? Tornadoes are spawned by severe thunderstorms, so being able to predict the path of thunderstorms is essential. If a thunderstorm is moving at 15 km/h in a direction 37° north of east, how far north does the thunderstorm move in 2.0 h? (i) 30 km; (ii) 24 km; (iii) 18 km; (iv) 12 km; (v) 9 km.

1 Units, Physical Quantities, and Vectors

Physics is one of the most fundamental of the sciences. Scientists of all disciplines use the ideas of physics, including chemists who study the structure of molecules, paleontologists who try to reconstruct how dinosaurs walked, and climatologists who study how human activities affect the atmosphere and oceans. Physics is also the foundation of all engineering and technology. No engineer could design a flat-screen TV, a prosthetic leg, or even a better mousetrap without first understanding the basic laws of physics.

The study of physics is also an adventure. You'll find it challenging, sometimes frustrating, occasionally painful, and often richly rewarding. If you've ever wondered why the sky is blue, how radio waves can travel through empty space, or how a satellite stays in orbit, you can find the answers by using fundamental physics. You'll come to see physics as a towering achievement of the human intellect in its quest to understand our world and ourselves.

In this opening chapter, we'll go over some important preliminaries that we'll need throughout our study. We'll discuss the nature of physical theory and the use of idealized models to represent physical systems. We'll introduce the systems of units used to describe physical quantities and discuss ways to describe the accuracy of a number. We'll look at examples of problems for which we can't (or don't want to) find a precise answer, but for which rough estimates can be useful and interesting. Finally, we'll study several aspects of vectors and vector algebra. We'll need vectors throughout our study of physics to help us describe and analyze physical quantities, such as velocity and force, that have direction as well as magnitude.

LEARNING OUTCOMES

In this chapter, you'll learn...

1.1 What a physical theory is.
1.2 The four steps you can use to solve any physics problem.
1.3 Three fundamental quantities of physics and the units physicists use to measure them.
1.4 How to work with units in your calculations.
1.5 How to keep track of significant figures in your calculations.
1.6 How to make rough, order-of-magnitude estimates.
1.7 The difference between scalars and vectors, and how to add and subtract vectors graphically.
1.8 What the components of a vector are and how to use them in calculations.
1.9 What unit vectors are and how to use them with components to describe vectors.
1.10 Two ways to multiply vectors: the scalar (dot) product and the vector (cross) product.

1.1 THE NATURE OF PHYSICS

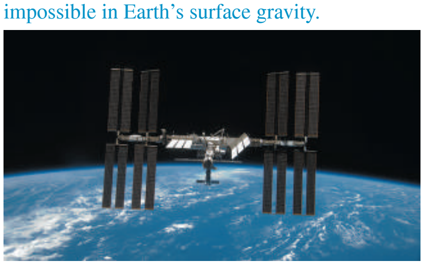
Physics is an *experimental* science. Physicists observe the phenomena of nature and try to find patterns that relate these phenomena. These patterns are called physical theories or, when they are very well established and widely used, physical laws or principles.

Figure **1.1** Two research laboratories.

(a) According to legend, Galileo investigated falling objects by dropping them from the Leaning Tower of Pisa, Italy, ...

... and he studied pendulum motion by observing the swinging chandelier in the adjacent cathedral.

(b) By doing experiments in apparent weightlessness on board the International Space Station, physicists have been able to make sensitive measurements that would be impossible in Earth's surface gravity.



CAUTION **The meaning of "theory"** A theory is *not* just a random thought or an unproven concept. Rather, a theory is an explanation of natural phenomena based on observation and accepted fundamental principles. An example is the well-established theory of biological evolution, which is the result of extensive research and observation by generations of biologists.

To develop a physical theory, a physicist has to ask appropriate questions, design experiments to try to answer the questions, and draw appropriate conclusions from the results. **Figure 1.1** shows two important facilities used for physics experiments.

Legend has it that Galileo Galilei (1564–1642) dropped light and heavy objects from the top of the Leaning Tower of Pisa (Fig. 1.1a) to find out whether their rates of fall were different. From examining the results of his experiments (which were actually much more sophisticated than in the legend), he deduced the theory that the acceleration of a freely falling object is independent of its weight.

The development of physical theories such as Galileo's often takes an indirect path, with blind alleys, wrong guesses, and the discarding of unsuccessful theories in favor of more promising ones. Physics is not simply a collection of facts and principles; it is also the *process* by which we arrive at general principles that describe how the physical universe behaves.

No theory is ever regarded as the ultimate truth. It's always possible that new observations will require that a theory be revised or discarded. Note that we can disprove a theory by finding behavior that is inconsistent with it, but we can never prove that a theory is always correct.

Getting back to Galileo, suppose we drop a feather and a cannonball. They certainly do *not* fall at the same rate. This does not mean that Galileo was wrong; it means that his theory was incomplete. If we drop the feather and the cannonball *in a vacuum* to eliminate the effects of the air, then they do fall at the same rate. Galileo's theory has a **range of validity:** It applies only to objects for which the force exerted by the air (due to air resistance and buoyancy) is much less than the weight. Objects like feathers or parachutes are clearly outside this range.

1.2 SOLVING PHYSICS PROBLEMS

At some point in their studies, almost all physics students find themselves thinking, "I understand the concepts, but I just can't solve the problems." But in physics, truly understanding a concept *means* being able to apply it to a variety of problems. Learning how to solve problems is absolutely essential; you don't *know* physics unless you can *do* physics.

How do you learn to solve physics problems? In every chapter of this book you'll find *Problem-Solving Strategies* that offer techniques for setting up and solving problems efficiently and accurately. Following each *Problem-Solving Strategy* are one or more worked *Examples* that show these techniques in action. (The *Problem-Solving Strategies* will also steer you away from some *incorrect* techniques that you may be tempted to use.) You'll also find additional examples that aren't associated with a particular *Problem-Solving Strategy*. In addition, at the end of each chapter you'll find a *Bridging Problem* that uses more than one of the key ideas from the chapter. Study these strategies and problems carefully, and work through each example for yourself on a piece of paper.

Different techniques are useful for solving different kinds of physics problems, which is why this book offers dozens of *Problem-Solving Strategies*. No matter what kind of problem you're dealing with, however, there are certain key steps that you'll always follow. (These same steps are equally useful for problems in math, engineering, chemistry, and many other fields.) In this book we've organized these steps into four stages of solving a problem.

All of the *Problem-Solving Strategies* and *Examples* in this book will follow these four steps. (In some cases we'll combine the first two or three steps.) We encourage you to follow these same steps when you solve problems yourself. You may find it useful to remember the acronym ***I SEE***—short for *Identify, Set up, Execute,* and *Evaluate.*

PROBLEM-SOLVING STRATEGY 1.1 Solving Physics Problems

IDENTIFY *the relevant concepts:*

- Use the physical conditions stated in the problem to help you decide which physics concepts are relevant.
- Identify the **target variables** of the problem—that is, the quantities whose values you're trying to find, such as the speed at which a projectile hits the ground, the intensity of a sound made by a siren, or the size of an image made by a lens.
- Identify the known quantities, as stated or implied in the problem. This step is essential whether the problem asks for an algebraic expression or a numerical answer.

SET UP *the problem*:

- Given the concepts, known quantities, and target variables that you found in the IDENTIFY step, choose the equations that you'll use to solve the problem and decide how you'll use them. Study the worked examples in this book for tips on how to select the proper equations. If this seems challenging, don't worry—you'll get better with practice!
- Make sure that the variables you have identified correlate exactly with those in the equations.
- If appropriate, draw a sketch of the situation described in the problem. (Graph paper and a ruler will help you make clear, useful sketches.)

EXECUTE *the solution*:

- Here's where you'll "do the math" with the equations that you selected in the SET UP step to solve for the target variables that you found in the IDENTIFY step. Study the worked examples to see what's involved in this step.

EVALUATE *your answer:*

- Check your answer from the SOLVE step to see if it's reasonable. (If you're calculating how high a thrown baseball goes, an answer of 1.0 mm is unreasonably small and an answer of 100 km is unreasonably large.) If your answer includes an algebraic expression, confirm that it correctly represents what would happen if the variables in it had very large or very small values.
- For future reference, make note of any answer that represents a quantity of particular significance. Ask yourself how you might answer a more general or more difficult version of the problem you have just solved.

Idealized Models

In everyday conversation we use the word "model" to mean either a small-scale replica, such as a model railroad, or a person who displays articles of clothing (or the absence thereof). In physics a **model** is a simplified version of a physical system that would be too complicated to analyze in full detail.

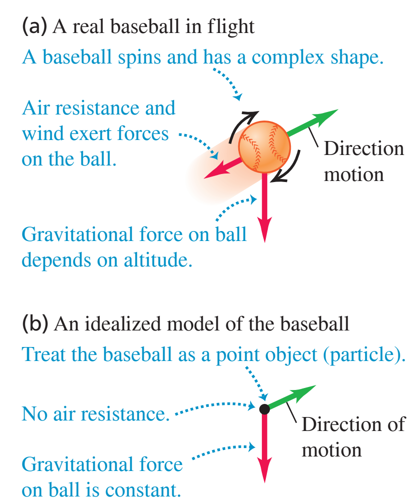
For example, suppose we want to analyze the motion of a thrown baseball (**Fig. 1.2a**). How complicated is this problem? The ball is not a perfect sphere (it has raised seams), and it spins as it moves through the air. Air resistance and wind influence its motion, the ball's weight varies a little as its altitude changes, and so on. If we try to include all these effects, the analysis gets hopelessly complicated. Instead, we invent a simplified version of the problem. We ignore the size, shape, and rotation of the ball by representing it as a point object, or **particle.** We ignore air resistance by making the ball move in a vacuum, and we make the weight constant. Now we have a problem that is simple enough to deal with (Fig. 1.2b). We'll analyze this model in detail in Chapter 3.

We have to overlook quite a few minor effects to make an idealized model, but we must be careful not to neglect too much. If we ignore the effects of gravity completely, then our model predicts that when we throw the ball up, it will go in a straight line and disappear into space. A useful model simplifies a problem enough to make it manageable, yet keeps its essential features.

The validity of the predictions we make using a model is limited by the validity of the model. For example, Galileo's prediction about falling objects (see Section 1.1) corresponds to an idealized model that does not include the effects of air resistance. This model works fairly well for a dropped cannonball, but not so well for a feather.

Idealized models play a crucial role throughout this book. Watch for them in discussions of physical theories and their applications to specific problems.

Figure 1.2 To simplify the analysis of (a) a baseball in flight, we use (b) an idealized model.



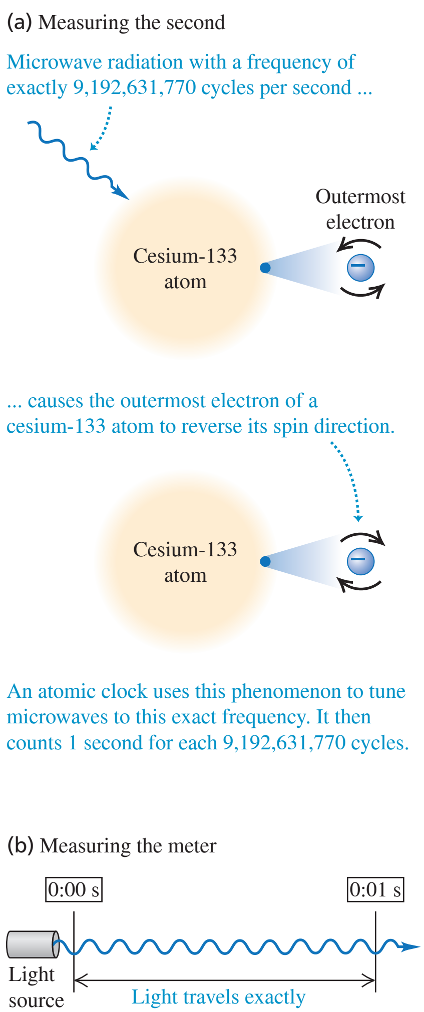
1.3 STANDARDS AND UNITS

As we learned in Section 1.1, physics is an experimental science. Experiments require measurements, and we generally use numbers to describe the results of measurements. Any number that is used to describe a physical phenomenon quantitatively is called a **physical quantity.** For example, two physical quantities that describe you are your weight and your height. Some physical quantities are so fundamental that we can define them only by describing how to measure them. Such a definition is called an **operational definition.** Two examples are measuring a distance by using a ruler and measuring a time interval by using a stopwatch. In other cases we define a physical quantity by describing how to calculate it from other quantities that we *can* measure. Thus we might define the average speed of a moving object as the distance traveled (measured with a ruler) divided by the time of travel (measured with a stopwatch).

When we measure a quantity, we always compare it with some reference standard. When we say that a basketball hoop is 3.05 meters above the ground, we mean that this distance is 3.05 times as long as a meter stick, which we define to be 1 meter long. Such a standard defines a **unit** of the quantity. The meter is a unit of distance, and the second is a unit of time. When we use a number to describe a physical quantity, we must always specify the unit that we are using; to describe a distance as simply "3.05" wouldn't mean anything.

To make accurate, reliable measurements, we need units of measurement that do not change and that can be duplicated by observers in various locations. The system of units used by scientists and engineers around the world is commonly called "the metric system," but since 1960 it has been known officially as the **International System,** or **SI** (the abbreviation for its French name, *Système International*). Appendix A gives a list of all SI units as well as definitions of the most fundamental units.

Figure **1.3** The measurements used to determine (a) the duration of a second and (b) the length of a meter. These measurements are useful for setting standards because they give the same results no matter where they are made.



Time

From 1889 until 1967, the unit of time was defined as a certain fraction of the mean solar day, the average time between successive arrivals of the sun at its highest point in the sky. The present standard, adopted in 1967, is much more precise. It is based on an atomic clock, which uses the energy difference between the two lowest energy states of the cesium atom (^{133}Cs). When bombarded by microwaves of precisely the proper frequency, cesium atoms undergo a transition from one of these states to the other. One **second** (abbreviated s) is defined as the time required for 9,192,631,770 cycles of this microwave radiation (**Fig. 1.3a**).

Length

In 1960 an atomic standard for the meter was also established, using the wavelength of the orange-red light emitted by excited atoms of krypton (^{86}Kr). From this length standard, the speed of light in vacuum was measured to be 299,792,458 m/s. In November 1983, the length standard was changed again so that the speed of light in vacuum was *defined* to be precisely 299,792,458 m/s. Hence the new definition of the **meter** (abbreviated m) is the distance that light travels in vacuum in 1/299,792,458 second (Fig. 1.3b). This modern definition provides a much more precise standard of length than the one based on a wavelength of light.

Mass

Until recently the unit of mass, the **kilogram** (abbreviated kg), was defined to be the mass of a metal cylinder kept at the International Bureau of Weights and Measures in France (**Fig. 1.4**). This was a very inconvenient standard to use. Since 2018 the value of the kilogram has been based on a fundamental constant of nature called *Planck's constant* (symbol h), whose defined value $h = 6.62607015 \times 10^{-34}\ \text{kg} \cdot \text{m}^2/\text{s}$ is related to those of the kilogram, meter, and second. Given the values of the meter and the second, the masses of objects can be experimentally determined in terms of h. (We'll explain the meaning of h in Chapter 28.) The *gram* (which is not a fundamental unit) is 0.001 kilogram.

Other *derived units* can be formed from the fundamental units. For example, the units of speed are meters per second, or m/s; these are the units of length (m) divided by the units of time (s).

Figure **1.4** Until 2018 a metal cylinder was used to define the value of the kilogram. (The one shown here, a copy of the one in France, was maintained by the U. S. National Institute of Standards and Technology.) Today the kilogram is defined in terms of one of the fundamental constants of nature.

Unit Prefixes

Once we have defined the fundamental units, it is easy to introduce larger and smaller units for the same physical quantities. In the metric system these other units are related to the fundamental units (or, in the case of mass, to the gram) by multiples of 10 or $\frac{1}{10}$. Thus one kilometer $(1\ \text{km})$ is 1000 meters, and one centimeter $(1\ \text{cm})$ is $\frac{1}{100}$ meter. We usually express multiples of 10 or $\frac{1}{10}$ in exponential notation: $1000 = 10^3$, $\frac{1}{1000} = 10^{-3}$, and so on. With this notation, $1\ \text{km} = 10^3\ \text{m}$ and $1\ \text{cm} = 10^{-2}\ \text{m}$.

The names of the additional units are derived by adding a **prefix** to the name of the fundamental unit. For example, the prefix "kilo-," abbreviated k, always means a unit larger by a factor of 1000; thus

$$\begin{aligned}
1\ \text{kilometer} &= 1\ \text{km} = 10^3\ \text{meters} = 10^3\ \text{m}\\
1\ \text{kilogram} &= 1\ \text{kg} = 10^3\ \text{grams} = 10^3\ \text{g}\\
1\ \text{kilowatt} &= 1\ \text{kW} = 10^3\ \text{watts} = 10^3\ \text{W}
\end{aligned}$$

A table in Appendix A lists the standard SI units, with their meanings and abbreviations.

Table 1.1 gives some examples of the use of multiples of 10 and their prefixes with the units of length, mass, and time. **Figure 1.5** (next page) shows how these prefixes are used to describe both large and small distances.

The British System

Finally, we mention the British system of units. These units are used in only the United States and a few other countries, and in most of these they are being replaced by SI units. British units are now officially defined in terms of SI units, as follows:

Length: 1 inch = 2.54 cm (exactly)

Force: 1 pound = 4.448221615260 newtons (exactly)

The newton, abbreviated N, is the SI unit of force. The British unit of time is the second, defined the same way as in SI. In physics, British units are used in mechanics and thermodynamics only; there is no British system of electrical units.

TABLE 1.1 Some Units of Length, Mass, and Time

Length	Mass	Time
1 nanometer = 1 nm = 10^{-9} m (a few times the size of the largest atom)	1 microgram = 1 μg = 10^{-6} g = 10^{-9} kg (mass of a very small dust particle)	1 nanosecond = 1 ns = 10^{-9} s (time for light to travel 0.3 m)
1 micrometer = 1 μm = 10^{-6} m (size of some bacteria and other cells)	1 milligram = 1 mg = 10^{-3} g = 10^{-6} kg (mass of a grain of salt)	1 microsecond = 1 μs = 10^{-6} s (time for space station to move 8 mm)
1 millimeter = 1 mm = 10^{-3} m (diameter of the point of a ballpoint pen)	1 gram = 1 g = 10^{-3} kg (mass of a paper clip)	1 millisecond = 1 ms = 10^{-3} s (time for a car moving at freeway speed to travel 3 cm)
1 centimeter = 1 cm = 10^{-2} m (diameter of your little finger)		
1 kilometer = 1 km = 10^3 m (distance in a 10 minute walk)		

Figure **1.5** Some typical lengths in the universe.

Note: (f) is a scanning tunneling microscope image of atoms on a crystal surface; (g) is an artist's impression.

(a) 10^{26} m Limit of the observable universe

(b) 10^{11} m Distance to the sun

(c) 10^{7} m Diameter of the earth

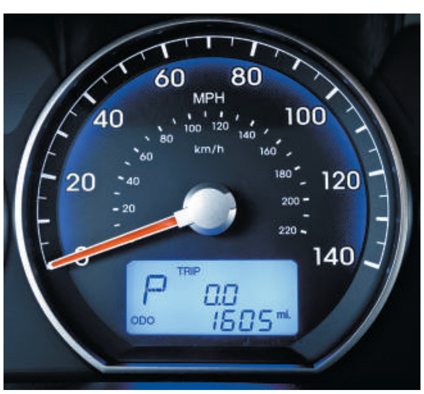
(d) 1 m Human dimensions

(e) 10^{-5} m Diameter of a red blood cell

(f) 10^{-10} m Radius of an atom

(g) 10^{-14} m Radius of an atomic nucleus

Figure **1.6** Many everyday items make use of both SI and British units. An example is this speedometer from a U.S.-built car, which shows the speed in both kilometers per hour (inner scale) and miles per hour (outer scale).



In this book we use SI units for all examples and problems, but we occasionally give approximate equivalents in British units. As you do problems using SI units, you may also wish to convert to the approximate British equivalents if they are more familiar to you (**Fig. 1.6**). But you should try to *think* in SI units as much as you can.

1.4 USING AND CONVERTING UNITS

We use equations to express relationships among physical quantities, represented by algebraic symbols. Each algebraic symbol always denotes both a number and a unit. For example, d might represent a distance of 10 m, t a time of 5 s, and v a speed of 2 m/s.

An equation must always be **dimensionally consistent.** You can't add apples and automobiles; two terms may be added or equated only if they have the same units. For example, if an object moving with constant speed v travels a distance d in a time t, these quantities are related by the equation

$$d = vt$$

If d is measured in meters, then the product vt must also be expressed in meters. Using the above numbers as an example, we may write

$$10\ \text{m} = \left(2\,\frac{\text{m}}{\cancel{\text{s}}}\right)(5\ \cancel{\text{s}})$$

Because the unit s in the denominator of m/s cancels, the product has units of meters, as it must. In calculations, units are treated just like algebraic symbols with respect to multiplication and division.

CAUTION **Always use units in calculations** Make it a habit to *always* write numbers with the correct units and carry the units through the calculation as in the example above. This provides a very useful check. If at some stage in a calculation you find that an equation or an expression has inconsistent units, you know you have made an error. In this book we'll *always* carry units through all calculations, and we strongly urge you to follow this practice when you solve problems.

PROBLEM-SOLVING STRATEGY 1.2 Unit Conversions

IDENTIFY *the relevant concepts:* In most cases, it's best to use the fundamental SI units (lengths in meters, masses in kilograms, and times in seconds) in every problem. If you need the answer to be in a different set of units (such as kilometers, grams, or hours), wait until the end of the problem to make the conversion.

SET UP *the problem* and **EXECUTE** *the solution:* Units are multiplied and divided just like ordinary algebraic symbols. This gives us an easy way to convert a quantity from one set of units to another: Express the same physical quantity in two different units and form an equality.

For example, when we say that 1 min = 60 s, we don't mean that the number 1 is equal to the number 60; rather, we mean that 1 min represents the same physical time interval as 60 s. For this reason, the ratio (1 min)/(60 s) equals 1, as does its reciprocal, (60 s)/(1 min). We may multiply a quantity by either of these factors (which we call *unit multipliers*) without changing that quantity's physical meaning. For example, to find the number of seconds in 3 min, we write

$$3\ \text{min} = (3\ \cancel{\text{min}})\left(\frac{60\ \text{s}}{1\ \cancel{\text{min}}}\right) = 180\ \text{s}$$

EVALUATE *your answer:* If you do your unit conversions correctly, unwanted units will cancel, as in the example above. If, instead, you had multiplied 3 min by (1 min)/(60 s), your result would have been the nonsensical $\frac{1}{20}\ \text{min}^2/\text{s}$. To be sure you convert units properly, include the units at *all* stages of the calculation.

Finally, check whether your answer is reasonable. For example, the result 3 min = 180 s is reasonable because the second is a smaller unit than the minute, so there are more seconds than minutes in the same time interval.

EXAMPLE 1.1 Converting speed units

The world land speed record of 763.0 mi/h was set on October 15, 1997, by Andy Green in the jet-engine car *Thrust SSC*. Express this speed in meters per second.

IDENTIFY, SET UP, and EXECUTE We need to convert the units of a speed from mi/h to m/s. We must therefore find unit multipliers that relate (i) miles to meters and (ii) hours to seconds. In Appendix E we find the equalities 1 mi = 1.609 km, 1 km = 1000 m, and 1 h = 3600 s. We set up the conversion as follows, which ensures that all the desired cancellations by division take place:

$$\begin{aligned} 763.0\ \text{mi/h} &= \left(763.0\ \frac{\cancel{\text{mi}}}{\cancel{\text{h}}}\right)\left(\frac{1.609\ \cancel{\text{km}}}{1\ \cancel{\text{mi}}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right) \\ &= 341.0\ \text{m/s} \end{aligned}$$

EVALUATE This example shows a useful rule of thumb: A speed expressed in m/s is a bit less than half the value expressed in mi/h, and a bit less than one-third the value expressed in km/h. For example, a normal freeway speed is about 30 m/s = 67 mi/h = 108 km/h, and a typical walking speed is about 1.4 m/s = 3.1 mi/h = 5.0 km/h.

KEYCONCEPT To convert units, multiply by an appropriate unit multiplier.

EXAMPLE 1.2 Converting volume units

One of the world's largest cut diamonds is the First Star of Africa (mounted in the British Royal Sceptre and kept in the Tower of London). Its volume is 1.84 cubic inches. What is its volume in cubic centimeters? In cubic meters?

IDENTIFY, SET UP, and EXECUTE Here we are to convert the units of a volume from cubic inches (in.^{3}) to both cubic centimeters (cm^3) and cubic meters (m^3). Appendix E gives us the equality 1 in. = 2.540 cm, from which we obtain 1 in.^{3} = (2.54 cm)^3. We then have

$$\begin{aligned} 1.84\ \text{in.}^3 &= (1.84\ \text{in.}^3)\left(\frac{2.54\ \text{cm}}{1\ \text{in.}}\right)^3 \\ &= (1.84)(2.54)^3\ \frac{\cancel{\text{in.}^3}\ \text{cm}^3}{\cancel{\text{in.}^3}} = 30.2\ \text{cm}^3 \end{aligned}$$

Appendix E also gives us 1 m = 100 cm, so

$$\begin{aligned} 30.2\ \text{cm}^3 &= (30.2\ \text{cm}^3)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^3 \\ &= (30.2)\left(\frac{1}{100}\right)^3 \frac{\cancel{\text{cm}^3}\ \text{m}^3}{\cancel{\text{cm}^3}} = 30.2 \times 10^{-6}\ \text{m}^3 \\ &= 3.02 \times 10^{-5}\ \text{m}^3 \end{aligned}$$

EVALUATE Following the pattern of these conversions, can you show that 1 in.^{3} ≈ 16 cm^3 and that 1 m^3 ≈ 60,000 in.^{3}?

KEYCONCEPT If the units of a quantity are a product of simpler units, such as m^3 = m × m × m, use a product of unit multipliers to convert these units.

Figure **1.7** This spectacular mishap was the result of a very small percent error—traveling a few meters too far at the end of a journey of hundreds of thousands of meters.

TABLE 1.2 Using Significant Figures

Multiplication or division:
Result can have no more significant figures than the factor with the fewest significant figures:

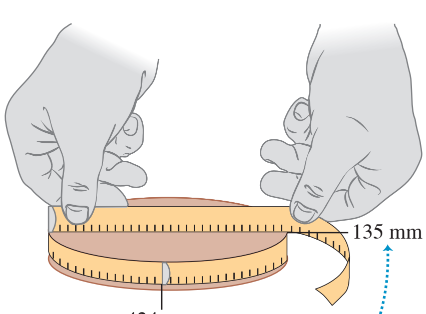
$$\frac{0.745 \times 2.2}{3.885} = 0.42$$

$$1.32578 \times 10^{7} \times 4.11 \times 10^{-3} = 5.45 \times 10^{4}$$

Addition or subtraction:
Number of significant figures is determined by the term with the largest uncertainty (i.e., fewest digits to the right of the decimal point):

$$27.153 + 138.2 - 11.74 = 153.6$$

Figure **1.8** Determining the value of π from the circumference and diameter of a circle.



The measured values have only three significant figures, so their calculated ratio (π) also has only three significant figures.

1.5 UNCERTAINTY AND SIGNIFICANT FIGURES

Measurements always have uncertainties. If you measure the thickness of the cover of a hardbound version of this book using an ordinary ruler, your measurement is reliable to only the nearest millimeter, and your result will be 3 mm. It would be *wrong* to state this result as 3.00 mm; given the limitations of the measuring device, you can't tell whether the actual thickness is 3.00 mm, 2.85 mm, or 3.11 mm. But if you use a micrometer caliper, a device that measures distances reliably to the nearest 0.01 mm, the result will be 2.91 mm. The distinction between the measurements with a ruler and with a caliper is in their **uncertainty**; the measurement with a caliper has a smaller uncertainty. The uncertainty is also called the **error** because it indicates the maximum difference there is likely to be between the measured value and the true value. The uncertainty or error of a measured value depends on the measurement technique used.

We often indicate the **accuracy** of a measured value—that is, how close it is likely to be to the true value—by writing the number, the symbol $\pm$, and a second number indicating the uncertainty of the measurement. If the diameter of a steel rod is given as 56.47 ± 0.02 mm, this means that the true value is likely to be within the range from 56.45 mm to 56.49 mm. In a commonly used shorthand notation, the number 1.6454(21) means 1.6454 ± 0.0021. The numbers in parentheses show the uncertainty in the final digits of the main number.

We can also express accuracy in terms of the maximum likely **fractional error** or **percent error** (also called *fractional uncertainty* and *percent uncertainty*). A resistor labeled "47 ohms $\pm$ 10%" probably has a true resistance that differs from 47 ohms by no more than 10% of 47 ohms—that is, by about 5 ohms. The resistance is probably between 42 and 52 ohms. For the diameter of the steel rod given above, the fractional error is $(0.02 \text{ mm})/(56.47 \text{ mm})$, or about 0.0004; the percent error is $(0.0004)(100\%)$, or about 0.04%. Even small percent errors can be very significant (**Fig. 1.7**).

In many cases the uncertainty of a number is not stated explicitly. Instead, the uncertainty is indicated by the number of meaningful digits, or **significant figures,** in the measured value. We gave the thickness of the cover of the book as 2.91 mm, which has three significant figures. By this we mean that the first two digits are known to be correct, while the third digit is uncertain. The last digit is in the hundredths place, so the uncertainty is about 0.01 mm. Two values with the *same* number of significant figures may have *different* uncertainties; a distance given as 137 km also has three significant figures, but the uncertainty is about 1 km. A distance given as 0.25 km has two significant figures (the zero to the left of the decimal point doesn't count); if given as 0.250 km, it has three significant figures.

When you use numbers that have uncertainties to compute other numbers, the computed numbers are also uncertain. When numbers are multiplied or divided, the result can have no more significant figures than the factor with the fewest significant figures has. For example, $3.1416 \times 2.34 \times 0.58 = 4.3$. When we add and subtract numbers, it's the location of the decimal point that matters, not the number of significant figures. For example, $123.62 + 8.9 = 132.5$. Although 123.62 has an uncertainty of about 0.01, 8.9 has an uncertainty of about 0.1. So their sum has an uncertainty of about 0.1 and should be written as 132.5, not 132.52. **Table 1.2** summarizes these rules for significant figures.

To apply these ideas, suppose you want to verify the value of π, the ratio of the circumference of a circle to its diameter. The true value of this ratio to ten digits is 3.141592654. To test this, you draw a large circle and measure its circumference and diameter to the nearest millimeter, obtaining the values 424 mm and 135 mm (**Fig. 1.8**). You enter these into your calculator and obtain the quotient $(424 \text{ mm})/(135 \text{ mm}) = 3.140740741$. This may seem to disagree with the true value of π, but keep in mind that each of your measurements has three significant figures, so your measured value of π can have only three significant figures. It should be stated simply as 3.14. Within the limit of three significant figures, your value does agree with the true value.

In the examples and problems in this book we usually give numerical values with three significant figures, so your answers should usually have no more than three significant figures. (Many numbers in the real world have even less accuracy. The speedometer in a car, for example, usually gives only two significant figures.) Even if you do the arithmetic with a calculator that displays ten digits, a ten-digit answer would misrepresent the accuracy of the results. Always round your final answer to keep only the correct number of significant figures or, in doubtful cases, one more at most. In Example 1.1 it would have been wrong to state the answer as 341.01861 m/s. Note that when you reduce such an answer to the appropriate number of significant figures, you must *round,* not *truncate.* Your calculator will tell you that the ratio of 525 m to 311 m is 1.688102894; to three significant figures, this is 1.69, not 1.68.

Here's a special note about calculations that involve multiple steps: As you work, it's helpful to keep extra significant figures in your calculations. Once you have your final answer, round it to the correct number of significant figures. This will give you the most accurate results.

When we work with very large or very small numbers, we can show significant figures much more easily by using **scientific notation,** sometimes called **powers-of-10 notation.** The distance from the earth to the moon is about 384,000,000 m, but writing the number in this form doesn't indicate the number of significant figures. Instead, we move the decimal point eight places to the left (corresponding to dividing by 10^8) and multiply by 10^8; that is,

$$384{,}000{,}000 \text{ m} = 3.84 \times 10^8 \text{ m}$$

In this form, it is clear that we have three significant figures. The number 4.00×10^{-7} also has three significant figures, even though two of them are zeros. Note that in scientific notation the usual practice is to express the quantity as a number between 1 and 10 multiplied by the appropriate power of 10.

When an integer or a fraction occurs in an algebraic equation, we treat that number as having no uncertainty at all. For example, in the equation $v_x{}^2 = v_{0x}{}^2 + 2a_x(x - x_0)$, which is Eq. (2.13) in Chapter 2, the coefficient 2 is *exactly* 2. We can consider this coefficient as having an infinite number of significant figures (2.000000 . . .). The same is true of the exponent 2 in $v_x{}^2$ and $v_{0x}{}^2$.

Finally, let's note that **precision** is not the same as *accuracy.* A cheap digital watch that gives the time as 10:35:17 a.m. is very *precise* (the time is given to the second), but if the watch runs several minutes slow, then this value isn't very *accurate.* On the other hand, a grandfather clock might be very accurate (that is, display the correct time), but if the clock has no second hand, it isn't very precise. A high-quality measurement is both precise *and* accurate.

EXAMPLE 1.3 Significant figures in multiplication

The rest energy E of an object with rest mass m is given by Albert Einstein's famous equation $E = mc^2$, where c is the speed of light in vacuum. Find E for an electron for which (to three significant figures) $m = 9.11 \times 10^{-31}$ kg. The SI unit for E is the joule (J); $1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$.

IDENTIFY and SET UP Our target variable is the energy E. We are given the value of the mass m; from Section 1.3 (or Appendix F) the speed of light is $c = 2.99792458 \times 10^8$ m/s.

EXECUTE Substituting the values of m and c into Einstein's equation, we find

$$\begin{aligned} E &= (9.11 \times 10^{-31} \text{ kg})(2.99792458 \times 10^8 \text{ m/s})^2 \\ &= (9.11)(2.99792458)^2(10^{-31})(10^8)^2 \text{ kg} \cdot \text{m}^2/\text{s}^2 \\ &= (81.87659678)(10^{[-31+(2\times 8)]}) \text{ kg} \cdot \text{m}^2/\text{s}^2 \\ &= 8.187659678 \times 10^{-14} \text{ kg} \cdot \text{m}^2/\text{s}^2 \end{aligned}$$

Since the value of m was given to only three significant figures, we must round this to

$$E = 8.19 \times 10^{-14} \text{ kg} \cdot \text{m}^2/\text{s}^2 = 8.19 \times 10^{-14} \text{ J}$$

EVALUATE While the rest energy contained in an electron may seem ridiculously small, on the atomic scale it is tremendous. Compare our answer to 10^{-19} J, the energy gained or lost by a single atom during a typical chemical reaction. The rest energy of an electron is about 1,000,000 times larger! (We'll discuss the significance of rest energy in Chapter 37.)

KEYCONCEPT When you are multiplying (or dividing) quantities, the result can have no more significant figures than the quantity with the fewest significant figures.

TEST YOUR UNDERSTANDING OF SECTION 1.5 The density of a material is equal to its mass divided by its volume. What is the density (in kg/m^3) of a rock of mass 1.80 kg and volume 6.0×10^{-4} m^3? (i) 3×10^3 kg/m^3; (ii) 3.0×10^3 kg/m^3; (iii) 3.00×10^3 kg/m^3; (iv) 3.000×10^3 kg/m^3; (v) any of these—all of these answers are mathematically equivalent.

ANSWER

(ii) Density $= (1.80 \text{ kg})/(6.0 \times 10^{-4} \text{ m}^3) = 3.0 \times 10^3 \text{ kg/m}^3$. When we multiply or divide, the number with the fewest significant figures controls the number of significant figures in the result.

1.6 ESTIMATES AND ORDERS OF MAGNITUDE

We have stressed the importance of knowing the accuracy of numbers that represent physical quantities. But even a very crude estimate of a quantity often gives us useful information. Sometimes we know how to calculate a certain quantity, but we have to guess at the data we need for the calculation. Or the calculation might be too complicated to carry out exactly, so we make rough approximations. In either case our result is also a guess, but such a guess can be useful even if it is uncertain by a factor of two, ten, or more. Such calculations are called **order-of-magnitude estimates.** The great Italian-American nuclear physicist Enrico Fermi (1901–1954) called them "back-of-the-envelope calculations."

Exercises 1.15 through 1.20 at the end of this chapter are of the estimating, or order-of-magnitude, variety. Most require guesswork for the needed input data. Don't try to look up a lot of data; make the best guesses you can. Even when they are off by a factor of ten, the results can be useful and interesting.

EXAMPLE 1.4 An order-of-magnitude estimate

You are writing an adventure novel in which the hero escapes with a billion dollars' worth of gold in his suitcase. Could anyone carry that much gold? Would it fit in a suitcase?

IDENTIFY, SET UP, and EXECUTE Gold sells for about \$1400 an ounce, or about \$100 for $\frac{1}{14}$ ounce. (The price per ounce has varied between \$200 and \$1900 over the past twenty years or so.) An ounce is about 30 grams, so \$100 worth of gold has a mass of about $\frac{1}{14}$ of 30 grams, or roughly 2 grams. A billion (10^9) dollars' worth of gold has a mass 10^7 times greater, about 2×10^7 (20 million) grams or 2×10^4 (20,000) kilograms. A thousand kilograms has a weight in British units of about a ton, so the suitcase weighs roughly 20 tons! No human could lift it.

Roughly what is the *volume* of this gold? The density of water is 10^3 kg/m^3; if gold, which is much denser than water, has a density 10 times greater, then 10^4 kg of gold fits into a volume of 1 m^3. So 10^9 dollars' worth of gold has a volume of 2 m^3, many times the volume of a suitcase.

EVALUATE Clearly your novel needs rewriting. Try the calculation again with a suitcase full of five-carat (1-gram) diamonds, each worth \$500,000. Would this work?

KEYCONCEPT To decide whether the numerical value of a quantity is reasonable, assess the quantity in terms of other quantities that you can estimate, even if only roughly.

TEST YOUR UNDERSTANDING OF SECTION 1.6 Can you estimate the total number of teeth in the mouths of all the students on your campus? (*Hint:* How many teeth are in your mouth? Count them!)

ANSWER

The answer depends on how many students are enrolled at your campus.

APPLICATION Scalar Temperature, Vector Wind The comfort level on a wintry day depends on the temperature, a scalar quantity that can be positive or negative (say, +5°C or −20°C) but has no direction. It also depends on the wind velocity, a vector quantity with both magnitude and direction (for example, 15 km/h from the west).

1.7 VECTORS AND VECTOR ADDITION

Some physical quantities, such as time, temperature, mass, and density, can be described completely by a single number with a unit. But many other important quantities in physics have a *direction* associated with them and cannot be described by a single number. A simple example is the motion of an airplane: We must say not only how fast the plane is moving but also in what direction. The speed of the airplane combined with its direction of motion constitute a quantity called *velocity.* Another example is *force,* which in physics means a push or pull exerted on an object. Giving a complete description of a force means describing both how hard the force pushes or pulls on the object and the direction of the push or pull.

When a physical quantity is described by a single number, we call it a **scalar quantity.** In contrast, a **vector quantity** has both a **magnitude** (the "how much" or "how big" part) and a direction in space. Calculations that combine scalar quantities use the operations of ordinary arithmetic. For example, 6 kg + 3 kg = 9 kg, or 4×2 s = 8 s. However, combining vectors requires a different set of operations.

To understand more about vectors and how they combine, we start with the simplest vector quantity, **displacement.** Displacement is a change in the position of an object.

Displacement is a vector quantity because we must state not only how far the object moves but also in what direction. Walking 3 km north from your front door doesn't get you to the same place as walking 3 km southeast; these two displacements have the same magnitude but different directions.

We usually represent a vector quantity such as displacement by a single letter, such as $\vec{A}$ in **Fig. 1.9a**. In this book we always print vector symbols in ***boldface italic type with an arrow above them.*** We do this to remind you that vector quantities have different properties from scalar quantities; the arrow is a reminder that vectors have direction. When you handwrite a symbol for a vector, *always* write it with an arrow on top. If you don't distinguish between scalar and vector quantities in your notation, you probably won't make the distinction in your thinking either, and confusion will result.

We always *draw* a vector as a line with an arrowhead at its tip. The length of the line shows the vector's magnitude, and the direction of the arrowhead shows the vector's direction. Displacement is always a straight-line segment directed from the starting point to the ending point, even though the object's actual path may be curved (Fig. 1.9b). Note that displacement is not related directly to the total *distance* traveled. If the object were to continue past P_2 and then return to P_1, the displacement for the entire trip would be *zero* (Fig. 1.9c).

If two vectors have the same direction, they are **parallel.** If they have the same magnitude *and* the same direction, they are *equal,* no matter where they are located in space. The vector $\vec{A}\,'$ from point P_3 to point P_4 in **Fig. 1.10** has the same length and direction as the vector $\vec{A}$ from P_1 to P_2. These two displacements are equal, even though they start at different points. We write this as $\vec{A}\,' = \vec{A}$ in Fig. 1.10; the boldface equals sign emphasizes that equality of two vector quantities is not the same relationship as equality of two scalar quantities. Two vector quantities are equal only when they have the same magnitude *and* the same direction.

Vector $\vec{B}$ in Fig. 1.10, however, is not equal to $\vec{A}$ because its direction is *opposite* that of $\vec{A}$. We define the **negative of a vector** as a vector having the same magnitude as the original vector but the *opposite* direction. The negative of vector quantity $\vec{A}$ is denoted as $-\vec{A}$, and we use a boldface minus sign to emphasize the vector nature of the quantities. If $\vec{A}$ is 87 m south, then $-\vec{A}$ is 87 m north. Thus we can write the relationship between $\vec{A}$ and $\vec{B}$ in Fig. 1.10 as $\vec{A} = -\vec{B}$ or $\vec{B} = -\vec{A}$. When two vectors $\vec{A}$ and $\vec{B}$ have opposite directions, whether their magnitudes are the same or not, we say that they are **antiparallel.**

We usually represent the *magnitude* of a vector quantity by the same letter used for the vector, but in *lightface italic type* with *no* arrow on top. For example, if displacement vector $\vec{A}$ is 87 m south, then $A = 87$ m. An alternative notation is the vector symbol with vertical bars on both sides:

$$(\text{Magnitude of } \vec{A}) = A = |\vec{A}| \qquad (1.1)$$

The magnitude of a vector quantity is a scalar quantity (a number) and is *always positive.* Note that a vector can never be equal to a scalar because they are different kinds of quantities. The expression "$\vec{A} = 6$ m" is just as wrong as "2 oranges = 3 apples"!

When we draw diagrams with vectors, it's best to use a scale similar to those used for maps. For example, a displacement of 5 km might be represented in a diagram by a vector 1 cm long, and a displacement of 10 km by a vector 2 cm long.

Vector Addition and Subtraction

Suppose a particle undergoes a displacement $\vec{A}$ followed by a second displacement $\vec{B}$. The final result is the same as if the particle had started at the same initial point and undergone a single displacement $\vec{C}$ (**Fig. 1.11a,** next page). We call displacement $\vec{C}$ the **vector sum,** or **resultant,** of displacements $\vec{A}$ and $\vec{B}$. We express this relationship symbolically as

$$\vec{C} = \vec{A} + \vec{B} \qquad (1.2)$$

The boldface plus sign emphasizes that adding two vector quantities requires a geometrical process and is not the same operation as adding two scalar quantities such as $2 + 3 = 5$. In vector addition we usually place the *tail* of the *second* vector at the *head,* or tip, of the *first* vector (Fig. 1.11a).

Figure **1.9** Displacement as a vector quantity.

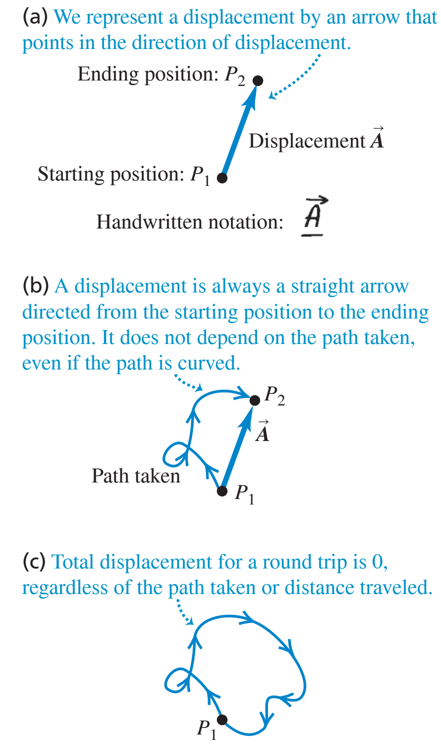


Figure **1.10** The meaning of vectors that have the same magnitude and the same or opposite direction.

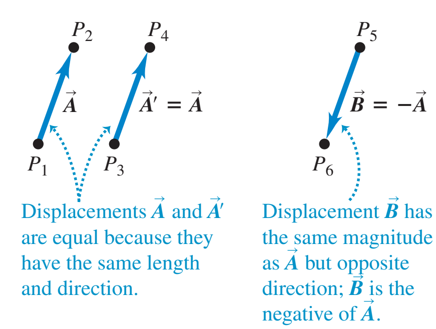
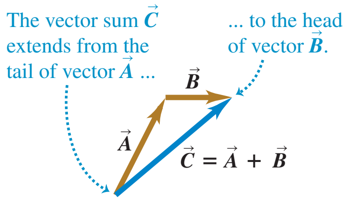
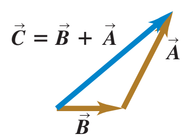


Figure **1.11** Three ways to add two vectors.

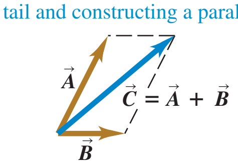
(a) We can add two vectors by placing them head to tail.



(b) Adding them in reverse order gives the same result: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. The order doesn't matter in vector addition.



(c) We can also add two vectors by placing them tail to tail and constructing a parallelogram.



If we make the displacements $\vec{A}$ and $\vec{B}$ in reverse order, with $\vec{B}$ first and $\vec{A}$ second, the result is the same (Fig. 1.11b). Thus

$$\vec{C} = \vec{B} + \vec{A} \quad \text{and} \quad \vec{A} + \vec{B} = \vec{B} + \vec{A} \tag{1.3}$$

This shows that the order of terms in a vector sum doesn't matter. In other words, vector addition obeys the *commutative* law.

Figure 1.11c shows another way to represent the vector sum: If we draw vectors $\vec{A}$ and $\vec{B}$ with their tails at the same point, vector $\vec{C}$ is the diagonal of a parallelogram constructed with $\vec{A}$ and $\vec{B}$ as two adjacent sides.

CAUTION **Magnitudes in vector addition** It's a common error to conclude that if $\vec{C} = \vec{A} + \vec{B}$, then magnitude C equals magnitude A plus magnitude B. In general, this conclusion is *wrong;* for the vectors shown in Fig. 1.11, $C < A + B$. The magnitude of $\vec{A} + \vec{B}$ depends on the magnitudes of $\vec{A}$ and $\vec{B}$ *and* on the angle between $\vec{A}$ and $\vec{B}$. Only in the special case in which $\vec{A}$ and $\vec{B}$ are *parallel* is the magnitude of $\vec{C} = \vec{A} + \vec{B}$ equal to the sum of the magnitudes of $\vec{A}$ and $\vec{B}$ (**Fig. 1.12a**). When the vectors are *antiparallel* (Fig. 1.12b), the magnitude of $\vec{C}$ equals the *difference* of the magnitudes of $\vec{A}$ and $\vec{B}$. Be careful to distinguish between scalar and vector quantities, and you'll avoid making errors about the magnitude of a vector sum.

Figure 1.13a shows *three* vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$. To find the vector sum of all three, in Fig. 1.13b we first add $\vec{A}$ and $\vec{B}$ to give a vector sum $\vec{D}$; we then add vectors $\vec{C}$ and $\vec{D}$ by the same process to obtain the vector sum $\vec{R}$:

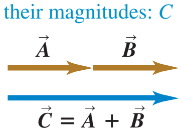
$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C} = \vec{D} + \vec{C}$$

Alternatively, we can first add $\vec{B}$ and $\vec{C}$ to obtain vector $\vec{E}$ (Fig. 1.13c), and then add $\vec{A}$ and $\vec{E}$ to obtain $\vec{R}$:

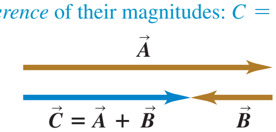
$$\vec{R} = \vec{A} + (\vec{B} + \vec{C}) = \vec{A} + \vec{E}$$

Figure **1.12** Adding vectors that are (a) parallel and (b) antiparallel.

(a) Only when vectors $\vec{A}$ and $\vec{B}$ are parallel does the magnitude of their vector sum $\vec{C}$ equal the sum of their magnitudes: $C = A + B$.



(b) When $\vec{A}$ and $\vec{B}$ are antiparallel, the magnitude of their vector sum $\vec{C}$ equals the *difference* of their magnitudes: $C = |A - B|$.



We don't even need to draw vectors $\vec{D}$ and $\vec{E}$; all we need to do is draw $\vec{A}$, $\vec{B}$, and $\vec{C}$ in succession, with the tail of each at the head of the one preceding it. The sum vector $\vec{R}$ extends from the tail of the first vector to the head of the last vector (Fig. 1.13d). The order makes no difference; Fig. 1.13e shows a different order, and you should try others. Vector addition obeys the *associative* law.

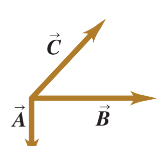
We can *subtract* vectors as well as add them. To see how, recall that vector $-\vec{A}$ has the same magnitude as $\vec{A}$ but the opposite direction. We define the difference $\vec{A} - \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ to be the vector sum of $\vec{A}$ and $-\vec{B}$:

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) \tag{1.4}$$

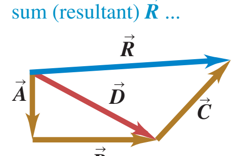
Figure 1.14 shows an example of vector subtraction.

Figure **1.13** Several constructions for finding the vector sum $\vec{A} + \vec{B} + \vec{C}$.

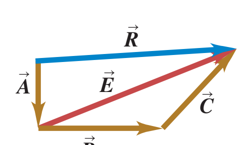
(a) To find the sum of these three vectors ...



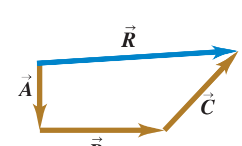
(b) ... add $\vec{A}$ and $\vec{B}$ to get $\vec{D}$ and then add $\vec{C}$ to $\vec{D}$ to get the final sum (resultant) $\vec{R}$ ...



(c) ... or add $\vec{B}$ and $\vec{C}$ to get $\vec{E}$ and then add $\vec{E}$ to $\vec{A}$ to get $\vec{R}$ ...



(d) ... or add $\vec{A}$, $\vec{B}$, and $\vec{C}$ to get $\vec{R}$ directly ...



(e) ... or add $\vec{A}$, $\vec{B}$, and $\vec{C}$ in any other order and still get $\vec{R}$.

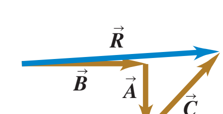
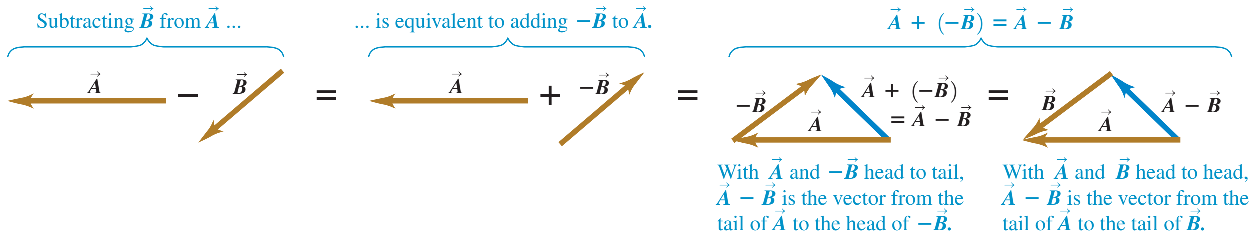


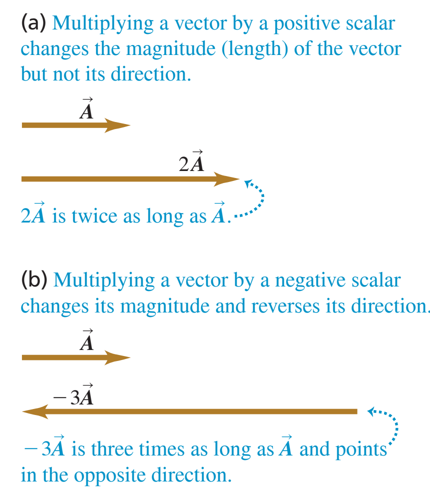
Figure 1.14 To construct the vector difference $\vec{A} - \vec{B}$, you can either place the tail of $-\vec{B}$ at the head of $\vec{A}$ or place the two vectors $\vec{A}$ and $\vec{B}$ head to head.



A vector quantity such as a displacement can be multiplied by a scalar quantity (an ordinary number). The displacement $2\vec{A}$ is a displacement (vector quantity) in the same direction as vector $\vec{A}$ but twice as long; this is the same as adding $\vec{A}$ to itself (**Fig. 1.15a**). In general, when we multiply a vector $\vec{A}$ by a scalar c, the result $c\vec{A}$ has magnitude $|c|A$ (the absolute value of c multiplied by the magnitude of vector $\vec{A}$). If c is positive, $c\vec{A}$ is in the same direction as $\vec{A}$; if c is negative, $c\vec{A}$ is in the direction opposite to $\vec{A}$. Thus $3\vec{A}$ is parallel to $\vec{A}$, while $-3\vec{A}$ is antiparallel to $\vec{A}$ (Fig. 1.15b).

A scalar used to multiply a vector can also be a physical quantity. For example, you may be familiar with the relationship $\vec{F} = m\vec{a}$; the net force $\vec{F}$ (a vector quantity) that acts on an object is equal to the product of the object's mass m (a scalar quantity) and its acceleration $\vec{a}$ (a vector quantity). The direction of $\vec{F}$ is the same as that of $\vec{a}$ because m is positive, and the magnitude of $\vec{F}$ is equal to the mass m multiplied by the magnitude of $\vec{a}$. The unit of force is the unit of mass multiplied by the unit of acceleration.

Figure 1.15 Multiplying a vector by a scalar.



EXAMPLE 1.5 Adding two vectors at right angles

A cross-country skier skis 1.00 km north and then 2.00 km east on a horizontal snowfield. How far and in what direction is she from the starting point?

IDENTIFY and SET UP The problem involves combining two displacements at right angles to each other. This vector addition amounts to solving a right triangle, so we can use the Pythagorean theorem and trigonometry. The target variables are the skier's straight-line distance and direction from her starting point. **Figure 1.16** is a scale diagram of the two displacements and the resultant net displacement. We denote the direction from the starting point by the angle ϕ (the Greek letter phi). The displacement appears to be a bit more than 2 km. Measuring the angle with a protractor indicates that ϕ is about 63°.

EXECUTE The distance from the starting point to the ending point is equal to the length of the hypotenuse:

$$\sqrt{(1.00\text{ km})^2 + (2.00\text{ km})^2} = 2.24\text{ km}$$

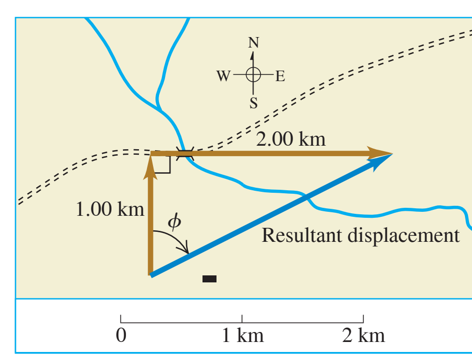
A little trigonometry (from Appendix B) allows us to find angle ϕ:

$$\tan\phi = \frac{\text{Opposite side}}{\text{Adjacent side}} = \frac{2.00\text{ km}}{1.00\text{ km}} = 2.00$$

$$\phi = \arctan 2.00 = 63.4°$$

We can describe the direction as 63.4° east of north or $90° - 63.4° = 26.6°$ north of east.

Figure 1.16 The vector diagram, drawn to scale, for a ski trip.



EVALUATE Our answers (2.24 km and $\phi = 63.4°$) are close to our predictions. In Section 1.8 we'll learn how to easily add two vectors *not* at right angles to each other.

KEYCONCEPT In every problem involving vector addition, draw the two vectors being added as well as the vector sum. The head-to-tail arrangement shown in Figs. 1.11a and 1.11b is easiest. This will help you to visualize the vectors and understand the direction of the vector sum. Drawing the vectors is equally important for problems involving vector subtraction (see Fig. 1.14).

TEST YOUR UNDERSTANDING OF SECTION 1.7 Two displacement vectors, $\vec{S}$ and $\vec{T}$, have magnitudes $S = 3$ m and $T = 4$ m. Which of the following could be the magnitude of the difference vector $\vec{S} - \vec{T}$? (There may be more than one correct answer.) (i) 9 m; (ii) 7 m; (iii) 5 m; (iv) 1 m; (v) 0 m; (vi) -1 m.

ANSWER

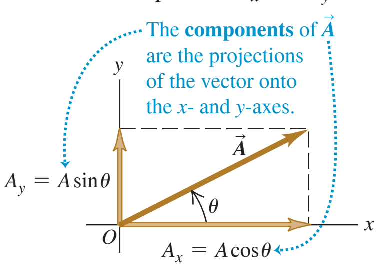
(ii), (iii), and (iv) Vector $-\vec{T}$ has the same magnitude as vector $\vec{T}$, so $\vec{S} - \vec{T} = \vec{S} + (-\vec{T})$ is the *sum* of one vector of magnitude 3 m and one of magnitude 4 m. This sum has magnitude 7 m if $\vec{S}$ and $-\vec{T}$ are parallel and magnitude 1 m if $\vec{S}$ and $-\vec{T}$ are antiparallel. The magnitude of $\vec{S} - \vec{T}$ is 5 m if $\vec{S}$ and $-\vec{T}$ are perpendicular, when vectors $\vec{S}$, $\vec{T}$, and $\vec{S} - \vec{T}$ form a 3–4–5 right triangle. Answer (i) is impossible because the magnitude of the sum of two vectors cannot be greater than the sum of the magnitudes; answer (v) is impossible because the sum of two vectors can be zero only if the two vectors are antiparallel and have the same magnitude; and answer (vi) is impossible because the magnitude of a vector cannot be negative.

1.8 COMPONENTS OF VECTORS

In Section 1.7 we added vectors by using a scale diagram and properties of right triangles. But calculations with right triangles work only when the two vectors are perpendicular. So we need a simple but general method for adding vectors. This is called the method of *components.*

To define what we mean by the components of a vector $\vec{A}$, we begin with a rectangular (Cartesian) coordinate system of axes (**Fig. 1.17**). If we think of $\vec{A}$ as a displacement vector, we can regard $\vec{A}$ as the sum of a displacement parallel to the x-axis and a displacement parallel to the y-axis. We use the numbers A_x and A_y to tell us how much displacement there is parallel to the x-axis and how much there is parallel to the y-axis, respectively. For example, if the $+x$-axis points east and the $+y$-axis points north, $\vec{A}$ in Fig. 1.17 could be the sum of a 2.00 m displacement to the east and a 1.00 m displacement to the north. Then $A_x = +2.00$ m and $A_y = +1.00$ m. We can use the same idea for any vectors, not just displacement vectors. The two numbers A_x and A_y are called the **components** of $\vec{A}$.

Figure **1.17** Representing a vector $\vec{A}$ in terms of its components A_x and A_y.

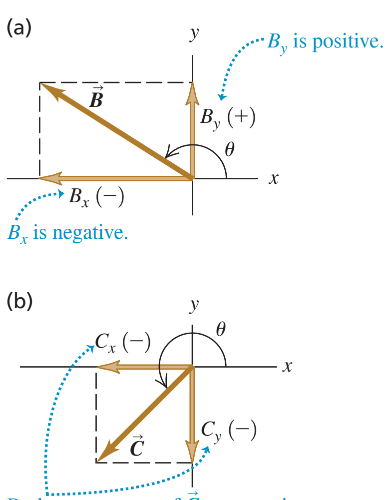


CAUTION **Components are not vectors** The components A_x and A_y of a vector $\vec{A}$ are numbers; they are *not* vectors themselves. This is why we print the symbols for components in lightface italic type with *no* arrow on top instead of in boldface italic with an arrow, which is reserved for vectors.

We can calculate the components of vector $\vec{A}$ if we know its magnitude A and its direction. We'll describe the direction of a vector by its angle relative to some reference direction. In Fig. 1.17 this reference direction is the positive x-axis, and the angle between vector $\vec{A}$ and the positive x-axis is θ (the Greek letter theta). Imagine that vector $\vec{A}$ originally lies along the $+x$-axis and that you then rotate it to its true direction, as indicated by the arrow in Fig. 1.17 on the arc for angle θ. If this rotation is from the $+x$-axis toward the $+y$-axis, as is the case in Fig. 1.17, then θ is *positive;* if the rotation is from the $+x$-axis toward the $-y$-axis, then θ is *negative.* Thus the $+y$-axis is at an angle of 90°, the $-x$-axis at 180°, and the $-y$-axis at 270° (or $-90°$). If θ is measured in this way, then from the definition of the trigonometric functions,

$$\frac{A_x}{A} = \cos\theta \quad \text{and} \quad \frac{A_y}{A} = \sin\theta$$
$$A_x = A\cos\theta \quad \text{and} \quad A_y = A\sin\theta \tag{1.5}$$
$$(\theta \text{ measured from the } +x\text{-axis, rotating toward the } +y\text{-axis})$$

Figure **1.18** The components of a vector may be positive or negative numbers.



In Fig. 1.17 A_x and A_y are positive. This is consistent with Eqs. (1.5); θ is in the first quadrant (between 0° and 90°), and both the cosine and the sine of an angle in this quadrant are positive. But in **Fig. 1.18a** the component B_x is negative and the component B_y is positive. (If the $+x$-axis points east and the $+y$-axis points north, $\vec{B}$ could represent a displacement of 2.00 m west and 1.00 m north. Since west is in the $-x$-direction and north is in the $+y$-direction, $B_x = -2.00$ m is negative and $B_y = +1.00$ m is positive.) Again, this is consistent with Eqs. (1.5); now θ is in the second quadrant, so $\cos\theta$ is negative and $\sin\theta$ is positive. In Fig. 1.18b both C_x and C_y are negative (both $\cos\theta$ and $\sin\theta$ are negative in the third quadrant).

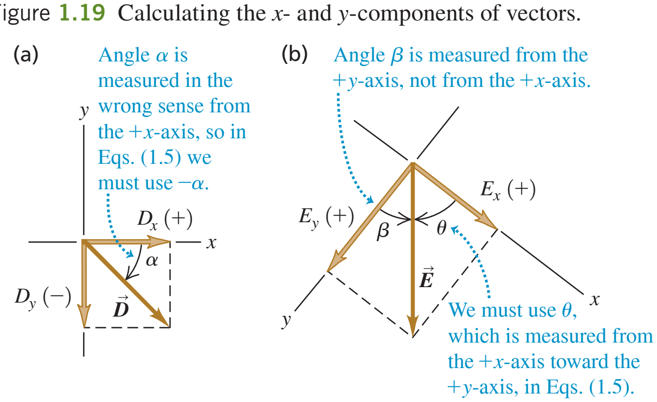
CAUTION **Relating a vector's magnitude and direction to its components** Equations (1.5) are correct *only* when the angle θ is measured from the positive x-axis. If the angle of the vector is given from a different reference direction or you use a different rotation direction, the relationships are different! Example 1.6 illustrates this point.

EXAMPLE 1.6 Finding components

(a) What are the x- and y-components of vector $\vec{D}$ in **Fig. 1.19a**? The magnitude of the vector is $D = 3.00$ m, and angle $\alpha = 45°$. (b) What are the x- and y-components of vector $\vec{E}$ in Fig. 1.19b? The magnitude of the vector is $E = 4.50$ m, and angle $\beta = 37.0°$.

IDENTIFY and SET UP We can use Eqs. (1.5) to find the components of these vectors, but we must be careful: Neither angle α nor β in Fig. 1.19 is measured from the $+x$-axis toward the $+y$-axis. We estimate from the figure that the lengths of both components in part (a) are roughly 2 m, and that those in part (b) are 3 m and 4 m. The figure indicates the signs of the components.

Figure 1.19 Calculating the x- and y-components of vectors.



EXECUTE (a) The angle α (the Greek letter alpha) between the positive x-axis and $\vec{D}$ is measured toward the *negative* y-axis. The angle we must use in Eqs. (1.5) is $\theta = -\alpha = -45°$. We then find

$$D_x = D\cos\theta = (3.00\text{ m})(\cos(-45°)) = +2.1\text{ m}$$
$$D_y = D\sin\theta = (3.00\text{ m})(\sin(-45°)) = -2.1\text{ m}$$

Had we carelessly substituted $+45°$ for θ in Eqs. (1.5), our result for D_y would have had the wrong sign.

(b) The x- and y-axes in Fig. 1.19b are at right angles, so it doesn't matter that they aren't horizontal and vertical, respectively. But we can't use the angle β (the Greek letter beta) in Eqs. (1.5), because β is measured from the $+y$-axis. Instead, we must use the angle $\theta = 90.0° - \beta = 90.0° - 37.0° = 53.0°$. Then we find

$$E_x = E\cos 53.0° = (4.50\text{ m})(\cos 53.0°) = +2.71\text{ m}$$
$$E_y = E\sin 53.0° = (4.50\text{ m})(\sin 53.0°) = +3.59\text{ m}$$

EVALUATE Our answers to both parts are close to our predictions. But why do the answers in part (a) correctly have only two significant figures?

KEYCONCEPT When you are finding the components of a vector, always use a diagram of the vector and the coordinate axes to guide your calculations.

Using Components to Do Vector Calculations

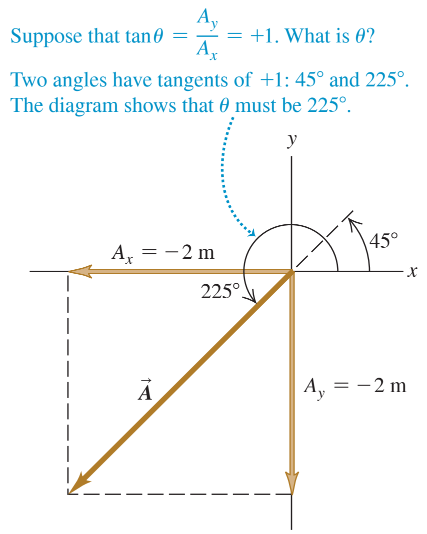
Using components makes it relatively easy to do various calculations involving vectors. Let's look at three important examples: finding a vector's magnitude and direction, multiplying a vector by a scalar, and calculating the vector sum of two or more vectors.

1. **Finding a vector's magnitude and direction from its components.** We can describe a vector completely by giving either its magnitude and direction or its x- and y-components. Equations (1.5) show how to find the components if we know the magnitude and direction. We can also reverse the process: We can find the magnitude and direction if we know the components. By applying the Pythagorean theorem to Fig. 1.17, we find that the magnitude of vector $\vec{A}$ is

$$A = \sqrt{A_x^2 + A_y^2} \tag{1.6}$$

(We always take the positive root.) Equation (1.6) is valid for any choice of x-axis and y-axis, as long as they are mutually perpendicular. The expression for the vector direction comes from the definition of the tangent of an angle. If θ is measured from

Figure **1.20** Drawing a sketch of a vector reveals the signs of its x- and y-components.



the positive x-axis, and a positive angle is measured toward the positive y-axis (as in Fig. 1.17), then

$$\tan\theta = \frac{A_y}{A_x} \qquad \text{and} \qquad \theta = \arctan\frac{A_y}{A_x} \tag{1.7}$$

We'll always use the notation arctan for the inverse tangent function (see Example 1.5 in Section 1.7). The notation $\tan^{-1}$ is also commonly used, and your calculator may have an INV or 2ND button to be used with the TAN button.

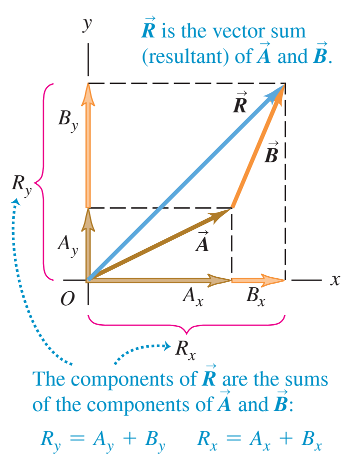
CAUTION **Finding the direction of a vector from its components** There's one complication in using Eqs. (1.7) to find θ: Any two angles that differ by 180° have the same tangent. For example, in **Fig. 1.20** the tangent of the angle θ is $\tan\theta = A_y/A_x = +1$. A calculator will tell you that $\theta = \tan^{-1}(+1) = 45°$. But the tangent of $180° + 45° = 225°$ is also equal to $+1$, so θ could also be 225° (which is actually the case in Fig. 1.20). *Always* draw a sketch like Fig. 1.20 to determine which of the two possibilities is correct.

2. **Multiplying a vector by a scalar.** If we multiply a vector $\vec{A}$ by a scalar c, each component of the product $\vec{D} = c\vec{A}$ is the product of c and the corresponding component of $\vec{A}$:

$$D_x = cA_x, \qquad D_y = cA_y \quad \text{(components of } \vec{D} = c\vec{A}\text{)} \tag{1.8}$$

For example, Eqs. (1.8) say that each component of the vector $2\vec{A}$ is twice as great as the corresponding component of $\vec{A}$, so $2\vec{A}$ is in the same direction as $\vec{A}$ but has twice the magnitude. Each component of the vector $-3\vec{A}$ is three times as great as the corresponding component of $\vec{A}$ but has the opposite sign, so $-3\vec{A}$ is in the opposite direction from $\vec{A}$ and has three times the magnitude. Hence Eqs. (1.8) are consistent with our discussion in Section 1.7 of multiplying a vector by a scalar (see Fig. 1.15).

Figure **1.21** Finding the vector sum (resultant) of $\vec{A}$ and $\vec{B}$ using components.



3. **Using components to calculate the vector sum (resultant) of two or more vectors. Figure 1.21** shows two vectors $\vec{A}$ and $\vec{B}$ and their vector sum $\vec{R}$, along with the x- and y-components of all three vectors. The x-component R_x of the vector sum is simply the sum $(A_x + B_x)$ of the x-components of the vectors being added. The same is true for the y-components. In symbols,

Each **component of** $\vec{R} = \vec{A} + \vec{B}$ ...

$$R_x = A_x + B_x, \qquad R_y = A_y + B_y \tag{1.9}$$

... is the sum of the corresponding components of $\vec{A}$ and $\vec{B}$.

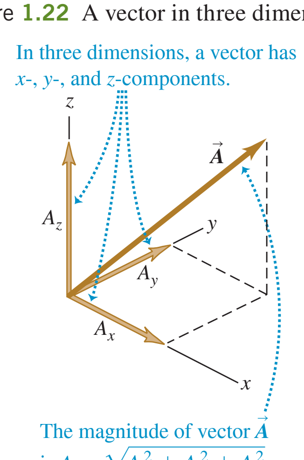
Figure 1.21 shows this result for the case in which the components A_x, A_y, B_x, and B_y are all positive. Draw additional diagrams to verify for yourself that Eqs. (1.9) are valid for *any* signs of the components of $\vec{A}$ and $\vec{B}$.

If we know the components of any two vectors $\vec{A}$ and $\vec{B}$, perhaps by using Eqs. (1.5), we can compute the components of the vector sum $\vec{R}$. Then if we need the magnitude and direction of $\vec{R}$, we can obtain them from Eqs. (1.6) and (1.7) with the A's replaced by R's.

We can use the same procedure to find the sum of any number of vectors. If $\vec{R}$ is the vector sum of $\vec{A}, \vec{B}, \vec{C}, \vec{D}, \vec{E}, \ldots$, the components of $\vec{R}$ are

$$\begin{aligned} R_x &= A_x + B_x + C_x + D_x + E_x + \cdots \\ R_y &= A_y + B_y + C_y + D_y + E_y + \cdots \end{aligned} \tag{1.10}$$

Figure **1.22** A vector in three dimensions.



We have talked about vectors that lie in the xy-plane only, but the component method works just as well for vectors having any direction in space. We can introduce a z-axis perpendicular to the xy-plane; then in general a vector $\vec{A}$ has components A_x, A_y, and A_z in the three coordinate directions. Its magnitude A is

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2} \tag{1.11}$$

Again, we always take the positive root **(Fig. 1.22)**. Also, Eqs. (1.10) for the vector sum $\vec{R}$ have a third component:

$$R_z = A_z + B_z + C_z + D_z + E_z + \cdots$$

We've focused on adding *displacement* vectors, but the method is applicable to all vector quantities. When we study the concept of force in Chapter 4, we'll find that forces are vectors that obey the same rules of vector addition.

PROBLEM-SOLVING STRATEGY 1.3 Vector Addition

IDENTIFY *the relevant concepts:* Decide what the target variable is. It may be the magnitude of the vector sum, the direction, or both.

SET UP *the problem:* Sketch the vectors being added, along with suitable coordinate axes. Place the tail of the first vector at the origin of the coordinates, place the tail of the second vector at the head of the first vector, and so on. Draw the vector sum $\vec{R}$ from the tail of the first vector (at the origin) to the head of the last vector. Use your sketch to estimate the magnitude and direction of $\vec{R}$. Select the equations you'll need: Eqs. (1.5) to obtain the components of the vectors given, if necessary, Eqs. (1.10) to obtain the components of the vector sum, Eq. (1.11) to obtain its magnitude, and Eqs. (1.7) to obtain its direction.

EXECUTE *the solution* as follows:

1. Find the x- and y-components of each individual vector and record your results in a table, as in Example 1.7 below. If a vector is described by a magnitude A and an angle θ, measured from the $+x$-axis toward the $+y$-axis, then its components are given by Eqs. (1.5):
$$A_x = A\cos\theta \qquad A_y = A\sin\theta$$
If the angles of the vectors are given in some other way, perhaps using a different reference direction, convert them to angles measured from the $+x$-axis as in Example 1.6.
2. Add the individual x-components algebraically (including signs) to find R_x, the x-component of the vector sum. Do the same for the y-components to find R_y. See Example 1.7.
3. Calculate the magnitude R and direction θ of the vector sum by using Eqs. (1.6) and (1.7):
$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \arctan\frac{R_y}{R_x}$$

EVALUATE *your answer:* Confirm that your results for the magnitude and direction of the vector sum agree with the estimates you made from your sketch. The value of θ that you find with a calculator may be off by 180°; your drawing will indicate the correct value. (See Example 1.7 below for an illustration of this.)

EXAMPLE 1.7 Using components to add vectors

WITH VARIATION PROBLEMS

Three players on a reality TV show are brought to the center of a large, flat field. Each is given a meter stick, a compass, a calculator, a shovel, and (in a different order for each contestant) the following three displacements:

$\vec{A}$: 72.4 m, 32.0° east of north

$\vec{B}$: 57.3 m, 36.0° south of west

$\vec{C}$: 17.8 m due south

The three displacements lead to the point in the field where the keys to a new Porsche are buried. Two players start measuring immediately, but the winner first *calculates* where to go. What does she calculate?

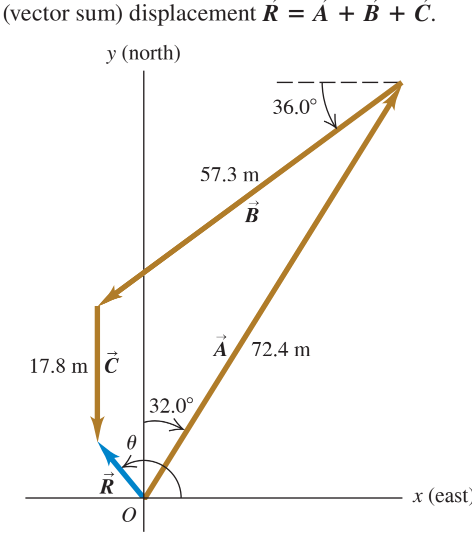
IDENTIFY and SET UP The goal is to find the sum (resultant) of the three displacements, so this is a problem in vector addition. See **Fig. 1.23**. We have chosen the $+x$-axis as east and the $+y$-axis as north. We estimate from the diagram that the vector sum $\vec{R}$ is about 10 m, 40° west of north (so θ is about 90° plus 40°, or about 130°).

EXECUTE The angles of the vectors, measured from the $+x$-axis toward the $+y$-axis, are $(90.0° - 32.0°) = 58.0°$, $(180.0° + 36.0°) = 216.0°$, and 270.0°, respectively. We may now use Eqs. (1.5) to find the components of $\vec{A}$:

$$A_x = A\cos\theta_A = (72.4\text{ m})(\cos 58.0°) = 38.37\text{ m}$$
$$A_y = A\sin\theta_A = (72.4\text{ m})(\sin 58.0°) = 61.40\text{ m}$$

We've kept an extra significant figure in the components; we'll round to the correct number of significant figures at the end of our calculation. The table at right shows the components of all the displacements, the addition of the components, and the other calculations from Eqs. (1.6) and (1.7).

Figure 1.23 Three successive displacements $\vec{A}$, $\vec{B}$, and $\vec{C}$ and the resultant (vector sum) displacement $\vec{R} = \vec{A} + \vec{B} + \vec{C}$.



Distance	Angle	x-component	y-component
$A = 72.4$ m	58.0°	38.37 m	61.40 m
$B = 57.3$ m	216.0°	−46.36 m	−33.68 m
$C = 17.8$ m	270.0°	0.00 m	−17.80 m
		$R_x = -7.99$ m	$R_y = 9.92$ m

$$R = \sqrt{(-7.99\text{ m})^2 + (9.92\text{ m})^2} = 12.7\text{ m}$$
$$\theta = \arctan\frac{9.92\text{ m}}{-7.99\text{ m}} = -51°$$

Continued

Comparing to angle θ in Fig. 1.23 shows that the calculated angle is clearly off by 180°. The correct value is $\theta = 180° + (-51°) = 129°$, or 39° west of north.

EVALUATE Our calculated answers for R and θ agree with our estimates. Notice how drawing the diagram in Fig. 1.23 made it easy to avoid a 180° error in the direction of the vector sum.

KEYCONCEPT When you are adding vectors, the x-component of the vector sum is equal to the sum of the x-components of the vectors being added, and likewise for the y-component. Always use a diagram to help determine the direction of the vector sum.

TEST YOUR UNDERSTANDING OF SECTION 1.8 Two vectors $\vec{A}$ and $\vec{B}$ lie in the xy-plane. (a) Can $\vec{A}$ have the same magnitude as $\vec{B}$ but different components? (b) Can $\vec{A}$ have the same components as $\vec{B}$ but a different magnitude?

ANSWER

(a) **yes**, (b) **no** Vectors $\vec{A}$ and $\vec{B}$ can have the same magnitude but different components if they point in different directions. If they have the same components, however, they are the same vector ($\vec{A} = \vec{B}$) and so must have the same magnitude.

1.9 UNIT VECTORS

A **unit vector** is a vector that has a magnitude of 1, with no units. Its only purpose is to *point*—that is, to describe a direction in space. Unit vectors provide a convenient notation for many expressions involving components of vectors. We'll always include a caret, or "hat" (^), in the symbol for a unit vector to distinguish it from ordinary vectors whose magnitude may or may not be equal to 1.

In an xy-coordinate system we can define a unit vector $\hat{\imath}$ that points in the direction of the positive x-axis and a unit vector $\hat{\jmath}$ that points in the direction of the positive y-axis (**Fig. 1.24a**). Then we can write a vector $\vec{A}$ in terms of its components as

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} \tag{1.12}$$

Equation (1.12) is a vector equation; each term, such as $A_x\hat{\imath}$, is a vector quantity (Fig. 1.24b).

Using unit vectors, we can express the vector sum $\vec{R}$ of two vectors $\vec{A}$ and $\vec{B}$ as follows:

$$\begin{aligned}
\vec{A} &= A_x\hat{\imath} + A_y\hat{\jmath}\\
\vec{B} &= B_x\hat{\imath} + B_y\hat{\jmath}\\
\vec{R} &= \vec{A} + \vec{B}\\
&= (A_x\hat{\imath} + A_y\hat{\jmath}) + (B_x\hat{\imath} + B_y\hat{\jmath})\\
&= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath}\\
&= R_x\hat{\imath} + R_y\hat{\jmath}
\end{aligned} \tag{1.13}$$

Equation (1.13) restates the content of Eqs. (1.9) in the form of a single vector equation rather than two component equations.

If not all of the vectors lie in the xy-plane, then we need a third component. We introduce a third unit vector $\hat{k}$ that points in the direction of the positive z-axis (**Fig. 1.25**). Then Eqs. (1.12) and (1.13) become

Any vector can be expressed in terms of its x-, y-, and z-components ...

$$\begin{aligned}
\vec{A} &= A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}\\
\vec{B} &= B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}
\end{aligned} \tag{1.14}$$

... and unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

$$\begin{aligned}
\vec{R} &= (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}\\
&= R_x\hat{\imath} + R_y\hat{\jmath} + R_z\hat{k}
\end{aligned} \tag{1.15}$$

Figure **1.24** (a) The unit vectors $\hat{\imath}$ and $\hat{\jmath}$. (b) Expressing a vector $\vec{A}$ in terms of its components.

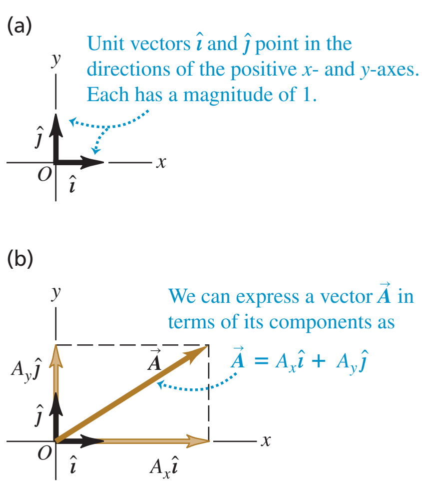
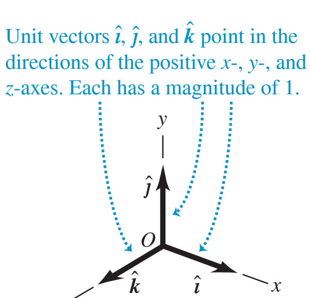


Figure **1.25** The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.



EXAMPLE 1.8 Using unit vectors

Given the two displacements

$$\vec{D} = (6.00\,\hat{\imath} + 3.00\,\hat{\jmath} - 1.00\hat{k})\text{ m} \quad \text{and}$$
$$\vec{E} = (4.00\,\hat{\imath} - 5.00\,\hat{\jmath} + 8.00\hat{k})\text{ m}$$

find the magnitude of the displacement $2\vec{D} - \vec{E}$.

IDENTIFY and SET UP We are to multiply vector $\vec{D}$ by 2 (a scalar) and subtract vector $\vec{E}$ from the result, so as to obtain the vector $\vec{F} = 2\vec{D} - \vec{E}$. Equation (1.8) says that to multiply $\vec{D}$ by 2, we multiply each of its components by 2. We can use Eq. (1.15) to do the subtraction; recall from Section 1.7 that subtracting a vector is the same as adding the negative of that vector.

EXECUTE We have

$$\begin{aligned}\vec{F} &= 2(6.00\hat{\imath} + 3.00\,\hat{\jmath} - 1.00\hat{k})\text{ m} - (4.00\hat{\imath} - 5.00\,\hat{\jmath} + 8.00\hat{k})\text{ m}\\ &= [(12.00 - 4.00)\hat{\imath} + (6.00 + 5.00)\,\hat{\jmath} + (-2.00 - 8.00)\hat{k}]\text{ m}\\ &= (8.00\hat{\imath} + 11.00\,\hat{\jmath} - 10.00\hat{k})\text{ m}\end{aligned}$$

From Eq. (1.11) the magnitude of $\vec{F}$ is

$$\begin{aligned}F &= \sqrt{F_x^2 + F_y^2 + F_z^2}\\ &= \sqrt{(8.00\text{ m})^2 + (11.00\text{ m})^2 + (-10.00\text{ m})^2}\\ &= 16.9\text{ m}\end{aligned}$$

EVALUATE Our answer is of the same order of magnitude as the larger components that appear in the sum. We wouldn't expect our answer to be much larger than this, but it could be much smaller.

KEYCONCEPT By using unit vectors, you can write a single equation for vector addition that incorporates the *x*-, *y*-, and *z*-components.

TEST YOUR UNDERSTANDING OF SECTION 1.9 Arrange the following vectors in order of their magnitude, with the vector of largest magnitude first. (i) $\vec{A} = (3\hat{\imath} + 5\hat{\jmath} - 2\hat{k})$ m; (ii) $\vec{B} = (-3\hat{\imath} + 5\hat{\jmath} - 2\hat{k})$ m; (iii) $\vec{C} = (3\hat{\imath} - 5\hat{\jmath} - 2\hat{k})$ m; (iv) $\vec{D} = (3\hat{\imath} + 5\hat{\jmath} + 2\hat{k})$ m.

ANSWER

❙ **All have the same magnitude.** Vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ point in different directions but have the same magnitude:

$$\begin{aligned}A = B = C = D &= \sqrt{(\pm 3\text{ m})^2 + (\pm 5\text{ m})^2 + (\pm 2\text{ m})^2}\\ &= \sqrt{9\text{ m}^2 + 25\text{ m}^2 + 4\text{ m}^2} = \sqrt{38\text{ m}^2} = 6.2\text{ m}\end{aligned}$$

1.10 PRODUCTS OF VECTORS

We saw how vector addition develops naturally from the problem of combining displacements. It will prove useful for calculations with many other vector quantities. We can also express many physical relationships by using *products* of vectors. Vectors are not ordinary numbers, so we can't directly apply ordinary multiplication to vectors. We'll define two different kinds of products of vectors. The first, called the *scalar product,* yields a result that is a scalar quantity. The second, the *vector product,* yields another vector.

Scalar Product

We denote the **scalar product** of two vectors $\vec{A}$ and $\vec{B}$ by $\vec{A}\cdot\vec{B}$. Because of this notation, the scalar product is also called the **dot product.** Although $\vec{A}$ and $\vec{B}$ are vectors, the quantity $\vec{A}\cdot\vec{B}$ is a scalar.

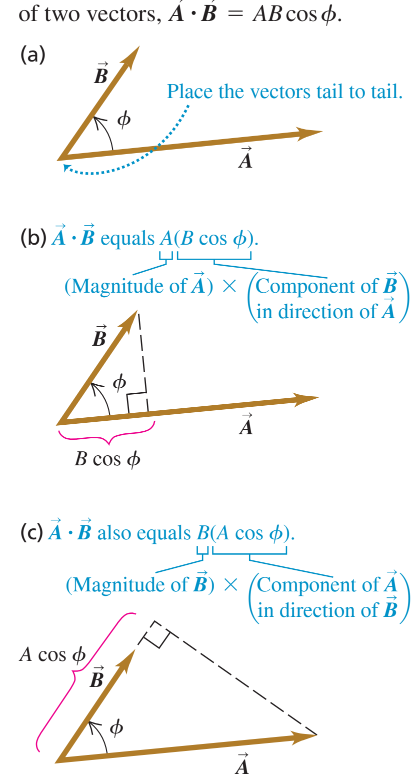
To define the scalar product $\vec{A}\cdot\vec{B}$ we draw the two vectors $\vec{A}$ and $\vec{B}$ with their tails at the same point (**Fig. 1.26a**). The angle ϕ (the Greek letter phi) between their directions ranges from 0° to 180°. Figure 1.26b shows the projection of vector $\vec{B}$ onto the direction of $\vec{A}$; this projection is the component of $\vec{B}$ in the direction of $\vec{A}$ and is equal to $B\cos\phi$. (We can take components along *any* direction that's convenient, not just the *x*- and *y*-axes.) We define $\vec{A}\cdot\vec{B}$ to be the magnitude of $\vec{A}$ multiplied by the component of $\vec{B}$ in the direction of $\vec{A}$, or

Scalar (dot) product of vectors $\vec{A}$ and $\vec{B}$ — Magnitudes of $\vec{A}$ and $\vec{B}$

$$\vec{A}\cdot\vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi \qquad (1.16)$$

Angle between $\vec{A}$ and $\vec{B}$ when placed tail to tail

Figure 1.26 Calculating the scalar product of two vectors, $\vec{A}\cdot\vec{B} = AB\cos\phi$.



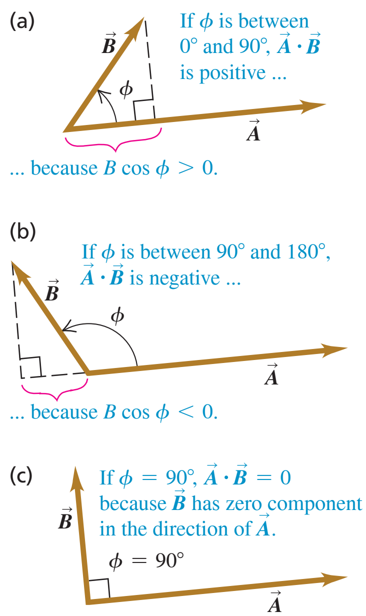


Figure **1.27** The scalar product $\vec{A}\cdot\vec{B} = AB\cos\phi$ can be positive, negative, or zero, depending on the angle between $\vec{A}$ and $\vec{B}$.

Alternatively, we can define $\vec{A}\cdot\vec{B}$ to be the magnitude of $\vec{B}$ multiplied by the component of $\vec{A}$ in the direction of $\vec{B}$, as in Fig. 1.26c. Hence $\vec{A}\cdot\vec{B} = B(A\cos\phi) = AB\cos\phi$, which is the same as Eq. (1.16).

The scalar product is a scalar quantity, not a vector, and it may be positive, negative, or zero. When ϕ is between 0° and 90°, $\cos\phi > 0$ and the scalar product is positive (**Fig. 1.27a**). When ϕ is between 90° and 180° so $\cos\phi < 0$, the component of $\vec{B}$ in the direction of $\vec{A}$ is negative, and $\vec{A}\cdot\vec{B}$ is negative (Fig. 1.27b). Finally, when $\phi = 90°$, $\vec{A}\cdot\vec{B} = 0$ (Fig. 1.27c). *The scalar product of two perpendicular vectors is always zero.*

For any two vectors $\vec{A}$ and $\vec{B}$, $AB\cos\phi = BA\cos\phi$. This means that $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$. The scalar product obeys the commutative law of multiplication; the order of the two vectors does not matter.

We'll use the scalar product in Chapter 6 to describe work done by a force. In later chapters we'll use the scalar product for a variety of purposes, from calculating electric potential to determining the effects that varying magnetic fields have on electric circuits.

Using Components to Calculate the Scalar Product

We can calculate the scalar product $\vec{A}\cdot\vec{B}$ directly if we know the x-, y-, and z-components of $\vec{A}$ and $\vec{B}$. To see how this is done, let's first work out the scalar products of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$. All unit vectors have magnitude 1 and are perpendicular to each other. Using Eq. (1.16), we find

$$\begin{aligned} \hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} &= (1)(1)\cos 0° = 1 \\ \hat{\imath}\cdot\hat{\jmath} = \hat{\imath}\cdot\hat{k} = \hat{\jmath}\cdot\hat{k} &= (1)(1)\cos 90° = 0 \end{aligned} \tag{1.17}$$

Now we express $\vec{A}$ and $\vec{B}$ in terms of their components, expand the product, and use these products of unit vectors:

$$\begin{aligned} \vec{A}\cdot\vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\ &= A_x\hat{\imath}\cdot B_x\hat{\imath} + A_x\hat{\imath}\cdot B_y\hat{\jmath} + A_x\hat{\imath}\cdot B_z\hat{k} \\ &\quad + A_y\hat{\jmath}\cdot B_x\hat{\imath} + A_y\hat{\jmath}\cdot B_y\hat{\jmath} + A_y\hat{\jmath}\cdot B_z\hat{k} \\ &\quad + A_z\hat{k}\cdot B_x\hat{\imath} + A_z\hat{k}\cdot B_y\hat{\jmath} + A_z\hat{k}\cdot B_z\hat{k} \\ &= A_xB_x\hat{\imath}\cdot\hat{\imath} + A_xB_y\hat{\imath}\cdot\hat{\jmath} + A_xB_z\hat{\imath}\cdot\hat{k} \\ &\quad + A_yB_x\hat{\jmath}\cdot\hat{\imath} + A_yB_y\hat{\jmath}\cdot\hat{\jmath} + A_yB_z\hat{\jmath}\cdot\hat{k} \\ &\quad + A_zB_x\hat{k}\cdot\hat{\imath} + A_zB_y\hat{k}\cdot\hat{\jmath} + A_zB_z\hat{k}\cdot\hat{k} \end{aligned} \tag{1.18}$$

From Eqs. (1.17) you can see that six of these nine terms are zero. The three that survive give

Scalar (dot) product of vectors $\vec{A}$ and $\vec{B}$ — Components of $\vec{A}$

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z \tag{1.19}$$

Components of $\vec{B}$

Thus *the scalar product of two vectors is the sum of the products of their respective components.*

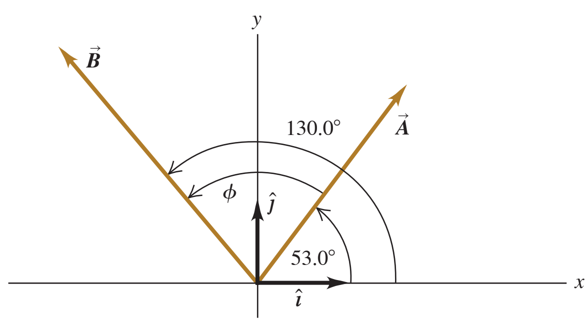
The scalar product gives a straightforward way to find the angle ϕ between any two vectors $\vec{A}$ and $\vec{B}$ whose components are known. In this case we can use Eq. (1.19) to find the scalar product of $\vec{A}$ and $\vec{B}$. Example 1.10 shows how to do this.

EXAMPLE 1.9 Calculating a scalar product — WITH VARIATION PROBLEMS

Find the scalar product $\vec{A} \cdot \vec{B}$ of the two vectors in **Fig. 1.28**. The magnitudes of the vectors are $A = 4.00$ and $B = 5.00$.

IDENTIFY and SET UP We can calculate the scalar product in two ways: using the magnitudes of the vectors and the angle between them (Eq. 1.16) and using the components of the vectors (Eq. 1.19). We'll do it both ways, and the results will check each other.

Figure **1.28** Two vectors $\vec{A}$ and $\vec{B}$ in two dimensions.



EXECUTE The angle between the two vectors $\vec{A}$ and $\vec{B}$ is $\phi = 130.0° - 53.0° = 77.0°$, so Eq. (1.16) gives us

$$\vec{A} \cdot \vec{B} = AB\cos\phi = (4.00)(5.00)\cos 77.0° = 4.50$$

To use Eq. (1.19), we must first find the components of the vectors. The angles of $\vec{A}$ and $\vec{B}$ are given with respect to the $+x$-axis and are measured in the sense from the $+x$-axis to the $+y$-axis, so we can use Eqs. (1.5):

$$A_x = (4.00)\cos 53.0° = 2.407$$
$$A_y = (4.00)\sin 53.0° = 3.195$$
$$B_x = (5.00)\cos 130.0° = -3.214$$
$$B_y = (5.00)\sin 130.0° = 3.830$$

As in Example 1.7, we keep an extra significant figure in the components and round at the end. Equation (1.19) now gives us

$$\begin{aligned}\vec{A} \cdot \vec{B} &= A_xB_x + A_yB_y + A_zB_z\\ &= (2.407)(-3.214) + (3.195)(3.830) + (0)(0) = 4.50\end{aligned}$$

EVALUATE Both methods give the same result, as they should.

KEYCONCEPT The scalar product $\vec{A} \cdot \vec{B}$ is a scalar (a number) that equals the sum of the products of the x-components, y-components, and z-components of $\vec{A}$ and $\vec{B}$.

EXAMPLE 1.10 Finding an angle with the scalar product — WITH VARIATION PROBLEMS

Find the angle between the vectors

$$\vec{A} = 2.00\hat{\imath} + 3.00\hat{\jmath} + 1.00\hat{k}$$

and

$$\vec{B} = -4.00\hat{\imath} + 2.00\hat{\jmath} - 1.00\hat{k}$$

IDENTIFY and SET UP We're given the x-, y-, and z-components of two vectors. Our target variable is the angle ϕ between them (**Fig. 1.29**). To find this, we'll solve Eq. (1.16), $\vec{A} \cdot \vec{B} = AB\cos\phi$, for ϕ in terms of the scalar product $\vec{A} \cdot \vec{B}$ and the magnitudes A and B. We can use Eq. (1.19) to evaluate the scalar product, $\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z$, and we can use Eq. (1.6) to find A and B.

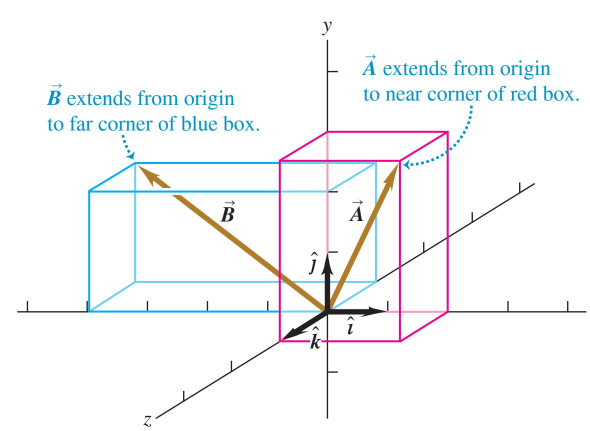
EXECUTE We solve Eq. (1.16) for $\cos\phi$ and use Eq. (1.19) to write $\vec{A} \cdot \vec{B}$:

$$\cos\phi = \frac{\vec{A} \cdot \vec{B}}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{AB}$$

We can use this formula to find the angle between *any* two vectors $\vec{A}$ and $\vec{B}$. Here we have $A_x = 2.00$, $A_y = 3.00$, and $A_z = 1.00$, and $B_x = -4.00$, $B_y = 2.00$, and $B_z = -1.00$. Thus

$$\begin{aligned}\vec{A} \cdot \vec{B} &= A_xB_x + A_yB_y + A_zB_z\\ &= (2.00)(-4.00) + (3.00)(2.00) + (1.00)(-1.00)\\ &= -3.00\\ A &= \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(2.00)^2 + (3.00)^2 + (1.00)^2}\\ &= \sqrt{14.00}\\ B &= \sqrt{B_x^2 + B_y^2 + B_z^2} = \sqrt{(-4.00)^2 + (2.00)^2 + (-1.00)^2}\\ &= \sqrt{21.00}\end{aligned}$$

Figure **1.29** Two vectors in three dimensions.



$$\cos\phi = \frac{A_xB_x + A_yB_y + A_zB_z}{AB} = \frac{-3.00}{\sqrt{14.00}\,\sqrt{21.00}} = -0.175$$

$$\phi = 100°$$

EVALUATE As a check on this result, note that the scalar product $\vec{A} \cdot \vec{B}$ is negative. This means that ϕ is between 90° and 180° (see Fig. 1.27), which agrees with our answer.

KEYCONCEPT You can find the angle ϕ between two vectors $\vec{A}$ and $\vec{B}$ whose components are known by first finding their scalar product, then using the equation $\vec{A} \cdot \vec{B} = AB\cos\phi$.

Figure **1.30** The vector product of (a) $\vec{A} \times \vec{B}$ and (b) $\vec{B} \times \vec{A}$.

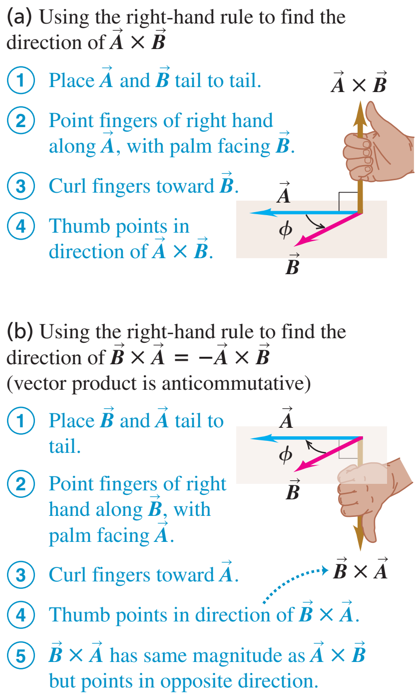
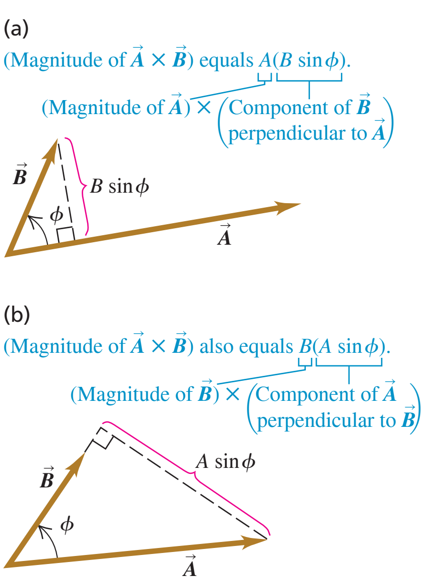


Figure **1.31** Calculating the magnitude $AB\sin\phi$ of the vector product of two vectors, $\vec{A} \times \vec{B}$.



Vector Product

We denote the **vector product** of two vectors $\vec{A}$ and $\vec{B}$, also called the **cross product,** by $\vec{A} \times \vec{B}$. As the name suggests, the vector product is itself a vector. We'll use this product in Chapter 10 to describe torque and angular momentum; in Chapters 27 and 28 we'll use it to describe magnetic fields and forces.

To define the vector product $\vec{A} \times \vec{B}$, we again draw the two vectors $\vec{A}$ and $\vec{B}$ with their tails at the same point (**Fig. 1.30a**). The two vectors then lie in a plane. We define the vector product to be a vector quantity with a direction perpendicular to this plane (that is, perpendicular to both $\vec{A}$ and $\vec{B}$) and a magnitude equal to $AB\sin\phi$. That is, if $\vec{C} = \vec{A} \times \vec{B}$, then

Magnitude of **vector (cross) product** of vectors $\vec{B}$ and $\vec{A}$

$$C = AB\sin\phi \tag{1.20}$$

Magnitudes of $\vec{A}$ and $\vec{B}$

Angle between $\vec{A}$ and $\vec{B}$ when placed tail to tail

We measure the angle ϕ from $\vec{A}$ toward $\vec{B}$ and take it to be the smaller of the two possible angles, so ϕ ranges from 0° to 180°. Then $\sin\phi \geq 0$ and C in Eq. (1.20) is never negative, as must be the case for a vector magnitude. Note that when $\vec{A}$ and $\vec{B}$ are parallel or antiparallel, $\phi = 0°$ or 180° and $C = 0$. That is, *the vector product of two parallel or antiparallel vectors is always zero.* In particular, *the vector product of any vector with itself is zero.*

CAUTION Vector product vs. scalar product Don't confuse the expression $AB\sin\phi$ for the magnitude of the vector product $\vec{A} \times \vec{B}$ with the similar expression $AB\cos\phi$ for the scalar product $\vec{A} \cdot \vec{B}$. To see the difference between these two expressions, imagine that we vary the angle between $\vec{A}$ and $\vec{B}$ while keeping their magnitudes constant. When $\vec{A}$ and $\vec{B}$ are parallel, the magnitude of the vector product will be zero and the scalar product will be maximum. When $\vec{A}$ and $\vec{B}$ are perpendicular, the magnitude of the vector product will be maximum and the scalar product will be zero.

There are always *two* directions perpendicular to a given plane, one on each side of the plane. We choose which of these is the direction of $\vec{A} \times \vec{B}$ as follows. Imagine rotating vector $\vec{A}$ about the perpendicular line until $\vec{A}$ is aligned with $\vec{B}$, choosing the smaller of the two possible angles between $\vec{A}$ and $\vec{B}$. Curl the fingers of your right hand around the perpendicular line so that your fingertips point in the direction of rotation; your thumb will then point in the direction of $\vec{A} \times \vec{B}$. Figure 1.30a shows this **right-hand rule** and describes a second way to think about this rule.

Similarly, we determine the direction of $\vec{B} \times \vec{A}$ by rotating $\vec{B}$ into $\vec{A}$ as in Fig. 1.30b. The result is a vector that is *opposite* to the vector $\vec{A} \times \vec{B}$. The vector product is *not* commutative but instead is *anticommutative:* For any two vectors $\vec{A}$ and $\vec{B}$,

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A} \tag{1.21}$$

Just as we did for the scalar product, we can give a geometrical interpretation of the magnitude of the vector product. In **Fig. 1.31a,** $B\sin\phi$ is the component of vector $\vec{B}$ that is *perpendicular* to the direction of vector $\vec{A}$. From Eq. (1.20) the magnitude of $\vec{A} \times \vec{B}$ equals the magnitude of $\vec{A}$ multiplied by the component of $\vec{B}$ that is perpendicular to $\vec{A}$. Figure 1.31b shows that the magnitude of $\vec{A} \times \vec{B}$ also equals the magnitude of $\vec{B}$ multiplied by the component of $\vec{A}$ that is perpendicular to $\vec{B}$. Note that Fig. 1.31 shows the case in which ϕ is between 0° and 90°; draw a similar diagram for ϕ between 90° and 180° to show that the same geometrical interpretation of the magnitude of $\vec{A} \times \vec{B}$ applies.

Using Components to Calculate the Vector Product

If we know the components of $\vec{A}$ and $\vec{B}$, we can calculate the components of the vector product by using a procedure similar to that for the scalar product. First we work out the multiplication table for unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, all three of which are perpendicular to each other (**Fig. 1.32a**). The vector product of any vector with itself is zero, so

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = \mathbf{0}$$

The boldface zero is a reminder that each product is a zero *vector*—that is, one with all components equal to zero and an undefined direction. Using Eqs. (1.20) and (1.21) and the right-hand rule, we find

$$\begin{aligned} \hat{\imath} \times \hat{\jmath} &= -\hat{\jmath} \times \hat{\imath} = \hat{k} \\ \hat{\jmath} \times \hat{k} &= -\hat{k} \times \hat{\jmath} = \hat{\imath} \\ \hat{k} \times \hat{\imath} &= -\hat{\imath} \times \hat{k} = \hat{\jmath} \end{aligned} \tag{1.22}$$

You can verify these equations by referring to Fig. 1.32a.

Next we express $\vec{A}$ and $\vec{B}$ in terms of their components and the corresponding unit vectors, and we expand the expression for the vector product:

$$\begin{aligned} \vec{A} \times \vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\ &= A_x\hat{\imath} \times B_x\hat{\imath} + A_x\hat{\imath} \times B_y\hat{\jmath} + A_x\hat{\imath} \times B_z\hat{k} \\ &\quad + A_y\hat{\jmath} \times B_x\hat{\imath} + A_y\hat{\jmath} \times B_y\hat{\jmath} + A_y\hat{\jmath} \times B_z\hat{k} \\ &\quad + A_z\hat{k} \times B_x\hat{\imath} + A_z\hat{k} \times B_y\hat{\jmath} + A_z\hat{k} \times B_z\hat{k} \end{aligned} \tag{1.23}$$

We can also rewrite the individual terms in Eq. (1.23) as $A_x\hat{\imath} \times B_y\hat{\jmath} = (A_xB_y)\hat{\imath} \times \hat{\jmath}$, and so on. Evaluating these by using the multiplication table for the unit vectors in Eqs. (1.22) and then grouping the terms, we get

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k} \tag{1.24}$$

If you compare Eq. (1.24) with Eq. (1.14), you'll see that the components of $\vec{C} = \vec{A} \times \vec{B}$ are

Components of vector (cross) product $\vec{A} \times \vec{B}$

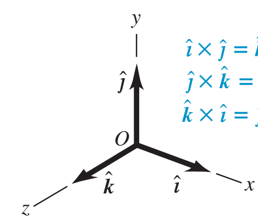
$$C_x = A_yB_z - A_zB_y \qquad C_y = A_zB_x - A_xB_z \qquad C_z = A_xB_y - A_yB_x \tag{1.25}$$

A_x, A_y, A_z = components of $\vec{A}$ $\qquad$ B_x, B_y, B_z = components of $\vec{B}$

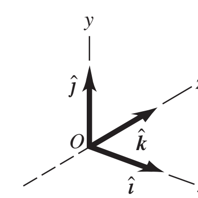
With the axis system of Fig. 1.32a, if we reverse the direction of the z-axis, we get the system shown in Fig. 1.32b. Then, as you may verify, the definition of the vector product gives $\hat{\imath} \times \hat{\jmath} = -\hat{k}$ instead of $\hat{\imath} \times \hat{\jmath} = \hat{k}$. In fact, all vector products of unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ would have signs opposite to those in Eqs. (1.22). So there are two kinds of coordinate systems, which differ in the signs of the vector products of unit vectors. An axis system in which $\hat{\imath} \times \hat{\jmath} = \hat{k}$, as in Fig. 1.32a, is called a **right-handed system.** The usual practice is to use *only* right-handed systems, and we'll follow that practice throughout this book.

Figure **1.32** (a) We'll always use a right-handed coordinate system, like this one. (b) We'll never use a left-handed coordinate system (in which $\hat{\imath} \times \hat{\jmath} = -\hat{k}$, and so on).

(a) A right-handed coordinate system



(b) A left-handed coordinate system; we will not use these.

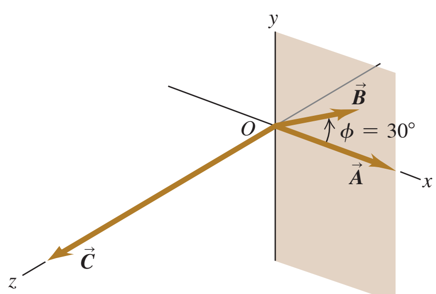


EXAMPLE 1.11 Calculating a vector product

Vector $\vec{A}$ has magnitude 6 units and is in the direction of the $+x$-axis. Vector $\vec{B}$ has magnitude 4 units and lies in the xy-plane, making an angle of 30° with the $+x$-axis (**Fig. 1.33**). Find the vector product $\vec{C} = \vec{A} \times \vec{B}$.

IDENTIFY and SET UP We'll find the vector product in two ways, which will provide a check of our calculations. First we'll use Eq. (1.20) and the right-hand rule; then we'll use Eqs. (1.25) to find the vector product by using components.

Figure 1.33 Vectors $\vec{A}$ and $\vec{B}$ and their vector product $\vec{C} = \vec{A} \times \vec{B}$. Vector $\vec{B}$ lies in the xy-plane.



EXECUTE From Eq. (1.20) the magnitude of the vector product is

$$AB\sin\phi = (6)(4)(\sin 30°) = 12$$

By the right-hand rule, the direction of $\vec{A} \times \vec{B}$ is along the $+z$-axis (the direction of the unit vector $\hat{k}$), so $\vec{C} = \vec{A} \times \vec{B} = 12\hat{k}$.

To use Eqs. (1.25), we first determine the components of $\vec{A}$ and $\vec{B}$. Note that $\vec{A}$ points along the x-axis, so its only nonzero component is A_x. For $\vec{B}$, Fig. 1.33 shows that $\phi = 30°$ is measured from the $+x$-axis toward the $+y$-axis, so we can use Eqs. (1.5):

$$A_x = 6 \qquad A_y = 0 \qquad A_z = 0$$
$$B_x = 4\cos 30° = 2\sqrt{3} \qquad B_y = 4\sin 30° = 2 \qquad B_z = 0$$

Then Eqs. (1.25) yield

$$C_x = (0)(0) - (0)(2) = 0$$
$$C_y = (0)(2\sqrt{3}) - (6)(0) = 0$$
$$C_z = (6)(2) - (0)(2\sqrt{3}) = 12$$

Thus again we have $\vec{C} = 12\hat{k}$.

EVALUATE Both methods give the same result. Depending on the situation, one or the other of the two approaches may be the more convenient one to use.

KEYCONCEPT The vector product $\vec{A} \times \vec{B}$ of two vectors is a third vector that is perpendicular to both $\vec{A}$ and $\vec{B}$. You can find the vector product either from the magnitudes of the two vectors, the angle between them, and the right-hand rule, or from the components of the two vectors.

TEST YOUR UNDERSTANDING OF SECTION 1.10 Vector $\vec{A}$ has magnitude 2 and vector $\vec{B}$ has magnitude 3. The angle ϕ between $\vec{A}$ and $\vec{B}$ is (i) 0°, (ii) 90°, or (iii) 180°. For each of the following situations, state what the value of ϕ must be. (In each situation there may be more than one correct answer.) (a) $\vec{A}\cdot\vec{B} = 0$; (b) $\vec{A} \times \vec{B} = \mathbf{0}$; (c) $\vec{A}\cdot\vec{B} = 6$; (d) $\vec{A}\cdot\vec{B} = -6$; (e) (magnitude of $\vec{A} \times \vec{B}$) $= 6$.

ANSWER

(a) (ii) $\phi = 90°$, (b) (i) $\phi = 0°$ or (iii) $\phi = 180°$, (c) (i) $\phi = 0°$, (d) (iii) $\phi = 180°$, (e) (ii) $\phi = 90°$ (a) The scalar product is zero only if $\vec{A}$ and $\vec{B}$ are perpendicular. (b) The vector product is zero only if $\vec{A}$ and $\vec{B}$ are parallel or antiparallel. (c) The scalar product is equal to the product of the magnitudes ($\vec{A}\cdot\vec{B} = AB$) only if $\vec{A}$ and $\vec{B}$ are parallel. (d) The scalar product is equal to the negative of the product of the magnitudes ($\vec{A}\cdot\vec{B} = -AB$) only if $\vec{A}$ and $\vec{B}$ are antiparallel. (e) The magnitude of the vector product is equal to the product of the magnitudes [(magnitude of $\vec{A} \times \vec{B}$) $= AB$] only if $\vec{A}$ and $\vec{B}$ are perpendicular.

CHAPTER 1 SUMMARY

Physical quantities and units: Three fundamental physical quantities are mass, length, and time. The corresponding fundamental SI units are the kilogram, the meter, and the second. Derived units for other physical quantities are products or quotients of the basic units. Equations must be dimensionally consistent; two terms can be added only when they have the same units. (See Examples 1.1 and 1.2.)

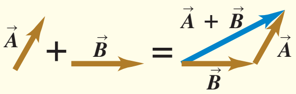
Significant figures: The accuracy of a measurement can be indicated by the number of significant figures or by a stated uncertainty. The significant figures in the result of a calculation are determined by the rules summarized in Table 1.2. When only crude estimates are available for input data, we can often make useful order-of-magnitude estimates. (See Examples 1.3 and 1.4.)

Significant figures in magenta

$$\pi = \frac{C}{2r} = \frac{0.424\text{ m}}{2(0.06750\text{ m})} = 3.14$$

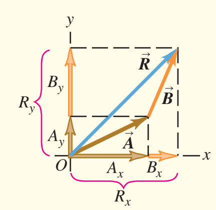
$$123.62 + 8.9 = 132.5$$

Scalars, vectors, and vector addition: Scalar quantities are numbers and combine according to the usual rules of arithmetic. Vector quantities have direction as well as magnitude and combine according to the rules of vector addition. The negative of a vector has the same magnitude but points in the opposite direction. (See Example 1.5.)



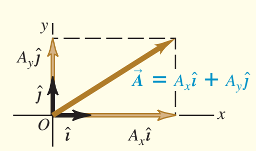
Vector components and vector addition: Vectors can be added by using components of vectors. The x-component of $\vec{R} = \vec{A} + \vec{B}$ is the sum of the x-components of $\vec{A}$ and $\vec{B}$, and likewise for the y- and z-components. (See Examples 1.6 and 1.7.)

$$R_x = A_x + B_x$$
$$R_y = A_y + B_y \qquad (1.9)$$
$$R_z = A_z + B_z$$



Unit vectors: Unit vectors describe directions in space. A unit vector has a magnitude of 1, with no units. The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, aligned with the x-, y-, and z-axes of a rectangular coordinate system, are especially useful. (See Example 1.8.)

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k} \qquad (1.14)$$

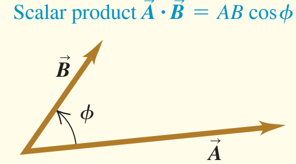


Scalar product: The scalar product $C = \vec{A} \cdot \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is a scalar quantity. It can be expressed in terms of the magnitudes of $\vec{A}$ and $\vec{B}$ and the angle ϕ between the two vectors, or in terms of the components of $\vec{A}$ and $\vec{B}$. The scalar product is commutative; $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$. The scalar product of two perpendicular vectors is zero. (See Examples 1.9 and 1.10.)

$$\vec{A} \cdot \vec{B} = AB\cos\phi = |\vec{A}||\vec{B}|\cos\phi \qquad (1.16)$$

$$\vec{A} \cdot \vec{B} = A_xB_x + A_yB_y + A_zB_z \qquad (1.19)$$

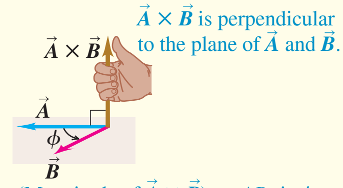
Scalar product $\vec{A} \cdot \vec{B} = AB\cos\phi$



Vector product: The vector product $\vec{C} = \vec{A} \times \vec{B}$ of two vectors $\vec{A}$ and $\vec{B}$ is a third vector $\vec{C}$. The magnitude of $\vec{A} \times \vec{B}$ depends on the magnitudes of $\vec{A}$ and $\vec{B}$ and the angle ϕ between the two vectors. The direction of $\vec{A} \times \vec{B}$ is perpendicular to the plane of the two vectors being multiplied, as given by the right-hand rule. The components of $\vec{C} = \vec{A} \times \vec{B}$ can be expressed in terms of the components of $\vec{A}$ and $\vec{B}$. The vector product is not commutative; $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$. The vector product of two parallel or antiparallel vectors is zero. (See Example 1.11.)

$$C = AB\sin\phi \qquad (1.20)$$

$$C_x = A_yB_z - A_zB_y$$
$$C_y = A_zB_x - A_xB_z \qquad (1.25)$$
$$C_z = A_xB_y - A_yB_x$$



(Magnitude of $\vec{A} \times \vec{B}$) $= AB\sin\phi$

Chapter 1 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLE 1.7 (Section 1.8) before attempting these problems.

VP1.7.1 Consider the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Example 1.7. If a fourth vector $\vec{D}$ is added to $\vec{A}+\vec{B}+\vec{C}$, the result is zero: $\vec{A}+\vec{B}+\vec{C}+\vec{D}=\mathbf{0}$. Find the magnitude and direction of $\vec{D}$. State the direction of $\vec{D}$ in terms of an angle measured counterclockwise from the positive x-axis, and state in which quadrant this angle lies.

VP1.7.2 Consider the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Example 1.7. Calculate the magnitude and direction of the vector $\vec{S}=\vec{A}-\vec{B}+\vec{C}$. State the direction of $\vec{S}$ in terms of an angle measured counterclockwise from the positive x-axis, and state in which quadrant this angle lies. (*Hint:* The components of $-\vec{B}$ are just the negatives of the components of $\vec{B}$.)

VP1.7.3 Consider the three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Example 1.7. (a) Find the components of the vector $\vec{T}=\vec{A}+\vec{B}+2\vec{C}$. (b) Find the magnitude and direction of $\vec{T}$. State the direction of $\vec{T}$ in terms of an angle measured counterclockwise from the positive x-axis, and state in which quadrant this angle lies.

VP1.7.4 A hiker undergoes the displacement $\vec{A}$ shown in Example 1.7. The hiker then undergoes a second displacement such that she ends up 38.0 m from her starting point, in a direction from her starting point that is 37.0° west of north. Find the magnitude and direction of this second displacement. State the direction in terms of an angle measured counterclockwise from the positive x-axis, and state in which quadrant this angle lies.

Be sure to review EXAMPLES 1.9 and 1.10 (Section 1.10) before attempting these problems.

VP1.10.1 Vector $\vec{A}$ has magnitude 5.00 and is at an angle of 36.9° south of east. Vector $\vec{B}$ has magnitude 6.40 and is at an angle of 20.0° west of north. (a) Choose the positive x-direction to the east and the positive y-direction to the north. Find the components of $\vec{A}$ and $\vec{B}$. (b) Calculate the scalar product $\vec{A}\cdot\vec{B}$.

VP1.10.2 Vector $\vec{C}$ has magnitude 6.50 and is at an angle of 55.0° measured counterclockwise from the $+x$-axis toward the $+y$-axis. Vector $\vec{D}$ has components $D_x=+4.80$ and $D_y=-8.40$. (a) Calculate the scalar product $\vec{C}\cdot\vec{D}$. (b) Find the angle ϕ between the vectors $\vec{C}$ and $\vec{D}$.

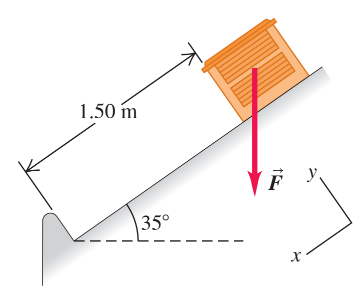
VP1.10.3 Vector $\vec{A}$ has components $A_x=-5.00$, $A_y=3.00$, and $A_z=0$. Vector $\vec{B}$ has components $B_x=2.50$, $B_y=4.00$, and $B_z=-1.50$. Find the angle between the two vectors.

VP1.10.4 If a force $\vec{F}$ acts on an object as that object moves through a displacement $\vec{s}$, the *work* done by that force equals the scalar product of $\vec{F}$ and $\vec{s}$: $W=\vec{F}\cdot\vec{s}$. A certain object moves through displacement $\vec{s}=(4.00\text{ m})\hat{\imath}+(5.00\text{ m})\hat{\jmath}$. As it moves it is acted on by force $\vec{F}$, which has x-component $F_x=-12.0\text{ N}$ (1 N = 1 newton is the SI unit of force). The work done by this force is $26.0\text{ N}\cdot\text{m}=26.0\text{ J}$ (1 J = 1 joule = 1 newton-meter is the SI unit of work). (a) Find the y-component of $\vec{F}$. (b) Find the angle between $\vec{F}$ and $\vec{s}$.

BRIDGING PROBLEM Vectors on the Roof

An air-conditioning unit is fastened to a roof that slopes at an angle of 35° above the horizontal (**Fig. 1.34**). Its weight is a force $\vec{F}$ on the air conditioner that is directed vertically downward. In order that the unit not crush the roof tiles, the component of the unit's weight perpendicular to the roof cannot exceed 425 N. (One newton, or 1 N, is the SI unit of force. It is equal to 0.2248 lb.) (a) What is the maximum allowed weight of the unit? (b) If the fasteners fail, the unit slides 1.50 m along the roof before it comes to a halt against a ledge. How much work does the weight force do on the unit during its slide if the unit has the weight calculated in part (a)? The work done by a force $\vec{F}$ on an object that undergoes a displacement $\vec{s}$ is $W=\vec{F}\cdot\vec{s}$.

Figure 1.34 An air-conditioning unit on a slanted roof.



SOLUTION GUIDE

IDENTIFY and SET UP

1. This problem involves vectors and components. What are the known quantities? Which aspect(s) of the weight vector (magnitude, direction, and/or particular components) represent the target variable for part (a)? Which aspect(s) must you know to solve part (b)?
2. Make a sketch based on Fig. 1.34. Draw the x- and y-axes, choosing the positive direction for each. Your axes don't have to be horizontal and vertical, but they do have to be mutually perpendicular. Figure 1.34 shows a convenient choice of axes: The x-axis is parallel to the slope of the roof.
3. Choose the equations you'll use to determine the target variables.

EXECUTE

4. Use the relationship between the magnitude and direction of a vector and its components to solve for the target variable in part (a). Be careful: Is 35° the correct angle to use in the equation? (*Hint:* Check your sketch.)
5. Make sure your answer has the correct number of significant figures.
6. Use the definition of the scalar product to solve for the target variable in part (b). Again, use the correct number of significant figures.

EVALUATE

7. Did your answer to part (a) include a vector component whose absolute value is greater than the magnitude of the vector? Is that possible?
8. There are two ways to find the scalar product of two vectors, one of which you used to solve part (b). Check your answer by repeating the calculation, using the other way. Do you get the same answer?

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q1.1 How many correct experiments do we need to disprove a theory? How many do we need to prove a theory? Explain.

Q1.2 Suppose you are asked to compute the tangent of 5.00 meters. Is this possible? Why or why not?

Q1.3 What is your height in centimeters? What is your weight in newtons?

Q1.4 The U.S. National Institute of Standards and Technology (NIST) maintains several accurate copies of the international standard kilogram. Even after careful cleaning, these national standard kilograms are gaining mass at an average rate of about $1\ \mu g/y$ (y = year) when compared every 10 years or so to the standard international kilogram. Does this apparent increase have any importance? Explain.

Q1.5 What physical phenomena (other than a pendulum or cesium clock) could you use to define a time standard?

Q1.6 Describe how you could measure the thickness of a sheet of paper with an ordinary ruler.

Q1.7 The quantity $\pi = 3.14159\ldots$ is a number with no dimensions, since it is a ratio of two lengths. Describe two or three other geometrical or physical quantities that are dimensionless.

Q1.8 What are the units of volume? Suppose another student tells you that a cylinder of radius r and height h has volume given by $\pi r^3 h$. Explain why this cannot be right.

Q1.9 Three archers each fire four arrows at a target. Joe's four arrows hit points that are spread around in a region that goes 10 cm above, 10 cm below, 10 cm to the left, and 10 cm to the right of the center of the target. All four of Moe's arrows hit within 1 cm of a point 20 cm from the center, and Flo's four arrows hit within 1 cm of the center. The contest judge says that one of the archers is precise but not accurate, another archer is accurate but not precise, and the third archer is both accurate and precise. Which description applies to which archer? Explain.

Q1.10 Is the vector $(\hat{\imath} + \hat{\jmath} + \hat{k})$ a unit vector? Is the vector $(3.0\hat{\imath} - 2.0\hat{\jmath})$ a unit vector? Justify your answers.

Q1.11 A circular racetrack has a radius of 500 m. What is the displacement of a bicyclist when she travels around the track from the north side to the south side? When she makes one complete circle around the track? Explain.

Q1.12 Can you find two vectors with different lengths that have a vector sum of zero? What length restrictions are required for three vectors to have a vector sum of zero? Explain.

Q1.13 The "direction of time" is said to proceed from past to future. Does this mean that time is a vector quantity? Explain.

Q1.14 Air traffic controllers give instructions called "vectors" to tell airline pilots in which direction they are to fly. If these are the only instructions given, is the name "vector" used correctly? Why or why not?

Q1.15 Can you find a vector quantity that has a magnitude of zero but components that are not zero? Explain. Can the magnitude of a vector be less than the magnitude of any of its components? Explain.

Q1.16 (a) Does it make sense to say that a vector is *negative*? Why? (b) Does it make sense to say that one vector is the negative of another? Why? Does your answer here contradict what you said in part (a)?

Q1.17 If $\vec{C} = \vec{A} + \vec{B}$, what must be true about the directions and magnitudes of $\vec{A}$ and $\vec{B}$ if $C = A + B$? What must be true about the directions and magnitudes of $\vec{A}$ and $\vec{B}$ if $C = 0$?

Q1.18 If $\vec{A}$ and $\vec{B}$ are nonzero vectors, is it possible for *both* $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$ to be zero? Explain.

Q1.19 What does $\vec{A} \cdot \vec{A}$, the scalar product of a vector with itself, give? What about $\vec{A} \times \vec{A}$, the vector product of a vector with itself?

Q1.20 Let $\vec{A}$ represent any nonzero vector. Why is $\vec{A}/A$ a unit vector, and what is its direction? If θ is the angle that $\vec{A}$ makes with the $+x$-axis, explain why $(\vec{A}/A) \cdot \hat{\imath}$ is called the *direction cosine* for that axis.

Q1.21 Figure 1.7 shows the result of an unacceptable error in the stopping position of a train. If a train travels 890 km from Berlin to Paris and then overshoots the end of the track by 10.0 m, what is the percent error in the total distance covered? Is it correct to write the total distance covered by the train as 890,010 m? Explain.

Q1.22 Which of the following are legitimate mathematical operations: (a) $\vec{A} \cdot (\vec{B} - \vec{C})$; (b) $(\vec{A} - \vec{B}) \times \vec{C}$; (c) $\vec{A} \cdot (\vec{B} \times \vec{C})$; (d) $\vec{A} \times (\vec{B} \times \vec{C})$; (e) $\vec{A} \times (\vec{B} \cdot \vec{C})$? In each case, give the reason for your answer.

Q1.23 Consider the vector products $\vec{A} \times (\vec{B} \times \vec{C})$ and $(\vec{A} \times \vec{B}) \times \vec{C}$. Give an example that illustrates the general rule that these two vector products do not have the same magnitude or direction. Can you choose vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ such that these two vector products *are* equal? If so, give an example.

Q1.24 Show that, no matter what $\vec{A}$ and $\vec{B}$ are, $\vec{A} \cdot (\vec{A} \times \vec{B}) = 0$. (*Hint:* Do not look for an elaborate mathematical proof. Consider the definition of the direction of the cross product.)

Q1.25 (a) If $\vec{A} \cdot \vec{B} = 0$, does it necessarily follow that $A = 0$ or $B = 0$? Explain. (b) If $\vec{A} \times \vec{B} = \mathbf{0}$, does it necessarily follow that $A = 0$ or $B = 0$? Explain.

Q1.26 If $\vec{A} = \mathbf{0}$ for a vector in the xy-plane, does it follow that $A_x = -A_y$? What *can* you say about A_x and A_y?

EXERCISES

Section 1.3 Standards and Units

Section 1.4 Using and Converting Units

1.1 • Starting with the definition 1 in. = 2.54 cm, find the number of (a) kilometers in 1.00 mile and (b) feet in 1.00 km.

1.2 •• According to the label on a bottle of salad dressing, the volume of the contents is 0.473 liter (L). Using only the conversions $1\ L = 1000\ cm^3$ and 1 in. = 2.54 cm, express this volume in cubic inches.

1.3 •• How many nanoseconds does it take light to travel 1.00 ft in vacuum? (This result is a useful quantity to remember.)

1.4 •• The density of gold is $19.3\ g/cm^3$. What is this value in kilograms per cubic meter?

1.5 • How many years older will you be 1.00 gigasecond from now? (Assume a 365-day year.)

1.6 • The following conversions occur frequently in physics and are very useful. (a) Use 1 mi = 5280 ft and 1 h = 3600 s to convert 60 mph to units of ft/s. (b) The acceleration of a freely falling object is $32\ ft/s^2$. Use 1 ft = 30.48 cm to express this acceleration in units of m/s^2. (c) The density of water is $1.0\ g/cm^3$. Convert this density to units of kg/m^3.

1.7 • A certain fuel-efficient hybrid car gets gasoline mileage of 55.0 mpg (miles per gallon). (a) If you are driving this car in Europe and want to compare its mileage with that of other European cars, express this mileage in km/L (L = liter). Use the conversion factors in Appendix E. (b) If this car's gas tank holds 45 L, how many tanks of gas will you use to drive 1500 km?

1.8 • **BIO** (a) The recommended daily allowance (RDA) of the trace metal magnesium is 410 mg/day for males. Express this quantity in μg/day. (b) For adults, the RDA of the amino acid lysine is 12 mg per kg of body weight. How many grams per day should a 75 kg adult receive? (c) A typical multivitamin tablet can contain 2.0 mg of vitamin B_2 (riboflavin), and the RDA is 0.0030 g/day. How many such tablets should a person take each day to get the proper amount of this vitamin, if he gets none from other sources? (d) The RDA for the trace element selenium is 0.000070 g/day. Express this dose in mg/day.

1.9 •• **Neptunium.** In the fall of 2002, scientists at Los Alamos National Laboratory determined that the critical mass of neptunium-237 is about 60 kg. The critical mass of a fissionable material is the minimum amount that must be brought together to start a nuclear chain reaction. Neptunium-237 has a density of 19.5 g/cm^3. What would be the radius of a sphere of this material that has a critical mass?

1.10 •• **BIO** **Bacteria.** Bacteria vary in size, but a diameter of 2.0 μm is not unusual. What are the volume (in cubic centimeters) and surface area (in square millimeters) of a spherical bacterium of that size? (Consult Appendix B for relevant formulas.)

Section 1.5 Uncertainty and Significant Figures

1.11 • With a wooden ruler, you measure the length of a rectangular piece of sheet metal to be 12 mm. With micrometer calipers, you measure the width of the rectangle to be 5.98 mm. Use the correct number of significant figures: What are (a) the area of the rectangle; (b) the ratio of the rectangle's width to its length; (c) the perimeter of the rectangle; (d) the difference between the length and the width; and (e) the ratio of the length to the width?

1.12 • The volume of a solid cylinder is given by $V = \pi r^2 h$, where r is the radius and h is the height. You measure the radius and height of a thin cylindrical wire and obtain the results $r = 0.036$ cm and $h = 12.1$ cm. What do your measurements give for the volume of the wire in mm^3? Use the correct number of significant figures in your answer.

1.13 •• A useful and easy-to-remember approximate value for the number of seconds in a year is $\pi \times 10^7$. Determine the percent error in this approximate value. (There are 365.24 days in one year.)

1.14 • Express each approximation of π to six significant figures: (a) 22/7 and (b) 355/113. (c) Are these approximations accurate to that precision?

Section 1.6 Estimates and Orders of Magnitude

1.15 •• **BIO** A rather ordinary middle-aged man is in the hospital for a routine checkup. The nurse writes "200" on the patient's medical chart but forgets to include the units. Which of these quantities could the 200 plausibly represent? The patient's (a) mass in kilograms; (b) height in meters; (c) height in centimeters; (d) height in millimeters; (e) age in months.

1.16 • How many gallons of gasoline are used in the United States in one day? Assume that there are two cars for every three people, that each car is driven an average of 10,000 miles per year, and that the average car gets 20 miles per gallon.

1.17 • In Wagner's opera *Das Rheingold,* the goddess Freia is ransomed for a pile of gold just tall enough and wide enough to hide her from sight. Estimate the monetary value of this pile. The density of gold is 19.3 g/cm^3, and take its value to be about \$40 per gram.

1.18 • **BIO** Four astronauts are in a spherical space station. (a) If, as is typical, each of them breathes about 500 cm^3 of air with each breath, approximately what volume of air (in cubic meters) do these astronauts breathe in a year? (b) What would the diameter (in meters) of the space station have to be to contain all this air?

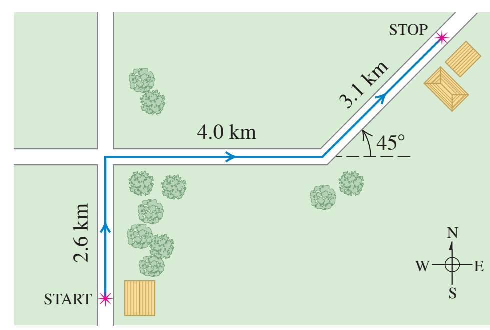
1.19 • You are using water to dilute small amounts of chemicals in the laboratory, drop by drop. How many drops of water are in a 1.0 L bottle? (*Hint:* Start by estimating the diameter of a drop of water.)

1.20 • **BIO** How many times does a human heart beat during a person's lifetime? How many gallons of blood does it pump? (Estimate that the heart pumps 50 cm^3 of blood with each beat.)

Section 1.7 Vectors and Vector Addition

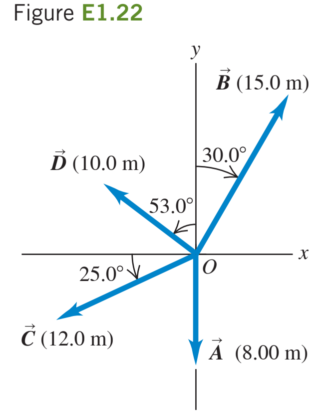
1.21 •• A postal employee drives a delivery truck along the route shown in **Fig. E1.21.** Determine the magnitude and direction of the resultant displacement by drawing a scale diagram. (See also Exercise 1.28 for a different approach.)

Figure **E1.21**



1.22 •• For the vectors $\vec{A}$ and $\vec{B}$ in **Fig. E1.22,** use a scale drawing to find the magnitude and direction of (a) the vector sum $\vec{A} + \vec{B}$ and (b) the vector difference $\vec{A} - \vec{B}$. Use your answers to find the magnitude and direction of (c) $-\vec{A} - \vec{B}$ and (d) $\vec{B} - \vec{A}$. (See also Exercise 1.29 for a different approach.)

Figure **E1.22**



1.23 •• A spelunker is surveying a cave. She follows a passage 180 m straight west, then 210 m in a direction 45° east of south, and then 280 m at 30° east of north. After a fourth displacement, she finds herself back where she started. Use a scale drawing to determine the magnitude and direction of the fourth displacement. (See also Problem 1.57 for a different approach.)

Section 1.8 Components of Vectors

1.24 •• Let θ be the angle that the vector $\vec{A}$ makes with the $+x$-axis, measured counterclockwise from that axis. Find angle θ for a vector that has these components: (a) $A_x = 2.00$ m, $A_y = -1.00$ m; (b) $A_x = 2.00$ m, $A_y = 1.00$ m; (c) $A_x = -2.00$ m, $A_y = 1.00$ m; (d) $A_x = -2.00$ m, $A_y = -1.00$ m.

1.25 • Compute the x- and y-components of the vectors $\vec{A}$, $\vec{B}$, $\vec{C}$, and $\vec{D}$ in Fig. E1.22.

1.26 • Vector $\vec{A}$ is in the direction 34.0° clockwise from the $-y$-axis. The x-component of $\vec{A}$ is $A_x = -16.0$ m. (a) What is the y-component of $\vec{A}$? (b) What is the magnitude of $\vec{A}$?

1.27 • Vector $\vec{A}$ has y-component $A_y = +9.60$ m. $\vec{A}$ makes an angle of 32.0° counterclockwise from the $+y$-axis. (a) What is the x-component of $\vec{A}$? (b) What is the magnitude of $\vec{A}$?

1.28 •• A postal employee drives a delivery truck over the route shown in Fig. E1.21. Use the method of components to determine the magnitude and direction of her resultant displacement. In a vector-addition diagram (roughly to scale), show that the resultant displacement found from your diagram is in qualitative agreement with the result you obtained by using the method of components.

1.29 • For the vectors $\vec{A}$ and $\vec{B}$ in Fig. E1.22, use the method of components to find the magnitude and direction of (a) the vector sum $\vec{A} + \vec{B}$; (b) the vector sum $\vec{B} + \vec{A}$; (c) the vector difference $\vec{A} - \vec{B}$; (d) the vector difference $\vec{B} - \vec{A}$.

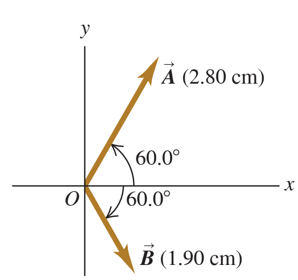
1.30 • Find the magnitude and direction of the vector represented by the following pairs of components: (a) $A_x = -8.60$ cm, $A_y = 5.20$ cm; (b) $A_x = -9.70$ m, $A_y = -2.45$ m; (c) $A_x = 7.75$ km, $A_y = -2.70$ km.

1.31 •• A disoriented physics professor drives 3.25 km north, then 2.20 km west, and then 1.50 km south. Find the magnitude and direction of the resultant displacement, using the method of components. In a vector-addition diagram (roughly to scale), show that the resultant displacement found from your diagram is in qualitative agreement with the result you obtained by using the method of components.

1.32 •• Vector $\vec{A}$ has magnitude 8.00 m and is in the xy-plane at an angle of 127° counterclockwise from the $+x$–axis (37° past the $+y$-axis). What are the magnitude and direction of vector $\vec{B}$ if the sum $\vec{A} + \vec{B}$ is in the $-y$-direction and has magnitude 12.0 m?

1.33 •• Vector $\vec{A}$ is 2.80 cm long and is 60.0° above the x-axis in the first quadrant. Vector $\vec{B}$ is 1.90 cm long and is 60.0° below the x-axis in the fourth quadrant (**Fig. E1.33**). Use components to find the magnitude and direction of (a) $\vec{A} + \vec{B}$; (b) $\vec{A} - \vec{B}$; (c) $\vec{B} - \vec{A}$. In each case, sketch the vector addition or subtraction and show that your numerical answers are in qualitative agreement with your sketch.

Figure **E1.33**



Section 1.9 Unit Vectors

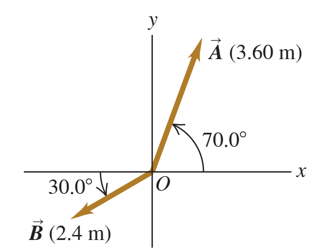
1.34 • In each case, find the x- and y-components of vector $\vec{A}$: (a) $\vec{A} = 5.0\hat{\imath} - 6.3\hat{\jmath}$; (b) $\vec{A} = 11.2\hat{\jmath} - 9.91\hat{\imath}$; (c) $\vec{A} = -15.0\hat{\imath} + 22.4\hat{\jmath}$; (d) $\vec{A} = 5.0\vec{B}$, where $\vec{B} = 4\hat{\imath} - 6\hat{\jmath}$.

1.35 •• Write each vector in Fig. E1.22 in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$.

1.36 •• Given two vectors $\vec{A} = 4.00\hat{\imath} + 7.00\hat{\jmath}$ and $\vec{B} = 5.00\hat{\imath} - 2.00\hat{\jmath}$, (a) find the magnitude of each vector; (b) use unit vectors to write an expression for the vector difference $\vec{A} - \vec{B}$; and (c) find the magnitude and direction of the vector difference $\vec{A} - \vec{B}$. (d) In a vector diagram show $\vec{A}$, $\vec{B}$, and $\vec{A} - \vec{B}$, and show that your diagram agrees qualitatively with your answer to part (c).

1.37 •• (a) Write each vector in **Fig. E1.37** in terms of the unit vectors $\hat{\imath}$ and $\hat{\jmath}$. (b) Use unit vectors to express vector $\vec{C}$, where $\vec{C} = 3.00\vec{A} - 4.00\vec{B}$. (c) Find the magnitude and direction of $\vec{C}$.

Figure **E1.37**



1.38 • You are given two vectors $\vec{A} = -3.00\hat{\imath} + 6.00\hat{\jmath}$ and $\vec{B} = 7.00\hat{\imath} + 2.00\hat{\jmath}$. Let counterclockwise angles be positive. (a) What angle does $\vec{A}$ make with the $+x$-axis? (b) What angle does $\vec{B}$ make with the $+x$-axis? (c) Vector $\vec{C}$ is the sum of $\vec{A}$ and $\vec{B}$, so $\vec{C} = \vec{A} + \vec{B}$. What angle does $\vec{C}$ make with the $+x$-axis?

1.39 • Given two vectors $\vec{A} = -2.00\hat{\imath} + 3.00\hat{\jmath} + 4.00\hat{k}$ and $\vec{B} = 3.00\hat{\imath} + 1.00\hat{\jmath} - 3.00\hat{k}$, (a) find the magnitude of each vector; (b) use unit vectors to write an expression for the vector difference $\vec{A} - \vec{B}$; and (c) find the magnitude of the vector difference $\vec{A} - \vec{B}$. Is this the same as the magnitude of $\vec{B} - \vec{A}$? Explain.

Section 1.10 Products of Vectors

1.40 •• (a) Find the scalar product of the vectors $\vec{A}$ and $\vec{B}$ given in Exercise 1.36. (b) Find the angle between these two vectors.

1.41 • For the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ in Fig. E1.22, find the scalar products (a) $\vec{A} \cdot \vec{B}$; (b) $\vec{B} \cdot \vec{C}$; (c) $\vec{A} \cdot \vec{C}$.

1.42 •• Find the vector product $\vec{A} \times \vec{B}$ (expressed in unit vectors) of the two vectors given in Exercise 1.36. What is the magnitude of the vector product?

1.43 •• Find the angle between each of these pairs of vectors:
(a) $\vec{A} = -2.00\hat{\imath} + 6.00\hat{\jmath}$ and $\vec{B} = 2.00\hat{\imath} - 3.00\hat{\jmath}$
(b) $\vec{A} = 3.00\hat{\imath} + 5.00\hat{\jmath}$ and $\vec{B} = 10.00\hat{\imath} + 6.00\hat{\jmath}$
(c) $\vec{A} = -4.00\hat{\imath} + 2.00\hat{\jmath}$ and $\vec{B} = 7.00\hat{\imath} + 14.00\hat{\jmath}$

1.44 • For the two vectors in Fig. E1.33, find the magnitude and direction of (a) the vector product $\vec{A} \times \vec{B}$; (b) the vector product $\vec{B} \times \vec{A}$.

1.45 • For the two vectors $\vec{A}$ and $\vec{D}$ in Fig. E1.22, find the magnitude and direction of (a) the vector product $\vec{A} \times \vec{D}$; (b) the vector product $\vec{D} \times \vec{A}$.

1.46 • For the two vectors $\vec{A}$ and $\vec{B}$ in Fig. E1.37, find (a) the scalar product $\vec{A} \cdot \vec{B}$; (b) the magnitude and direction of the vector product $\vec{A} \times \vec{B}$.

1.47 •• The vector product of vectors $\vec{A}$ and $\vec{B}$ has magnitude 16.0 m^2 and is in the $+z$-direction. If vector $\vec{A}$ has magnitude 8.0 m and is in the $-x$-direction, what are the magnitude and direction of vector $\vec{B}$ if it has no x-component?

1.48 • The angle between two vectors is θ. (a) If $\theta = 30.0°$, which has the greater magnitude: the scalar product or the vector product of the two vectors? (b) For what value (or values) of θ are the magnitudes of the scalar product and the vector product equal?

PROBLEMS

1.49 •• **White Dwarfs and Neutron Stars.** Recall that density is mass divided by volume, and consult Appendix B as needed. (a) Calculate the average density of the earth in g/cm^3, assuming our planet is a perfect sphere. (b) In about 5 billion years, at the end of its lifetime, our sun will end up as a white dwarf that has about the same mass as it does now but is reduced to about 15,000 km in diameter. What will be its density at that stage? (c) A neutron star is the remnant of certain supernovae (explosions of giant stars). Typically, neutron stars are about 20 km in diameter and have about the same mass as our sun. What is a typical neutron star density in g/cm^3?

1.50 •• The Hydrogen Maser. A maser is a laser-type device that produces electromagnetic waves with frequencies in the microwave and radio-wave bands of the electromagnetic spectrum. You can use the radio waves generated by a hydrogen maser as a standard of frequency. The frequency of these waves is 1,420,405,751.786 hertz. (A hertz is another name for one cycle per second.) A clock controlled by a hydrogen maser is off by only 1 s in 100,000 years. For the following questions, use only three significant figures. (The large number of significant figures given for the frequency simply illustrates the remarkable accuracy to which it has been measured.) (a) What is the time for one cycle of the radio wave? (b) How many cycles occur in 1 h? (c) How many cycles would have occurred during the age of the earth, which is estimated to be 4.6×10^9 years? (d) By how many seconds would a hydrogen maser clock be off after a time interval equal to the age of the earth?

1.51 •• An Earthlike Planet. In January 2006 astronomers reported the discovery of a planet, comparable in size to the earth, orbiting another star and having a mass about 5.5 times the earth's mass. It is believed to consist of a mixture of rock and ice, similar to Neptune. If this planet has the same density as Neptune $(1.76\ \text{g/cm}^3)$, what is its radius expressed (a) in kilometers and (b) as a multiple of earth's radius? Consult Appendix F for astronomical data.

1.52 ••• A rectangular piece of aluminum is 7.60 ± 0.01 cm long and 1.90 ± 0.01 cm wide. (a) Find the area of the rectangle and the uncertainty in the area. (b) Verify that the fractional uncertainty in the area is equal to the sum of the fractional uncertainties in the length and in the width. (This is a general result.)

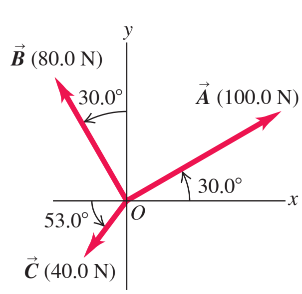
1.53 • BIO Estimate the number of atoms in your body. (*Hint:* Based on what you know about biology and chemistry, what are the most common types of atom in your body? What is the mass of each type of atom? Appendix D gives the atomic masses of different elements, measured in atomic mass units; you can find the value of an atomic mass unit, or 1 u, in Appendix E.)

1.54 • BIO Biological tissues are typically made up of 98% water. Given that the density of water is $1.0 \times 10^3\ \text{kg/m}^3$, estimate the mass of (a) the heart of an adult human; (b) a cell with a diameter of 0.5 μm; (c) a honeybee.

1.55 •• Vector $\vec{A} = 3.0\hat{\imath} - 4.0\hat{k}$. (a) Construct a unit vector that is parallel to $\vec{A}$. (b) Construct a unit vector that is antiparallel to $\vec{A}$. (c) Construct two unit vectors that are perpendicular to $\vec{A}$ and that have no y-component.

1.56 •• Three horizontal ropes pull on a large stone stuck in the ground, producing the vector forces $\vec{A}$, $\vec{B}$, and $\vec{C}$ shown in **Fig. P1.56.** Find the magnitude and direction of a fourth force on the stone that will make the vector sum of the four forces zero.

Figure **P1.56**



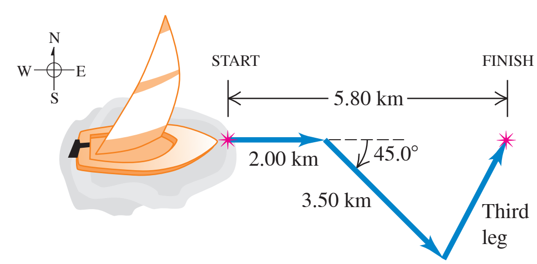
1.57 ••• As noted in Exercise 1.23, a spelunker is surveying a cave. She follows a passage 180 m straight west, then 210 m in a direction 45° east of south, and then 280 m at 30° east of north. After a fourth displacement, she finds herself back where she started. Use the method of components to determine the magnitude and direction of the fourth displacement. Draw the vector-addition diagram and show that it is in qualitative agreement with your numerical solution.

1.58 ••• Emergency Landing. A plane leaves the airport in Galisteo and flies 170 km at 68.0° east of north; then it changes direction to fly 230 km at 36.0° south of east, after which it makes an immediate emergency landing in a pasture. When the airport sends out a rescue crew, in which direction and how far should this crew fly to go directly to this plane?

1.59 •• A charged object with electric charge q produces an electric field. The SI unit for electric field is N/C, where N is the SI unit for force and C is the SI unit for charge. If at point P there are electric fields from two or more charged objects, then the resultant field is the vector sum of the fields from each object. At point P the electric field $\vec{E}_1$ from charge q_1 is 450 N/C in the $+y$-direction, and the electric field $\vec{E}_2$ from charge q_2 is 600 N/C in the direction 36.9° from the $-y$-axis toward the $-x$-axis. What are the magnitude and direction of the resultant field $\vec{E} = \vec{E}_1 + \vec{E}_2$ at point P due to these two charges?

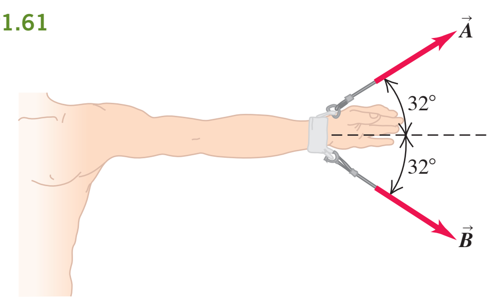
1.60 •• A sailor in a small sailboat encounters shifting winds. She sails 2.00 km east, next 3.50 km southeast, and then an additional distance in an unknown direction. Her final position is 5.80 km directly east of the starting point (**Fig. P1.60**). Find the magnitude and direction of the third leg of the journey. Draw the vector-addition diagram and show that it is in qualitative agreement with your numerical solution.

Figure **P1.60**



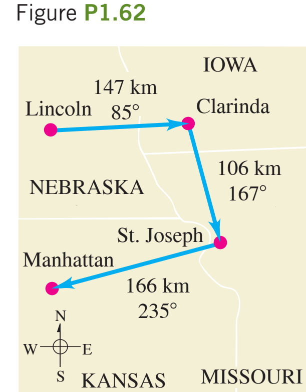
1.61 ••• BIO Dislocated Shoulder. A patient with a dislocated shoulder is put into a traction apparatus as shown in **Fig. P1.61.** The pulls $\vec{A}$ and $\vec{B}$ have equal magnitudes and must combine to produce an outward traction force of 12.8 N on the patient's arm. How large should these pulls be?

Figure **P1.61**



1.62 ••• On a training flight, a student pilot flies from Lincoln, Nebraska, to Clarinda, Iowa, next to St. Joseph, Missouri, and then to Manhattan, Kansas (**Fig. P1.62**). The directions are shown relative to north: 0° is north, 90° is east, 180° is south, and 270° is west. Use the method of components to find (a) the distance she has to fly from Manhattan to get back to Lincoln, and (b) the direction (relative to north) she must fly to get there. Illustrate your solutions with a vector diagram.

Figure **P1.62**



1.63 •• You leave the airport in College Station and fly 23.0 km in a direction 34.0° south of east. You then fly 46.0 km due north. How far and in what direction must you then fly to reach a private landing strip that is 32.0 km due west of the College Station airport?

1.64 ••• **Getting Back.** An explorer in Antarctica leaves his shelter during a whiteout. He takes 40 steps northeast, next 80 steps at 60° north of west, and then 50 steps due south. Assume all of his steps are equal in length. (a) Sketch, roughly to scale, the three vectors and their resultant. (b) Save the explorer from becoming hopelessly lost by giving him the displacement, calculated by using the method of components, that will return him to his shelter.

1.65 •• As a test of orienteering skills, your physics class holds a contest in a large, open field. Each contestant is told to travel 20.8 m due north from the starting point, then 38.0 m due east, and finally 18.0 m in the direction 33.0° west of south. After the specified displacements, a contestant will find a silver dollar hidden under a rock. The winner is the person who takes the shortest time to reach the location of the silver dollar. Remembering what you learned in class, you run on a straight line from the starting point to the hidden coin. How far and in what direction do you run?

1.66 • You are standing on a street corner with your friend. You then travel 14.0 m due west across the street and into your apartment building. You travel in the elevator 22.0 m upward to your floor, walk 12.0 m north to the door of your apartment, and then walk 6.0 m due east to your balcony that overlooks the street. Your friend is standing where you left her. Now how far are you from your friend?

1.67 •• You are lost at night in a large, open field. Your GPS tells you that you are 122.0 m from your truck, in a direction 58.0° east of south. You walk 72.0 m due west along a ditch. How much farther, and in what direction, must you walk to reach your truck?

1.68 ••• You live in a town where the streets are straight but are in a variety of directions. On Saturday you go from your apartment to the grocery store by driving 0.60 km due north and then 1.40 km in the direction 60.0° west of north. On Sunday you again travel from your apartment to the same store but this time by driving 0.80 km in the direction 50.0° north of west and then in a straight line to the store. (a) How far is the store from your apartment? (b) On which day do you travel the greater distance, and how much farther do you travel? Or, do you travel the same distance on each route to the store?

1.69 •• While following a treasure map, you start at an old oak tree. You first walk 825 m directly south, then turn and walk 1.25 km at 30.0° west of north, and finally walk 1.00 km at 32.0° north of east, where you find the treasure: a biography of Isaac Newton! (a) To return to the old oak tree, in what direction should you head and how far will you walk? Use components to solve this problem. (b) To see whether your calculation in part (a) is reasonable, compare it with a graphical solution drawn roughly to scale.

1.70 •• A fence post is 52.0 m from where you are standing, in a direction 37.0° north of east. A second fence post is due south from you. How far are you from the second post if the distance between the two posts is 68.0 m?

1.71 •• A dog in an open field runs 12.0 m east and then 28.0 m in a direction 50.0° west of north. In what direction and how far must the dog then run to end up 10.0 m south of her original starting point?

1.72 ••• Ricardo and Jane are standing under a tree in the middle of a pasture. An argument ensues, and they walk away in different directions. Ricardo walks 26.0 m in a direction 60.0° west of north. Jane walks 16.0 m in a direction 30.0° south of west. They then stop and turn to face each other. (a) What is the distance between them? (b) In what direction should Ricardo walk to go directly toward Jane?

1.73 ••• You are camping with Joe and Karl. Since all three of you like your privacy, you don't pitch your tents close together. Joe's tent is 21.0 m from yours, in the direction 23.0° south of east. Karl's tent is 32.0 m from yours, in the direction 37.0° north of east. What is the distance between Karl's tent and Joe's tent?

1.74 •• **Bond Angle in Methane.** In the methane molecule, CH_4, each hydrogen atom is at a corner of a regular tetrahedron with the carbon atom at the center. In coordinates for which one of the C—H bonds is in the direction of $\hat{\imath} + \hat{\jmath} + \hat{k}$, an adjacent C—H bond is in the $\hat{\imath} - \hat{\jmath} - \hat{k}$ direction. Calculate the angle between these two bonds.

1.75 •• The work W done by a constant force $\vec{F}$ on an object that undergoes displacement $\vec{s}$ from point 1 to point 2 is $W = \vec{F} \cdot \vec{s}$. For F in newtons (N) and s in meters (m), W is in joules (J). If, during a displacement of the object, $\vec{F}$ has constant direction 60.0° above the $-x$-axis and constant magnitude 5.00 N and if the displacement is 0.800 m in the $+x$-direction, what is the work done by the force $\vec{F}$?

1.76 •• Magnetic fields are produced by moving charges and exert forces on moving charges. When a particle with charge q is moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$, the force $\vec{F}$ that the field exerts on the particle is given by $\vec{F} = q\vec{v} \times \vec{B}$. The SI units are as follows: For charge it is the coulomb (C), for magnetic field it is tesla (T), for force it is newton (N), and for velocity it is m/s. If $q = -8.00 \times 10^{-6}$ C, $\vec{v}$ is 3.00×10^4 m/s in the $+x$-direction, and $\vec{B}$ is 5.00 T in the $-y$-direction, what are the magnitude and direction of the force that the magnetic field exerts on the charged particle?

1.77 •• Vectors $\vec{A}$ and $\vec{B}$ have scalar product -6.00, and their vector product has magnitude $+9.00$. What is the angle between these two vectors?

1.78 •• Torque is a vector quantity that specifies the effectiveness of a force in causing the rotation of an object. The torque that a force $\vec{F}$ exerts on a rigid object depends on the point where the force acts and on the location of the axis of rotation. If $\vec{r}$ is the length vector from the axis to the point of application of the force, then the torque is $\vec{r} \times \vec{F}$. If $\vec{F}$ is 22.0 N in the $-y$-direction and if $\vec{r}$ is in the xy-plane at an angle of 36° from the $+y$-axis toward the $-x$-axis and has magnitude 4.0 m, what are the magnitude and direction of the torque exerted by $\vec{F}$?

1.79 •• Vector $\vec{A} = a\hat{\imath} - b\hat{k}$ and vector $\vec{B} = -c\hat{\jmath} + d\hat{k}$. (a) In terms of the positive scalar quantities a, b, c, and d, what are $\vec{A} \cdot \vec{B}$ and $\vec{A} \times \vec{B}$? (b) If $c = 0$, what is the magnitude of $\vec{A} \cdot \vec{B}$ and what are the magnitude and direction of $\vec{A} \times \vec{B}$? Does your result for the direction for $\vec{A} \times \vec{B}$ agree with the result you get if you sketch $\vec{A}$ and $\vec{B}$ in the xz-plane and apply the right-hand rule? The scalar product can be described as the magnitude of $\vec{B}$ times the component of $\vec{A}$ that is parallel to $\vec{B}$. Does this agree with your result? The magnitude of the vector product can be described as the magnitude of $\vec{B}$ times the component of $\vec{A}$ that is perpendicular to $\vec{B}$. Does this agree with your result?

1.80 •• Vectors $\vec{A}$ and $\vec{B}$ are in the xy-plane. Vector $\vec{A}$ is in the $+x$-direction, and the direction of vector $\vec{B}$ is at an angle θ from the $+x$-axis measured toward the $+y$-axis. (a) If θ is in the range $0° \leq \theta \leq 180°$, for what two values of θ does the scalar product $\vec{A} \cdot \vec{B}$ have its maximum magnitude? For each of these values of θ, what is the magnitude of the vector product $\vec{A} \times \vec{B}$? (b) If θ is in the range $0° \leq \theta \leq 180°$, for what value of θ does the vector product $\vec{A} \times \vec{B}$ have its maximum value? For this value of θ, what is the magnitude of the scalar product $\vec{A} \cdot \vec{B}$? (c) What is the angle θ in the range $0° \leq \theta \leq 180°$ for which $\vec{A} \cdot \vec{B}$ is twice $|\vec{A} \times \vec{B}|$?

1.81 •• Vector $\vec{A}$ has magnitude 12.0 m, and vector $\vec{B}$ has magnitude 16.0 m. The scalar product $\vec{A} \cdot \vec{B}$ is 112.0 m^2. What is the magnitude of the vector product between these two vectors?

1.82 •• Vector $\vec{A}$ has magnitude 5.00 m and lies in the *xy*-plane in a direction 53.0° from the $+x$-axis axis measured toward the $+y$-axis. Vector $\vec{B}$ has magnitude 8.00 m and a direction you can adjust. (a) You want the vector product $\vec{A} \times \vec{B}$ to have a positive *z*-component of the largest possible magnitude. What direction should you select for vector $\vec{B}$? (b) What is the direction of $\vec{B}$ for which $\vec{A} \times \vec{B}$ has the most negative *z*-component? (c) What are the two directions of $\vec{B}$ for which $\vec{A} \times \vec{B}$ is zero?

1.83 •• The scalar product of vectors $\vec{A}$ and $\vec{B}$ is $+48.0\ \text{m}^2$. Vector $\vec{A}$ has magnitude 9.00 m and direction 28.0° west of south. If vector $\vec{B}$ has direction 39.0° south of east, what is the magnitude of $\vec{B}$?

1.84 ••• Obtain a *unit vector* perpendicular to the two vectors given in Exercise 1.39.

1.85 •• You are given vectors $\vec{A} = 5.0\hat{\imath} - 6.5\hat{\jmath}$ and $\vec{B} = 3.5\hat{\imath} - 7.0\hat{\jmath}$. A third vector, $\vec{C}$, lies in the *xy*-plane. Vector $\vec{C}$ is perpendicular to vector $\vec{A}$, and the scalar product of $\vec{C}$ with $\vec{B}$ is 15.0. From this information, find the components of vector $\vec{C}$.

1.86 •• Two vectors $\vec{A}$ and $\vec{B}$ have magnitudes $A = 3.00$ and $B = 3.00$. Their vector product is $\vec{A} \times \vec{B} = -5.00\hat{k} + 2.00\hat{\imath}$. What is the angle between $\vec{A}$ and $\vec{B}$?

1.87 ••• DATA You are a team leader at a pharmaceutical company. Several technicians are preparing samples, and you want to compare the densities of the samples (density = mass/volume) by using the mass and volume values they have reported. Unfortunately, you did not specify what units to use. The technicians used a variety of units in reporting their values, as shown in the following table.

Sample ID	Mass	Volume
A	8.00 g	$1.67 \times 10^{-6}\ \text{m}^3$
B	6.00 μg	$9.38 \times 10^{6}\ \mu\text{m}^3$
C	8.00 mg	$2.50 \times 10^{-3}\ \text{cm}^3$
D	9.00×10^{-4} kg	$2.81 \times 10^{3}\ \text{mm}^3$
E	9.00×10^{4} ng	$1.41 \times 10^{-2}\ \text{mm}^3$
F	6.00×10^{-2} mg	$1.25 \times 10^{8}\ \mu\text{m}^3$

List the sample IDs in order of increasing density of the sample.

1.88 ••• DATA You are a mechanical engineer working for a manufacturing company. Two forces, $\vec{F}_1$ and $\vec{F}_2$, act on a component part of a piece of equipment. Your boss asked you to find the magnitude of the larger of these two forces. You can vary the angle between $\vec{F}_1$ and $\vec{F}_2$ from 0° to 90° while the magnitude of each force stays constant. And, you can measure the magnitude of the resultant force they produce (their vector sum), but you cannot directly measure the magnitude of each separate force. You measure the magnitude of the resultant force for four angles θ between the directions of the two forces as follows:

θ	Resultant force (N)
0.0°	8.00
45.0°	7.43
60.0°	7.00
90.0°	5.83

(a) What is the magnitude of the larger of the two forces? (b) When the equipment is used on the production line, the angle between the two forces is 30.0°. What is the magnitude of the resultant force in this case?

1.89 ••• DATA **Navigating in the Solar System.** The *Mars Polar Lander* spacecraft was launched on January 3, 1999. On December 3, 1999, the day *Mars Polar Lander* impacted the Martian surface at high velocity and probably disintegrated, the positions of the earth and Mars were given by these coordinates:

	x	*y*	*z*
Earth	0.3182 AU	0.9329 AU	−0.0000 AU
Mars	1.3087 AU	−0.4423 AU	−0.0414 AU

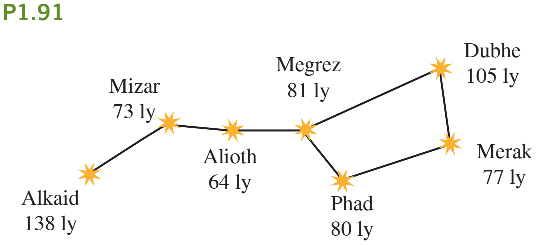
With these coordinates, the sun is at the origin and the earth's orbit is in the *xy*-plane. The earth passes through the $+x$-axis once a year on the autumnal equinox, the first day of autumn in the northern hemisphere (on or about September 22). One AU, or *astronomical unit,* is equal to 1.496×10^8 km, the average distance from the earth to the sun. (a) Draw the positions of the sun, the earth, and Mars on December 3, 1999. (b) Find these distances in AU on December 3, 1999: from (i) the sun to the earth; (ii) the sun to Mars; (iii) the earth to Mars. (c) As seen from the earth, what was the angle between the direction to the sun and the direction to Mars on December 3, 1999? (d) Explain whether Mars was visible from your current location at midnight on December 3, 1999. (When it is midnight, the sun is on the opposite side of the earth from you.)

CHALLENGE PROBLEMS

1.90 ••• **Completed Pass.** The football team at Enormous State University (ESU) uses vector displacements to record its plays, with the origin taken to be the position of the ball before the play starts. In a certain pass play, the receiver starts at $+1.0\hat{\imath} - 5.0\hat{\jmath}$, where the units are yards, $\hat{\imath}$ is to the right, and $\hat{\jmath}$ is downfield. Subsequent displacements of the receiver are $+9.0\hat{\imath}$ (he is in motion before the snap), $+11.0\hat{\jmath}$ (breaks downfield), $-6.0\hat{\imath} + 4.0\hat{\jmath}$ (zigs), and $+12.0\hat{\imath} + 18.0\hat{\jmath}$ (zags). Meanwhile, the quarterback has dropped straight back to a position $-7.0\hat{\jmath}$. How far and in which direction must the quarterback throw the ball? (Like the coach, you'll be well advised to diagram the situation before solving this numerically.)

1.91 ••• **Navigating in the Big Dipper.** All of the stars of the Big Dipper (part of the constellation Ursa Major) may appear to be the same distance from the earth, but in fact they are very far from each other. **Figure P1.91** shows the distances from the earth to each of these stars. The distances are given in light-years (ly), the distance that light travels in one year. One light-year equals 9.461×10^{15} m. (a) Alkaid and Merak are 25.6° apart in the earth's sky. In a diagram, show the relative positions of Alkaid, Merak, and our sun. Find the distance in light-years from Alkaid to Merak. (b) To an inhabitant of a planet orbiting Merak, how many degrees apart in the sky would Alkaid and our sun be?

Figure **P1.91**



MCAT-STYLE PASSAGE PROBLEMS

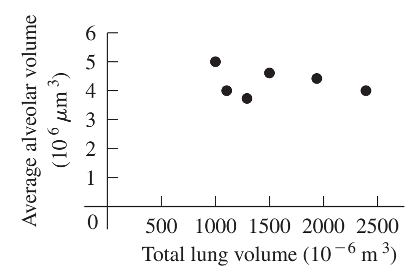
BIO Calculating Lung Volume in Humans. In humans, oxygen and carbon dioxide are exchanged in the blood within many small sacs called alveoli in the lungs. Alveoli provide a large surface area for gas exchange. Recent careful measurements show that the total number of alveoli in a typical pair of lungs is about 480×10^6 and that the average volume of a single alveolus is $4.2 \times 10^6\ \mu m^3$. (The volume of a sphere is $V = \frac{4}{3}\pi r^3$, and the area of a sphere is $A = 4\pi r^2$.)

1.92 What is total volume of the gas-exchanging region of the lungs? (a) 2000 μm^3; (b) 2 m^3; (c) 2.0 L; (d) 120 L.

1.93 If we assume that alveoli are spherical, what is the diameter of a typical alveolus? (a) 0.20 mm; (b) 2 mm; (c) 20 mm; (d) 200 mm.

1.94 Individuals vary considerably in total lung volume. **Figure P1.94** shows the results of measuring the total lung volume and average alveolar volume of six individuals. From these data, what can you infer about the relationship among alveolar size, total lung volume, and number of alveoli per individual? As the total volume of the lungs increases, (a) the number and volume of individual alveoli increase; (b) the number of alveoli increases and the volume of individual alveoli decreases; (c) the volume of the individual alveoli remains constant and the number of alveoli increases; (d) both the number of alveoli and the volume of individual alveoli remain constant.

Figure **P1.94**



ANSWERS

Chapter Opening Question ?

(iii) Take the $+x$-axis to point east and the $+y$-axis to point north. Then we need to find the y-component of the velocity vector, which has magnitude $v = 15$ km/h and is at an angle $\theta = 37°$ measured from the $+x$-axis toward the $+y$-axis. From Eqs. (1.5) we have $v_y = v \sin\theta = (15\text{ km/h})\sin 37° = 9.0$ km/h. So the thunderstorm moves 9.0 km north in 1 h and 18 km north in 2 h.

Key Example VARIATION Problems

VP1.7.1 $D = 12.7$ m, $\theta = -51° = 309°$ (fourth quadrant)
VP1.7.2 $S = 115$ m, $\theta = 42°$ (first quadrant)
VP1.7.3 (a) $T_x = -7.99$ m, $T_y = -7.88$ m (b) $T = 11.2$ m, $\theta = 225°$ (third quadrant)
VP1.7.4 68.7 m, $\theta = 207°$ (third quadrant)
VP1.10.1 (a) $A_x = 4.00$, $A_y = -3.00$, $B_x = -2.19$, $B_y = 6.01$ (b) $\vec{A} \cdot \vec{B} = -26.8$
VP1.10.2 (a) $\vec{C} \cdot \vec{D} = -26.8$ (b) $\phi = 115°$
VP1.10.3 $\phi = 91°$
VP1.10.4 (a) 14.8 N (b) 77.7°

Bridging Problem

(a) 5.2×10^2 N
(b) 4.5×10^2 N·m

? A typical runner gains speed gradually during the course of a sprinting foot race and then slows down after crossing the finish line. In which part of the motion is it accurate to say that the runner is accelerating? (i) During the race; (ii) after the runner crosses the finish line; (iii) both (i) and (ii); (iv) neither (i) nor (ii); (v) answer depends on how rapidly the runner gains speed during the race.

2 Motion Along a Straight Line

LEARNING OUTCOMES

In this chapter, you'll learn...

2.1 How the ideas of displacement and average velocity help us describe straight-line motion.

2.2 The meaning of instantaneous velocity; the difference between velocity and speed.

2.3 How to use average acceleration and instantaneous acceleration to describe changes in velocity.

2.4 How to use equations and graphs to solve problems that involve straight-line motion with constant acceleration.

2.5 How to solve problems in which an object is falling freely under the influence of gravity alone.

2.6 How to analyze straight-line motion when the acceleration is not constant.

You'll need to review...

1.7 The displacement vector.

1.8 Components of a vector.

What distance must an airliner travel down a runway before it reaches takeoff speed? When you throw a baseball straight up in the air, how high does it go? When a glass slips from your hand, how much time do you have to catch it before it hits the floor? These are the kinds of questions you'll learn to answer in this chapter. *Mechanics* is the study of the relationships among force, matter, and motion. In this chapter and the next we'll study *kinematics,* the part of mechanics that enables us to describe motion. Later we'll study *dynamics,* which helps us understand why objects move in different ways.

In this chapter we'll concentrate on the simplest kind of motion: an object moving along a straight line. To describe this motion, we introduce the physical quantities *velocity* and *acceleration*. In physics these quantities have definitions that are more precise and slightly different from the ones used in everyday language. Both velocity and acceleration are *vectors:* As you learned in Chapter 1, this means that they have both magnitude and direction. Our concern in this chapter is with motion along a straight line only, so we won't need the full mathematics of vectors just yet. But using vectors will be essential in Chapter 3 when we consider motion in two or three dimensions.

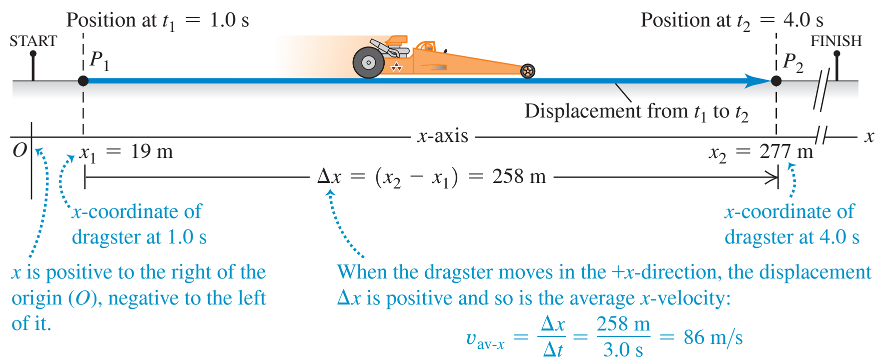
We'll develop simple equations to describe straight-line motion in the important special case when acceleration is constant. An example is the motion of a freely falling object. We'll also consider situations in which acceleration varies during the motion; in this case, it's necessary to use integration to describe the motion. (If you haven't studied integration yet, Section 2.6 is optional.)

2.1 DISPLACEMENT, TIME, AND AVERAGE VELOCITY

Suppose a drag racer drives her dragster along a straight track (**Fig. 2.1**). To study the dragster's motion, we need a coordinate system. We choose the x-axis to lie along the dragster's straight-line path, with the origin O at the starting line. We also choose a point on the dragster, such as its front end, and represent the entire dragster by that point. Hence we treat the dragster as a **particle.**

A useful way to describe the motion of this particle is in terms of the change in its coordinate x over a time interval. Suppose that 1.0 s after the start the front of the dragster is at point P_1, 19 m from the origin, and 4.0 s after the start it is at point P_2, 277 m

Figure **2.1** Positions of a dragster at two times during its run.



from the origin. The *displacement* of the particle is a vector that points from P_1 to P_2 (see Section 1.7). Figure 2.1 shows that this vector points along the x-axis. The x-component (see Section 1.8) of the displacement is the change in the value of x, $(277 \text{ m} - 19 \text{ m}) = 258$ m, that took place during the time interval of $(4.0 \text{ s} - 1.0 \text{ s}) = 3.0$ s. We define the dragster's **average velocity** during this time interval as a *vector* whose x-component is the change in x divided by the time interval: $(258 \text{ m})/(3.0 \text{ s}) = 86 \text{ m/s}$.

In general, the average velocity depends on the particular time interval chosen. For a 3.0 s time interval *before* the start of the race, the dragster is at rest at the starting line and has zero displacement, so its average velocity for this time interval is zero.

Let's generalize the concept of average velocity. At time t_1 the dragster is at point P_1, with coordinate x_1, and at time t_2 it is at point P_2, with coordinate x_2. The displacement of the dragster during the time interval from t_1 to t_2 is the vector from P_1 to P_2. The x-component of the displacement, denoted Δx, is the change in the coordinate x:

$$\Delta x = x_2 - x_1 \tag{2.1}$$

The dragster moves along the x-axis only, so the y- and z-components of the displacement are equal to zero.

The x-component of average velocity, or the **average x-velocity,** is the x-component of displacement, Δx, divided by the time interval Δt during which the displacement occurs. We use the symbol $v_{\text{av-}x}$ for average x-velocity (the subscript "av" signifies average value, and the subscript x indicates that this is the x-component):

Average x-velocity of a particle in **straight-line motion** during time interval from t_1 to t_2

x-component of the particle's displacement

$$v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \tag{2.2}$$

Final x-coordinate minus initial x-coordinate

Time interval

Final time minus initial time

As an example, for the dragster in Fig. 2.1, $x_1 = 19$ m, $x_2 = 277$ m, $t_1 = 1.0$ s, and $t_2 = 4.0$ s. So Eq. (2.2) gives

$$v_{\text{av-}x} = \frac{277 \text{ m} - 19 \text{ m}}{4.0 \text{ s} - 1.0 \text{ s}} = \frac{258 \text{ m}}{3.0 \text{ s}} = 86 \text{ m/s}$$

The average x-velocity of the dragster is positive. This means that during the time interval, the coordinate x increased and the dragster moved in the positive x-direction (to the right in Fig. 2.1).

If a particle moves in the *negative* x-direction during a time interval, its average velocity for that time interval is negative. For example, suppose an official's truck moves to the left along the track (**Fig. 2.2**, next page). The truck is at $x_1 = 277$ m at $t_1 = 16.0$ s and is at $x_2 = 19$ m at $t_2 = 25.0$ s. Then $\Delta x = (19 \text{ m} - 277 \text{ m}) -258$ m and $\Delta t = (25.0 \text{ s} - 16.0 \text{ s}) = 9.0$ s. The x-component of average velocity is $v_{\text{av-}x} = \Delta x/\Delta t = (-258 \text{ m})/(9.0 \text{ s}) = -29$ m/s. **Table 2.1** lists some simple rules for deciding whether the x-velocity is positive or negative.

CAUTION The meaning of Δx Note that Δx is *not* the product of Δ and x; it is a single symbol that means "the change in quantity x." We use the Greek capital letter Δ (delta) to represent a *change* in a quantity, equal to the *final* value of the quantity minus the *initial* value—never the reverse. Likewise, the time interval from t_1 to t_2 is Δt, the change in t: $\Delta t = t_2 - t_1$ (final time minus initial time).

TABLE 2.1 Rules for the Sign of x-Velocity

If x-coordinate is:	. . . x-velocity is:
Positive & increasing (getting more positive)	Positive: Particle is moving in $+x$-direction
Positive & decreasing (getting less positive)	Negative: Particle is moving in $-x$-direction
Negative & increasing (getting less negative)	Positive: Particle is moving in $+x$-direction
Negative & decreasing (getting more negative)	Negative: Particle is moving in $-x$-direction

Note: These rules apply to both the average x-velocity $v_{\text{av-}x}$ and the instantaneous x-velocity v_x (to be discussed in Section 2.2).

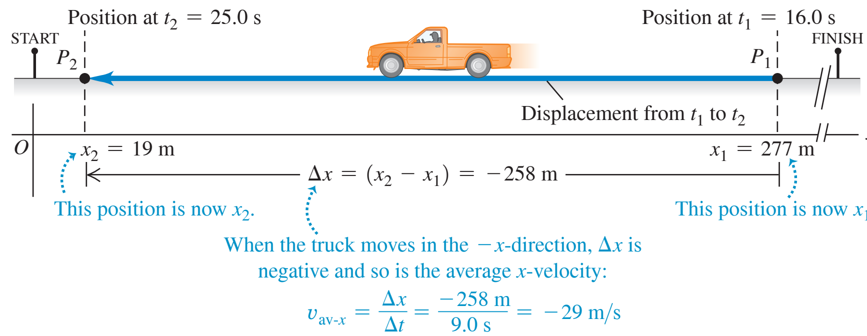


Figure **2.2** Positions of an official's truck at two times during its motion. The points P_1 and P_2 now indicate the positions of the truck, not the dragster, and so are the reverse of Fig. 2.1.

CAUTION The sign of average *x*-velocity In our example positive $v_{av\text{-}x}$ means motion to the right, as in Fig. 2.1, and negative $v_{av\text{-}x}$ means motion to the left, as in Fig. 2.2. But that's *only* because we chose the $+x$-direction to be to the right. Had we chosen the $+x$-direction to be to the left, the average x-velocity $v_{av\text{-}x}$ would have been negative for the dragster moving to the right and positive for the truck moving to the left. In many problems the direction of the coordinate axis is yours to choose. Once you've made your choice, you *must* take it into account when interpreting the signs of $v_{av\text{-}x}$ and other quantities that describe motion!

With straight-line motion we sometimes call Δx simply the displacement and $v_{av\text{-}x}$ simply the average velocity. But remember that these are the x-components of vector quantities that, in this special case, have *only* x-components. In Chapter 3, displacement, velocity, and acceleration vectors will have two or three nonzero components.

Figure 2.3 is a graph of the dragster's position as a function of time—that is, an ***x-t* graph.** The curve in the figure *does not* represent the dragster's path; as Fig. 2.1 shows, the path is a straight line. Rather, the graph represents how the dragster's position changes with time. The points p_1 and p_2 on the graph correspond to the points P_1 and P_2 along the dragster's path. Line p_1p_2 is the hypotenuse of a right triangle with vertical side $\Delta x = x_2 - x_1$ and horizontal side $\Delta t = t_2 - t_1$. The average x-velocity $v_{av\text{-}x} = \Delta x/\Delta t$ of the dragster equals the *slope* of the line p_1p_2—that is, the ratio of the triangle's vertical side Δx to its horizontal side Δt. (The slope has units of meters divided by seconds, or m/s, the correct units for average x-velocity.)

The average x-velocity depends on only the total displacement $\Delta x = x_2 - x_1$ that occurs during the time interval $\Delta t = t_2 - t_1$, not on what happens during the time interval. At time t_1 a motorcycle might have raced past the dragster at point P_1 in Fig. 2.1, then slowed down to pass through point P_2 at the same time t_2 as the dragster. Both vehicles have the same displacement during the same time interval and so have the same average x-velocity.

If distance is given in meters and time in seconds, average velocity is measured in meters per second, or m/s (**Table 2.2**). Other common units of velocity are kilometers per hour (km/h), feet per second (ft/s), miles per hour (mi/h), and knots (1 knot = 1 nautical mile/h = 6080 ft/h).

TABLE 2.2 Typical Velocity Magnitudes

A snail's pace	10^{-3} m/s
A brisk walk	2 m/s
Fastest human	11 m/s
Freeway speeds	30 m/s
Fastest car	341 m/s
Random motion of air molecules	500 m/s
Fastest airplane	1000 m/s
Orbiting communications satellite	3000 m/s
Average speed of an electron in a hydrogen atom	2×10^6 m/s
Light traveling in vacuum	3×10^8 m/s

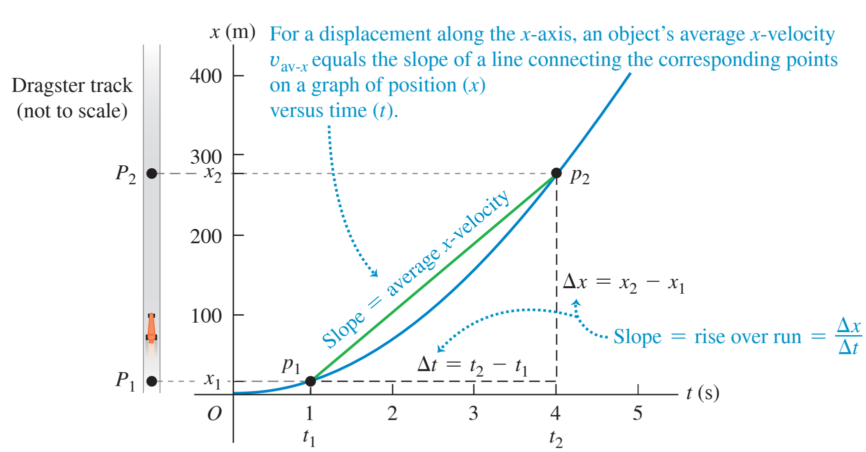


Figure **2.3** A graph of the position of a dragster as a function of time.

TEST YOUR UNDERSTANDING OF SECTION 2.1 Five cars, A, B, C, D, and E, each take a trip that lasts one hour. The positive x-direction is to the east. (i) A travels 50 km due east. (ii) B travels 50 km due west. (iii) C travels 60 km due east, then turns around and travels 10 km due west. (iv) D travels 70 km due east. (v) E travels 20 km due west, then turns around and travels 20 km due east. (a) Rank the five trips in order of average x-velocity from most positive to most negative. (b) Which trips, if any, have the same average x-velocity? (c) For which trip, if any, is the average x-velocity equal to zero?

ANSWERS

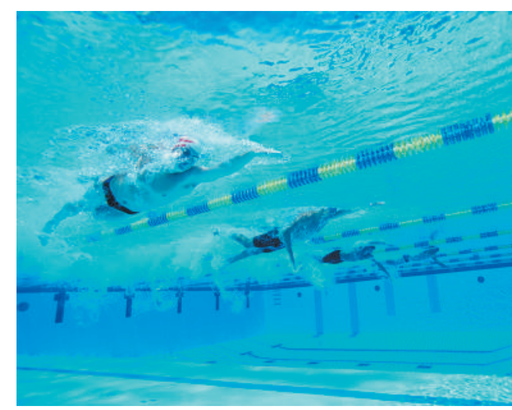
(a): **(iv), (i) and (iii) (tie), (v), (ii)**; (b): **(i) and (iii)**; (c): **(v)** In (a) the average x-velocity is $v_{\text{av-}x} = \Delta x/\Delta t$. For all five trips, $\Delta t = 1$ h. For the individual trips, (i) $\Delta x = +50$ km, $v_{\text{av-}x} = +50$ km/h. (ii) $\Delta x = -50$ km, $v_{\text{av-}x} = -50$ km/h; (iii) $\Delta x = 60$ km $- 10$ km $= +50$ km, $v_{\text{av-}x} = +50$ km/h; (iv) $\Delta x = +70$ km, $v_{\text{av-}x} = +70$ km/h; (v) $\Delta x = -20$ km $+ 20$ km $= 0$, $v_{\text{av-}x} = 0$. In (b) both have $v_{\text{av-}x} = +50$ km/h.

2.2 INSTANTANEOUS VELOCITY

Sometimes average velocity is all you need to know about a particle's motion. For example, a race along a straight line is really a competition to see whose average velocity, $v_{\text{av-}x}$, has the greatest magnitude. The prize goes to the competitor who can travel the displacement Δx from the start to the finish line in the shortest time interval, Δt (**Fig. 2.4**).

But the average velocity of a particle during a time interval can't tell us how fast, or in what direction, the particle was moving at any given time during the interval. For that we need to know the **instantaneous velocity,** or the velocity at a specific instant of time or specific point along the path.

Figure **2.4** The winner of a 50 m swimming race is the swimmer whose average velocity has the greatest magnitude—that is, the swimmer who traverses a displacement Δx of 50 m in the shortest elapsed time Δt.



CAUTION **How long is an instant?** You might use the phrase "It lasted just an instant" to refer to something that spanned a very short time interval. But in physics an instant has no duration at all; it refers to a single value of time.

To find the instantaneous velocity of the dragster in Fig. 2.1 at point P_1, we move point P_2 closer and closer to point P_1 and compute the average velocity $v_{\text{av-}x} = \Delta x/\Delta t$ over the ever-shorter displacement and time interval. Both Δx and Δt become very small, but their ratio does not necessarily become small. In the language of calculus, the limit of $\Delta x/\Delta t$ as Δt approaches zero is called the **derivative** of x with respect to t and is written dx/dt. We use the symbol v_x, with no "av" subscript, for the instantaneous velocity along the x-axis, or the **instantaneous *x*-velocity:**

The **instantaneous** ***x*-velocity** of a particle in **straight-line motion** ...

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.3)$$

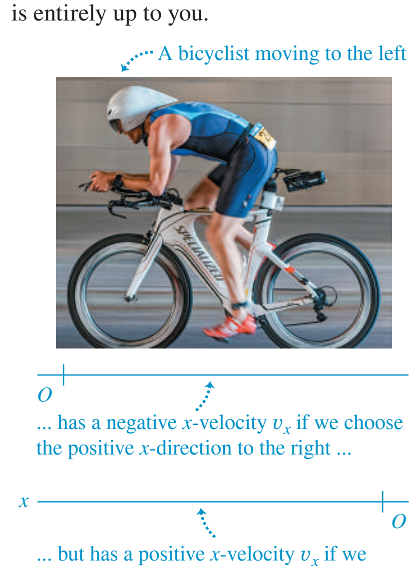
... equals the limit of the particle's average x-velocity as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's x-coordinate.

The time interval Δt is always positive, so v_x has the same algebraic sign as Δx. A positive value of v_x means that x is increasing and the motion is in the positive x-direction; a negative value of v_x means that x is decreasing and the motion is in the negative x-direction. An object can have positive x and negative v_x, or the reverse; x tells us where the object is, while v_x tells us how it's moving (**Fig. 2.5**). The rules that we presented in Table 2.1 (Section 2.1) for the sign of average x-velocity $v_{\text{av-}x}$ also apply to the sign of instantaneous x-velocity v_x.

Instantaneous velocity, like average velocity, is a vector; Eq. (2.3) defines its x-component. In straight-line motion, all other components of instantaneous velocity are zero. In this case we often call v_x simply the instantaneous velocity. (In Chapter 3 we'll deal with the general case in which the instantaneous velocity can have nonzero x-, y-, and z-components.) When we use the term "velocity," we'll always mean instantaneous rather than average velocity.

Figure **2.5** In any problem involving straight-line motion, the choice of which direction is positive and which is negative is entirely up to you.



"Velocity" and "speed" are used interchangeably in everyday language, but they have distinct definitions in physics. We use the term **speed** to denote distance traveled divided by time, on either an average or an instantaneous basis. Instantaneous *speed,* for which we use the symbol v with *no* subscripts, measures how fast a particle is moving; instantaneous *velocity* measures how fast *and* in what direction it's moving. Instantaneous speed is the magnitude of instantaneous velocity and so can never be negative. For example, a particle with instantaneous velocity $v_x = 25$ m/s and a second particle with $v_x = -25$ m/s are moving in opposite directions at the same instantaneous speed 25 m/s.

CAUTION **Average speed and average velocity** Average speed is *not* the magnitude of average velocity. When César Cielo set a world record in 2009 by swimming 100.0 m in 46.91 s, his average speed was (100.0 m)/(46.91 s) = 2.132 m/s. But because he swam two lengths in a 50 m pool, he started and ended at the same point and so had zero total displacement and zero average *velocity!* Both average speed and instantaneous speed are scalars, not vectors, because these quantities contain no information about direction.

EXAMPLE 2.1 Average and instantaneous velocities

A cheetah is crouched 20 m to the east of a vehicle (**Fig. 2.6a**). At time $t = 0$ the cheetah begins to run due east toward an antelope that is 50 m to the east of the vehicle. During the first 2.0 s of the chase, the cheetah's x-coordinate varies with time according to the equation $x = 20\text{ m} + (5.0\text{ m/s}^2)t^2$. (a) Find the cheetah's displacement between $t_1 = 1.0$ s and $t_2 = 2.0$ s. (b) Find its average velocity during that interval. (c) Find its instantaneous velocity at $t_1 = 1.0$ s by taking $\Delta t = 0.1$ s, then 0.01 s, then 0.001 s. (d) Derive an expression for the cheetah's instantaneous velocity as a function of time, and use it to find v_x at $t = 1.0$ s and $t = 2.0$ s.

IDENTIFY and SET UP Figure 2.6b shows our sketch of the cheetah's motion. We use Eq. (2.1) for displacement, Eq. (2.2) for average velocity, and Eq. (2.3) for instantaneous velocity.

EXECUTE (a) At $t_1 = 1.0$ s and $t_2 = 2.0$ s the cheetah's positions x_1 and x_2 are

$$x_1 = 20\text{ m} + (5.0\text{ m/s}^2)(1.0\text{ s})^2 = 25\text{ m}$$
$$x_2 = 20\text{ m} + (5.0\text{ m/s}^2)(2.0\text{ s})^2 = 40\text{ m}$$

The displacement during this 1.0 s interval is

$$\Delta x = x_2 - x_1 = 40\text{ m} - 25\text{ m} = 15\text{ m}$$

(b) The average x-velocity during this interval is

$$v_{\text{av-}x} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{40\text{ m} - 25\text{ m}}{2.0\text{ s} - 1.0\text{ s}} = \frac{15\text{ m}}{1.0\text{ s}} = 15\text{ m/s}$$

(c) With $\Delta t = 0.1$ s the time interval is from $t_1 = 1.0$ s to a new $t_2 = 1.1$ s. At t_2 the position is

$$x_2 = 20\text{ m} + (5.0\text{ m/s}^2)(1.1\text{ s})^2 = 26.05\text{ m}$$

The average x-velocity during this 0.1 s interval is

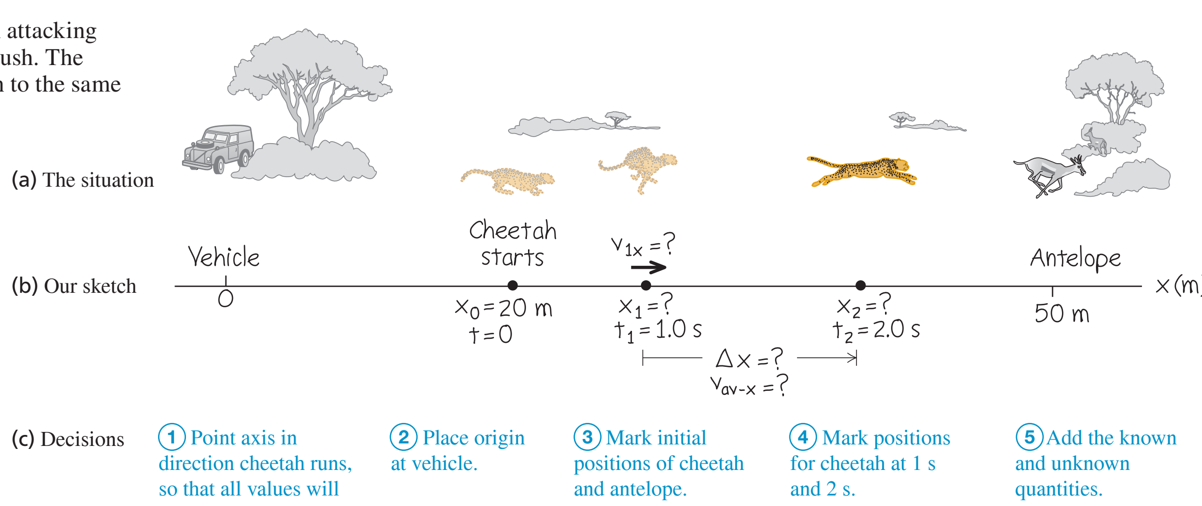
$$v_{\text{av-}x} = \frac{26.05\text{ m} - 25\text{ m}}{1.1\text{ s} - 1.0\text{ s}} = 10.5\text{ m/s}$$

Following this pattern, you can calculate the average x-velocities for 0.01 s and 0.001 s intervals: The results are 10.05 m/s and 10.005 m/s. As Δt gets smaller, the average x-velocity gets closer to 10.0 m/s, so we conclude that the instantaneous x-velocity at $t = 1.0$ s is 10.0 m/s. (We suspended the rules for significant-figure counting in these calculations.)

(d) From Eq. (2.3) the instantaneous x-velocity is $v_x = dx/dt$. The derivative of a constant is zero and the derivative of t^2 is $2t$, so

$$v_x = \frac{dx}{dt} = \frac{d}{dt}[20\text{ m} + (5.0\text{ m/s}^2)t^2]$$
$$= 0 + (5.0\text{ m/s}^2)(2t) = (10\text{ m/s}^2)t$$

Figure **2.6** A cheetah attacking an antelope from ambush. The animals are not drawn to the same scale as the axis.



At $t = 1.0$ s, this yields $v_x = 10$ m/s, as we found in part (c); at $t = 2.0$ s, $v_x = 20$ m/s.

EVALUATE Our results show that the cheetah picked up speed from $t = 0$ (when it was at rest) to $t = 1.0$ s $(v_x = 10\text{ m/s})$ to $t = 2.0$ s $(v_x = 20\text{ m/s})$. This makes sense; the cheetah covered only 5 m during the interval $t = 0$ to $t = 1.0$ s, but it covered 15 m during the interval $t = 1.0$ s to $t = 2.0$ s.

KEYCONCEPT To calculate the average velocity of an object in straight-line motion, first find its displacement (final coordinate minus initial coordinate) during a time interval. Then divide by the time interval. To calculate the object's instantaneous velocity (its average velocity over an infinitesimally short time interval), take the derivative of its position with respect to time.

Finding Velocity on an *x-t* Graph

We can also find the x-velocity of a particle from the graph of its position as a function of time. Suppose we want to find the x-velocity of the dragster in Fig. 2.1 at point P_1. As point P_2 in Fig. 2.1 approaches point P_1, point p_2 in the x-t graphs of **Figs. 2.7a** and 2.7b approaches point p_1 and the average x-velocity is calculated over shorter time intervals Δt. In the limit that $\Delta t \rightarrow 0$, shown in Fig. 2.7c, the slope of the line p_1p_2 equals the slope of the line tangent to the curve at point p_1. Thus, *on a graph of position as a function of time for straight-line motion, the instantaneous x-velocity at any point is equal to the slope of the tangent to the curve at that point.*

If the tangent to the x-t curve slopes upward to the right, as in Fig. 2.7c, then its slope is positive, the x-velocity is positive, and the motion is in the positive x-direction. If the tangent slopes downward to the right, the slopes of the x-t graph and the x-velocity are negative, and the motion is in the negative x-direction. When the tangent is horizontal, the slope and the x-velocity are zero. **Figure 2.8** illustrates these three possibilities.

Figure 2.7 Using an x-t graph to go from (a), (b) average x-velocity to (c) instantaneous x-velocity v_x. In (c) we find the slope of the tangent to the x-t curve by dividing any vertical interval (with distance units) along the tangent by the corresponding horizontal interval (with time units).

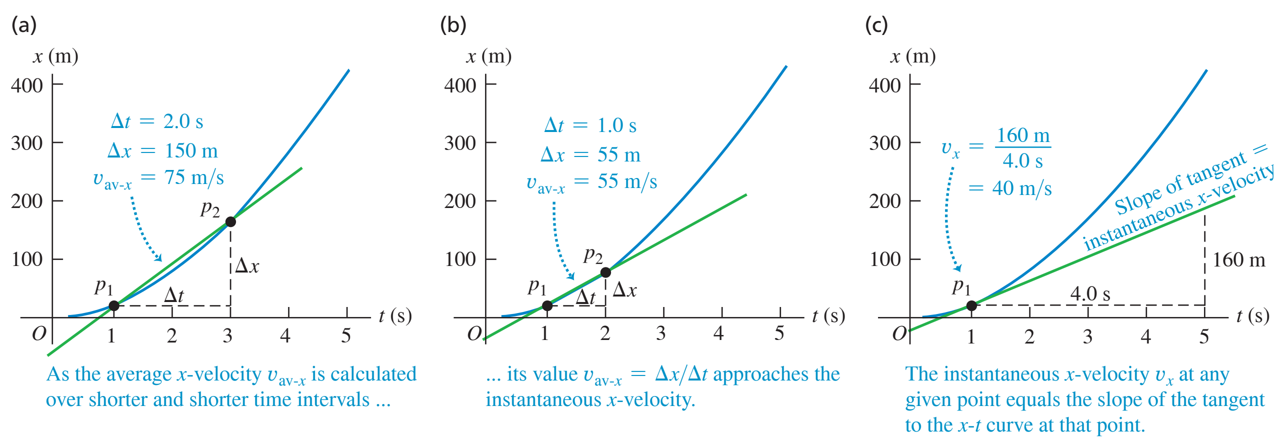


Figure 2.8 (a) The x-t graph of the motion of a particular particle. (b) A motion diagram showing the position and velocity of the particle at each of the times labeled on the x-t graph.

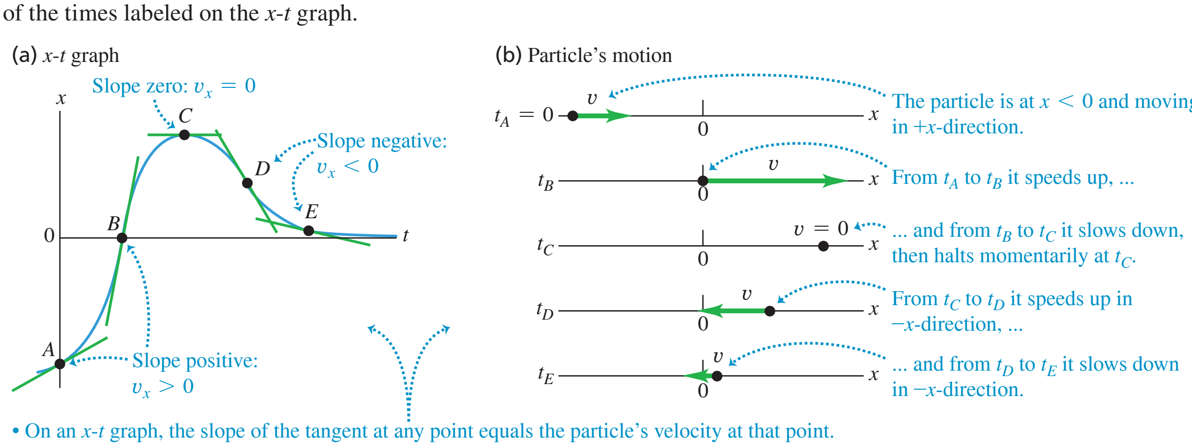
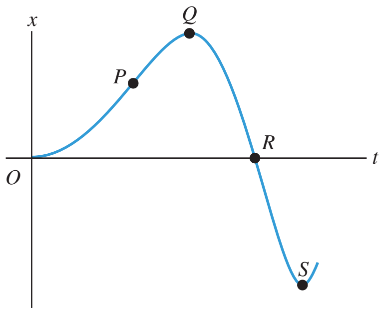


Figure 2.8 depicts the motion of a particle in two ways: as (a) an x-t graph and (b) a **motion diagram** that shows the particle's position at various instants (like frames from a video of the particle's motion) as well as arrows to represent the particle's velocity at each instant. We'll use both x-t graphs and motion diagrams in this chapter to represent motion. You'll find it helpful to draw *both* an x-t graph and a motion diagram when you solve any problem involving motion.

Figure **2.9** An x-t graph for a particle.



TEST YOUR UNDERSTANDING OF SECTION 2.2 **Figure 2.9** is an x-t graph of the motion of a particle. (a) Rank the values of the particle's x-velocity v_x at points P, Q, R, and S from most positive to most negative. (b) At which points is v_x positive? (c) At which points is v_x negative? (d) At which points is v_x zero? (e) Rank the values of the particle's *speed* at points P, Q, R, and S from fastest to slowest.

ANSWER

(a) ***P*, *Q*, and *S* (tie), *R*** The x-velocity is (b) positive when the slope of the x-t graph is positive (***P***), (c) negative when the slope is negative (***R***), and (d) zero when the slope is zero (***Q* and *S***). (e) ***R*, *P*, *Q*, and *S* (tie)** The speed is greatest when the slope of the x-t graph is steepest (either positive or negative) and zero when the slope is zero.

2.3 AVERAGE AND INSTANTANEOUS ACCELERATION

Just as velocity describes the rate of change of position with time, *acceleration* describes the rate of change of velocity with time. Like velocity, acceleration is a vector quantity. When the motion is along a straight line, its only nonzero component is along that line. In everyday language, acceleration refers only to speeding up; in physics, acceleration refers to *any* kind of velocity change, so we say an object accelerates if it is either speeding up or slowing down.

Average Acceleration

Let's consider again a particle moving along the x-axis. Suppose that at time t_1 the particle is at point P_1 and has x-component of (instantaneous) velocity v_{1x}, and at a later time t_2 it is at point P_2 and has x-component of velocity v_{2x}. So the x-component of velocity changes by an amount $\Delta v_x = v_{2x} - v_{1x}$ during the time interval $\Delta t = t_2 - t_1$. As the particle moves from P_1 to P_2, its **average acceleration** is a vector quantity whose x-component $a_{\text{av-}x}$ (called the **average *x*-acceleration**) equals Δv_x, the change in the x-component of velocity, divided by the time interval Δt:

Average *x*-acceleration of a particle in **straight-line motion** during time interval from t_1 to t_2

Change in x-component of the particle's velocity

$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \tag{2.4}$$

Final x-velocity minus initial x-velocity

Time interval

Final time minus initial time

For straight-line motion along the x-axis we'll often call $a_{\text{av-}x}$ simply the average acceleration. (We'll encounter the other components of the average acceleration vector in Chapter 3.)

If we express velocity in meters per second and time in seconds, then average acceleration is in meters per second per second. This is usually written as m/s^2 and is read "meters per second squared."

CAUTION **Don't confuse velocity and acceleration** Velocity describes how an object's position changes with time; it tells us how fast and in what direction the object moves. Acceleration describes how the velocity changes with time; it tells us how the speed and direction of motion change. Another difference is that you can *feel* acceleration but you can't feel velocity. If you're a passenger in a car that accelerates forward and gains speed, you feel pushed backward in your seat; if it accelerates backward and loses speed, you feel pushed forward. If the velocity is constant and there's no acceleration, you feel neither sensation. (We'll explain these sensations in Chapter 4.)

EXAMPLE 2.2 Average acceleration

An astronaut has left an orbiting spacecraft to test a new personal maneuvering unit. As she moves along a straight line, her partner on the spacecraft measures her velocity every 2.0 s, starting at time $t = 1.0$ s:

t	v_x	t	v_x
1.0 s	0.8 m/s	9.0 s	−0.4 m/s
3.0 s	1.2 m/s	11.0 s	−1.0 m/s
5.0 s	1.6 m/s	13.0 s	−1.6 m/s
7.0 s	1.2 m/s	15.0 s	−0.8 m/s

Find the average x-acceleration, and state whether the speed of the astronaut increases or decreases over each of these 2.0 s time intervals: (a) $t_1 = 1.0$ s to $t_2 = 3.0$ s; (b) $t_1 = 5.0$ s to $t_2 = 7.0$ s; (c) $t_1 = 9.0$ s to $t_2 = 11.0$ s; (d) $t_1 = 13.0$ s to $t_2 = 15.0$ s.

IDENTIFY and SET UP We'll use Eq. (2.4) to determine the average acceleration $a_{\text{av-}x}$ from the change in velocity over each time interval. To find the changes in speed, we'll use the idea that speed v is the magnitude of the instantaneous velocity v_x.

The upper part of **Fig. 2.10** is our graph of the x-velocity as a function of time. On this v_x-t graph, the slope of the line connecting the endpoints of each interval is the average x-acceleration $a_{\text{av-}x} = \Delta v_x/\Delta t$ for that interval. The four slopes (and thus the *signs* of the average accelerations) are, from left to right, positive, negative, negative, and positive. The third and fourth slopes (and thus the average accelerations themselves) have greater magnitude than the first and second.

EXECUTE Using Eq. (2.4), we find:

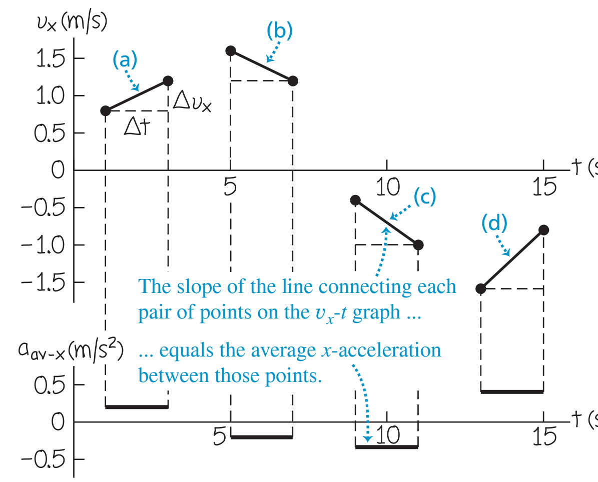
(a) $a_{\text{av-}x} = (1.2\ \text{m/s} - 0.8\ \text{m/s})/(3.0\ \text{s} - 1.0\ \text{s}) = 0.2\ \text{m/s}^2$. The speed (magnitude of instantaneous x-velocity) increases from 0.8 m/s to 1.2 m/s.

(b) $a_{\text{av-}x} = (1.2\ \text{m/s} - 1.6\ \text{m/s})/(7.0\ \text{s} - 5.0\ \text{s}) = -0.2\ \text{m/s}^2$. The speed decreases from 1.6 m/s to 1.2 m/s.

(c) $a_{\text{av-}x} = [-1.0\ \text{m/s} - (-0.4\ \text{m/s})]/(11.0\ \text{s} - 9.0\ \text{s}) = -0.3\ \text{m/s}^2$. The speed increases from 0.4 m/s to 1.0 m/s.

(d) $a_{\text{av-}x} = [-0.8\ \text{m/s} - (-1.6\ \text{m/s})]/(15.0\ \text{s} - 13.0\ \text{s}) = 0.4\ \text{m/s}^2$. The speed decreases from 1.6 m/s to 0.8 m/s.

Figure **2.10** Our graphs of x-velocity versus time (top) and average x-acceleration versus time (bottom) for the astronaut.



In the lower part of Fig. 2.10, we graph the values of $a_{\text{av-}x}$.

EVALUATE The signs and relative magnitudes of the average accelerations agree with our qualitative predictions.

Notice that when the average x-acceleration has the *same* algebraic sign as the initial velocity, as in intervals (a) and (c), the astronaut goes faster. When $a_{\text{av-}x}$ has the *opposite* algebraic sign from the initial velocity, as in intervals (b) and (d), she slows down. Thus positive x-acceleration means speeding up if the x-velocity is positive [interval (a)] but slowing down if the x-velocity is negative [interval (d)]. Similarly, negative x-acceleration means speeding up if the x-velocity is negative [interval (c)] but slowing down if the x-velocity is positive [interval (b)].

KEYCONCEPT To calculate the average acceleration of an object in straight-line motion, first find the change in its velocity (final velocity minus initial velocity) during a time interval. Then divide by the time interval.

Instantaneous Acceleration

We can now define **instantaneous acceleration** by following the same procedure that we used to define instantaneous velocity. Suppose a race car driver is driving along a straightaway as shown in **Fig. 2.11**. To define the instantaneous acceleration at point P_1, we take point P_2 in Fig. 2.11 to be closer and closer to P_1 so that the average acceleration is computed over shorter and shorter time intervals. Thus

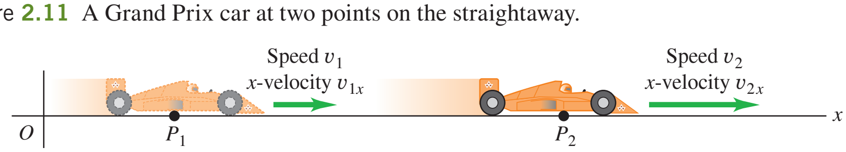
The **instantaneous** **x-acceleration** of a particle in **straight-line motion** ...

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad (2.5)$$

... equals the limit of the particle's average x-acceleration as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's x-velocity.

Figure **2.11** A Grand Prix car at two points on the straightaway.



In Eq. (2.5) a_x is the x-component of the acceleration vector, which we call the **instantaneous x-acceleration;** in straight-line motion, all other components of this vector are zero. From now on, when we use the term "acceleration," we'll always mean instantaneous acceleration, not average acceleration.

EXAMPLE 2.3 Average and instantaneous accelerations

Suppose the x-velocity v_x of the car in Fig. 2.11 at any time t is given by the equation

$$v_x = 60\text{ m/s} + (0.50\text{ m/s}^3)t^2$$

(a) Find the change in x-velocity of the car in the time interval $t_1 = 1.0$ s to $t_2 = 3.0$ s. (b) Find the average x-acceleration in this time interval. (c) Find the instantaneous x-acceleration at time $t_1 = 1.0$ s by taking Δt to be first 0.1 s, then 0.01 s, then 0.001 s. (d) Derive an expression for the instantaneous x-acceleration as a function of time, and use it to find a_x at $t = 1.0$ s and $t = 3.0$ s.

IDENTIFY and SET UP This example is analogous to Example 2.1 in Section 2.2. In that example we found the average x-velocity from the change in position over shorter and shorter time intervals, and we obtained an expression for the instantaneous x-velocity by differentiating the position as a function of time. In this example we have an exact parallel. Using Eq. (2.4), we'll find the average *x-acceleration* from the change in *x-velocity* over a time interval. Likewise, using Eq. (2.5), we'll obtain an expression for the instantaneous *x-acceleration* by differentiating the *x-velocity* as a function of time.

EXECUTE (a) Before we can apply Eq. (2.4), we must find the x-velocity at each time from the given equation. At $t_1 = 1.0$ s and $t_2 = 3.0$ s, the velocities are

$$v_{1x} = 60\text{ m/s} + (0.50\text{ m/s}^3)(1.0\text{ s})^2 = 60.5\text{ m/s}$$
$$v_{2x} = 60\text{ m/s} + (0.50\text{ m/s}^3)(3.0\text{ s})^2 = 64.5\text{ m/s}$$

The change in x-velocity Δv_x between $t_1 = 1.0$ s and $t_2 = 3.0$ s is

$$\Delta v_x = v_{2x} - v_{1x} = 64.5\text{ m/s} - 60.5\text{ m/s} = 4.0\text{ m/s}$$

(b) The average x-acceleration during this time interval of duration $t_2 - t_1 = 2.0$ s is

$$a_{\text{av-}x} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} = \frac{4.0\text{ m/s}}{2.0\text{ s}} = 2.0\text{ m/s}^2$$

During this time interval the x-velocity and average x-acceleration have the same algebraic sign (in this case, positive), and the car speeds up.

(c) When $\Delta t = 0.1$ s, we have $t_2 = 1.1$ s. Proceeding as before, we find

$$v_{2x} = 60\text{ m/s} + (0.50\text{ m/s}^3)(1.1\text{ s})^2 = 60.605\text{ m/s}$$
$$\Delta v_x = 0.105\text{ m/s}$$
$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{0.105\text{ m/s}}{0.1\text{ s}} = 1.05\text{ m/s}^2$$

You should follow this pattern to calculate $a_{\text{av-}x}$ for $\Delta t = 0.01$ s and $\Delta t = 0.001$ s; the results are $a_{\text{av-}x} = 1.005\text{ m/s}^2$ and $a_{\text{av-}x} = 1.0005\text{ m/s}^2$, respectively. As Δt gets smaller, the average x-acceleration gets closer to 1.0 m/s^2, so the instantaneous x-acceleration at $t = 1.0$ s is 1.0 m/s^2.

(d) By Eq. (2.5) the instantaneous x-acceleration is $a_x = dv_x/dt$. The derivative of a constant is zero and the derivative of t^2 is $2t$, so

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}[60\text{ m/s} + (0.50\text{ m/s}^3)t^2 = (0.50\text{ m/s}^3)(2t) = (1.0\text{ m/s}^3)t$$

When $t = 1.0$ s,

$$a_x = (1.0\text{ m/s}^3)(1.0\text{ s}) = 1.0\text{ m/s}^2$$

When $t = 3.0$ s,

$$a_x = (1.0\text{ m/s}^3)(3.0\text{ s}) = 3.0\text{ m/s}^2$$

EVALUATE Neither of the values we found in part (d) is equal to the average x-acceleration found in part (b). That's because the car's instantaneous x-acceleration varies with time. The rate of change of acceleration with time is sometimes called the "jerk."

KEYCONCEPT To calculate an object's instantaneous acceleration (its average acceleration over an infinitesimally short time interval), take the derivative of its velocity with respect to time.

Finding Acceleration on a v_x-t Graph or an x-t Graph

In Section 2.2 we interpreted average and instantaneous x-velocity in terms of the slope of a graph of position versus time. In the same way, we can interpret average and instantaneous x-acceleration by using a graph of instantaneous velocity v_x versus time t—that is, a **v_x-t graph** (**Fig. 2.12**). Points p_1 and p_2 on the graph correspond to points P_1 and P_2 in Fig. 2.11. The average x-acceleration $a_{\text{av-}x} = \Delta v_x/\Delta t$ during this interval is the slope of the line p_1p_2.

As point P_2 in Fig. 2.11 approaches point P_1, point p_2 in the v_x-t graph of Fig. 2.12 approaches point p_1, and the slope of the line p_1p_2 approaches the slope of the line tangent to the curve at point p_1. Thus, *on a graph of x-velocity as a function of time, the instantaneous x-acceleration at any point is equal to the slope of the tangent to the curve at that point.* Tangents drawn at different points along the curve in Fig. 2.12 have different slopes, so the instantaneous x-acceleration varies with time.

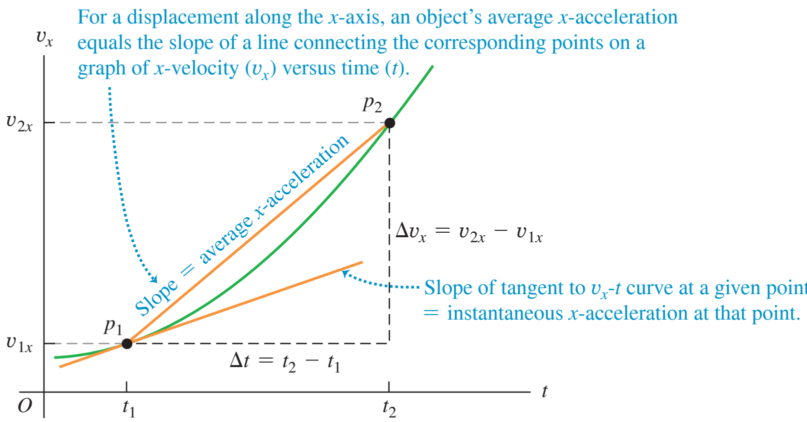


Figure **2.12** A v_x-t graph of the motion in Fig. 2.11.

CAUTION **Signs of *x*-acceleration and *x*-velocity** The algebraic sign of the *x*-acceleration does *not* tell you whether an object is speeding up or slowing down. You must compare the signs of the *x*-velocity and the *x*-acceleration. As we saw in Example 2.2, when v_x and a_x have the *same* sign, the object is speeding up. When v_x and a_x have *opposite* signs, the object is slowing down. **Table 2.3** summarizes these rules, and **Fig. 2.13** illustrates some of them.

The term "deceleration" is sometimes used for a decrease in speed. Because it may mean positive or negative a_x, depending on the sign of v_x, we avoid this term.

We can also learn about the acceleration of an object from a graph of its *position* versus time. Because $a_x = dv_x/dt$ and $v_x = dx/dt$, we can write

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \tag{2.6}$$

That is, a_x is the second derivative of x with respect to t. The second derivative of any function is directly related to the *concavity* or *curvature* of the graph of that function (**Fig. 2.14**, next page). At a point where the x-t graph is concave up (curved upward), such as point A or E in Fig. 2.14a, the *x*-acceleration is positive and v_x is increasing. At a point where the x-t graph is concave down (curved downward), such as point C in Fig. 2.14a, the *x*-acceleration is negative and v_x is decreasing. At a point where the x-t graph has no curvature, such as the inflection points B and D in Fig. 2.14a, the *x*-acceleration is zero and the velocity is not changing.

Examining the curvature of an x-t graph is an easy way to identify the *sign* of acceleration. This technique is less helpful for determining numerical values of acceleration because the curvature of a graph is hard to measure accurately.

TABLE 2.3 Rules for the Sign of *x*-Acceleration

If *x*-velocity is:	... *x*-acceleration is:
Positive & increasing (getting more positive)	Positive: Particle is moving in +*x*-direction & speeding up
Positive & decreasing (getting less positive)	Negative: Particle is moving in +*x*-direction & slowing down
Negative & increasing (getting less negative)	Positive: Particle is moving in −*x*-direction & slowing down
Negative & decreasing (getting more negative)	Negative: Particle is moving in −*x*-direction & speeding up

Note: These rules apply to both the average *x*-acceleration $a_{\text{av-}x}$ and the instantaneous *x*-acceleration a_x.

Figure **2.13** (a) The v_x-t graph of the motion of a different particle from that shown in Fig. 2.8. (b) A motion diagram showing the position, velocity, and acceleration of the particle at each of the times labeled on the v_x-t graph.

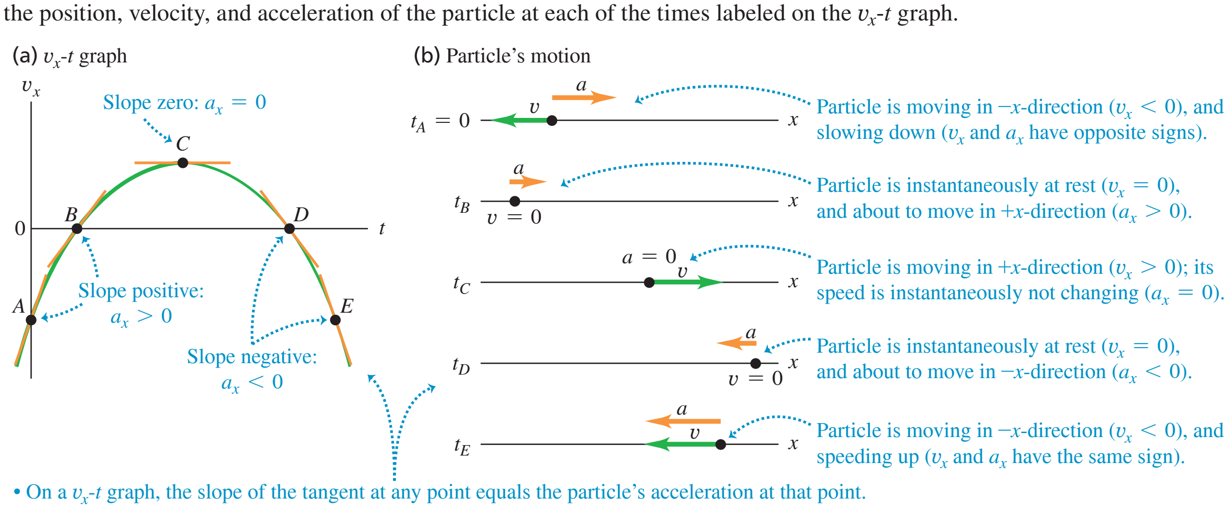


Figure **2.14** (a) The same x-t graph as shown in Fig. 2.8a. (b) A motion diagram showing the position, velocity, and acceleration of the particle at each of the times labeled on the x-t graph.

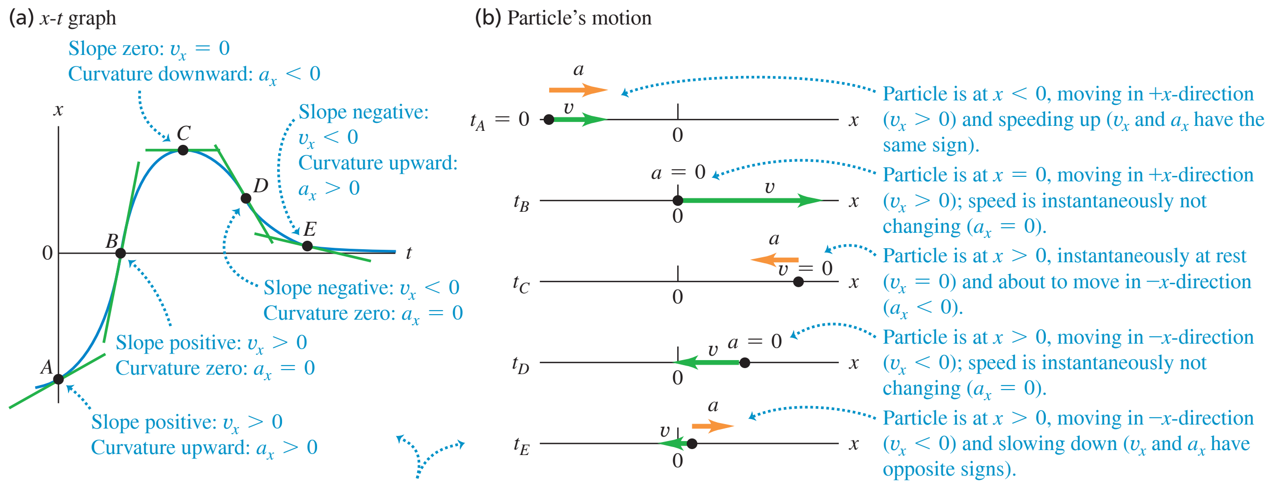


Table 2.4 summarizes what you can learn from the x-t graph and v_x-t graph of the straight-line motion of a particle.

TABLE 2.4 What x-t Graphs and v_x-t Graphs Tell You

	On an x-t graph	**On a v_x-t graph**
The *value* of the graph at a given time tells you ...	The coordinate x at that time	The velocity v_x at that time
The *slope* of the graph at a given time tells you ...	The velocity v_x at that time	The acceleration a_x at that time
The *concavity* or *curvature* of the graph at a given time tells you ...	The acceleration a_x at that time	If the acceleration a_x is changing at that time

TEST YOUR UNDERSTANDING OF SECTION 2.3 Look again at the x-t graph in Fig. 2.9 at the end of Section 2.2. (a) At which of the points P, Q, R, and S is the x-acceleration a_x positive? (b) At which points is the x-acceleration negative? (c) At which points does the x-acceleration appear to be zero? (d) At each point state whether the velocity is increasing, decreasing, or not changing.

ANSWER

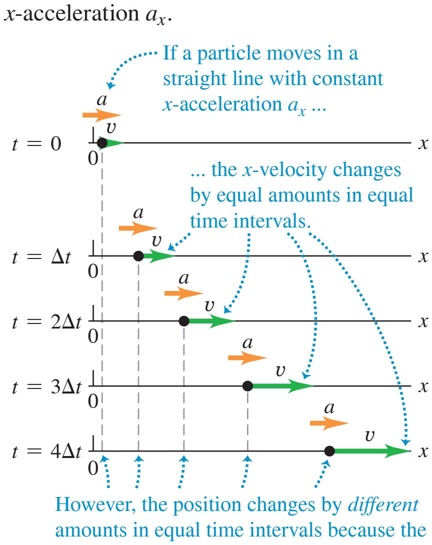
(a) ***S***, where the x-t graph is curved upward (concave up). (b) ***Q***, where the x-t graph is curved downward (concave down). (c) ***P* and *R***, where the x-t graph is not curved either up or down. (d) At P, $a_x = 0$ (velocity is **not changing**); at Q, $a_x < 0$ (velocity is **decreasing**, i.e., changing from positive to zero to negative); at R, $a_x = 0$ (velocity is **not changing**); and at S, $a_x > 0$ (velocity is **increasing**, i.e., changing from negative to zero to positive).

2.4 MOTION WITH CONSTANT ACCELERATION

The simplest kind of accelerated motion is straight-line motion with *constant* acceleration. In this case the velocity changes at the same rate throughout the motion. As an example, a falling object has a constant acceleration if the effects of the air are not important. The same is true for an object sliding on an incline or along a rough horizontal surface, or for an airplane being catapulted from the deck of an aircraft carrier.

Figure 2.15 is a motion diagram showing the position, velocity, and acceleration of a particle moving with constant acceleration. **Figures 2.16** and **2.17** depict this same motion in the form of graphs. Since the x-acceleration is constant, the **a_x-t graph** (graph of x-acceleration versus time) in Fig. 2.16 is a horizontal line. The graph of x-velocity versus time, or v_x-t graph, has a constant *slope* because the acceleration is constant, so this graph is a straight line (Fig. 2.17).

Figure **2.15** A motion diagram for a particle moving in a straight line in the positive x-direction with constant positive x-acceleration a_x.



When the x-acceleration a_x is constant, the average x-acceleration $a_{\text{av-}x}$ for any time interval is the same as a_x. This makes it easy to derive equations for the position x and the x-velocity v_x as functions of time. To find an equation for v_x, we first replace $a_{\text{av-}x}$ in Eq. (2.4) by a_x:

$$a_x = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \tag{2.7}$$

Now we let $t_1 = 0$ and let t_2 be any later time t. We use the symbol v_{0x} for the *initial x-velocity* at time $t = 0$; the x-velocity at the later time t is v_x. Then Eq. (2.7) becomes

$$a_x = \frac{v_x - v_{0x}}{t - 0} \qquad \text{or}$$

x-velocity at time t of a particle with **constant x-acceleration**

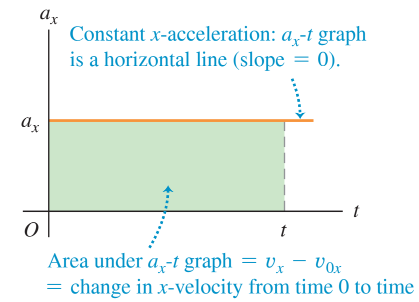
x-velocity of the particle at time 0

$$v_x = v_{0x} + a_x t \tag{2.8}$$

Constant x-acceleration of the particle

Time

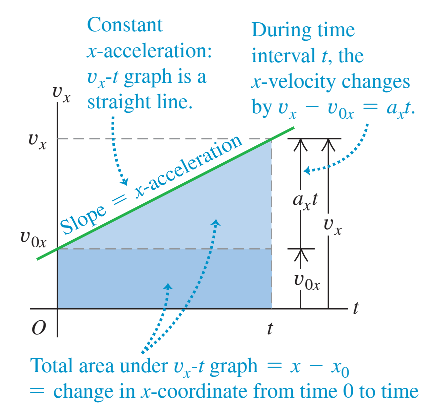
Figure **2.16** An acceleration-time (a_x-t) graph of straight-line motion with constant positive x-acceleration a_x.



In Eq. (2.8) the term $a_x t$ is the product of the constant rate of change of x-velocity, a_x, and the time interval t. Therefore it equals the *total* change in x-velocity from $t = 0$ to time t. The x-velocity v_x at any time t then equals the initial x-velocity v_{0x} (at $t = 0$) plus the change in x-velocity $a_x t$ (Fig. 2.17).

Equation (2.8) also says that the change in x-velocity $v_x - v_{0x}$ of the particle between $t = 0$ and any later time t equals the *area* under the a_x-t graph between those two times. You can verify this from Fig. 2.16: Under this graph is a rectangle of vertical side a_x, horizontal side t, and area $a_x t$. From Eq. (2.8) the area $a_x t$ is indeed equal to the change in velocity $v_x - v_{0x}$. In Section 2.6 we'll show that even if the x-acceleration is not constant, the change in x-velocity during a time interval is still equal to the area under the a_x-t curve, although then Eq. (2.8) does not apply.

Figure **2.17** A velocity-time (v_x-t) graph of straight-line motion with constant positive x-acceleration a_x. The initial x-velocity v_{0x} is also positive in this case.



Next we'll derive an equation for the position x as a function of time when the x-acceleration is constant. To do this, we use two different expressions for the average x-velocity $v_{\text{av-}x}$ during the interval from $t = 0$ to any later time t. The first expression comes from the definition of $v_{\text{av-}x}$, Eq. (2.2), which is true whether or not the acceleration is constant. The position at time $t = 0$, called the *initial position*, is x_0. The position at time t is simply x. Thus for the time interval $\Delta t = t - 0$ the displacement is $\Delta x = x - x_0$, and Eq. (2.2) gives

$$v_{\text{av-}x} = \frac{x - x_0}{t} \tag{2.9}$$

To find a second expression for $v_{\text{av-}x}$, note that the x-velocity changes at a constant rate if the x-acceleration is constant. In this case the average x-velocity for the time interval from 0 to t is simply the average of the x-velocities at the beginning and end of the interval:

$$v_{\text{av-}x} = \tfrac{1}{2}(v_{0x} + v_x) \quad \text{(constant } x\text{-acceleration only)} \tag{2.10}$$

[Equation (2.10) is *not* true if the x-acceleration varies during the time interval.] We also know that with constant x-acceleration, the x-velocity v_x at any time t is given by Eq. (2.8). Substituting that expression for v_x into Eq. (2.10), we find

$$\begin{aligned} v_{\text{av-}x} &= \tfrac{1}{2}(v_{0x} + v_{0x} + a_x t) \\ &= v_{0x} + \tfrac{1}{2}a_x t \end{aligned} \quad \text{(constant } x\text{-acceleration only)} \tag{2.11}$$

Finally, we set Eqs. (2.9) and (2.11) equal to each other and simplify:

$$v_{0x} + \tfrac{1}{2}a_x t = \frac{x - x_0}{t} \qquad \text{or}$$

BIO APPLICATION Testing Humans at High Accelerations In experiments carried out by the U.S. Air Force in the 1940s and 1950s, humans riding a rocket sled could withstand accelerations as great as 440 m/s^2. The first three photos in this sequence show Air Force physician John Stapp speeding up from rest to 188 m/s (678 km/h = 421 mi/h) in just 5 s. Photos 4–6 show the even greater magnitude of acceleration as the rocket sled braked to a halt.

Position of the particle at time 0

Time

Position at time t of a particle with **constant** ***x*-acceleration**

$$x = x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \qquad (2.12)$$

x-velocity of the particle at time 0

Constant x-acceleration of the particle

Equation (2.12) tells us that the particle's position at time t is the sum of three terms: its initial position at $t = 0$, x_0, plus the displacement $v_{0x}t$ it would have if its x-velocity remained equal to its initial value, plus an additional displacement $\frac{1}{2}a_x t^2$ caused by the change in x-velocity.

A graph of Eq. (2.12)—that is, an x-t graph for motion with constant x-acceleration **(Fig. 2.18a)**—is always a *parabola*. Figure 2.18b shows such a graph. The curve intercepts the vertical axis (x-axis) at x_0, the position at $t = 0$. The slope of the tangent at $t = 0$ equals v_{0x}, the initial x-velocity, and the slope of the tangent at any time t equals the x-velocity v_x at that time. The slope and x-velocity are continuously increasing, so the x-acceleration a_x is positive and the graph in Fig. 2.18b is concave up (it curves upward). If a_x is negative, the x-t graph is a parabola that is concave down (has a downward curvature).

If there is zero x-acceleration, the x-t graph is a straight line; if there is a constant x-acceleration, the additional $\frac{1}{2}a_x t^2$ term in Eq. (2.12) for x as a function of t curves the graph into a parabola **(Fig. 2.19a)**. Similarly, if there is zero x-acceleration, the v_x-t graph is a horizontal line (the x-velocity is constant). Adding a constant x-acceleration in Eq. (2.8) gives a slope to the graph (Fig. 2.19b).

Here's another way to derive Eq. (2.12). Just as the change in x-velocity of the particle equals the area under the a_x-t graph, the displacement (change in position) equals the area under the v_x-t graph. So the displacement $x - x_0$ of the particle between $t = 0$ and any later time t equals the area under the v_x-t graph between those times. In Fig. 2.17 we divide the area under the graph into a dark-colored rectangle (vertical side v_{0x}, horizontal side t, and area $v_{0x}t$) and a light-colored right triangle (vertical side $a_x t$, horizontal side t, and area $\frac{1}{2}(a_x t)(t) = \frac{1}{2}a_x t^2$). The total area under the v_x-t graph is $x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$, in accord with Eq. (2.12).

It's often useful to have a relationship for position, x-velocity, and (constant) x-acceleration that does not involve time. To obtain this, we first solve Eq. (2.8) for t and then substitute the resulting expression into Eq. (2.12):

$$t = \frac{v_x - v_{0x}}{a_x}$$

$$x = x_0 + v_{0x}\left(\frac{v_x - v_{0x}}{a_x}\right) + \frac{1}{2}a_x\left(\frac{v_x - v_{0x}}{a_x}\right)^2$$

We transfer the term x_0 to the left side, multiply through by $2a_x$, and simplify:

$$2a_x(x - x_0) = 2v_{0x}v_x - 2v_{0x}{}^2 + v_x^2 - 2v_{0x}v_x + v_{0x}{}^2$$

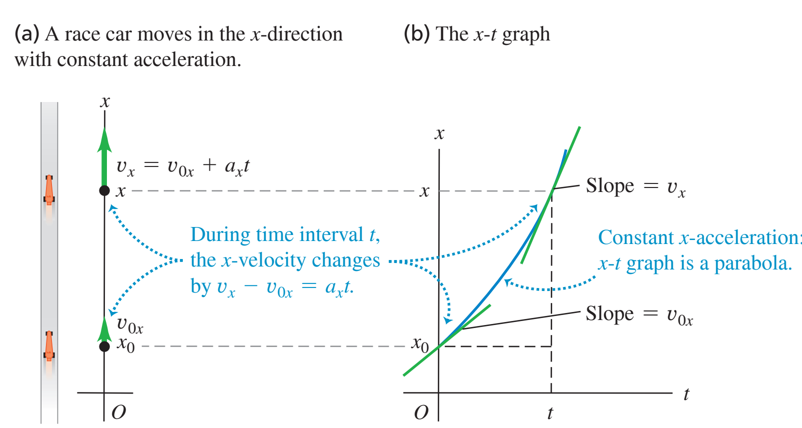


Figure **2.18** (a) Straight-line motion with constant acceleration. (b) A position-time (x-t) graph for this motion (the same motion as is shown in Figs. 2.15, 2.16, and 2.17). For this motion the initial position x_0, the initial velocity v_{0x}, and the acceleration a_x are all positive.

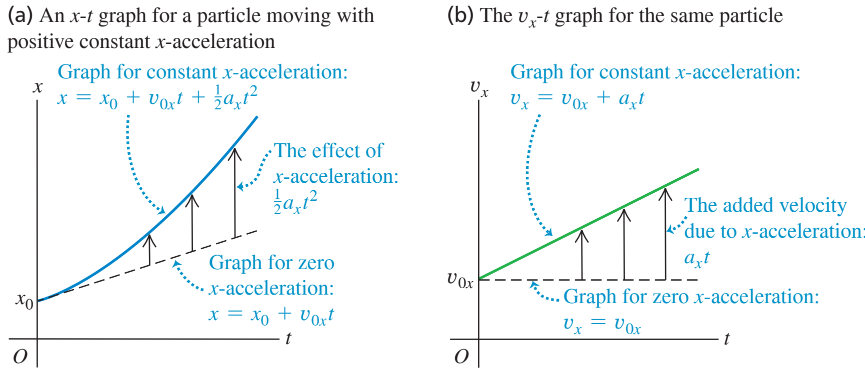


Figure **2.19** (a) How a constant x-acceleration affects a particle's (a) x-t graph and (b) v_x-t graph.

Finally,

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad (2.13)$$

x-velocity at time t of a particle with **constant x-acceleration**
x-velocity of the particle at time 0
Constant x-acceleration of the particle
Position of the particle at time t
Position of the particle at time 0

We can get one more useful relationship by equating the two expressions for $v_{\text{av-}x}$, Eqs. (2.9) and (2.10), and multiplying through by t:

$$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t \qquad (2.14)$$

Position at time t of a particle with **constant x-acceleration**
Position of the particle at time 0
Time
x-velocity of the particle at time 0
x-velocity of the particle at time t

Note that Eq. (2.14) does not contain the x-acceleration a_x. This equation can be handy when a_x is constant but its value is unknown.

Equations (2.8), (2.12), (2.13), and (2.14) are the *equations of motion with constant acceleration* (**Table 2.5**). By using these equations, we can solve *any* problem involving straight-line motion of a particle with constant acceleration.

For the particular case of motion with constant x-acceleration depicted in Fig. 2.15 and graphed in Figs. 2.16, 2.17, and 2.18, the values of x_0, v_{0x}, and a_x are all positive. We recommend that you redraw these figures for cases in which one, two, or all three of these quantities are negative.

TABLE 2.5 Equations of Motion with Constant Acceleration

Equation		**Includes Quantities**			
$v_x = v_{0x} + a_x t$	(2.8)	t		v_x	a_x
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	(2.12)	t	x		a_x
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	(2.13)		x	v_x	a_x
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$	(2.14)	t	x	v_x	

PROBLEM-SOLVING STRATEGY 2.1 Motion with Constant Acceleration

IDENTIFY *the relevant concepts:* In most straight-line motion problems, you can use the constant-acceleration Equations (2.8), (2.12), (2.13), and (2.14). If you encounter a situation in which the acceleration *isn't* constant, you'll need a different approach (see Section 2.6).

SET UP *the problem* using the following steps:

1. Read the problem carefully. Make a motion diagram showing the location of the particle at the times of interest. Decide where to place the origin of coordinates and which axis direction is positive. It's often helpful to place the particle at the origin at time $t = 0$; then $x_0 = 0$. Your choice of the positive axis direction automatically determines the positive directions for x-velocity and x-acceleration. If x is positive to the right of the origin, then v_x and a_x are also positive toward the right.
2. Identify the physical quantities (times, positions, velocities, and accelerations) that appear in Eqs. (2.8), (2.12), (2.13), and (2.14) and assign them appropriate symbols: t, x, x_0, v_x, v_{0x}, and a_x, or symbols related to those. Translate the prose into physics: "*When* does the particle arrive at its highest point" means "What is the value of t when x has its maximum value?" In Example 2.4, "Where is he when his speed is 25 m/s?" means "What is the value of x when $v_x = 25$ m/s?" Be alert for implicit information. For example, "A car sits at a stop light" usually means $v_{0x} = 0$.
3. List the quantities such as x, x_0, v_x, v_{0x}, a_x, and t. Some of them will be known and some will be unknown. Write down the values of the known quantities, and decide which of the unknowns are the target variables. Make note of the *absence* of any of the quantities that appear in the four constant-acceleration equations.

Continued

4. Use Table 2.5 to identify the applicable equations. (These are often the equations that don't include any of the absent quantities that you identified in step 3.) Usually you'll find a single equation that contains only one of the target variables. Sometimes you must find two equations, each containing the same two unknowns.
5. Sketch graphs corresponding to the applicable equations. The v_x-t graph of Eq. (2.8) is a straight line with slope a_x. The x-t graph of Eq. (2.12) is a parabola that's concave up if a_x is positive and concave down if a_x is negative.
6. On the basis of your experience with such problems, and taking account of what your sketched graphs tell you, make any qualitative and quantitative predictions you can about the solution.

EXECUTE *the solution:* If a single equation applies, solve it for the target variable, *using symbols only*; then substitute the known values and calculate the value of the target variable. If you have two equations in two unknowns, solve them simultaneously for the target variables.

EVALUATE *your answer:* Take a hard look at your results to see whether they make sense. Are they within the general range of values that you expected?

EXAMPLE 2.4 Constant-acceleration calculations

A motorcyclist heading east through a small town accelerates at a constant 4.0 m/s^2 after he leaves the city limits (**Fig. 2.20**). At time $t = 0$ he is 5.0 m east of the city-limits signpost while he moves east at 15 m/s. (a) Find his position and velocity at $t = 2.0$ s. (b) Where is he when his speed is 25 m/s?

IDENTIFY and SET UP The x-acceleration is constant, so we can use the constant-acceleration equations. We take the signpost as the origin of coordinates ($x = 0$) and choose the positive x-axis to point east (see Fig. 2.20, which is also a motion diagram). The known variables are the initial position and velocity, $x_0 = 5.0$ m and $v_{0x} = 15$ m/s, and the acceleration, $a_x = 4.0 \text{ m/s}^2$. The unknown target variables in part (a) are the values of the position x and the x-velocity v_x at $t = 2.0$ s; the target variable in part (b) is the value of x when $v_x = 25$ m/s.

EXECUTE (a) Since we know the values of x_0, v_{0x}, and a_x, Table 2.5 tells us that we can find the position x at $t = 2.0$ s by using Eq. (2.12) and the x-velocity v_x at this time by using Eq. (2.8):

$$\begin{aligned} x &= x_0 + v_{0x}t + \tfrac{1}{2}a_x t^2 \\ &= 5.0 \text{ m} + (15 \text{ m/s})(2.0 \text{ s}) + \tfrac{1}{2}(4.0 \text{ m/s}^2)(2.0 \text{ s})^2 = 43 \text{ m} \\ v_x &= v_{0x} + a_x t \\ &= 15 \text{ m/s} + (4.0 \text{ m/s}^2)(2.0 \text{ s}) = 23 \text{ m/s} \end{aligned}$$

(b) We want to find the value of x when $v_x = 25$ m/s, but we don't know the time when the motorcycle has this velocity. Table 2.5 tells us that we should use Eq. (2.13), which involves x, v_x, and a_x but does *not* involve t:

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

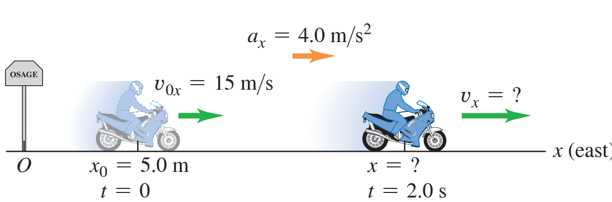
Solving for x and substituting the known values, we find

$$\begin{aligned} x &= x_0 + \frac{v_x^2 - v_{0x}^2}{2a_x} \\ &= 5.0 \text{ m} + \frac{(25 \text{ m/s})^2 - (15 \text{ m/s})^2}{2(4.0 \text{ m/s}^2)} = 55 \text{ m} \end{aligned}$$

EVALUATE You can check the result in part (b) by first using Eq. (2.8), $v_x = v_{0x} + a_x t$, to find the time at which $v_x = 25$ m/s, which turns out to be $t = 25$ s. You can then use Eq. (2.12), $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$, to solve for x. You should find $x = 55$ m, the same answer as above. That's the long way to solve the problem, though. The method we used in part (b) is much more efficient.

KEYCONCEPT By using one or more of the four equations in Table 2.5, you can solve any problem involving straight-line motion with constant acceleration.

Figure 2.20 A motorcyclist traveling with constant acceleration.



EXAMPLE 2.5 Two objects with different accelerations

WITH VARIATION PROBLEMS

A motorist traveling at a constant 15 m/s (54 km/h, or about 34 mi/h) passes a school crossing where the speed limit is 10 m/s (36 km/h, or about 22 mi/h). Just as the motorist passes the school-crossing sign, a police officer on a motorcycle stopped there starts in pursuit with constant acceleration 3.0 m/s^2 (**Fig. 2.21a**). (a) How much time elapses before the officer passes the motorist? At that time, (b) what is the officer's speed and (c) how far has each vehicle traveled?

IDENTIFY and SET UP Both the officer and the motorist move with constant acceleration (equal to zero for the motorist), so we can use the constant-acceleration formulas. We take the origin at the sign, so $x_0 = 0$ for both, and we take the positive direction to the right. Let x_P and x_M represent the positions of the police officer and the motorist at any time. Their initial velocities are $v_{P0x} = 0$ and $v_{M0x} = 15$ m/s, and their accelerations are $a_{Px} = 3.0 \text{ m/s}^2$ and $a_{Mx} = 0$. Our target variable in part (a) is the time when the officer and motorist are at the same position x; Table 2.5 tells us that Eq. (2.12) is useful for this part. In part (b) we'll use Eq. (2.8) to find the officer's speed v (the magnitude of her velocity) at the time found in part (a). In part (c) we'll use Eq. (2.12) again to find the position of either vehicle at this same time.

Figure 2.21 (a) Motion with constant acceleration overtaking motion with constant velocity. (b) A graph of x versus t for each vehicle.

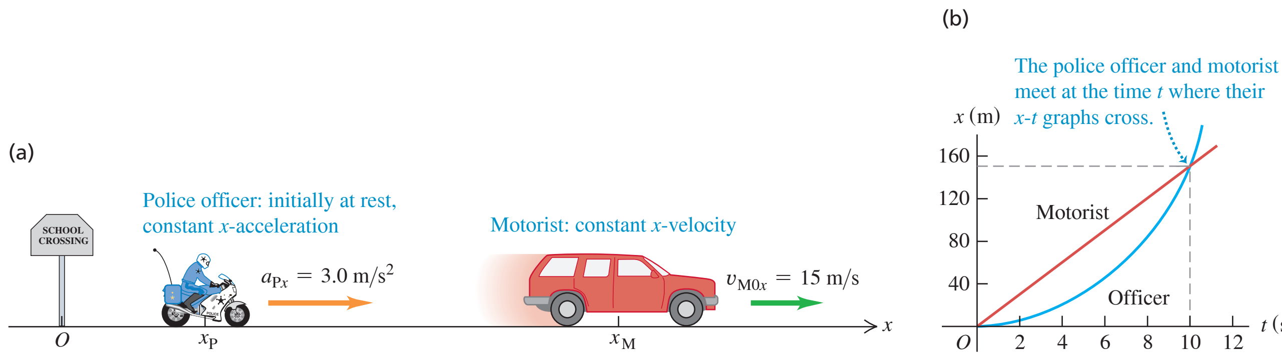


Figure 2.21b shows an x-t graph for both vehicles. The straight line represents the motorist's motion, $x_M = x_{M0} + v_{M0x}t = v_{M0x}t$. The graph for the officer's motion is the right half of a parabola with upward curvature:

$$x_P = x_{P0} + v_{P0x}t + \tfrac{1}{2}a_{Px}t^2 = \tfrac{1}{2}a_{Px}t^2$$

A good sketch shows that the officer and motorist are at the same position ($x_P = x_M$) at about $t = 10$ s, at which time both have traveled about 150 m from the sign.

EXECUTE (a) To find the value of the time t at which the motorist and police officer are at the same position, we set $x_P = x_M$ by equating the expressions above and solving that equation for t:

$$v_{M0x}t = \tfrac{1}{2}a_{Px}t^2$$

$$t = 0 \qquad \text{or} \qquad t = \frac{2v_{M0x}}{a_{Px}} = \frac{2(15\ \text{m/s})}{3.0\ \text{m/s}^2} = 10\ \text{s}$$

Both vehicles have the same x-coordinate at *two* times, as Fig. 2.21b indicates. At $t = 0$ the motorist passes the officer; at $t = 10$ s the officer passes the motorist.

(b) We want the magnitude of the officer's x-velocity v_{Px} at the time t found in part (a). Substituting the values of v_{P0x} and a_{Px} into Eq. (2.8) along with $t = 10$ s from part (a), we find

$$v_{Px} = v_{P0x} + a_{Px}t = 0 + (3.0\ \text{m/s}^2)(10\ \text{s}) = 30\ \text{m/s}$$

The officer's speed is the absolute value of this, which is also 30 m/s.

(c) In 10 s the motorist travels a distance

$$x_M = v_{M0x}t = (15\ \text{m/s})(10\ \text{s}) = 150\ \text{m}$$

and the officer travels

$$x_P = \tfrac{1}{2}a_{Px}t^2 = \tfrac{1}{2}(3.0\ \text{m/s}^2)(10\ \text{s})^2 = 150\ \text{m}$$

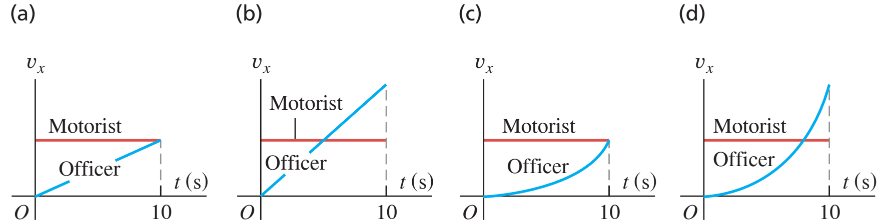
This verifies that they have gone equal distances after 10 s.

EVALUATE Our results in parts (a) and (c) agree with our estimates from our sketch. Note that when the officer passes the motorist, they do *not* have the same velocity: The motorist is moving at 15 m/s and the officer is moving at 30 m/s. You can also see this from Fig. 2.21b. Where the two x-t curves cross, their slopes (equal to the values of v_x for the two vehicles) are different.

Is it just coincidence that when the two vehicles are at the same position, the officer is going twice the speed of the motorist? Equation (2.14), $x - x_0 = \tfrac{1}{2}(v_{0x} + v_x)t$, gives the answer. The motorist has constant velocity, so $v_{M0x} = v_{Mx}$, and the motorist's displacement $x - x_0$ in time t is $v_{M0x}t$. Because $v_{P0x} = 0$, in the same time t the officer's displacement is $\tfrac{1}{2}v_{Px}t$. The two vehicles have the same displacement in the same amount of time, so $v_{M0x}t = \tfrac{1}{2}v_{Px}t$ and $v_{Px} = 2v_{M0x}$—that is, the officer has exactly twice the motorist's velocity. This is true no matter what the value of the officer's acceleration.

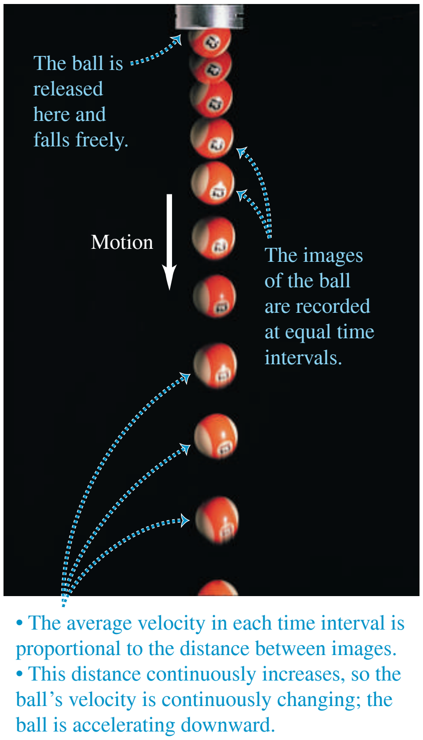
KEYCONCEPT In straight-line motion, one object meets or passes another at the time when the two objects have the same coordinate x (and so their x-t graphs cross). The objects can have different velocities at that time.

TEST YOUR UNDERSTANDING OF SECTION 2.4 Four possible v_x-t graphs are shown for the two vehicles in Example 2.5. Which graph is correct?



ANSWER **(b)** The officer's x-acceleration is constant, so her v_x-t graph is a straight line. The motorcycle is moving faster than the car when the two vehicles meet at $t = 10$ s.

Figure 2.22 Multiflash photo of a freely falling ball.



2.5 FREELY FALLING OBJECTS

The most familiar example of motion with (nearly) constant acceleration is an object falling under the influence of the earth's gravitational attraction. Such motion has held the attention of philosophers and scientists since ancient times. In the fourth century B.C., Aristotle thought (erroneously) that heavy objects fall faster than light objects, in proportion to their weight. Nineteen centuries later, Galileo (see Section 1.1) argued that an object should fall with a downward acceleration that is constant and independent of its weight.

Experiment shows that if the effects of the air can be ignored, Galileo is right; all objects at a particular location fall with the same downward acceleration, regardless of their size or weight. If in addition the distance of the fall is small compared with the radius of the earth, and if we ignore small effects due to the earth's rotation, the acceleration is constant. The idealized motion that results under all of these assumptions is called **free fall,** although it includes rising as well as falling motion. (In Chapter 3 we'll extend the discussion of free fall to include the motion of projectiles, which move both vertically and horizontally.)

Figure 2.22 is a photograph of a falling ball made with a stroboscopic light source that produces a series of short, intense flashes at equal time intervals. As each flash occurs, an image of the ball at that instant is recorded on the photograph. The increasing spacing between successive images in Fig. 2.22 indicates that the ball is accelerating downward. Careful measurement shows that the velocity change is the same in each time interval, so the acceleration of the freely falling ball is constant.

The constant acceleration of a freely falling object is called the **acceleration due to gravity,** and we denote its magnitude with the letter g. We'll frequently use the approximate value of g at or near the earth's surface:

$$g = 9.80\ \text{m/s}^2 = 980\ \text{cm/s}^2 = 32.2\ \text{ft/s}^2 \quad \text{(approximate value near the earth's surface)}$$

The exact value varies with location, so we'll often give the value of g at the earth's surface to only two significant figures as $9.8\ \text{m/s}^2$. On the moon's surface, the acceleration due to gravity is caused by the attractive force of the moon rather than the earth, and $g = 1.6\ \text{m/s}^2$. Near the surface of the sun, $g = 270\ \text{m/s}^2$.

CAUTION **Don't confuse speed, velocity, and acceleration in free fall** Speed can never be negative; velocity can be positive or negative, depending on the direction of motion. In free fall, speed and velocity change continuously but acceleration (the rate of change of velocity) is constant and downward.

CAUTION **g is always a positive number** Because g is the *magnitude* of a vector quantity, it is always a *positive* number. If you take the positive y-direction to be upward, as we do in most situations involving free fall, the y-component of the acceleration is negative and equal to $-g$. Be careful with the sign of g, or you'll have trouble with free-fall problems.

In the following examples we use the constant-acceleration equations developed in Section 2.4. Review Problem-Solving Strategy 2.1 in that section before you study the next examples.

EXAMPLE 2.6 A freely falling coin

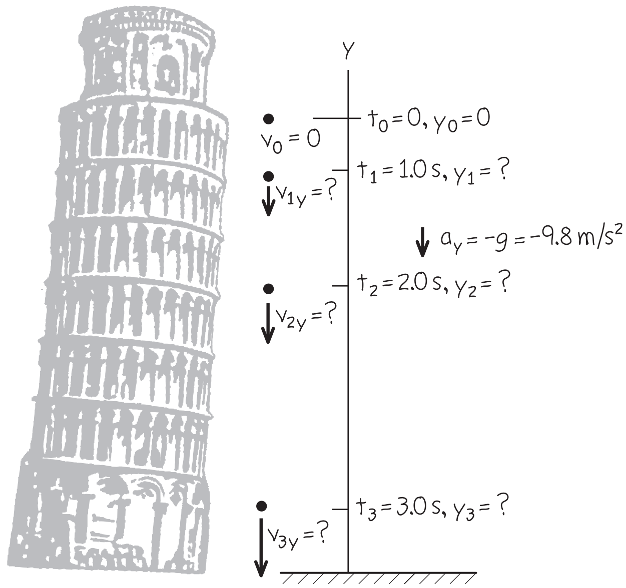
A one-euro coin is dropped from the Leaning Tower of Pisa and falls freely from rest. What are its position and velocity after 1.0 s, 2.0 s, and 3.0 s? Ignore air resistance.

IDENTIFY and SET UP "Falls freely" means "falls with constant acceleration due to gravity," so we can use the constant-acceleration equations. The right side of **Fig. 2.23** shows our motion diagram for the coin. The motion is vertical, so we use a vertical coordinate axis and call the coordinate y instead of x. We take the origin at the starting point and the *upward* direction as positive. Both the initial coordinate y_0 and initial y-velocity v_{0y} are zero. The y-acceleration is downward (in the negative y-direction), so $a_y = -g = -9.8\ \text{m/s}^2$. (Remember that g is a positive quantity.) Our target variables are the values of y and v_y at the three given times. To find these, we use Eqs. (2.12) and (2.8) with x replaced by y. Our choice of the upward direction as positive means that all positions and velocities we calculate will be negative.

EXECUTE At a time t after the coin is dropped, its position and y-velocity are

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = 0 + 0 + \tfrac{1}{2}(-g)t^2 = (-4.9\ \text{m/s}^2)t^2$$
$$v_y = v_{0y} + a_y t = 0 + (-g)t = (-9.8\ \text{m/s}^2)t$$

Figure 2.23 A coin freely falling from rest.



When $t = 1.0$ s, $y = (-4.9 \text{ m/s}^2)(1.0 \text{ s})^2 = -4.9$ m and $v_y = (-9.8 \text{ m/s}^2)(1.0 \text{ s}) = -9.8$ m/s; after 1.0 s, the coin is 4.9 m below the origin (y is negative) and has a downward velocity (v_y is negative) with magnitude 9.8 m/s.

We can find the positions and y-velocities at 2.0 s and 3.0 s in the same way. The results are $y = -20$ m and $v_y = -20$ m/s at $t = 2.0$ s, and $y = -44$ m and $v_y = -29$ m/s at $t = 3.0$ s.

EVALUATE All our answers are negative, as we expected. If we had chosen the positive y-axis to point downward, the acceleration would have been $a_y = +g$ and all our answers would have been positive.

KEYCONCEPT By using one or more of the four equations in Table 2.5 with x replaced by y, the positive y-direction chosen to be upward, and acceleration $a_y = -g$, you can solve any free-fall problem.

EXAMPLE 2.7 Up-and-down motion in free fall

WITH VARIATION PROBLEMS

You throw a ball vertically upward from the roof of a tall building. The ball leaves your hand at a point even with the roof railing with an upward speed of 15.0 m/s; the ball is then in free fall. (We ignore air resistance.) On its way back down, it just misses the railing. Find (a) the ball's position and velocity 1.00 s and 4.00 s after leaving your hand; (b) the ball's velocity when it is 5.00 m above the railing; (c) the maximum height reached; (d) the ball's acceleration when it is at its maximum height.

IDENTIFY and SET UP The words "in free fall" mean that the acceleration is due to gravity, which is constant. Our target variables are position [in parts (a) and (c)], velocity [in parts (a) and (b)], and acceleration [in part (d)]. We take the origin at the point where the ball leaves your hand, and take the positive direction to be upward (**Fig. 2.24**). The initial position y_0 is zero, the initial y-velocity v_{0y} is $+15.0$ m/s, and the y-acceleration is $a_y = -g = -9.80 \text{ m/s}^2$. In part (a), as in Example 2.6, we'll use Eqs. (2.12) and (2.8) to find the position and velocity as functions of time. In part (b) we must find the velocity at a given *position* (no time is given), so we'll use Eq. (2.13).

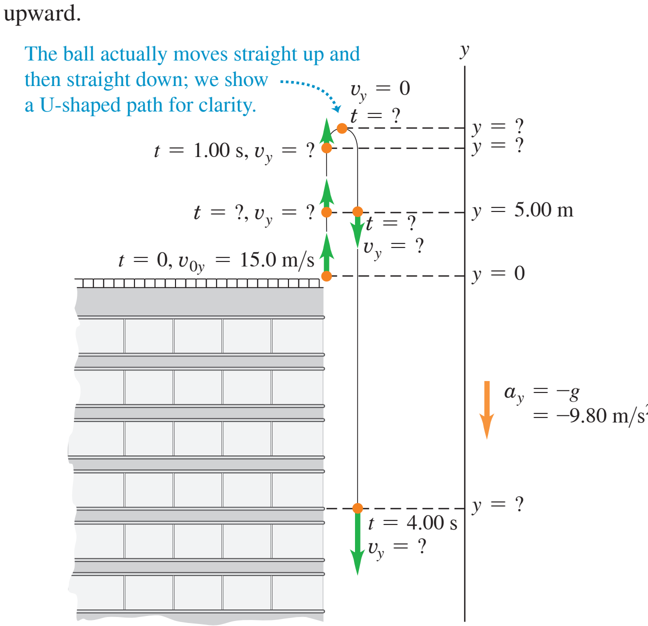
Figure 2.25 (next page) shows the y-t and v_y-t graphs for the ball. The y-t graph is a concave-down parabola that rises and then falls, and the v_y-t graph is a downward-sloping straight line. Note that the ball's velocity is zero when it is at its highest point.

EXECUTE (a) The position and y-velocity at time t are given by Eqs. (2.12) and (2.8) with x's replaced by y's:

$$\begin{aligned} y &= y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = y_0 + v_{0y}t + \tfrac{1}{2}(-g)t^2 \\ &= (0) + (15.0 \text{ m/s})t + \tfrac{1}{2}(-9.80 \text{ m/s}^2)t^2 \\ v_y &= v_{0y} + a_y t = v_{0y} + (-g)t \\ &= 15.0 \text{ m/s} + (-9.80 \text{ m/s}^2)t \end{aligned}$$

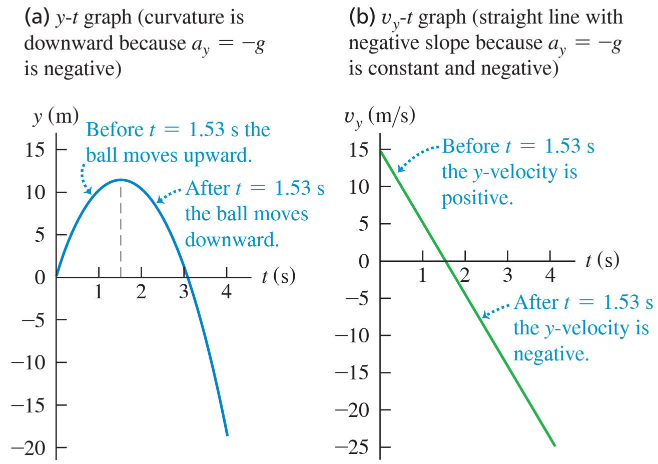
When $t = 1.00$ s, these equations give $y = +10.1$ m and $v_y = +5.2$ m/s. That is, the ball is 10.1 m above the origin (y is positive) and moving upward (v_y is positive) with a speed of 5.2 m/s. This is less than the initial speed because the ball slows as it ascends. When $t = 4.00$ s, those equations give $y = -18.4$ m and $v_y = -24.2$ m/s. The ball has passed its highest point and is 18.4 m *below* the origin (y is negative). It is moving *downward* (v_y is negative) with a speed of 24.2 m/s.

Figure 2.24 Position and velocity of a ball thrown vertically upward.



Continued

Figure **2.25** (a) Position and (b) velocity as functions of time for a ball thrown upward with an initial speed of 15.0 m/s.



(b) The y-velocity at any position y is given by Eq. (2.13) with x's replaced by y's:

$$\begin{aligned} v_y^2 &= v_{0y}^2 + 2a_y(y - y_0) = v_{0y}^2 + 2(-g)(y - 0) \\ &= (15.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)y \end{aligned}$$

When the ball is 5.00 m above the origin we have $y = +5.00$ m, so

$$\begin{aligned} v_y^2 &= (15.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(5.00 \text{ m}) = 127 \text{ m}^2/\text{s}^2 \\ v_y &= \pm 11.3 \text{ m/s} \end{aligned}$$

We get *two* values of v_y because the ball passes through the point $y = +5.00$ m twice, once on the way up (so v_y is positive) and once on the way down (so v_y is negative) (see Fig. 2.24). Note that the speed of the ball is 11.3 m/s each time it passes through this point.

(c) At the instant at which the ball reaches its maximum height y_1, its y-velocity is momentarily zero: $v_y = 0$. We use Eq. (2.13) to find y_1. With $v_y = 0$, $y_0 = 0$, and $a_y = -g$, we get

$$\begin{aligned} 0 &= v_{0y}^2 + 2(-g)(y_1 - 0) \\ y_1 &= \frac{v_{0y}^2}{2g} = \frac{(15.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = +11.5 \text{ m} \end{aligned}$$

(d) **CAUTION** **A free-fall misconception** It's a common misconception that at the highest point of free-fall motion, where the velocity is zero, the acceleration is also zero. If this were so, once the ball reached the highest point it would hang there suspended in midair! Remember that acceleration is the rate of change of velocity, and the ball's velocity is continuously changing. At every point, including the highest point, and at any velocity, including zero, the acceleration in free fall is always $a_y = -g = -9.80 \text{ m/s}^2$.

EVALUATE A useful way to check any free-fall problem is to draw the y-t and v_y-t graphs, as we did in Fig. 2.25. Note that these are graphs of Eqs. (2.12) and (2.8), respectively. Given the initial position, initial velocity, and acceleration, you can easily create these graphs by using a graphing calculator app or an online math program.

KEYCONCEPT If a freely falling object passes a given point at two different times, once moving upward and once moving downward, its speed will be the same at both times.

EXAMPLE 2.8 Two solutions or one?

WITH VARIATION PROBLEMS

At what time after being released has the ball in Example 2.7 fallen 5.00 m below the roof railing?

IDENTIFY and SET UP We treat this as in Example 2.7, so y_0, v_{0y}, and $a_y = -g$ have the same values as there. Now, however, the target variable is the time at which the ball is at $y = -5.00$ m. The best equation to use is Eq. (2.12) with a_y replaced by $-g$, which gives the position y as a function of time t:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}(-g)t^2$$

This is a *quadratic* equation for t, which we want to solve for the value of t when $y = -5.00$ m.

EXECUTE We rearrange the equation so that it has the standard form of a quadratic equation for an unknown x, $Ax^2 + Bx + C = 0$:

$$\left(\tfrac{1}{2}g\right)t^2 + (-v_{0y})t + (y - y_0) = At^2 + Bt + C = 0$$

By comparison, we identify $A = \frac{1}{2}g$, $B = -v_{0y}$, and $C = y - y_0$. The quadratic formula (see Appendix B) tells us that this equation has *two* solutions:

$$\begin{aligned} t &= \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \\ &= \frac{-(-v_{0y}) \pm \sqrt{(-v_{0y})^2 - 4\left(\frac{1}{2}g\right)(y - y_0)}}{2\left(\frac{1}{2}g\right)} \\ &= \frac{v_{0y} \pm \sqrt{v_{0y}^2 - 2g(y - y_0)}}{g} \end{aligned}$$

Substituting the values $y_0 = 0$, $v_{0y} = +15.0$ m/s, $g = 9.80 \text{ m/s}^2$, and $y = -5.00$ m, we find

$$t = \frac{(15.0 \text{ m/s}) \pm \sqrt{(15.0 \text{ m/s})^2 - 2(9.80 \text{ m/s}^2)(-5.00 \text{ m} - 0)}}{9.80 \text{ m/s}^2}$$

You can confirm that the numerical answers are $t = +3.36$ s and $t = -0.30$ s. The answer $t = -0.30$ s doesn't make physical sense, since it refers to a time *before* the ball left your hand at $t = 0$. So the correct answer is $t = +3.36$ s.

EVALUATE Why did we get a second, fictitious solution? The explanation is that constant-acceleration equations like Eq. (2.12) are based on the assumption that the acceleration is constant for *all* values of time, whether positive, negative, or zero. Hence the solution $t = -0.30$ s refers to an imaginary moment when a freely falling ball was 5.00 m below the roof railing and rising to meet your hand. Since the ball didn't leave your hand and go into free fall until $t = 0$, this result is pure fiction.

Repeat these calculations to find the times when the ball is 5.00 m *above* the origin ($y = +5.00$ m). The two answers are $t = +0.38$ s and $t = +2.68$ s. Both are positive values of t, and both refer to the real motion of the ball after leaving your hand. At the earlier time the ball passes through $y = +5.00$ m moving upward; at the later time it passes through this point moving downward. [Compare this with part (b) of Example 2.7, and again refer to Fig. 2.25a.]

You should also solve for the times when $y = +15.0$ m. In this case, both solutions involve the square root of a negative number, so there are *no* real solutions. Again Fig. 2.25a shows why; we found in part

(c) of Example 2.7 that the ball's maximum height is $y = +11.5\text{ m}$, so it *never* reaches $y = +15.0\text{ m}$. While a quadratic equation such as Eq. (2.12) always has two solutions, in some situations one or both of the solutions aren't physically reasonable.

In this example we encountered a quadratic equation for the case of free fall. But Eq. (2.12) is a quadratic equation that applies to *all* cases of straight-line motion with constant acceleration, so you'll need to exercise the same care in solving many problems of this kind.

KEYCONCEPT When the acceleration of an object is constant, its position as a function of time is given by a quadratic equation. Inspect the solutions to this equation to determine what they tell you about the problem you're solving.

TEST YOUR UNDERSTANDING OF SECTION 2.5 If you toss a ball upward with a certain initial speed, it falls freely and reaches a maximum height h a time t after it leaves your hand. (a) If you throw the ball upward with double the initial speed, what new maximum height does the ball reach? (i) $h\sqrt{2}$; (ii) $2h$; (iii) $4h$; (iv) $8h$; (v) $16h$. (b) If you throw the ball upward with double the initial speed, how long does it take to reach its new maximum height? (i) $t/2$; (ii) $t/\sqrt{2}$; (iii) t; (iv) $t\sqrt{2}$; (v) $2t$.

ANSWER

(a) **(iii)** Use Eq. (2.13) with x replaced by y and $a_y = -g$: $v_y^2 = v_{0y}^2 - 2g(y - y_0)$. The starting height is $y_0 = 0$ and the y-velocity at the maximum height $y = h$ is $v_y = 0$, so $0 = v_{0y}^2 - 2gh$ and $h = v_{0y}^2/2g$. If the initial y-velocity is increased by a factor of 2, the maximum height increases by a factor of $2^2 = 4$ and the ball goes to height $4h$. (b) **(v)** Use Eq. (2.8) with x replaced by y and $a_y = -g$: $v_y = v_{0y} - gt$. The y-velocity at the maximum height is $v_y = 0$, so $0 = v_{0y} - gt$ and $t = v_{0y}/g$. If the initial y-velocity is increased by a factor of 2, the time to reach the maximum height increases by a factor of 2 and becomes $2t$.

2.6 VELOCITY AND POSITION BY INTEGRATION

This section is intended for students who have already learned a little integral calculus. In Section 2.4 we analyzed the special case of straight-line motion with constant acceleration. When a_x is not constant, as is frequently the case, the equations that we derived in that section are no longer valid (**Fig. 2.26**). But even when a_x varies with time, we can still use the relationship $v_x = dx/dt$ to find the x-velocity v_x as a function of time if the position x is a known function of time. And we can still use $a_x = dv_x/dt$ to find the x-acceleration a_x as a function of time if the x-velocity v_x is a known function of time.

In many situations, however, position and velocity are not known functions of time, while acceleration is (**Fig. 2.27**). How can we find the position and velocity in straight-line motion from the acceleration function $a_x(t)$?

Figure **2.26** When you push a car's accelerator pedal to the floorboard, the resulting acceleration is *not* constant: The greater the car's speed, the more slowly it gains additional speed. A typical car takes twice as long to accelerate from 50 km/h to 100 km/h as it does to accelerate from 0 to 50 km/h.

Figure **2.27** The inertial navigation system (INS) on board a long-range airliner keeps track of the airliner's acceleration. Given the airliner's initial position and velocity before takeoff, the INS uses the acceleration data to calculate the airliner's position and velocity throughout the flight.

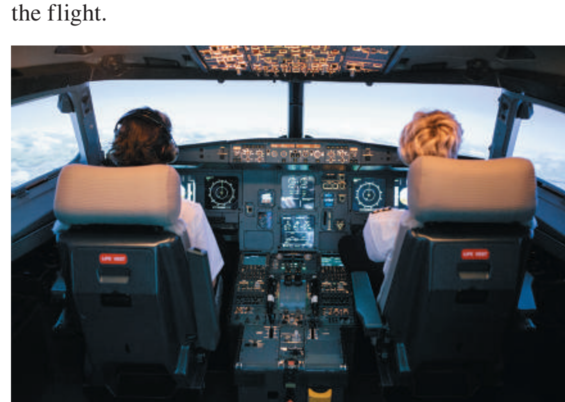


Figure **2.28** An a_x-t graph for a particle whose x-acceleration is not constant.

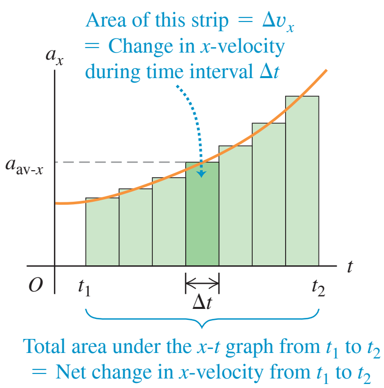


Figure 2.28 is a graph of x-acceleration versus time for a particle whose acceleration is not constant. We can divide the time interval between times t_1 and t_2 into many smaller subintervals, calling a typical one Δt. Let the average x-acceleration during Δt be $a_{\text{av-}x}$. From Eq. (2.4) the change in x-velocity Δv_x during Δt is

$$\Delta v_x = a_{\text{av-}x}\,\Delta t$$

Graphically, Δv_x equals the area of the shaded strip with height $a_{\text{av-}x}$ and width Δt—that is, the area under the curve between the left and right sides of Δt. The total change in x-velocity from t_1 to t_2 is the sum of the x-velocity changes Δv_x in the small subintervals. So the total x-velocity change is represented graphically by the *total* area under the a_x-t curve between the vertical lines t_1 and t_2. (In Section 2.4 we showed this for the special case in which a_x is constant.)

In the limit that all the Δt's become very small and they become very large in number, the value of $a_{\text{av-}x}$ for the interval from any time t to $t + \Delta t$ approaches the instantaneous x-acceleration a_x at time t. In this limit, the area under the a_x-t curve is the *integral* of a_x (which is in general a function of t) from t_1 to t_2. If v_{1x} is the x-velocity of the particle at time t_1 and v_{2x} is the velocity at time t_2, then

$$v_{2x} - v_{1x} = \int_{v_{1x}}^{v_{2x}} dv_x = \int_{t_1}^{t_2} a_x\,dt \tag{2.15}$$

The change in the x-velocity v_x is the time integral of the x-acceleration a_x.

We can carry out exactly the same procedure with the curve of x-velocity versus time. If x_1 is a particle's position at time t_1 and x_2 is its position at time t_2, from Eq. (2.2) the displacement Δx during a small time interval Δt is equal to $v_{\text{av-}x}\,\Delta t$, where $v_{\text{av-}x}$ is the average x-velocity during Δt. The total displacement $x_2 - x_1$ during the interval $t_2 - t_1$ is given by

$$x_2 - x_1 = \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v_x\,dt \tag{2.16}$$

The change in position x—that is, the displacement—is the time integral of x-velocity v_x. Graphically, the displacement between times t_1 and t_2 is the area under the v_x-t curve between those two times. [This is the same result that we obtained in Section 2.4 for the special case in which v_x is given by Eq. (2.8).]

If $t_1 = 0$ and t_2 is any later time t, and if x_0 and v_{0x} are the position and velocity, respectively, at time $t = 0$, then we can rewrite Eqs. (2.15) and (2.16) as

x-velocity of a particle at time t … x-velocity of the particle at time 0

$$v_x = v_{0x} + \underbrace{\int_0^t a_x\,dt}_{\text{Integral of the } x\text{-acceleration of the particle from time 0 to time } t} \tag{2.17}$$

Position of a particle at time t … Position of the particle at time 0

$$x = x_0 + \underbrace{\int_0^t v_x\,dt}_{\text{Integral of the } x\text{-velocity of the particle from time 0 to time } t} \tag{2.18}$$

If we know the x-acceleration a_x as a function of time and we know the initial velocity v_{0x}, we can use Eq. (2.17) to find the x-velocity v_x at any time; that is, we can find v_x as a function of time. Once we know this function, and given the initial position x_0, we can use Eq. (2.18) to find the position x at any time.

EXAMPLE 2.9 Motion with changing acceleration

Sally is driving along a straight highway. At $t = 0$, when she is moving at 10 m/s in the positive x-direction, she passes a signpost at $x = 50$ m. Her x-acceleration as a function of time is

$$a_x = 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t$$

(a) Find her x-velocity v_x and position x as functions of time. (b) When is her x-velocity greatest? (c) What is that maximum x-velocity? (d) Where is the car when it reaches that maximum x-velocity?

IDENTIFY and SET UP The x-acceleration is a function of time, so we *cannot* use the constant-acceleration formulas of Section 2.4. Instead, we use Eq. (2.17) to obtain an expression for v_x as a function of time, and then use that result in Eq. (2.18) to find an expression for x as a function of t. We'll then be able to answer a variety of questions about the motion.

EXECUTE (a) At $t = 0$, Sally's position is $x_0 = 50$ m and her x-velocity is $v_{0x} = 10$ m/s. To use Eq. (2.17), we note that the integral of t^n (except for $n = -1$) is $\int t^n\,dt = \frac{1}{n+1}t^{n+1}$. Hence

$$\begin{aligned} v_x &= 10\ \text{m/s} + \int_0^t [2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t]\,dt \\ &= 10\ \text{m/s} + (2.0\ \text{m/s}^2)t - \tfrac{1}{2}(0.10\ \text{m/s}^3)t^2 \end{aligned}$$

Now we use Eq. (2.18) to find x as a function of t:

$$\begin{aligned} x &= 50\ \text{m} + \int_0^t [10\ \text{m/s} + (2.0\ \text{m/s}^2)t - \tfrac{1}{2}(0.10\ \text{m/s}^3)t^2]\,dt \\ &= 50\ \text{m} + (10\ \text{m/s})t + \tfrac{1}{2}(2.0\ \text{m/s}^2)t^2 - \tfrac{1}{6}(0.10\ \text{m/s}^3)t^3 \end{aligned}$$

Figure 2.29 shows graphs of a_x, v_x, and x as functions of time as given by the previous equations. Note that for any time t, the slope of the v_x-t graph equals the value of a_x and the slope of the x-t graph equals the value of v_x.

(b) The maximum value of v_x occurs when the x-velocity stops increasing and begins to decrease. At that instant, $dv_x/dt = a_x = 0$. So we set the expression for a_x equal to zero and solve for t:

$$\begin{aligned} 0 &= 2.0\ \text{m/s}^2 - (0.10\ \text{m/s}^3)t \\ t &= \frac{2.0\ \text{m/s}^2}{0.10\ \text{m/s}^3} = 20\ \text{s} \end{aligned}$$

(c) We find the maximum x-velocity by substituting $t = 20$ s, the time from part (b) when velocity is maximum, into the equation for v_x from part (a):

$$\begin{aligned} v_{\text{max-}x} &= 10\ \text{m/s} + (2.0\ \text{m/s}^2)(20\ \text{s}) - \tfrac{1}{2}(0.10\ \text{m/s}^3)(20\ \text{s})^2 \\ &= 30\ \text{m/s} \end{aligned}$$

(d) To find the car's position at the time that we found in part (b), we substitute $t = 20$ s into the expression for x from part (a):

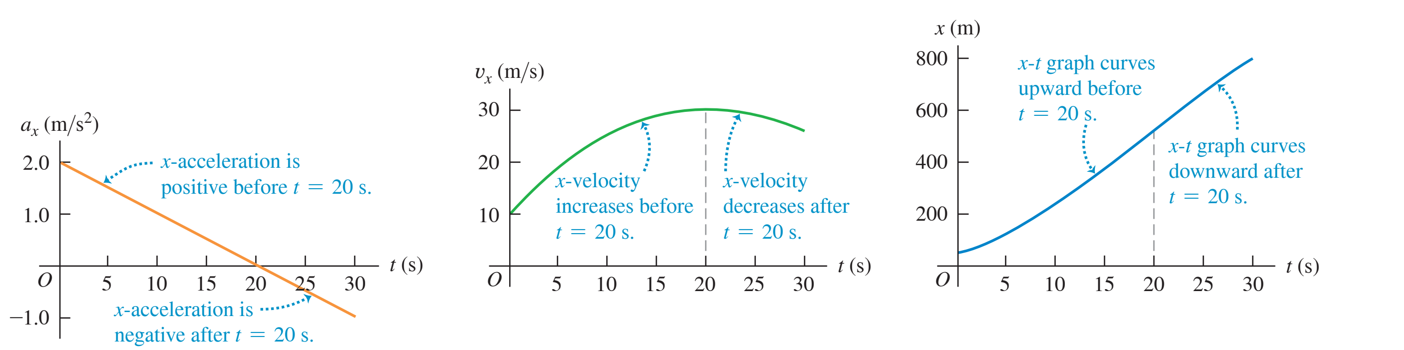
$$\begin{aligned} x &= 50\ \text{m} + (10\ \text{m/s})(20\ \text{s}) + \tfrac{1}{2}(2.0\ \text{m/s}^2)(20\ \text{s})^2 - \tfrac{1}{6}(0.10\ \text{m/s}^3)(20\ \text{s})^3 \\ &= 517\ \text{m} \end{aligned}$$

EVALUATE Figure 2.29 helps us interpret our results. The left-hand graph shows that a_x is positive between $t = 0$ and $t = 20$ s and negative after that. It is zero at $t = 20$ s, the time at which v_x is maximum (the high point in the middle graph). The car speeds up until $t = 20$ s (because v_x and a_x have the same sign) and slows down after $t = 20$ s (because v_x and a_x have opposite signs).

Since v_x is maximum at $t = 20$ s, the x-t graph (the right-hand graph in Fig. 2.29) has its maximum positive slope at this time. Note that the x-t graph is concave up (curved upward) from $t = 0$ to $t = 20$ s, when a_x is positive. The graph is concave down (curved downward) after $t = 20$ s, when a_x is negative.

KEYCONCEPT If the acceleration in straight-line motion is not constant but is a known function of time, you can find the velocity and position as functions of time by integration.

Figure **2.29** The position, velocity, and acceleration of the car in Example 2.9 as functions of time. Can you show that if this motion continues, the car will stop at $t = 44.5$ s?



TEST YOUR UNDERSTANDING OF SECTION 2.6 If the x-acceleration a_x of an object moving in straight-line motion is increasing with time, will the v_x-t graph be (i) a straight line, (ii) concave up (i.e., with an upward curvature), or (iii) concave down (i.e., with a downward curvature)?

ANSWER

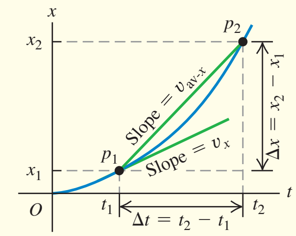
| **(ii)** The acceleration a_x is equal to the slope of the v_x-t graph. If a_x is increasing, the slope of the v_x-t graph is also increasing and the graph is concave up.

CHAPTER 2 SUMMARY

Straight-line motion, average and instantaneous *x*-velocity: When a particle moves along a straight line, we describe its position with respect to an origin O by means of a coordinate such as x. The particle's average x-velocity $v_{av\text{-}x}$ during a time interval $\Delta t = t_2 - t_1$ is equal to its displacement $\Delta x = x_2 - x_1$ divided by Δt. The instantaneous x-velocity v_x at any time t is equal to the average x-velocity over the time interval from t to $t + \Delta t$ in the limit that Δt goes to zero. Equivalently, v_x is the derivative of the position function with respect to time. (See Example 2.1.)

$$v_{av\text{-}x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1} \qquad (2.2)$$

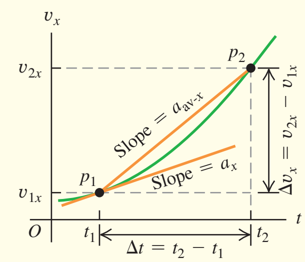
$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.3)$$



Average and instantaneous *x*-acceleration: The average x-acceleration $a_{av\text{-}x}$ during a time interval Δt is equal to the change in velocity $\Delta v_x = v_{2x} - v_{1x}$ during that time interval divided by Δt. The instantaneous x-acceleration a_x is the limit of $a_{av\text{-}x}$ as Δt goes to zero, or the derivative of v_x with respect to t. (See Examples 2.2 and 2.3.)

$$a_{av\text{-}x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{2x} - v_{1x}}{t_2 - t_1} \qquad (2.4)$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad (2.5)$$



Straight-line motion with constant acceleration: When the x-acceleration is constant, four equations relate the position x and the x-velocity v_x at any time t to the initial position x_0, the initial x-velocity v_{0x} (both measured at time $t = 0$), and the x-acceleration a_x. (See Examples 2.4 and 2.5.)

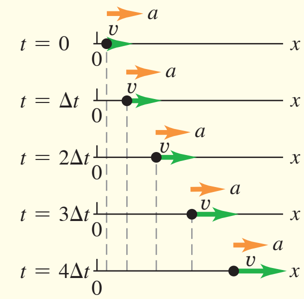
Constant x-acceleration only:

$$v_x = v_{0x} + a_x t \qquad (2.8)$$

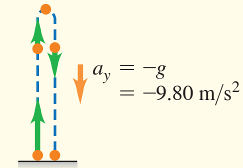
$$x = x_0 + v_{0x} t + \tfrac{1}{2} a_x t^2 \qquad (2.12)$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0) \qquad (2.13)$$

$$x - x_0 = \tfrac{1}{2}(v_{0x} + v_x)t \qquad (2.14)$$



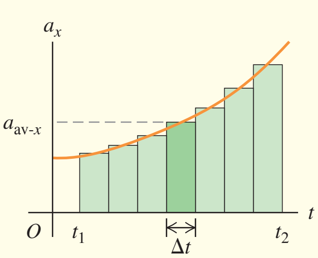
Freely falling objects: Free fall (vertical motion without air resistance, so only gravity affects the motion) is a case of motion with constant acceleration. The magnitude of the acceleration due to gravity is a positive quantity, g. The acceleration of an object in free fall is always downward. (See Examples 2.6–2.8.)



Straight-line motion with varying acceleration: When the acceleration is not constant but is a known function of time, we can find the velocity and position as functions of time by integrating the acceleration function. (See Example 2.9.)

$$v_x = v_{0x} + \int_0^t a_x\, dt \qquad (2.17)$$

$$x = x_0 + \int_0^t v_x\, dt \qquad (2.18)$$



Chapter 2 Media Assets

For assigned homework and other learning materials, go to Mastering Physics.

GUIDED PRACTICE

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLE 2.5 (Section 2.4) before attempting these problems.

VP2.5.1 A sports car starts from rest at an intersection and accelerates toward the east on a straight road at 8.0 m/s^2. Just as the sports car starts to move, a bus traveling east at a constant 18 m/s on the same straight road passes the sports car. When the sports car catches up with and passes the bus, (a) how much time has elapsed and (b) how far has the sports car traveled?

VP2.5.2 A car is traveling on a straight road at a constant 30.0 m/s, which is faster than the speed limit. Just as the car passes a police motorcycle that is stopped at the side of the road, the motorcycle accelerates forward in pursuit. The motorcycle passes the car 12.5 s after starting from rest. (a) What is the acceleration of the motorcycle (assumed to be constant)? (b) How far does the motorcycle travel before it passes the car?

VP2.5.3 A police car is traveling north on a straight road at a constant 20.0 m/s. An SUV traveling north at 30.0 m/s passes the police car. The driver of the SUV suspects he may be exceeding the speed limit, so just as he passes the police car he lets the SUV slow down at a constant 1.80 m/s^2. (a) How much time elapses from when the SUV passes the police car to when the police car passes the SUV? (b) What distance does the SUV travel during this time? (c) What is the speed of the SUV when the police car passes it?

VP2.5.4 At $t = 0$ a truck starts from rest at $x = 0$ and speeds up in the positive x-direction on a straight road with acceleration a_T. At the same time, $t = 0$, a car is at $x = 0$ and traveling in the positive x-direction with speed v_C. The car has a constant negative x-acceleration: $a_{\text{car-}x} = -a_C$, where a_C is a positive quantity. (a) At what time does the truck pass the car? (b) At what x-coordinate does the truck pass the car?

Be sure to review EXAMPLE 2.7 (Section 2.5) before attempting these problems.

VP2.7.1 You throw a ball straight up from the edge of a cliff. It leaves your hand moving at 12.0 m/s. Air resistance can be neglected. Take the positive y-direction to be upward, and choose $y = 0$ to be the point where the ball leaves your hand. Find the ball's position and velocity (a) 0.300 s after it leaves your hand and (b) 2.60 s after it leaves your hand. At each time state whether the ball is above or below your hand and whether it is moving upward or downward.

VP2.7.2 You throw a stone straight down from the top of a tall tower. It leaves your hand moving at 8.00 m/s. Air resistance can be neglected. Take the positive y-direction to be upward, and choose $y = 0$ to be the point where the stone leaves your hand. (a) Find the stone's position and velocity 1.50 s after it leaves your hand. (b) Find the stone's velocity when it is 8.00 m below your hand.

VP2.7.3 You throw a football straight up. Air resistance can be neglected. (a) When the football is 4.00 m above where it left your hand, it is moving upward at 0.500 m/s. What was the speed of the football when it left your hand? (b) How much time elapses from when the football leaves your hand until it is 4.00 m above your hand?

VP2.7.4 You throw a tennis ball straight up. Air resistance can be neglected. (a) The maximum height above your hand that the ball reaches is H. At what speed does the ball leave your hand? (b) What is the speed of the ball when it is a height $H/2$ above your hand? Express your answer as a fraction of the speed at which it left your hand. (c) At what height above your hand is the speed of the ball half as great as when it left your hand? Express your answer in terms of H.

Be sure to review EXAMPLE 2.8 (Section 2.5) before attempting these problems.

VP2.8.1 You throw a rock straight up from the edge of a cliff. It leaves your hand at time $t = 0$ moving at 12.0 m/s. Air resistance can be neglected. (a) Find both times at which the rock is 4.00 m above where it left your hand. (b) Find the time when the rock is 4.00 m below where it left your hand.

VP2.8.2 You throw a basketball straight down from the roof of a building. The basketball leaves your hand at time $t = 0$ moving at 9.00 m/s. Air resistance can be neglected. Find the time when the ball is 5.00 m below where it left your hand.

VP2.8.3 You throw an apple straight up. The apple leaves your hand at time $t = 0$ moving at 5.50 m/s. Air resistance can be neglected. (a) How many times (two, one, or none) does the apple pass through a point 1.30 m above your hand? If the apple does pass through this point, at what times t does it do so, and is the apple moving upward or downward at each of these times? (b) How many times (two, one, or none) does the apple pass through a point 1.80 m above your hand? If the apple does pass through this point, at what times t does it do so, and is the apple moving upward or downward at each of these times?

VP2.8.4 You throw an orange straight up. The orange leaves your hand at time $t = 0$ moving at speed v_0. Air resistance can be neglected. (a) At what time(s) is the orange at a height $v_0^2/2g$ above the point where it left your hand? At these time(s) is the orange moving upward, downward, or neither? (b) At what time(s) is the orange at a height $3v_0^2/8g$ above the point where it left your hand? At these time(s) is the orange moving upward, downward, or neither?

BRIDGING PROBLEM The Fall of a Superhero

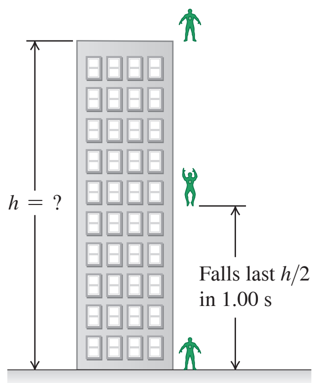
The superhero Green Lantern steps from the top of a tall building. He falls freely from rest to the ground, falling half the total distance to the ground during the last 1.00 s of his fall (**Fig. 2.30**). What is the height h of the building?

SOLUTION GUIDE

IDENTIFY and SET UP

1. You're told that Green Lantern falls freely from rest. What does this imply about his acceleration? About his initial velocity?
2. Choose the direction of the positive y-axis. It's easiest to make the same choice we used for freely falling objects in Section 2.5.
3. You can divide Green Lantern's fall into two parts: from the top of the building to the halfway point and from the halfway point to the ground. You know that the second part of the fall lasts 1.00 s. Decide what you would need to know about Green Lantern's motion at the halfway point in order to solve for the target variable h. Then choose two equations, one for the first part of the fall and one for the second part, that you'll use

Figure 2.30 Green Lantern in free fall.



together to find an expression for h. (There are several pairs of equations that you could choose.)

EXECUTE

4. Use your two equations to solve for the height h. Heights are always positive numbers, so your answer should be positive.

EVALUATE

5. To check your answer for h, use one of the free-fall equations to find how long it takes Green Lantern to fall (i) from the top of the building to half the height and (ii) from the top of the building to the ground. If your answer for h is correct, time (ii) should be 1.00 s greater than time (i). If it isn't, go back and look for errors in how you found h.

PROBLEMS

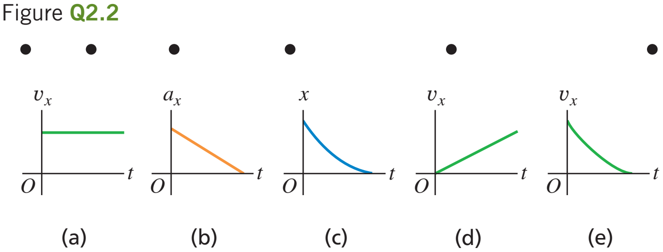
•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q2.1 Does the speedometer of a car measure speed or velocity? Explain.

Q2.2 The black dots at the top of **Fig. Q2.2** represent a series of high-speed photographs of an insect flying in a straight line from left to right (in the positive x-direction). Which of the graphs in Fig. Q2.2 most plausibly depicts this insect's motion?

Figure Q2.2



Q2.3 Can an object with constant acceleration reverse its direction of travel? Can it reverse its direction *twice*? In both cases, explain your reasoning.

Q2.4 Under what conditions is average velocity equal to instantaneous velocity?

Q2.5 Is it possible for an object to be (a) slowing down while its acceleration is increasing in magnitude; (b) speeding up while its acceleration is decreasing? In both cases, explain your reasoning.

Q2.6 Under what conditions does the magnitude of the average velocity equal the average speed?

Q2.7 When a Dodge Viper is at Elwood's Car Wash, a BMW Z3 is at Elm and Main. Later, when the Dodge reaches Elm and Main, the BMW reaches Elwood's Car Wash. How are the cars' average velocities between these two times related?

Q2.8 A driver in Massachusetts was sent to traffic court for speeding. The evidence against the driver was that a policewoman observed the driver's car alongside a second car at a certain moment, and the policewoman had already clocked the second car going faster than the speed limit. The driver argued, "The second car was passing me. I was not speeding." The judge ruled against the driver because, in the judge's words, "If two cars were side by side, both of you were speeding." If you were a lawyer representing the accused driver, how would you argue this case?

Q2.9 Can you have zero displacement and nonzero average velocity? Zero displacement and nonzero velocity? Illustrate your answers on an x-t graph.

Q2.10 Can you have zero acceleration and nonzero velocity? Use a v_x-t graph to explain.

Q2.11 Can you have zero velocity and nonzero average acceleration? Zero velocity and nonzero acceleration? Use a v_x-t graph to explain, and give an example of such motion.

Q2.12 An automobile is traveling west. Can it have a velocity toward the west and at the same time have an acceleration toward the east? Under what circumstances?

Q2.13 The official's truck in Fig. 2.2 is at $x_1 = 277$ m at $t_1 = 16.0$ s and is at $x_2 = 19$ m at $t_2 = 25.0$ s. (a) Sketch *two* different possible x-t graphs for the motion of the truck. (b) Does the average velocity $v_{\text{av-}x}$ during the time interval from t_1 to t_2 have the same value for both of your graphs? Why or why not?

Q2.14 Under constant acceleration the average velocity of a particle is half the sum of its initial and final velocities. Is this still true if the acceleration is *not* constant? Explain.

Q2.15 You throw a baseball straight up in the air so that it rises to a maximum height much greater than your height. Is the magnitude of the ball's acceleration greater while it is being thrown or after it leaves your hand? Explain.

Q2.16 Prove these statements: (a) As long as you can ignore the effects of the air, if you throw anything vertically upward, it will have the same speed when it returns to the release point as when it was released. (b) The time of flight will be twice the time it takes to get to its highest point.

Q2.17 A dripping water faucet steadily releases drops 1.0 s apart. As these drops fall, does the distance between them increase, decrease, or remain the same? Prove your answer.

Q2.18 If you know the initial position and initial velocity of a vehicle and have a record of the acceleration at each instant, can you compute the vehicle's position after a certain time? If so, explain how this might be done.

Q2.19 From the top of a tall building, you throw one ball straight up with speed v_0 and one ball straight down with speed v_0. (a) Which ball has the greater speed when it reaches the ground? (b) Which ball gets to the ground first? (c) Which ball has a greater displacement when it reaches the ground? (d) Which ball has traveled the greater distance when it hits the ground?

Q2.20 You run due east at a constant speed of 3.00 m/s for a distance of 120.0 m and then continue running east at a constant speed of 5.00 m/s for another 120.0 m. For the total 240.0 m run, is your average velocity 4.00 m/s, greater than 4.00 m/s, or less than 4.00 m/s? Explain.

Q2.21 An object is thrown straight up into the air and feels no air resistance. How can the object have an acceleration when it has stopped moving at its highest point?

Q2.22 When you drop an object from a certain height, it takes time T to reach the ground with no air resistance. If you dropped it from three times that height, how long (in terms of T) would it take to reach the ground?

EXERCISES

Section 2.1 Displacement, Time, and Average Velocity

2.1 • A car travels in the $+x$-direction on a straight and level road. For the first 4.00 s of its motion, the average velocity of the car is $v_{\text{av-}x} = 6.25$ m/s. How far does the car travel in 4.00 s?

2.2 •• In an experiment, a shearwater (a seabird) was taken from its nest, flown 5150 km away, and released. The bird found its way back to its nest 13.5 days after release. If we place the origin at the nest and extend the $+x$-axis to the release point, what was the bird's average velocity in m/s (a) for the return flight and (b) for the whole episode, from leaving the nest to returning?

2.3 •• **Trip Home.** You normally drive on the freeway between San Diego and Los Angeles at an average speed of 105 km/h (65 mi/h), and the trip takes 1 h and 50 min. On a Friday afternoon, however, heavy traffic slows you down and you drive the same distance at an average speed of only 70 km/h (43 mi/h). How much longer does the trip take?

2.4 •• **From Pillar to Post.** Starting from a pillar, you run 200 m east (the $+x$-direction) at an average speed of 5.0 m/s and then run 280 m west at an average speed of 4.0 m/s to a post. Calculate (a) your average speed from pillar to post and (b) your average velocity from pillar to post.

2.5 • Starting from the front door of a ranch house, you walk 60.0 m due east to a windmill, turn around, and then slowly walk 40.0 m west to a bench, where you sit and watch the sunrise. It takes you 28.0 s to walk from the house to the windmill and then 36.0 s to walk from the windmill to the bench. For the entire trip from the front door to the bench, what are your (a) average velocity and (b) average speed?

2.6 •• A Honda Civic travels in a straight line along a road. The car's distance x from a stop sign is given as a function of time t by the equation $x(t) = \alpha t^2 - \beta t^3$, where $\alpha = 1.50\ \text{m/s}^2$ and $\beta = 0.0500\ \text{m/s}^3$. Calculate the average velocity of the car for each time interval: (a) $t = 0$ to $t = 2.00$ s; (b) $t = 0$ to $t = 4.00$ s; (c) $t = 2.00$ s to $t = 4.00$ s.

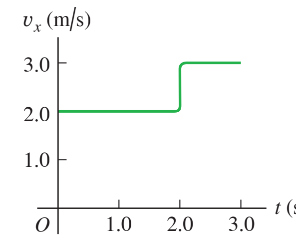
Section 2.2 Instantaneous Velocity

2.7 • CALC A car is stopped at a traffic light. It then travels along a straight road such that its distance from the light is given by $x(t) = bt^2 - ct^3$, where $b = 2.40\ \text{m/s}^2$ and $c = 0.120\ \text{m/s}^3$. (a) Calculate the average velocity of the car for the time interval $t = 0$ to $t = 10.0$ s. (b) Calculate the instantaneous velocity of the car at $t = 0$, $t = 5.0$ s, and $t = 10.0$ s. (c) How long after starting from rest is the car again at rest?

2.8 • CALC A bird is flying due east. Its distance from a tall building is given by $x(t) = 28.0\ \text{m} + (12.4\ \text{m/s})t - (0.0450\ \text{m/s}^3)t^3$. What is the instantaneous velocity of the bird when $t = 8.00$ s?

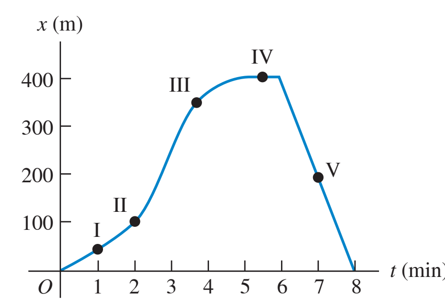
2.9 •• A ball moves in a straight line (the x-axis). The graph in **Fig. E2.9** shows this ball's velocity as a function of time. (a) What are the ball's average speed and average velocity during the first 3.0 s? (b) Suppose that the ball moved in such a way that the graph segment after 2.0 s was -3.0 m/s instead of $+3.0$ m/s. Find the ball's average speed and average velocity in this case.

Figure E2.9



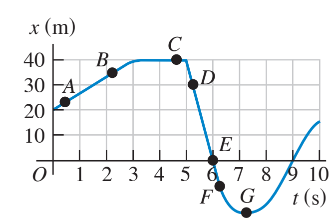
2.10 •• A physics professor leaves her house and walks along the sidewalk toward campus. After 5 min it starts to rain, and she returns home. Her distance from her house as a function of time is shown in **Fig. E2.10**. At which of the labeled points is her velocity (a) zero? (b) constant and positive? (c) constant and negative? (d) increasing in magnitude? (e) decreasing in magnitude?

Figure E2.10



2.11 •• A test car travels in a straight line along the x-axis. The graph in **Fig. E2.11** shows the car's position x as a function of time. Find its instantaneous velocity at points A through G.

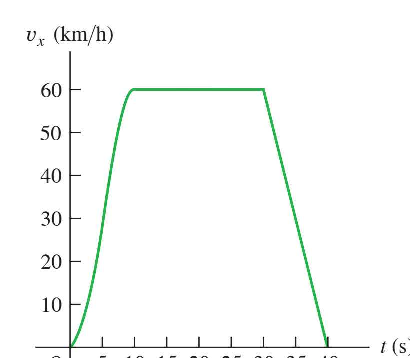
Figure E2.11



Section 2.3 Average and Instantaneous Acceleration

2.12 • **Figure E2.12** shows the velocity of a solar-powered car as a function of time. The driver accelerates from a stop sign, cruises for 20 s at a constant speed of 60 km/h, and then brakes to come to a stop 40 s after leaving the stop sign. (a) Compute the average acceleration during these time intervals: (i) $t = 0$ to $t = 10$ s; (ii) $t = 30$ s to $t = 40$ s; (iii) $t = 10$ s to $t = 30$ s; (iv) $t = 0$ to $t = 40$ s. (b) What is the instantaneous acceleration at $t = 20$ s and at $t = 35$ s?

Figure E2.12



2.13 • **CALC** A turtle crawls along a straight line, which we'll call the x-axis with the positive direction to the right. The equation for the turtle's position as a function of time is $x(t) = 50.0\text{ cm} + (2.00\text{ cm/s})t - (0.0625\text{ cm/s}^2)t^2$. (a) Find the turtle's initial velocity, initial position, and initial acceleration. (b) At what time t is the velocity of the turtle zero? (c) How long after starting does it take the turtle to return to its starting point? (d) At what times t is the turtle a distance of 10.0 cm from its starting point? What is the velocity (magnitude and direction) of the turtle at each of those times? (e) Sketch graphs of x versus t, v_x versus t, and a_x versus t, for the time interval $t = 0$ to $t = 40$ s.

2.14 •• **CALC** A race car starts from rest and travels east along a straight and level track. For the first 5.0 s of the car's motion, the eastward component of the car's velocity is given by $v_x(t) = (0.860\text{ m/s}^3)t^2$. What is the acceleration of the car when $v_x = 12.0$ m/s?

2.15 • **CALC** A car's velocity as a function of time is given by $v_x(t) = \alpha + \beta t^2$, where $\alpha = 3.00$ m/s and $\beta = 0.100\text{ m/s}^3$. (a) Calculate the average acceleration for the time interval $t = 0$ to $t = 5.00$ s. (b) Calculate the instantaneous acceleration for $t = 0$ and $t = 5.00$ s. (c) Draw v_x-t and a_x-t graphs for the car's motion between $t = 0$ and $t = 5.00$ s.

2.16 • An astronaut has left the International Space Station to test a new space scooter. Her partner measures the following velocity changes, each taking place in a 10 s interval. What are the magnitude, the algebraic sign, and the direction of the average acceleration in each interval? Assume that the positive direction is to the right. (a) At the beginning of the interval, the astronaut is moving toward the right along the x-axis at 15.0 m/s, and at the end of the interval she is moving toward the right at 5.0 m/s. (b) At the beginning she is moving toward the left at 5.0 m/s, and at the end she is moving toward the left at 15.0 m/s. (c) At the beginning she is moving toward the right at 15.0 m/s, and at the end she is moving toward the left at 15.0 m/s.

2.17 •• **CALC** The position of the front bumper of a test car under microprocessor control is given by $x(t) = 2.17\text{ m} + (4.80\text{ m/s}^2)t^2 - (0.100\text{ m/s}^6)t^6$. (a) Find its position and acceleration at the instants when the car has zero velocity. (b) Draw x-t, v_x-t, and a_x-t graphs for the motion of the bumper between $t = 0$ and $t = 2.00$ s.

Section 2.4 Motion with Constant Acceleration

2.18 • Estimate the distance in feet that your car travels on the entrance ramp to a freeway as it accelerates from 30 mph to the freeway speed of 70 mph. During this motion what is the average acceleration of the car?

2.19 •• An antelope moving with constant acceleration covers the distance between two points 70.0 m apart in 6.00 s. Its speed as it passes the second point is 15.0 m/s. What are (a) its speed at the first point and (b) its acceleration?

2.20 •• **A Tennis Serve.** In the fastest measured tennis serve, the ball left the racquet at 73.14 m/s. A served tennis ball is typically in contact with the racquet for 30.0 ms and starts from rest. Assume constant acceleration. (a) What was the ball's acceleration during this serve? (b) How far did the ball travel during the serve?

2.21 • **A Fast Pitch.** The fastest measured pitched baseball left the pitcher's hand at a speed of 45.0 m/s. If the pitcher was in contact with the ball over a distance of 1.50 m and produced constant acceleration, (a) what acceleration did he give the ball, and (b) how much time did it take him to pitch it?

2.22 • You are traveling on an interstate highway at the posted speed limit of 70 mph when you see that the traffic in front of you has stopped due to an accident up ahead. You step on your brakes to slow down as quickly as possible. (a) Estimate how many seconds it takes you to slow down to 30 mph. What is the magnitude of the average acceleration of the car while it is slowing down? With this same average acceleration, (b) how much longer would it take you to stop, and (c) what total distance would you travel from when you first apply the brakes until the car stops?

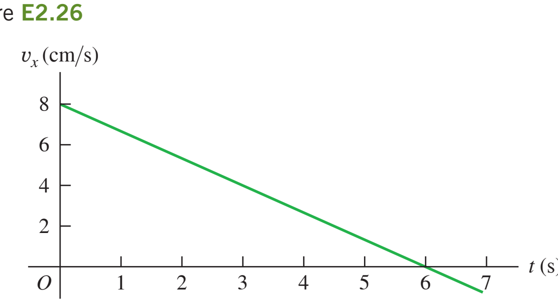
2.23 •• **BIO** **Automobile Airbags.** The human body can survive an acceleration trauma incident (sudden stop) if the magnitude of the acceleration is less than 250 m/s^2. If you are in an automobile accident with an initial speed of 105 km/h (65 mi/h) and are stopped by an airbag that inflates from the dashboard, over what minimum distance must the airbag stop you for you to survive the crash?

2.24 • **Entering the Freeway.** A car sits on an entrance ramp to a freeway, waiting for a break in the traffic. Then the driver accelerates with constant acceleration along the ramp and onto the freeway. The car starts from rest, moves in a straight line, and has a speed of 20 m/s (45 mi/h) when it reaches the end of the 120-m-long ramp. (a) What is the acceleration of the car? (b) How much time does it take the car to travel the length of the ramp? (c) The traffic on the freeway is moving at a constant speed of 20 m/s.What distance does the traffic travel while the car is moving the length of the ramp?

2.25 • **BIO** **Airbag Injuries.** During an auto accident, the vehicle's airbags deploy and slow down the passengers more gently than if they had hit the windshield or steering wheel. According to safety standards, airbags produce a maximum acceleration of $60g$ that lasts for only 36 ms (or less). How far (in meters) does a person travel in coming to a complete stop in 36 ms at a constant acceleration of $60g$?

2.26 •• A cat walks in a straight line, which we shall call the x-axis, with the positive direction to the right. As an observant physicist, you make measurements of this cat's motion and construct a graph of the feline's velocity as a function of time (**Fig. E2.26**). (a) Find the cat's velocity at $t = 4.0$ s and at $t = 7.0$ s. (b) What is the cat's acceleration at $t = 3.0$ s? At $t = 6.0$ s? At $t = 7.0$ s? (c) What distance does the cat move during the first 4.5 s? From $t = 0$ to $t = 7.0$ s? (d) Assuming that the cat started at the origin, sketch clear graphs of the cat's acceleration and position as functions of time.

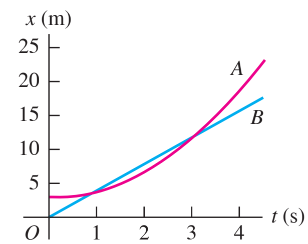
Figure E2.26



2.27 • **BIO** **Are We Martians?** It has been suggested, and not facetiously, that life might have originated on Mars and been carried to the earth when a meteor hit Mars and blasted pieces of rock (perhaps containing primitive life) free of the Martian surface. Astronomers know that many Martian rocks have come to the earth this way. (For instance, search the Internet for "ALH 84001.") One objection to this idea is that microbes would have had to undergo an enormous lethal acceleration during the impact. Let us investigate how large such an acceleration might be. To escape Mars, rock fragments would have to reach its escape velocity of 5.0 km/s, and that would most likely happen over a distance of about 4.0 m during the meteor impact. (a) What would be the acceleration (in m/s^2 and g's) of such a rock fragment, if the acceleration is constant? (b) How long would this acceleration last? (c) In tests, scientists have found that over 40% of *Bacillus subtilis* bacteria survived after an acceleration of $450{,}000g$. In light of your answer to part (a), can we rule out the hypothesis that life might have been blasted from Mars to the earth?

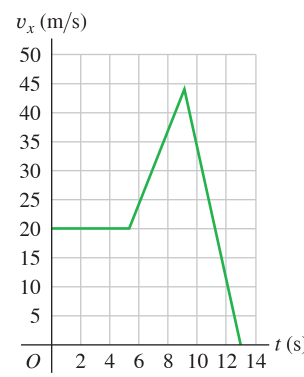
2.28 • Two cars, A and B, move along the x-axis. **Figure E2.28** is a graph of the positions of A and B versus time. (a) In motion diagrams (like Figs. 2.13b and 2.14b), show the position, velocity, and acceleration of each of the two cars at $t = 0$, $t = 1$ s, and $t = 3$ s. (b) At what time(s), if any, do A and B have the same position? (c) Graph velocity versus time for both A and B. (d) At what time(s), if any, do A and B have the same velocity? (e) At what time(s), if any, does car A pass car B? (f) At what time(s), if any, does car B pass car A?

Figure **E2.28**



2.29 •• The graph in **Fig. E2.29** shows the velocity of a motorcycle police officer plotted as a function of time. (a) Find the instantaneous acceleration at $t = 3$ s, $t = 7$ s, and $t = 11$ s. (b) How far does the officer go in the first 5 s? The first 9 s? The first 13 s?

Figure **E2.29**



2.30 •• A small block has constant acceleration as it slides down a frictionless incline. The block is released from rest at the top of the incline, and its speed after it has traveled 6.80 m to the bottom of the incline is 3.80 m/s. What is the speed of the block when it is 3.40 m from the top of the incline?

2.31 •• (a) If a flea can jump straight up to a height of 0.440 m, what is its initial speed as it leaves the ground? (b) How long is it in the air?

2.32 •• A small rock is thrown vertically upward with a speed of 22.0 m/s from the edge of the roof of a 30.0-m-tall building. The rock doesn't hit the building on its way back down and lands on the street below. Ignore air resistance. (a) What is the speed of the rock just before it hits the street? (b) How much time elapses from when the rock is thrown until it hits the street?

Section 2.5 Freely Falling Objects

2.33 • A juggler throws a bowling pin straight up with an initial speed of 8.20 m/s. How much time elapses until the bowling pin returns to the juggler's hand?

2.34 •• You throw a glob of putty straight up toward the ceiling, which is 3.60 m above the point where the putty leaves your hand. The initial speed of the putty as it leaves your hand is 9.50 m/s. (a) What is the speed of the putty just before it strikes the ceiling? (b) How much time from when it leaves your hand does it take the putty to reach the ceiling?

2.35 •• A tennis ball on Mars, where the acceleration due to gravity is $0.379g$ and air resistance is negligible, is hit directly upward and returns to the same level 8.5 s later. (a) How high above its original point did the ball go? (b) How fast was it moving just after it was hit? (c) Sketch graphs of the ball's vertical position, vertical velocity, and vertical acceleration as functions of time while it's in the Martian air.

2.36 •• Estimate the maximum height in feet that you can throw a baseball straight up. (a) For this height, how long after the ball leaves your hand does it return to your hand? (b) Estimate the distance in feet that the ball moves while you are throwing it—that is, the distance from where the ball is when you start your throw until it leaves your hand. Calculate the average acceleration in m/s^2 that the ball has while it is being thrown, as it moves from rest to the point where it leaves your hand.

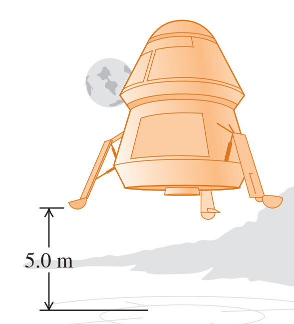
2.37 •• A rock is thrown straight up with an initial speed of 24.0 m/s. Neglect air resistance. (a) At $t = 1.0$ s, what are the directions of the velocity and acceleration of the rock? Is the speed of the rock increasing or decreasing? (b) At $t = 3.0$ s, what are the directions of the velocity and acceleration of the rock? Is the speed of the rock increasing or decreasing?

2.38 •• A brick is dropped (zero initial speed) from the roof of a building. The brick strikes the ground in 1.90 s. You may ignore air resistance, so the brick is in free fall. (a) How tall, in meters, is the building? (b) What is the magnitude of the brick's velocity just before it reaches the ground? (c) Sketch a_y-t, v_y-t, and y-t graphs for the motion of the brick.

2.39 •• **A Simple Reaction-Time Test.** A meter stick is held vertically above your hand, with the lower end between your thumb and first finger. When you see the meter stick released, you grab it with those two fingers. You can calculate your reaction time from the distance the meter stick falls, read directly from the point where your fingers grabbed it. (a) Derive a relationship for your reaction time in terms of this measured distance, d. (b) If the measured distance is 17.6 cm, what is your reaction time?

2.40 •• **Touchdown on the Moon.** A lunar lander is making its descent to Moon Base I (**Fig. E2.40**). The lander descends slowly under the retro-thrust of its descent engine. The engine is cut off when the lander is 5.0 m above the surface and has a downward speed of 0.8 m/s. With the engine off, the lander is in free fall. What is the speed of the lander just before it touches the surface? The acceleration due to gravity on the moon is 1.6 m/s^2.

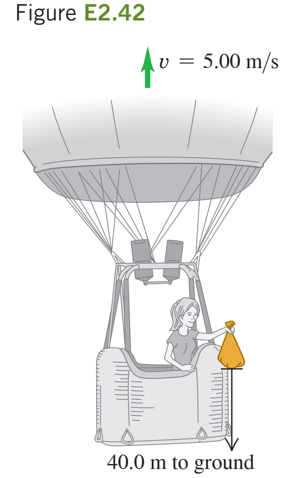
Figure **E2.40**



2.41 •• **Launch Failure.** A 7500 kg rocket blasts off vertically from the launch pad with a constant upward acceleration of 2.25 m/s^2 and feels no appreciable air resistance. When it has reached a height of 525 m, its engines suddenly fail; the only force acting on it is now gravity. (a) What is the maximum height this rocket will reach above the launch pad? (b) How much time will elapse after engine failure before the rocket comes crashing down to the launch pad, and how fast will it be moving just before it crashes? (c) Sketch a_y-t, v_y-t, and y-t graphs of the rocket's motion from the instant of blast-off to the instant just before it strikes the launch pad.

2.42 •• A hot-air balloonist, rising vertically with a constant velocity of magnitude 5.00 m/s, releases a sandbag at an instant when the balloon is 40.0 m above the ground (**Fig. E2.42**). After the sandbag is released, it is in free fall. (a) Compute the position and velocity of the sandbag at 0.250 s and 1.00 s after its release. (b) How many seconds after its release does the bag strike the ground? (c) With what magnitude of velocity does it strike the ground? (d) What is the greatest height above the ground that the sandbag reaches? (e) Sketch a_y-t, v_y-t, and y-t graphs for the motion.

Figure **E2.42**



2.43 • You throw a rock straight up and find that it returns to your hand 3.60 s after it left your hand. Neglect air resistance. What was the maximum height above your hand that the rock reached?

2.44 • An egg is thrown nearly vertically upward from a point near the cornice of a tall building. The egg just misses the cornice on the way down and passes a point 30.0 m below its starting point 5.00 s after it leaves the thrower's hand. Ignore air resistance. (a) What is the initial speed of the egg? (b) How high does it rise above its starting point? (c) What is the magnitude of its velocity at the highest point? (d) What are the magnitude and direction of its acceleration at the highest point? (e) Sketch a_y-t, v_y-t, and y-t graphs for the motion of the egg.

2.45 •• A 15 kg rock is dropped from rest on the earth and reaches the ground in 1.75 s. When it is dropped from the same height on Saturn's satellite Enceladus, the rock reaches the ground in 18.6 s. What is the acceleration due to gravity on Enceladus?

2.46 • A large boulder is ejected vertically upward from a volcano with an initial speed of 40.0 m/s. Ignore air resistance. (a) At what time after being ejected is the boulder moving at 20.0 m/s upward? (b) At what time is it moving at 20.0 m/s downward? (c) When is the displacement of the boulder from its initial position zero? (d) When is the velocity of the boulder zero? (e) What are the magnitude and direction of the acceleration while the boulder is (i) moving upward? (ii) Moving downward? (iii) At the highest point? (f) Sketch a_y-t, v_y-t, and y-t graphs for the motion.

2.47 •• You throw a small rock straight up from the edge of a highway bridge that crosses a river. The rock passes you on its way down, 6.00 s after it was thrown. What is the speed of the rock just before it reaches the water 28.0 m below the point where the rock left your hand? Ignore air resistance.

Section 2.6 Velocity and Position by Integration

2.48 •• Consider the motion described by the v_x-t graph of Fig. E2.26. (a) Calculate the area under the graph between $t = 0$ and $t = 6.0$ s. (b) For the time interval $t = 0$ to $t = 6.0$ s, what is the magnitude of the average velocity of the cat? (c) Use constant-acceleration equations to calculate the distance the cat travels in this time interval. How does your result compare to the area you calculated in part (a)?

2.49 • **CALC** A rocket starts from rest and moves upward from the surface of the earth. For the first 10.0 s of its motion, the vertical acceleration of the rocket is given by $a_y = (2.80\ \text{m/s}^3)t$, where the $+y$-direction is upward. (a) What is the height of the rocket above the surface of the earth at $t = 10.0$ s? (b) What is the speed of the rocket when it is 325 m above the surface of the earth?

2.50 •• **CALC** A small object moves along the x-axis with acceleration $a_x(t) = -(0.0320\ \text{m/s}^3)(15.0\ \text{s} - t)$. At $t = 0$ the object is at $x = -14.0$ m and has velocity $v_{0x} = 8.00$ m/s. What is the x-coordinate of the object when $t = 10.0$ s?

2.51 •• **CALC** The acceleration of a motorcycle is given by $a_x(t) = At - Bt^2$, where $A = 1.50\ \text{m/s}^3$ and $B = 0.120\ \text{m/s}^4$. The motorcycle is at rest at the origin at time $t = 0$. (a) Find its position and velocity as functions of time. (b) Calculate the maximum velocity it attains.

2.52 •• **CALC** The acceleration of a bus is given by $a_x(t) = \alpha t$, where $\alpha = 1.2\ \text{m/s}^3$. (a) If the bus's velocity at time $t = 1.0$ s is 5.0 m/s, what is its velocity at time $t = 2.0$ s? (b) If the bus's position at time $t = 1.0$ s is 6.0 m, what is its position at time $t = 2.0$ s? (c) Sketch a_y-t, v_y-t, and x-t graphs for the motion.

PROBLEMS

2.53 • **BIO** A typical male sprinter can maintain his maximum acceleration for 2.0 s, and his maximum speed is 10 m/s. After he reaches this maximum speed, his acceleration becomes zero, and then he runs at constant speed. Assume that his acceleration is constant during the first 2.0 s of the race, that he starts from rest, and that he runs in a straight line. (a) How far has the sprinter run when he reaches his maximum speed? (b) What is the magnitude of his average velocity for a race of these lengths: (i) 50.0 m; (ii) 100.0 m; (iii) 200.0 m?

2.54 • **CALC** A lunar lander is descending toward the moon's surface. Until the lander reaches the surface, its height above the surface of the moon is given by $y(t) = b - ct + dt^2$, where $b = 800$ m is the initial height of the lander above the surface, $c = 60.0$ m/s, and $d = 1.05\ \text{m/s}^2$. (a) What is the initial velocity of the lander, at $t = 0$? (b) What is the velocity of the lander just before it reaches the lunar surface?

2.55 ••• **Earthquake Analysis.** Earthquakes produce several types of shock waves. The most well known are the P-waves (P for *primary* or *pressure*) and the S-waves (S for *secondary* or *shear*). In the earth's crust, P-waves travel at about 6.5 km/s and S-waves move at about 3.5 km/s. The time delay between the arrival of these two waves at a seismic recording station tells geologists how far away an earthquake occurred. If the time delay is 33 s, how far from the seismic station did the earthquake occur?

2.56 •• You throw a small rock straight up with initial speed V_0 from the edge of the roof of a building that is a distance H above the ground. The rock travels upward to a maximum height in time T_{max}, misses the edge of the roof on its way down, and reaches the ground in time T_{total} after it was thrown. Neglect air resistance. If the total time the rock is in the air is three times the time it takes it to reach its maximum height, so $T_{\text{total}} = 3T_{\text{max}}$, then in terms of H what must be the value of V_0?

2.57 ••• A rocket carrying a satellite is accelerating straight up from the earth's surface. At 1.15 s after liftoff, the rocket clears the top of its launch platform, 63 m above the ground. After an additional 4.75 s, it is 1.00 km above the ground. Calculate the magnitude of the average velocity of the rocket for (a) the 4.75 s part of its flight and (b) the first 5.90 s of its flight.

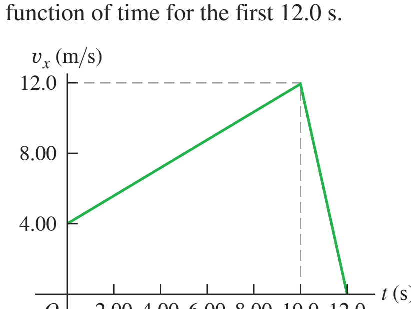
2.58 •• A block moving on a horizontal surface is at $x = 0$ when $t = 0$ and is sliding east with a speed of 12.0 m/s. Because of a net force acting on the block, it has a constant acceleration with direction west and magnitude $2.00\ \text{m/s}^2$. The block travels east, slows down, reverses direction, and then travels west with increasing speed. (a) At what value of t is the block again at $x = 0$? (b) What is the maximum distance east of $x = 0$ that the rock reaches, and how long does it take the rock to reach this point?

2.59 •• A block is sliding with constant acceleration down an incline. The block starts from rest at $t = 0$ and has speed 3.00 m/s after it has traveled a distance 8.00 m from its starting point. (a) What is the speed of the block when it is a distance of 16.0 m from its $t = 0$ starting point? (b) How long does it take the block to slide 16.0 m from its starting point?

2.60 ••• A subway train starts from rest at a station and accelerates at a rate of $1.60\ \text{m/s}^2$ for 14.0 s. It runs at constant speed for 70.0 s and slows down at a rate of $3.50\ \text{m/s}^2$ until it stops at the next station. Find the *total* distance covered.

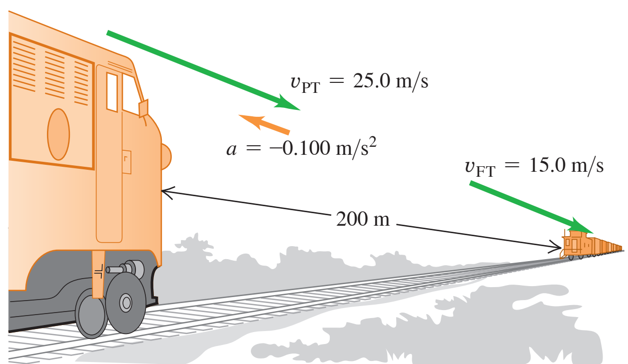
2.61 • A gazelle is running in a straight line (the x-axis). The graph in **Fig. P2.61** shows this animal's velocity as a function of time. During the first 12.0 s, find (a) the total distance moved and (b) the displacement of the gazelle. (c) Sketch an a_x-t graph showing this gazelle's acceleration as a function of time for the first 12.0 s.

Figure **P2.61**



2.62 •• Collision. The engineer of a passenger train traveling at 25.0 m/s sights a freight train whose caboose is 200 m ahead on the same track (**Fig. P2.62**). The freight train is traveling at 15.0 m/s in the same direction as the passenger train. The engineer of the passenger train immediately applies the brakes, causing a constant acceleration of 0.100 m/s^2 in a direction opposite to the train's velocity, while the freight train continues with constant speed. Take $x = 0$ at the location of the front of the passenger train when the engineer applies the brakes. (a) Will the cows nearby witness a collision? (b) If so, where will it take place? (c) On a single graph, sketch the positions of the front of the passenger train and the back of the freight train.

Figure **P2.62**



2.63 ••• A ball starts from rest and rolls down a hill with uniform acceleration, traveling 200 m during the second 5.0 s of its motion. How far did it roll during the first 5.0 s of motion?

2.64 •• A rock moving in the $+x$-direction with speed 16.0 m/s has a net force applied to it at time $t = 0$, and this produces a constant acceleration in the $-x$-direction that has magnitude 4.00 m/s^2. For what three times t after the force is applied is the rock a distance of 24.0 m from its position at $t = 0$? For each of these three values of t, what is the velocity (magnitude and direction) of the rock?

2.65 • A car and a truck start from rest at the same instant, with the car initially at some distance behind the truck. The truck has a constant acceleration of 2.10 m/s^2, and the car has an acceleration of 3.40 m/s^2. The car overtakes the truck after the truck has moved 60.0 m. (a) How much time does it take the car to overtake the truck? (b) How far was the car behind the truck initially? (c) What is the speed of each when they are abreast? (d) On a single graph, sketch the position of each vehicle as a function of time. Take $x = 0$ at the initial location of the truck.

2.66 •• You are standing at rest at a bus stop. A bus moving at a constant speed of 5.00 m/s passes you. When the rear of the bus is 12.0 m past you, you realize that it is your bus, so you start to run toward it with a constant acceleration of 0.960 m/s^2. How far would you have to run before you catch up with the rear of the bus, and how fast must you be running then? Would an average college student be physically able to accomplish this?

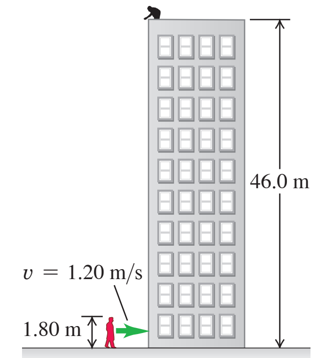
2.67 •• A sprinter runs a 100 m dash in 12.0 s. She starts from rest with a constant acceleration a_x for 3.0 s and then runs with constant speed for the remainder of the race. What is the value of a_x?

2.68 •• CALC An object's velocity is measured to be $v_x(t) = \alpha - \beta t^2$, where $\alpha = 4.00$ m/s and $\beta = 2.00$ m/s^3. At $t = 0$ the object is at $x = 0$. (a) Calculate the object's position and acceleration as functions of time. (b) What is the object's maximum *positive* displacement from the origin?

2.69 •• CALC An object is moving along the x-axis. At $t = 0$ it is at $x = 0$. Its x-component of velocity v_x as a function of time is given by $v_x(t) = \alpha t - \beta t^3$, where $\alpha = 8.0$ m/s^2 and $\beta = 4.0$ m/s^4. (a) At what nonzero time t is the object again at $x = 0$? (b) At the time calculated in part (a), what are the velocity and acceleration of the object (magnitude and direction)?

2.70 • Egg Drop. You are on the roof of the physics building, 46.0 m above the ground (**Fig. P2.70**). Your physics professor, who is 1.80 m tall, is walking alongside the building at a constant speed of 1.20 m/s. If you wish to drop an egg on your professor's head, where should the professor be when you release the egg? Assume that the egg is in free fall.

Figure **P2.70**



2.71 ••• CALC The acceleration of a particle is given by $a_x(t) = -2.00\text{ m/s}^2 + (3.00\text{ m/s}^3)t$. (a) Find the initial velocity v_{0x} such that the particle will have the same x-coordinate at $t = 4.00$ s as it had at $t = 0$. (b) What will be the velocity at $t = 4.00$ s?

2.72 •• A small rock is thrown straight up with initial speed v_0 from the edge of the roof of a building with height H. The rock travels upward and then downward to the ground at the base of the building. Let $+y$ be upward, and neglect air resistance. (a) For the rock's motion from the roof to the ground, what is the vertical component $v_{\text{av-}y}$ of its average velocity? Is this quantity positive or negative? Explain. (b) What does your expression for $v_{\text{av-}y}$ give in the limit that H is zero? Explain. (c) Show that your result in part (a) agrees with Eq. (2.10).

2.73 •• A watermelon is dropped from the edge of the roof of a building and falls to the ground. You are standing on the sidewalk and see the watermelon falling when it is 30.0 m above the ground. Then 1.50 s after you first spot it, the watermelon lands at your feet. What is the height of the building? Neglect air resistance.

2.74 ••• A flowerpot falls off a windowsill and passes the window of the story below. Ignore air resistance. It takes the pot 0.380 s to pass from the top to the bottom of this window, which is 1.90 m high. How far is the top of the window below the windowsill from which the flowerpot fell?

2.75 ••• Look Out Below. Sam heaves a 16 lb shot straight up, giving it a constant upward acceleration from rest of 35.0 m/s^2 for 64.0 cm. He releases it 2.20 m above the ground. Ignore air resistance. (a) What is the speed of the shot when Sam releases it? (b) How high above the ground does it go? (c) How much time does he have to get out of its way before it returns to the height of the top of his head, 1.83 m above the ground?

2.76 ••• A Multistage Rocket. In the first stage of a two-stage rocket, the rocket is fired from the launch pad starting from rest but with a constant acceleration of 3.50 m/s^2 upward. At 25.0 s after launch, the second stage fires for 10.0 s, which boosts the rocket's velocity to 132.5 m/s upward at 35.0 s after launch. This firing uses up all of the fuel, however, so after the second stage has finished firing, the only force acting on the rocket is gravity. Ignore air resistance. (a) Find the maximum height that the stage-two rocket reaches above the launch pad. (b) How much time after the end of the stage-two firing will it take for the rocket to fall back to the launch pad? (c) How fast will the stage-two rocket be moving just as it reaches the launch pad?

2.77 •• Two stones are thrown vertically upward from the ground, one with three times the initial speed of the other. (a) If the faster stone takes 10 s to return to the ground, how long will it take the slower stone to return? (b) If the slower stone reaches a maximum height of H, how high (in terms of H) will the faster stone go? Assume free fall.

2.78 ••• During your summer internship for an aerospace company, you are asked to design a small research rocket. The rocket is to be launched from rest from the earth's surface and is to reach a maximum height of 960 m above the earth's surface. The rocket's engines give the rocket an upward acceleration of $16.0\ \text{m/s}^2$ during the time T that they fire. After the engines shut off, the rocket is in free fall. Ignore air resistance. What must be the value of T in order for the rocket to reach the required altitude?

2.79 ••• A helicopter carrying Dr. Evil takes off with a constant upward acceleration of $5.0\ \text{m/s}^2$. Secret agent Austin Powers jumps on just as the helicopter lifts off the ground. After the two men struggle for 10.0 s, Powers shuts off the engine and steps out of the helicopter. Assume that the helicopter is in free fall after its engine is shut off, and ignore the effects of air resistance. (a) What is the maximum height above ground reached by the helicopter? (b) Powers deploys a jet pack strapped on his back 7.0 s after leaving the helicopter, and then he has a constant downward acceleration with magnitude $2.0\ \text{m/s}^2$. How far is Powers above the ground when the helicopter crashes into the ground?

2.80 •• **Cliff Height.** You are climbing in the High Sierra when you suddenly find yourself at the edge of a fog-shrouded cliff. To find the height of this cliff, you drop a rock from the top; 8.00 s later you hear the sound of the rock hitting the ground at the foot of the cliff. (a) If you ignore air resistance, how high is the cliff if the speed of sound is 330 m/s? (b) Suppose you had ignored the time it takes the sound to reach you. In that case, would you have overestimated or underestimated the height of the cliff? Explain.

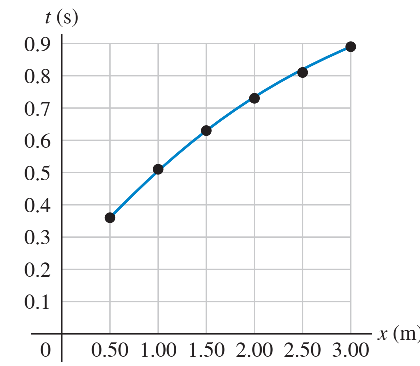
2.81 •• CALC An object is moving along the x-axis. At $t = 0$ it has velocity $v_{0x} = 20.0\ \text{m/s}$. Starting at time $t = 0$ it has acceleration $a_x = -Ct$, where C has units of m/s^3. (a) What is the value of C if the object stops in 8.00 s after $t = 0$? (b) For the value of C calculated in part (a), how far does the object travel during the 8.00 s?

2.82 •• A ball is thrown straight up from the ground with speed v_0. At the same instant, a second ball is dropped from rest from a height H, directly above the point where the first ball was thrown upward. There is no air resistance. (a) Find the time at which the two balls collide. (b) Find the value of H in terms of v_0 and g such that at the instant when the balls collide, the first ball is at the highest point of its motion.

2.83 • CALC Cars A and B travel in a straight line. The distance of A from the starting point is given as a function of time by $x_A(t) = \alpha t + \beta t^2$, with $\alpha = 2.60\ \text{m/s}$ and $\beta = 1.20\ \text{m/s}^2$. The distance of B from the starting point is $x_B(t) = \gamma t^2 - \delta t^3$, with $\gamma = 2.80\ \text{m/s}^2$ and $\delta = 0.20\ \text{m/s}^3$. (a) Which car is ahead just after the two cars leave the starting point? (b) At what time(s) are the cars at the same point? (c) At what time(s) is the distance from A to B neither increasing nor decreasing? (d) At what time(s) do A and B have the same acceleration?

2.84 •• DATA In your physics lab you release a small glider from rest at various points on a long, frictionless air track that is inclined at an angle θ above the horizontal. With an electronic photocell, you measure the time t it takes the glider to slide a distance x from the release point to the bottom of the track. Your measurements are given in **Fig. P2.84**, which shows a second-order polynomial (quadratic) fit to the plotted data. You are asked to find the glider's acceleration, which is assumed to be constant. There is some error in each measurement, so instead of using a single set of x and t values, you can be more accurate if you use graphical methods and obtain your measured value of the acceleration from the graph. (a) How can you re-graph the data so that the data points fall close to a straight line? (*Hint:* You might want to plot x or t, or both, raised to some power.) (b) Construct the graph you described in part (a) and find the equation for the straight line that is the best fit to the data points. (c) Use the straight-line fit from part (b) to calculate the acceleration of the glider. (d) The glider is released at a distance $x = 1.35$ m from the bottom of the track. Use the acceleration value you obtained in part (c) to calculate the speed of the glider when it reaches the bottom of the track.

Figure **P2.84**



2.85 •• DATA In a physics lab experiment, you release a small steel ball at various heights above the ground and measure the ball's speed just before it strikes the ground. You plot your data on a graph that has the release height (in meters) on the vertical axis and the square of the final speed (in m^2/s^2) on the horizontal axis. In this graph your data points lie close to a straight line. (a) Using $g = 9.80\ \text{m/s}^2$ and ignoring the effect of air resistance, what is the numerical value of the slope of this straight line? (Include the correct units.) The presence of air resistance reduces the magnitude of the downward acceleration, and the effect of air resistance increases as the speed of the object increases. You repeat the experiment, but this time with a tennis ball as the object being dropped. Air resistance now has a noticeable effect on the data. (b) Is the final speed for a given release height higher than, lower than, or the same as when you ignored air resistance? (c) Is the graph of the release height versus the square of the final speed still a straight line? Sketch the qualitative shape of the graph when air resistance is present.

2.86 ••• DATA A model car starts from rest and travels in a straight line. A smartphone mounted on the car has an app that transmits the magnitude of the car's acceleration (measured by an accelerometer) every second. The results are given in the table:

Time (s)	Acceleration (m/s^2)
0	5.95
1.00	5.52
2.00	5.08
3.00	4.55
4.00	3.96
5.00	3.40

Each measured value has some experimental error. (a) Plot acceleration versus time and find the equation for the straight line that gives the best fit to the data. (b) Use the equation for $a(t)$ that you found in part (a) to calculate $v(t)$, the speed of the car as a function of time. Sketch the graph of v versus t. Is this graph a straight line? (c) Use your result from part (b) to calculate the speed of the car at $t = 5.00$ s. (d) Calculate the distance the car travels between $t = 0$ and $t = 5.00$ s.

CHALLENGE PROBLEMS

2.87 ••• In the vertical jump, an athlete starts from a crouch and jumps upward as high as possible. Even the best athletes spend little more than 1.00 s in the air (their "hang time"). Treat the athlete as a particle and let y_{max} be his maximum height above the floor. To explain why he seems to hang in the air, calculate the ratio of the time he is above $y_{max}/2$ to the time it takes him to go from the floor to that height. Ignore air resistance.

2.88 ••• **Catching the Bus.** A student is running at her top speed of 5.0 m/s to catch a bus, which is stopped at the bus stop. When the student is still 40.0 m from the bus, it starts to pull away, moving with a constant acceleration of 0.170 m/s^2. (a) For how much time and what distance does the student have to run at 5.0 m/s before she overtakes the bus? (b) When she reaches the bus, how fast is the bus traveling? (c) Sketch an *x-t* graph for both the student and the bus. Take $x = 0$ at the initial position of the student. (d) The equations you used in part (a) to find the time have a second solution, corresponding to a later time for which the student and bus are again at the same place if they continue their specified motions. Explain the significance of this second solution. How fast is the bus traveling at this point? (e) If the student's top speed is 3.5 m/s, will she catch the bus? (f) What is the *minimum* speed the student must have to just catch up with the bus? For what time and what distance does she have to run in that case?

2.89 ••• A ball is thrown straight up from the edge of the roof of a building. A second ball is dropped from the roof 1.00 s later. Ignore air resistance. (a) If the height of the building is 20.0 m, what must the initial speed of the first ball be if both are to hit the ground at the same time? On the same graph, sketch the positions of both balls as a function of time, measured from when the first ball is thrown. Consider the same situation, but now let the initial speed v_0 of the first ball be given and treat the height h of the building as an unknown. (b) What must the height of the building be for both balls to reach the ground at the same time if (i) v_0 is 6.0 m/s and (ii) v_0 is 9.5 m/s? (c) If v_0 is greater than some value v_{max}, no value of h exists that allows both balls to hit the ground at the same time. Solve for v_{max}. The value v_{max} has a simple physical interpretation. What is it? (d) If v_0 is less than some value v_{min}, no value of h exists that allows both balls to hit the ground at the same time. Solve for v_{min}. The value v_{min} also has a simple physical interpretation. What is it?

MCAT-STYLE PASSAGE PROBLEMS

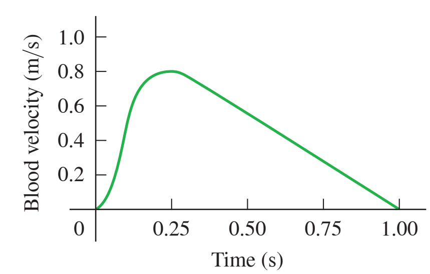
BIO Blood Flow in the Heart. The human circulatory system is closed—that is, the blood pumped out of the left ventricle of the heart into the arteries is constrained to a series of continuous, branching vessels as it passes through the capillaries and then into the veins as it returns to the heart. The blood in each of the heart's four chambers comes briefly to rest before it is ejected by contraction of the heart muscle.

2.90 If the contraction of the left ventricle lasts 250 ms and the speed of blood flow in the aorta (the large artery leaving the heart) is 0.80 m/s at the end of the contraction, what is the average acceleration of a red blood cell as it leaves the heart? (a) 310 m/s^2; (b) 31 m/s^2; (c) 3.2 m/s^2; (d) 0.32 m/s^2.

2.91 If the aorta (diameter d_a) branches into two equal-sized arteries with a combined area equal to that of the aorta, what is the diameter of one of the branches? (a) $\sqrt{d_a}$; (b) $d_a/\sqrt{2}$; (c) $2d_a$; (d) $d_a/2$.

2.92 The velocity of blood in the aorta can be measured directly with ultrasound techniques. A typical graph of blood velocity versus time during a single heartbeat is shown in **Fig. P2.92**. Which statement is the best interpretation of this graph? (a) The blood flow changes direction at about 0.25 s; (b) the speed of the blood flow begins to decrease at about 0.10 s; (c) the acceleration of the blood is greatest in magnitude at about 0.25 s; (d) the acceleration of the blood is greatest in magnitude at about 0.10 s.

Figure **P2.92**



ANSWERS

Chapter Opening Question ?

(iii) Acceleration refers to *any* change in velocity, including both speeding up and slowing down.

Key Example VARIATION Problems

VP2.5.1 (a) 4.5 s (b) 81 m
VP2.5.2 (a) 4.80 m/s^2 (b) 375 m
VP2.5.3 (a) 11.1 s (b) 222 m (c) 10.0 m/s
VP2.5.4 (a) $t = 2v_C/(a_T + a_C)$ (b) $x = 2a_T v_C^2/(a_T + a_C)^2$
VP2.7.1 (a) $y = 3.16$ m, $v_y = 9.06$ m/s, above your hand and moving upward (b) $y = -1.92$ m, $v_y = -13.5$ m/s, below your hand and moving downward
VP2.7.2 (a) $y = -23.0$ m, $v_y = -22.7$ m/s (b) $v_y = -14.9$ m/s
VP2.7.3 (a) 8.87 m/s (b) 0.854 s
VP2.7.4 (a) $\sqrt{2gH}$ (b) $\sqrt{gH} = 1/\sqrt{2} = 0.707$ times the speed at which it left your hand (c) $3H/4$
VP2.8.1 (a) 0.398 s and 2.05 s (b) 2.75 s
VP2.8.2 0.447 s
VP2.8.3 (a) two; $t =$ 0.338 s (moving upward) and $t =$ 0.784 s (moving downward) (b) none
VP2.8.4 (a) $t = v_0/g$ (neither upward nor downward) (b) $t = v_0/2g$ (upward), $t = 3v_0/2g$ (downward)

Bridging Problem

$h = 57.1$ m

? If a cyclist is going around a curve at constant speed, is he accelerating? If so, what is the direction of his acceleration? (i) No; (ii) yes, in the direction of his motion; (iii) yes, toward the inside of the curve; (iv) yes, toward the outside of the curve; (v) yes, but in some other direction.

3 Motion in Two or Three Dimensions

LEARNING OUTCOMES

In this chapter, you'll learn...

3.1 How to use vectors to represent the position and velocity of a particle in two or three dimensions.

3.2 How to find the vector acceleration of a particle, why a particle can have an acceleration even if its speed is constant, and how to interpret the components of acceleration parallel and perpendicular to a particle's path.

3.3 How to solve problems that involve the curved path followed by a projectile.

3.4 How to analyze motion in a circular path, with either constant speed or varying speed.

3.5 How to relate the velocities of a moving object as seen from two different frames of reference.

You'll need to review...

2.1 Average *x*-velocity.

2.2 Instantaneous *x*-velocity.

2.3 Average and instantaneous *x*-acceleration.

2.4 Straight-line motion with constant acceleration.

2.5 The motion of freely falling objects.

What determines where a batted baseball lands? How do you describe the motion of a roller coaster car along a curved track or the flight of a circling hawk? Which hits the ground first: a baseball that you simply drop or one that you throw horizontally?

We can't answer these kinds of questions by using the techniques of Chapter 2, in which particles moved only along a straight line. Instead, we need to extend our descriptions of motion to two- and three-dimensional situations. We'll still use the vector quantities displacement, velocity, and acceleration, but now these quantities will no longer lie along a single line. We'll find that several important kinds of motion take place in two dimensions only—that is, in a *plane*.

We also need to consider how the motion of a particle is described by different observers who are moving relative to each other. The concept of *relative velocity* will play an important role later in the book when we explore electromagnetic phenomena and when we introduce Einstein's special theory of relativity.

This chapter merges the vector mathematics of Chapter 1 with the kinematic language of Chapter 2. As before, we're concerned with describing motion, not with analyzing its causes. But the language you learn here will be an essential tool in later chapters when we study the relationship between force and motion.

3.1 POSITION AND VELOCITY VECTORS

Let's see how to describe a particle's motion in space. If the particle is at a point P at a certain instant, the **position vector** $\vec{r}$ of the particle at this instant is a vector that goes from the origin of the coordinate system to point P (**Fig. 3.1**). The Cartesian coordinates x, y, and z of point P are the x-, y-, and z-components of vector $\vec{r}$. Using the unit vectors we introduced in Section 1.9, we can write

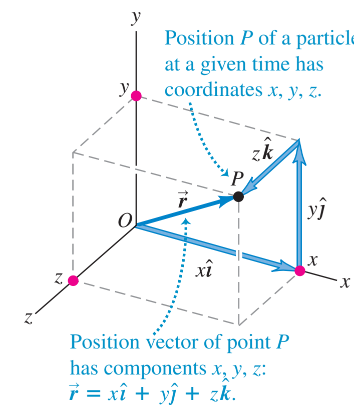
Position vector of a particle at a given instant

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \tag{3.1}$$

Coordinates of particle's position

Unit vectors in x-, y-, and z-directions

Figure 3.1 The position vector $\vec{r}$ from origin O to point P has components x, y, and z.



During a time interval Δt the particle moves from P_1, where its position vector is $\vec{r}_1$, to P_2, where its position vector is $\vec{r}_2$. The change in position (the displacement) during this interval is $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath} + (z_2 - z_1)\hat{k}$. We define the **average velocity** $\vec{v}_{av}$ during this interval in the same way we did in Chapter 2 for straight-line motion, as the displacement divided by the time interval (**Fig. 3.2**):

Change in the particle's position vector

Average velocity vector of a particle during time interval from t_1 to t_2

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} \tag{3.2}$$

Final position minus initial position

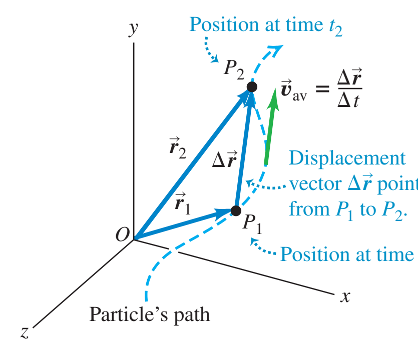
Time interval

Final time minus initial time

Dividing a vector by a scalar is a special case of *multiplying* a vector by a scalar, described in Section 1.7; the average velocity $\vec{v}_{av}$ is equal to the displacement vector $\Delta\vec{r}$ multiplied by $1/\Delta t$. Note that the x-component of Eq. (3.2) is $v_{av\text{-}x} = (x_2 - x_1)/(t_2 - t_1) = \Delta x/\Delta t$. This is just Eq. (2.2), the expression for average x-velocity that we found in Section 2.1 for one-dimensional motion.

We now define **instantaneous velocity** just as we did in Chapter 2: It equals the instantaneous rate of change of position with time. The key difference is that both position $\vec{r}$ and instantaneous velocity $\vec{v}$ are now vectors:

Figure 3.2 The average velocity $\vec{v}_{av}$ between points P_1 and P_2 has the same direction as the displacement $\Delta\vec{r}$.



The **instantaneous velocity vector** of a particle ...

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \tag{3.3}$$

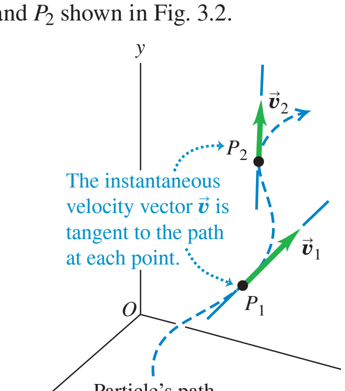
... equals the limit of its average velocity vector as the time interval approaches zero ...

... and equals the instantaneous rate of change of its position vector.

At any instant, the *magnitude* of $\vec{v}$ is the *speed* v of the particle at that instant, and the *direction* of $\vec{v}$ is the direction in which the particle is moving at that instant.

As $\Delta t \to 0$, points P_1 and P_2 in Fig. 3.2 move closer and closer together. In this limit, the vector $\Delta\vec{r}$ becomes tangent to the path. The direction of $\Delta\vec{r}$ in this limit is also the direction of $\vec{v}$. So *at every point along the path, the instantaneous velocity vector is tangent to the path at that point* (**Fig. 3.3**).

Figure 3.3 The vectors $\vec{v}_1$ and $\vec{v}_2$ are the instantaneous velocities at the points P_1 and P_2 shown in Fig. 3.2.



It's often easiest to calculate the instantaneous velocity vector by using components. During any displacement $\Delta\vec{r}$, the changes Δx, Δy, and Δz in the three coordinates of the particle are the *components* of $\Delta\vec{r}$. It follows that the components v_x, v_y, and v_z of the instantaneous velocity $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$ are simply the time derivatives of the coordinates x, y, and z:

Each **component of** a particle's **instantaneous velocity vector** ...

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt} \tag{3.4}$$

... equals the instantaneous rate of change of its corresponding coordinate.

The x-component of $\vec{v}$ is $v_x = dx/dt$, which is the same as Eq. (2.3) for straight-line motion (see Section 2.2). Hence Eq. (3.4) is a direct extension of instantaneous velocity to motion in three dimensions.

We can also get Eq. (3.4) by taking the derivative of Eq. (3.1). The unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$ don't depend on time, so their derivatives are zero and we find

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k} \tag{3.5}$$

This shows again that the components of $\vec{v}$ are dx/dt, dy/dt, and dz/dt.

The magnitude of the instantaneous velocity vector $\vec{v}$—that is, the speed—is given in terms of the components v_x, v_y, and v_z by the Pythagorean relationship:

$$|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2} \tag{3.6}$$

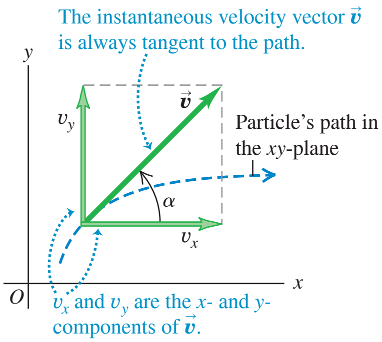


Figure 3.4 The two velocity components for motion in the xy-plane.

Figure 3.4 shows the situation when the particle moves in the xy-plane. In this case, z and v_z are zero. Then the speed (the magnitude of $\vec{v}$) is

$$v = \sqrt{v_x^2 + v_y^2}$$

and the direction of the instantaneous velocity $\vec{v}$ is given by angle α (the Greek letter alpha) in the figure. We see that

$$\tan\alpha = \frac{v_y}{v_x} \tag{3.7}$$

(We use α for the direction of the instantaneous velocity vector to avoid confusion with the direction θ of the *position* vector of the particle.)

From now on, when we use the word "velocity," we'll always mean the *instantaneous* velocity vector $\vec{v}$ (rather than the average velocity vector). Usually, we won't even bother to call $\vec{v}$ a vector; it's up to you to remember that velocity is a vector quantity with both magnitude and direction.

EXAMPLE 3.1 Calculating average and instantaneous velocity

A robotic vehicle, or rover, is exploring the surface of Mars. The stationary Mars lander is the origin of coordinates, and the surrounding Martian surface lies in the xy-plane. The rover, which we represent as a point, has x- and y-coordinates that vary with time:

$$x = 2.0\ \text{m} - (0.25\ \text{m/s}^2)t^2$$
$$y = (1.0\ \text{m/s})t + (0.025\ \text{m/s}^3)t^3$$

(a) Find the rover's coordinates and distance from the lander at $t = 2.0$ s. (b) Find the rover's displacement and average velocity vectors for the interval $t = 0.0$ s to $t = 2.0$ s. (c) Find a general expression for the rover's instantaneous velocity vector $\vec{v}$. Express $\vec{v}$ at $t = 2.0$ s in component form and in terms of magnitude and direction.

IDENTIFY and SET UP This problem involves motion in two dimensions, so we must use the vector equations obtained in this section. **Figure 3.5** shows the rover's path (dashed line). We'll use Eq. (3.1) for position $\vec{r}$, the expression $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ for displacement, Eq. (3.2) for average velocity, and Eqs. (3.5), (3.6), and (3.7) for instantaneous velocity and its magnitude and direction.

EXECUTE (a) At $t = 2.0$ s the rover's coordinates are

$$x = 2.0\ \text{m} - (0.25\ \text{m/s}^2)(2.0\ \text{s})^2 = 1.0\ \text{m}$$
$$y = (1.0\ \text{m/s})(2.0\ \text{s}) + (0.025\ \text{m/s}^3)(2.0\ \text{s})^3 = 2.2\ \text{m}$$

The rover's distance from the origin at this time is

$$r = \sqrt{x^2 + y^2} = \sqrt{(1.0\ \text{m})^2 + (2.2\ \text{m})^2} = 2.4\ \text{m}$$

(b) To find the displacement and average velocity over the given time interval, we first express the position vector $\vec{r}$ as a function of time t. From Eq. (3.1) this is

$$\vec{r} = x\hat{\imath} + y\hat{\jmath}$$
$$= [2.0\ \text{m} - (0.25\ \text{m/s}^2)t^2]\hat{\imath} + [(1.0\ \text{m/s})t + (0.025\ \text{m/s}^3)t^3]\hat{\jmath}$$

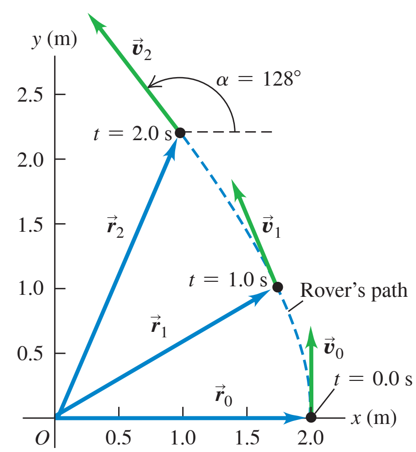
At $t = 0.0$ s the position vector $\vec{r}_0$ is

$$\vec{r}_0 = (2.0\ \text{m})\hat{\imath} + (0.0\ \text{m})\hat{\jmath}$$

From part (a), the position vector $\vec{r}_2$ at $t = 2.0$ s is

$$\vec{r}_2 = (1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath}$$

Figure 3.5 At $t = 0.0$ s the rover has position vector $\vec{r}_0$ and instantaneous velocity vector $\vec{v}_0$. Likewise, $\vec{r}_1$ and $\vec{v}_1$ are the vectors at $t = 1.0$ s; $\vec{r}_2$ and $\vec{v}_2$ are the vectors at $t = 2.0$ s.



The displacement from $t = 0.0$ s to $t = 2.0$ s is therefore

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_0 = (1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath} - (2.0\ \text{m})\hat{\imath}$$
$$= (-1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath}$$

During this interval the rover moves 1.0 m in the negative x-direction and 2.2 m in the positive y-direction. From Eq. (3.2), the average velocity over this interval is the displacement divided by the elapsed time:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{(-1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath}}{2.0\ \text{s} - 0.0\ \text{s}}$$
$$= (-0.50\ \text{m/s})\hat{\imath} + (1.1\ \text{m/s})\hat{\jmath}$$

The components of this average velocity are $v_{av\text{-}x} = -0.50$ m/s and $v_{av\text{-}y} = 1.1$ m/s.

(c) From Eq. (3.4) the components of *instantaneous* velocity are the time derivatives of the coordinates:

$$v_x = \frac{dx}{dt} = (-0.25\ \text{m/s}^2)(2t)$$
$$v_y = \frac{dy}{dt} = 1.0\ \text{m/s} + (0.025\ \text{m/s}^3)(3t^2)$$

Hence the instantaneous velocity vector is

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath}$$
$$= (-0.50\text{ m/s}^2)t\hat{\imath} + [1.0\text{ m/s} + (0.075\text{ m/s}^3)t^2]\,\hat{\jmath}$$

At $t = 2.0$ s the velocity vector $\vec{v}_2$ has components

$$v_{2x} = (-0.50\text{ m/s}^2)(2.0\text{ s}) = -1.0\text{ m/s}$$
$$v_{2y} = 1.0\text{ m/s} + (0.075\text{ m/s}^3)(2.0\text{ s})^2 = 1.3\text{ m/s}$$

The magnitude of the instantaneous velocity (that is, the speed) at $t = 2.0$ s is

$$v_2 = \sqrt{v_{2x}{}^2 + v_{2y}{}^2} = \sqrt{(-1.0\text{ m/s})^2 + (1.3\text{ m/s})^2} = 1.6\text{ m/s}$$

Figure 3.5 shows the direction of velocity vector $\vec{v}_2$, which is at an angle α between 90° and 180° with respect to the positive x-axis. From Eq. (3.7) we have

$$\arctan\frac{v_y}{v_x} = \arctan\frac{1.3\text{ m/s}}{-1.0\text{ m/s}} = -52°$$

This is off by 180°; the correct value is $\alpha = 180° - 52° = 128°$, or 38° west of north.

EVALUATE Compare the components of *average* velocity from part (b) for the interval from $t = 0.0$ s to $t = 2.0$ s ($v_{\text{av-}x} = -0.50$ m/s, $v_{\text{av-}y} = 1.1$ m/s) with the components of *instantaneous* velocity at $t = 2.0$ s from part (c) ($v_{2x} = -1.0$ m/s, $v_{2y} = 1.3$ m/s). Just as in one dimension, the average velocity vector $\vec{v}_{av}$ over an interval is in general *not* equal to the instantaneous velocity $\vec{v}$ at the end of the interval (see Example 2.1).

Figure 3.5 shows the position vectors $\vec{r}$ and instantaneous velocity vectors $\vec{v}$ at $t = 0.0$ s, 1.0 s, and 2.0 s. (Calculate these quantities for $t = 0.0$ s and $t = 1.0$ s.) Notice that $\vec{v}$ is tangent to the path at every point. The magnitude of $\vec{v}$ increases as the rover moves, which means that its speed is increasing.

KEYCONCEPT To calculate the average velocity vector of an object, first find its displacement vector during a time interval. Then divide by the time interval. To calculate the object's instantaneous velocity vector (its average velocity vector over an infinitesimally short time interval), take the derivative of its position vector with respect to time.

TEST YOUR UNDERSTANDING OF SECTION 3.1 In which of these situations *would* the average velocity vector $\vec{v}_{av}$ over an interval be equal to the instantaneous velocity $\vec{v}$ at the end of the interval? (i) An object moving along a curved path at constant speed; (ii) an object moving along a curved path and speeding up; (iii) an object moving along a straight line at constant speed; (iv) an object moving along a straight line and speeding up.

ANSWER

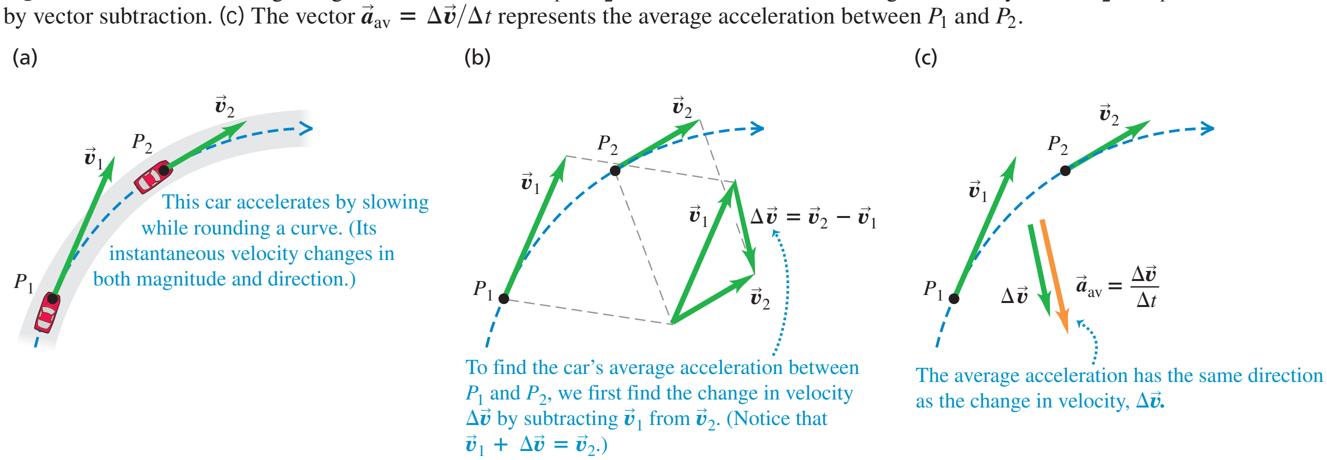
(iii) If the instantaneous velocity $\vec{v}$ is constant over an interval, its value at any point (including the end of the interval) is the same as the average velocity $\vec{v}_{av}$ over the interval. In (i) and (ii) the direction of $\vec{v}$ at the end of the interval is tangent to the path at that point, while the direction of $\vec{v}_{av}$ points from the beginning of the path to its end (in the direction of the net displacement). In (iv) both $\vec{v}$ and $\vec{v}_{av}$ are directed along the straight line, but $\vec{v}$ has a greater magnitude because the speed has been increasing.

3.2 THE ACCELERATION VECTOR

Now let's consider the *acceleration* of a particle moving in space. Just as for motion in a straight line, acceleration describes how the velocity of the particle changes. But since we now treat velocity as a vector, acceleration will describe changes in the velocity magnitude (that is, the speed) *and* changes in the direction of velocity (that is, the direction in which the particle is moving).

In **Fig. 3.6a**, a car (treated as a particle) is moving along a curved road. Vectors $\vec{v}_1$ and $\vec{v}_2$ represent the car's instantaneous velocities at time t_1, when the car is at point

Figure 3.6 (a) A car moving along a curved road from P_1 to P_2. (b) How to obtain the change in velocity $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ by vector subtraction. (c) The vector $\vec{a}_{av} = \Delta\vec{v}/\Delta t$ represents the average acceleration between P_1 and P_2.



P_1, and at time t_2, when the car is at point P_2. During the time interval from t_1 to t_2, the *vector change in velocity* is $\vec{v}_2 - \vec{v}_1 = \Delta\vec{v}$, so $\vec{v}_2 = \vec{v}_1 + \Delta\vec{v}$ (Fig. 3.6b). The **average acceleration** $\vec{a}_{av}$ of the car during this time interval is the velocity change divided by the time interval $t_2 - t_1 = \Delta t$:

Change in the particle's velocity

Average acceleration vector of a particle during time interval from t_1 to t_2

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} \qquad (3.8)$$

Final velocity minus initial velocity

Time interval

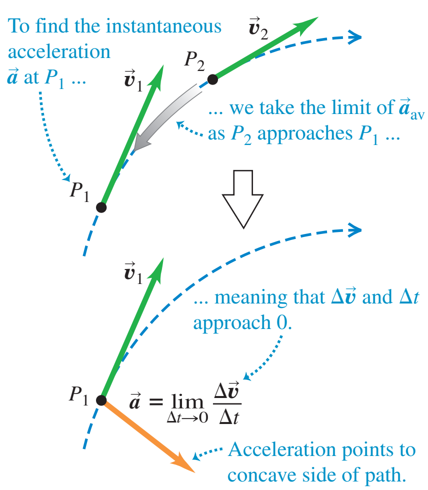
Final time minus initial time

Average acceleration is a *vector* quantity in the same direction as $\Delta\vec{v}$ (Fig. 3.6c). The x-component of Eq. (3.8) is $a_{av\text{-}x} = (v_{2x} - v_{1x})/(t_2 - t_1) = \Delta v_x/\Delta t$, which is just Eq. (2.4) for average acceleration in straight-line motion.

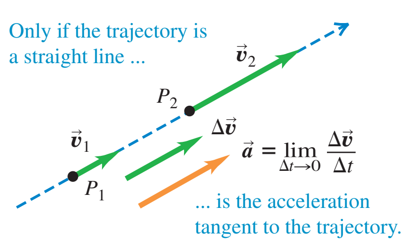
As in Chapter 2, we define the **instantaneous acceleration** $\vec{a}$ (a *vector* quantity) at point P_1 as the limit of the average acceleration vector when point P_2 approaches point P_1, so both $\Delta\vec{v}$ and Δt approach zero (**Fig. 3.7**):

Figure 3.7 (a) Instantaneous acceleration $\vec{a}$ at point P_1 in Fig. 3.6. (b) Instantaneous acceleration for motion along a straight line.

(a) Acceleration: curved trajectory



(b) Acceleration: straight-line trajectory



The **instantaneous acceleration vector** of a particle ...

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \qquad (3.9)$$

... equals the limit of its average acceleration vector as the time interval approaches zero ...

... and equals the instantaneous rate of change of its velocity vector.

The velocity vector $\vec{v}$ is always tangent to the particle's path, but the instantaneous acceleration vector $\vec{a}$ does *not* have to be tangent to the path. If the path is curved, $\vec{a}$ points toward the concave side of the path—that is, toward the inside of any turn that the particle is making (Fig. 3.7a). The acceleration is tangent to the path only if the particle moves in a straight line (Fig. 3.7b).

CAUTION **Any particle following a curved path is accelerating** When a particle is moving in a curved path, it always has nonzero acceleration, even when it moves with constant speed. This conclusion is contrary to the everyday use of the word "acceleration" to mean that speed is increasing. The more precise definition given in Eq. (3.9) shows that there is a nonzero acceleration whenever the velocity vector changes in *any* way, whether there is a change of speed, direction, or both.

BIO APPLICATION Horses on a Curved Path By leaning to the side and hitting the ground with their hooves at an angle, these horses give themselves the sideways acceleration necessary to make a sharp change in direction.

To convince yourself that a particle is accelerating as it moves on a curved path with constant speed, think of your sensations when you ride in a car. When the car accelerates, you tend to move inside the car in a direction *opposite* to the car's acceleration. (In Chapter 4 we'll learn why this is so.) Thus you tend to slide toward the back of the car when it accelerates forward (speeds up) and toward the front of the car when it accelerates backward (slows down). If the car makes a turn on a level road, you tend to slide toward the outside of the turn; hence the car is accelerating toward the inside of the turn.

We'll usually be interested in instantaneous acceleration, not average acceleration. From now on, we'll use the term "acceleration" to mean the instantaneous acceleration vector $\vec{a}$.

Each component of the acceleration vector $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$ is the derivative of the corresponding component of velocity:

Each **component of** a particle's **instantaneous acceleration vector** ...

$$a_x = \frac{dv_x}{dt} \qquad a_y = \frac{dv_y}{dt} \qquad a_z = \frac{dv_z}{dt} \qquad (3.10)$$

... equals the instantaneous rate of change of its corresponding velocity component.

In terms of unit vectors,

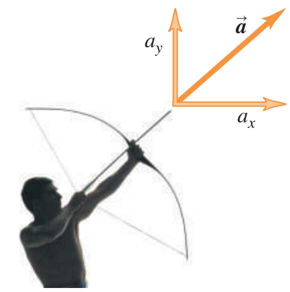
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k} \tag{3.11}$$

The x-component of Eqs. (3.10) and (3.11), $a_x = dv_x/dt$, is just Eq. (2.5) for instantaneous acceleration in one dimension. **Figure 3.8** shows an example of an acceleration vector that has both x- and y-components.

Since each component of velocity is the derivative of the corresponding coordinate, we can express the components a_x, a_y, and a_z of the acceleration vector $\vec{a}$ as

$$a_x = \frac{d^2x}{dt^2} \qquad a_y = \frac{d^2y}{dt^2} \qquad a_z = \frac{d^2z}{dt^2} \tag{3.12}$$

Figure 3.8 When the fingers release the arrow, its acceleration vector has a horizontal component (a_x) and a vertical component (a_y).



EXAMPLE 3.2 Calculating average and instantaneous acceleration

Let's return to the motions of the Mars rover in Example 3.1. (a) Find the components of the average acceleration for the interval $t = 0.0$ s to $t = 2.0$ s. (b) Find the instantaneous acceleration at $t = 2.0$ s.

IDENTIFY and SET UP In Example 3.1 we found the components of the rover's instantaneous velocity at any time t:

$$v_x = \frac{dx}{dt} = (-0.25\ \text{m/s}^2)(2t) = (-0.50\ \text{m/s}^2)t$$

$$v_y = \frac{dy}{dt} = 1.0\ \text{m/s} + (0.025\ \text{m/s}^3)(3t^2) = 1.0\ \text{m/s} + (0.075\ \text{m/s}^3)t^2$$

We'll use the vector relationships among velocity, average acceleration, and instantaneous acceleration. In part (a) we determine the values of v_x and v_y at the beginning and end of the interval and then use Eq. (3.8) to calculate the components of the average acceleration. In part (b) we obtain expressions for the instantaneous acceleration components at any time t by taking the time derivatives of the velocity components as in Eqs. (3.10).

EXECUTE (a) In Example 3.1 we found that at $t = 0.0$ s the velocity components are

$$v_x = 0.0\ \text{m/s} \qquad v_y = 1.0\ \text{m/s}$$

and that at $t = 2.0$ s the components are

$$v_x = -1.0\ \text{m/s} \qquad v_y = 1.3\ \text{m/s}$$

Thus the components of average acceleration in the interval $t = 0.0$ s to $t = 2.0$ s are

$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{-1.0\ \text{m/s} - 0.0\ \text{m/s}}{2.0\ \text{s} - 0.0\ \text{s}} = -0.50\ \text{m/s}^2$$

$$a_{\text{av-}y} = \frac{\Delta v_y}{\Delta t} = \frac{1.3\ \text{m/s} - 1.0\ \text{m/s}}{2.0\ \text{s} - 0.0\ \text{s}} = 0.15\ \text{m/s}^2$$

(b) Using Eqs. (3.10), we find

$$a_x = \frac{dv_x}{dt} = -0.50\ \text{m/s}^2 \qquad a_y = \frac{dv_y}{dt} = (0.075\ \text{m/s}^3)(2t)$$

Hence the instantaneous acceleration vector $\vec{a}$ at time t is

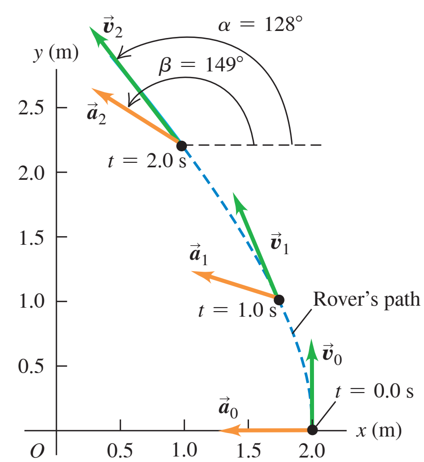
$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = (-0.50\ \text{m/s}^2)\hat{\imath} + (0.15\ \text{m/s}^3)t\hat{\jmath}$$

At $t = 2.0$ s the components of acceleration and the acceleration vector are

$$a_x = -0.50\ \text{m/s}^2 \qquad a_y = (0.15\ \text{m/s}^3)(2.0\ \text{s}) = 0.30\ \text{m/s}^2$$

$$\vec{a} = (-0.50\ \text{m/s}^2)\hat{\imath} + (0.30\ \text{m/s}^2)\hat{\jmath}$$

Figure 3.9 The path of the robotic rover, showing the velocity and acceleration at $t = 0.0$ s ($\vec{v}_0$ and $\vec{a}_0$), $t = 1.0$ s ($\vec{v}_1$ and $\vec{a}_1$), and $t = 2.0$ s ($\vec{v}_2$ and $\vec{a}_2$).



The magnitude of acceleration at this time is

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.50\ \text{m/s}^2)^2 + (0.30\ \text{m/s}^2)^2} = 0.58\ \text{m/s}^2$$

A sketch of this vector (**Fig. 3.9**) shows that the direction angle β of $\vec{a}$ with respect to the positive x-axis is between 90° and 180°. From Eq. (3.7) we have

$$\arctan\frac{a_y}{a_x} = \arctan\frac{0.30\ \text{m/s}^2}{-0.50\ \text{m/s}^2} = -31°$$

Hence $\beta = 180° + (-31°) = 149°$.

EVALUATE Figure 3.9 shows the rover's path and the velocity and acceleration vectors at $t = 0.0$ s, 1.0 s, and 2.0 s. (Use the results of part (b) to calculate the instantaneous acceleration at $t = 0.0$ s and $t = 1.0$ s for yourself.) Note that $\vec{v}$ and $\vec{a}$ are *not* in the same direction at any of these times. The velocity vector $\vec{v}$ is tangent to the path at each point (as is always the case), and the acceleration vector $\vec{a}$ points toward the concave side of the path.

KEYCONCEPT To calculate the average acceleration vector of an object, first find the change in its velocity vector (final velocity minus initial velocity) during a time interval. Then divide by the time interval. To calculate the object's instantaneous acceleration vector (its average velocity vector over an infinitesimally short time interval), take the derivative of its velocity vector with respect to time.

Figure 3.10 The acceleration can be resolved into a component $a_{\parallel}$ parallel to the path (that is, along the tangent to the path) and a component $a_{\perp}$ perpendicular to the path (that is, along the normal to the path).

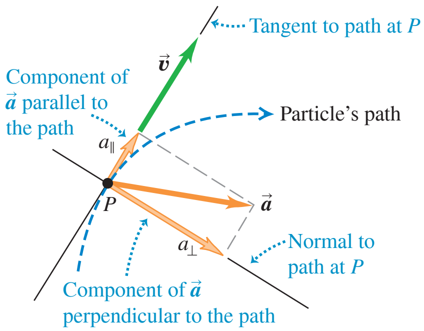
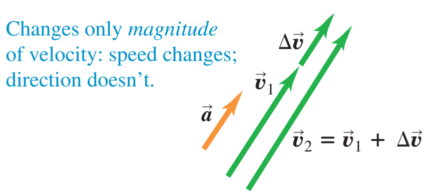
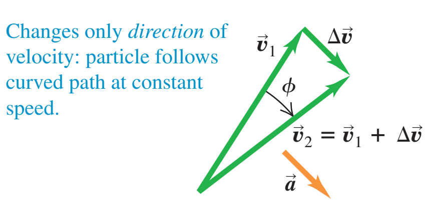


Figure 3.11 The effect of acceleration directed (a) parallel to and (b) perpendicular to a particle's velocity.

(a) Acceleration parallel to velocity



(b) Acceleration perpendicular to velocity



Parallel and Perpendicular Components of Acceleration

Equations (3.10) tell us about the components of a particle's instantaneous acceleration vector $\vec{a}$ along the x-, y-, and z-axes. Another useful way to think about $\vec{a}$ is in terms of one component *parallel* to the particle's path and to its velocity $\vec{v}$, and one component *perpendicular* to the path and to $\vec{v}$ (**Fig. 3.10**). That's because the parallel component $a_{\parallel}$ tells us about changes in the particle's *speed,* while the perpendicular component $a_{\perp}$ tells us about changes in the particle's *direction of motion*. To see why the parallel and perpendicular components of $\vec{a}$ have these properties, let's consider two special cases.

In **Fig. 3.11a** the acceleration vector is in the same direction as the velocity $\vec{v}_1$, so $\vec{a}$ has only a parallel component $a_{\parallel}$ (that is, $a_{\perp} = 0$). The velocity change $\Delta\vec{v}$ during a small time interval Δt is in the same direction as $\vec{a}$ and hence in the same direction as $\vec{v}_1$. The velocity $\vec{v}_2$ at the end of Δt is in the same direction as $\vec{v}_1$ but has greater magnitude. Hence during the time interval Δt the particle in Fig. 3.11a moved in a straight line with increasing speed (compare Fig. 3.7b).

In Fig. 3.11b the acceleration is *perpendicular* to the velocity, so $\vec{a}$ has only a perpendicular component $a_{\perp}$ (that is, $a_{\parallel} = 0$). In a small time interval Δt, the velocity change $\Delta\vec{v}$ is very nearly perpendicular to $\vec{v}_1$, and so $\vec{v}_1$ and $\vec{v}_2$ have different directions. As the time interval Δt approaches zero, the angle ϕ in the figure also approaches zero, $\Delta\vec{v}$ becomes perpendicular to *both* $\vec{v}_1$ and $\vec{v}_2$, and $\vec{v}_1$ and $\vec{v}_2$ have the same magnitude. In other words, the speed of the particle stays the same, but the direction of motion changes and the path of the particle curves.

In the most general case, the acceleration $\vec{a}$ has *both* components parallel and perpendicular to the velocity $\vec{v}$, as in Fig. 3.10. Then the particle's speed will change (described by the parallel component $a_{\parallel}$) *and* its direction of motion will change (described by the perpendicular component $a_{\perp}$).

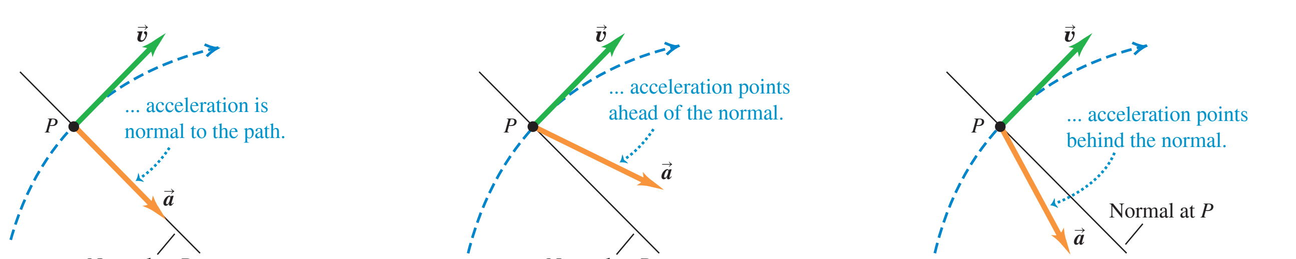
Figure 3.12 shows a particle moving along a curved path for three situations: constant speed, increasing speed, and decreasing speed. If the speed is constant, $\vec{a}$ is perpendicular, or *normal,* to the path and to $\vec{v}$ and points toward the concave side of the path (Fig. 3.12a). If the speed is increasing, there is still a perpendicular component of $\vec{a}$, but there is also a parallel component with the same direction as $\vec{v}$ (Fig. 3.12b). Then $\vec{a}$ points ahead of the normal to the path. (This was the case in Example 3.2.) If the speed is decreasing, the parallel component has the direction opposite to $\vec{v}$, and $\vec{a}$ points behind the normal to the path (Fig. 3.12c; compare Fig. 3.7a). We'll use these ideas again in Section 3.4 when we study the special case of motion in a circle.

Figure 3.12 Velocity and acceleration vectors for a particle moving through a point P on a curved path with (a) constant speed, (b) increasing speed, and (c) decreasing speed.

(a) When speed is constant along a curved path ...

(b) When speed is increasing along a curved path ...

(c) When speed is decreasing along a curved path ...



EXAMPLE 3.3 Calculating parallel and perpendicular components of acceleration

For the rover of Examples 3.1 and 3.2, find the parallel and perpendicular components of the acceleration at $t = 2.0$ s.

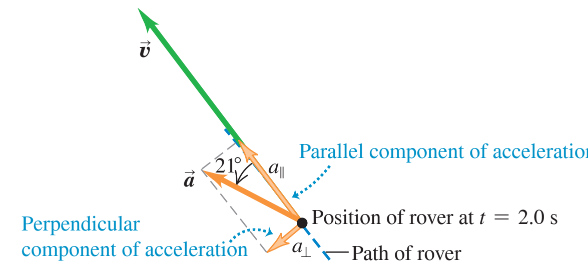
IDENTIFY and SET UP We want to find the components of the acceleration vector $\vec{a}$ that are parallel and perpendicular to velocity vector $\vec{v}$. We found the directions of $\vec{v}$ and $\vec{a}$ in Examples 3.1 and 3.2, respectively; Fig. 3.9 shows the results. From these directions we can find the angle between the two vectors and the components of $\vec{a}$ with respect to the direction of $\vec{v}$.

EXECUTE From Example 3.2, at $t = 2.0$ s the particle has an acceleration of magnitude $0.58\ \text{m/s}^2$ at an angle of 149° with respect to the positive x-axis. In Example 3.1 we found that at this time the velocity vector is at an angle of 128° with respect to the positive x-axis. The angle between $\vec{a}$ and $\vec{v}$ is therefore $149° - 128° = 21°$ (**Fig. 3.13**). Hence the components of acceleration parallel and perpendicular to $\vec{v}$ are

$$a_{\parallel} = a \cos 21° = (0.58\ \text{m/s}^2)\cos 21° = 0.54\ \text{m/s}^2$$
$$a_{\perp} = a \sin 21° = (0.58\ \text{m/s}^2)\sin 21° = 0.21\ \text{m/s}^2$$

EVALUATE The parallel component $a_{\parallel}$ is positive (in the same direction as $\vec{v}$), which means that the speed is increasing at this instant. The value $a_{\parallel} = +0.54\ \text{m/s}^2$ tells us that the speed is increasing at this instant at a rate of 0.54 m/s per second. The perpendicular component $a_{\perp}$ is not zero, which means that at this instant the rover is turning—that is, it is changing direction and following a curved path.

Figure 3.13 The parallel and perpendicular components of the acceleration of the rover at $t = 2.0$ s.



KEYCONCEPT If an object's *speed* is changing, there is a component of its acceleration vector *parallel* to its velocity vector. If an object's *direction of motion* is changing—that is, it is turning—there is a component of its acceleration vector *perpendicular* to its velocity vector and toward the inside of the turn.

CONCEPTUAL EXAMPLE 3.4 Acceleration of a skier

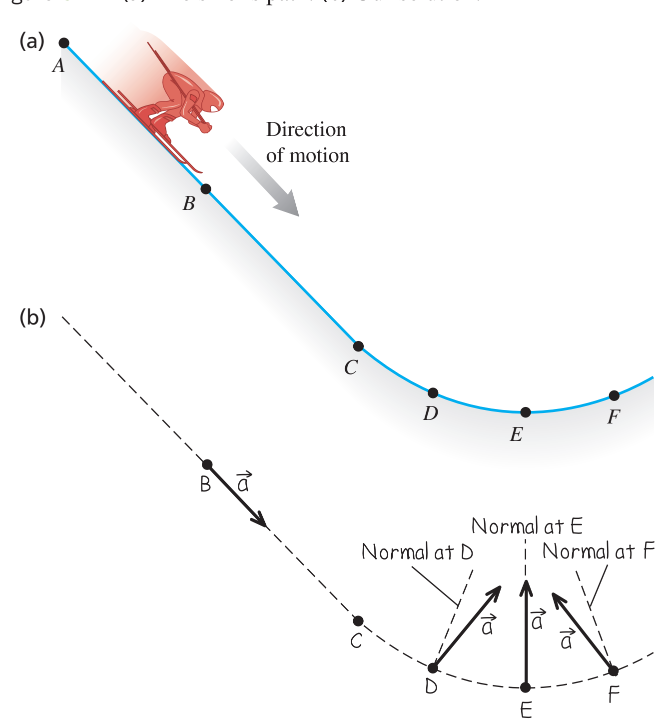
A skier moves along a ski-jump ramp (**Fig. 3.14a**). The ramp is straight from point A to point C and curved from point C onward. The skier speeds up as she moves downhill from point A to point E, where her speed is maximum. She slows down after passing point E. Draw the direction of the acceleration vector at each of the points B, D, E, and F.

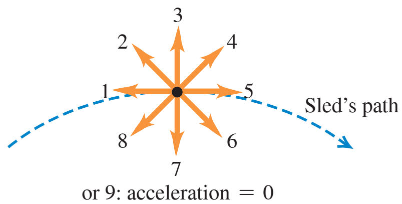
SOLUTION Figure 3.14b shows our solution. At point B the skier is moving in a straight line with increasing speed, so her acceleration points downhill, in the same direction as her velocity. At points D, E, and F the skier is moving along a curved path, so her acceleration has a component perpendicular to the path (toward the concave side of the path) at each of these points. At point D there is also an acceleration component in the direction of her motion because she is speeding up. So the acceleration vector points *ahead* of the normal to her path at point D. At point E, the skier's speed is instantaneously not changing; her speed is maximum at this point, so its derivative is zero. There is therefore no parallel component of $\vec{a}$, and the acceleration is perpendicular to her motion. At point F there is an acceleration component *opposite to* the direction of her motion because she's slowing down. The acceleration vector therefore points *behind* the normal to her path.

In the next section we'll consider the skier's acceleration after she flies off the ramp.

KEYCONCEPT If a moving object is turning (changing direction), its acceleration vector points ahead of the normal to its path if it is speeding up, behind the normal if it is slowing down, and along the normal if its speed is instantaneously not changing.

Figure 3.14 (a) The skier's path. (b) Our solution.





TEST YOUR UNDERSTANDING OF SECTION 3.2 A sled travels over the crest of a snow-covered hill. The sled slows down as it climbs up one side of the hill and gains speed as it descends on the other side. Which of the vectors (1 through 9) in the figure correctly shows the direction of the sled's acceleration at the crest? (Choice 9 is that the acceleration is zero.)

ANSWER

Vector 7 At the high point of the sled's path, the speed is minimum. At that point the speed is neither increasing nor decreasing, and the parallel component of the acceleration (that is, the horizontal component) is zero. The acceleration has only a perpendicular component toward the inside of the sled's curved path. In other words, the acceleration is downward.

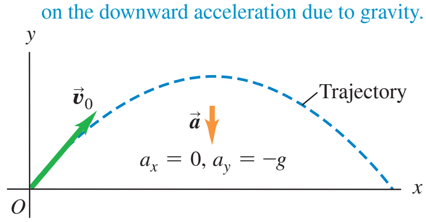
3.3 PROJECTILE MOTION

A **projectile** is any object that is given an initial velocity and then follows a path determined entirely by the effects of gravitational acceleration and air resistance. A batted baseball, a thrown football, and a bullet shot from a rifle are all projectiles. The path followed by a projectile is called its **trajectory.**

To analyze the motion of a projectile, we'll use an idealized model. We'll represent the projectile as a particle with an acceleration (due to gravity) that is constant in both magnitude and direction. We'll ignore the effects of air resistance and the curvature and rotation of the earth. This model has limitations, however: We have to consider the earth's curvature when we study the flight of long-range missiles, and air resistance is of crucial importance to a sky diver. Nevertheless, we can learn a lot from analysis of this simple model. For the remainder of this chapter the phrase "projectile motion" will imply that we're ignoring air resistance. In Chapter 5 we'll see what happens when air resistance cannot be ignored.

Figure **3.15** The trajectory of an idealized projectile.

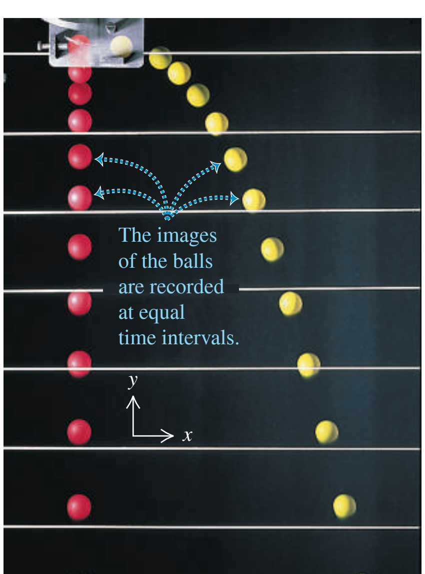
- A projectile moves in a vertical plane that contains the initial velocity vector $\vec{v}_0$.
- Its trajectory depends only on $\vec{v}_0$ and on the downward acceleration due to gravity.



Projectile motion is always confined to a vertical plane determined by the direction of the initial velocity (**Fig. 3.15**). This is because the acceleration due to gravity is purely vertical; gravity can't accelerate the projectile sideways. Thus projectile motion is *two-dimensional.* We'll call the plane of motion the xy-coordinate plane, with the x-axis horizontal and the y-axis vertically upward.

The key to analyzing projectile motion is that we can treat the x- and y-coordinates separately. **Figure 3.16** illustrates this for two projectiles: a red ball dropped from rest and a yellow ball projected horizontally from the same height. The figure shows that the horizontal motion of the yellow projectile has *no* effect on its vertical motion. For both projectiles, the x-component of acceleration is zero and the y-component is constant and equal to $-g$. (By definition, g is always positive; with our choice of coordinate directions, a_y is negative.) So *we can analyze projectile motion as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.*

Figure **3.16** The red ball is dropped from rest, and the yellow ball is simultaneously projected horizontally.



- At any time the two balls have different x-coordinates and x-velocities but the same y-coordinate, y-velocity, and y-acceleration.
- The horizontal motion of the yellow ball has no effect on its vertical motion.

We can then express all the vector relationships for the projectile's position, velocity, and acceleration by separate equations for the horizontal and vertical components. The components of $\vec{a}$ are

$$a_x = 0 \qquad a_y = -g \quad \text{(projectile motion, no air resistance)} \tag{3.13}$$

Since both the x-acceleration and y-acceleration are constant, we can use Eqs. (2.8), (2.12), (2.13), and (2.14) directly. Suppose that at time $t = 0$ our particle is at the point (x_0, y_0) and its initial velocity at this time has components v_{0x} and v_{0y}. The components of acceleration are $a_x = 0$, $a_y = -g$. Considering the x-motion first, we substitute 0 for a_x in Eqs. (2.8) and (2.12). We find

$$v_x = v_{0x} \tag{3.14}$$

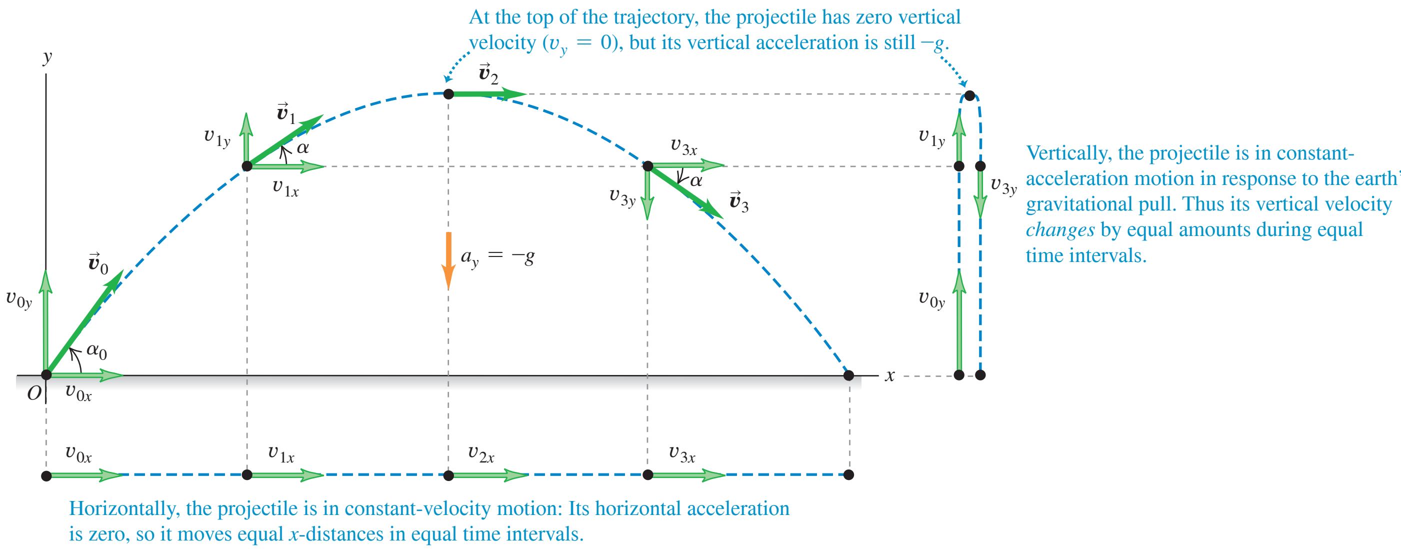
$$x = x_0 + v_{0x}t \tag{3.15}$$

For the y-motion we substitute y for x, v_y for v_x, v_{0y} for v_{0x}, and $a_y = -g$ for a_x:

$$v_y = v_{0y} - gt \tag{3.16}$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \tag{3.17}$$

Figure **3.17** If air resistance is negligible, the trajectory of a projectile is a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.



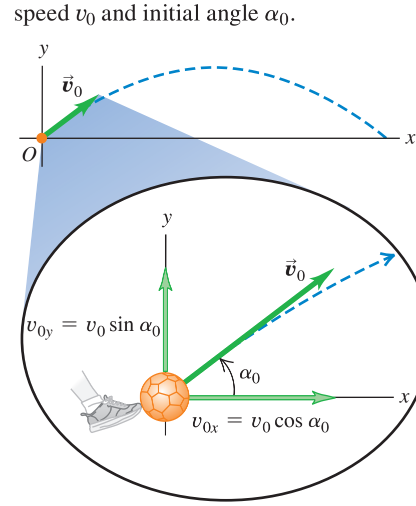
It's usually simplest to take the initial position (at $t = 0$) as the origin; then $x_0 = y_0 = 0$. This might be the position of a ball at the instant it leaves the hand of the person who throws it or the position of a bullet at the instant it leaves the gun barrel.

Figure 3.17 shows the trajectory of a projectile that starts at (or passes through) the origin at time $t = 0$, along with its position, velocity, and velocity components at equal time intervals. The x-velocity v_x is constant; the y-velocity v_y changes by equal amounts in equal times, just as if the projectile were launched vertically with the same initial y-velocity.

We can also represent the initial velocity $\vec{v}_0$ by its magnitude v_0 (the initial speed) and its angle α_0 with the positive x-axis (**Fig. 3.18**). In terms of these quantities, the components v_{0x} and v_{0y} of the initial velocity are

$$v_{0x} = v_0 \cos\alpha_0 \qquad v_{0y} = v_0 \sin\alpha_0 \tag{3.18}$$

Figure **3.18** The initial velocity components v_{0x} and v_{0y} of a projectile (such as a kicked soccer ball) are related to the initial speed v_0 and initial angle α_0.



If we substitute Eqs. (3.18) into Eqs. (3.14) through (3.17) and set $x_0 = y_0 = 0$, we get the following equations. They describe the position and velocity of the projectile in Fig. 3.17 at any time t:

Coordinates at time t of a **projectile** (positive y-direction is upward, and $x = y = 0$ at $t = 0$)

$$x = (v_0 \cos\alpha_0)t \tag{3.19}$$

$$y = (v_0 \sin\alpha_0)t - \tfrac{1}{2}gt^2 \tag{3.20}$$

Speed at $t = 0$ — Direction at $t = 0$ — Time — Acceleration due to gravity: Note $g > 0$.

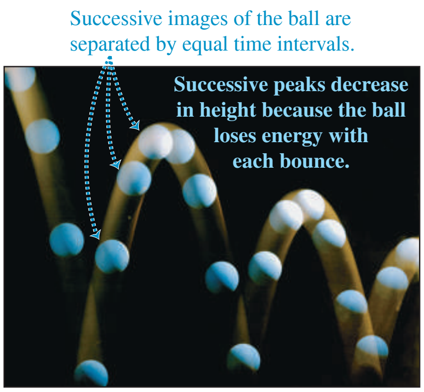
Velocity components at time t of a **projectile** (positive y-direction is upward)

$$v_x = v_0 \cos\alpha_0 \tag{3.21}$$

$$v_y = v_0 \sin\alpha_0 - gt \tag{3.22}$$

Speed at $t = 0$ — Direction at $t = 0$ — Time

Figure 3.19 The nearly parabolic trajectories of a bouncing ball.



We can get a lot of information from Eqs. (3.19) through (3.22). For example, the distance r from the origin to the projectile at any time t is

$$r = \sqrt{x^2 + y^2} \tag{3.23}$$

The projectile's speed (the magnitude of its velocity) at any time is

$$v = \sqrt{v_x^2 + v_y^2} \tag{3.24}$$

The *direction* of the velocity, in terms of the angle α it makes with the positive x-direction (see Fig. 3.17), is

$$\tan\alpha = \frac{v_y}{v_x} \tag{3.25}$$

The velocity vector $\vec{v}$ is tangent to the trajectory at each point.

We can derive an equation for the trajectory's shape in terms of x and y by eliminating t. From Eqs. (3.19) and (3.20), we find $t = x/(v_0 \cos\alpha_0)$ and

$$y = (\tan\alpha_0)x - \frac{g}{2v_0^2 \cos^2\alpha_0}x^2 \tag{3.26}$$

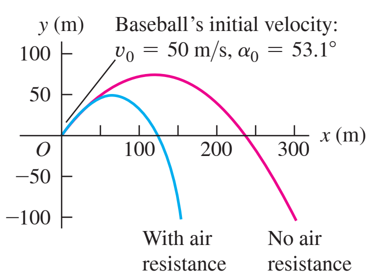
Don't worry about the details of this equation; the important point is its general form. Since v_0, $\tan\alpha_0$, $\cos\alpha_0$, and g are constants, Eq. (3.26) has the form

$$y = bx - cx^2$$

where b and c are constants. This is the equation of a *parabola*. In our simple model of projectile motion, the trajectory is always a parabola (**Fig. 3.19**).

When air resistance *isn't* negligible and has to be included, calculating the trajectory becomes a lot more complicated; the effects of air resistance depend on velocity, so the acceleration is no longer constant. **Figure 3.20** shows a computer simulation of the trajectory of a baseball both without air resistance and with air resistance proportional to the square of the baseball's speed. We see that air resistance has a very large effect; the projectile does not travel as far or as high, and the trajectory is no longer a parabola.

Figure 3.20 Air resistance has a large cumulative effect on the motion of a baseball. In this simulation we allow the baseball to fall below the height from which it was thrown (for example, the baseball could have been thrown from a cliff).



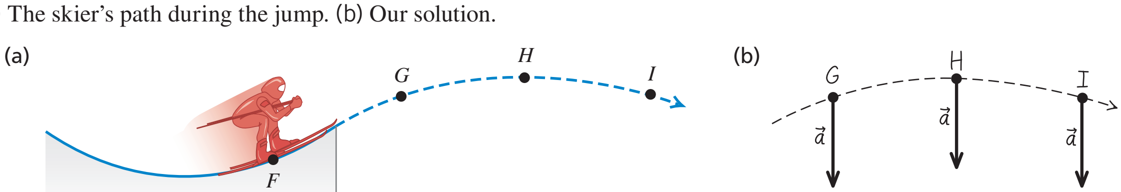
CONCEPTUAL EXAMPLE 3.5 Acceleration of a skier, continued

Let's consider again the skier in Conceptual Example 3.4. What is her acceleration at each of the points G, H, and I in **Fig. 3.21a** *after* she flies off the ramp? Neglect air resistance.

SOLUTION Figure 3.21b shows our answer. The skier's acceleration changed from point to point while she was on the ramp. But as soon as she leaves the ramp, she becomes a projectile. So at points G, H, and I, and indeed at *all* points after she leaves the ramp, the skier's acceleration points vertically downward and has magnitude g.

KEYCONCEPT No matter how complicated the acceleration of a particle before it becomes a projectile, its acceleration as a projectile is given by $a_x = 0$, $a_y = -g$.

Figure 3.21 (a) The skier's path during the jump. (b) Our solution.



PROBLEM-SOLVING STRATEGY 3.1 Projectile Motion

NOTE: The strategies we used in Sections 2.4 and 2.5 for straight-line, constant-acceleration problems are also useful here.

IDENTIFY *the relevant concepts:* The key concept is that throughout projectile motion, the acceleration is downward and has a constant magnitude g. Projectile-motion equations don't apply to *throwing* a ball, because during the throw the ball is acted on by both the thrower's hand and gravity. These equations apply only *after* the ball leaves the thrower's hand.

SET UP *the problem* using the following steps:

1. Define your coordinate system and make a sketch showing your axes. It's almost always best to make the x-axis horizontal and the y-axis vertical, and to choose the origin to be where the object first becomes a projectile (for example, where a ball leaves the thrower's hand). Then the components of acceleration are $a_x = 0$ and $a_y = -g$, as in Eq. (3.13); the initial position is $x_0 = y_0 = 0$; and you can use Eqs. (3.19) through (3.22). (If you choose a different origin or axes, you'll have to modify these equations.)
2. List the unknown and known quantities, and decide which unknowns are your target variables. For example, you might be given the initial velocity (either the components or the magnitude and direction) and asked to find the coordinates and velocity components at some later time. Make sure that you have as many equations as there are target variables to be found. In addition to Eqs. (3.19) through (3.22), Eqs. (3.23) through (3.26) may be useful.
3. State the problem in words and then translate those words into symbols. For example, *when* does the particle arrive at a certain point? (That is, at what value of t?) *Where* is the particle when its velocity has a certain value? (That is, what are the values of x and y when v_x or v_y has the specified value?) Since $v_y = 0$ at the highest point in a trajectory, the question "When does the projectile reach its highest point?" translates into "What is the value of t when $v_y = 0$?" Similarly, "When does the projectile return to its initial elevation?" translates into "What is the value of t when $y = y_0$?"

EXECUTE *the solution:* Find the target variables using the equations you chose. Resist the temptation to break the trajectory into segments and analyze each segment separately. You don't have to start all over when the projectile reaches its highest point! It's almost always easier to use the same axes and time scale throughout the problem. If you need numerical values, use $g = 9.80\ \text{m/s}^2$. Remember that g is positive!

EVALUATE *your answer:* Do your results make sense? Do the numerical values seem reasonable?

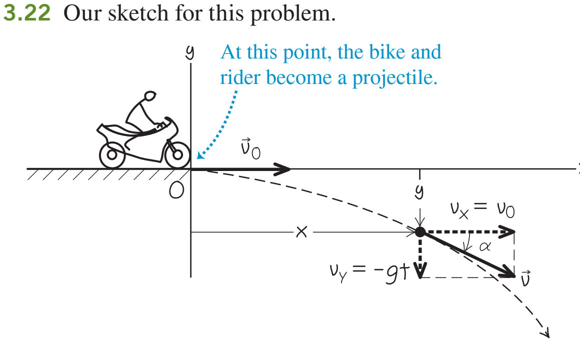
EXAMPLE 3.6 An object projected horizontally

WITH VARIATION PROBLEMS

A motorcycle stunt rider rides off the edge of a cliff. Just at the edge his velocity is horizontal, with magnitude 9.0 m/s. Find the motorcycle's position, distance from the edge of the cliff, and velocity 0.50 s after it leaves the edge of the cliff. Ignore air resistance.

IDENTIFY and SET UP **Figure 3.22** shows our sketch of the trajectory of motorcycle and rider. He is in projectile motion as soon as he leaves the edge of the cliff, which we take to be the origin (so $x_0 = y_0 = 0$). His initial velocity $\vec{v}_0$ at the edge of the cliff is horizontal (that is, $\alpha_0 = 0$), so its components are $v_{0x} = v_0 \cos\alpha_0 = 9.0\ \text{m/s}$ and $v_{0y} = v_0 \sin\alpha_0 = 0$. To find the motorcycle's position at $t = 0.50$ s, we use Eqs. (3.19) and (3.20); we then find the distance from the origin using Eq. (3.23). Finally, we use Eqs. (3.21) and (3.22) to find the velocity components at $t = 0.50$ s.

Figure **3.22** Our sketch for this problem.



EXECUTE From Eqs. (3.19) and (3.20), the motorcycle's x- and y-coordinates at $t = 0.50$ s are

$$x = v_{0x}t = (9.0\ \text{m/s})(0.50\ \text{s}) = 4.5\ \text{m}$$
$$y = -\tfrac{1}{2}gt^2 = -\tfrac{1}{2}(9.80\ \text{m/s}^2)(0.50\ \text{s})^2 = -1.2\ \text{m}$$

The negative value of y shows that the motorcycle is below its starting point.

From Eq. (3.23), the motorcycle's distance from the origin at $t = 0.50$ s is

$$r = \sqrt{x^2 + y^2} = \sqrt{(4.5\ \text{m})^2 + (-1.2\ \text{m})^2} = 4.7\ \text{m}$$

From Eqs. (3.21) and (3.22), the velocity components at $t = 0.50$ s are

$$v_x = v_{0x} = 9.0\ \text{m/s}$$
$$v_y = -gt = (-9.80\ \text{m/s}^2)(0.50\ \text{s}) = -4.9\ \text{m/s}$$

The motorcycle has the same horizontal velocity v_x as when it left the cliff at $t = 0$, but in addition there is a downward (negative) vertical velocity v_y. The velocity vector at $t = 0.50$ s is

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} = (9.0\ \text{m/s})\hat{\imath} + (-4.9\ \text{m/s})\hat{\jmath}$$

From Eqs. (3.24) and (3.25), at $t = 0.50$ s the velocity has magnitude v and angle α given by

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(9.0\ \text{m/s})^2 + (-4.9\ \text{m/s})^2} = 10.2\ \text{m/s}$$
$$\alpha = \arctan\frac{v_y}{v_x} = \arctan\left(\frac{-4.9\ \text{m/s}}{9.0\ \text{m/s}}\right) = -29^\circ$$

The motorcycle is moving at 10.2 m/s in a direction 29° below the horizontal.

EVALUATE Just as in Fig. 3.17, the motorcycle's horizontal motion is unchanged by gravity; the motorcycle continues to move horizontally at 9.0 m/s, covering 4.5 m in 0.50 s. The motorcycle initially has zero vertical velocity, so it falls vertically just like an object released from rest and descends a distance $\tfrac{1}{2}gt^2 = 1.2$ m in 0.50 s.

KEYCONCEPT The motion of a projectile is a combination of motion with constant velocity in the horizontal x-direction and motion with constant downward acceleration in the vertical y-direction.

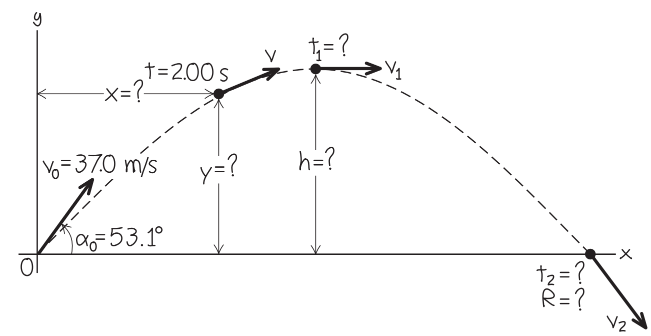
EXAMPLE 3.7 Height and range of a projectile I: A batted baseball

WITH VARIATION PROBLEMS

A batter hits a baseball so that it leaves the bat at speed $v_0 = 37.0\ \text{m/s}$ at an angle $\alpha_0 = 53.1°$. (a) Find the position of the ball and its velocity (magnitude and direction) at $t = 2.00\ \text{s}$. (b) Find the time when the ball reaches the highest point of its flight, and its height h at this time. (c) Find the *horizontal range R*—that is, the horizontal distance from the starting point to where the ball hits the ground—and the ball's velocity just before it hits.

IDENTIFY and SET UP As Fig. 3.20 shows, air resistance strongly affects the motion of a baseball. For simplicity, however, we'll ignore air resistance here and use the projectile-motion equations to describe the motion. The ball leaves the bat at $t = 0$ a meter or so above ground level, but we'll ignore this distance and assume that it starts at ground level ($y_0 = 0$). **Figure 3.23** shows our sketch of the ball's trajectory. We'll use the same coordinate system as in Figs. 3.17 and 3.18, so we can use Eqs. (3.19) through (3.22). Our target variables are (a) the position and velocity of the ball 2.00 s after it leaves the bat, (b) the time t when the ball is at its maximum height (that is, when $v_y = 0$) and the y-coordinate at this time, and (c) the x-coordinate when the ball returns to ground level ($y = 0$) and the ball's vertical component of velocity then.

Figure 3.23 Our sketch for this problem.



EXECUTE (a) We want to find x, y, v_x, and v_y at $t = 2.00\ \text{s}$. The initial velocity of the ball has components

$$v_{0x} = v_0 \cos\alpha_0 = (37.0\ \text{m/s})\cos 53.1° = 22.2\ \text{m/s}$$
$$v_{0y} = v_0 \sin\alpha_0 = (37.0\ \text{m/s})\sin 53.1° = 29.6\ \text{m/s}$$

From Eqs. (3.19) through (3.22),

$$x = v_{0x}t = (22.2\ \text{m/s})(2.00\ \text{s}) = 44.4\ \text{m}$$
$$y = v_{0y}t - \tfrac{1}{2}gt^2$$
$$= (29.6\ \text{m/s})(2.00\ \text{s}) - \tfrac{1}{2}(9.80\ \text{m/s}^2)(2.00\ \text{s})^2 = 39.6\ \text{m}$$
$$v_x = v_{0x} = 22.2\ \text{m/s}$$
$$v_y = v_{0y} - gt = 29.6\ \text{m/s} - (9.80\ \text{m/s}^2)(2.00\ \text{s}) = 10.0\ \text{m/s}$$

The y-component of velocity is positive at $t = 2.00\ \text{s}$, so the ball is still moving upward (Fig. 3.23). From Eqs. (3.24) and (3.25), the magnitude and direction of the velocity are

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(22.2\ \text{m/s})^2 + (10.0\ \text{m/s})^2} = 24.4\ \text{m/s}$$
$$\alpha = \arctan\left(\frac{10.0\ \text{m/s}}{22.2\ \text{m/s}}\right) = \arctan 0.450 = 24.2°$$

The ball is moving at 24.4 m/s in a direction 24.2° above the horizontal.

(b) At the highest point, the vertical velocity v_y is zero. Call the time when this happens t_1; then

$$v_y = v_{0y} - gt_1 = 0$$
$$t_1 = \frac{v_{0y}}{g} = \frac{29.6\ \text{m/s}}{9.80\ \text{m/s}^2} = 3.02\ \text{s}$$

The height h at the highest point is the value of y at time t_1:

$$h = v_{0y}t_1 - \tfrac{1}{2}gt_1^2$$
$$= (29.6\ \text{m/s})(3.02\ \text{s}) - \tfrac{1}{2}(9.80\ \text{m/s}^2)(3.02\ \text{s})^2 = 44.7\ \text{m}$$

(c) We'll find the horizontal range in two steps. First, we find the time t_2 when $y = 0$ (the ball is at ground level):

$$y = 0 = v_{0y}t_2 - \tfrac{1}{2}gt_2^2 = t_2\left(v_{0y} - \tfrac{1}{2}gt_2\right)$$

This is a quadratic equation for t_2. It has two roots:

$$t_2 = 0 \qquad \text{and} \qquad t_2 = \frac{2v_{0y}}{g} = \frac{2(29.6\ \text{m/s})}{9.80\ \text{m/s}^2} = 6.04\ \text{s}$$

The ball is at $y = 0$ at both times. The ball *leaves* the ground at $t_2 = 0$, and it hits the ground at $t_2 = 2v_{0y}/g = 6.04\ \text{s}$.

The horizontal range R is the value of x when the ball returns to the ground at $t_2 = 6.04\ \text{s}$:

$$R = v_{0x}t_2 = (22.2\ \text{m/s})(6.04\ \text{s}) = 134\ \text{m}$$

The vertical component of velocity when the ball hits the ground is

$$v_y = v_{0y} - gt_2 = 29.6\ \text{m/s} - (9.80\ \text{m/s}^2)(6.04\ \text{s}) = -29.6\ \text{m/s}$$

That is, v_y has the same magnitude as the initial vertical velocity v_{0y} but the opposite direction (down). Since v_x is constant, the angle $\alpha = -53.1°$ (below the horizontal) at this point is the negative of the initial angle $\alpha_0 = 53.1°$.

EVALUATE As a check on our results, we can also find the maximum height in part (b) by applying the constant-acceleration formula Eq. (2.13) to the y-motion:

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) = v_{0y}^2 - 2g(y - y_0)$$

At the highest point, $v_y = 0$ and $y = h$. Solve this equation for h; you should get the answer that we obtained in part (b). (Do you?)

Note that the time to hit the ground, $t_2 = 6.04\ \text{s}$, is exactly twice the time to reach the highest point, $t_1 = 3.02\ \text{s}$. Hence the time of descent equals the time of ascent. This is *always* true if the starting point and end point are at the same elevation and if air resistance can be ignored.

Note also that $h = 44.7\ \text{m}$ in part (b) is comparable to the 61.0 m height above second base of the roof at Marlins Park in Miami, and the horizontal range $R = 134\ \text{m}$ in part (c) is greater than the 99.7 m distance from home plate to the right-field fence at Safeco Field in Seattle. In reality, due to air resistance (which we have ignored) a batted ball with the initial speed and angle we've used here won't go as high or as far as we've calculated (see Fig. 3.20).

KEYCONCEPT You can solve most projectile problems by using the equations for x, y, v_x, and v_y as functions of time. The highest point of a projectile's motion occurs at the time its vertical component of velocity is zero.

EXAMPLE 3.8 Height and range of a projectile II: Maximum height, maximum range

WITH VARIATION PROBLEMS

Find the maximum height h and horizontal range R (see Fig. 3.23) of a projectile launched with speed v_0 at an initial angle α_0 between 0 and 90°. For a given v_0, what value of α_0 gives maximum height? What value gives maximum horizontal range?

IDENTIFY and SET UP This is almost the same as parts (b) and (c) of Example 3.7, except that now we want general expressions for h and R. We also want the values of α_0 that give the maximum values of h and R. In part (b) of Example 3.7 we found that the projectile reaches the high point of its trajectory (so that $v_y = 0$) at time $t_1 = v_{0y}/g$, and in part (c) we found that the projectile returns to its starting height (so that $y = y_0$) at time $t_2 = 2v_{0y}/g = 2t_1$. We'll use Eq. (3.20) to find the y-coordinate h at t_1 and Eq. (3.19) to find the x-coordinate R at time t_2. We'll express our answers in terms of the launch speed v_0 and launch angle α_0 by using Eqs. (3.18).

EXECUTE From Eqs. (3.18), $v_{0x} = v_0 \cos\alpha_0$ and $v_{0y} = v_0 \sin\alpha_0$. Hence we can write the time t_1 when $v_y = 0$ as

$$t_1 = \frac{v_{0y}}{g} = \frac{v_0 \sin\alpha_0}{g}$$

Equation (3.20) gives the height $y = h$ at this time:

$$h = (v_0 \sin\alpha_0)\left(\frac{v_0 \sin\alpha_0}{g}\right) - \tfrac{1}{2}g\left(\frac{v_0 \sin\alpha_0}{g}\right)^2 = \frac{v_0^2 \sin^2\alpha_0}{2g}$$

For a given launch speed v_0, the maximum value of h occurs for $\sin\alpha_0 = 1$ and $\alpha_0 = 90°$—that is, when the projectile is launched straight up. (If it is launched horizontally, as in Example 3.6, $\alpha_0 = 0$ and the maximum height is zero!)

The time t_2 when the projectile hits the ground is

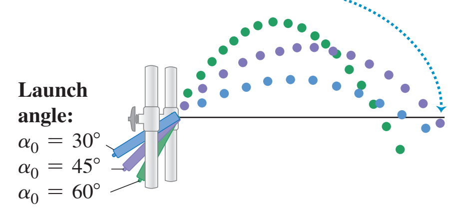
$$t_2 = \frac{2v_{0y}}{g} = \frac{2v_0 \sin\alpha_0}{g}$$

The horizontal range R is the value of x at this time. From Eq. (3.19), this is

$$R = (v_0 \cos\alpha_0)t_2 = (v_0 \cos\alpha_0)\frac{2v_0 \sin\alpha_0}{g} = \frac{v_0^2 \sin 2\alpha_0}{g}$$

(We used the trigonometric identity $2\sin\alpha_0 \cos\alpha_0 = \sin 2\alpha_0$, found in Appendix B.) The maximum value of $\sin 2\alpha_0$ is 1; this occurs when $2\alpha_0 = 90°$, or $\alpha_0 = 45°$. This angle gives the maximum range for a given initial speed if air resistance can be ignored.

Figure 3.24 A launch angle of 45° gives the maximum horizontal range. The range is shorter with launch angles of 30° and 60°.



EVALUATE **Figure 3.24** is based on a composite photograph of three trajectories of a ball projected from a small spring gun at angles of 30°, 45°, and 60°. The initial speed v_0 is approximately the same in all three cases. The horizontal range is greatest for the 45° angle. The ranges are nearly the same for the 30° and 60° angles: Can you prove that for a given value of v_0 the range is the same for both an initial angle α_0 and an initial angle $90° - \alpha_0$? (This is not the case in Fig. 3.24 due to air resistance.)

CAUTION **Height and range of a projectile** We don't recommend memorizing the above expressions for h and R. They are applicable only in the special circumstances we've described. In particular, you can use the expression for the range R *only* when launch and landing heights are equal. There are many end-of-chapter problems to which these equations do *not* apply.

KEYCONCEPT When you solve physics problems in general, and projectile problems in particular, it's best to use symbols rather than numbers as far into the solution as possible. This allows you to better explore and understand your result.

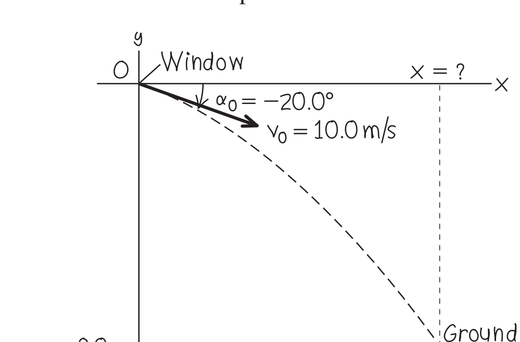
EXAMPLE 3.9 Different initial and final heights

WITH VARIATION PROBLEMS

You throw a ball from your window 8.0 m above the ground. When the ball leaves your hand, it is moving at 10.0 m/s at an angle of 20.0° below the horizontal. How far horizontally from your window will the ball hit the ground? Ignore air resistance.

IDENTIFY and SET UP As in Examples 3.7 and 3.8, we want to find the horizontal coordinate of a projectile when it is at a given y-value. The difference here is that this value of y is *not* the same as the initial value. We again choose the x-axis to be horizontal and the y-axis to be upward, and place the origin of coordinates at the point where the ball leaves your hand (**Fig. 3.25**). We have $v_0 = 10.0$ m/s and $\alpha_0 = -20.0°$ (the angle is negative because the initial velocity is below the horizontal). Our target variable is the value of x when the ball reaches the ground at $y = -8.0$ m. We'll use Eq. (3.20) to find the time t when this happens and then use Eq. (3.19) to find the value of x at this time.

Figure 3.25 Our sketch for this problem.



Continued

EXECUTE To determine t, we rewrite Eq. (3.20) in the standard form for a quadratic equation for t:

$$\tfrac{1}{2}gt^2 - (v_0 \sin\alpha_0)t + y = 0$$

The roots of this equation are

$$\begin{aligned} t &= \frac{v_0\sin\alpha_0 \pm \sqrt{(-v_0\sin\alpha_0)^2 - 4\left(\tfrac{1}{2}g\right)y}}{2\left(\tfrac{1}{2}g\right)} \\ &= \frac{v_0\sin\alpha_0 \pm \sqrt{v_0^2\sin^2\alpha_0 - 2gy}}{g} \\ &= \frac{\left[\begin{array}{l}(10.0\text{ m/s})\sin(-20.0^\circ) \\ \pm\sqrt{(10.0\text{ m/s})^2\sin^2(-20.0^\circ) - 2(9.80\text{ m/s}^2)(-8.0\text{ m})}\end{array}\right]}{9.80\text{ m/s}^2} \\ &= -1.7\text{ s} \quad \text{or} \quad 0.98\text{ s} \end{aligned}$$

We discard the negative root, since it refers to a time before the ball left your hand. The positive root tells us that the ball reaches the ground at $t = 0.98$ s. From Eq. (3.19), the ball's x-coordinate at that time is

$$x = (v_0\cos\alpha_0)t = (10.0\text{ m/s})[\cos(-20^\circ)](0.98\text{ s}) = 9.2\text{ m}$$

The ball hits the ground a horizontal distance of 9.2 m from your window.

EVALUATE The root $t = -1.7$ s is an example of a "fictional" solution to a quadratic equation. We discussed these in Example 2.8 in Section 2.5; review that discussion.

KEYCONCEPT A projectile's vertical coordinate y as a function of time is given by a quadratic equation, which in general has more than one solution. Take care to select the solution that's appropriate for the problem you're solving.

EXAMPLE 3.10 The zookeeper and the monkey

A monkey escapes from the zoo and climbs a tree. After failing to entice the monkey down, the zookeeper fires a tranquilizer dart directly at the monkey (**Fig. 3.26**). The monkey lets go at the instant the dart leaves the gun. Show that the dart will *always* hit the monkey, provided that the dart reaches the monkey before he hits the ground and runs away.

IDENTIFY and SET UP We have *two* objects in projectile motion: the dart and the monkey. They have different initial positions and initial velocities, but they go into projectile motion at the same time $t = 0$. We'll first use Eq. (3.19) to find an expression for the time t when the x-coordinates x_{monkey} and x_{dart} are equal. Then we'll use that expression in Eq. (3.20) to see whether y_{monkey} and y_{dart} are also equal at this time; if they are, the dart hits the monkey. We make the usual choice for the x- and y-directions, and place the origin of coordinates at the muzzle of the tranquilizer gun (Fig. 3.26).

EXECUTE The monkey drops straight down, so $x_{\text{monkey}} = d$ at all times. From Eq. (3.19), $x_{\text{dart}} = (v_0\cos\alpha_0)t$. We solve for the time t when these x-coordinates are equal:

$$d = (v_0\cos\alpha_0)t \qquad \text{so} \qquad t = \frac{d}{v_0\cos\alpha_0}$$

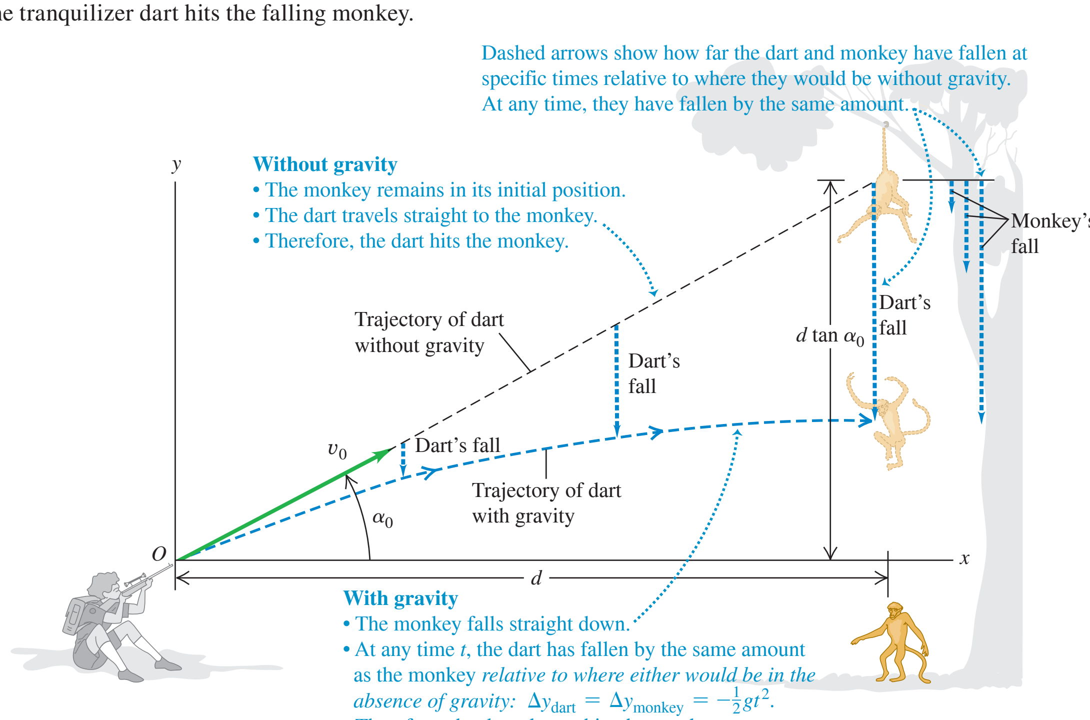
We must now show that $y_{\text{monkey}} = y_{\text{dart}}$ at this time. The monkey is in one-dimensional free fall; its position at any time is given by Eq. (2.12), with appropriate symbol changes. Figure 3.26 shows that the monkey's initial height above the dart-gun's muzzle is $y_{\text{monkey-0}} = d\tan\alpha_0$, so

$$y_{\text{monkey}} = d\tan\alpha_0 - \tfrac{1}{2}gt^2$$

From Eq. (3.20),

$$y_{\text{dart}} = (v_0\sin\alpha_0)t - \tfrac{1}{2}gt^2$$

Figure **3.26** The tranquilizer dart hits the falling monkey.



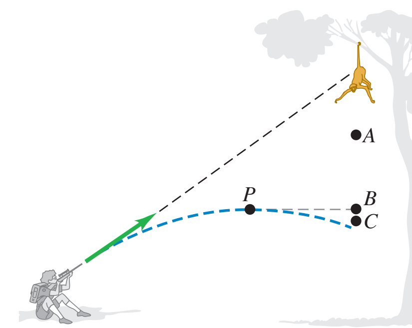
Comparing these two equations, we see that we'll have $y_{\text{monkey}} = y_{\text{dart}}$ (and a hit) if $d \tan\alpha_0 = (v_0 \sin\alpha_0)t$ when the two x-coordinates are equal. To show that this happens, we replace t with $d/(v_0 \cos\alpha_0)$, the time when $x_{\text{monkey}} = x_{\text{dart}}$. Sure enough,

$$(v_0 \sin\alpha_0)t = (v_0 \sin\alpha_0)\frac{d}{v_0 \cos\alpha_0} = d \tan\alpha_0$$

EVALUATE We've proved that the y-coordinates of the dart and the monkey are equal at the same time that their x-coordinates are equal; a dart aimed at the monkey *always* hits it, no matter what v_0 is (provided the monkey doesn't hit the ground first). This result is independent of the value of g, the acceleration due to gravity. With no gravity ($g = 0$), the monkey would remain motionless, and the dart would travel in a straight line to hit him. With gravity, both fall the same distance $\frac{1}{2}gt^2$ below their $t = 0$ positions, and the dart still hits the monkey (Fig. 3.26).

KEYCONCEPT It can be useful to think of a projectile as following a straight-line path that's pulled downward by gravity a distance $\frac{1}{2}gt^2$ in a time t.

TEST YOUR UNDERSTANDING OF SECTION 3.3 In Example 3.10, suppose the tranquilizer dart has a relatively low muzzle velocity so that the dart reaches a maximum height at a point P before striking the monkey, as shown in the figure. When the dart is at point P, will the monkey be (i) at point A (higher than P), (ii) at point B (at the same height as P), or (iii) at point C (lower than P)? Ignore air resistance.



ANSWER

(i) If there were no gravity ($g = 0$), the monkey would not fall and the dart would follow a straight-line path (shown as a dashed line). The effect of gravity is to make both the monkey and the dart fall the same distance $\frac{1}{2}gt^2$ below their $g = 0$ positions. Point A is the same distance below the monkey's initial position as point P is below the dashed straight line, so point A is where we would find the monkey at the time in question.

3.4 MOTION IN A CIRCLE

When a particle moves along a curved path, the direction of its velocity changes. As we saw in Section 3.2, this means that the particle *must* have a component of acceleration perpendicular to the path, even if its speed is constant (see Fig. 3.11b). In this section we'll calculate the acceleration for the important special case of motion in a circle.

Uniform Circular Motion

When a particle moves in a circle with *constant speed,* the motion is called **uniform circular motion.** A car rounding a curve with constant radius at constant speed, a satellite moving in a circular orbit, and an ice skater skating in a circle with constant speed are all examples of uniform circular motion (**Fig. 3.27a**; compare Fig. 3.12a). There is no component of acceleration parallel (tangent) to the path; otherwise, the speed would change. The acceleration vector is perpendicular (normal) to the path and hence directed inward (never outward!) toward the center of the circular path. This causes the direction of the velocity to change without changing the speed.

Figure **3.27** A car moving along a circular path. If the car is in uniform circular motion as in (a), the speed is constant and the acceleration is directed toward the center of the circular path (compare Fig. 3.12).

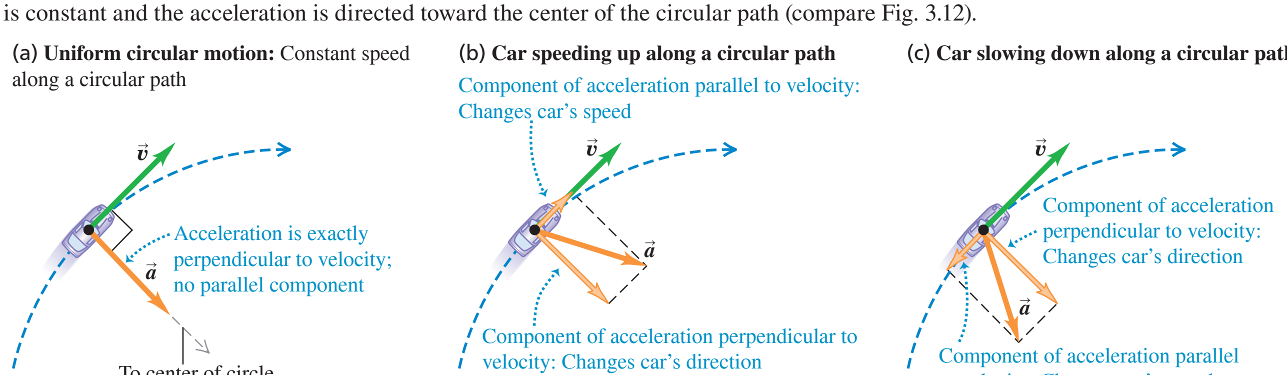
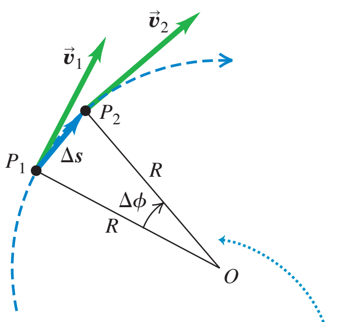
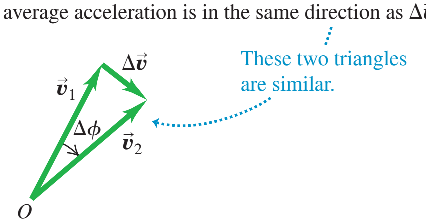


Figure **3.28** Finding the velocity change $\Delta\vec{v}$, average acceleration $\vec{a}_{av}$, and instantaneous acceleration $\vec{a}_{rad}$ for a particle moving in a circle with constant speed.

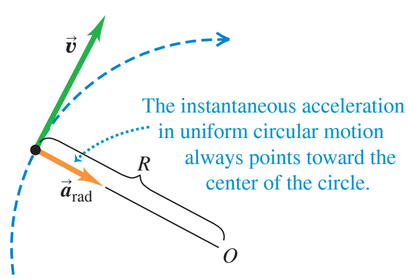
(a) A particle moves a distance Δs at constant speed along a circular path.



(b) The corresponding change in velocity $\Delta\vec{v}$. The average acceleration is in the same direction as $\Delta\vec{v}$.



(c) The instantaneous acceleration



We can find a simple expression for the magnitude of the acceleration in uniform circular motion. We begin with **Fig. 3.28a**, which shows a particle moving with constant speed in a circular path of radius R with center at O. The particle moves a distance Δs from P_1 to P_2 in a time interval Δt. Figure 3.28b shows the vector change in velocity $\Delta\vec{v}$ during this interval.

The angles labeled $\Delta\phi$ in Figs. 3.28a and 3.28b are the same because $\vec{v}_1$ is perpendicular to the line OP_1 and $\vec{v}_2$ is perpendicular to the line OP_2. Hence the triangles in Figs. 3.28a and 3.28b are *similar*. The ratios of corresponding sides of similar triangles are equal, so

$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{R} \qquad \text{or} \qquad |\Delta\vec{v}| = \frac{v_1}{R}\Delta s$$

The magnitude a_{av} of the average acceleration during Δt is therefore

$$a_{av} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v_1}{R}\frac{\Delta s}{\Delta t}$$

The magnitude a of the *instantaneous* acceleration $\vec{a}$ at point P_1 is the limit of this expression as we take point P_2 closer and closer to point P_1:

$$a = \lim_{\Delta t \to 0} \frac{v_1}{R}\frac{\Delta s}{\Delta t} = \frac{v_1}{R}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}$$

If the time interval Δt is short, Δs is the distance the particle moves along its curved path. So the limit of $\Delta s/\Delta t$ is the speed v_1 at point P_1. Also, P_1 can be any point on the path, so we can drop the subscript and let v represent the speed at any point. Then

Magnitude of acceleration of an object in **uniform circular motion**

$$a_{rad} = \frac{v^2}{R} \qquad (3.27)$$

Speed of object

Radius of object's circular path

The subscript "rad" is a reminder that the direction of the instantaneous acceleration at each point is always along a radius of the circle (toward the center of the circle; see Figs. 3.27a and 3.28c). So *in uniform circular motion, the magnitude a_{rad} of the instantaneous acceleration is equal to the square of the speed v divided by the radius R of the circle. Its direction is perpendicular to $\vec{v}$ and inward along the radius* (**Fig. 3.29a**). Because the acceleration in uniform circular motion is along the radius, we often call it **radial acceleration.**

Because the acceleration in uniform circular motion is always directed toward the center of the circle, it is sometimes called **centripetal acceleration.** The word "centripetal" is derived from two Greek words meaning "seeking the center."

APPLICATION The Moons of Jupiter

Each of the three moons of Jupiter shown in this image from the Hubble Space Telescope—Io, Europa, and Callisto—moves around Jupiter in a nearly circular orbit at a nearly constant speed. The larger the radius R of a moon's orbit, the slower the speed v at which the moon moves and the smaller its centripetal acceleration $a_{rad} = v^2/R$.

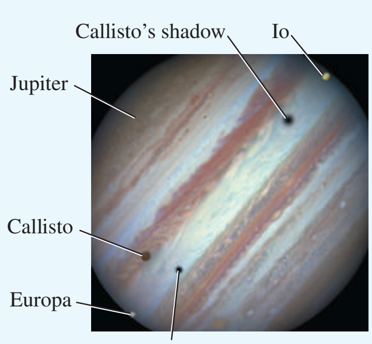
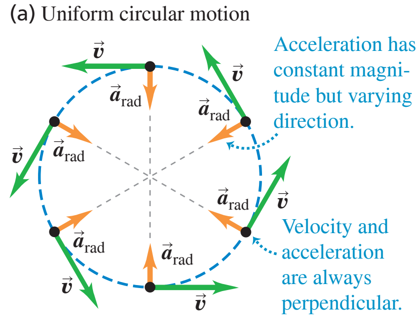
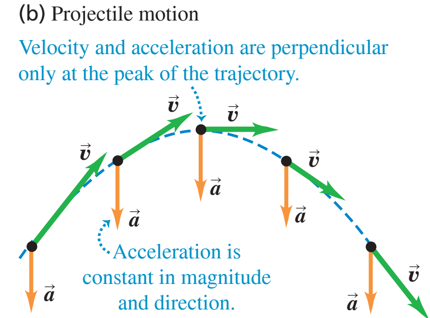


Figure **3.29** Acceleration and velocity (a) for a particle in uniform circular motion and (b) for a projectile with no air resistance.

(a) Uniform circular motion



(b) Projectile motion



CAUTION **Uniform circular motion vs. projectile motion** Notice the differences between acceleration in uniform circular motion (Fig. 3.29a) and acceleration in projectile motion (Fig. 3.29b). It's true that in both kinds of motion the *magnitude* of acceleration is the same at all times. However, in uniform circular motion the *direction* of $\vec{a}$ changes continuously—it always points toward the center of the circle and is always perpendicular to the velocity $\vec{v}$. In projectile motion, the direction of $\vec{a}$ remains the same at all times and is perpendicular to $\vec{v}$ only at the highest point of the trajectory.

We can also express the magnitude of the acceleration in uniform circular motion in terms of the **period** T of the motion, the time for one revolution (one complete trip around the circle). In a time T the particle travels a distance equal to the circumference $2\pi R$ of the circle, so its speed is

$$v = \frac{2\pi R}{T} \tag{3.28}$$

When we substitute this into Eq. (3.27), we obtain the alternative expression

Magnitude of acceleration of an object in **uniform circular motion**

$$a_{\text{rad}} = \frac{4\pi^2 R}{T^2} \tag{3.29}$$

Radius of object's circular path

Period of motion

EXAMPLE 3.11 Centripetal acceleration on a curved road

WITH VARIATION PROBLEMS

An Aston Martin V12 Vantage sports car has a "lateral acceleration" of $0.97g = (0.97)(9.8\ \text{m/s}^2) = 9.5\ \text{m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h, or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

IDENTIFY, SET UP, and EXECUTE The car is in uniform circular motion because it's moving at a constant speed along a curve that is a segment of a circle. Hence we can use Eq. (3.27) to solve for the target variable R in terms of the given centripetal acceleration a_{rad} and speed v:

$$R = \frac{v^2}{a_{\text{rad}}} = \frac{(40\ \text{m/s})^2}{9.5\ \text{m/s}^2} = 170\ \text{m (about 560 ft)}$$

This is the *minimum* turning radius because a_{rad} is the *maximum* centripetal acceleration.

EVALUATE The minimum turning radius R is proportional to the *square* of the speed, so even a small reduction in speed can make R substantially smaller. For example, reducing v by 20% (from 40 m/s to 32 m/s) would decrease R by 36% (from 170 m to 109 m).

Another way to make the minimum turning radius smaller is to *bank* the curve. We'll investigate this option in Chapter 5.

KEYCONCEPT For uniform circular motion at a given speed, decreasing the radius increases the centripetal acceleration.

EXAMPLE 3.12 Centripetal acceleration on a carnival ride

WITH VARIATION PROBLEMS

Passengers on a carnival ride move at constant speed in a horizontal circle of radius 5.0 m, making a complete circle in 4.0 s. What is their acceleration?

IDENTIFY and SET UP The speed is constant, so this is uniform circular motion. We are given the radius $R = 5.0$ m and the period $T = 4.0$ s, so we can use Eq. (3.29) to calculate the acceleration directly, or we can calculate the speed v by using Eq. (3.28) and then find the acceleration by using Eq. (3.27).

EXECUTE From Eq. (3.29),

$$a_{\text{rad}} = \frac{4\pi^2(5.0\ \text{m})}{(4.0\ \text{s})^2} = 12\ \text{m/s}^2 = 1.3g$$

EVALUATE We can check this answer by using the second, roundabout approach. From Eq. (3.28), the speed is

$$v = \frac{2\pi R}{T} = \frac{2\pi(5.0\ \text{m})}{4.0\ \text{s}} = 7.9\ \text{m/s}$$

The centripetal acceleration is then

$$a_{\text{rad}} = \frac{v^2}{R} = \frac{(7.9\ \text{m/s})^2}{5.0\ \text{m}} = 12\ \text{m/s}^2$$

As in Fig. 3.29a, the direction of $\vec{a}$ is always toward the center of the circle. The magnitude of $\vec{a}$ is relatively mild as carnival rides go; some roller coasters subject their passengers to accelerations as great as $4g$. To produce this acceleration with a carnival ride of radius 5.0 m would require a shorter period T and hence a faster speed v than in this example.

KEYCONCEPT For uniform circular motion with a given radius, decreasing the period increases the speed and the centripetal acceleration.

APPLICATION Watch Out: Tight Curves Ahead! These roller coaster cars are in nonuniform circular motion: They slow down and speed up as they move around a vertical loop. The large accelerations involved in traveling at high speed around a tight loop mean extra stress on the passengers' circulatory systems, which is why people with cardiac conditions are cautioned against going on such rides.

Nonuniform Circular Motion

We have assumed throughout this section that the particle's speed is constant as it goes around the circle. If the speed varies, we call the motion **nonuniform circular motion.** In nonuniform circular motion, Eq. (3.27) still gives the *radial* component of acceleration $a_{\text{rad}} = v^2/R$, which is always *perpendicular* to the instantaneous velocity and directed toward the center of the circle. But since the speed v has different values at different points in the motion, the value of a_{rad} is not constant. The radial (centripetal) acceleration is greatest at the point in the circle where the speed is greatest.

In nonuniform circular motion there is also a component of acceleration that is *parallel* to the instantaneous velocity (see Figs. 3.27b and 3.27c). This is the component $a_{\parallel}$ that we discussed in Section 3.2; here we call this component $a_{\tan}$ to emphasize that it is *tangent* to the circle. This component, called the **tangential acceleration** $a_{\tan}$, is equal to the rate of change of *speed*. Thus

$$a_{\text{rad}} = \frac{v^2}{R} \quad \text{and} \quad a_{\tan} = \frac{d|\vec{v}|}{dt} \quad \text{(nonuniform circular motion)} \tag{3.30}$$

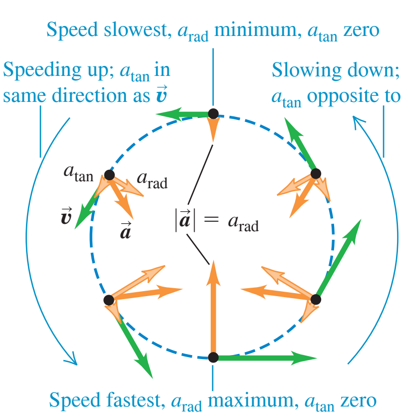
The tangential component is in the same direction as the velocity if the particle is speeding up, and in the opposite direction if the particle is slowing down (**Fig. 3.30**). If the particle's speed is constant, $a_{\tan} = 0$.

CAUTION Uniform vs. nonuniform circular motion The two quantities

$$\frac{d|\vec{v}|}{dt} \quad \text{and} \quad \left|\frac{d\vec{v}}{dt}\right|$$

are *not* the same. The first, equal to the tangential acceleration, is the rate of change of speed; it is zero whenever a particle moves with constant speed, even when its direction of motion changes (such as in *uniform* circular motion). The second is the magnitude of the vector acceleration; it is zero only when the particle's acceleration *vector* is zero—that is, when the particle moves in a straight line with constant speed. In *uniform* circular motion $|d\vec{v}/dt| = a_{\text{rad}} = v^2/r$; in *nonuniform* circular motion there is also a tangential component of acceleration, so $|d\vec{v}/dt| = \sqrt{a_{\text{rad}}^2 + a_{\tan}^2}$. ❙

Figure 3.30 A particle moving in a vertical loop with a varying speed, like a roller coaster car.



TEST YOUR UNDERSTANDING OF SECTION 3.4 Suppose that the particle in Fig. 3.30 experiences four times the acceleration at the bottom of the loop as it does at the top of the loop. Compared to its speed at the top of the loop, is its speed at the bottom of the loop (i) $\sqrt{2}$ times as great; (ii) 2 times as great; (iii) $2\sqrt{2}$ times as great; (iv) 4 times as great; or (v) 16 times as great?

ANSWER

❙ **(ii)** At both the top and bottom of the loop, the acceleration is purely radial and is given by Eq. (3.27). Radius R is the same at both points, so the difference in acceleration is due purely to differences in speed. Since a_{rad} is proportional to the square of v, the speed must be twice as great at the bottom of the loop as at the top.

3.5 RELATIVE VELOCITY

If you stand next to a one-way highway, all the cars appear to be moving forward. But if you're driving in the fast lane on that highway, slower cars appear to be moving backward. In general, when two observers measure the velocity of the same object, they get different results if one observer is moving relative to the other. The velocity seen by a particular observer is called the velocity *relative* to that observer, or simply **relative velocity.** In many situations relative velocity is extremely important (**Fig. 3.31**).

We'll first consider relative velocity along a straight line and then generalize to relative velocity in a plane.

Relative Velocity in One Dimension

A passenger walks with a velocity of $+1.0\text{ m/s}$ along the aisle of a train that is moving with a velocity of $+3.0\text{ m/s}$ (**Fig. 3.32a**). What is the passenger's velocity? It's a simple enough question, but it has no single answer. As seen by a second passenger sitting in the train, she is moving at $+1.0\text{ m/s}$. A person on a bicycle standing beside the train sees the walking passenger moving at $+1.0\text{ m/s} + 3.0\text{ m/s} = +4.0\text{ m/s}$. An observer in another train going in the opposite direction would give still another answer. We have to specify which observer we mean, and we speak of the velocity *relative* to a particular observer. The walking passenger's velocity relative to the train is $+1.0\text{ m/s}$, her velocity relative to the cyclist is $+4.0\text{ m/s}$, and so on. Each observer, equipped in principle with a meter stick and a stopwatch, forms what we call a **frame of reference**. Thus a frame of reference is a coordinate system plus a time scale.

Let's use the symbol A for the cyclist's frame of reference (at rest with respect to the ground) and the symbol B for the frame of reference of the moving train. In straight-line motion the position of a point P relative to frame A is given by $x_{P/A}$ (the position of P with respect to A), and the position of P relative to frame B is given by $x_{P/B}$ (Fig. 3.32b). The position of the origin of B with respect to the origin of A is $x_{B/A}$. Figure 3.32b shows that

$$x_{P/A} = x_{P/B} + x_{B/A} \tag{3.31}$$

In words, the coordinate of P relative to A equals the coordinate of P relative to B plus the coordinate of B relative to A.

The x-velocity of P relative to frame A, denoted by $v_{P/A\text{-}x}$, is the derivative of $x_{P/A}$ with respect to time. We can find the other velocities in the same way. So the time derivative of Eq. (3.31) gives us a relationship among the various velocities:

$$\frac{dx_{P/A}}{dt} = \frac{dx_{P/B}}{dt} + \frac{dx_{B/A}}{dt} \quad \text{or}$$

Relative velocity along a line:

$$\underbrace{v_{P/A\text{-}x}}_{x\text{-velocity of } P \text{ relative to } A} = \underbrace{v_{P/B\text{-}x}}_{x\text{-velocity of } P \text{ relative to } B} + \underbrace{v_{B/A\text{-}x}}_{x\text{-velocity of } B \text{ relative to } A} \tag{3.32}$$

Getting back to the passenger on the train in Fig. 3.32a, we see that A is the cyclist's frame of reference, B is the frame of reference of the train, and point P represents the passenger. Using the above notation, we have

$$v_{P/B\text{-}x} = +1.0\text{ m/s} \qquad v_{B/A\text{-}x} = +3.0\text{ m/s}$$

From Eq. (3.32) the passenger's velocity $v_{P/A\text{-}x}$ relative to the cyclist is

$$v_{P/A\text{-}x} = +1.0\text{ m/s} + 3.0\text{ m/s} = +4.0\text{ m/s}$$

as we already knew.

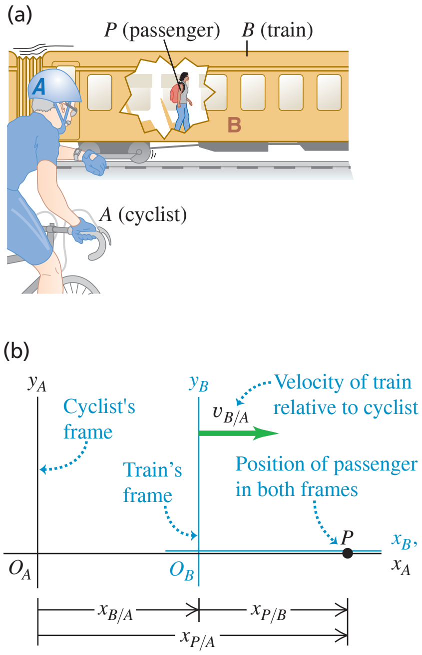
In this example, both velocities are toward the right, and we have taken this as the positive x-direction. If the passenger walks toward the *left* relative to the train, then $v_{P/B\text{-}x} = -1.0\text{ m/s}$, and her x-velocity relative to the cyclist is $v_{P/A\text{-}x} = -1.0\text{ m/s} + 3.0\text{ m/s} = +2.0\text{ m/s}$. The sum in Eq. (3.32) is always an algebraic sum, and any or all of the x-velocities may be negative.

When the passenger looks out the window, the stationary cyclist on the ground appears to her to be moving backward; we call the cyclist's velocity relative to her $v_{A/P\text{-}x}$. This is just the negative of the *passenger's* velocity relative to the *cyclist*, $v_{P/A\text{-}x}$. In general, if A and B are any two points or frames of reference,

$$v_{A/B\text{-}x} = -v_{B/A\text{-}x} \tag{3.33}$$

Figure **3.31** Airshow pilots face a complicated problem involving relative velocities. They must keep track of their motion relative to the air (to maintain enough airflow over the wings to sustain lift), relative to each other (to keep a tight formation without colliding), and relative to their audience (to remain in sight of the spectators).

Figure **3.32** (a) A passenger walking in a train. (b) The position of the passenger relative to the cyclist's frame of reference and the train's frame of reference.



PROBLEM-SOLVING STRATEGY 3.2 Relative Velocity

IDENTIFY *the relevant concepts:* Whenever you see the phrase "velocity relative to" or "velocity with respect to," it's likely that the concepts of relative velocity will be helpful.

SET UP *the problem:* Sketch and label each frame of reference in the problem. Each moving object has its own frame of reference; in addition, you'll almost always have to include the frame of reference of the earth's surface. (Statements such as "The car is traveling north at 90 km/h" implicitly refer to the car's velocity relative to the surface of the earth.) Use the labels to help identify the target variable. For example, if you want to find the x-velocity of a car (C) with respect to a bus (B), your target variable is $v_{C/B\text{-}x}$.

EXECUTE *the solution:* Solve for the target variable using Eq. (3.32). (If the velocities aren't along the same direction, you'll need to use the vector form of this equation, derived later in this section.) It's important to note the order of the double subscripts in Eq. (3.32): $v_{B/A\text{-}x}$ means "x-velocity of B relative to A." These subscripts obey a kind of algebra. If we regard each one as a fraction, then the fraction on the left side is the *product* of the fractions on the right side: $P/A = (P/B)(B/A)$. You can apply this rule to any number of frames of reference. For example, if there are three frames of reference A, B, and C, Eq. (3.32) becomes

$$v_{P/A\text{-}x} = v_{P/C\text{-}x} + v_{C/B\text{-}x} + v_{B/A\text{-}x}$$

EVALUATE *your answer:* Be on the lookout for stray minus signs in your answer. If the target variable is the x-velocity of a car relative to a bus $(v_{C/B\text{-}x})$, make sure that you haven't accidentally calculated the x-velocity of the *bus* relative to the *car* $(v_{B/C\text{-}x})$. If you've made this mistake, you can recover by using Eq. (3.33).

EXAMPLE 3.13 Relative velocity on a straight road

WITH VARIATION PROBLEMS

You drive north on a straight two-lane road at a constant 88 km/h. A truck in the other lane approaches you at a constant 104 km/h (**Fig. 3.33**). Find (a) the truck's velocity relative to you and (b) your velocity relative to the truck. (c) How do the relative velocities change after you and the truck pass each other? Treat this as a one-dimensional problem.

IDENTIFY and SET UP In this problem about relative velocities along a line, there are three reference frames: you (Y), the truck (T), and the earth's surface (E). Let the positive x-direction be north (Fig. 3.33). Then your x-velocity relative to the earth is $v_{Y/E\text{-}x} = +88$ km/h. The truck is initially approaching you, so it is moving south and its x-velocity with respect to the earth is $v_{T/E\text{-}x} = -104$ km/h. The target variables in parts (a) and (b) are $v_{T/Y\text{-}x}$ and $v_{Y/T\text{-}x}$, respectively. We'll use Eq. (3.32) to find the first target variable and Eq. (3.33) to find the second.

EXECUTE (a) To find $v_{T/Y\text{-}x}$, we write Eq. (3.32) for the known $v_{T/E\text{-}x}$ and rearrange:

$$v_{T/E\text{-}x} = v_{T/Y\text{-}x} + v_{Y/E\text{-}x}$$
$$v_{T/Y\text{-}x} = v_{T/E\text{-}x} - v_{Y/E\text{-}x} = -104\text{ km/h} - 88\text{ km/h} = -192\text{ km/h}$$

The truck is moving at 192 km/h in the negative x-direction (south) relative to you.

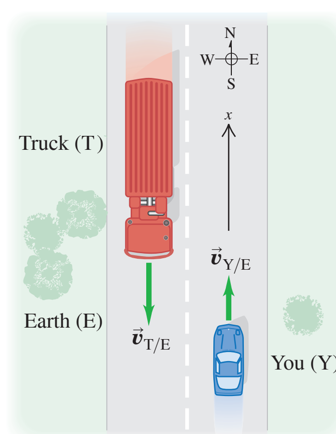
(b) From Eq. (3.33),

$$v_{Y/T\text{-}x} = -v_{T/Y\text{-}x} = -(-192\text{ km/h}) = +192\text{ km/h}$$

You are moving at 192 km/h in the positive x-direction (north) relative to the truck.

(c) The relative velocities do *not* change after you and the truck pass each other. The relative *positions* of the objects don't matter. After it passes you, the truck is still moving at 192 km/h toward the south relative to you, even though it is now moving away from you instead of toward you.

Figure 3.33 Reference frames for you and the truck.



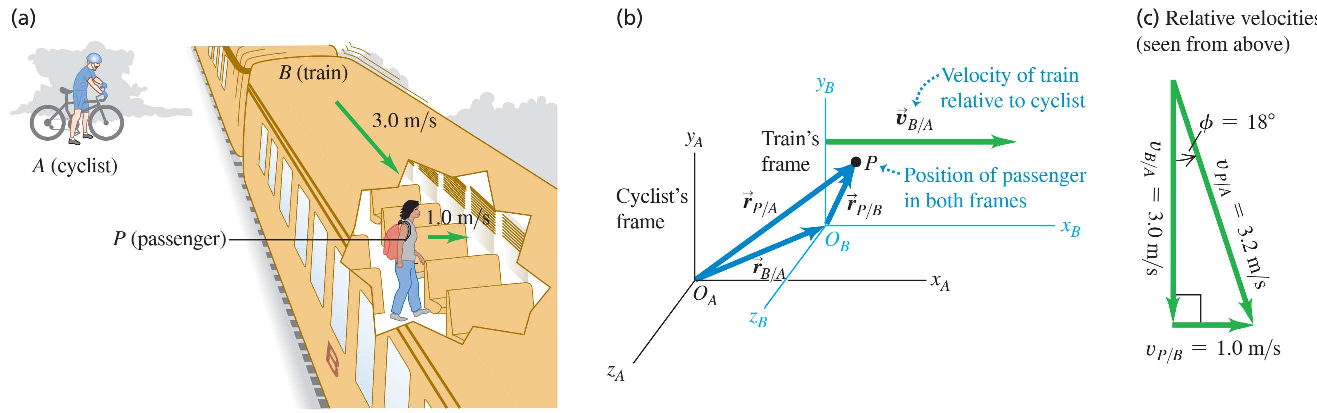
EVALUATE To check your answer in part (b), use Eq. (3.32) directly in the form $v_{Y/T\text{-}x} = v_{Y/E\text{-}x} + v_{E/T\text{-}x}$. (The x-velocity of the earth with respect to the truck is the opposite of the x-velocity of the truck with respect to the earth: $v_{E/T\text{-}x} = -v_{T/E\text{-}x}$.) Do you get the same result?

KEYCONCEPT To solve problems involving relative velocity along a line, use Eq. (3.32) and pay careful attention to the subscripts for the frames of reference in the problem.

Relative Velocity in Two or Three Dimensions

Let's extend the concept of relative velocity to include motion in a plane or in space. Suppose that the passenger in Fig. 3.32a is walking not down the aisle of the railroad car but from one side of the car to the other, with a speed of 1.0 m/s (**Fig. 3.34a**). We can again describe the passenger's position P in two frames of reference: A for the stationary ground observer and B for the moving train. But instead of coordinates x, we use position

Figure **3.34** (a) A passenger walking across a railroad car. (b) Position of the passenger relative to the cyclist's frame and the train's frame. (c) Vector diagram for the velocity of the passenger relative to the ground (the cyclist's frame), $\vec{v}_{P/A}$.



vectors $\vec{r}$ because the problem is now two-dimensional. Then, as Fig. 3.34b shows,

$$\vec{r}_{P/A} = \vec{r}_{P/B} + \vec{r}_{B/A} \tag{3.34}$$

Just as we did before, we take the time derivative of this equation to get a relationship among the various velocities; the velocity of P relative to A is $\vec{v}_{P/A} = d\vec{r}_{P/A}/dt$ and so on for the other velocities. We get

Relative velocity in space:

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A} \tag{3.35}$$

Velocity of P relative to A — Velocity of P relative to B — Velocity of B relative to A

Equation (3.35) is known as the *Galilean velocity transformation*. It relates the velocity of an object P with respect to frame A and its velocity with respect to frame B ($\vec{v}_{P/A}$ and $\vec{v}_{P/B}$, respectively) to the velocity of frame B with respect to frame A ($\vec{v}_{B/A}$). If all three of these velocities lie along the same line, then Eq. (3.35) reduces to Eq. (3.32) for the components of the velocities along that line.

If the train is moving at $v_{B/A} = 3.0$ m/s relative to the ground and the passenger is moving at $v_{P/B} = 1.0$ m/s relative to the train, then the passenger's velocity vector $\vec{v}_{P/A}$ relative to the ground is as shown in Fig. 3.34c. The Pythagorean theorem then gives us

$$v_{P/A} = \sqrt{(3.0 \text{ m/s})^2 + (1.0 \text{ m/s})^2} = \sqrt{10 \text{ m}^2/\text{s}^2} = 3.2 \text{ m/s}$$

Figure 3.34c also shows that the *direction* of the passenger's velocity vector relative to the ground makes an angle ϕ with the train's velocity vector $\vec{v}_{B/A}$, where

$$\tan\phi = \frac{v_{P/B}}{v_{B/A}} = \frac{1.0 \text{ m/s}}{3.0 \text{ m/s}} \quad \text{and} \quad \phi = 18°$$

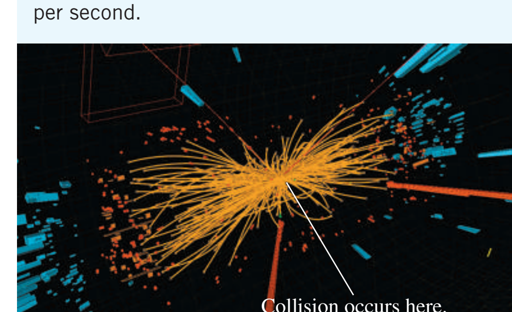
As in the case of motion along a straight line, we have the general rule that if A and B are *any* two points or frames of reference,

$$\vec{v}_{A/B} = -\vec{v}_{B/A} \tag{3.36}$$

The velocity of the passenger relative to the train is the negative of the velocity of the train relative to the passenger, and so on.

In the early 20th century Albert Einstein showed that Eq. (3.35) has to be modified when speeds approach the speed of light, denoted by c. It turns out that if the passenger in Fig. 3.32a could walk down the aisle at $0.30c$ and the train could move at $0.90c$, then her speed relative to the ground would be not $1.20c$ but $0.94c$; nothing can travel faster than light! We'll return to Einstein and his *special theory of relativity* in Chapter 37.

APPLICATION Relative Velocities near the Speed of Light This image shows a spray of subatomic particles produced by the head-on collision of two protons moving in opposite directions. Relative to the laboratory, before the collision each proton is moving only 11 m/s slower than the speed of light, $c = 3.00 \times 10^8$ m/s. According to the Galilean velocity transformation, the velocity of one proton relative to the other should be approximately $c + c = 2c$. But Einstein's special theory of relativity shows that the relative velocity of the two protons must be slower than the speed of light: In fact it is equal to c minus 0.19 millimeter per second.



EXAMPLE 3.14 Flying in a crosswind

WITH VARIATION PROBLEMS

An airplane's compass indicates that it is headed due north, and its airspeed indicator shows that it is moving through the air at 240 km/h. If there is a 100 km/h wind from west to east, what is the velocity of the airplane relative to the earth?

IDENTIFY and SET UP This problem involves velocities in two dimensions (northward and eastward), so it is a relative-velocity problem using vectors. We are given the magnitude and direction of the velocity of the plane (P) relative to the air (A). We are also given the magnitude and direction of the wind velocity, which is the velocity of the air A with respect to the earth (E):

$$\vec{v}_{\mathrm{P/A}} = 240 \text{ km/h} \qquad \text{due north}$$

$$\vec{v}_{\mathrm{A/E}} = 100 \text{ km/h} \qquad \text{due east}$$

We'll use Eq. (3.35) to find our target variables: the magnitude and direction of velocity $\vec{v}_{\mathrm{P/E}}$ of the plane relative to the earth.

EXECUTE From Eq. (3.35) we have

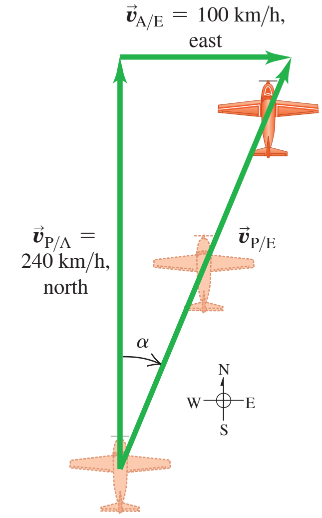
$$\vec{v}_{\mathrm{P/E}} = \vec{v}_{\mathrm{P/A}} + \vec{v}_{\mathrm{A/E}}$$

Figure 3.35 shows that the three relative velocities constitute a right-triangle vector addition; the unknowns are the speed $v_{\mathrm{P/E}}$ and the angle α. We find

$$v_{\mathrm{P/E}} = \sqrt{(240 \text{ km/h})^2 + (100 \text{ km/h})^2} = 260 \text{ km/h}$$

$$\alpha = \arctan\left(\frac{100 \text{ km/h}}{240 \text{ km/h}}\right) = 23^\circ \text{ E of N}$$

Figure 3.35 The plane is pointed north, but the wind blows east, giving the resultant velocity $\vec{v}_{\mathrm{P/E}}$ relative to the earth.



EVALUATE You can check the results by taking measurements on the scale drawing in Fig. 3.35. The crosswind increases the speed of the airplane relative to the earth, but pushes the airplane off course.

KEYCONCEPT To solve problems involving relative velocity in a plane or in space, use Eq. (3.35). Pay careful attention to the subscripts for the frames of reference in the problem.

EXAMPLE 3.15 Correcting for a crosswind

WITH VARIATION PROBLEMS

With wind and airspeed as in Example 3.14, in what direction should the pilot head to travel due north? What will be her velocity relative to the earth?

IDENTIFY and SET UP Like Example 3.14, this is a relative-velocity problem with vectors. **Figure 3.36** is a scale drawing of the situation. Again the vectors add in accordance with Eq. (3.35) and form a right triangle:

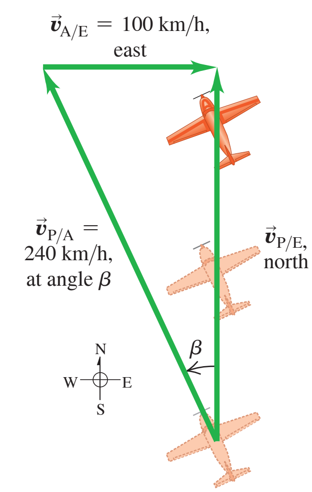
$$\vec{v}_{\mathrm{P/E}} = \vec{v}_{\mathrm{P/A}} + \vec{v}_{\mathrm{A/E}}$$

As Fig. 3.36 shows, the pilot points the nose of the airplane at an angle β into the wind to compensate for the crosswind. This angle, which tells us the direction of the vector $\vec{v}_{\mathrm{P/A}}$ (the velocity of the airplane relative to the air), is one of our target variables. The other target variable is the speed of the airplane over the ground, which is the magnitude of the vector $\vec{v}_{\mathrm{P/E}}$ (the velocity of the airplane relative to the earth). The known and unknown quantities are

$$\vec{v}_{\mathrm{P/E}} = \text{magnitude unknown} \qquad \text{due north}$$
$$\vec{v}_{\mathrm{P/A}} = 240 \text{ km/h} \qquad \text{direction unknown}$$
$$\vec{v}_{\mathrm{A/E}} = 100 \text{ km/h} \qquad \text{due east}$$

We'll solve for the target variables by using Fig. 3.36 and trigonometry.

Figure 3.36 The pilot must point the plane in the direction of the vector $\vec{v}_{\mathrm{P/A}}$ to travel due north relative to the earth.



EXECUTE From Fig. 3.36 the speed $v_{P/E}$ and the angle β are

$$v_{P/E} = \sqrt{(240\ \text{km/h})^2 - (100\ \text{km/h})^2} = 218\ \text{km/h}$$

$$\beta = \arcsin\left(\frac{100\ \text{km/h}}{240\ \text{km/h}}\right) = 25^\circ$$

The pilot should point the airplane 25° west of north, and her ground speed is then 218 km/h.

EVALUATE There were two target variables—the magnitude of a vector and the direction of a vector—in both this example and Example 3.14. In Example 3.14 the magnitude and direction referred to the *same* vector ($\vec{v}_{P/E}$); here they refer to *different* vectors ($\vec{v}_{P/E}$ and $\vec{v}_{P/A}$).

While we expect a *headwind* to reduce an airplane's speed relative to the ground, this example shows that a *crosswind* does, too. That's an unfortunate fact of aeronautical life.

KEYCONCEPT The vector equation for relative velocity in a plane, Eq. (3.35), allows you to solve for two unknowns, such as an unknown vector magnitude and an unknown direction.

TEST YOUR UNDERSTANDING OF SECTION 3.5 Suppose the nose of an airplane is pointed due east and the airplane has an airspeed of 150 km/h. Due to the wind, the airplane is moving due *north* relative to the ground and its speed relative to the ground is 150 km/h. What is the velocity of the air relative to the earth? (i) 150 km/h from east to west; (ii) 150 km/h from south to north; (iii) 150 km/h from southeast to northwest; (iv) 212 km/h from east to west; (v) 212 km/h from south to north; (vi) 212 km/h from southeast to northwest; (vii) there is no possible wind velocity that could cause this.

ANSWER

(vi) The effect of the wind is to cancel the airplane's eastward motion and give it a northward motion. So the velocity of the air relative to the ground (the wind velocity) must have one 150 km/h component to the west and one 150 km/h component to the north. The combination of these is a vector of magnitude $\sqrt{(150\ \text{km/h})^2 + (150\ \text{km/h})^2} = 212\ \text{km/h}$ that points to the northwest.

CHAPTER 3 SUMMARY

Position, velocity, and acceleration vectors: The position vector $\vec{r}$ of a point P in space is the vector from the origin to P. Its components are the coordinates x, y, and z.

The average velocity vector $\vec{v}_{av}$ during the time interval Δt is the displacement $\Delta\vec{r}$ (the change in position vector $\vec{r}$) divided by Δt. The instantaneous velocity vector $\vec{v}$ is the time derivative of $\vec{r}$, and its components are the time derivatives of x, y, and z. The instantaneous speed is the magnitude of $\vec{v}$. The velocity $\vec{v}$ of a particle is always tangent to the particle's path. (See Example 3.1.)

The average acceleration vector $\vec{a}_{av}$ during the time interval Δt equals $\Delta\vec{v}$ (the change in velocity vector $\vec{v}$) divided by Δt. The instantaneous acceleration vector $\vec{a}$ is the time derivative of $\vec{v}$, and its components are the time derivatives of v_x, v_y, and v_z. (See Example 3.2.)

The component of acceleration parallel to the direction of the instantaneous velocity affects the speed, while the component of $\vec{a}$ perpendicular to $\vec{v}$ affects the direction of motion. (See Examples 3.3 and 3.4.)

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \tag{3.1}$$

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t} \tag{3.2}$$

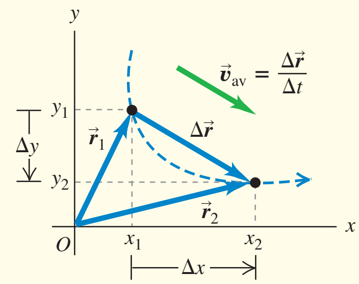
$$\vec{v} = \lim_{\Delta t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} \tag{3.3}$$

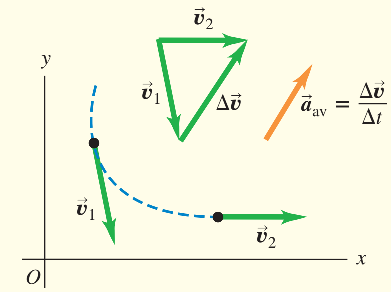
$$v_x = \frac{dx}{dt} \quad v_y = \frac{dy}{dt} \quad v_z = \frac{dz}{dt} \tag{3.4}$$

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t} \tag{3.8}$$

$$\vec{a} = \lim_{\Delta t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} \tag{3.9}$$

$$a_x = \frac{dv_x}{dt} \qquad a_y = \frac{dv_y}{dt} \qquad a_z = \frac{dv_z}{dt} \tag{3.10}$$





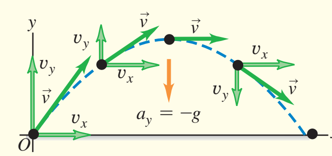
Projectile motion: In projectile motion with no air resistance, $a_x = 0$ and $a_y = -g$. The coordinates and velocity components are simple functions of time, and the shape of the path is always a parabola. We usually choose the origin to be at the initial position of the projectile. (See Examples 3.5–3.10.)

$$x = (v_0\cos\alpha_0)t \tag{3.19}$$

$$y = (v_0\sin\alpha_0)t - \tfrac{1}{2}gt^2 \tag{3.20}$$

$$v_x = v_0\cos\alpha_0 \tag{3.21}$$

$$v_y = v_0\sin\alpha_0 - gt \tag{3.22}$$

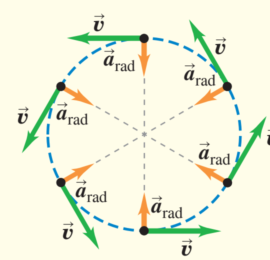


Uniform and nonuniform circular motion: When a particle moves in a circular path of radius R with constant speed v (uniform circular motion), its acceleration $\vec{a}$ is directed toward the center of the circle and perpendicular to $\vec{v}$. The magnitude a_{rad} of this radial acceleration can be expressed in terms of v and R or in terms of R and the period T (the time for one revolution), where $v = 2\pi R/T$. (See Examples 3.11 and 3.12.)

If the speed is not constant in circular motion (nonuniform circular motion), there is still a radial component of $\vec{a}$ given by Eq. (3.27) or (3.29), but there is also a component of $\vec{a}$ parallel (tangential) to the path. This tangential component is equal to the rate of change of speed, dv/dt.

$$a_{rad} = \frac{v^2}{R} \tag{3.27}$$

$$a_{rad} = \frac{4\pi^2 R}{T^2} \tag{3.29}$$



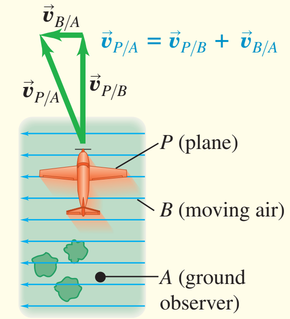
Relative velocity: When an object P moves relative to an object (or reference frame) B, and B moves relative to an object (or reference frame) A, we denote the velocity of P relative to B by $\vec{v}_{P/B}$, the velocity of P relative to A by $\vec{v}_{P/A}$, and the velocity of B relative to A by $\vec{v}_{B/A}$. If these velocities are all along the same line, their components along that line are related by Eq. (3.32). More generally, these velocities are related by Eq. (3.35). (See Examples 3.13–3.15.)

$$v_{P/A\text{-}x} = v_{P/B\text{-}x} + v_{B/A\text{-}x} \tag{3.32}$$

(relative velocity along a line)

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A} \tag{3.35}$$

(relative velocity in space)



Chapter 3 Media Assets

For assigned homework and other learning materials, go to Mastering Physics.

GUIDED PRACTICE

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLES 3.6, 3.7, 3.8, and 3.9 (Section 3.3) before attempting these problems. In all problems, ignore air resistance.

VP3.9.1 You launch a projectile from level ground at a speed of 25.0 m/s and an angle of 36.9° above the horizontal. (a) How long after it is launched does the projectile reach its maximum height above the ground, and what is that maximum height? (b) How long after the projectile is launched does it return to ground level, and how far from its launch point does it land?

VP3.9.2 You throw a baseball at an angle of 30.0° above the horizontal. It reaches the highest point of its trajectory 1.05 s later. (a) At what speed does the baseball leave your hand? (b) What is the maximum height above the launch point that the baseball reaches?

VP3.9.3 You toss a walnut at a speed of 15.0 m/s at an angle of 50.0° above the horizontal. The launch point is on the roof of a building that is 20.0 m above the ground. (a) How long after it is launched does the walnut reach the ground? (b) How far does the walnut travel horizontally from launch point to landing point? (c) What are the horizontal and vertical components of the walnut's velocity just before it reaches the ground?

VP3.9.4 You use a slingshot to launch a potato horizontally from the edge of a cliff with speed v_0. The acceleration due to gravity is g. Take the origin at the launch point. (a) How long after you launch the potato has it moved as far horizontally from the launch point as it has moved vertically? What are the coordinates of the potato at this time? (b) How long after you launch the potato is it moving in a direction exactly 45° below the horizontal? What are the coordinates of the potato at this time?

Be sure to review EXAMPLES 3.11 and 3.12 (Section 3.4) before attempting these problems.

VP3.12.1 A cyclist going around a circular track at 10.0 m/s has a centripetal acceleration of $5.00\,\text{m/s}^2$. What is the radius of the curve?

VP3.12.2 A race car is moving at 40.0 m/s around a circular racetrack of radius 265 m. Calculate (a) the period of the motion and (b) the car's centripetal acceleration.

VP3.12.3 The wheel of a stationary exercise bicycle at your gym makes one rotation in 0.670 s. Consider two points on this wheel: Point P is 10.0 cm from the rotation axis, and point Q is 20.0 cm from the rotation axis. Find (a) the speed of each point on the spinning wheel and (b) the centripetal acceleration of each point. (c) For points on this spinning wheel, as the distance from the axis increases, does the speed increase or decrease? Does the centripetal acceleration increase or decrease?

VP3.12.4 The planets Venus, Earth, and Mars all move in approximately circular orbits around the sun. Use the data in the table to find (a) the speed of each planet in its orbit and (b) the centripetal acceleration of each planet. (c) As the size of a planet's orbit increases, does the speed increase or decrease? Does the centripetal acceleration increase or decrease?

Planet	Orbital radius (m)	Orbital period (days)
Venus	1.08×10^{11}	225
Earth	1.50×10^{11}	365
Mars	2.28×10^{11}	687

VP3.12.5 Object A is moving at speed v in a circle of radius R. Object B is moving at speed $2v$ in a circle of radius $R/2$. (a) What is the ratio of the period of object A to the period of object B? (b) What is the ratio of the centripetal acceleration of object A to the centripetal acceleration of object B?

Be sure to review EXAMPLES 3.13, 3.14, and 3.15 (Section 3.5) before attempting these problems.

VP3.15.1 A police car in a high-speed chase is traveling north on a two-lane highway at 35.0 m/s. In the southbound lane of the same highway, an SUV is moving at 18.0 m/s. Take the positive x-direction to be toward the north. Find the x-velocity of (a) the police car relative to the SUV and (b) the SUV relative to the police car.

VP3.15.2 Race cars A and B are driving on the same circular racetrack at the same speed of 45.0 m/s. At a given instant car A is on the north side of the track moving eastward and car B is on the south side of the track moving westward. Find the velocity vector (magnitude and direction) of (a) car A relative to car B and (b) car B relative to car A. (c) Does the relative velocity have a component along the line connecting the two cars? Are the two cars approaching each other, moving away from each other, or neither?

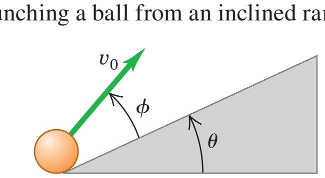
VP3.15.3 Two vehicles approach an intersection: a truck moving eastbound at 16.0 m/s and an SUV moving southbound at 20.0 m/s. Find the velocity vector (magnitude and direction) of (a) the truck relative to the SUV and (b) the SUV relative to the truck.

VP3.15.4 A jet is flying due north relative to the ground. The speed of the jet relative to the ground is 155 m/s. The wind at the jet's altitude is 40.0 m/s toward the northeast (45.0° north of east). Find the speed of the jet relative to the air (its airspeed) and the direction in which the pilot of the jet must point the plane so that it travels due north relative to the ground.

BRIDGING PROBLEM Launching Up an Incline

You fire a ball with an initial speed v_0 at an angle ϕ above the surface of an incline, which is itself inclined at an angle θ above the horizontal (**Fig. 3.37**). (a) Find the distance, measured along the incline, from the launch point to the point when the ball strikes the incline. (b) What angle ϕ gives the maximum range, measured along the incline? Ignore air resistance.

Figure 3.37 Launching a ball from an inclined ramp.



SOLUTION GUIDE

IDENTIFY and SET UP

1. Since there's no air resistance, this is a problem in projectile motion. The goal is to find the point where the ball's parabolic trajectory intersects the incline.

Continued

2. Choose the x- and y-axes and the position of the origin. When in doubt, use the suggestions given in Problem-Solving Strategy 3.1 in Section 3.3.
3. In the projectile equations in Section 3.3, the launch angle α_0 is measured from the horizontal. What is this angle in terms of θ and ϕ? What are the initial x- and y-components of the ball's initial velocity?
4. You'll need to write an equation that relates x and y for points along the incline. What is this equation? (This takes just geometry and trigonometry, not physics.)

EXECUTE

5. Write the equations for the x-coordinate and y-coordinate of the ball as functions of time t.
6. When the ball hits the incline, x and y are related by the equation that you found in step 4. Based on this, at what time t does the ball hit the incline?
7. Based on your answer from step 6, at what coordinates x and y does the ball land on the incline? How far is this point from the launch point?
8. What value of ϕ gives the *maximum* distance from the launch point to the landing point? (Use your knowledge of calculus.)

EVALUATE

9. Check your answers for the case $\theta = 0$, which corresponds to the incline being horizontal rather than tilted. (You already know the answers for this case. Do you know why?)

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q3.1 A simple pendulum (a mass swinging at the end of a string) swings back and forth in a circular arc. What is the direction of the acceleration of the mass when it is at the ends of the swing? At the midpoint? In each case, explain how you obtained your answer.

Q3.2 Redraw Fig. 3.11a if $\vec{a}$ is antiparallel to $\vec{v}_1$. Does the particle move in a straight line? What happens to its speed?

Q3.3 A projectile moves in a parabolic path without air resistance. Is there any point at which $\vec{a}$ is parallel to $\vec{v}$? Perpendicular to $\vec{v}$? Explain.

Q3.4 A book slides off a horizontal tabletop. As it leaves the table's edge, the book has a horizontal velocity of magnitude v_0. The book strikes the floor in time t. If the initial velocity of the book is doubled to $2v_0$, what happens to (a) the time the book is in the air, (b) the horizontal distance the book travels while it is in the air, and (c) the speed of the book just before it reaches the floor? In particular, does each of these quantities stay the same, double, or change in another way? Explain.

Q3.5 At the instant that you fire a bullet horizontally from a rifle, you drop a bullet from the height of the gun barrel. If there is no air resistance, which bullet hits the level ground first? Explain.

Q3.6 A package falls out of an airplane that is flying in a straight line at a constant altitude and speed. If you ignore air resistance, what would be the path of the package as observed by the pilot? As observed by a person on the ground?

Q3.7 Sketch the six graphs of the x- and y-components of position, velocity, and acceleration versus time for projectile motion with $x_0 = y_0 = 0$ and $0 < \alpha_0 < 90°$.

Q3.8 If a jumping frog can give itself the same initial speed regardless of the direction in which it jumps (forward or straight up), how is the maximum vertical height to which it can jump related to its maximum horizontal range $R_{max} = v_0^2/g$?

Q3.9 A projectile is fired upward at an angle θ above the horizontal with an initial speed v_0. At its maximum height, what are its velocity vector, its speed, and its acceleration vector?

Q3.10 In uniform circular motion, what are the *average* velocity and *average* acceleration for one revolution? Explain.

Q3.11 In uniform circular motion, how does the acceleration change when the speed is increased by a factor of 3? When the radius is decreased by a factor of 2?

Q3.12 In uniform circular motion, the acceleration is perpendicular to the velocity at every instant. Is this true when the motion is not uniform—that is, when the speed is not constant?

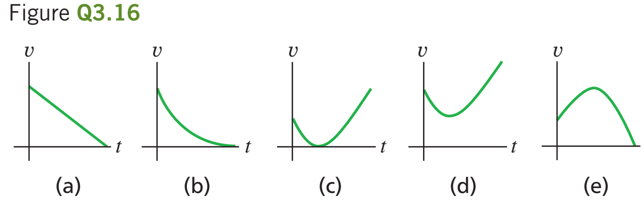
Q3.13 Raindrops hitting the side windows of a car in motion often leave diagonal streaks even if there is no wind. Why? Is the explanation the same or different for diagonal streaks on the windshield?

Q3.14 In a rainstorm with a strong wind, what determines the best position in which to hold an umbrella?

Q3.15 You are on the west bank of a river that is flowing north with a speed of 1.2 m/s. Your swimming speed relative to the water is 1.5 m/s, and the river is 60 m wide. What is your path relative to the earth that allows you to cross the river in the shortest time? Explain your reasoning.

Q3.16 A stone is thrown into the air at an angle above the horizontal and feels negligible air resistance. Which graph in **Fig. Q3.16** best depicts the stone's *speed* v as a function of time t while it is in the air?

Figure **Q3.16**



EXERCISES

Section 3.1 Position and Velocity Vectors

3.1 • A squirrel has x- and y-coordinates (1.1 m, 3.4 m) at time $t_1 = 0$ and coordinates (5.3 m, −0.5 m) at time $t_2 = 3.0$ s. For this time interval, find (a) the components of the average velocity, and (b) the magnitude and direction of the average velocity.

3.2 • A rhinoceros is at the origin of coordinates at time $t_1 = 0$. For the time interval from $t_1 = 0$ to $t_2 = 12.0$ s, the rhino's average velocity has x-component -3.8 m/s and y-component 4.9 m/s. At time $t_2 = 12.0$ s, (a) what are the x- and y-coordinates of the rhino? (b) How far is the rhino from the origin?

3.3 •• **CALC** A web page designer creates an animation in which a dot on a computer screen has position

$$\vec{r} = [4.0 \text{ cm} + (2.5 \text{ cm/s}^2)t^2]\hat{\imath} + (5.0 \text{ cm/s})t\hat{\jmath}$$

(a) Find the magnitude and direction of the dot's average velocity between $t = 0$ and $t = 2.0$ s. (b) Find the magnitude and direction of the instantaneous velocity at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s. (c) Sketch the dot's trajectory from $t = 0$ to $t = 2.0$ s, and show the velocities calculated in part (b).

3.4 • **CALC** The position of a squirrel running in a park is given by $\vec{r} = [(0.280 \text{ m/s})t + (0.0360 \text{ m/s}^2)t^2]\hat{\imath} + (0.0190 \text{ m/s}^3)t^3\hat{\jmath}$. (a) What are $v_x(t)$ and $v_y(t)$, the x- and y-components of the velocity of the squirrel, as functions of time? (b) At $t = 5.00$ s, how far is the squirrel from its initial position? (c) At $t = 5.00$ s, what are the magnitude and direction of the squirrel's velocity?

Section 3.2 The Acceleration Vector

3.5 • A jet plane is flying at a constant altitude. At time $t_1 = 0$, it has components of velocity $v_x = 90$ m/s, $v_y = 110$ m/s. At time $t_2 = 30.0$ s, the components are $v_x = -170$ m/s, $v_y = 40$ m/s. (a) Sketch the velocity vectors at t_1 and t_2. How do these two vectors differ? For this time interval calculate (b) the components of the average acceleration, and (c) the magnitude and direction of the average acceleration.

3.6 •• A dog running in an open field has components of velocity $v_x = 2.6$ m/s and $v_y = -1.8$ m/s at $t_1 = 10.0$ s. For the time interval from $t_1 = 10.0$ s to $t_2 = 20.0$ s, the average acceleration of the dog has magnitude 0.45 m/s^2 and direction 31.0° measured from the $+x$-axis toward the $+y$-axis. At $t_2 = 20.0$ s, (a) what are the x- and y-components of the dog's velocity? (b) What are the magnitude and direction of the dog's velocity? (c) Sketch the velocity vectors at t_1 and t_2. How do these two vectors differ?

3.7 •• **CALC** The coordinates of a bird flying in the xy-plane are given by $x(t) = \alpha t$ and $y(t) = 3.0 \text{ m} - \beta t^2$, where $\alpha = 2.4$ m/s and $\beta = 1.2 \text{ m/s}^2$. (a) Sketch the path of the bird between $t = 0$ and $t = 2.0$ s. (b) Calculate the velocity and acceleration vectors of the bird as functions of time. (c) Calculate the magnitude and direction of the bird's velocity and acceleration at $t = 2.0$ s. (d) Sketch the velocity and acceleration vectors at $t = 2.0$ s. At this instant, is the bird's speed increasing, decreasing, or not changing? Is the bird turning? If so, in what direction?

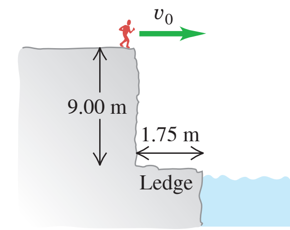
3.8 • **CALC** A remote-controlled car is moving in a vacant parking lot. The velocity of the car as a function of time is given by $\vec{v} = [5.00 \text{ m/s} - (0.0180 \text{ m/s}^3)t^2]\hat{\imath} + [2.00 \text{ m/s} + (0.550 \text{ m/s}^2)t]\hat{\jmath}$. (a) What are $a_x(t)$ and $a_y(t)$, the x- and y-components of the car's velocity as functions of time? (b) What are the magnitude and direction of the car's velocity at $t = 8.00$ s? (b) What are the magnitude and direction of the car's acceleration at $t = 8.00$ s?

Section 3.3 Projectile Motion

3.9 • A physics book slides off a horizontal tabletop with a speed of 1.10 m/s. It strikes the floor in 0.480 s. Ignore air resistance. Find (a) the height of the tabletop above the floor; (b) the horizontal distance from the edge of the table to the point where the book strikes the floor; (c) the horizontal and vertical components of the book's velocity, and the magnitude and direction of its velocity, just before the book reaches the floor. (d) Draw x-t, y-t, v_x-t, and v_y-t graphs for the motion.

3.10 •• A daring 510 N swimmer dives off a cliff with a running horizontal leap, as shown in **Fig. E3.10**. What must her minimum speed be just as she leaves the top of the cliff so that she will miss the ledge at the bottom, which is 1.75 m wide and 9.00 m below the top of the cliff?

Figure **E3.10**



3.11 • Crickets Chirpy and Milada jump from the top of a vertical cliff. Chirpy drops downward and reaches the ground in 2.70 s, while Milada jumps horizontally with an initial speed of 95.0 cm/s. How far from the base of the cliff will Milada hit the ground? Ignore air resistance.

3.12 • A rookie quarterback throws a football with an initial upward velocity component of 12.0 m/s and a horizontal velocity component of 20.0 m/s. Ignore air resistance. (a) How much time is required for the football to reach the highest point of the trajectory? (b) How high is this point? (c) How much time (after it is thrown) is required for the football to return to its original level? How does this compare with the time calculated in part (a) (d) How far has the football traveled horizontally during this time? (e) Draw x-t, y-t, v_x-t, and v_y-t graphs for the motion.

3.13 •• **Leaping the River I.** During a storm, a car traveling on a level horizontal road comes upon a bridge that has washed out. The driver must get to the other side, so he decides to try leaping the river with his car. The side of the road the car is on is 21.3 m above the river, while the opposite side is only 1.8 m above the river. The river itself is a raging torrent 48.0 m wide. (a) How fast should the car be traveling at the time it leaves the road in order just to clear the river and land safely on the opposite side? (b) What is the speed of the car just before it lands on the other side?

3.14 • **BIO The Champion Jumper of the Insect World.** The froghopper, *Philaenus spumarius,* holds the world record for insect jumps. When leaping at an angle of 58.0° above the horizontal, some of the tiny critters have reached a maximum height of 58.7 cm above the level ground. (See *Nature,* Vol. 424, July 31, 2003, p. 509.) Neglect air resistance in answering the following. (a) What was the takeoff speed for such a leap? (b) What horizontal distance did the froghopper cover for this world-record leap?

3.15 •• Inside a starship at rest on the earth, a ball rolls off the top of a horizontal table and lands a distance D from the foot of the table. This starship now lands on the unexplored Planet X. The commander, Captain Curious, rolls the same ball off the same table with the same initial speed as on earth and finds that it lands a distance $2.76D$ from the foot of the table. What is the acceleration due to gravity on Planet X?

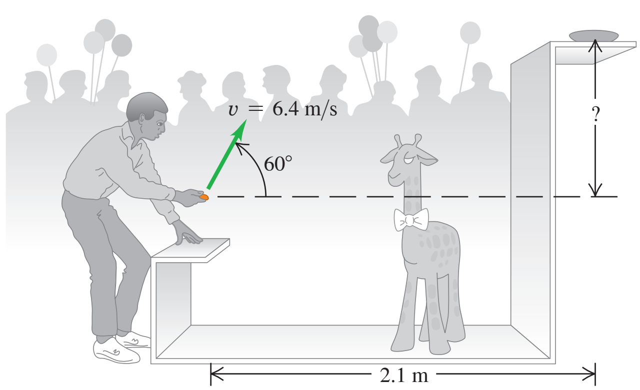
3.16 • On level ground a shell is fired with an initial velocity of 40.0 m/s at 60.0° above the horizontal and feels no appreciable air resistance. (a) Find the horizontal and vertical components of the shell's initial velocity. (b) How long does it take the shell to reach its highest point? (c) Find its maximum height above the ground. (d) How far from its firing point does the shell land? (e) At its highest point, find the horizontal and vertical components of its acceleration and velocity.

3.17 • A major leaguer hits a baseball so that it leaves the bat at a speed of 30.0 m/s and at an angle of 36.9° above the horizontal. Ignore air resistance. (a) At what *two* times is the baseball at a height of 10.0 m above the point at which it left the bat? (b) Calculate the horizontal and vertical components of the baseball's velocity at each of the two times calculated in part (a). (c) What are the magnitude and direction of the baseball's velocity when it returns to the level at which it left the bat?

3.18 • A shot putter releases the shot some distance above the level ground with a velocity of 12.0 m/s, 51.0° above the horizontal. The shot hits the ground 2.08 s later. Ignore air resistance. (a) What are the components of the shot's acceleration while in flight? (b) What are the components of the shot's velocity at the beginning and at the end of its trajectory? (c) How far did she throw the shot horizontally? (d) Why does the expression for R in Example 3.8 *not* give the correct answer for part (c)? (e) How high was the shot above the ground when she released it? (f) Draw x-t, y-t, v_x-t, and v_y-t graphs for the motion.

3.19 •• **Win the Prize.** In a carnival booth, you can win a stuffed giraffe if you toss a quarter into a small dish. The dish is on a shelf above the point where the quarter leaves your hand and is a horizontal distance of 2.1 m from this point (**Fig. E3.19**). If you toss the coin with a velocity of 6.4 m/s at an angle of 60° above the horizontal, the coin will land in the dish. Ignore air resistance. (a) What is the height of the shelf above the point where the quarter leaves your hand? (b) What is the vertical component of the velocity of the quarter just before it lands in the dish?

Figure **E3.19**



3.20 • Firefighters use a high-pressure hose to shoot a stream of water at a burning building. The water has a speed of 25.0 m/s as it leaves the end of the hose and then exhibits projectile motion. The firefighters adjust the angle of elevation α of the hose until the water takes 3.00 s to reach a building 45.0 m away. Ignore air resistance; assume that the end of the hose is at ground level. (a) Find α. (b) Find the speed and acceleration of the water at the highest point in its trajectory. (c) How high above the ground does the water strike the building, and how fast is it moving just before it hits the building?

3.21 •• A man stands on the roof of a 15.0-m-tall building and throws a rock with a speed of 30.0 m/s at an angle of 33.0° above the horizontal. Ignore air resistance. Calculate (a) the maximum height above the roof that the rock reaches; (b) the speed of the rock just before it strikes the ground; and (c) the horizontal range from the base of the building to the point where the rock strikes the ground. (d) Draw x-t, y-t, v_x-t, and v_y-t graphs for the motion.

3.22 •• At $t = 0$ a rock is projected from ground level with a speed of 15.0 m/s and at an angle of 53.0° above the horizontal. Neglect air resistance. At what two times t is the rock 5.00 m above the ground? At each of these two times, what are the horizontal and vertical components of the velocity of the rock? Let v_{0x} and v_{0y} be in the positive x- and y-directions, respectively.

3.23 • Estimate the maximum horizontal distance that you can throw a baseball. (a) Based on your estimate, what is the speed of the baseball as it leaves your hand? (b) If you could throw the baseball straight up at the same speed as in part (a), how high would it go? (c) The value of g on Mars is 3.7 m/s^2. What horizontal distance can you throw a baseball on Mars if you throw it with the same initial speed as in part (a)?

Section 3.4 Motion in a Circle

3.24 • Merry-go-rounds are a common ride in park playgrounds. The ride is a horizontal disk that rotates about a vertical axis at their center. A typical size is a diameter of 12 ft. A rider sits at the outer edge of the disk and holds onto a metal bar while someone pushes on the ride to make it rotate. Estimate a typical time for one rotation. (a) For your estimated time, what is the speed of the rider, in m/s? (b) What is the rider's radial acceleration, in m/s^2? (c) What is the rider's radial acceleration if the time for one rotation is halved?

3.25 • The earth has a radius of 6380 km and turns around once on its axis in 24 h. (a) What is the radial acceleration of an object at the earth's equator? Give your answer in m/s^2 and as a fraction of g. (b) If a_{rad} at the equator is greater than g, objects will fly off the earth's surface and into space. (We'll see the reason for this in Chapter 5.) What would the period of the earth's rotation have to be for this to occur?

3.26 •• **BIO** **Dizziness.** Our balance is maintained, at least in part, by the endolymph fluid in the inner ear. Spinning displaces this fluid, causing dizziness. Suppose that a skater is spinning very fast at 3.0 revolutions per second about a vertical axis through the center of his head. Take the inner ear to be approximately 7.0 cm from the axis of spin. (The distance varies from person to person.) What is the radial acceleration (in m/s^2 and in g's) of the endolymph fluid?

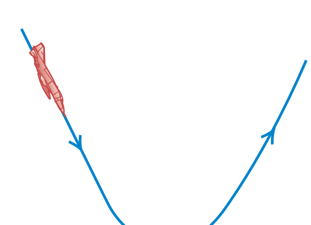
3.27 • A small ball is attached to the lower end of a 0.800-m-long string, and the other end of the string is tied to a horizontal rod. The string makes a constant angle of 37.0° with the vertical as the ball moves at a constant speed in a horizontal circle. If it takes the ball 0.600 s to complete one revolution, what is the magnitude of the radial acceleration of the ball?

3.28 •• A model of a helicopter rotor has four blades, each 3.40 m long from the central shaft to the blade tip. The model is rotated in a wind tunnel at 550 rev/min. (a) What is the linear speed of the blade tip, in m/s? (b) What is the radial acceleration of the blade tip expressed as a multiple of g?

3.29 **BIO** **Pilot Blackout in a Power Dive.** A jet plane comes in for a downward dive as shown in **Fig. E3.29.** The bottom part of the path is a quarter circle with a radius of curvature of 280 m. According to medical tests, pilots will lose consciousness when they pull out of a dive at an upward acceleration greater than 5.5g. At what minimum speed (in m/s and in mph) will the pilot black out during this dive?

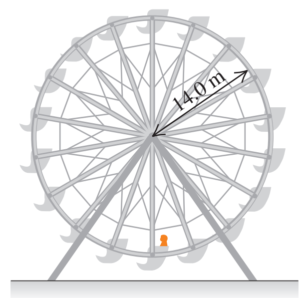
3.30 • An object moves in a horizontal circle at constant speed v (in units of m/s). It takes the object T seconds to complete one revolution. Derive an expression that gives the radial acceleration of the ball in terms of v and T, but not r. (a) If the speed doubles, by what factor must the period T change if a_{rad} is to remain unchanged? (b) If the radius doubles, by what factor must the period change to keep a_{rad} the same?

Figure **E3.29**



3.31 • A Ferris wheel with radius 14.0 m is turning about a horizontal axis through its center (**Fig. E3.31**). The linear speed of a passenger on the rim is constant and equal to 6.00 m/s. What are the magnitude and direction of the passenger's acceleration as she passes through (a) the lowest point in her circular motion and (b) the highest point in her circular motion? (c) How much time does it take the Ferris wheel to make one revolution?

Figure **E3.31**



3.32 • A roller coaster car moves in a vertical circle of radius R. At the top of the circle the car has speed v_1, and at the bottom of the circle it has speed v_2, where $v_2 > v_1$. (a) When the car is at the top of its circular path, what is the direction of its radial acceleration, $a_{\text{rad, top}}$? (b) When the car is at the bottom of its circular path, what is the direction of its radial acceleration, $a_{\text{rad, bottom}}$? (c) In terms of v_1 and v_2, what is the ratio $a_{\text{rad, bottom}}/a_{\text{rad, top}}$?

3.33 •• BIO **Hypergravity.** At its Ames Research Center, NASA uses its large "20 G" centrifuge to test the effects of very large accelerations ("hypergravity") on test pilots and astronauts. In this device, an arm 8.84 m long rotates about one end in a horizontal plane, and an astronaut is strapped in at the other end. Suppose that he is aligned along the centrifuge's arm with his head at the outermost end. The maximum sustained acceleration to which humans are subjected in this device is typically $12.5g$. (a) How fast must the astronaut's head be moving to experience this maximum acceleration? (b) What is the *difference* between the acceleration of his head and feet if the astronaut is 2.00 m tall? (c) How fast in rpm (rev/min) is the arm turning to produce the maximum sustained acceleration?

3.34 • The radius of the earth's orbit around the sun (assumed to be circular) is 1.50×10^8 km, and the earth travels around this orbit in 365 days. (a) What is the magnitude of the orbital velocity of the earth, in m/s? (b) What is the magnitude of the radial acceleration of the earth toward the sun, in m/s^2? (c) Repeat parts (a) and (b) for the motion of the planet Mercury (orbit radius $= 5.79 \times 10^7$ km, orbital period $= 88.0$ days).

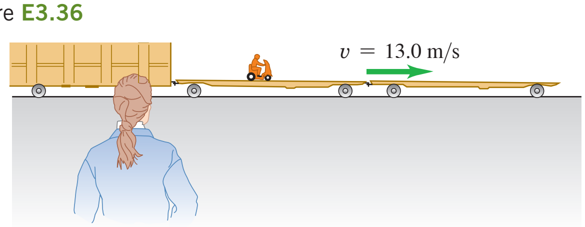
Section 3.5 Relative Velocity

3.35 • A "moving sidewalk" in an airport terminal moves at 1.0 m/s and is 35.0 m long. If a woman steps on at one end and walks at 1.5 m/s relative to the moving sidewalk, how much time does it take her to reach the opposite end if she walks (a) in the same direction the sidewalk is moving? (b) In the opposite direction?

3.36 • A railroad flatcar is traveling to the right at a speed of 13.0 m/s relative to an observer standing on the ground. Someone is riding a motor scooter on the flatcar (**Fig. E3.36**). What is the velocity (magnitude and direction) of the scooter relative to the flatcar if the scooter's velocity relative to the observer on the ground is (a) 18.0 m/s to the right? (b) 3.0 m/s to the left? (c) zero?

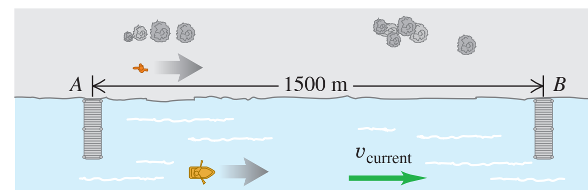
3.37 •• A canoe has a velocity of 0.40 m/s southeast relative to the earth. The canoe is on a river that is flowing 0.50 m/s east relative to the earth. Find the velocity (magnitude and direction) of the canoe relative to the river.

Figure **E3.36**



3.38 • Two piers, A and B, are located on a river; B is 1500 m downstream from A (**Fig. E3.38**). Two friends must make round trips from pier A to pier B and return. One rows a boat at a constant speed of 4.00 km/h relative to the water; the other walks on the shore at a constant speed of 4.00 km/h. The velocity of the river is 2.80 km/h in the direction from A to B. How much time does it take each person to make the round trip?

Figure **E3.38**



3.39 •• BIO **Bird Migration.** Canada geese migrate essentially along a north–south direction for well over a thousand kilometers in some cases, traveling at speeds up to about 100 km/h. If one goose is flying at 100 km/h relative to the air but a 40 km/h wind is blowing from west to east, (a) at what angle relative to the north–south direction should this bird head to travel directly southward relative to the ground? (b) How long will it take the goose to cover a ground distance of 500 km from north to south? (*Note:* Even on cloudy nights, many birds can navigate by using the earth's magnetic field to fix the north–south direction.)

3.40 •• The nose of an ultralight plane is pointed due south, and its airspeed indicator shows 35 m/s. The plane is in a 10 m/s wind blowing toward the southwest relative to the earth. (a) In a vector-addition diagram, show the relationship of $\vec{v}_{P/E}$ (the velocity of the plane relative to the earth) to the two given vectors. (b) Let x be east and y be north, and find the components of $\vec{v}_{P/E}$. (c) Find the magnitude and direction of $\vec{v}_{P/E}$.

3.41 • **Crossing the River I.** A river flows due south with a speed of 2.0 m/s. You steer a motorboat across the river; your velocity relative to the water is 4.2 m/s due east. The river is 500 m wide. (a) What is your velocity (magnitude and direction) relative to the earth? (b) How much time is required to cross the river? (c) How far south of your starting point will you reach the opposite bank?

3.42 • **Crossing the River II.** (a) In which direction should the motorboat in Exercise 3.41 head to reach a point on the opposite bank directly east from your starting point? (The boat's speed relative to the water remains 4.2 m/s.) (b) What is the velocity of the boat relative to the earth? (c) How much time is required to cross the river?

3.43 •• An airplane pilot wishes to fly due west. A wind of 80.0 km/h (about 50 mi/h) is blowing toward the south. (a) If the airspeed of the plane (its speed in still air) is 320.0 km/h (about 200 mi/h), in which direction should the pilot head? (b) What is the speed of the plane over the ground? Draw a vector diagram.

PROBLEMS

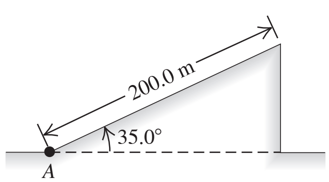
3.44 •• CP CALC A dog in an open field is at rest under a tree at time $t = 0$ and then runs with acceleration $\vec{a}(t) = (0.400\ \text{m/s}^2)\hat{\imath} - (0.180\ \text{m/s}^3)t\hat{\jmath}$. How far is the dog from the tree 8.00 s after it starts to run?

3.45 •• CALC If $\vec{r} = bt^2\hat{\imath} + ct^3\hat{\jmath}$, where b and c are positive constants, when does the velocity vector make an angle of 45.0° with the x- and y-axes?

3.46 ••• CALC A faulty model rocket moves in the xy-plane (the positive y-direction is vertically upward). The rocket's acceleration has components $a_x(t) = \alpha t^2$ and $a_y(t) = \beta - \gamma t$, where $\alpha = 2.50\ \text{m/s}^4$, $\beta = 9.00\ \text{m/s}^2$, and $\gamma = 1.40\ \text{m/s}^3$. At $t = 0$ the rocket is at the origin and has velocity $\vec{v}_0 = v_{0x}\hat{\imath} + v_{0y}\hat{\jmath}$ with $v_{0x} = 1.00\ \text{m/s}$ and $v_{0y} = 7.00\ \text{m/s}$. (a) Calculate the velocity and position vectors as functions of time. (b) What is the maximum height reached by the rocket? (c) What is the horizontal displacement of the rocket when it returns to $y = 0$?

3.47 ••• CP A test rocket starting from rest at point A is launched by accelerating it along a 200.0 m incline at $1.90\ \text{m/s}^2$ (**Fig. P3.47**). The incline rises at 35.0° above the horizontal, and at the instant the rocket leaves it, the engines turn off and the rocket is subject to gravity only (ignore air resistance). Find (a) the maximum height above the ground that the rocket reaches, and (b) the rocket's greatest horizontal range beyond point A.

Figure P3.47



3.48 •• CALC The position of a dragonfly that is flying parallel to the ground is given as a function of time by $\vec{r} = [2.90\ \text{m} + (0.0900\ \text{m/s}^2)t^2]\hat{\imath} - (0.0150\ \text{m/s}^3)t^3\hat{\jmath}$. (a) At what value of t does the velocity vector of the dragonfly make an angle of 30.0° clockwise from the $+x$-axis? (b) At the time calculated in part (a), what are the magnitude and direction of the dragonfly's acceleration vector?

3.49 •• In fighting forest fires, airplanes work in support of ground crews by dropping water on the fires. For practice, a pilot drops a canister of red dye, hoping to hit a target on the ground below. If the plane is flying in a horizontal path 90.0 m above the ground and has a speed of 64.0 m/s (143 mi/h), at what horizontal distance from the target should the pilot release the canister? Ignore air resistance.

3.50 •• CALC A bird flies in the xy-plane with a velocity vector given by $\vec{v} = (\alpha - \beta t^2)\hat{\imath} + \gamma t\hat{\jmath}$, with $\alpha = 2.4\ \text{m/s}$, $\beta = 1.6\ \text{m/s}^3$, and $\gamma = 4.0\ \text{m/s}^2$. The positive y-direction is vertically upward. At $t = 0$ the bird is at the origin. (a) Calculate the position and acceleration vectors of the bird as functions of time. (b) What is the bird's altitude (y-coordinate) as it flies over $x = 0$ for the first time after $t = 0$?

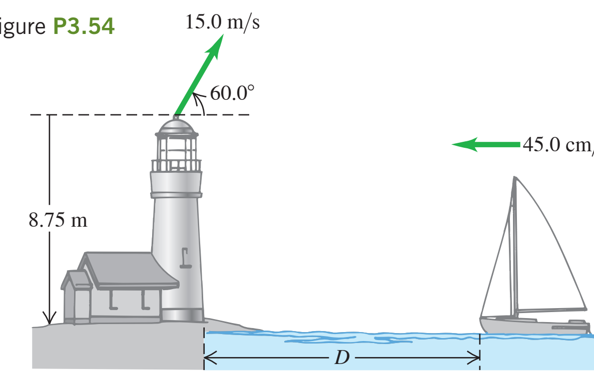
3.51 ••• A movie stuntwoman drops from a helicopter that is 30.0 m above the ground and moving with a constant velocity whose components are 10.0 m/s upward and 15.0 m/s horizontal and toward the south. Ignore air resistance. (a) Where on the ground (relative to the position of the helicopter when she drops) should the stuntwoman have placed foam mats to break her fall? (b) Draw x-t, y-t, v_x-t, and v_y-t graphs of her motion.

3.52 •• A cannon, located 60.0 m from the base of a vertical 25.0-m-tall cliff, shoots a 15 kg shell at 43.0° above the horizontal toward the cliff. (a) What must the minimum muzzle velocity be for the shell to clear the top of the cliff? (b) The ground at the top of the cliff is level, with a constant elevation of 25.0 m above the cannon. Under the conditions of part (a), how far does the shell land past the edge of the cliff?

3.53 • CP CALC A toy rocket is launched with an initial velocity of 12.0 m/s in the horizontal direction from the roof of a 30.0-m-tall building. The rocket's engine produces a horizontal acceleration of $(1.60\ \text{m/s}^3)t$, in the same direction as the initial velocity, but in the vertical direction the acceleration is g, downward. Ignore air resistance. What horizontal distance does the rocket travel before reaching the ground?

3.54 ••• An important piece of landing equipment must be thrown to a ship, which is moving at 45.0 cm/s, before the ship can dock. This equipment is thrown at 15.0 m/s at 60.0° above the horizontal from the top of a tower at the edge of the water, 8.75 m above the ship's deck (**Fig. P3.54**). For this equipment to land at the front of the ship, at what distance D from the dock should the ship be when the equipment is thrown? Ignore air resistance.

Figure P3.54

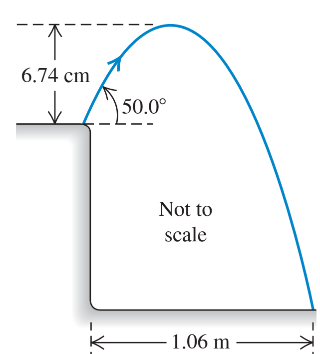


3.55 •• A baseball thrown at an angle of 60.0° above the horizontal strikes a building 18.0 m away at a point 8.00 m above the point from which it is thrown. Ignore air resistance. (a) Find the magnitude of the ball's initial velocity (the velocity with which the ball is thrown). (b) Find the magnitude and direction of the velocity of the ball just before it strikes the building.

3.56 •• An Errand of Mercy. An airplane is dropping bales of hay to cattle stranded in a blizzard on the Great Plains. The pilot releases the bales at 150 m above the level ground when the plane is flying at 75 m/s in a direction 55° above the horizontal. How far in front of the cattle should the pilot release the hay so that the bales land at the point where the cattle are stranded?

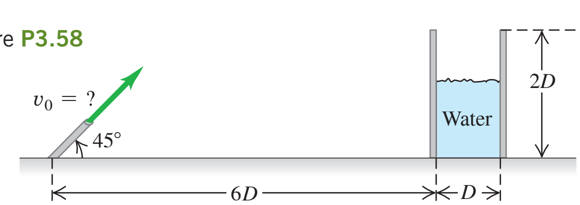
3.57 •• A grasshopper leaps into the air from the edge of a vertical cliff, as shown in **Fig. P3.57.** Find (a) the initial speed of the grasshopper and (b) the height of the cliff.

Figure P3.57



3.58 ••• A water hose is used to fill a large cylindrical storage tank of diameter D and height $2D$. The hose shoots the water at 45° above the horizontal from the same level as the base of the tank and is a distance $6D$ away (**Fig. P3.58**). For what *range* of launch speeds (v_0) will the water enter the tank? Ignore air resistance, and express your answer in terms of D and g.

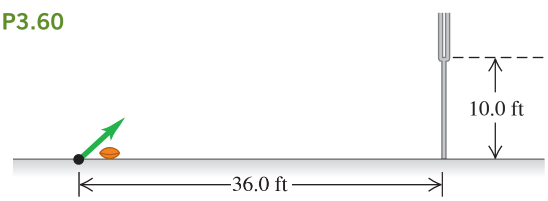
Figure P3.58



3.59 •• An object is projected with initial speed v_0 from the edge of the roof of a building that has height H. The initial velocity of the object makes an angle α_0 with the horizontal. Neglect air resistance. (a) If α_0 is 90°, so that the object is thrown straight up (but misses the roof on the way down), what is the speed v of the object just before it strikes the ground? (b) If $\alpha_0 = -90°$, so that the object is thrown straight down, what is its speed just before it strikes the ground? (c) Derive an expression for the speed v of the object just before it strikes the ground for general α_0. (d) The final speed v equals v_1 when α_0 equals α_1. If α_0 is increased, does v increase, decrease, or stay the same?

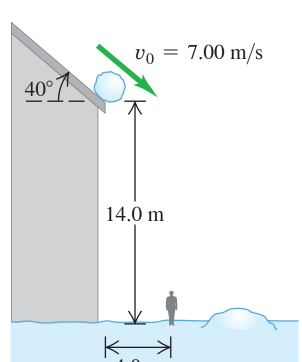
3.60 •• **Kicking an Extra Point.** In Canadian football, after a touchdown the team has the opportunity to earn one more point by kicking the ball over the bar between the goal posts. The bar is 10.0 ft above the ground, and the ball is kicked from ground level, 36.0 ft horizontally from the bar (**Fig. P3.60**). Football regulations are stated in English units, but convert them to SI units for this problem. (a) There is a minimum angle above the ground such that if the ball is launched below this angle, it can never clear the bar, no matter how fast it is kicked. What is this angle? (b) If the ball is kicked at 45.0° above the horizontal, what must its initial speed be if it is just to clear the bar? Express your answer in m/s and in km/h.

Figure P3.60



3.61 ••• **Look Out!** A snowball rolls off a barn roof that slopes downward at an angle of 40° (**Fig. P3.61**). The edge of the roof is 14.0 m above the ground, and the snowball has a speed of 7.00 m/s as it rolls off the roof. Ignore air resistance. (a) How far from the edge of the barn does the snowball strike the ground if it doesn't strike anything else while falling? (b) Draw x-t, y-t, v_x-t, and v_y-t graphs for the motion in part (a). (c) A man 1.9 m tall is standing 4.0 m from the edge of the barn. Will the snowball hit him?

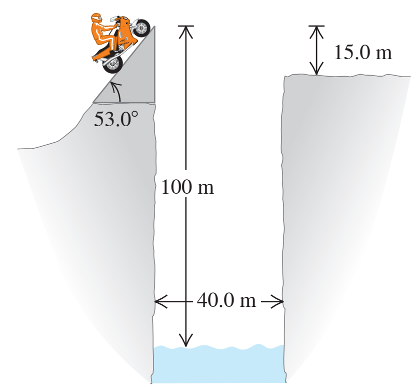
Figure P3.61



3.62 • A 2.7 kg ball is thrown upward with an initial speed of 20.0 m/s from the edge of a 45.0-m-high cliff. At the instant the ball is thrown, a woman starts running away from the base of the cliff with a constant speed of 6.00 m/s. The woman runs in a straight line on level ground. Ignore air resistance on the ball. (a) At what angle above the horizontal should the ball be thrown so that the runner will catch it just before it hits the ground, and how far does she run before she catches the ball? (b) Carefully sketch the ball's trajectory as viewed by (i) a person at rest on the ground and (ii) the runner.

3.63 •• **Leaping the River II.** A physics professor did daredevil stunts in his spare time. His last stunt was an attempt to jump across a river on a motorcycle (**Fig. P3.63**). The takeoff ramp was inclined at 53.0°, the river was 40.0 m wide, and the far bank was 15.0 m lower than the top of the ramp. The river itself was 100 m below the ramp. Ignore air resistance. (a) What should his speed have been at the top of the ramp to have just made it to the edge of the far bank? (b) If his speed was only half the value found in part (a), where did he land?

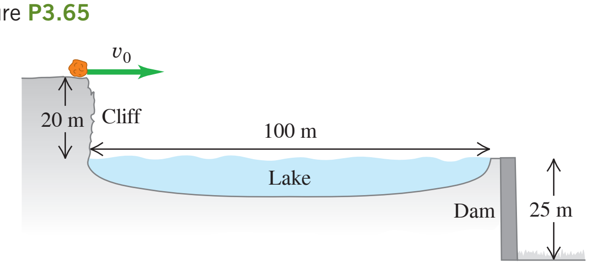
Figure P3.63



3.64 •• **Tossing Your Lunch.** Henrietta is jogging on the sidewalk at 3.05 m/s on the way to her physics class. Bruce realizes that she forgot her bag of bagels, so he runs to the window, which is 38.0 m above the street level and directly above the sidewalk, to throw the bag to her. He throws it horizontally 9.00 s after she has passed below the window, and she catches it on the run. Ignore air resistance. (a) With what initial speed must Bruce throw the bagels so that Henrietta can catch the bag just before it hits the ground? (b) Where is Henrietta when she catches the bagels?

3.65 • A 76.0 kg rock is rolling horizontally at the top of a vertical cliff that is 20 m above the surface of a lake (**Fig. P3.65**). The top of the vertical face of a dam is located 100 m from the foot of the cliff, with the top of the dam level with the surface of the water in the lake. A level plain is 25 m below the top of the dam. (a) What must be the minimum speed of the rock just as it leaves the cliff so that it will reach the plain without striking the dam? (b) How far from the foot of the dam does the rock hit the plain?

Figure P3.65



3.66 •• A firefighting crew uses a water cannon that shoots water at 25.0 m/s at a fixed angle of 53.0° above the horizontal. The firefighters want to direct the water at a blaze that is 10.0 m above ground level. How far from the building should they position their cannon? There are *two* possibilities; can you get them both? (*Hint:* Start with a sketch showing the trajectory of the water.)

3.67 •• On a level football field a football is projected from ground level. It has speed 8.0 m/s when it is at its maximum height. It travels a horizontal distance of 50.0 m. Neglect air resistance. How long is the ball in the air?

3.68 ••• You are standing on a loading dock at the top of a flat ramp that is at a constant angle α_0 below the horizontal. You slide a small box horizontally off the loading dock with speed v_0 in a direction so that it lands on the ramp. How far vertically downward does the box travel before it strikes the ramp?

3.69 ••• In the middle of the night you are standing a horizontal distance of 14.0 m from the high fence that surrounds the estate of your rich uncle. The top of the fence is 5.00 m above the ground. You have taped an important message to a rock that you want to throw over the fence. The ground is level, and the width of the fence is small enough to be ignored. You throw the rock from a height of 1.60 m above the ground and at an angle of 56.0° above the horizontal. (a) What minimum initial speed must the rock have as it leaves your hand to clear the top of the fence? (b) For the initial velocity calculated in part (a), what horizontal distance beyond the fence will the rock land on the ground?

3.70 ••• A small object is projected from level ground with an initial velocity of magnitude 16.0 m/s and directed at an angle of 60.0° above the horizontal. (a) What is the horizontal displacement of the object when it is at its maximum height? How does your result compare to the horizontal range R of the object? (b) What is the vertical displacement of the object when its horizontal displacement is 80.0% of its horizontal range R? How does your result compare to the maximum height h_{max} reached by the object? (c) For when the object has horizontal displacement $x - x_0 = \alpha R$, where α is a positive constant, derive an expression (in terms of α) for $(y - y_0)/h_{max}$. Your result should not depend on the initial velocity or the angle of projection. Show that your expression gives the correct result when $\alpha = 0.80$, as is the case in part (b). Also show that your expression gives the correct result for $\alpha = 0$, $\alpha = 0.50$, and $\alpha = 1.0$.

3.71 •• An airplane pilot sets a compass course due west and maintains an airspeed of 220 km/h. After flying for 0.500 h, she finds herself over a town 120 km west and 20 km south of her starting point. (a) Find the wind velocity (magnitude and direction). (b) If the wind velocity is 40 km/h due south, in what direction should the pilot set her course to travel due west? Use the same airspeed of 220 km/h.

3.72 •• **Raindrops.** When a train's velocity is 12.0 m/s eastward, raindrops that are falling vertically with respect to the earth make traces that are inclined 30.0° to the vertical on the windows of the train. (a) What is the horizontal component of a drop's velocity with respect to the earth? With respect to the train? (b) What is the magnitude of the velocity of the raindrop with respect to the earth? With respect to the train?

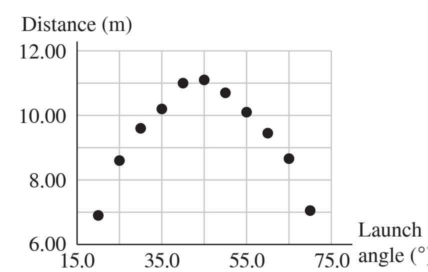
3.73 ••• In a World Cup soccer match, Juan is running due north toward the goal with a speed of 8.00 m/s relative to the ground. A teammate passes the ball to him. The ball has a speed of 12.0 m/s and is moving in a direction 37.0° east of north, relative to the ground. What are the magnitude and direction of the ball's velocity relative to Juan?

3.74 •• A shortstop is running due east as he throws a baseball to the catcher, who is standing at home plate. The velocity of the baseball relative to the shortstop is 6.00 m/s in the direction due south, and the speed of the baseball relative to the catcher is 9.00 m/s. What is the speed of the shortstop relative to the ground when he throws the ball?

3.75 •• Two soccer players, Mia and Alice, are running as Alice passes the ball to Mia. Mia is running due north with a speed of 6.00 m/s. The velocity of the ball relative to Mia is 5.00 m/s in a direction 30.0° east of south. What are the magnitude and direction of the velocity of the ball relative to the ground?

3.76 •• **DATA** A spring-gun projects a small rock from the ground with speed v_0 at an angle θ_0 above the ground. You have been asked to determine v_0. From the way the spring-gun is constructed, you know that to a good approximation v_0 is independent of the launch angle. You go to a level, open field, select a launch angle, and measure the horizontal distance the rock travels. You use $g = 9.80\ \text{m/s}^2$ and ignore the small height of the end of the spring-gun's barrel above the ground. Since your measurement includes some uncertainty in values measured for the launch angle and for the horizontal range, you repeat the measurement for several launch angles and obtain the results given in **Fig. 3.76**. You ignore air resistance because there is no wind and the rock is small and heavy. (a) Select a way to represent the data well as a straight line. (b) Use the slope of the best straight-line fit to your data from part (a) to calculate v_0. (c) When the launch angle is 36.9°, what maximum height above the ground does the rock reach?

Figure **P3.76**



3.77 •• **DATA** You have constructed a hair-spray-powered potato gun and want to find the muzzle speed v_0 of the potatoes, the speed they have as they leave the end of the gun barrel. You use the same amount of hair spray each time you fire the gun, and you have confirmed by repeated firings at the same height that the muzzle speed is approximately the same for each firing. You climb on a microwave relay tower (with permission, of course) to launch the potatoes horizontally from different heights above the ground. Your friend measures the height of the gun barrel above the ground and the range R of each potato. You obtain the data in the table.

Launch height h	Horizontal range R
2.00 m	10.4 m
6.00 m	17.1 m
9.00 m	21.3 m
12.00 m	25.8 m

Each of the values of h and R has some measurement error: The muzzle speed is not precisely the same each time, and the barrel isn't precisely horizontal. So you use all of the measurements to get the best estimate of v_0. No wind is blowing, so you decide to ignore air resistance. You use $g = 9.80\ \text{m/s}^2$ in your analysis. (a) Select a way to represent the data well as a straight line. (b) Use the slope of the best-fit line from part (a) to calculate the average value of v_0. (c) What would be the horizontal range of a potato that is fired from ground level at an angle of 30.0° above the horizontal? Use the value of v_0 that you calculated in part (b).

3.78 ••• **DATA** You are a member of a geological team in Central Africa. Your team comes upon a wide river that is flowing east. You must determine the width of the river and the current speed (the speed of the water relative to the earth). You have a small boat with an outboard motor. By measuring the time it takes to cross a pond where the water isn't flowing, you have calibrated the throttle settings to the speed of the boat in still water. You set the throttle so that the speed of the boat relative to the river is a constant 6.00 m/s. Traveling due north across the river, you reach the opposite bank in 20.1 s. For the return trip, you change the throttle setting so that the speed of the boat relative to the water is 9.00 m/s. You travel due south from one bank to the other and cross the river in 11.2 s. (a) How wide is the river, and what is the current speed? (b) With the throttle set so that the speed of the boat relative to the water is 6.00 m/s, what is the shortest time in which you could cross the river, and where on the far bank would you land?

CHALLENGE PROBLEMS

3.79 ••• **CALC** A projectile thrown from a point *P* moves in such a way that its distance from *P* is always increasing. Find the maximum angle above the horizontal with which the projectile could have been thrown. Ignore air resistance.

3.80 ••• Two students are canoeing on a river. While heading upstream, they accidentally drop an empty bottle overboard. They then continue paddling for 60 minutes, reaching a point 2.0 km farther upstream. At this point they realize that the bottle is missing and, driven by ecological awareness, they turn around and head downstream. They catch up with and retrieve the bottle (which has been moving along with the current) 5.0 km downstream from the turnaround point. (a) Assuming a constant paddling effort throughout, how fast is the river flowing? (b) What would the canoe speed in a still lake be for the same paddling effort?

3.81 ••• **CP** A rocket designed to place small payloads into orbit is carried to an altitude of 12.0 km above sea level by a converted airliner. When the airliner is flying in a straight line at a constant speed of 850 km/h, the rocket is dropped. After the drop, the airliner maintains the same altitude and speed and continues to fly in a straight line. The rocket falls for a brief time, after which its rocket motor turns on. Once that motor is on, the combined effects of thrust and gravity give the rocket a constant acceleration of magnitude 3.00g directed at an angle of 30.0° above the horizontal. For safety, the rocket should be at least 1.00 km in front of the airliner when it climbs through the airliner's altitude. Your job is to determine the minimum time that the rocket must fall before its engine starts. Ignore air resistance. Your answer should include (i) a diagram showing the flight paths of both the rocket and the airliner, labeled at several points with vectors for their velocities and accelerations; (ii) an x-t graph showing the motions of both the rocket and the airliner; and (iii) a y-t graph showing the motions of both the rocket and the airliner. In the diagram and the graphs, indicate when the rocket is dropped, when the rocket motor turns on, and when the rocket climbs through the altitude of the airliner.

MCAT-STYLE PASSAGE PROBLEMS

BIO Ballistic Seed Dispersal. Some plants disperse their seeds when the fruit splits and contracts, propelling the seeds through the air. The trajectory of these seeds can be determined with a high-speed camera. In an experiment on one type of plant, seeds are projected at 20 cm above ground level with initial speeds between 2.3 m/s and 4.6 m/s. The launch angle is measured from the horizontal, with +90° corresponding to an initial velocity straight up and −90° straight down.

3.82 The experiment is designed so that the seeds move no more than 0.20 mm between photographic frames. What minimum frame rate for the high-speed camera is needed to achieve this? (a) 250 frames/s; (b) 2500 frames/s; (c) 25,000 frames/s; (d) 250,000 frames/s.

3.83 About how long does it take a seed launched at 90° at the highest possible initial speed to reach its maximum height? Ignore air resistance. (a) 0.23 s; (b) 0.47 s; (c) 1.0 s; (d) 2.3 s.

3.84 If a seed is launched at an angle of 0° with the maximum initial speed, how far from the plant will it land? Ignore air resistance, and assume that the ground is flat. (a) 20 cm; (b) 93 cm; (c) 2.2 m; (d) 4.6 m.

3.85 A large number of seeds are observed, and their initial launch angles are recorded. The range of projection angles is found to be −51° to 75°, with a mean of 31°. Approximately 65% of the seeds are launched between 6° and 56°. (See W. J. Garrison et al., "Ballistic seed projection in two herbaceous species," *Amer. J. Bot.,* Sept. 2000, 87:9, 1257–64.) Which of these hypotheses is best supported by the data? Seeds are preferentially launched (a) at angles that maximize the height they travel above the plant; (b) at angles below the horizontal in order to drive the seeds into the ground with more force; (c) at angles that maximize the horizontal distance the seeds travel from the plant; (d) at angles that minimize the time the seeds spend exposed to the air.

ANSWERS

Chapter Opening Question ?

(iii) A cyclist going around a curve at constant speed has an acceleration directed toward the inside of the curve (see Section 3.2, especially Fig. 3.12a).

Key Example VARIATION Problems

VP3.9.1 (a) 1.53 s, 11.5 m (b) 3.06 s, 61.2 m
VP3.9.2 (a) 20.6 m/s (b) 5.40 m
VP3.9.3 (a) 3.51 s (b) 33.8 m (c) $v_x = 9.64$ m/s, $v_y = -22.9$ m/s
VP3.9.4 (a) $t = 2v_0/g$, $x = 2v_0^2/g$, $y = -2v_0^2/g$ (b) $t = v_0/g$, $x = v_0^2/g$, $y = -v_0^2/2g$
VP3.12.1 20.0 m
VP3.12.2 (a) 41.6 s (b) 6.04 m/s^2
VP3.12.3 (a) P: 0.938 m/s, Q: 1.88 m/s (b) P: 8.79 m/s^2, Q: 17.6 m/s^2 (c) increase, increase
VP3.12.4 (a) Venus: 3.49×10^4 m/s, Earth: 2.99×10^4 m/s, Mars: 2.41×10^4 m/s (b) Venus: 1.13×10^{-2} m/s^2, Earth: 5.95×10^{-3} m/s^2, Mars: 2.55×10^{-3} m/s^2 (c) decrease, decrease
VP3.12.5 (a) $T_A/T_B = 4$ (b) $a_{\text{rad}, A}/a_{\text{rad}, B} = 1/8$
VP3.15.1 (a) +53.0 m/s (b) −53.0 m/s
VP3.15.2 (a) 90.0 m/s eastward (b) 90.0 m/s westward (c) neither
VP3.15.3 (a) 25.6 m/s, 51.3° north of east (b) 25.6 m/s, 51.3° south of west
VP3.15.4 130 m/s, 12.6° west of north

Bridging Problem

(a) $R = \dfrac{2v_0^2}{g}\dfrac{\cos(\theta + \phi)\sin\phi}{\cos^2\theta}$ (b) $\phi = 45° - \dfrac{\theta}{2}$

? Under what circumstances does the barbell push on the weightlifter just as hard as she pushes on the barbell? (i) When she holds the barbell stationary; (ii) when she raises the barbell; (iii) when she lowers the barbell; (iv) two of (i), (ii), and (iii); (v) all of (i), (ii), and (iii); (vi) none of these.

4 Newton's Laws of Motion

LEARNING OUTCOMES

In this chapter, you'll learn...

4.1 What the concept of force means in physics, why forces are vectors, and the significance of the net force on an object.
4.2 What happens when the net external force on an object is zero, and the significance of inertial frames of reference.
4.3 How the acceleration of an object is determined by the net external force on the object and the object's mass.
4.4 The difference between the mass of an object and its weight.
4.5 How the forces that two objects exert on each other are related.
4.6 How to use a free-body diagram to help analyze the forces on an object.

You'll need to review...

1.7 Vectors and vector addition.
1.8 Vector components.
2.4 Straight-line motion with constant acceleration.
2.5 The motion of freely falling objects.
3.2 Acceleration as a vector.
3.4 Uniform circular motion.
3.5 Relative velocity.

We've seen in the last two chapters how to use *kinematics* to describe motion in one, two, or three dimensions. But what *causes* objects to move the way that they do? For example, why does a dropped feather fall more slowly than a dropped baseball? Why do you feel pushed backward in a car that accelerates forward? The answers to such questions take us into the subject of **dynamics,** the relationship of motion to the forces that cause it.

The principles of dynamics were clearly stated for the first time by Sir Isaac Newton (1642–1727); today we call them **Newton's laws of motion.** Newton did not *derive* the laws of motion, but rather *deduced* them from a multitude of experiments performed by other scientists, especially Galileo Galilei (who died the year Newton was born). Newton's laws are the foundation of **classical mechanics** (also called **Newtonian mechanics**); using them, we can understand most familiar kinds of motion. Newton's laws need modification only for situations involving extremely high speeds (near the speed of light) or very small sizes (such as within the atom).

Newton's laws are very simple to state, yet many students find these laws difficult to grasp and to work with. The reason is that before studying physics, you've spent years walking, throwing balls, pushing boxes, and doing dozens of things that involve motion. Along the way, you've developed a set of "common sense" ideas about motion and its causes. But many of these "common sense" ideas don't stand up to logical analysis. A big part of the job of this chapter—and of the rest of our study of physics—is helping you recognize how "common sense" ideas can sometimes lead you astray, and how to adjust your understanding of the physical world to make it consistent with what experiments tell us.

4.1 FORCE AND INTERACTIONS

A **force** is a push or a pull. More precisely, a force is an *interaction* between two objects or between an object and its environment (**Fig. 4.1**). That's why we always refer to the force that one object *exerts* on a second object. When you push on a car that is stuck in the snow, you exert a force on the car; a steel cable exerts a force on the beam it is hoisting at a construction site; and so on. As Fig. 4.1 shows, force is a *vector* quantity; you can push or pull an object in different directions.

When a force involves direct contact between two objects, such as a push or pull that you exert on an object with your hand, we call it a **contact force. Figures 4.2a,** 4.2b, and 4.2c show three common types of contact forces. The **normal force** (Fig. 4.2a) is exerted on an object by any surface with which it is in contact. The adjective "normal" means that the force always acts *perpendicular* to the surface of contact, no matter what the angle of that surface. By contrast, the **friction force** (Fig. 4.2b) exerted on an object by a surface acts *parallel* to the surface, in the direction that opposes sliding. The pulling force exerted by a stretched rope or cord on an object to which it's attached is called a **tension force** (Fig. 4.2c). When you tug on your dog's leash, the force that pulls on her collar is a tension force.

In addition to contact forces, there are **long-range forces** that act even when the objects are separated by empty space. The force between two magnets is an example of a long-range force, as is the force of gravity (Fig. 4.2d); the earth pulls a dropped object toward it even though there is no direct contact between the object and the earth. The gravitational force that the earth exerts on your body is called your **weight.**

To describe a force vector $\vec{F}$, we need to describe the *direction* in which it acts as well as its *magnitude,* the quantity that describes "how much" or "how hard" the force pushes or pulls. The SI unit of the magnitude of force is the *newton,* abbreviated N. (We'll give a precise definition of the newton in Section 4.3.) **Table 4.1** lists some typical force magnitudes.

A common instrument for measuring force magnitudes is the *spring balance*. It consists of a coil spring enclosed in a case with a pointer attached to one end. When forces are applied to the ends of the spring, it stretches by an amount that depends on the force. We can make a scale for the pointer by using a number of identical objects with weights of exactly 1 N each. When one, two, or more of these are suspended simultaneously from the balance, the total force stretching the spring is 1 N, 2 N, and so on, and we can label the corresponding positions of the pointer 1 N, 2 N, and so on. Then we can use this instrument to measure the magnitude of an unknown force. We can also make a similar instrument that measures pushes instead of pulls.

Figure 4.3 (next page) shows a spring balance being used to measure a pull or push that we apply to a box. In each case we draw a vector to represent the applied force. The length of the vector shows the magnitude; the longer the vector, the greater the force magnitude.

Superposition of Forces

When you hold a ball in your hand to throw it, at least two forces act on it: the push of your hand and the downward pull of gravity. Experiment shows that when two forces $\vec{F}_1$ and $\vec{F}_2$ act at the same time at the same point on an object (**Fig. 4.4**, next page), the effect on the object's motion is the same as if a single force $\vec{R}$ were acting equal to the vector sum, or resultant, of the original forces: $\vec{R} = \vec{F}_1 + \vec{F}_2$. More generally, *any number of*

TABLE 4.1 Typical Force Magnitudes

Sun's gravitational force on the earth	3.5×10^{22} N
Weight of a large blue whale	1.9×10^{6} N
Maximum pulling force of a locomotive	8.9×10^{5} N
Weight of a 250 lb linebacker	1.1×10^{3} N
Weight of a medium apple	1 N
Weight of the smallest insect eggs	2×10^{-6} N
Electric attraction between the proton and the electron in a hydrogen atom	8.2×10^{-8} N
Weight of a very small bacterium	1×10^{-18} N
Weight of a hydrogen atom	1.6×10^{-26} N
Weight of an electron	8.9×10^{-30} N
Gravitational attraction between the proton and the electron in a hydrogen atom	3.6×10^{-47} N

Figure **4.1** Some properties of forces.

- A force is a push or a pull.
- A force is an interaction between two objects or between an object and its environment.
- A force is a vector quantity, with magnitude and direction.

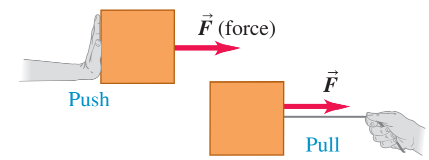
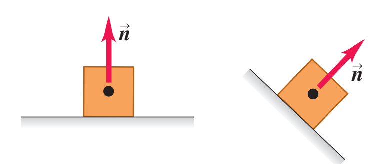
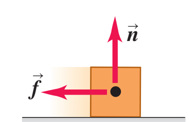


Figure **4.2** Four common types of forces.

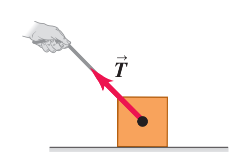
(a) **Normal force $\vec{n}$:** When an object rests or pushes on a surface, the surface exerts a push on it that is directed perpendicular to the surface.



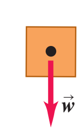
(b) **Friction force $\vec{f}$:** In addition to the normal force, a surface may exert a friction force on an object, directed parallel to the surface.



(c) **Tension force $\vec{T}$:** A pulling force exerted on an object by a rope, cord, etc.



(d) **Weight $\vec{w}$:** The pull of gravity on an object is a long-range force (a force that acts over a distance).



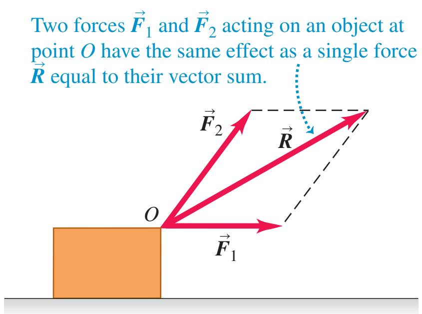


Figure 4.4 Superposition of forces.

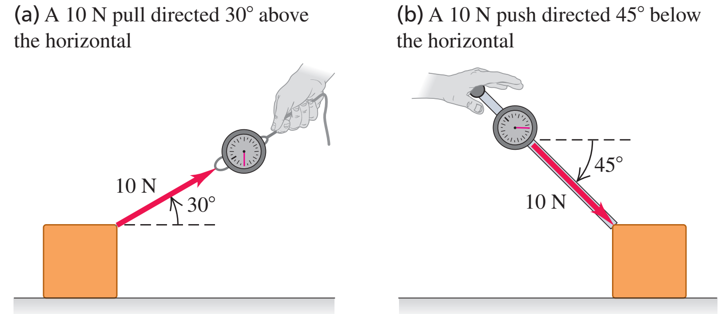


Figure 4.3 Using a vector arrow to denote the force that we exert when (a) pulling a block with a string or (b) pushing a block with a stick.

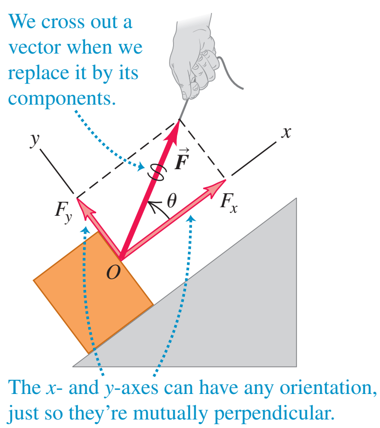


Figure 4.5 F_x and F_y are the components of $\vec{F}$ parallel and perpendicular to the sloping surface of the inclined plane.

forces applied at a point on an object have the same effect as a single force equal to the vector sum of the forces. This important principle is called **superposition of forces.**

Since forces are vector quantities and add like vectors, we can use all of the rules of vector mathematics that we learned in Chapter 1 to solve problems that involve vectors. This would be a good time to review the rules for vector addition presented in Sections 1.7 and 1.8.

We learned in Section 1.8 that it's easiest to add vectors by using components. That's why we often describe a force $\vec{F}$ in terms of its x- and y-components F_x and F_y. Note that the x- and y-coordinate axes do *not* have to be horizontal and vertical, respectively. As an example, **Fig. 4.5** shows a crate being pulled up a ramp by a force $\vec{F}$. In this situation it's most convenient to choose one axis to be parallel to the ramp and the other to be perpendicular to the ramp. For the case shown in Fig. 4.5, both F_x and F_y are positive; in other situations, depending on your choice of axes and the orientation of the force $\vec{F}$, either F_x or F_y may be negative or zero.

CAUTION **Using a wiggly line in force diagrams** In Fig. 4.5 we draw a wiggly line through the force vector $\vec{F}$ to show that we have replaced it by its x- and y-components. Otherwise, the diagram would include the same force twice. We'll draw such a wiggly line in any force diagram where a force is replaced by its components. We encourage you to do the same in your own diagrams!

We'll often need to find the vector sum (resultant) of *all* forces acting on an object. We call this the **net force** acting on the object. We'll use the Greek letter Σ (capital sigma, equivalent to the Roman S) as a shorthand notation for a sum. If the forces are labeled $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, and so on, we can write

The net force acting on an object ...

$$\vec{R} = \Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots \qquad (4.1)$$

... is the vector sum, or resultant, of all individual forces acting on that object.

We read $\Sigma\vec{F}$ as "the vector sum of the forces" or "the net force." The x-component of the net force is the sum of the x-components of the individual forces, and likewise for the y-component (**Fig. 4.6**):

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad (4.2)$$

Each component may be positive or negative, so be careful with signs when you evaluate these sums.

Once we have R_x and R_y we can find the magnitude and direction of the net force $\vec{R} = \Sigma\vec{F}$ acting on the object. The magnitude is

$$R = \sqrt{R_x^2 + R_y^2}$$

and the angle θ between $\vec{R}$ and the $+x$-axis can be found from the relationship $\tan\theta = R_y/R_x$. The components R_x and R_y may be positive, negative, or zero, and the angle θ may be in any of the four quadrants.

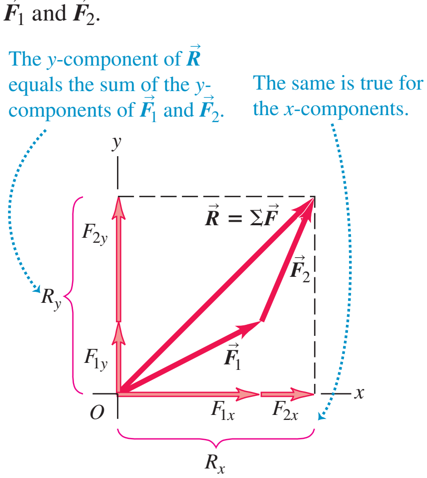


Figure 4.6 Finding the components of the vector sum (resultant) $\vec{R}$ of two forces $\vec{F}_1$ and $\vec{F}_2$.

In three-dimensional problems, forces may also have z-components; then we add the equation $R_z = \sum F_z$ to Eqs. (4.2). The magnitude of the net force is then

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

EXAMPLE 4.1 Superposition of forces: Finding the net force

WITH VARIATION PROBLEMS

Three professional wrestlers are fighting over a champion's belt. **Figure 4.7a** shows the horizontal force each wrestler applies to the belt, as viewed from above. The forces have magnitudes $F_1 = 50\text{ N}$, $F_2 = 120\text{ N}$, and $F_3 = 250\text{ N}$. Find the x- and y-components of the net force on the belt, and find its magnitude and direction.

IDENTIFY and SET UP This is a problem in vector addition in which the vectors happen to represent forces. To find the x- and y-components of the net force $\vec{\boldsymbol{R}}$, we'll use the component method of vector addition expressed by Eqs. (4.2). Once we know the components of $\vec{\boldsymbol{R}}$, we can find its magnitude and direction.

EXECUTE Figure 4.7a shows that force $\vec{\boldsymbol{F}}_1$ (magnitude 50 N) points in the positive x-direction. Hence it has a positive x-component and zero y-component:

$$F_{1x} = 50\text{ N} \qquad F_{1y} = 0\text{ N}$$

Force $\vec{\boldsymbol{F}}_2$ points in the negative y-direction and so has zero x-component and a negative y-component:

$$F_{2x} = 0\text{ N} \qquad F_{2y} = -120\text{ N}$$

Force $\vec{\boldsymbol{F}}_3$ doesn't point along either the x-direction or the y-direction: Figure 4.7a shows that its x-component is negative and its y-component is positive. The angle between $\vec{\boldsymbol{F}}_3$ and the negative x-axis is 53°, so the absolute value of its x-component is equal to the magnitude of $\vec{\boldsymbol{F}}_3$ times the cosine of 53°. The absolute value of the y-component is therefore the magnitude of $\vec{\boldsymbol{F}}_3$ times the sine of 53°. Keeping track of the signs, we find the components of $\vec{\boldsymbol{F}}_3$ are

$$F_{3x} = -(250\text{ N})\cos 53° = -150\text{ N} \qquad F_{3y} = (250\text{ N})\sin 53° = 200\text{ N}$$

From Eqs. (4.2) the components of the net force $\vec{\boldsymbol{R}} = \sum\vec{\boldsymbol{F}}$ are

$$R_x = F_{1x} + F_{2x} + F_{3x} = 50\text{ N} + 0\text{ N} + (-150\text{ N}) = -100\text{ N}$$

$$R_y = F_{1y} + F_{2y} + F_{3y} = 0\text{ N} + (-120\text{ N}) + 200\text{ N} = 80\text{ N}$$

The net force has a negative x-component and a positive y-component, as Fig. 4.7b shows.

The magnitude of $\vec{\boldsymbol{R}}$ is

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-100\text{ N})^2 + (80\text{ N})^2} = 128\text{ N}$$

To find the angle between the net force and the $+x$-axis, we use Eq. (1.7):

$$\theta = \arctan\frac{R_y}{R_x} = \arctan\left(\frac{80\text{ N}}{-100\text{ N}}\right) = \arctan(-0.80)$$

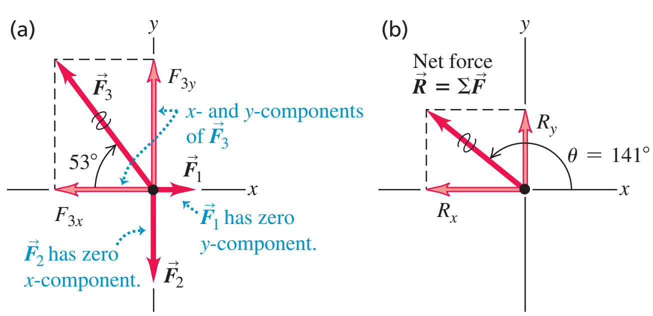
The arctangent of -0.80 is $-39°$, but Fig. 4.7b shows that the net force lies in the second quadrant. Hence the correct solution is $\theta = -39° + 180° = 141°$.

EVALUATE The net force is *not* zero. Wrestler 3 exerts the greatest force on the belt, $F_3 = 250\text{ N}$, and will walk away with it when the struggle ends.

You should check the direction of $\vec{\boldsymbol{R}}$ by adding the vectors $\vec{\boldsymbol{F}}_1$, $\vec{\boldsymbol{F}}_2$, and $\vec{\boldsymbol{F}}_3$ graphically. Does your drawing show that $\vec{\boldsymbol{R}} = \vec{\boldsymbol{F}}_1 + \vec{\boldsymbol{F}}_2 + \vec{\boldsymbol{F}}_3$ points in the second quadrant as we found?

KEYCONCEPT The net force is the vector sum of all of the individual forces that act on the object. It can be specified by its components or by its magnitude and direction.

Figure 4.7 (a) Three forces acting on a belt. (b) The net force $\vec{\boldsymbol{R}} = \sum\vec{\boldsymbol{F}}$ and its components.



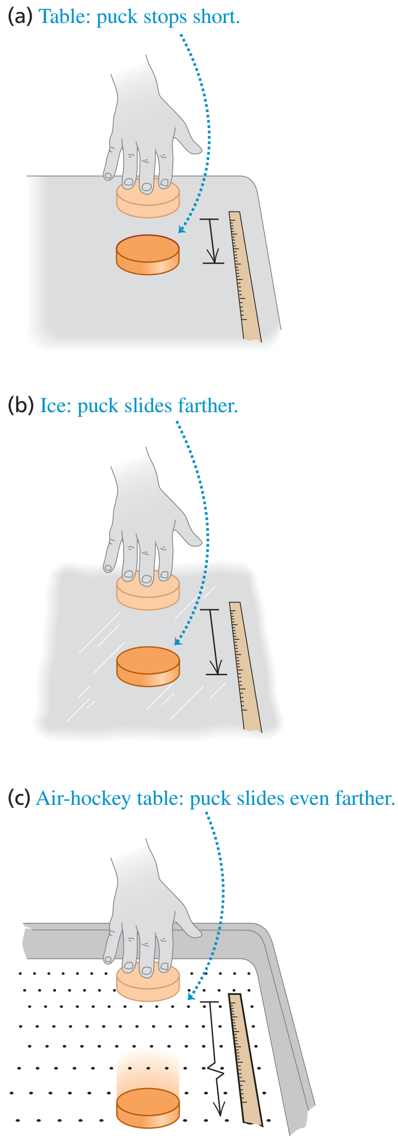
TEST YOUR UNDERSTANDING OF SECTION 4.1 Figure 4.5 shows a force $\vec{\boldsymbol{F}}$ acting on a crate. (a) With the x- and y-axes shown in the figure, is the x-component of the *gravitational* force that the earth exerts on the crate (the crate's weight) positive, negative, or zero? (b) What about the y-component?

ANSWER (a) **negative**, (b) **negative**. The gravitational force on the crate points straight downward. In Fig. 4.5 the x-axis points up and to the right, and the y-axis points up and to the left. With this choice of axes, the gravitational force has both an x-component and a y-component, and both are negative.

4.2 NEWTON'S FIRST LAW

How do the forces acting on an object affect that object's motion? Let's first note that it's impossible for an object to affect its own motion by exerting a force on itself. If that were possible, you could lift yourself to the ceiling by pulling up on your belt! The forces that affect an object's motion are **external forces**, those forces exerted on the object by other

Figure 4.8 The slicker the surface, the farther a puck slides after being given an initial velocity. On an air-hockey table (c) the friction force is practically zero, so the puck continues with almost constant velocity.



objects in its environment. So the question we must answer is this: How do the external forces that act on an object affect its motion?

To begin to answer this question, let's first consider what happens when the net external force on an object is *zero*. You would almost certainly agree that if an object is at rest, and if no net external force acts on it (that is, no net push or pull from other objects), that object will remain at rest. But what if there is zero net external force acting on an object in *motion*?

To see what happens in this case, suppose you slide a hockey puck along a horizontal tabletop, applying a horizontal force to it with your hand (**Fig. 4.8a**). After you stop pushing, the puck does *not* continue to move indefinitely; it slows down and stops. To keep it moving, you have to keep pushing (that is, applying a force). You might come to the "common sense" conclusion that objects in motion naturally come to rest and that a force is required to sustain motion.

But now imagine pushing the puck across a smooth surface of ice (Fig. 4.8b). After you quit pushing, the puck will slide a lot farther before it stops. Put it on an air-hockey table, where it floats on a thin cushion of air, and it moves still farther (Fig. 4.8c). In each case, what slows the puck down is *friction,* an interaction between the lower surface of the puck and the surface on which it slides. Each surface exerts a friction force on the puck that resists the puck's motion; the difference in the three cases is the magnitude of the friction force. The ice exerts less friction than the tabletop, so the puck travels farther. The gas molecules of the air-hockey table exert the least friction of all. If we could eliminate friction completely, the puck would never slow down, and we would need no force at all to keep the puck moving once it had been started. Thus the "common sense" idea that a force is required to sustain motion is *incorrect.*

Experiments like the ones we've just described show that when *no* net external force acts on an object, the object either remains at rest *or* moves with constant velocity in a straight line. Once an object has been set in motion, no net external force is needed to keep it moving. We call this observation *Newton's first law of motion:*

NEWTON'S FIRST LAW OF MOTION An object acted on by no net external force has a constant velocity (which may be zero) and zero acceleration.

The tendency of an object to keep moving once it is set in motion is called **inertia.** You use inertia when you try to get ketchup out of a bottle by shaking it. First you start the bottle (and the ketchup inside) moving forward; when you jerk the bottle back, the ketchup tends to keep moving forward and, you hope, ends up on your burger. Inertia is also the tendency of an object at rest to remain at rest. You may have seen a tablecloth yanked out from under a table setting without breaking anything. The force on the table setting isn't great enough to make it move appreciably during the short time it takes to pull the tablecloth away.

It's important to note that the *net* external force is what matters in Newton's first law. For example, a physics book at rest on a horizontal tabletop has two forces acting on it: an upward supporting force, or normal force, exerted by the tabletop (see Fig. 4.2a) and the downward force of the earth's gravity (see Fig. 4.2d). The upward push of the surface is just as great as the downward pull of gravity, so the *net* external force acting on the book (that is, the vector sum of the two forces) is zero. In agreement with Newton's first law, if the book is at rest on the tabletop, it remains at rest. The same principle applies to a hockey puck sliding on a horizontal, frictionless surface: The vector sum of the upward push of the surface and the downward pull of gravity is zero. Once the puck is in motion, it continues to move with constant velocity because the *net* external force acting on it is zero.

Here's another example. Suppose a hockey puck rests on a horizontal surface with negligible friction, such as an air-hockey table or a slab of wet ice. If the puck is initially at rest and a single horizontal force $\vec{F}_1$ acts on it (**Fig. 4.9a**), the puck starts to move. If the puck is in motion to begin with, the force changes its speed, its direction, or both, depending on the direction of the force. In this case the net external force is equal to $\vec{F}_1$, which is *not* zero. (There are also two vertical forces: the earth's gravitational attraction and the upward normal force exerted by the surface. But as we mentioned earlier, these two forces cancel.)

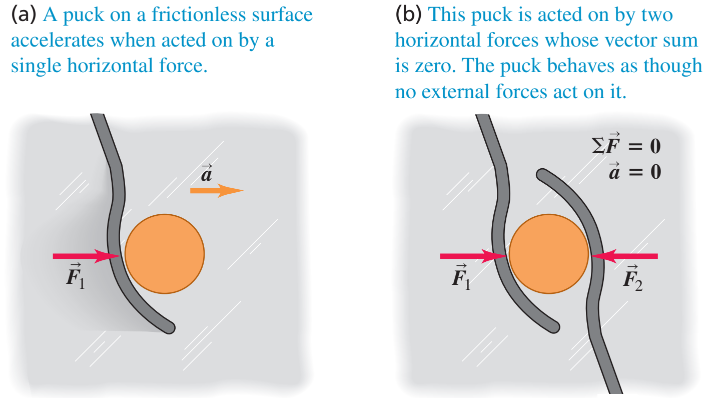


Figure 4.9 (a) A hockey puck accelerates in the direction of a net applied force $\vec{F}_1$. (b) When the net external force is zero, the acceleration is zero, and the puck is in equilibrium.

Now suppose we apply a second force, $\vec{F}_2$ (Fig. 4.9b), equal in magnitude to $\vec{F}_1$ but opposite in direction. The two forces are negatives of each other, $\vec{F}_2 = -\vec{F}_1$, and their vector sum is zero:

$$\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 = \vec{F}_1 + (-\vec{F}_1) = 0$$

Again, we find that if the object is at rest at the start, it remains at rest; if it is initially moving, it continues to move in the same direction with constant speed. These results show that in Newton's first law, *zero net external force is equivalent to no external force at all*. This is just the principle of superposition of forces that we saw in Section 4.1.

When an object is either at rest or moving with constant velocity (in a straight line with constant speed), we say that the object is in **equilibrium.** For an object to be in equilibrium, it must be acted on by no forces, or by several forces such that their vector sum—that is, the net external force—is zero:

Newton's first law: Net external force on an object ... $\Sigma\vec{F} = 0$ ... must be zero if the object is in **equilibrium**. (4.3)

We're assuming that the object can be represented adequately as a point particle. When the object has finite size, we also have to consider *where* on the object the forces are applied. We'll return to this point in Chapter 11.

APPLICATION Sledding with Newton's First Law The downward force of gravity acting on the child and sled is balanced by an upward normal force exerted by the ground. The adult's foot exerts a forward force that balances the backward force of friction on the sled. Hence there is no net external force on the child and sled, and they slide with a constant velocity.

CONCEPTUAL EXAMPLE 4.2 Using Newton's first law I

In the classic 1950 science-fiction film *Rocketship X-M*, a spaceship is moving in the vacuum of outer space, far from any star or planet, when its engine dies. As a result, the spaceship slows down and stops. What does Newton's first law say about this scene?

SOLUTION No external forces act on the spaceship after the engine dies, so according to Newton's first law it will *not* stop but will continue to move in a straight line with constant speed. Some science-fiction movies are based on accurate science; this is not one of them.

KEYCONCEPT If the net external force on an object is zero, the object either remains at rest or keeps moving at a constant velocity.

CONCEPTUAL EXAMPLE 4.3 Using Newton's first law II

You are driving a Porsche 918 Spyder on a straight testing track at a constant speed of 250 km/h. You pass a 1971 Volkswagen Beetle doing a constant 75 km/h. On which car is the net external force greater?

SOLUTION The key word in this question is "net." Both cars are in equilibrium because their velocities are constant; Newton's first law therefore says that the *net* external force on each car is *zero*.

This seems to contradict the "common sense" idea that the faster car must have a greater force pushing it. Thanks to your Porsche's high-power engine, it's true that the track exerts a greater forward force on your Porsche than it does on the Volkswagen. But a *backward* force also acts on each car due to road friction and air resistance. When the car is traveling with constant velocity, the vector sum of the forward and backward forces is zero. There is a greater backward force on the fast-moving Porsche than on the slow-moving Volkswagen, which is why the Porsche's engine must be more powerful than that of the Volkswagen.

KEYCONCEPT If an object either remains at rest or keeps moving at a constant velocity, the net external force on the object is zero.

Inertial Frames of Reference

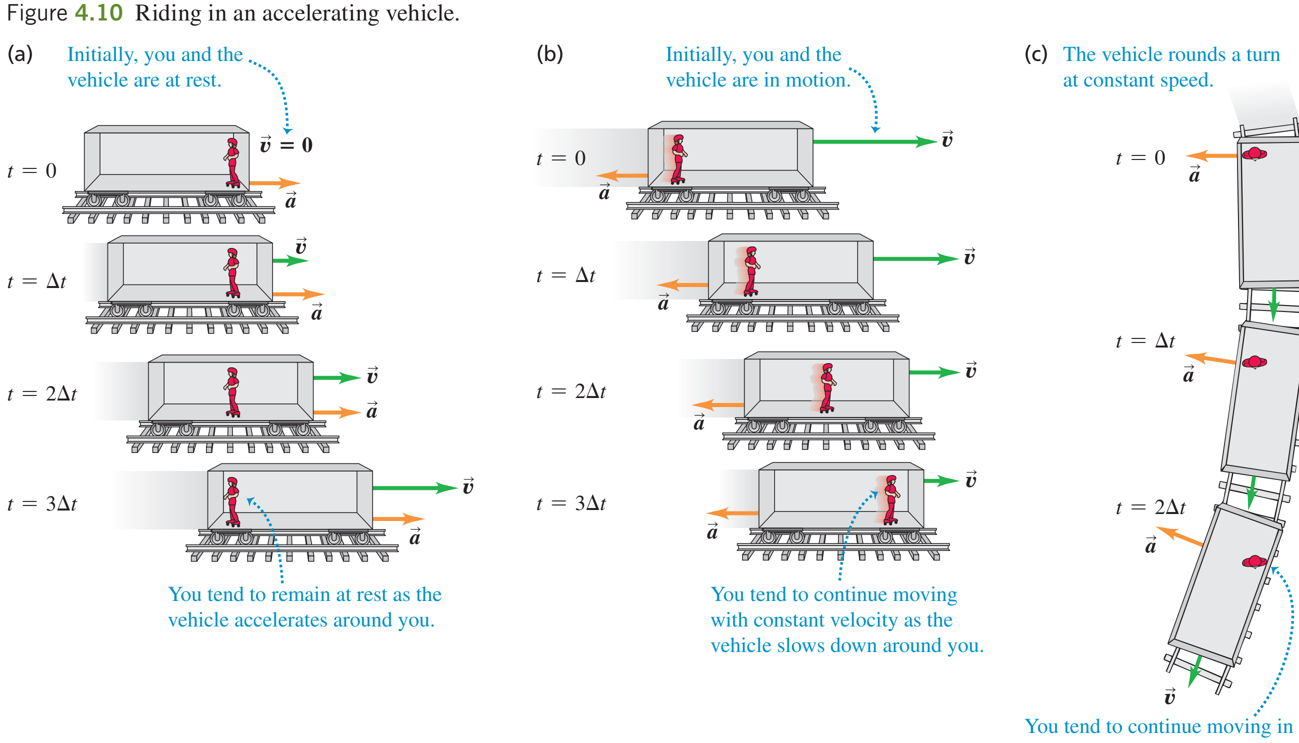
In discussing relative velocity in Section 3.5, we introduced the concept of *frame of reference*. This concept is central to Newton's laws of motion. Suppose you are in a bus that is traveling on a straight road and speeding up. If you could stand in the aisle on roller skates, you would start moving *backward* relative to the bus as the bus gains speed. If instead the bus was slowing to a stop, you would start moving *forward* down the aisle. In either case, it looks as though Newton's first law is not obeyed; there is no net external force acting on you, yet your velocity changes. What's wrong?

The point is that the bus is accelerating with respect to the earth and is *not* a suitable frame of reference for Newton's first law. This law is valid in some frames of reference and not valid in others. A frame of reference in which Newton's first law *is* valid is called an **inertial frame of reference.** The earth is at least approximately an inertial frame of reference, but the bus is not. (The earth is not a completely inertial frame, owing to the acceleration associated with its rotation and its motion around the sun. These effects are quite small, however; see Exercises 3.25 and 3.34.) Because Newton's first law is used to define what we mean by an inertial frame of reference, it is sometimes called the *law of inertia*.

Figure 4.10 helps us understand what you experience when riding in a vehicle that's accelerating. In Fig. 4.10a, a vehicle is initially at rest and then begins to accelerate to the right. A passenger standing on roller skates (which nearly eliminate the effects of friction) has virtually no net external force acting on her, so she tends to remain at rest relative to the inertial frame of the earth. As the vehicle accelerates around her, she moves backward relative to the vehicle. In the same way, a passenger in a vehicle that is slowing down tends to continue moving with constant velocity relative to the earth, and so moves forward relative to the vehicle (Fig. 4.10b). A vehicle is also accelerating if it moves at a constant speed but is turning (Fig. 4.10c). In this case a passenger tends to continue moving relative to the earth at constant speed in a straight line; relative to the vehicle, the passenger moves to the side of the vehicle on the outside of the turn.

In each case shown in Fig. 4.10, an observer in the vehicle's frame of reference might be tempted to conclude that there *is* a net external force acting on the passenger, since the passenger's velocity *relative to the vehicle* changes in each case. This conclusion is simply

Figure 4.10 Riding in an accelerating vehicle.



wrong; the net external force on the passenger is indeed zero. The vehicle observer's mistake is in trying to apply Newton's first law in the vehicle's frame of reference, which is *not* an inertial frame and in which Newton's first law isn't valid (**Fig. 4.11**). In this book we'll use *only* inertial frames of reference.

We've mentioned only one (approximately) inertial frame of reference: the earth's surface. But there are many inertial frames. If we have an inertial frame of reference A, in which Newton's first law is obeyed, then *any* second frame of reference B will also be inertial if it moves relative to A with constant velocity $\vec{v}_{B/A}$. We can prove this by using the relative-velocity relationship Eq. (3.35) from Section 3.5:

$$\vec{v}_{P/A} = \vec{v}_{P/B} + \vec{v}_{B/A}$$

Suppose that P is an object that moves with constant velocity $\vec{v}_{P/A}$ with respect to an inertial frame A. By Newton's first law the net external force on this object is zero. The velocity of P relative to another frame B has a different value, $\vec{v}_{P/B} = \vec{v}_{P/A} - \vec{v}_{B/A}$. But if the relative velocity $\vec{v}_{B/A}$ of the two frames is constant, then $\vec{v}_{P/B}$ is constant as well. Thus B is also an inertial frame; the velocity of P in this frame is constant, and the net external force on P is zero, so Newton's first law is obeyed in B. Observers in frames A and B will disagree about the velocity of P, but they will agree that P has a constant velocity (zero acceleration) and has zero net external force acting on it.

There is no single inertial frame of reference that is preferred over all others for formulating Newton's laws. If one frame is inertial, then every other frame moving relative to it with constant velocity is also inertial. Viewed in this light, the state of rest and the state of motion with constant velocity are not very different; both occur when the vector sum of forces acting on the object is zero.

Figure 4.11 From the frame of reference of the car, it seems as though a force is pushing the crash test dummies forward as the car comes to a sudden stop. But there is really no such force: As the car stops, the dummies keep moving forward as a consequence of Newton's first law.

TEST YOUR UNDERSTANDING OF SECTION 4.2 In which of the following situations is there zero net external force on the object? (i) An airplane flying due north at a steady 120 m/s and at a constant altitude; (ii) a car driving straight up a hill with a 3° slope at a constant 90 km/h; (iii) a hawk circling at a constant 20 km/h at a constant height of 15 m above an open field; (iv) a box with slick, frictionless surfaces in the back of a truck as the truck accelerates forward on a level road at 5 m/s^2.

ANSWER

(i), (ii), and (iv) In (i), (ii), and (iv) the object is not accelerating, so the net external force on the object is zero. [In (iv), if the truck starts from rest, the box remains stationary as seen in the inertial reference frame of the ground as the truck accelerates forward, like the person on skates in Fig. 4.10a.] In (iii), the hawk is moving in a circle; hence it is accelerating and is *not* in equilibrium.

4.3 NEWTON'S SECOND LAW

Newton's first law tells us that when an object is acted on by zero net external force, the object moves with constant velocity and zero acceleration. In **Fig. 4.12a** (next page), a hockey puck is sliding to the right on wet ice. There is negligible friction, so there are no horizontal forces acting on the puck; the downward force of gravity and the upward normal force exerted by the ice surface sum to zero. So the net external force $\sum\vec{F}$ acting on the puck is zero, the puck has zero acceleration, and its velocity is constant.

But what happens when the net external force is *not* zero? In Fig. 4.12b we apply a constant horizontal force to a sliding puck in the same direction that the puck is moving. Then $\sum\vec{F}$ is constant and in the same horizontal direction as $\vec{v}$. We find that during the time the force is acting, the velocity of the puck changes at a constant rate; that is, the puck moves with constant acceleration. The speed of the puck increases, so the acceleration $\vec{a}$ is in the same direction as $\vec{v}$ and $\sum\vec{F}$.

In Fig. 4.12c we reverse the direction of the force on the puck so that $\sum\vec{F}$ acts opposite to $\vec{v}$. In this case as well, the puck has an acceleration; the puck moves more and more slowly to the right. The acceleration $\vec{a}$ in this case is to the left, in the same direction as $\sum\vec{F}$. As in the previous case, experiment shows that the acceleration is constant if $\sum\vec{F}$ is constant.

We conclude that *a net external force acting on an object causes the object to accelerate in the same direction as the net external force*. If the magnitude of the net external force is constant, as in Fig. 4.12b and Fig. 4.12c, then so is the magnitude of the acceleration.

Figure **4.12** Using a hockey puck on a frictionless surface to explore the relationship between the net external force $\sum\vec{F}$ on an object and the resulting acceleration $\vec{a}$ of the object.

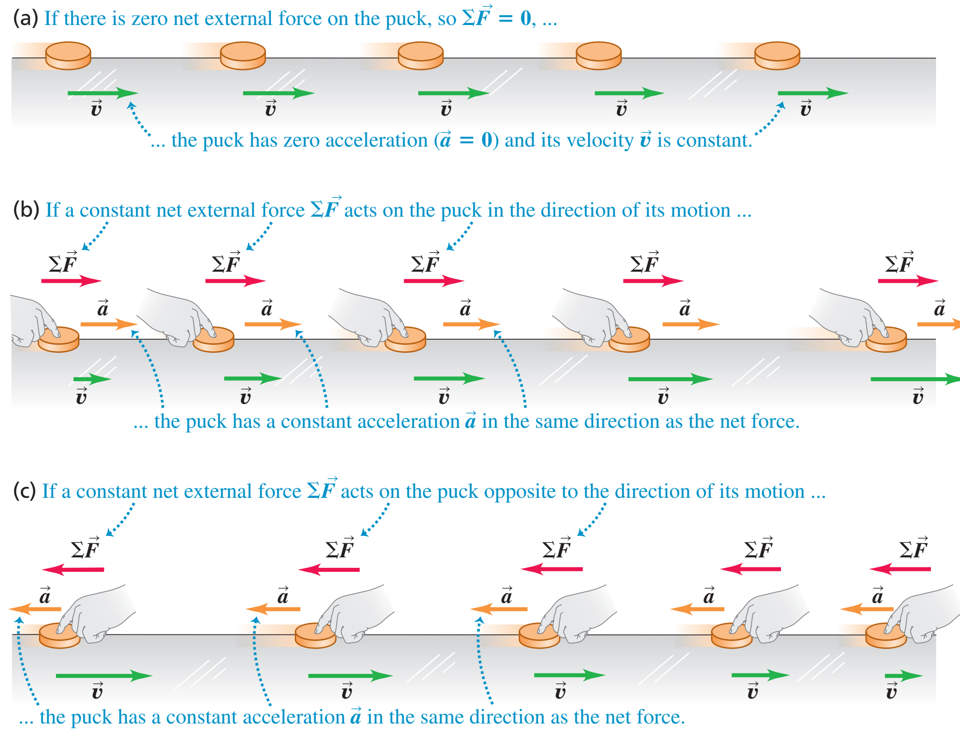
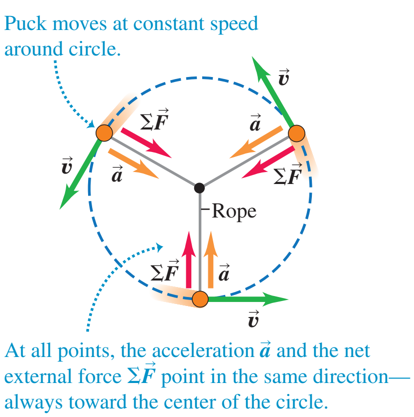


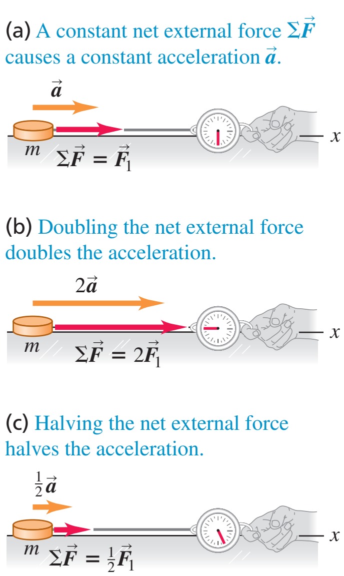
Figure **4.13** A top view of a hockey puck in uniform circular motion on a frictionless horizontal surface.



These conclusions about net external force and acceleration also apply to an object moving along a curved path. For example, **Fig. 4.13** shows a hockey puck moving in a horizontal circle on an ice surface of negligible friction. A rope is attached to the puck and to a stick in the ice, and this rope exerts an inward tension force of constant magnitude on the puck. The net external force and acceleration are both constant in magnitude and directed toward the center of the circle. The speed of the puck is constant, so this is uniform circular motion (see Section 3.4).

Figure 4.14a shows another experiment involving acceleration and net external force. We apply a constant horizontal force to a puck on a frictionless horizontal surface, using the spring balance described in Section 4.1 with the spring stretched a constant amount. As in Figs. 4.12b and Figs. 4.12c, this horizontal force equals the net external force on the puck. If we change the magnitude of the net external force, the acceleration changes in the same proportion. Doubling the net external force doubles the acceleration (Fig. 4.14b), halving the net external force halves the acceleration (Fig. 4.14c), and so on. Many such experiments show that *for any given object, the magnitude of the acceleration is directly proportional to the magnitude of the net external force acting on the object.*

Figure **4.14** The magnitude of an object's acceleration $\vec{a}$ is directly proportional to the magnitude of the net external force $\sum\vec{F}$ acting on the object of mass m.



Mass and Force

Our results mean that for a given object, the *ratio* of the magnitude $|\Sigma\vec{F}|$ of the net external force to the magnitude $a = |\vec{a}|$ of the acceleration is constant, regardless of the magnitude of the net external force. We call this ratio the *inertial mass,* or simply the **mass,** of the object and denote it by m. That is,

$$m = \frac{|\Sigma\vec{F}|}{a} \quad \text{or} \quad |\Sigma\vec{F}| = ma \quad \text{or} \quad a = \frac{|\Sigma\vec{F}|}{m} \tag{4.4}$$

Mass is a quantitative measure of inertia, which we discussed in Section 4.2. The last of the equations in Eqs. (4.4) says that the greater an object's mass, the more the object "resists" being accelerated. When you hold a piece of fruit in your hand at the supermarket and move it slightly up and down to estimate its heft, you're applying a force and seeing how much the fruit accelerates up and down in response. If a force causes a large acceleration, the fruit has a small mass; if the same force causes only a small acceleration, the fruit has a large mass. In the same way, if you hit a table-tennis ball and then a basketball with the same force, the basketball has much smaller acceleration because it has much greater mass.

The SI unit of mass is the **kilogram.** We mentioned in Section 1.3 that the kilogram is officially defined in terms of the definitions of the second and the meter, as well as the value of a fundamental quantity called Planck's constant. We can use this definition:

One newton is the amount of net external force that gives an acceleration of 1 meter per second squared to an object with a mass of 1 kilogram.

This definition allows us to calibrate the spring balances and other instruments used to measure forces. Because of the way we have defined the newton, it is related to the units of mass, length, and time. For Eqs. (4.4) to be dimensionally consistent, it must be true that

$$1 \text{ newton} = (1 \text{ kilogram})(1 \text{ meter per second squared})$$

or

$$1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$$

We'll use this relationship many times in the next few chapters, so keep it in mind.

Here's an application of Eqs. (4.4). Suppose we apply a constant net external force $\sum \vec{F}$ to an object of known mass m_1 and we find an acceleration of magnitude a_1 (**Fig. 4.15a**). We then apply the same force to another object of unknown mass m_2, and we find an acceleration of magnitude a_2 (Fig. 4.15b). Then, according to Eqs. (4.4),

$$m_1 a_1 = m_2 a_2$$

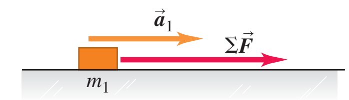
$$\frac{m_2}{m_1} = \frac{a_1}{a_2} \qquad \text{(same net external force)} \tag{4.5}$$

For the same net external force, the ratio of the masses of two objects is the inverse of the ratio of their accelerations. In principle we could use Eq. (4.5) to measure an unknown mass m_2, but it is usually easier to determine mass indirectly by measuring the object's *weight*. We'll return to this point in Section 4.4.

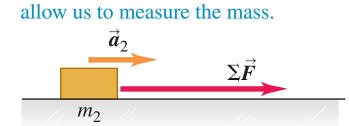
When two objects with masses m_1 and m_2 are fastened together, we find that the mass of the composite object is always $m_1 + m_2$ (Fig. 4.15c). This additive property of mass may seem obvious, but it has to be verified experimentally. Ultimately, the mass of an object is related to the number of protons, electrons, and neutrons it contains. This wouldn't be a good way to *define* mass because there is no practical way to count these particles. But the concept of mass is the most fundamental way to characterize the quantity of matter in an object.

Figure **4.15** For a given net external force $\sum \vec{F}$ acting an object, the acceleration is inversely proportional to the mass of the object. Masses add like ordinary scalars.

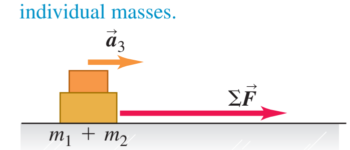
(a) A known net external force $\sum \vec{F}$ causes an object with mass m_1 to have an acceleration $\vec{a}_1$.



(b) Applying the same net external force $\sum \vec{F}$ to a second object and noting the acceleration allow us to measure the mass.



(c) When the two objects are fastened together, the same method shows that their composite mass is the sum of their individual masses.



Stating Newton's Second Law

Experiment shows that the *net* external force on an object is what causes that object to accelerate. If a combination of forces $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$, and so on is applied to an object, the object will have the same acceleration vector $\vec{a}$ as when only a single force is applied, if that single force is equal to the vector sum $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots$. In other words, the principle of superposition of forces (see Fig. 4.4) also holds true when the net external force is not zero and the object is accelerating.

Equations (4.4) relate the magnitude of the net external force on an object to the magnitude of the acceleration that it produces. We have also seen that the direction of the net external force is the same as the direction of the acceleration, whether the object's path is straight or curved. Finally, we've seen that the forces that affect an object's motion are *external* forces, those exerted on the object by other objects in its environment. Newton wrapped up all these results into a single concise statement that we now call *Newton's second law of motion:*

NEWTON'S SECOND LAW OF MOTION If a net external force acts on an object, the object accelerates. The direction of acceleration is the same as the direction of the net external force. The mass of the object times the acceleration vector of the object equals the net external force vector.

Figure **4.16** The design of high-performance motorcycles depends fundamentally on Newton's second law. To maximize the forward acceleration, the designer makes the motorcycle as light as possible (that is, minimizes the mass) and uses the most powerful engine possible (thus maximizing the forward force).

In symbols,

Newton's second law: If there is a net external force on an object ...

$$\sum \vec{F} = m\vec{a} \tag{4.6}$$

Mass of object

... the object accelerates in the same direction as the net external force.

An alternative statement is that the acceleration of an object is equal to the net external force acting on the object divided by the object's mass:

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

Newton's second law is a fundamental law of nature, the basic relationship between force and motion. Most of the remainder of this chapter and all of the next are devoted to learning how to apply this principle in various situations.

Equation (4.6) has many practical applications (**Fig. 4.16**). You've actually been using it all your life to measure your body's acceleration. In your inner ear, microscopic hair cells are attached to a gelatinous substance that holds tiny crystals of calcium carbonate called *otoliths*. When your body accelerates, the hair cells pull the otoliths along with the rest of your body and sense the magnitude and direction of the force that they exert. By Newton's second law, the acceleration of the otoliths—and hence that of your body as a whole—is proportional to this force and has the same direction. In this way, you can sense the magnitude and direction of your acceleration even with your eyes closed!

Using Newton's Second Law

At least four aspects of Newton's second law deserve special attention. First, Eq. (4.6) is a *vector* equation. Usually we'll use it in component form, with a separate equation for each component of force and the corresponding component of acceleration:

Newton's second law: Each component of the net external force on an object ...

$$\sum F_x = ma_x \qquad \sum F_y = ma_y \qquad \sum F_z = ma_z \tag{4.7}$$

... equals the object's mass times the corresponding acceleration component.

This set of component equations is equivalent to the single vector Eq. (4.6).

Second, the statement of Newton's second law refers to *external* forces. As an example, how a kicked soccer ball moves isn't affected by the *internal* forces that hold the pieces of the ball together. That's why only external forces are included in the sum $\sum \vec{F}$ in Eqs. (4.6) and (4.7).

Third, Eqs. (4.6) and (4.7) are valid only when the mass m is *constant*. It's easy to think of systems whose masses change, such as a leaking tank truck or a moving railroad car being loaded with coal. Such systems are better handled by using the concept of momentum; we'll get to that in Chapter 8.

Finally, Newton's second law is valid in inertial frames of reference *only,* just like the first law. It's not valid in the reference frame of any of the accelerating vehicles in Fig. 4.10; relative to any of these frames, the passenger accelerates even though the net external force on the passenger is zero. We'll usually treat the earth as an adequate approximation to an inertial frame, although because of its rotation and orbital motion it is not precisely inertial.

CAUTION **$m\vec{a}$ is not a force** Even though the vector $m\vec{a}$ is equal to the vector sum $\sum \vec{F}$ of all the forces acting on the object, the vector $m\vec{a}$ is *not* a force. Acceleration is the *result* of the net external force; it is not a force itself. It's "common sense" to think that a "force of acceleration" pushes you back into your seat when your car accelerates forward from rest. But *there is no such force;* instead, your inertia causes you to tend to stay at rest relative to the earth, and the car accelerates around you (see Fig. 4.10a). The "common sense" confusion arises from trying to apply Newton's second law where it isn't valid—in the noninertial reference frame of an accelerating car. We'll always examine motion relative to *inertial* frames of reference only, and we strongly recommend that you do the same in solving problems.

In learning how to use Newton's second law, we'll begin in this chapter with examples of straight-line motion. Then in Chapter 5 we'll consider more general kinds of motion and develop more detailed problem-solving strategies.

EXAMPLE 4.4 Newton's second law I: Determining acceleration from force

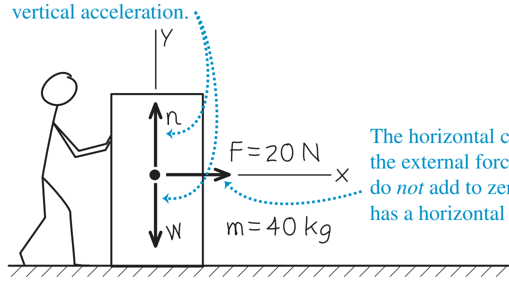
WITH VARIATION PROBLEMS

A worker applies a constant horizontal force with magnitude 20 N to a box with mass 40 kg resting on a level, freshly waxed floor with negligible friction. What is the acceleration of the box?

IDENTIFY and SET UP This problem involves force and acceleration, so we'll use Newton's second law. That means we'll have to find the net external force acting on the box and set it equal to the mass of the box multiplied by its acceleration. In this example, the acceleration is our target variable.

In *any* problem involving forces, to find the *net* external force we must first identify all of the *individual* external forces that act on the object in question. (Remember that the net external force is the vector sum of these individual forces.) To identify these forces, we'll use the idea that two broad categories of forces act on an object like the box in **Fig. 4.17**: the *weight* of the object $\vec{w}$—that is, the downward gravitational force exerted by the earth—and *contact forces*, which are forces exerted by other objects that the object in question is touching. Two objects are touching the box—the worker's hands and the floor—and both exert contact forces on the box. The worker's hands exert a horizontal force $\vec{F}$ of magnitude 20 N. The floor exerts an upward supporting force; as in Section 4.1, we call this a *normal* force $\vec{n}$ because it acts perpendicular to the surface of contact. (Remember that "normal" is a synonym for "perpendicular." It does not mean the opposite of "abnormal"!) If friction were present, the floor would also exert a friction force on the box; we'll ignore this here, since we're told that friction is negligible. Figure 4.17 shows these three external forces that act on the box.

Figure 4.17 Our sketch for this problem.



Just as we did for the forces in Example 4.1 (Section 4.1), we'll find the vector sum of these external forces using components. That's why the second step in any problem involving forces is choosing a coordinate system for finding vector components. It's usually convenient to take one axis either along or opposite the direction of the object's acceleration, which in this case is horizontal. Hence we take the $+x$-axis to be in the direction of the applied horizontal force (which is the direction in which the box accelerates) and the $+y$-axis to be upward. In most force problems that you'll encounter (including this one), the force vectors all lie in a plane, so the z-axis isn't used.

The box doesn't move vertically, so the y-acceleration is zero: $a_y = 0$. Our target variable is the x-acceleration, a_x. We'll find it by using Newton's second law in component form, Eqs. (4.7).

EXECUTE The force $\vec{F}$ exerted by the worker has a positive x-component and zero y-component (so $F_x = F = 20\text{ N}$, $F_y = 0$); the normal force $\vec{n}$ has zero x-component and an upward, positive y-component (so $n_x = 0, n_y = n$); and the weight $\vec{w}$ has zero x-component and a downward, negative y-component (so $w_x = 0, w_y = -w$). From Newton's second law, Eqs. (4.7),

$$\sum F_x = F + 0 + 0 = F = 20\text{ N} = ma_x$$

$$\sum F_y = 0 + n - w = ma_y = 0$$

From the first equation, the x-component of acceleration is

$$a_x = \frac{\sum F_x}{m} = \frac{20\text{ N}}{40\text{ kg}} = \frac{20\text{ kg}\cdot\text{m/s}^2}{40\text{ kg}} = 0.50\text{ m/s}^2$$

EVALUATE The net external force is constant, so the acceleration in the $+x$-direction is also constant. If we know the initial position and velocity of the box, we can find its position and velocity at any later time from the constant-acceleration equations of Chapter 2.

To determine a_x, we didn't need the y-component of Newton's second law from Eqs. (4.7), $\sum F_y = ma_y$. Can you use this equation to show that the magnitude n of the normal force in this situation is equal to the weight of the box?

KEYCONCEPT In problems involving forces and acceleration, first identify all of the external forces acting on an object, then choose a coordinate system. Find the vector sum of the external forces, and then set it equal to the mass of the object times the acceleration.

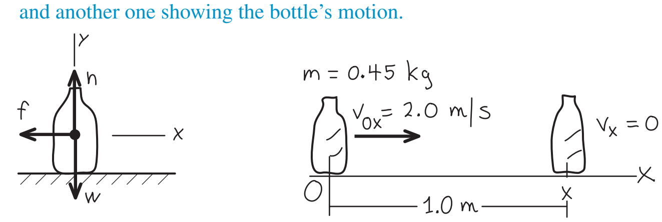
EXAMPLE 4.5 Newton's second law II: Determining force from acceleration

WITH VARIATION PROBLEMS

A waitress shoves a ketchup bottle with mass 0.45 kg to her right along a smooth, level lunch counter. The bottle leaves her hand moving at 2.0 m/s, then slows down as it slides because of a constant horizontal friction force exerted on it by the countertop. It slides for 1.0 m before coming to rest. What are the magnitude and direction of the friction force acting on the bottle?

IDENTIFY and SET UP This problem involves forces and acceleration (the slowing of the ketchup bottle), so we'll use Newton's second law to solve it. As in Example 4.4, we identify the external forces acting on the bottle and choose a coordinate system (**Fig. 4.18**). And as in

Figure 4.18 Our sketch for this problem.



Example 4.4, we have a downward gravitational force $\vec{w}$ and an upward normal force $\vec{n}$ exerted by the countertop. The countertop also exerts a friction force $\vec{f}$; this slows the bottle down, so its direction must be opposite the direction of the bottle's velocity (see Fig. 4.12c). We choose the $+x$-axis to be in the direction that the bottle slides, and take the origin to be where the bottle leaves the waitress's hand.

Our target variable is the magnitude f of the friction force. We'll find it by using the x-component of Newton's second law from Eqs. (4.7). We aren't told the x-component of the bottle's acceleration, a_x, but we know that it's constant because the friction force that causes the acceleration is constant. Hence we can use a constant-acceleration formula from Section 2.4 to calculate a_x. We know the bottle's initial and final x-coordinates $(x_0 = 0$ and $x = 1.0\text{ m})$ and its initial and final x-velocity ($v_{0x} = 2.0$ m/s and $v_x = 0$), so the easiest equation to use is Eq. (2.13), $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$.

EXECUTE We solve Eq. (2.13) for a_x:

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)} = \frac{(0\text{ m/s})^2 - (2.0\text{ m/s})^2}{2(1.0\text{ m} - 0\text{ m})} = -2.0\text{ m/s}^2$$

The negative sign means that the bottle's acceleration is toward the *left* in Fig. 4.18, opposite to its velocity; this makes sense because the bottle is slowing down. As in Example 4.4, neither the normal force nor the weight has an x-component. That means the net external force in the x-direction is just the x-component $-f$ of the friction force:

$$\begin{aligned}\sum F_x &= -f = ma_x = (0.45\text{ kg})(-2.0\text{ m/s}^2)\\ &= -0.90\text{ kg}\cdot\text{m/s}^2 = -0.90\text{ N}\end{aligned}$$

The negative sign shows that the net external force on the bottle is toward the left. The *magnitude* of the friction force is $f = 0.90$ N.

EVALUATE As a check on the result, try repeating the calculation with the $+x$-axis to the *left* in Fig. 4.18. You'll find that $\sum F_x$ is equal to $+f = +0.90$ N (because the friction force is now in the $+x$-direction), and again you'll find $f = 0.90$ N. The answers for the *magnitudes* of forces don't depend on the choice of coordinate axes!

We didn't write the y-component of Newton's second law in this example. You should do this and show that it's the same as in Example 4.4, so again the normal force and the weight have the same magnitude.

CAUTION Normal force and weight don't always have the same magnitude Be careful that you *never* assume automatically that the normal force $\vec{n}$ and the weight $\vec{w}$ have the same magnitude! Although that is the case in this example and the preceding one, we'll see many examples in Chapter 5 and later where the magnitude of the normal force is *not* equal to the weight.

KEYCONCEPT In problems involving forces in which you're given velocity, time, and/or displacement data, you'll need to use the equations for motion with constant acceleration as well as Newton's second law.

Some Notes on Units

A few words about units are in order. In the cgs metric system (not used in this book), the unit of mass is the gram, equal to 10^{-3} kg, and the unit of distance is the centimeter, equal to 10^{-2} m. The cgs unit of force is called the *dyne:*

$$1\text{ dyne} = 1\text{ g}\cdot\text{cm/s}^2 = 10^{-5}\text{ N}$$

In the British system, the unit of force is the *pound* (or pound-force) and the unit of mass is the *slug* (**Fig. 4.19**). The unit of acceleration is 1 foot per second squared, so

$$1\text{ pound} = 1\text{ slug}\cdot\text{ft/s}^2$$

The official definition of the pound is

$$1\text{ pound} = 4.448221615260\text{ newtons}$$

It is handy to remember that a pound is about 4.4 N and a newton is about 0.22 pound. Another useful fact: An object with a mass of 1 kg has a weight of about 2.2 lb at the earth's surface.

Table 4.2 lists the units of force, mass, and acceleration in the three systems.

Figure **4.19** Despite its name, the British unit of mass has nothing to do with the type of slug shown here. A common garden slug has a mass of about 15 grams, or about 10^{-3} slug.

TABLE 4.2 Units of Force, Mass, and Acceleration

System of Units	Force	Mass	Acceleration
SI	newton (N)	kilogram (kg)	m/s^2
cgs	dyne (dyn)	gram (g)	cm/s^2
British	pound (lb)	slug	ft/s^2

TEST YOUR UNDERSTANDING OF SECTION 4.3 Rank the following situations in order of the magnitude of the object's acceleration, from lowest to highest. Are there any cases that have the same magnitude of acceleration? (i) A 2.0 kg object acted on by a 2.0 N net force; (ii) a 2.0 kg object acted on by an 8.0 N net force; (iii) an 8.0 kg object acted on by a 2.0 N net force; (iv) an 8.0 kg object acted on by a 8.0 N net force.

ANSWER

(iii), (i) and (iv) (tie), (ii) The acceleration is equal to the net force divided by the mass. Hence the magnitude of the acceleration in each situation is
(i) $a = (2.0\ \text{N})/(2.0\ \text{kg}) = 1.0\ \text{m/s}^2$; (ii) $a = (8.0\ \text{N})/(2.0\ \text{N}) = 4.0\ \text{m/s}^2$;
(iii) $a = (2.0\ \text{N})/(8.0\ \text{kg}) = 0.25\ \text{m/s}^2$; (iv) $a = (8.0\ \text{N})/(8.0\ \text{kg}) = 1.0\ \text{m/s}^2$.

4.4 MASS AND WEIGHT

The *weight* of an object is the gravitational force that the earth exerts on the object. (If you are on another planet, your weight is the gravitational force that planet exerts on you.) Unfortunately, the terms "mass" and "weight" are often misused and interchanged in everyday conversation. It's absolutely essential for you to understand clearly the distinctions between these two physical quantities.

Mass characterizes the *inertial* properties of an object. Mass is what keeps the table setting on the table when you yank the tablecloth out from under it. The greater the mass, the greater the force needed to cause a given acceleration; this is reflected in Newton's second law, $\sum\vec{F} = m\vec{a}$.

Weight, on the other hand, is a *force* exerted on an object by the pull of the earth. Mass and weight are related: Objects that have large mass also have large weight. A large stone is hard to throw because of its large *mass,* and hard to lift off the ground because of its large *weight.*

To understand the relationship between mass and weight, note that a freely falling object has an acceleration of magnitude g (see Section 2.5). Newton's second law tells us that a force must act to produce this acceleration. If a 1 kg object falls with an acceleration of $9.8\ \text{m/s}^2$, the required force has magnitude

$$F = ma = (1\ \text{kg})(9.8\ \text{m/s}^2) = 9.8\ \text{kg}\cdot\text{m/s}^2 = 9.8\ \text{N}$$

The force that makes the object accelerate downward is its weight. Any object near the surface of the earth that has a mass of 1 kg *must* have a weight of 9.8 N to give it the acceleration we observe when it is in free fall. More generally,

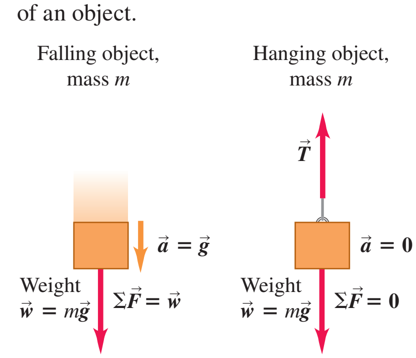
$$w = mg \tag{4.8}$$

Magnitude of weight of an object (w); Mass of object (m); Magnitude of acceleration due to gravity (g)

Hence the magnitude w of an object's weight is directly proportional to its mass m. The weight of an object is a force, a vector quantity, and we can write Eq. (4.8) as a vector equation (**Fig. 4.20**):

$$\vec{w} = m\vec{g} \tag{4.9}$$

Figure 4.20 Relating the mass and weight of an object.



- The relationship of mass to weight: $\vec{w} = m\vec{g}$.
- This relationship is the same whether an object is falling or stationary.

Remember that g is the *magnitude* of $\vec{g}$, the acceleration due to gravity, so g is always a positive number, by definition. Thus w, given by Eq. (4.8), is the *magnitude* of the weight and is also always positive.

CAUTION An object's weight acts at all times When keeping track of the external forces on an object, remember that the weight is present *all the time,* whether the object is in free fall or not. If we suspend an object from a rope, it is in equilibrium and its acceleration is zero. But its weight, given by Eq. (4.9), is still pulling down on it (Fig. 4.20). In this case the rope pulls up on the object, applying an upward force. The *vector sum* of the external forces is zero, but the weight still acts.