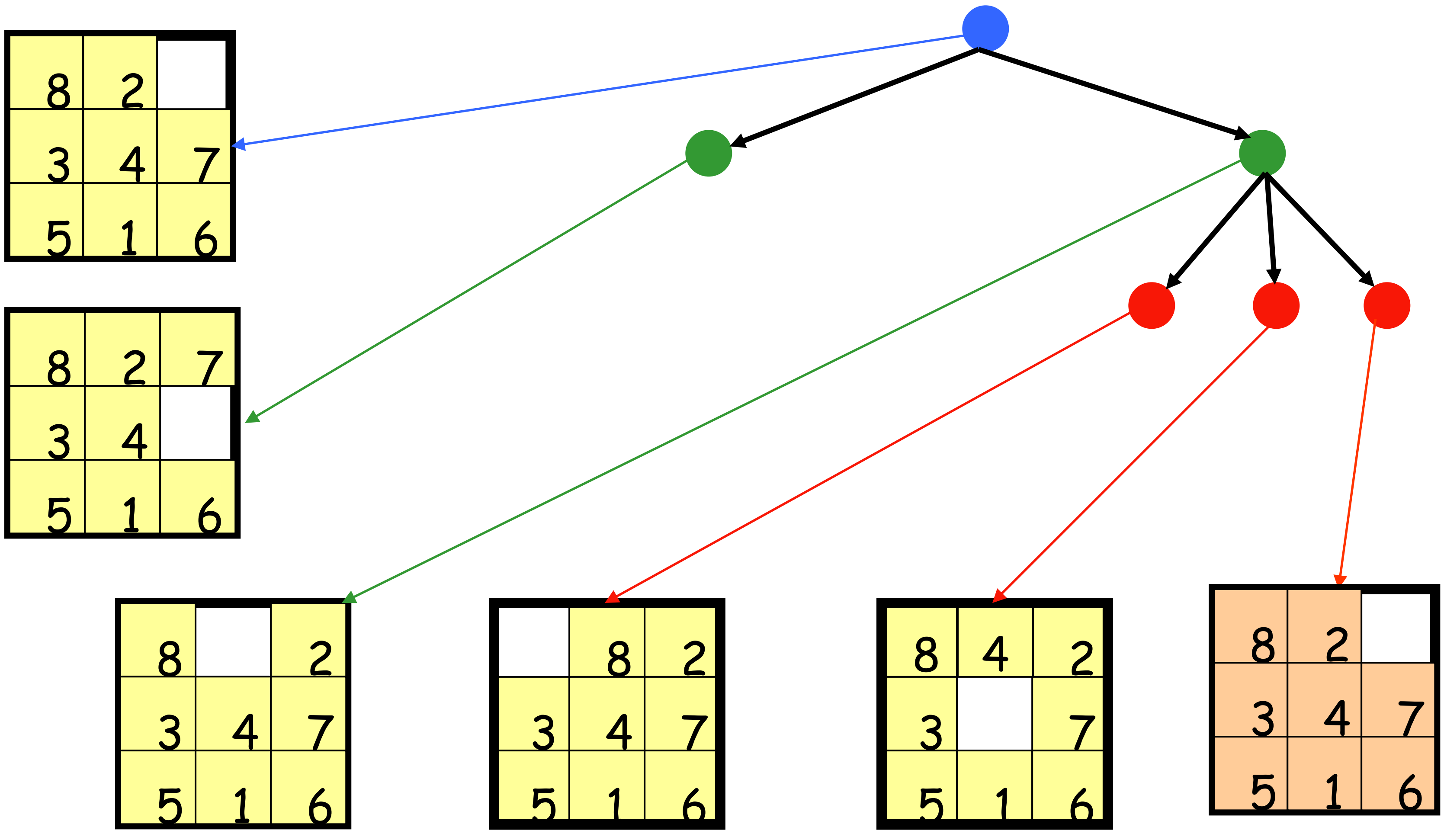
Search Tree



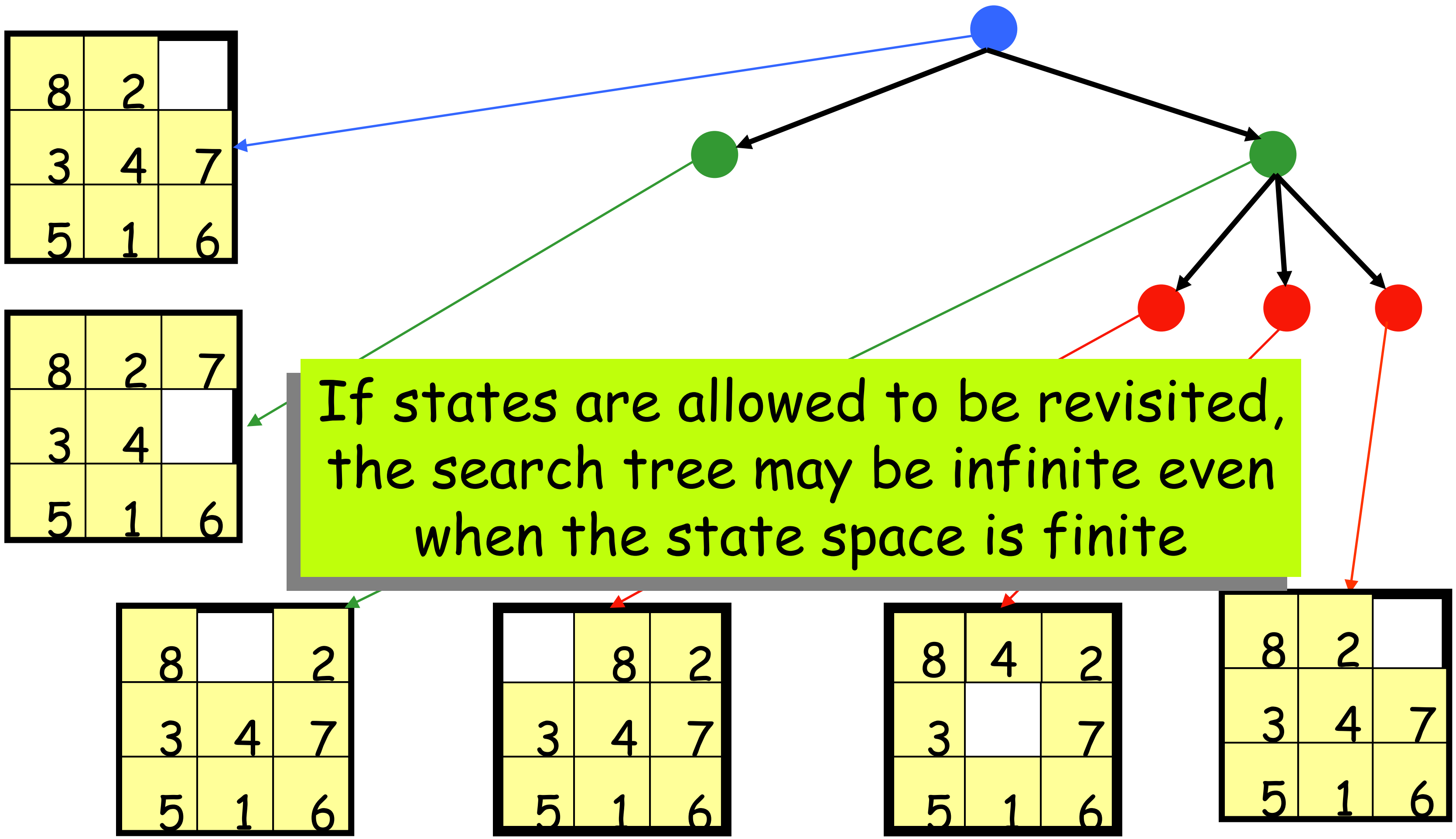


Note that some states may be visited multiple times

Search Nodes and States

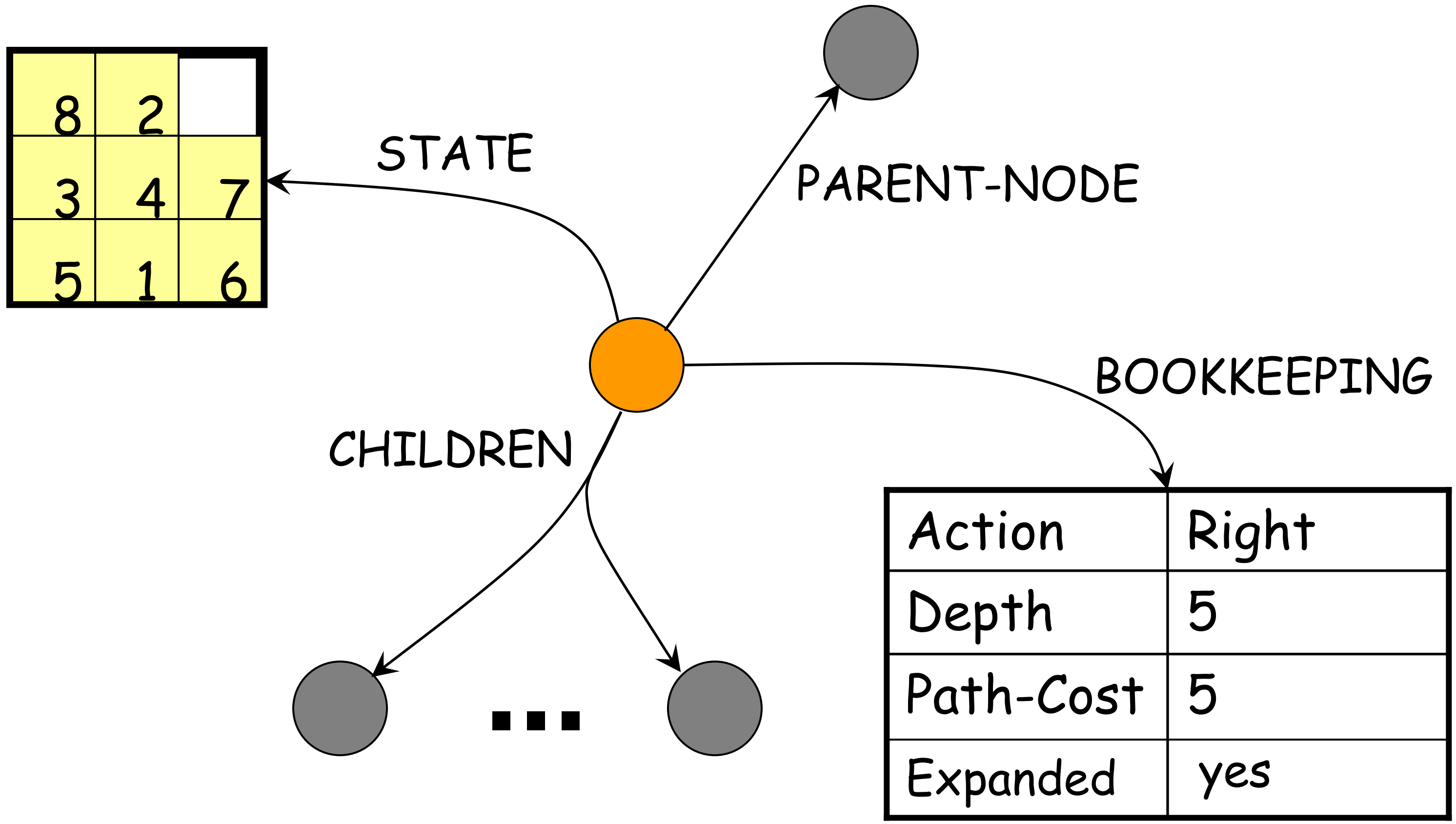


Search Nodes and States



If states are allowed to be revisited, the search tree may be infinite even when the state space is finite

Data Structure of a Node



Depth of a node N
= length of path from root to N

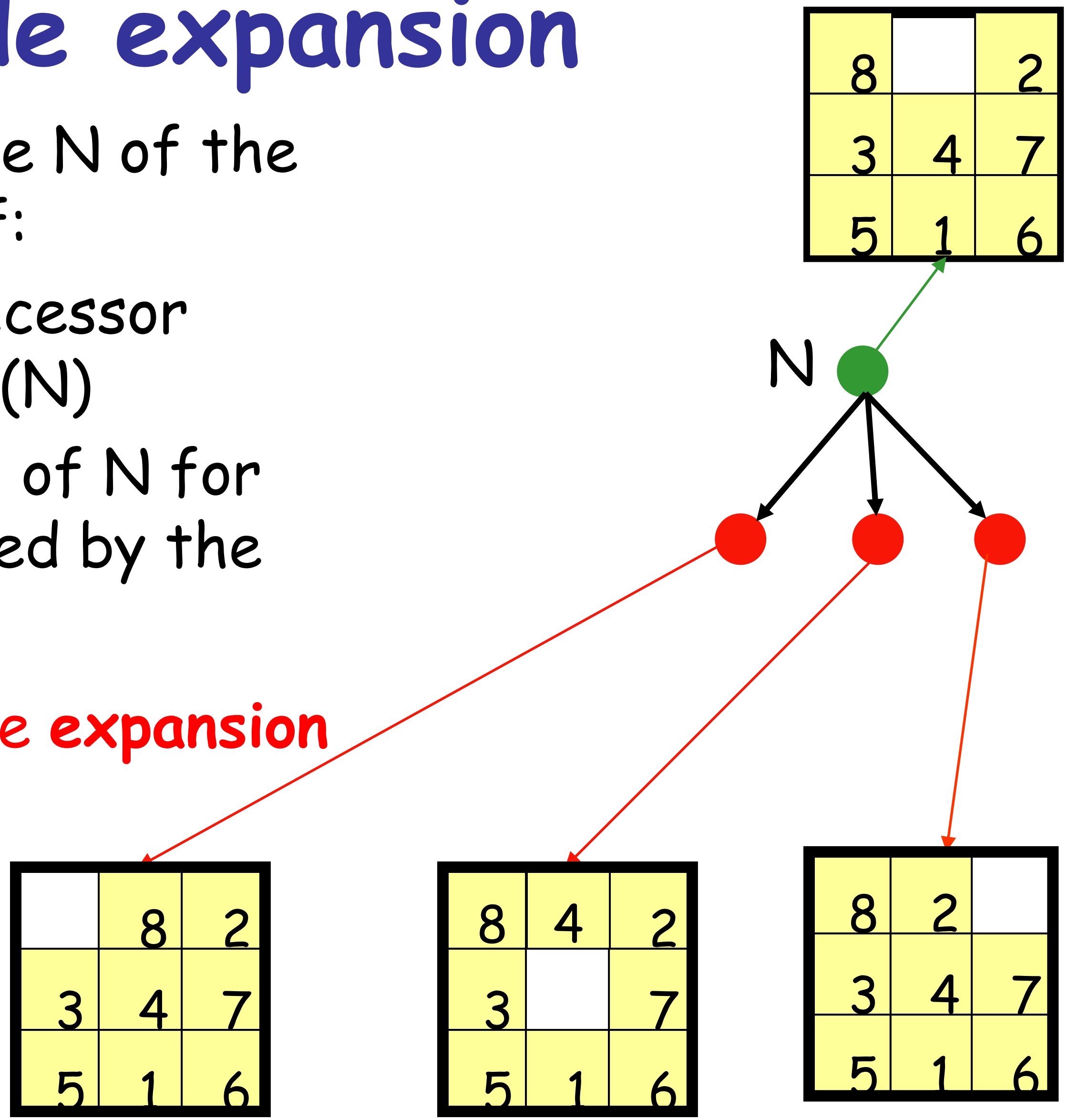
(depth of the root = 0)

Node expansion

The expansion of a node N of the search tree consists of:

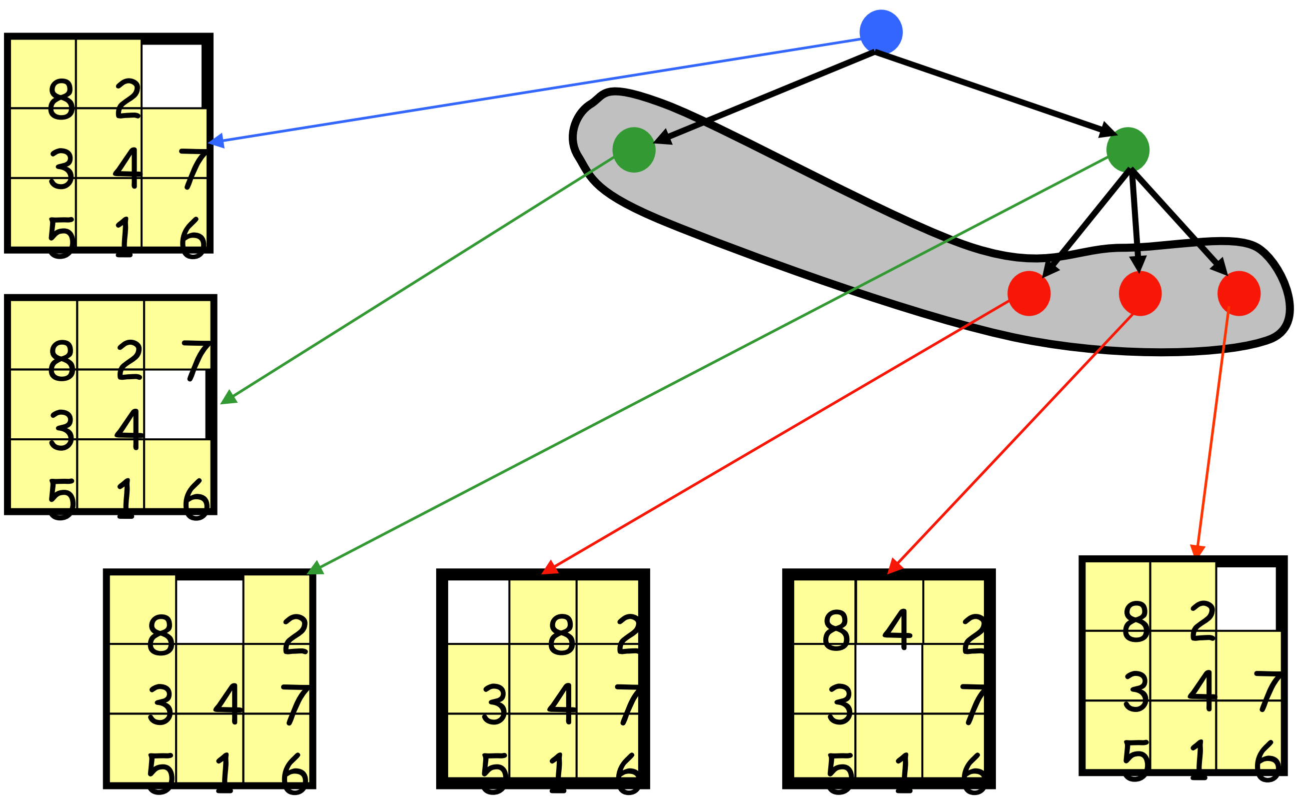
1) Evaluating the successor function on STATE(N)
2) Generating a child of N for each state returned by the function

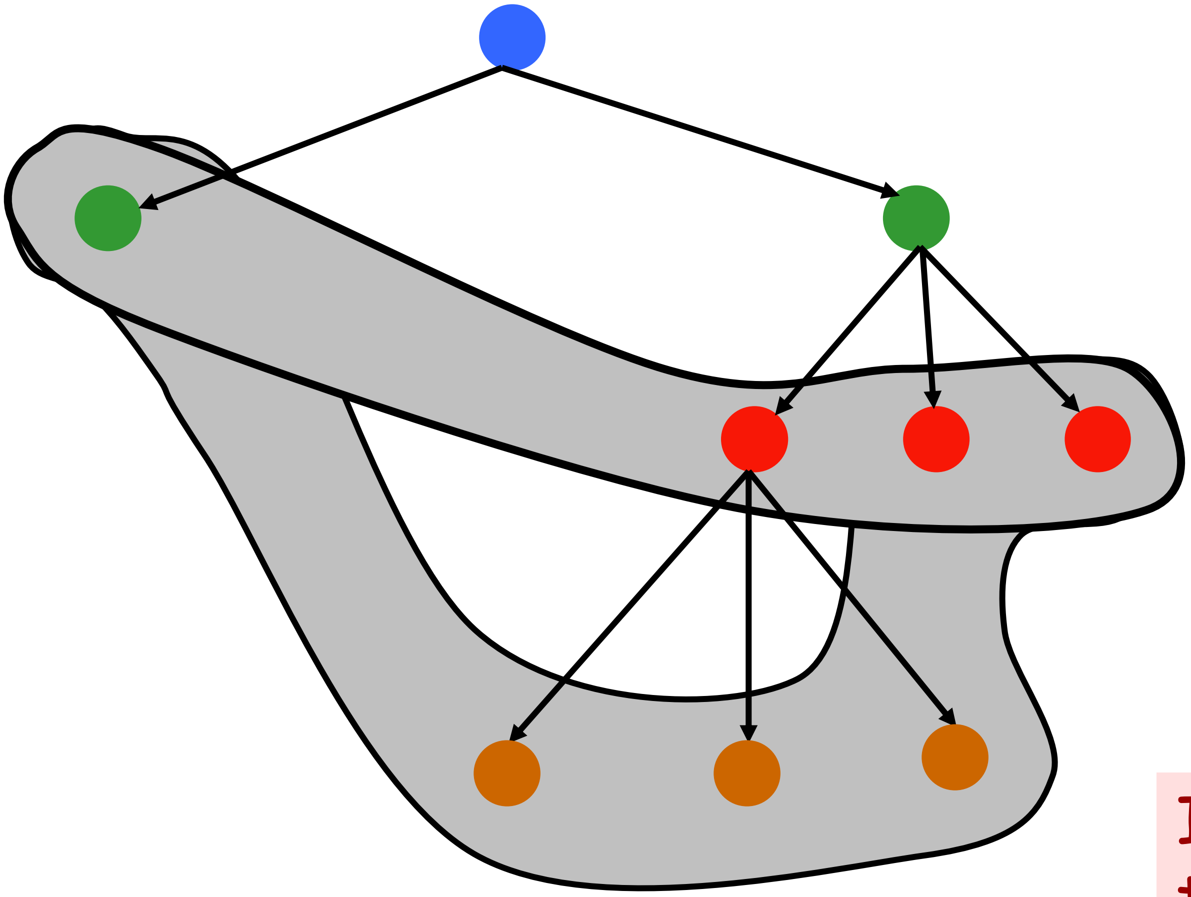
node **generation** ≠ node **expansion**



Fringe of Search Tree

- The fringe is the set of all search nodes that haven't been expanded yet





Is it identical to the set of leaves?

Search Strategy

- The fringe is the set of all search nodes that haven't been expanded yet
- The fringe is implemented as a priority queue FRINGE
  - INSERT(node,FRINGE)
  - REMOVE(FRINGE)
- The ordering of the nodes in FRINGE defines the search strategy

Search Algorithm #1

SEARCH#1

1. If GOAL?(initial-state) then return initial-state
2. INSERT(initial-node,FRINGE)
3. Repeat:
   a. If empty(FRINGE) then return failure
   b. N ← REMOVE(FRINGE)
   c. s ← STATE(N)
   d. For every state s' in SUCCESSORS(s)
      i. Create a new node N' as a child of N
      ii. If GOAL?(s') then return path or goal state
      iii. INSERT(N',FRINGE)

Expansion of N

Performance Measures

- Completeness
  A search algorithm is complete if it finds a solution whenever one exists
  [What about the case when no solution exists?]
- Optimality
  A search algorithm is optimal if it returns a minimum-cost path whenever a solution exists
- Complexity
  It measures the time and amount of memory required by the algorithm

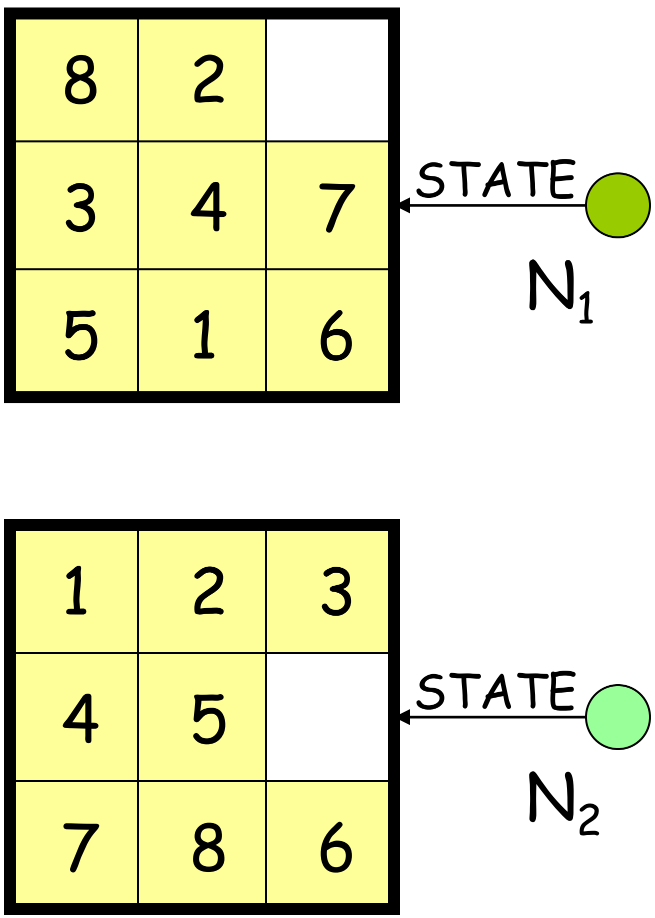
Blind vs. Heuristic Strategies

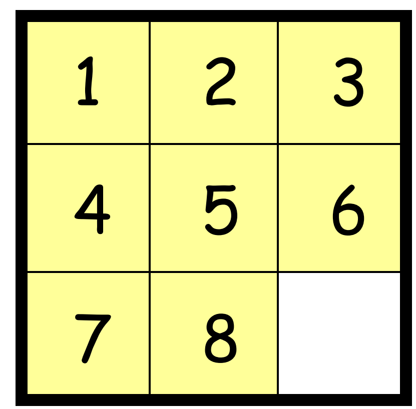
- Blind (or un-informed) strategies do not exploit state descriptions to order FRINGE. They only exploit the positions of the nodes in the search tree

- Heuristic (or informed) strategies exploit state descriptions to order FRINGE (the most "promising" nodes are placed at the beginning of FRINGE)

Example

For a blind strategy, N_1 and N_2 are just two nodes (at some position in the search tree)

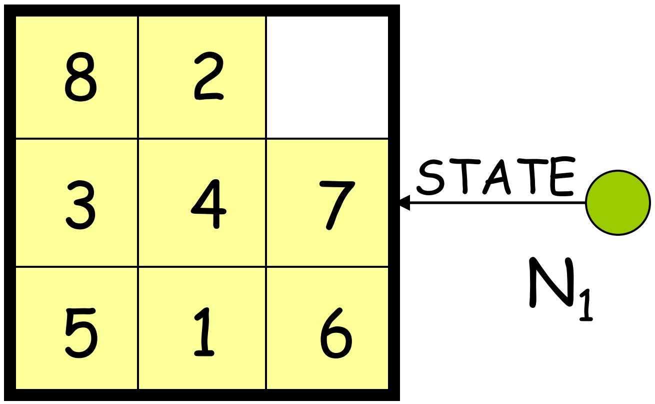


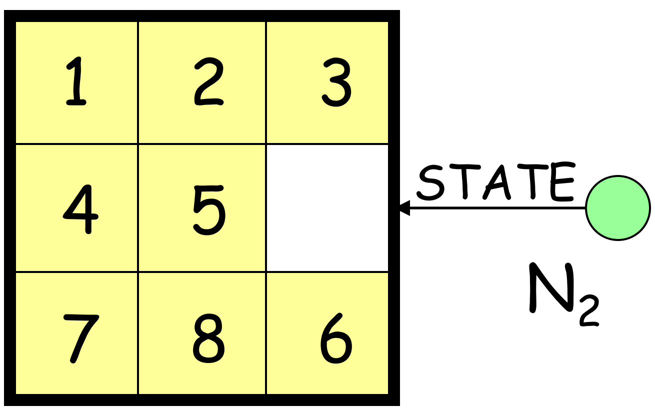


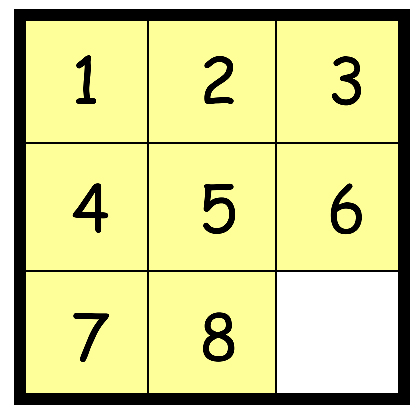
Goal state

Example

For a heuristic strategy counting the number of misplaced tiles, N_2 is more promising than N_1







Goal state

Remark

- Some search problems, such as the (n^2-1)-puzzle, are NP-hard
- One can't expect to solve all instances of such problems in less than exponential time (in n)
- One may still strive to solve each instance as efficiently as possible

→ This is the purpose of the search strategy

Blind Strategies

- Breadth-first
  - Bidirectional
- Depth-first
  - Depth-limited
  - Iterative deepening

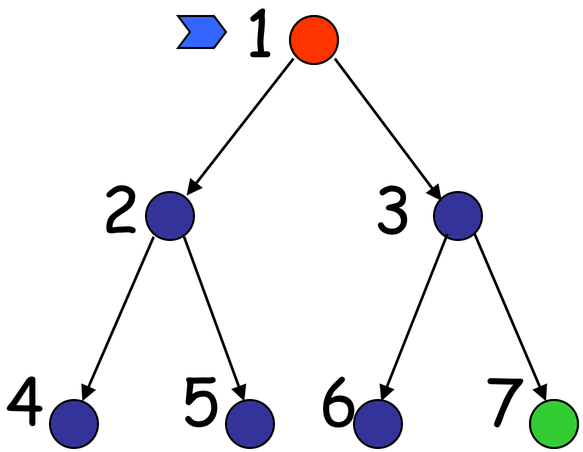
Arc cost = 1

- Uniform-Cost
  (variant of breadth-first)

Arc cost
$= c(\text{action}) \geq \varepsilon > 0$

Breadth-First Strategy

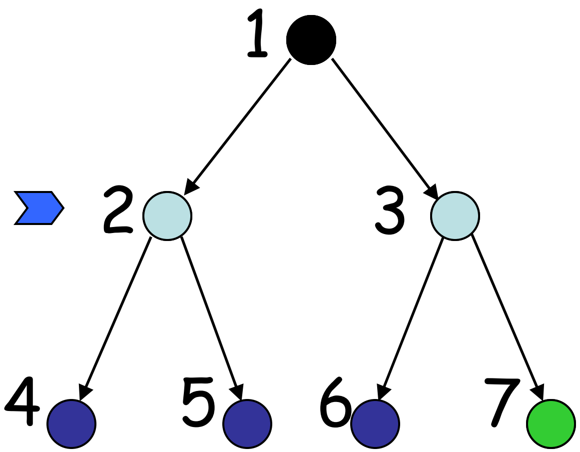
New nodes are inserted at the end of FRINGE



FRINGE = (1)

Breadth-First Strategy

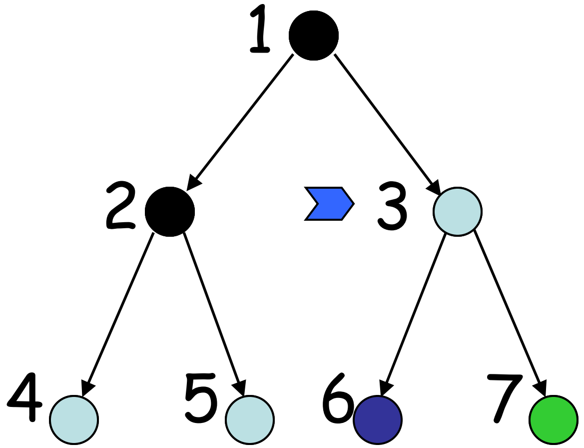
New nodes are inserted at the end of FRINGE



FRINGE = (2, 3)

Breadth-First Strategy

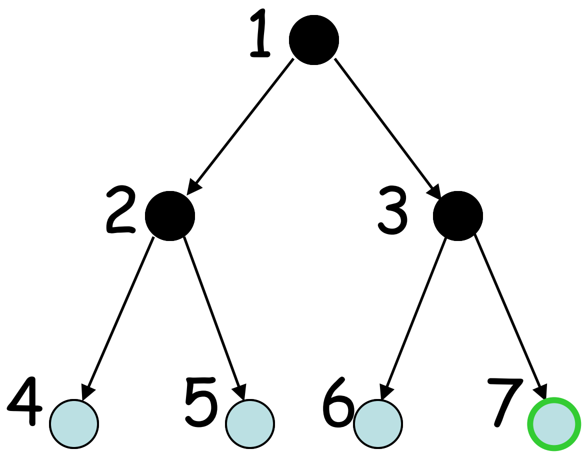
New nodes are inserted at the end of FRINGE



FRINGE = (3, 4, 5)

Breadth-First Strategy

New nodes are inserted at the end of FRINGE



FRINGE = (4, 5, 6, 7)

Important Parameters

1) Maximum number of successors of any state

   → branching factor b of the search tree

2) Minimal length (≠ cost) of a path between the initial and a goal state

   → depth d of the shallowest goal node in the search tree

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete? Not complete?
  - Optimal? Not optimal?

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete
  - Optimal if step cost is 1
- Number of nodes generated:

  ???

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete
  - Optimal if step cost is 1
- Number of nodes generated:
  $1 + b + b^2 + \ldots + b^d \quad = \quad ???$

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- Breadth-first search is:
  - Complete
  - Optimal if step cost is 1
- Number of nodes generated:
  $1 + b + b^2 + \ldots + b^d = (b^{d+1}-1)/(b-1) = O(b^d)$
- → Time and space complexity is $O(b^d)$

Big O Notation

$g(n) = O(f(n))$ if there exist two positive constants a and N such that:

for all $n > N$: $g(n) \leq a \times f(n)$

Time and Memory Requirements

d	# Nodes	Time	Memory
2	111	.01 msec	11 Kbytes
4	11,111	1 msec	1 Mbyte
6	$\sim 10^6$	1 sec	100 Mb
8	$\sim 10^8$	100 sec	10 Gbytes
10	$\sim 10^{10}$	2.8 hours	1 Tbyte
12	$\sim 10^{12}$	11.6 days	100 Tbytes
14	$\sim 10^{14}$	3.2 years	10,000 Tbytes

Assumptions: b = 10; 1,000,000 nodes/sec; 100bytes/node

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14	$\sim 10^{14}$	3.2 years	10,000 Tbytes

Assumptions: b = 10; 1,000,000 nodes/sec; 100bytes/node

Remark

If a problem has no solution, breadth-first may run for ever (if the state space is infinite or states can be revisited arbitrary many times)

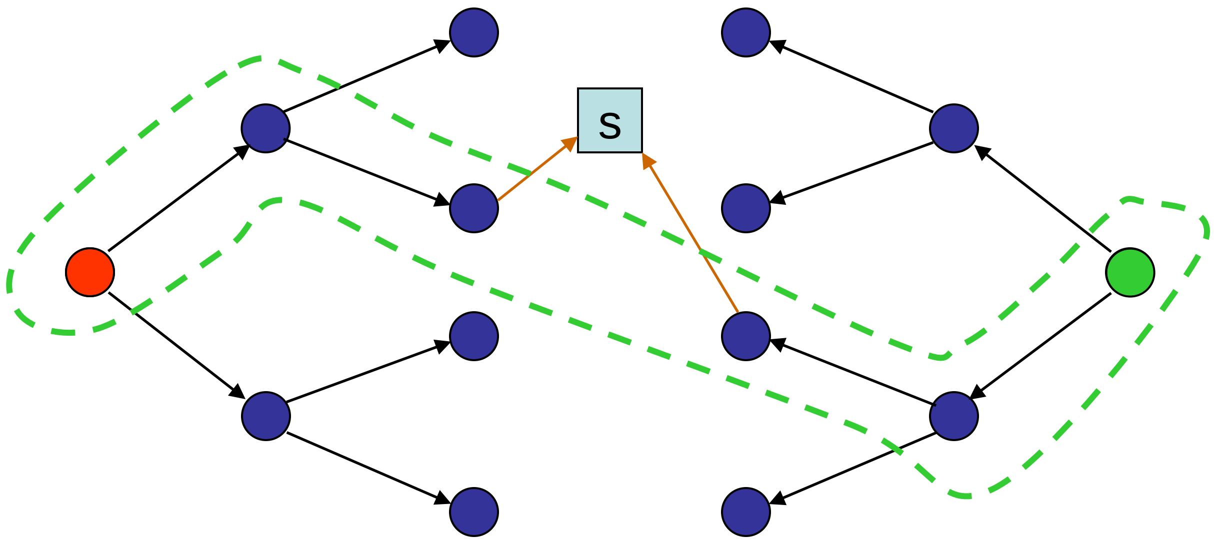
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

?

1	2	3	4
5	6	7	8
9	10	11	12
13	15	14	

Bidirectional Strategy

2 fringe queues: FRINGE1 and FRINGE2

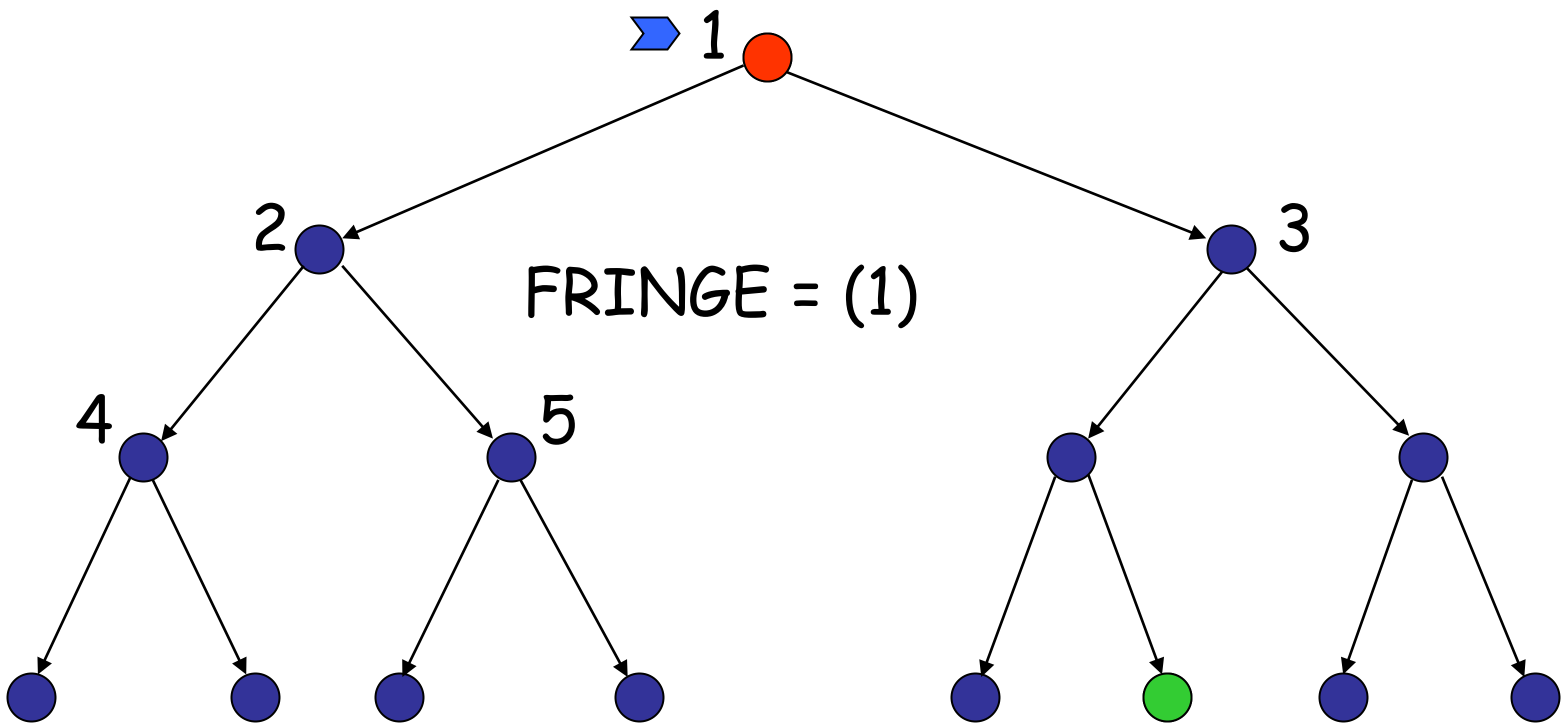


Time and space complexity is $O(b^{d/2}) << O(b^d)$ if both trees have the same branching factor b

Question: What happens if the branching factor is different in each direction?

Depth-First Strategy

New nodes are inserted at the front of FRINGE



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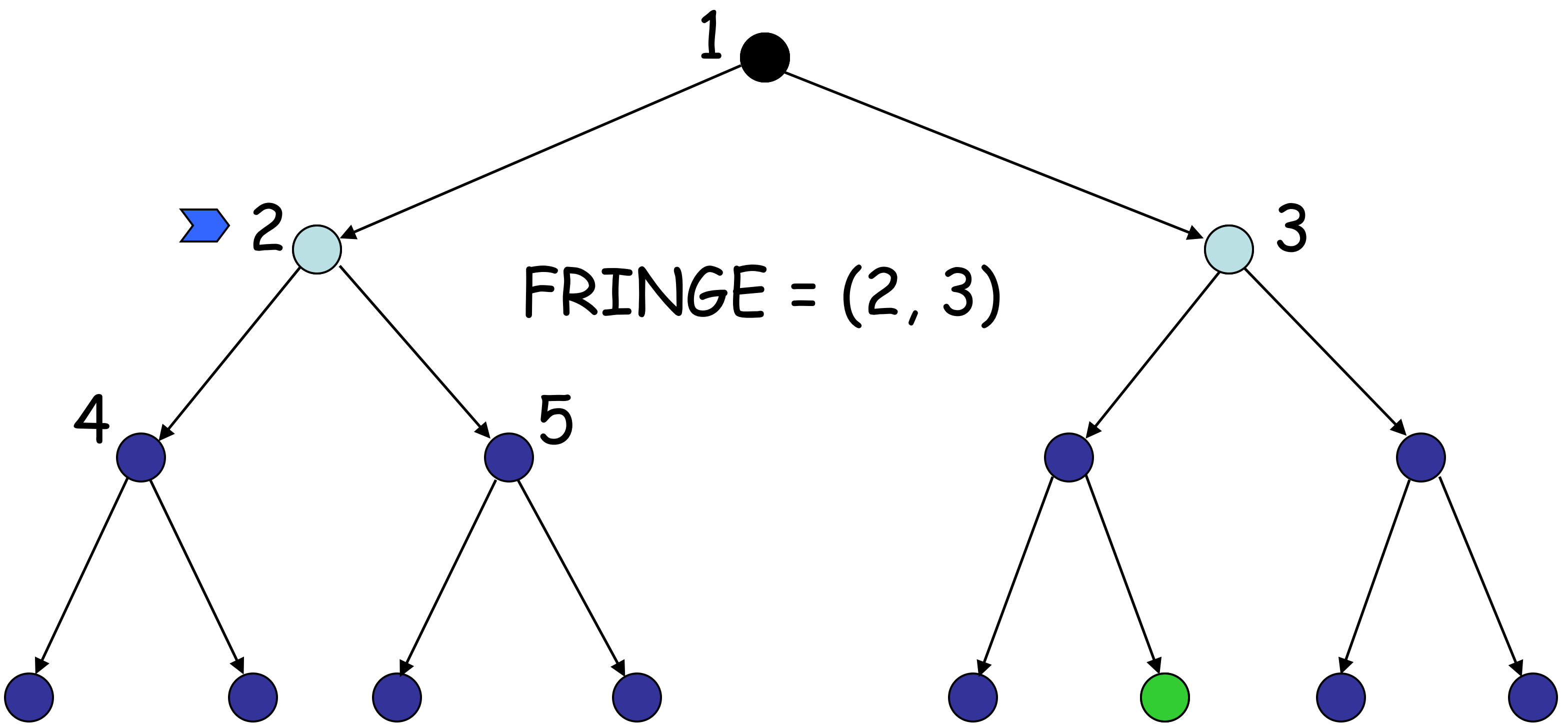

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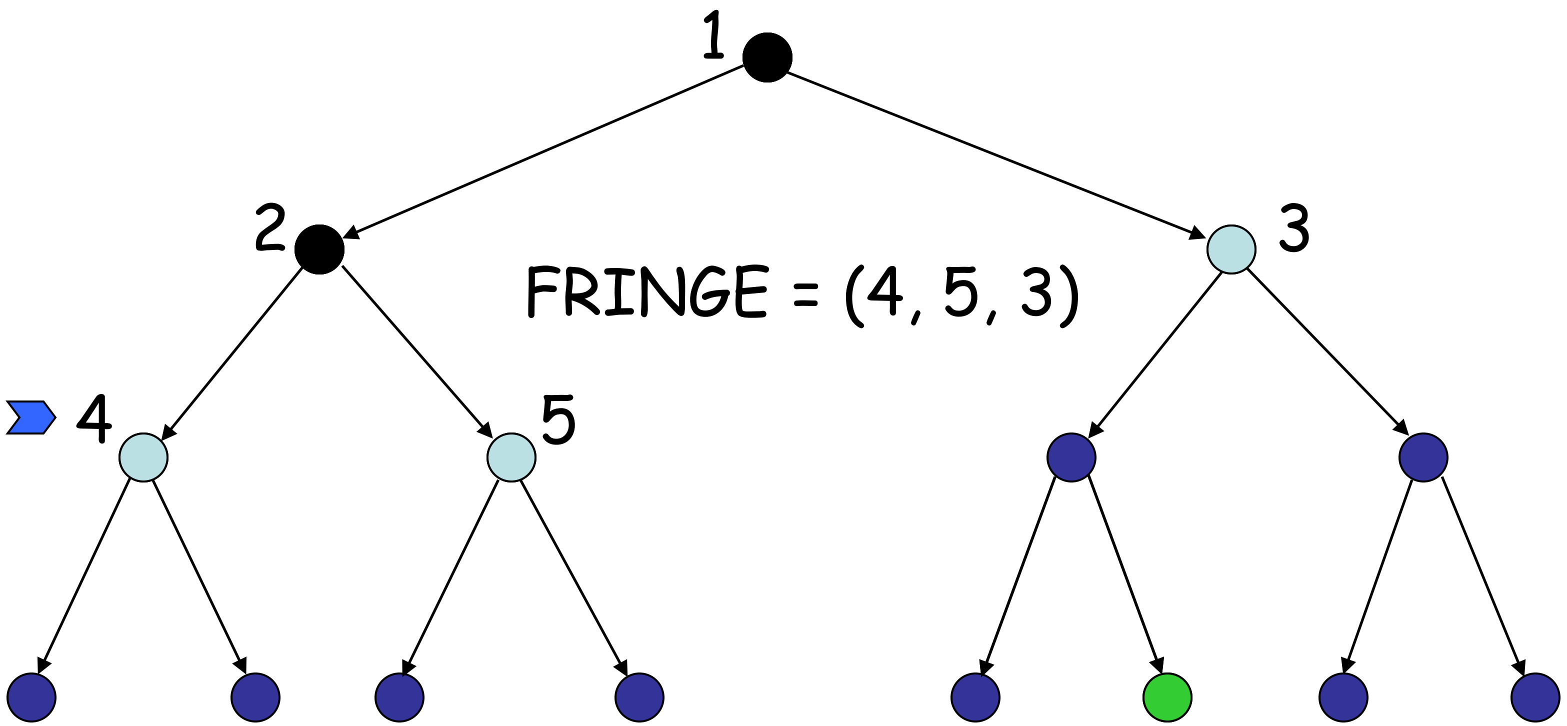

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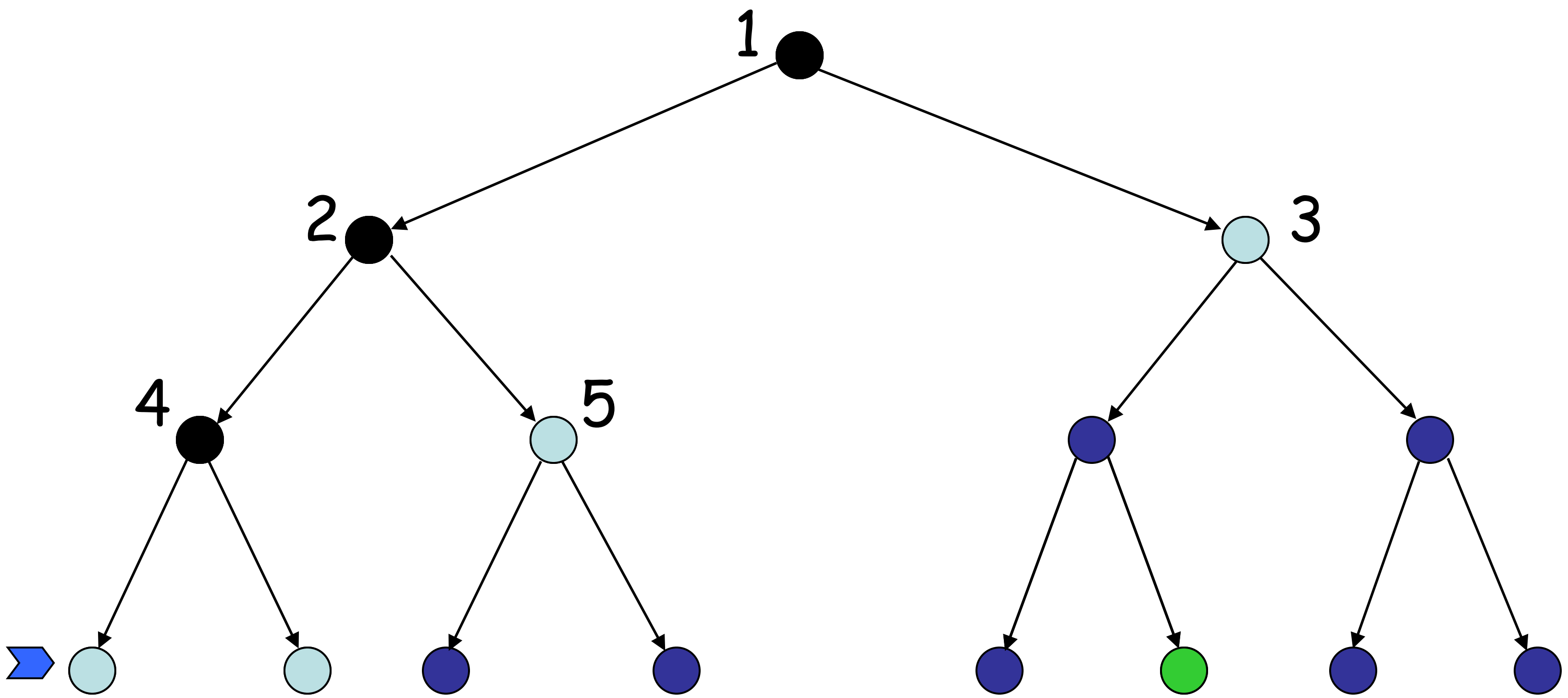

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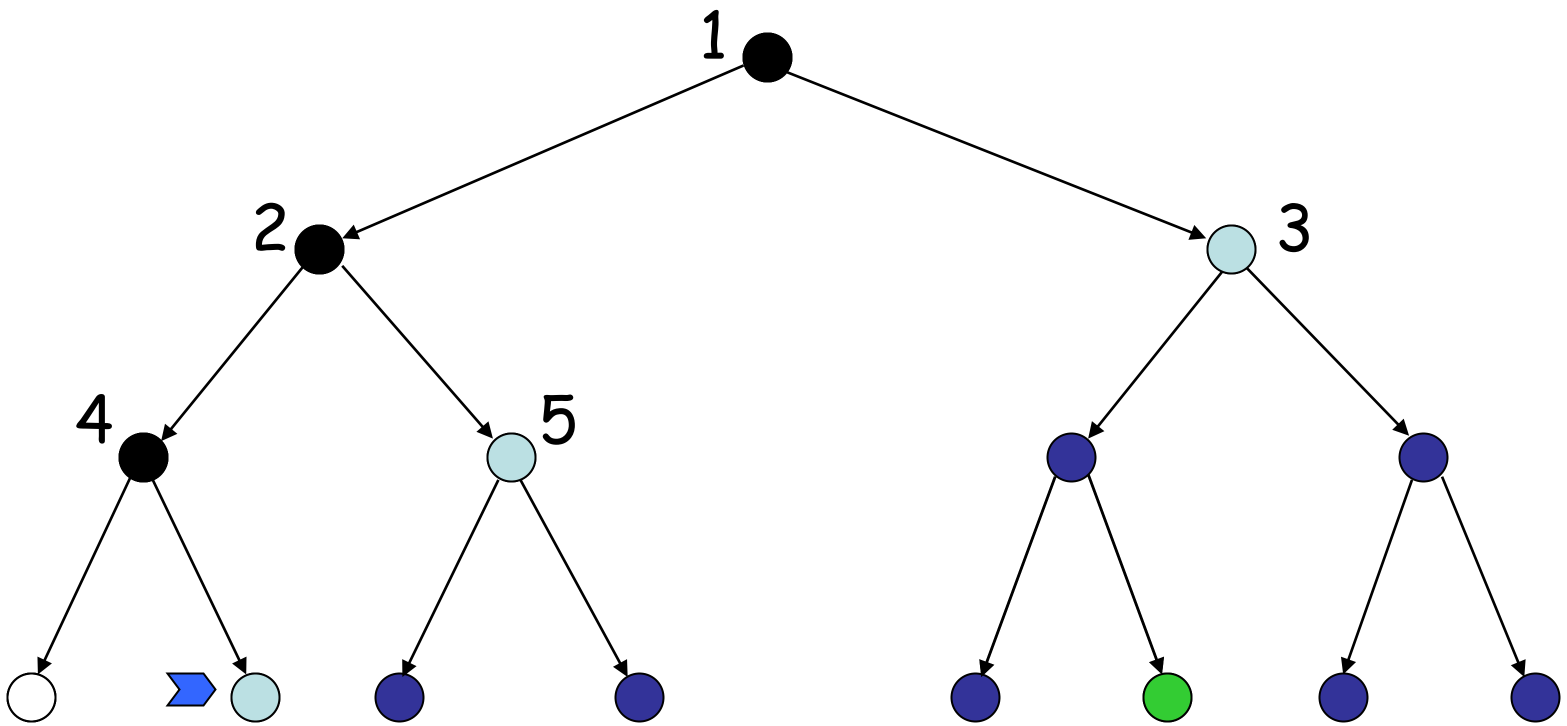

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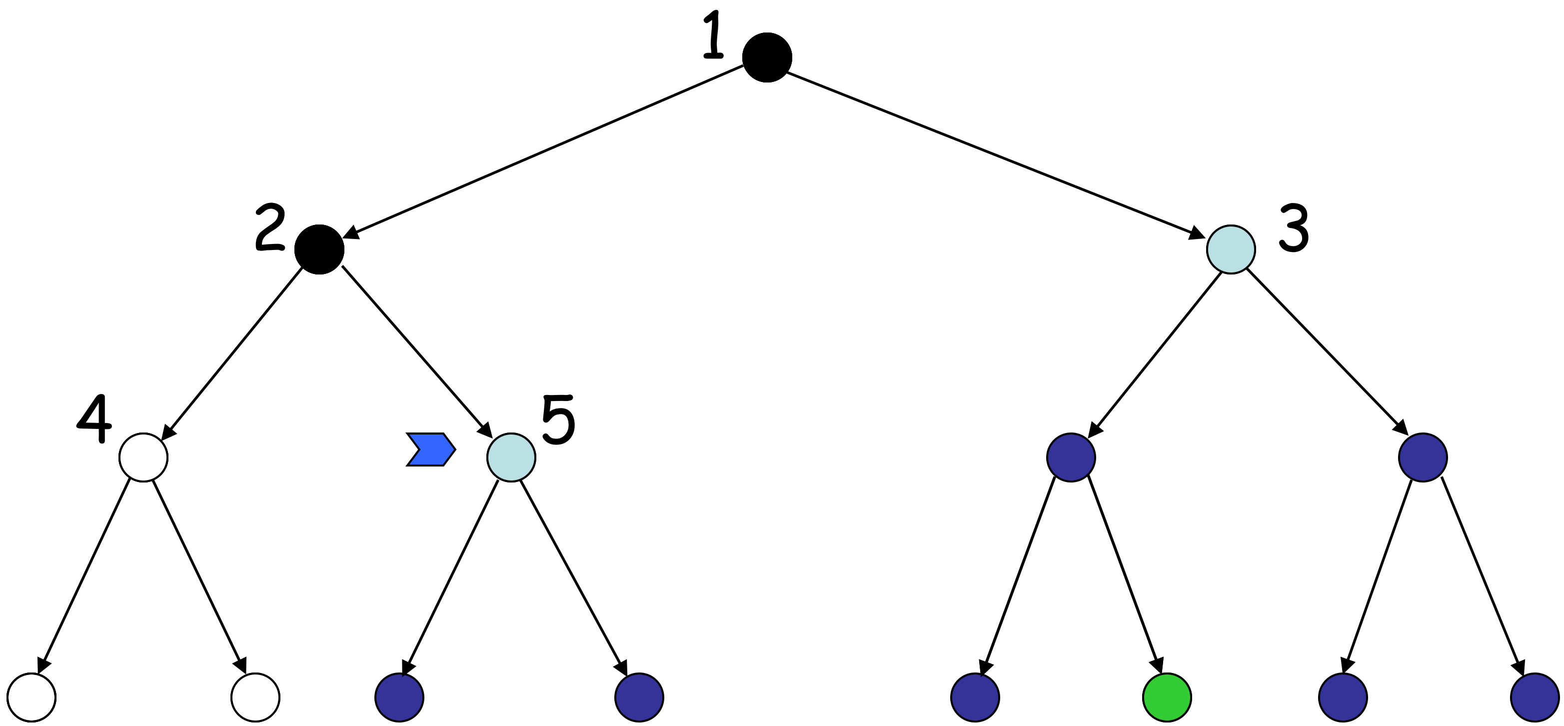

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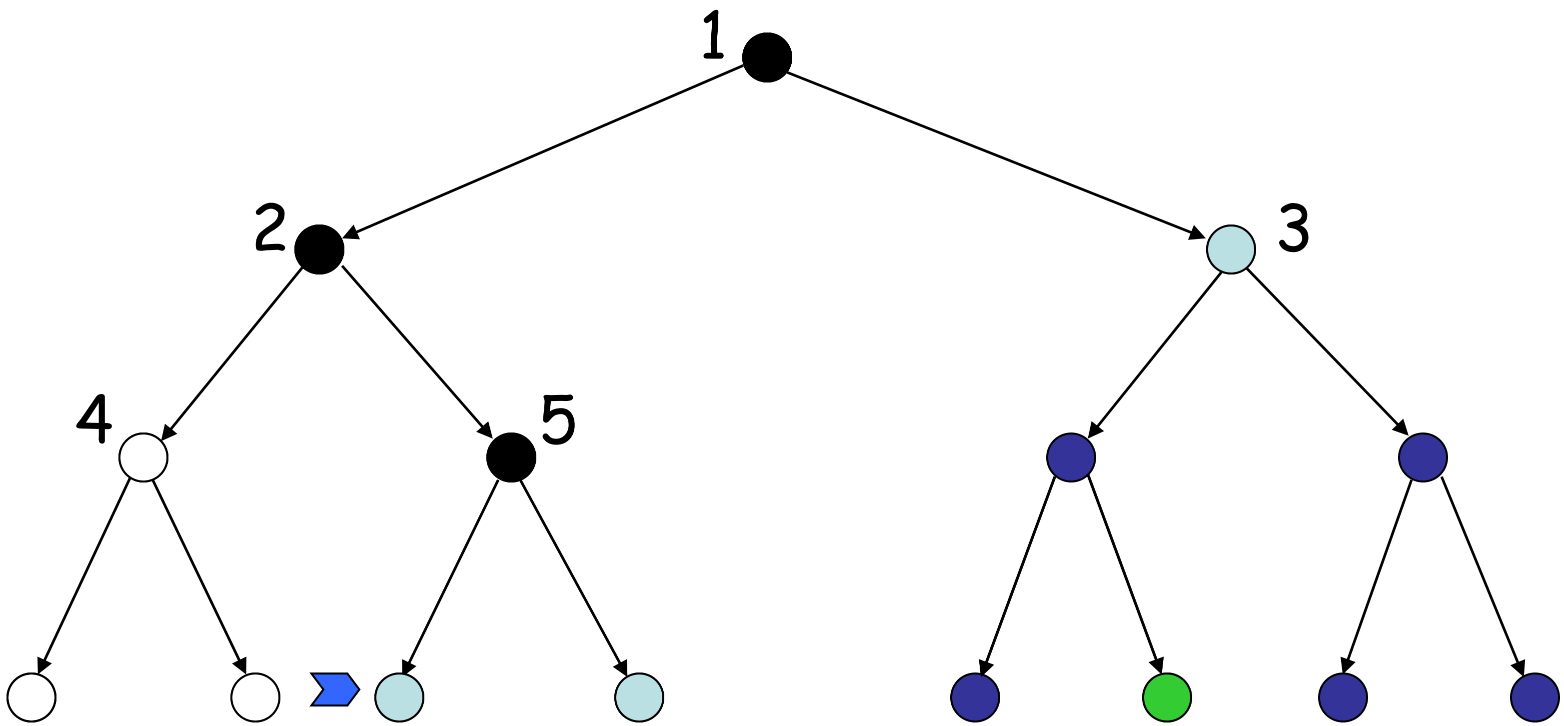

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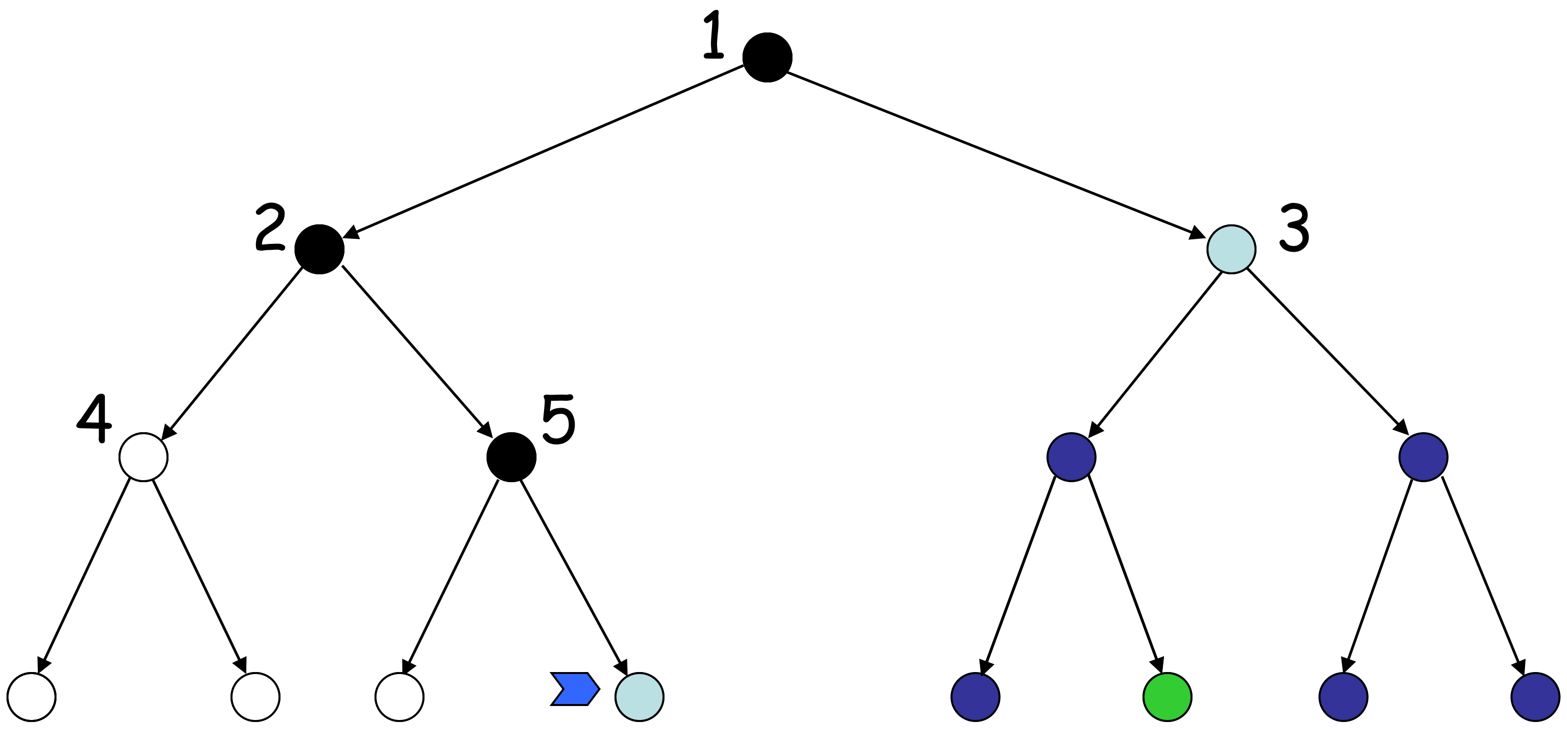

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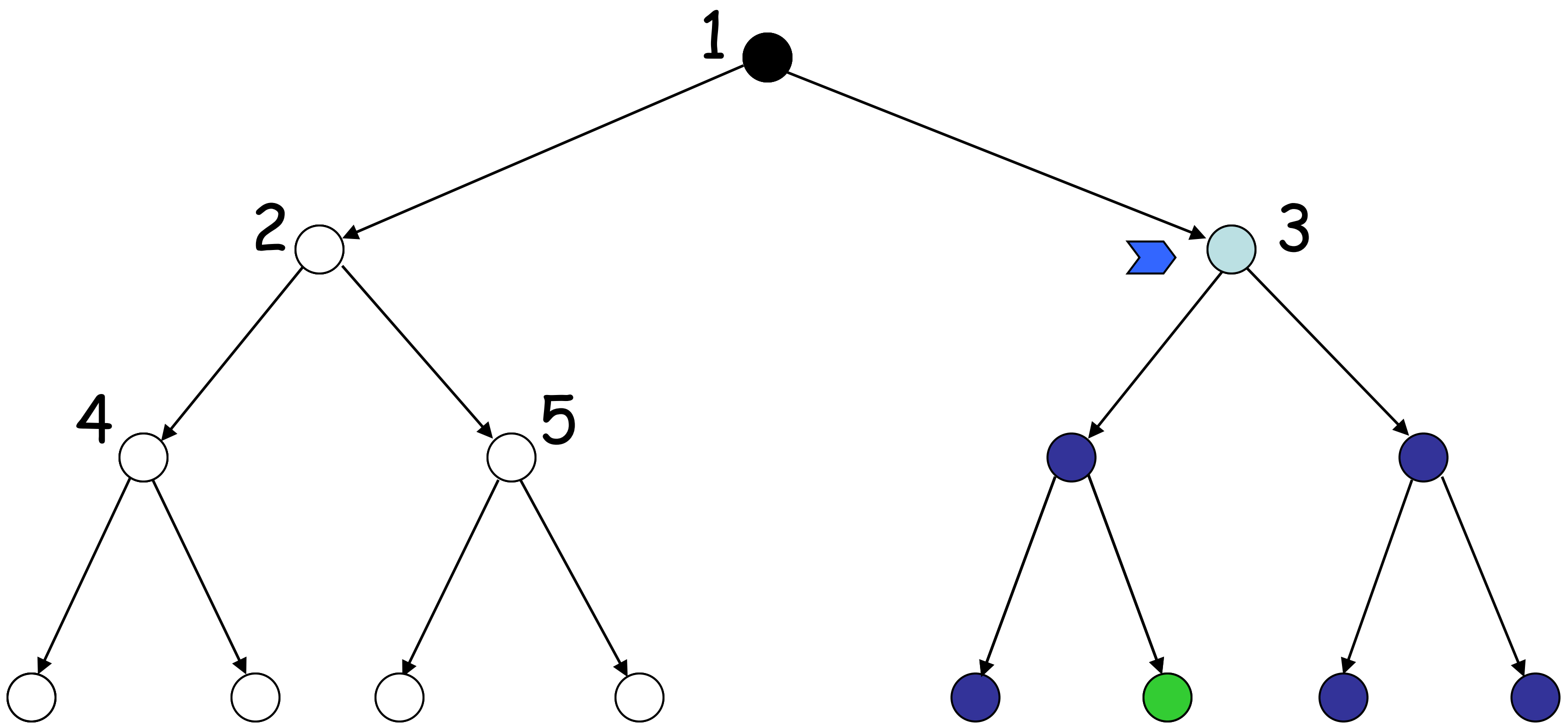

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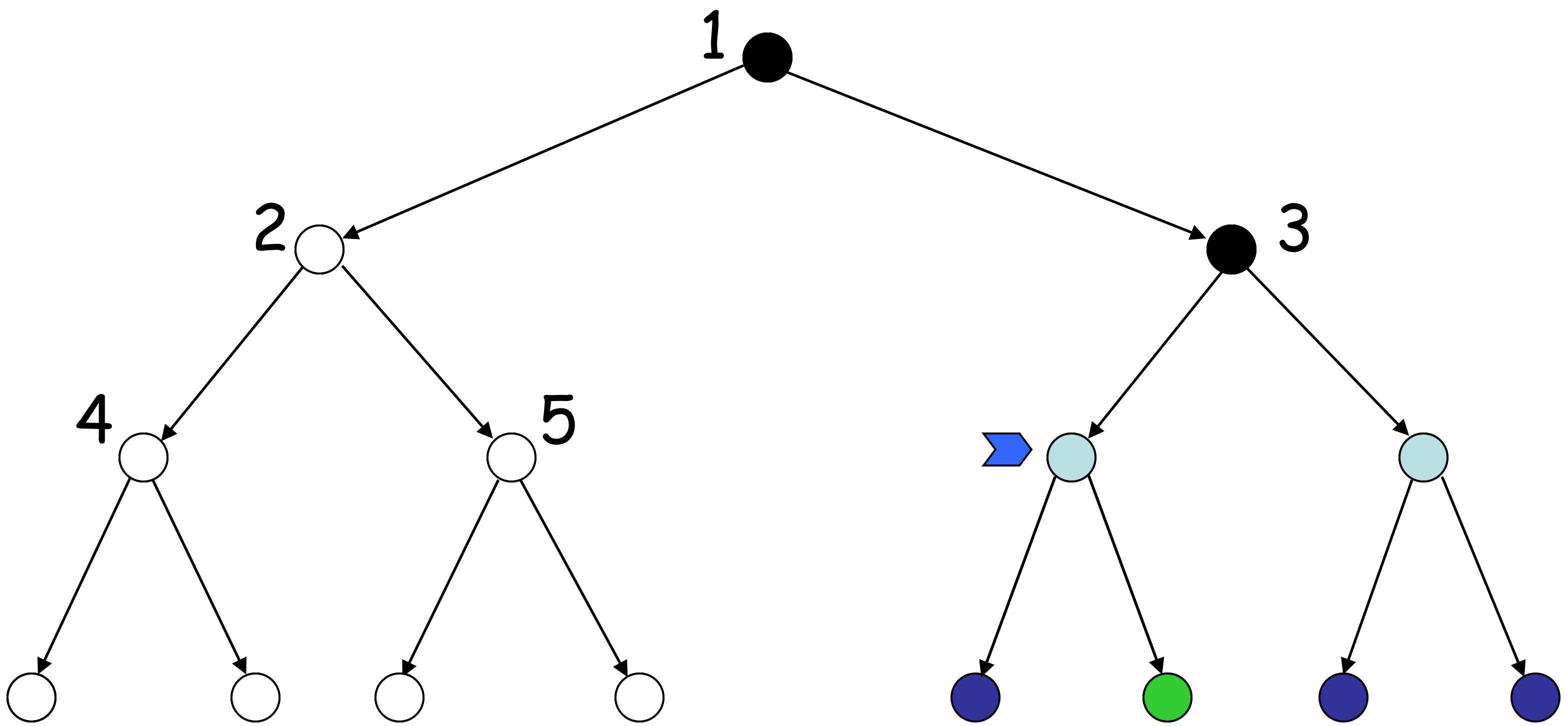

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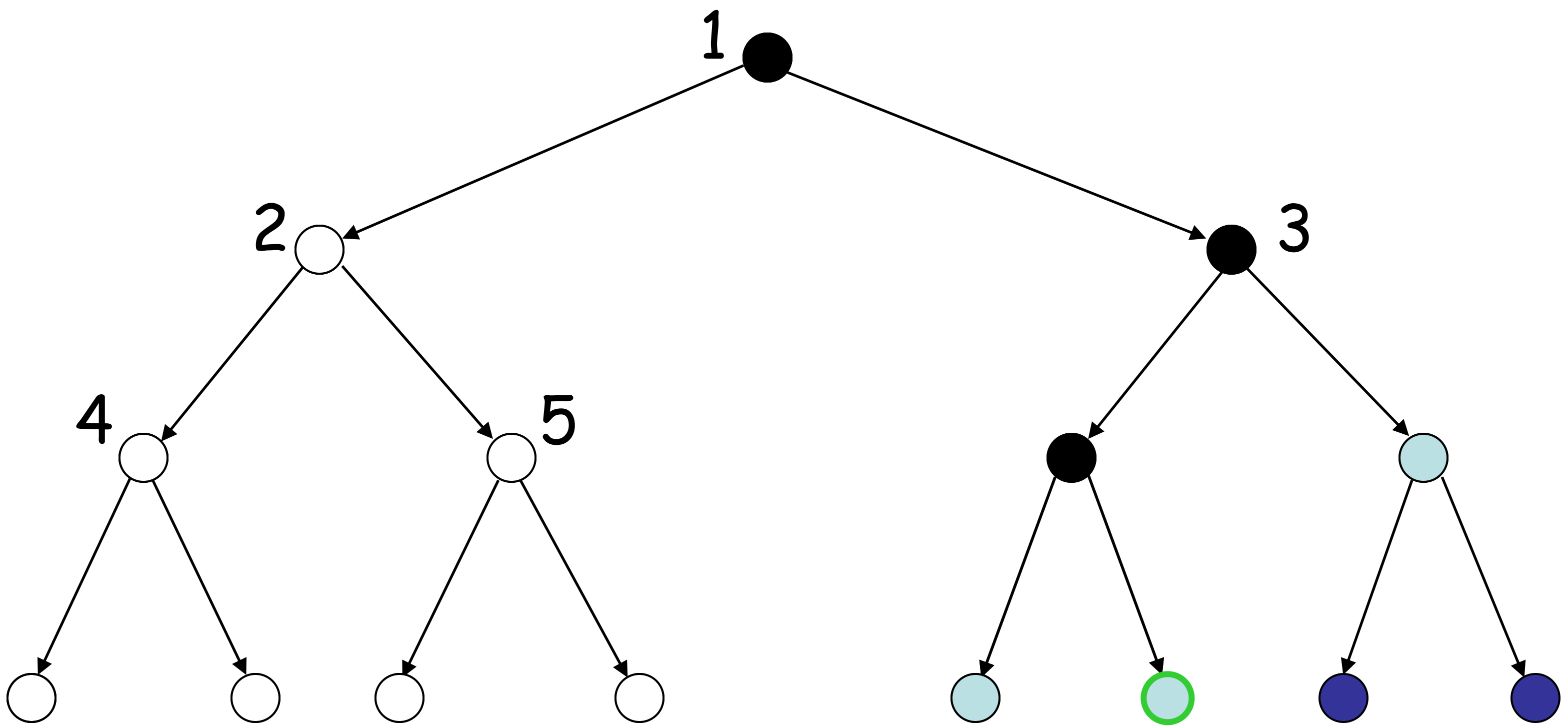


Evaluation

- b: branching factor
- d: depth of shallowest goal node
- m: maximal depth of a leaf node
- Depth-first search is:
  - Complete?
  - Optimal?

Evaluation

- b: branching factor
- d: depth of shallowest goal node
- m: maximal depth of a leaf node
- Depth-first search is:
  - Complete only for finite search tree
  - Not optimal
- Number of nodes generated (worst case):
  $1 + b + b^2 + \ldots + b^m = O(b^m)$
- Time complexity is $O(b^m)$
- Space complexity is $O(bm)$ [or $O(m)$]

[Reminder: Breadth-first requires $O(b^d)$ time and space]

Depth-Limited Search

- Depth-first with depth cutoff k (depth at which nodes are not expanded)
- Three possible outcomes:
  - Solution
  - Failure (no solution)
  - Cutoff (no solution within cutoff)

Iterative Deepening Search

Provides the best of both breadth-first and depth-first search

Main idea: Totally horrifying !

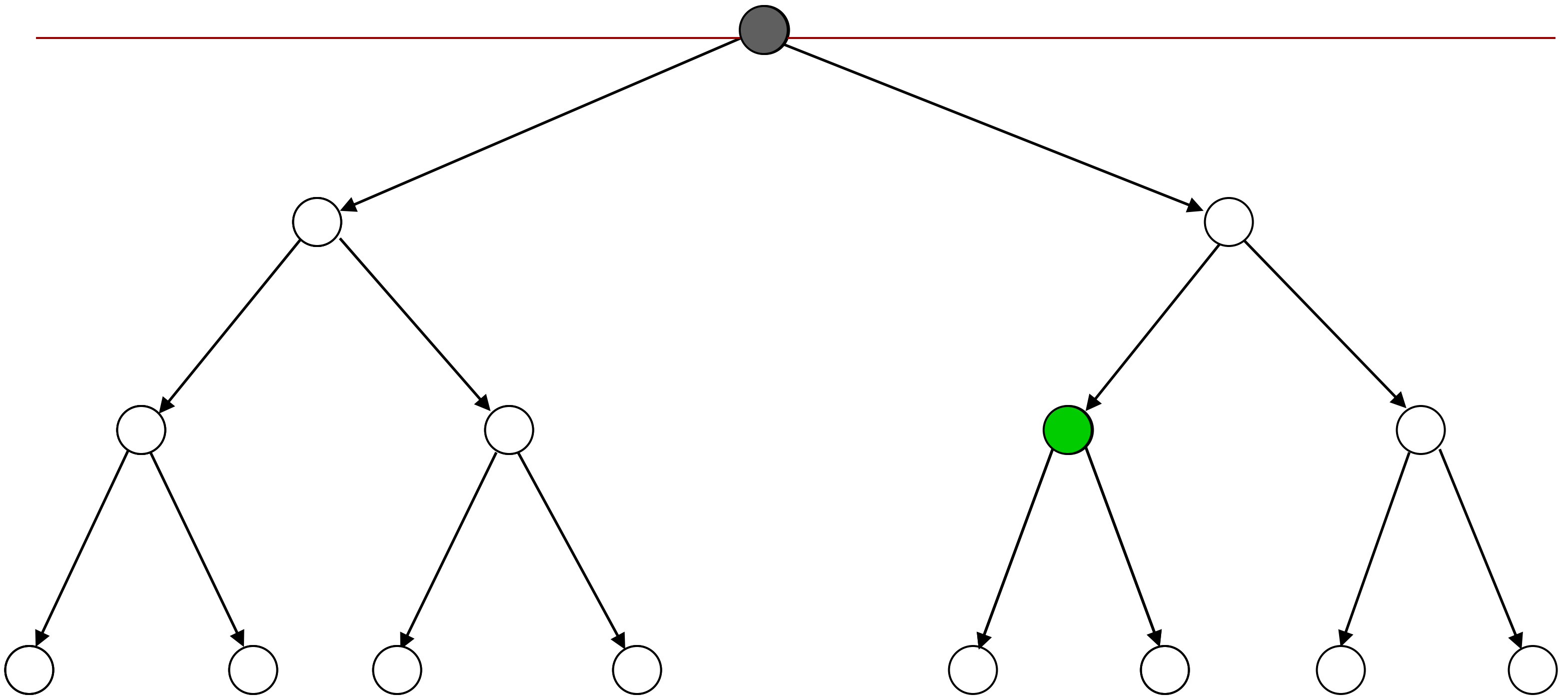
IDS

For k = 0, 1, 2, … do:

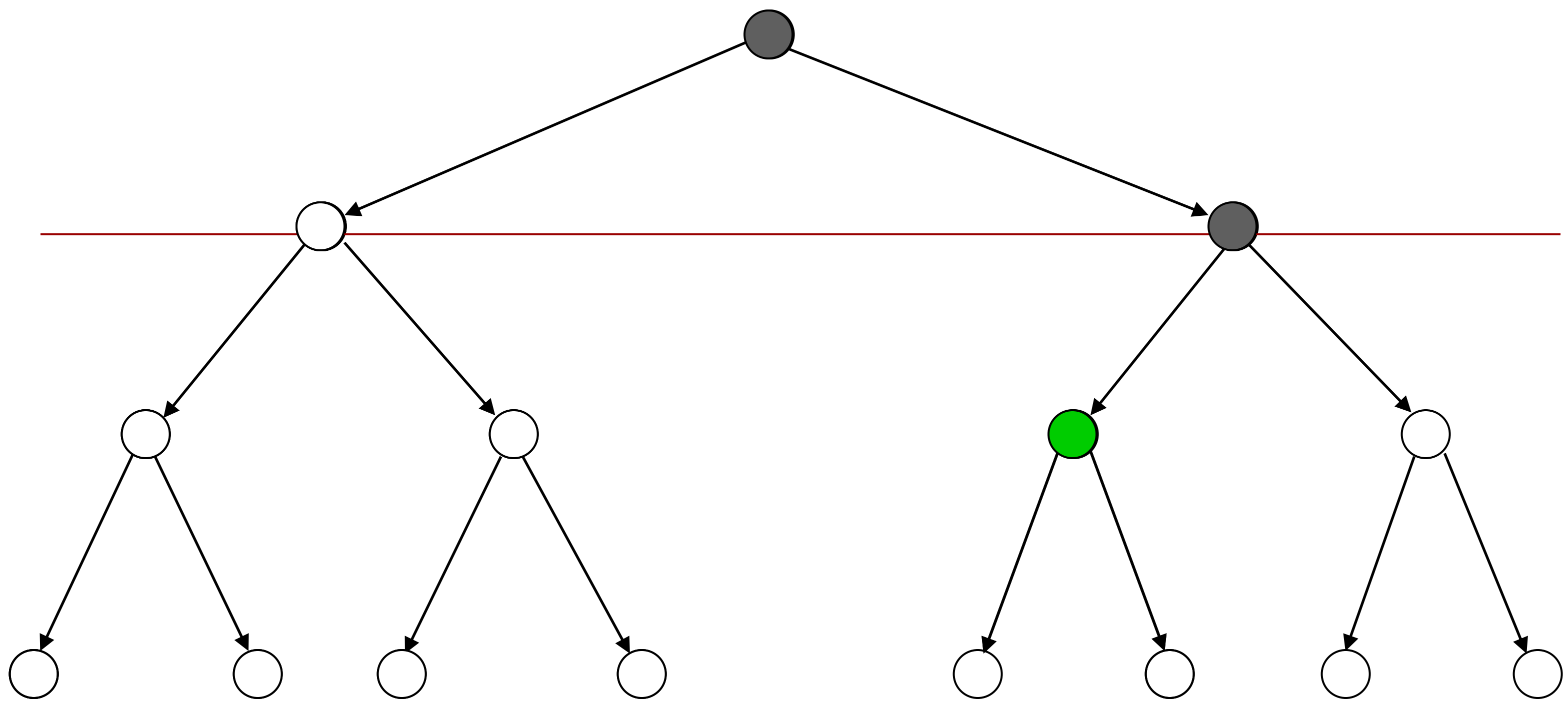
Perform depth-first search with depth cutoff k

(i.e., only generate nodes with depth $\leq$ k)

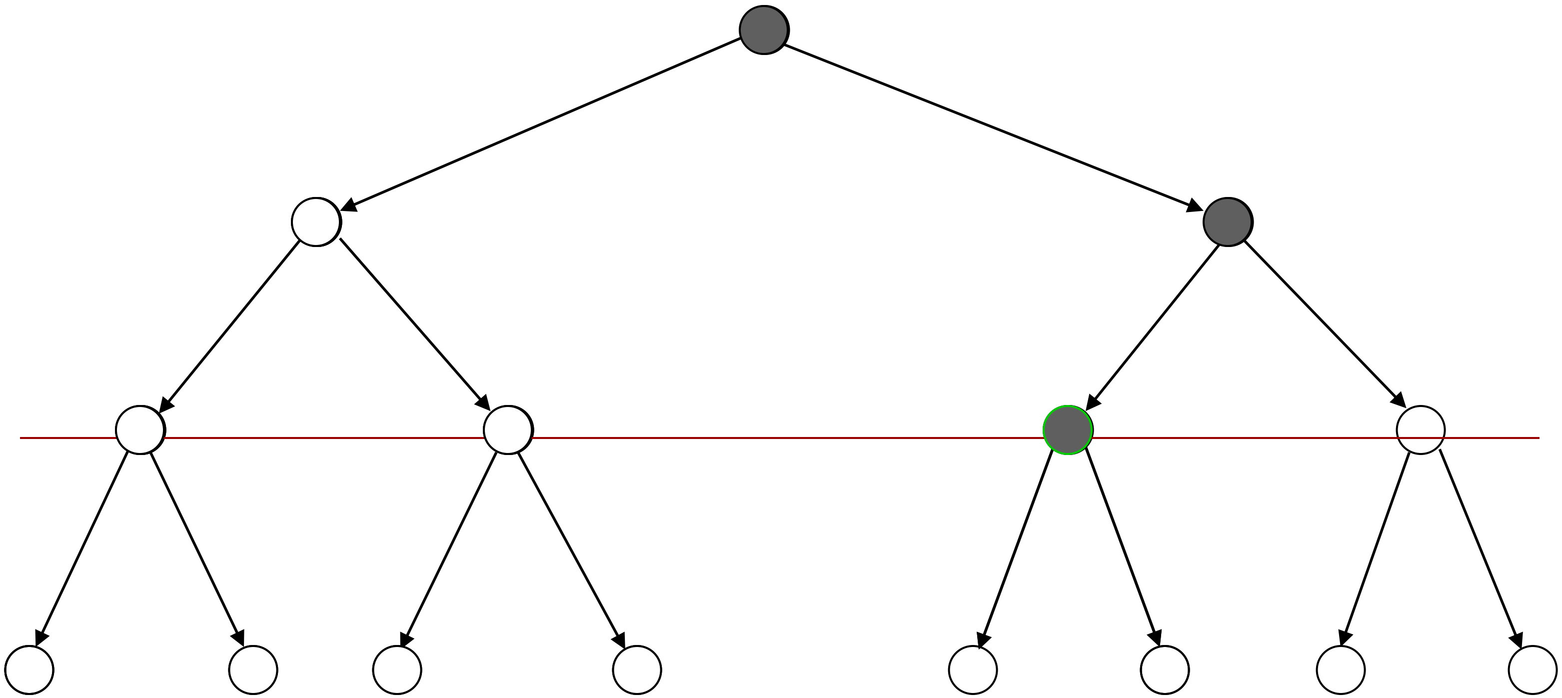
Iterative Deepening



Iterative Deepening



Iterative Deepening



Performance

- Iterative deepening search is:
  - Complete
  - Optimal if step cost =1
- Time complexity is:

  $(d+1)(1) + db + (d-1)b^2 + \ldots + (1)\, b^d = O(b^d)$
- Space complexity is: $O(bd)$ or $O(d)$

Calculation

$db + (d-1)b^2 + \ldots + (1)\, b^d$

$= b^d + 2b^{d-1} + 3b^{d-2} + \ldots + db$

$= (1 + 2b^{-1} + 3b^{-2} + \ldots + db^{-d}) \times b^d$

$\leq \left(\sum_{i=1,\ldots,\infty} ib^{(1-i)}\right) \times b^d = b^d\,(b/(b-1))^2$

Number of Generated Nodes (Breadth-First & Iterative Deepening)

d = 5 and b = 2

BF	ID
1	$1 \times 6 = 6$
2	$2 \times 5 = 10$
4	$4 \times 4 = 16$
8	$8 \times 3 = 24$
16	$16 \times 2 = 32$
32	$32 \times 1 = 32$
63	120

120/63 ~ 2

Number of Generated Nodes (Breadth-First & Iterative Deepening)

d = 5 and b = 10

BF	ID
1	6
10	50
100	400
1,000	3,000
10,000	20,000
100,000	100,000
111,111	123,456

123,456/111,111 ~ 1.111

Comparison of Strategies

- Breadth-first is complete and optimal, but has high space complexity
- Depth-first is space efficient, but is neither complete, nor optimal
- Iterative deepening is complete and optimal, with the same space complexity as depth-first and almost the same time complexity as breadth-first

Quiz: Would IDS + bi-directional search be a good combination?

Revisited States

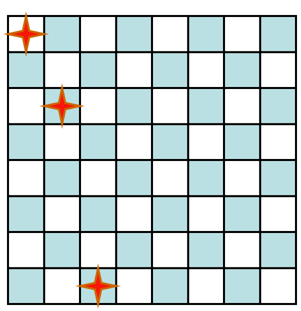
No

Few

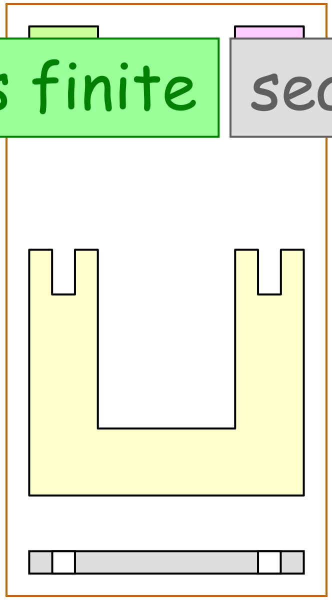
Many

search tree is finite

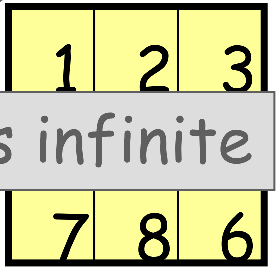
search tree is infinite

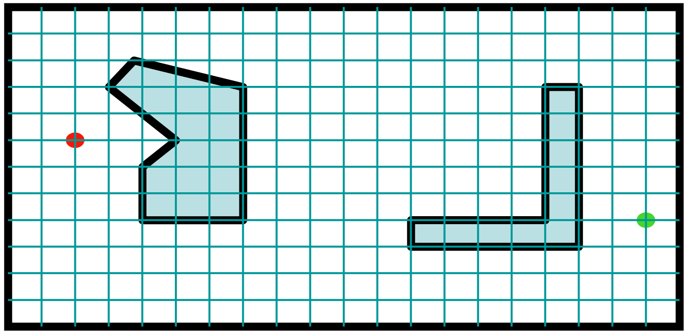


8-queens



assembly planning





8-puzzle and robot navigation

Avoiding Revisited States

- Requires comparing state descriptions
- Breadth-first search:
  - Store all states associated with **generated** nodes in VISITED
  - If the state of a new node is in VISITED, then discard the node

Avoiding Revisited States

- Requires comparing state descriptions
- Breadth-first search:
  - Store all states associated with **generated** nodes in VISITED
  - If the state of a new node is in VISITED, then discard the node

Implemented as hash-table
or as explicit data structure with flags

Avoiding Revisited States

- Depth-first search:

  Solution 1:

  - Store all states associated with nodes in current path in VISITED
  - If the state of a new node is in VISITED, then discard the node

  → ??

Avoiding Revisited States

- Depth-first search:

  Solution 1:
  - Store all states associated with nodes in current path in VISITED
  - If the state of a new node is in VISITED, then discard the node

  → Only avoids loops

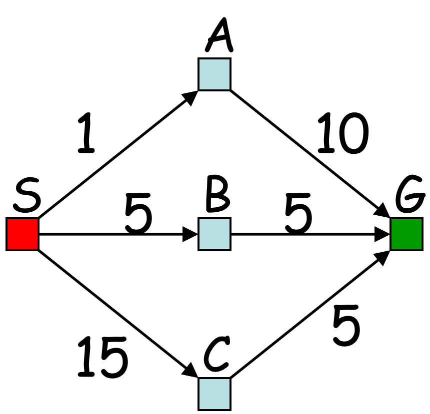
  Solution 2:
  - Store all generated states in VISITED
  - If the state of a new node is in VISITED, then discard the node

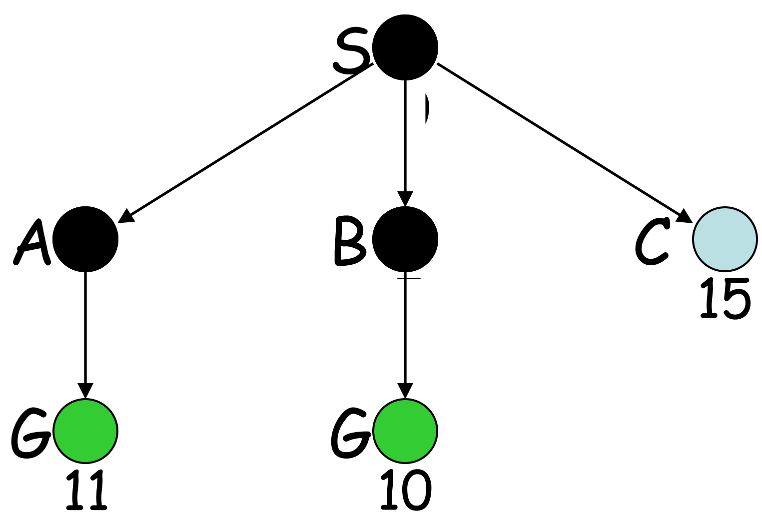
  → Same space complexity as breadth-first !

Uniform-Cost Search

- Each arc has some cost $c \geq \varepsilon > 0$
- The cost of the path to each node N is

  $g(N) = \Sigma$ costs of arcs
- The goal is to generate a solution path of minimal cost
- The nodes N in the queue FRINGE are sorted in increasing g(N)





- Need to modify search algorithm

Search Algorithm #2

The goal test is applied to a node when this node is **expanded**, not when it is generated.

SEARCH#2

1. INSERT(initial-node,FRINGE)
2. Repeat:
   a. If empty(FRINGE) then return failure
   b. N ← REMOVE(FRINGE)
   c. s ← STATE(N)
   d. If GOAL?(s) then return path or goal state
   e. For every state s' in SUCCESSORS(s)
      i. Create a node N' as a successor of N
      ii. INSERT(N',FRINGE)

Avoiding Revisited States in Uniform-Cost Search

- For any state S, when the first node N such that STATE(N) = S is expanded, the path to N is the best path from the initial state to S
- So:
  - When a node is **expanded**, store its state into CLOSED
  - When a new node N is generated:
    - If STATE(N) is in CLOSED, discard N
    - If there exits a node N' in the fringe such that STATE(N') = STATE(N), discard the node — N or N' — with the highest-cost path