**TEST YOUR UNDERSTANDING OF SECTION 4.3** Rank the following situations in order of the magnitude of the object's acceleration, from lowest to highest. Are there any cases that have the same magnitude of acceleration? (i) A 2.0 kg object acted on by a 2.0 N net force; (ii) a 2.0 kg object acted on by an 8.0 N net force; (iii) an 8.0 kg object acted on by a 2.0 N net force; (iv) an 8.0 kg object acted on by a 8.0 N net force.

**ANSWER**

**(iii), (i) and (iv) (tie), (ii)** The acceleration is equal to the net force divided by the mass. Hence the magnitude of the acceleration in each situation is (i) $a = (2.0\text{ N})/(2.0\text{ kg}) = 1.0\text{ m/s}^2$; (ii) $a = (8.0\text{ N})/(2.0\text{ N}) = 4.0\text{ m/s}^2$; (iii) $a = (2.0\text{ N})/(8.0\text{ kg}) = 0.25\text{ m/s}^2$; (iv) $a = (8.0\text{ N})/(8.0\text{ kg}) = 1.0\text{ m/s}^2$.

## 4.4 MASS AND WEIGHT

The *weight* of an object is the gravitational force that the earth exerts on the object. (If you are on another planet, your weight is the gravitational force that planet exerts on you.) Unfortunately, the terms "mass" and "weight" are often misused and interchanged in everyday conversation. It's absolutely essential for you to understand clearly the distinctions between these two physical quantities.

Mass characterizes the *inertial* properties of an object. Mass is what keeps the table setting on the table when you yank the tablecloth out from under it. The greater the mass, the greater the force needed to cause a given acceleration; this is reflected in Newton's second law, $\sum\vec{F} = m\vec{a}$.

Weight, on the other hand, is a *force* exerted on an object by the pull of the earth. Mass and weight are related: Objects that have large mass also have large weight. A large stone is hard to throw because of its large *mass,* and hard to lift off the ground because of its large *weight.*

To understand the relationship between mass and weight, note that a freely falling object has an acceleration of magnitude $g$ (see Section 2.5). Newton's second law tells us that a force must act to produce this acceleration. If a 1 kg object falls with an acceleration of 9.8 m/s$^2$, the required force has magnitude

$$F = ma = (1\text{ kg})(9.8\text{ m/s}^2) = 9.8\text{ kg}\cdot\text{m/s}^2 = 9.8\text{ N}$$

The force that makes the object accelerate downward is its weight. Any object near the surface of the earth that has a mass of 1 kg *must* have a weight of 9.8 N to give it the acceleration we observe when it is in free fall. More generally,

$$\underbrace{w}_{\text{Magnitude of weight of an object}} = \underbrace{m}_{\text{Mass of object}}\underbrace{g}_{\text{Magnitude of acceleration due to gravity}} \tag{4.8}$$

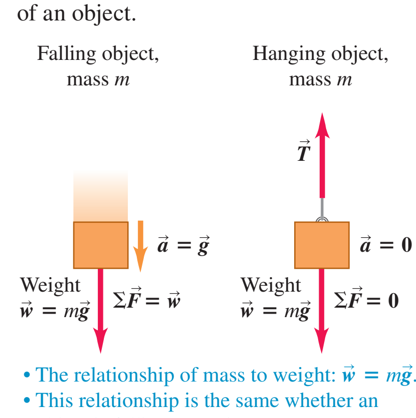
Hence the magnitude $w$ of an object's weight is directly proportional to its mass $m$. The weight of an object is a force, a vector quantity, and we can write Eq. (4.8) as a vector equation (**Fig. 4.20**):

$$\vec{w} = m\vec{g} \tag{4.9}$$

Remember that $g$ is the *magnitude* of $\vec{g}$, the acceleration due to gravity, so $g$ is always a positive number, by definition. Thus $w$, given by Eq. (4.8), is the *magnitude* of the weight and is also always positive.

**CAUTION An object's weight acts at all times** When keeping track of the external forces on an object, remember that the weight is present *all the time,* whether the object is in free fall or not. If we suspend an object from a rope, it is in equilibrium and its acceleration is zero. But its weight, given by Eq. (4.9), is still pulling down on it (Fig. 4.20). In this case the rope pulls up on the object, applying an upward force. The *vector sum* of the external forces is zero, but the weight still acts.

Figure 4.20 Relating the mass and weight of an object.



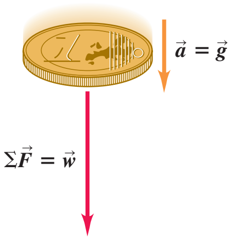
## CONCEPTUAL EXAMPLE 4.6 Net external force and acceleration in free fall

In Example 2.6 of Section 2.5, a one-euro coin was dropped from rest from the Leaning Tower of Pisa. If the coin falls freely, so that the effects of the air are negligible, how does the net external force on the coin vary as it falls?

**SOLUTION** In free fall, the acceleration $\vec{a}$ of the coin is constant and equal to $\vec{g}$. Hence by Newton's second law the net external force $\sum \vec{F} = m\vec{a}$ is also constant and equal to $m\vec{g}$, which is the coin's weight $\vec{w}$ (**Fig. 4.21**). The coin's velocity changes as it falls, but the net external force acting on it is constant.

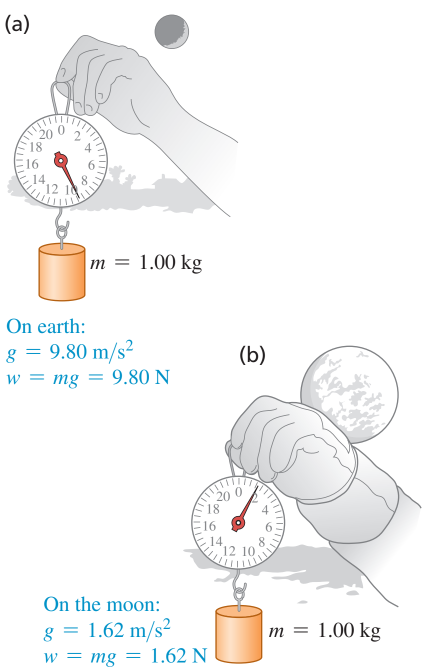
The net external force on a freely falling coin is constant even if you initially toss it upward. The force that your hand exerts on the coin to toss it is a contact force, and it disappears the instant the coin leaves your hand. From then on, the only force acting on the coin is its weight $\vec{w}$.

Figure 4.21 The acceleration of a freely falling object is constant, and so is the net external force acting on the object.




**KEYCONCEPT** The gravitational force on an object (its weight) does not depend on how the object is moving.

Figure 4.22 The weight of a 1 kilogram mass (a) on earth and (b) on the moon.




## Variation of *g* with Location

We'll use $g = 9.80\ \text{m/s}^2$ for problems set on the earth (or, if the other data in the problem are given to only two significant figures, $g = 9.8\ \text{m/s}^2$). In fact, the value of $g$ varies somewhat from point to point on the earth's surface—from about 9.78 to $9.82\ \text{m/s}^2$—because the earth is not perfectly spherical and because of effects due to its rotation. At a point where $g = 9.80\ \text{m/s}^2$, the weight of a standard kilogram is $w = 9.80$ N. At a different point, where $g = 9.78\ \text{m/s}^2$, the weight is $w = 9.78$ N but the mass is still 1 kg. The weight of an object varies from one location to another; the mass does not.

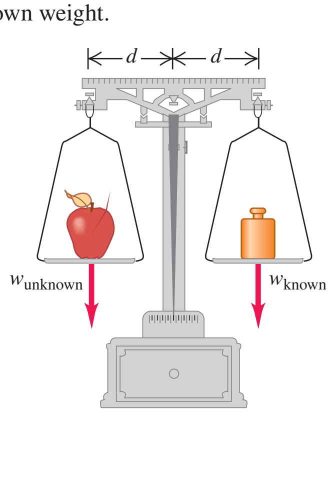
If we take a standard kilogram to the surface of the moon, where the value of $g$ is $1.62\ \text{m/s}^2$, its weight is 1.62 N but its mass is still 1 kg (**Fig. 4.22**). An 80.0 kg astronaut has a weight on earth of $(80.0\ \text{kg})(9.80\ \text{m/s}^2) = 784$ N, but on the moon the astronaut's weight would be only $(80.0\ \text{kg})(1.62\ \text{m/s}^2) = 130$ N. In Chapter 13 we'll see how to calculate the value of $g$ at the surface of the moon or on other worlds.

## Measuring Mass and Weight

In Section 4.3 we described a way to compare masses by comparing their accelerations when they are subjected to the same net external force. Usually, however, the easiest way to measure the mass of an object is to measure its weight, often by comparing with a standard. Equation (4.8) says that two objects that have the same weight at a particular location also have the same mass. We can compare weights very precisely; the familiar equal-arm balance (**Fig. 4.23**) can determine with great precision (up to 1 part in $10^6$) when the weights of two objects are equal and hence when their masses are equal.

The concept of mass plays two rather different roles in mechanics. The weight of an object (the gravitational force acting on it) is proportional to its mass as stated in the equation $w = mg$; we call the property related to gravitational interactions *gravitational mass*. On the other hand, we call the inertial property that appears in Newton's second law $(\Sigma\vec{F} = m\vec{a})$ the *inertial mass*. If these two quantities were different, the acceleration due to gravity might well be different for different objects. However, extraordinarily precise experiments have established that in fact the two *are* the same to a precision of better than one part in $10^{12}$.

Figure 4.23 An equal-arm balance determines the mass of an object (such as an apple) by comparing its weight to a known weight.




**CAUTION Don't confuse mass and weight** The SI units for mass and weight are often misused in everyday life. For example, it's incorrect to say "This box weighs 6 kg." What this really means is that the *mass* of the box, probably determined indirectly by *weighing,* is 6 kg. Avoid this sloppy usage in your own work! In SI units, weight (a force) is measured in newtons, while mass is measured in kilograms.

### EXAMPLE 4.7 Mass and weight

A $2.45 \times 10^4$ N truck traveling in the $+x$-direction makes an emergency stop; the $x$-component of the net external force acting on it is $-1.83 \times 10^4$ N. What is its acceleration?

**IDENTIFY and SET UP** Our target variable is the $x$-component of the truck's acceleration, $a_x$. We use the $x$-component portion of Newton's second law, Eqs. (4.7), to relate force and acceleration. To do this, we need to know the truck's mass. The newton is a unit for force, however, so $2.49 \times 10^4$ N is the truck's *weight,* not its mass. Hence we'll first use Eq. (4.8) to determine the truck's mass from its weight. The truck has a positive $x$-velocity and is slowing down, so its $x$-acceleration will be negative.

**EXECUTE** The mass of the truck is

$$m = \frac{w}{g} = \frac{2.45 \times 10^4\ \text{N}}{9.80\ \text{m/s}^2} = \frac{2.45 \times 10^4\ \text{kg} \cdot \text{m/s}^2}{9.80\ \text{m/s}^2} = 2540\ \text{kg}$$

Then $\sum F_x = ma_x$ gives

$$a_x = \frac{\sum F_x}{m} = \frac{-1.83 \times 10^4\ \text{N}}{2540\ \text{kg}} = \frac{-1.83 \times 10^4\ \text{kg} \cdot \text{m/s}^2}{2540\ \text{kg}} = -7.20\ \text{m/s}^2$$

**EVALUATE** The negative sign means that the acceleration vector points in the negative $x$-direction, as we expected. The magnitude of this acceleration is pretty high; passengers in this truck will experience a lot of rearward force from their seat belts.

**KEYCONCEPT** In problems involving Newton's second law, make sure that for $m$ you use the mass of the object, *not* its weight.

**TEST YOUR UNDERSTANDING OF SECTION 4.4** Suppose an astronaut landed on a planet where $g = 19.6\ \text{m/s}^2$. Compared to earth, would it be easier, harder, or just as easy for her to walk around? Would it be easier, harder, or just as easy for her to catch a ball that is moving horizontally at 12 m/s? (Assume that the astronaut's spacesuit is a lightweight model that doesn't impede her movements in any way.)

**ANSWER**

It would take twice the effort for the astronaut to walk around because her weight on the planet would be twice as much as on the earth. But it would be just as easy to catch a ball moving horizontally. The ball's *mass* is the same as on earth, so the horizontal force the astronaut would have to exert to bring it to a stop (i.e., to give it the same acceleration) would also be the same as on earth.

## 4.5 NEWTON'S THIRD LAW

A force acting on an object is always the result of its interaction with another object, so forces always come in pairs. You can't pull on a doorknob without the doorknob pulling back on you. When you kick a football, your foot exerts a forward force on the ball, but you also feel the force the ball exerts back on your foot.

In each of these cases, the force that you exert on the other object is in the opposite direction to the force that object exerts on you. Experiments show that whenever two objects interact, the two forces that they exert on each other are always *equal in magnitude* and *opposite in direction*. This fact is called *Newton's third law of motion:*

**NEWTON'S THIRD LAW OF MOTION If object $A$ exerts a force on object $B$ (an "action"), then object $B$ exerts a force on object $A$ (a "reaction"). These two forces have the same magnitude but are opposite in direction. These two forces act on *different* objects.**

For example, in **Fig. 4.24** $\vec{F}_{A \text{ on } B}$ is the force applied *by* object $A$ (first subscript) *on* object $B$ (second subscript), and $\vec{F}_{B \text{ on } A}$ is the force applied *by* object $B$ (first subscript) *on* object $A$ (second subscript). In equation form,

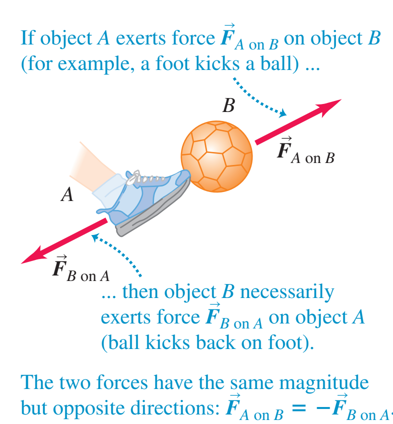
**Newton's third law:** When two objects $A$ and $B$ exert forces on each other ...

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A} \qquad (4.10)$$

... the two forces have the same magnitude but opposite directions.

*Note*: The two forces act on *different* objects.

Figure **4.24** Newton's third law of motion.



**APPLICATION Blame Newton's Laws** This car stopped because of Newton's second and third laws. During the impact, the car exerted a force on the tree; in accordance with the third law, the tree exerted an equally strong force back on the car. In accordance with the second law, the force of the tree on the car gave the car an acceleration that changed its velocity to zero.

It doesn't matter whether one object is inanimate (like the soccer ball in Fig. 4.24) and the other is not (like the kicker's foot): They necessarily exert forces on each other that obey Eq. (4.10).

In the statement of Newton's third law, "action" and "reaction" are the two opposite forces (in Fig. 4.24, $\vec{F}_{A \text{ on } B}$ and $\vec{F}_{B \text{ on } A}$); we sometimes refer to them as an **action–reaction pair.** This is *not* meant to imply any cause-and-effect relationship; we can consider either force as the "action" and the other as the "reaction." We often say simply that the forces are "equal and opposite," meaning that they have equal magnitudes and opposite directions.

**CAUTION The two forces in an action–reaction pair act on different objects** We stress that the two forces described in Newton's third law act on *different* objects. This is important when you solve problems involving Newton's first or second law, which involve the forces that act *on* an object. For instance, the net external force on the soccer ball in Fig. 4.24 is the vector sum of the weight of the ball and the force $\vec{F}_{A \text{ on } B}$ exerted by kicker $A$ on the ball $B$. You wouldn't include the force $\vec{F}_{B \text{ on } A}$ because this force acts on the kicker $A$, *not* on the ball.

In Fig. 4.24 the action and reaction forces are *contact* forces that are present only when the two objects are touching. But Newton's third law also applies to *long-range* forces that do not require physical contact, such as the force of gravitational attraction. A table-tennis ball exerts an upward gravitational force on the earth that's equal in magnitude to the downward gravitational force the earth exerts on the ball. When you drop the ball, both the ball and the earth accelerate toward each other. The net force on each object has the same magnitude, but the earth's acceleration is microscopically small because its mass is so great. Nevertheless, it does move!

**CAUTION Contact forces need contact** If your fingers push on an object, the force you exert acts only when your fingers and the object are in contact. Once contact is broken, the force is no longer present even if the object is still moving.

## CONCEPTUAL EXAMPLE 4.8 Which force is greater?

After your sports car breaks down, you start to push it to the nearest repair shop. While the car is starting to move, how does the force you exert on the car compare to the force the car exerts on you? How do these forces compare when you are pushing the car along at a constant speed?

**SOLUTION** Newton's third law says that in *both* cases, the force you exert on the car is equal in magnitude and opposite in direction to the force the car exerts on you. It's true that you have to push harder to get the car going than to keep it going. But no matter how hard you push on the car, the car pushes just as hard back on you. Newton's third law gives the same result whether the two objects are at rest, moving with constant velocity, or accelerating.

You may wonder how the car "knows" to push back on you with the same magnitude of force that you exert on it. It may help to visualize the forces you and the car exert on each other as interactions between the atoms at the surface of your hand and the atoms at the surface of the car. These interactions are analogous to miniature springs between adjacent atoms, and a compressed spring exerts equally strong forces on both of its ends.

**KEYCONCEPT** No matter how two interacting objects are moving, the forces that they exert on each other always have the same magnitude and point in opposite directions.

## CONCEPTUAL EXAMPLE 4.9 Newton's third law I: Objects at rest

An apple sits at rest on a table, in equilibrium. What forces act on the apple? What is the reaction force to each of the forces acting on the apple? What are the action–reaction pairs?

**SOLUTION** **Figure 4.25a** shows the forces acting on the apple. $\vec{F}_{\text{earth on apple}}$ is the weight of the apple—that is, the downward gravitational force exerted *by* the earth *on* the apple. Similarly, $\vec{F}_{\text{table on apple}}$ is the upward normal force exerted *by* the table *on* the apple.

Figure 4.25b shows one of the action–reaction pairs involving the apple. As the earth pulls down on the apple, with force $\vec{F}_{\text{earth on apple}}$, the apple exerts an equally strong upward pull on the earth $\vec{F}_{\text{apple on earth}}$. By Newton's third law (Eq. 4.10) we have

$$\vec{F}_{\text{apple on earth}} = -\vec{F}_{\text{earth on apple}}$$

Figure 4.25c shows the other action–reaction pair involving the apple. The table pushes up on the apple with force $\vec{F}_{\text{table on apple}}$; the corresponding reaction is the downward force $\vec{F}_{\text{apple on table}}$ exerted by the apple on the table. For this action–reaction pair we have

$$\vec{F}_{\text{apple on table}} = -\vec{F}_{\text{table on apple}}$$

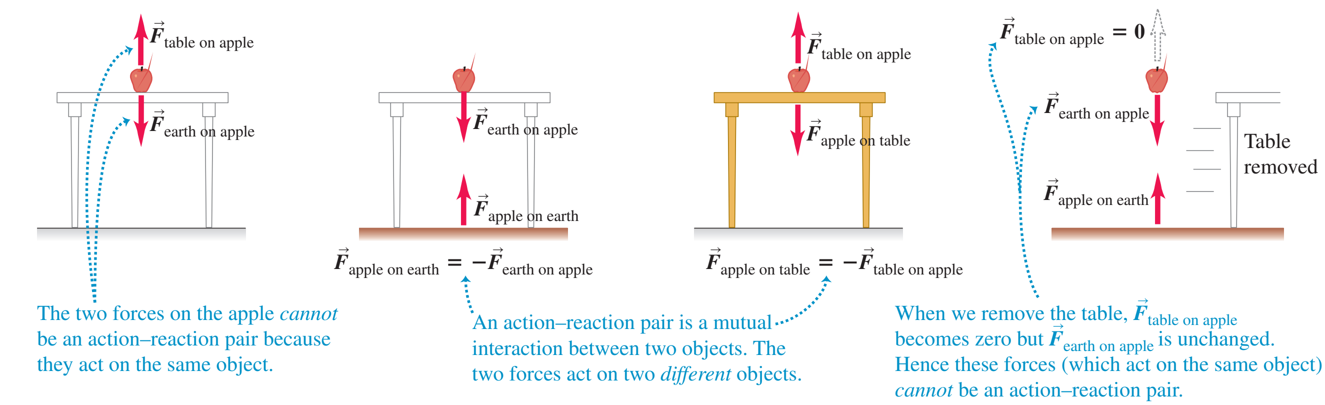
Figure **4.25** Identifying action–reaction pairs.

(a) The forces acting on the apple

(b) The action–reaction pair for the interaction between the apple and the earth

(c) The action–reaction pair for the interaction between the apple and the table

(d) We eliminate the force of the table on the apple.




The two forces acting on the apple in Fig. 4.25a, $\vec{F}_{\text{table on apple}}$ and $\vec{F}_{\text{earth on apple}}$, are *not* an action–reaction pair, despite being equal in magnitude and opposite in direction. They do not represent the mutual interaction of two objects; they are two different forces acting on the *same* object. Figure 4.25d shows another way to see this. If we suddenly yank the table out from under the apple, the forces $\vec{F}_{\text{apple on table}}$ and $\vec{F}_{\text{table on apple}}$ suddenly become zero, but $\vec{F}_{\text{apple on earth}}$ and $\vec{F}_{\text{earth on apple}}$ are unchanged (the gravitational interaction is still present). Because $\vec{F}_{\text{table on apple}}$ is now zero, it can't be the negative of the nonzero $\vec{F}_{\text{earth on apple}}$, and these two forces can't be an action–reaction pair. *The two forces in an action–reaction pair* ***never*** *act on the same object.*

**KEYCONCEPT** The two forces in an action–reaction pair always act on two different objects.

## CONCEPTUAL EXAMPLE 4.10 Newton's third law II: Objects in motion

A stonemason drags a marble block across a floor by pulling on a rope attached to the block (**Fig. 4.26a**). The block is not necessarily in equilibrium. What are the forces that correspond to the interactions between the block, rope, and mason? What are the action–reaction pairs?

**SOLUTION** We'll use the subscripts B for the block, R for the rope, and M for the mason. In Fig. 4.26b $\vec{F}_{\text{M on R}}$ is the force exerted by the *mason* on the *rope*, and the corresponding reaction is the force $\vec{F}_{\text{R on M}}$ exerted by the *rope* on the *mason*. Similarly, $\vec{F}_{\text{R on B}}$ is the force exerted by the *rope* on the *block,* and the corresponding reaction is the force $\vec{F}_{\text{B on R}}$ exerted by the *block* on the *rope*. The forces in each action–reaction pair are equal and opposite:

$$\vec{F}_{\text{R on M}} = -\vec{F}_{\text{M on R}} \quad \text{and} \quad \vec{F}_{\text{B on R}} = -\vec{F}_{\text{R on B}}$$

Forces $\vec{F}_{\text{M on R}}$ and $\vec{F}_{\text{B on R}}$ (Fig. 4.26c) are *not* an action–reaction pair because both of these forces act on the *same* object (the rope); an action and its reaction *must* always act on *different* objects. Furthermore, the forces $\vec{F}_{\text{M on R}}$ and $\vec{F}_{\text{B on R}}$ are not necessarily equal in magnitude. Applying Newton's second law to the rope, we get

$$\sum\vec{F} = \vec{F}_{\text{M on R}} + \vec{F}_{\text{B on R}} = m_{\text{R}}\vec{a}_{\text{R}}$$

If the block and rope are accelerating (speeding up or slowing down), the rope is not in equilibrium, and $\vec{F}_{\text{M on R}}$ must have a different magnitude than $\vec{F}_{\text{B on R}}$. By contrast, the action–reaction forces $\vec{F}_{\text{M on R}}$ and $\vec{F}_{\text{R on M}}$ are always equal in magnitude, as are $\vec{F}_{\text{R on B}}$ and $\vec{F}_{\text{B on R}}$. Newton's third law holds whether or not the objects are accelerating.

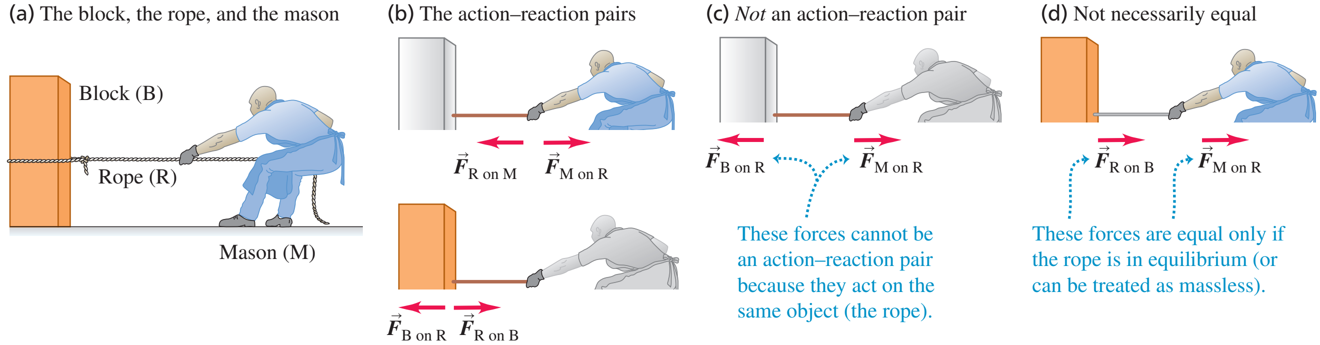
Figure **4.26** Identifying the interaction forces when a mason pulls on a rope attached to a block.

(a) The block, the rope, and the mason

(b) The action–reaction pairs

(c) *Not* an action–reaction pair

(d) Not necessarily equal




*Continued*

In the special case in which the rope is in equilibrium, the forces $\vec{F}_{\text{M on R}}$ and $\vec{F}_{\text{B on R}}$ are equal in magnitude and opposite in direction. But this is an example of Newton's *first* law, not his third; these are two forces on the same object, not forces of two objects on each other. Another way to look at this is that in equilibrium, $\vec{a}_{\text{R}} = \mathbf{0}$ in the previous equation. Then $\vec{F}_{\text{B on R}} = -\vec{F}_{\text{M on R}}$ because of Newton's first law.

Another special case is if the rope is accelerating but has negligibly small mass compared to that of the block or the mason. In this case, $m_{\text{R}} = 0$ in the previous equation, so again $\vec{F}_{\text{B on R}} = -\vec{F}_{\text{M on R}}$. Since Newton's third law says that $\vec{F}_{\text{B on R}}$ *always* equals $-\vec{F}_{\text{R on B}}$ (they are an action–reaction pair), in this "massless-rope" case $\vec{F}_{\text{R on B}}$ also equals $\vec{F}_{\text{M on R}}$.

For both the "massless-rope" case and a rope in equilibrium, the rope exerts the same force on the block as the mason exerts on the rope (Fig. 4.26d). Hence we can think of the rope as "transmitting" to the block the force the mason exerts on the rope. But remember that this is true *only* when the rope has negligibly small mass or is in equilibrium.

**KEYCONCEPT** In problems that involve more than one object, use Newton's third law to relate the forces that the objects exert on each other.

## CONCEPTUAL EXAMPLE 4.11 A Newton's third law paradox?

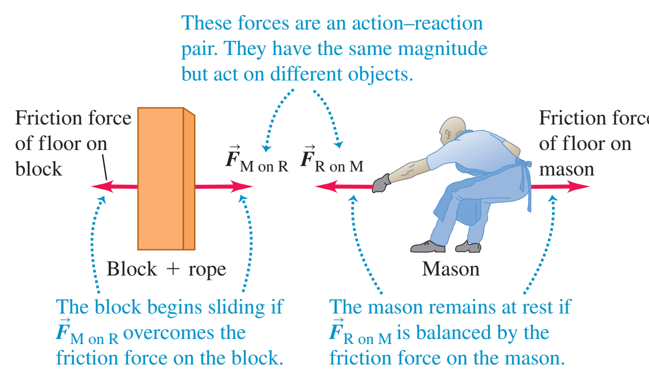
We saw in Conceptual Example 4.10 that the stonemason pulls as hard on the rope–block combination as that combination pulls back on him. Why, then, does the block move while the stonemason remains stationary?

**SOLUTION** To resolve this seeming paradox, keep in mind the difference between Newton's *second* and *third* laws. The only forces involved in Newton's second law are those that act *on* a given object. The vector sum of these forces determines the object's acceleration, if any. By contrast, Newton's third law relates the forces that two *different* objects exert on *each other*. The third law alone tells you nothing about the motion of either object.

If the rope–block combination is initially at rest, it begins to slide if the stonemason exerts a force on the rope $\vec{F}_{\text{M on R}}$ that is *greater* in magnitude than the friction force that the floor exerts on the block (**Fig. 4.27**). Then there is a net external force to the right on the rope–block combination, and it accelerates to the right. By contrast, the stonemason *doesn't* move because the net external force acting on him is *zero*. His shoes have nonskid soles that don't slip on the floor, so the friction force that the floor exerts on him is strong enough to balance the pull of the rope on him, $\vec{F}_{\text{R on M}}$. (Both the block and the stonemason also experience a downward force of gravity and an upward normal force exerted by the floor. These forces balance each other, and so don't affect the motion of the block or the mason.)

Once the block is moving, the stonemason doesn't need to pull as hard; he must exert only enough force to balance the friction force on the block. Then the net external force on the moving block is zero, and by Newton's first law the block continues to move toward the mason at a constant velocity.

Figure **4.27** The horizontal forces acting on the block–rope combination (left) and the mason (right). (The vertical forces are not shown.)




So the block accelerates but the stonemason doesn't because different amounts of friction act on them. If the floor were freshly waxed, so that there was little friction between the floor and the stonemason's shoes, pulling on the rope might start the block sliding to the right *and* start him sliding to the left.

Here's the moral: When analyzing the motion of an object, remember that only the forces acting *on* an object determine its motion. From this perspective, Newton's third law is merely a tool that can help you determine what those forces are.

**KEYCONCEPT** The motion of an object depends on the forces that are exerted on it, not the forces that it exerts on other objects.

An object that has pulling forces applied at its ends, such as the rope in Fig. 4.26, is said to be in *tension*. The **tension** at any point along the rope is the magnitude of the force acting at that point (see Fig. 4.2c). In Fig. 4.26b the tension at the right end of the rope is the magnitude of $\vec{F}_{\text{M on R}}$ (or of $\vec{F}_{\text{R on M}}$), and the tension at the left end is the magnitude of $\vec{F}_{\text{B on R}}$ (or of $\vec{F}_{\text{R on B}}$). If the rope is in equilibrium and if no forces act except at its ends, the net external force on the rope is zero and the tension is the *same* at both ends and throughout the rope. Thus, if the magnitudes of $\vec{F}_{\text{B on R}}$ and $\vec{F}_{\text{M on R}}$ are 50 N each, the tension in the rope is 50 N (*not* 100 N). The same is true if we can regard the rope as "massless" (that is, if its mass is small compared to that of the objects to which it's attached).

We emphasize once again that the two forces in an action–reaction pair *never* act on the same object. Remembering this fact can help you avoid confusion about action–reaction pairs and Newton's third law.

**TEST YOUR UNDERSTANDING OF SECTION 4.5** You are driving a car on a country road when a mosquito splatters on the windshield. Which has the greater magnitude: the force that the car exerted on the mosquito or the force that the mosquito exerted on the car? Or are the magnitudes the same? If they are different, how can you reconcile this fact with Newton's third law? If they are equal, why is the mosquito splattered while the car is undamaged?

**ANSWER**

By Newton's third law, the two forces have equal magnitude. Because the car has much greater mass than the mosquito, it undergoes only a tiny, imperceptible acceleration in response to the force of the impact. By contrast, the mosquito, with its minuscule mass, undergoes a catastrophically large acceleration.

## 4.6 FREE-BODY DIAGRAMS

Newton's three laws of motion contain all the basic principles we need to solve a wide variety of problems in mechanics. These laws are very simple in form, but the process of applying them to specific situations can pose real challenges. In this brief section we'll point out three key ideas and techniques to use in any problems involving Newton's laws. You'll learn others in Chapter 5, which also extends the use of Newton's laws to cover more complex situations.

1. *Newton's first and second laws apply to a specific object.* Whenever you use Newton's first law, $\sum \vec{F} = \mathbf{0}$, for an equilibrium situation or Newton's second law, $\sum \vec{F} = m\vec{a}$, for a nonequilibrium situation, you must decide at the beginning to which object you are referring. This decision may sound trivial, but it isn't.
2. *Only forces acting on the object matter.* The sum $\sum \vec{F}$ includes all the forces that act *on* the object in question. Hence, once you've chosen the object to analyze, you have to identify all the forces acting on it. Don't confuse the forces acting on a object with the forces exerted by that object on some other object. For example, to analyze a person walking, you would include in $\sum \vec{F}$ the force that the ground exerts on the person as he walks, but *not* the force that the person exerts on the ground (**Fig. 4.28**). These forces form an action–reaction pair and are related by Newton's third law, but only the member of the pair that acts on the object you're working with goes into $\sum \vec{F}$.
3. *Free-body diagrams are essential to help identify the relevant forces.* A **free-body diagram** shows the chosen object by itself, "free" of its surroundings, with vectors drawn to show the magnitudes and directions of all the forces that act on the object. (Here "body" is another word for "object.") We've shown free-body diagrams in Figs. 4.17, 4.18, 4.20, and 4.25a. Be careful to include all the forces acting *on* the object, but be equally careful *not* to include any forces that the object exerts on any other object. In particular, the two forces in an action–reaction pair must *never* appear in the same free-body diagram because they never act on the same object. Furthermore, never include forces that a object exerts on itself, since these can't affect the object's motion.

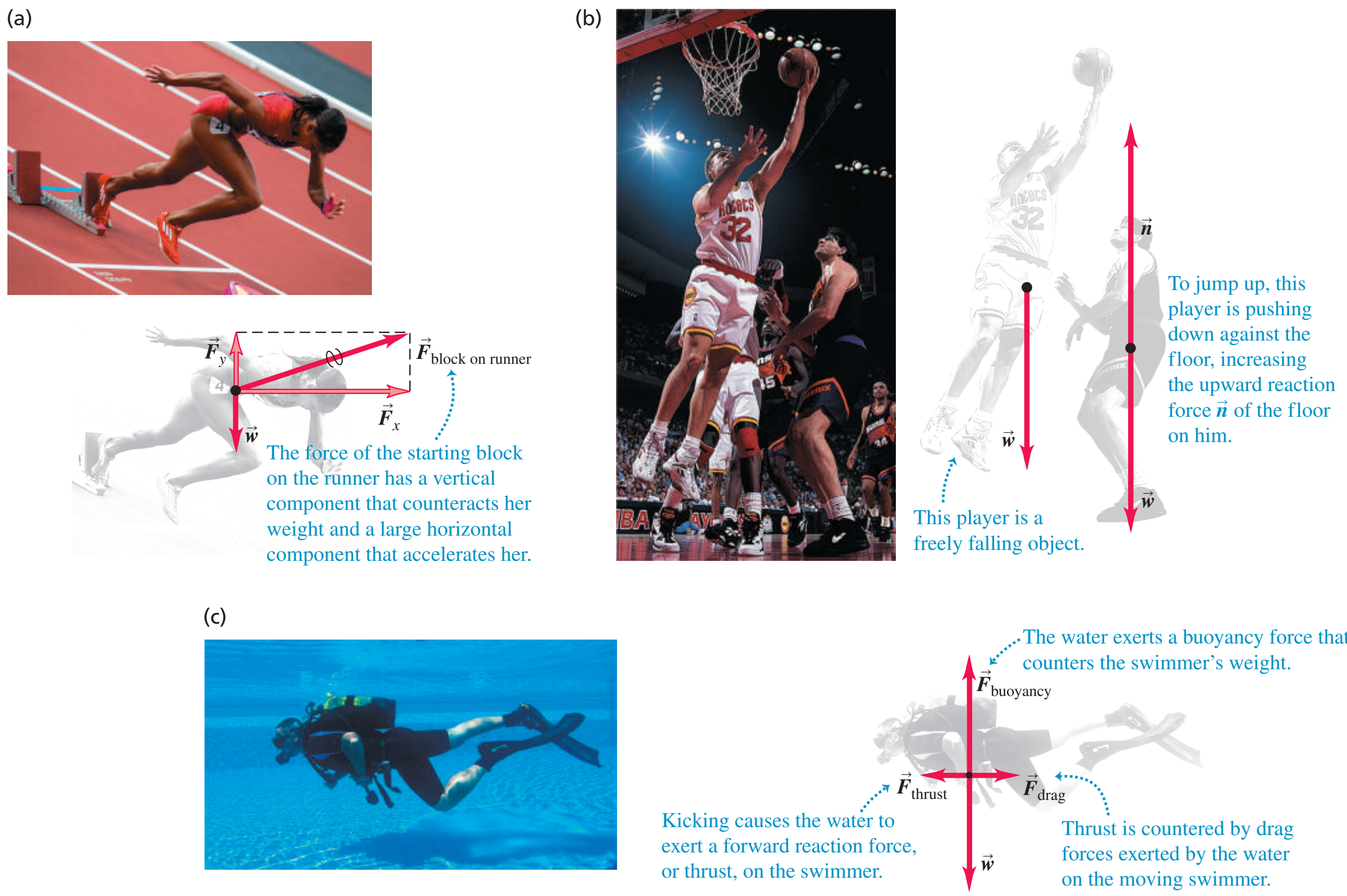
Figure **4.28** The simple act of walking depends crucially on Newton's third law. To start moving forward, you push backward on the ground with your foot. As a reaction, the ground pushes forward on your foot (and hence on your body as a whole) with a force of the same magnitude. This *external* force provided by the ground is what accelerates your body forward.

When a problem involves more than one object, you have to take the problem apart and draw a separate free-body diagram for each object. For example, Fig. 4.26c shows a separate free-body diagram for the rope in the case in which the rope is considered massless (so that no gravitational force acts on it). Figure 4.27 also shows diagrams for the block and the mason, but these are *not* complete free-body diagrams because they don't show all the forces acting on each object. (We left out the vertical forces—the weight force exerted by the earth and the upward normal force exerted by the floor.)

In **Fig. 4.29** (next page) we present three real-life situations and the corresponding complete free-body diagrams. Note that in each situation a person exerts a force on something in his or her surroundings, but the force that shows up in the person's free-body diagram is the surroundings pushing back *on* the person.

**CAUTION** **Forces in free-body diagrams** For a free-body diagram to be complete, you *must* be able to answer this question for each force: What other object is applying this force? If you can't answer that question, you may be dealing with a nonexistent force. Avoid nonexistent forces such as "the force of acceleration" or "the $m\vec{a}$ force," discussed in Section 4.3.

Figure **4.29** Examples of free-body diagrams. Each free-body diagram shows all of the external forces that act on the object in question.




**TEST YOUR UNDERSTANDING OF SECTION 4.6** The buoyancy force shown in Fig. 4.29c is one half of an action–reaction pair. What force is the other half of this pair? (i) The weight of the swimmer; (ii) the forward thrust force; (iii) the backward drag force; (iv) the downward force that the swimmer exerts on the water; (v) the backward force that the swimmer exerts on the water by kicking.

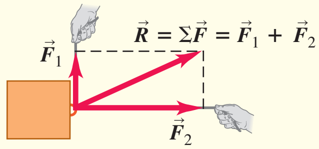
**ANSWER**

**(iv)** The buoyancy force is an *upward* force that the *water* exerts on the *swimmer*. By Newton's third law, the other half of the action–reaction pair is a *downward* force that the *swimmer* exerts on the *water* and has the same magnitude as the buoyancy force. It's true that the weight of the swimmer is also downward and has the same magnitude as the buoyancy force; however, the weight acts on the same object (the swimmer) as the buoyancy force, and so these forces aren't an action–reaction pair.

# CHAPTER 4 SUMMARY

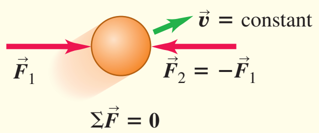
**Force as a vector:** Force is a quantitative measure of the interaction between two objects. It is a vector quantity. When several external forces act on an object, the effect on its motion is the same as if a single force, equal to the vector sum (resultant) of the forces, acts on the object. (See Example 4.1.)

$$\vec{R} = \Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots \quad (4.1)$$




**The net external force on an object and Newton's first law:** Newton's first law states that when the vector sum of all external forces acting on a object (the *net external force*) is zero, the object is in equilibrium and has zero acceleration. If the object is initially at rest, it remains at rest; if it is initially in motion, it continues to move with constant velocity. This law is valid in inertial frames of reference only. (See Examples 4.2 and 4.3.)

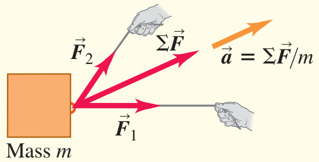
$$\Sigma\vec{F} = \mathbf{0} \quad (4.3)$$




**Mass, acceleration, and Newton's second law:** The inertial properties of an object are characterized by its *mass*. Newton's second law states that the acceleration of an object under the action of a given set of external forces is directly proportional to the vector sum of the forces (the *net force*) and inversely proportional to the mass of the object. Like Newton's first law, this law is valid in inertial frames of reference only. In SI units, the unit of force is the newton (N), equal to 1 $\text{kg} \cdot \text{m/s}^2$. (See Examples 4.4 and 4.5.)

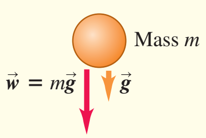
$$\Sigma\vec{F} = m\vec{a} \quad (4.6)$$

$$\Sigma F_x = ma_x \qquad \Sigma F_y = ma_y \qquad \Sigma F_z = ma_z \quad (4.7)$$



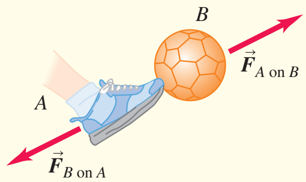

**Weight:** The weight $\vec{w}$ of an object is the gravitational force exerted on it by the earth. Weight is a vector quantity. The magnitude of the weight of an object at any specific location is equal to the product of its mass $m$ and the magnitude of the acceleration due to gravity $g$ at that location. The weight of an object depends on its location; its mass does not. (See Examples 4.6 and 4.7.)

$$w = mg \quad (4.8)$$




**Newton's third law and action–reaction pairs:** Newton's third law states that when two objects interact, they exert forces on each other that are equal in magnitude and opposite in direction. These forces are called action and reaction forces. Each of these two forces acts on only one of the two objects; they never act on the same object. (See Examples 4.8–4.11.)

$$\vec{F}_{A\text{ on }B} = -\vec{F}_{B\text{ on }A} \quad (4.10)$$




Chapter 4 Media Assets

## GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

### KEY EXAMPLE VARIATION PROBLEMS

**Be sure to review EXAMPLE 4.1 (Section 4.1) before attempting these problems.**

**VP4.1.1** Three professional wrestlers are fighting over a champion's belt, and each exerts a force on the belt. Wrestler 1 exerts a force $F_1 = 40.0$ N in the $+x$-direction, wrestler 2 exerts a force $F_2 = 80.0$ N in the $-y$-direction, and wrestler 3 exerts a force $F_3 = 60.0$ N at an angle of 36.9° counterclockwise from the $+x$-direction. Find the $x$- and $y$-components of the net external force on the belt, and find the force's magnitude and direction.

**VP4.1.2** Three forces act on a statue. Force $\vec{F}_1$ (magnitude 45.0 N) points in the $+x$-direction, force $\vec{F}_2$ (magnitude 105 N) points in the $+y$-direction, and force $\vec{F}_3$ (magnitude 235 N) is at an angle of 36.9° from the $-x$-direction and 53.1° from the $+y$-direction. Find the $x$- and $y$-components of the net external force on the statue, and find the force's magnitude and direction.

**VP4.1.3** An eagle descends steeply onto its prey. Its weight (the gravitational force on the eagle), of magnitude 60.0 N, points downward in the $-y$-direction. The lift force exerted on the eagle's wings by the air, also of magnitude 60.0 N, is at an angle of 20.0° from the vertical (the $+y$-direction) and 70.0° from the $+x$-direction. The drag force (air resistance) exerted on the eagle by the air has magnitude 15.0 N and is at an angle of 20.0° from the $-x$-direction and 70.0° from the $+y$-direction. Find the $x$- and $y$-components of the net external force on the eagle, and find the force's magnitude and direction.

**VP4.1.4** A box containing pizza sits on a table. Ernesto, who sits due east of the pizza box, pulls the box toward him with a force of 35.0 N. Kamala, who sits due north of the pizza box, pulls the box toward her with a 50.0 N force. Tsuroku also sits at the table and pulls the box toward her so that the net external force on the box is 24.0 N in a direction 30.0° south of west. Take the $+x$-direction to be due east and the $+y$-direction to be due north. Find the $x$- and $y$-components of the force that Tsuroku exerts, and find the force's magnitude and direction.

**Be sure to review EXAMPLE 4.4 (Section 4.3) before attempting these problems.**

**VP4.4.1** A box of books with mass 55 kg rests on the level floor of the campus bookstore. The floor is freshly waxed and has negligible friction. A bookstore worker applies a constant horizontal force with magnitude 25 N to the box. What is the magnitude of the acceleration of the box?

**VP4.4.2** A block of cheese of mass 2.0 kg sits on a freshly waxed, essentially frictionless table. You apply a constant horizontal force of 0.50 N to the cheese. (a) Name the three external forces that act on the cheese and what exerts each force. (b) What is the magnitude of the acceleration of the cheese?

**VP4.4.3** In a game of ice hockey, you use a hockey stick to hit a puck of mass 0.16 kg that slides on essentially frictionless ice. During the hit you exert a constant horizontal force on the puck that gives it an acceleration of 75 $m/s^2$ for a fraction of a second. (a) During the hit, what is the magnitude of the horizontal force that you exert on the puck? (b) How does the magnitude of the normal force due to the ice compare to the weight of the puck?

**VP4.4.4** A plate of cafeteria food is on a horizontal table. You push it away from you with a constant horizontal force of 14.0 N. The plate has a mass of 0.800 kg, and during the push it has an acceleration of 12.0 $m/s^2$ in the direction you are pushing it. (a) What is the magnitude of the net external force on the plate during the push? (b) What are the magnitude and direction of the friction force that the table exerts on the plate during the push?

**Be sure to review EXAMPLE 4.5 (Section 4.3) before attempting these problems.**

**VP4.5.1** On a winter day a child of mass 20.0 kg slides on a horizontal sidewalk covered in ice. Initially she is moving at 3.00 m/s, but due to friction she comes to a halt in 2.25 m. What is the magnitude of the constant friction force that acts on her as she slides?

**VP4.5.2** An airliner of mass $1.70 \times 10^5$ kg lands at a speed of 75.0 m/s. As it travels along the runway, the combined effects of air resistance, friction from the tires, and reverse thrust from the engines produce a constant force of $2.90 \times 10^5$ N opposite to the airliner's motion. What distance along the runway does the airliner travel before coming to a halt?

**VP4.5.3** A truck of mass $2.40 \times 10^3$ kg is moving at 25.0 m/s. When the driver applies the brakes, the truck comes a stop after traveling 48.0 m. (a) How much time is required for the truck to stop? (b) What is the magnitude of the truck's constant acceleration as it slows down? (c) What is the magnitude of the constant braking force that acts on the truck as it slows down?

**VP4.5.4** A car of mass $1.15 \times 10^3$ kg is stalled on a horizontal road. You and your friends give the car a constant, forward, horizontal push. There is friction between the car and the road. (a) Name the four external forces that act on the car as you and your friends push it and what exerts each force. (You can regard the combined push from you and your friends as a single force.) (b) The combined force that you and your friends exert has magnitude $8.00 \times 10^2$ N, and starting from rest the car reaches a speed of 1.40 m/s after you have pushed it 5.00 m. Find the magnitude of the constant friction force that acts on the car.

## BRIDGING PROBLEM Links in a Chain

A student suspends a chain consisting of three links, each of mass $m = 0.250$ kg, from a light rope. The rope is attached to the top link of the chain, which does not swing. She pulls upward on the rope, so that the rope applies an upward force of 9.00 N to the chain. (a) Draw a free-body diagram for the entire chain, considered as an object, and one for each of the three links. (b) Use the diagrams of part (a) and Newton's laws to find (i) the acceleration of the chain, (ii) the force exerted by the top link on the middle link, and (iii) the force exerted by the middle link on the bottom link. Treat the rope as massless.

### SOLUTION GUIDE

### IDENTIFY and SET UP

1. There are four objects of interest in this problem: the chain as a whole and the three individual links. For each of these four objects, make a list of the external forces that act on it. Besides the force of gravity, your list should include only forces exerted by other objects that *touch* the object in question.
2. Some of the forces in your lists form action–reaction pairs (one pair is the force of the top link on the middle link and the force of the middle link on the top link). Identify all such pairs.
3. Use your lists to help you draw a free-body diagram for each of the four objects. Choose the coordinate axes.
4. Use your lists to decide how many unknowns there are in this problem. Which of these are target variables?

### EXECUTE

5. Write a Newton's second law equation for each of the four objects, and write a Newton's third law equation for each action–reaction pair. You should have at least as many equations as there are unknowns (see step 4). Do you?
6. Solve the equations for the target variables.

### EVALUATE

7. You can check your results by substituting them back into the equations from step 5. This is especially important to do if you ended up with more equations in step 5 than you used in step 6.
8. Rank the force of the rope on the chain, the force of the top link on the middle link, and the force of the middle link on the bottom link in order from smallest to largest magnitude. Does this ranking make sense? Explain.
9. Repeat the problem for the case in which the upward force that the rope exerts on the chain is only 7.35 N. Is the ranking in step 8 the same? Does this make sense?

## PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

### DISCUSSION QUESTIONS

**Q4.1** Can an object be in equilibrium when only one force acts on it? Explain.

**Q4.2** A ball thrown straight up has zero velocity at its highest point. Is the ball in equilibrium at this point? Why or why not?

**Q4.3** A helium balloon hovers in midair, neither ascending nor descending. Is it in equilibrium? What forces act on it?

**Q4.4** When you fly in an airplane at night in smooth air, you have no sensation of motion, even though the plane may be moving at 800 km/h (500 mi/h). Why?

**Q4.5** If the two ends of a rope in equilibrium are pulled with forces of equal magnitude and opposite directions, why isn't the total tension in the rope zero?

**Q4.6** You tie a brick to the end of a rope and whirl the brick around you in a horizontal circle. Describe the path of the brick after you suddenly let go of the rope.

**Q4.7** When a car stops suddenly, the passengers tend to move forward relative to their seats. Why? When a car makes a sharp turn, the passengers tend to slide to one side of the car. Why?

**Q4.8** Some people say that the "force of inertia" (or "force of momentum") throws the passengers forward when a car brakes sharply. What is wrong with this explanation?

**Q4.9** A passenger in a moving bus with no windows notices that a ball that has been at rest in the aisle suddenly starts to move toward the rear of the bus. Think of two possible explanations, and devise a way to decide which is correct.

**Q4.10** Suppose you chose the fundamental physical quantities to be force, length, and time instead of mass, length, and time. What would be the units of mass in terms of those fundamental quantities?

**Q4.11** Why is the earth only approximately an inertial reference frame?

**Q4.12** Does Newton's second law hold true for an observer in a van as it speeds up, slows down, or rounds a corner? Explain.

**Q4.13** Some students refer to the quantity $m\vec{a}$ as "the force of acceleration." Is it correct to refer to this quantity as a force? If so, what exerts this force? If not, what is a better description of this quantity?

**Q4.14** The acceleration of a falling object is measured in an elevator that is traveling upward at a constant speed of 9.8 m/s. What value is obtained?

**Q4.15** You can play catch with a softball in a bus moving with constant speed on a straight road, just as though the bus were at rest. Is this still possible when the bus is making a turn at constant speed on a level road? Why or why not?

**Q4.16** Students sometimes say that the force of gravity on an object is 9.8 $m/s^2$. What is wrong with this view?

**Q4.17** Why can it hurt your foot more to kick a big rock than a small pebble? *Must* the big rock hurt more? Explain.

**Q4.18** "It's not the fall that hurts you; it's the sudden stop at the bottom." Translate this saying into the language of Newton's laws of motion.

**Q4.19** A person can dive into water from a height of 10 m without injury, but a person who jumps off the roof of a 10-m-tall building and lands on a concrete street is likely to be seriously injured. Why is there a difference?

**Q4.20** Why are cars designed to crumple in front and back for safety? Why not for side collisions and rollovers?

**Q4.21** When a string barely strong enough lifts a heavy weight, it can lift the weight by a steady pull; but if you jerk the string, it will break. Explain in terms of Newton's laws of motion.

**Q4.22** A large crate is suspended from the end of a vertical rope. Is the tension in the rope greater when the crate is at rest or when it is moving upward at constant speed? If the crate is traveling upward, is the tension in the rope greater when the crate is speeding up or when it is slowing down? In each case, explain in terms of Newton's laws of motion.

**Q4.23** Which feels a greater pull due to the earth's gravity: a 10 kg stone or a 20 kg stone? If you drop the two stones, why doesn't the 20 kg stone fall with twice the acceleration of the 10 kg stone? Explain.

**Q4.24** Why is it incorrect to say that 1.0 kg *equals* 2.2 lb?

**Q4.25** A horse is hitched to a wagon. Since the wagon pulls back on the horse just as hard as the horse pulls on the wagon, why doesn't the wagon remain in equilibrium, no matter how hard the horse pulls?

**Q4.26** True or false? You exert a push $P$ on an object and it pushes back on you with a force $F$. If the object is moving at constant velocity, then $F$ is equal to $P$, but if the object is being accelerated, then $P$ must be greater than $F$.

**Q4.27** A large truck and a small compact car have a head-on collision. During the collision, the truck exerts a force $\vec{F}_{\text{T on C}}$ on the car, and the car exerts a force $\vec{F}_{\text{C on T}}$ on the truck. Which force has the larger magnitude, or are they the same? Does your answer depend on how fast each vehicle was moving before the collision? Why or why not?

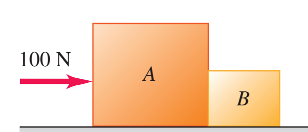
**Q4.28** When a car comes to a stop on a level highway, what force causes it to slow down? When the car increases its speed on the same highway, what force causes it to speed up? Explain.

**Q4.29** A small compact car is pushing a large van that has broken down, and they travel along the road with equal velocities and accelerations. While the car is speeding up, is the force it exerts on the van larger than, smaller than, or the same magnitude as the force the van exerts on it? Which vehicle has the larger net force on it, or are the net forces the same? Explain.

**Q4.30** Consider a tug-of-war between two people who pull in opposite directions on the ends of a rope. By Newton's third law, the force that $A$ exerts on $B$ is just as great as the force that $B$ exerts on $A$. So what determines who wins? (*Hint:* Draw a free-body diagram showing all the forces that act on each person.)

**Q4.31** Boxes $A$ and $B$ are in contact on a horizontal, frictionless surface. You push on box $A$ with a horizontal 100 N force (**Fig. Q4.31**). Box $A$ weighs 150 N, and box $B$ weighs 50 N. Is the force that box $A$ exerts on box $B$ equal to 100 N, greater than 100 N, or less than 100 N? Explain.

Figure **Q4.31**




**Q4.32** A manual for student pilots contains this passage: "When an airplane flies at a steady altitude, neither climbing nor descending, the upward lift force from the wings equals the plane's weight. When the plane is climbing at a steady rate, the upward lift is greater than the weight; when the plane is descending at a steady rate, the upward lift is less than the weight." Are these statements correct? Explain.

**Q4.33** If your hands are wet and no towel is handy, you can remove some of the excess water by shaking them. Why does this work?

**Q4.34** If you squat down (such as when you examine the books on a bottom shelf) and then suddenly get up, you may temporarily feel light-headed. What do Newton's laws of motion have to say about why this happens?

**Q4.35** When a car is hit from behind, the occupants may experience whiplash. Use Newton's laws of motion to explain what causes this result.

**Q4.36** In a head-on auto collision, passengers who are not wearing seat belts may be thrown through the windshield. Use Newton's laws of motion to explain why this happens.

**Q4.37** In a head-on collision between a compact 1000 kg car and a large 2500 kg car, which one experiences the greater force? Explain. Which one experiences the greater acceleration? Explain why. Why are passengers in the small car more likely to be injured than those in the large car, even when the two car bodies are equally strong?

**Q4.38** Suppose you are in a rocket with no windows, traveling in deep space far from other objects. Without looking outside the rocket or making any contact with the outside world, explain how you could determine whether the rocket is (a) moving forward at a constant 80% of the speed of light and (b) accelerating in the forward direction.

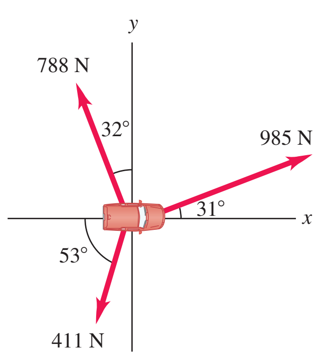
## EXERCISES

### Section 4.1 Force and Interactions

**4.1** •• Two dogs pull horizontally on ropes attached to a post; the angle between the ropes is 60.0°. If Rover exerts a force of 270 N and Fido exerts a force of 300 N, find the magnitude of the resultant force and the angle it makes with Rover's rope.

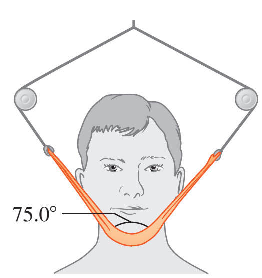
**4.2** • To extricate an SUV stuck in the mud, workmen use three horizontal ropes, producing the force vectors shown in **Fig. E4.2.** (a) Find the $x$- and $y$-components of each of the three pulls. (b) Use the components to find the magnitude and direction of the resultant of the three pulls.

Figure **E4.2**



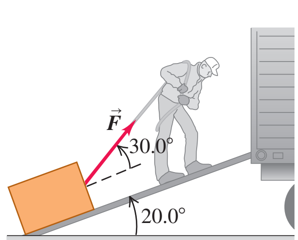

**4.3** • **BIO** **Jaw Injury.** Due to a jaw injury, a patient must wear a strap (**Fig. E4.3**) that produces a net upward force of 5.00 N on his chin. The tension is the same throughout the strap. To what tension must the strap be adjusted to provide the necessary upward force?

Figure **E4.3**




**4.4** • A man is dragging a trunk up the loading ramp of a mover's truck. The ramp has a slope angle of 20.0°, and the man pulls upward with a force $\vec{F}$ whose direction makes an angle of 30.0° with the ramp (**Fig. E4.4**). (a) How large a force $\vec{F}$ is necessary for the component $F_x$ parallel to the ramp to be 90.0 N? (b) How large will the component $F_y$ perpendicular to the ramp be then?

Figure **E4.4**




**4.5** • Forces $\vec{F}_1$ and $\vec{F}_2$ act at a point. The magnitude of $\vec{F}_1$ is 9.00 N, and its direction is 60.0° above the $x$-axis in the second quadrant. The magnitude of $\vec{F}_2$ is 6.00 N, and its direction is 53.1° below the $x$-axis in the third quadrant. (a) What are the $x$- and $y$-components of the resultant force? (b) What is the magnitude of the resultant force?

### Section 4.3 Newton's Second Law

**4.6** • An electron $(\text{mass} = 9.11 \times 10^{-31}\ \text{kg})$ leaves one end of a TV picture tube with zero initial speed and travels in a straight line to the accelerating grid, which is 1.80 cm away. It reaches the grid with a speed of $3.00 \times 10^6$ m/s. If the accelerating force is constant, compute (a) the acceleration; (b) the time to reach the grid; and (c) the net force, in newtons. Ignore the gravitational force on the electron.

**4.7** •• A 68.5 kg skater moving initially at 2.40 m/s on rough horizontal ice comes to rest uniformly in 3.52 s due to friction from the ice. What force does friction exert on the skater?

**4.8** •• You walk into an elevator, step onto a scale, and push the "up" button. You recall that your normal weight is 625 N. Draw a free-body diagram. (a) When the elevator has an upward acceleration of magnitude 2.50 m/s$^2$, what does the scale read? (b) If you hold a 3.85 kg package by a light vertical string, what will be the tension in this string when the elevator accelerates as in part (a)?

**4.9** • A box rests on a frozen pond, which serves as a frictionless horizontal surface. If a fisherman applies a horizontal force with magnitude 48.0 N to the box and produces an acceleration of magnitude 2.20 m/s$^2$, what is the mass of the box?

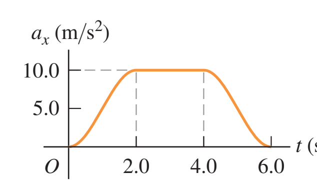
**4.10** •• A dockworker applies a constant horizontal force of 80.0 N to a block of ice on a smooth horizontal floor. The frictional force is negligible. The block starts from rest and moves 11.0 m in 5.00 s. (a) What is the mass of the block of ice? (b) If the worker stops pushing at the end of 5.00 s, how far does the block move in the next 5.00 s?

**4.11** • A hockey puck with mass 0.160 kg is at rest at the origin $(x = 0)$ on the horizontal, frictionless surface of the rink. At time $t = 0$ a player applies a force of 0.250 N to the puck, parallel to the $x$-axis; she continues to apply this force until $t = 2.00$ s. (a) What are the position and speed of the puck at $t = 2.00$ s? (b) If the same force is again applied at $t = 5.00$ s, what are the position and speed of the puck at $t = 7.00$ s?

**4.12** • A crate with mass 32.5 kg initially at rest on a warehouse floor is acted on by a net horizontal force of 14.0 N. (a) What acceleration is produced? (b) How far does the crate travel in 10.0 s? (c) What is its speed at the end of 10.0 s?

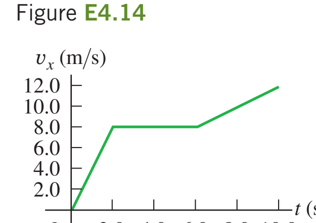
**4.13** • A 4.50 kg experimental cart undergoes an acceleration in a straight line (the $x$-axis). The graph in **Fig. E4.13** shows this acceleration as a function of time. (a) Find the maximum net force on this cart. When does this maximum force occur? (b) During what times is the net force on the cart a constant? (c) When is the net force equal to zero?

Figure **E4.13**




**4.14** • A 2.75 kg cat moves in a straight line (the $x$-axis). **Figure E4.14** shows a graph of the $x$-component of this cat's velocity as a function of time. (a) Find the maximum net force on this cat. When does this force occur? (b) When is the net force on the cat equal to zero? (c) What is the net force at time 8.5 s?

Figure **E4.14**



**4.15** • A small 8.00 kg rocket burns fuel that exerts a time-varying upward force on the rocket (assume constant mass) as the rocket moves upward from the launch pad. This force obeys the equation $F = A + Bt^2$. Measurements show that at $t = 0$, the force is 100.0 N, and at the end of the first 2.00 s, it is 150.0 N. (a) Find the constants $A$ and $B$, including their SI units. (b) Find the *net* force on this rocket and its acceleration (i) the instant after the fuel ignites and (ii) 3.00 s after the fuel ignites. (c) Suppose that you were using this rocket in outer space, far from all gravity. What would its acceleration be 3.00 s after fuel ignition?

## Section 4.4 Mass and Weight

**4.16** • An astronaut's pack weighs 17.5 N when she is on the earth but only 3.24 N when she is at the surface of a moon. (a) What is the acceleration due to gravity on this moon? (b) What is the mass of the pack on this moon?

**4.17** • Superman throws a 2400 N boulder at an adversary. What horizontal force must Superman apply to the boulder to give it a horizontal acceleration of 12.0 $m/s^2$?

**4.18** • **BIO** (a) An ordinary flea has a mass of 210 $\mu$g. How many newtons does it weigh? (b) The mass of a typical froghopper is 12.3 mg. How many newtons does it weigh? (c) A house cat typically weighs 45 N. How many pounds does it weigh, and what is its mass in kilograms?

**4.19** • At the surface of Jupiter's moon Io, the acceleration due to gravity is $g = 1.81$ $m/s^2$. A watermelon weighs 44.0 N at the surface of the earth. (a) What is the watermelon's mass on the earth's surface? (b) What would be its mass and weight on the surface of Io?

## Section 4.5 Newton's Third Law

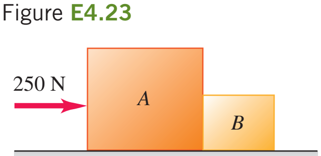
**4.20** • Estimate the mass in kilograms and the weight in pounds of a typical sumo wrestler. How do your estimates for the wrestler compare to your estimates of the average mass and weight of the students in your physics class? Do a web search if necessary to help make the estimates. In your solution list what values you assume for the quantities you use in making your estimates.

**4.21** • **BIO** World-class sprinters can accelerate out of the starting blocks with an acceleration that is nearly horizontal and has magnitude 15 $m/s^2$. How much horizontal force must a 55 kg sprinter exert on the starting blocks to produce this acceleration? Which object exerts the force that propels the sprinter: the blocks or the sprinter herself?

**4.22** • A small car of mass 380 kg is pushing a large truck of mass 900 kg due east on a level road. The car exerts a horizontal force of 1600 N on the truck. What is the magnitude of the force that the truck exerts on the car?

**4.23** •• Boxes $A$ and $B$ are in contact on a horizontal, frictionless surface (**Fig. E4.23**). Box $A$ has mass 20.0 kg and box $B$ has mass 5.0 kg. A horizontal force of 250 N is exerted on box $A$. What is the magnitude of the force that box $A$ exerts on box $B$?

Figure **E4.23**



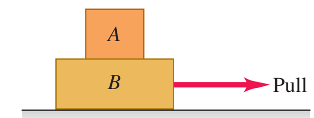

**4.24** •• The upward normal force exerted by the floor is 620 N on an elevator passenger who weighs 650 N. What are the reaction forces to these two forces? Is the passenger accelerating? If so, what are the magnitude and direction of the acceleration?

**4.25** •• A student of mass 45 kg jumps off a high diving board. What is the acceleration of the earth toward her as she accelerates toward the earth with an acceleration of 9.8 $m/s^2$? Use $6.0 \times 10^{24}$ kg for the mass of the earth, and assume that the net force on the earth is the force of gravity she exerts on it.

## Section 4.6 Free-Body Diagrams

**4.26** •• You pull horizontally on block $B$ in **Fig. E4.26,** causing both blocks to move together as a unit. For this moving system, make a carefully labeled free-body diagram of block $A$ if (a) the table is frictionless and (b) there is friction between block $B$ and the table and the pull is equal in magnitude to the friction force on block $B$ due to the table.

Figure **E4.26**




**4.27** •• Crates $A$ and $B$ sit at rest side by side on a frictionless horizontal surface. They have masses $m_A$ and $m_B$, respectively. When a horizontal force $\vec{F}$ is applied to crate $A$, the two crates move off to the right. (a) Draw clearly labeled free-body diagrams for crate $A$ and for crate $B$. Indicate which pairs of forces, if any, are third-law action–reaction pairs. (b) If the magnitude of $\vec{F}$ is less than the total weight of the two crates, will it cause the crates to move? Explain.

**4.28** •• **CP** A .22 caliber rifle bullet traveling at 350 m/s strikes a large tree and penetrates it to a depth of 0.130 m. The mass of the bullet is 1.80 g. Assume a constant retarding force. (a) How much time is required for the bullet to stop? (b) What force, in newtons, does the tree exert on the bullet?

**4.29** • A ball is hanging from a long string that is tied to the ceiling of a train car traveling eastward on horizontal tracks. An observer inside the train car sees the ball hang motionless. Draw a clearly labeled free-body diagram for the ball if (a) the train has a uniform velocity and (b) the train is speeding up uniformly. Is the net force on the ball zero in either case? Explain.

**4.30** •• A chair of mass 12.0 kg is sitting on the horizontal floor; the floor is not frictionless. You push on the chair with a force $F = 40.0$ N that is directed at an angle of 37.0° below the horizontal, and the chair slides along the floor. (a) Draw a clearly labeled free-body diagram for the chair. (b) Use your diagram and Newton's laws to calculate the normal force that the floor exerts on the chair.

## PROBLEMS

**4.31** •• **CP** Estimate the average force that a major-league pitcher exerts on the baseball when he throws a fastball. Express your answer in pounds. In your solution, list the quantities for which you estimate values and any assumptions you make. Do a web search to help determine the values you use in making your estimates.

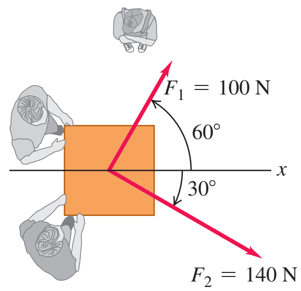
**4.32** •• **CP** You have just landed on Planet X. You release a 100 g ball from rest from a height of 10.0 m and measure that it takes 3.40 s to reach the ground. Ignore any force on the ball from the atmosphere of the planet. How much does the 100 g ball weigh on the surface of Planet X?

**4.33** •• **CP** A 5.60 kg bucket of water is accelerated upward by a cord of negligible mass whose breaking strength is 75.0 N. If the bucket starts from rest, what is the minimum time required to raise the bucket a vertical distance of 12.0 m without breaking the cord?

**4.34** •• Block $A$ rests on top of block $B$ as shown in Fig. E4.26. The table is frictionless but there is friction (a horizontal force) between blocks $A$ and $B$. Block $B$ has mass 6.00 kg and block $A$ has mass 2.00 kg. If the horizontal pull applied to block $B$ equals 12.0 N, then block $B$ has an acceleration of 1.80 $m/s^2$. What is the acceleration of block $A$?

**4.35** •• Two adults and a child want to push a wheeled cart in the direction marked $x$ in **Fig. P4.35** (next page). The two adults push with horizontal forces $\vec{F}_1$ and $\vec{F}_2$ as shown. (a) Find the magnitude and direction of the *smallest* force that the child should exert. Ignore the effects of friction. (b) If the child exerts the minimum force found in part (a), the cart accelerates at 2.0 $m/s^2$ in the $+x$-direction. What is the weight of the cart?

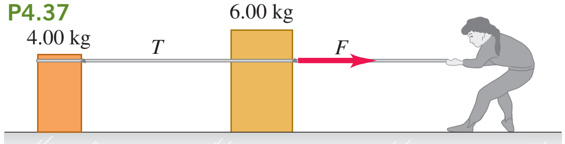
Figure **P4.35**




**4.36** ••• **CP** An advertisement claims that a particular automobile can "stop on a dime." What net force would be necessary to stop a 850 kg automobile traveling initially at 45.0 km/h in a distance equal to the diameter of a dime, 1.8 cm?

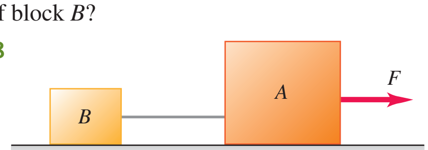
**4.37** • Two crates, one with mass 4.00 kg and the other with mass 6.00 kg, sit on the frictionless surface of a frozen pond, connected by a light rope (**Fig. P4.37**). A woman wearing golf shoes (for traction) pulls horizontally on the 6.00 kg crate with a force $F$ that gives the crate an acceleration of 2.50 $m/s^2$. (a) What is the acceleration of the 4.00 kg crate? (b) Draw a free-body diagram for the 4.00 kg crate. Use that diagram and Newton's second law to find the tension $T$ in the rope that connects the two crates. (c) Draw a free-body diagram for the 6.00 kg crate. What is the direction of the net force on the 6.00 kg crate? Which is larger in magnitude, $T$ or $F$? (d) Use part (c) and Newton's second law to calculate the magnitude of $F$.

Figure **P4.37**




**4.38** •• **CP** Two blocks connected by a light horizontal rope sit at rest on a horizontal, frictionless surface. Block $A$ has mass 15.0 kg, and block $B$ has mass $m$. A constant horizontal force $F = 60.0$ N is applied to block $A$ (**Fig. P4.38**). In the first 5.00 s after the force is applied, block $A$ moves 18.0 m to the right. (a) While the blocks are moving, what is the tension $T$ in the rope that connects the two blocks? (b) What is the mass of block $B$?

Figure **P4.38**




**4.39** • **CALC** To study damage to aircraft that collide with large birds, you design a test gun that will accelerate chicken-sized objects so that their displacement along the gun barrel is given by $x = (9.0 \times 10^3\ \mathrm{m/s^2})t^2 - (8.0 \times 10^4\ \mathrm{m/s^3})t^3$. The object leaves the end of the barrel at $t = 0.025$ s. (a) How long must the gun barrel be? (b) What will be the speed of the objects as they leave the end of the barrel? (c) What net force must be exerted on a 1.50 kg object at (i) $t = 0$ and (ii) $t = 0.025$ s?

**4.40** •• **CP** On a test flight a rocket with mass 400 kg blasts off from the surface of the earth. The rocket engines apply a constant upward force $F$ until the rocket reaches a height of 100 m and then they shut off. If the rocket is to reach a maximum height of 400 m above the surface of the earth, what value of $F$ is required? Assume the change in the rocket's mass is negligible.

**4.41** •• **CP** After an annual checkup, you leave your physician's office, where you weighed 683 N. You then get into an elevator that, conveniently, has a scale. Find the magnitude and direction of the elevator's acceleration if the scale reads (a) 725 N and (b) 595 N.

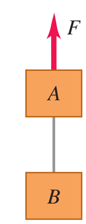
**4.42** • A loaded elevator with very worn cables has a total mass of 2200 kg, and the cables can withstand a maximum tension of 28,000 N. (a) Draw the free-body force diagram for the elevator. In terms of the forces on your diagram, what is the net force on the elevator? Apply Newton's second law to the elevator and find the maximum upward acceleration for the elevator if the cables are not to break. (b) What would be the answer to part (a) if the elevator were on the moon, where $g = 1.62\ \mathrm{m/s^2}$?

**4.43** •• **CP** A batter swings at a baseball (mass 0.145 kg) that is moving horizontally toward him at a speed of 40.0 m/s. He hits a line drive with the ball moving away from him horizontally at 50.0 m/s just after it leaves the bat. If the bat and ball are in contact for 8.00 ms, what is the average force that the bat applies to the ball?

**4.44** •• **CP CALC** An object with mass $m$ is moving along the $x$-axis according to the equation $x(t) = \alpha t^2 - 2\beta t$, where $\alpha$ and $\beta$ are positive constants. What is the magnitude of the net force on the object at time $t = 0$?

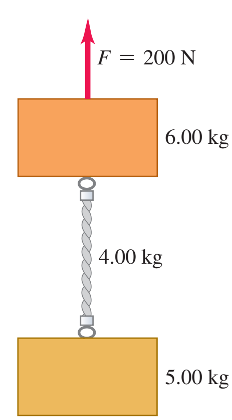
**4.45** •• **CP** Boxes $A$ and $B$ are connected to each end of a light vertical rope (**Fig. P4.45**). A constant upward force $F = 80.0$ N is applied to box $A$. Starting from rest, box $B$ descends 12.0 m in 4.00 s. The tension in the rope connecting the two boxes is 36.0 N. What are the masses of (a) box $B$, (b) box $A$?

Figure **P4.45**




**4.46** •• The two blocks in **Fig. P4.46** are connected by a heavy uniform rope with a mass of 4.00 kg. An upward force of 200 N is applied as shown. (a) Draw three free-body diagrams: one for the 6.00 kg block, one for the 4.00 kg rope, and another one for the 5.00 kg block. For each force, indicate what object exerts that force. (b) What is the acceleration of the system? (c) What is the tension at the top of the heavy rope? (d) What is the tension at the midpoint of the rope?

Figure **P4.46**




**4.47** ••• **CP** A small rocket with mass 20.0 kg is moving in free fall toward the earth. Air resistance can be neglected. When the rocket is 80.0 m above the surface of the earth, it is moving downward with a speed of 30.0 m/s. At that instant the rocket engines start to fire and produce a constant upward force $F$ on the rocket. Assume the change in the rocket's mass is negligible. What is the value of $F$ if the rocket's speed becomes zero just as it reaches the surface of the earth, for a soft landing? (*Hint:* The net force on the rocket is the combination of the upward force $F$ from the engines and the downward weight of the rocket.)

**4.48** •• **CP** **Extraterrestrial Physics.** You have landed on an unknown planet, Newtonia, and want to know what objects weigh there. When you push a certain tool, starting from rest, on a frictionless horizontal surface with a 12.0 N force, the tool moves 16.0 m in the first 2.00 s. You next observe that if you release this tool from rest at 10.0 m above the ground, it takes 2.58 s to reach the ground. What does the tool weigh on Newtonia, and what does it weigh on earth?

**4.49 •• CP CALC** A mysterious rocket-propelled object of mass 45.0 kg is initially at rest in the middle of the horizontal, frictionless surface of an ice-covered lake. Then a force directed east and with magnitude $F(t) = (16.8\ \text{N/s})t$ is applied. How far does the object travel in the first 5.00 s after the force is applied?

**4.50 •• CP** Starting at time $t = 0$, net force $F_1$ is applied to an object that is initially at rest. (a) If the force remains constant with magnitude $F_1$ while the object moves a distance $d$, the final speed of the object is $v_1$. What is the final speed $v_2$ (in terms of $v_1$) if the net force is $F_2 = 2F_1$ and the object moves the same distance $d$ while the force is being applied? (b) If the force $F_1$ remains constant while it is applied for a time $T$, the final speed of the object is $v_1$. What is the final speed $v_2$ (in terms of $v_1$) if the applied force is $F_2 = 2F_1$ and is constant while it is applied for the same time $T$? In a later chapter we'll call force times distance work and force times *time* impulse and associate work and impulse with the change in speed.)

**4.51 •• DATA** The table* gives automobile performance data for a few types of cars:

| Make and Model (Year) | Mass (kg) | Time (s) to go from 0 to 60 mph |
|---|---|---|
| Alpha Romeo 4C (2013) | 895 | 4.4 |
| Honda Civic 2.0i (2011) | 1320 | 6.4 |
| Ferrari F430 (2004) | 1435 | 3.9 |
| Ford Focus RS500 (2010) | 1468 | 5.4 |
| Volvo S60 (2013) | 1650 | 7.2 |

*Source: www.autosnout.com

(a) During an acceleration of 0 to 60 mph, which car has the largest average net force acting on it? The smallest? (b) During this acceleration, for which car would the average net force on a 72.0 kg passenger be the largest? The smallest? (c) When the Ferrari F430 accelerates from 0 to 100 mph in 8.6 s, what is the average net force acting on it? How does this net force compare with the average net force during the acceleration from 0 to 60 mph? Explain why these average net forces might differ. (d) Discuss why a car has a top speed. What is the net force on the Ferrari F430 when it is traveling at its top speed, 196 mph?

**4.52 ••• CALC** The position of a training helicopter (weight $2.75 \times 10^5$ N) in a test is given by $\hat{\boldsymbol{r}} = (0.020\ \text{m/s}^3)t^3\hat{\imath} + (2.2\ \text{m/s})t\hat{\jmath} - (0.060\ \text{m/s}^2)t^2\hat{\boldsymbol{k}}$. Find the net force on the helicopter at $t = 5.0$ s.

**4.53 •• DATA** You are a Starfleet captain going boldly where no one has gone before. You land on a distant planet and visit an engineering testing lab. In one experiment a short, light rope is attached to the top of a block and a constant upward force $F$ is applied to the free end of the rope. The block has mass $m$ and is initially at rest. As $F$ is varied, the time for the block to move upward 8.00 m is measured. The values that you collected are given in the table:

| $F$ (N) | Time (s) |
|---|---|
| 250 | 3.3 |
| 300 | 2.2 |
| 350 | 1.7 |
| 400 | 1.5 |
| 450 | 1.3 |
| 500 | 1.2 |

(a) Plot $F$ versus the acceleration $a$ of the block. (b) Use your graph to determine the mass $m$ of the block and the acceleration of gravity $g$ at the surface of the planet. Note that even on that planet, measured values contain some experimental error.

**4.54 •• DATA** An 8.00 kg box sits on a level floor. You give the box a sharp push and find that it travels 8.22 m in 2.8 s before coming to rest again. (a) You measure that with a different push the box traveled 4.20 m in 2.0 s. Do you think the box has a constant acceleration as it slows down? Explain your reasoning. (b) You add books to the box to increase its mass. Repeating the experiment, you give the box a push and measure how long it takes the box to come to rest and how far the box travels. The results, including the initial experiment with no added mass, are given in the table:

| Added Mass (kg) | Distance (m) | Time (s) |
|---|---|---|
| 0 | 8.22 | 2.8 |
| 3.00 | 10.75 | 3.2 |
| 7.00 | 9.45 | 3.0 |
| 12.0 | 7.10 | 2.6 |

In each case, did your push give the box the same initial speed? What is the ratio between the greatest initial speed and the smallest initial speed for these four cases? (c) Is the average horizontal force $f$ exerted on the box by the floor the same in each case? Graph the magnitude of force $f$ versus the total mass $m$ of the box plus its contents, and use your graph to determine an equation for $f$ as a function of $m$.

## CHALLENGE PROBLEMS

**4.55 ••• CP CALC** A block of mass 2.00 kg is initially at rest at $x = 0$ on a slippery horizontal surface for which there is no friction. Starting at time $t = 0$, a horizontal force $F_x(t) = \beta - \alpha t$ is applied to the block, where $\alpha = 6.00$ N/s anwd $\beta = 4.00$ N. (a) What is the largest positive value of $x$ reached by the block? How long does it take the block to reach this point, starting from $t = 0$, and what is the magnitude of the force when the block is at this value of $x$? (b) How long from $t = 0$ does it take the block to return to $x = 0$, and what is its speed at this point?

**4.56 ••• CALC** An object of mass $m$ is at rest in equilibrium at the origin. At $t = 0$ a new force $\vec{\boldsymbol{F}}(t)$ is applied that has components

$$F_x(t) = k_1 + k_2 y \qquad F_y(t) = k_3 t$$

where $k_1$, $k_2$, and $k_3$ are constants. Calculate the position $\vec{\boldsymbol{r}}(t)$ and velocity $\vec{\boldsymbol{v}}(t)$ vectors as functions of time.

## MCAT-STYLE PASSAGE PROBLEMS

**BIO Forces on a Dancer's Body.** Dancers experience large forces associated with the jumps they make. For example, when a dancer lands after a vertical jump, the force exerted on the head by the neck must exceed the head's weight by enough to cause the head to slow down and come to rest. The head is about 9.4% of a typical person's mass. Video analysis of a 65 kg dancer landing after a vertical jump shows that her head decelerates from 4.0 m/s to rest in a time of 0.20 s.

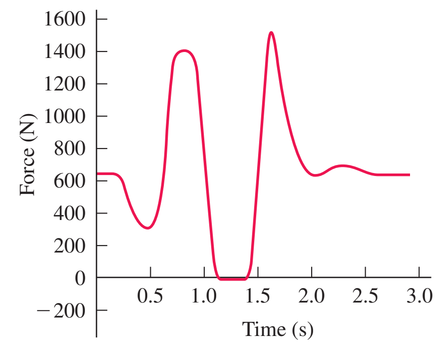
**4.57** What is the magnitude of the average force that her neck exerts on her head during the landing? (a) 0 N; (b) 60 N; (c) 120 N; (d) 180 N.

**4.58** Compared with the force her neck exerts on her head during the landing, the force her head exerts on her neck is (a) the same; (b) greater; (c) smaller; (d) greater during the first half of the landing and smaller during the second half of the landing.

**4.59** While the dancer is in the air and holding a fixed pose, what is the magnitude of the force her neck exerts on her head? (a) 0 N; (b) 60 N; (c) 120 N; (d) 180 N.

**4.60** The forces on a dancer can be measured directly when a dancer performs a jump on a force plate that measures the force between her feet and the ground. A graph of force versus time throughout a vertical jump performed on a force plate is shown in **Fig. P4.60.** What is happening at 0.4 s? The dancer is (a) bending her legs so that her body is accelerating downward; (b) pushing her body up with her legs and is almost ready to leave the ground; (c) in the air and at the top of her jump; (d) landing and her feet have just touched the ground.

Figure **P4.60**




## ANSWERS

### Chapter Opening Question ?

**(v)** Newton's third law tells us that the barbell pushes on the weightlifter just as hard as the weightlifter pushes on the barbell in *all* circumstances, no matter how the barbell is moving. However, the magnitude of the force that the weightlifter exerts is different in different circumstances. This force magnitude is equal to the weight of the barbell when the barbell is stationary, moving upward at a constant speed, or moving downward at a constant speed; it is greater than the weight of the barbell when the barbell accelerates upward; and it is less than the weight of the barbell when the barbell accelerates downward. But in each case the push of the barbell on the weightlifter has exactly the same magnitude as the push of the weightlifter on the barbell.

### Key Example VARIATION Problems

**VP4.1.1** $\sum F_x = 88.0\text{ N}$, $\sum F_y = -44.0\text{ N}$, $F = 98.4\text{ N}$, angle $= 26.6°$ clockwise from the $+x$-direction

**VP4.1.2** $\sum F_x = -143\text{ N}$, $\sum F_y = 246\text{ N}$, $F = 285\text{ N}$, angle $= 120°$ counterclockwise from the $+x$-direction

**VP4.1.3** $\sum F_x = 6.4\text{ N}$, $\sum F_y = 1.5\text{ N}$, $F = 6.6\text{ N}$, angle $= 13°$ counterclockwise from the $+x$-direction

**VP4.1.4** $F_{\text{Tsuroku},x} = -55.8\text{ N}$, $F_{\text{Tsuroku},y} = -62.0\text{ N}$, magnitude $F_{\text{Tsuroku}} = 83.4\text{ N}$, angle $= 48.0°$ south of west

**VP4.4.1** $0.45\text{ m/s}^2$

**VP4.4.2** (a) normal force, exerted by the floor; weight or gravitational force, exerted by the earth; horizontal force, exerted by your hand (b) $0.25\text{ m/s}^2$

**VP4.4.3** (a) 12 N (b) equal

**VP4.4.4** (a) 9.60 N (b) 4.4 N, in the direction opposite to your push

**VP4.5.1** 40.0 N

**VP4.5.2** $1.65 \times 10^3\text{ m}$

**VP4.5.3** (a) 3.84 s (b) $6.51\text{ m/s}^2$ (c) $1.56 \times 10^4\text{ N}$

**VP4.5.4** (a) gravity (the earth), normal force (the road), friction (the road), push (you and your friends) (b) 575 N

### Bridging Problem

(a) *See the Video Tutor Solution in Mastering Physics.*
(b) (i) $2.20\text{ m/s}^2$; (ii) 6.00 N; (iii) 3.00 N

**?** Each of the seeds being blown off the head of a dandelion (genus *Taraxacum*) has a feathery structure called a pappus. The pappus acts like a parachute and enables the seed to be borne by the wind and drift gently to the ground. If a seed with its pappus descends straight down at a steady speed, which force acting on the seed has a greater magnitude? (i) The force of gravity; (ii) the upward force exerted by the air; (iii) both forces have the same magnitude; (iv) it depends on the speed at which the seed descends.

# 5 Applying Newton's Laws

We saw in Chapter 4 that Newton's three laws of motion, the foundation of classical mechanics, can be stated very simply. But *applying* these laws to situations such as an iceboat skating across a frozen lake, a toboggan sliding down a hill, or an airplane making a steep turn requires analytical skills and problem-solving technique. In this chapter we'll help you extend the problem-solving skills you began to develop in Chapter 4.

We'll begin with equilibrium problems, in which we analyze the forces that act on an object that is at rest or moving with constant velocity. We'll then consider objects that are not in equilibrium. For these we'll have to take account of the relationship between forces and acceleration. We'll learn how to describe and analyze the contact force that acts on an object when it rests on or slides over a surface. We'll also analyze the forces that act on an object that moves in a circle with constant speed. We close the chapter with a brief look at the fundamental nature of force and the classes of forces found in our physical universe.

## LEARNING OUTCOMES

***In this chapter, you'll learn...***

- **5.1** How to use Newton's first law to solve problems involving the forces that act on an object in equilibrium.
- **5.2** How to use Newton's second law to solve problems involving the forces that act on an accelerating object.
- **5.3** The nature of the different types of friction forces—static friction, kinetic friction, rolling friction, and fluid resistance—and how to solve problems that involve these forces.
- **5.4** How to solve problems involving the forces that act on an object moving along a circular path.
- **5.5** The key properties of the four fundamental forces of nature.

***You'll need to review...***

- **1.8** Determining the components of a vector from its magnitude and direction.
- **2.4** Straight-line motion with constant acceleration.
- **3.3** Projectile motion.
- **3.4** Uniform and nonuniform circular motion.
- **4.1** Superposition of forces.
- **4.2** Newton's first law.
- **4.3** Newton's second law.
- **4.4** Mass and weight.
- **4.5** Newton's third law.

## 5.1 USING NEWTON'S FIRST LAW: PARTICLES IN EQUILIBRIUM

We learned in Chapter 4 that an object is in *equilibrium* when it is at rest or moving with constant velocity in an inertial frame of reference. A hanging lamp, a kitchen table, an airplane flying straight and level at a constant speed—all are examples of objects in equilibrium. In this section we consider only the equilibrium of an object that can be modeled as a particle. (In Chapter 11 we'll see how to analyze an object in equilibrium that can't be represented adequately as a particle, such as a bridge that's supported at various points along its span.) The essential physical principle is Newton's first law:

**Newton's first law:** *Net* force on an object ... $\sum \vec{F} = \mathbf{0}$ ... must be *zero* for an **object in equilibrium.**

Sum of $x$-components of force on object must be zero. Sum of $y$-components of force on object must be zero.

$$\sum F_x = 0 \qquad \sum F_y = 0 \qquad (5.1)$$

This section is about using Newton's first law to solve problems dealing with objects in equilibrium. Some of these problems may seem complicated, but remember that *all* problems involving particles in equilibrium are done in the same way. Problem-Solving Strategy 5.1 details the steps you need to follow for any and all such problems. Study this strategy carefully, look at how it's applied in the worked-out examples, and try to apply it when you solve assigned problems.

## PROBLEM-SOLVING STRATEGY 5.1 Newton's First Law: Equilibrium of a Particle

**IDENTIFY** *the relevant concepts:* You must use Newton's *first* law, Eqs. (5.1), for any problem that involves forces acting on an object in equilibrium—that is, either at rest or moving with constant velocity. A car is in equilibrium when it's parked, but also when it's traveling down a straight road at a steady speed.

If the problem involves more than one object and the objects interact with each other, you'll also need to use Newton's *third* law. This law allows you to relate the force that one object exerts on a second object to the force that the second object exerts on the first one.

Identify the target variable(s). Common target variables in equilibrium problems include the magnitude and direction (angle) of one of the forces, or the components of a force.

**SET UP** *the problem* by using the following steps:

1. Draw a very simple sketch of the physical situation, showing dimensions and angles. You don't have to be an artist!
2. Draw a free-body diagram for each object that is in equilibrium. For now, we consider the object as a particle, so you can represent it as a large dot. In your free-body diagram, *do not* include the other objects that interact with it, such as a surface it may be resting on or a rope pulling on it.
3. Ask yourself what is interacting with the object by contact or in any other way. On your free-body diagram, draw a force vector for each interaction. Label each force with a symbol for the *magnitude* of the force. If you know the angle at which a force is directed, draw the angle accurately and label it. Include the object's weight, unless the object has negligible mass. If the mass is given, use $w = mg$ to find the weight. A surface in contact with the object exerts a normal force perpendicular to the surface and possibly a friction force parallel to the surface. A rope or chain exerts a pull (never a push) in a direction along its length.
4. *Do not* show in the free-body diagram any forces exerted *by* the object on any other object. The sums in Eqs. (5.1) include only forces that act *on* the object. For each force on the object, ask yourself "What other object causes that force?" If you can't answer that question, you may be imagining a force that isn't there.
5. Choose a set of coordinate axes and include them in your free-body diagram. (If there is more than one object in the problem, choose axes for each object separately.) Label the positive direction for each axis. If an object rests or slides on a plane surface, for simplicity choose axes that are parallel and perpendicular to this surface, even when the plane is tilted.

**EXECUTE** *the solution* as follows:

1. Find the components of each force along each of the object's coordinate axes. Draw a wiggly line through each force vector that has been replaced by its components, so you don't count it twice. The *magnitude* of a force is always positive, but its *components* may be positive or negative.
2. Set the sum of all $x$-components of force equal to zero. In a separate equation, set the sum of all $y$-components equal to zero. (*Never* add $x$- and $y$-components in a single equation.)
3. If there are two or more objects, repeat all of the above steps for each object. If the objects interact with each other, use Newton's third law to relate the forces they exert on each other.
4. Make sure that you have as many independent equations as the number of unknown quantities. Then solve these equations to obtain the target variables.

**EVALUATE** *your answer:* Look at your results and ask whether they make sense. When the result is a symbolic expression or formula, check to see that your formula works for any special cases (particular values or extreme cases for the various quantities) for which you can guess what the results ought to be.

## EXAMPLE 5.1 One-dimensional equilibrium: Tension in a massless rope

A gymnast with mass $m_G = 50.0$ kg suspends herself from the lower end of a hanging rope of negligible mass. The upper end of the rope is attached to the gymnasium ceiling. (a) What is the gymnast's weight? (b) What force (magnitude and direction) does the rope exert on her? (c) What is the tension at the top of the rope?

**IDENTIFY and SET UP** The gymnast and the rope are in equilibrium, so we can apply Newton's first law to both objects. We'll use Newton's third law to relate the forces that they exert on each other. The target variables are the gymnast's weight, $w_G$; the force that the bottom of the rope exerts on the gymnast (call it $T_{R\text{ on }G}$); and the force that the ceiling exerts on the top of the rope (call it $T_{C\text{ on }R}$). **Figure 5.1** shows our sketch of the situation and free-body diagrams for the gymnast and for the rope. We take the positive $y$-axis to be upward in each

Figure 5.1 Our sketches for this problem.

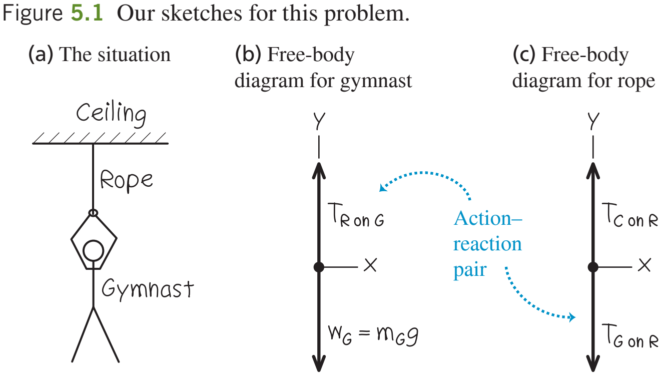


diagram. Each force acts in the vertical direction and so has only a $y$-component.

The forces $T_{\text{R on G}}$ (the upward force of the rope on the gymnast, Fig. 5.1b) and $T_{\text{G on R}}$ (the downward force of the gymnast on the rope, Fig. 5.1c) form an action–reaction pair. By Newton's third law, they must have the same magnitude.

Note that Fig. 5.1c includes only the forces that act *on* the rope. In particular, it doesn't include the force that the *rope* exerts on the *ceiling* (compare the discussion of the apple in Conceptual Example 4.9 in Section 4.5).

**EXECUTE** (a) The magnitude of the gymnast's weight is the product of her mass and the acceleration due to gravity, $g$:

$$w_{\text{G}} = m_{\text{G}}g = (50.0\ \text{kg})(9.80\ \text{m/s}^2) = 490\ \text{N}$$

(b) The gravitational force on the gymnast (her weight) points in the negative $y$-direction, so its $y$-component is $-w_{\text{G}}$. The upward force of the rope on the gymnast has unknown magnitude $T_{\text{R on G}}$ and positive $y$-component $+T_{\text{R on G}}$. We find this by using Newton's first law from Eqs. (5.1):

$$\text{Gymnast:} \quad \sum F_y = T_{\text{R on G}} + (-w_{\text{G}}) = 0 \quad \text{so}$$

$$T_{\text{R on G}} = w_{\text{G}} = 490\ \text{N}$$

The rope pulls *up* on the gymnast with a force $T_{\text{R on G}}$ of magnitude 490 N. (By Newton's third law, the gymnast pulls *down* on the rope with a force of the same magnitude, $T_{\text{G on R}} = 490$ N.)

(c) We have assumed that the rope is weightless, so the only forces on it are those exerted by the ceiling (upward force of unknown magnitude $T_{\text{C on R}}$) and by the gymnast (downward force of magnitude $T_{\text{G on R}} = 490$ N). From Newton's first law, the *net* vertical force on the rope in equilibrium must be zero:

$$\text{Rope:} \quad \sum F_y = T_{\text{C on R}} + (-T_{\text{G on R}}) = 0 \quad \text{so}$$

$$T_{\text{C on R}} = T_{\text{G on R}} = 490\ \text{N}$$

**EVALUATE** The *tension* at any point in the rope is the magnitude of the force that acts at that point. For this weightless rope, the tension $T_{\text{G on R}}$ at the lower end has the same value as the tension $T_{\text{C on R}}$ at the upper end. For such an ideal weightless rope, the tension has the same value at any point along the rope's length. (See the discussion in Conceptual Example 4.10 in Section 4.5.)

**KEYCONCEPT** The sum of all the external forces on an object in equilibrium is zero. The tension has the same value at either end of a rope or string of negligible mass.

## EXAMPLE 5.2 One-dimensional equilibrium: Tension in a rope with mass

Find the tension at each end of the rope in Example 5.1 if the weight of the rope is 120 N.

**IDENTIFY and SET UP** As in Example 5.1, the target variables are the magnitudes $T_{\text{G on R}}$ and $T_{\text{C on R}}$ of the forces that act at the bottom and top of the rope, respectively. Once again, we'll apply Newton's first law to the gymnast and to the rope, and use Newton's third law to relate the forces that the gymnast and rope exert on each other. Again we draw separate free-body diagrams for the gymnast (**Fig. 5.2a**) and the rope (Fig. 5.2b). There is now a *third* force acting on the rope, however: the weight of the rope, of magnitude $w_{\text{R}} = 120$ N.

**EXECUTE** The gymnast's free-body diagram is the same as in Example 5.1, so her equilibrium condition is also the same. From Newton's third law, $T_{\text{R on G}} = T_{\text{G on R}}$, and we again have

$$\text{Gymnast:} \quad \sum F_y = T_{\text{R on G}} + (-w_{\text{G}}) = 0 \quad \text{so}$$

$$T_{\text{R on G}} = T_{\text{G on R}} = w_{\text{G}} = 490\ \text{N}$$

The equilibrium condition $\sum F_y = 0$ for the rope is now

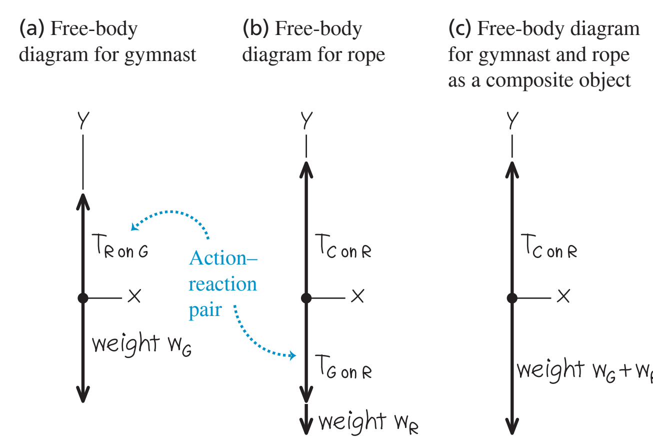
$$\text{Rope:} \quad \sum F_y = T_{\text{C on R}} + (-T_{\text{G on R}}) + (-w_{\text{R}}) = 0$$

Note that the $y$-component of $T_{\text{C on R}}$ is positive because it points in the $+y$-direction, but the $y$-components of both $T_{\text{G on R}}$ and $w_{\text{R}}$ are negative. We solve for $T_{\text{C on R}}$ and substitute the values $T_{\text{G on R}} = T_{\text{R on G}} = 490$ N and $w_{\text{R}} = 120$ N:

$$T_{\text{C on R}} = T_{\text{G on R}} + w_{\text{R}} = 490\ \text{N} + 120\ \text{N} = 610\ \text{N}$$

**EVALUATE** When we include the weight of the rope, the tension is *different* at the rope's two ends: 610 N at the top and 490 N at the bottom. The force $T_{\text{C on R}} = 610$ N exerted by the ceiling has to hold up both the 490 N weight of the gymnast and the 120 N weight of the rope.

Figure 5.2 Our sketches for this problem, including the weight of the rope.




To see this more clearly, we draw a free-body diagram for a composite object consisting of the gymnast and rope together (Fig. 5.2c). Only two external forces act on this composite object: the force $T_{\text{C on R}}$ exerted by the ceiling and the total weight $w_{\text{G}} + w_{\text{R}} = 490\ \text{N} + 120\ \text{N} = 610$ N. (The forces $T_{\text{G on R}}$ and $T_{\text{R on G}}$ are *internal* to the composite object. Newton's first law applies only to *external* forces, so these internal forces play no role.) Hence Newton's first law applied to this composite object is

$$\text{Composite object:} \quad \sum F_y = T_{\text{C on R}} + [-(w_{\text{G}} + w_{\text{R}})] = 0$$

and so $T_{\text{C on R}} = w_{\text{G}} + w_{\text{R}} = 610$ N.

Treating the gymnast and rope as a composite object is simpler, but we can't find the tension $T_{\text{G on R}}$ at the bottom of the rope by this method.

**KEYCONCEPT** If there's more than one object in a problem that involves Newton's laws, the safest approach is to treat each object separately.

## EXAMPLE 5.3 Two-dimensional equilibrium

In **Fig. 5.3a**, a car engine with weight $w$ hangs from a chain that is linked at ring $O$ to two other chains, one fastened to the ceiling and the other to the wall. Find expressions for the tension in each of the three chains in terms of $w$. The weights of the ring and chains are negligible compared with the weight of the engine.

**IDENTIFY and SET UP** The target variables are the tension magnitudes $T_1$, $T_2$, and $T_3$ in the three chains (Fig. 5.3a). All the objects are in equilibrium, so we'll use Newton's first law. We need three independent equations, one for each target variable. However, applying Newton's first law in component form to just one object gives only *two* equations [the $x$- and $y$-equations in Eqs. (5.1)]. So we'll have to consider more than one object in equilibrium. We'll look at the engine (which is acted on by $T_1$) and the ring (which is attached to all three chains and so is acted on by all three tensions).

Figures 5.3b and 5.3c show our free-body diagrams and choice of coordinate axes. Two forces act on the engine: its weight $w$ and the upward force $T_1$ exerted by the vertical chain. Three forces act on the ring: the tensions from the vertical chain ($T_1$), the horizontal chain ($T_2$), and the slanted chain ($T_3$). Because the vertical chain has negligible weight, it exerts forces of the same magnitude $T_1$ at both of its ends (see Example 5.1). (If the weight of this chain were not negligible, these two forces would have different magnitudes; see Example 5.2.) The weight of the ring is also negligible, so it isn't included in Fig. 5.3c.

**EXECUTE** The forces acting on the engine are along the $y$-axis only, so Newton's first law [Eqs. (5.1)] says

$$\text{Engine:}\qquad \sum F_y = T_1 + (-w) = 0 \qquad \text{and} \qquad T_1 = w$$

The horizontal and slanted chains don't exert forces on the engine itself because they are not attached to it. These forces do appear when we apply Newton's first law to the ring, however. In the free-body diagram for the ring (Fig. 5.3c), remember that $T_1$, $T_2$, and $T_3$ are the *magnitudes* of the forces. We resolve the force with magnitude $T_3$ into its $x$- and $y$-components. Applying Newton's first law in component form to the ring gives us the two equations

$$\text{Ring:}\qquad \sum F_x = T_3 \cos 60^\circ + (-T_2) = 0$$

$$\text{Ring:}\qquad \sum F_y = T_3 \sin 60^\circ + (-T_1) = 0$$

Because $T_1 = w$ (from the engine equation), we can rewrite the second ring equation as

$$T_3 = \frac{T_1}{\sin 60^\circ} = \frac{w}{\sin 60^\circ} = 1.2w$$

We can now use this result in the first ring equation:

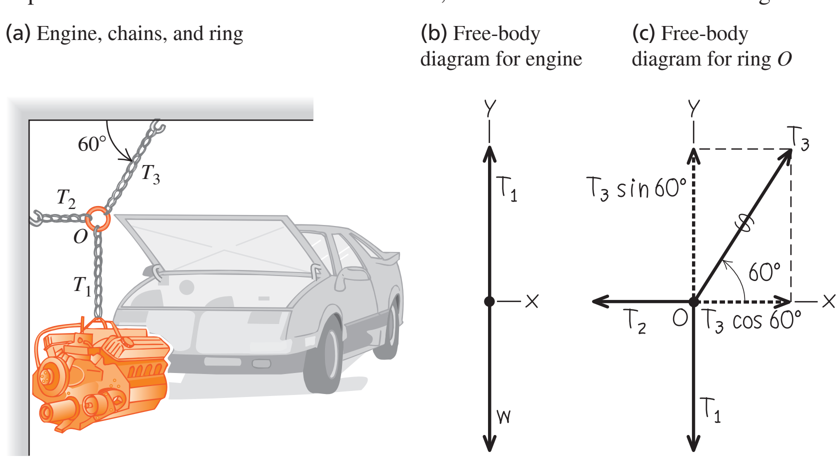
$$T_2 = T_3 \cos 60^\circ = w\frac{\cos 60^\circ}{\sin 60^\circ} = 0.58w$$

**EVALUATE** The chain attached to the ceiling exerts a force on the ring with a *vertical* component equal to $T_1$, which in turn is equal to $w$. But this force also has a horizontal component, so its magnitude $T_3$ is somewhat greater than $w$. This chain is under the greatest tension and is the one most susceptible to breaking.

To get enough equations to solve this problem, we had to consider not only the forces on the engine but also the forces acting on a second object (the ring connecting the chains). Situations like this are fairly common in equilibrium problems, so keep this technique in mind.

**KEYCONCEPT** In two-dimensional problems that involve forces, always write *two* force equations for each object: one for the $x$-components of the forces and one for the $y$-components of the forces.

Figure **5.3** Our sketches for this problem. Note that with our choice of axes, all but one of the forces lie along either the $x$-axis or the $y$-axis.



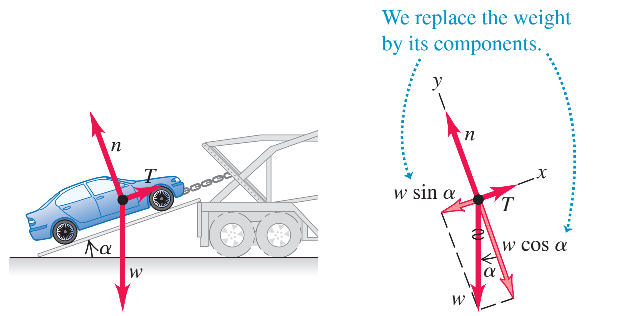

## EXAMPLE 5.4 An inclined plane

A car of weight $w$ rests on a slanted ramp attached to a trailer (**Fig. 5.4a**). Only a cable running from the trailer to the car prevents the car from rolling off the ramp. (The car's brakes are off and its transmission is in neutral.) Find the tension in the cable and the force that the ramp exerts on the car's tires.

**IDENTIFY** The car is in equilibrium, so we use Newton's first law. The ramp exerts a separate force on each of the car's tires, but for simplicity we lump these forces into a single force. For a further simplification, we'll neglect any friction force the ramp exerts on the tires (see Fig. 4.2b). Hence the ramp exerts only a force on the car that is *perpendicular* to the ramp. As in Section 4.1, we call this force the *normal* force (see Fig. 4.2a). The two target variables are the magnitude $T$ of the tension in the cable and the magnitude $n$ of the normal force.

**SET UP** Figure 5.4 shows the three forces acting on the car: its weight (magnitude $w$), the tension in the cable (magnitude $T$), and the normal force (magnitude $n$). Note that the angle $\alpha$ between the ramp and the

Figure **5.4** A cable holds a car at rest on a ramp.




horizontal is equal to the angle $\alpha$ between the weight vector $\vec{w}$ and the downward normal to the plane of the ramp. Note also that we choose the $x$- and $y$-axes to be parallel and perpendicular to the ramp so that we need to resolve only one force (the weight) into $x$- and $y$-components. If we had chosen axes that were horizontal and vertical, we'd have to resolve both the normal force and the tension into components.

**EXECUTE** To write down the $x$- and $y$-components of Newton's first law, we must first find the components of the weight. One complication is that the angle $\alpha$ in Fig. 5.4b is *not* measured from the $+x$-axis toward the $+y$-axis. Hence we *cannot* use Eqs. (1.5) directly to find the components. (You may want to review Section 1.8 to make sure that you understand this important point.)

One way to find the components of $\vec{w}$ is to consider the right triangles in Fig. 5.4b. The sine of $\alpha$ is the magnitude of the $x$-component of $\vec{w}$ (that is, the side of the triangle opposite $\alpha$) divided by the magnitude $w$ (the hypotenuse of the triangle). Similarly, the cosine of $\alpha$ is the magnitude of the $y$-component (the side of the triangle adjacent to $\alpha$) divided by $w$. Both components are negative, so $w_x = -w\sin\alpha$ and $w_y = -w\cos\alpha$.

Another approach is to recognize that one component of $\vec{w}$ must involve $\sin\alpha$ while the other component involves $\cos\alpha$. To decide which is which, draw the free-body diagram so that the angle $\alpha$ is noticeably smaller or larger than 45°. (You'll have to fight the natural tendency to draw such angles as being close to 45°.) We've drawn Fig. 5.4b so that $\alpha$ is smaller than 45°, so $\sin\alpha$ is less than $\cos\alpha$. The figure shows that the $x$-component of $\vec{w}$ is smaller than the $y$-component, so the $x$-component must involve $\sin\alpha$ and the $y$-component must involve $\cos\alpha$. We again find $w_x = -w\sin\alpha$ and $w_y = -w\cos\alpha$.

In Fig. 5.4b we draw a wiggly line through the original vector representing the weight to remind us not to count it twice. Newton's first law gives us

$$\sum F_x = T + (-w\sin\alpha) = 0$$
$$\sum F_y = n + (-w\cos\alpha) = 0$$

(Remember that $T$, $w$, and $n$ are all *magnitudes* of vectors and are therefore all positive.) Solving these equations for $T$ and $n$, we find

$$T = w\sin\alpha$$
$$n = w\cos\alpha$$

**EVALUATE** Our answers for $T$ and $n$ depend on the value of $\alpha$. To check this dependence, let's look at some special cases. If the ramp is horizontal ($\alpha = 0$), we get $T = 0$ and $n = w$: No cable tension $T$ is needed to hold the car, and the normal force $n$ is equal in magnitude to the weight. If the ramp is vertical ($\alpha = 90°$), we get $T = w$ and $n = 0$: The cable tension $T$ supports all of the car's weight, and there's nothing pushing the car against the ramp.

**CAUTION** **Normal force and weight may not be equal** It's a common error to assume that the normal-force magnitude $n$ equals the weight $w$. Our result shows that this is *not* always the case. Always treat $n$ as a variable and solve for its value, as we've done here.

**?** How would the answers for $T$ and $n$ be affected if the car were being pulled up the ramp at a constant speed? This, too, is an equilibrium situation, since the car's velocity is constant. So the calculation is the same, and $T$ and $n$ have the same values as when the car is at rest. (It's true that $T$ must be greater than $w\sin\alpha$ to *start* the car moving up the ramp, but that's not what we asked.)

**KEYCONCEPT** In two-dimensional equilibrium problems, choose the coordinate axes so that as many forces as possible lie along either the $x$-axis or the $y$-axis.

## EXAMPLE 5.5 Equilibrium of objects connected by cable and pulley

WITH VARIATION PROBLEMS

Your firm needs to haul granite blocks up a 15° slope out of a quarry and to lower dirt into the quarry to fill the holes. You design a system in which a granite block on a cart with steel wheels (weight $w_1$, including both block and cart) is pulled uphill on steel rails by a dirt-filled bucket (weight $w_2$, including both dirt and bucket) that descends vertically into the quarry (**Fig. 5.5a**, next page). How must the weights $w_1$ and $w_2$ be related in order for the system to move with constant speed? Ignore friction in the pulley and wheels, and ignore the weight of the cable.

**IDENTIFY and SET UP** The cart and bucket each move with a constant velocity (in a straight line at constant speed). Hence each object is in equilibrium, and we can apply Newton's first law to each. Our target is an expression relating the weights $w_1$ and $w_2$.

Figure 5.5b shows our idealized model for the system, and Figs. 5.5c and 5.5d show our free-body diagrams. The two forces on the bucket are its weight $w_2$ and an upward tension exerted by the cable. As for the car on the ramp in Example 5.4, three forces act on the cart: its weight $w_1$, a normal force of magnitude $n$ exerted by the rails, and a tension force from the cable. Since we're assuming that the cable has negligible weight, the tension forces that the cable exerts on the cart and on the bucket have the same magnitude $T$. (We're ignoring friction, so we assume that the rails exert no force on the cart parallel to the incline.) Note that we are free to orient the axes differently for each object; the choices shown are the most convenient ones. We find the components of the weight force in the same way that we did in Example 5.4. (Compare Fig. 5.5d with Fig. 5.4b.)

**EXECUTE** Applying $\sum F_y = 0$ to the bucket in Fig. 5.5c, we find

$$\sum F_y = T + (-w_2) = 0 \qquad \text{so} \qquad T = w_2$$

Applying $\sum F_x = 0$ to the cart (and block) in Fig. 5.5d, we get

$$\sum F_x = T + (-w_1 \sin 15°) = 0 \qquad \text{so} \qquad T = w_1 \sin 15°$$

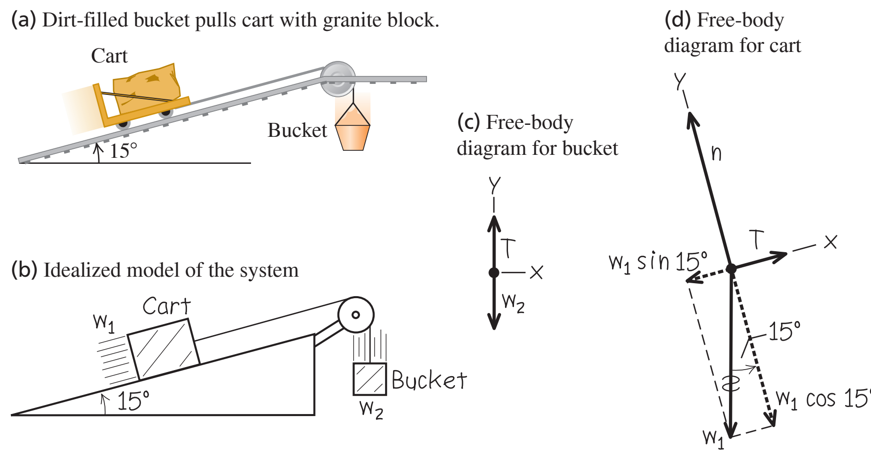
Equating the two expressions for $T$, we find

$$w_2 = w_1 \sin 15° = 0.26 w_1$$

**EVALUATE** Our analysis doesn't depend at all on the direction in which the cart and bucket move. Hence the system can move with constant speed in *either* direction if the weight of the dirt and bucket is 26% of

*Continued*

Figure **5.5** Our sketches for this problem.




the weight of the granite block and cart. What would happen if $w_2$ were greater than $0.26w_1$? If it were less than $0.26w_1$?

Notice that we didn't need the equation $\sum F_y = 0$ for the cart and block. Can you use this to show that $n = w_1 \cos 15°$?

**KEYCONCEPT** If there's more than one object in a problem involving forces, you are free to choose different $x$- and $y$-axes for each object to make it easier to find force components.

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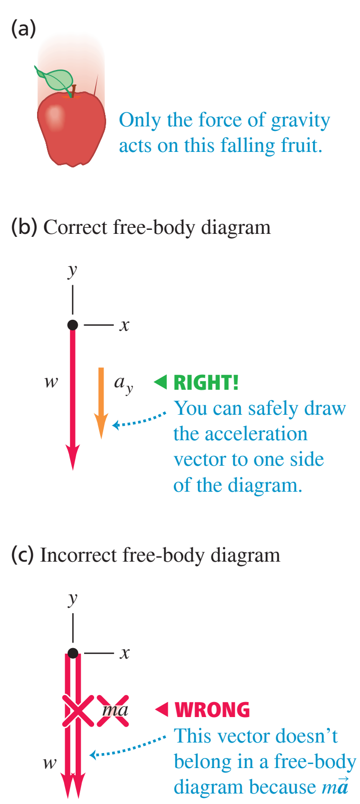
**TEST YOUR UNDERSTANDING OF SECTION 5.1** A traffic light of weight $w$ hangs from two lightweight cables, one on each side of the light. Each cable hangs at a 45° angle from the horizontal. What is the tension in each cable? (i) $w/2$; (ii) $w/\sqrt{2}$; (iii) $w$; (iv) $w\sqrt{2}$; (v) $2w$.

**ANSWER**

**(ii)** The two cables are arranged symmetrically, so the tension in either cable has the same magnitude $T$. The vertical component of the tension from each cable is $T \sin 45°$ (or, equivalently, $T \cos 45°$), so Newton's first law applied to the vertical forces tells us that $2T \sin 45° - w = 0$. Hence $T = w/(2 \sin 45°) = w/\sqrt{2} = 0.71w$. Each cable supports half of the weight of the traffic light, but the tension is greater than $w/2$ because only the vertical component of the tension counteracts the weight.

Figure **5.6** Correct and incorrect free-body diagrams for a falling object.




## 5.2 USING NEWTON'S SECOND LAW: DYNAMICS OF PARTICLES

We are now ready to discuss *dynamics* problems. In these problems, we apply Newton's second law to objects on which the net force is *not* zero. These objects are *not* in equilibrium and hence are accelerating:

**Newton's second law:** If the *net* force on an object is not zero ... $\sum \vec{F} = m\vec{a}$ ... the object has *acceleration* in the same direction as the net force. Mass of object

$$\sum \vec{F} = m\vec{a} \qquad (5.2)$$

Each component of the net force on the object ... $\sum F_x = ma_x \quad \sum F_y = ma_y$ ... equals the object's mass times the corresponding acceleration component.

**CAUTION** $m\vec{a}$ **doesn't belong in free-body diagrams** Remember that the quantity $m\vec{a}$ is the *result* of forces acting on an object, *not* a force itself. When you draw the free-body diagram for an accelerating object (like the fruit in **Fig. 5.6a**), *never* include the "$m\vec{a}$ force" because *there is no such force* (Fig. 5.6c). Review Section 4.3 if you're not clear on this point. Sometimes we draw the acceleration vector $\vec{a}$ *alongside* a free-body diagram, as in Fig. 5.6b. But we *never* draw the acceleration vector with its tail touching the object (a position reserved exclusively for forces that act on the object).

The following problem-solving strategy is very similar to Problem-Solving Strategy 5.1 for equilibrium problems in Section 5.1. Study it carefully, watch how we apply it in our examples, and use it when you tackle the end-of-chapter problems. You can use this strategy to solve *any* dynamics problem.

## PROBLEM-SOLVING STRATEGY 5.2 Newton's Second Law: Dynamics of Particles

**IDENTIFY** *the relevant concepts:* You have to use Newton's second law, Eqs. (5.2), for *any* problem that involves forces acting on an accelerating object.

Identify the target variable—usually an acceleration or a force. If the target variable is something else, you'll need to select another concept to use. For example, suppose the target variable is how fast a sled is moving when it reaches the bottom of a hill. Newton's second law will let you find the sled's acceleration; you'll then use the constant-acceleration relationships from Section 2.4 to find velocity from acceleration.

**SET UP** *the problem* by using the following steps:

1. Draw a simple sketch of the situation that shows each moving object. For each object, draw a free-body diagram that shows all the forces acting *on* the object. [The sums in Eqs. (5.2) include the forces that act on the object, *not* the forces that it exerts on anything else.] Make sure you can answer the question "What other object is applying this force?" for each force in your diagram. Never include the quantity $m\vec{a}$ in your free-body diagram; it's not a force!
2. Label each force with an algebraic symbol for the force's *magnitude*. Usually, one of the forces will be the object's weight; it's usually best to label this as $w = mg$.
3. Choose your $x$- and $y$-coordinate axes for each object, and show them in its free-body diagram. Indicate the positive direction for each axis. If you know the direction of the acceleration, it usually simplifies things to take one positive axis along that direction. If your problem involves two or more objects that accelerate in different directions, you can use a different set of axes for each object.
4. In addition to Newton's second law, $\sum\vec{F} = m\vec{a}$, identify any other equations you might need. For example, you might need one or more of the equations for motion with constant acceleration. If more than one object is involved, there may be relationships among their motions; for example, they may be connected by a rope. Express any such relationships as equations relating the accelerations of the various objects.

**EXECUTE** *the solution* as follows:

1. For each object, determine the components of the forces along each of the object's coordinate axes. When you represent a force in terms of its components, draw a wiggly line through the original force vector to remind you not to include it twice.
2. List all of the known and unknown quantities. In your list, identify the target variable or variables.
3. For each object, write a separate equation for each component of Newton's second law, as in Eqs. (5.2). Write any additional equations that you identified in step 4 of "Set Up." (You need as many equations as there are target variables.)
4. Do the easy part—the math! Solve the equations to find the target variable(s).

**EVALUATE** *your answer:* Does your answer have the correct units? (When appropriate, use the conversion $1\text{ N} = 1\text{ kg}\cdot\text{m/s}^2$.) Does it have the correct algebraic sign? When possible, consider particular values or extreme cases of quantities and compare the results with your intuitive expectations. Ask, "Does this result make sense?"

## EXAMPLE 5.6 Straight-line motion with a constant force

An iceboat is at rest on a frictionless horizontal surface (**Fig. 5.7a**). Due to the blowing wind, 4.0 s after the iceboat is released, it is moving to the right at 6.0 m/s (about 22 km/h, or 13 mi/h). What constant horizontal force $F_W$ does the wind exert on the iceboat? The combined mass of iceboat and rider is 200 kg.

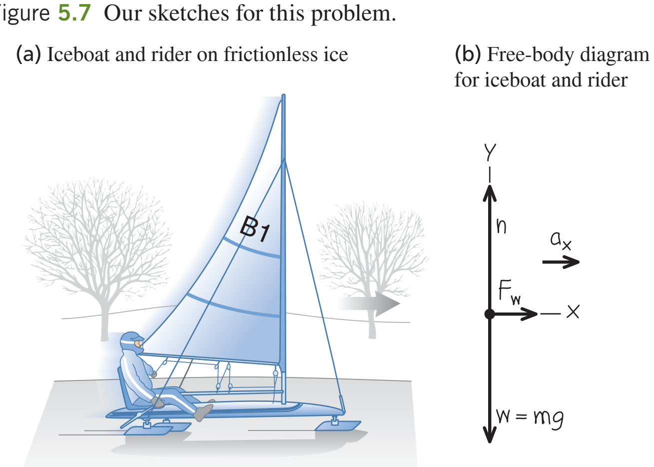
**IDENTIFY and SET UP** Our target variable is one of the forces ($F_W$) acting on the accelerating iceboat, so we need to use Newton's second law. The forces acting on the iceboat and rider (considered as a unit) are the weight $w$, the normal force $n$ exerted by the surface, and the horizontal force $F_W$. Figure 5.7b shows the free-body diagram. The net force and hence the acceleration are to the right, so we chose the positive $x$-axis in this direction. The acceleration isn't given; we'll need to find it. Since the wind is assumed to exert a constant force, the resulting acceleration is constant and we can use one of the constant-acceleration formulas from Section 2.4.

The iceboat starts at rest (its initial $x$-velocity is $v_{0x} = 0$) and it attains an $x$-velocity $v_x = 6.0\text{ m/s}$ after an elapsed time $t = 4.0\text{ s}$. To relate the $x$-acceleration $a_x$ to these quantities we use Eq. (2.8), $v_x = v_{0x} + a_x t$. There is no vertical acceleration, so we expect that the normal force on the iceboat is equal in magnitude to the iceboat's weight.

**EXECUTE** The *known* quantities are the mass $m = 200\text{ kg}$, the initial and final $x$-velocities $v_{0x} = 0$ and $v_x = 6.0\text{ m/s}$, and the elapsed time $t = 4.0\text{ s}$. There are three *unknown* quantities: the acceleration $a_x$, the normal force $n$, and the horizontal force $F_W$. Hence we need three equations.

The first two equations are the $x$- and $y$-equations for Newton's second law, Eqs. (5.2). The force $F_W$ is in the positive $x$-direction, while

Figure 5.7 Our sketches for this problem.




*Continued*

the forces $n$ and $w = mg$ are in the positive and negative $y$-directions, respectively. Hence we have

$$\sum F_x = F_W = ma_x$$
$$\sum F_y = n + (-mg) = 0 \qquad \text{so} \qquad n = mg$$

The third equation is Eq. (2.8) for constant acceleration:

$$v_x = v_{0x} + a_x t$$

To find $F_W$, we first solve this third equation for $a_x$ and then substitute the result into the $\sum F_x$ equation:

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{6.0\ \text{m/s} - 0}{4.0\ \text{s}} = 1.5\ \text{m/s}^2$$
$$F_W = ma_x = (200\ \text{kg})(1.5\ \text{m/s}^2) = 300\ \text{kg} \cdot \text{m/s}^2$$

Since $1\ \text{kg} \cdot \text{m/s}^2 = 1\ \text{N}$, the final answer is

$$F_W = 300\ \text{N (about 67 lb)}$$

**EVALUATE** Our answers for $F_W$ and $n$ have the correct units for a force, and (as expected) the magnitude $n$ of the normal force is equal to $mg$. Does it seem reasonable that the force $F_W$ is substantially *less* than the weight of the boat, $mg$?

**KEYCONCEPT** For problems in which an object is accelerating, it's usually best to choose one positive axis to be in the direction of the acceleration.

## EXAMPLE 5.7 Straight-line motion with friction

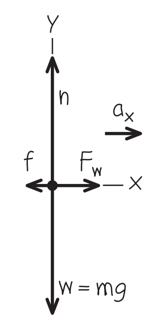
Suppose a constant horizontal friction force with magnitude 100 N opposes the motion of the iceboat in Example 5.6. In this case, what constant force $F_W$ must the wind exert on the iceboat to cause the same constant $x$-acceleration $a_x = 1.5\ \text{m/s}^2$?

**IDENTIFY and SET UP** Again the target variable is $F_W$. We are given the $x$-acceleration, so to find $F_W$ all we need is Newton's second law. **Figure 5.8** shows our new free-body diagram. The only difference from Fig. 5.7b is the addition of the friction force $\vec{f}$, which points in the negative $x$-direction (opposite the motion). Because the wind must now overcome the friction force to yield the same acceleration as in Example 5.6, we expect our answer for $F_W$ to be greater than the 300 N we found there.

**EXECUTE** Two forces now have $x$-components: the force of the wind ($x$-component $+F_W$) and the friction force ($x$-component $-f$). The $x$-component of Newton's second law gives

$$\sum F_x = F_W + (-f) = ma_x$$
$$F_W = ma_x + f = (200\ \text{kg})(1.5\ \text{m/s}^2) + (100\ \text{N}) = 400\ \text{N}$$

Figure 5.8 Our free-body diagram for the iceboat and rider with friction force $\vec{f}$ opposing the motion.




**EVALUATE** The required value of $F_W$ is 100 N greater than in Example 5.6 because the wind must now push against an additional 100 N friction force.

**KEYCONCEPT** When friction is present, the friction force on an object is always in the direction that opposes sliding.

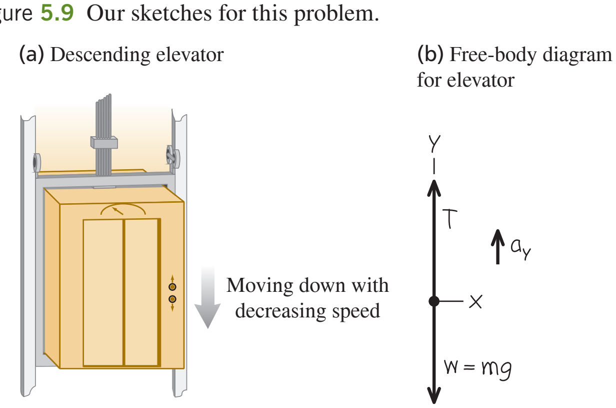
## EXAMPLE 5.8 Tension in an elevator cable

An elevator and its load have a combined mass of 800 kg (**Fig. 5.9a**). The elevator is initially moving downward at 10.0 m/s; it slows to a stop with constant acceleration in a distance of 25.0 m. What is the tension $T$ in the supporting cable while the elevator is being brought to rest?

**IDENTIFY and SET UP** The target variable is the tension $T$, which we'll find by using Newton's second law. As in Example 5.6, we'll use a constant-acceleration formula to determine the acceleration. Our free-body diagram (Fig. 5.9b) shows two forces acting on the elevator: its weight $w$ and the tension force $T$ of the cable. The elevator is moving downward with decreasing speed, so its acceleration is upward; we chose the positive $y$-axis to be upward.

The elevator is moving in the negative $y$-direction, so both its initial $y$-velocity $v_{0y}$ and its $y$-displacement $y - y_0$ are negative: $v_{0y} = -10.0\ \text{m/s}$ and $y - y_0 = -25.0\ \text{m}$. The final $y$-velocity is $v_y = 0$.

Figure 5.9 Our sketches for this problem.



To find the $y$-acceleration $a_y$ from this information, we'll use Eq. (2.13) in the form $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$. Once we have $a_y$, we'll substitute it into the $y$-component of Newton's second law from Eqs. (5.2) and solve for $T$. The net force must be upward to give an upward acceleration, so we expect $T$ to be greater than the weight $w = mg = (800\text{ kg})(9.80\text{ m/s}^2) = 7840\text{ N}$.

**EXECUTE** First let's write out Newton's second law. The tension force acts upward and the weight acts downward, so

$$\sum F_y = T + (-w) = ma_y$$

We solve for the target variable $T$:

$$T = w + ma_y = mg + ma_y = m(g + a_y)$$

To determine $a_y$, we rewrite the constant-acceleration equation $v_y^2 = v_{0y}^2 + 2a_y(y - y_0)$:

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(y - y_0)} = \frac{(0)^2 - (-10.0\text{ m/s})^2}{2(-25.0\text{ m})} = +2.00\text{ m/s}^2$$

The acceleration is upward (positive), just as it should be.

Now we can substitute the acceleration into the equation for the tension:

$$T = m(g + a_y) = (800\text{ kg})(9.80\text{ m/s}^2 + 2.00\text{ m/s}^2) = 9440\text{ N}$$

**EVALUATE** The tension is greater than the weight, as expected. Can you see that we would get the same answers for $a_y$ and $T$ if the elevator were moving *upward* and *gaining* speed at a rate of $2.00\text{ m/s}^2$?

**KEYCONCEPT** If an object suspended from a cable (or rope or string) is accelerating vertically, the tension in the cable is *not* equal to the weight of the object.

## EXAMPLE 5.9 Apparent weight in an accelerating elevator

A 50.0 kg woman stands on a bathroom scale while riding in the elevator in Example 5.8. While the elevator is moving downward with decreasing speed, what is the reading on the scale?

**IDENTIFY and SET UP** The scale (**Fig. 5.10a**) reads the magnitude of the downward force exerted *by* the woman *on* the scale. By Newton's third law, this equals the magnitude of the upward normal force exerted *by* the scale *on* the woman. Hence our target variable is the magnitude $n$ of the normal force. We'll find $n$ by applying Newton's second law to the woman. We already know her acceleration; it's the same as the acceleration of the elevator, which we calculated in Example 5.8.

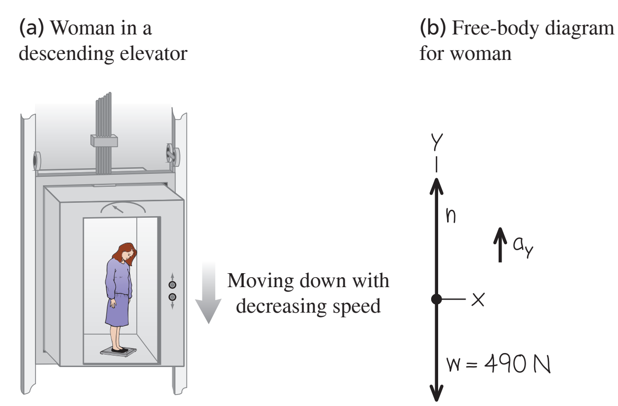
Figure 5.10b shows our free-body diagram for the woman. The forces acting on her are the normal force $n$ exerted by the scale and her weight $w = mg = (50.0\text{ kg})(9.80\text{ m/s}^2) = 490\text{ N}$. (The tension force, which played a major role in Example 5.8, doesn't appear here because it doesn't act on the woman.) From Example 5.8, the $y$-acceleration of the elevator and of the woman is $a_y = +2.00\text{ m/s}^2$. As in Example 5.8, the upward force on the object accelerating upward (in this case, the normal force on the woman) will have to be greater than the object's weight to produce the upward acceleration.

**EXECUTE** Newton's second law gives

$$\sum F_y = n + (-mg) = ma_y$$
$$n = mg + ma_y = m(g + a_y)$$
$$= (50.0\text{ kg})(9.80\text{ m/s}^2 + 2.00\text{ m/s}^2) = 590\text{ N}$$

**EVALUATE** Our answer for $n$ means that while the elevator is stopping, the scale pushes up on the woman with a force of 590 N. By Newton's third law, she pushes down on the scale with the same force. So the scale reads 590 N, which is 100 N more than her actual weight. The scale reading is called the passenger's **apparent weight.** The woman *feels* the floor pushing up harder on her feet than when the elevator is stationary or moving with constant velocity.

Figure 5.10 Our sketches for this problem.




What would the woman feel if the elevator were accelerating *downward,* so that $a_y = -2.00\text{ m/s}^2$? This would be the case if the elevator were moving upward with decreasing speed or moving downward with increasing speed. To find the answer for this situation, we just insert the new value of $a_y$ in our equation for $n$:

$$n = m(g + a_y) = (50.0\text{ kg})[9.80\text{ m/s}^2 + (-2.00\text{ m/s}^2)] = 390\text{ N}$$

Now the woman would feel as though she weighs only 390 N, or 100 N *less* than her actual weight $w$.

You can feel these effects yourself; try taking a few steps in an elevator that is coming to a stop after descending (when your apparent weight is greater than $w$) or coming to a stop after ascending (when your apparent weight is less than $w$).

**KEYCONCEPT** When you are riding in an accelerating vehicle such as an elevator, your apparent weight (the normal force that the vehicle exerts on you) is in general *not* equal to your actual weight.

Figure 5.11 Astronauts in orbit feel "weightless" because they have the same acceleration as their spacecraft. They are *not* outside the pull of the earth's gravity. (We'll discuss the motions of orbiting objects in detail in Chapter 12.)

## Apparent Weight and Apparent Weightlessness

Let's generalize the result of Example 5.9. When a passenger with mass $m$ rides in an elevator with $y$-acceleration $a_y$, a scale shows the passenger's apparent weight to be

$$n = m(g + a_y)$$

When the elevator is accelerating upward, $a_y$ is positive and $n$ is greater than the passenger's weight $w = mg$. When the elevator is accelerating downward, $a_y$ is negative and $n$ is less than the weight. If the passenger doesn't know the elevator is accelerating, she may feel as though her weight is changing; indeed, this is just what the scale shows.

The extreme case occurs when the elevator has a downward acceleration $a_y = -g$—that is, when it is in free fall. In that case $n = 0$ and the passenger *seems* to be weightless. Similarly, an astronaut orbiting the earth with a spacecraft experiences *apparent weightlessness* (**Fig. 5.11**). In each case, the person is not truly weightless because a gravitational force still acts. But the person's *sensations* in this free-fall condition are exactly the same as though the person were in outer space with no gravitational force at all. In both cases the person and the vehicle (elevator or spacecraft) fall together with the same acceleration $g$, so nothing pushes the person against the floor or walls of the vehicle.

### EXAMPLE 5.10 Acceleration down a hill

A toboggan loaded with students (total weight $w$) slides down a snow-covered hill that slopes at a constant angle $\alpha$. The toboggan is well waxed, so there is virtually no friction. What is its acceleration?

**IDENTIFY and SET UP** Our target variable is the acceleration, which we'll find by using Newton's second law. There is no friction, so only two forces act on the toboggan: its weight $w$ and the normal force $n$ exerted by the hill.

**Figure 5.12** shows our sketch and free-body diagram. We take axes parallel and perpendicular to the surface of the hill, so that the acceleration (which is parallel to the hill) is along the positive $x$-direction.

**EXECUTE** The normal force has only a $y$-component, but the weight has both $x$- and $y$-components: $w_x = w\sin\alpha$ and $w_y = -w\cos\alpha$. (In Example 5.4 we had $w_x = -w\sin\alpha$. The difference is that the positive $x$-axis was uphill in Example 5.4 but is downhill in Fig. 5.12b.) The wiggly line in Fig. 5.12b reminds us that we have resolved the weight into its components. The acceleration is purely in the $+x$-direction, so $a_y = 0$. Newton's second law in component form from Eqs. (5.2) then tells us that

$$\sum F_x = w\sin\alpha = ma_x$$

$$\sum F_y = n - w\cos\alpha = ma_y = 0$$

Since $w = mg$, the $x$-component equation gives $mg\sin\alpha = ma_x$, or

$$a_x = g\sin\alpha$$

Note that we didn't need the $y$-component equation to find the acceleration. That's part of the beauty of choosing the $x$-axis to lie along the acceleration direction! The $y$-equation tells us the magnitude of the normal force exerted by the hill on the toboggan:

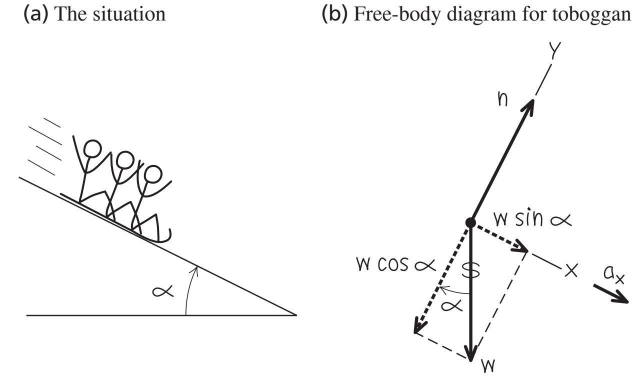
$$n = w\cos\alpha = mg\cos\alpha$$

**EVALUATE** Notice that the normal force $n$ is not equal to the toboggan's weight (compare Example 5.4). Notice also that the mass $m$ does not appear in our result for the acceleration. That's because the downhill force on the toboggan (a component of the weight) is proportional to $m$, so the mass cancels out when we use $\sum F_x = ma_x$ to calculate $a_x$. Hence *any* toboggan, regardless of its mass, slides down a frictionless hill with acceleration $g\sin\alpha$.

If the plane is horizontal, $\alpha = 0$ and $a_x = 0$ (the toboggan does not accelerate); if the plane is vertical, $\alpha = 90°$ and $a_x = g$ (the toboggan is in free fall).

Figure 5.12 Our sketches for this problem.

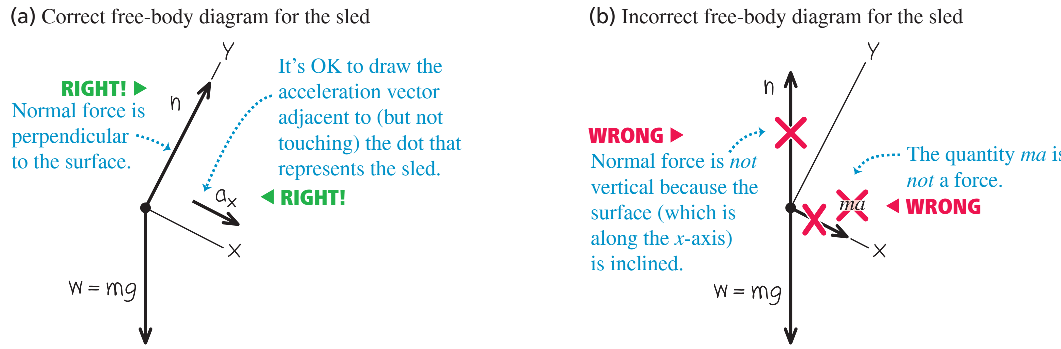
(a) The situation (b) Free-body diagram for toboggan




**CAUTION** **Common free-body diagram errors** **Figure 5.13** shows both the correct way (Fig. 5.13a) and a common *incorrect* way (Fig. 5.13b) to draw the free-body diagram for the toboggan. The diagram in Fig. 5.13b is wrong for two reasons: The normal force must be drawn perpendicular to the surface (remember, "normal" means perpendicular), and there's no such thing as the "$m\vec{a}$ force."

**KEYCONCEPT** In problems with an object on an incline, it's usually best to take the positive $x$-direction for that object to be down the incline. The force of gravity will then have both an $x$-component and a $y$-component.

Figure **5.13** Correct and incorrect free-body diagrams for a toboggan on a frictionless hill.




## EXAMPLE 5.11 Two objects with the same acceleration

You push a 1.00 kg food tray through the cafeteria line with a constant 9.0 N force. The tray pushes a 0.50 kg milk carton (**Fig. 5.14a**). The tray and carton slide on a horizontal surface so greasy that friction can be ignored. Find the acceleration of the tray and carton and the horizontal force that the tray exerts on the carton.

**IDENTIFY and SET UP** Our *two* target variables are the acceleration of the tray–carton system and the force of the tray on the carton. We'll use Newton's second law to get two equations, one for each target variable. We set up and solve the problem in two ways.

*Method 1:* We treat the carton (mass $m_C$) and tray (mass $m_T$) as separate objects, each with its own free-body diagram (Figs. 5.14b and 5.14c). The force $F$ that you exert on the tray doesn't appear in the free-body diagram for the carton, which is accelerated by the force (of magnitude $F_{\text{T on C}}$) exerted on it by the tray. By Newton's third law, the carton exerts a force of equal magnitude on the tray: $F_{\text{C on T}} = F_{\text{T on C}}$. We take the acceleration to be in the positive $x$-direction; both the tray and milk carton move with the same $x$-acceleration $a_x$.

*Method 2:* We treat the tray and milk carton as a composite object of mass $m = m_T + m_C = 1.50$ kg (Fig. 5.14d). The only horizontal force acting on this object is the force $F$ that you exert. The forces $F_{\text{T on C}}$ and $F_{\text{C on T}}$ don't come into play because they're *internal* to this composite object, and Newton's second law tells us that only *external* forces affect an object's acceleration (see Section 4.3). To find the magnitude $F_{\text{T on C}}$ we'll again apply Newton's second law to the carton, as in Method 1.

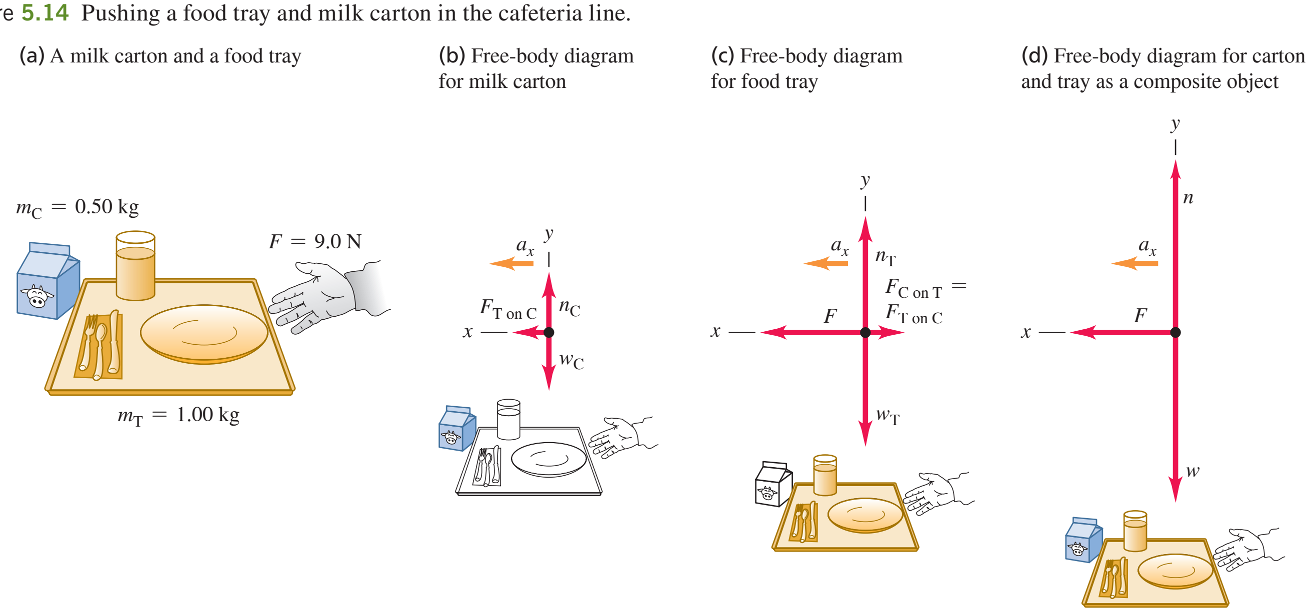
**EXECUTE** *Method 1:* The $x$-component equations of Newton's second law are

$$\text{Tray:} \quad \sum F_x = F - F_{\text{C on T}} = F - F_{\text{T on C}} = m_T a_x$$

$$\text{Carton:} \quad \sum F_x = F_{\text{T on C}} = m_C a_x$$

These are two simultaneous equations for the two target variables $a_x$ and $F_{\text{T on C}}$. (Two equations are all we need, which means that the

Figure **5.14** Pushing a food tray and milk carton in the cafeteria line.




*Continued*

$y$-components don't play a role in this example.) An easy way to solve the two equations for $a_x$ is to add them; this eliminates $F_{\text{T on C}}$, giving

$$F = m_T a_x + m_C a_x = (m_T + m_C)a_x$$

We solve this equation for $a_x$:

$$a_x = \frac{F}{m_T + m_C} = \frac{9.0 \text{ N}}{1.00 \text{ kg} + 0.50 \text{ kg}} = 6.0 \text{ m/s}^2 = 0.61g$$

Substituting this value into the carton equation gives

$$F_{\text{T on C}} = m_C a_x = (0.50 \text{ kg})(6.0 \text{ m/s}^2) = 3.0 \text{ N}$$

*Method 2:* The $x$-component of Newton's second law for the composite object of mass $m$ is

$$\sum F_x = F = ma_x$$

The acceleration of this composite object is

$$a_x = \frac{F}{m} = \frac{9.0 \text{ N}}{1.50 \text{ kg}} = 6.0 \text{ m/s}^2$$

Then, looking at the milk carton by itself, we see that to give it an acceleration of 6.0 m/s$^2$ requires that the tray exert a force

$$F_{\text{T on C}} = m_C a_x = (0.50 \text{ kg})(6.0 \text{ m/s}^2) = 3.0 \text{ N}$$

**EVALUATE** The answers are the same with both methods. To check the answers, note that there are different forces on the two sides of the tray: $F = 9.0$ N on the right and $F_{\text{C on T}} = 3.0$ N on the left. The net horizontal force on the tray is $F - F_{\text{C on T}} = 6.0$ N, exactly enough to accelerate a 1.00 kg tray at 6.0 m/s$^2$.

Treating two objects as a single, composite object works *only* if the two objects have the same magnitude *and* direction of acceleration. If the accelerations are different we must treat the two objects separately, as in the next example.

**KEYCONCEPT** When two objects are touching each other, the free-body diagram for each object must include the force exerted on it by the other object.

## EXAMPLE 5.12 Two objects with the same magnitude of acceleration

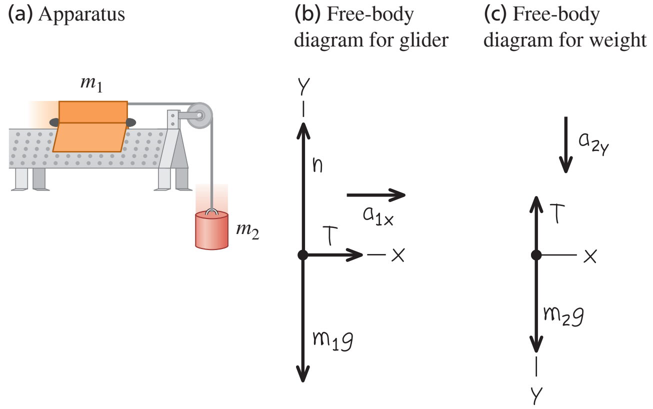
**Figure 5.15a** shows an air-track glider with mass $m_1$ moving on a level, frictionless air track in the physics lab. The glider is connected to a lab weight with mass $m_2$ by a light, flexible, nonstretching string that passes over a stationary, frictionless pulley. Find the acceleration of each object and the tension in the string.

**IDENTIFY and SET UP** The glider and weight are accelerating, so again we must use Newton's second law. Our three target variables are the tension $T$ in the string and the accelerations of the two objects.

The two objects move in different directions—one horizontal, one vertical—so we can't consider them to be a single unit as we did the objects in Example 5.11. Figures 5.15b and 5.15c show our free-body diagrams and coordinate systems. It's convenient to have both objects accelerate in the positive axis directions, so we chose the positive $y$-direction for the lab weight to be downward.

We consider the string to be massless and to slide over the pulley without friction, so the tension $T$ in the string is the same throughout and it applies a force of the same magnitude $T$ to each object. (You may want to review Conceptual Example 4.10, in which we discussed the tension force exerted by a massless rope.) The weights are $m_1g$ and $m_2g$.

Figure 5.15 Our sketches for this problem.




While the *directions* of the two accelerations are different, their *magnitudes* are the same. (That's because the string doesn't stretch, so the two objects must move equal distances in equal times and their speeds at any instant must be equal. When the speeds change, they change at the same rate, so the accelerations of the two objects must have the same magnitude $a$.) We can express this relationship as $a_{1x} = a_{2y} = a$, which means that we have only *two* target variables: $a$ and the tension $T$.

What results do we expect? If $m_1 = 0$ (or, approximately, for $m_1$ much less than $m_2$) the lab weight will fall freely with acceleration $g$, and the tension in the string will be zero. For $m_2 = 0$ (or, approximately, for $m_2$ much less than $m_1$) we expect zero acceleration and zero tension.

**EXECUTE** Newton's second law gives

$$\text{Glider:} \quad \sum F_x = T = m_1 a_{1x} = m_1 a$$

$$\text{Glider:} \quad \sum F_y = n + (-m_1 g) = m_1 a_{1y} = 0$$

$$\text{Lab weight:} \quad \sum F_y = m_2 g + (-T) = m_2 a_{2y} = m_2 a$$

(There are no forces on the lab weight in the $x$-direction.) In these equations we've used $a_{1y} = 0$ (the glider doesn't accelerate vertically) and $a_{1x} = a_{2y} = a$.

The $x$-equation for the glider and the equation for the lab weight give us two simultaneous equations for $T$ and $a$:

$$\text{Glider:} \quad T = m_1 a$$

$$\text{Lab weight:} \quad m_2 g - T = m_2 a$$

We add the two equations to eliminate $T$, giving

$$m_2 g = m_1 a + m_2 a = (m_1 + m_2)a$$

and so the magnitude of each object's acceleration is

$$a = \frac{m_2}{m_1 + m_2} g$$

Substituting this back into the glider equation $T = m_1 a$, we get

$$T = \frac{m_1 m_2}{m_1 + m_2} g$$

**EVALUATE** The acceleration is in general less than $g$, as you might expect; the string tension keeps the lab weight from falling freely. The tension $T$ is *not* equal to the weight $m_2 g$ of the lab weight, but is *less* by a factor of $m_1/(m_1 + m_2)$. If $T$ *were* equal to $m_2 g$, then the lab weight would be in equilibrium, and it isn't.

As predicted, the acceleration is equal to $g$ for $m_1 = 0$ and equal to zero for $m_2 = 0$, and $T = 0$ for either $m_1 = 0$ or $m_2 = 0$.

**CAUTION Tension and weight may not be equal** It's a common mistake to assume that if an object is attached to a vertical string, the string tension must be equal to the object's weight. That was the case in Example 5.5, where the acceleration was zero, but it's not the case in this example! (Tension and weight were also not equal for the accelerating elevator in Example 5.8.) The only safe approach is *always* to treat the tension as a variable, as we did here.

**KEYCONCEPT** If two objects are connected by a string under tension, both objects have the same magnitude of acceleration but may accelerate in different directions. Choose the positive $x$-direction for each object to be in the direction of its acceleration.

**TEST YOUR UNDERSTANDING OF SECTION 5.2** Suppose you hold the glider in Example 5.12 so that it and the weight are initially at rest. You give the glider a push to the left in Fig. 5.15a and then release it. The string remains taut as the glider moves to the left, comes instantaneously to rest, then moves to the right. At the instant the glider has zero velocity, what is the tension in the string? (i) Greater than in Example 5.12; (ii) the same as in Example 5.12; (iii) less than in Example 5.12 but greater than zero; (iv) zero.

**ANSWER** **(ii)** No matter what the instantaneous velocity of the glider, its acceleration is constant and has the value found in Example 5.12. In the same way, the acceleration of an object in free fall is the same whether it is ascending, descending, or at the high point of its motion (see Section 2.5).

## 5.3 FRICTION FORCES

We've seen several problems in which an object rests or slides on a surface that exerts forces on the object. Whenever two objects interact by direct contact (touching) of their surfaces, we describe the interaction in terms of *contact forces*. The normal force is one example of a contact force; in this section we'll look in detail at another contact force, the force of friction.

Friction is important in many aspects of everyday life. The oil in a car engine minimizes friction between moving parts, but without friction between the tires and the road we couldn't drive or turn the car. Air drag—the friction force exerted by the air on an object moving through it—decreases automotive fuel economy but makes parachutes work. Without friction, nails would pull out and most forms of animal locomotion would be impossible (**Fig. 5.16**).

### Kinetic and Static Friction

When you try to slide a heavy box of books across the floor, the box doesn't move at all unless you push with a certain minimum force. Once the box starts moving, you can usually keep it moving with less force than you needed to get it started. If you take some of the books out, you need less force to get it started or keep it moving. What can we say in general about this behavior?

First, when an object rests or slides on a surface, we can think of the surface as exerting a single contact force on the object, with force components perpendicular and parallel to the surface (**Fig. 5.17**, next page). The perpendicular component vector is the normal force, denoted by $\vec{n}$. The component vector parallel to the surface (and perpendicular to $\vec{n}$) is the **friction force,** denoted by $\vec{f}$. If the surface is frictionless, then $\vec{f}$ is zero but there is still a normal force. (Frictionless surfaces are an unattainable idealization, like a massless rope. But we can approximate a surface as frictionless if the effects of friction are negligibly small.) The direction of the friction force is always such as to oppose relative motion of the two surfaces.

Figure 5.16 There is friction between the feet of this caterpillar (the larval stage of a butterfly of the family Papilionidae) and the surfaces over which it walks. Without friction, the caterpillar could not move forward or climb over obstacles.

Figure **5.17** When a block is pushed or pulled over a surface, the surface exerts a contact force on it.

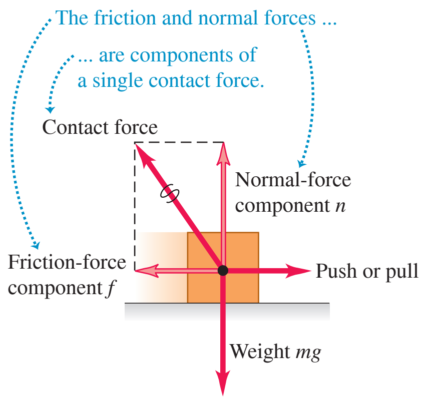



Figure **5.18** A microscopic view of the friction and normal forces.

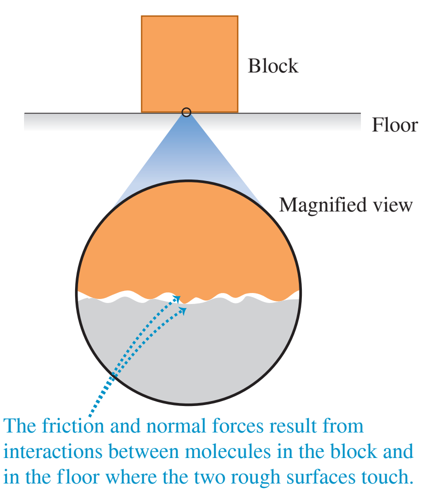



**TABLE 5.1** Approximate Coefficients of Friction

| Materials | Coefficient of Static Friction, $\mu_s$ | Coefficient of Kinetic Friction, $\mu_k$ |
|---|---|---|
| Steel on steel | 0.74 | 0.57 |
| Aluminum on steel | 0.61 | 0.47 |
| Copper on steel | 0.53 | 0.36 |
| Brass on steel | 0.51 | 0.44 |
| Zinc on cast iron | 0.85 | 0.21 |
| Copper on cast iron | 1.05 | 0.29 |
| Glass on glass | 0.94 | 0.40 |
| Copper on glass | 0.68 | 0.53 |
| Teflon on Teflon | 0.04 | 0.04 |
| Teflon on steel | 0.04 | 0.04 |
| Rubber on concrete (dry) | 1.0 | 0.8 |
| Rubber on concrete (wet) | 0.30 | 0.25 |

The kind of friction that acts when an object slides over a surface is called a **kinetic friction force** $\vec{f}_k$. The adjective "kinetic" and the subscript "k" remind us that the two surfaces are moving relative to each other. The *magnitude* of the kinetic friction force usually increases when the normal force increases. This is why it takes more force to slide a full box of books across the floor than an empty one. Automotive brakes use the same principle: The harder the brake pads are squeezed against the rotating brake discs, the greater the braking effect. In many cases the magnitude of the kinetic friction force $f_k$ is found experimentally to be approximately *proportional* to the magnitude $n$ of the normal force:

$$\underbrace{f_k}_{\text{Magnitude of kinetic friction force}} = \overbrace{\mu_k}^{\text{Coefficient of kinetic friction}} \underbrace{n}_{\text{Magnitude of normal force}} \tag{5.3}$$

Here $\mu_k$ (pronounced "mu-sub-k") is a constant called the **coefficient of kinetic friction.** The more slippery the surface, the smaller this coefficient. Because it is a quotient of two force magnitudes, $\mu_k$ is a pure number without units.

**CAUTION** **Friction and normal forces are always perpendicular** Remember that Eq. (5.3) is *not* a vector equation because $\vec{f}_k$ and $\vec{n}$ are always perpendicular. Rather, it is a scalar relationship between the magnitudes of the two forces.

Equation (5.3) is only an approximate representation of a complex phenomenon. On a microscopic level, friction and normal forces result from the intermolecular forces (electrical in nature) between two rough surfaces at points where they come into contact (**Fig. 5.18**). As a box slides over the floor, bonds between the two surfaces form and break, and the total number of such bonds varies. Hence the kinetic friction force is not perfectly constant. Smoothing the surfaces can actually increase friction, since more molecules can interact and bond; bringing two smooth surfaces of the same metal together can cause a "cold weld." Lubricating oils work because an oil film between two surfaces (such as the pistons and cylinder walls in a car engine) prevents them from coming into actual contact.

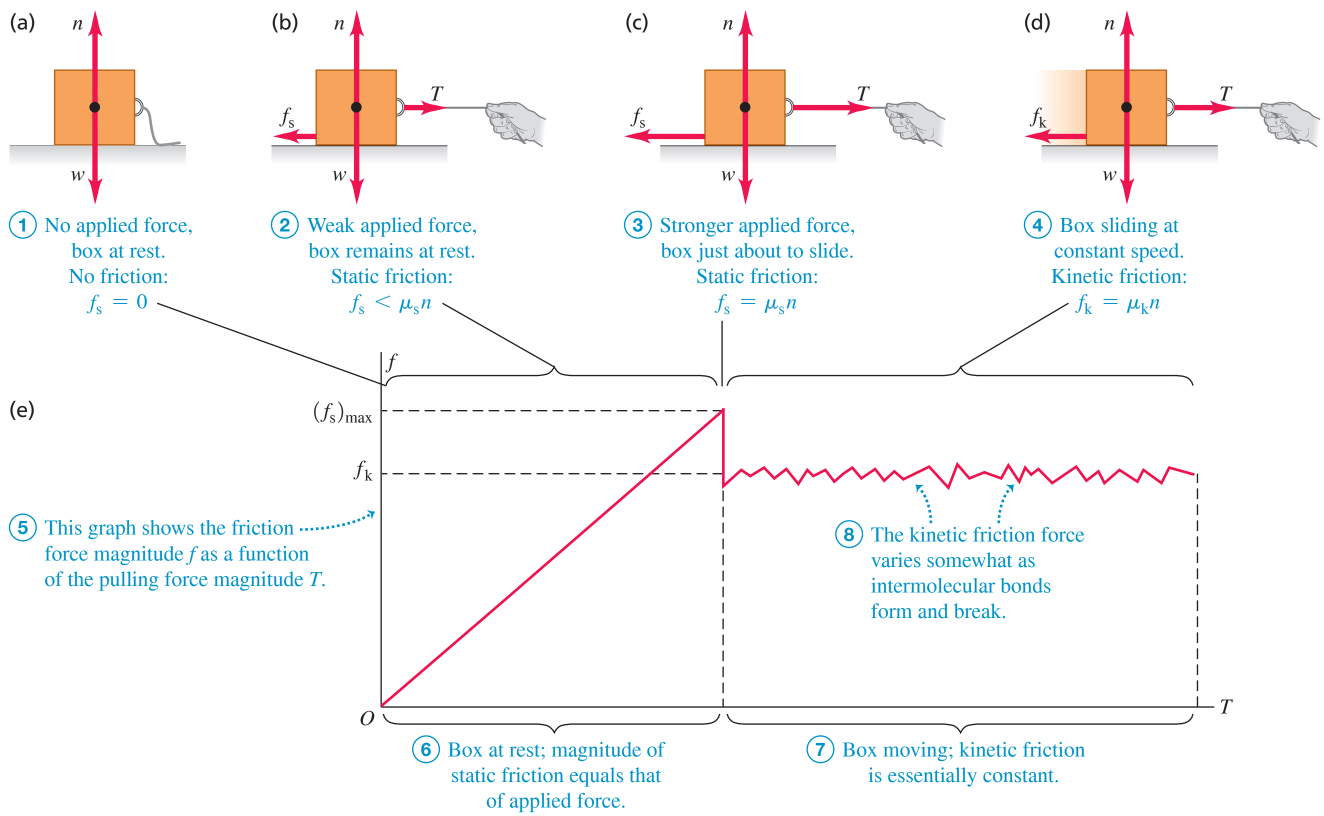
**Table 5.1** lists some representative values of $\mu_k$. Although these values are given with two significant figures, they are only approximate, since friction forces can also depend on the speed of the object relative to the surface. For now we'll ignore this effect and assume that $\mu_k$ and $f_k$ are independent of speed, in order to concentrate on the simplest cases. Table 5.1 also lists coefficients of static friction; we'll define these shortly.

Friction forces may also act when there is *no* relative motion. If you try to slide a box across the floor, the box may not move at all because the floor exerts an equal and opposite friction force on the box. This is called a **static friction force** $\vec{f}_s$.

In **Fig. 5.19a**, the box is at rest, in equilibrium, under the action of its weight $\vec{w}$ and the upward normal force $\vec{n}$. The normal force is equal in magnitude to the weight ($n = w$) and is exerted on the box by the floor. Now we tie a rope to the box (Fig. 5.19b) and gradually increase the tension $T$ in the rope. At first the box remains at rest because the force of static friction $f_s$ also increases and stays equal in magnitude to $T$.

At some point $T$ becomes greater than the maximum static friction force $f_s$ the surface can exert. Then the box "breaks loose" and starts to slide. Figure 5.19c shows the forces when $T$ is at this critical value. For a given pair of surfaces the maximum value of $f_s$ depends on the normal force. Experiment shows that in many cases this maximum value, called $(f_s)_{max}$, is approximately *proportional* to $n$; we call the proportionality factor $\mu_s$ the **coefficient of static friction.** Table 5.1 lists some representative values of $\mu_s$. In a particular situation, the actual force of static friction can have any magnitude between zero (when there is no other force parallel to the surface) and a maximum value given by $\mu_s n$:

Figure 5.19 When there is no relative motion, the magnitude of the static friction force $f_s$ is less than or equal to $\mu_s n$. When there is relative motion, the magnitude of the kinetic friction force $f_k$ equals $\mu_k n$.




**Magnitude of static friction force**

$$f_s \le (f_s)_{max} = \mu_s n \quad (5.4)$$

Maximum static friction force

Coefficient of static friction

Magnitude of normal force

Like Eq. (5.3), this is a relationship between magnitudes, *not* a vector relationship. The equality sign holds only when the applied force $T$ has reached the critical value at which motion is about to start (Fig. 5.19c). When $T$ is less than this value (Fig. 5.19b), the inequality sign holds. In that case we have to use the equilibrium conditions $(\sum \vec{F} = \mathbf{0})$ to find $f_s$. If there is no applied force $(T = 0)$ as in Fig. 5.19a, then there is no static friction force either $(f_s = 0)$.

As soon as the box starts to slide (Fig. 5.19d), the friction force usually *decreases* (Fig. 5.19e); it's easier to keep the box moving than to start it moving. Hence the coefficient of kinetic friction is usually *less* than the coefficient of static friction for any given pair of surfaces, as Table 5.1 shows.

In some situations the surfaces will alternately stick (static friction) and slip (kinetic friction). This is what causes the horrible sound made by chalk held at the wrong angle on a blackboard and the shriek of tires sliding on asphalt pavement. A more positive example is the motion of a violin bow against the string.

In the linear air tracks used in physics laboratories, gliders move with very little friction because they are supported on a layer of air. The friction force is velocity dependent, but at typical speeds the effective coefficient of friction is of the order of 0.001.

**APPLICATION Static Friction and Windshield Wipers** The squeak of windshield wipers on dry glass is a stick-slip phenomenon. The moving wiper blade sticks to the glass momentarily, then slides when the force applied to the blade by the wiper motor overcomes the maximum force of static friction. When the glass is wet from rain or windshield cleaning solution, friction is reduced and the wiper blade doesn't stick.

## EXAMPLE 5.13 Friction in horizontal motion

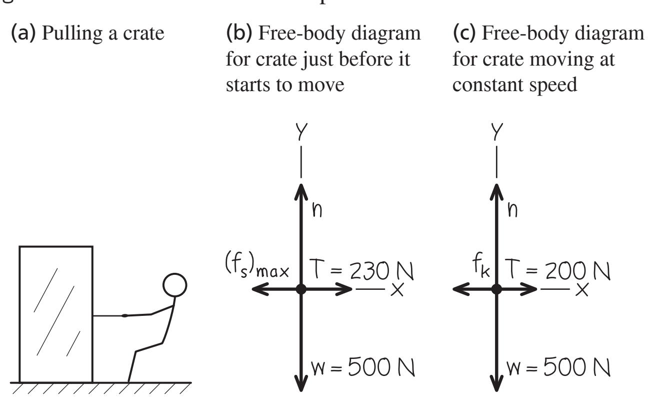
WITH VARIATION PROBLEMS

You want to move a 500 N crate across a level floor. To start the crate moving, you have to pull with a 230 N horizontal force. Once the crate starts to move, you can keep it moving at constant velocity with only 200 N. What are the coefficients of static and kinetic friction?

**IDENTIFY and SET UP** The crate is in equilibrium both when it is at rest and when it is moving at constant velocity, so we use Newton's first law, as expressed by Eqs. (5.1). We use Eqs. (5.3) and (5.4) to find the target variables $\mu_s$ and $\mu_k$.

**Figures 5.20a** and 5.20b show our sketch and free-body diagram for the instant just before the crate starts to move, when the static friction force has its maximum possible value $(f_s)_{max} = \mu_s n$. Once the crate is moving, the friction force changes to its kinetic form (Fig. 5.20c). In both situations, four forces act on the crate: the downward weight (magnitude $w = 500$ N), the upward normal force (magnitude $n$) exerted by the floor, a tension force (magnitude $T$) to the right exerted by the rope, and a friction force to the left exerted by the floor.

Figure **5.20** Our sketches for this problem.




Because the rope in Fig. 5.20a is in equilibrium, the tension is the same at both ends. Hence the tension force that the rope exerts on the crate has the same magnitude as the force you exert on the rope. Since it's easier to keep the crate moving than to start it moving, we expect that $\mu_k < \mu_s$.

**EXECUTE** Just before the crate starts to move (Fig. 5.20b), we have from Eqs. (5.1)

$$\sum F_x = T + (-(f_s)_{max}) = 0 \quad \text{so} \quad (f_s)_{max} = T = 230\text{ N}$$

$$\sum F_y = n + (-w) = 0 \quad \text{so} \quad n = w = 500\text{ N}$$

Now we solve Eq. (5.4), $(f_s)_{max} = \mu_s n$, for the value of $\mu_s$:

$$\mu_s = \frac{(f_s)_{max}}{n} = \frac{230\text{ N}}{500\text{ N}} = 0.46$$

After the crate starts to move (Fig. 5.20c) we have

$$\sum F_x = T + (-f_k) = 0 \quad \text{so} \quad f_k = T = 200\text{ N}$$

$$\sum F_y = n + (-w) = 0 \quad \text{so} \quad n = w = 500\text{ N}$$

Using $f_k = \mu_k n$ from Eq. (5.3), we find

$$\mu_k = \frac{f_k}{n} = \frac{200\text{ N}}{500\text{ N}} = 0.40$$

**EVALUATE** As expected, the coefficient of kinetic friction is less than the coefficient of static friction.

**KEYCONCEPT** For any object, the *maximum* magnitude of the *static* friction force and the magnitude of the *kinetic* friction force are proportional to the magnitude of the normal force on that object.

## EXAMPLE 5.14 Static friction can be less than the maximum

WITH VARIATION PROBLEMS

In Example 5.13, what is the friction force if the crate is at rest on the surface and a horizontal force of 50 N is applied to it?

**IDENTIFY and SET UP** The applied force is less than the maximum force of static friction, $(f_s)_{max} = 230$ N. Hence the crate remains at rest and the net force acting on it is zero. The target variable is the magnitude $f_s$ of the friction force. The free-body diagram is the same as in Fig. 5.20b, but with $(f_s)_{max}$ replaced by $f_s$ and $T = 230$ N replaced by $T = 50$ N.

**EXECUTE** From the equilibrium conditions, Eqs. (5.1), we have

$$\sum F_x = T + (-f_s) = 0 \quad \text{so} \quad f_s = T = 50\text{ N}$$

**EVALUATE** The friction force can prevent motion for any horizontal applied force up to $(f_s)_{max} = \mu_s n = 230$ N. Below that value, $f_s$ has the same magnitude as the applied force.

**KEYCONCEPT** The magnitude $f_s$ of the static friction force on an object at rest does *not* have to equal the maximum magnitude $\mu_s n$. The actual value of $f_s$ depends on the other forces acting on the object; you can find this value using Newton's first law.

## EXAMPLE 5.15 Minimizing kinetic friction

WITH VARIATION PROBLEMS

In Example 5.13, suppose you move the crate by pulling upward on the rope at an angle of 30° above the horizontal. How hard must you pull to keep it moving with constant velocity? Assume that $\mu_k = 0.40$.

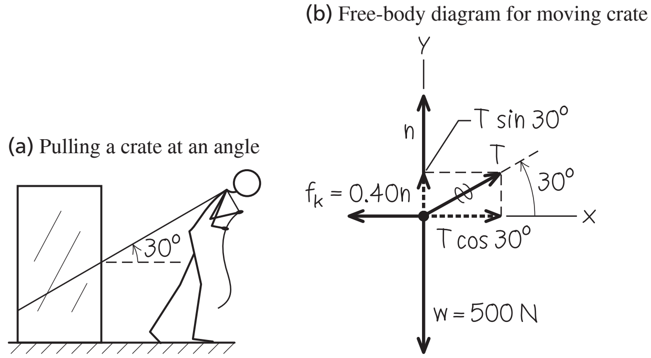
**IDENTIFY and SET UP** The crate is in equilibrium because its velocity is constant, so we again apply Newton's first law. Since the crate is in motion, the floor exerts a *kinetic* friction force. The target variable is the magnitude $T$ of the tension force.

**Figure 5.21** shows our sketch and free-body diagram. The kinetic friction force $f_k$ is still equal to $\mu_k n$, but now the normal force $n$ is *not* equal in magnitude to the crate's weight. The force exerted by the rope has a vertical component that tends to lift the crate off the floor; this *reduces* $n$ and so reduces $f_k$.

**EXECUTE** From the equilibrium conditions and Eq. (5.3), $f_k = \mu_k n$, we have

$$\sum F_x = T\cos 30° + (-f_k) = 0 \quad \text{so} \quad T\cos 30° = \mu_k n$$
$$\sum F_y = T\sin 30° + n + (-w) = 0 \quad \text{so} \quad n = w - T\sin 30°$$

Figure **5.21** Our sketches for this problem.




These are two equations for the two unknown quantities $T$ and $n$. One way to find $T$ is to substitute the expression for $n$ in the second equation into the first equation and then solve the resulting equation for $T$:

$$T\cos 30° = \mu_k(w - T\sin 30°)$$
$$T = \frac{\mu_k w}{\cos 30° + \mu_k \sin 30°} = 188 \text{ N}$$

We can substitute this result into either of the original equations to obtain $n$. If we use the second equation, we get

$$n = w - T\sin 30° = (500 \text{ N}) - (188 \text{ N})\sin 30° = 406 \text{ N}$$

**EVALUATE** As expected, the normal force is less than the 500 N weight of the box. It turns out that the tension required to keep the crate moving at constant speed is a little less than the 200 N force needed when you pulled horizontally in Example 5.13. Can you find an angle where the required pull is *minimum*?

**KEYCONCEPT** In problems that involve kinetic friction, you'll always need at least three equations: two from Newton's first or second law in component form, and Eq. (5.3) for kinetic friction, $f_k = \mu_k n$.

## EXAMPLE 5.16 Toboggan ride with friction I

Let's go back to the toboggan we studied in Example 5.10. The wax has worn off, so there is now a nonzero coefficient of kinetic friction $\mu_k$. The slope has just the right angle to make the toboggan slide with constant velocity. Find this angle in terms of $w$ and $\mu_k$.

**IDENTIFY and SET UP** Our target variable is the slope angle $\alpha$. The toboggan is in equilibrium because its velocity is constant, so we use Newton's first law in the form of Eqs. (5.1).

Three forces act on the toboggan: its weight, the normal force, and the kinetic friction force. The motion is downhill, so the friction force (which opposes the motion) is directed uphill. **Figure 5.22** shows our sketch and free-body diagram (compare Fig. 5.12b in Example 5.10). From Eq. (5.3), the magnitude of the kinetic friction force is $f_k = \mu_k n$. We expect that the greater the value of $\mu_k$, the steeper will be the required slope.

**EXECUTE** The equilibrium conditions are

$$\sum F_x = w\sin\alpha + (-f_k) = w\sin\alpha - \mu_k n = 0$$
$$\sum F_y = n + (-w\cos\alpha) = 0$$

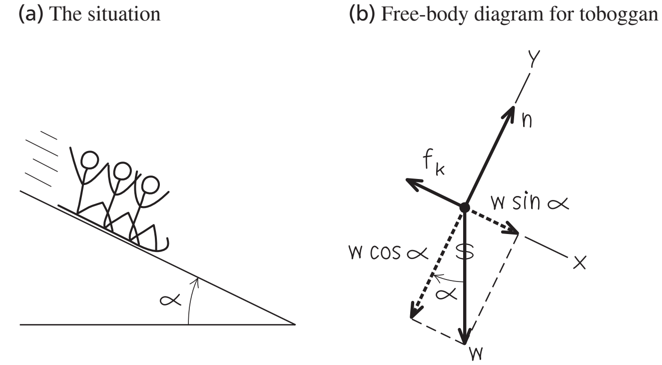
Rearranging these two equations, we get

$$\mu_k n = w\sin\alpha \quad \text{and} \quad n = w\cos\alpha$$

As in Example 5.10, the normal force is *not* equal to the weight. We eliminate $n$ by dividing the first of these equations by the second, with the result

$$\mu_k = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha \quad \text{so} \quad \alpha = \arctan \mu_k$$

Figure **5.22** Our sketches for this problem.




**EVALUATE** The weight $w$ doesn't appear in this expression. *Any* toboggan, regardless of its weight, slides down an incline with constant speed if the coefficient of kinetic friction equals the tangent of the slope angle of the incline. The arctangent function increases as its argument increases, so it's indeed true that the slope angle $\alpha$ for constant speed increases as $\mu_k$ increases.

**KEYCONCEPT** When kinetic friction of magnitude $f_k = \mu_k n$ is present for an object on an incline, the magnitude $n$ of the normal force on the object is *not* equal to the object's weight.

## EXAMPLE 5.17 Toboggan ride with friction II

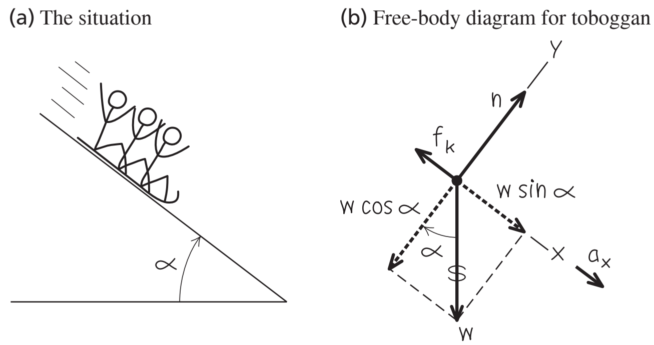
The same toboggan with the same coefficient of friction as in Example 5.16 *accelerates* down a steeper hill. Derive an expression for the acceleration in terms of $g$, $\alpha$, $\mu_k$, and $w$.

**IDENTIFY and SET UP** The toboggan is accelerating, so we must use Newton's second law as given in Eqs. (5.2). Our target variable is the downhill acceleration.

Our sketch and free-body diagram (**Fig. 5.23**, next page) are almost the same as for Example 5.16. The toboggan's $y$-component of acceleration $a_y$ is still zero but the $x$-component $a_x$ is not, so we've drawn $w \sin\alpha$, the downhill component of weight, as a longer vector than the (uphill) friction force.

*Continued*

Figure **5.23** Our sketches for this problem.




**EXECUTE** It's convenient to express the weight as $w = mg$. Then Newton's second law in component form says

$$\sum F_x = mg\sin\alpha + (-f_k) = ma_x$$
$$\sum F_y = n + (-mg\cos\alpha) = 0$$

From the second equation and Eq. (5.3) we get an expression for $f_k$:

$$n = mg\cos\alpha$$
$$f_k = \mu_k n = \mu_k mg\cos\alpha$$

We substitute this into the $x$-component equation and solve for $a_x$:

$$mg\sin\alpha + (-\mu_k mg\cos\alpha) = ma_x$$
$$a_x = g(\sin\alpha - \mu_k\cos\alpha)$$

**EVALUATE** As for the frictionless toboggan in Example 5.10, the acceleration doesn't depend on the mass $m$ of the toboggan. That's because all of the forces that act on the toboggan (weight, normal force, and kinetic friction force) are proportional to $m$.

Let's check some special cases. If the hill is vertical ($\alpha = 90°$) so that $\sin\alpha = 1$ and $\cos\alpha = 0$, we have $a_x = g$ (the toboggan falls freely). For a certain value of $\alpha$ the acceleration is zero; this happens if

$$\sin\alpha = \mu_k\cos\alpha \qquad \text{and} \qquad \mu_k = \tan\alpha$$

This agrees with our result for the constant-velocity toboggan in Example 5.16. If the angle is even smaller, $\mu_k\cos\alpha$ is greater than $\sin\alpha$ and $a_x$ is *negative;* if we give the toboggan an initial downhill push to start it moving, it will slow down and stop. Finally, if the hill is frictionless so that $\mu_k = 0$, we retrieve the result of Example 5.10: $a_x = g\sin\alpha$.

Notice that we started with a simple problem (Example 5.10) and extended it to more and more general situations. The general result we found in this example includes *all* the previous ones as special cases. Don't memorize this result, but do make sure you understand how we obtained it and what it means.

Suppose instead we give the toboggan an initial push *up* the hill. The direction of the kinetic friction force is now reversed, so the acceleration is different from the downhill value. It turns out that the expression for $a_x$ is the same as for downhill motion except that the minus sign becomes plus. Can you show this?

**KEYCONCEPT** The magnitude $f_k = \mu_k n$ of the kinetic friction force is the same whether or not the object is accelerating.

## Rolling Friction

It's a lot easier to move a loaded filing cabinet across a horizontal floor by using a cart with wheels than by sliding it. How much easier? We can define a **coefficient of rolling friction** $\mu_r$, which is the horizontal force needed for constant speed on a flat surface divided by the upward normal force exerted by the surface. Transportation engineers call $\mu_r$ the *tractive resistance*. Typical values of $\mu_r$ are 0.002 to 0.003 for steel wheels on steel rails and 0.01 to 0.02 for rubber tires on concrete. These values show one reason railroad trains are generally much more fuel efficient than highway trucks.

## Fluid Resistance and Terminal Speed

Sticking your hand out the window of a fast-moving car will convince you of the existence of **fluid resistance,** the force that a fluid (a gas or liquid) exerts on an object moving through it. The moving object exerts a force on the fluid to push it out of the way. By Newton's third law, the fluid pushes back on the object with an equal and opposite force.

The *direction* of the fluid resistance force acting on an object is always opposite the direction of the object's velocity relative to the fluid. The *magnitude* of the fluid resistance force usually increases with the speed of the object through the fluid. This is very different from the kinetic friction force between two surfaces in contact, which we can usually regard as independent of speed. For small objects moving at very low speeds, the magnitude $f$ of the fluid resistance force is approximately proportional to the object's speed $v$:

$$f = kv \qquad \text{(fluid resistance at low speed)} \tag{5.5}$$

where $k$ is a proportionality constant that depends on the shape and size of the object and the properties of the fluid. Equation (5.5) is appropriate for dust particles falling in air or a ball bearing falling in oil. For larger objects moving through air at the speed of a tossed tennis ball or faster, the resisting force is approximately proportional to $v^2$ rather than to $v$.

It is then called **air drag** or simply *drag*. Airplanes, falling raindrops, and bicyclists all experience air drag. In this case we replace Eq. (5.5) by

$$f = Dv^2 \qquad \text{(fluid resistance at high speed)} \tag{5.6}$$

Because of the $v^2$ dependence, air drag increases rapidly with increasing speed. The air drag on a typical car is negligible at low speeds but comparable to or greater than rolling resistance at highway speeds. The value of $D$ depends on the shape and size of the object and on the density of the air. You should verify that the units of the constant $k$ in Eq. (5.5) are $\text{N} \cdot \text{s/m}$ or kg/s, and that the units of the constant $D$ in Eq. (5.6) are $\text{N} \cdot \text{s}^2/\text{m}^2$ or kg/m.

Because of the effects of fluid resistance, an object falling in a fluid does *not* have a constant acceleration. To describe its motion, we can't use the constant-acceleration relationships from Chapter 2; instead, we have to start over with Newton's second law. As an example, suppose you drop a metal ball at the surface of a bucket of oil and let it fall to the bottom (**Fig. 5.24a**). The fluid resistance force in this situation is given by Eq. (5.5). What are the acceleration, velocity, and position of the metal ball as functions of time?

Figure 5.24b shows the free-body diagram. We take the positive $y$-direction to be downward and neglect any force associated with buoyancy in the oil. Since the ball is moving downward, its speed $v$ is equal to its $y$-velocity $v_y$ and the fluid resistance force is in the $-y$-direction. There are no $x$-components, so Newton's second law gives

$$\sum F_y = mg + (-kv_y) = ma_y \tag{5.7}$$

When the ball first starts to move, $v_y = 0$, the resisting force is zero and the initial acceleration is $a_y = g$. As the speed increases, the resisting force also increases, until finally it is equal in magnitude to the weight. At this time $mg - kv_y = 0$, the acceleration is zero, and there is no further increase in speed. The final speed $v_t$, called the **terminal speed,** is given by $mg - kv_t = 0$, or

$$v_t = \frac{mg}{k} \qquad \text{(terminal speed, fluid resistance } f = kv\text{)} \tag{5.8}$$

**Figure 5.25** shows how the acceleration, velocity, and position vary with time. As time goes by, the acceleration approaches zero and the velocity approaches $v_t$ (remember that we chose the positive $y$-direction to be down). The slope of the graph of $y$ versus $t$ becomes constant as the velocity becomes constant.

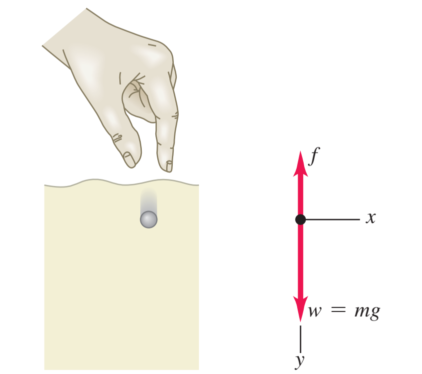
To see how the graphs in Fig. 5.25 are derived, we must find the relationship between velocity and time during the interval before the terminal speed is reached. We go back to Newton's second law for the falling ball, Eq. (5.7), which we rewrite with $a_y = dv_y/dt$:

$$m\frac{dv_y}{dt} = mg - kv_y$$

Figure **5.24** Motion with fluid resistance.

(a) Metal ball falling through oil

(b) Free-body diagram for ball in oil




**BIO APPLICATION Pollen and Fluid Resistance** These spiky spheres are pollen grains from the ragweed flower (*Ambrosia artemisiifolia*) and a common cause of hay fever. Because of their small radius (about 10 $\mu$m = 0.01 mm), when they are released into the air the fluid resistance force on them is proportional to their speed. The terminal speed given by Eq. (5.8) is only about 1 cm/s. Hence even a moderate wind can keep pollen grains aloft and carry them substantial distances from their source.

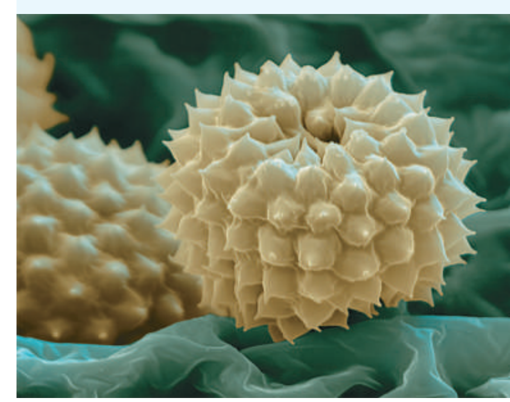


Figure **5.25** Graphs of the motion of an object falling without fluid resistance and with fluid resistance proportional to the speed.

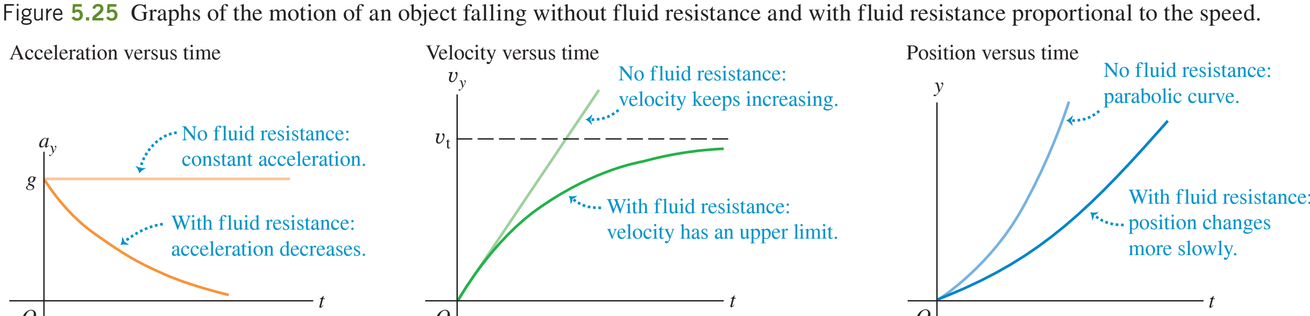


Figure **5.26** (a) Air drag and terminal speed. (b) By changing the positions of their arms and legs while falling, skydivers can change the value of the constant $D$ in Eq. (5.6) and hence adjust the terminal speed of their fall [Eq. (5.12)].

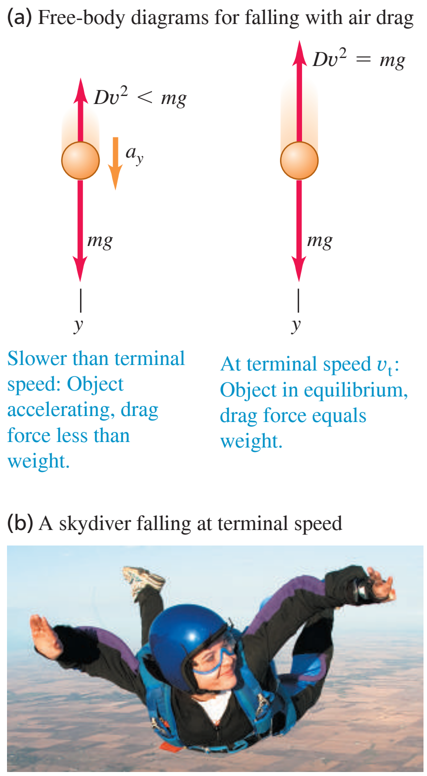
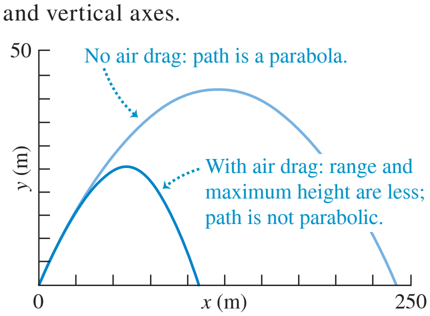



Figure **5.27** Computer-generated trajectories of a baseball launched at 50 m/s at 35° above the horizontal. Note that the scales are different on the horizontal and vertical axes.




After rearranging terms and replacing $mg/k$ by $v_t$, we integrate both sides, noting that $v_y = 0$ when $t = 0$:

$$\int_0^v \frac{dv_y}{v_y - v_t} = -\frac{k}{m}\int_0^t dt$$

which integrates to

$$\ln\frac{v_t - v_y}{v_t} = -\frac{k}{m}t \qquad \text{or} \qquad 1 - \frac{v_y}{v_t} = e^{-(k/m)t}$$

and finally

$$v_y = v_t\left[1 - e^{-(k/m)t}\right] \tag{5.9}$$

Note that $v_y$ becomes equal to the terminal speed $v_t$ only in the limit that $t \rightarrow \infty$; the ball cannot attain terminal speed in any finite length of time.

The derivative of $v_y$ in Eq. (5.9) gives $a_y$ as a function of time, and the integral of $v_y$ gives $y$ as a function of time. We leave the derivations for you to complete; the results are

$$a_y = ge^{-(k/m)t} \tag{5.10}$$

$$y = v_t\left[t - \frac{m}{k}\left(1 - e^{-(k/m)t}\right)\right] \tag{5.11}$$

Now look again at Fig. 5.25, which shows graphs of these three relationships.

In deriving the terminal speed in Eq. (5.8), we assumed that the fluid resistance force is proportional to the speed. For an object falling through the air at high speeds, so that the fluid resistance is equal to $Dv^2$ as in Eq. (5.6), the terminal speed is reached when $Dv^2$ equals the weight $mg$ (**Fig. 5.26a**). You can show that the terminal speed $v_t$ is given by

$$v_t = \sqrt{\frac{mg}{D}} \qquad \text{(terminal speed, fluid resistance } f = Dv^2\text{)} \tag{5.12}$$

This expression for terminal speed explains why heavy objects in air tend to fall faster than light objects. Two objects that have the same physical size but different mass (say, a table-tennis ball and a lead ball with the same radius) have the same value of $D$ but different values of $m$. The more massive object has a higher terminal speed and falls faster. The same idea explains why a sheet of paper falls faster if you first crumple it into a ball; the mass $m$ is the same, but the smaller size makes $D$ smaller (less air drag for a given speed) and $v_t$ larger. Skydivers use the same principle to control their descent (Fig. 5.26b).

**Figure 5.27** shows the trajectories of a baseball with and without air drag, assuming a coefficient $D = 1.3 \times 10^{-3}$ kg/m (appropriate for a batted ball at sea level). Both the range of the baseball and the maximum height reached are substantially smaller than the zero-drag calculation would lead you to believe. Hence the baseball trajectory we calculated in Example 3.7 (Section 3.3) by ignoring air drag is unrealistic. Air drag is an important part of the game of baseball!

## EXAMPLE 5.18 Terminal speed of a skydiver

For a human body falling through air in a spread-eagle position (Fig. 5.26b), the numerical value of the constant $D$ in Eq. (5.6) is about 0.25 kg/m. Find the terminal speed for a 50 kg skydiver.

**IDENTIFY and SET UP** This example uses the relationship among terminal speed, mass, and drag coefficient. We use Eq. (5.12) to find the target variable $v_t$.

**EXECUTE** We find for $m = 50$ kg:

$$v_t = \sqrt{\frac{mg}{D}} = \sqrt{\frac{(50\text{ kg})(9.8\text{ m/s}^2)}{0.25\text{ kg/m}}}$$

$$= 44\text{ m/s (about 160 km/h, or 99 mi/h)}$$

**EVALUATE** The terminal speed is proportional to the square root of the skydiver's mass. A skydiver with the same drag coefficient $D$ but twice the mass would have a terminal speed $\sqrt{2} = 1.41$ times greater, or 63 m/s. (A more massive skydiver would also have more frontal area and hence a larger drag coefficient, so his terminal speed would be a bit less than 63 m/s.) Even the 50 kg skydiver's terminal speed is quite high, so skydives don't last very long. A drop from 2800 m (9200 ft) to the surface at the terminal speed takes only $(2800\ \text{m})/(44\ \text{m/s}) = 64\ \text{s}$.

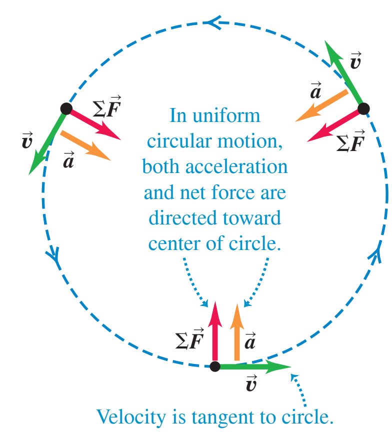
When the skydiver deploys the parachute, the value of $D$ increases greatly. Hence the terminal speed of the skydiver with parachute decreases dramatically to a much lower value.

**KEYCONCEPT** A falling object reaches its terminal speed when the upward force of fluid resistance equals the downward force of gravity. Depending on the object's speed, use either Eq. (5.8) or Eq. (5.12) to find the terminal speed.

**TEST YOUR UNDERSTANDING OF SECTION 5.3** Consider a box that is placed on different surfaces. (a) In which situation(s) is *no* friction force acting on the box? (b) In which situation(s) is a *static* friction force acting on the box? (c) In which situation(s) is a *kinetic* friction force acting on the box? (i) The box is at rest on a rough horizontal surface. (ii) The box is at rest on a rough tilted surface. (iii) The box is on the rough-surfaced flat bed of a truck that is moving at a constant velocity on a straight, level road, and the box remains in place in the middle of the truck bed. (iv) The box is on the rough-surfaced flat bed of a truck that is speeding up on a straight, level road, and the box remains in place in the middle of the truck bed. (v) The box is on the rough-surfaced flat bed of a truck that is climbing a hill, and the box is sliding toward the back of the truck.

**ANSWER** (a): **(i)**, **(iii)**; (b): **(ii)**, **(iv)**; (c): **(v)** In situations (i) and (iii) the box is not accelerating (so the net force on it must be zero) and no other force is acting parallel to the horizontal surface; hence no friction force is needed to prevent sliding. In situations (ii) and (iv) the box would start to slide over the surface if no friction were present, so a static friction force must act to prevent this. In situation (v) the box is sliding over a rough surface, so a kinetic friction force acts on it.

Figure **5.28** Net force, acceleration, and velocity in uniform circular motion.




## 5.4 DYNAMICS OF CIRCULAR MOTION

We talked about uniform circular motion in Section 3.4. We showed that when a particle moves in a circular path with constant speed, the particle's acceleration has a constant magnitude $a_{\text{rad}}$ given by

$$a_{\text{rad}} = \frac{v^2}{R} \tag{5.13}$$

Magnitude of acceleration of an object in **uniform circular motion**; Speed of object; Radius of object's circular path

The subscript "rad" is a reminder that at each point the acceleration points radially inward toward the center of the circle, perpendicular to the instantaneous velocity. We explained in Section 3.4 why this acceleration is often called *centripetal acceleration* or *radial acceleration*.

We can also express the centripetal acceleration $a_{\text{rad}}$ in terms of the *period* $T$, the time for one revolution:

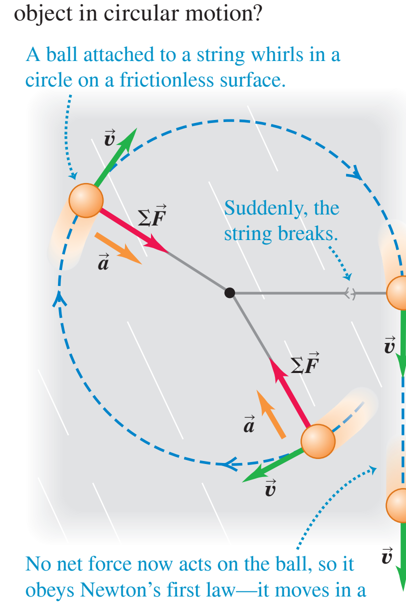
$$T = \frac{2\pi R}{v} \tag{5.14}$$

In terms of the period, $a_{\text{rad}}$ is

$$a_{\text{rad}} = \frac{4\pi^2 R}{T^2} \tag{5.15}$$

Magnitude of acceleration of an object in **uniform circular motion**; Radius of object's circular path; Period of motion

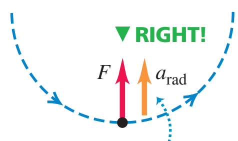
Figure **5.29** What happens if the inward radial force suddenly ceases to act on an object in circular motion?



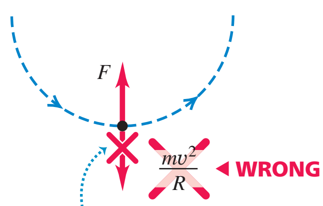

Uniform circular motion, like all other motion of a particle, is governed by Newton's second law. To make the particle accelerate toward the center of the circle, the net force $\Sigma\vec{F}$ on the particle must always be directed toward the center (**Fig. 5.28**). The magnitude of the acceleration is constant, so the magnitude $F_{\text{net}}$ of the net force must also be constant. If the inward net force stops acting, the particle flies off in a straight line tangent to the circle (**Fig. 5.29**).

Figure 5.30 Right and wrong ways to depict uniform circular motion.

(a) Correct free-body diagram




(b) Incorrect free-body diagram




The magnitude of the radial acceleration is given by $a_{rad} = v^2/R$, so the magnitude $F_{net}$ of the net force on a particle with mass $m$ in uniform circular motion must be

$$F_{net} = ma_{rad} = m\frac{v^2}{R} \qquad \text{(uniform circular motion)} \qquad (5.16)$$

Uniform circular motion can result from *any* combination of forces, just so the net force $\sum\vec{F}$ is always directed toward the center of the circle and has a constant magnitude. Note that the object need not move around a complete circle: Equation (5.16) is valid for *any* path that can be regarded as part of a circular arc.

**CAUTION** **Avoid using "centrifugal force"** **Figure 5.30** shows a correct free-body diagram for uniform circular motion (Fig. 5.30a) and an *incorrect* diagram (Fig. 5.30b). Figure 5.30b is incorrect because it includes an extra outward force of magnitude $m(v^2/R)$ to "keep the object out there" or to "keep it in equilibrium." There are three reasons not to include such an outward force, called *centrifugal force* ("centrifugal" means "fleeing from the center"). First, the object does *not* "stay out there": It is in constant motion around its circular path. Because its velocity is constantly changing in direction, the object accelerates and is *not* in equilibrium. Second, if there *were* an outward force that balanced the inward force, the net force would be zero and the object would move in a straight line, not a circle (Fig. 5.29). Third, the quantity $m(v^2/R)$ is *not* a force; it corresponds to the $m\vec{a}$ side of $\sum\vec{F} = m\vec{a}$ and does not appear in $\sum\vec{F}$ (Fig. 5.30a). It's true that when you ride in a car that goes around a circular path, you tend to slide to the outside of the turn as though there was a "centrifugal force." But we saw in Section 4.2 that what happens is that you tend to keep moving in a straight line, and the outer side of the car "runs into" you as the car turns (Fig. 4.10c). *In an inertial frame of reference there is no such thing as "centrifugal force."* We won't mention this term again, and we strongly advise you to avoid it.

## EXAMPLE 5.19 Force in uniform circular motion

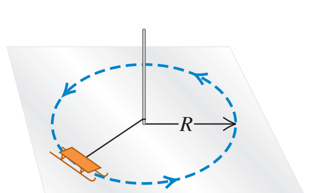
A sled with a mass of 25.0 kg rests on a horizontal sheet of essentially frictionless ice. It is attached by a 5.00 m rope to a post set in the ice. Once given a push, the sled revolves uniformly in a circle around the post (**Fig. 5.31a**). If the sled makes five complete revolutions every minute, find the force $F$ exerted on it by the rope.

**IDENTIFY and SET UP** The sled is in uniform circular motion, so it has a constant radial acceleration. We'll apply Newton's second law to the sled to find the magnitude $F$ of the force exerted by the rope (our target variable).

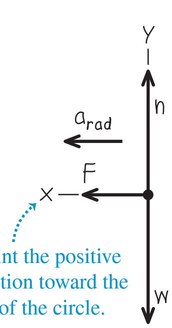
Figure 5.31b shows our free-body diagram for the sled. The acceleration has only an $x$-component; this is toward the center of the circle, so we denote it as $a_{rad}$. The acceleration isn't given, so we'll need to determine its value by using Eq. (5.13) or Eq. (5.15).

Figure 5.31 (a) The situation. (b) Our free-body diagram.

(a) A sled in uniform circular motion




(b) Free-body diagram for sled




**EXECUTE** The force $F$ appears in Newton's second law for the $x$-direction:

$$\sum F_x = F = ma_{rad}$$

We can find the centripetal acceleration $a_{rad}$ by using Eq. (5.15). The sled moves in a circle of radius $R = 5.00\text{ m}$ with a period $T = (60.0\text{ s})/(5\text{ rev}) = 12.0\text{ s}$, so

$$a_{rad} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(5.00\text{ m})}{(12.0\text{ s})^2} = 1.37\text{ m/s}^2$$

The magnitude $F$ of the force exerted by the rope is then

$$F = ma_{rad} = (25.0\text{ kg})(1.37\text{ m/s}^2) = 34.3\text{ kg}\cdot\text{m/s}^2 = 34.3\text{ N}$$

**EVALUATE** You can check our value for $a_{rad}$ by first using Eq. (5.14), $v = 2\pi R/T$, to find the speed and then using $a_{rad} = v^2/R$ from Eq. (5.13). Do you get the same result?

A greater force would be needed if the sled moved around the circle at a higher speed $v$. In fact, if $v$ were doubled while $R$ remained the same, $F$ would be four times greater. Can you show this? How would $F$ change if $v$ remained the same but the radius $R$ were doubled?

**KEYCONCEPT** In problems that involve forces on an object in uniform circular motion, take the positive $x$-direction to be toward the center of the circle. The net force component in that direction is equal to the object's mass times its radial acceleration.

## EXAMPLE 5.20 A conical pendulum

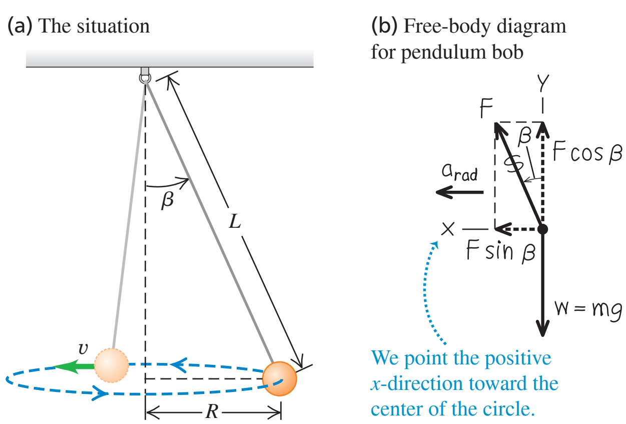
WITH VARIATION PROBLEMS

An inventor designs a pendulum clock using a bob with mass $m$ at the end of a thin wire of length $L$. Instead of swinging back and forth, the bob is to move in a horizontal circle at constant speed $v$, with the wire making a fixed angle $\beta$ with the vertical direction (**Fig. 5.32a**). This is called a *conical pendulum* because the suspending wire traces out a cone. Find the tension $F$ in the wire and the period $T$ (the time for one revolution of the bob).

**IDENTIFY and SET UP** To find our target variables, the tension $F$ and period $T$, we need two equations. These will be the horizontal and vertical components of Newton's second law applied to the bob. We'll find the radial acceleration of the bob from one of the circular motion equations.

Figure 5.32b shows our free-body diagram and coordinate system for the bob at a particular instant. There are just two forces on the bob: the weight $mg$ and the tension $F$ in the wire. Note that the center of the circular path is in the same horizontal plane as the bob, *not* at the top end of the wire. The horizontal component of tension is the force that produces the radial acceleration $a_{rad}$.

Figure **5.32** (a) The situation. (b) Our free-body diagram.




**EXECUTE** The bob has zero vertical acceleration; the horizontal acceleration is toward the center of the circle, which is why we use the symbol $a_{rad}$. Newton's second law, Eqs. (5.2), says

$$\sum F_x = F\sin\beta = ma_{rad}$$

$$\sum F_y = F\cos\beta + (-mg) = 0$$

These are two equations for the two unknowns $F$ and $\beta$. The equation for $\sum F_y$ gives $F = mg/\cos\beta$; that's our target expression for $F$ in terms of $\beta$. Substituting this result into the equation for $\sum F_x$ and using $\sin\beta/\cos\beta = \tan\beta$, we find

$$a_{rad} = g\tan\beta$$

To relate $\beta$ to the period $T$, we use Eq. (5.15) for $a_{rad}$, solve for $T$, and insert $a_{rad} = g\tan\beta$:

$$a_{rad} = \frac{4\pi^2 R}{T^2} \quad \text{so} \quad T^2 = \frac{4\pi^2 R}{a_{rad}}$$

$$T = 2\pi\sqrt{\frac{R}{g\tan\beta}}$$

Figure 5.32a shows that $R = L\sin\beta$. We substitute this and use $\sin\beta/\tan\beta = \cos\beta$:

$$T = 2\pi\sqrt{\frac{L\cos\beta}{g}}$$

**EVALUATE** For a given length $L$, as the angle $\beta$ increases, $\cos\beta$ decreases, the period $T$ becomes smaller, and the tension $F = mg/\cos\beta$ increases. The angle can never be 90°, however; this would require that $T = 0$, $F = \infty$, and $v = \infty$. A conical pendulum would not make a very good clock because the period depends on the angle $\beta$ in such a direct way.

**KEYCONCEPT** In uniform circular motion, any kind of force (or component of a force) can produce the radial acceleration.

## EXAMPLE 5.21 Rounding a flat curve

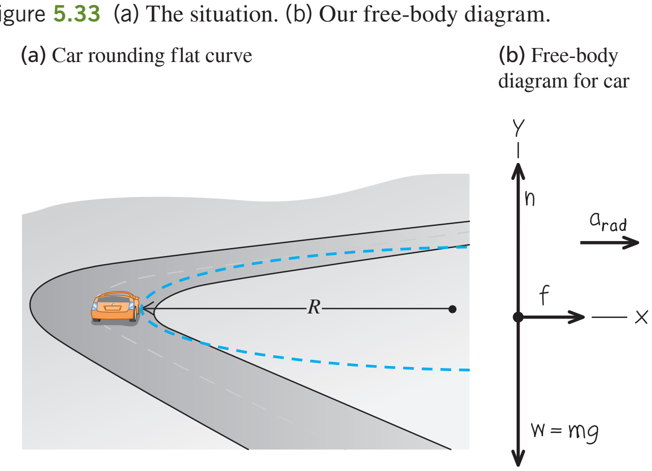
WITH VARIATION PROBLEMS

The sports car in Example 3.11 (Section 3.4) is rounding a flat, unbanked curve with radius $R$ (**Fig. 5.33a**). If the coefficient of static friction between tires and road is $\mu_s$, what is the maximum speed $v_{max}$ at which the driver can take the curve without sliding?

**IDENTIFY and SET UP** The car's acceleration as it rounds the curve has magnitude $a_{rad} = v^2/R$. Hence the maximum speed $v_{max}$ (our target variable) corresponds to the maximum acceleration $a_{rad}$ and to the maximum horizontal force on the car toward the center of its circular path. The only horizontal force acting on the car is the friction force exerted by the road. So to solve this problem we'll need Newton's second law, the equations of uniform circular motion, and our knowledge of the friction force from Section 5.3.

The free-body diagram in Fig. 5.33b includes the car's weight $w = mg$ and the two forces exerted by the road: the normal force $n$ and the horizontal friction force $f$. The friction force must point toward the center of the circular path in order to cause the radial acceleration. The car doesn't slide toward or away from the center of the circle, so the friction force is *static* friction, with a maximum magnitude $f_{max} = \mu_s n$ [see Eq. (5.4)].

Figure **5.33** (a) The situation. (b) Our free-body diagram.




*Continued*

**EXECUTE** The acceleration toward the center of the circular path is $a_{rad} = v^2/R$. There is no vertical acceleration. Thus

$$\sum F_x = f = ma_{rad} = m\frac{v^2}{R}$$

$$\sum F_y = n + (-mg) = 0$$

The second equation shows that $n = mg$. The first equation shows that the friction force *needed* to keep the car moving in its circular path increases with the car's speed. But the maximum friction force *available* is $f_{max} = \mu_s n = \mu_s mg$, and this determines the car's maximum speed. Substituting $\mu_s mg$ for $f$ and $v_{max}$ for $v$ in the first equation, we find

$$\mu_s mg = m\frac{v_{max}{}^2}{R} \qquad \text{so} \qquad v_{max} = \sqrt{\mu_s gR}$$

As an example, if $\mu_s = 0.96$ and $R = 230$ m, we have

$$v_{max} = \sqrt{(0.96)(9.8\ \text{m/s}^2)(230\ \text{m})} = 47\ \text{m/s}$$

or about 170 km/h (100 mi/h). This is the maximum speed for this radius.

**EVALUATE** If the car's speed is slower than $v_{max} = \sqrt{\mu_s gR}$, the required friction force is less than the maximum value $f_{max} = \mu_s mg$, and the car can easily make the curve. If we try to take the curve going *faster* than $v_{max}$, we'll skid. We could still go in a circle without skidding at this higher speed, but the radius would have to be larger.

The maximum centripetal acceleration (called the "lateral acceleration" in Example 3.11) is equal to $\mu_s g$. That's why it's best to take curves at less than the posted speed limit if the road is wet or icy, either of which can reduce the value of $\mu_s$ and hence $\mu_s g$.

**KEYCONCEPT** For a vehicle following a curved path on a level road, the radial acceleration is produced by the static friction force exerted by the road on the vehicle.

## EXAMPLE 5.22 Rounding a banked curve

WITH VARIATION PROBLEMS

For a car traveling at a certain speed, it is possible to bank a curve at just the right angle so that no friction is needed to maintain the car's turning radius. Then a car can safely round the curve even on wet ice. (Bobsled racing depends on this idea.) Your engineering firm plans to rebuild the curve in Example 5.21 so that a car moving at a chosen speed $v$ can safely make the turn even with no friction (**Fig. 5.34a**). At what angle $\beta$ should the curve be banked?

**IDENTIFY and SET UP** With no friction, the only forces acting on the car are its weight and the normal force. Because the road is banked, the normal force (which acts perpendicular to the road surface) has a horizontal component. This component causes the car's horizontal acceleration toward the center of the car's circular path. We'll use Newton's second law to find the target variable $\beta$.

Our free-body diagram (Fig. 5.34b) is very similar to the diagram for the conical pendulum in Example 5.20 (Fig. 5.32b). The normal force acting on the car plays the role of the tension force exerted by the wire on the pendulum bob.

**EXECUTE** The normal force $\vec{n}$ is perpendicular to the roadway and is at an angle $\beta$ with the vertical (Fig. 5.34b). Thus it has a vertical component $n\cos\beta$ and a horizontal component $n\sin\beta$. The acceleration in the $x$-direction is the centripetal acceleration $a_{rad} = v^2/R$; there is no acceleration in the $y$-direction. Thus the equations of Newton's second law are

$$\sum F_x = n\sin\beta = ma_{rad}$$

$$\sum F_y = n\cos\beta + (-mg) = 0$$

From the $\sum F_y$ equation, $n = mg/\cos\beta$. Substituting this into the $\sum F_x$ equation and using $a_{rad} = v^2/R$, we get an expression for the bank angle:

$$\tan\beta = \frac{a_{rad}}{g} = \frac{v^2}{gR} \qquad \text{so} \qquad \beta = \arctan\frac{v^2}{gR}$$

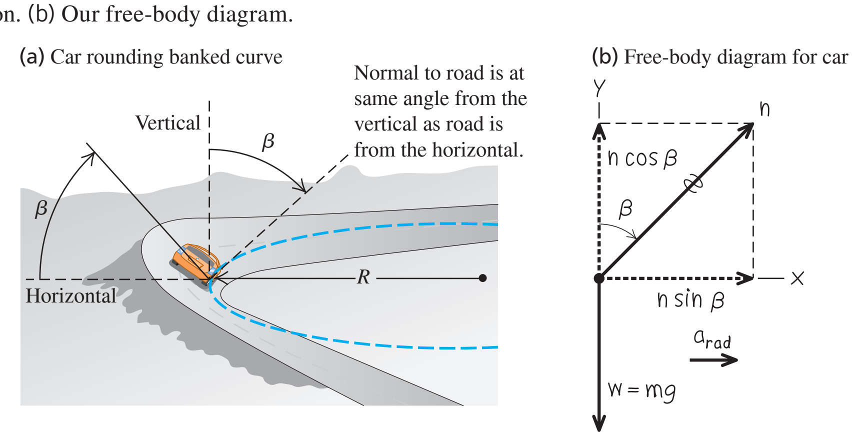
**EVALUATE** The bank angle depends on both the speed and the radius. For a given radius, no one angle is correct for all speeds. In the design of highways and railroads, curves are often banked for the average speed of the traffic over them. If $R = 230$ m and $v = 25$ m/s (equal to a highway speed of 88 km/h, or 55 mi/h), then

$$\beta = \arctan\frac{(25\ \text{m/s})^2}{(9.8\ \text{m/s}^2)(230\ \text{m})} = 15°$$

This is within the range of bank angles actually used in highways.

**KEYCONCEPT** The normal force exerted by the road on a vehicle can provide the radial acceleration needed for the vehicle to follow a curved path, provided the road is banked at the correct angle for the vehicle's speed.

Figure 5.34 (a) The situation. (b) Our free-body diagram.

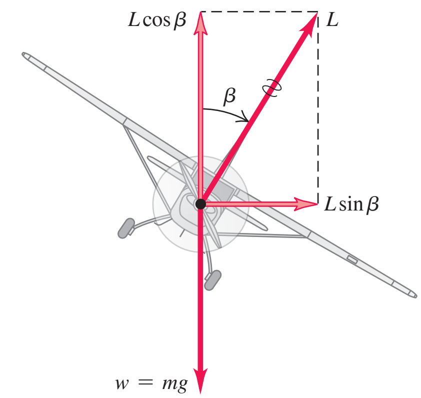


## Banked Curves and the Flight of Airplanes

The results of Example 5.22 also apply to an airplane when it makes a turn in level flight (**Fig. 5.35**). When an airplane is flying in a straight line at a constant speed and at a steady altitude, the airplane's weight is exactly balanced by the lift force $\vec{L}$ exerted by the air. (The upward lift force that the air exerts on the wings is a reaction to the downward push the wings exert on the air as they move through it.) To make the airplane turn, the pilot banks the airplane to one side so that the lift force has a horizontal component, as Fig. 5.35 shows. (The pilot also changes the angle at which the wings "bite" into the air so that the vertical component of lift continues to balance the weight.) The bank angle is related to the airplane's speed $v$ and the radius $R$ of the turn by the same expression as in Example 5.22: $\tan\beta = v^2/gR$. For an airplane to make a tight turn (small $R$) at high speed (large $v$), $\tan\beta$ must be large and the required bank angle $\beta$ must approach 90°.

We can also apply the results of Example 5.22 to the *pilot* of an airplane. The free-body diagram for the pilot of the airplane is exactly as shown in Fig. 5.34b; the normal force $n = mg/\cos\beta$ is exerted on the pilot by the seat. As in Example 5.9, $n$ is equal to the apparent weight of the pilot, which is greater than the pilot's true weight $mg$. In a tight turn with a large bank angle $\beta$, the pilot's apparent weight can be tremendous: $n = 5.8mg$ at $\beta = 80°$ and $n = 9.6mg$ at $\beta = 84°$. Pilots black out in such tight turns because the apparent weight of their blood increases by the same factor, and the human heart isn't strong enough to pump such apparently "heavy" blood to the brain.

Figure 5.35 An airplane banks to one side in order to turn in that direction. The vertical component of the lift force $\vec{L}$ balances the force of gravity; the horizontal component of $\vec{L}$ causes the acceleration $v^2/R$.




## Motion in a Vertical Circle

In Examples 5.19, 5.20, 5.21, and 5.22 the object moved in a horizontal circle. Motion in a *vertical* circle is no different in principle, but the weight of the object has to be treated carefully. The following example shows what we mean.

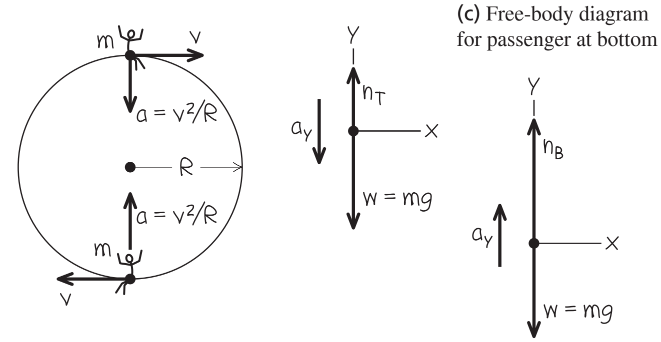
### EXAMPLE 5.23 Uniform circular motion in a vertical circle

A passenger on a carnival Ferris wheel moves in a vertical circle of radius $R$ with constant speed $v$. The seat remains upright during the motion. Find expressions for the force the seat exerts on the passenger when at the top of the circle and when at the bottom.

**IDENTIFY and SET UP** The target variables are $n_T$, the upward normal force the seat applies to the passenger at the top of the circle, and $n_B$, the normal force at the bottom. We'll find these by using Newton's second law and the uniform circular motion equations.

**Figure 5.36a** shows the passenger's velocity and acceleration at the two positions. The acceleration always points toward the center of the circle—downward at the top of the circle and upward at the bottom of the circle. At each position the only forces acting are vertical: the upward normal force and the downward force of gravity. Hence we need only the vertical component of Newton's second law. Figures 5.36b and 5.36c show free-body diagrams for the two positions. We take the positive $y$-direction as upward in both cases (that is, *opposite* the direction of the acceleration at the top of the circle).

Figure 5.36 Our sketches for this problem.




**EXECUTE** At the top the acceleration has magnitude $v^2/R$, but its vertical component is negative because its direction is downward. Hence $a_y = -v^2/R$ and Newton's second law tells us that

$$\text{Top:}\quad \sum F_y = n_T + (-mg) = -m\frac{v^2}{R} \quad \text{or}$$

$$n_T = mg\left(1 - \frac{v^2}{gR}\right)$$

At the bottom the acceleration is upward, so $a_y = +v^2/R$ and Newton's second law says

$$\text{Bottom:}\quad \sum F_y = n_B + (-mg) = +m\frac{v^2}{R} \quad \text{or}$$

$$n_B = mg\left(1 + \frac{v^2}{gR}\right)$$

**EVALUATE** Our result for $n_T$ tells us that at the top of the Ferris wheel, the upward force the seat applies to the passenger is *smaller* in magnitude than the passenger's weight $w = mg$. If the ride goes fast enough that $g - v^2/R$ becomes zero, the seat applies *no* force, and the passenger is about to become airborne. If $v$ becomes still larger, $n_T$ becomes negative; this means that a *downward* force (such as from a seat belt) is needed to keep the passenger in the seat. By contrast, the normal force

*Continued*

$n_B$ at the bottom is always *greater* than the passenger's weight. You feel the seat pushing up on you more firmly than when you are at rest. You can see that $n_T$ and $n_B$ are the values of the passenger's *apparent weight* at the top and bottom of the circle (see Section 5.2).

**KEYCONCEPT** Even when an object moves with varying speed along a circular path, at any point along the path the net force component toward the center of the circle equals the object's mass times its radial acceleration.

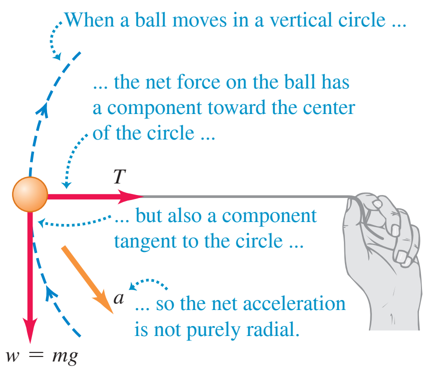



Figure 5.37 A ball moving in a vertical circle.

When we tie a string to an object and whirl it in a vertical circle, the analysis in Example 5.23 isn't directly applicable. The reason is that $v$ is *not* constant in this case; except at the top and bottom of the circle, the net force (and hence the acceleration) does *not* point toward the center of the circle (**Fig. 5.37**). So both $\sum \vec{F}$ and $\vec{a}$ have a component tangent to the circle, which means that the speed changes. Hence this is a case of *nonuniform* circular motion (see Section 3.4). Even worse, we can't use the constant-acceleration formulas to relate the speeds at various points because *neither* the magnitude nor the direction of the acceleration is constant. The speed relationships we need are best obtained by using the concept of energy. We'll consider such problems in Chapter 7.

**TEST YOUR UNDERSTANDING OF SECTION 5.4** Satellites are held in orbit by the force of our planet's gravitational attraction. A satellite in a small-radius orbit moves at a higher speed than a satellite in an orbit of large radius. Based on this information, what can you conclude about the earth's gravitational attraction for the satellite? (i) It increases with increasing distance from the earth. (ii) It is the same at all distances from the earth. (iii) It decreases with increasing distance from the earth. (iv) This information by itself isn't enough to answer the question.

**ANSWER**

**(iii)** A satellite of mass $m$ orbiting the earth at speed $v$ in an orbit of radius $r$ has an acceleration of magnitude $v^2/r$, so the net force acting on it from the earth's gravity has magnitude $F = mv^2/r$. The farther the satellite is from the earth, the greater the value of $r$, the smaller the value of $v$, and hence the smaller the values of $v^2/r$ and of $F$. In other words, the earth's gravitational force decreases with increasing distance.

**BIO APPLICATION Circular Motion in a Centrifuge** An important tool in medicine and biological research is the ultracentrifuge, a device that makes use of the dynamics of circular motion. A tube is filled with a solvent that contains various small particles (for example, blood containing platelets and white and red blood cells). The tube is inserted into the centrifuge, which then spins at thousands of revolutions per minute. The solvent provides the inward force that keeps the particles in circular motion. The particles slowly drift away from the rotation axis within the solvent. Because the drift rate depends on the particle size and density, particles of different types become separated in the tube, making analysis much easier.

## 5.5 THE FUNDAMENTAL FORCES OF NATURE

We have discussed several kinds of forces—including weight, tension, friction, fluid resistance, and the normal force—and we'll encounter others as we continue our study of physics. How many kinds of forces are there? Our best understanding is that all forces are expressions of just four distinct classes of *fundamental* forces, or interactions between particles (**Fig. 5.38**). Two are familiar in everyday experience. The other two involve interactions between subatomic particles that we cannot observe with the unaided senses.

**Gravitational interactions** include the familiar force of your *weight,* which results from the earth's gravitational attraction acting on you. The mutual gravitational attraction of various parts of the earth for each other holds our planet together, and likewise for the other planets (Fig. 5.38a). Newton recognized that the sun's gravitational attraction for the earth keeps our planet in its nearly circular orbit around the sun. In Chapter 13 we'll study gravitational interactions in more detail, including their vital role in the motions of planets and satellites.

The second familiar class of forces, **electromagnetic interactions,** includes electric and magnetic forces. If you run a comb through your hair, the comb ends up with an electric charge; you can use the electric force exerted by this charge to pick up bits of paper. All atoms contain positive and negative electric charge, so atoms and molecules can exert electric forces on one another. Contact forces, including the normal force, friction, and fluid resistance, are the result of electrical interactions between atoms on the surface of an object and atoms in its surroundings (Fig. 5.38b). *Magnetic* forces, such as those between magnets or between a magnet and a piece of iron, are actually the result of electric charges in motion. For example, an electromagnet causes magnetic interactions because electric charges move through its wires. We'll study electromagnetic interactions in detail in the second half of this book.

On the atomic or molecular scale, gravitational forces play no role because electric forces are enormously stronger: The electrical repulsion between two protons is stronger than their gravitational attraction by a factor of about $10^{35}$. But in objects of astronomical


Figure **5.38** Examples of the fundamental interactions in nature.

size, positive and negative charges are usually present in nearly equal amounts, and the resulting electrical interactions nearly cancel out. Gravitational interactions are thus the dominant influence in the motion of planets and in the internal structure of stars.

The other two classes of interactions are less familiar. One, the **strong interaction,** is responsible for holding the nucleus of an atom together (Fig. 5.38c). Nuclei contain electrically neutral neutrons and positively charged protons. The electric force between charged protons tries to push them apart; the strong attractive force between nuclear particles counteracts this repulsion and makes the nucleus stable. In this context the strong interaction is also called the *strong nuclear force*. It has much shorter range than electrical interactions, but within its range it is much stronger. Without the strong interaction, the nuclei of atoms essential to life, such as carbon (six protons, six neutrons) and oxygen (eight protons, eight neutrons), would not exist and you would not be reading these words!

Finally, there is the **weak interaction.** Its range is so short that it plays a role only on the scale of the nucleus or smaller. The weak interaction is responsible for a common form of radioactivity called beta decay, in which a neutron in a radioactive nucleus is transformed into a proton while ejecting an electron and a nearly massless particle called an antineutrino. The weak interaction between the antineutrino and ordinary matter is so feeble that an antineutrino could easily penetrate a wall of lead a million kilometers thick!

An important application of the weak interaction is *radiocarbon dating,* a technique that enables scientists to determine the ages of many biological specimens (Fig. 5.38d). Naturally occurring carbon includes atoms of both carbon-12 (with six protons and six neutrons in the nucleus) and carbon-14 (with two additional neutrons). Living organisms take in carbon atoms of both kinds from their environment but stop doing so when they die. The weak interaction makes carbon-14 nuclei unstable—one of the neutrons changes to a proton, an electron, and an antineutrino—and these nuclei decay at a known rate. By measuring the fraction of carbon-14 that is left in an organism's remains, scientists can determine how long ago the organism died.

In the 1960s physicists developed a theory that described the electromagnetic and weak interactions as aspects of a single *electroweak* interaction. This theory has passed every experimental test to which it has been put. Encouraged by this success, physicists have made similar attempts to describe the strong, electromagnetic, and weak interactions in terms of a single *grand unified theory* (GUT) and have taken steps toward a possible unification of all interactions into a *theory of everything* (TOE). Such theories are still speculative, and there are many unanswered questions in this very active field of current research.

# CHAPTER 5 SUMMARY

**Using Newton's first law:** When an object is in equilibrium in an inertial frame of reference—that is, either at rest or moving with constant velocity—the vector sum of forces acting on it must be zero (Newton's first law). Free-body diagrams are essential in identifying the forces that act on the object being considered.

Newton's third law (action and reaction) is also frequently needed in equilibrium problems. The two forces in an action–reaction pair *never* act on the same object. (See Examples 5.1–5.5.)

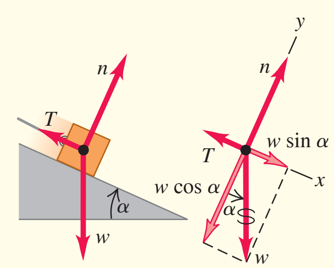
The normal force exerted on an object by a surface is *not* always equal to the object's weight. (See Example 5.4.)

Vector form:

$$\sum \vec{F} = \mathbf{0} \tag{5.1}$$

Component form:

$$\sum F_x = 0 \quad \sum F_y = 0$$




**Using Newton's second law:** If the vector sum of forces on an object is *not* zero, the object accelerates. The acceleration is related to the net force by Newton's second law.

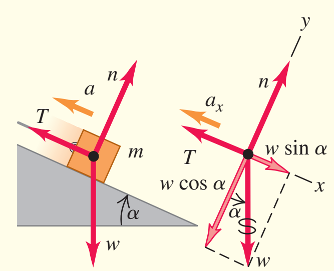
Just as for equilibrium problems, free-body diagrams are essential for solving problems involving Newton's second law, and the normal force exerted on an object is not always equal to its weight. (See Examples 5.6–5.12.)

Vector form:

$$\sum \vec{F} = m\vec{a} \tag{5.2}$$

Component form:

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$




**Friction and fluid resistance:** The contact force between two objects can always be represented in terms of a normal force $\vec{n}$ perpendicular to the surface of contact and a friction force $\vec{f}$ parallel to the surface.

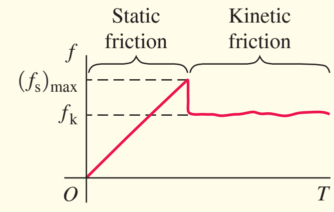
When an object is sliding over the surface, the friction force is called *kinetic* friction. Its magnitude $f_k$ is approximately equal to the normal force magnitude $n$ multiplied by the coefficient of kinetic friction $\mu_k$.

Magnitude of kinetic friction force:

$$f_k = \mu_k n \tag{5.3}$$

Magnitude of static friction force:

$$f_s \le (f_s)_{max} = \mu_s n \tag{5.4}$$



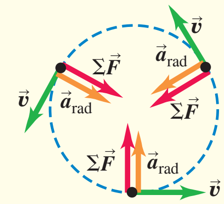

When an object is *not* moving relative to a surface, the friction force is called *static* friction. The *maximum* possible static friction force is approximately equal to the magnitude $n$ of the normal force multiplied by the coefficient of static friction $\mu_s$. The *actual* static friction force may be anything from zero to this maximum value, depending on the situation. Usually $\mu_s$ is greater than $\mu_k$ for a given pair of surfaces in contact. (See Examples 5.13–5.17.)

Rolling friction is similar to kinetic friction, but the force of fluid resistance depends on the speed of an object through a fluid. (See Example 5.18.)

**Forces in circular motion:** In uniform circular motion, the acceleration vector is directed toward the center of the circle. The motion is governed by Newton's second law, $\sum \vec{F} = m\vec{a}$. (See Examples 5.19–5.23.)

Acceleration in uniform circular motion:

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{4\pi^2 R}{T^2} \qquad (5.13), (5.15)$$




Chapter 5 Media Assets

## GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

### KEY EXAMPLE VARIATION PROBLEMS

**Be sure to review EXAMPLE 5.5 (Section 5.1) before attempting these problems. In all problems, ignore air resistance.**

**VP5.5.1** In a modified version of the cart and bucket in Fig. 5.5a, the angle of the slope is 36.9° and the bucket weighs 255 N. The cart moves up the incline and the bucket moves downward, both at constant speed. The cable has negligible mass, and there is no friction. (a) What is the weight of the cart? (b) What is the tension in the cable?

**VP5.5.2** You increase the angle of the slope in Fig. 5.5a to 25.0° and use a different cart and a different bucket. You observe that the cart and bucket remain at rest when released and that the tension in the cable of negligible mass is 155 N. There is no friction. (a) What is the weight of the cart? (b) What is the *combined* weight of the cart and bucket?

**VP5.5.3** You construct a version of the cart and bucket in Fig. 5.5a, but with a slope whose angle can be adjusted. You use a cart of mass 175 kg and a bucket of mass 65.0 kg. The cable has negligible mass, and there is no friction. (a) What must be the angle of the slope so that the cart moves downhill at a constant speed and the bucket moves upward at the same constant speed? (b) With this choice of angle, what will be the tension in the cable?

**VP5.5.4** In the situation shown in Fig. 5.5a, let $\theta$ be the angle of the slope and suppose there *is* friction between the cart and the track. You find that if the cart and bucket each have the same weight $w$, they remain at rest when released. In this case, what is the magnitude of the friction force on the cart? Is it less than, greater than, or equal to $w$?

**Be sure to review EXAMPLES 5.13, 5.14, and 5.15 (Section 5.3) before attempting these problems.**

**VP5.15.1** You pull on a crate using a rope as in Fig. 5.21a, except the rope is at an angle of 20.0° above the horizontal. The weight of the crate is 325 N, and the coefficient of kinetic friction between the crate and the floor is 0.250. (a) What must be the tension in the rope to make the crate move at a constant velocity? (b) What is the normal force that the floor exerts on the crate?

**VP5.15.2** You pull on a large box using a rope as in Fig. 5.21a, except the rope is at an angle of 15.0° *below* the horizontal. The weight of the box is 325 N, and the coefficient of kinetic friction between the box and the floor is 0.250. (a) What must be the tension in the rope to make the box move at a constant velocity? (b) What is the normal force that the floor exerts on the box?

**VP5.15.3** You are using a lightweight rope to pull a sled along level ground. The sled weighs 475 N, the coefficient of kinetic friction between the sled and the ground is 0.200, the rope is at an angle of 12.0° above the horizontal, and you pull on the rope with a force of 125 N. (a) Find the normal force that the ground exerts on the sled. (b) Find the acceleration of the sled. Is the sled speeding up or slowing down?

**VP5.15.4** A large box of mass $m$ sits on a horizontal floor. You attach a lightweight rope to this box, hold the rope at an angle $\theta$ above the horizontal, and pull. You find that the minimum tension you can apply to the rope in order to make the box start moving is $T_{\text{min}}$. Find the coefficient of static friction between the floor and the box.

**Be sure to review EXAMPLES 5.20, 5.21, and 5.22 (Section 5.4) before attempting these problems.**

**VP5.22.1** You make a conical pendulum (see Fig. 5.32a) using a string of length 0.800 m and a bob of mass 0.250 kg. When the bob is moving in a circle at a constant speed, the string is at an angle of 20.0° from the vertical. (a) What is the radius of the circle around which the bob moves? (b) How much time does it take the bob to complete one circle? (c) What is the tension in the string?

**VP5.22.2** A competition cyclist rides at a constant 12.5 m/s around a curve that is banked at 40.0°. The cyclist and her bicycle have a combined mass of 64.0 kg. (a) What must be the radius of her turn if there is to be no friction force pushing her either up or down the banked curve? (b) What is the magnitude of her acceleration? (c) What is the magnitude of the normal force that the surface of the banked curve exerts on the bicycle?

**VP5.22.3** An aerobatic airplane flying at a constant 80.0 m/s makes a horizontal turn of radius 175 m. The pilot has mass 80.0 kg. (a) What is the bank angle of the airplane? (b) What is the pilot's apparent weight during the turn? How many times greater than his actual weight is this?

**VP5.22.4** A sports car moves around a banked curve at just the right constant speed $v$ so that no friction is needed to make the turn. During the turn, the driver (mass $m$) feels as though she weighs $x$ times her actual weight. (a) Find the magnitude of the *net* force on the driver during the turn in terms of $m$, $g$, and $x$. (b) Find the radius of the turn in terms of $v$, $g$, and $x$.

## BRIDGING PROBLEM In a Rotating Cone

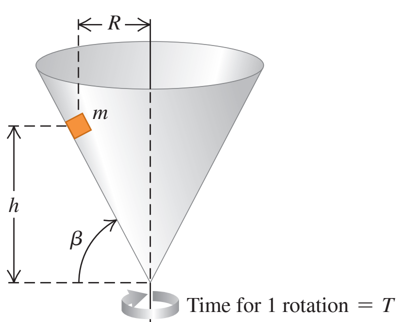
A small block with mass $m$ is placed inside an inverted cone that is rotating about a vertical axis such that the time for one revolution of the cone is $T$ (**Fig. 5.39**). The walls of the cone make an angle $\beta$ with the horizontal. The coefficient of static friction between the block and the cone is $\mu_s$. If the block is to remain at a constant height $h$ above the apex of the cone, what are (a) the maximum value of $T$ and (b) the minimum value of $T$? (That is, find expressions for $T_{max}$ and $T_{min}$ in terms of $\beta$ and $h$.)

### SOLUTION GUIDE

### IDENTIFY and SET UP

1. Although we want the block not to slide up or down on the inside of the cone, this is *not* an equilibrium problem. The block rotates with the cone and is in uniform circular motion, so it has an acceleration directed toward the center of its circular path.
2. Identify the forces on the block. What is the direction of the friction force when the cone is rotating as slowly as possible, so $T$ has its maximum value $T_{max}$? What is the direction of the friction force when the cone is rotating as rapidly as possible, so $T$ has its minimum value $T_{min}$? In these situations does the static friction force have its *maximum* magnitude? Why or why not?
3. Draw a free-body diagram for the block when the cone is rotating with $T = T_{max}$ and a free-body diagram when the cone is rotating with $T = T_{min}$. Choose coordinate axes, and remember that it's usually easiest to choose one of the axes to be in the direction of the acceleration.
4. What is the radius of the circular path that the block follows? Express this in terms of $\beta$ and $h$.
5. List the unknown quantities, and decide which of these are the target variables.

Figure **5.39** A block inside a spinning cone.




### EXECUTE

6. Write Newton's second law in component form for the case in which the cone is rotating with $T = T_{max}$. Write the acceleration in terms of $T_{max}$, $\beta$, and $h$, and write the static friction force in terms of the normal force $n$.
7. Solve these equations for the target variable $T_{max}$.
8. Repeat steps 6 and 7 for the case in which the cone is rotating with $T = T_{min}$, and solve for the target variable $T_{min}$.

### EVALUATE

9. You'll end up with some fairly complicated expressions for $T_{max}$ and $T_{min}$, so check them over carefully. Do they have the correct units? Is the minimum time $T_{min}$ less than the maximum time $T_{max}$, as it must be?
10. What do your expressions for $T_{max}$ and $T_{min}$ become if $\mu_s = 0$? Check your results by comparing them with Example 5.22 in Section 5.4.

# PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

[Always assume that pulleys are frictionless and massless and that strings and cords are massless, unless otherwise noted.]

## DISCUSSION QUESTIONS

**Q5.1** A man sits in a seat that is hanging from a rope. The rope passes over a pulley suspended from the ceiling, and the man holds the other end of the rope in his hands. What is the tension in the rope, and what force does the seat exert on him? Draw a free-body force diagram for the man.

**Q5.2** "In general, the normal force is not equal to the weight." Give an example in which these two forces are equal in magnitude, and at least two examples in which they are not.

**Q5.3** A clothesline hangs between two poles. No matter how tightly the line is stretched, it sags a little at the center. Explain why.

**Q5.4** You drive a car up a steep hill at constant speed. Discuss all of the forces that act on the car. What pushes it up the hill?

**Q5.5** For medical reasons, astronauts in outer space must determine their body mass at regular intervals. Devise a scheme for measuring body mass in an apparently weightless environment.

**Q5.6** To push a box up a ramp, which requires less force: pushing horizontally or pushing parallel to the ramp? Why?

**Q5.7** A woman in an elevator lets go of her briefcase, but it does not fall to the floor. How is the elevator moving?

**Q5.8** A block rests on an inclined plane with enough friction to prevent it from sliding down. To start the block moving, is it easier to push it up the plane or down the plane? Why?

**Q5.9** A crate slides up an inclined ramp and then slides down the ramp after momentarily stopping near the top. There is kinetic friction between the surface of the ramp and the crate. Which is greater? (i) The crate's acceleration going up the ramp; (ii) the crate's acceleration going down the ramp; (iii) both are the same. Explain.

**Q5.10** A crate of books rests on a level floor. To move it along the floor at a constant velocity, why do you exert less force if you pull it at an angle $\theta$ above the horizontal than if you push it at the same angle below the horizontal?

**Q5.11** In a world without friction, which of the following activities could you do (or not do)? Explain your reasoning. (a) Drive around an unbanked highway curve; (b) jump into the air; (c) start walking on a horizontal sidewalk; (d) climb a vertical ladder; (e) change lanes while you drive.

**Q5.12** When you stand with bare feet in a wet bathtub, the grip feels fairly secure, and yet a catastrophic slip is quite possible. Explain this in terms of the two coefficients of friction.

**Q5.13** You are pushing a large crate from the back of a freight elevator to the front as the elevator is moving to the next floor. In which situation is the force you must apply to move the crate the least, and in which is it the greatest: when the elevator is accelerating upward, when it is accelerating downward, or when it is traveling at constant speed? Explain.

**Q5.14** It is often said that "friction always opposes motion." Give at least one example in which (a) static friction *causes* motion, and (b) kinetic friction *causes* motion.

**Q5.15** If there is a net force on a particle in uniform circular motion, why doesn't the particle's speed change?

**Q5.16** A curve in a road has a bank angle calculated and posted for 80 km/h. However, the road is covered with ice, so you cautiously plan to drive slower than this limit. What might happen to your car? Why?

**Q5.17** You swing a ball on the end of a lightweight string in a horizontal circle at constant speed. Can the string ever be truly horizontal? If not, would it slope above the horizontal or below the horizontal? Why?

**Q5.18** The centrifugal force is not included in the free-body diagrams of Figs. 5.34b and 5.35. Explain why not.

**Q5.19** A professor swings a rubber stopper in a horizontal circle on the end of a string in front of his class. He tells Caroline, in the front row, that he is going to let the string go when the stopper is directly in front of her face. Should Caroline worry?

**Q5.20** To keep the forces on the riders within allowable limits, many loop-the-loop roller coaster rides are designed so that the loop is not a perfect circle but instead has a larger radius of curvature at the bottom than at the top. Explain.

**Q5.21** A tennis ball drops from rest at the top of a tall glass cylinder—first with the air pumped out of the cylinder so that there is no air resistance, and again after the air has been readmitted to the cylinder. You examine multiflash photographs of the two drops. Can you tell which photo belongs to which drop? If so, how?

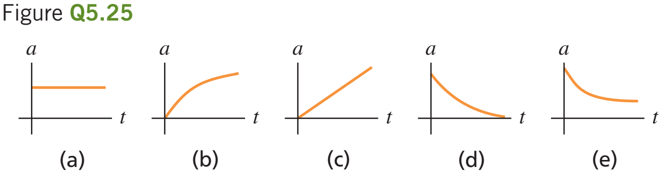
**Q5.22** You throw a baseball straight upward with speed $v_0$. When the ball returns to the point from where you threw it, how does its speed compare to $v_0$ (a) in the absence of air resistance and (b) in the presence of air resistance? Explain.

**Q5.23** You throw a baseball straight upward. If you do *not* ignore air resistance, how does the time required for the ball to reach its maximum height compare to the time required for it to fall from its maximum height back down to the height from which you threw it? Explain.

**Q5.24** You have two identical tennis balls and fill one with water. You release both balls simultaneously from the top of a tall building. If air resistance is negligible, which ball will strike the ground first? Explain. What if air resistance is *not* negligible?

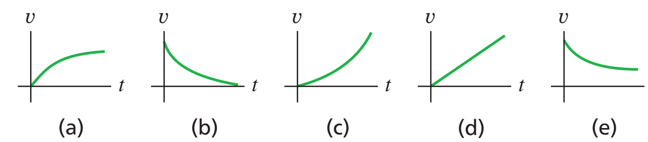
**Q5.25** A ball is dropped from rest and feels air resistance as it falls. Which of the graphs in **Fig. Q5.25** best represents its acceleration as a function of time?

Figure **Q5.25**




**Q5.26** A ball is dropped from rest and feels air resistance as it falls. Which of the graphs in **Fig. Q5.26** best represents its vertical velocity component as a function of time, if the $+y$-direction is taken to be downward?

Figure **Q5.26**




**Q5.27** When a batted baseball moves with air drag, when does the ball travel a greater horizontal distance? (i) While climbing to its maximum height; (ii) while descending from its maximum height back to the ground; (iii) the same for both. Explain in terms of the forces acting on the ball.

**Q5.28** "A ball is thrown from the edge of a high cliff. Regardless of the angle at which it is thrown, due to air resistance, the ball will eventually end up moving vertically downward." Justify this statement.

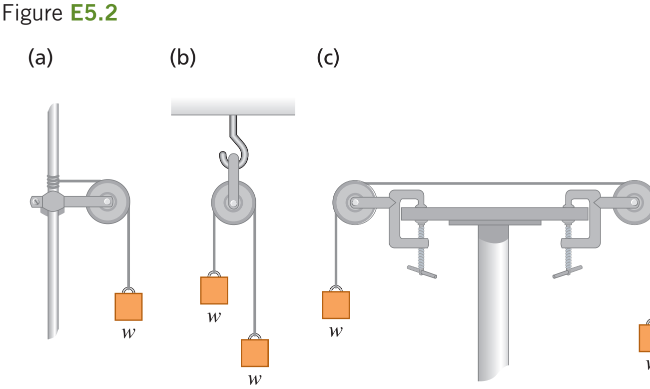
## EXERCISES

### Section 5.1 Using Newton's First Law: Particles in Equilibrium

**5.1** • Two 25.0 N weights are suspended at opposite ends of a rope that passes over a light, frictionless pulley. The pulley is attached to a chain from the ceiling. (a) What is the tension in the rope? (b) What is the tension in the chain?

**5.2** • In **Fig. E5.2** each of the suspended blocks has weight $w$. The pulleys are frictionless, and the ropes have negligible weight. In each case, draw a free-body diagram and calculate the tension $T$ in the rope in terms of $w$.

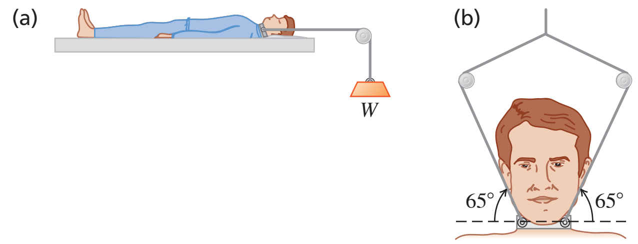
Figure **E5.2**




**5.3** • A 75.0 kg wrecking ball hangs from a uniform, heavy-duty chain of mass 26.0 kg. (a) Find the maximum and minimum tensions in the chain. (b) What is the tension at a point three-fourths of the way up from the bottom of the chain?

**5.4** •• **BIO** **Injuries to the Spinal Column.** In the treatment of spine injuries, it is often necessary to provide tension along the spinal column to stretch the backbone. One device for doing this is the Stryker frame (**Fig. E5.4a**, next page). A weight $W$ is attached to the patient (sometimes around a neck collar, Fig. E5.4b), and friction between the person's body and the bed prevents sliding. (a) If the coefficient of static friction between a 78.5 kg patient's body and the bed is 0.75, what is the maximum traction force along the spinal column that $W$ can provide without causing the patient to slide? (b) Under the conditions of maximum traction, what is the tension in each cable attached to the neck collar?

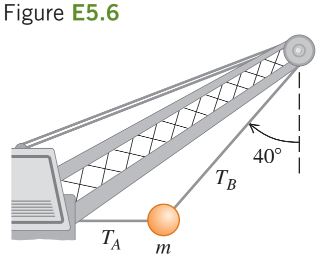
Figure **E5.4**




**5.5** •• A picture frame hung against a wall is suspended by two wires attached to its upper corners. If the two wires make the same angle with the vertical, what must this angle be if the tension in each wire is equal to 0.75 of the weight of the frame? (Ignore any friction between the wall and the picture frame.)

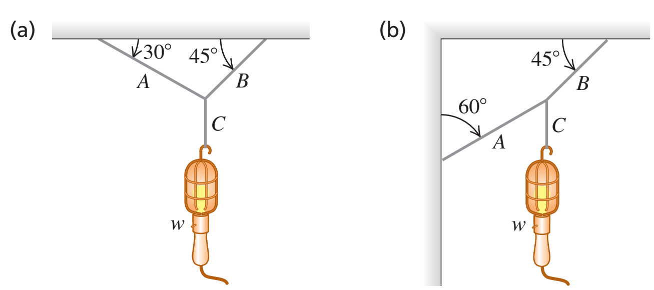
**5.6** •• A large wrecking ball is held in place by two light steel cables (**Fig. E5.6**). If the mass $m$ of the wrecking ball is 3620 kg, what are (a) the tension $T_B$ in the cable that makes an angle of 40° with the vertical and (b) the tension $T_A$ in the horizontal cable?

Figure **E5.6**



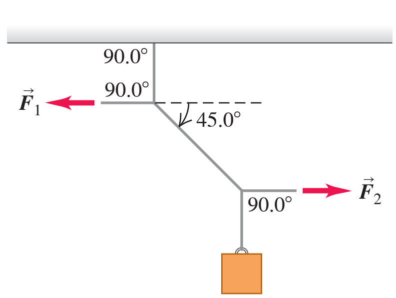

**5.7** •• Find the tension in each cord in **Fig. E5.7** if the weight of the suspended object is $w$.

Figure **E5.7**




**5.8** •• In **Fig. E5.8** the weight $w$ is 60.0 N. (a) What is the tension in the diagonal string? (b) Find the magnitudes of the horizontal forces $\vec{F}_1$ and $\vec{F}_2$ that must be applied to hold the system in the position shown.

Figure **E5.8**




**5.9** •• A man pushes on a piano with mass 180 kg; it slides at constant velocity down a ramp that is inclined at 19.0° above the horizontal floor. Neglect any friction acting on the piano. Calculate the magnitude of the force applied by the man if he pushes (a) parallel to the incline and (b) parallel to the floor.

## Section 5.2 Using Newton's Second Law: Dynamics of Particles

**5.10** •• **Apparent Weight.** A 550 N physics student stands on a bathroom scale in an elevator that is supported by a cable. The combined mass of student plus elevator is 850 kg. As the elevator starts moving, the scale reads 450 N. (a) Find the acceleration of the elevator (magnitude and direction). (b) What is the acceleration if the scale reads 670 N? (c) If the scale reads zero, should the student worry? Explain. (d) What is the tension in the cable in parts (a) and (c)?

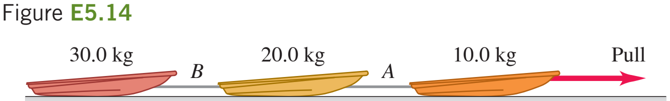
**5.11** •• **BIO Stay Awake!** An astronaut is inside a $2.25 \times 10^6$ kg rocket that is blasting off vertically from the launch pad. You want this rocket to reach the speed of sound (331 m/s) as quickly as possible, but astronauts are in danger of blacking out at an acceleration greater than $4g$. (a) What is the maximum initial thrust this rocket's engines can have but just barely avoid blackout? Start with a free-body diagram of the rocket. (b) What force, in terms of the astronaut's weight $w$, does the rocket exert on her? Start with a free-body diagram of the astronaut. (c) What is the shortest time it can take the rocket to reach the speed of sound?

**5.12** •• A rocket of initial mass 125 kg (including all the contents) has an engine that produces a constant vertical force (the *thrust*) of 1720 N. Inside this rocket, a 15.5 N electric power supply rests on the floor. (a) Find the initial acceleration of the rocket. (b) When the rocket initially accelerates, how hard does the floor push on the power supply? (*Hint:* Start with a free-body diagram for the power supply.)

**5.13** •• **CP *Genesis* Crash.** On September 8, 2004, the *Genesis* spacecraft crashed in the Utah desert because its parachute did not open. The 210 kg capsule hit the ground at 311 km/h and penetrated the soil to a depth of 81.0 cm. (a) What was its acceleration (in m/s$^2$ and in $g$'s), assumed to be constant, during the crash? (b) What force did the ground exert on the capsule during the crash? Express the force in newtons and as a multiple of the capsule's weight. (c) How long did this force last?

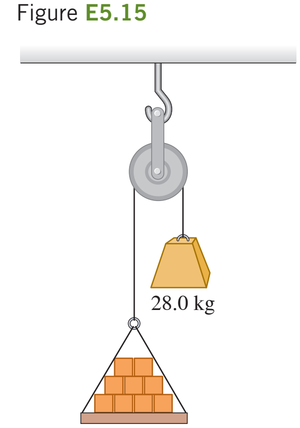
**5.14** • Three sleds are being pulled horizontally on frictionless horizontal ice using horizontal ropes (**Fig. E5.14**). The pull is of magnitude 190 N. Find (a) the acceleration of the system and (b) the tension in ropes $A$ and $B$.

Figure **E5.14**




**5.15** •• **Atwood's Machine.** A 15.0 kg load of bricks hangs from one end of a rope that passes over a small, frictionless pulley. A 28.0 kg counterweight is suspended from the other end of the rope (**Fig. E5.15**). The system is released from rest. (a) Draw two free-body diagrams, one for the load of bricks and one for the counterweight. (b) What is the magnitude of the upward acceleration of the load of bricks? (c) What is the tension in the rope while the load is moving? How does the tension compare to the weight of the load of bricks? To the weight of the counterweight?

Figure **E5.15**



**5.16** •• **CP** An 8.00 kg block of ice, released from rest at the top of a 1.50-m-long frictionless ramp, slides downhill, reaching a speed of 2.50 m/s at the bottom. (a) What is the angle between the ramp and the horizontal? (b) What would be the speed of the ice at the bottom if the motion were opposed by a constant friction force of 10.0 N parallel to the surface of the ramp?

**5.17** •• A light rope is attached to a block with mass 4.00 kg that rests on a frictionless, horizontal surface. The horizontal rope passes over a frictionless, massless pulley, and a block with mass $m$ is suspended from the other end. When the blocks are released, the tension in the rope is 15.0 N. (a) Draw two free-body diagrams: one for each block. (b) What is the acceleration of either block? (c) Find $m$. (d) How does the tension compare to the weight of the hanging block?

**5.18** •• **CP** **Runway Design.** A transport plane takes off from a level landing field with two gliders in tow, one behind the other. The mass of each glider is 700 kg, and the total resistance (air drag plus friction with the runway) on each may be assumed constant and equal to 2500 N. The tension in the towrope between the transport plane and the first glider is not to exceed 12,000 N. (a) If a speed of 40 m/s is required for takeoff, what minimum length of runway is needed? (b) What is the tension in the towrope between the two gliders while they are accelerating for the takeoff?

**5.19** •• **CP** **BIO** **Force During a Jump.** When jumping straight up from a crouched position, an average person can reach a maximum height of about 60 cm. During the jump, the person's body from the knees up typically rises a distance of around 50 cm. To keep the calculations simple and yet get a reasonable result, assume that the *entire body* rises this much during the jump. (a) With what initial speed does the person leave the ground to reach a height of 60 cm? (b) Draw a free-body diagram of the person during the jump. (c) In terms of this jumper's weight $w$, what force does the ground exert on him or her during the jump?

**5.20** **CP CALC** A 2540 kg test rocket is launched vertically from the launch pad. Its fuel (of negligible mass) provides a thrust force such that its vertical velocity as a function of time is given by $v(t) = At + Bt^2$, where $A$ and $B$ are constants and time is measured from the instant the fuel is ignited. The rocket has an upward acceleration of 1.50 m/s$^2$ at the instant of ignition and, 1.00 s later, an upward velocity of 2.00 m/s. (a) Determine $A$ and $B$, including their SI units. (b) At 4.00 s after fuel ignition, what is the acceleration of the rocket, and (c) what thrust force does the burning fuel exert on it, assuming no air resistance? Express the thrust in newtons and as a multiple of the rocket's weight. (d) What was the initial thrust due to the fuel?

**5.21** •• **CP CALC** A 2.00 kg box is moving to the right with speed 9.00 m/s on a horizontal, frictionless surface. At $t = 0$ a horizontal force is applied to the box. The force is directed to the left and has magnitude $F(t) = (6.00\ \text{N/s}^2)t^2$. (a) What distance does the box move from its position at $t = 0$ before its speed is reduced to zero? (b) If the force continues to be applied, what is the speed of the box at $t = 3.00$ s?

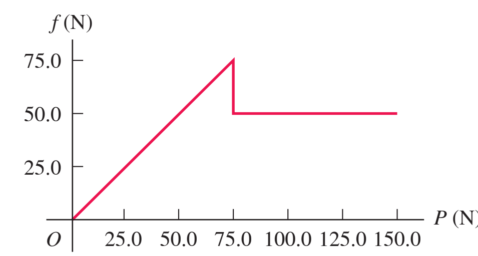
**5.22** •• **CP CALC** A 5.00 kg crate is suspended from the end of a short vertical rope of negligible mass. An upward force $F(t)$ is applied to the end of the rope, and the height of the crate above its initial position is given by $y(t) = (2.80\ \text{m/s})t + (0.610\ \text{m/s}^3)t^3$. What is the magnitude of $F$ when $t = 4.00$ s?

## Section 5.3 Friction Forces

**5.23** • **BIO** **The Trendelenburg Position.** After emergencies with major blood loss, a patient is placed in the Trendelenburg position, in which the foot of the bed is raised to get maximum blood flow to the brain. If the coefficient of static friction between a typical patient and the bedsheets is 1.20, what is the maximum angle at which the bed can be tilted with respect to the floor before the patient begins to slide?

**5.24** • In a laboratory experiment on friction, a 135 N block resting on a rough horizontal table is pulled by a horizontal wire. The pull gradually increases until the block begins to move and continues to increase thereafter. **Figure E5.24** shows a graph of the friction force on this block as a function of the pull. (a) Identify the regions of the graph where static friction and kinetic friction occur. (b) Find the coefficients of static friction and kinetic friction between the block and the table. (c) Why does the graph slant upward at first but then level out? (d) What would the graph look like if a 135 N brick were placed on the block, and what would the coefficients of friction be?

Figure **E5.24**




**5.25** •• **CP** A stockroom worker pushes a box with mass 16.8 kg on a horizontal surface with a constant speed of 3.50 m/s. The coefficient of kinetic friction between the box and the surface is 0.20. (a) What horizontal force must the worker apply to maintain the motion? (b) If the force calculated in part (a) is removed, how far does the box slide before coming to rest?

**5.26** •• A box of bananas weighing 40.0 N rests on a horizontal surface. The coefficient of static friction between the box and the surface is 0.40, and the coefficient of kinetic friction is 0.20. (a) If no horizontal force is applied to the box and the box is at rest, how large is the friction force exerted on it? (b) What is the magnitude of the friction force if a monkey applies a horizontal force of 6.0 N to the box and the box is initially at rest? (c) What minimum horizontal force must the monkey apply to start the box in motion? (d) What minimum horizontal force must the monkey apply to keep the box moving at constant velocity once it has been started? (e) If the monkey applies a horizontal force of 18.0 N, what is the magnitude of the friction force and what is the box's acceleration?

**5.27** •• A 45.0 kg crate of tools rests on a horizontal floor. You exert a gradually increasing horizontal push on it, and the crate just begins to move when your force exceeds 313 N. Then you must reduce your push to 208 N to keep it moving at a steady 25.0 cm/s. (a) What are the coefficients of static and kinetic friction between the crate and the floor? (b) What push must you exert to give it an acceleration of 1.10 m/s$^2$? (c) Suppose you were performing the same experiment on the moon, where the acceleration due to gravity is 1.62 m/s$^2$. (i) What magnitude push would cause it to move? (ii) What would its acceleration be if you maintained the push in part (b)?

**5.28** • Consider the heaviest box that you can push at constant speed across a level floor, where the coefficient of kinetic friction is 0.50, and estimate the maximum horizontal force that you can apply to the box. A box sits on a ramp that is inclined at an angle of 60° above the horizontal. The coefficient of kinetic friction between the box and the ramp is 0.50. If you apply the same magnitude force, now parallel to the ramp, that you applied to the box on the floor, what is the heaviest box (in pounds) that you can push up the ramp at constant speed? (In both cases assume you can give enough extra push to get the box started moving.)

**5.29** • **CP** Estimate the height of a typical playground slide and the angle its surface makes with the horizontal. (a) Some children like to slide down while sitting on a sheet of wax paper. This makes the friction force exerted by the slide very small. If a child starts from rest and we take the friction force to be zero, what is the speed of the child when he reaches the bottom of the slide? (b) If the child doesn't use the wax paper, his speed at the bottom is half the value calculated in part (a). What is the coefficient of kinetic friction between the child and the slide when wax paper isn't used? (c) A child wearing a different sort of clothing than the first child climbs the ladder to the top of the slide, sits on the slide, lets go of the handrail, and remains at rest. What is the minimum possible value for the coefficient of static friction between this child and the surface of the slide?

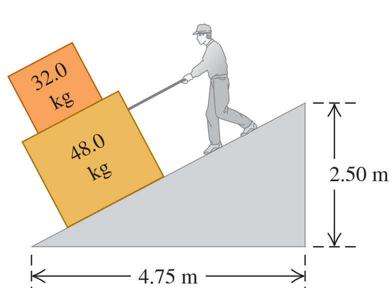
**5.30** •• Some sliding rocks approach the base of a hill with a speed of 12 m/s. The hill rises at 36° above the horizontal and has coefficients of kinetic friction and static friction of 0.45 and 0.65, respectively, with these rocks. (a) Find the acceleration of the rocks as they slide up the hill. (b) Once a rock reaches its highest point, will it stay there or slide down the hill? If it stays, show why. If it slides, find its acceleration on the way down.

**5.31** •• A box with mass 10.0 kg moves on a ramp that is inclined at an angle of 55.0° above the horizontal. The coefficient of kinetic friction between the box and the ramp surface is $\mu_k = 0.300$. Calculate the magnitude of the acceleration of the box if you push on the box with a constant force $F = 120.0$ N that is parallel to the ramp surface and (a) directed down the ramp, moving the box down the ramp; (b) directed up the ramp, moving the box up the ramp.

**5.32** •• A pickup truck is carrying a toolbox, but the rear gate of the truck is missing. The toolbox will slide out if it is set moving. The coefficients of kinetic friction and static friction between the box and the level bed of the truck are 0.355 and 0.650, respectively. Starting from rest, what is the shortest time this truck could accelerate uniformly to 30.0 m/s without causing the box to slide? Draw a free-body diagram of the toolbox.

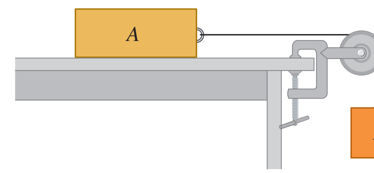
**5.33** •• You are lowering two boxes, one on top of the other, down a ramp by pulling on a rope parallel to the surface of the ramp (**Fig. E5.33**). Both boxes move together at a constant speed of 15.0 cm/s. The coefficient of kinetic friction between the ramp and the lower box is 0.444, and the coefficient of static friction between the two boxes is 0.800. (a) What force do you need to exert to accomplish this? (b) What are the magnitude and direction of the friction force on the upper box?

Figure **E5.33**




**5.34** •• Consider the system shown in **Fig. E5.34.** Block $A$ weighs 45.0 N, and block $B$ weighs 25.0 N. Once block $B$ is set into downward motion, it descends at a constant speed. (a) Calculate the coefficient of kinetic friction between block $A$ and the tabletop. (b) A cat, also of weight 45.0 N, falls asleep on top of block $A$. If block $B$ is now set into downward motion, what is its acceleration (magnitude and direction)?

Figure **E5.34**

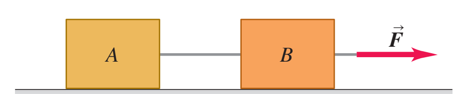



**5.35** •• **CP** **Stopping Distance.** (a) If the coefficient of kinetic friction between tires and dry pavement is 0.80, what is the shortest distance in which you can stop a car by locking the brakes when the car is traveling at 28.7 m/s (about 65 mi/h)? (b) On wet pavement the coefficient of kinetic friction may be only 0.25. How fast should you drive on wet pavement to be able to stop in the same distance as in part (a)? (*Note:* Locking the brakes is *not* the safest way to stop.)

**5.36** •• **CP** A 25.0 kg box of textbooks rests on a loading ramp that makes an angle $\alpha$ with the horizontal. The coefficient of kinetic friction is 0.25, and the coefficient of static friction is 0.35. (a) As $\alpha$ is increased, find the minimum angle at which the box starts to slip. (b) At this angle, find the acceleration once the box has begun to move. (c) At this angle, how fast will the box be moving after it has slid 5.0 m along the loading ramp?

**5.37** • Two crates connected by a rope lie on a horizontal surface (**Fig. E5.37**). Crate $A$ has mass $m_A$, and crate $B$ has mass $m_B$. The coefficient of kinetic friction between each crate and the surface is $\mu_k$. The crates are pulled to the right at constant velocity by a horizontal force $\vec{F}$. Draw one or more free-body diagrams to calculate the following in terms of $m_A$, $m_B$, and $\mu_k$: (a) the magnitude of $\vec{F}$ and (b) the tension in the rope connecting the blocks.

Figure **E5.37**




**5.38** •• A box with mass $m$ is dragged across a level floor with coefficient of kinetic friction $\mu_k$ by a rope that is pulled upward at an angle $\theta$ above the horizontal with a force of magnitude $F$. (a) In terms of $m$, $\mu_k$, $\theta$, and $g$, obtain an expression for the magnitude of the force required to move the box with constant speed. (b) Knowing that you are studying physics, a CPR instructor asks you how much force it would take to slide a 90 kg patient across a floor at constant speed by pulling on him at an angle of 25° above the horizontal. By dragging weights wrapped in an old pair of pants down the hall with a spring balance, you find that $\mu_k = 0.35$. Use the result of part (a) to answer the instructor's question.

**5.39** •• **CP** As shown in Fig. E5.34, block $A$ (mass 2.25 kg) rests on a tabletop. It is connected by a horizontal cord passing over a light, frictionless pulley to a hanging block $B$ (mass 1.30 kg). The coefficient of kinetic friction between block $A$ and the tabletop is 0.450. The blocks are released then from rest. Draw one or more free-body diagrams to find (a) the speed of each block after they move 3.00 cm and (b) the tension in the cord.

**5.40** •• You throw a baseball straight upward. The drag force is proportional to $v^2$. In terms of $g$, what is the $y$-component of the ball's acceleration when the ball's speed is half its terminal speed and (a) it is moving up? (b) It is moving back down?

**5.41** •• A large crate with mass $m$ rests on a horizontal floor. The coefficients of friction between the crate and the floor are $\mu_s$ and $\mu_k$. A woman pushes downward with a force $\vec{F}$ on the crate at an angle $\theta$ below the horizontal. (a) What magnitude of force $\vec{F}$ is required to keep the crate moving at constant velocity? (b) If $\mu_s$ is greater than some critical value, the woman cannot start the crate moving no matter how hard she pushes. Calculate this critical value of $\mu_s$.

## Section 5.4 Dynamics of Circular Motion

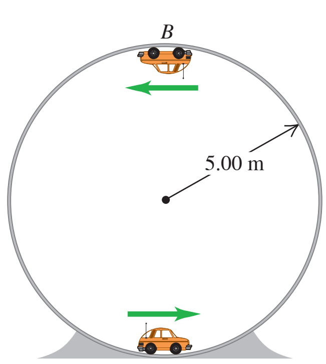
**5.42** • You are sitting on the edge of a horizontal disk (for example, a playground merry-go-round) that has radius 3.00 m and is rotating at a constant rate about a vertical axis. (a) If the coefficient of static friction between you and the surface of the disk is 0.400, what is the minimum time for one revolution of the disk if you are not to slide off? (b) Your friend's weight is half yours. If the coefficient of static friction for him is the same as for you, what is the minimum time for one revolution if he is not to slide off?

**5.43** • A stone with mass 0.80 kg is attached to one end of a string 0.90 m long. The string will break if its tension exceeds 60.0 N. The stone is whirled in a horizontal circle on a frictionless tabletop; the other end of the string remains fixed. (a) Draw a free-body diagram of the stone. (b) Find the maximum speed the stone can attain without the string breaking.

**5.44** • BIO **Force on a Skater's Wrist.** A 52 kg ice skater spins about a vertical axis through her body with her arms horizontally outstretched; she makes 2.0 turns each second. The distance from one hand to the other is 1.50 m. Biometric measurements indicate that each hand typically makes up about 1.25% of body weight. (a) Draw a free-body diagram of one of the skater's hands. (b) What horizontal force must her wrist exert on her hand? (c) Express the force in part (b) as a multiple of the weight of her hand.

**5.45** •• A small remote-controlled car with mass 1.60 kg moves at a constant speed of $v = 12.0\ \text{m/s}$ in a track formed by a vertical circle inside a hollow metal cylinder that has a radius of 5.00 m (**Fig. E5.45**). What is the magnitude of the normal force exerted on the car by the walls of the cylinder at (a) point $A$ (bottom of the track) and (b) point $B$ (top of the track)?

Figure **E5.45**




**5.46** •• A small car with mass 0.800 kg travels at constant speed on the inside of a track that is a vertical circle with radius 5.00 m (Fig. E5.45). If the normal force exerted by the track on the car when it is at the top of the track (point $B$) is 6.00 N, what is the normal force on the car when it is at the bottom of the track (point $A$)?

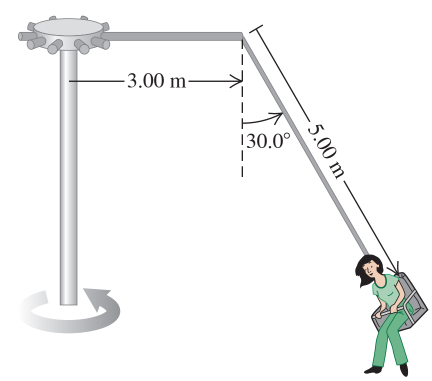
**5.47** • A small model car with mass $m$ travels at constant speed on the inside of a track that is a vertical circle with radius 5.00 m (Fig. E5.45). If the normal force exerted by the track on the car when it is at the bottom of the track (point $A$) is equal to $2.50mg$, how much time does it take the car to complete one revolution around the track?

**5.48** • A flat (unbanked) curve on a highway has a radius of 170.0 m. A car rounds the curve at a speed of 25.0 m/s. (a) What is the minimum coefficient of static friction that will prevent sliding? (b) Suppose that the highway is icy and the coefficient of static friction between the tires and pavement is only one-third of what you found in part (a). What should be the maximum speed of the car so that it can round the curve safely?

**5.49** •• A 1125 kg car and a 2250 kg pickup truck approach a curve on a highway that has a radius of 225 m. (a) At what angle should the highway engineer bank this curve so that vehicles traveling at 65.0 mi/h can safely round it regardless of the condition of their tires? Should the heavy truck go slower than the lighter car? (b) As the car and truck round the curve at 65.0 mi/h, find the normal force on each one due to the highway surface.

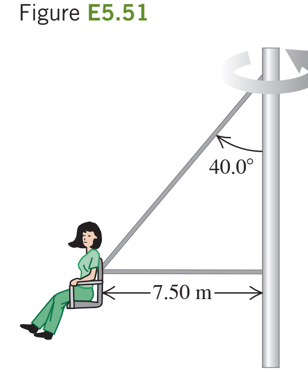
**5.50** •• The "Giant Swing" at a county fair consists of a vertical central shaft with a number of horizontal arms attached at its upper end. Each arm supports a seat suspended from a cable 5.00 m long, and the upper end of the cable is fastened to the arm at a point 3.00 m from the central shaft (**Fig. E5.50**). (a) Find the time of one revolution of the swing if the cable supporting a seat makes an angle of 30.0° with the vertical. (b) Does the angle depend on the weight of the passenger for a given rate of revolution?

Figure **E5.50**




**5.51** •• In another version of the "Giant Swing" (see Exercise 5.50), the seat is connected to two cables, one of which is horizontal (**Fig. E5.51**). The seat swings in a horizontal circle at a rate of 28.0 rpm (rev/min). If the seat weighs 255 N and an 825 N person is sitting in it, find the tension in each cable.

Figure **E5.51**




**5.52** • A steel ball with mass $m$ is suspended from the ceiling at the bottom end of a light, 15.0-m-long rope. The ball swings back and forth like a pendulum. When the ball is at its lowest point and the rope is vertical, the tension in the rope is three times the weight of the ball, so $T = 3mg$. (a) What is the speed of the ball as it swings through this point? (b) What is the speed of the ball if $T = mg$ at this point, where the rope is vertical?

**5.53 ••** **Rotating Space Stations.** One problem for humans living in outer space is that they are apparently weightless. One way around this problem is to design a space station that spins about its center at a constant rate. This creates "artificial gravity" at the outside rim of the station. (a) If the diameter of the space station is 800 m, how many revolutions per minute are needed for the "artificial gravity" acceleration to be $9.80\ \text{m/s}^2$? (b) If the space station is a waiting area for travelers going to Mars, it might be desirable to simulate the acceleration due to gravity on the Martian surface $(3.70\ \text{m/s}^2)$. How many revolutions per minute are needed in this case?

**5.54 •** The Cosmo Clock 21 Ferris wheel in Yokohama, Japan, has a diameter of 100 m. Its name comes from its 60 arms, each of which can function as a second hand (so that it makes one revolution every 60.0 s). (a) Find the speed of the passengers when the Ferris wheel is rotating at this rate. (b) A passenger weighs 882 N at the weight-guessing booth on the ground. What is his apparent weight at the highest and at the lowest point on the Ferris wheel? (c) What would be the time for one revolution if the passenger's apparent weight at the highest point were zero? (d) What then would be the passenger's apparent weight at the lowest point?

**5.55 ••** A small rock with mass $m$ is attached to a light string of length $L$ and whirled in a vertical circle of radius $R$. (a) What is the minimum speed $v$ at the rock's highest point for which it stays in a circular path? (b) If the speed at the rock's lowest point in its circular path is twice the value found in part (a), what is the tension in the string when the rock is at this point?

**5.56 ••** A 50.0 kg stunt pilot who has been diving her airplane vertically pulls out of the dive by changing her course to a circle in a vertical plane. (a) If the plane's speed at the lowest point of the circle is 95.0 m/s, what is the minimum radius of the circle so that the acceleration at this point will not exceed $4.00g$? (b) What is the apparent weight of the pilot at the lowest point of the pullout?

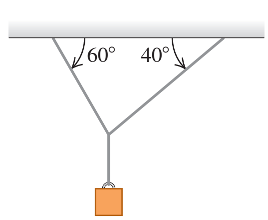
**5.57 •** **Stay Dry!** You tie a cord to a pail of water and swing the pail in a vertical circle of radius 0.600 m. What minimum speed must you give the pail at the highest point of the circle to avoid spilling water?

**5.58 ••** A bowling ball weighing 71.2 N (16.0 lb) is attached to the ceiling by a 3.80 m rope. The ball is pulled to one side and released; it then swings back and forth as a pendulum. As the rope swings through the vertical, the speed of the bowling ball is 4.20 m/s. At this instant, what are (a) the acceleration of the bowling ball, in magnitude and direction, and (b) the tension in the rope?

## PROBLEMS

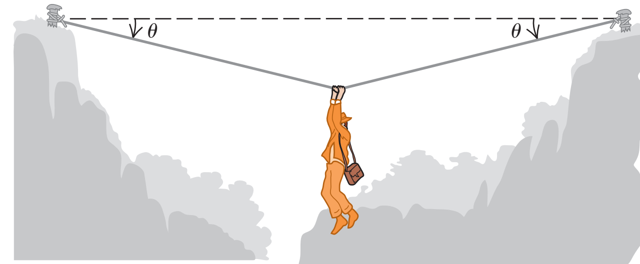
**5.59 •••** Two ropes are connected to a steel cable that supports a hanging weight (**Fig. P5.59**). (a) Draw a free-body diagram showing all of the forces acting at the knot that connects the two ropes to the steel cable. Based on your diagram, which of the two ropes will have the greater tension? (b) If the maximum tension either rope can sustain without breaking is 5000 N, determine the maximum value of the hanging weight that these ropes can safely support. Ignore the weight of the ropes and of the steel cable.

Figure P5.59



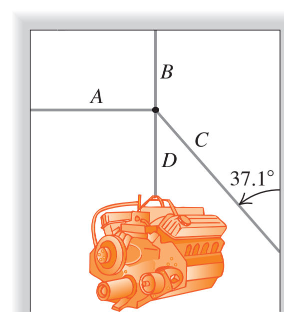

**5.60 ••** An adventurous archaeologist crosses between two rock cliffs by slowly going hand over hand along a rope stretched between the cliffs. He stops to rest at the middle of the rope (**Fig. P5.60**). The rope will break if the tension in it exceeds $2.50 \times 10^4$ N, and our hero's mass is 90.0 kg. (a) If the angle $\theta$ is 10.0°, what is the tension in the rope? (b) What is the smallest value $\theta$ can have if the rope is not to break?

Figure P5.60



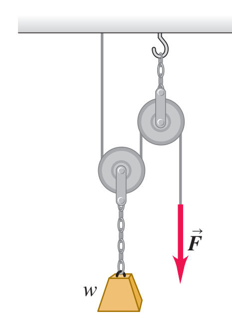

**5.61 ••** In a repair shop a truck engine that has mass 409 kg is held in place by four light cables (**Fig. P5.61**). Cable $A$ is horizontal, cables $B$ and $D$ are vertical, and cable $C$ makes an angle of 37.1° with a vertical wall. If the tension in cable $A$ is 722 N, what are the tensions in cables $B$ and $C$?

Figure P5.61




**5.62 ••** In **Fig. P5.62** a worker lifts a weight $w$ by pulling down on a rope with a force $\vec{F}$. The upper pulley is attached to the ceiling by a chain, and the lower pulley is attached to the weight by another chain. Draw one or more free-body diagrams to find the tension in each chain and the magnitude of $\vec{F}$, in terms of $w$, if the weight is lifted at constant speed. Assume that the rope, pulleys, and chains have negligible weights.

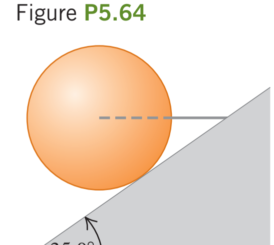
Figure P5.62




**5.63 •• CP** A small block sits at one end of a flat board that is 3.00 m long. The coefficients of friction between the block and the board are $\mu_s = 0.600$ and $\mu_k = 0.400$. The end of the board where the block sits is slowly raised until the angle the board makes with the horizontal is $\alpha_0$, and then the block starts to slide down the board. If the angle is kept equal to $\alpha_0$ as the block slides, what is the speed of the block when it reaches the bottom of the board?

**5.64 •••** A horizontal wire holds a solid uniform ball of mass $m$ in place on a tilted ramp that rises 35.0° above the horizontal. The surface of this ramp is perfectly smooth, and the wire is directed away from the center of the ball (**Fig. P5.64**). (a) Draw a free-body diagram of the ball. (b) How hard does the surface of the ramp push on the ball? (c) What is the tension in the wire?

Figure P5.64



**5.65 •• CP** A box of mass 12.0 kg sits at rest on a horizontal surface. The coefficient of kinetic friction between the surface and the box is 0.300. The box is initially at rest, and then a constant force of magnitude $F$ and direction 37.0° below the horizontal is applied to the box; the box slides along the surface. (a) What is $F$ if the box has a speed of 6.00 m/s after traveling a distance of 8.00 m? (b) What is $F$ if the surface is frictionless and all the other quantities are the same? (c) What is $F$ if all the quantities are the same as in part (a) but the force applied to the box is horizontal?

**5.66 •• CP** A box is sliding with a constant speed of 4.00 m/s in the $+x$-direction on a horizontal, frictionless surface. At $x = 0$ the box encounters a rough patch of the surface, and then the surface becomes even rougher. Between $x = 0$ and $x = 2.00$ m, the coefficient of kinetic friction between the box and the surface is 0.200; between $x = 2.00$ m and $x = 4.00$ m, it is 0.400. (a) What is the $x$-coordinate of the point where the box comes to rest? (b) How much time does it take the box to come to rest after it first encounters the rough patch at $x = 0$?

**5.67 •• CP BIO Forces During Chin-ups.** When you do a chin-up, you raise your chin just over a bar (the chinning bar), supporting yourself with only your arms. Typically, the body below the arms is raised by about 30 cm in a time of 1.0 s, starting from rest. Assume that the entire body of a 680 N person doing chin-ups is raised by 30 cm, and that half the 1.0 s is spent accelerating upward and the other half accelerating downward, uniformly in both cases. Draw a free-body diagram of the person's body, and use it to find the force his arms must exert on him during the accelerating part of the chin-up.

**5.68 •• CP CALC** A 2.00 kg box is suspended from the end of a light vertical rope. A time-dependent force is applied to the upper end of the rope, and the box moves upward with a velocity magnitude that varies in time according to $v(t) = (2.00\ \text{m/s}^2)t + (0.600\ \text{m/s}^3)t^2$. What is the tension in the rope when the velocity of the box is 9.00 m/s?

**5.69 ••• CALC** A 3.00 kg box that is several hundred meters above the earth's surface is suspended from the end of a short vertical rope of negligible mass. A time-dependent upward force is applied to the upper end of the rope and results in a tension in the rope of $T(t) = (36.0\ \text{N/s})t$. The box is at rest at $t = 0$. The only forces on the box are the tension in the rope and gravity. (a) What is the velocity of the box at (i) $t = 1.00$ s and (ii) $t = 3.00$ s? (b) What is the maximum distance that the box descends below its initial position? (c) At what value of $t$ does the box return to its initial position?

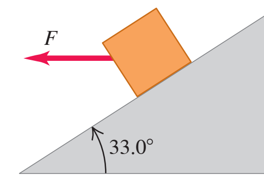
**5.70 •• CP** A 5.00 kg box sits at rest at the bottom of a ramp that is 8.00 m long and is inclined at 30.0° above the horizontal. The coefficient of kinetic friction is $\mu_k = 0.40$, and the coefficient of static friction is $\mu_s = 0.43$. What constant force $F$, applied parallel to the surface of the ramp, is required to push the box to the top of the ramp in a time of 6.00 s?

**5.71 ••** When a crate with mass 25.0 kg is placed on a ramp that is inclined at an angle $\alpha$ below the horizontal, it slides down the ramp with an acceleration of 4.9 m/s$^2$. The ramp is not frictionless. To increase the acceleration of the crate, a downward vertical force $\vec{F}$ is applied to the top of the crate. What must $F$ be in order to increase the acceleration of the crate so that it is 9.8 m/s$^2$? How does the value of $F$ that you calculate compare to the weight of the crate?

**5.72 ••** A large crate is at rest on a horizontal floor. The coefficient of static friction between the crate and the floor is 0.400. A force $\vec{F}$ is applied to the crate in a direction 30.0° above the horizontal. The minimum value of $F$ required to get the crate to start sliding is 380 N. What is the mass of the crate?

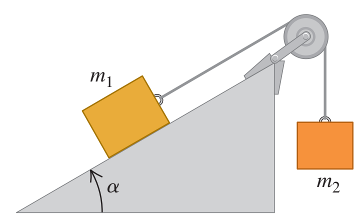
**5.73 •• CP** An 8.00 kg box sits on a ramp that is inclined at 33.0° above the horizontal. The coefficient of kinetic friction between the box and the surface of the ramp is $\mu_k = 0.300$. A constant *horizontal* force $F = 26.0$ N is applied to the box (**Fig. P5.73**), and the box moves down the ramp. If the box is initially at rest, what is its speed 2.00 s after the force is applied?

Figure **P5.73**



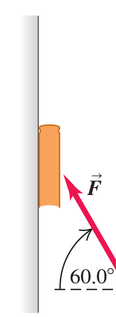

**5.74 •• CP** In **Fig. P5.74,** $m_1 = 20.0$ kg and $\alpha = 53.1°$. The coefficient of kinetic friction between the block of mass $m_1$ and the incline is $\mu_k = 0.40$. What must be the mass $m_2$ of the hanging block if it is to descend 12.0 m in the first 3.00 s after the system is released from rest?

Figure **P5.74**



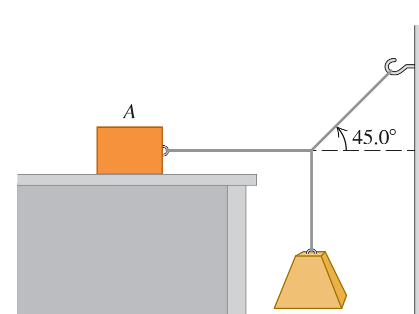

**5.75 •• CP** You place a book of mass 5.00 kg against a vertical wall. You apply a constant force $\vec{F}$ to the book, where $F = 96.0$ N and the force is at an angle of 60.0° above the horizontal (**Fig. P5.75**). The coefficient of kinetic friction between the book and the wall is 0.300. If the book is initially at rest, what is its speed after it has traveled 0.400 m up the wall?

Figure **P5.75**




**5.76 ••** Block $A$ in **Fig. P5.76** weighs 60.0 N. The coefficient of static friction between the block and the surface on which it rests is 0.25. The weight $w$ is 12.0 N, and the system is in equilibrium. (a) Find the friction force exerted on block $A$. (b) Find the maximum weight $w$ for which the system will remain in equilibrium.

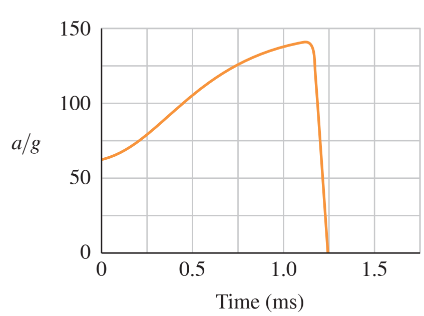
Figure **P5.76**



**5.77 ••** A block with mass $m_1$ is placed on an inclined plane with slope angle $\alpha$ and is connected to a hanging block with mass $m_2$ by a cord passing over a small, frictionless pulley (Fig. P5.74). The coefficient of static friction is $\mu_s$, and the coefficient of kinetic friction is $\mu_k$. (a) Find the value of $m_2$ for which the block of mass $m_1$ moves up the plane at constant speed once it is set in motion. (b) Find the value of $m_2$ for which the block of mass $m_1$ moves down the plane at constant speed once it is set in motion. (c) For what range of values of $m_2$ will the blocks remain at rest if they are released from rest?

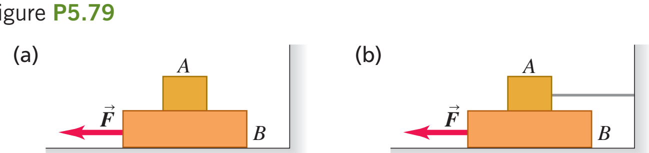
**5.78 •• DATA BIO The Flying Leap of a Flea.** High-speed motion pictures (3500 frames/second) of a jumping 210 $\mu$g flea yielded the data to plot the flea's acceleration as a function of time, as shown in **Fig. P5.78.** (See "The Flying Leap of the Flea," by M. Rothschild et al., *Scientific American,* November 1973.) This flea was about 2 mm long and jumped at a nearly vertical takeoff angle. Using the graph, (a) find the *initial* net external force on the flea. How does it compare to the flea's weight? (b) Find the *maximum* net external force on this jumping flea. When does this maximum force occur? (c) Use the graph to find the flea's maximum speed.

Figure P5.78




**5.79 ••** Block $A$ in **Fig. P5.79** weighs 1.20 N, and block $B$ weighs 3.60 N. The coefficient of kinetic friction between all surfaces is 0.300. Find the magnitude of the horizontal force $\vec{F}$ necessary to drag block $B$ to the left at constant speed (a) if $A$ rests on $B$ and moves with it (Fig. P5.79a), (b) if $A$ is held at rest (Fig. P5.79b).

Figure P5.79




**5.80 ••• CP Elevator Design.** You are designing an elevator for a hospital. The force exerted on a passenger by the floor of the elevator is not to exceed 1.60 times the passenger's weight. The elevator accelerates upward with constant acceleration for a distance of 3.0 m and then starts to slow down. What is the maximum speed of the elevator?

**5.81 ••• CP CALC** You are standing on a bathroom scale in an elevator in a tall building. Your mass is 64 kg. The elevator starts from rest and travels upward with a speed that varies with time according to $v(t) = (3.0\ \text{m/s}^2)t + (0.20\ \text{m/s}^3)t^2$. When $t = 4.0$ s, what is the reading on the bathroom scale?

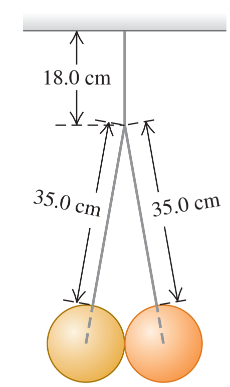
**5.82 •• CP** Consider the system shown in Fig. E5.34. As in Exercise 5.34, block $A$ weighs 45.0 N and block $B$ weighs 25.0 N. The system is released from rest, and block $B$ has speed 3.30 m/s after it has descended 2.00 m. (a) While the blocks are moving, what is the tension in the rope connecting the blocks? (b) What is the coefficient of kinetic friction between block $A$ and the tabletop?

**5.83 •• CP** Two blocks are suspended from opposite ends of a light rope that passes over a light, frictionless pulley. One block has mass $m_1$ and the other has mass $m_2$, where $m_2 > m_1$. The two blocks are released from rest, and the block with mass $m_2$ moves downward 5.00 m in 2.00 s after being released. While the blocks are moving, the tension in the rope is 16.0 N. Calculate $m_1$ and $m_2$.

**5.84 •••** If the coefficient of static friction between a table and a uniform, massive rope is $\mu_s$, what fraction of the rope can hang over the edge of the table without the rope sliding?

**5.85 •••** Two identical 15.0 kg balls, each 25.0 cm in diameter, are suspended by two 35.0 cm wires (**Fig. P5.85**). The entire apparatus is supported by a single 18.0 cm wire, and the surfaces of the balls are perfectly smooth. (a) Find the tension in each of the three wires. (b) How hard does each ball push on the other one?

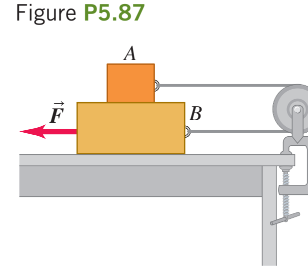
Figure P5.85




**5.86 • CP Traffic Court.** You are called as an expert witness in a trial for a traffic violation. The facts are these: A driver slammed on his brakes and came to a stop with constant acceleration. Measurements of his tires and the skid marks on the pavement indicate that he locked his car's wheels, the car traveled 192 ft before stopping, and the coefficient of kinetic friction between the road and his tires was 0.750. He was charged with speeding in a 45 mi/h zone but pleads innocent. What is your conclusion: guilty or innocent? How fast was he going when he hit his brakes?

**5.87 •••** Block $A$ in **Fig. P5.87** weighs 1.90 N, and block $B$ weighs 4.20 N. The coefficient of kinetic friction between all surfaces is 0.30. Find the magnitude of the horizontal force $\vec{F}$ necessary to drag block $B$ to the left at constant speed if $A$ and $B$ are connected by a light, flexible cord passing around a fixed, frictionless pulley.

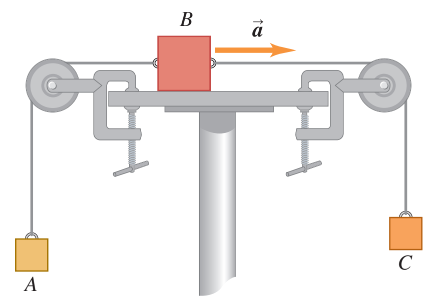
Figure P5.87




**5.88 ••** Block $B$ has mass 5.00 kg and sits at rest on a horizontal, frictionless surface. Block $A$ has mass 2.00 kg and sits at rest on top of block $B$. The coefficient of static friction between the two blocks is 0.400. A horizontal force $\vec{P}$ is then applied to block $A$. What is the largest value $P$ can have and the blocks move together with equal accelerations?

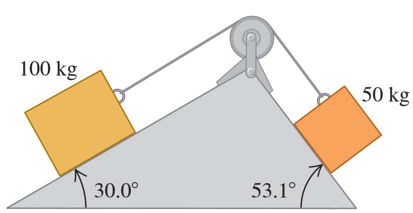
**5.89 ••** Block $A$ in **Fig. P5.89** has mass 4.00 kg, and block $B$ has mass 12.0 kg. The coefficient of kinetic friction between block $B$ and the horizontal surface is 0.25. (a) What is the mass of block $C$ if block $B$ is moving to the right and speeding up with an acceleration of 2.00 m/s$^2$? (b) What is the tension in each cord when block $B$ has this acceleration?

Figure **P5.89**



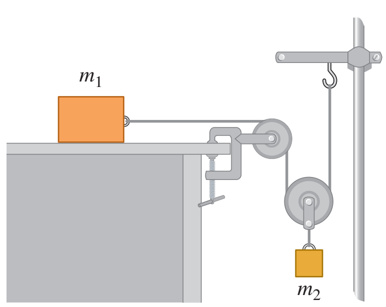

**5.90** •• Two blocks connected by a cord passing over a small, frictionless pulley rest on frictionless planes (**Fig. P5.90**). (a) Which way will the system move when the blocks are released from rest? (b) What is the acceleration of the blocks? (c) What is the tension in the cord?

Figure **P5.90**



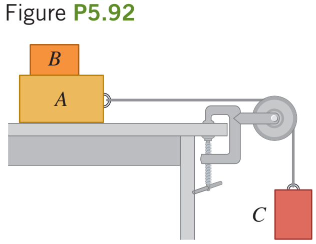

**5.91** •• In terms of $m_1$, $m_2$, and $g$, find the acceleration of each block in **Fig. P5.91.** There is no friction anywhere in the system.

Figure **P5.91**




**5.92** ••• Block *B*, with mass 5.00 kg, rests on block *A*, with mass 8.00 kg, which in turn is on a horizontal tabletop (**Fig. P5.92**). There is no friction between block *A* and the tabletop, but the coefficient of static friction between blocks *A* and *B* is 0.750. A light string attached to block *A* passes over a frictionless, massless pulley, and block *C* is suspended from the other end of the string. What is the largest mass that block *C* can have so that blocks *A* and *B* still slide together when the system is released from rest?

Figure **P5.92**

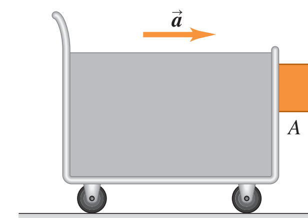



**5.93** •• Consider two blocks connected as shown in Fig. P5.87. Block *A* has mass 2.00 kg, and block *B* has mass 5.00 kg. The table on which *B* sits is frictionless, the cord connecting the blocks is light and flexible, and the pulley is light and frictionless. The horizontal force $\vec{F}$ has magnitude $F = 20.0$ N, and block *B* moves to the left with an acceleration of 1.50 m/s$^2$. (a) What is the tension in the cord that connects the two blocks? (b) What is the coefficient of kinetic friction that one block exerts on the other?

**5.94** •• **Friction in an Elevator.** You are riding in an elevator on the way to the 18th floor of your dormitory. The elevator is accelerating upward with $a = 1.90$ m/s$^2$. Beside you is the box containing your new computer; the box and its contents have a total mass of 36.0 kg. While the elevator is accelerating upward, you push horizontally on the box to slide it at constant speed toward the elevator door. If the coefficient of kinetic friction between the box and the elevator floor is $\mu_k = 0.32$, what magnitude of force must you apply?

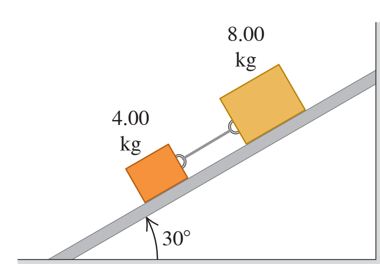
**5.95** • A block is placed against the vertical front of a cart (**Fig. P5.95**). What acceleration must the cart have so that block *A* does not fall? The coefficient of static friction between the block and the cart is $\mu_s$. How would an observer on the cart describe the behavior of the block?

Figure **P5.95**



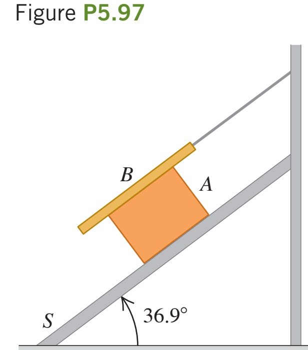

**5.96** ••• Two blocks, with masses 4.00 kg and 8.00 kg, are connected by a string and slide down a 30.0° inclined plane (**Fig. P5.96**). The coefficient of kinetic friction between the 4.00 kg block and the plane is 0.25; that between the 8.00 kg block and the plane is 0.35. Calculate (a) the acceleration of each block and (b) the tension in the string. (c) What happens if the positions of the blocks are reversed, so that the 4.00 kg block is uphill from the 8.00 kg block?

Figure **P5.96**




**5.97** ••• Block *A*, with weight $3w$, slides down an inclined plane *S* of slope angle 36.9° at a constant speed while plank *B*, with weight $w$, rests on top of *A*. The plank is attached by a cord to the wall (**Fig. P5.97**). (a) Draw a diagram of all the forces acting on block *A*. (b) If the coefficient of kinetic friction is the same between *A* and *B* and between *S* and *A*, determine its value.

Figure **P5.97**



**5.98 ••** Jack sits in the chair of a Ferris wheel that is rotating at a constant 0.100 rev/s. As Jack passes through the highest point of his circular path, the upward force that the chair exerts on him is equal to one-fourth of his weight. What is the radius of the circle in which Jack travels? Treat him as a point mass.

**5.99 ••** On the ride "Spindletop" at the amusement park Six Flags Over Texas, people stood against the inner wall of a hollow vertical cylinder with radius 2.5 m. The cylinder started to rotate, and when it reached a constant rotation rate of 0.60 rev/s, the floor dropped about 0.5 m. The people remained pinned against the wall without touching the floor. (a) Draw a force diagram for a person on this ride after the floor has dropped. (b) What minimum coefficient of static friction was required for the person not to slide downward to the new position of the floor? (c) Does your answer in part (b) depend on the person's mass? (*Note:* When such a ride is over, the cylinder is slowly brought to rest. As it slows down, people slide down the wall to the floor.)

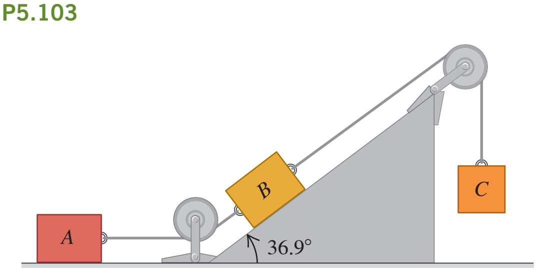
**5.100 ••** A large piece of ice breaks loose from the roof of a chalet at a ski resort and slides across the snow-covered terrain. It passes over the top of a hill in a path that has the shape of the arc of a circle of radius $R$. What is the speed $v$ of the piece of ice as it passes over the top of the hill if the normal force exerted on it at this point is half its weight?

**5.101 •••** A racetrack curve has radius 90.0 m and is banked at an angle of 18.0°. The coefficient of static friction between the tires and the roadway is 0.400. A race car with mass 1200 kg rounds the curve with the maximum speed to avoid skidding. (a) As the car rounds the curve, what is the normal force exerted on it by the road? What are the car's (b) radial acceleration and (c) speed?

**5.102 •••** A racetrack curve has radius 120.0 m and is banked at an angle of 18.0°. The coefficient of static friction between the tires and the roadway is 0.300. A race car with mass 900 kg rounds the curve with the minimum speed needed to not slide down the banking. (a) As the car rounds the curve, what is the normal force exerted on it by the road? (b) What is the car's speed?

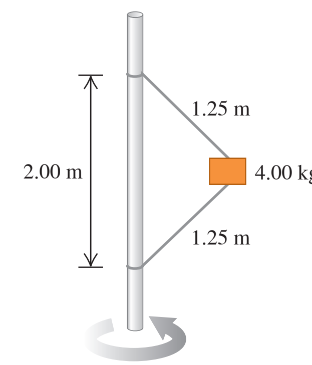
**5.103 •••** Blocks $A$, $B$, and $C$ are placed as in **Fig. P5.103** and connected by ropes of negligible mass. Both $A$ and $B$ weigh 25.0 N each, and the coefficient of kinetic friction between each block and the surface is 0.35. Block $C$ descends with constant velocity. (a) Draw separate free-body diagrams showing the forces acting on $A$ and on $B$. (b) Find the tension in the rope connecting blocks $A$ and $B$. (c) What is the weight of block $C$? (d) If the rope connecting $A$ and $B$ were cut, what would be the acceleration of $C$?

Figure **P5.103**




**5.104 •••** A 4.00 kg block is attached to a vertical rod by means of two strings. When the system rotates about the axis of the rod, the strings are extended as shown in **Fig. P5.104** and the tension in the upper string is 80.0 N. (a) What is the tension in the lower cord? (b) How many revolutions per minute does the system make? (c) Find the number of revolutions per minute at which the lower cord just goes slack. (d) Explain what happens if the number of revolutions per minute is less than that in part (c).

Figure **P5.104**

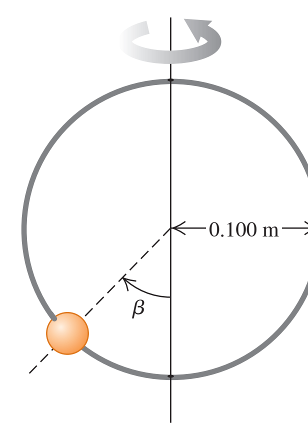



**5.105 •• CALC** You throw a rock downward into water with a speed of $3mg/k$, where $k$ is the coefficient in Eq. (5.5). Assume that the relationship between fluid resistance and speed is as given in Eq. (5.5), and calculate the speed of the rock as a function of time.

**5.106 •••** A box with mass $m$ sits at the bottom of a long ramp that is sloped upward at an angle $\alpha$ above the horizontal. You give the box a quick shove, and after it leaves your hands it is moving up the ramp with an initial speed $v_0$. The box travels a distance $d$ up the ramp and then slides back down. When it returns to its starting point, the speed of the box is half the speed it started with; it has speed $v_0/2$. What is the coefficient of kinetic friction between the box and the ramp? (Your answer should depend on only $\alpha$.)

**5.107 ••** A small bead can slide without friction on a circular hoop that is in a vertical plane and has a radius of 0.100 m. The hoop rotates at a constant rate of 4.00 rev/s about a vertical diameter (**Fig. P5.107**). (a) Find the angle $\beta$ at which the bead is in vertical equilibrium. (It has a radial acceleration toward the axis.) (b) Is it possible for the bead to "ride" at the same elevation as the center of the hoop? (c) What will happen if the hoop rotates at 1.00 rev/s?

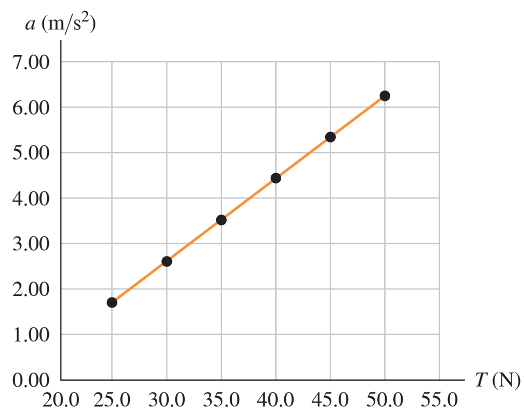
Figure **P5.107**




**5.108 ••** A physics major is working to pay her college tuition by performing in a traveling carnival. She rides a motorcycle inside a hollow, transparent plastic sphere. After gaining sufficient speed, she travels in a vertical circle with radius 13.0 m. She has mass 70.0 kg, and her motorcycle has mass 40.0 kg. (a) What minimum speed must she have at the top of the circle for the motorcycle tires to remain in contact with the sphere? (b) At the bottom of the circle, her speed is twice the value calculated in part (a). What is the magnitude of the normal force exerted on the motorcycle by the sphere at this point?

**5.109 •• DATA** In your physics lab, a block of mass m is at rest on a horizontal surface. You attach a light cord to the block and apply a horizontal force to the free end of the cord. You find that the block remains at rest until the tension $T$ in the cord exceeds 20.0 N. For $T > 20.0$ N, you measure the acceleration of the block when $T$ is maintained at a constant value, and you plot the results (**Fig. P5.109**). The equation for the straight line that best fits your data is $a = [0.182\ \text{m}/(\text{N}\cdot\text{s}^2)]T - 2.842\ \text{m/s}^2$. For this block and surface, what are (a) the coefficient of static friction and (b) the coefficient of kinetic friction? (c) If the experiment were done on the earth's moon, where $g$ is much smaller than on the earth, would the graph of $a$ versus $T$ still be fit well by a straight line? If so, how would the slope and intercept of the line differ from the values in Fig. P5.109? Or, would each of them be the same?

Figure **P5.109**




**5.110 •• DATA** A road heading due east passes over a small hill. You drive a car of mass $m$ at constant speed $v$ over the top of the hill, where the shape of the roadway is well approximated as an arc of a circle with radius $R$. Sensors have been placed on the road surface there to measure the downward force that cars exert on the surface at various speeds. The table gives values of this force versus speed for your car:

| **Speed (m/s)** | 6.00 | 8.00 | 10.0 | 12.0 | 14.0 | 16.0 |
|---|---|---|---|---|---|---|
| **Force (N)** | 8100 | 7690 | 7050 | 6100 | 5200 | 4200 |

Treat the car as a particle. (a) Plot the values in such a way that they are well fitted by a straight line. You might need to raise the speed, the force, or both to some power. (b) Use your graph from part (a) to calculate $m$ and $R$. (c) What maximum speed can the car have at the top of the hill and still not lose contact with the road?

**5.111 •• DATA** You are an engineer working for a manufacturing company. You are designing a mechanism that uses a cable to drag heavy metal blocks a distance of 8.00 m along a ramp that is sloped at 40.0° above the horizontal. The coefficient of kinetic friction between these blocks and the incline is $\mu_k = 0.350$. Each block has a mass of 2170 kg. The block will be placed on the bottom of the ramp, the cable will be attached, and the block will then be given just enough of a momentary push to overcome static friction. The block is then to accelerate at a constant rate to move the 8.00 m in 4.20 s. The cable is made of wire rope and is parallel to the ramp surface. The table gives the breaking strength of the cable as a function of its diameter; the safe load tension, which is 20% of the breaking strength; and the mass per meter of the cable:

| Cable Diameter (in.) | Breaking Strength (kN) | Safe Load (kN) | Mass per Meter (kg/m) |
|---|---|---|---|
| $\frac{1}{4}$ | 24.4 | 4.89 | 0.16 |
| $\frac{3}{8}$ | 54.3 | 10.9 | 0.36 |
| $\frac{1}{2}$ | 95.2 | 19.0 | 0.63 |
| $\frac{5}{8}$ | 149 | 29.7 | 0.98 |
| $\frac{3}{4}$ | 212 | 42.3 | 1.41 |
| $\frac{7}{8}$ | 286 | 57.4 | 1.92 |
| 1 | 372 | 74.3 | 2.50 |

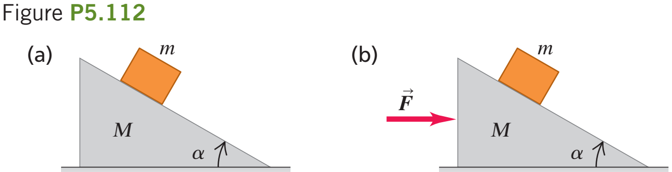
Source: www.engineeringtoolbox.com

(a) What is the minimum diameter of the cable that can be used to pull a block up the ramp without exceeding the safe load value of the tension in the cable? Ignore the mass of the cable, and select the diameter from those listed in the table. (b) You need to know safe load values for diameters that aren't in the table, so you hypothesize that the breaking strength and safe load limit are proportional to the cross-sectional area of the cable. Draw a graph that tests this hypothesis, and discuss its accuracy. What is your estimate of the safe load value for a cable with diameter $\frac{9}{16}$ in.? (c) The coefficient of static friction between the crate and the ramp is $\mu_s = 0.620$, which is nearly twice the value of the coefficient of kinetic friction. If the machinery jams and the block stops in the middle of the ramp, what is the tension in the cable? Is it larger or smaller than the value when the block is moving? (d) Is the actual tension in the cable, at its upper end, larger or smaller than the value calculated when you ignore the mass of the cable? If the cable is 9.00 m long, how accurate is it to ignore the cable's mass?

## CHALLENGE PROBLEMS

**5.112 ••• Moving Wedge.** A wedge with mass $M$ rests on a frictionless, horizontal tabletop. A block with mass $m$ is placed on the wedge (**Fig. P5.112a**). There is no friction between the block and the wedge. The system is released from rest. (a) Calculate the acceleration of the wedge and the horizontal and vertical components of the acceleration of the block. (b) Do your answers to part (a) reduce to the correct results when $M$ is very large? (c) As seen by a stationary observer, what is the shape of the trajectory of the block?

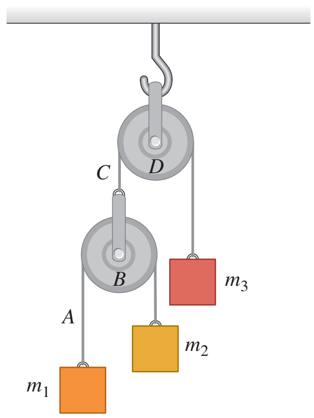
Figure **P5.112**




**5.113 •••** A wedge with mass $M$ rests on a frictionless, horizontal tabletop. A block with mass $m$ is placed on the wedge, and a horizontal force $\vec{F}$ is applied to the wedge (Fig. P5.112b). What must the magnitude of $\vec{F}$ be if the block is to remain at a constant height above the tabletop?

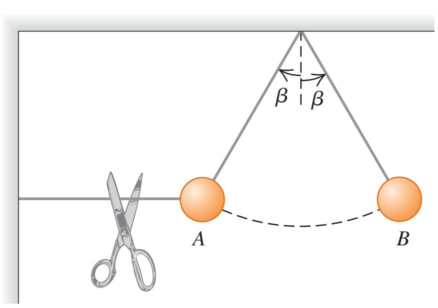
**5.114 ••• Double Atwood's Machine.** In **Fig. P5.114** masses $m_1$ and $m_2$ are connected by a light string $A$ over a light, frictionless pulley $B$. The axle of pulley $B$ is connected by a light string $C$ over a light, frictionless pulley $D$ to a mass $m_3$. Pulley $D$ is suspended from the ceiling by an attachment to its axle. The system is released from rest. In terms of $m_1$, $m_2$, $m_3$, and $g$, what are (a) the acceleration of block $m_3$; (b) the acceleration of pulley $B$; (c) the acceleration of block $m_1$; (d) the acceleration of block $m_2$; (e) the tension in string $A$; (f) the tension in string $C$? (g) What do your expressions give for the special case of $m_1 = m_2$ and $m_3 = m_1 + m_2$? Is this reasonable?

Figure P5.114




**5.115 •••** A ball is held at rest at position $A$ in **Fig. P5.115** by two light strings. The horizontal string is cut, and the ball starts swinging as a pendulum. Position $B$ is the farthest to the right that the ball can go as it swings back and forth. What is the ratio of the tension in the supporting string at $B$ to its value at $A$ before the string was cut?

Figure P5.115




## MCAT-STYLE PASSAGE PROBLEMS

**Friction and Climbing Shoes.** Shoes made for the sports of bouldering and rock climbing are designed to provide a great deal of friction between the foot and the surface of the ground. Such shoes on smooth rock might have a coefficient of static friction of 1.2 and a coefficient of kinetic friction of 0.90.

**5.116** For a person wearing these shoes, what's the maximum angle (with respect to the horizontal) of a smooth rock that can be walked on without slipping? (a) 42°; (b) 50°; (c) 64°; (d) larger than 90°.

**5.117** If the person steps onto a smooth rock surface that's inclined at an angle large enough that these shoes begin to slip, what will happen? (a) She will slide a short distance and stop; (b) she will accelerate down the surface; (c) she will slide down the surface at constant speed; (d) we can't tell what will happen without knowing her mass.

**5.118** A person wearing these shoes stands on a smooth, horizontal rock. She pushes against the ground to begin running. What is the maximum horizontal acceleration she can have without slipping? (a) $0.20g$; (b) $0.75g$; (c) $0.90g$; (d) $1.2g$.

# ANSWERS

## Chapter Opening Question ?

**(iii)** The upward force exerted by the air has the same magnitude as the force of gravity. Although the seed and pappus are descending, their vertical velocity is constant, so their vertical acceleration is zero. According to Newton's first law, the net vertical force on the seed and pappus must also be zero. The individual vertical forces must balance.

## Key Example √ARIATION Problems

**VP5.5.1** (a) 425 N (b) 255 N
**VP5.5.2** (a) 367 N (b) 522 N
**VP5.5.3** (a) 21.8° (b) 637 N
**VP5.5.4** $w(1 - \sin\theta)$, less than $w$
**VP5.15.1** (a) 79.3 N (b) 298 N
**VP5.15.2** (a) 90.2 N (b) 348 N
**VP5.15.3** (a) 449 N (b) 0.670 m/s$^2$, speeding up
**VP5.15.4** $\mu_s = (T_{\min}\cos\theta)/(mg - T_{\min}\sin\theta)$
**VP5.22.1** (a) 0.274 m (b) 1.74 s (c) 2.61 N
**VP5.22.2** (a) 19.0 m (b) 8.22 m/s$^2$ (c) 819 N
**VP5.22.3** (a) 75.0° (b) $3.03 \times 10^3$ N, 3.86 times greater
**VP5.22.4** (a) $mg\sqrt{x^2 - 1}$ (b) $\dfrac{v^2}{g\sqrt{x^2 - 1}}$

## Bridging Problem

(a) $T_{\max} = 2\pi\sqrt{\dfrac{h(\cos\beta + \mu_s\sin\beta)}{g\tan\beta(\sin\beta - \mu_s\cos\beta)}}$

(b) $T_{\min} = 2\pi\sqrt{\dfrac{h(\cos\beta - \mu_s\sin\beta)}{g\tan\beta(\sin\beta + \mu_s\cos\beta)}}$