? A baseball pitcher does work with his throwing arm to give the ball a property called kinetic energy, which depends on the ball's mass and speed. Which has the greatest kinetic energy? (i) A ball of mass 0.145 kg moving at 20.0 m/s; (ii) a smaller ball of mass 0.0145 kg moving at 200 m/s; (iii) a larger ball of mass 1.45 kg moving at 2.00 m/s; (iv) all three balls have the same kinetic energy; (v) it depends on the directions in which the balls move.

6 Work and Kinetic Energy

LEARNING OUTCOMES

In this chapter, you'll learn...

6.1 What it means for a force to do work on an object, and how to calculate the amount of work done.

6.2 The definition of the kinetic energy (energy of motion) of an object, and how the total work done on an object changes the object's kinetic energy.

6.3 How to use the relationship between total work and change in kinetic energy when the forces are not constant, the object follows a curved path, or both.

6.4 How to solve problems involving power (the rate of doing work).

You'll need to review...

1.10 The scalar product (or dot product) of two vectors.

2.4 Straight-line motion with constant acceleration.

4.3 Newton's second law.

4.5 Newton's third law.

5.1, 5.2 Using components to find the net force.

Suppose you try to find the speed of an arrow that has been shot from a bow. You apply Newton's laws and all the problem-solving techniques that we've learned, but you run across a major stumbling block: After the archer releases the arrow, the bow string exerts a *varying* force that depends on the arrow's position. As a result, the simple methods that we've learned aren't enough to calculate the speed. Never fear; we aren't by any means finished with mechanics, and there are other methods for dealing with such problems.

The new method that we're about to introduce uses the ideas of *work* and *energy*. The importance of the energy idea stems from the *principle of conservation of energy*: Energy is a quantity that can be converted from one form to another but cannot be created or destroyed. In an automobile engine, chemical energy stored in the fuel is converted partially to the energy of the automobile's motion and partially to thermal energy. In a microwave oven, electromagnetic energy obtained from your power company is converted to thermal energy of the food being cooked. In these and all other processes, the *total* energy—the sum of all energy present in all different forms—remains the same. No exception has ever been found.

We'll use the energy idea throughout the rest of this book to study a tremendous range of physical phenomena. This idea will help you understand how automotive engines work, how a camera's flash unit can produce a short burst of light, and the meaning of Einstein's famous equation $E = mc^2$.

In this chapter, though, our concentration will be on mechanics. We'll learn about one important form of energy called *kinetic energy,* or energy of motion, and how it relates to the concept of *work*. We'll also consider *power,* which is the time rate of doing work. In Chapter 7 we'll expand these ideas into a deeper understanding of the concepts of energy and the conservation of energy.

6.1 WORK

You'd probably agree that it's hard work to pull a heavy sofa across the room, to lift a stack of encyclopedias from the floor to a high shelf, or to push a stalled car off the road. Indeed, all of these examples agree with the everyday meaning of "work"—any activity that requires muscular or mental effort.

Figure 6.1 These people are doing work as they push on the car because they exert a force on the car as it moves.

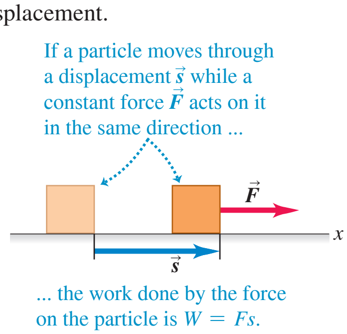
In physics, work has a much more precise definition. By making use of this definition we'll find that in any motion, no matter how complicated, the total work done on a particle by all forces that act on it equals the change in its *kinetic energy*—a quantity that's related to the particle's mass and speed. This relationship holds even when the forces acting on the particle aren't constant, a situation that can be difficult or impossible to handle with the techniques you learned in Chapters 4 and 5. The ideas of work and kinetic energy enable us to solve problems in mechanics that we could not have attempted before.

In this section we'll see how work is defined and how to calculate work in a variety of situations involving *constant* forces. Later in this chapter we'll relate work and kinetic energy, and then apply these ideas to problems in which the forces are *not* constant.

The three examples of work described above—pulling a sofa, lifting encyclopedias, and pushing a car—have something in common. In each case you do work by exerting a *force* on an object while that object *moves* from one place to another—that is, undergoes a *displacement* (**Fig. 6.1**). You do more work if the force is greater (you push harder on the car) or if the displacement is greater (you push the car farther down the road).

The physicist's definition of work is based on these observations. Consider an object that undergoes a displacement of magnitude s along a straight line. (For now, we'll assume that any object we discuss can be treated as a particle so that we can ignore any rotation or changes in shape of the object.) While the object moves, a constant force $\vec{F}$ acts on it in the same direction as the displacement $\vec{s}$ (**Fig. 6.2**). We define the **work** W done by this constant force under these circumstances as the product of the force magnitude F and the displacement magnitude s:

Figure 6.2 The work done by a constant force acting in the same direction as the displacement.



$$W = Fs \qquad \text{(constant force in direction of straight-line displacement)} \qquad (6.1)$$

The work done on the object is greater if either the force F or the displacement s is greater, in agreement with our observations above.

CAUTION **Work = *W*, weight = *w*** Don't confuse uppercase W (work) with lowercase w (weight). Though the symbols are similar, work and weight are different quantities.

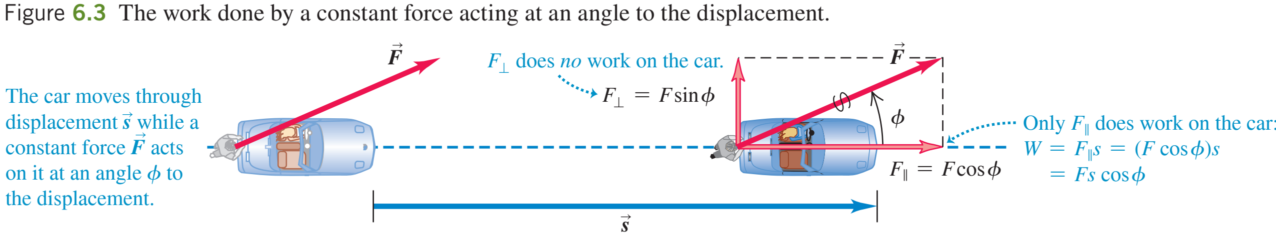
The SI unit of work is the **joule** (abbreviated J, pronounced "jool," and named in honor of the 19th-century English physicist James Prescott Joule). From Eq. (6.1) we see that in any system of units, the unit of work is the unit of force multiplied by the unit of distance. In SI units the unit of force is the newton and the unit of distance is the meter, so 1 joule is equivalent to 1 *newton-meter* $(\text{N} \cdot \text{m})$:

$$1 \text{ joule} = (1 \text{ newton})(1 \text{ meter}) \quad \text{or} \quad 1 \text{ J} = 1 \text{ N} \cdot \text{m}$$

If you lift an object with a weight of 1 N (about the weight of a medium-sized apple) a distance of 1 m at a constant speed, you exert a 1 N force on the object in the same direction as its 1 m displacement and so do 1 J of work on it.

As an illustration of Eq. (6.1), think of a person pushing a stalled car. If he pushes the car through a displacement $\vec{s}$ with a constant force $\vec{F}$ in the direction of motion, the amount of work he does on the car is given by Eq. (6.1): $W = Fs$. But what if the person pushes at an angle ϕ to the car's displacement (**Fig. 6.3**)? Then $\vec{F}$ has a component $F_{\parallel} = F\cos\phi$ in the direction of the displacement $\vec{s}$ and a component $F_{\perp} = F\sin\phi$ that acts perpendicular to $\vec{s}$. (Other forces must act on the car so that it moves along $\vec{s}$, not

Figure 6.3 The work done by a constant force acting at an angle to the displacement.



in the direction of $\vec{F}$. We're interested in only the work that the person does, however, so we'll consider only the force he exerts.) Only the parallel component $F_\parallel$ is effective in moving the car, so we define the work as the product of this force component and the magnitude of the displacement. Hence $W = F_\parallel s = (F\cos\phi)s$, or

Work done on a particle by **constant force** $\vec{F}$ during **straight-line displacement** $\vec{s}$

Magnitude of $\vec{F}$ · Angle between $\vec{F}$ and $\vec{s}$ · Magnitude of $\vec{s}$

$$W = Fs\cos\phi \tag{6.2}$$

If $\phi = 0$, so that $\vec{F}$ and $\vec{s}$ are in the same direction, then $\cos\phi = 1$ and we are back to Eq. (6.1).

Equation (6.2) has the form of the *scalar product* of two vectors, which we introduced in Section 1.10: $\vec{A}\cdot\vec{B} = AB\cos\phi$. You may want to review that definition. Hence we can write Equation (6.2) more compactly as

Work done on a particle by **constant force** $\vec{F}$ during **straight-line displacement** $\vec{s}$

$$W = \underbrace{\vec{F}\cdot\vec{s}}_{\text{Scalar product (dot product) of vectors } \vec{F} \text{ and } \vec{s}} \tag{6.3}$$

CAUTION **Work is a scalar** An essential point: Work is a *scalar* quantity, even though it's calculated from two vector quantities (force and displacement). A 5 N force toward the east acting on an object that moves 6 m to the east does the same amount of work as a 5 N force toward the north acting on an object that moves 6 m to the north.

BIO APPLICATION Work and Muscle Fibers Our ability to do work with our bodies comes from our skeletal muscles. The fiberlike cells of skeletal muscle, shown in this micrograph, can shorten, causing the muscle as a whole to contract and to exert force on the tendons to which it attaches. Muscle can exert a force of about 0.3 N per square millimeter of cross-sectional area: The greater the cross-sectional area, the more fibers the muscle has and the more force it can exert when it contracts.

EXAMPLE 6.1 Work done by a constant force

WITH VARIATION PROBLEMS

(a) Steve exerts a steady force of magnitude 210 N (about 47 lb) on the stalled car in Fig. 6.3 as he pushes it a distance of 18 m. The car also has a flat tire, so to make the car track straight Steve must push at an angle of 30° to the direction of motion. How much work does Steve do? (b) In a helpful mood, Steve pushes a second stalled car with a steady force $\vec{F} = (160\text{ N})\hat{\imath} - (40\text{ N})\hat{\jmath}$. The displacement of the car is $\vec{s} = (14\text{ m})\hat{\imath} + (11\text{ m})\hat{\jmath}$. How much work does Steve do in this case?

IDENTIFY and SET UP In both parts (a) and (b), the target variable is the work W done by Steve. In each case the force is constant and the displacement is along a straight line, so we can use Eq. (6.2) or (6.3). The angle between $\vec{F}$ and $\vec{s}$ is given in part (a), so we can apply Eq. (6.2) directly. In part (b) both $\vec{F}$ and $\vec{s}$ are given in terms of components, so it's best to calculate the scalar product by using Eq. (1.19): $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$.

EXECUTE (a) From Eq. (6.2),

$$W = Fs\cos\phi = (210\text{ N})(18\text{ m})\cos 30° = 3.3\times 10^3\text{ J}$$

(b) The components of $\vec{F}$ are $F_x = 160\text{ N}$ and $F_y = -40\text{ N}$, and the components of $\vec{s}$ are $x = 14\text{ m}$ and $y = 11\text{ m}$. (There are no z-components for either vector.) Hence, using Eqs. (1.19) and (6.3), we have

$$\begin{aligned} W &= \vec{F}\cdot\vec{s} = F_x x + F_y y \\ &= (160\text{ N})(14\text{ m}) + (-40\text{ N})(11\text{ m}) = 1.8\times 10^3\text{ J} \end{aligned}$$

EVALUATE In each case the work that Steve does is more than 1000 J. This shows that 1 joule is a rather small amount of work.

KEYCONCEPT To find the work W done by a constant force $\vec{F}$ acting on an object that undergoes a straight-line displacement $\vec{s}$, calculate the scalar product of these two vectors: $\boldsymbol{W = \vec{F}\cdot\vec{s}}$.

Work: Positive, Negative, or Zero

In Example 6.1 the work done in pushing the cars was positive. But it's important to understand that work can also be negative or zero. This is the essential way in which work as defined in physics differs from the "everyday" definition of work. When the force has a component in the *same direction* as the displacement (ϕ between 0° and 90°), $\cos\phi$ in Eq. (6.2) is positive and the work W is *positive* (**Fig. 6.4a**, next page). When the force has a component *opposite* to the displacement (ϕ between 90° and 180°), $\cos\phi$ is negative and the work is *negative* (Fig. 6.4b). When the force is *perpendicular* to the displacement, $\phi = 90°$ and the work done by the force is *zero* (Fig. 6.4c). The cases of zero work and negative work bear closer examination, so let's look at some examples.

Figure **6.4** A constant force $\vec{F}$ can do positive, negative, or zero work depending on the angle between $\vec{F}$ and the displacement $\vec{s}$.

Direction of Force (or Force Component)	Situation	Force Diagram
(a) **Force $\vec{F}$ has a component in direction of displacement:** $W = F_{\parallel}s = (F\cos\phi)s$ Work is *positive*.		
(b) **Force $\vec{F}$ has a component opposite to direction of displacement:** $W = F_{\parallel}s = (F\cos\phi)s$ Work is *negative* (because $F\cos\phi$ is negative for $90° < \phi < 180°$).		
(c) **Force $\vec{F}$ (or force component $F_{\perp}$) is perpendicular to direction of displacement:** The force (or force component) does *no* work on the object.		

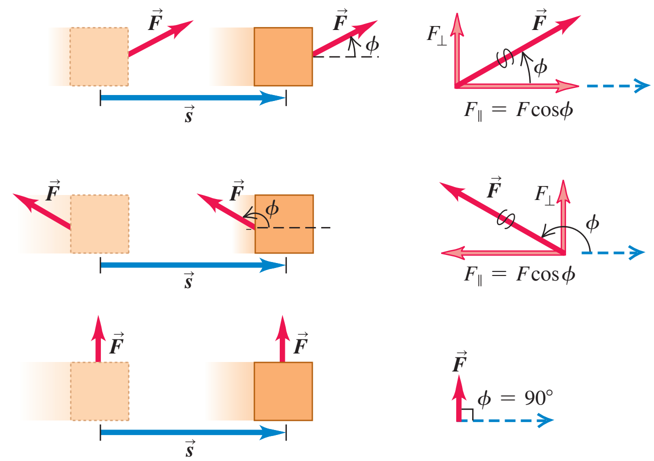
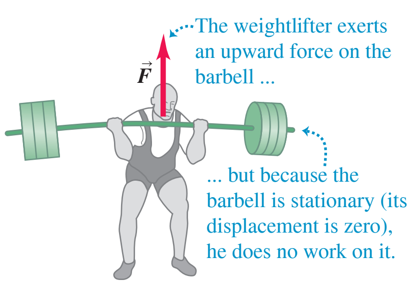


Figure **6.5** A weightlifter does no work on a barbell as long as he holds it stationary.



There are many situations in which forces act but do zero work. You might think it's "hard work" to hold a barbell motionless in the air for 5 minutes (**Fig. 6.5**). But in fact, you aren't doing any work on the barbell because there is no displacement. (Holding the barbell requires you to keep the muscles of your arms contracted, and this consumes energy stored in carbohydrates and fat within your body. As these energy stores are used up, your muscles feel fatigued even though you do no work on the barbell.) Even when you carry a book while you walk with constant velocity on a level floor, you do no work on the book. It has a displacement, but the (vertical) supporting force that you exert on the book has no component in the direction of the (horizontal) motion. Then $\phi = 90°$ in Eq. (6.2), and $\cos\phi = 0$. When an object slides along a surface, the work done on the object by the normal force is zero; and when a ball on a string moves in uniform circular motion, the work done on the ball by the tension in the string is also zero. In both cases the work is zero because the force has no component in the direction of motion.

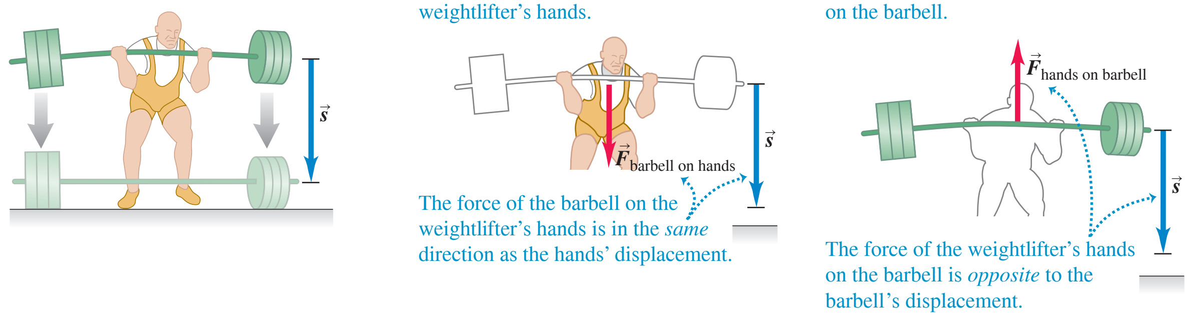
What does it mean to do *negative* work? The answer comes from Newton's third law of motion. When a weightlifter lowers a barbell as in **Fig. 6.6a**, his hands and the barbell move together with the same displacement $\vec{s}$. The barbell exerts a force $\vec{F}_{\text{barbell on hands}}$ on his hands in the same direction as the hands' displacement, so the work done by the *barbell* on his *hands* is positive (Fig. 6.6b). But by Newton's third law the weightlifter's hands exert an equal and opposite force $\vec{F}_{\text{hands on barbell}} = -\vec{F}_{\text{barbell on hands}}$ on the barbell (Fig. 6.6c). This force, which keeps the barbell from crashing to the floor, acts opposite to the barbell's displacement. Thus the work done by his *hands* on the *barbell* is negative.

Figure **6.6** This weightlifter's hands do negative work on a barbell as the barbell does positive work on his hands.

(a) A weightlifter lowers a barbell to the floor.

(b) The barbell does *positive* work on the weightlifter's hands.

(c) The weightlifter's hands do *negative* work on the barbell.



Because the weightlifter's hands and the barbell have the same displacement, the work that his hands do on the barbell is just the negative of the work that the barbell does on his hands. In general, when one object does negative work on a second object, the second object does an equal amount of *positive* work on the first object.

As a final note, you should review Fig. 6.4 to help remember when work is positive, when it is zero, and when it is negative.

CAUTION **Keep track of who's doing the work** We always speak of work done *on* a particular object *by* a specific force. Always specify exactly what force is doing the work. When you lift a book, you exert an upward force on it and the book's displacement is upward, so the work done by the lifting force on the book is positive. But the work done by the *gravitational* force (weight) on a book being lifted is *negative* because the downward gravitational force is opposite to the upward displacement.

Total Work

How do we calculate work when *several* forces act on an object? One way is to use Eq. (6.2) or (6.3) to compute the work done by each separate force. Then, because work is a scalar quantity, the *total* work W_{tot} done on the object by all the forces is the algebraic sum of the quantities of work done by the individual forces. An alternative way to find the total work W_{tot} is to compute the vector sum of the forces (that is, the net force) and then use this vector sum as $\vec{F}$ in Eq. (6.2) or (6.3). The following example illustrates both of these techniques.

EXAMPLE 6.2 Work done by several forces

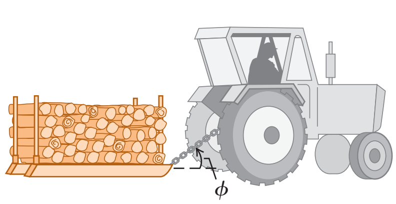
WITH VARIATION PROBLEMS

A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground (**Fig. 6.7a**). The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000 N force at an angle of 36.9° above the horizontal. A 3500 N friction force opposes the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces.

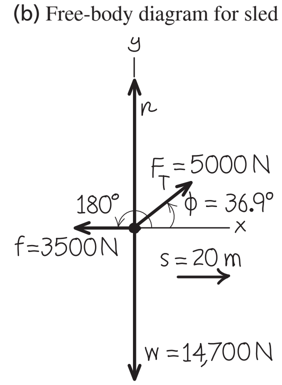
IDENTIFY and SET UP We'll find the total work in two ways: (1) by adding the work done on the sled by each force and (2) by finding the work done by the net force on the sled. We first draw a free-body diagram showing all of the forces acting on the sled, and we choose a coordinate system (Fig. 6.7b). Each of these forces—weight, normal force, force of the tractor, and friction force—is constant, the sled's displacement is along a straight line, and we know the angle between the displacement (in the positive x-direction) and each force. Hence we can use Eq. (6.2) to calculate the work each force does.

Figure 6.7 Calculating the work done on a sled of firewood being pulled by a tractor.

(a)



(b) Free-body diagram for sled



As in Chapter 5, we'll find the net force by adding the components of the four forces. Newton's second law tells us that because the sled's motion is purely horizontal, the net force can have only a horizontal component.

EXECUTE (1) The work W_w done by the weight is zero because its direction is perpendicular to the displacement (compare Fig. 6.4c). For the same reason, the work W_n done by the normal force is also zero. (Note that we don't need to calculate the magnitude n to conclude this.) So $W_w = W_n = 0$.

That leaves the work W_T done by the force F_T exerted by the tractor and the work W_f done by the friction force f. From Eq. (6.2),

$$W_T = F_T s \cos 36.9° = (5000 \text{ N})(20 \text{ m})(0.800) = 80{,}000 \text{ N}\cdot\text{m}$$
$$= 80 \text{ kJ}$$

The friction force $\vec{f}$ is opposite to the displacement, so for this force $\phi = 180°$ and $\cos\phi = -1$. Again from Eq. (6.2),

$$W_f = fs \cos 180° = (3500 \text{ N})(20 \text{ m})(-1) = -70{,}000 \text{ N}\cdot\text{m}$$
$$= -70 \text{ kJ}$$

The total work W_{tot} done on the sled by all forces is the *algebraic* sum of the work done by the individual forces:

$$W_{tot} = W_w + W_n + W_T + W_f = 0 + 0 + 80 \text{ kJ} + (-70 \text{ kJ})$$
$$= 10 \text{ kJ}$$

(2) In the second approach, we first find the *vector* sum of all the forces (the net force) and then use it to compute the total work. It's easiest to find the net force by using components. From Fig. 6.7b,

$$\sum F_x = F_T \cos\phi + (-f) = (5000 \text{ N})\cos 36.9° - 3500 \text{ N}$$
$$= 500 \text{ N}$$
$$\sum F_y = F_T \sin\phi + n + (-w)$$
$$= (5000 \text{ N})\sin 36.9° + n - 14{,}700 \text{ N}$$

Continued

We don't need the second equation; we know that the y-component of force is perpendicular to the displacement, so it does no work. Besides, there is no y-component of acceleration, so $\sum F_y$ must be zero anyway. The total work is therefore the work done by the total x-component:

$$W_{tot} = (\sum \vec{F}) \cdot \vec{s} = (\sum F_x)s = (500\ \text{N})(20\ \text{m}) = 10{,}000\ \text{J}$$
$$= 10\ \text{kJ}$$

EVALUATE We get the same result for W_{tot} with either method, as we should. Note that the net force in the x-direction is *not* zero, and so the sled must accelerate as it moves. In Section 6.2 we'll return to this example and see how to use the concept of work to explore the sled's changes of speed.

KEYCONCEPT To find the total work done on a moving object, calculate the sum of the amounts of work done by each force that acts on the object. The total work also equals the work done by the *net* force on the object.

TEST YOUR UNDERSTANDING OF SECTION 6.1 An electron moves in a straight line toward the east with a constant speed of 8×10^7 m/s. It has electric, magnetic, and gravitational forces acting on it. During a 1 m displacement, the total work done on the electron is (i) positive; (ii) negative; (iii) zero; (iv) not enough information is given.

ANSWER

(iii) The electron has constant velocity, so its acceleration is zero and (by Newton's second law) the net force on the electron is also zero. Therefore the total work done by all the forces (equal to the work done by the net force) must be zero as well. The individual forces may do nonzero work, but that's not what the question asks.

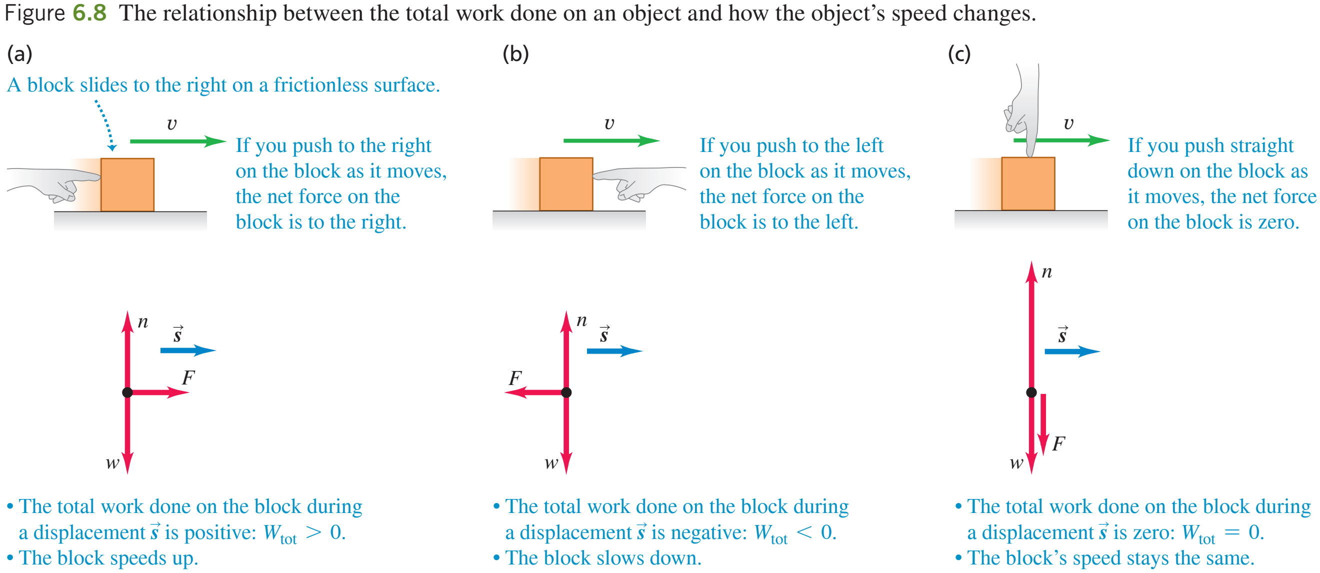
6.2 KINETIC ENERGY AND THE WORK–ENERGY THEOREM

The total work done on an object by external forces is related to the object's displacement—that is, to changes in its position. But the total work is also related to changes in the *speed* of the object. To see this, consider **Fig. 6.8**, which shows a block sliding on a frictionless table. The forces acting on the block are its weight $\vec{w}$, the normal force $\vec{n}$, and the force $\vec{F}$ exerted on it by the hand.

In Fig. 6.8a the net force on the block is in the direction of its motion. From Newton's second law, this means that the block speeds up; from Eq. (6.1), this also means that the total work W_{tot} done on the block is positive. The total work is *negative* in Fig. 6.8b because the net force opposes the displacement; in this case the block slows down. The net force is zero in Fig. 6.8c, so the speed of the block stays the same and the total work done on the block is zero. We can conclude that *when a particle undergoes a displacement, it speeds up if* $W_{tot} > 0$, *slows down if* $W_{tot} < 0$, *and maintains the same speed if* $W_{tot} = 0$.

Let's make this more quantitative. In **Fig. 6.9** (next page) a particle with mass m moves along the x-axis under the action of a constant net force with magnitude F that points in the positive x-direction. The particle's acceleration is constant and given by Newton's second law

Figure 6.8 The relationship between the total work done on an object and how the object's speed changes.



(Section 4.3): $F = ma_x$. As the particle moves from point x_1 to x_2, it undergoes a displacement $s = x_2 - x_1$ and its speed changes from v_1 to v_2. Using a constant-acceleration equation from Section 2.4, Eq. (2.13), and replacing v_{0x} by v_1, v_x by v_2, and $(x - x_0)$ by s, we have

$$v_2^2 = v_1^2 + 2a_x s$$

$$a_x = \frac{v_2^2 - v_1^2}{2s}$$

When we multiply this equation by m and equate ma_x to the net force F, we find

$$F = ma_x = m\frac{v_2^2 - v_1^2}{2s} \qquad \text{and}$$

$$Fs = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \tag{6.4}$$

In Eq. (6.4) the product Fs is the work done by the net force F and thus is equal to the total work W_{tot} done by all the forces acting on the particle. The quantity $\frac{1}{2}mv^2$ is called the **kinetic energy** K of the particle:

Kinetic energy of a particle: $K = \frac{1}{2}mv^2$ (6.5)
Mass of particle: m
Speed of particle: v

Like work, the kinetic energy of a particle is a scalar quantity; it depends on only the particle's mass and speed, not its direction of motion (**Fig. 6.10**). Kinetic energy can never be negative, and it is zero only when the particle is at rest. **?**

We can now interpret Eq. (6.4) in terms of work and kinetic energy. The first term on the right side of Eq. (6.4) is $K_2 = \frac{1}{2}mv_2^2$, the final kinetic energy of the particle (that is, after the displacement). The second term is the initial kinetic energy, $K_1 = \frac{1}{2}mv_1^2$, and the difference between these terms is the *change* in kinetic energy. So Eq. (6.4) says:

Work–energy theorem: Work done by the net force on a particle equals the change in the particle's kinetic energy.

$$\underbrace{W_{tot}}_{\substack{\text{Total work done on particle} \\ = \text{work done by net force}}} = \underbrace{K_2}_{\text{Final kinetic energy}} - \underbrace{K_1}_{\text{Initial kinetic energy}} = \underbrace{\Delta K}_{\text{Change in kinetic energy}} \tag{6.6}$$

This **work–energy theorem** agrees with our observations about the block in Fig. 6.8. When W_{tot} is *positive,* the kinetic energy *increases* (the final kinetic energy K_2 is greater than the initial kinetic energy K_1) and the particle is going faster at the end of the displacement than at the beginning. When W_{tot} is *negative,* the kinetic energy *decreases* (K_2 is less than K_1) and the speed is less after the displacement. When $W_{tot} = 0$, the kinetic energy stays the same $(K_1 = K_2)$ and the speed is unchanged. Note that the work–energy theorem by itself tells us only about changes in *speed,* not velocity, since the kinetic energy doesn't depend on the direction of motion.

From Eq. (6.4) or Eq. (6.6), kinetic energy and work must have the same units. Hence the joule is the SI unit of both work and kinetic energy (and, as we'll see later, of all kinds of energy). To verify this, note that in SI the quantity $K = \frac{1}{2}mv^2$ has units $\text{kg} \cdot (\text{m/s})^2$ or $\text{kg} \cdot \text{m}^2/\text{s}^2$; we recall that $1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2$, so

$$1\ \text{J} = 1\ \text{N} \cdot \text{m} = 1\ (\text{kg} \cdot \text{m/s}^2) \cdot \text{m} = 1\ \text{kg} \cdot \text{m}^2/\text{s}^2$$

Because we used Newton's laws in deriving the work–energy theorem, we can use this theorem only in an inertial frame of reference. Note that the work–energy theorem is valid in *any* inertial frame, but the values of W_{tot} and $K_2 - K_1$ may differ from one inertial frame to another (because the displacement and speed of an object may be different in different frames).

We've derived the work–energy theorem for the special case of straight-line motion with constant forces, and in the following examples we'll apply it to this special case only. We'll find in the next section that the theorem is valid even when the forces are not constant and the particle's trajectory is curved.

Figure 6.9 A constant net force $\vec{F}$ does work on a moving object.

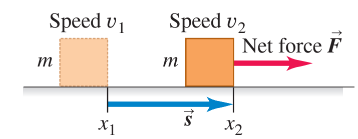
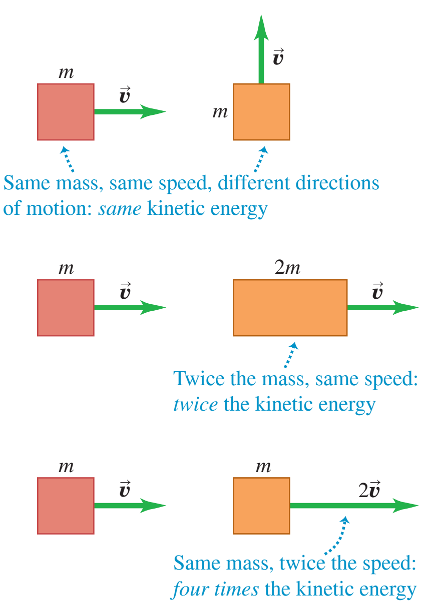


Figure 6.10 Comparing the kinetic energy $K = \frac{1}{2}mv^2$ of different objects.



PROBLEM-SOLVING STRATEGY 6.1 Work and Kinetic Energy

IDENTIFY *the relevant concepts:* The work–energy theorem, $W_{tot} = K_2 - K_1$, is extremely useful when you want to relate an object's speed v_1 at one point in its motion to its speed v_2 at a different point. (It's less useful for problems that involve the *time* it takes an object to go from point 1 to point 2 because the work–energy theorem doesn't involve time at all. For such problems it's usually best to use the relationships among time, position, velocity, and acceleration described in Chapters 2 and 3.)

SET UP *the problem* using the following steps:

1. Identify the initial and final positions of the object, and draw a free-body diagram showing all the forces that act on the object.
2. Choose a coordinate system. (If the motion is along a straight line, it's usually easiest to have both the initial and final positions lie along one of the axes.)
3. List the unknown and known quantities, and decide which unknowns are your target variables. The target variable may be the object's initial or final speed, the magnitude of one of the forces acting on the object, or the object's displacement.

EXECUTE *the solution:* Calculate the work W done by each force. If the force is constant and the displacement is a straight line, you can use Eq. (6.2) or Eq. (6.3). (Later in this chapter we'll see how to handle varying forces and curved trajectories.) Be sure to check signs; W must be positive if the force has a component in the direction of the displacement, negative if the force has a component opposite to the displacement, and zero if the force and displacement are perpendicular.

Add the amounts of work done by each force to find the total work W_{tot}. Sometimes it's easier to calculate the vector sum of the forces (the net force) and then find the work done by the net force; this value is also equal to W_{tot}.

Write expressions for the initial and final kinetic energies, K_1 and K_2. Note that kinetic energy involves *mass,* not *weight*; if you are given the object's weight, use $w = mg$ to find the mass.

Finally, use Eq. (6.6), $W_{tot} = K_2 - K_1$, and Eq. (6.5), $K = \frac{1}{2}mv^2$, to solve for the target variable. Remember that the right-hand side of Eq. (6.6) represents the change of the object's kinetic energy between points 1 and 2; that is, it is the *final* kinetic energy minus the *initial* kinetic energy, never the other way around. (If you can predict the sign of W_{tot}, you can predict whether the object speeds up or slows down.)

EVALUATE *your answer:* Check whether your answer makes sense. Remember that kinetic energy $K = \frac{1}{2}mv^2$ can never be negative. If you come up with a negative value of K, perhaps you interchanged the initial and final kinetic energies in $W_{tot} = K_2 - K_1$ or made a sign error in one of the work calculations.

EXAMPLE 6.3 Using work and energy to calculate speed

WITH VARIATION PROBLEMS

Let's look again at the sled in Fig. 6.7 and our results from Example 6.2. Suppose the sled's initial speed v_1 is 2.0 m/s. What is the speed of the sled after it moves 20 m?

IDENTIFY and SET UP We'll use the work–energy theorem, Eq. (6.6), $W_{tot} = K_2 - K_1$, since we are given the initial speed $v_1 = 2.0$ m/s and want to find the final speed v_2. **Figure 6.11** shows our sketch of the situation. The motion is in the positive x-direction. In Example 6.2 we calculated the total work done by all the forces: $W_{tot} = 10$ kJ. Hence the kinetic energy of the sled and its load must increase by 10 kJ, and the speed of the sled must also increase.

EXECUTE To write expressions for the initial and final kinetic energies, we need the mass of the sled and load. The combined *weight* is 14,700 N, so the mass is

$$m = \frac{w}{g} = \frac{14{,}700\ \text{N}}{9.8\ \text{m/s}^2} = 1500\ \text{kg}$$

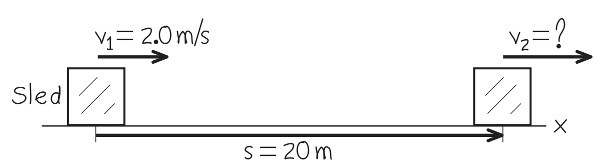
Then the initial kinetic energy K_1 is

$$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(1500\ \text{kg})(2.0\ \text{m/s})^2 = 3000\ \text{kg}\cdot\text{m}^2/\text{s}^2 = 3000\ \text{J}$$

The final kinetic energy K_2 is

$$K_2 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}(1500\ \text{kg})v_2^2$$

Figure 6.11 Our sketch for this problem.



The work–energy theorem, Eq. (6.6), gives

$$K_2 = K_1 + W_{tot} = 3000\ \text{J} + 10{,}000\ \text{J} = 13{,}000\ \text{J}$$

Setting these two expressions for K_2 equal, substituting $1\ \text{J} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$, and solving for the final speed v_2, we find

$$v_2 = 4.2\ \text{m/s}$$

EVALUATE The total work is positive, so the kinetic energy increases $(K_2 > K_1)$ and the speed increases $(v_2 > v_1)$.

This problem can also be solved without the work–energy theorem. We can find the acceleration from $\sum\vec{F} = m\vec{a}$ and then use the equations of motion for constant acceleration to find v_2. Since the acceleration is along the x-axis,

$$a = a_x = \frac{\sum F_x}{m} = \frac{500\ \text{N}}{1500\ \text{kg}} = 0.333\ \text{m/s}^2$$

Then, using Eq. (2.13), we have

$$v_2^2 = v_1^2 + 2as = (2.0\ \text{m/s})^2 + 2(0.333\ \text{m/s}^2)(20\ \text{m}) = 17.3\ \text{m}^2/\text{s}^2$$
$$v_2 = 4.2\ \text{m/s}$$

This is the same result we obtained with the work–energy approach, but there we avoided the intermediate step of finding the acceleration. You'll find several other examples in this chapter and the next that *can* be done without using energy considerations but that are easier when energy methods are used. When a problem can be done by two methods, doing it by both methods (as we did here) is a good way to check your work.

KEYCONCEPT You can use the work–energy theorem to easily relate the initial and final speeds of an object that moves while being acted on by constant forces.

EXAMPLE 6.4 Forces on a hammerhead

WITH VARIATION PROBLEMS

The 200 kg steel hammerhead of a pile driver is lifted 3.00 m above the top of a vertical I-beam being driven into the ground (**Fig. 6.12a**). The hammerhead is then dropped, driving the I-beam 7.4 cm deeper into the ground. The vertical guide rails exert a constant 60 N friction force on the hammerhead. Use the work–energy theorem to find (a) the speed of the hammerhead just as it hits the I-beam and (b) the average force the hammerhead exerts on the I-beam. Ignore the effects of the air.

IDENTIFY We'll use the work–energy theorem to relate the hammerhead's speed at different locations and the forces acting on it. There are *three* locations of interest: point 1, where the hammerhead starts from rest; point 2, where it first contacts the I-beam; and point 3, where the hammerhead and I-beam come to a halt (Fig. 6.12a). The two target variables are the hammerhead's speed at point 2 and the average force the hammerhead exerts between points 2 and 3. Hence we'll apply the work–energy theorem twice: once for the motion from 1 to 2, and once for the motion from 2 to 3.

SET UP Figure 6.12b shows the vertical forces on the hammerhead as it falls from point 1 to point 2. (We can ignore any horizontal forces that may be present because they do no work as the hammerhead moves vertically.) For this part of the motion, our target variable is the hammerhead's final speed v_2.

Figure 6.12c shows the vertical forces on the hammerhead during the motion from point 2 to point 3. In addition to the forces shown in Fig. 6.12b, the I-beam exerts an upward normal force of magnitude n on the hammerhead. This force actually varies as the hammerhead comes to a halt, but for simplicity we'll treat n as a constant. Hence n represents the *average* value of this upward force during the motion. Our target variable for this part of the motion is the force that the *hammerhead* exerts on the I-beam; it is the reaction force to the normal force exerted by the I-beam, so by Newton's third law its magnitude is also n.

EXECUTE (a) From point 1 to point 2, the vertical forces are the downward weight $w = mg = (200\text{ kg})(9.8\text{ m/s}^2) = 1960\text{ N}$ and the upward friction force $f = 60\text{ N}$. Thus the net downward force is $w - f = 1900\text{ N}$. The displacement of the hammerhead from point 1 to point 2 is downward and equal to $s_{12} = 3.00\text{ m}$. The total work done on the hammerhead between point 1 and point 2 is then

$$W_{\text{tot}} = (w - f)s_{12} = (1900\text{ N})(3.00\text{ m}) = 5700\text{ J}$$

At point 1 the hammerhead is at rest, so its initial kinetic energy K_1 is zero. Hence the kinetic energy K_2 at point 2 equals the total work done on the hammerhead between points 1 and 2:

$$W_{\text{tot}} = K_2 - K_1 = K_2 - 0 = \tfrac{1}{2}mv_2{}^2 - 0$$

$$v_2 = \sqrt{\frac{2W_{\text{tot}}}{m}} = \sqrt{\frac{2(5700\text{ J})}{200\text{ kg}}} = 7.55\text{ m/s}$$

This is the hammerhead's speed at point 2, just as it hits the I-beam.

(b) As the hammerhead moves downward from point 2 to point 3, its displacement is $s_{23} = 7.4\text{ cm} = 0.074\text{ m}$ and the net downward force acting on it is $w - f - n$ (Fig. 6.12c). The total work done on the hammerhead during this displacement is

$$W_{\text{tot}} = (w - f - n)s_{23}$$

The initial kinetic energy for this part of the motion is K_2, which from part (a) equals 5700 J. The final kinetic energy is $K_3 = 0$ (the hammerhead ends at rest). From the work–energy theorem,

$$W_{\text{tot}} = (w - f - n)s_{23} = K_3 - K_2$$

$$n = w - f - \frac{K_3 - K_2}{s_{23}}$$

$$= 1960\text{ N} - 60\text{ N} - \frac{0\text{ J} - 5700\text{ J}}{0.074\text{ m}} = 79{,}000\text{ N}$$

The downward force that the hammerhead exerts on the I-beam has this same magnitude, 79,000 N (about 9 tons)—more than 40 times the weight of the hammerhead.

EVALUATE The net change in the hammerhead's kinetic energy from point 1 to point 3 is zero; a relatively small net force does positive work over a large distance, and then a much larger net force does negative work over a much smaller distance. The same thing happens if you speed up your car gradually and then drive it into a brick wall. The very large force needed to reduce the kinetic energy to zero over a short distance is what does the damage to your car—and possibly to you.

KEYCONCEPT If you know the initial and final speeds of an object that moves over a given straight-line distance, the work–energy theorem lets you calculate the net force that causes the change in speed.

Figure 6.12 (a) A pile driver pounds an I-beam into the ground. (b), (c) Free-body diagrams. Vector lengths are not to scale.

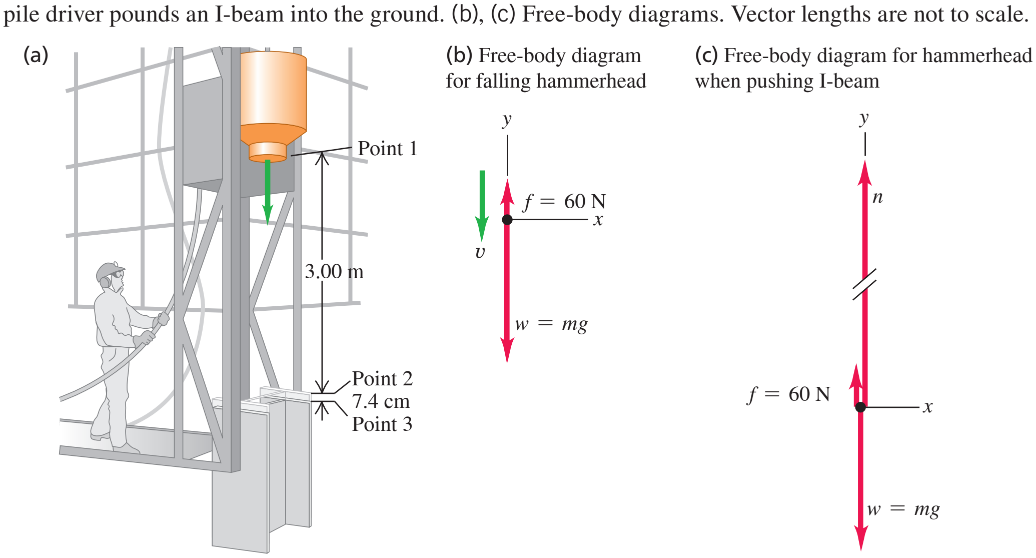
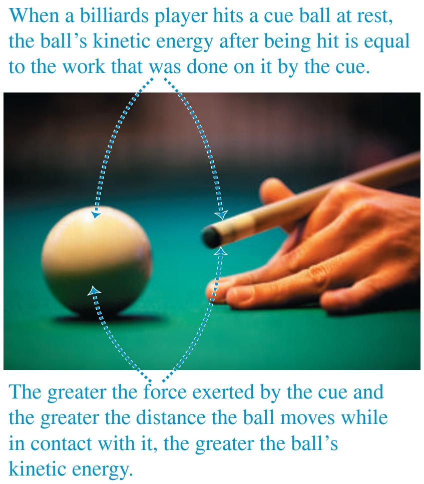


Figure **6.13** Imparting kinetic energy to a cue ball.



The Meaning of Kinetic Energy

Example 6.4 gives insight into the physical meaning of kinetic energy. The hammerhead is dropped from rest, and its kinetic energy when it hits the I-beam equals the total work done on it up to that point by the net force. This result is true in general: To accelerate a particle of mass m from rest (zero kinetic energy) up to a speed v, the total work done on it must equal the change in kinetic energy from zero to $K = \frac{1}{2}mv^2$:

$$W_{\text{tot}} = K - 0 = K$$

So *the kinetic energy of a particle is equal to the total work that was done to accelerate it from rest to its present speed* (**Fig. 6.13**). The definition $K = \frac{1}{2}mv^2$, Eq. (6.5), wasn't chosen at random; it's the *only* definition that agrees with this interpretation of kinetic energy.

In the second part of Example 6.4 the kinetic energy of the hammerhead did work on the I-beam and drove it into the ground. This gives us another interpretation of kinetic energy: *The kinetic energy of a particle is equal to the total work that particle can do in the process of being brought to rest.* This is why you pull your hand and arm backward when you catch a ball. As the ball comes to rest, it does an amount of work (force times distance) on your hand equal to the ball's initial kinetic energy. By pulling your hand back, you maximize the distance over which the force acts and so minimize the force on your hand.

CONCEPTUAL EXAMPLE 6.5 Comparing kinetic energies

Two iceboats like the one in Example 5.6 (Section 5.2) hold a race on a frictionless horizontal lake (**Fig. 6.14**). The two iceboats have masses m and $2m$. The iceboats have identical sails, so the wind exerts the same constant force $\vec{F}$ on each iceboat. They start from rest and cross the finish line a distance s away. Which iceboat crosses the finish line with greater kinetic energy?

SOLUTION If you use the definition of kinetic energy, $K = \frac{1}{2}mv^2$, Eq. (6.5), the answer to this problem isn't obvious. The iceboat of mass $2m$ has greater mass, so you might guess that it has greater kinetic energy at the finish line. But the lighter iceboat, of mass m, has greater acceleration and crosses the finish line with a greater speed, so you might guess that *this* iceboat has the greater kinetic energy. How can we decide?

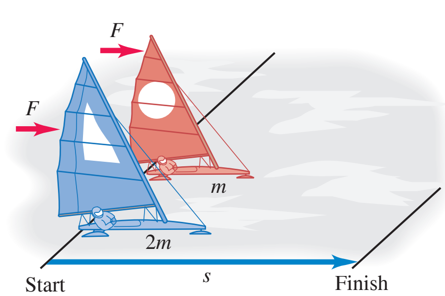
The key is to remember that *the kinetic energy of a particle is equal to the total work done to accelerate it from rest*. Both iceboats travel the same distance s from rest, and only the horizontal force F in the direction of motion does work on either iceboat. Hence the total work done between the starting line and the finish line is the *same* for each iceboat, $W_{\text{tot}} = Fs$. At the finish line, each iceboat has a kinetic energy equal to the work W_{tot} done on it, because each iceboat started from rest. So both iceboats have the *same* kinetic energy at the finish line!

You might think this is a "trick" question, but it isn't. If you really understand the meanings of quantities such as kinetic energy, you can solve problems more easily and with better insight.

Notice that we didn't need to know anything about how much time each iceboat took to reach the finish line. This is because the work–energy theorem makes no direct reference to time, only to displacement. In fact the iceboat of mass m has greater acceleration and so takes less time to reach the finish line than does the iceboat of mass $2m$.

KEYCONCEPT The kinetic energy of an object with speed v equals the amount of work you must do to accelerate it from rest to speed v.

Figure **6.14** A race between iceboats.



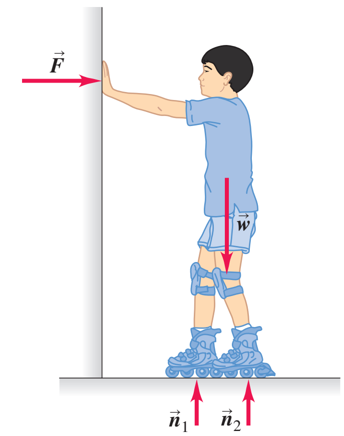
Work and Kinetic Energy in Composite Systems

In this section we've been careful to apply the work–energy theorem only to objects that we can represent as *particles*—that is, as moving point masses. New subtleties appear for more complex systems that have to be represented as many particles with different motions. We can't go into these subtleties in detail in this chapter, but here's an example.

Suppose a boy stands on frictionless roller skates on a level surface, facing a rigid wall (**Fig. 6.15**). He pushes against the wall, which makes him move to the right. The forces acting on him are his weight $\vec{w}$, the upward normal forces $\vec{n}_1$ and $\vec{n}_2$ exerted by the ground on his skates, and the horizontal force $\vec{F}$ exerted on him by the wall. There is no vertical displacement, so $\vec{w}$, $\vec{n}_1$, and $\vec{n}_2$ do no work. Force $\vec{F}$ accelerates him to the right, but the parts of his body where that force is applied (the boy's hands) do not move while the force acts. Thus the force $\vec{F}$ also does no work. Where, then, does the boy's kinetic energy come from?

The explanation is that it's not adequate to represent the boy as a single point mass. Different parts of the boy's body have different motions; his hands remain stationary against the wall while his torso is moving away from the wall. The various parts of his body interact with each other, and one part can exert forces and do work on another part. Therefore the *total* kinetic energy of this *composite* system of body parts can change, even though no work is done by forces applied by objects (such as the wall) that are outside the system. In Chapter 8 we'll consider further the motion of a collection of particles that interact with each other. We'll discover that just as for the boy in this example, the total kinetic energy of such a system can change even when no work is done on any part of the system by anything outside it.

Figure **6.15** The external forces acting on a skater pushing off a wall. The work done by these forces is zero, but the skater's kinetic energy changes nonetheless.



TEST YOUR UNDERSTANDING OF SECTION 6.2 Rank the following objects in order of their kinetic energy, from least to greatest. (i) A 2.0 kg object moving at 5.0 m/s; (ii) a 1.0 kg object that initially was at rest and then had 30 J of work done on it; (iii) a 1.0 kg object that initially was moving at 4.0 m/s and then had 20 J of work done on it; (iv) a 2.0 kg object that initially was moving at 10 m/s and then did 80 J of work on another object.

ANSWER

(iv), (i), (iii), (ii) Object (i) has kinetic energy $K = \frac{1}{2}mv^2 = \frac{1}{2}(2.0\text{ kg})(5.0\text{ m/s})^2 = 25\text{ J}$. Object (ii) had zero kinetic energy initially and then had 30 J of work done on it, so its final kinetic energy is $K_2 = K_1 + W = 0 + 30\text{ J} = 30\text{ J}$. Object (iii) had initial kinetic energy $K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(1.0\text{ kg})(4.0\text{ m/s})^2 = 8.0\text{ J}$ and then had 20 J of work done on it, so its final kinetic energy is $K_2 = K_1 + W = 8.0\text{ J} + 20\text{ J} = 28\text{ J}$. Object (iv) had initial kinetic energy $K_1 = \frac{1}{2}mv_1^2 = \frac{1}{2}(2.0\text{ kg})(10\text{ m/s})^2 = 100\text{ J}$; when it did 80 J of work on another object, the other object did -80 J of work on object (iv), so the final kinetic energy of object (iv) is $K_2 = K_1 + W = 100\text{ J} + (-80\text{ J}) = 20\text{ J}$.

6.3 WORK AND ENERGY WITH VARYING FORCES

So far we've considered work done by *constant forces* only. But what happens when you stretch a spring? The more you stretch it, the harder you have to pull, so the force you exert is *not* constant as the spring is stretched. We've also restricted our discussion to *straight-line* motion. There are many situations in which an object moves along a curved path and is acted on by a force that varies in magnitude, direction, or both. We need to be able to compute the work done by the force in these more general cases. Fortunately, the work–energy theorem holds true even when forces are varying and when the object's path is not straight.

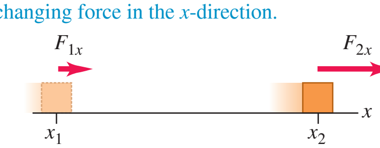
Work Done by a Varying Force, Straight-Line Motion

To add only one complication at a time, let's consider straight-line motion along the x-axis with a force whose x-component F_x may change as the object moves. (A real-life example is driving a car along a straight road with stop signs, so the driver has to alternately step on the gas and apply the brakes.) Suppose a particle moves along the x-axis from point x_1 to x_2 (**Fig. 6.16a**). Figure 6.16b is a graph of the x-component of force as a function of the particle's coordinate x. To find the work done by this force, we divide the total displacement into narrow segments Δx_a, Δx_b, and so on (Fig. 6.16c). We approximate the work done by the force during segment Δx_a as the average x-component of force F_{ax} in that segment multiplied by the x-displacement Δx_a. We do this for each segment and then add the results for all the segments. The work done by the force in the total displacement from x_1 to x_2 is approximately

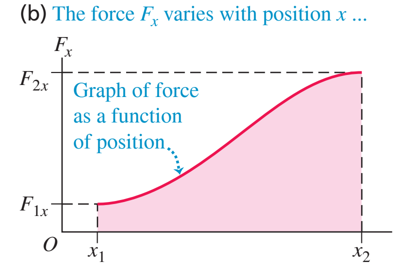
$$W = F_{ax}\Delta x_a + F_{bx}\Delta x_b + \cdots$$

Figure **6.16** Calculating the work done by a varying force F_x in the x-direction as a particle moves from x_1 to x_2.

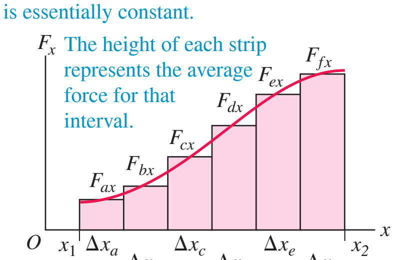
(a) A particle moves from x_1 to x_2 in response to a changing force in the x-direction.



(b) The force F_x varies with position x ...



(c) ... but over a short displacement Δx, the force is essentially constant.



In the limit that the number of segments becomes very large and the width of each becomes very small, this sum becomes the *integral* of F_x from x_1 to x_2:

Work done on a particle by a **varying x-component of force F_x** during **straight-line displacement** along x-axis

$$W = \int_{x_1}^{x_2} F_x\,dx \qquad (6.7)$$

Upper limit = final position
Integral of x-component of force
Lower limit = initial position

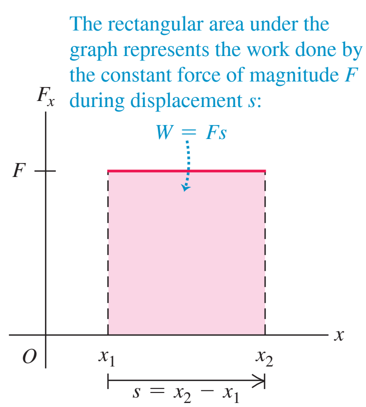
Note that $F_{ax}\,\Delta x_a$ represents the *area* of the first vertical strip in Fig. 6.16c and that the integral in Eq. (6.7) represents the area under the curve of Fig. 6.16b between x_1 and x_2. *On such a graph of force as a function of position, the total work done by the force is represented by the area under the curve between the initial and final positions.* Alternatively, the work W equals the average force that acts over the entire displacement, multiplied by the displacement.

In the special case that F_x, the x-component of the force, is constant, we can take it outside the integral in Eq. (6.7):

$$W = \int_{x_1}^{x_2} F_x\,dx = F_x \int_{x_1}^{x_2} dx = F_x(x_2 - x_1) \qquad \text{(constant force)}$$

But $x_2 - x_1 = s$, the total displacement of the particle. So in the case of a constant force F, Eq. (6.7) says that $W = Fs$, in agreement with Eq. (6.1). The interpretation of work as the area under the curve of F_x as a function of x also holds for a constant force: $W = Fs$ is the area of a rectangle of height F and width s (**Fig. 6.17**).

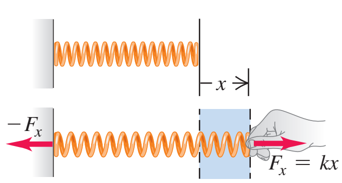
Figure **6.17** The work done by a constant force F in the x-direction as a particle moves from x_1 to x_2.



Now let's apply these ideas to the stretched spring. To keep a spring stretched beyond its unstretched length by an amount x, we have to apply a force of equal magnitude at each end (**Fig. 6.18**). If the elongation x is not too great, the force we apply to the right-hand end has an x-component directly proportional to x:

$$F_x = kx \qquad \text{(force required to stretch a spring)} \qquad (6.8)$$

Figure **6.18** The force needed to stretch an ideal spring is proportional to the spring's elongation: $F_x = kx$.

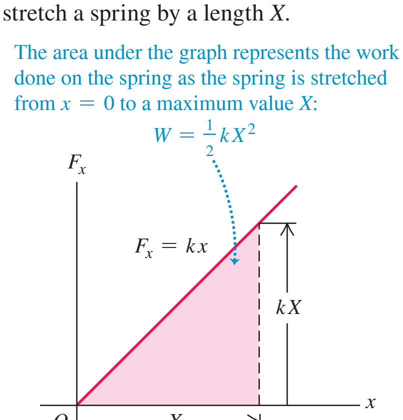


where k is a constant called the **force constant** (or spring constant) of the spring. The units of k are force divided by distance: N/m in SI units. A floppy toy spring such as a Slinky™ has a force constant of about 1 N/m; for the much stiffer springs in an automobile's suspension, k is about 10^5 N/m. The observation that force is directly proportional to elongation for elongations that are not too great was made by Robert Hooke in 1678 and is known as **Hooke's law.** It really shouldn't be called a "law," since it's a statement about a specific device and not a fundamental law of nature. Real springs don't always obey Eq. (6.8) precisely, but it's still a useful idealized model. We'll discuss Hooke's law more fully in Chapter 11.

To stretch a spring, we must do work. We apply equal and opposite forces to the ends of the spring and gradually increase the forces. We hold the left end stationary, so the force we apply at this end does no work. The force at the moving end *does* do work. **Figure 6.19** is a graph of F_x as a function of x, the elongation of the spring. The work done by this force when the elongation goes from zero to a maximum value X is

$$W = \int_0^X F_x\,dx = \int_0^X kx\,dx = \tfrac{1}{2}kX^2 \qquad (6.9)$$

Figure **6.19** Calculating the work done to stretch a spring by a length X.



We can also obtain this result graphically. The area of the shaded triangle in Fig. 6.19, representing the total work done by the force, is equal to half the product of the base and altitude, or

$$W = \tfrac{1}{2}(X)(kX) = \tfrac{1}{2}kX^2$$

This equation also says that the work is the *average* force $kX/2$ multiplied by the total displacement X. We see that the total work is proportional to the *square* of the final elongation X. To stretch an ideal spring by 2 cm, you must do four times as much work as is needed to stretch it by 1 cm.

Equation (6.9) assumes that the spring was originally unstretched. If initially the spring is already stretched a distance x_1, the work we must do to stretch it to a greater elongation x_2 (**Fig. 6.20a**) is

$$W = \int_{x_1}^{x_2} F_x\,dx = \int_{x_1}^{x_2} kx\,dx = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2 \qquad (6.10)$$

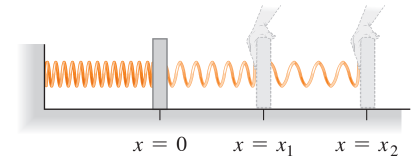
Use your knowledge of geometry to convince yourself that the trapezoidal area under the graph in Fig. 6.20b is given by the expression in Eq. (6.10).

If the spring has spaces between the coils when it is unstretched, then it can also be compressed, and Hooke's law holds for compression as well as stretching. In this case the force and displacement are in the opposite directions from those shown in Fig. 6.18, so both F_x and x in Eq. (6.8) are negative. Since both F_x and x are reversed, the force again is in the same direction as the displacement, and the work done by F_x is again positive. So the total work is still given by Eq. (6.9) or (6.10), even when X is negative or either or both of x_1 and x_2 are negative.

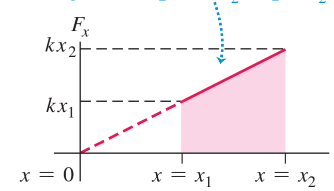
CAUTION **Work done *on* a spring vs. work done *by* a spring** Equation (6.10) gives the work that *you* must do *on* a spring to change its length. If you stretch a spring that's originally relaxed, then $x_1 = 0$, $x_2 > 0$, and $W > 0$: The force you apply to one end of the spring is in the same direction as the displacement, and the work you do is positive. By contrast, the work that the *spring* does on whatever it's attached to is given by the *negative* of Eq. (6.10). Thus, as you pull on the spring, the spring does negative work on you.

Figure **6.20** Calculating the work done to stretch a spring from one elongation to a greater one.

(a) Stretching a spring from elongation x_1 to elongation x_2



(b) Force-versus-distance graph

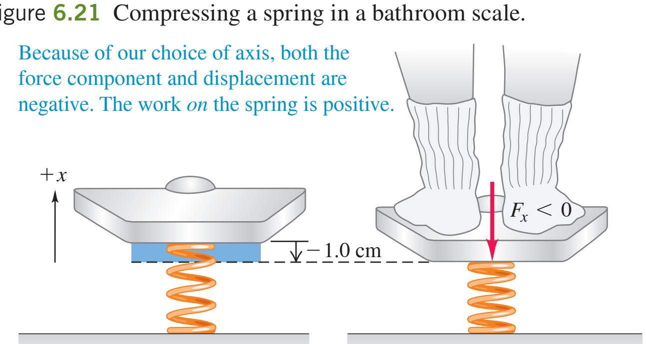


EXAMPLE 6.6 Work done on a spring scale

A woman weighing 600 N steps on a bathroom scale that contains a stiff spring (**Fig. 6.21**). In equilibrium, the spring is compressed 1.0 cm under her weight. Find the force constant of the spring and the total work done on it during the compression.

IDENTIFY and SET UP In equilibrium the upward force exerted by the spring balances the downward force of the woman's weight. We'll use this principle and Eq. (6.8) to determine the force constant k, and we'll use Eq. (6.10) to calculate the work W that the woman does on the spring to compress it. We take positive values of x to correspond to elongation (upward in Fig. 6.21), so that both the displacement of the end of the spring (x) and the x-component of the force that the woman exerts on it (F_x) are negative. The applied force and the displacement are in the same direction, so the work done on the spring will be positive.

Figure **6.21** Compressing a spring in a bathroom scale.



EXECUTE The top of the spring is displaced by $x = -1.0\text{ cm} = -0.010\text{ m}$, and the woman exerts a force $F_x = -600\text{ N}$ on the spring. From Eq. (6.8) the force constant is then

$$k = \frac{F_x}{x} = \frac{-600\text{ N}}{-0.010\text{ m}} = 6.0 \times 10^4\text{ N/m}$$

Then, using $x_1 = 0$ and $x_2 = -0.010\text{ m}$ in Eq. (6.10), we have

$$W = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$$
$$= \tfrac{1}{2}(6.0 \times 10^4\text{ N/m})(-0.010\text{ m})^2 - 0 = 3.0\text{ J}$$

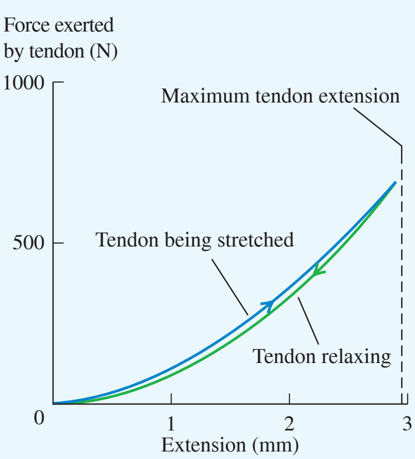
EVALUATE The work done is positive, as expected. Our arbitrary choice of the positive direction has no effect on the answer for W. You can test this by taking the positive x-direction to be downward, corresponding to compression. Do you get the same values for k and W as we found here?

KEYCONCEPT You must use Eq. (6.10) to calculate the work done by the nonconstant force that a spring exerts.

Work–Energy Theorem for Straight-Line Motion, Varying Forces

In Section 6.2 we derived the work–energy theorem, $W_{tot} = K_2 - K_1$, for the special case of straight-line motion with a constant net force. We can now prove that this

BIO APPLICATION Tendons Are Nonideal Springs Muscles exert forces via the tendons that attach them to bones. A tendon consists of long, stiff, elastic collagen fibers. The graph shows how the tendon from the hind leg of a wallaby (a small kangaroo-like marsupial) stretches in response to an applied force. The tendon does not exhibit the simple, straight-line behavior of an ideal spring, so the work it does has to be found by integration [Eq. (6.7)]. The tendon exerts less force while relaxing than while stretching. As a result, the relaxing tendon does only about 93% of the work that was done to stretch it.



theorem is true even when the force varies with position. As in Section 6.2, let's consider a particle that undergoes a displacement x while being acted on by a net force with x-component F_x, which we now allow to vary. Just as in Fig. 6.16, we divide the total displacement x into a large number of small segments Δx. We can apply the work–energy theorem, Eq. (6.6), to each segment because the value of F_x in each small segment is approximately constant. The change in kinetic energy in segment Δx_a is equal to the work $F_{ax}\Delta x_a$, and so on. The total change of kinetic energy is the sum of the changes in the individual segments, and thus is equal to the total work done on the particle during the entire displacement. So $W_{\text{tot}} = \Delta K$ holds for varying forces as well as for constant ones.

Here's an alternative derivation of the work–energy theorem for a force that may vary with position. It involves making a change of variable from x to v_x in the work integral. Note first that the acceleration a of the particle can be expressed in various ways, using $a_x = dv_x/dt$, $v_x = dx/dt$, and the chain rule for derivatives:

$$a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = v_x\frac{dv_x}{dx} \tag{6.11}$$

From this result, Eq. (6.7) tells us that the total work done by the *net* force F_x is

$$W_{\text{tot}} = \int_{x_1}^{x_2} F_x\,dx = \int_{x_1}^{x_2} ma_x\,dx = \int_{x_1}^{x_2} mv_x\frac{dv_x}{dx}\,dx \tag{6.12}$$

Now $(dv_x/dx)dx$ is the change in velocity dv_x during the displacement dx, so we can make that substitution in Eq. (6.12). This changes the integration variable from x to v_x, so we change the limits from x_1 and x_2 to the corresponding x-velocities v_1 and v_2:

$$W_{\text{tot}} = \int_{v_1}^{v_2} mv_x\,dv_x$$

The integral of $v_x\,dv_x$ is just $v_x^2/2$. Substituting the upper and lower limits, we finally find

$$W_{\text{tot}} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \tag{6.13}$$

This is the same as Eq. (6.6), so the work–energy theorem is valid even without the assumption that the net force is constant.

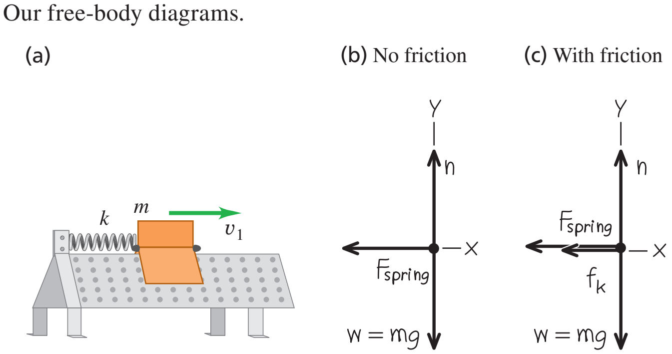
EXAMPLE 6.7 Motion with a varying force

WITH VARIATION PROBLEMS

An air-track glider of mass 0.100 kg is attached to the end of a horizontal air track by a spring with force constant 20.0 N/m (**Fig. 6.22a**). Initially the spring is unstretched and the glider is moving at 1.50 m/s to the right. Find the maximum distance d that the glider moves to the right (a) if the air track is turned on, so that there is no friction, and (b) if the air is turned off, so that there is kinetic friction with coefficient $\mu_k = 0.47$.

IDENTIFY and SET UP The force exerted by the spring is not constant, so we *cannot* use the constant-acceleration formulas of Chapter 2 to solve this problem. Instead, we'll use the work–energy theorem, since the total work done involves the distance moved (our target variable). In Figs. 6.22b and 6.22c we choose the positive x-direction to be to the right (in the direction of the glider's motion). We take $x = 0$ at the glider's initial position (where the spring is unstretched) and $x = d$ (the target variable) at the position where the glider stops. The motion

Figure **6.22** (a) A glider attached to an air track by a spring. (b), (c) Our free-body diagrams.



is purely horizontal, so only the horizontal forces do work. Note that Eq. (6.10) gives the work done by the *glider* on the *spring* as it stretches; to use the work–energy theorem we need the work done by the *spring* on the *glider*, which is the negative of Eq. (6.10). We expect the glider to move farther without friction than with friction.

EXECUTE (a) Equation (6.10) says that as the glider moves from $x_1 = 0$ to $x_2 = d$, it does an amount of work $W = \frac{1}{2}kd^2 - \frac{1}{2}k(0)^2 = \frac{1}{2}kd^2$ on the spring. The amount of work that the *spring* does on the *glider* is the negative of this, $-\frac{1}{2}kd^2$. The spring stretches until the glider comes instantaneously to rest, so the final kinetic energy K_2 is zero. The initial kinetic energy is $\frac{1}{2}mv_1^2$, where $v_1 = 1.50\text{ m/s}$ is the glider's initial speed. From the work–energy theorem,

$$-\tfrac{1}{2}kd^2 = 0 - \tfrac{1}{2}mv_1^2$$

We solve for the distance d the glider moves:

$$d = v_1\sqrt{\frac{m}{k}} = (1.50\text{ m/s})\sqrt{\frac{0.100\text{ kg}}{20.0\text{ N/m}}} = 0.106\text{ m} = 10.6\text{ cm}$$

The stretched spring subsequently pulls the glider back to the left, so the glider is at rest only instantaneously.

(b) If the air is turned off, we must include the work done by the kinetic friction force. The normal force n is equal in magnitude to the weight of the glider, since the track is horizontal and there are no other vertical forces. Hence the kinetic friction force has constant magnitude $f_k = \mu_k n = \mu_k mg$. The friction force is directed opposite to the displacement, so the work done by friction is

$$W_{\text{fric}} = f_k d\cos 180° = -f_k d = -\mu_k mgd$$

The total work is the sum of W_{fric} and the work done by the spring, $-\frac{1}{2}kd^2$. The work–energy theorem then says that

$$-\mu_k mgd - \tfrac{1}{2}kd^2 = 0 - \tfrac{1}{2}mv_1^2 \qquad \text{or}$$

$$\tfrac{1}{2}kd^2 + \mu_k mgd - \tfrac{1}{2}mv_1^2 = 0$$

This is a quadratic equation for d. The solutions are

$$d = -\frac{\mu_k mg}{k} \pm \sqrt{\left(\frac{\mu_k mg}{k}\right)^2 + \frac{mv_1^2}{k}}$$

We have

$$\frac{\mu_k mg}{k} = \frac{(0.47)(0.100\text{ kg})(9.80\text{ m/s}^2)}{20.0\text{ N/m}} = 0.02303\text{ m}$$

$$\frac{mv_1^2}{k} = \frac{(0.100\text{ kg})(1.50\text{ m/s})^2}{20.0\text{ N/m}} = 0.01125\text{ m}^2$$

so

$$d = -(0.02303\text{ m}) \pm \sqrt{(0.02303\text{ m})^2 + 0.01125\text{ m}^2}$$
$$= 0.086\text{ m} \quad \text{or} \quad -0.132\text{ m}$$

The quantity d is a positive displacement, so only the positive value of d makes sense. Thus with friction the glider moves a distance $d = 0.086\text{ m} = 8.6\text{ cm}$.

EVALUATE If we set $\mu_k = 0$, our algebraic solution for d in part (b) reduces to $d = v_1\sqrt{m/k}$, the zero-friction result from part (a). With friction, the glider goes a shorter distance. Again the glider stops instantaneously, and again the spring force pulls it toward the left; whether it moves or not depends on how great the *static* friction force is. How large would the coefficient of static friction μ_s have to be to keep the glider from springing back to the left?

KEYCONCEPT The work–energy theorem also allows you to solve problems with *varying* forces, such as the force exerted by a spring.

Work–Energy Theorem for Motion Along a Curve

We can generalize our definition of work further to include a force that varies in direction as well as magnitude, and a displacement that lies along a curved path. **Figure 6.23a** shows a particle moving from P_1 to P_2 along a curve. We divide the curve between these points into many infinitesimal vector displacements, and we call a typical one of these $d\vec{l}$. Each $d\vec{l}$ is tangent to the path at its position. Let $\vec{F}$ be the force at a typical point along the path, and let ϕ be the angle between $\vec{F}$ and $d\vec{l}$ at this point. Then the small element of work dW done on the particle during the displacement $d\vec{l}$ may be written as

$$dW = \vec{F}\cdot d\vec{l} = F\cos\phi\, dl = F_{\parallel}\, dl$$

where $F_{\parallel} = F\cos\phi$ is the component of $\vec{F}$ in the direction parallel to $d\vec{l}$ (Fig. 6.23b). The work done by $\vec{F}$ on the particle as it moves from P_1 to P_2 is

Upper limit = final position

Scalar product (dot product) of $\vec{F}$ and displacement $d\vec{l}$

$$W = \int_{P_1}^{P_2} \vec{F}\cdot d\vec{l} = \int_{P_1}^{P_2} F\cos\phi\, dl = \int_{P_1}^{P_2} F_{\parallel}\, dl \qquad (6.14)$$

Work done on a particle by a **varying force** $\vec{F}$ along a **curved path**

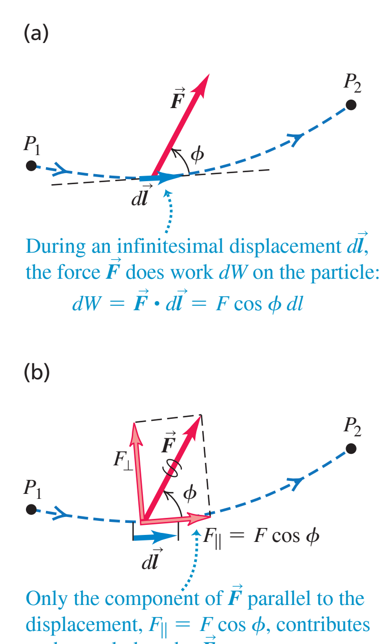
Lower limit = initial position

Angle between $\vec{F}$ and $d\vec{l}$

Component of $\vec{F}$ parallel to $d\vec{l}$

The integral in Eq. (6.14) (shown in three versions) is called a *line integral*. We'll see shortly how to evaluate an integral of this kind.

Figure **6.23** A particle moves along a curved path from point P_1 to P_2, acted on by a force $\vec{F}$ that varies in magnitude and direction.



We can now show that the work–energy theorem, Eq. (6.6), holds true even with varying forces and a displacement along a curved path. The force $\vec{F}$ is essentially constant over any given infinitesimal segment $d\vec{l}$ of the path, so we can apply the work–energy theorem for straight-line motion to that segment. Thus the change in the particle's kinetic energy K over that segment equals the work $dW = F_{\parallel}\,dl = \vec{F}\cdot d\vec{l}$ done on the particle. Adding up these infinitesimal quantities of work from all the segments along the whole path gives the total work done, Eq. (6.14), which equals the total change in kinetic energy over the whole path. So $W_{tot} = \Delta K = K_2 - K_1$ is true *in general,* no matter what the path and no matter what the character of the forces. This can be proved more rigorously by using steps like those in Eq. (6.11) through (6.13).

Note that only the component of the net force parallel to the path, $F_{\parallel}$, does work on the particle, so only this component can change the speed and kinetic energy of the particle. The component perpendicular to the path, $F_{\perp} = F\sin\phi$, has no effect on the particle's speed; it acts only to change the particle's direction.

To evaluate the line integral in Eq. (6.14) in a specific problem, we need some sort of detailed description of the path and of the way in which $\vec{F}$ varies along the path. We usually express the line integral in terms of some scalar variable, as in the following example.

EXAMPLE 6.8 Motion on a curved path

WITH VARIATION PROBLEMS

At a family picnic you are appointed to push your obnoxious cousin Throckmorton in a swing (**Fig. 6.24a**). His weight is w, the length of the chains is R, and you push Throcky until the chains make an angle θ_0 with the vertical. To do this, you exert a varying horizontal force $\vec{F}$ that starts at zero and gradually increases just enough that Throcky and the swing move very slowly and remain very nearly in equilibrium throughout the process. (a) What is the total work done on Throcky by all forces? (b) What is the work done by the tension T in the chains? (c) What is the work you do by exerting force $\vec{F}$? (Ignore the weight of the chains and seat.)

IDENTIFY and SET UP The motion is along a curve, so we'll use Eq. (6.14) to calculate the work done by the net force, by the tension force, and by the force $\vec{F}$. Figure 6.24b shows our free-body diagram and coordinate system for some arbitrary point in Throcky's motion. We have replaced the sum of the tensions in the two chains with a single tension T.

EXECUTE (a) There are two ways to find the total work done during the motion: (1) by calculating the work done by each force and then adding those quantities, and (2) by calculating the work done by the net force. The second approach is far easier here because Throcky is nearly in equilibrium at every point. Hence the net force on him is zero, the integral of the net force in Eq. (6.14) is zero, and the total work done on him is zero.

(b) It's also easy to find the work done by the chain tension T because this force is perpendicular to the direction of motion at all points along the path. Hence at all points the angle between the chain tension and the displacement vector $d\vec{l}$ is 90° and the scalar product in Eq. (6.14) is zero. Thus the chain tension does zero work.

(c) To compute the work done by $\vec{F}$, we need to calculate the line integral in Eq. (6.14). Inside the integral is the quantity $F\cos\phi\,dl$; let's see how to express each term in this quantity.

Figure 6.24a shows that the angle between $\vec{F}$ and $d\vec{l}$ is θ, so we replace ϕ in Eq. (6.14) with θ. The value of θ changes as Throcky moves.

To find the magnitude F of force $\vec{F}$, note that the net force on Throcky is zero (he is nearly in equilibrium at all points), so $\sum F_x = 0$ and $\sum F_y = 0$. From Fig. 6.24b,

$$\sum F_x = F + (-T\sin\theta) = 0 \qquad \sum F_y = T\cos\theta + (-w) = 0$$

If you eliminate T from these two equations, you can show that $F = w\tan\theta$. As the angle θ increases, the tangent increases and F increases (you have to push harder).

To find the magnitude dl of the infinitesimal displacement $d\vec{l}$, note that Throcky moves through a circular arc of radius R (Fig. 6.24a). The arc length s equals the radius R multiplied by the length θ (in radians): $s = R\theta$. Therefore the displacement $d\vec{l}$ corresponding to a small change of angle $d\theta$ has a magnitude $dl = ds = R\,d\theta$.

When we put all the pieces together, the integral in Eq. (6.14) becomes

$$W = \int_{P_1}^{P_2} F\cos\phi\,dl = \int_0^{\theta_0} (w\tan\theta)\cos\theta(R\,d\theta) = \int_0^{\theta_0} wR\sin\theta\,d\theta$$

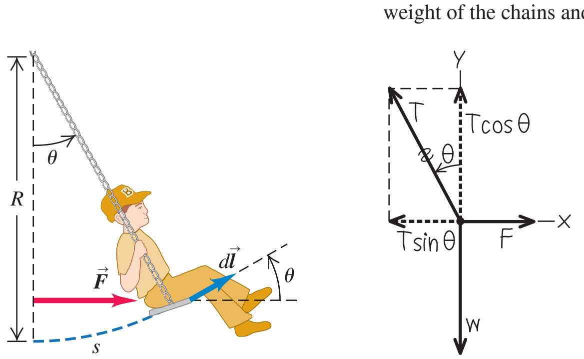
(Recall that $\tan\theta = \sin\theta/\cos\theta$, so $\tan\theta\cos\theta = \sin\theta$.) We've converted the *line* integral into an *ordinary* integral in terms of the angle θ. The limits of integration are from the starting position at $\theta = 0$ to the final position at $\theta = \theta_0$. The final result is

$$W = wR\int_0^{\theta_0}\sin\theta\,d\theta = -wR\cos\theta\Big|_0^{\theta_0} = -wR(\cos\theta_0 - 1)$$
$$= wR(1 - \cos\theta_0)$$

Figure 6.24 (a) Pushing cousin Throckmorton in a swing. (b) Our free-body diagram.

(a)

(b) Free-body diagram for Throckmorton (neglecting the weight of the chains and seat)



EVALUATE If $\theta_0 = 0$, there is no displacement; then $\cos\theta_0 = 1$ and $W = 0$, as we should expect. As θ_0 increases, $\cos\theta_0$ decreases and $W = wR(1 - \cos\theta_0)$ increases. So the farther along the arc you push Throcky, the more work you do. You can confirm that the quantity $R(1 - \cos\theta_0)$ is equal to h, the increase in Throcky's height during the displacement. So the work that you do to raise Throcky is just equal to his weight multiplied by the height that you raise him.

We can check our results by calculating the work done by the force of gravity $\vec{w}$. From part (a) the total work done on Throcky is zero, and from part (b) the work done by tension is zero. So gravity must do a negative amount of work that just balances the positive work done by the force $\vec{F}$ that we calculated in part (c).

For variety, let's calculate the work done by gravity by using the form of Eq. (6.14) that involves the quantity $\vec{F} \cdot d\vec{l}$, and express the force $\vec{w}$ and displacement $d\vec{l}$ in terms of their x- and y-components. The force of gravity has zero x-component and a y-component of $-w$. Figure 6.24a shows that $d\vec{l}$ has a magnitude of ds, an x-component of $ds\cos\theta$, and a y-component of $ds\sin\theta$.

So

$$\vec{w} = \hat{\jmath}(-w)$$
$$d\vec{l} = \hat{\imath}(ds\cos\theta) + \hat{\jmath}(ds\sin\theta)$$

Use Eq. (1.19) to calculate the scalar product $\vec{w} \cdot d\vec{l}$:

$$\vec{w} \cdot d\vec{l} = (-w)(ds\sin\theta) = -w\sin\theta\, ds$$

Using $ds = R\,d\theta$, we find the work done by the force of gravity:

$$\int_{P_1}^{P_2} \vec{w} \cdot d\vec{l} = \int_0^{\theta_0} (-w\sin\theta)R\,d\theta = -wR\int_0^{\theta_0} \sin\theta\,d\theta = -wR(1 - \cos\theta_0)$$

The work done by gravity is indeed the negative of the work done by force $\vec{F}$ that we calculated in part (c). Gravity does negative work because the force pulls downward while Throcky moves upward.

As we saw earlier, $R(1 - \cos\theta_0)$ is equal to h, the increase in Throcky's height during the displacement. So the work done by gravity along the curved path is $-mgh$, the *same* work that gravity would have done if Throcky had moved *straight upward* a distance h. This is an example of a more general result that we'll prove in Section 7.1.

KEYCONCEPT The work–energy theorem can help you solve problems in which an object follows a curved path. Take care in calculating the work done on such a path.

TEST YOUR UNDERSTANDING OF SECTION 6.3 In Example 5.20 (Section 5.4) we examined a conical pendulum. The speed of the pendulum bob remains constant as it travels around the circle shown in Fig. 5.32a. (a) Over one complete circle, how much work does the tension force F do on the bob? (i) A positive amount; (ii) a negative amount; (iii) zero. (b) Over one complete circle, how much work does the weight do on the bob? (i) A positive amount; (ii) a negative amount; (iii) zero.

ANSWER

(a) **(iii)** (b) **(iii)** At any point during the pendulum bob's motion, both the tension force and the weight act perpendicular to the motion—that is, perpendicular to an infinitesimal displacement $d\vec{l}$ of the bob. (In Fig. 5.32b, the displacement $d\vec{l}$ would be directed outward from the plane of the free-body diagram.) Hence for either force the scalar product inside the integral in Eq. (6.14) is $\vec{F} \cdot d\vec{l} = 0$, and the work done along any part of the circular path (including a complete circle) is $W = \int \vec{F} \cdot d\vec{l} = 0$.

6.4 POWER

The definition of work makes no reference to the passage of time. If you lift a barbell weighing 100 N through a vertical distance of 1.0 m at constant velocity, you do $(100\text{ N})(1.0\text{ m}) = 100$ J of work whether it takes you 1 second, 1 hour, or 1 year to do it. But often we need to know how quickly work is done. We describe this in terms of *power*. In ordinary conversation the word "power" is often synonymous with "energy" or "force." In physics we use a much more precise definition: **Power** is the time *rate* at which work is done. Like work and energy, power is a scalar quantity.

The average work done per unit time, or **average power** P_{av}, is defined to be

$$\underbrace{P_{av}}_{\text{Average power during time interval }\Delta t} = \frac{\overbrace{\Delta W}^{\text{Work done during time interval}}}{\underbrace{\Delta t}_{\text{Duration of time interval}}} \qquad (6.15)$$

The rate at which work is done might not be constant. We define **instantaneous power** P as the quotient in Eq. (6.15) as Δt approaches zero:

$$\underbrace{P}_{\text{Instantaneous power}} = \underbrace{\lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t}}_{\text{Average power over infinitesimally short time interval}} = \underbrace{\frac{dW}{dt}}_{\text{Time rate of doing work}} \qquad (6.16)$$

BIO APPLICATION Muscle Power Skeletal muscles provide the power that makes animals move. Muscle fibers that rely on anaerobic metabolism do not require oxygen; they produce large amounts of power but are useful for short sprints only. Muscle fibers that metabolize aerobically use oxygen and produce smaller amounts of power for long intervals. Both fiber types are visible in a fish fillet: The pale (anaerobic) muscle is used for brief bursts of speed, while the darker (aerobic) muscle is used for sustained swimming.

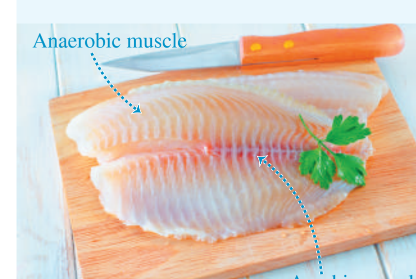


Figure 6.25 The same amount of work is done in both of these situations, but the power (the rate at which work is done) is different.

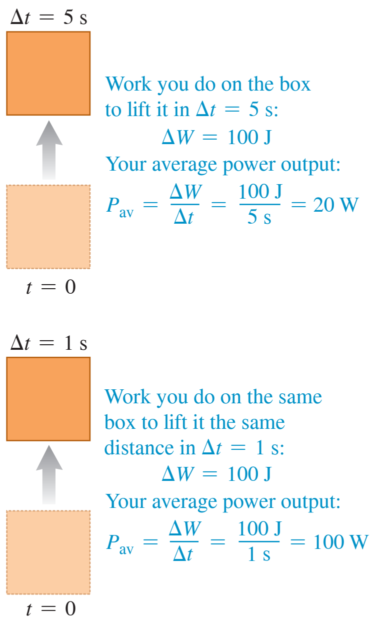


Figure 6.26 A one-horsepower (746 W) propulsion system.

The SI unit of power is the **watt** (W), named for the English inventor James Watt. One watt equals 1 joule per second: $1\text{ W} = 1\text{ J/s}$ (**Fig. 6.25**). The kilowatt $(1\text{ kW} = 10^3\text{ W})$ and the megawatt $(1\text{ MW} = 10^6\text{ W})$ are also commonly used.

Another common unit of power is the *horsepower* (hp) (**Fig. 6.26**). The value of this unit derives from experiments by James Watt, who measured that in one minute a horse could do an amount of work equivalent to lifting 33,000 pounds (lb) a distance of 1 foot (ft), or 33,000 ft·lb. Thus 1 hp = 33,000 ft·lb/min. Using 1 ft = 0.3048 m, 1 lb = 4.448 N, and 1 min = 60 s, we can show that

$$1\text{ hp} = 746\text{ W} = 0.746\text{ kW}$$

The watt is a familiar unit of *electrical* power; a 100 W light bulb converts 100 J of electrical energy into light and heat each second. But there's nothing inherently electrical about a watt. A light bulb could be rated in horsepower, and an engine can be rated in kilowatts.

The *kilowatt-hour* $(\text{kW}\cdot\text{h})$ is the usual commercial unit of electrical energy. One kilowatt-hour is the total work done in 1 hour (3600 s) when the power is 1 kilowatt (10^3 J/s), so

$$1\text{ kW}\cdot\text{h} = (10^3\text{ J/s})(3600\text{ s}) = 3.6\times10^6\text{ J} = 3.6\text{ MJ}$$

The kilowatt-hour is a unit of *work* or *energy,* not power.

In mechanics we can also express power in terms of force and velocity. Suppose that a force $\vec{F}$ acts on an object while it undergoes a vector displacement $\Delta\vec{s}$. If $F_\parallel$ is the component of $\vec{F}$ tangent to the path (parallel to $\Delta\vec{s}$), then the work done by the force is $\Delta W = F_\parallel \Delta s$. The average power is

$$P_{av} = \frac{F_\parallel \Delta s}{\Delta t} = F_\parallel \frac{\Delta s}{\Delta t} = F_\parallel v_{av} \qquad (6.17)$$

Instantaneous power P is the limit of this expression as $\Delta t \rightarrow 0$:

$$P = F_\parallel v \qquad (6.18)$$

where v is the magnitude of the instantaneous velocity. We can also express Eq. (6.18) in terms of the scalar product:

Instantaneous power for a force doing work on a particle

$$P = \vec{F}\cdot\vec{v} \qquad (6.19)$$

Force that acts on particle
Velocity of particle

EXAMPLE 6.9 Force and power

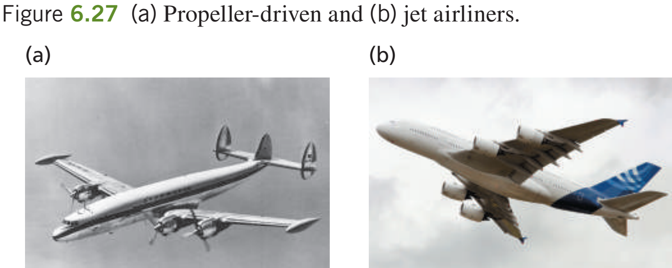
Each of the four jet engines on an Airbus A380 airliner develops a thrust (a forward force on the airliner) of 322,000 N (72,000 lb). When the airplane is flying at 250 m/s (900 km/h, or roughly 560 mi/h), what horsepower does each engine develop?

IDENTIFY, SET UP, and EXECUTE Our target variable is the instantaneous power P, which is the rate at which the thrust does work. We use Eq. (6.18). The thrust is in the direction of motion, so $F_\parallel$ is just equal to the thrust. At $v = 250$ m/s, the power developed by each engine is

$$P = F_\parallel v = (3.22\times10^5\text{ N})(250\text{ m/s}) = 8.05\times10^7\text{ W}$$
$$= (8.05\times10^7\text{ W})\frac{1\text{ hp}}{746\text{ W}} = 108{,}000\text{ hp}$$

EVALUATE The speed of modern airliners is directly related to the power of their engines (**Fig. 6.27**). The largest propeller-driven airliners of the 1950s had engines that each developed about 3400 hp $(2.5\times10^6\text{ W})$, giving them maximum speeds of about 600 km/h (370 mi/h). Each engine on an Airbus A380 develops more than 30 times more power, enabling it to fly at about 900 km/h (560 mi/h) and to carry a much heavier load.

Figure 6.27 (a) Propeller-driven and (b) jet airliners.



If the engines are at maximum thrust while the airliner is at rest on the ground so that $v = 0$, the engines develop *zero* power. Force and power are not the same thing!

KEYCONCEPT To find the power of a force acting on a moving object, multiply the component of force in the direction of motion by the object's speed.

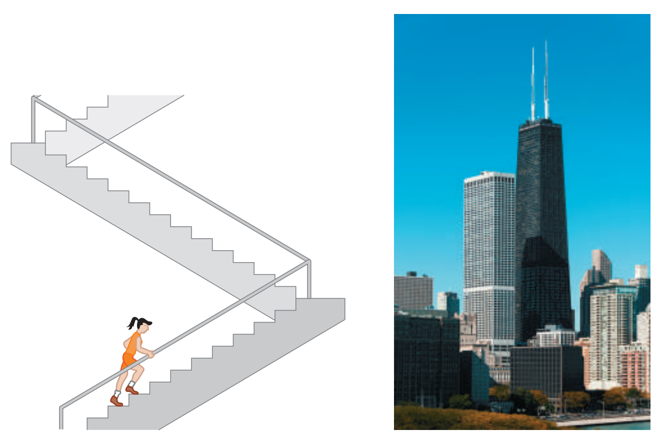
EXAMPLE 6.10 A "power climb"

A 50.0 kg marathon runner runs up the stairs to the top of Chicago's 443-m-tall Willis Tower, the second tallest building in the United States (**Fig. 6.28**). To lift herself to the top in 15.0 minutes, what must be her average power output? Express your answer in watts, in kilowatts, and in horsepower.

IDENTIFY and SET UP We'll treat the runner as a particle of mass m. Her average power output P_{av} must be enough to lift her at constant speed against gravity.

We can find P_{av} in two ways: (1) by determining how much work she must do and dividing that quantity by the elapsed time, as in Eq. (6.15), or (2) by calculating the average upward force she must exert (in the direction of the climb) and multiplying that quantity by her upward velocity, as in Eq. (6.17).

Figure 6.28 How much power is required to run up the stairs of Chicago's Willis Tower in 15 minutes?



EXECUTE (1) As in Example 6.8, lifting a mass m against gravity requires an amount of work equal to the weight mg multiplied by the height h it is lifted. Hence the work the runner must do is

$$W = mgh = (50.0\ \text{kg})(9.80\ \text{m/s}^2)(443\ \text{m}) = 2.17 \times 10^5\ \text{J}$$

She does this work in a time 15.0 min = 900 s, so from Eq. (6.15) the average power is

$$P_{av} = \frac{2.17 \times 10^5\ \text{J}}{900\ \text{s}} = 241\ \text{W} = 0.241\ \text{kW} = 0.323\ \text{hp}$$

(2) The force exerted is vertical and the average vertical component of velocity is $(443\ \text{m})/(900\ \text{s}) = 0.492\ \text{m/s}$, so from Eq. (6.17) the average power is

$$\begin{aligned} P_{av} &= F_{\parallel} v_{av} = (mg)v_{av} \\ &= (50.0\ \text{kg})(9.80\ \text{m/s}^2)(0.492\ \text{m/s}) = 241\ \text{W} \end{aligned}$$

which is the same result as before.

EVALUATE The runner's *total* power output will be several times greater than 241 W. The reason is that the runner isn't really a particle but a collection of parts that exert forces on each other and do work, such as the work done to inhale and exhale and to make her arms and legs swing. What we've calculated is only the part of her power output that lifts her to the top of the building.

KEYCONCEPT To calculate average power (the average rate of doing work), divide the work done by the time required to do that work.

TEST YOUR UNDERSTANDING OF SECTION 6.4 The air surrounding an airplane in flight exerts a drag force that acts opposite to the airplane's motion. When the Airbus A380 in Example 6.9 is flying in a straight line at a constant altitude at a constant 250 m/s, what is the rate at which the drag force does work on it? (i) 432,000 hp; (ii) 108,000 hp; (iii) 0; (iv) −108,000 hp; (v) −432,000 hp.

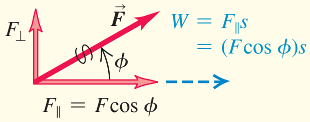
ANSWER

(v) The airliner has a constant horizontal velocity, so the net horizontal force on it must be zero. Hence the backward drag force must have the same magnitude as the forward force due to the combined thrust of the four engines. This means that the drag force must do *negative* work on the airplane at the same rate that the combined thrust force does *positive* work. The combined thrust does work at a rate of $4(108{,}000\ \text{hp}) = 432{,}000$ hp, so the drag force must do work at a rate of −432,000 hp.

CHAPTER 6 SUMMARY

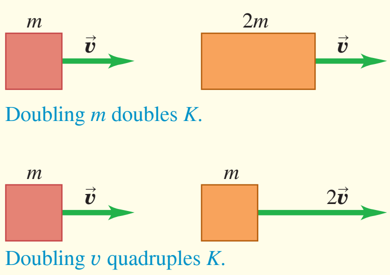
Work done by a force: When a constant force $\vec{F}$ acts on a particle that undergoes a straight-line displacement $\vec{s}$, the work done by the force on the particle is defined to be the scalar product of $\vec{F}$ and $\vec{s}$. The unit of work in SI units is 1 joule = 1 newton-meter ($1\ \text{J} = 1\ \text{N}\cdot\text{m}$). Work is a scalar quantity; it can be positive or negative, but it has no direction in space. (See Examples 6.1 and 6.2.)

$$W = \vec{F} \cdot \vec{s} = Fs\cos\phi$$
$$\phi = \text{angle between } \vec{F} \text{ and } \vec{s} \qquad (6.2), (6.3)$$



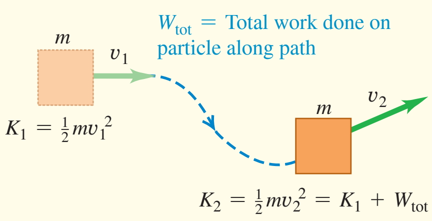
Kinetic energy: The kinetic energy K of a particle equals the amount of work required to accelerate the particle from rest to speed v. It is also equal to the amount of work the particle can do in the process of being brought to rest. Kinetic energy is a scalar that has no direction in space; it is always positive or zero. Its units are the same as the units of work: $1\ \text{J} = 1\ \text{N}\cdot\text{m} = 1\ \text{kg}\cdot\text{m}^2/\text{s}^2$.

$$K = \tfrac{1}{2}mv^2 \qquad (6.5)$$



The work–energy theorem: When forces act on a particle while it undergoes a displacement, the particle's kinetic energy changes by an amount equal to the total work done on the particle by all the forces. This relationship, called the work–energy theorem, is valid whether the forces are constant or varying and whether the particle moves along a straight or curved path. It is applicable only to objects that can be treated as particles. (See Examples 6.3–6.5.)

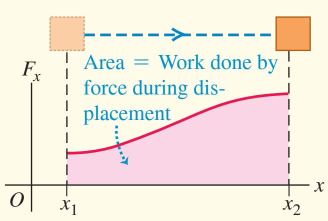
$$W_{\text{tot}} = K_2 - K_1 = \Delta K \qquad (6.6)$$



Work done by a varying force or on a curved path: When a force varies during a straight-line displacement, the work done by the force is given by an integral, Eq. (6.7). (See Examples 6.6 and 6.7.) When a particle follows a curved path, the work done on it by a force $\vec{F}$ is given by an integral that involves the angle ϕ between the force and the displacement. This expression is valid even if the force magnitude and the angle ϕ vary during the displacement. (See Example 6.8.)

$$W = \int_{x_1}^{x_2} F_x\,dx \qquad (6.7)$$

$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l}$$
$$= \int_{P_1}^{P_2} F\cos\phi\,dl = \int_{P_1}^{P_2} F_\|\,dl \qquad (6.14)$$

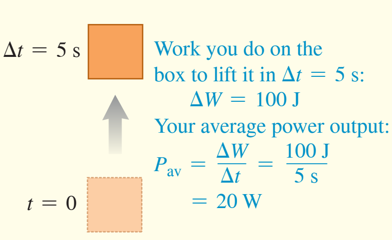


Power: Power is the time rate of doing work. The average power P_{av} is the amount of work ΔW done in time Δt divided by that time. The instantaneous power is the limit of the average power as Δt goes to zero. When a force $\vec{F}$ acts on a particle moving with velocity $\vec{v}$, the instantaneous power (the rate at which the force does work) is the scalar product of $\vec{F}$ and $\vec{v}$. Like work and kinetic energy, power is a scalar quantity. The SI unit of power is 1 watt = 1 joule/second ($1\ \text{W} = 1\ \text{J/s}$). (See Examples 6.9 and 6.10.)

$$P_{\text{av}} = \frac{\Delta W}{\Delta t} \qquad (6.15)$$

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} \qquad (6.16)$$

$$P = \vec{F} \cdot \vec{v} \qquad (6.19)$$



Chapter 6 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLES 6.1 and 6.2 (Section 6.1) before attempting these problems.

VP6.2.1 As a football player moves in a straight line [displacement $(3.00\text{ m})\hat{\imath} - (6.50\text{ m})\hat{\jmath}$], an opponent exerts a constant force $(126\text{ N})\hat{\imath} + (168\text{ N})\hat{\jmath}$ on him. (a) How much work does the opponent do on the football player? (b) How much work does the football player do on the opponent?

VP6.2.2 You push a stalled car with a constant force of 215 N as it moves a distance 8.40 m in a straight line. The amount of work that you do in this process is 1.47×10^3 J. What is the angle between the direction of your push and the direction of the car's motion?

VP6.2.3 A block of mass 15.0 kg slides down a ramp inclined at 28.0° above **the horiz**ontal. As it slides, a kinetic friction force of 30.0 N parallel to the ramp acts on it. If the block slides for 3.00 m along the ramp, find (a) the work done on the block by friction, (b) the work done on the block by the force of gravity, (c) the work done on the block by the normal force, and (d) the total work done on the block.

VP6.2.4 Three students are fighting over a T-shirt. Student 1 exerts a constant force $\vec{F}_1 = F_0\hat{\imath}$ on the shirt, student 2 exerts a constant force $\vec{F}_2 = -3F_0\,\hat{\jmath}$ and student 3 exerts a constant force $\vec{F}_3 = -4F_0\,\hat{\imath} + G\hat{\jmath}$. (In these expressions F_0 and G are positive constants with units of force.) As the three students exert these forces, the T-shirt undergoes a straight-line displacemen $2d\hat{\imath} + d\hat{\jmath}$ where d is a positive constant with units of distance. (a) Find the work done on the T-shirt by each student. (b) What must be the value of G in order for the total work to be equal to zero?

Be sure to review EXAMPLES 6.3 and 6.4 (Section 6.2) before attempting these problems.

VP6.4.1 A nail is partially inserted into a block of wood, with a length of 0.0300 m protruding above the top of the block. To hammer the nail in the rest of the way, you drop a 20.0 kg metal cylinder onto it. The cylinder rides on vertical tracks that exert an upward friction force of 16.0 N on the cylinder as it falls. You release the cylinder from rest at a height of 1.50 m above the top of the nail. The cylinder comes to rest on top of the block of wood, with the nail fully inside the block. Use the work–energy theorem to find (a) the speed of the cylinder just as it hits the nail and (b) the average force the cylinder exerts on the nail while pushing it into the block. Ignore the effects of the air.

VP6.4.2 You are using a rope to lift a 14.5 kg crate of fruit. Initially you are lifting the crate at 0.500 m/s. You then increase the tension in the rope to 175 N and lift the crate an additional 1.25 m. During this 1.25 m motion, how much work is done on the crate by (a) the tension force, (b) the force of gravity, and (c) the net force? (d) What are the kinetic energy and speed of the crate after being lifted the additional 1.25 m?

VP6.4.3 A helicopter of mass 1.40×10^3 kg is descending vertically at 3.00 m/s. The pilot increases the upward thrust provided by the main rotor so that the vertical speed decreases to 0.450 m/s as the helicopter descends 2.00 m. (a) What are the initial and final kinetic energies of the helicopter? (b) As the helicopter descends this 2.00 m distance, how much total work is done on it? How much work is done on it by gravity? How much work is done on it by the upward thrust force? (c) What is the magnitude of the upward thrust force (assumed constant)?

VP6.4.4 A block of mass m is released from rest on a ramp that is inclined at an angle θ from the horizontal. The coefficient of kinetic friction between the block and the ramp is μ_k. The block slides a distance d along the ramp until it reaches the bottom. (a) How much work is done on the block by the force of gravity? (b) How much work is done on the block by the friction force? (c) Use the work–energy theorem to find the speed of the block just as it reaches the bottom of the ramp.

Be sure to review EXAMPLES 6.7 and 6.8 (Section 6.3) before attempting these problems.

VP6.8.1 An air-track glider of mass 0.150 kg is attached to the end of a horizontal air track by a spring with force constant 30.0 N/m (see Fig. 6.22a). Initially the spring is unstretched and the glider is moving at 1.25 m/s to the right. Find the maximum distance d that the glider moves to the right (a) if the air track is turned on, so that there is no friction, and (b) if the air is turned off, so that there is kinetic friction with coefficient $\mu_k = 0.320$.

VP6.8.2 An air-track glider of mass m is attached to the end of a horizontal air track by a spring with force constant k as in Fig. 6.22a. The air track is turned off, so there is friction between the glider and the track. Initially the spring is unstretched, but unlike the situation in Fig. 6.22a, the glider is initially moving to the *left* at speed v_1 The glider moves a distance d to the left before coming momentarily to rest. Use the work–energy theorem to find the coefficient of kinetic friction between the glider and the track.

VP6.8.3 A pendulum is made up of a small sphere of mass 0.500 kg attached to a string of length 0.750 m. The sphere is swinging back and forth between point A, where the string is at the maximum angle of 35.0° to the left of vertical, and point C, where the string is at the maximum angle of 35.0° to the right of vertical. The string is vertical when the sphere is at point B. Calculate how much work the force of gravity does on the sphere (a) from A to B, (b) from B to C, and (c) from A to C.

VP6.8.4 A spider of mass m is swinging back and forth at the end of a strand of silk of length L. During the spider's swing the strand makes a maximum angle of θ with the vertical. What is the speed of the spider at the low point of its motion, when the strand of silk is vertical?

BRIDGING PROBLEM A Spring That Disobeys Hooke's Law

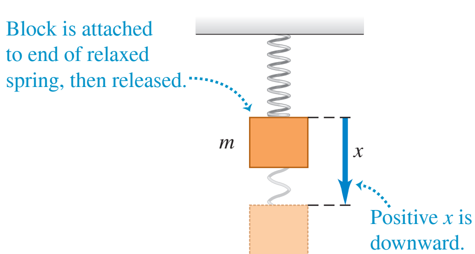
Consider a hanging spring of negligible mass that does *not* obey Hooke's law. When the spring is pulled downward by a distance x, the spring exerts an upward force of magnitude αx^2, where α is a positive constant. Initially the hanging spring is relaxed (not extended). We then attach a block of mass m to the spring and release the block. The block stretches the spring as it falls (**Fig. 6.29**). (a) How fast is the block moving when it has fallen a distance x_1? (b) At what rate does the spring do work on the block at this point? (c) Find the maximum distance x_2 that the spring stretches. (d) Will the block *remain* at the point found in part (c)?

SOLUTION GUIDE

IDENTIFY and SET UP

1. The spring force in this problem isn't constant, so you have to use the work–energy theorem. You'll also need Eq. (6.7) to find the work done by the spring over a given displacement.
2. Draw a free-body diagram for the block, including your choice of coordinate axes. Note that x represents how far the spring is *stretched,* so choose the positive x-direction to be downward, as in Fig. 6.29. On your coordinate axis, label the points $x = x_1$ and $x = x_2$.

Figure **6.29** The block is attached to a spring that does not obey Hooke's law.



3. Make a list of the unknown quantities, and decide which of these are the target variables.

EXECUTE

4. Calculate the work done on the block by the spring as the block falls an arbitrary distance x. (The integral isn't a difficult one. Use Appendix B if you need a reminder.) Is the work done by the spring positive, negative, or zero?
5. Calculate the work done on the block by any other forces as the block falls an arbitrary distance x. Is this work positive, negative, or zero?
6. Use the work–energy theorem to find the target variables. (You'll also need an equation for power.) *Hint:* When the spring is at its maximum stretch, what is the speed of the block?
7. To answer part (d), consider the *net* force that acts on the block when it is at the point found in part (c).

EVALUATE

8. We learned in Section 2.5 that after an object dropped from rest has fallen freely a distance x_1, its speed is $\sqrt{2gx_1}$. Use this to decide whether your answer in part (a) makes sense. In addition, ask yourself whether the algebraic sign of your answer in part (b) makes sense.
9. Find the value of x where the net force on the block would be zero. How does this compare to your result for x_2? Is this consistent with your answer in part (d)?

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q6.1 The sign of many physical quantities depends on the choice of coordinates. For example, a_y for free-fall motion can be negative or positive, depending on whether we choose upward or downward as positive. Is the same true of work? In other words, can we make positive work negative by a different choice of coordinates? Explain.

Q6.2 An elevator is hoisted by its cables at constant speed. Is the total work done on the elevator positive, negative, or zero? Explain.

Q6.3 A rope tied to an object is pulled, causing the object to accelerate. But according to Newton's third law, the object pulls back on the rope with a force of equal magnitude and opposite direction. Is the total work done then zero? If so, how can the object's kinetic energy change? Explain.

Q6.4 If it takes total work W to give an object a speed v and kinetic energy K, starting from rest, what will be the object's speed (in terms of v) and kinetic energy (in terms of K) if we do twice as much work on it, again starting from rest?

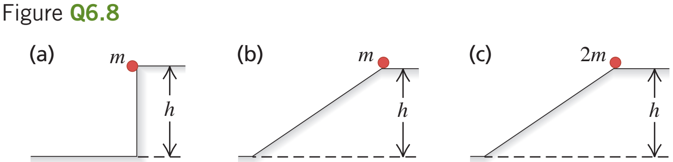
Q6.5 If there is a net nonzero force on a moving object, can the total work done on the object be zero? Explain, using an example.

Q6.6 In Example 5.5 (Section 5.1), how does the work done on the bucket by the tension in the cable compare with the work done on the cart by the tension in the cable?

Q6.7 In the conical pendulum of Example 5.20 (Section 5.4), which of the forces do work on the bob while it is swinging?

Q6.8 For the cases shown in **Fig. Q6.8,** the object is released from rest at the top and feels no friction or air resistance. In which (if any) cases will the mass have (i) the greatest speed at the bottom and (ii) the most work done on it by the time it reaches the bottom?

Figure **Q6.8**



Q6.9 A force $\vec{F}$ is in the x-direction and has a magnitude that depends on x. Sketch a possible graph of F versus x such that the force does zero work on an object that moves from x_1 to x_2, even though the force magnitude is not zero at all x in this range.

Q6.10 Does a car's kinetic energy change more when the car speeds up from 10 to 15 m/s or from 15 to 20 m/s? Explain.

Q6.11 A falling brick has a mass of 1.5 kg and is moving straight downward with a speed of 5.0 m/s. A 1.5 kg physics book is sliding across the floor with a speed of 5.0 m/s. A 1.5 kg melon is traveling with a horizontal velocity component 3.0 m/s to the right and a vertical component 4.0 m/s upward. Do all of these objects have the same velocity? Do all of them have the same kinetic energy? For both questions, give your reasoning.

Q6.12 Can the *total* work done on an object during a displacement be negative? Explain. If the total work is negative, can its magnitude be larger than the initial kinetic energy of the object? Explain.

Q6.13 A net force acts on an object and accelerates it from rest to a speed v_1. In doing so, the force does an amount of work W_1. By what factor must the work done on the object be increased to produce three times the final speed, with the object again starting from rest?

Q6.14 A truck speeding down the highway has a lot of kinetic energy relative to a stopped state trooper but no kinetic energy relative to the truck driver. In these two frames of reference, is the same amount of work required to stop the truck? Explain.

Q6.15 You are holding a briefcase by the handle, with your arm straight down by your side. Does the force your hand exerts do work on the briefcase when (a) you walk at a constant speed down a horizontal hallway and (b) you ride an escalator from the first to second floor of a building? In both cases justify your answer.

Q6.16 When a book slides along a tabletop, the force of friction does negative work on it. Can friction ever do *positive* work? Explain. (*Hint:* Think of a box in the back of an accelerating truck.)

Q6.17 Time yourself while running up a flight of steps, and compute the average rate at which you do work against the force of gravity. Express your answer in watts and in horsepower.

Q6.18 **Fractured Physics.** Many terms from physics are badly misused in everyday language. In both cases, explain the errors involved. (a) A *strong* person is called *powerful*. What is wrong with this use of *power*? (b) When a worker carries a bag of concrete along a level construction site, people say he did a lot of *work*. Did he?

Q6.19 An advertisement for a portable electrical generating unit claims that the unit's diesel engine produces 28,000 hp to drive an electrical generator that produces 30 MW of electrical power. Is this possible? Explain.

Q6.20 A car speeds up while the engine delivers constant power. Is the acceleration greater at the beginning of this process or at the end? Explain.

Q6.21 Consider a graph of instantaneous power versus time, with the vertical P-axis starting at $P = 0$. What is the physical significance of the area under the P-versus-t curve between vertical lines at t_1 and t_2? How could you find the average power from the graph? Draw a P-versus-t curve that consists of two straight-line sections and for which the peak power is equal to twice the average power.

Q6.22 A nonzero net force acts on an object. Is it possible for any of the following quantities to be constant: the object's (a) speed; (b) velocity; (c) kinetic energy?

Q6.23 When a certain force is applied to an ideal spring, the spring stretches a distance x from its unstretched length and does work W. If instead twice the force is applied, what distance (in terms of x) does the spring stretch from its unstretched length, and how much work (in terms of W) is required to stretch it this distance?

Q6.24 If work W is required to stretch an ideal spring a distance x from its unstretched length, what work (in terms of W) is required to stretch the spring an *additional* distance x?

EXERCISES

Section 6.1 Work

6.1 • You push your physics book 1.50 m along a horizontal tabletop with a horizontal push of 2.40 N while the opposing force of friction is 0.600 N. How much work does each of the following forces do on the book: (a) your 2.40 N push, (b) the friction force, (c) the normal force from the tabletop, and (d) gravity? (e) What is the net work done on the book?

6.2 • Using a cable with a tension of 1350 N, a tow truck pulls a car 5.00 km along a horizontal roadway. (a) How much work does the cable do on the car if it pulls horizontally? If it pulls at 35.0° above the horizontal? (b) How much work does the cable do on the tow truck in both cases of part (a)? (c) How much work does gravity do on the car in part (a)?

6.3 • A factory worker pushes a 30.0 kg crate a distance of 4.5 m along a level floor at constant velocity by pushing horizontally on it. The coefficient of kinetic friction between the crate and the floor is 0.25. (a) What magnitude of force must the worker apply? (b) How much work is done on the crate by this force? (c) How much work is done on the crate by friction? (d) How much work is done on the crate by the normal force? By gravity? (e) What is the total work done on the crate?

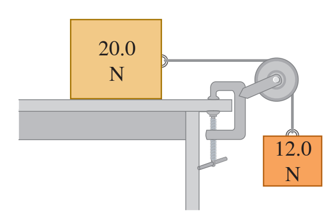
6.4 •• Suppose the worker in Exercise 6.3 pushes downward at an angle of 30° below the horizontal. (a) What magnitude of force must the worker apply to move the crate at constant velocity? (b) How much work is done on the crate by this force when the crate is pushed a distance of 4.5 m? (c) How much work is done on the crate by friction during this displacement? (d) How much work is done on the crate by the normal force? By gravity? (e) What is the total work done on the crate?

6.5 •• A 75.0 kg painter climbs a ladder that is 2.75 m long and leans against a vertical wall. The ladder makes a 30.0° angle with the wall. (a) How much work does gravity do on the painter? (b) Does the answer to part (a) depend on whether the painter climbs at constant speed or accelerates up the ladder?

6.6 •• Two tugboats pull a disabled supertanker. Each tug exerts a constant force of 1.80×10^6 N, one 14° west of north and the other 14° east of north, as they pull the tanker 0.75 km toward the north. What is the total work they do on the supertanker?

6.7 • Two blocks are connected by a very light string passing over a massless and frictionless pulley (**Fig. E6.7**). Traveling at constant speed, the 20.0 N block moves 75.0 cm to the right and the 12.0 N block moves 75.0 cm downward. (a) How much work is done on the 12.0 N block by (i) gravity and (ii) the tension in the string? (b) How much work is done on the 20.0 N block by (i) gravity, (ii) the tension in the string, (iii) friction, and (iv) the normal force? (c) Find the total work done on each block.

Figure **E6.7**



6.8 •• A loaded grocery cart is rolling across a parking lot in a strong wind. You apply a constant force $\vec{F} = (30\text{ N})\hat{\imath} - (40\text{ N})\hat{\jmath}$ to the cart as it undergoes a displacement $\vec{s} = (-9.0\text{ m})\hat{\imath} - (3.0\text{ m})\hat{\jmath}$. How much work does the force you apply do on the grocery cart?

6.9 • Your physics book is resting in front of you on a horizontal table in the campus library. You push the book over to your friend, who is seated at the other side of the table, 0.400 m north and 0.300 m east of you. If you push the book in a straight line to your friend, friction does −4.8 J of work on the book. If instead you push the book 0.400 m due north and then 0.300 m due east, how much work is done by friction?

6.10 •• A 12.0 kg package in a mail-sorting room slides 2.00 m down a chute that is inclined at 53.0° below the horizontal. The coefficient of kinetic friction between the package and the chute's surface is 0.40. Calculate the work done on the package by (a) friction, (b) gravity, and (c) the normal force. (d) What is the net work done on the package?

6.11 • A force $\vec{F}$ that is at an angle 60° above the horizontal is applied to a box that moves on a horizontal frictionless surface, and the force does work W as the box moves a distance d. (a) At what angle above the horizontal would the force have to be directed in order for twice the work to be done for the same displacement of the box? (b) If the angle is kept at 60° and the box is initially at rest, by what factor would F have to be increased to double the final speed of the box after moving distance d?

6.12 •• A boxed 10.0 kg computer monitor is dragged by friction 5.50 m upward along a conveyor belt inclined at an angle of 36.9° above the horizontal. If the monitor's speed is a constant 2.10 cm/s, how much work is done on the monitor by (a) friction, (b) gravity, and (c) the normal force of the conveyor belt?

6.13 •• A large crate sits on the floor of a warehouse. Paul and Bob apply constant horizontal forces to the crate. The force applied by Paul has magnitude 48.0 N and direction 61.0° south of west. How much work does Paul's force do during a displacement of the crate that is 12.0 m in the direction 22.0° east of north?

6.14 •• You apply a constant force $\vec{F} = (-68.0\text{ N})\hat{\imath} + (36.0\text{ N})\hat{\jmath}$ to a 380 kg car as the car travels 48.0 m in a direction that is 240.0° counterclockwise from the $+x$-axis. How much work does the force you apply do on the car?

6.15 •• On a farm, you are pushing on a stubborn pig with a constant horizontal force with magnitude 30.0 N and direction 37.0° counterclockwise from the $+x$-axis. How much work does this force do during a displacement of the pig that is (a) $\vec{s} = (5.00\text{ m})\hat{\imath}$; (b) $\vec{s} = -(6.00\text{ m})\hat{\jmath}$; (c) $\vec{s} = -(2.00\text{ m})\hat{\imath} + (4.00\text{ m})\hat{\jmath}$?

Section 6.2 Kinetic Energy and the Work–Energy Theorem

6.16 •• A 1.50 kg book is sliding along a rough horizontal surface. At point A it is moving at 3.21 m/s, and at point B it has slowed to 1.25 m/s. (a) How much total work was done on the book between A and B? (b) If -0.750 J of total work is done on the book from B to C, how fast is it moving at point C? (c) How fast would it be moving at C if $+0.750$ J of total work was done on it from B to C?

6.17 •• BIO **Animal Energy.** Adult cheetahs, the fastest of the great cats, have a mass of about 70 kg and have been clocked to run at up to 72 mi/h (32 m/s). (a) How many joules of kinetic energy does such a swift cheetah have? (b) By what factor would its kinetic energy change if its speed were doubled?

6.18 •• A baseball has a mass of 0.145 kg. (a) In batting practice a batter hits a ball that is sitting at rest on top of a post. The ball leaves the post with a horizontal speed of 30.0 m/s. How much work did the force applied by the bat do on the ball? (b) During a game the same batter swings at a ball thrown by the pitcher and hits a line drive. Just before the ball is hit it is traveling at a speed of 20.0 m/s, and just after it is hit it is traveling in the opposite direction at a speed of 30.0 m/s. What is the total work done on the baseball by the force exerted by the bat? (c) How do the results of parts (a) and (b) compare? Explain.

6.19 • **Meteor Crater.** About 50,000 years ago, a meteor crashed into the earth near present-day Flagstaff, Arizona. Measurements from 2005 estimate that this meteor had a mass of about 1.4×10^8 kg (around 150,000 tons) and hit the ground at a speed of 12 km/s. (a) How much kinetic energy did this meteor deliver to the ground? (b) How does this energy compare to the energy released by a 1.0 megaton nuclear bomb? (A megaton bomb releases the same amount of energy as a million tons of TNT, and 1.0 ton of TNT releases 4.184×10^9 J of energy.)

6.20 • A 4.80 kg watermelon is dropped from rest from the roof of an 18.0 m-tall building and feels no appreciable air resistance. (a) Calculate the work done by gravity on the watermelon during its displacement from the roof to the ground. (b) Just before it strikes the ground, what are the watermelon's (i) kinetic energy and (ii) speed? (c) Which of the answers in parts (a) and (b) would be *different* if there were appreciable air resistance?

6.21 • CP You are pushing a large box across a frictionless floor by applying a constant horizontal force. If the box starts at rest, you have to do work W_1 in order for the box to travel a distance d in time t. How much work would you have to do, in terms of W_1, to make the box go the same distance in half the time?

6.22 •• Use the work–energy theorem to solve each of these problems. You can use Newton's laws to check your answers. (a) A skier moving at 5.00 m/s encounters a long, rough horizontal patch of snow having a coefficient of kinetic friction of 0.220 with her skis. How far does she travel on this patch before stopping? (b) Suppose the rough patch in part (a) was only 2.90 m long. How fast would the skier be moving when she reached the end of the patch? (c) At the base of a frictionless icy hill that rises at 25.0° above the horizontal, a toboggan has a speed of 12.0 m/s toward the hill. How high vertically above the base will it go before stopping?

6.23 •• You are a member of an Alpine Rescue Team. You must project a box of supplies up an incline of constant slope angle α so that it reaches a stranded skier who is a vertical distance h above the bottom of the incline. The incline is slippery, but there is some friction present, with kinetic friction coefficient μ_k. Use the work–energy theorem to calculate the minimum speed you must give the box at the bottom of the incline so that it will reach the skier. Express your answer in terms of g, h, μ_k, and α.

6.24 •• You throw a 3.00 N rock vertically into the air from ground level. You observe that when it is 15.0 m above the ground, it is traveling at 25.0 m/s upward. Use the work–energy theorem to find (a) the rock's speed just as it left the ground and (b) its maximum height.

6.25 • A sled with mass 12.00 kg moves in a straight line on a frictionless, horizontal surface. At one point in its path, its speed is 4.00 m/s; after it has traveled 2.50 m beyond this point, its speed is 6.00 m/s. Use the work–energy theorem to find the net force acting on the sled, assuming that this force is constant and that it acts in the direction of the sled's motion.

6.26 •• A soccer ball with mass 0.420 kg is initially moving with speed 2.00 m/s. A soccer player kicks the ball, exerting a constant force of magnitude 40.0 N in the same direction as the ball's motion. Over what distance must the player's foot be in contact with the ball to increase the ball's speed to 6.00 m/s?

6.27 • A 12-pack of Omni-Cola (mass 4.30 kg) is initially at rest on a horizontal floor. It is then pushed in a straight line for 1.20 m by a trained dog that exerts a horizontal force with magnitude 36.0 N. Use the work–energy theorem to find the final speed of the 12-pack if (a) there is no friction between the 12-pack and the floor, and (b) the coefficient of kinetic friction between the 12-pack and the floor is 0.30.

6.28 •• A block of ice with mass 2.00 kg slides 1.35 m down an inclined plane that slopes downward at an angle of 36.9° below the horizontal. If the block of ice starts from rest, what is its final speed? Ignore friction.

6.29 • Object A has 27 J of kinetic energy. Object B has one-quarter the mass of object A. (a) If object B also has 27 J of kinetic energy, is it moving faster or slower than object A? By what factor? (b) By what factor does the speed of each object change if total work -18 J is done on each?

6.30 •• A 30.0 kg crate is initially moving with a velocity that has magnitude 3.90 m/s in a direction 37.0° west of north. How much work must be done on the crate to change its velocity to 5.62 m/s in a direction 63.0° south of east?

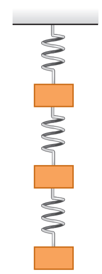
6.31 • **Stopping Distance.** A car is traveling on a level road with speed v_0 at the instant when the brakes lock, so that the tires slide rather than roll. (a) Use the work–energy theorem to calculate the minimum stopping distance of the car in terms of v_0, g, and the coefficient of kinetic friction μ_k between the tires and the road. (b) By what factor would the minimum stopping distance change if (i) the coefficient of kinetic friction were doubled, or (ii) the initial speed were doubled, or (iii) both the coefficient of kinetic friction and the initial speed were doubled?

Section 6.3 Work and Energy with Varying Forces

6.32 •• To stretch an ideal spring 3.00 cm from its unstretched length, 12.0 J of work must be done. (a) What is the force constant of this spring? (b) What magnitude force is needed to stretch the spring 3.00 cm from its unstretched length? (c) How much work must be done to compress this spring 4.00 cm from its unstretched length, and what force is needed to compress it this distance?

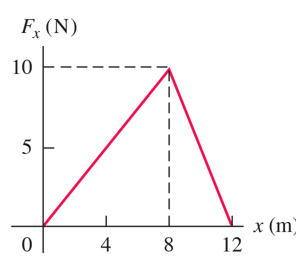
6.33 • Three identical 8.50 kg masses are hung by three identical springs (**Fig. E6.33**). Each spring has a force constant of 7.80 kN/m and was 12.0 cm long before any masses were attached to it. (a) Draw a free-body diagram of each mass. (b) How long is each spring when hanging as shown? (*Hint:* First isolate only the bottom mass. Then treat the bottom two masses as a system. Finally, treat all three masses as a system.)

Figure E6.33



6.34 • A child applies a force $\vec{F}$ parallel to the x-axis to a 10.0 kg sled moving on the frozen surface of a small pond. As the child controls the speed of the sled, the x-component of the force she applies varies with the x-coordinate of the sled as shown in **Fig. E6.34.** Calculate the work done by $\vec{F}$ when the sled moves (a) from $x = 0$ to $x = 8.0$ m; (b) from $x = 8.0$ m to $x = 12.0$ m; (c) from $x = 0$ to 12.0 m.

Figure E6.34



6.35 •• Suppose the sled in Exercise 6.34 is initially at rest at $x = 0$. Use the work–energy theorem to find the speed of the sled at (a) $x = 8.0$ m and (b) $x = 12.0$ m. Ignore friction between the sled and the surface of the pond.

6.36 •• A spring of force constant 300.0 N/m and unstretched length 0.240 m is stretched by two forces, pulling in opposite directions at opposite ends of the spring, that increase to 15.0 N. How long will the spring now be, and how much work was required to stretch it that distance?

6.37 •• A 6.0 kg box moving at 3.0 m/s on a horizontal, frictionless surface runs into one end of a light horizontal spring of force constant 75 N/cm that is fixed at the other end. Use the work–energy theorem to find the maximum compression of the spring.

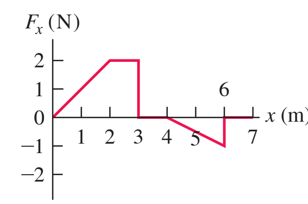
6.38 •• **Leg Presses.** As part of your daily workout, you lie on your back and push with your feet against a platform attached to two stiff ideal springs arranged side by side so that they are parallel to each other. When you push the platform, you compress the springs. You do 80.0 J of work when you compress the springs 0.200 m from their uncompressed length. (a) What magnitude of force must you apply to hold the platform in this position? (b) How much *additional* work must you do to move the platform 0.200 m *farther,* and what maximum force must you apply?

6.39 •• (a) In Example 6.7 (Section 6.3) it was calculated that with the air track turned off, the glider travels 8.6 cm before it stops instantaneously. How large would the coefficient of static friction μ_s have to be to keep the glider from springing back to the left? (b) If the coefficient of static friction between the glider and the track is $\mu_s = 0.60$, what is the maximum initial speed v_1 that the glider can be given and still remain at rest after it stops instantaneously? With the air track turned off, the coefficient of kinetic friction is $\mu_k = 0.47$.

6.40 • A 4.00 kg block of ice is placed against one end of a horizontal spring that is fixed at the other end, has force constant $k = 200$ N/m and is compressed 0.025 m. The spring is released and accelerates the block along a horizontal surface. Ignore friction and the mass of the spring. (a) Calculate the work done on the block by the spring during the motion of the block from its initial position to where the spring has returned to its uncompressed length. (b) What is the speed of the block after it leaves the spring?

6.41 • A force $\vec{F}$ is applied to a 2.0 kg, radio-controlled model car parallel to the x-axis as it moves along a straight track. The x-component of the force varies with the x-coordinate of the car (**Fig. E6.41**). Calculate the work done by the force $\vec{F}$ when the car moves from (a) $x = 0$ to $x = 3.0$ m; (b) $x = 3.0$ m to $x = 4.0$ m; (c) $x = 4.0$ m to $x = 7.0$ m; (d) $x = 0$ to $x = 7.0$ m; (e) $x = 7.0$ m to $x = 2.0$ m.

Figure E6.41



6.42 • Suppose the 2.0 kg model car in Exercise 6.41 is initially at rest at $x = 0$ and $\vec{F}$ is the net force acting on it. Use the work–energy theorem to find the speed of the car at (a) $x = 3.0$ m; (b) $x = 4.0$ m; (c) $x = 7.0$ m.

6.43 •• At a waterpark, sleds with riders are sent along a slippery, horizontal surface by the release of a large compressed spring. The spring, with force constant $k = 40.0$ N/cm and negligible mass, rests on the frictionless horizontal surface. One end is in contact with a stationary wall. A sled and rider with total mass 70.0 kg are pushed against the other end, compressing the spring 0.375 m. The sled is then released with zero initial velocity. What is the sled's speed when the spring (a) returns to its uncompressed length and (b) is still compressed 0.200 m?

6.44 •• A small glider is placed against a compressed spring at the bottom of an air track that slopes upward at an angle of 40.0° above the horizontal. The glider has mass 0.0900 kg. The spring has $k = 640$ N/m and negligible mass. When the spring is released, the glider travels a maximum distance of 1.80 m along the air track before sliding back down. Before reaching this maximum distance, the glider loses contact with the spring. (a) What distance was the spring originally compressed? (b) When the glider has traveled along the air track 0.80 m from its initial position against the compressed spring, is it still in contact with the spring? What is the kinetic energy of the glider at this point?

6.45 •• CALC A force in the $+x$-direction with magnitude $F(x) = 18.0\text{ N} - (0.530\text{ N/m})x$ is applied to a 6.00 kg box that is sitting on the horizontal, frictionless surface of a frozen lake. $F(x)$ is the only horizontal force on the box. If the box is initially at rest at $x = 0$, what is its speed after it has traveled 14.0 m?

Section 6.4 Power

6.46 •• A crate on a motorized cart starts from rest and moves with a constant eastward acceleration of $a = 2.80\text{ m/s}^2$. A worker assists the cart by pushing on the crate with a force that is eastward and has magnitude that depends on time according to $F(t) = (5.40\text{ N/s})t$. What is the instantaneous power supplied by this force at $t = 5.00$ s?

6.47 • How many joules of energy does a 100 watt light bulb use per hour? How fast would a 70 kg person have to run to have that amount of kinetic energy?

6.48 •• BIO **Should You Walk or Run?** It is 5.0 km from your home to the physics lab. As part of your physical fitness program, you could run that distance at 10 km/h (which uses up energy at the rate of 700 W), or you could walk it leisurely at 3.0 km/h (which uses energy at 290 W). Which choice would burn up more energy, and how much energy (in joules) would it burn? Why does the more intense exercise burn up less energy than the less intense exercise?

6.49 • Estimate how many 30 lb bags of mulch an average student in your physics class can load into the bed of a pickup truck in 5.0 min. The truck bed is 4.0 ft off the ground. If you assume that the magnitude of the work done on each bag by the student equals the magnitude of the work done on the bag by gravity when the bag is lifted into the truck, what is the average power output of the student? Express your result in watts and in horsepower.

6.50 •• A 20.0 kg rock is sliding on a rough, horizontal surface at 8.00 m/s and eventually stops due to friction. The coefficient of kinetic friction between the rock and the surface is 0.200. What average power is produced by friction as the rock stops?

6.51 • A student walks up three flights of stairs, a vertical height of about 50 ft. Estimate the student's weight to be the average for students in your physics class. If the magnitude of the average rate at which the gravity force does work on the student equals 500 W, how long would it take the student to travel up the three flights of stairs?

6.52 •• When its 75 kW (100 hp) engine is generating full power, a small single-engine airplane with mass 700 kg gains altitude at a rate of 2.5 m/s (150 m/min, or 500 ft/min). What fraction of the engine power is being used to make the airplane climb? (The remainder is used to overcome the effects of air resistance and of inefficiencies in the propeller and engine.)

6.53 •• **Working Like a Horse.** Your job is to lift 30 kg crates a vertical distance of 0.90 m from the ground onto the bed of a truck. How many crates would you have to load onto the truck in 1 minute (a) for the average power output you use to lift the crates to equal 0.50 hp; (b) for an average power output of 100 W?

6.54 •• An elevator has mass 600 kg, not including passengers. The elevator is designed to ascend, at constant speed, a vertical distance of 20.0 m (five floors) in 16.0 s, and it is driven by a motor that can provide up to 40 hp to the elevator. What is the maximum number of passengers that can ride in the elevator? Assume that an average passenger has mass 65.0 kg.

6.55 •• A ski tow operates on a 15.0° slope of length 300 m. The rope moves at 12.0 km/h and provides power for 50 riders at one time, with an average mass per rider of 70.0 kg. Estimate the power required to operate the tow.

6.56 • You are applying a constant horizontal force $\vec{F} = (-8.00\text{ N})\hat{\imath} + (3.00\text{ N})\hat{\jmath}$ to a crate that is sliding on a factory floor. At the instant that the velocity of the crate is $\vec{v} = (3.20\text{ m/s})\hat{\imath} + (2.20\text{ m/s})\hat{\jmath}$, what is the instantaneous power supplied by this force?

6.57 • BIO While hovering, a typical flying insect applies an average force equal to twice its weight during each downward stroke. Take the mass of the insect to be 10 g, and assume the wings move an average downward distance of 1.0 cm during each stroke. Assuming 100 downward strokes per second, estimate the average power output of the insect.

PROBLEMS

6.58 ••• CALC A balky cow is leaving the barn as you try harder and harder to push her back in. In coordinates with the origin at the barn door, the cow walks from $x = 0$ to $x = 6.9$ m as you apply a force with x-component $F_x = -[20.0\text{ N} + (3.0\text{ N/m})x]$. How much work does the force you apply do on the cow during this displacement?

6.59 • A luggage handler pulls a 20.0 kg suitcase up a ramp inclined at 32.0° above the horizontal by a force $\vec{F}$ of magnitude 160 N that acts parallel to the ramp. The coefficient of kinetic friction between the ramp and the incline is $\mu_k = 0.300$. If the suitcase travels 3.80 m along the ramp, calculate (a) the work done on the suitcase by $\vec{F}$; (b) the work done on the suitcase by the gravitational force; (c) the work done on the suitcase by the normal force; (d) the work done on the suitcase by the friction force; (e) the total work done on the suitcase. (f) If the speed of the suitcase is zero at the bottom of the ramp, what is its speed after it has traveled 3.80 m along the ramp?

6.60 •• CP A can of beans that has mass M is launched by a spring-powered device from level ground. The can is launched at an angle of α_0 above the horizontal and is in the air for time T before it returns to the ground. Air resistance can be neglected. (a) How much work was done on the can by the launching device? (b) How much work is done on the can if it is launched at the same angle α_0 but stays in the air twice as long? How does your result compare to the answer to part (a)?

6.61 •• A 5.00 kg block is released from rest on a ramp that is inclined at an angle of 60.0° below the horizontal. The initial position of the block is a vertical distance of 2.00 m above the bottom of the ramp. (a) If the speed of the block is 5.00 m/s when it reaches the bottom of the ramp, what was the work done on it by the friction force? (b) If the angle of the ramp is changed but the block is released from a point that is still 2.00 m above the base of the ramp, both the magnitude of the friction force and the distance along the ramp that the block travels change. If the angle of the incline is changed to 50.0°, does the magnitude of the work done by the friction force increase or decrease compared to the value calculated in part (a)? (c) How much work is done by friction when the ramp angle is 50.0°?

6.62 •• A block of mass m is released from rest at the top of an incline that makes an angle α with the horizontal. The coefficient of kinetic friction between the block and incline is μ_k. The top of the incline is a vertical distance h above the bottom of the incline. Derive an expression for the work W_f done on the block by friction as it travels from the top of the incline to the bottom. When α is decreased, does the magnitude of W_f increase or decrease?

6.63 ••• Consider the blocks in Exercise 6.7 as they move 75.0 cm. Find the total work done on each one (a) if there is no friction between the table and the 20.0 N block, and (b) if $\mu_s = 0.500$ and $\mu_k = 0.325$ between the table and the 20.0 N block.

6.64 •• A 5.00 kg package slides 2.80 m down a long ramp that is inclined at 24.0° below the horizontal. The coefficient of kinetic friction between the package and the ramp is $\mu_k = 0.310$. Calculate (a) the work done on the package by friction; (b) the work done on the package by gravity; (c) the work done on the package by the normal force; (d) the total work done on the package. (e) If the package has a speed of 2.20 m/s at the top of the ramp, what is its speed after it has slid 2.80 m down the ramp?

6.65 •• CP BIO **Whiplash Injuries.** When a car is hit from behind, its passengers undergo sudden forward acceleration, which can cause a severe neck injury known as *whiplash.* During normal acceleration, the neck muscles play a large role in accelerating the head so that the bones are not injured. But during a very sudden acceleration, the muscles do not react immediately because they are flexible; most of the accelerating force is provided by the neck bones. Experiments have shown that these bones will fracture if they absorb more than 8.0 J of energy. (a) If a car waiting at a stoplight is rear-ended in a collision that lasts for 10.0 ms, what is the greatest speed this car and its driver can reach without breaking neck bones if the driver's head has a mass of 5.0 kg (which is about right for a 70 kg person)? Express your answer in m/s and in mi/h. (b) What is the acceleration of the passengers during the collision in part (a), and how large a force is acting to accelerate their heads? Express the acceleration in m/s^2 and in g's.

6.66 •• CALC A net force along the x-axis that has x-component $F_x = -12.0\text{ N} + (0.300\text{ N/m}^2)x^2$ is applied to a 5.00 kg object that is initially at the origin and moving in the $-x$-direction with a speed of 6.00 m/s. What is the speed of the object when it reaches the point $x = 5.00$ m?

6.67 • CALC Varying Coefficient of Friction. A box is sliding with a speed of 4.50 m/s on a horizontal surface when, at point P, it encounters a rough section. The coefficient of friction there is not constant; it starts at 0.100 at P and increases linearly with distance past P, reaching a value of 0.600 at 12.5 m past point P. (a) Use the work–energy theorem to find how far this box slides before stopping. (b) What is the coefficient of friction at the stopping point? (c) How far would the box have slid if the friction coefficient didn't increase but instead had the constant value of 0.100?

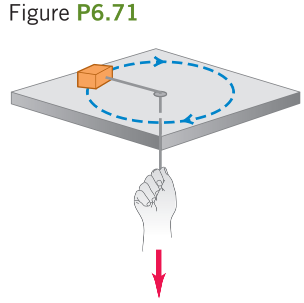
6.68 •• CALC Consider a spring that does not obey Hooke's law very faithfully. One end of the spring is fixed. To keep the spring stretched or compressed an amount x, a force along the x-axis with x-component $F_x = kx - bx^2 + cx^3$ must be applied to the free end. Here $k = 100$ N/m, $b = 700\text{ N/m}^2$, and $c = 12{,}000\text{ N/m}^3$. Note that $x > 0$ when the spring is stretched and $x < 0$ when it is compressed. How much work must be done (a) to stretch this spring by 0.050 m from its unstretched length? (b) To *compress* this spring by 0.050 m from its unstretched length? (c) Is it easier to stretch or compress this spring? Explain why in terms of the dependence of F_x on x. (Many real springs behave qualitatively in the same way.)

6.69 •• A net horizontal force $\vec{F}$ is applied to a box with mass M that is on a horizontal, frictionless surface. The box is initially at rest and then moves in the direction of the force. After the box has moved a distance D, the work that the constant force has done on it is W_D and the speed of the box is V. The equation $P = Fv$ tells us that the instantaneous rate at which $\vec{F}$ is doing work on the box depends on the speed of the box. (a) At the point in the motion of the box where the force has done half the total work, and so has done work $W_D/2$ on the box that started from rest, in terms of V what is the speed of the box? Is the speed at this point less than, equal to, or greater than half the final speed? (b) When the box has reached half its final speed, so its speed is $V/2$, how much work has been done on the box? Express your answer in terms of W_D. Is the amount of work done to produce this speed less than, equal to, or greater than half the work W_D done for the full displacement D?

6.70 •• You weigh 530 N. Your bathroom scale contains a light but very stiff ideal spring. When you stand at rest on the scale, the spring is compressed 1.80 cm. Your 180 N dog then gently jumps into your arms. How much work is done by the spring as the two of you are brought to rest by friction?

6.71 •• CP A small block with a mass of 0.0600 kg is attached to a cord passing through a hole in a frictionless, horizontal surface (**Fig. P6.71**). The block is originally revolving at a distance of 0.40 m from the hole with a speed of 0.70 m/s. The cord is then pulled from below, shortening the radius of the circle in which the block revolves to 0.10 m. At this new distance, the speed of the block is 2.80 m/s. (a) What is the tension in the cord in the original situation, when the block has speed $v = 0.70$ m/s? (b) What is the tension in the cord in the final situation, when the block has speed $v = 2.80$ m/s? (c) How much work was done by the person who pulled on the cord?

Figure **P6.71**

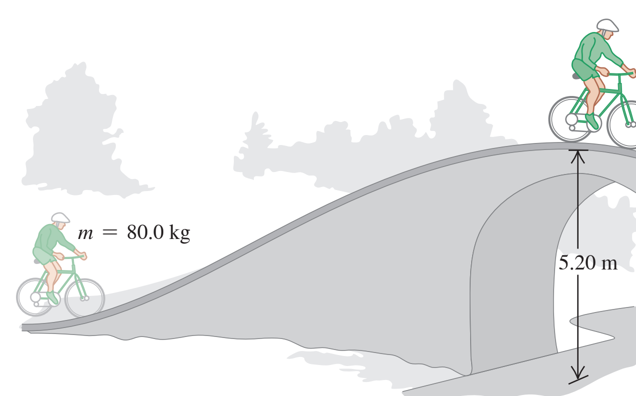


6.72 •• CALC Proton Bombardment. A proton with mass 1.67×10^{-27} kg is propelled at an initial speed of 3.00×10^5 m/s directly toward a uranium nucleus 5.00 m away. The proton is repelled by the uranium nucleus with a force of magnitude $F = \alpha/x^2$, where x is the separation between the two objects and $\alpha = 2.12 \times 10^{-26}\text{ N}\cdot\text{m}^2$. Assume that the uranium nucleus remains at rest. (a) What is the speed of the proton when it is 8.00×10^{-10} m from the uranium nucleus? (b) As the proton approaches the uranium nucleus, the repulsive force slows down the proton until it comes momentarily to rest, after which the proton moves away from the uranium nucleus. How close to the uranium nucleus does the proton get? (c) What is the speed of the proton when it is again 5.00 m away from the uranium nucleus?

6.73 •• You are asked to design spring bumpers for the walls of a parking garage. A freely rolling 1200 kg car moving at 0.65 m/s is to compress the spring no more than 0.090 m before stopping. What should be the force constant of the spring? Assume that the spring has negligible mass.

6.74 •• You and your bicycle have combined mass 80.0 kg. When you reach the base of a bridge, you are traveling along the road at 5.00 m/s (**Fig. P6.74**). At the top of the bridge, you have climbed a vertical distance of 5.20 m and slowed to 1.50 m/s. Ignore work done by friction and any inefficiency in the bike or your legs. (a) What is the total work done on you and your bicycle when you go from the base to the top of the bridge? (b) How much work have you done with the force you apply to the pedals?

Figure **P6.74**



6.75 ••• A 2.50 kg textbook is forced against one end of a horizontal spring of negligible mass that is fixed at the other end and has force constant 250 N/m, compressing the spring a distance of 0.250 m. When released, the textbook slides on a horizontal tabletop with coefficient of kinetic friction $\mu_k = 0.30$. Use the work–energy theorem to find how far the textbook moves from its initial position before it comes to rest.

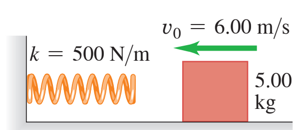
6.76 •• The spring of a spring gun has force constant $k = 400$ N/m and negligible mass. The spring is compressed 6.00 cm, and a ball with mass 0.0300 kg is placed in the horizontal barrel against the compressed spring. The spring is then released, and the ball is propelled out the barrel of the gun. The barrel is 6.00 cm long, so the ball leaves the barrel at the same point that it loses contact with the spring. The gun is held so that the barrel is horizontal. (a) Calculate the speed with which the ball leaves the barrel if you can ignore friction. (b) Calculate the speed of the ball as it leaves the barrel if a constant resisting force of 6.00 N acts on the ball as it moves along the barrel. (c) For the situation in part (b), at what position along the barrel does the ball have the greatest speed, and what is that speed? (In this case, the maximum speed does not occur at the end of the barrel.)

6.77 •• One end of a horizontal spring with force constant 130.0 N/m is attached to a vertical wall. A 4.00 kg block sitting on the floor is placed against the spring. The coefficient of kinetic friction between the block and the floor is $\mu_k = 0.400$. You apply a constant force $\vec{F}$ to the block. $\vec{F}$ has magnitude $F = 82.0$ N and is directed toward the wall. At the instant that the spring is compressed 80.0 cm, what are (a) the speed of the block, and (b) the magnitude and direction of the block's acceleration?

6.78 •• One end of a horizontal spring with force constant 76.0 N/m is attached to a vertical post. A 2.00 kg block of frictionless ice is attached to the other end and rests on the floor. The spring is initially neither stretched nor compressed. A constant horizontal force of 54.0 N is then applied to the block, in the direction away from the post. (a) What is the speed of the block when the spring is stretched 0.400 m? (b) At that instant, what are the magnitude and direction of the acceleration of the block?

6.79 • A 5.00 kg block is moving at $v_0 = 6.00$ m/s along a frictionless, horizontal surface toward a spring with force constant $k = 500$ N/m that is attached to a wall (**Fig. P6.79**). The spring has negligible mass. (a) Find the maximum distance the spring will be compressed. (b) If the spring is to compress by no more than 0.150 m, what should be the maximum value of v_0?

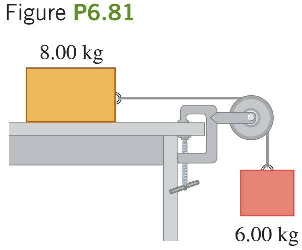
Figure **P6.79**



6.80 ••• A physics professor is pushed up a ramp inclined upward at 30.0° above the horizontal as she sits in her desk chair, which slides on frictionless rollers. The combined mass of the professor and chair is 85.0 kg. She is pushed 2.50 m along the incline by a group of students who together exert a constant horizontal force of 600 N. The professor's speed at the bottom of the ramp is 2.00 m/s. Use the work–energy theorem to find her speed at the top of the ramp.

6.81 •• Consider the system shown in **Fig. P6.81.** The rope and pulley have negligible mass, and the pulley is frictionless. Initially the 6.00 kg block is moving downward and the 8.00 kg block is moving to the right, both with a speed of 0.900 m/s. The blocks come to rest after moving 2.00 m. Use the work–energy theorem to calculate the coefficient of kinetic friction between the 8.00 kg block and the tabletop.

Figure **P6.81**



6.82 •• Consider the system shown in Fig. P6.81. The rope and pulley have negligible mass, and the pulley is frictionless. The coefficient of kinetic friction between the 8.00 kg block and the tabletop is $\mu_k = 0.250$. The blocks are released from rest. Use energy methods to calculate the speed of the 6.00 kg block after it has descended 1.50 m.

6.83 •• On an essentially frictionless, horizontal ice rink, a skater moving at 3.0 m/s encounters a rough patch that reduces her speed to 1.65 m/s due to a friction force that is 25% of her weight. Use the work–energy theorem to find the length of this rough patch.

6.84 •• **BIO** All birds, independent of their size, must maintain a power output of 10–25 watts per kilogram of object mass in order to fly by flapping their wings. (a) The Andean giant hummingbird (*Patagona gigas*) has mass 70 g and flaps its wings 10 times per second while hovering. Estimate the amount of work done by such a hummingbird in each wingbeat. (b) A 70 kg athlete can maintain a power output of 1.4 kW for no more than a few seconds; the *steady* power output of a typical athlete is only 500 W or so. Is it possible for a human-powered aircraft to fly for extended periods by flapping its wings? Explain.

6.85 •• A pump is required to lift 800 kg of water (about 210 gallons) per minute from a well 14.0 m deep and eject it with a speed of 18.0 m/s. (a) How much work is done per minute in lifting the water? (b) How much work is done in giving the water the kinetic energy it has when ejected? (c) What must be the power output of the pump?

6.86 •• The upper end of a light rope of length $L = 0.600$ m is attached to the ceiling, and a small steel ball with mass $m = 0.200$ kg is suspended from the lower end of the rope. Initially the ball is at rest and the rope is vertical. Then a force $\vec{F}$ with constant magnitude $F = 0.760$ N and a direction that is maintained tangential to the path of the ball is applied and the ball moves in an arc of a circle of radius L. What is the speed of the ball when the rope makes an angle $\alpha = 37.0°$ with the vertical?

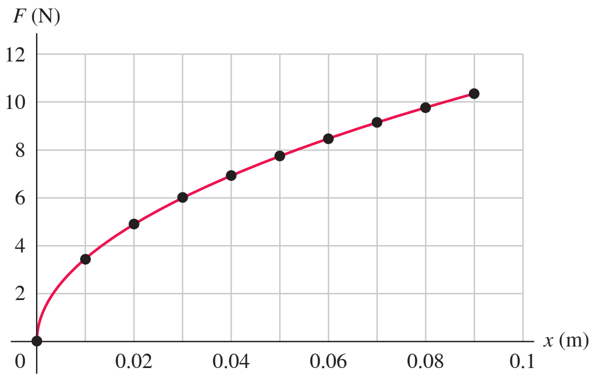
6.87 •• Consider the system of two blocks shown in Fig. P6.81, but with a different friction force on the 8.00 kg block. The blocks are released from rest. While the two blocks are moving, the tension in the light rope that connects them is 37.0 N. (a) During a 0.800 m downward displacement of the 6.00 kg block, how much work has been done on it by gravity? By the tension T in the rope? Use the work–energy theorem to find the speed of the 6.00 kg block after it has descended 0.800 m. (b) During the 0.800 m displacement of the 6.00 kg block, what is the total work done on the 8.00 kg block? During this motion how much work was done on the 8.00 kg block by the tension T in the cord? By the friction force exerted on the 8.00 kg block? (c) If the work–energy theorem is applied to the two blocks considered together as a composite system, use the theorem to find the net work done on the system during the 0.800 m downward displacement of the 6.00 kg block. How much work was done on the system of two blocks by gravity? By friction? By the tension in the rope?

6.88 • **CALC** An object has several forces acting on it. One of these forces is $\vec{F} = \alpha xy\hat{\imath}$, a force in the x-direction whose magnitude depends on the position of the object, with $\alpha = 2.50$ N/m^2. Calculate the work done on the object by this force for the following displacements of the object: (a) The object starts at the point $(x = 0, y = 3.00\text{ m})$ and moves parallel to the x-axis to the point $(x = 2.00\text{ m}, y = 3.00\text{ m})$. (b) The object starts at the point $(x = 2.00\text{ m}, y = 0)$ and moves in the y-direction to the point $(x = 2.00\text{ m}, y = 3.00\text{ m})$. (c) The object starts at the origin and moves on the line $y = 1.5x$ to the point $(x = 2.00\text{ m}, y = 3.00\text{ m})$.

6.89 • **BIO** **Power of the Human Heart.** The human heart is a powerful and extremely reliable pump. Each day it takes in and discharges about 7500 L of blood. Assume that the work done by the heart is equal to the work required to lift this amount of blood a height equal to that of the average American woman (1.63 m). The density (mass per unit volume) of blood is 1.05×10^3 kg/m^3. (a) How much work does the heart do in a day? (b) What is the heart's power output in watts?

6.90 •• **DATA** **Figure P6.90** shows the results of measuring the force F exerted on both ends of a rubber band to stretch it a distance x from its unstretched position. (Source: www.sciencebuddies.org) The data points are well fit by the equation $F = 33.55x^{0.4871}$, where F is in newtons and x is in meters. (a) Does this rubber band obey Hooke's law over the range of x shown in the graph? Explain. (b) The stiffness of a spring that obeys Hooke's law is measured by the value of its force constant k, where $k = F/x$. This can be written as $k = dF/dx$ to emphasize the quantities that are changing. Define $k_{eff} = dF/dx$ and calculate k_{eff} as a function of x for this rubber band. For a spring that obeys Hooke's law, k_{eff} is constant, independent of x. Does the stiffness of this band, as measured by k_{eff}, increase or decrease as x is increased, within the range of the data? (c) How much work must be done to stretch the rubber band from $x = 0$ to $x = 0.0400$ m? From $x = 0.0400$ m to $x = 0.0800$ m? (d) One end of the rubber band is attached to a stationary vertical rod, and the band is stretched horizontally 0.0800 m from its unstretched length. A 0.300 kg object on a horizontal, frictionless surface is attached to the free end of the rubber band and released from rest. What is the speed of the object after it has traveled 0.0400 m?

Figure **P6.90**

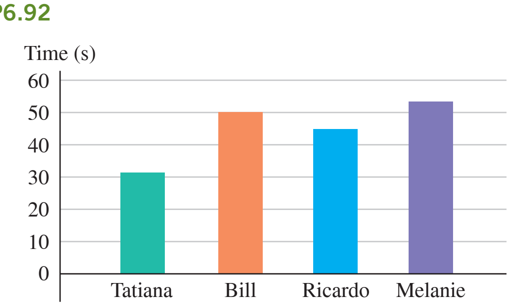


6.91 ••• DATA In a physics lab experiment, one end of a horizontal spring that obeys Hooke's law is attached to a wall. The spring is compressed 0.400 m, and a block with mass 0.300 kg is attached to it. The spring is then released, and the block moves along a horizontal surface. Electronic sensors measure the speed v of the block after it has traveled a distance d from its initial position against the compressed spring. The measured values are listed in the table. (a) The data show that the speed v of the block increases and then decreases as the spring returns to its unstretched length. Explain why this happens, in terms of the work done on the block by the forces that act on it. (b) Use the work–energy theorem to derive an expression for v^2 in terms of d. (c) Use a computer graphing program (for example, Excel or Matlab) to graph the data as v^2 (vertical axis) versus d (horizontal axis). The equation that you derived in part (b) should show that v^2 is a quadratic function of d, so, in your graph, fit the data by a second-order polynomial (quadratic) and have the graphing program display the equation for this trendline. Use that equation to find the block's maximum speed v and the value of d at which this speed occurs. (d) By comparing the equation from the graphing program to the formula you derived in part (b), calculate the force constant k for the spring and the coefficient of kinetic friction for the friction force that the surface exerts on the block.

d (m)	v (m/s)
0	0
0.05	0.85
0.10	1.11
0.15	1.24
0.25	1.26
0.30	1.14
0.35	0.90
0.40	0.36

6.92 •• DATA For a physics lab experiment, four classmates run up the stairs from the basement to the top floor of their physics building—a vertical distance of 16.0 m. The classmates and their masses are: Tatiana, 50.2 kg; Bill, 68.2 kg; Ricardo, 81.8 kg; and Melanie, 59.1 kg. The time it takes each of them is shown in **Fig. P6.92.** (a) Considering only the work done against gravity, which person had the largest average power output? The smallest? (b) Chang is very fit and has mass 62.3 kg. If his average power output is 1.00 hp, how many seconds does it take him to run up the stairs?

Figure **P6.92**

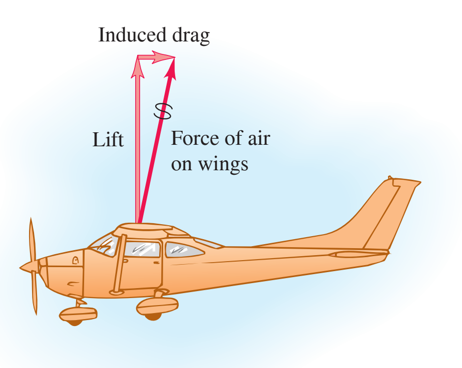


CHALLENGE PROBLEMS

6.93 ••• CALC A Spring with Mass. We usually ignore the kinetic energy of the moving coils of a spring, but let's try to get a reasonable approximation to this. Consider a spring of mass M, equilibrium length L_0, and force constant k. The work done to stretch or compress the spring by a distance L is $\frac{1}{2}kX^2$, where $X = L - L_0$. Consider a spring, as described above, that has one end fixed and the other end moving with speed v. Assume that the speed of points along the length of the spring varies linearly with distance l from the fixed end. Assume also that the mass M of the spring is distributed uniformly along the length of the spring. (a) Calculate the kinetic energy of the spring in terms of M and v. (*Hint:* Divide the spring into pieces of length dl; find the speed of each piece in terms of l, v, and L; find the mass of each piece in terms of dl, M, and L; and integrate from 0 to L. The result is *not* $\frac{1}{2}Mv^2$, since not all of the spring moves with the same speed.) In a spring gun, a spring of mass 0.243 kg and force constant 3200 N/m is compressed 2.50 cm from its unstretched length. When the trigger is pulled, the spring pushes horizontally on a 0.053 kg ball. The work done by friction is negligible. Calculate the ball's speed when the spring reaches its uncompressed length (b) ignoring the mass of the spring and (c) including, using the results of part (a), the mass of the spring. (d) In part (c), what is the final kinetic energy of the ball and of the spring?

6.94 ••• CALC An airplane in flight is subject to an air resistance force proportional to the square of its speed v. But there is an additional resistive force because the airplane has wings. Air flowing over the wings is pushed down and slightly forward, so from Newton's third law the air exerts a force on the wings and airplane that is up and slightly backward (**Fig. P6.94**). The upward force is the lift force that keeps the airplane aloft, and the backward force is called *induced drag.* At flying speeds, induced drag is inversely proportional to v^2, so the total air resistance force can be expressed by $F_{air} = \alpha v^2 + \beta/v^2$, where α and β are positive constants that depend on the shape and size of the airplane and the density of the air. For a Cessna 150, a small single-engine airplane, $\alpha = 0.30\ \mathrm{N \cdot s^2/m^2}$ and $\beta = 3.5 \times 10^5\ \mathrm{N \cdot m^2/s^2}$. In steady flight, the engine must provide a forward force that exactly balances the air resistance force. (a) Calculate the speed (in km/h) at which this airplane will have the maximum *range* (that is, travel the greatest distance) for a given quantity of fuel. (b) Calculate the speed (in km/h) for which the airplane will have the maximum *endurance* (that is, remain in the air the longest time).

Figure **P6.94**



MCAT-STYLE PASSAGE PROBLEMS

BIO Energy of locomotion. On flat ground, a 70 kg person requires about 300 W of metabolic power to walk at a steady pace of 5.0 km/h (1.4 m/s). Using the same metabolic power output, that person can bicycle over the same ground at 15 km/h.

6.95 Based on the given data, how does the energy used in biking 1 km compare with that used in walking 1 km? Biking takes (a) $\frac{1}{3}$ of the energy of walking the same distance; (b) the same energy as walking the same distance; (c) 3 times the energy of walking the same distance; (d) 9 times the energy of walking the same distance.

6.96 A 70 kg person walks at a steady pace of 5.0 km/h on a treadmill at a 5.0% grade. (That is, the vertical distance covered is 5.0% of the horizontal distance covered.) If we assume the metabolic power required is equal to that required for walking on a flat surface plus the rate of doing work for the vertical climb, how much power is required? (a) 300 W; (b) 315 W; (c) 350 W; (d) 370 W.

6.97 How many times greater is the kinetic energy of the person when biking than when walking? Ignore the mass of the bike. (a) 1.7; (b) 3; (c) 6; (d) 9.

ANSWERS

Chapter Opening Question ?

(ii) The expression for kinetic energy is $K = \frac{1}{2}mv^2$. If we calculate K for the three balls, we find (i) $K = \frac{1}{2}(0.145\text{ kg}) \times (20.0\text{ m/s})^2 = 29.0\text{ kg}\cdot\text{m}^2/\text{s}^2 = 29.0\text{ J}$, (ii) $K = \frac{1}{2}(0.0145\text{ kg}) \times (200\text{ m/s})^2 = 290\text{ J}$, and (iii) $K = \frac{1}{2}(1.45\text{ kg})(2.00\text{ m/s})^2 = 2.90\text{ J}$. The smaller ball has the least mass of all three, but it also has the greatest speed and so the most kinetic energy. Since kinetic energy is a scalar, it does not depend on the direction of motion.

Key Example ✓ARIATION Problems

VP6.2.1 (a) -714 J (b) $+714$ J
VP6.2.2 35.5°
VP6.2.3 (a) -90.0 J (b) 207 J (c) zero (d) 117 J
VP6.2.4 (a) Student 1: $2F_0d$; student 2: $-3F_0d$; student 3: $-8F_0d + Gd$ (b) $9F_0$
VP6.4.1 (a) 5.20 m/s (b) 9.18×10^3 N
VP6.4.2 (a) 219 J (b) -178 J (c) 41 J (d) 43 J, 2.4 m/s
VP6.4.3 (a) Initial: 6.30×10^3 J; final: 142 J (b) Total: -6.16×10^3 J; gravity: 2.74×10^4 J; thrust: -3.36×10^4 J (c) 1.68×10^4 N
VP6.4.4 (a) $mgd\sin\theta$ (b) $-\mu_k mgd\cos\theta$ (c) $\sqrt{2gd(\sin\theta - \mu_k\cos\theta)}$
VP6.8.1 (a) 8.84 cm (b) 7.41 cm
VP6.8.2 $(mv^2 - kd^2)/2mgd$
VP6.8.3 (a) $+0.665$ J (b) -0.665 J (c) zero
VP6.8.4 $\sqrt{2gL(1 - \cos\theta)}$

Bridging Problem

(a) $v_1 = \sqrt{\dfrac{2}{m}\left(mgx_1 - \frac{1}{3}\alpha x_1^3\right)} = \sqrt{2gx_1 - \dfrac{2\alpha x_1^3}{3m}}$

(b) $P = -F_{\text{spring-1}}v_1 = -\alpha x_1^2\sqrt{2gx_1 - \dfrac{2\alpha x_1^3}{3m}}$

(c) $x_2 = \sqrt{\dfrac{3mg}{\alpha}}$ (d) no

? As this sandhill crane (*Grus canadensis*) glides in to a landing, it descends along a straight-line path at a constant speed. During the glide, what happens to the total mechanical energy (the sum of kinetic energy and gravitational potential energy)? (i) It stays the same; (ii) it increases due to the effect of gravity; (iii) it increases due to the effect of the air; (iv) it decreases due to the effect of gravity; (v) it decreases due to the effect of the air.

7 Potential Energy and Energy Conservation

When a diver jumps off a high board into a swimming pool, she hits the water moving pretty fast, with a lot of kinetic energy—energy of *motion*. Where does that energy come from? The answer we learned in Chapter 6 was that the gravitational force does work on the diver as she falls, and her kinetic energy increases by an amount equal to the work done.

However, there's a useful alternative way to think about work and kinetic energy. This new approach uses the idea of *potential energy*, which is associated with the *position* of a system rather than with its motion. In this approach, there is *gravitational potential energy* even when the diver is at rest on the high board. As she falls, this potential energy is *transformed* into her kinetic energy.

If the diver bounces on the end of the board before she jumps, the bent board stores a second kind of potential energy called *elastic potential energy*. We'll discuss elastic potential energy of simple systems such as a stretched or compressed spring. (An important third kind of potential energy is associated with the forces between electrically charged objects. We'll return to this in Chapter 23.)

We'll prove that in some cases the sum of a system's kinetic and potential energies, called the *total mechanical energy* of the system, is constant during the motion of the system. This will lead us to the general statement of the *law of conservation of energy,* one of the most fundamental principles in all of science.

LEARNING OUTCOMES

In this chapter, you'll learn...

7.1 How to use the concept of gravitational potential energy in problems that involve vertical motion.

7.2 How to use the concept of elastic potential energy in problems that involve a moving object attached to a stretched or compressed spring.

7.3 The distinction between conservative and nonconservative forces, and how to solve problems in which both kinds of forces act on a moving object.

7.4 How to calculate the properties of a conservative force if you know the corresponding potential-energy function.

7.5 How to use energy diagrams to understand how an object moves in a straight line under the influence of a conservative force.

You'll need to review...

5.3 Kinetic friction and fluid resistance.

5.4 Dynamics of circular motion.

6.1, 6.2 Work and the work–energy theorem.

6.3 Work done by an ideal spring.

7.1 GRAVITATIONAL POTENTIAL ENERGY

In many situations it seems as though energy has been stored in a system, to be recovered later. For example, you must do work to lift a heavy stone over your head. It seems reasonable that in hoisting the stone into the air you are storing energy in the system, energy that is later converted into kinetic energy when you let the stone fall.

Figure **7.1** The greater the height of a basketball, the greater the associated gravitational potential energy. As the basketball descends, gravitational potential energy is converted to kinetic energy and the basketball's speed increases.

This example points to the idea of an energy associated with the *position* of objects in a system. This kind of energy is a measure of the *potential* or *possibility* for work to be done; if you raise a stone into the air, there is a potential for the gravitational force to do work on it, but only if you allow the stone to fall to the ground. For this reason, energy associated with position is called **potential energy.** The potential energy associated with an object's weight and its height above the ground is called *gravitational potential energy* (**Fig. 7.1**).

We now have *two* ways to describe what happens when an object falls without air resistance. One way, which we learned in Chapter 6, is to say that a falling object's kinetic energy increases because the force of the earth's gravity does work on the object. The other way is to say that the kinetic energy increases as the gravitational potential energy decreases. Later in this section we'll use the work–energy theorem to show that these two descriptions are equivalent.

Let's derive the expression for gravitational potential energy. Suppose an object with mass m moves along the (vertical) y-axis, as in **Fig. 7.2**. The forces acting on it are its weight, with magnitude $w = mg$, and possibly some other forces; we call the vector sum (resultant) of all the other forces $\vec{F}_{\text{other}}$. We'll assume that the object stays close enough to the earth's surface that the weight is constant. (We'll find in Chapter 13 that weight decreases with altitude.) We want to find the work done by the weight when the object moves downward from a height y_1 above the origin to a lower height y_2 (Fig. 7.2a). The weight and displacement are in the same direction, so the work W_{grav} done on the object by its weight is positive:

$$W_{\text{grav}} = Fs = w(y_1 - y_2) = mgy_1 - mgy_2 \tag{7.1}$$

This expression also gives the correct work when the object moves *upward* and y_2 is greater than y_1 (Fig. 7.2b). In that case the quantity $(y_1 - y_2)$ is negative, and W_{grav} is negative because the weight and displacement are opposite in direction.

Equation (7.1) shows that we can express W_{grav} in terms of the values of the quantity mgy at the beginning and end of the displacement. This quantity is called the **gravitational potential energy,** U_{grav}:

Gravitational potential energy associated with a particle

Vertical coordinate of particle (y increases if particle moves upward)

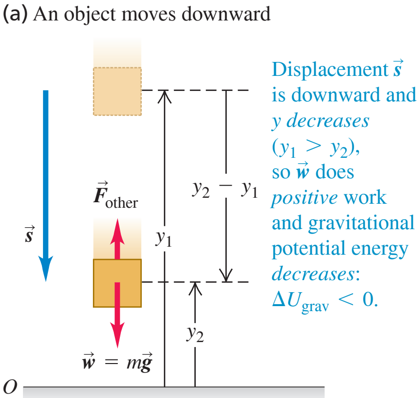
$$U_{\text{grav}} = mgy \tag{7.2}$$

Mass of particle

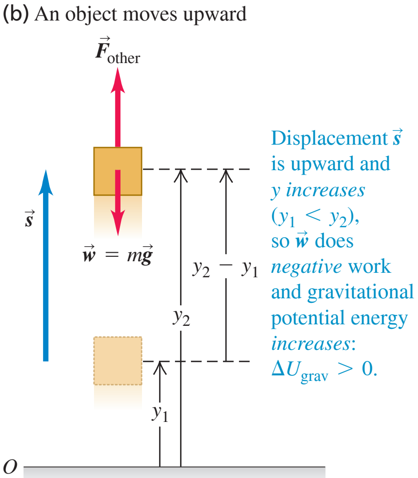
Acceleration due to gravity

Figure **7.2** When an object moves vertically from an initial height y_1 to a final height y_2, the gravitational force $\vec{w}$ does work and the gravitational potential energy changes.

(a) An object moves downward



(b) An object moves upward



Its initial value is $U_{\text{grav},1} = mgy_1$ and its final value is $U_{\text{grav},2} = mgy_2$. The change in U_{grav} is the final value minus the initial value, or $\Delta U_{\text{grav}} = U_{\text{grav},2} - U_{\text{grav},1}$. Using Eq. (7.2), we can rewrite Eq. (7.1) for the work done by the gravitational force during the displacement from y_1 to y_2:

$$W_{\text{grav}} = U_{\text{grav},1} - U_{\text{grav},2} = -(U_{\text{grav},2} - U_{\text{grav},1}) = -\Delta U_{\text{grav}}$$

or

Work done by the gravitational force on a particle ...

... equals the **negative of the change in the gravitational potential energy.**

$$W_{\text{grav}} = mgy_1 - mgy_2 = U_{\text{grav},1} - U_{\text{grav},2} = -\Delta U_{\text{grav}} \tag{7.3}$$

Mass of particle

Acceleration due to gravity

Initial and final vertical coordinates of particle

The negative sign in front of ΔU_{grav} is *essential*. When the object moves up, y increases, the work done by the gravitational force is negative, and the gravitational potential energy increases $(\Delta U_{\text{grav}} > 0)$. When the object moves down, y decreases, the gravitational force does positive work, and the gravitational potential energy decreases $(\Delta U_{\text{grav}} < 0)$. It's like drawing money out of the bank (decreasing U_{grav}) and spending it (doing positive work). The unit of potential energy is the joule (J), the same unit as is used for work.

CAUTION **To what object does gravitational potential energy "belong"?** It is *not* correct to call $U_{grav} = mgy$ the "gravitational potential energy of the object." The reason is that U_{grav} is a *shared* property of the object and the earth. The value of U_{grav} increases if the earth stays fixed and the object moves upward, away from the earth; it also increases if the object stays fixed and the earth is moved away from it. Notice that the formula $U_{grav} = mgy$ involves characteristics of both the object (its mass m) and the earth (the value of g).

Conservation of Total Mechanical Energy (Gravitational Forces Only)

To see what gravitational potential energy is good for, suppose an object's weight is the *only* force acting on it, so $\vec{F}_{other} = \mathbf{0}$. The object is then falling freely with no air resistance and can be moving either up or down. Let its speed at point y_1 be v_1 and let its speed at y_2 be v_2. The work–energy theorem, Eq. (6.6), says that the total work done on the object equals the change in the object's kinetic energy: $W_{tot} = \Delta K = K_2 - K_1$. If gravity is the only force that acts, then from Eq. (7.3), $W_{tot} = W_{grav} = -\Delta U_{grav} = U_{grav,1} - U_{grav,2}$. Putting these together, we get

$$\Delta K = -\Delta U_{grav} \qquad \text{or} \qquad K_2 - K_1 = U_{grav,1} - U_{grav,2}$$

which we can rewrite as

If only the gravitational force does work, total mechanical energy is conserved:

Initial kinetic energy $K_1 = \frac{1}{2}mv_1^2$

Initial gravitational potential energy $U_{grav,1} = mgy_1$

$$K_1 + U_{grav,1} = K_2 + U_{grav,2} \tag{7.4}$$

Final kinetic energy $K_2 = \frac{1}{2}mv_2^2$

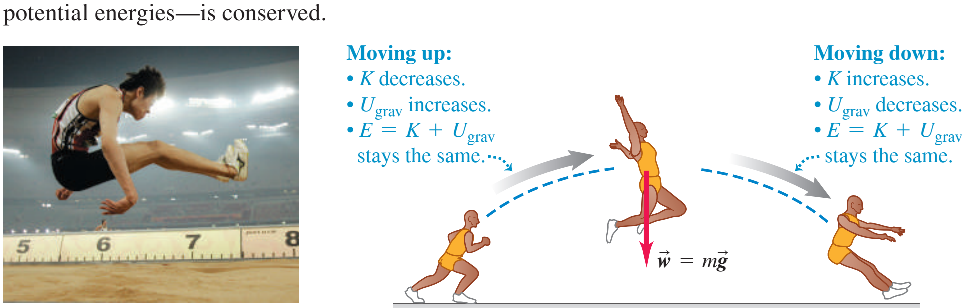
Final gravitational potential energy $U_{grav,2} = mgy_2$

The sum $K + U_{grav}$ of kinetic and potential energies is called E, the **total mechanical energy of the system.** By "system" we mean the object of mass m and the earth considered together, because gravitational potential energy U is a shared property of both objects. Then $E_1 = K_1 + U_{grav,1}$ is the total mechanical energy at y_1 and $E_2 = K_2 + U_{grav,2}$ is the total mechanical energy at y_2. Equation (7.4) says that when the object's weight is the only force doing work on it, $E_1 = E_2$. That is, E is constant; it has the same value at y_1 and y_2. But since positions y_1 and y_2 are arbitrary points in the motion of the object, the total mechanical energy E has the same value at *all* points during the motion:

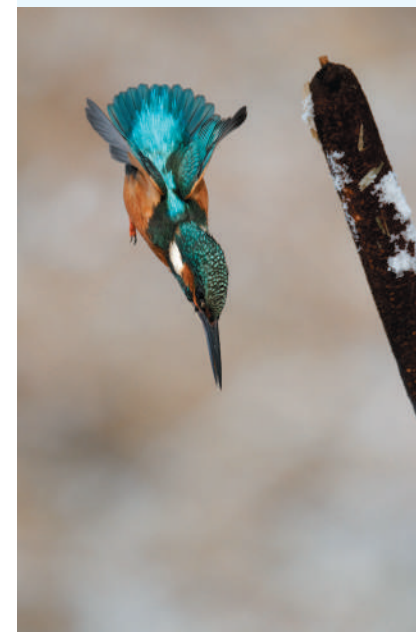
$$E = K + U_{grav} = \text{constant} \qquad \text{(if only gravity does work)}$$

A quantity that always has the same value is called a *conserved* quantity. *When only the force of gravity does work, the total mechanical energy is constant—that is, it is conserved* (**Fig. 7.3**). This is our first example of the **conservation of total mechanical energy.**

Figure 7.3 While this athlete is in midair, only gravity does work on him (if we neglect the minor effects of air resistance). Total mechanical energy E—the sum of kinetic and gravitational potential energies—is conserved.



BIO APPLICATION Converting Gravitational Potential Energy to Kinetic Energy When a kingfisher (*Alcedo atthis*) spots a tasty fish, the bird dives from its perch with its wings tucked in to minimize air resistance. Effectively the only force acting on the diving kingfisher is the force of gravity, so the total mechanical energy is conserved: The gravitational potential energy lost as the kingfisher descends is converted into the bird's kinetic energy.



When we throw a ball into the air, its speed decreases on the way up as kinetic energy is converted to potential energy: $\Delta K < 0$ and $\Delta U_{grav} > 0$. On the way back down, potential energy is converted back to kinetic energy and the ball's speed increases: $\Delta K > 0$ and $\Delta U_{grav} < 0$. But the *total* mechanical energy (kinetic plus potential) is the same at every point in the motion, provided that no force other than gravity does work on the ball (that is, air resistance must be negligible). It's still true that the gravitational force does work on the object as it moves up or down, but we no longer have to calculate work directly; keeping track of changes in the value of U_{grav} takes care of this completely.

Equation (7.4) is also valid if forces other than gravity are present but do *not* do work. We'll see a situation of this kind later, in Example 7.4.

CAUTION **Choose "zero height" to be wherever you like** When working with gravitational potential energy, we may choose any height to be $y = 0$. If we shift the origin for y, the values of y_1 and y_2 change, as do the values of $U_{grav,1}$ and $U_{grav,2}$. But this shift has no effect on the *difference* in height $y_2 - y_1$ or on the *difference* in gravitational potential energy $U_{grav,2} - U_{grav,1} = mg(y_2 - y_1)$. As Example 7.1 shows, the physically significant quantity is not the value of U_{grav} at a particular point but the *difference* in U_{grav} between two points. We can define U_{grav} to be zero at whatever point we choose.

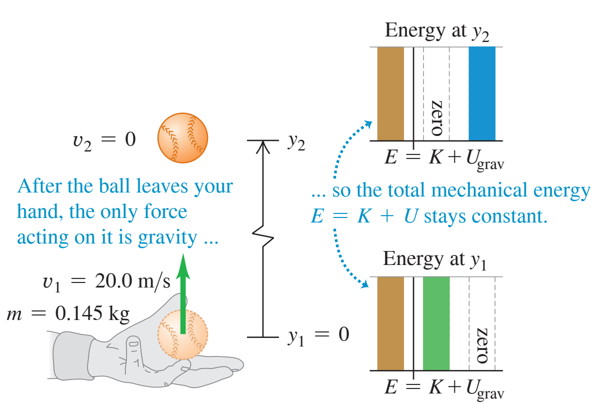
EXAMPLE 7.1 Height of a baseball from energy conservation

WITH VARIATION PROBLEMS

You throw a 0.145 kg baseball straight up, giving it an initial velocity of magnitude 20.0 m/s. Find how high it goes, ignoring air resistance.

IDENTIFY and SET UP After the ball leaves your hand, only gravity does work on it. Hence total mechanical energy is conserved, and we can use Eq. (7.4). We take point 1 to be where the ball leaves your hand and point 2 to be where it reaches its maximum height. As in Fig. 7.2, we take the positive y-direction to be upward. The ball's speed at point 1 is $v_1 = 20.0$ m/s; at its maximum height it is instantaneously at rest, so $v_2 = 0$. We take the origin at point 1, so $y_1 = 0$ (**Fig. 7.4**). Our target variable, the distance the ball moves vertically between the two points, is the displacement $y_2 - y_1 = y_2 - 0 = y_2$.

Figure **7.4** After a baseball leaves your hand, total mechanical energy $E = K + U$ is conserved.



EXECUTE We have $y_1 = 0$, $U_{grav,1} = mgy_1 = 0$, and $K_2 = \frac{1}{2}mv_2^2 = 0$. Then Eq. (7.4), $K_1 + U_{grav,1} = K_2 + U_{grav,2}$, becomes

$$K_1 = U_{grav,2}$$

As the energy bar graphs in Fig. 7.4 show, this equation says that the kinetic energy of the ball at point 1 is completely converted to gravitational potential energy at point 2. We substitute $K_1 = \frac{1}{2}mv_1^2$ and $U_{grav,2} = mgy_2$ and solve for y_2:

$$\frac{1}{2}mv_1^2 = mgy_2$$

$$y_2 = \frac{v_1^2}{2g} = \frac{(20.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 20.4\ \text{m}$$

EVALUATE As a check, use the given value of v_1 and our result for y_2 to calculate the kinetic energy at point 1 and the gravitational potential energy at point 2. You should find that these are equal: $K_1 = \frac{1}{2}mv_1^2 = 29.0$ J and $U_{grav,2} = mgy_2 = 29.0$ J. Note that we could have found the result $y_2 = v_1^2/2g$ by using Eq. (2.13) in the form $v_{2y}^2 = v_{1y}^2 - 2g(y_2 - y_1)$.

What if we put the origin somewhere else—for example, 5.0 m below point 1, so that $y_1 = 5.0$ m? Then the total mechanical energy at point 1 is part kinetic and part potential; at point 2 it's still purely potential because $v_2 = 0$. You'll find that this choice of origin yields $y_2 = 25.4$ m, but again $y_2 - y_1 = 20.4$ m. In problems like this, you are free to choose the height at which $U_{grav} = 0$. The physics doesn't depend on your choice.

KEYCONCEPT Total mechanical energy (the sum of kinetic energy and gravitational potential energy) is conserved when only the force of gravity does work.

When Forces Other Than Gravity Do Work

If other forces act on the object in addition to its weight, then $\vec{F}_{other}$ in Fig. 7.2 is *not* zero. For the pile driver described in Example 6.4 (Section 6.2), the force applied by the hoisting cable and the friction with the vertical guide rails are examples of forces that might be included in $\vec{F}_{\text{other}}$. The gravitational work W_{grav} is still given by Eq. (7.3), but the total work W_{tot} is then the sum of W_{grav} and the work done by $\vec{F}_{\text{other}}$. We'll call this additional work W_{other}, so the total work done by all forces is $W_{\text{tot}} = W_{\text{grav}} + W_{\text{other}}$. Equating this to the change in kinetic energy, we have

$$W_{\text{other}} + W_{\text{grav}} = K_2 - K_1 \tag{7.5}$$

Also, from Eq. (7.3), $W_{\text{grav}} = U_{\text{grav},1} - U_{\text{grav},2}$, so Eq. (7.5) becomes

$$W_{\text{other}} + U_{\text{grav},1} - U_{\text{grav},2} = K_2 - K_1$$

which we can rearrange in the form

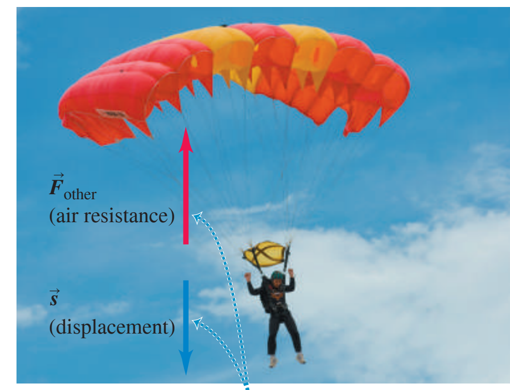
$$K_1 + U_{\text{grav},1} + W_{\text{other}} = K_2 + U_{\text{grav},2} \quad \text{(if forces other than gravity do work)} \tag{7.6}$$

We can use the expressions for the various energy terms to rewrite Eq. (7.6):

$$\tfrac{1}{2}mv_1^2 + mgy_1 + W_{\text{other}} = \tfrac{1}{2}mv_2^2 + mgy_2 \quad \text{(if forces other than gravity do work)} \tag{7.7}$$

The meaning of Eqs. (7.6) and (7.7) is this: *The work done by all forces other than the gravitational force equals the change in the total mechanical energy $E = K + U_{\text{grav}}$ of the system, where U_{grav} is the gravitational potential energy.* When W_{other} is positive, E increases and $K_2 + U_{\text{grav},2}$ is greater than $K_1 + U_{\text{grav},1}$. When W_{other} is negative, E decreases (**Fig. 7.5**). In the special case in which no forces other than the object's weight do work, $W_{\text{other}} = 0$. The total mechanical energy is then constant, and we are back to Eq. (7.4).

Figure **7.5** As this parachutist moves downward at a constant speed, the upward force of air resistance does negative work W_{other} on him. Hence the total mechanical energy $E = K + U$ decreases.



- $\vec{F}_{\text{other}}$ and $\vec{s}$ are opposite, so $W_{\text{other}} < 0$.
- Hence $E = K + U_{\text{grav}}$ must decrease.
- The parachutist's speed remains constant, so K is constant.
- The parachutist descends, so U_{grav} decreases.

PROBLEM-SOLVING STRATEGY 7.1 Problems Using Total Mechanical Energy I

IDENTIFY *the relevant concepts:* Decide whether the problem should be solved by energy methods, by using $\sum\vec{F} = m\vec{a}$ directly, or by a combination of these. The energy approach is best when the problem involves varying forces or motion along a curved path (discussed later in this section). If the problem involves elapsed time, the energy approach is usually *not* the best choice because it doesn't involve time directly.

SET UP *the problem* using the following steps:

1. When using the energy approach, first identify the initial and final states (the positions and velocities) of the objects in question. Use the subscript 1 for the initial state and the subscript 2 for the final state. Draw sketches showing these states.
2. Define a coordinate system, and choose the level at which $y = 0$. Choose the positive y-direction to be upward. (The equations in this section require this.)
3. Identify any forces that do work on each object and that *cannot* be described in terms of potential energy. (So far, this means any forces other than gravity. In Section 7.2 we'll see that the work done by an ideal spring can also be expressed as a change in potential energy.) Sketch a free-body diagram for each object.
4. List the unknown and known quantities, including the coordinates and velocities at each point. Identify the target variables.

EXECUTE *the solution:* Write expressions for the initial and final kinetic and potential energies K_1, K_2, $U_{\text{grav},1}$, and $U_{\text{grav},2}$. If no other forces do work, use Eq. (7.4). If there are other forces that do work, use Eq. (7.6). Draw bar graphs showing the initial and final values of K, $U_{\text{grav},1}$, and $E = K + U_{\text{grav}}$. Then solve to find your target variables.

EVALUATE *your answer:* Check whether your answer makes physical sense. Remember that the gravitational work is included in ΔU_{grav}, so do not include it in W_{other}.

EXAMPLE 7.2 Work and energy in throwing a baseball

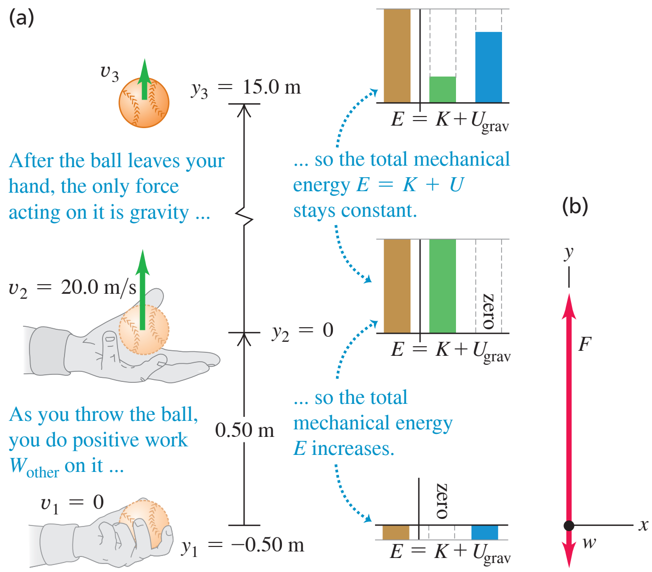
WITH VARIATION PROBLEMS

In Example 7.1 suppose your hand moves upward by 0.50 m while you are throwing the ball. The ball leaves your hand with an upward velocity of 20.0 m/s. (a) Find the magnitude of the force (assumed constant) that your hand exerts on the ball. (b) Find the speed of the ball at a point 15.0 m above the point where it leaves your hand. Ignore air resistance.

IDENTIFY and SET UP In Example 7.1 only gravity did work. Here we must include the nongravitational, "other" work done by your hand. **Figure 7.6** (next page) shows a diagram of the situation, including a free-body diagram for the ball while it is being thrown. We let point 1 be where your hand begins to move, point 2 be where the ball leaves your hand, and point 3 be where the ball is 15.0 m above

Continued

Figure 7.6 (a) Applying energy ideas to a ball thrown vertically upward. (b) Free-body diagram for the ball as you throw it.



point 2. The nongravitational force $\vec{F}$ of your hand acts only between points 1 and 2. Using the same coordinate system as in Example 7.1, we have $y_1 = -0.50$ m, $y_2 = 0$, and $y_3 = 15.0$ m. The ball starts at rest at point 1, so $v_1 = 0$, and the ball's speed as it leaves your hand is $v_2 = 20.0$ m/s. Our target variables are (a) the magnitude F of the force of your hand and (b) the magnitude of the ball's velocity v_{3y} at point 3.

EXECUTE (a) To determine F, we'll first use Eq. (7.6) to calculate the work W_{other} done by this force. We have

$$K_1 = 0$$
$$U_{grav,1} = mgy_1 = (0.145\text{ kg})(9.80\text{ m/s}^2)(-0.50\text{ m}) = -0.71\text{ J}$$
$$K_2 = \tfrac{1}{2}mv_2{}^2 = \tfrac{1}{2}(0.145\text{ kg})(20.0\text{ m/s})^2 = 29.0\text{ J}$$
$$U_{grav,2} = mgy_2 = (0.145\text{ kg})(9.80\text{ m/s}^2)(0) = 0$$

(Don't worry that $U_{grav,1}$ is less than zero; all that matters is the *difference* in potential energy from one point to another.) From Eq. (7.6),

$$K_1 + U_{grav,1} + W_{other} = K_2 + U_{grav,2}$$
$$W_{other} = (K_2 - K_1) + (U_{grav,2} - U_{grav,1})$$
$$= (29.0\text{ J} - 0) + [0 - (-0.71\text{ J})] = 29.7\text{ J}$$

But since $\vec{F}$ is constant and upward, the work done by $\vec{F}$ equals the force magnitude times the displacement: $W_{other} = F(y_2 - y_1)$. So

$$F = \frac{W_{other}}{y_2 - y_1} = \frac{29.7\text{ J}}{0.50\text{ m}} = 59\text{ N}$$

This is more than 40 times the weight of the ball (1.42 N).

(b) To find v_{3y}, note that between points 2 and 3 only gravity acts on the ball. So between these points the total mechanical energy is conserved and $W_{other} = 0$. From Eq. (7.4), we can solve for K_3 and from that solve for v_{3y}:

$$K_2 + U_{grav,2} = K_3 + U_{grav,3}$$
$$U_{grav,3} = mgy_3 = (0.145\text{ kg})(9.80\text{ m/s}^2)(15.0\text{ m}) = 21.3\text{ J}$$
$$K_3 = (K_2 + U_{grav,2}) - U_{grav,3}$$
$$= (29.0\text{ J} + 0) - 21.3\text{ J} = 7.7\text{ J}$$

Since $K_3 = \frac{1}{2}mv_{3y}{}^2$ we find

$$v_{3y} = \pm\sqrt{\frac{2K_3}{m}} = \pm\sqrt{\frac{2(7.7\text{ J})}{0.145\text{ kg}}} = \pm 10\text{ m/s}$$

The plus-or-minus sign reminds us that the ball passes point 3 on the way up and again on the way down. The ball's kinetic energy $K_3 = 7.7$ J at point 3, and hence its speed at that point, doesn't depend on the direction the ball is moving. The velocity v_{3y} is positive (+10 m/s) when the ball is moving up and negative (−10 m/s) when it is moving down; the speed v_3 is 10 m/s in either case.

EVALUATE In Example 7.1 we found that the ball reaches a maximum height $y = 20.4$ m. At that point all of the kinetic energy it had when it left your hand at $y = 0$ has been converted to gravitational potential energy. At $y = 15.0$ m, the ball is about three-fourths of the way to its maximum height, so about three-fourths of its total mechanical energy should be in the form of potential energy. Can you verify this from our results for K_3 and $U_{grav,3}$?

KEYCONCEPT When a force that cannot be described in terms of potential energy does work W_{other}, the final value of the total mechanical energy equals the initial value of the mechanical energy plus W_{other}.

Gravitational Potential Energy for Motion Along a Curved Path

In our first two examples the object moved along a straight vertical line. What happens when the path is slanted or curved (**Fig. 7.7a**)? The object is acted on by the gravitational force $\vec{w} = m\vec{g}$ and possibly by other forces whose resultant we call $\vec{F}_{other}$. To find the work W_{grav} done by the gravitational force during this displacement, we divide the path into small segments $\Delta\vec{s}$; Fig. 7.7b shows a typical segment. The work done by the gravitational force over this segment is the scalar product of the force and the displacement. In terms of unit vectors, the force is $\vec{w} = m\vec{g} = -mg\hat{j}$ and the displacement is $\Delta\vec{s} = \Delta x\hat{i} + \Delta y\hat{j}$, so

$$W_{grav} = \vec{w}\cdot\Delta\vec{s} = -mg\hat{j}\cdot(\Delta x\hat{i} + \Delta y\hat{j}) = -mg\Delta y$$

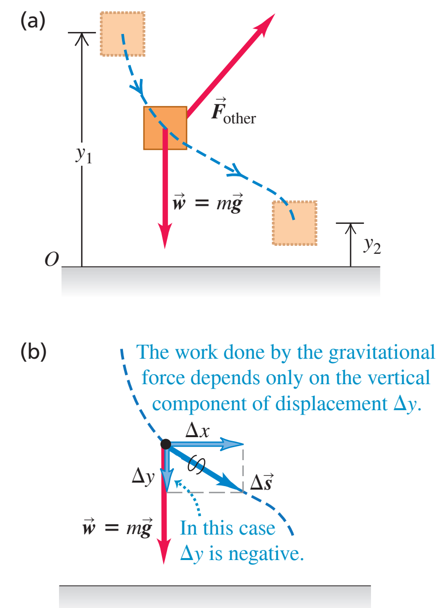
The work done by gravity is the same as though the object had been displaced vertically a distance Δy, with no horizontal displacement. This is true for every segment, so the *total* work done by the gravitational force is $-mg$ multiplied by the *total* vertical displacement $(y_2 - y_1)$:

$$W_{grav} = -mg(y_2 - y_1) = mgy_1 - mgy_2 = U_{grav,1} - U_{grav,2}$$

This is the same as Eq. (7.1) or (7.3), in which we assumed a purely vertical path. So even if the path an object follows between two points is curved, the total work done by the gravitational force depends on only the difference in height between the two points of the path. This work is unaffected by any horizontal motion that may occur. So *we can use the same expression for gravitational potential energy whether the object's path is curved or straight.*

CAUTION **With gravitational potential energy, only the change in height matters** The change in gravitational potential energy along a curved path depends only on the difference between the final and initial heights, not on the shape of the path. If gravity is the only force that does work along a curved path, then the total mechanical energy is conserved.

Figure 7.7 Calculating the change in gravitational potential energy for a displacement along a curved path.



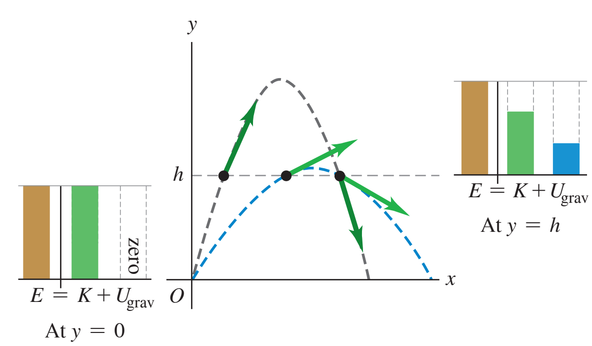
CONCEPTUAL EXAMPLE 7.3 Energy in projectile motion

A batter hits two identical baseballs with the same initial speed and from the same initial height but at different initial angles. Prove that both balls have the same speed at any height h if air resistance can be ignored.

SOLUTION The only force acting on each ball after it is hit is its weight. Hence the total mechanical energy for each ball is constant. **Figure 7.8** shows the trajectories of two balls batted at the same height with the same initial speed, and thus the same total mechanical energy, but with different initial angles. At all points at the same height the potential energy is the same. Thus the kinetic energy at this height must be the same for both balls, and the speeds are the same.

KEYCONCEPT The gravitational potential energy of an object depends on its height, not on the path the object took to reach that height.

Figure 7.8 For the same initial speed and initial height, the speed of a projectile at a given elevation h is always the same, if we ignore air resistance.



EXAMPLE 7.4 Speed at the bottom of a vertical circle

WITH VARIATION PROBLEMS

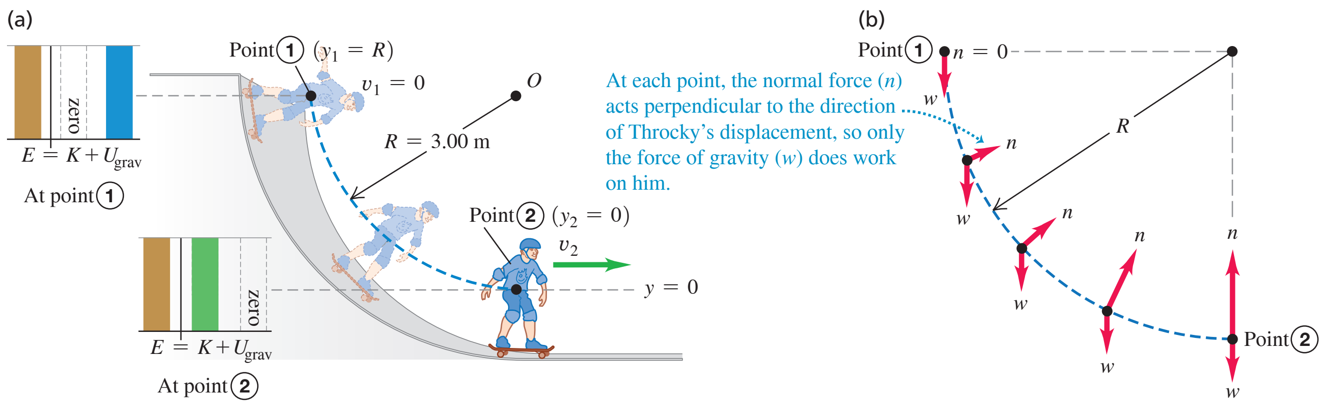
Your cousin Throckmorton skateboards from rest down a curved, frictionless ramp. If we treat Throcky and his skateboard as a particle, he moves through a quarter-circle with radius $R = 3.00$ m (**Fig. 7.9**, next page). Throcky and his skateboard have a total mass of 25.0 kg. (a) Find his speed at the bottom of the ramp. (b) Find the normal force that acts on him at the bottom of the curve.

IDENTIFY We can't use the constant-acceleration equations of Chapter 2 because Throcky's acceleration isn't constant; the slope decreases as he descends. Instead, we'll use the energy approach. Throcky moves along a circular arc, so we'll also use what we learned about circular motion in Section 5.4.

SET UP The only forces on Throcky are his weight and the normal force $\vec{n}$ exerted by the ramp (Fig. 7.9b). Although $\vec{n}$ acts all along the path, it does zero work because $\vec{n}$ is perpendicular to Throcky's displacement at every point. Hence $W_{other} = 0$ and the total mechanical energy is conserved. We treat Throcky as a particle located at the center of his body, take point 1 at the particle's starting point, and take point 2 (which we let be $y = 0$) at the particle's low point. We take the positive y-direction upward; then $y_1 = R$ and $y_2 = 0$. Throcky starts at rest at the top, so $v_1 = 0$. In part (a) our target variable is his speed v_2 at the bottom; in part (b) the target variable is the magnitude n of the normal force at point 2. To find n, we'll use Newton's second law and the relationship $a = v^2/R$.

Continued

Figure 7.9 (a) Throcky skateboarding down a frictionless circular ramp. The total mechanical energy is constant. (b) Free-body diagrams for Throcky and his skateboard at various points on the ramp.



EXECUTE (a) The various energy quantities are

$$K_1 = 0 \qquad U_{\text{grav},1} = mgR$$
$$K_2 = \tfrac{1}{2}mv_2^2 \qquad U_{\text{grav},2} = 0$$

From conservation of total mechanical energy, Eq. (7.4),

$$K_1 + U_{\text{grav},1} = K_2 + U_{\text{grav},2}$$
$$0 + mgR = \tfrac{1}{2}mv_2^2 + 0$$
$$v_2 = \sqrt{2gR} = \sqrt{2(9.80\ \text{m/s}^2)(3.00\ \text{m})} = 7.67\ \text{m/s}$$

This answer doesn't depend on the ramp being circular; Throcky would have the same speed $v_2 = \sqrt{2gR}$ at the bottom of any ramp of height R, no matter what its shape.

(b) To use Newton's second law to find n at point 2, we need the free-body diagram at that point (Fig. 7.9b). At point 2, Throcky is moving at speed $v_2 = \sqrt{2gR}$ in a circle of radius R; his acceleration is toward the center of the circle and has magnitude

$$a_{\text{rad}} = \frac{v_2^2}{R} = \frac{2gR}{R} = 2g$$

The y-component of Newton's second law is

$$\Sigma F_y = n + (-w) = ma_{\text{rad}} = 2mg$$
$$n = w + 2mg = 3mg$$
$$= 3(25.0\ \text{kg})(9.80\ \text{m/s}^2) = 735\ \text{N}$$

At point 2 the normal force is three times Throcky's weight. This result doesn't depend on the radius R of the ramp. We saw in Examples 5.9 and 5.23 that the magnitude of n is the *apparent weight,* so at the bottom of the *curved part* of the ramp Throcky feels as though he weighs three times his true weight mg. But when he reaches the *horizontal* part of the ramp, immediately to the right of point 2, the normal force decreases to $w = mg$ and thereafter Throcky feels his true weight again. Can you see why?

EVALUATE This example shows a general rule about the role of forces in problems in which we use energy techniques: What matters is not simply whether a force *acts,* but whether that force *does work.* If the force does no work, like the normal force $\vec{n}$ here, then it does not appear in Eqs. (7.4) and (7.6).

KEYCONCEPT If one of the forces that acts on a moving object is always perpendicular to the object's path, that force does no work on the object and plays no role in the equation for total mechanical energy.

EXAMPLE 7.5 A vertical circle with friction

WITH VARIATION PROBLEMS

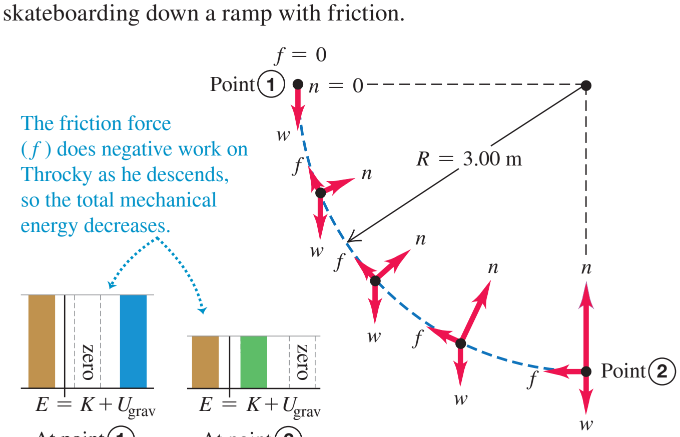
Suppose that the ramp of Example 7.4 is not frictionless and that Throcky's speed at the bottom is only 6.00 m/s, not the 7.67 m/s we found there. What work was done on him by the friction force?

IDENTIFY and SET UP The setup is the same as in Example 7.4. **Figure 7.10** shows that again the normal force does no work, but now there is a friction force $\vec{f}$ that *does* do work W_f. Hence the nongravitational work W_{other} done on Throcky between points 1 and 2 is equal to W_f and is not zero. Our target variable is $W_f = W_{\text{other}}$, which we'll find by using Eq. (7.6). Since $\vec{f}$ points opposite to Throcky's motion, W_f is negative.

EXECUTE The energy quantities are

$$K_1 = 0$$
$$U_{\text{grav},1} = mgR = (25.0\ \text{kg})(9.80\ \text{m/s}^2)(3.00\ \text{m}) = 735\ \text{J}$$
$$K_2 = \tfrac{1}{2}mv_2^2 = \tfrac{1}{2}(25.0\ \text{kg})(6.00\ \text{m/s})^2 = 450\ \text{J}$$
$$U_{\text{grav},2} = 0$$

Figure 7.10 Energy bar graphs and free-body diagrams for Throcky skateboarding down a ramp with friction.



From Eq. (7.6),

$$\begin{aligned} W_f = W_{other} &= K_2 + U_{grav,2} - K_1 - U_{grav,1} \\ &= 450\text{ J} + 0 - 0 - 735\text{ J} = -285\text{ J} \end{aligned}$$

The work done by the friction force is −285 J, and the total mechanical energy *decreases* by 285 J.

EVALUATE Our result for W_f is negative. Can you see from the free-body diagrams in Fig. 7.10 why this must be so?

It would be very difficult to apply Newton's second law, $\sum \vec{F} = m\vec{a}$, directly to this problem because the normal and friction forces and the acceleration are continuously changing in both magnitude and direction as Throcky descends. The energy approach, by contrast, relates the motions at the top and bottom of the ramp without involving the details of the motion in between.

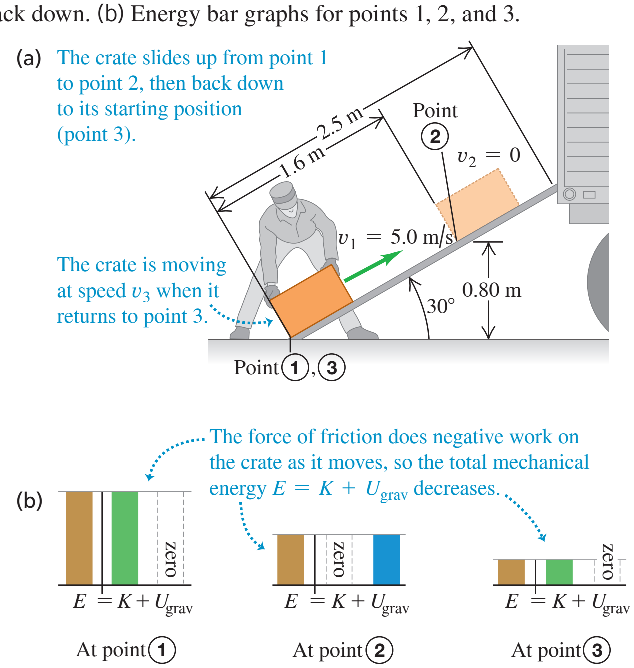
KEYCONCEPT Whether an object's path is straight or curved, the relationship is the same among the initial total mechanical energy, the final total mechanical energy, and the work done by forces other than gravity.

EXAMPLE 7.6 An inclined plane with friction

We want to slide a 12 kg crate up a 2.5-m-long ramp inclined at 30°. A worker, ignoring friction, calculates that he can do this by giving it an initial speed of 5.0 m/s at the bottom and letting it go. But friction is *not* negligible; the crate slides only 1.6 m up the ramp, stops, and slides back down (**Fig. 7.11a**). (a) Find the magnitude of the friction force acting on the crate, assuming that it is constant. (b) How fast is the crate moving when it reaches the bottom of the ramp?

IDENTIFY and SET UP The friction force does work on the crate as it slides from point 1, at the bottom of the ramp, to point 2, where the crate stops instantaneously ($v_2 = 0$). Friction also does work as the crate returns to the bottom of the ramp, which we'll call point 3 (Fig. 7.11a). We take the positive y-direction upward. We take $y = 0$ (and hence $U_{grav} = 0$) to be at ground level (point 1), so $y_1 = 0$, $y_2 = (1.6\text{ m})\sin 30° = 0.80\text{ m}$, and $y_3 = 0$. We are given $v_1 = 5.0\text{ m/s}$. In part (a) our target variable is f, the magnitude of the friction force as the crate slides up; we'll find this by using the energy approach. In part (b) our target variable is v_3, the crate's speed at the bottom of the ramp. We'll calculate the work done by friction as the crate slides back down, then use the energy approach to find v_3.

Figure 7.11 (a) A crate slides partway up the ramp, stops, and slides back down. (b) Energy bar graphs for points 1, 2, and 3.



EXECUTE (a) The energy quantities are

$$\begin{aligned} K_1 &= \tfrac{1}{2}(12\text{ kg})(5.0\text{ m/s})^2 = 150\text{ J} \\ U_{grav,1} &= 0 \\ K_2 &= 0 \\ U_{grav,2} &= (12\text{ kg})(9.8\text{ m/s}^2)(0.80\text{ m}) = 94\text{ J} \\ W_{other} &= -fs \end{aligned}$$

Here $s = 1.6$ m. Using Eq. (7.6), we find

$$\begin{aligned} K_1 + U_{grav,1} + W_{other} &= K_2 + U_{grav,2} \\ W_{other} = -fs &= (K_2 + U_{grav,2}) - (K_1 + U_{grav,1}) \\ &= (0 + 94\text{ J}) - (150\text{ J} + 0) = -56\text{ J} = -fs \\ f = \frac{W_{other}}{s} &= \frac{56\text{ J}}{1.6\text{ m}} = 35\text{ N} \end{aligned}$$

The friction force of 35 N, acting over 1.6 m, causes the total mechanical energy of the crate to decrease from 150 J to 94 J (Fig. 7.11b).

(b) As the crate moves from point 2 to point 3, the work done by friction has the same negative value as from point 1 to point 2. (Both the friction force and the displacement reverse direction, but their magnitudes don't change.) The total work done by friction between points 1 and 3 is therefore

$$W_{other} = W_{fric} = -2fs = -2(56\text{ J}) = -112\text{ J}$$

From part (a), $K_1 = 150$ J and $U_{grav,1} = 0$; in addition, $U_{grav,3} = 0$ since $y_3 = 0$. Equation (7.6) then gives

$$\begin{aligned} K_1 + U_{grav,1} + W_{other} &= K_3 + U_{grav,3} \\ K_3 &= K_1 + U_{grav,1} - U_{grav,3} + W_{other} \\ &= 150\text{ J} + 0 - 0 + (-112\text{ J}) = 38\text{ J} \end{aligned}$$

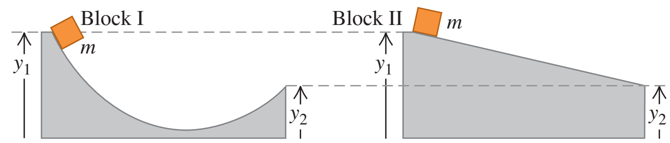
The crate returns to the bottom of the ramp with only 38 J of the original 150 J of total mechanical energy (Fig. 7.11b). Since $K_3 = \frac{1}{2}mv_3^2$,

$$v_3 = \sqrt{\frac{2K_3}{m}} = \sqrt{\frac{2(38\text{ J})}{12\text{ kg}}} = 2.5\text{ m/s}$$

EVALUATE Energy is lost due to friction, so the crate's speed $v_3 = 2.5$ m/s when it returns to the bottom of the ramp is less than the speed $v_1 = 5.0$ m/s at which it left that point. In part (b) we applied Eq. (7.6) to points 1 and 3, considering the round trip as a whole. Alternatively, we could have considered the second part of the motion by itself and applied Eq. (7.6) to points 2 and 3. Try it; do you get the same result for v_3?

KEYCONCEPT For straight-line motion problems in which the forces are constant in each stage of the motion, you can use the total mechanical energy to find the magnitude of an unknown force that does work.

TEST YOUR UNDERSTANDING OF SECTION 7.1 The figure shows two frictionless ramps. The heights y_1 and y_2 are the same for both ramps. If a block of mass m is released from rest at the left-hand end of each ramp, which block arrives at the right-hand end with the greater speed? (i) Block I; (ii) block II; (iii) the speed is the same for both blocks.



ANSWER

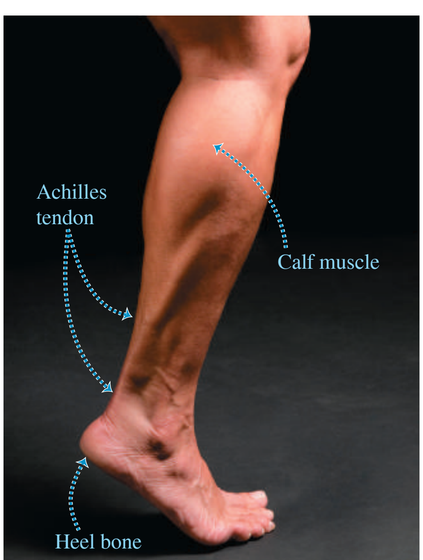
(iii) The initial kinetic energy $K_1 = 0$, the initial potential energy $U_1 = mgy_1$, and the final potential energy $U_2 = mgy_2$ are the same for both blocks. Total mechanical energy is conserved in both cases, so the final kinetic energy $K_2 = \frac{1}{2}mv_2^2$ is also the same for both blocks. Hence the speed at the right-hand end is the *same* in both cases!

7.2 ELASTIC POTENTIAL ENERGY

In many situations we encounter potential energy that is not gravitational in nature. One example is a rubber-band slingshot. Work is done on the rubber band by the force that stretches it, and that work is stored in the rubber band until you let it go. Then the rubber band gives kinetic energy to the projectile.

This is the same pattern we saw with the baseball in Example 7.2: Do work on the system to store energy, which can later be converted to kinetic energy. We'll describe the process of storing energy in a deformable object such as a spring or rubber band in terms of *elastic potential energy* (**Fig. 7.12**). An object is called *elastic* if it returns to its original shape and size after being deformed.

Figure 7.12 The Achilles tendon, which runs along the back of the ankle to the heel bone, acts like a natural spring. When it stretches and then relaxes, this tendon stores and then releases elastic potential energy. This spring action reduces the amount of work your leg muscles must do as you run.



To be specific, we'll consider storing energy in an ideal spring, like the ones we discussed in Section 6.3. To keep such an ideal spring stretched by a distance x, we must exert a force $F = kx$, where k is the force constant of the spring. Many elastic objects show this same direct proportionality between force $\vec{F}$ and displacement x, provided that x is sufficiently small.

Let's proceed just as we did for gravitational potential energy. We begin with the work done by the elastic (spring) force and then combine this with the work–energy theorem. The difference is that gravitational potential energy is a shared property of an object and the earth, but elastic potential energy is stored in just the spring (or other deformable object).

Figure 7.13 shows the ideal spring from Fig. 6.18 but with its left end held stationary and its right end attached to a block with mass m that can move along the x-axis. In Fig. 7.13a the block is at $x = 0$ when the spring is neither stretched nor compressed. We move the block to one side, thereby stretching or compressing the spring, then let it go. As the block moves from a different position x_1 to a different position x_2, how much work does the elastic (spring) force do on the block?

We found in Section 6.3 that the work we must do *on* the spring to move one end from an elongation x_1 to a different elongation x_2 is

$$W = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2 \qquad \text{(work done } on \text{ a spring)} \qquad (7.8)$$

where k is the force constant of the spring. If we stretch the spring farther, we do positive work on the spring; if we let the spring relax while holding one end, we do negative work on it. This expression for work is also correct when the spring is compressed such that x_1, x_2, or both are negative. Now, from Newton's third law the work done *by* the spring is just the negative of the work done *on* the spring. So by changing the signs in Eq. (7.8), we find that in a displacement from x_1 to x_2 the spring does an amount of work W_{el} given by

$$W_{el} = \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2 \qquad \text{(work done } by \text{ a spring)} \qquad (7.9)$$

The subscript "el" stands for *elastic*. When both x_1 and x_2 are positive and $x_2 > x_1$ (Fig. 7.13b), the spring does negative work on the block, which moves in the $+x$-direction while the spring pulls on it in the $-x$-direction. The spring stretches farther, and the block slows down. When both x_1 and x_2 are positive and $x_2 < x_1$ (Fig. 7.13c), the spring does positive work as it relaxes and the block speeds up. If the spring can be compressed as well as stretched, x_1, x_2, or both may be negative, but the expression for W_{el} is still valid. In Fig. 7.13d, both x_1 and x_2 are negative, but x_2 is less negative than x_1; the compressed spring does positive work as it relaxes, speeding the block up.

Just as for gravitational work, we can express Eq. (7.9) for the work done by the spring in terms of a quantity at the beginning and end of the displacement. This quantity is $\frac{1}{2}kx^2$, and we define it to be the **elastic potential energy:**

Force constant of spring

Elastic potential energy stored in a spring

$$U_{el} = \frac{1}{2}kx^2 \qquad (7.10)$$

Elongation of spring ($x > 0$ if stretched, $x < 0$ if compressed)

Figure 7.14 is a graph of Eq. (7.10). As for all other energy and work quantities, the unit of U_{el} is the joule (J); to see this from Eq. (7.10), recall that the units of k are N/m and that 1 N·m = 1 J. We can now use Eq. (7.10) to rewrite Eq. (7.9) for the work W_{el} done by the spring:

Work done by the elastic force ... ... equals the **negative of the change in elastic potential energy.**

$$W_{el} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = U_{el,1} - U_{el,2} = -\Delta U_{el} \qquad (7.11)$$

Force constant of spring Initial and final elongations of spring

When a stretched spring is stretched farther, as in Fig. 7.13b, W_{el} is negative and U_{el} increases; more elastic potential energy is stored in the spring. When a stretched spring relaxes, as in Fig. 7.13c, x decreases, W_{el} is positive, and U_{el} decreases; the spring loses elastic potential energy. Figure 7.14 shows that U_{el} is positive for both positive and negative x values; Eqs. (7.10) and (7.11) are valid for both cases. The more a spring is compressed or stretched, the greater its elastic potential energy.

CAUTION **Gravitational potential energy vs. elastic potential energy** An important difference between gravitational potential energy $U_{grav} = mgy$ and elastic potential energy $U_{el} = \frac{1}{2}kx^2$ is that we *cannot* choose $x = 0$ to be wherever we wish. In Eq. (7.10), $x = 0$ *must* be the position at which the spring is neither stretched nor compressed. At that position, both its elastic potential energy and the force that it exerts are zero.

The work–energy theorem says that $W_{tot} = K_2 - K_1$, no matter what kind of forces are acting on an object. If the elastic force is the *only* force that does work on the object, then

$$W_{tot} = W_{el} = U_{el,1} - U_{el,2}$$

and so

If only the elastic force does work, total mechanical energy is conserved:

Initial kinetic energy $K_1 = \frac{1}{2}mv_1^2$ Initial elastic potential energy $U_{el,1} = \frac{1}{2}kx_1^2$

$$K_1 + U_{el,1} = K_2 + U_{el,2} \qquad (7.12)$$

Final kinetic energy $K_2 = \frac{1}{2}mv_2^2$ Final elastic potential energy $U_{el,2} = \frac{1}{2}kx_2^2$

In this case the total mechanical energy $E = K + U_{el}$—the sum of kinetic and *elastic* potential energies—is *conserved.* An example of this is the motion of the block in Fig. 7.13, provided the horizontal surface is frictionless so no force does work other than that exerted by the spring.

For Eq. (7.12) to be strictly correct, the ideal spring that we've been discussing must also be *massless.* If the spring has mass, it also has kinetic energy as the coils of the

Figure **7.13** Calculating the work done by a spring attached to a block on a horizontal surface. The quantity x is the extension or compression of the spring.

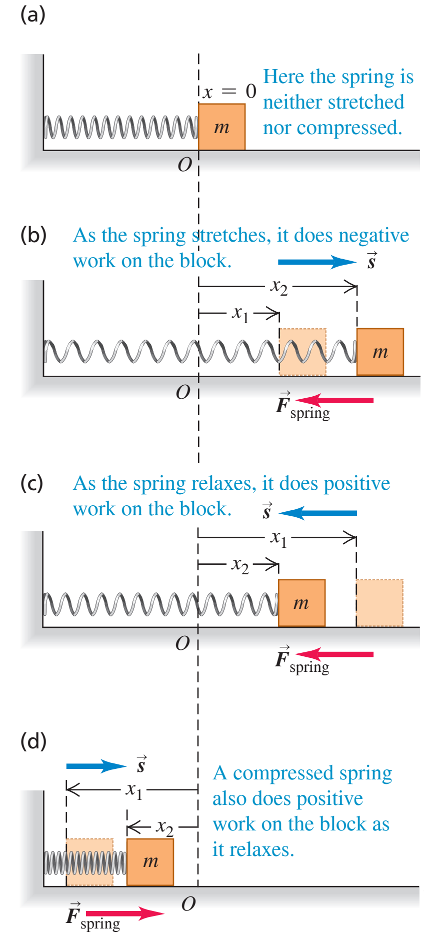
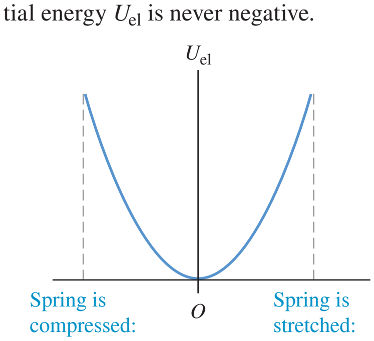
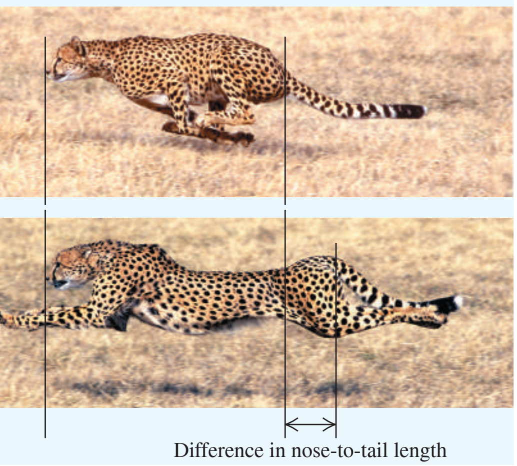


Figure **7.14** The graph of elastic potential energy for an ideal spring is a parabola: $U_{el} = \frac{1}{2}kx^2$, where x is the extension or compression of the spring. Elastic potential energy U_{el} is never negative.



BIO APPLICATION Elastic Potential Energy of a Cheetah When a cheetah (*Acinonyx jubatus*) gallops, its back flexes and extends dramatically. Flexion of the back stretches tendons and muscles along the top of the spine and also compresses the spine, storing elastic potential energy. When the cheetah launches into its next bound, this energy is released, enabling the cheetah to run more efficiently.



spring move back and forth. We can ignore the kinetic energy of the spring if its mass is much less than the mass m of the object attached to the spring. For instance, a typical automobile has a mass of 1200 kg or more. The springs in its suspension have masses of only a few kilograms, so their mass can be ignored if we want to study how a car bounces on its suspension.

Situations with Both Gravitational and Elastic Potential Energy

Equation (7.12) is valid when the only potential energy in the system is elastic potential energy. What happens when we have *both* gravitational and elastic forces, such as a block attached to the lower end of a vertically hanging spring? And what if work is also done by other forces that *cannot* be described in terms of potential energy, such as the force of air resistance on a moving block? Then the total work is the sum of the work done by the gravitational force (W_{grav}), the work done by the elastic force (W_{el}), and the work done by other forces (W_{other}): $W_{tot} = W_{grav} + W_{el} + W_{other}$. The work–energy theorem then gives

$$W_{grav} + W_{el} + W_{other} = K_2 - K_1$$

The work done by the gravitational force is $W_{grav} = U_{grav,1} - U_{grav,2}$ and the work done by the spring is $W_{el} = U_{el,1} - U_{el,2}$. Hence we can rewrite the work–energy theorem for this most general case as

$$K_1 + U_{grav,1} + U_{el,1} + W_{other} = K_2 + U_{grav,2} + U_{el,2} \quad \text{(valid in general)} \qquad (7.13)$$

or, equivalently,

General relationship for kinetic energy and potential energy:

$$\underbrace{K_1}_{\text{Initial kinetic energy}} + \underbrace{U_1}_{\text{Initial potential energy of all kinds}} + \underbrace{W_{other}}_{\text{Work done by other forces (not associated with potential energy)}} = \underbrace{K_2}_{\text{Final kinetic energy}} + \underbrace{U_2}_{\text{Final potential energy of all kinds}} \qquad (7.14)$$

where $U = U_{grav} + U_{el} = mgy + \frac{1}{2}kx^2$ is the *sum* of gravitational potential energy and elastic potential energy. We call U simply "the potential energy."

Equation (7.14) is *the most general statement* of the relationship among kinetic energy, potential energy, and work done by other forces. It says:

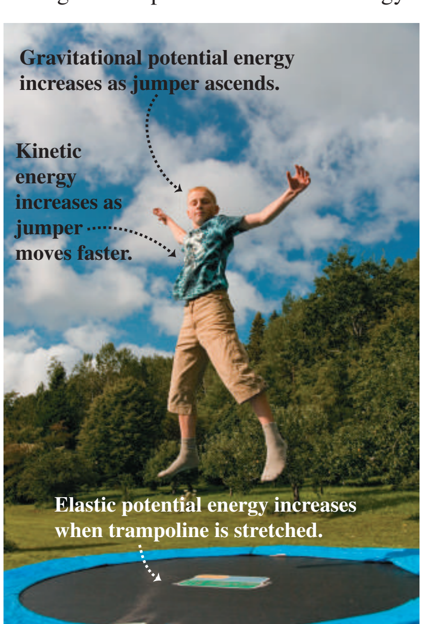
The work done by all forces other than the gravitational force or elastic force equals the change in the total mechanical energy $E = K + U$ of the system.

The "system" is made up of the object of mass m, the earth with which it interacts through the gravitational force, and the spring of force constant k.

If W_{other} is positive, $E = K + U$ increases; if W_{other} is negative, E decreases. If the gravitational and elastic forces are the *only* forces that do work on the object, then $W_{other} = 0$ and the total mechanical energy $E = K + U$ is conserved. [Compare Eq. (7.14) to Eqs. (7.6) and (7.7), which include gravitational potential energy but not elastic potential energy.]

Trampoline jumping (**Fig. 7.15**) involves transformations among kinetic energy, elastic potential energy, and gravitational potential energy. As the jumper descends through the air from the high point of the bounce, gravitational potential energy U_{grav} decreases and kinetic energy K increases. Once the jumper touches the trampoline, some of the total mechanical energy goes into elastic potential energy U_{el} stored in the trampoline's springs. At the lowest point of the trajectory (U_{grav} is minimum), the jumper comes to a momentary halt ($K = 0$) and the springs are maximally stretched (U_{el} is maximum). The springs then convert their energy back into K and U_{grav}, propelling the jumper upward.

Figure **7.15** Trampoline jumping involves an interplay among kinetic energy, gravitational potential energy, and elastic potential energy. Due to air resistance and friction forces within the trampoline, total mechanical energy is not conserved. That's why the bouncing eventually stops unless the jumper does work with his or her legs to compensate for the lost energy.



PROBLEM-SOLVING STRATEGY 7.2 Problems Using Total Mechanical Energy II

Problem-Solving Strategy 7.1 (Section 7.1) is useful in solving problems that involve elastic forces as well as gravitational forces. The only new wrinkle is that the potential energy U now includes the elastic potential energy $U_{el} = \frac{1}{2}kx^2$, where x is the displacement of the spring *from its unstretched length*. The work done by the gravitational and elastic forces is accounted for by their potential energies; the work done by other forces, W_{other}, must still be included separately.

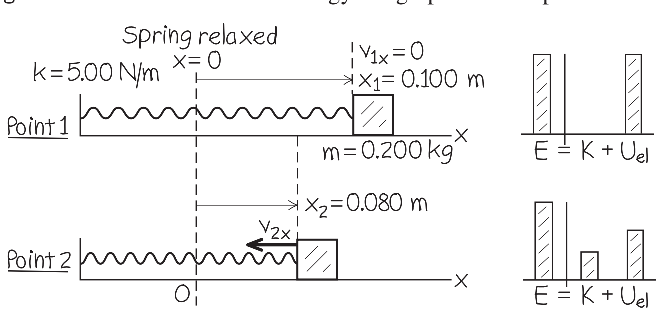
EXAMPLE 7.7 Motion with elastic potential energy

WITH VARIATION PROBLEMS

A glider with mass $m = 0.200$ kg sits on a frictionless, horizontal air track, connected to a spring with force constant $k = 5.00$ N/m. You pull on the glider, stretching the spring 0.100 m, and release it from rest. The glider moves back toward its equilibrium position ($x = 0$). What is its x-velocity when $x = 0.080$ m?

IDENTIFY and SET UP There is no friction, so total mechanical energy is conserved. As the glider starts to move, elastic potential energy is converted to kinetic energy. The glider remains at the same height throughout the motion, so gravitational potential energy is not a factor and $U = U_{el} = \frac{1}{2}kx^2$. **Figure 7.16** shows our sketches. Only the spring force does work on the glider, so $W_{other} = 0$ in Eq. (7.14). We designate the point where the glider is released as point 1 (that is, $x_1 = 0.100$ m) and $x_2 = 0.080$ m as point 2. We are given $v_{1x} = 0$; our target variable is v_{2x}.

Figure 7.16 Our sketches and energy bar graphs for this problem.



EXECUTE The energy quantities are

$$K_1 = \tfrac{1}{2}mv_{1x}^2 = \tfrac{1}{2}(0.200\text{ kg})(0)^2 = 0$$
$$U_1 = \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}(5.00\text{ N/m})(0.100\text{ m})^2 = 0.0250\text{ J}$$
$$K_2 = \tfrac{1}{2}mv_{2x}^2$$
$$U_2 = \tfrac{1}{2}kx_2^2 = \tfrac{1}{2}(5.00\text{ N/m})(0.080\text{ m})^2 = 0.0160\text{ J}$$

We use Eq. (7.14) with $W_{other} = 0$ to solve for K_2 and then find v_{2x}:

$$K_2 = K_1 + U_1 - U_2 = 0 + 0.0250\text{ J} - 0.0160\text{ J} = 0.0090\text{ J}$$

$$v_{2x} = \pm\sqrt{\frac{2K_2}{m}} = \pm\sqrt{\frac{2(0.0090\text{ J})}{0.200\text{ kg}}} = \pm 0.30\text{ m/s}$$

We choose the negative root because the glider is moving in the $-x$-direction. Our answer is $v_{2x} = -0.30$ m/s.

EVALUATE Eventually the spring will reverse the glider's motion, pushing it back in the $+x$-direction (see Fig. 7.13d). The solution $v_{2x} = +0.30$ m/s tells us that when the glider passes through $x = 0.080$ m on this return trip, its speed will be 0.30 m/s, just as when it passed through this point while moving to the left.

KEYCONCEPT You can use elastic potential energy to describe the work done by an ideal spring that obeys Hooke's law.

EXAMPLE 7.8 Motion with elastic potential energy and work done by other forces

WITH VARIATION PROBLEMS

Suppose the glider in Example 7.7 is initially at rest at $x = 0$, with the spring unstretched. You then push on the glider with a constant force $\vec{F}$ (magnitude 0.610 N) in the $+x$-direction. What is the glider's velocity when it has moved to $x = 0.100$ m?

IDENTIFY and SET UP Although the force $\vec{F}$ you apply is constant, the spring force isn't, so the acceleration of the glider won't be constant. Total mechanical energy is not conserved because of the work done by force $\vec{F}$, so W_{other} in Eq. (7.14) is not zero. As in Example 7.7, we ignore gravitational potential energy because the glider's height doesn't change. Hence we again have $U = U_{el} = \frac{1}{2}kx^2$. This time, we let point 1 be at $x_1 = 0$, where the velocity is $v_{1x} = 0$, and let point 2 be at $x = 0.100$ m. The glider's displacement is then $\Delta x = x_2 - x_1 = 0.100$ m. Our target variable is v_{2x}, the velocity at point 2.

EXECUTE Force $\vec{F}$ is constant and in the same direction as the displacement, so the work done by this force is $F\Delta x$. Then the energy quantities are

$$K_1 = 0$$
$$U_1 = \tfrac{1}{2}kx_1^2 = 0$$
$$K_2 = \tfrac{1}{2}mv_{2x}^2$$
$$U_2 = \tfrac{1}{2}kx_2^2 = \tfrac{1}{2}(5.00\text{ N/m})(0.100\text{ m})^2 = 0.0250\text{ J}$$
$$W_{other} = F\Delta x = (0.610\text{ N})(0.100\text{ m}) = 0.0610\text{ J}$$

Continued

The initial total mechanical energy is zero; the work done by $\vec{F}$ increases the total mechanical energy to 0.0610 J, of which $U_2 = 0.0250$ J is elastic potential energy. The remainder is kinetic energy. From Eq. (7.14),

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

$$K_2 = K_1 + U_1 + W_{\text{other}} - U_2$$

$$= 0 + 0 + 0.0610 \text{ J} - 0.0250 \text{ J} = 0.0360 \text{ J}$$

$$v_{2x} = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.0360 \text{ J})}{0.200 \text{ kg}}} = 0.60 \text{ m/s}$$

We choose the positive square root because the glider is moving in the $+x$-direction.

EVALUATE What would be different if we disconnected the glider from the spring? Then only $\vec{F}$ would do work, there would be zero elastic potential energy at all times, and Eq. (7.14) would give us

$$K_2 = K_1 + W_{\text{other}} = 0 + 0.0610 \text{ J}$$

$$v_{2x} = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.0610 \text{ J})}{0.200 \text{ kg}}} = 0.78 \text{ m/s}$$

Our answer $v_{2x} = 0.60$ m/s is less than 0.78 m/s because the spring does negative work on the glider as it stretches (see Fig. 7.13b).

If you stop pushing on the glider when it reaches $x = 0.100$ m, only the spring force does work on it thereafter. Hence for $x > 0.100$ m, the total mechanical energy $E = K + U = 0.0610$ J is constant. As the spring continues to stretch, the glider slows down and the kinetic energy K decreases as the potential energy increases. The glider comes to rest at some point $x = x_3$, at which the kinetic energy is zero and the potential energy $U = U_{\text{el}} = \frac{1}{2}kx_3^2$ equals the total mechanical energy 0.0610 J. Can you show that $x_3 = 0.156$ m? (It moves an additional 0.056 m after you stop pushing.) If there is no friction, will the glider remain at rest?

KEYCONCEPT You can solve problems that involve elastic potential energy by using the same steps as for problems that involve gravitational potential energy, even when work is done by other forces.

EXAMPLE 7.9 Motion with gravitational, elastic, and friction forces

WITH VARIATION PROBLEMS

A 2000 kg (19,600 N) elevator with broken cables in a test rig is falling at 4.00 m/s when it contacts a cushioning spring at the bottom of the shaft. The spring is intended to stop the elevator, compressing 2.00 m as it does so (**Fig. 7.17**). During the motion a safety clamp applies a constant 17,000 N friction force to the elevator. What is the necessary force constant k for the spring?

IDENTIFY and SET UP We'll use the energy approach and Eq. (7.14) to determine k, which appears in the expression for elastic potential energy. This problem involves *both* gravitational and elastic potential energies. Total mechanical energy is not conserved because the friction force does negative work W_{other} on the elevator. We take point 1 as the position of the bottom of the elevator when it contacts the spring, and point 2 as its position when it stops. We choose the origin to be at point 1, so $y_1 = 0$ and $y_2 = -2.00$ m. With this choice the coordinate of the upper end of the spring after contact is the same as the coordinate of the elevator, so the elastic potential energy at any point between points 1 and 2 is $U_{\text{el}} = \frac{1}{2}ky^2$. The gravitational potential energy is $U_{\text{grav}} = mgy$ as usual. We know the initial and final speeds of the elevator and the magnitude of the friction force, so the only unknown is the force constant k (our target variable).

EXECUTE The elevator's initial speed is $v_1 = 4.00$ m/s, so its initial kinetic energy is

$$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(2000 \text{ kg})(4.00 \text{ m/s})^2 = 16{,}000 \text{ J}$$

The elevator stops at point 2, so $K_2 = 0$. At point 1 the potential energy $U_1 = U_{\text{grav}} + U_{\text{el}}$ is zero; U_{grav} is zero because $y_1 = 0$, and $U_{\text{el}} = 0$ because the spring is uncompressed. At point 2 there are both gravitational and elastic potential energies, so

$$U_2 = mgy_2 + \tfrac{1}{2}ky_2^2$$

The gravitational potential energy at point 2 is

$$mgy_2 = (2000 \text{ kg})(9.80 \text{ m/s}^2)(-2.00 \text{ m}) = -39{,}200 \text{ J}$$

The "other" force is the constant 17,000 N friction force. It acts opposite to the 2.00 m displacement, so

$$W_{\text{other}} = -(17{,}000 \text{ N})(2.00 \text{ m}) = -34{,}000 \text{ J}$$

We put these terms into Eq. (7.14), $K_1 + U_1 + W_{\text{other}} = K_2 + U_2$:

$$K_1 + 0 + W_{\text{other}} = 0 + (mgy_2 + \tfrac{1}{2}ky_2^2)$$

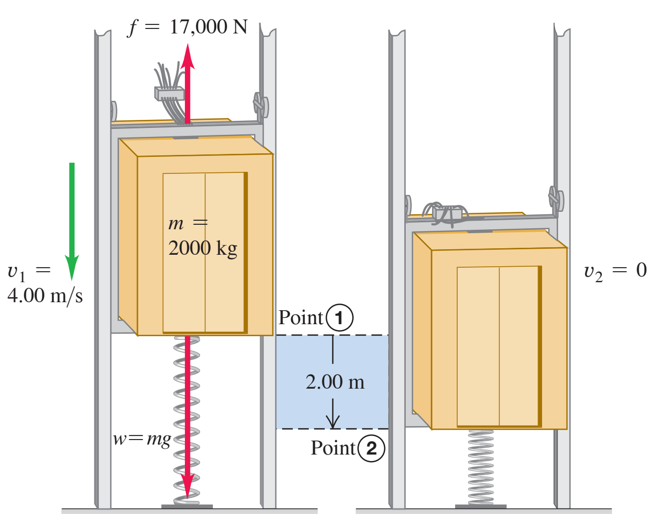
$$k = \frac{2(K_1 + W_{\text{other}} - mgy_2)}{y_2^2}$$

$$= \frac{2[16{,}000 \text{ J} + (-34{,}000 \text{ J}) - (-39{,}200 \text{ J})]}{(-2.00 \text{ m})^2}$$

$$= 1.06 \times 10^4 \text{ N/m}$$

This is about one-tenth the force constant of a spring in an automobile suspension.

Figure 7.17 The fall of an elevator is stopped by a spring and by a constant friction force.



EVALUATE There might seem to be a paradox here. The elastic potential energy at point 2 is

$$\tfrac{1}{2}ky_2^2 = \tfrac{1}{2}(1.06 \times 10^4\ \text{N/m})(-2.00\ \text{m})^2 = 21{,}200\ \text{J}$$

This is *more* than the total mechanical energy at point 1:

$$E_1 = K_1 + U_1 = 16{,}000\ \text{J} + 0 = 16{,}000\ \text{J}$$

But the friction force *decreased* the total mechanical energy of the system by 34,000 J between points 1 and 2. Did energy appear from nowhere? No. At point 2, which is below the origin, there is also *negative* gravitational potential energy $mgy_2 = -39{,}200$ J. The total mechanical energy at point 2 is therefore not 21,200 J but

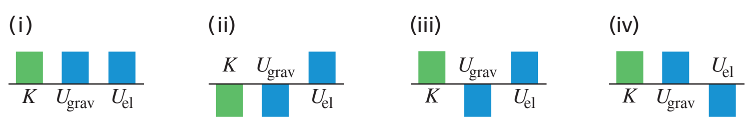
$$\begin{aligned} E_2 &= K_2 + U_2 = 0 + \tfrac{1}{2}ky_2^2 + mgy_2 \\ &= 0 + 21{,}200\ \text{J} + (-39{,}200\ \text{J}) = -18{,}000\ \text{J} \end{aligned}$$

This is just the initial total mechanical energy of 16,000 J minus 34,000 J lost to friction.

Will the elevator stay at the bottom of the shaft? At point 2 the compressed spring exerts an upward force of magnitude $F_{\text{spring}} = (1.06 \times 10^4\ \text{N/m})(2.00\ \text{m}) = 21{,}200$ N, while the downward force of gravity is only $w = mg = (2000\ \text{kg})(9.80\ \text{m/s}^2) = 19{,}600$ N. If there were no friction, there would be a net upward force of 21,200 N − 19,600 N = 1600 N, and the elevator would rebound. But the safety clamp can exert a kinetic friction force of 17,000 N, and it can presumably exert a maximum static friction force greater than that. Hence the clamp will keep the elevator from rebounding.

KEYCONCEPT For problems in which you use an energy approach to analyze an object that both changes height and interacts with an ideal spring, you must include both gravitational potential energy and elastic potential energy.

TEST YOUR UNDERSTANDING OF SECTION 7.2 Consider the situation in Example 7.9 at the instant when the elevator is still moving downward and the spring is compressed by 1.00 m. Which of the energy bar graphs in the figure most accurately shows the kinetic energy K, gravitational potential energy U_{grav}, and elastic potential energy U_{el} at this instant?



ANSWER

(iii) The elevator is still moving downward, so the kinetic energy K is positive (remember that K can never be negative); the elevator is below point 1, so $y < 0$ and $U_{\text{grav}} < 0$; and the spring is compressed, so $U_{\text{el}} > 0$.

7.3 CONSERVATIVE AND NONCONSERVATIVE FORCES

In our discussions of potential energy we have talked about "storing" kinetic energy by converting it to potential energy, with the idea that we can retrieve it again as kinetic energy. For example, when you throw a ball up in the air, it slows down as kinetic energy is converted to gravitational potential energy. But on the way down the ball speeds up as potential energy is converted back to kinetic energy. If there is no air resistance, the ball is moving just as fast when you catch it as when you threw it.

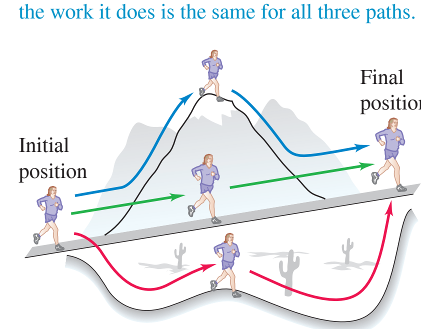
Another example is a glider moving on a frictionless horizontal air track that runs into a spring bumper. The glider compresses the spring and then bounces back. If there is no friction, the glider ends up with the same speed and kinetic energy it had before the collision. Again, there is a two-way conversion from kinetic to potential energy and back. In both cases the total mechanical energy, kinetic plus potential, is constant or *conserved* during the motion.

Conservative Forces

A force that offers this opportunity of two-way conversion between kinetic and potential energies is called a **conservative force.** We have seen two examples of conservative forces: the gravitational force and the spring force. (Later in this book we'll study another conservative force, the electric force between charged objects.) An essential feature of conservative forces is that their work is always *reversible*. Anything that we deposit in the energy "bank" can later be withdrawn without loss. Another important aspect of conservative forces is that if an object follows different paths from point 1 to point 2, the work done by a conservative force is the same for all of these paths (**Fig. 7.18**). For example, if an object stays close to the surface of the earth, the gravitational force $m\vec{g}$ is independent

Figure **7.18** The work done by a conservative force such as gravity depends on only the endpoints of a path, not the specific path taken between those points.

Because the gravitational force is conservative, the work it does is the same for all three paths.



of height, and the work done by this force depends on only the change in height. If the object moves around a closed path, ending at the same height where it started, the *total* work done by the gravitational force is always zero.

In summary, the work done by a conservative force has four properties:

1. It can be expressed as the difference between the initial and final values of a *potential-energy* function.
2. It is reversible.
3. It is independent of the path of the object and depends on only the starting and ending points.
4. When the starting and ending points are the same, the total work is zero.

When the *only* forces that do work are conservative forces, the total mechanical energy $E = K + U$ is constant.

Nonconservative Forces

Not all forces are conservative. Consider the friction force acting on the crate sliding on a ramp in Example 7.6 (Section 7.1). When the crate slides up and then back down to the starting point, the total work done on it by the friction force is *not* zero. When the direction of motion reverses, so does the friction force, and friction does *negative* work in *both* directions. Friction also acts when a car with its brakes locked skids with decreasing speed (and decreasing kinetic energy). The lost kinetic energy can't be recovered by reversing the motion or in any other way, and total mechanical energy is *not* conserved. So there is *no* potential-energy function for the friction force.

In the same way, the force of fluid resistance (see Section 5.3) is not conservative. If you throw a ball up in the air, air resistance does negative work on the ball while it's rising *and* while it's descending. The ball returns to your hand with less speed and less kinetic energy than when it left, and there is no way to get back the lost mechanical energy.

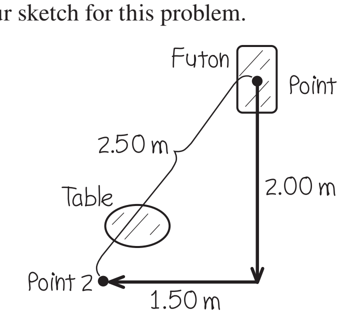
A force that is not conservative is called a **nonconservative force.** The work done by a nonconservative force *cannot* be represented by a potential-energy function. Some nonconservative forces, like kinetic friction or fluid resistance, cause mechanical energy to be lost or dissipated; a force of this kind is called a **dissipative force.** There are also nonconservative forces that *increase* mechanical energy. The fragments of an exploding firecracker fly off with very large kinetic energy, thanks to a chemical reaction of gunpowder with oxygen. The forces unleashed by this reaction are nonconservative because the process is not reversible. (The fragments never spontaneously reassemble themselves into a complete firecracker!)

EXAMPLE 7.10 Frictional work depends on the path

You are rearranging your furniture and wish to move a 40.0 kg futon 2.50 m across the room. A heavy coffee table, which you don't want to move, blocks this straight-line path. Instead, you slide the futon along a dogleg path; the doglegs are 2.00 m and 1.50 m long. How much more work must you do to push the futon along the dogleg path than along the straight-line path? The coefficient of kinetic friction is $\mu_k = 0.200$.

IDENTIFY and SET UP Here both you and friction do work on the futon, so we must use the energy relationship that includes "other" forces. We'll use this relationship to find a connection between the work that *you* do and the work that *friction* does. **Figure 7.19** shows our sketch. The futon is at rest at both point 1 and point 2, so $K_1 = K_2 = 0$. There is no elastic potential energy (there are no springs), and the gravitational potential energy does not change because the futon moves only horizontally, so $U_1 = U_2$. From Eq. (7.14) it follows that $W_{other} = 0$. That "other" work done on the futon is the sum of the positive work you do,

Figure 7.19 Our sketch for this problem.



W_{you}, and the negative work done by friction, W_{fric}. Since the sum of these is zero, we have

$$W_{you} = -W_{fric}$$

So we can calculate the work done by friction to determine W_{you}.

EXECUTE The floor is horizontal, so the normal force on the futon equals its weight mg and the magnitude of the friction force is $f_k = \mu_k n = \mu_k mg$. The work you do over each path is then

$$\begin{aligned} W_{you} &= -W_{fric} = -(-f_k s) = +\mu_k mgs \\ &= (0.200)(40.0\text{ kg})(9.80\text{ m/s}^2)(2.50\text{ m}) \\ &= 196\text{ J} \qquad \text{(straight-line path)} \\ W_{you} &= -W_{fric} = +\mu_k mgs \\ &= (0.200)(40.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m} + 1.50\text{ m}) \\ &= 274\text{ J} \qquad \text{(dogleg path)} \end{aligned}$$

The extra work you must do is $274\text{ J} - 196\text{ J} = 78\text{ J}$.

EVALUATE Friction does different amounts of work on the futon, -196 J and -274 J, on these different paths between points 1 and 2. Hence friction is a *nonconservative* force.

KEYCONCEPT The work done by a nonconservative force on an object that moves between two points depends on the path that the object follows. Unlike for conservative forces, you *cannot* express the work done by a nonconservative force in terms of a change in potential energy.

EXAMPLE 7.11 Conservative or nonconservative?

In a region of space the force on an electron is $\vec{F} = Cx\hat{j}$, where C is a positive constant. The electron moves around a square loop in the xy-plane (**Fig. 7.20**). Calculate the work done on the electron by force $\vec{F}$ during a counterclockwise trip around the square. Is this force conservative or nonconservative?

IDENTIFY and SET UP Force $\vec{F}$ is not constant and in general is not in the same direction as the displacement. To calculate the work done by $\vec{F}$, we'll use the general expression Eq. (6.14):

$$W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{l}$$

where $d\vec{l}$ is an infinitesimal displacement. We'll calculate the work done on each leg of the square separately, and add the results to find the work done on the round trip. If this round-trip work is zero, force $\vec{F}$ is conservative and can be represented by a potential-energy function.

EXECUTE On the first leg, from $(0, 0)$ to $(L, 0)$, the force is everywhere perpendicular to the displacement. So $\vec{F} \cdot d\vec{l} = 0$, and the work done on the first leg is $W_1 = 0$. The force has the same value $\vec{F} = CL\hat{j}$ everywhere on the second leg, from $(L, 0)$ to (L, L). The displacement on this leg is in the $+y$-direction, so $d\vec{l} = dy\hat{j}$ and

$$\vec{F} \cdot d\vec{l} = CL\hat{j} \cdot dy\hat{j} = CL\,dy$$

The work done on the second leg is then

$$W_2 = \int_{(L,0)}^{(L,L)} \vec{F} \cdot d\vec{l} = \int_{y=0}^{y=L} CL\,dy = CL\int_0^L dy = CL^2$$

On the third leg, from (L, L) to $(0, L)$, $\vec{F}$ is again perpendicular to the displacement and so $W_3 = 0$. The force is zero on the final leg, from $(0, L)$ to $(0, 0)$, so $W_4 = 0$. The work done by $\vec{F}$ on the round trip is therefore

$$W = W_1 + W_2 + W_3 + W_4 = 0 + CL^2 + 0 + 0 = CL^2$$

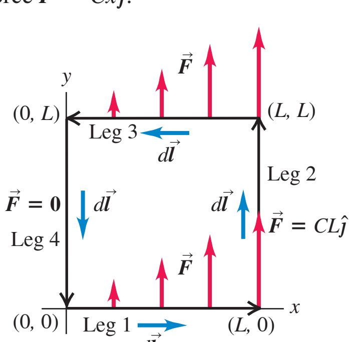
The starting and ending points are the same, but the total work done by $\vec{F}$ is not zero. This is a *nonconservative* force; it *cannot* be represented by a potential-energy function.

EVALUATE Because $W > 0$, the total mechanical energy *increases* as the electron goes around the loop. This is actually what happens in an electric generating plant: A loop of wire is moved through a magnetic field, which gives rise to a nonconservative force similar to the one here. Electrons in the wire gain energy as they move around the loop, and this energy is carried via transmission lines to the consumer. (We'll discuss this in Chapter 29.)

If the electron went *clockwise* around the loop, $\vec{F}$ would be unaffected but the direction of each infinitesimal displacement $d\vec{l}$ would be reversed. Thus the sign of work would also reverse, and the work for a clockwise round trip would be $W = -CL^2$. This is a different behavior than the nonconservative friction force. The work done by friction on an object that slides in any direction over a stationary surface is always negative (see Example 7.6 in Section 7.1).

KEYCONCEPT The work done on an object that makes a complete trip around a closed path is zero if the force is conservative, but nonzero if the force is nonconservative.

Figure 7.20 An electron moving around a square loop while being acted on by the force $\vec{F} = Cx\hat{j}$.



APPLICATION Nonconservative Forces and Internal Energy in a Tire An automobile tire deforms and flexes like a spring as it rolls, but it is not an ideal spring: Nonconservative internal friction forces act within the rubber. As a result, mechanical energy is lost and converted to internal energy of the tire. Thus the temperature of a tire increases as it rolls, which causes the pressure of the air inside the tire to increase as well. That's why tire pressures are best checked before the car is driven, when the tire is cold.

The Law of Conservation of Energy

Nonconservative forces cannot be represented in terms of potential energy. But we can describe the effects of these forces in terms of kinds of energy other than kinetic or potential energy. When a car with locked brakes skids to a stop, both the tires and the road surface become hotter. The energy associated with this change in the state of the materials is called **internal energy.** Raising the temperature of an object increases its internal energy; lowering the object's temperature decreases its internal energy.

To see the significance of internal energy, let's consider a block sliding on a rough surface. Friction does *negative* work on the block as it slides, and the change in internal energy of the block and surface (both of which get hotter) is *positive*. Careful experiments show that the increase in the internal energy is *exactly* equal to the absolute value of the work done by friction. In other words,

$$\Delta U_{\text{int}} = -W_{\text{other}}$$

where ΔU_{int} is the change in internal energy. We substitute this into Eq. (7.14):

$$K_1 + U_1 - \Delta U_{\text{int}} = K_2 + U_2$$

Writing $\Delta K = K_2 - K_1$ and $\Delta U = U_2 - U_1$, we can finally express this as

Law of conservation of energy:

$$\underbrace{\Delta K}_{\text{Change in kinetic energy}} + \underbrace{\Delta U}_{\text{Change in potential energy}} + \underbrace{\Delta U_{\text{int}}}_{\text{Change in internal energy}} = 0 \qquad (7.15)$$

This remarkable statement is the general form of the **law of conservation of energy.** In a given process, the kinetic energy, potential energy, and internal energy of a system may all change. But the *sum* of those changes is always zero. If there is a decrease in one form of energy, it is made up for by an increase in the other forms (**Fig. 7.21**). When we expand our definition of energy to include internal energy, Eq. (7.15) says: *Energy is never created or destroyed; it only changes form.* No exception to this rule has ever been found.

The concept of work has been banished from Eq. (7.15); instead, it suggests that we think purely in terms of the conversion of energy from one form to another. For example, when you throw a baseball straight up, you convert a portion of the internal energy of your molecules to kinetic energy of the baseball. This is converted to gravitational potential energy as the ball climbs and back to kinetic energy as the ball falls. If there is air resistance, part of the energy is used to heat up the air and the ball and increase their internal energy. Energy is converted back to the kinetic form as the ball falls. If you catch the ball in your hand, whatever energy was not lost to the air once again becomes internal energy; the ball and your hand are now warmer than they were at the beginning.

In Chapters 19 and 20, we'll study the relationship of internal energy to temperature changes, heat, and work. This is the heart of the area of physics called *thermodynamics*.

Figure 7.21 The battery pack in this radio-controlled helicopter contains 2.4×10^4 J of electric energy. When this energy is used up, the internal energy of the battery pack decreases by this amount, so $\Delta U_{\text{int}} = -2.4 \times 10^4$ J. This energy can be converted to kinetic energy to make the rotor blades and helicopter go faster, or to gravitational potential energy to make the helicopter climb.

CONCEPTUAL EXAMPLE 7.12 Work done by friction

Let's return to Example 7.5 (Section 7.1), in which Throcky skateboards down a curved ramp. He starts with zero kinetic energy and 735 J of potential energy, and at the bottom he has 450 J of kinetic energy and zero potential energy; hence $\Delta K = +450$ J and $\Delta U = -735$ J. The work $W_{\text{other}} = W_{\text{fric}}$ done by the friction forces is -285 J, so the change in internal energy is $\Delta U_{\text{int}} = -W_{\text{other}} = +285$ J. The skateboard wheels and bearings and the ramp all get a little warmer. In accordance with Eq. (7.15), the sum of the energy changes equals zero:

$$\Delta K + \Delta U + \Delta U_{\text{int}} = +450\text{ J} + (-735\text{ J}) + 285\text{ J} = 0$$

The total energy of the system (including internal, nonmechanical forms of energy) is conserved.

KEYCONCEPT In any physical process, energy is never created or destroyed; it is merely converted among the forms of kinetic energy, potential energy, and internal energy.

TEST YOUR UNDERSTANDING OF SECTION 7.3 In a hydroelectric generating station, falling water is used to drive turbines ("water wheels"), which in turn run electric generators. Compared to the amount of gravitational potential energy released by the falling water, how much electrical energy is produced? (i) The same; (ii) more; (iii) less.

ANSWER

(iii) Because of friction in the turbines and between the water and turbines, some of the potential energy goes into raising the temperatures of the water and the mechanism.

7.4 FORCE AND POTENTIAL ENERGY

For the two kinds of conservative forces (gravitational and elastic) we have studied, we started with a description of the behavior of the *force* and derived from that an expression for the *potential energy*. For example, for an object with mass m in a uniform gravitational field, the gravitational force is $F_y = -mg$. We found that the corresponding potential energy is $U(y) = mgy$. The force that an ideal spring exerts on an object is $F_x = -kx$, and the corresponding potential-energy function is $U(x) = \frac{1}{2}kx^2$.

In studying physics, however, you'll encounter situations in which you are given an expression for the *potential energy* as a function of position and have to find the corresponding *force*. We'll see several examples of this kind when we study electric forces later in this book: It's often far easier to calculate the electric potential energy first and then determine the corresponding electric force afterward.

Here's how we find the force that corresponds to a given potential-energy expression. First let's consider motion along a straight line, with coordinate x. We denote the x-component of force, a function of x, by $F_x(x)$ and the potential energy as $U(x)$. This notation reminds us that both F_x and U are *functions* of x. Now we recall that in any displacement, the work W done by a conservative force equals the negative of the change ΔU in potential energy:

$$W = -\Delta U$$

Let's apply this to a small displacement Δx. The work done by the force $F_x(x)$ during this displacement is approximately equal to $F_x(x)\Delta x$. We have to say "approximately" because $F_x(x)$ may vary a little over the interval Δx. So

$$F_x(x)\Delta x = -\Delta U \qquad \text{and} \qquad F_x(x) = -\frac{\Delta U}{\Delta x}$$

You can probably see what's coming. We take the limit as $\Delta x \rightarrow 0$; in this limit, the variation of F_x becomes negligible, and we have the exact relationship

Force from potential energy: In **one-dimensional motion,** the value of a conservative force at point x ...

$$F_x(x) = -\frac{dU(x)}{dx} \qquad (7.16)$$

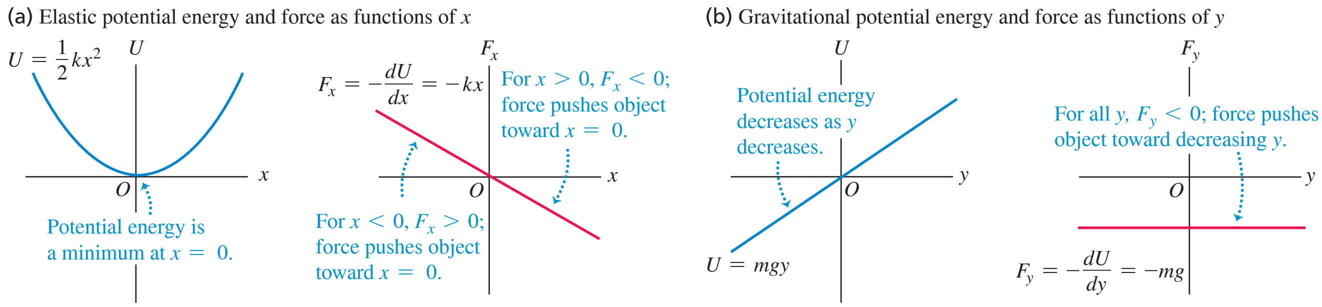
... is the negative of the derivative at x of the associated potential-energy function.

This result makes sense; in regions where $U(x)$ changes most rapidly with x (that is, where $dU(x)/dx$ is large), the greatest amount of work is done during a given displacement, and this corresponds to a large force magnitude. Also, when $F_x(x)$ is in the positive x-direction, $U(x)$ *decreases* with increasing x. So $F_x(x)$ and $dU(x)/dx$ should indeed have opposite signs. The physical meaning of Eq. (7.16) is that *a conservative force always acts to push the system toward lower potential energy.*

As a check, let's consider the function for elastic potential energy, $U(x) = \frac{1}{2}kx^2$. Substituting this into Eq. (7.16) yields

$$F_x(x) = -\frac{d}{dx}\left(\tfrac{1}{2}kx^2\right) = -kx$$

Figure 7.22 A conservative force is the negative derivative of the corresponding potential energy.



which is the correct expression for the force exerted by an ideal spring (**Fig. 7.22a**). Similarly, for gravitational potential energy we have $U(y) = mgy$; taking care to change x to y for the choice of axis, we get $F_y = -dU/dy = -d(mgy)/dy = -mg$, which is the correct expression for gravitational force (Fig. 7.22b).

EXAMPLE 7.13 An electric force and its potential energy

An electrically charged particle is held at rest at the point $x = 0$; a second particle with equal charge is free to move along the positive x-axis. The potential energy of the system is $U(x) = C/x$, where C is a positive constant that depends on the magnitude of the charges. Derive an expression for the x-component of force acting on the movable particle as a function of its position.

IDENTIFY and SET UP We are given the potential-energy function $U(x)$. We'll find the corresponding force function by using Eq. (7.16), $F_x(x) = -dU(x)/dx$.

EXECUTE The derivative of $1/x$ with respect to x is $-1/x^2$. So for $x > 0$ the force on the movable charged particle is

$$F_x(x) = -\frac{dU(x)}{dx} = -C\left(-\frac{1}{x^2}\right) = \frac{C}{x^2}$$

EVALUATE The x-component of force is positive, corresponding to a repulsion between like electric charges. Both the potential energy and the force are very large when the particles are close together (small x), and both get smaller as the particles move farther apart (large x). The force pushes the movable particle toward large positive values of x, where the potential energy is lower. (We'll study electric forces in detail in Chapter 21.)

KEYCONCEPT For motion in one dimension, the force associated with a potential-energy function equals the negative derivative of that function with respect to position.

Force and Potential Energy in Three Dimensions

We can extend this analysis to three dimensions for a particle that may move in the x-, y-, or z-direction, or all at once, under the action of a conservative force that has components F_x, F_y, and F_z. Each component of force may be a function of the coordinates x, y, and z. The potential-energy function U is also a function of all three space coordinates. The potential-energy change ΔU when the particle moves a small distance Δx in the x-direction is again given by $-F_x\,\Delta x$; it doesn't depend on F_y and F_z, which represent force components that are perpendicular to the displacement and do no work. So we again have the approximate relationship

$$F_x = -\frac{\Delta U}{\Delta x}$$

We determine the y- and z-components in exactly the same way:

$$F_y = -\frac{\Delta U}{\Delta y} \qquad F_z = -\frac{\Delta U}{\Delta z}$$

To make these relationships exact, we take the limits $\Delta x \to 0$, $\Delta y \to 0$, and $\Delta z \to 0$ so that these ratios become derivatives. Because U may be a function of all three coordinates, we need to remember that when we calculate each of these derivatives, only one coordinate changes at a time. We compute the derivative of U with respect to x by assuming that y and z are constant and only x varies, and so on. Such a derivative is called a *partial derivative*. The usual notation for a partial derivative is $\partial U/\partial x$ and so on; the symbol ∂ is a modified d. So we write

Force from potential energy: In **three-dimensional motion,** the value at a given point of each component of a conservative force ...

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial z} \tag{7.17}$$

... is the negative of the partial derivative at that point of the associated potential-energy function.

We can use unit vectors to write a single compact vector expression for the force $\vec{F}$:

Force from potential energy: The vector value of a conservative force at a given point ...

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right) = -\vec{\nabla}U \tag{7.18}$$

... is the negative of the gradient at that point of the associated potential-energy function.

In Eq. (7.18) we take the partial derivative of U with respect to each coordinate, multiply by the corresponding unit vector, and then take the vector sum. This operation is called the **gradient** of U and is often abbreviated as $\vec{\nabla}U$.

As a check, let's substitute into Eq. (7.18) the function $U = mgy$ for gravitational potential energy:

$$\vec{F} = -\vec{\nabla}(mgy) = -\left(\frac{\partial(mgy)}{\partial x}\hat{\imath} + \frac{\partial(mgy)}{\partial y}\hat{\jmath} + \frac{\partial(mgy)}{\partial z}\hat{k}\right) = (-mg)\hat{\jmath}$$

This is just the familiar expression for the gravitational force.

APPLICATION Topography and Potential Energy Gradient The greater the elevation of a hiker in Canada's Banff National Park, the greater the gravitational potential energy U_{grav}. Think of an x-axis that runs horizontally from west to east and a y-axis that runs horizontally from south to north. Then the function $U_{grav}(x, y)$ tells us the elevation as a function of position in the park. Where the mountains have steep slopes, $\vec{F} = -\vec{\nabla}U_{grav}$ has a large magnitude and there's a strong force pushing you along the mountain's surface toward a region of lower elevation (and hence lower U_{grav}). There's zero force along the surface of the lake, which is all at the same elevation. Hence U_{grav} is constant at all points on the lake surface, and $\vec{F} = -\vec{\nabla}U_{grav} = 0$.

EXAMPLE 7.14 Force and potential energy in two dimensions

A puck with coordinates x and y slides on a level, frictionless air-hockey table. It is acted on by a conservative force described by the potential-energy function

$$U(x, y) = \tfrac{1}{2}k(x^2 + y^2)$$

Note that $r = \sqrt{x^2 + y^2}$ is the distance on the table surface from the puck to the origin. Find a vector expression for the force acting on the puck, and find an expression for the magnitude of the force.

IDENTIFY and SET UP Starting with the function $U(x, y)$, we need to find the vector components and magnitude of the corresponding force $\vec{F}$. We'll use Eq. (7.18) to find the components. The function U doesn't depend on z, so the partial derivative of U with respect to z is $\partial U/\partial z = 0$ and the force has no z-component. We'll determine the magnitude F of the force by using $F = \sqrt{F_x{}^2 + F_y{}^2}$.

EXECUTE The x- and y-components of $\vec{F}$ are

$$F_x = -\frac{\partial U}{\partial x} = -kx \qquad F_y = -\frac{\partial U}{\partial y} = -ky$$

From Eq. (7.18), the vector expression for the force is

$$\vec{F} = (-kx)\hat{\imath} + (-ky)\hat{\jmath} = -k(x\hat{\imath} + y\hat{\jmath})$$

The magnitude of the force is

$$F = \sqrt{(-kx)^2 + (-ky)^2} = k\sqrt{x^2 + y^2} = kr$$

EVALUATE Because $x\hat{\imath} + y\hat{\jmath}$ is just the position vector $\vec{r}$ of the particle, we can rewrite our result as $\vec{F} = -k\vec{r}$. This represents a force that is opposite in direction to the particle's position vector— that is, a force directed toward the origin, $r = 0$. This is the force that would be exerted on the puck if it were attached to one end of a spring that obeys Hooke's law and has a negligibly small unstretched length compared to the other distances in the problem. (The other end is attached to the air-hockey table at $r = 0$.)

To check our result, note that $U = \tfrac{1}{2}kr^2$. We can find the force from this expression using Eq. (7.16) with x replaced by r:

$$F_r = -\frac{dU}{dr} = -\frac{d}{dr}\left(\tfrac{1}{2}kr^2\right) = -kr$$

As we found above, the force has magnitude kr; the minus sign indicates that the force is toward the origin (at $r = 0$).

KEYCONCEPT For motion in two or three dimensions, the force associated with a potential-energy function equals the negative gradient of that function.

TEST YOUR UNDERSTANDING OF SECTION 7.4 A particle moving along the x-axis is acted on by a conservative force F_x. At a certain point, the force is zero. (a) Which of the following statements about the value of the potential-energy function $U(x)$ at that point is correct? (i) $U(x) = 0$; (ii) $U(x) > 0$; (iii) $U(x) < 0$; (iv) not enough information is given to decide. (b) Which of the following statements about the value of the derivative of $U(x)$ at that point is correct? (i) $dU(x)/dx = 0$; (ii) $dU(x)/dx > 0$; (iii) $dU(x)/dx < 0$; (iv) not enough information is given to decide.

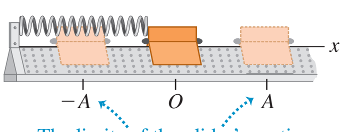
ANSWER

(a) **(iv)**, (b) **(i)** If $F_x = 0$ at a point, then the derivative of $U(x)$ must be zero at that point because $F_x = -dU(x)/dx$. However, this tells us absolutely nothing about the *value* of $U(x)$ at that point.

7.5 ENERGY DIAGRAMS

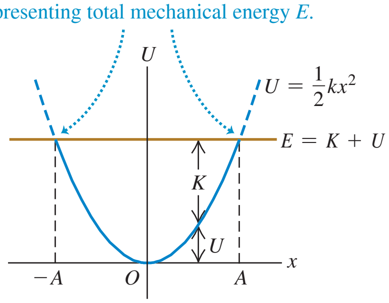
Figure **7.23** (a) A glider on an air track. The spring exerts a force $F_x = -kx$. (b) The potential-energy function.

(a)



(b)

On the graph, the limits of motion are the points where the U curve intersects the horizontal line representing total mechanical energy E.



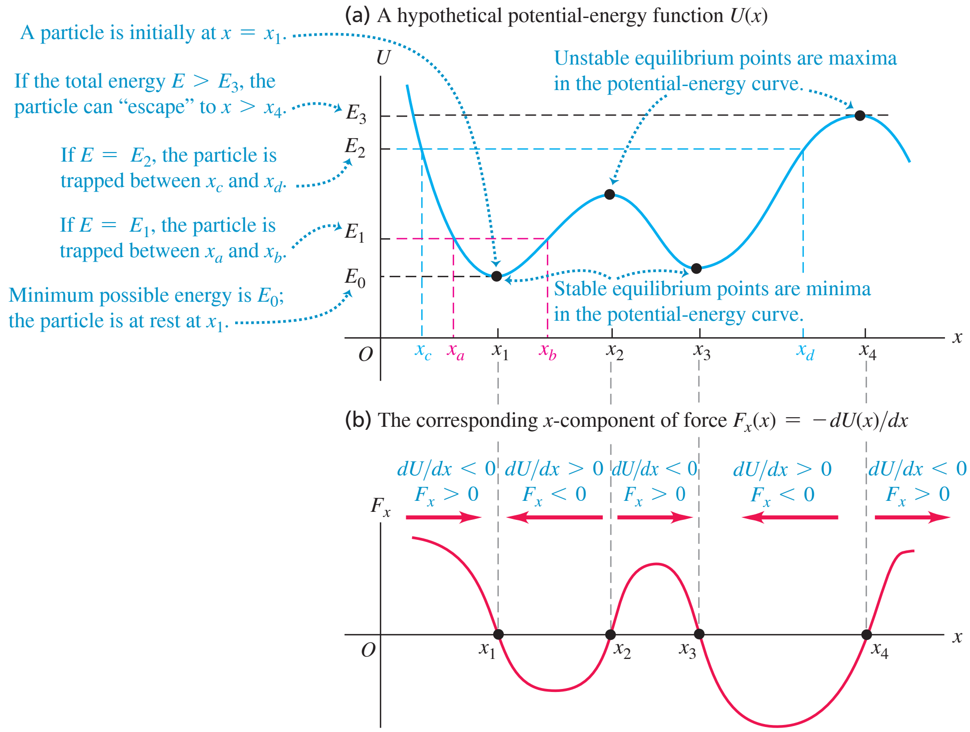
When a particle moves along a straight line under the action of a conservative force, we can get a lot of insight into its possible motions by looking at the graph of the potential-energy function $U(x)$. **Figure 7.23a** shows a glider with mass m that moves along the x-axis on an air track. The spring exerts on the glider a force with x-component $F_x = -kx$. Figure 7.23b is a graph of the corresponding potential-energy function $U(x) = \frac{1}{2}kx^2$. If the elastic force of the spring is the *only* horizontal force acting on the glider, the total mechanical energy $E = K + U$ is constant, independent of x. A graph of E as a function of x is thus a straight horizontal line. We use the term **energy diagram** for a graph like this, which shows both the potential-energy function $U(x)$ and the energy of the particle subjected to the force that corresponds to $U(x)$.

The vertical distance between the U and E graphs at each point represents the difference $E - U$, equal to the kinetic energy K at that point. We see that K is greatest at $x = 0$. It is zero at the values of x where the two graphs cross, labeled A and $-A$ in Fig. 7.23b. Thus the speed v is greatest at $x = 0$, and it is zero at $x = \pm A$, the points of *maximum* possible displacement from $x = 0$ for a given value of the total energy E. The potential energy U can never be greater than the total energy E; if it were, K would be negative, and that's impossible. The motion is a back-and-forth oscillation between the points $x = A$ and $x = -A$.

From Eq. (7.16), at each point the force F_x on the glider is equal to the negative of the slope of the $U(x)$ curve: $F_x = -dU/dx$ (see Fig. 7.22a). When the particle is at $x = 0$, the slope and the force are zero, so this is an *equilibrium* position. When x is positive, the slope of the $U(x)$ curve is positive and the force F_x is negative, directed toward the origin. When x is negative, the slope is negative and F_x is positive, again directed toward the origin. Such a force is called a *restoring force;* when the glider is displaced to either side of $x = 0$, the force tends to "restore" it back to $x = 0$. An analogous situation is a marble rolling around in a round-bottomed bowl. We say that $x = 0$ is a point of **stable equilibrium.** More generally, *any minimum in a potential-energy curve is a stable equilibrium position.*

Figure 7.24a shows a hypothetical but more general potential-energy function $U(x)$. Figure 7.24b shows the corresponding force $F_x = -dU/dx$. Points x_1 and x_3 are stable equilibrium points. At both points, F_x is zero because the slope of the $U(x)$ curve is zero. When the particle is displaced to either side, the force pushes back toward the equilibrium point. The slope of the $U(x)$ curve is also zero at points x_2 and x_4, and these are also equilibrium points. But when the particle is displaced a little to the right of either point, the slope of the $U(x)$ curve becomes negative, corresponding to a positive F_x that tends to push the particle still farther from the point. When the particle is displaced a little to the left, F_x is negative, again pushing away from equilibrium. This is analogous to a marble rolling on the top of a bowling ball. Points x_2 and x_4 are called **unstable equilibrium** points; *any maximum in a potential-energy curve is an unstable equilibrium position.*

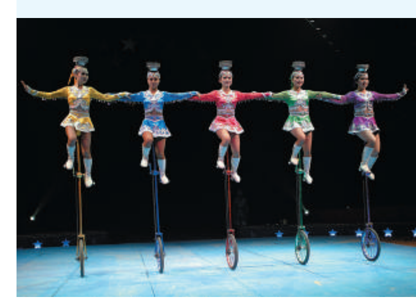
Figure **7.24** The maxima and minima of a potential-energy function $U(x)$ correspond to points where $F_x = 0$.



CAUTION **Potential energy and the direction of a conservative force** The direction of the force on an object is *not* determined by the sign of the potential energy U. Rather, it's the sign of $F_x = -dU/dx$ that matters. The physically significant quantity is the *difference* in the values of U between two points (Section 7.1), which is what the derivative $F_x = -dU/dx$ measures. You can always add a constant to the potential-energy function without changing the physics.

If the total energy is E_1 and the particle is initially near x_1, it can move only in the region between x_a and x_b determined by the intersection of the E_1 and U graphs (Fig. 7.24a). Again, U cannot be greater than E_1 because K can't be negative. We speak of the particle as moving in a *potential well,* and x_a and x_b are the *turning points* of the particle's motion (since at these points, the particle stops and reverses direction). If we increase the total energy to the level E_2, the particle can move over a wider range, from x_c to x_d. If the total energy is greater than E_3, the particle can "escape" and move to indefinitely large values of x. At the other extreme, E_0 represents the minimum total energy the system can have.

APPLICATION Acrobats in Equilibrium Each of these acrobats is in *unstable* equilibrium. The gravitational potential energy is lower no matter which way an acrobat tips, so if she begins to fall she will keep on falling. Staying balanced requires the acrobats' constant attention.



TEST YOUR UNDERSTANDING OF SECTION 7.5 The curve in Fig. 7.24b has a maximum at a point between x_2 and x_3. Which statement correctly describes the particle's acceleration (with magnitude a) at this point? (i) $a_x = 0$. (ii) The particle accelerates in the $+x$-direction, so $a_x > 0$; a is less than at any other point between x_2 and x_3. (iii) The particle accelerates in the $+x$-direction, so $a_x > 0$; a is greater than at any other point between x_2 and x_3. (iv) The particle accelerates in the $-x$-direction, so $a_x < 0$; a is less than at any other point between x_2 and x_3. (v) The particle accelerates in the $-x$-direction, so $a_x < 0$; a is greater than at any other point between x_2 and x_3.

ANSWER

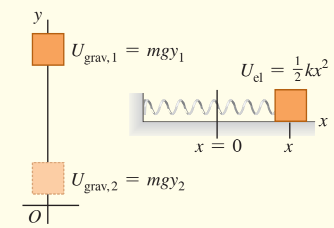
(iii) Figure 7.24b shows the x-component of force, F_x. Where this is maximum (most positive), the x-component of force and the x-acceleration have more positive values than at adjacent values of x.

CHAPTER 7 SUMMARY

Gravitational potential energy and elastic potential energy: The work done on a particle by a constant gravitational force can be represented as a change in the gravitational potential energy, $U_{\text{grav}} = mgy$. This energy is a shared property of the particle and the earth. A potential energy is also associated with the elastic force $F_x = -kx$ exerted by an ideal spring, where x is the amount of stretch or compression. The work done by this force can be represented as a change in the elastic potential energy of the spring, $U_{\text{el}} = \frac{1}{2}kx^2$.

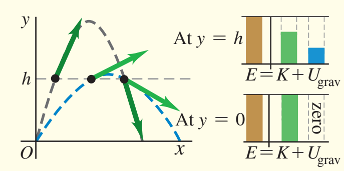
$$\begin{aligned} W_{\text{grav}} &= mgy_1 - mgy_2 \\ &= U_{\text{grav},1} - U_{\text{grav},2} \\ &= -\Delta U_{\text{grav}} \end{aligned} \qquad (7.2),\ (7.3)$$

$$\begin{aligned} W_{\text{el}} &= \tfrac{1}{2}kx_1^2 - \tfrac{1}{2}kx_2^2 \\ &= U_{\text{el},1} - U_{\text{el},2} = -\Delta U_{\text{el}} \end{aligned} \qquad (7.10),\ (7.11)$$



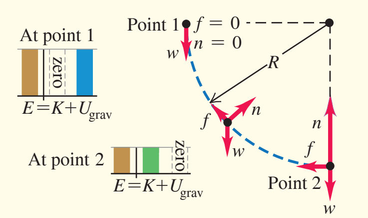
When total mechanical energy is conserved: The total potential energy U is the sum of the gravitational and elastic potential energies: $U = U_{\text{grav}} + U_{\text{el}}$. If no forces other than the gravitational and elastic forces do work on a particle, the sum of kinetic and potential energies is conserved. This sum $E = K + U$ is called the total mechanical energy. (See Examples 7.1, 7.3, 7.4, and 7.7.)

$$K_1 + U_1 = K_2 + U_2 \qquad (7.4),\ (7.12)$$



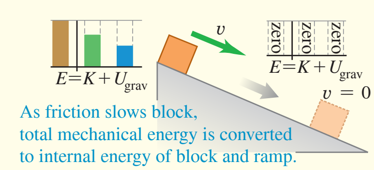
When total mechanical energy is not conserved: When forces other than the gravitational and elastic forces do work on a particle, the work W_{other} done by these other forces equals the change in total mechanical energy (kinetic energy plus total potential energy). (See Examples 7.2, 7.5, 7.6, 7.8, and 7.9.)

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2 \qquad (7.14)$$



Conservative forces, nonconservative forces, and the law of conservation of energy: All forces are either conservative or nonconservative. A conservative force is one for which the work–kinetic energy relationship is completely reversible. The work of a conservative force can always be represented by a potential-energy function, but the work of a nonconservative force cannot. The work done by nonconservative forces manifests itself as changes in the internal energy of objects. The sum of kinetic, potential, and internal energies is always conserved. (See Examples 7.10–7.12.)

$$\Delta K + \Delta U + \Delta U_{\text{int}} = 0 \qquad (7.15)$$

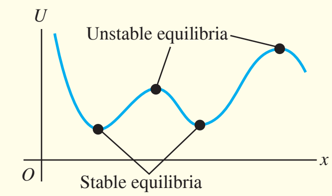


Determining force from potential energy: For motion along a straight line, a conservative force $F_x(x)$ is the negative derivative of its associated potential-energy function U. In three dimensions, the components of a conservative force are negative partial derivatives of U. (See Examples 7.13 and 7.14.)

$$F_x(x) = -\frac{dU(x)}{dx} \qquad (7.16)$$

$$F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \quad F_z = -\frac{\partial U}{\partial z} \qquad (7.17)$$

$$\begin{aligned} \vec{F} &= -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right) \\ &= -\vec{\nabla} U \end{aligned} \qquad (7.18)$$



Chapter 7 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLES 7.1 and 7.2 (Section 7.1) before attempting these problems.

VP7.2.1 You throw a baseball (mass 0.145 kg) vertically upward. It leaves your hand moving at 12.0 m/s. Air resistance can be neglected. At what height above your hand does the ball have (a) half as much upward velocity, (b) half as much kinetic energy as when it left your hand?

VP7.2.2 You toss a rock of mass m vertically upward. Air resistance can be neglected. The rock reaches a maximum height h above your hand. What is the speed of the rock when it is at height (a) $h/4$ and (b) $3h/4$?

VP7.2.3 You throw a tennis ball (mass 0.0570 kg) vertically upward. It leaves your hand moving at 15.0 m/s. Air resistance cannot be neglected, and the ball reaches a maximum height of 8.00 m. (a) By how much does the total mechanical energy decrease from when the ball leaves your hand to when it reaches its maximum height? (b) What is the magnitude of the average force of air resistance?

VP7.2.4 You catch a volleyball (mass 0.270 kg) that is moving downward at 7.50 m/s. In stopping the ball, your hands and the volleyball descend together a distance of 0.150 m. (a) How much work do your hands do on the volleyball in the process of stopping it? (b) What is the magnitude of the force (assumed constant) that your hands exert on the volleyball?

Be sure to review EXAMPLES 7.4 and 7.5 (Section 7.1) before attempting these problems.

VP7.5.1 A well-greased, essentially frictionless, metal bowl has the shape of a hemisphere of radius 0.150 m. You place a pat of butter of mass 5.00×10^{-3} kg at the rim of the bowl and let it slide to the bottom of the bowl. (a) What is the speed of the pat of butter when it reaches the bottom of the bowl? (b) At the bottom of the bowl, what is the force that the bowl exerts on the pat of butter? How does this compare to the weight of the pat?

VP7.5.2 A snowboarder and her board (combined mass 40.0 kg) are moving at 9.30 m/s at the bottom of a curved ditch. (a) If friction can be ignored, what is the maximum vertical distance that she can travel up the sides of the ditch? Does this answer depend on the shape of the ditch? (b) The snowboarder finds that, due to friction, the maximum vertical distance she can travel up the sides of the ramp is 3.50 m. How much work did the force of friction do on her?

VP7.5.3 A pendulum is made of a small sphere of mass 0.250 kg attached to a lightweight string 1.20 m in length. As the pendulum swings back and forth, the maximum angle that the string makes with the vertical is 34.0°. Friction can be ignored. At the low point of the sphere's trajectory, what are (a) the kinetic energy of the sphere and (b) the tension in the string?

VP7.5.4 You are testing a new roller coaster ride in which a car of mass m moves around a vertical circle of radius R. In one test, the car starts at the bottom of the circle (point A) with initial kinetic energy K_i. When the car reaches the top of the circle (point B), its kinetic energy is $\frac{1}{4}K_i$, and its gravitational potential energy has increased by $\frac{1}{2}K_i$. (a) What was the speed of the car at point A, in terms of g and R? (b) How much work was done on the car by the force of friction as it moved from point A to point B, in terms of m, g, and R? (c) What was the magnitude of the friction force (assumed to be constant throughout the motion), in terms of m and g?

Be sure to review EXAMPLES 7.7, 7.8, and 7.9 (Section 7.2) before attempting these problems.

VP7.9.1 A glider of mass 0.240 kg is on a frictionless, horizontal track, attached to a horizontal spring of force constant 6.00 N/m. Initially the spring (whose other end is fixed) is stretched by 0.100 m and the attached glider is moving at 0.400 m/s in the direction that causes the spring to stretch farther. (a) What is the total mechanical energy (kinetic energy plus elastic potential energy) of the system? (b) When the glider comes momentarily to rest, by what distance is the spring stretched?

VP7.9.2 A glider of mass 0.240 kg is on a horizontal track, attached to a horizontal spring of force constant 6.00 N/m. There is friction between the track and the glider. Initially the spring (whose other end is fixed) is stretched by 0.100 m and the attached glider is moving at 0.400 m/s in the direction that causes the spring to stretch farther. The glider comes momentarily to rest when the spring is stretched by 0.112 m. (a) How much work does the force of friction do on the glider as the stretch of the spring increases from 0.100 m to 0.112 m? (b) What is the coefficient of kinetic friction between the glider and the track?

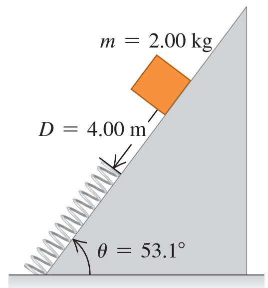
VP7.9.3 A lightweight vertical spring of force constant k has its lower end mounted on a table. You compress the spring by a distance d, place a block of mass m on top of the compressed spring, and then release the block. The spring launches the block upward, and the block rises to a maximum height some distance above the now-relaxed spring. (a) Find the speed of the block just as it loses contact with the spring. (b) Find the total vertical distance that the block travels from when it is first released to when it reaches its maximum height.

VP7.9.4 A cylinder of mass m is free to slide in a vertical tube. The kinetic friction force between the cylinder and the walls of the tube has magnitude f. You attach the upper end of a lightweight vertical spring of force constant k to the cap at the top of the tube, and attach the lower end of the spring to the top of the cylinder. Initially the cylinder is at rest and the spring is relaxed. You then release the cylinder. What vertical distance will the cylinder descend before it comes momentarily to rest?

BRIDGING PROBLEM A Spring and Friction on an Incline

A 2.00 kg package is released on a 53.1° incline, 4.00 m from a long spring with force constant 1.20×10^2 N/m that is attached at the bottom of the incline (**Fig. 7.25**). The coefficients of friction between the package and incline are $\mu_s = 0.400$ and $\mu_k = 0.200$. The mass of the spring is negligible. (a) What is the maximum compression of the spring? (b) The package rebounds up the incline. When it stops again, how close does it get to its original position? (c) What is the change in the internal energy of the package and incline from the point at which the package is released until it rebounds to its maximum height?

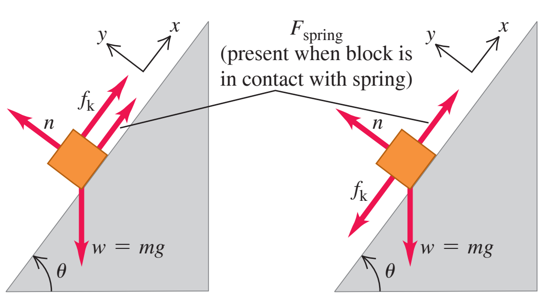
Figure 7.25 The initial situation.



SOLUTION GUIDE

IDENTIFY and SET UP

1. This problem involves the gravitational force, a spring force, and the friction force, as well as the normal force that acts on the package. Since the spring force isn't constant, you'll have to use energy methods. Is total mechanical energy conserved during any part of the motion? Why or why not?
2. Draw free-body diagrams for the package as it is sliding down the incline and sliding back up the incline. Include your choice of coordinate axes (see below). (*Hint:* If you choose $x = 0$ to be at the end of the uncompressed spring, you'll be able to use $U_{el} = \frac{1}{2}kx^2$ for the elastic potential energy of the spring.)



3. Label the three critical points in the package's motion: its starting position, its position when it comes to rest with the spring maximally compressed, and its position when it has rebounded as far as possible up the incline. (*Hint:* You can assume that the package is no longer in contact with the spring at the last of these positions. If this turns out to be incorrect, you'll calculate a value of x that tells you the spring is still partially compressed at this point.)
4. List the unknown quantities and decide which of these are the target variables.

EXECUTE

5. Find the magnitude of the friction force that acts on the package. Does the magnitude of this force depend on whether the package is moving up or down the incline, or on whether the package is in contact with the spring? Does the *direction* of the friction force depend on any of these?
6. Write the general energy equation for the motion of the package between the first two points you labeled in step 3. Use this equation to solve for the distance that the spring is compressed when the package is at its lowest point. (*Hint:* You'll have to solve a quadratic equation. To decide which of the two solutions of this equation is the correct one, remember that the distance the spring is compressed is positive.)
7. Write the general energy equation for the motion of the package between the second and third points you labeled in step 3. Use this equation to solve for how far the package rebounds.
8. Calculate the change in internal energy for the package's trip down and back up the incline. Remember that the amount the internal energy *increases* is equal to the amount the total mechanical energy *decreases*.

EVALUATE

9. Was it correct to assume in part (b) that the package is no longer in contact with the spring when it reaches its maximum rebound height?
10. Check your result for part (c) by finding the total work done by the force of friction over the entire trip. Is this in accordance with your result from step 8?

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q7.1 A baseball is thrown straight up with initial speed v_0. If air resistance cannot be ignored, when the ball returns to its initial height its speed is less than v_0. Explain why, using energy concepts.

Q7.2 A projectile has the same initial kinetic energy no matter what the angle of projection. Why doesn't it rise to the same maximum height in each case?

Q7.3 An object is released from rest at the top of a ramp. If the ramp is frictionless, does the object's speed at the bottom of the ramp depend on the shape of the ramp or just on its height? Explain. What if the ramp is *not* frictionless?

Q7.4 An egg is released from rest from the roof of a building and falls to the ground. Its fall is observed by a student on the roof of the building, who uses coordinates with origin at the roof, and by a student on the ground, who uses coordinates with origin at the ground. Do the values the two students assign to the following quantities match each other: initial gravitational potential energy, final gravitational potential energy, change in gravitational potential energy, and kinetic energy of the egg just before it strikes the ground? Explain.

Q7.5 A physics teacher had a bowling ball suspended from a very long rope attached to the high ceiling of a large lecture hall. To illustrate his faith in conservation of energy, he would back up to one side of the stage, pull the ball far to one side until the taut rope brought it just to the end of his nose, and then release it. The massive ball would swing in a mighty arc across the stage and then return to stop momentarily just in front of the nose of the stationary, unflinching teacher. However, one day after the demonstration he looked up just in time to see a student at the other side of the stage *push* the ball away from his nose as he tried to duplicate the demonstration. Tell the rest of the story, and explain the reason for the potentially tragic outcome.

Q7.6 Is it possible for a friction force to *increase* the total mechanical energy of a system? If so, give examples.

Q7.7 A woman bounces on a trampoline, going a little higher with each bounce. Explain how she increases the total mechanical energy.

Q7.8 **Fractured Physics.** People often call their electric bill a *power* bill, yet the quantity on which the bill is based is expressed in *kilowatt-hours.* What are people really being billed for?

Q7.9 (a) A book is lifted upward a vertical distance of 0.800 m. During this displacement, does the gravitational force acting on the book do positive work or negative work? Does the gravitational potential energy of the book increase or decrease? (b) A can of beans is released from rest and falls downward a vertical distance of 2.00 m. During this displacement, does the gravitational force acting on the can do positive work or negative work? Does the gravitational potential energy of the can increase or decrease?

Q7.10 (a) A block of wood is pushed against a spring, which is compressed 0.080 m. Does the force on the block exerted by the spring do positive or negative work? Does the potential energy stored in the spring increase or decrease? (b) A block of wood is placed against a vertical spring that is compressed 6.00 cm. The spring is released and pushes the block upward. From the point where the spring is compressed 6.00 cm to where it is compressed 2.00 cm from its equilibrium length and the block has moved 4.00 cm upward, does the spring force do positive or negative work on the block? During this motion, does the potential energy stored in the spring increase or decrease?

Q7.11 A 1.0 kg stone and a 10.0 kg stone are released from rest at the same height above the ground. Ignore air resistance. Which of these statements about the stones are true? Justify each answer. (a) Both have the same initial gravitational potential energy. (b) Both will have the same acceleration as they fall. (c) Both will have the same speed when they reach the ground. (d) Both will have the same kinetic energy when they reach the ground.

Q7.12 Two objects with different masses are launched vertically into the air by placing them on identical compressed springs and then releasing the springs. The two springs are compressed by the same amount before launching. Ignore air resistance and the masses of the springs. Which of these statements about the masses are true? Justify each answer. (a) Both reach the same maximum height. (b) At their maximum height, both have the same gravitational potential energy, if the initial gravitational potential of each mass is taken to be zero.

Q7.13 When people are cold, they often rub their hands together to warm up. How does doing this produce heat? Where does the heat come from?

Q7.14 A box slides down a ramp and work is done on the box by the forces of gravity and friction. Can the work of each of these forces be expressed in terms of the change in a potential-energy function? For each force explain why or why not.

Q7.15 In physical terms, explain why friction is a nonconservative force. Does it store energy for future use?

Q7.16 Since only changes in potential energy are important in any problem, a student decides to let the elastic potential energy of a spring be zero when the spring is stretched a distance x_1. The student decides, therefore, to let $U = \frac{1}{2}k(x - x_1)^2$. Is this correct? Explain.

Q7.17 Figure 7.22a shows the potential-energy function for the force $F_x = -kx$. Sketch the potential-energy function for the force $F_x = +kx$. For this force, is $x = 0$ a point of equilibrium? Is this equilibrium stable or unstable? Explain.

Q7.18 Figure 7.22b shows the potential-energy function associated with the gravitational force between an object and the earth. Use this graph to explain why objects always fall toward the earth when they are released.

Q7.19 For a system of two particles we often let the potential energy for the force between the particles approach zero as the separation of the particles approaches infinity. If this choice is made, explain why the potential energy at noninfinite separation is positive if the particles repel one another and negative if they attract.

Q7.20 Explain why the points $x = A$ and $x = -A$ in Fig. 7.23b are called *turning points.* How are the values of E and U related at a turning point?

Q7.21 A particle is in *neutral equilibrium* if the net force on it is zero and remains zero if the particle is displaced slightly in any direction. Sketch the potential-energy function near a point of neutral equilibrium for the case of one-dimensional motion. Give an example of an object in neutral equilibrium.

Q7.22 The net force on a particle of mass m has the potential-energy function graphed in Fig. 7.24a. If the total energy is E_1, graph the speed v of the particle versus its position x. At what value of x is the speed greatest? Sketch v versus x if the total energy is E_2.

Q7.23 The potential-energy function for a force $\vec{F}$ is $U = \alpha x^3$, where α is a positive constant. What is the direction of $\vec{F}$?

EXERCISES

Section 7.1 Gravitational Potential Energy

7.1 • In one day, a 75 kg mountain climber ascends from the 1500 m level on a vertical cliff to the top at 2400 m. The next day, she descends from the top to the base of the cliff, which is at an elevation of 1350 m. What is her change in gravitational potential energy (a) on the first day and (b) on the second day?

7.2 • **BIO** **How High Can We Jump?** The maximum height a typical human can jump from a crouched start is about 60 cm. By how much does the gravitational potential energy increase for a 72 kg person in such a jump? Where does this energy come from?

7.3 •• **CP** A 90.0 kg mail bag hangs by a vertical rope 3.5 m long. A postal worker then displaces the bag to a position 2.0 m sideways from its original position, always keeping the rope taut. (a) What horizontal force is necessary to hold the bag in the new position? (b) As the bag is moved to this position, how much work is done (i) by the rope and (ii) by the worker?

7.4 •• **BIO** **Food Calories.** The *food calorie,* equal to 4186 J, is a measure of how much energy is released when the body metabolizes food. A certain fruit-and-cereal bar contains 140 food calories. (a) If a 65 kg hiker eats one bar, how high a mountain must he climb to "work off" the calories, assuming that all the food energy goes into increasing gravitational potential energy? (b) If, as is typical, only 20% of the food calories go into mechanical energy, what would be the answer to part (a)? (*Note:* In this and all other problems, we are assuming that 100% of the food calories that are eaten are absorbed and used by the body. This is not true. A person's "metabolic efficiency" is the percentage of calories eaten that are actually used; the body eliminates the rest. Metabolic efficiency varies considerably from person to person.)

7.5 • A baseball is thrown from the roof of a 22.0-m-tall building with an initial velocity of magnitude 12.0 m/s and directed at an angle of 53.1° above the horizontal. (a) What is the speed of the ball just before it strikes the ground? Use energy methods and ignore air resistance. (b) What is the answer for part (a) if the initial velocity is at an angle of 53.1° *below* the horizontal? (c) If the effects of air resistance are included, will part (a) or (b) give the higher speed?

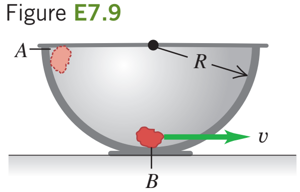
7.6 •• A crate of mass M starts from rest at the top of a frictionless ramp inclined at an angle α above the horizontal. Find its speed at the bottom of the ramp, a distance d from where it started. Do this in two ways: Take the level at which the potential energy is zero to be (a) at the bottom of the ramp with y positive upward, and (b) at the top of the ramp with y positive upward. (c) Why didn't the normal force enter into your solution?

7.7 •• **BIO** **Human Energy vs. Insect Energy.** For its size, the common flea is one of the most accomplished jumpers in the animal world. A 2.0-mm-long, 0.50 mg flea can reach a height of 20 cm in a single leap. (a) Ignoring air drag, what is the takeoff speed of such a flea? (b) Calculate the kinetic energy of this flea at takeoff and its kinetic energy per kilogram of mass. (c) If a 65 kg, 2.0-m-tall human could jump to the same height compared with his length as the flea jumps compared with its length, how high could the human jump, and what takeoff speed would the man need? (d) Most humans can jump no more than 60 cm from a crouched start. What is the kinetic energy per kilogram of mass at takeoff for such a 65 kg person? (e) Where does the flea store the energy that allows it to make sudden leaps?

7.8 • Estimate the maximum speed you can achieve while running a 100 m dash. Treat yourself as a point particle. (a) At this speed, what is your kinetic energy? (b) To what height above the ground would you have to climb in a tree to increase your gravitational potential energy by an amount equal to the kinetic energy you calculated in part (a)?

7.9 •• **CP** A small rock with mass 0.20 kg is released from rest at point A, which is at the top edge of a large, hemispherical bowl with radius $R = 0.50$ m (**Fig. E7.9**). Assume that the size of the rock is small compared to R, so that the rock can be treated as a particle, and assume that the rock slides rather than rolls. The work done by friction on the rock when it moves from point A to point B at the bottom of the bowl has magnitude 0.22 J. (a) Between points A and B, how much work is done on the rock by (i) the normal force and (ii) gravity? (b) What is the speed of the rock as it reaches point B? (c) Of the three forces acting on the rock as it slides down the bowl, which (if any) are constant and which are not? Explain. (d) Just as the rock reaches point B, what is the normal force on it due to the bottom of the bowl?

Figure **E7.9**



7.10 •• A 25.0 kg child plays on a swing having support ropes that are 2.20 m long. Her brother pulls her back until the ropes are 42.0° from the vertical and releases her from rest. (a) What is her potential energy just as she is released, compared with the potential energy at the bottom of the swing's motion? (b) How fast will she be moving at the bottom? (c) How much work does the tension in the ropes do as she swings from the initial position to the bottom of the motion?

7.11 •• You are testing a new amusement park roller coaster with an empty car of mass 120 kg. One part of the track is a vertical loop with radius 12.0 m. At the bottom of the loop (point A) the car has speed 25.0 m/s, and at the top of the loop (point B) it has speed 8.0 m/s. As the car rolls from point A to point B, how much work is done by friction?

7.12 • **Tarzan and Jane.** Tarzan, in one tree, sights Jane in another tree. He grabs the end of a vine with length 20 m that makes an angle of 45° with the vertical, steps off his tree limb, and swings down and then up to Jane's open arms. When he arrives, his vine makes an angle of 30° with the vertical. Determine whether he gives her a tender embrace or knocks her off her limb by calculating Tarzan's speed just before he reaches Jane. Ignore air resistance and the mass of the vine.

7.13 •• Two blocks are attached to either end of a light rope that passes over a light, frictionless pulley suspended from the ceiling. One block has mass 8.00 kg, and the other has mass 6.00 kg. The blocks are released from rest. (a) For a 0.200 m downward displacement of the 8.00 kg block, what is the change in the gravitational potential energy associated with each block? (b) If the tension in the rope is T, how much work is done on each block by the rope? (c) Apply conservation of energy to the system that includes both blocks. During the 0.200 m downward displacement, what is the total work done on the system by the tension in the rope? What is the change in gravitational potential energy associated with the system? Use energy conservation to find the speed of the 8.00 kg block after it has descended 0.200 m.

Section 7.2 Elastic Potential Energy

7.14 •• An ideal spring of negligible mass is 12.00 cm long when nothing is attached to it. When you hang a 3.15 kg weight from it, you measure its length to be 13.40 cm. If you wanted to store 10.0 J of potential energy in this spring, what would be its *total* length? Assume that it continues to obey Hooke's law.

7.15 •• A force of 520 N keeps a certain ideal spring stretched a distance of 0.200 m. (a) What is the potential energy of the spring when it is stretched 0.200 m? (b) What is its potential energy when it is compressed 5.00 cm?

7.16 • **BIO** **Tendons.** Tendons are strong elastic fibers that attach muscles to bones. To a reasonable approximation, they obey Hooke's law. In laboratory tests on a particular tendon, it was found that, when a 250 g object was hung from it, the tendon stretched 1.23 cm. (a) Find the force constant of this tendon in N/m. (b) Because of its thickness, the maximum tension this tendon can support without rupturing is 138 N. By how much can the tendon stretch without rupturing, and how much energy is stored in it at that point?

7.17 • An ideal spring stores potential energy U_0 when it is compressed a distance x_0 from its uncompressed length. (a) In terms of U_0, how much energy does the spring store when it is compressed (i) twice as much and (ii) half as much? (b) In terms of x_0, how much must the spring be compressed from its uncompressed length to store (i) twice as much energy and (ii) half as much energy?

7.18 • A small block of mass m on a horizontal frictionless surface is attached to a horizontal spring that has force constant k. The block is pushed against the spring, compressing the spring a distance d. The block is released, and it moves back and forth on the end of the spring. During its motion, what is the maximum speed of the block?

7.19 •• A spring of negligible mass has force constant $k = 800$ N/m. (a) How far must the spring be compressed for 1.20 J of potential energy to be stored in it? (b) You place the spring vertically with one end on the floor. You then lay a 1.60 kg book on top of the spring and release the book from rest. Find the maximum distance the spring will be compressed.

7.20 • A 1.20 kg piece of cheese is placed on a vertical spring of negligible mass and force constant $k = 1800$ N/m that is compressed 15.0 cm. When the spring is released, how high does the cheese rise from this initial position? (The cheese and the spring are *not* attached.)

7.21 •• A spring of negligible mass has force constant $k = 1600$ N/m. (a) How far must the spring be compressed for 3.20 J of potential energy to be stored in it? (b) You place the spring vertically with one end on the floor. You then drop a 1.20 kg book onto it from a height of 0.800 m above the top of the spring. Find the maximum distance the spring will be compressed.

7.22 •• (a) For the elevator of Example 7.9 (Section 7.2), what is the speed of the elevator after it has moved downward 1.00 m from point 1 in Fig. 7.17? (b) When the elevator is 1.00 m below point 1 in Fig. 7.17, what is its acceleration?

7.23 •• A 2.50 kg mass is pushed against a horizontal spring of force constant 25.0 N/cm on a frictionless air table. The spring is attached to the tabletop, and the mass is not attached to the spring in any way. When the spring has been compressed enough to store 11.5 J of potential energy in it, the mass is suddenly released from rest. (a) Find the greatest speed the mass reaches. When does this occur? (b) What is the greatest acceleration of the mass, and when does it occur?

7.24 •• A 2.50 kg block on a horizontal floor is attached to a horizontal spring that is initially compressed 0.0300 m. The spring has force constant 840 N/m. The coefficient of kinetic friction between the floor and the block is $\mu_k = 0.40$. The block and spring are released from rest, and the block slides along the floor. What is the speed of the block when it has moved a distance of 0.0200 m from its initial position? (At this point the spring is compressed 0.0100 m.)

7.25 •• You are asked to design a spring that will give a 1160 kg satellite a speed of 2.50 m/s relative to an orbiting space shuttle. Your spring is to give the satellite a maximum acceleration of $5.00g$. The spring's mass, the recoil kinetic energy of the shuttle, and changes in gravitational potential energy will all be negligible. (a) What must the force constant of the spring be? (b) What distance must the spring be compressed?

7.26 • It takes a force of 5.00 N to stretch an ideal spring 2.00 cm. (a) What force does it take to stretch the spring an additional 4.00 cm? (b) By what factor does the stored elastic potential energy increase when the spring, originally stretched 2.00 cm, is stretched 4.00 cm more?

Section 7.3 Conservative and Nonconservative Forces

7.27 • A 0.60 kg book slides on a horizontal table. The kinetic friction force on the book has magnitude 1.8 N. (a) How much work is done on the book by friction during a displacement of 3.0 m to the left? (b) The book now slides 3.0 m to the right, returning to its starting point. During this second 3.0 m displacement, how much work is done on the book by friction? (c) What is the total work done on the book by friction during the complete round trip? (d) On the basis of your answer to part (c), would you say that the friction force is conservative or nonconservative? Explain.

7.28 •• CALC In an experiment, one of the forces exerted on a proton is $\vec{F} = -\alpha x^2 \hat{\imath}$, where $\alpha = 12\ \text{N/m}^2$. (a) How much work does $\vec{F}$ do when the proton moves along the straight-line path from the point $(0.10\ \text{m}, 0)$ to the point $(0.10\ \text{m}, 0.40\ \text{m})$? (b) Along the straight-line path from the point $(0.10\ \text{m}, 0)$ to the point $(0.30\ \text{m}, 0)$? (c) Along the straight-line path from the point $(0.30\ \text{m}, 0)$ to the point $(0.10\ \text{m}, 0)$? (d) Is the force $\vec{F}$ conservative? Explain. If $\vec{F}$ is conservative, what is the potential-energy function for it? Let $U = 0$ when $x = 0$.

7.29 •• A 62.0 kg skier is moving at 6.50 m/s on a frictionless, horizontal, snow-covered plateau when she encounters a rough patch 4.20 m long. The coefficient of kinetic friction between this patch and her skis is 0.300. After crossing the rough patch and returning to friction-free snow, she skis down an icy, frictionless hill 2.50 m high. (a) How fast is the skier moving when she gets to the bottom of the hill? (b) How much internal energy was generated in crossing the rough patch?

7.30 • While a roofer is working on a roof that slants at 36° above the horizontal, he accidentally nudges his 85.0 N toolbox, causing it to start sliding downward from rest. If it starts 4.25 m from the lower edge of the roof, how fast will the toolbox be moving just as it reaches the edge of the roof if the kinetic friction force on it is 22.0 N?

Section 7.4 Force and Potential Energy

7.31 •• CALC A force parallel to the x-axis acts on a particle moving along the x-axis. This force produces potential energy $U(x)$ given by $U(x) = \alpha x^4$, where $\alpha = 0.630\ \text{J/m}^4$. What is the force (magnitude and direction) when the particle is at $x = -0.800$ m?

7.32 •• CALC The potential energy of a pair of hydrogen atoms separated by a large distance x is given by $U(x) = -C_6/x^6$, where C_6 is a positive constant. What is the force that one atom exerts on the other? Is this force attractive or repulsive?

7.33 •• CALC A small block with mass 0.0400 kg is moving in the xy-plane. The net force on the block is described by the potential-energy function $U(x, y) = (5.80\ \text{J/m}^2)x^2 - (3.60\ \text{J/m}^3)y^3$. What are the magnitude and direction of the acceleration of the block when it is at the point $(x = 0.300\ \text{m}, y = 0.600\ \text{m})$?

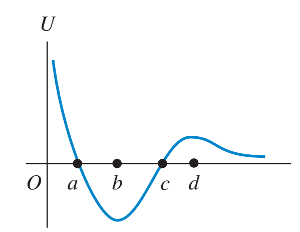
7.34 •• CALC An object moving in the xy-plane is acted on by a conservative force described by the potential-energy function $U(x, y) = \alpha[(1/x^2) + (1/y^2)]$, where α is a positive constant. Derive an expression for the force expressed in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

Section 7.5 Energy Diagrams

7.35 • CALC The potential energy of two atoms in a diatomic molecule is approximated by $U(r) = (a/r^{12}) - (b/r^6)$, where r is the spacing between atoms and a and b are positive constants. (a) Find the force $F(r)$ on one atom as a function of r. Draw two graphs: one of $U(r)$ versus r and one of $F(r)$ versus r. (b) Find the equilibrium distance between the two atoms. Is this equilibrium stable? (c) Suppose the distance between the two atoms is equal to the equilibrium distance found in part (b). What minimum energy must be added to the molecule to *dissociate* it—that is, to separate the two atoms to an infinite distance apart? This is called the *dissociation energy* of the molecule. (d) For the molecule CO, the equilibrium distance between the carbon and oxygen atoms is 1.13×10^{-10} m and the dissociation energy is 1.54×10^{-18} J per molecule. Find the values of the constants a and b.

7.36 • A marble moves along the x-axis. The potential-energy function is shown in **Fig. E7.36**. (a) At which of the labeled x-coordinates is the force on the marble zero? (b) Which of the labeled x-coordinates is a position of stable equilibrium? (c) Which of the labeled x-coordinates is a position of unstable equilibrium?

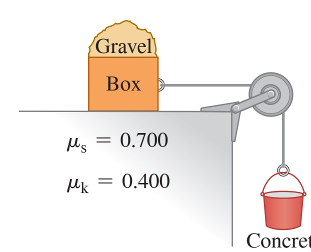
Figure E7.36



PROBLEMS

7.37 ••• At a construction site, a 65.0 kg bucket of concrete hangs from a light (but strong) cable that passes over a light, friction-free pulley and is connected to an 80.0 kg box on a horizontal roof (**Fig. P7.37**). The cable pulls horizontally on the box, and a 50.0 kg bag of gravel rests on top of the box. The coefficients of friction between the box and roof are shown. (a) Find the friction force on the bag of gravel and on the box. (b) Suddenly a worker picks up the bag of gravel. Use energy conservation to find the speed of the bucket after it has descended 2.00 m from rest. (Use Newton's laws to check your answer.)

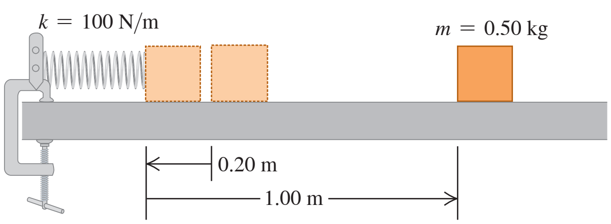
Figure P7.37



7.38 •• CP Estimate the maximum horizontal distance you can throw a baseball ($m = 0.145$ kg) if you throw it at an angle of $\alpha_0 = 45°$ above the horizontal in order to achieve the maximum range. (a) What is the kinetic energy of the baseball just after it leaves your hand? Ignore air resistance and the small distance the ball is above the ground when it leaves your hand. Take the zero of potential energy to be at the ground. (b) At the ball's maximum height, what fraction of its total mechanical energy is kinetic energy and what fraction is gravitational potential energy? (c) If you throw the baseball at an initial angle of 60° above the horizontal, at its maximum height what fraction of its total energy is kinetic energy and what fraction is gravitational potential energy? (d) What fraction of the total mechanical energy is kinetic energy at the maximum height in the limiting cases of $\alpha_0 = 0°$ and $\alpha_0 = 90°$?

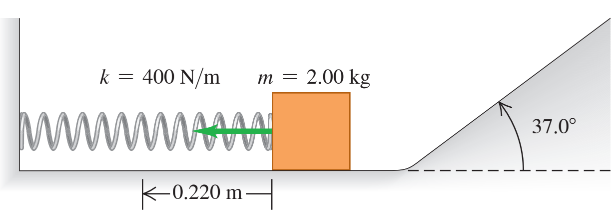
7.39 • A block with mass 0.50 kg is forced against a horizontal spring of negligible mass, compressing the spring a distance of 0.20 m (**Fig. P7.39**). When released, the block moves on a horizontal tabletop for 1.00 m before coming to rest. The force constant k is 100 N/m. What is the coefficient of kinetic friction μ_k between the block and the tabletop?

Figure **P7.39**



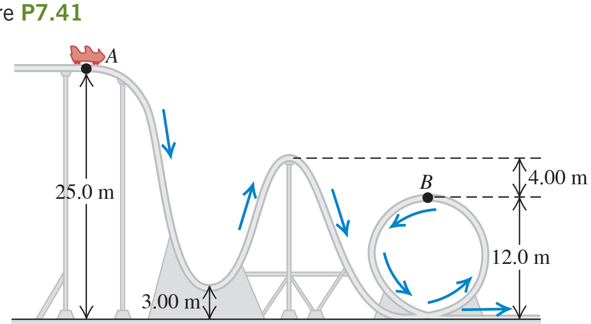
7.40 • A 2.00 kg block is pushed against a spring with negligible mass and force constant $k = 400$ N/m, compressing it 0.220 m. When the block is released, it moves along a frictionless, horizontal surface and then up a frictionless incline with slope 37.0° (**Fig. P7.40**). (a) What is the speed of the block as it slides along the horizontal surface after having left the spring? (b) How far does the block travel up the incline before starting to slide back down?

Figure **P7.40**



7.41 •• A 350 kg roller coaster car starts from rest at point A and slides down a frictionless loop-the-loop (**Fig. P7.41**). The car's wheels are designed to stay on the track. (a) How fast is this roller coaster car moving at point B? (b) How hard does it press against the track at point B?

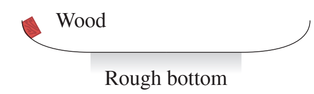
Figure **P7.41**



7.42 •• **CP** A small rock with mass m is released from rest at the inside rim of a large, hemispherical bowl (point A) that has radius R, as shown in Fig. E7.9. If the normal force exerted on the rock as it slides through its lowest point (point B) is twice the weight of the rock, how much work did friction do on the rock as it moved from A to B? Express your answer in terms of m, R, and g.

7.43 •• A 2.0 kg piece of wood slides on a curved surface (**Fig. P7.43**). The sides of the surface are perfectly smooth, but the rough horizontal bottom is 30 m long and has a kinetic friction coefficient of 0.20 with the wood. The piece of wood starts from rest 4.0 m above the rough bottom. (a) Where will this wood eventually come to rest? (b) For the motion from the initial release until the piece of wood comes to rest, what is the total amount of work done by friction?

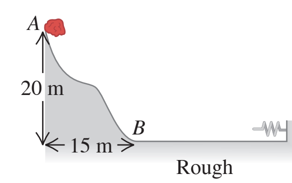
Figure **P7.43**



7.44 •• **CP** A small block with mass m slides without friction on the inside of a vertical circular track that has radius R. What minimum speed must the block have at the bottom of its path if it is not to fall off the track at the top of its path?

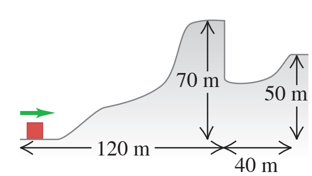
7.45 •• A 15.0 kg stone slides down a snow-covered hill (**Fig. P7.45**), leaving point A at a speed of 10.0 m/s. There is no friction on the hill between points A and B, but there is friction on the level ground at the bottom of the hill, between B and the wall. After entering the rough horizontal region, the stone travels 100 m and then runs into a very long, light spring with force constant 2.00 N/m. The coefficients of kinetic and static friction between the stone and the horizontal ground are 0.20 and 0.80, respectively. (a) What is the speed of the stone when it reaches point B? (b) How far will the stone compress the spring? (c) Will the stone move again after it has been stopped by the spring?

Figure **P7.45**



7.46 •• **CP** A 2.8 kg block slides over the smooth, icy hill shown in **Fig. P7.46**. The top of the hill is horizontal and 70 m higher than its base. What minimum speed must the block have at the base of the 70 m hill to pass over the pit at the far (right-hand) side of that hill?

Figure **P7.46**



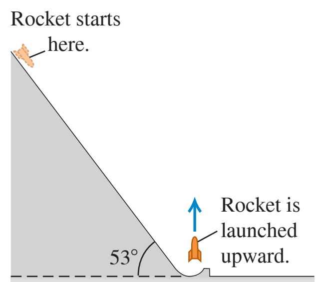
7.47 •• A small box with mass 0.600 kg is placed against a compressed spring at the bottom of an incline that slopes upward at 37.0° above the horizontal. The other end of the spring is attached to a wall. The coefficient of kinetic friction between the box and the surface of the incline is $\mu_k = 0.400$. The spring is released and the box travels up the incline, leaving the spring behind. What minimum elastic potential energy must be stored initially in the spring if the box is to travel 2.00 m from its initial position to the top of the incline?

7.48 ••• You are designing a delivery ramp for crates containing exercise equipment. The 1470 N crates will move at 1.8 m/s at the top of a ramp that slopes downward at 22.0°. The ramp exerts a 515 N kinetic friction force on each crate, and the maximum static friction force also has this value. Each crate will compress a spring at the bottom of the ramp and will come to rest after traveling a total distance of 5.0 m along the ramp. Once stopped, a crate must not rebound back up the ramp. Calculate the largest force constant of the spring that will be needed to meet the design criteria.

7.49 ••• The Great Sandini is a 60 kg circus performer who is shot from a cannon (actually a spring gun). You don't find many men of his caliber, so you help him design a new gun. This new gun has a very large spring with a very small mass and a force constant of 1100 N/m that he will compress with a force of 4400 N. The inside of the gun barrel is coated with Teflon, so the average friction force will be only 40 N during the 4.0 m he moves in the barrel. At what speed will he emerge from the end of the barrel, 2.5 m above his initial rest position?

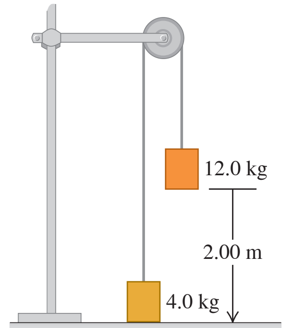
7.50 •• A 1500 kg rocket is to be launched with an initial upward speed of 50.0 m/s. In order to assist its engines, the engineers will start it from rest on a ramp that rises 53° above the horizontal (**Fig. P7.50**). At the bottom, the ramp turns upward and launches the rocket vertically. The engines provide a constant forward thrust of 2000 N, and friction with the ramp surface is a constant 500 N. How far from the base of the ramp should the rocket start, as measured along the surface of the ramp?

Figure **P7.50**



7.51 •• A system of two paint buckets connected by a lightweight rope is released from rest with the 12.0 kg bucket 2.00 m above the floor (**Fig. P7.51**). Use the principle of conservation of energy to find the speed with which this bucket strikes the floor. Ignore friction and the mass of the pulley.

Figure **P7.51**



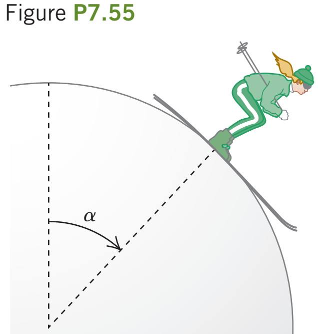
7.52 •• A block with mass $m =$ 0.200 kg is placed against a compressed spring at the bottom of a ramp that is at an angle of 53.0° above the horizontal. The spring has 8.00 J of elastic potential energy stored in it. The spring is released, and the block moves up the incline. After the block has traveled a distance of 3.00 m, its speed is 4.00 m/s. What is the magnitude of the friction force that the ramp exerts on the block while the block is moving?

7.53 •• **CP** A 0.300 kg potato is tied to a string with length 2.50 m, and the other end of the string is tied to a rigid support. The potato is held straight out horizontally from the point of support, with the string pulled taut, and is then released. (a) What is the speed of the potato at the lowest point of its motion? (b) What is the tension in the string at this point?

7.54 •• A 60.0 kg skier starts from rest at the top of a ski slope 65.0 m high. (a) If friction forces do −10.5 kJ of work on her as she descends, how fast is she going at the bottom of the slope? (b) Now moving horizontally, the skier crosses a patch of soft snow where $\mu_k = 0.20$. If the patch is 82.0 m wide and the average force of air resistance on the skier is 160 N, how fast is she going after crossing the patch? (c) The skier hits a snowdrift and penetrates 2.5 m into it before coming to a stop. What is the average force exerted on her by the snowdrift as it stops her?

7.55 • **CP** A skier starts at the top of a very large, frictionless snowball, with a very small initial speed, and skis straight down the side (**Fig. P7.55**). At what point does she lose contact with the snowball and fly off at a tangent? That is, at the instant she loses contact with the snowball, what angle α does a radial line from the center of the snowball to the skier make with the vertical?

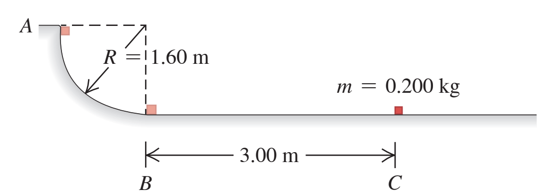
Figure **P7.55**



7.56 •• A block with mass 0.400 kg is on a horizontal frictionless surface and is attached to a horizontal compressed spring that has force constant $k = 200$ N/m. The other end of the spring is attached to a wall. The block is released, and it moves back and forth on the end of the spring. During this motion the block has speed 3.00 m/s when the spring is stretched 0.160 m. (a) During the motion of the block, what is its maximum speed? (b) During the block's motion, what is the maximum distance the spring is compressed from its equilibrium position? (c) When the spring has its maximum compression, what is the speed of the block and what is the magnitude of the acceleration of the block?

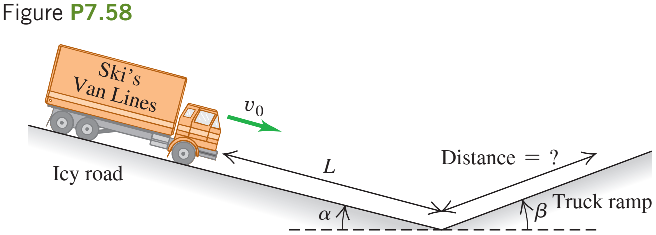
7.57 •• In a truck-loading station at a post office, a small 0.200 kg package is released from rest at point A on a track that is one-quarter of a circle with radius 1.60 m (**Fig. P7.57**). The size of the package is much less than 1.60 m, so the package can be treated as a particle. It slides down the track and reaches point B with a speed of 4.80 m/s. From point B, it slides on a level surface a distance of 3.00 m to point C, where it comes to rest. (a) What is the coefficient of kinetic friction on the horizontal surface? (b) How much work is done on the package by friction as it slides down the circular arc from A to B?

Figure **P7.57**



7.58 ••• A truck with mass m has a brake failure while going down an icy mountain road of constant downward slope angle α (**Fig. P7.58**). Initially the truck is moving downhill at speed v_0. After careening downhill a distance L with negligible friction, the truck driver steers the runaway vehicle onto a runaway truck ramp of constant upward slope angle β. The truck ramp has a soft sand surface for which the coefficient of rolling friction is μ_r. What is the distance that the truck moves up the ramp before coming to a halt? Solve by energy methods.

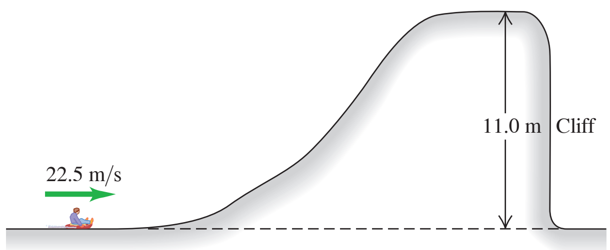
Figure **P7.58**



7.59 •• **CALC** A certain spring found *not* to obey Hooke's law exerts a restoring force $F_x(x) = -\alpha x - \beta x^2$ if it is stretched or compressed, where $\alpha = 60.0$ N/m and $\beta = 18.0$ N/m^2. The mass of the spring is negligible. (a) Calculate the potential-energy function $U(x)$ for this spring. Let $U = 0$ when $x = 0$. (b) An object with mass 0.900 kg on a frictionless, horizontal surface is attached to this spring, pulled a distance 1.00 m to the right (the $+x$-direction) to stretch the spring, and released. What is the speed of the object when it is 0.50 m to the right of the $x = 0$ equilibrium position?

7.60 •• **CP** A sled with rider having a combined mass of 125 kg travels over a perfectly smooth icy hill (**Fig. 7.60**). How far does the sled land from the foot of the cliff?

Figure **P7.60**



7.61 •• **CALC** A conservative force $\vec{F}$ is in the $+x$-direction and has magnitude $F(x) = \alpha/(x + x_0)^2$, where $\alpha = 0.800\ \text{N} \cdot \text{m}^2$ and $x_0 = 0.200$ m. (a) What is the potential-energy function $U(x)$ for this force? Let $U(x) \rightarrow 0$ as $x \rightarrow \infty$. (b) An object with mass $m = 0.500$ kg is released from rest at $x = 0$ and moves in the $+x$-direction. If $\vec{F}$ is the only force acting on the object, what is the object's speed when it reaches $x = 0.400$ m?

7.62 •• **CP** A light rope of length 1.40 m is attached to the ceiling. A small steel ball with mass 0.200 kg swings on the lower end of the rope as a pendulum. As the ball swings back and forth, the angle θ between the rope and the vertical direction has a maximum value of 37.0°. (a) What is the tension in the rope when $\theta = 37.0°$? (b) What is the tension when $\theta = 25.0°$?

7.63 •• A 0.150 kg block of ice is placed against a horizontal, compressed spring mounted on a horizontal tabletop that is 1.20 m above the floor. The spring has force constant 1900 N/m and is initially compressed 0.045 m. The mass of the spring is negligible. The spring is released, and the block slides along the table, goes off the edge, and travels to the floor. If there is negligible friction between the block of ice and the tabletop, what is the speed of the block of ice when it reaches the floor?

7.64 •• If a fish is attached to a vertical spring and slowly lowered to its equilibrium position, it is found to stretch the spring by an amount d. If the same fish is attached to the end of the unstretched spring and then allowed to fall from rest, through what maximum distance does it stretch the spring? (*Hint:* Calculate the force constant of the spring in terms of the distance d and the mass m of the fish.)

7.65 ••• **CALC** You are an industrial engineer with a shipping company. As part of the package-handling system, a small box with mass 1.60 kg is placed against a light spring that is compressed 0.280 m. The spring, whose other end is attached to a wall, has force constant $k = 45.0$ N/m. The spring and box are released from rest, and the box travels along a horizontal surface for which the coefficient of kinetic friction with the box is $\mu_k = 0.300$. When the box has traveled 0.280 m and the spring has reached its equilibrium length, the box loses contact with the spring. (a) What is the speed of the box at the instant when it leaves the spring? (b) What is the maximum speed of the box during its motion?

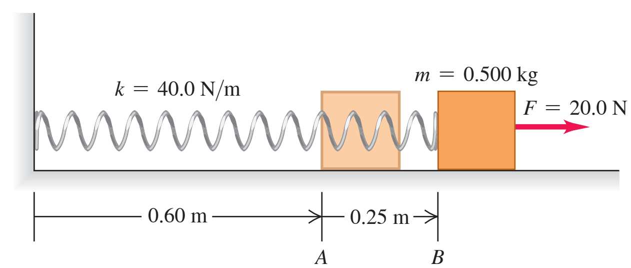
7.66 •• A basket of negligible weight hangs from a vertical spring scale of force constant 1500 N/m. (a) If you suddenly put a 3.0 kg adobe brick in the basket, find the maximum distance that the spring will stretch. (b) If, instead, you release the brick from 1.0 m above the basket, by how much will the spring stretch at its maximum elongation?

7.67 ••• **CALC** A 3.00 kg fish is attached to the lower end of a vertical spring that has negligible mass and force constant 900 N/m. The spring initially is neither stretched nor compressed. The fish is released from rest. (a) What is its speed after it has descended 0.0500 m from its initial position? (b) What is the maximum speed of the fish as it descends?

7.68 •• **CP** To test a slide at an amusement park, a block of wood with mass 3.00 kg is released at the top of the slide and slides down to the horizontal section at the end, a vertical distance of 23.0 m below the starting point. The block flies off the ramp in a horizontal direction and then lands on the ground after traveling through the air 30.0 m horizontally and 40.0 m downward. Neglect air resistance. How much work does friction do on the block as it slides down the ramp?

7.69 • A 0.500 kg block, attached to a spring with length 0.60 m and force constant 40.0 N/m, is at rest with the back of the block at point A on a frictionless, horizontal air table (**Fig. P7.69**). The mass of the spring is negligible. You move the block to the right along the surface by pulling with a constant 20.0 N horizontal force. (a) What is the block's speed when the back of the block reaches point B, which is 0.25 m to the right of point A? (b) When the back of the block reaches point B, you let go of the block. In the subsequent motion, how close does the block get to the wall where the left end of the spring is attached?

Figure **P7.69**



7.70 ••• **CP** A small block with mass 0.0400 kg slides in a vertical circle of radius $R = 0.500$ m on the inside of a circular track. During one of the revolutions of the block, when the block is at the bottom of its path, point A, the normal force exerted on the block by the track has magnitude 3.95 N. In this same revolution, when the block reaches the top of its path, point B, the normal force exerted on the block has magnitude 0.680 N. How much work is done on the block by friction during the motion of the block from point A to point B?

7.71 ••• **CP** A small block with mass 0.0500 kg slides in a vertical circle of radius $R = 0.800$ m on the inside of a circular track. There is no friction between the track and the block. At the bottom of the block's path, the normal force the track exerts on the block has magnitude 3.40 N. What is the magnitude of the normal force that the track exerts on the block when it is at the top of its path?

7.72 •• **CP Pendulum.** A small rock with mass 0.12 kg is fastened to a massless string with length 0.80 m to form a pendulum. The pendulum is swinging so as to make a maximum angle of 45° with the vertical. Air resistance is negligible. (a) What is the speed of the rock when the string passes through the vertical position? What is the tension in the string (b) when it makes an angle of 45° with the vertical, (c) as it passes through the vertical?

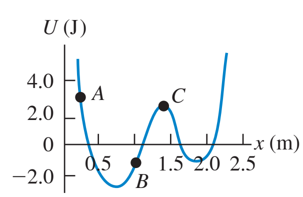
7.73 ••• A wooden block with mass 1.50 kg is placed against a compressed spring at the bottom of an incline of slope 30.0° (point A). When the spring is released, it projects the block up the incline. At point B, a distance of 6.00 m up the incline from A, the block is moving up the incline at 7.00 m/s and is no longer in contact with the spring. The coefficient of kinetic friction between the block and the incline is $\mu_k = 0.50$. The mass of the spring is negligible. Calculate the amount of potential energy that was initially stored in the spring.

7.74 •• CALC A small object with mass $m = 0.0900$ kg moves along the $+x$-axis. The only force on the object is a conservative force that has the potential-energy function $U(x) = -\alpha x^2 + \beta x^3$, where $\alpha = 2.00\ \text{J/m}^2$ and $\beta = 0.300\ \text{J/m}^3$. The object is released from rest at small x. When the object is at $x = 4.00$ m, what are its (a) speed and (b) acceleration (magnitude and direction)? (c) What is the maximum value of x reached by the object during its motion?

7.75 ••• CALC A cutting tool under microprocessor control has several forces acting on it. One force is $\vec{F} = -\alpha xy^2\hat{\jmath}$, a force in the negative y-direction whose magnitude depends on the position of the tool. For $\alpha = 2.50\ \text{N/m}^3$, consider the displacement of the tool from the origin to the point ($x = 3.00$ m, $y = 3.00$ m). (a) Calculate the work done on the tool by $\vec{F}$ if this displacement is along the straight line $y = x$ that connects these two points. (b) Calculate the work done on the tool by $\vec{F}$ if the tool is first moved out along the x-axis to the point ($x = 3.00$ m, $y = 0$) and then moved parallel to the y-axis to the point ($x = 3.00$ m, $y = 3.00$ m). (c) Compare the work done by $\vec{F}$ along these two paths. Is $\vec{F}$ conservative or nonconservative? Explain.

7.76 • A particle moves along the x-axis while acted on by a single conservative force parallel to the x-axis. The force corresponds to the potential-energy function graphed in **Fig. P7.76**. The particle is released from rest at point A. (a) What is the direction of the force on the particle when it is at point A? (b) At point B? (c) At what value of x is the kinetic energy of the particle a maximum? (d) What is the force on the particle when it is at point C? (e) What is the largest value of x reached by the particle during its motion? (f) What value or values of x correspond to points of stable equilibrium? (g) Of unstable equilibrium?

Figure **P7.76**



7.77 •• DATA You are designing a pendulum for a science museum. The pendulum is made by attaching a brass sphere with mass m to the lower end of a long, light metal wire of (unknown) length L. A device near the top of the wire measures the tension in the wire and transmits that information to your laptop computer. When the wire is vertical and the sphere is at rest, the sphere's center is 0.800 m above the floor and the tension in the wire is 265 N. Keeping the wire taut, you then pull the sphere to one side (using a ladder if necessary) and gently release it. You record the height h of the center of the sphere above the floor at the point where the sphere is released and the tension T in the wire as the sphere swings through its lowest point. You collect your results:

***h* (m)**	0.800	2.00	4.00	6.00	8.00	10.0	12.0
***T* (N)**	265	274	298	313	330	348	371

Assume that the sphere can be treated as a point mass, ignore the mass of the wire, and assume that total mechanical energy is conserved through each measurement. (a) Plot T versus h, and use this graph to calculate L. (b) If the breaking strength of the wire is 822 N, from what maximum height h can the sphere be released if the tension in the wire is not to exceed half the breaking strength? (c) The pendulum is swinging when you leave at the end of the day. You lock the museum doors, and no one enters the building until you return the next morning. You find that the sphere is hanging at rest. Using energy considerations, how can you explain this behavior?

7.78 •• DATA A long ramp made of cast iron is sloped at a constant angle $\theta = 52.0°$ above the horizontal. Small blocks, each with mass 0.42 kg but made of different materials, are released from rest at a vertical height h above the bottom of the ramp. In each case the coefficient of static friction is small enough that the blocks start to slide down the ramp as soon as they are released. You are asked to find h so that each block will have a speed of 4.00 m/s when it reaches the bottom of the ramp. You are given these coefficients of sliding (kinetic) friction for different pairs of materials:

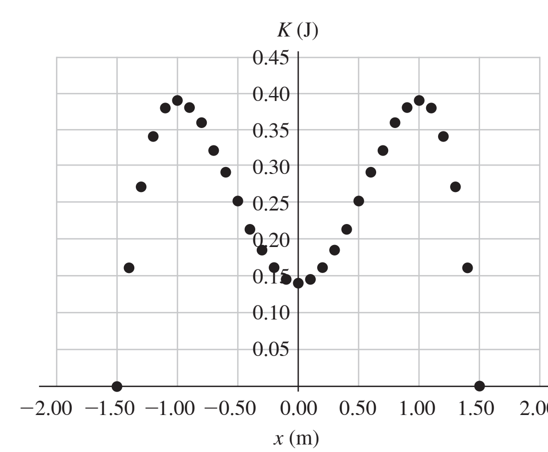
Material 1	**Material 2**	**Coefficient of Sliding Friction**
Cast iron	Cast iron	0.15
Cast iron	Copper	0.29
Cast iron	Lead	0.43
Cast iron	Zinc	0.85

Source: www.engineershandbook.com

(a) Use work and energy considerations to find the required value of h if the block is made from (i) cast iron; (ii) copper; (iii) zinc. (b) What is the required value of h for the copper block if its mass is doubled to 0.84 kg? (c) For a given block, if θ is increased while h is kept the same, does the speed v of the block at the bottom of the ramp increase, decrease, or stay the same?

7.79 •• DATA A single conservative force $F(x)$ acts on a small sphere of mass m while the sphere moves along the x-axis. You release the sphere from rest at $x = -1.50$ m. As the sphere moves, you measure its velocity as a function of position. You use the velocity data to calculate the kinetic energy K; **Fig. P7.79** shows your data. (a) Let $U(x)$ be the potential-energy function for $F(x)$. Is $U(x)$ symmetric about $x = 0$? [If so, then $U(x) = U(-x)$.] (b) If you set $U = 0$ at $x = 0$, what is the value of U at $x = -1.50$ m? (c) Sketch $U(x)$. (d) At what values of x (if any) is $F = 0$? (e) For what range of values of x between $x = -1.50$ m and $x = +1.50$ m is F positive? Negative? (f) If you release the sphere from rest at $x = -1.30$ m, what is the largest value of x that it reaches during its motion? The largest value of kinetic energy that it has during its motion?

Figure **P7.79**



CHALLENGE PROBLEM

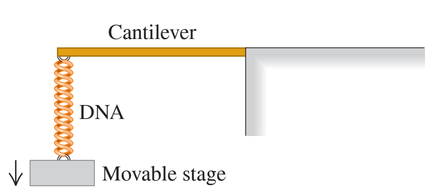
7.80 ••• **CALC** A proton with mass m moves in one dimension. The potential-energy function is $U(x) = (\alpha/x^2) - (\beta/x)$, where α and β are positive constants. The proton is released from rest at $x_0 = \alpha/\beta$. (a) Show that $U(x)$ can be written as

$$U(x) = \frac{\alpha}{x_0^2}\left[\left(\frac{x_0}{x}\right)^2 - \frac{x_0}{x}\right]$$

Graph $U(x)$. Calculate $U(x_0)$ and thereby locate the point x_0 on the graph. (b) Calculate $v(x)$, the speed of the proton as a function of position. Graph $v(x)$ and give a qualitative description of the motion. (c) For what value of x is the speed of the proton a maximum? What is the value of that maximum speed? (d) What is the force on the proton at the point in part (c)? (e) Let the proton be released instead at $x_1 = 3\alpha/\beta$. Locate the point x_1 on the graph of $U(x)$. Calculate $v(x)$ and give a qualitative description of the motion. (f) For each release point ($x = x_0$ and $x = x_1$), what are the maximum and minimum values of x reached during the motion?

MCAT-STYLE PASSAGE PROBLEMS

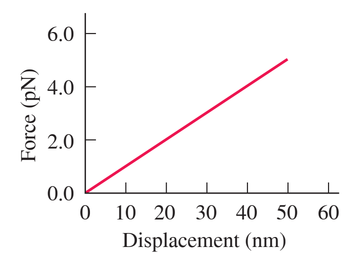
BIO The DNA Spring. A DNA molecule, with its double-helix structure, can in some situations behave like a spring. Measuring the force required to stretch single DNA molecules under various conditions can provide information about the biophysical properties of DNA. A technique for measuring the stretching force makes use of a very small cantilever, which consists of a beam that is supported at one end and is free to move at the other end, like a tiny diving board. The cantilever is constructed so that it obeys Hooke's law—that is, the displacement of its free end is proportional to the force applied to it. Because different cantilevers have different force constants, the cantilever's response must first be calibrated by applying a known force and determining the resulting deflection of the cantilever. Then one end of a DNA molecule is attached to the free end of the cantilever, and the other end of the DNA molecule is attached to a small stage that can be moved away from the cantilever, stretching the DNA. The stretched DNA pulls on the cantilever, deflecting the end of the cantilever very slightly. The measured deflection is then used to determine the force on the DNA molecule.



7.81 During the calibration process, the cantilever is observed to deflect by 0.10 nm when a force of 3.0 pN is applied to it. What deflection of the cantilever would correspond to a force of 6.0 pN? (a) 0.07 nm; (b) 0.14 nm; (c) 0.20 nm; (d) 0.40 nm.

7.82 A segment of DNA is put in place and stretched. **Figure P7.82** shows a graph of the force exerted on the DNA as a function of the displacement of the stage. Based on this graph, which statement is the best interpretation of the DNA's behavior over this range of displacements? The DNA (a) does not follow Hooke's law, because its force constant increases as the force on it increases; (b) follows Hooke's law and has a force constant of about 0.1 pN/nm; (c) follows Hooke's law and has a force constant of about 10 pN/nm; (d) does not follow Hooke's law, because its force constant decreases as the force on it increases.

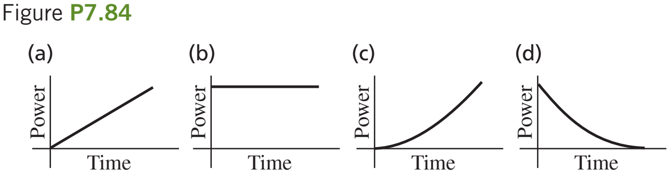
Figure P7.82



7.83 Based on Fig. P7.82, how much elastic potential energy is stored in the DNA when it is stretched 50 nm? (a) 2.5×10^{-19} J; (b) 1.2×10^{-19} J; (c) 5.0×10^{-12} J; (d) 2.5×10^{-12} J.

7.84 The stage moves at a constant speed while stretching the DNA. Which of the graphs in **Fig. P7.84** best represents the power supplied to the stage versus time?

Figure P7.84



ANSWERS

Chapter Opening Question ?

(v) As the crane descends, air resistance directed opposite to the bird's motion prevents its speed from increasing. Because the crane's speed stays the same, its kinetic energy K remains constant, but the gravitational potential energy U_{grav} decreases as the crane descends. Hence the total mechanical energy $E = K + U_{grav}$ decreases. The lost mechanical energy goes into warming the crane's skin (that is, an increase in the crane's internal energy) and stirring up the air through which the crane passes (an increase in the internal energy of the air). See Section 7.3.

Key Example VARIATION Problems

VP7.2.1 (a) 5.51 m (b) 3.67 m
VP7.2.2 (a) $\sqrt{3gh/2}$ (b) $\sqrt{gh/2}$
VP7.2.3 (a) 1.94 J (b) 0.243 N
VP7.2.4 (a) −7.99 J (b) 53.3 N
VP7.5.1 (a) 1.71 m/s (b) 0.147 N, or 3.00 times the weight
VP7.5.2 (a) 4.41 m; no (b) −358 J
VP7.5.3 (a) 0.503 J (b) 3.29 N
VP7.5.4 (a) $\sqrt{8gR}$ (b) $-mgR$ (c) mg/π
VP7.9.1 (a) 0.0492 J (b) 0.128 m
VP7.9.2 (a) −0.0116 J (b) 0.410
VP7.9.3 (a) $\sqrt{(kd^2/m) - 2gx}$ (b) $kd^2/2mg$
VP7.9.4 $2(mg - f)/k$

Bridging Problem

(a) 1.06 m (b) 1.32 m (c) 20.7 J