ASSOCIATION ANALYSIS

Association Rule Mining

association analysis: useful for discovering interesting relationships (Association Rules) hidden in large data sets

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

Market-Basket transactions

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Example of Association Rules

$\{Diaper\} \rightarrow \{Beer\}$,

Implication means co-occurrence, not causality!

Problem Definition

Binary Representation

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

TID	Bread	Milk	Diapers	Beer	Eggs	Cola
1	1	1	0	0	0	0
2	1	0	1	1	1	0
3	0	1	1	1	0	1
4	1	1	1	1	0	0
5	1	1	1	0	0	1

$$I = \{i_1, i_2, \ldots, i_d\}$$

$$T = \{t_1, t_2, \ldots, t_N\}$$

Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
  - transaction *tj* contains an itemset
- **Support count** (σ)
  - Frequency of occurrence of an itemset
  - E.g. σ({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Definition: Association Rule

- Association Rule
  - An implication expression of the form $X \rightarrow Y$, where X and Y are itemsets
  - Example:
    {Milk, Diaper} → {Beer}

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

- Rule Evaluation Metrics
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

$$\text{Support}, s(X \longrightarrow Y) = \frac{\sigma(X \cup Y)}{N}.$$

$$\text{Confidence}, c(X \longrightarrow Y) = \frac{\sigma(X \cup Y)}{\sigma(X)}.$$

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \text{Beer}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

Mining Association Rules

- ✓ a rule that has very low support may occur simply by chance
- ✓ Confidence measures the reliability of the inference made by a rule

TID	*Items*
1	**Bread, Milk**
2	**Bread, Diaper, Beer, Eggs**
3	**Milk, Diaper, Beer, Coke**
4	**Bread, Milk, Diaper, Beer**
5	**Bread, Milk, Diaper, Coke**

Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

Observations:

- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold

$$R = 3^d - 2^{d+1} + 1$$

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

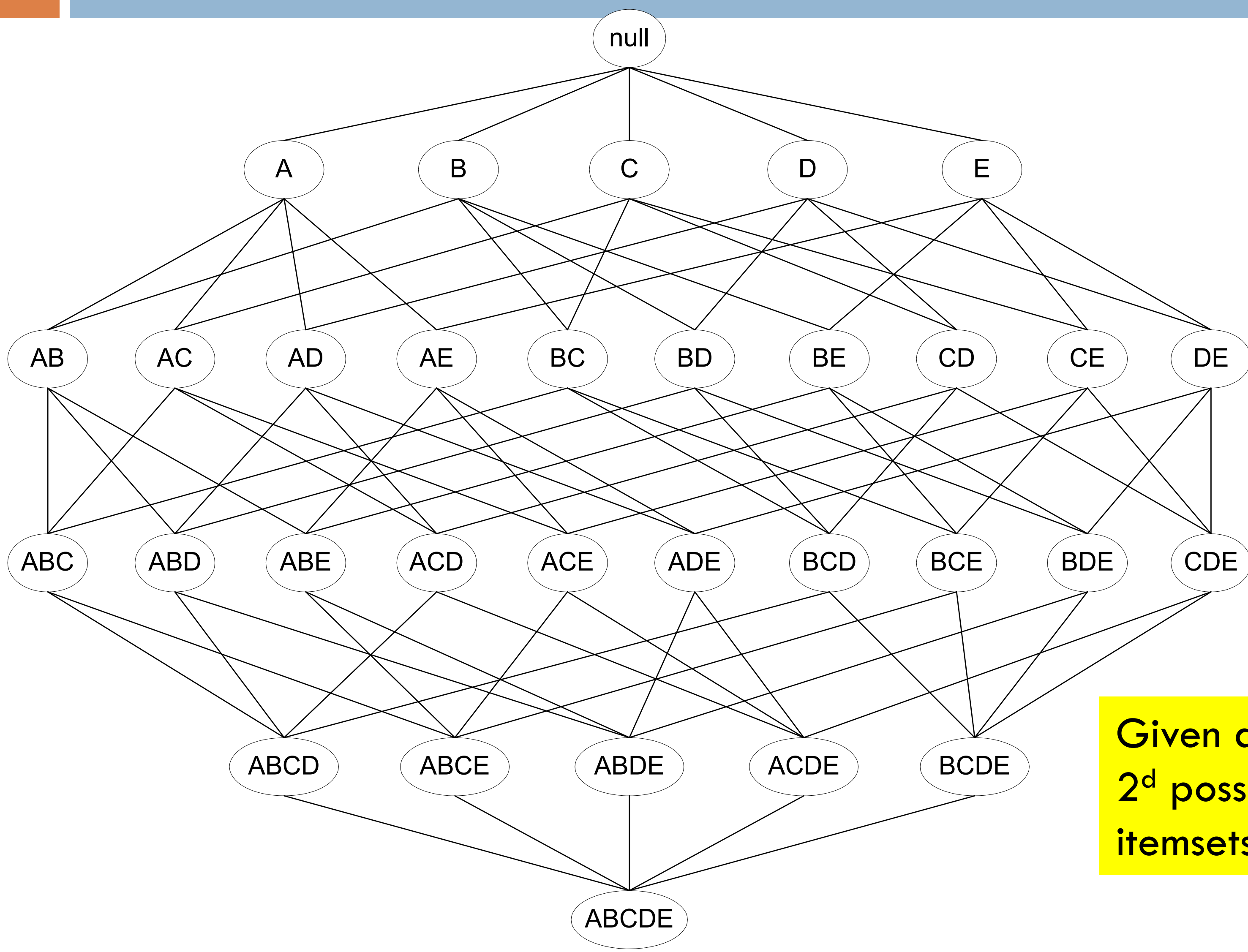
  $\Rightarrow$ Computationally prohibitive!

Mining Association Rules

If the itemset is infrequent, then all candidate rules can be pruned immediately without compute their confidence values

- Two-step approach:
  1. Frequent Itemset Generation
     - Generate all itemsets whose support $\geq$ minsup
  2. Rule Generation
     - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset
- Frequent itemset generation is still computationally expensive

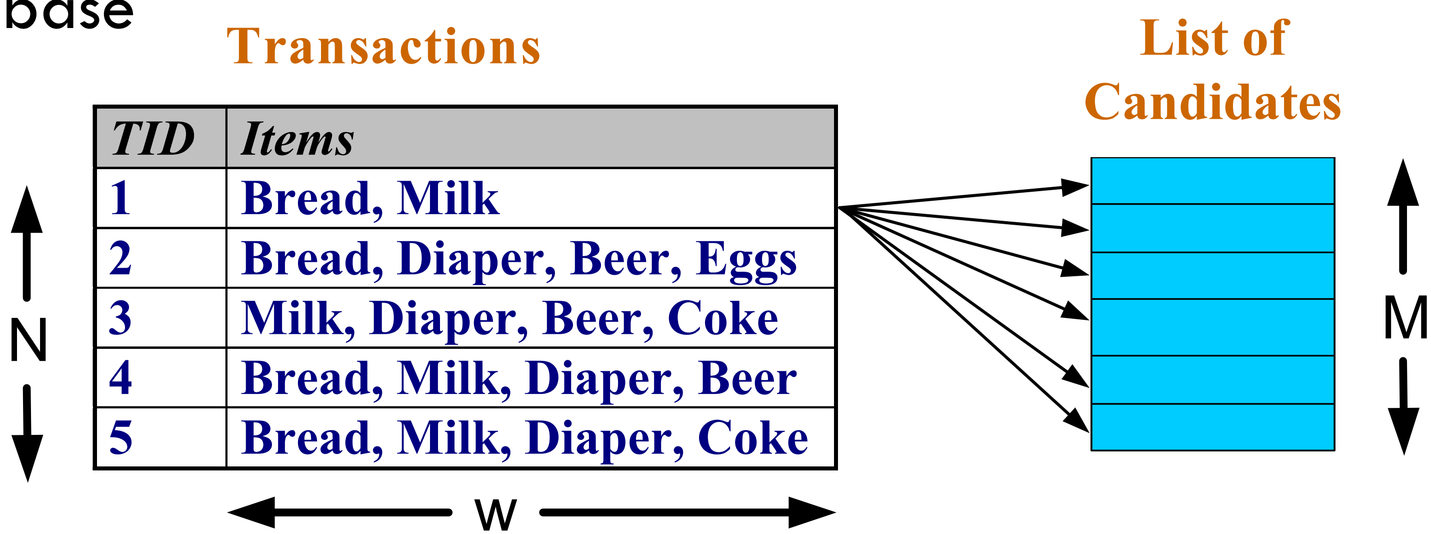
Frequent Itemset Generation



Given d items, there are 2^d possible candidate itemsets

Frequent Itemset Generation

- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database



TID	*Items*
1	Bread, Milk
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5	Bread, Milk, Diaper, Coke

  - Match each transaction against every candidate
  - Expensive!!!

Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M

- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

Reducing Number of Candidates

Reducing Number of Candidates

- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent

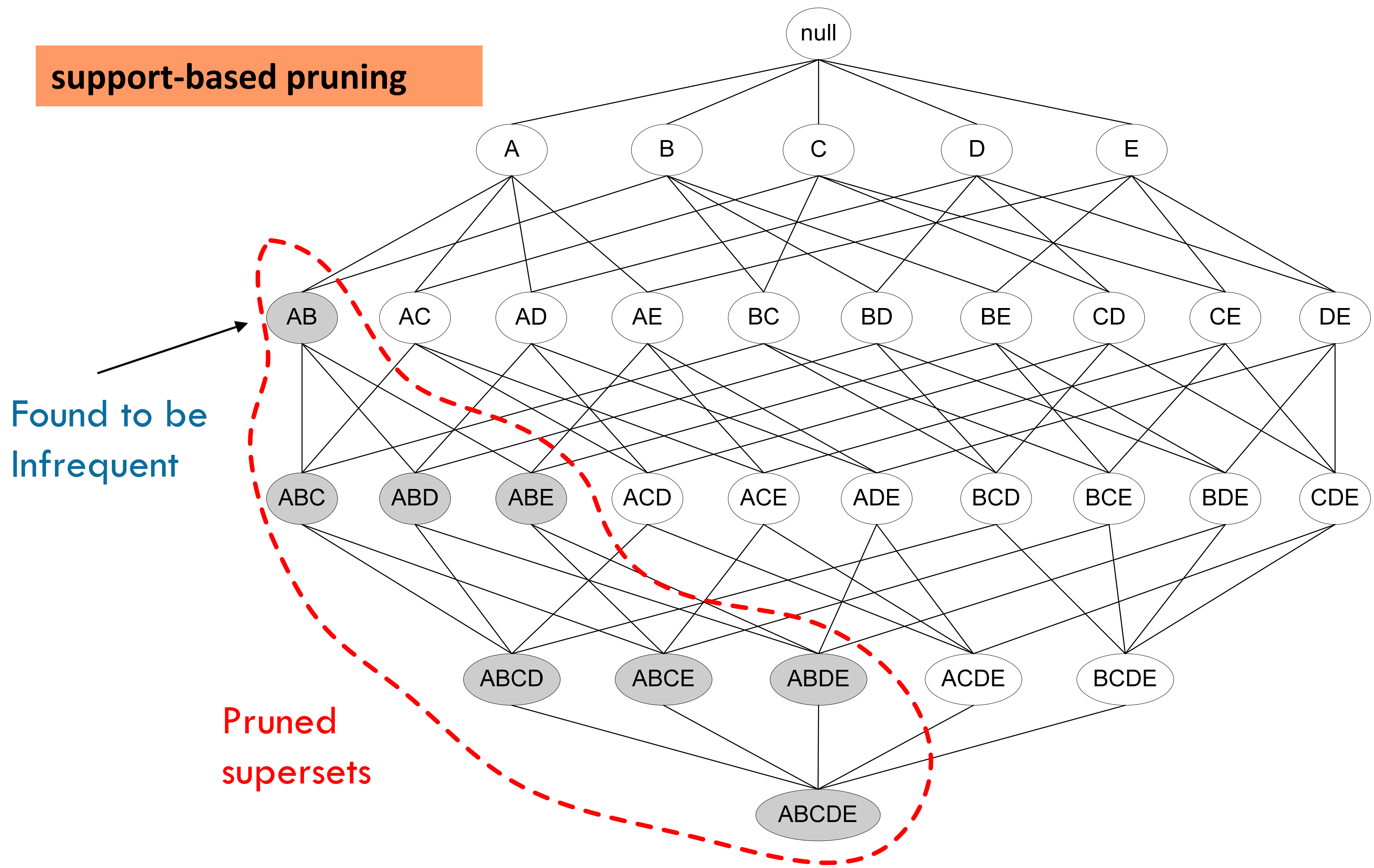
- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - Support of an itemset never exceeds the support of its subsets

Illustrating Apriori Principle

support-based pruning



Illustrating Apriori Principle

Minimum Support=0.6(3)

Item	Count
Bread	4
Coke	2
Milk	4
Beer	3
Diaper	4
Eggs	1

Items (1-itemsets)

Itemset	Count
{Bread,Milk}	3
{Bread,Beer}	2
{Bread,Diaper}	3
{Milk,Beer}	2
{Milk,Diaper}	3
{Beer,Diaper}	3

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Triplets (3-itemsets)

Itemset	Count
{Bread,Milk,Diaper}	3

Apriori Algorithm

- Method:
  - Let k=1
  - Generate frequent itemsets of length 1
  - Repeat until no new frequent itemsets are identified
    - Generate length (k+1) candidate itemsets from length k frequent itemsets
    - Prune candidate itemsets containing subsets of length k that are infrequent
    - Count the support of each candidate by scanning the DB
    - Eliminate candidates that are infrequent, leaving only those that are frequent
    - k=k+1

$F_{k-1} \times F_1$ Method

Frequent 2-itemset

Itemset
{Beer, Diapers}
{Bread, Diapers}
{Bread, Milk}
{Diapers, Milk}

Frequent 1-itemset

Item
Beer
Bread
Diapers
Milk

Candidate Generation

Itemset
{Beer, Diapers, Bread}
{Beer, Diapers, Milk}
{Bread, Diapers, Milk}
{Bread, Milk, Beer}

Candidate Pruning

Itemset
{Bread, Diapers, Milk}

$F_{k-1} \times F_{k-1}$ Method

$$A = \{a_1, a_2, \ldots, a_{k-1}\} \text{ and } B = \{b_1, b_2, \ldots, b_{k-1}\}$$

$$a_i = b_i \ (\text{for } i = 1, 2, \ldots, k-2) \text{ and } a_{k-1} \neq b_{k-1}$$

Frequent
2-itemset

Itemset
{Beer, Diapers}
{Bread, Diapers}
{Bread, Milk}
{Diapers, Milk}

Frequent
2-itemset

Itemset
{Beer, Diapers}
{Bread, Diapers}
{Bread, Milk}
{Diapers, Milk}

Candidate
Generation

Itemset
{Bread, Diapers, Milk}

Candidate
Pruning

Itemset
{Bread, Diapers, Milk}

Reducing Number of Comparisons

Reducing Number of Comparisons

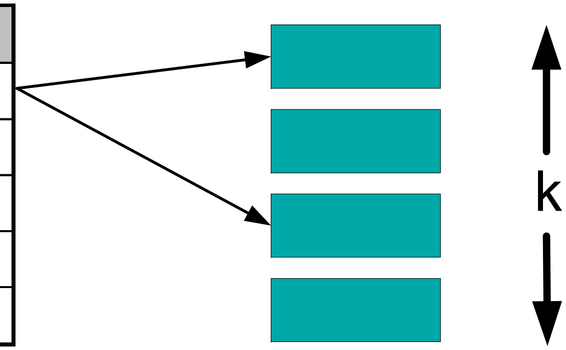
- Candidate counting:
  - Scan the database of transactions to determine the support of each candidate itemset
  - To reduce the number of comparisons, store the candidates in a hash structure
    - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets

Transactions

N

TID	*Items*
1	Bread, Milk
2	Bread, Diaper, Beer, Eggs
3	Milk, Diaper, Beer, Coke
4	Bread, Milk, Diaper, Beer
5	Bread, Milk, Diaper, Coke

Hash Structure



Buckets

Generate Hash Tree

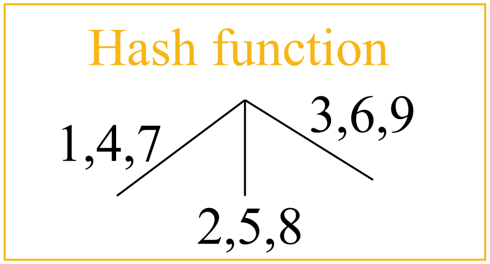
Suppose you have 15 candidate itemsets of length 3:

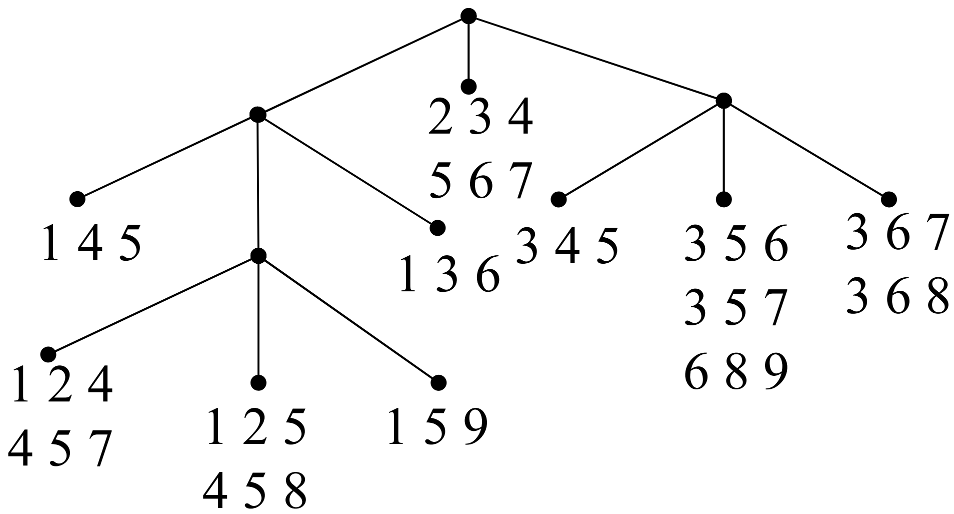
{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

You need:

• Hash function

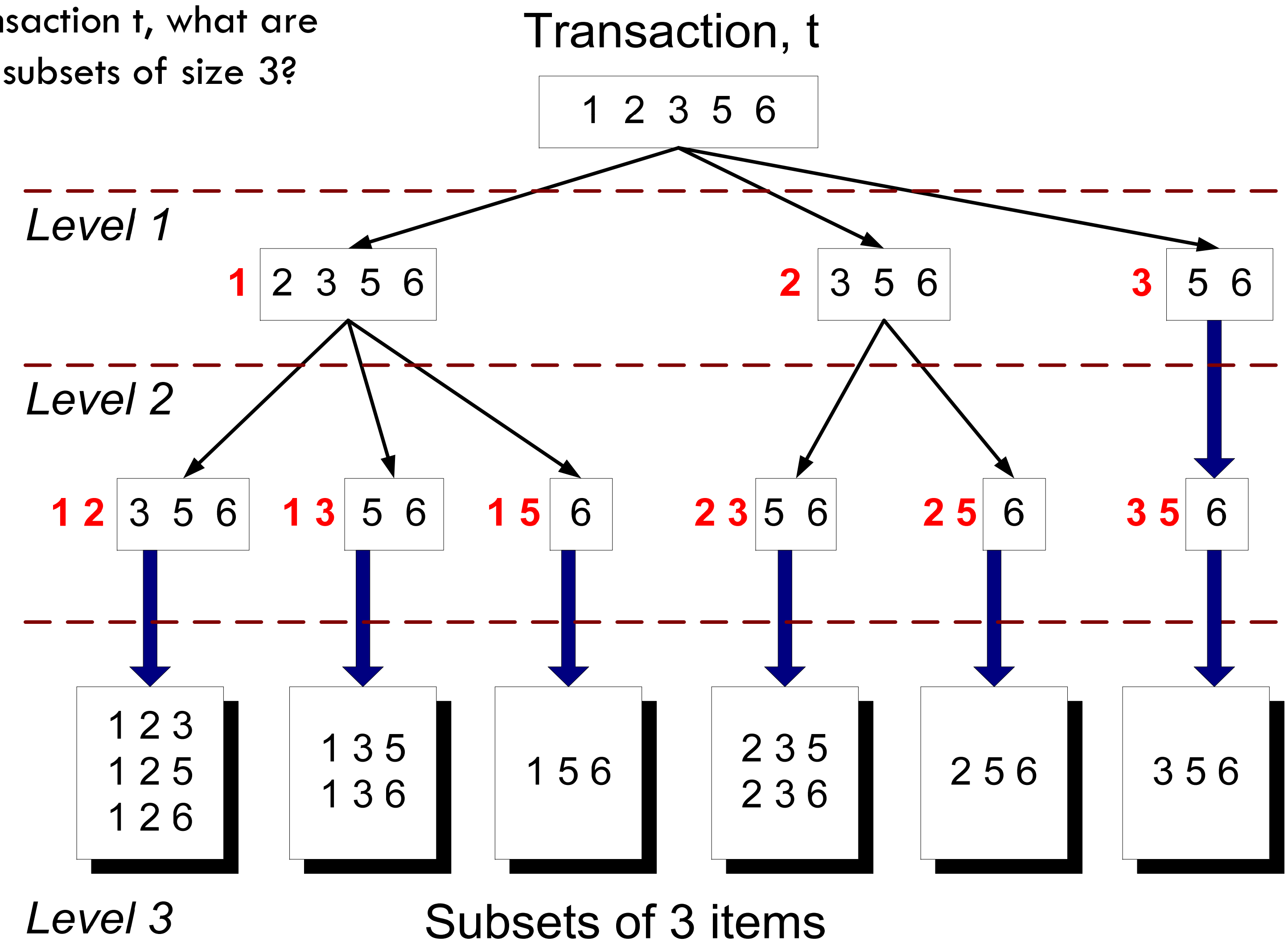
• Max leaf size: max number of itemsets stored in a leaf node (if number of candidate itemsets exceeds max leaf size, split the node)



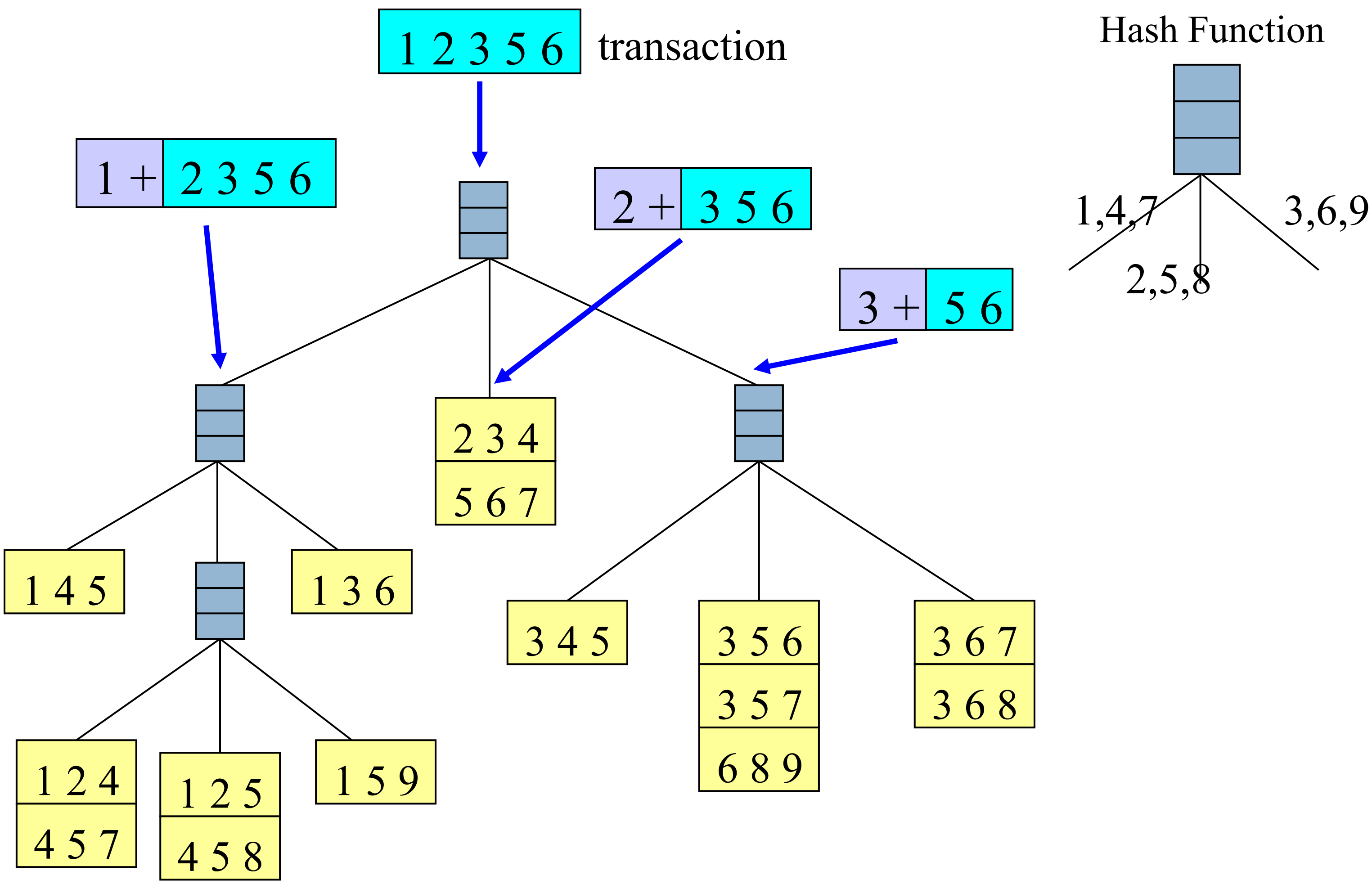


Subset Operation

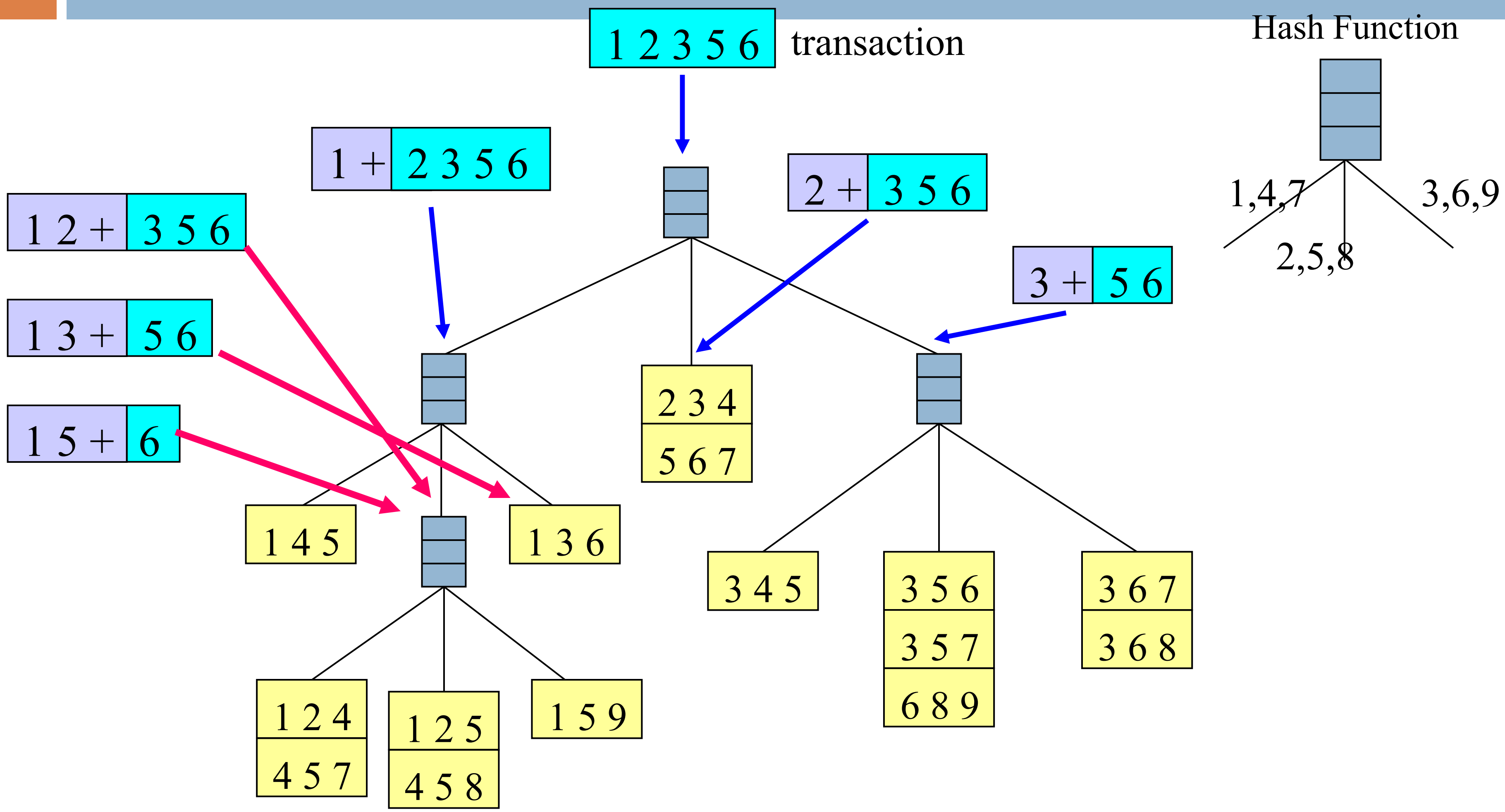
Given a transaction t, what are the possible subsets of size 3?



Subset Operation Using Hash Tree



Subset Operation Using Hash Tree



Rule Generation

- Given a frequent itemset L, find all non-empty subsets $f \subset L$ such that $f \rightarrow L - f$ satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

$ABC \rightarrow D$,	$ABD \rightarrow C$,	$ACD \rightarrow B$,	$BCD \rightarrow A$,
$A \rightarrow BCD$,	$B \rightarrow ACD$,	$C \rightarrow ABD$,	$D \rightarrow ABC$
$AB \rightarrow CD$,	$AC \rightarrow BD$,	$AD \rightarrow BC$,	$BC \rightarrow AD$,
$BD \rightarrow AC$,	$CD \rightarrow AB$,		

- If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \emptyset$ and $\emptyset \rightarrow L$)

Rule Generation

Rule Generation

- confidence of rules generated from the same itemset has an anti-monotone property
  - E.g., Suppose {A,B,C,D} is a frequent 4-itemset:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

frequent itemset

Theorem 6.2. *If a rule $X \longrightarrow Y - X$ does not satisfy the confidence threshold, then any rule $X' \longrightarrow Y - X'$, where X' is a subset of X, must not satisfy the confidence threshold as well.*

$$X \longrightarrow Y - X \qquad \sigma(Y)/\sigma(X)$$

$$X' \longrightarrow Y - X' \qquad \sigma(Y)/\sigma(X')$$

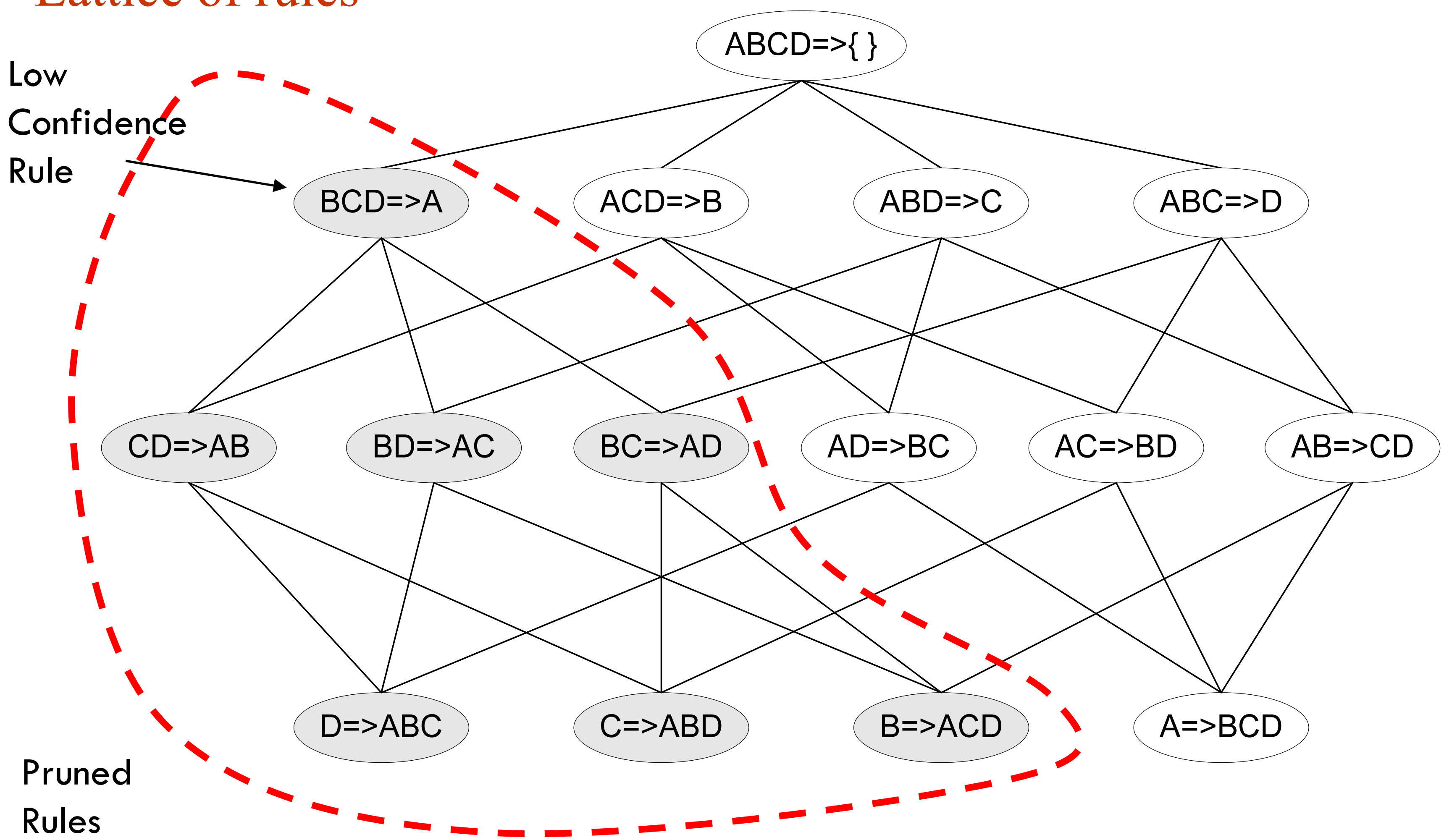
$$\sigma(X') \geq \sigma(X)$$

Rule Generation in *Apriori* Algorithm

- level-wise approach for generating association rules
- all the high-confidence rules that have only one item in the rule consequent are extracted
- These rules are then used to generate new candidate rules

Rule Generation for Apriori Algorithm

Lattice of rules



FP-growth Algorithm

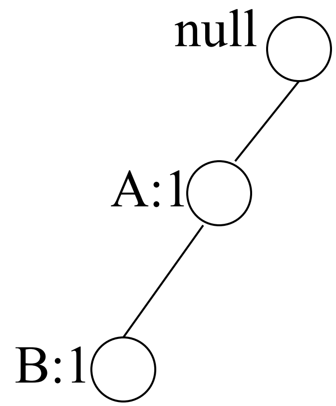
FP-growth Algorithm

- Use a compressed representation of the database using an FP-tree
- Once an FP-tree has been constructed, it uses a recursive divide-and-conquer approach to mine the frequent itemsets
- determine the support count of each item
- Infrequent items are discarded
- frequent items are sorted in decreasing support counts

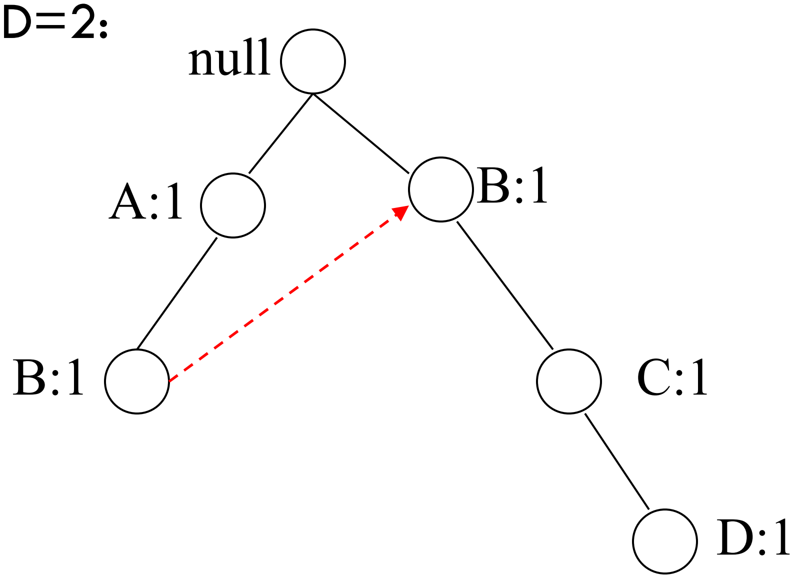
FP-tree construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

After reading TID=1:



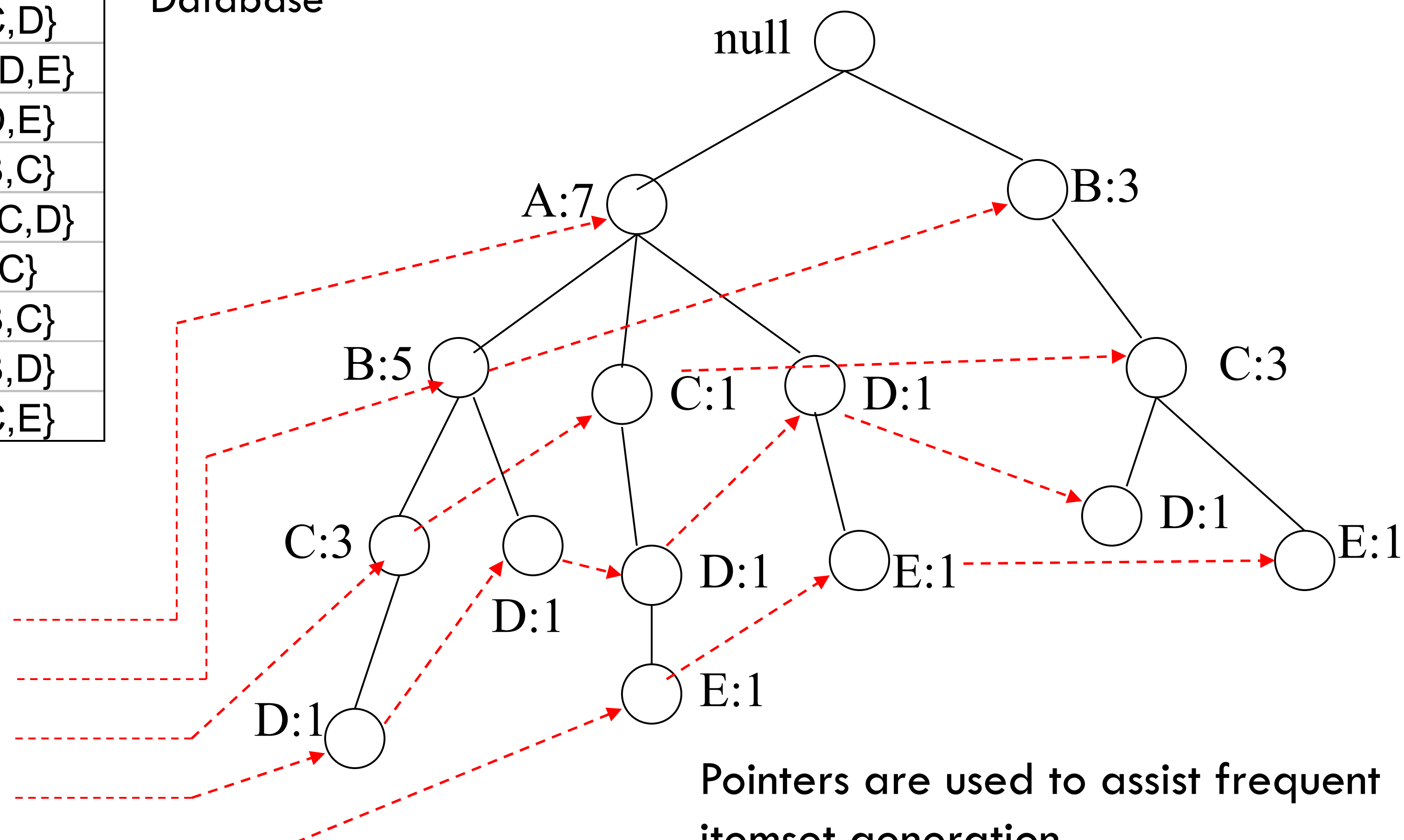
After reading TID=2:



FP-Tree Construction

TID	Items
1	{A,B}
2	{B,C,D}
3	{A,C,D,E}
4	{A,D,E}
5	{A,B,C}
6	{A,B,C,D}
7	{B,C}
8	{A,B,C}
9	{A,B,D}
10	{B,C,E}

Transaction Database



Pointers are used to assist frequent itemset generation

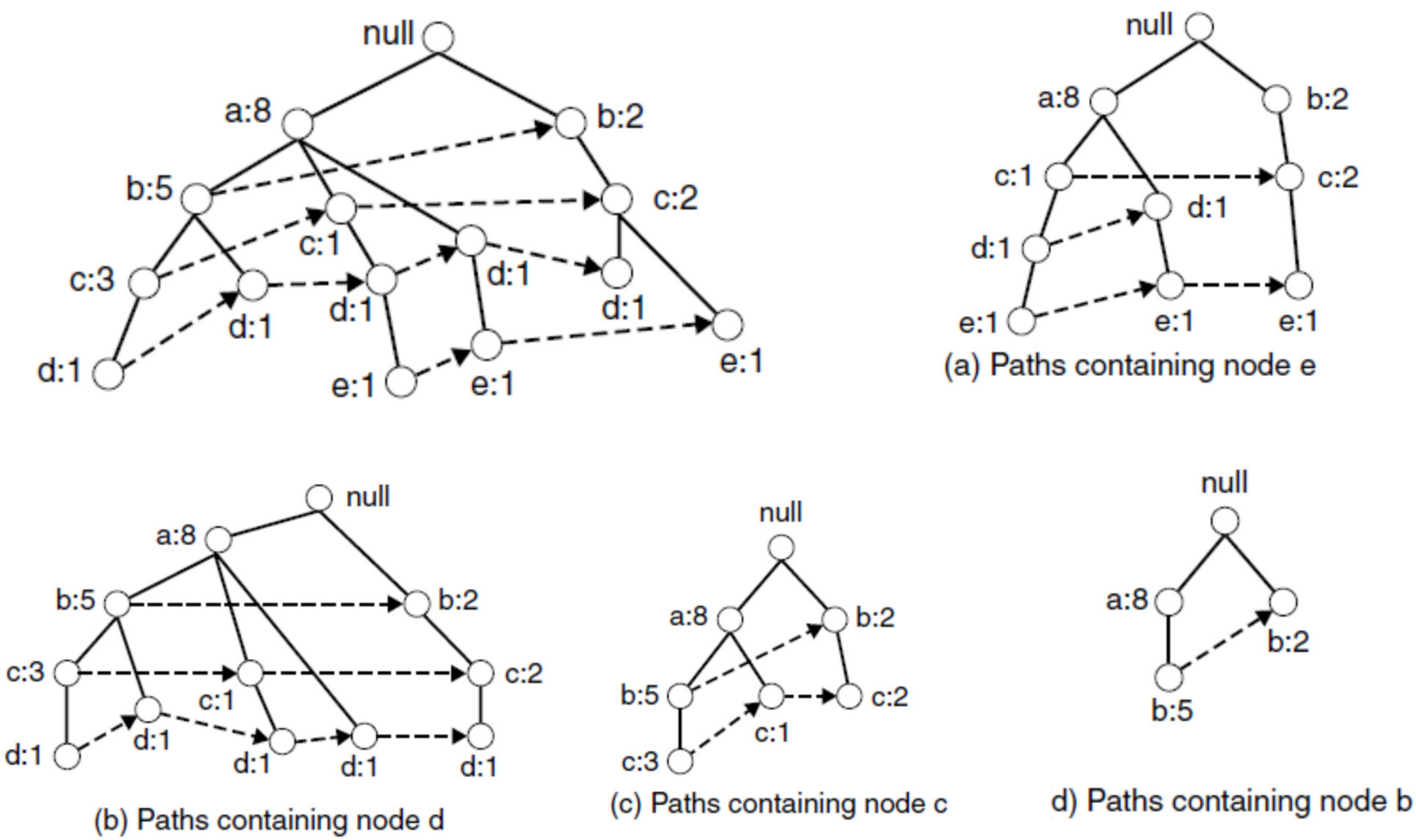
FP-Growth Algorithm

- ✓ FP-growth is an algorithm that generates frequent itemsets from an FP-tree by exploring the tree in a bottom-up fashion
- ✓ algorithm looks for frequent itemsets ending in e first, followed by d, c, b, and finally, a.

Table 6.6. The list of frequent itemsets ordered by their corresponding suffixes.

Suffix	Frequent Itemsets
e	{e}, {d,e}, {a,d,e}, {c,e},{a,e}
d	{d}, {c,d}, {b,c,d}, {a,c,d}, {b,d}, {a,b,d}, {a,d}
c	{c}, {b,c}, {a,b,c}, {a,c}
b	{b}, {a,b}
a	{a}

FP-Growth Algorithm



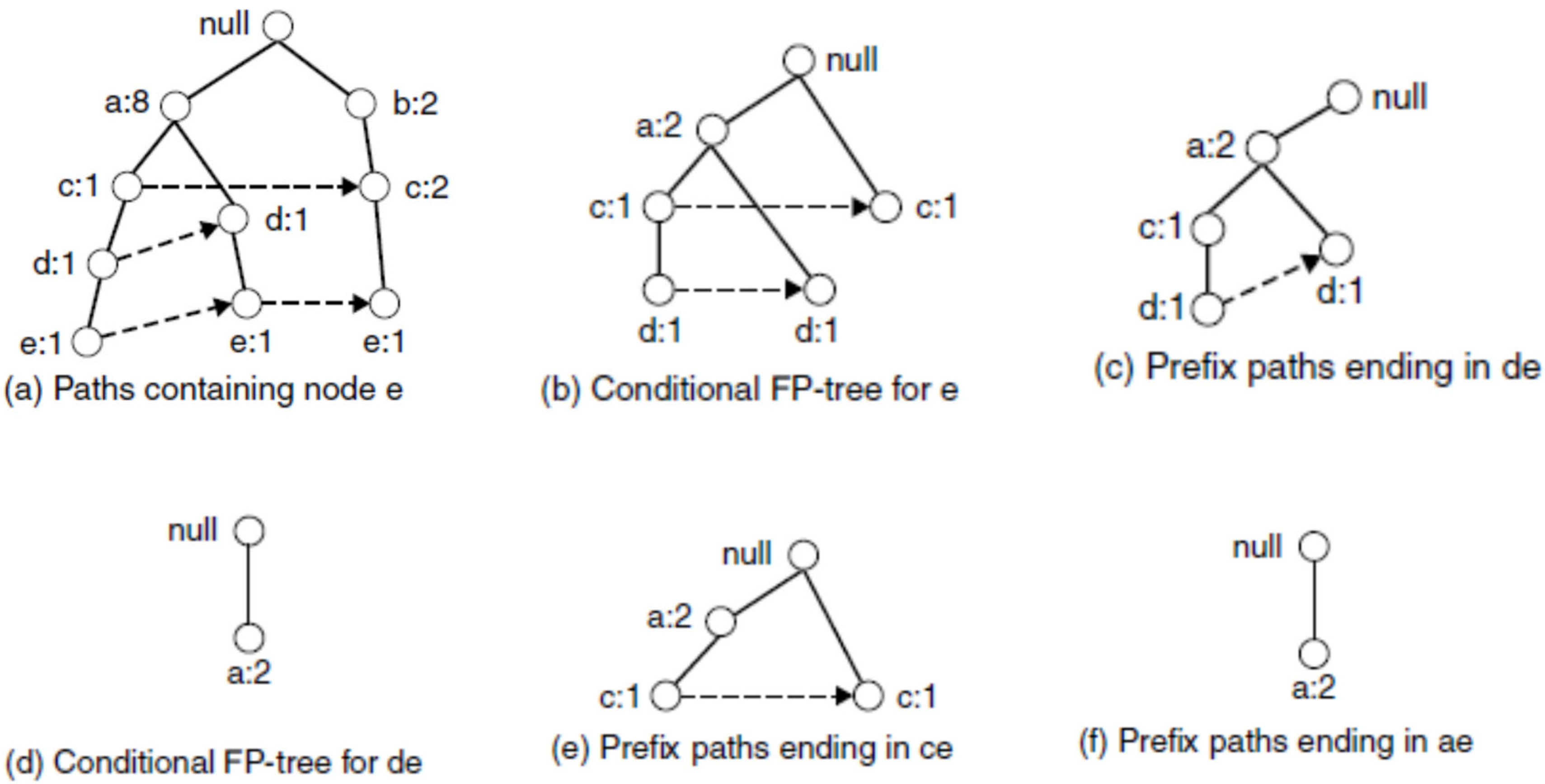
(a) Paths containing node e

(b) Paths containing node d

(c) Paths containing node c

d) Paths containing node b

FP-Growth Algorithm



(a) Paths containing node e

(b) Conditional FP-tree for e

(c) Prefix paths ending in de

(d) Conditional FP-tree for de

(e) Prefix paths ending in ce

(f) Prefix paths ending in ae

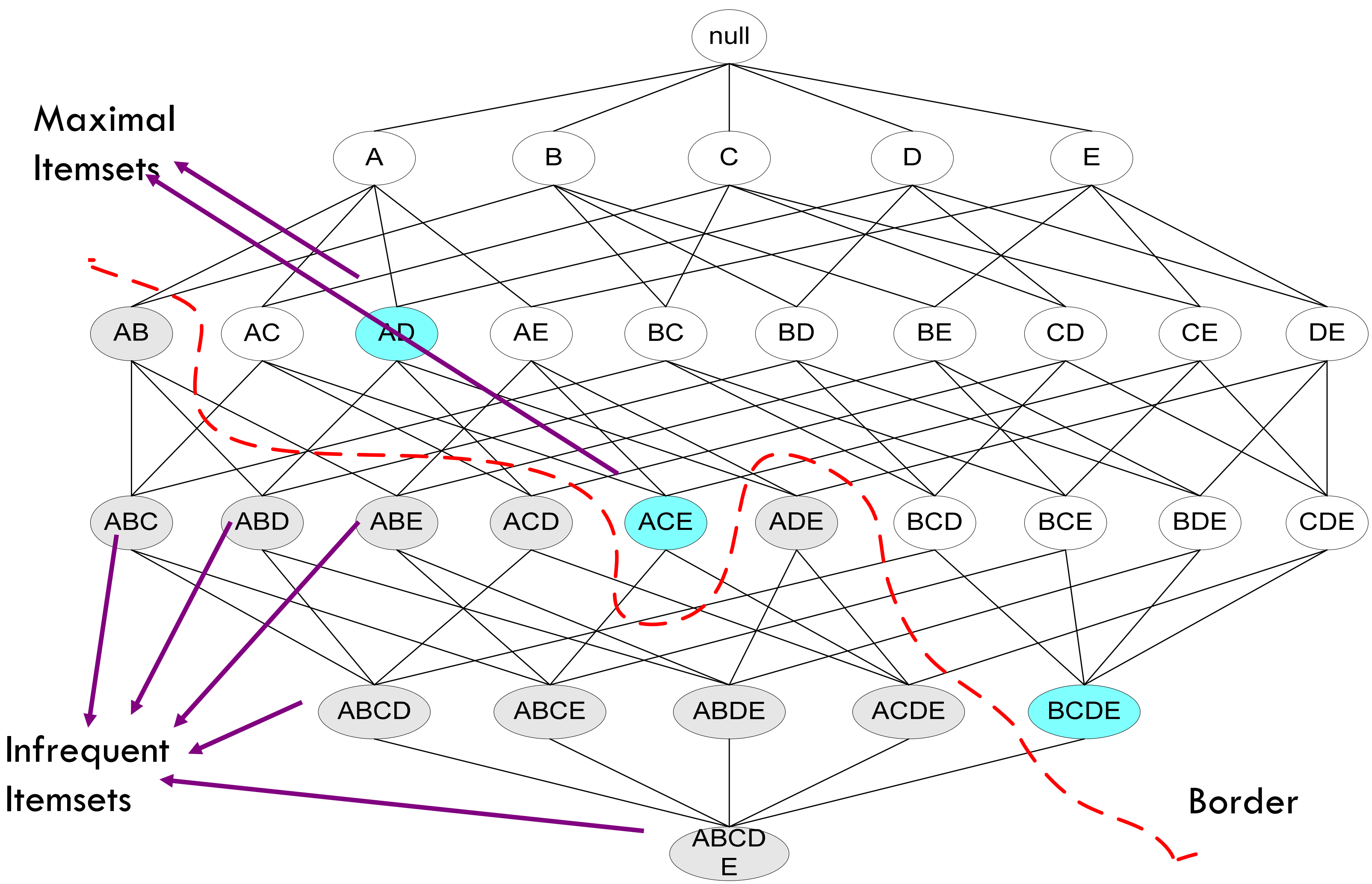
Compact Representation of frequent itemsets

Maximal Frequent Itemset

- number of frequent itemsets produced from a transaction data set can be very large
- identify a small representative set of itemsets from which all other frequent itemsets can be derived
- Two such representations
  - maximal frequent itemsets
  - closed frequent itemsets.

An itemset is maximal frequent if none of its immediate supersets is frequent

Maximal Frequent Itemset



Maximal Frequent Itemset

- ✓ Maximal frequent itemsets effectively provide a compact representation of frequent itemsets
- ✓ they form the smallest set of itemsets from which all frequent itemsets can be derived
- ✓ an efficient algorithm exists to explicitly find the maximal frequent itemsets without having to enumerate all their subsets
- ✓ Despite providing a compact representation, maximal frequent itemsets do not contain the support information of their subsets

a minimal representation of frequent itemsets that preserves the support information

Closed Itemset

- An itemset is closed if none of its immediate supersets has the same support as the itemset

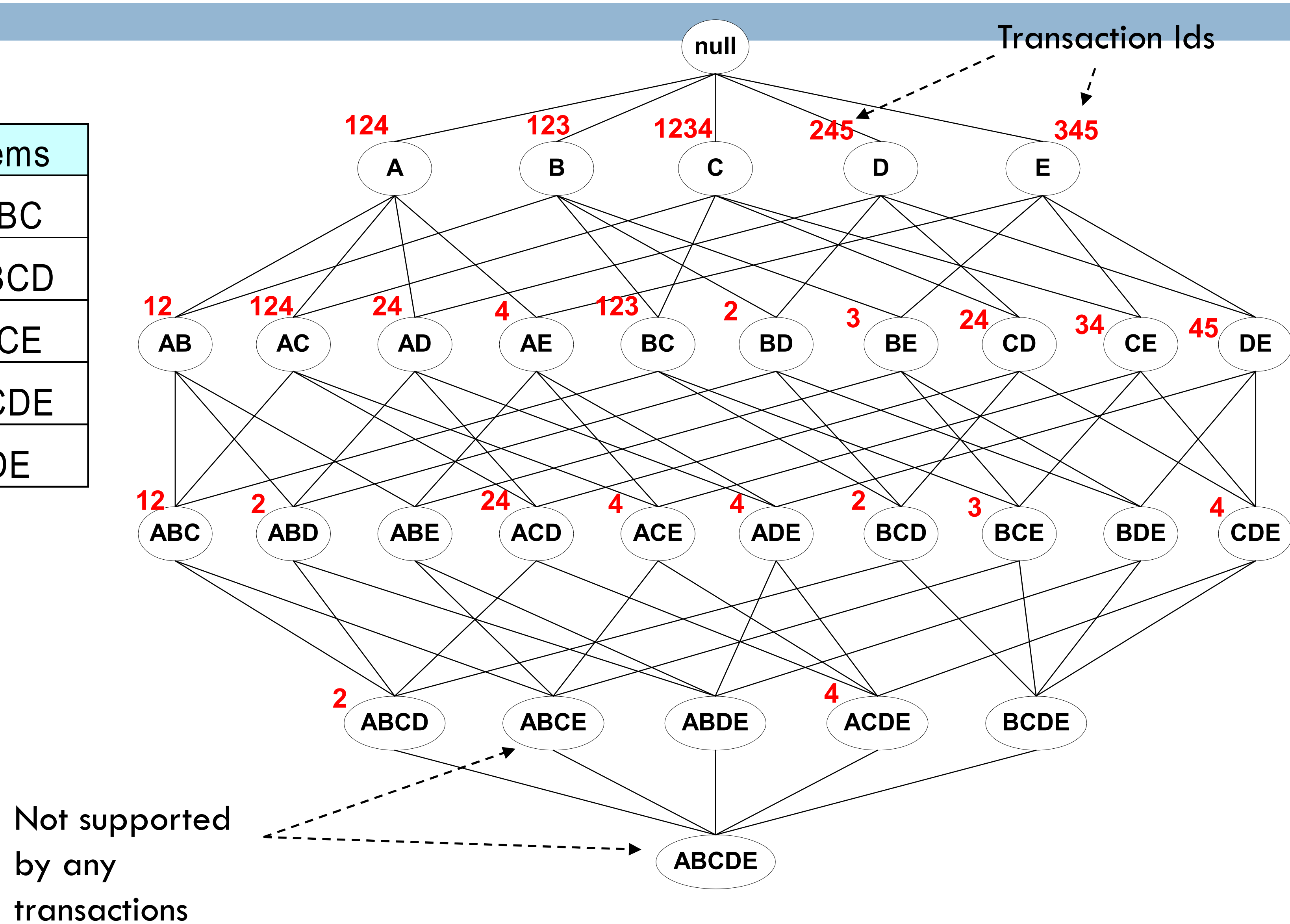
TID	Items
1	{A,B}
2	{B,C,D}
3	{A,B,C,D}
4	{A,B,D}
5	{A,B,C,D}

Itemset	Support
{A}	4
{B}	5
{C}	3
{D}	4
{A,B}	4
{A,C}	2
{A,D}	3
{B,C}	3
{B,D}	4
{C,D}	3

Itemset	Support
{A,B,C}	2
{A,B,D}	3
{A,C,D}	2
{B,C,D}	3
{A,B,C,D}	2

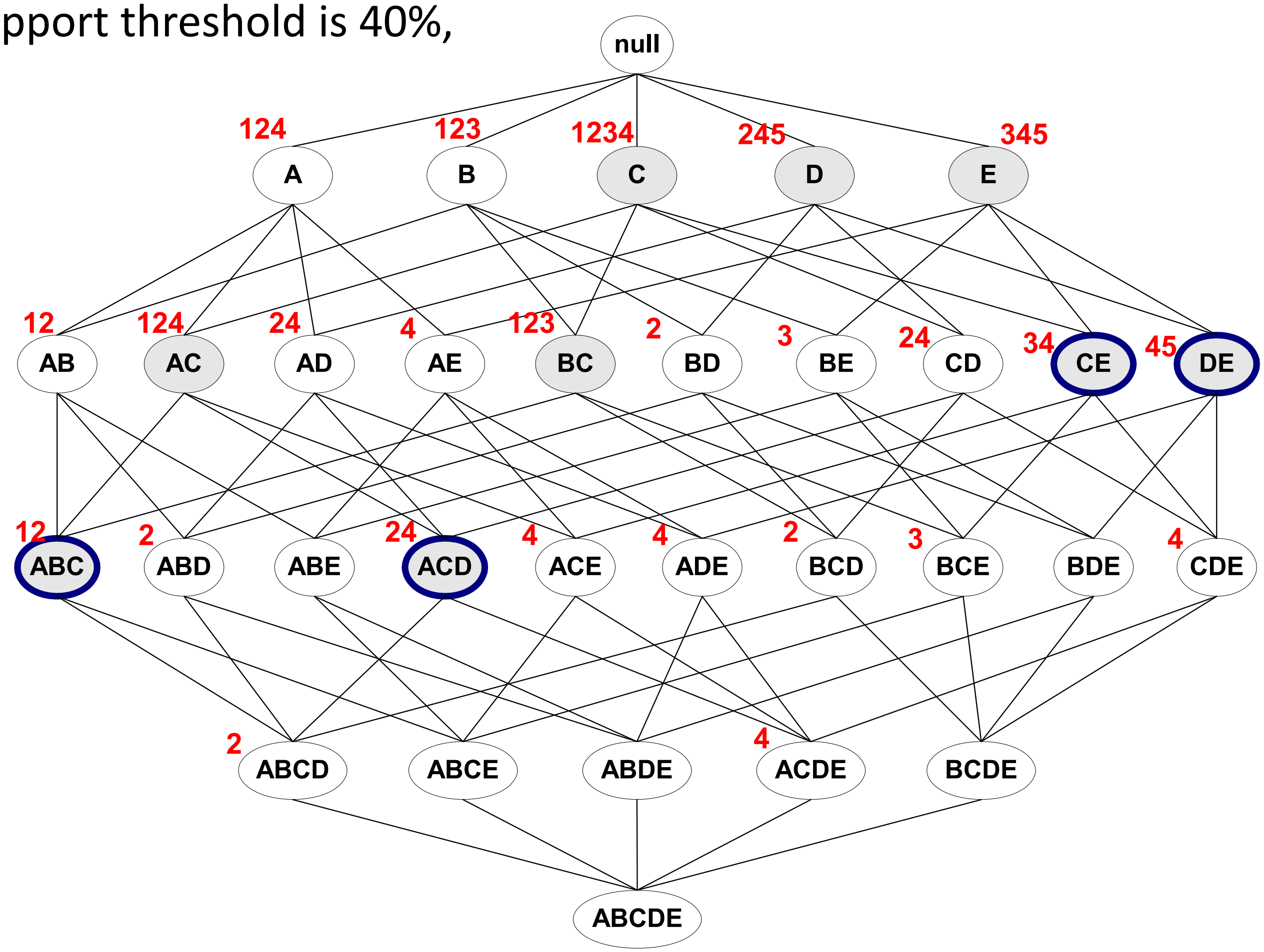
Closed Itemsets

TID	Items
1	ABC
2	ABCD
3	BCE
4	ACDE
5	DE

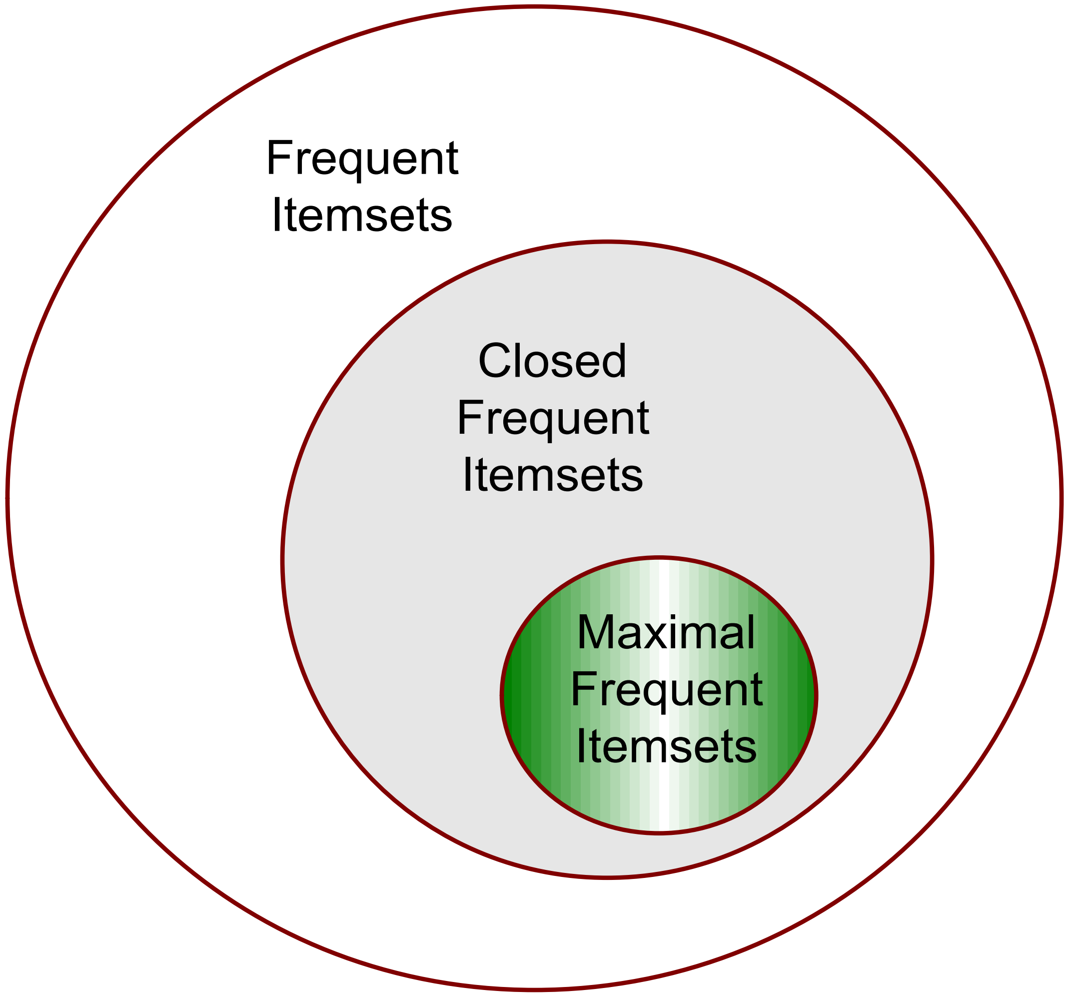


Frequent Closed Itemsets

assuming that the support threshold is 40%,



Maximal vs Closed Itemsets



Frequent Closed Itemsets

Use closed frequent itemsets to determine the support counts for the non-closed