? Hailstorms such as the one that broke the tiles on this roof cause billions of dollars of damage every year in North America alone. Which would do the greater amount of damage? (i) A 0.2 kg hailstone that hits at speed 15 m/s; (ii) a 0.1 kg hailstone that hits at speed 30 m/s; (iii) both would do the same amount of damage.

8 Momentum, Impulse, and Collisions

Many questions involving forces can't be answered by directly applying Newton's second law, $\sum \vec{F} = m\vec{a}$. For example, when a truck collides head-on with a compact car, what determines which way the wreckage moves after the collision? In playing pool, how do you decide how to aim the cue ball in order to knock the eight ball into the pocket? And when a meteorite collides with the earth, how much of the meteorite's kinetic energy is released in the impact?

All of these questions involve forces about which we know very little: the forces between the car and the truck, between the two pool balls, or between the meteorite and the earth. Remarkably, we'll find in this chapter that we don't have to know *anything* about these forces to answer questions of this kind!

Our approach uses two new concepts, *momentum* and *impulse,* and a new conservation law, *conservation of momentum*. This conservation law is every bit as important as the law of conservation of energy. The law of conservation of momentum is valid even in situations in which Newton's laws are inadequate, such as objects moving at very high speeds (near the speed of light) or objects on a very small scale (such as the constituents of atoms). Within the domain of Newtonian mechanics, conservation of momentum enables us to analyze many situations that would be very difficult if we tried to use Newton's laws directly. Among these are *collision* problems, in which two objects collide and can exert very large forces on each other for a short time. We'll also use momentum ideas to solve problems in which an object's mass changes as it moves, including the important special case of a rocket (which loses mass as it expends fuel).

LEARNING OUTCOMES

In this chapter, you'll learn...

- **8.1** The meaning of the momentum of a particle, and how the impulse of the net external force acting on a particle causes its momentum to change.
- **8.2** The circumstances under which the total momentum of a system of particles is constant (conserved).
- **8.3** How to use momentum conservation to solve problems in which two objects collide with each other, and what the differences are among elastic, inelastic, and completely inelastic collisions.
- **8.4** How to analyze what happens in the important special case of an elastic collision.
- **8.5** What's meant by the center of mass of a system, and what determines how the center of mass moves.
- **8.6** How to analyze situations such as rocket propulsion in which the mass of an object changes as it moves.

You'll need to review...

- **3.5** Relative velocity.
- **4.2** Inertial frames of reference.
- **6.1, 6.2** Work, kinetic energy, and the work–energy theorem.
- **6.3** Work done by an ideal spring.

8.1 MOMENTUM AND IMPULSE

In Section 6.2 we re-expressed Newton's second law for a particle, $\sum \vec{F} = m\vec{a}$, in terms of the work–energy theorem. This theorem helped us tackle a great number of problems and led us to the law of conservation of energy. Let's return to $\sum \vec{F} = m\vec{a}$ and see yet another useful way to restate this fundamental law.

Newton's Second Law in Terms of Momentum

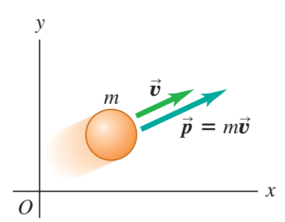
Consider a particle of constant mass m. Because $\vec{a} = d\vec{v}/dt$, we can write Newton's second law for this particle as

$$\sum\vec{F} = m\frac{d\vec{v}}{dt} = \frac{d}{dt}(m\vec{v}) \tag{8.1}$$

We can move the mass m inside the derivative because it is constant. Thus Newton's second law says that the net external force $\sum\vec{F}$ acting on a particle equals the time rate of change of the product of the particle's mass and velocity. We'll call this product the **momentum,** or **linear momentum,** of the particle:

Momentum of a particle (a vector quantity) $\vec{p} = m\vec{v}$ — Particle mass; Particle velocity (8.2)

Figure 8.1 The velocity and momentum vectors of a particle.



Momentum $\vec{p}$ is a vector quantity; a particle's momentum has the same direction as its velocity $\vec{v}$.

The greater the mass m and speed v of a particle, the greater is its magnitude of momentum mv. Keep in mind that momentum is a *vector* quantity with the same direction as the particle's velocity (**Fig. 8.1**). A car driving north at 20 m/s and an identical car driving east at 20 m/s have the same *magnitude* of momentum (mv) but different momentum *vectors* ($m\vec{v}$) because their directions are different.

We often express the momentum of a particle in terms of its components. If the particle has velocity components v_x, v_y, and v_z, then its momentum components p_x, p_y, and p_z (which we also call the *x-momentum, y-momentum,* and *z-momentum*) are

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z \tag{8.3}$$

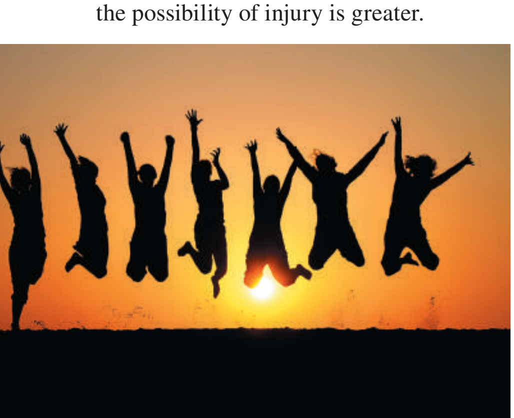
These three component equations are equivalent to Eq. (8.2).

The units of the magnitude of momentum are units of mass times speed; the SI units of momentum are kg · m/s. The plural of momentum is "momenta."

Let's now substitute the definition of momentum, Eq. (8.2), into Eq. (8.1):

Newton's second law in terms of momentum: The net external force acting on a particle ... $\sum\vec{F} = \dfrac{d\vec{p}}{dt}$ ... equals the rate of change of the particle's momentum. (8.4)

Figure 8.2 When you land after jumping upward, your momentum changes from a downward value to zero. It's best to land with your knees bent so that your legs can flex: You then take a relatively long time to stop, and the force that the ground exerts on your legs is small. If you land with your legs extended, you stop in a short time, the force on your legs is larger, and the possibility of injury is greater.



The net external force (vector sum of all forces) acting on a particle equals the time rate of change of momentum of the particle. This, not $\sum\vec{F} = m\vec{a}$, is the form in which Newton originally stated his second law (although he called momentum the "quantity of motion"). This law is valid only in inertial frames of reference (see Section 4.2). As Eq. (8.4) shows, a rapid change in momentum requires a large net external force, while a gradual change in momentum requires a smaller net external force (**Fig. 8.2**).

The Impulse–Momentum Theorem

Both a particle's momentum $\vec{p} = m\vec{v}$ and its kinetic energy $K = \frac{1}{2}mv^2$ depend on the mass and velocity of the particle. What is the fundamental difference between these two quantities? A purely mathematical answer is that momentum is a vector whose magnitude is proportional to speed, while kinetic energy is a scalar proportional to the speed squared. But to see the *physical* difference between momentum and kinetic energy, we must first define a quantity closely related to momentum called *impulse.*

Let's first consider a particle acted on by a *constant* net external force $\sum\vec{F}$ during a time interval Δt from t_1 to t_2. The **impulse** of the net external force, denoted by $\vec{J}$, is defined to be the product of the net external force and the time interval:

Impulse of a **constant net external force** $\vec{J} = \sum\vec{F}(t_2 - t_1) = \sum\vec{F}\Delta t$ — Constant net external force; Time interval over which net external force acts (8.5)

Impulse is a vector quantity; its direction is the same as the net external force $\sum\vec{F}$. The SI unit of impulse is the newton-second (N · s). Because $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$, an alternative set of units for impulse is kg · m/s, the same as for momentum.

To see what impulse is good for, let's go back to Newton's second law as restated in terms of momentum, Eq. (8.4). If the net external force $\sum\vec{F}$ is constant, then $d\vec{p}/dt$ is also constant. In that case, $d\vec{p}/dt$ is equal to the *total* change in momentum $\vec{p}_2 - \vec{p}_1$ during the time interval $t_2 - t_1$, divided by the interval:

$$\sum\vec{F} = \frac{\vec{p}_2 - \vec{p}_1}{t_2 - t_1}$$

Multiplying this equation by $(t_2 - t_1)$, we have

$$\sum\vec{F}(t_2 - t_1) = \vec{p}_2 - \vec{p}_1$$

Comparing with Eq. (8.5), we end up with

Impulse–momentum theorem: The impulse of the net external force on a particle during a time interval equals the change in momentum of that particle during that interval:

Impulse of net external force over a time interval

$$\vec{J} = \vec{p}_2 - \vec{p}_1 = \Delta\vec{p} \qquad (8.6)$$

Change in momentum

Final momentum

Initial momentum

The impulse–momentum theorem also holds when forces are not constant. To see this, we integrate both sides of Newton's second law $\sum\vec{F} = d\vec{p}/dt$ over time between the limits t_1 and t_2:

$$\int_{t_1}^{t_2} \sum\vec{F}\,dt = \int_{t_1}^{t_2} \frac{d\vec{p}}{dt}\,dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} = \vec{p}_2 - \vec{p}_1$$

We see from Eq. (8.6) that the integral on the left is the impulse of the net external force:

Impulse of a **general net external force** (either constant or varying)

$$\vec{J} = \int_{t_1}^{t_2} \sum\vec{F}\,dt \qquad (8.7)$$

Upper limit = final time

Time integral of net external force

Lower limit = initial time

If the net external force $\sum\vec{F}$ is constant, the integral in Eq. (8.7) reduces to Eq. (8.5). We can define an *average* net external force $\vec{F}_{av}$ such that even when $\sum\vec{F}$ is not constant, the impulse $\vec{J}$ is given by

$$\vec{J} = \vec{F}_{av}(t_2 - t_1) \qquad (8.8)$$

When $\sum\vec{F}$ is constant, $\sum\vec{F} = \vec{F}_{av}$ and Eq. (8.8) reduces to Eq. (8.5).

Figure 8.3a shows the x-component of net external force $\sum F_x$ as a function of time during a collision. This might represent the force on a soccer ball that is in contact with a player's foot from time t_1 to t_2. The x-component of impulse during this interval is represented by the red area under the curve between t_1 and t_2. This area is equal to the green rectangular area bounded by t_1, t_2, and $(F_{av})_x$, so $(F_{av})_x(t_2 - t_1)$ is equal to the impulse of the actual time-varying force during the same interval. Note that a large force acting for a short time can have the same impulse as a smaller force acting for a longer time if the areas under the force–time curves are the same (Fig. 8.3b). We used this idea in Fig. 8.2: A small force acting for a relatively long time (as when you land with your legs bent) has the same effect as a larger force acting for a short time (as when you land stiff-legged). Automotive air bags use the same principle (**Fig. 8.4**, next page).

BIO APPLICATION Woodpecker Impulse The pileated woodpecker (*Dryocopus pileatus*) has been known to strike its beak against a tree up to 20 times a second and up to 12,000 times a day. The impact force can be as much as 1200 times the weight of the bird's head. Because the impact lasts such a short time, the impulse—the net external force during the impact multiplied by the duration of the impact—is relatively small. (The woodpecker has a thick skull of spongy bone as well as shock-absorbing cartilage at the base of the lower jaw, and so avoids injury.)

Figure **8.3** The meaning of the area under a graph of $\sum F_x$ versus t.

(a)

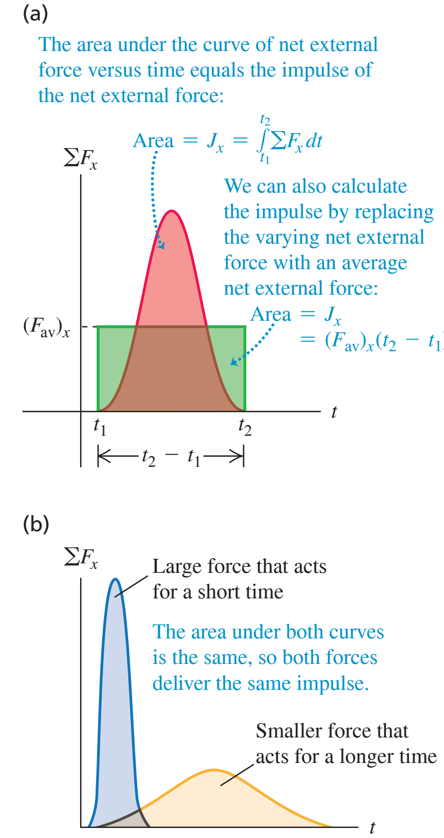
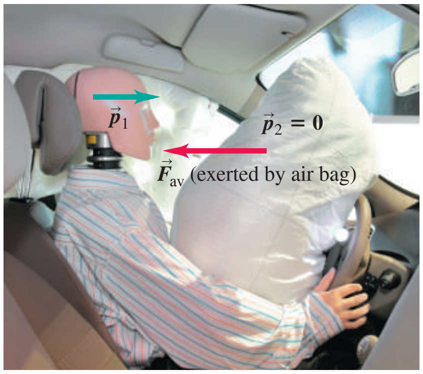


Figure 8.4 The impulse–momentum theorem explains how air bags reduce the chance of injury by minimizing the force on an occupant of a car.



- Impulse–momentum theorem: $\vec{J} = \vec{p}_2 - \vec{p}_1 = \vec{F}_{av}\Delta t$
- Impulse is the same no matter how the driver is brought to rest (so $\vec{p}_2 = \mathbf{0}$).
- Compared to striking the steering wheel, striking the air bag brings the driver to rest over a longer time interval Δt.
- Hence with an air bag, average force $\vec{F}_{av}$ on the driver is less.

Both impulse and momentum are vector quantities, and Eqs. (8.5)–(8.8) are vector equations. It's often easiest to use them in component form:

$$J_x = \int_{t_1}^{t_2} \sum F_x dt = (F_{av})_x(t_2 - t_1) = p_{2x} - p_{1x} = mv_{2x} - mv_{1x}$$
$$J_y = \int_{t_1}^{t_2} \sum F_y dt = (F_{av})_y(t_2 - t_1) = p_{2y} - p_{1y} = mv_{2y} - mv_{1y} \qquad (8.9)$$

and similarly for the z-component.

Momentum and Kinetic Energy Compared

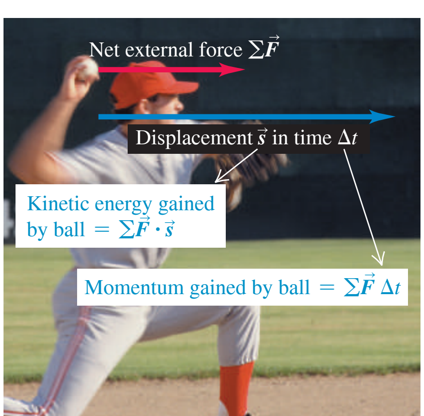
We can now see the fundamental difference between momentum and kinetic energy. The impulse–momentum theorem, $\vec{J} = \vec{p}_2 - \vec{p}_1$, says that changes in a particle's momentum are due to impulse, which depends on the *time* over which the net external force acts. By contrast, the work–energy theorem, $W_{tot} = K_2 - K_1$, tells us that kinetic energy changes when work is done on a particle; the total work depends on the *distance* over which the net external force acts.

Let's consider a particle that starts from rest at t_1 so that $\vec{v}_1 = \mathbf{0}$. Its initial momentum is $\vec{p}_1 = m\vec{v}_1 = \mathbf{0}$, and its initial kinetic energy is $K_1 = \frac{1}{2}mv_1^2 = 0$. Now let a constant net external force equal to $\vec{F}$ act on that particle from time t_1 until time t_2. During this interval, the particle moves a distance s in the direction of the force. From Eq. (8.6), the particle's momentum at time t_2 is

$$\vec{p}_2 = \vec{p}_1 + \vec{J} = \vec{J}$$

where $\vec{J} = \vec{F}(t_2 - t_1)$ is the impulse that acts on the particle. So *the momentum of a particle equals the impulse that accelerated it from rest to its present speed;* impulse is the product of the net external force that accelerated the particle and the *time* required for the acceleration. By comparison, the kinetic energy of the particle at t_2 is $K_2 = W_{tot} = Fs$, the total *work* done on the particle to accelerate it from rest. The total work is the product of the net external force and the *distance* required to accelerate the particle (**Fig. 8.5**).

Figure 8.5 The *kinetic energy* of a pitched baseball is equal to the work the pitcher does on it (force multiplied by the distance the ball moves during the throw). The *momentum* of the ball is equal to the impulse the pitcher imparts to it (force multiplied by the time it took to bring the ball up to speed).



Here's an application of the distinction between momentum and kinetic energy. Which is easier to catch: a 0.50 kg ball moving at 4.0 m/s or a 0.10 kg ball moving at 20 m/s? Both balls have the same magnitude of momentum, $p = mv = (0.50\text{ kg}) \times (4.0\text{ m/s}) = (0.10\text{ kg})(20\text{ m/s}) = 2.0\text{ kg}\cdot\text{m/s}$. However, the two balls have different values of kinetic energy $K = \frac{1}{2}mv^2$: The large, slow-moving ball has $K = 4.0$ J, while the small, fast-moving ball has $K = 20$ J. Since the momentum is the same for both balls, both require the same *impulse* to be brought to rest. But stopping the 0.10 kg ball with your hand requires five times more *work* than stopping the 0.50 kg ball because the smaller ball has five times more kinetic energy. For a given force that you exert with your hand, it takes the same amount of time (the duration of the catch) to stop either ball, but your hand and arm will be pushed back five times farther if you choose to catch the small, fast-moving ball. To minimize arm strain, you should choose to catch the 0.50 kg ball with its lower kinetic energy.

Both the impulse–momentum and work–energy theorems rest on the foundation of Newton's laws. They are *integral* principles, relating the motion at two different times separated by a finite interval. By contrast, Newton's second law itself (in either of the forms $\sum\vec{F} = m\vec{a}$ or $\sum\vec{F} = d\vec{p}/dt$) is a *differential* principle that concerns the rate of change of velocity or momentum at each instant.

CONCEPTUAL EXAMPLE 8.1 Momentum versus kinetic energy

Consider again the race described in Conceptual Example 6.5 (Section 6.2) between two iceboats on a frictionless frozen lake. The boats have masses m and $2m$, and the wind exerts the same constant horizontal force $\vec{F}$ on each boat (see Fig. 6.14). The boats start from rest and cross the finish line a distance s away. Which boat crosses the finish line with greater momentum?

SOLUTION In Conceptual Example 6.5 we asked how the *kinetic energies* of the boats compare when they cross the finish line. We answered this by remembering that *an object's kinetic energy equals the total work done to accelerate it from rest*. Both boats started from rest, and the total work done was the same for both boats (because the net external force and the displacement were the same for both). Hence both boats had the same kinetic energy at the finish line.

Similarly, to compare the *momenta* of the boats we use the idea that *the momentum of each boat equals the impulse that accelerated it from rest*. As in Conceptual Example 6.5, the net external force on each boat equals the constant horizontal wind force $\vec{F}$. Let Δt be the time a boat takes to reach the finish line, so that the impulse on the boat during that time is $\vec{J} = \vec{F}\Delta t$. Since the boat starts from rest, this equals the boat's momentum $\vec{p}$ at the finish line:

$$\vec{p} = \vec{F}\Delta t$$

Both boats are subjected to the same force $\vec{F}$, but they take different times Δt to reach the finish line. The boat of mass $2m$ accelerates more slowly and takes a longer time to travel the distance s; thus there is a greater impulse on this boat between the starting and finish lines. So the boat of mass $2m$ crosses the finish line with a greater magnitude of momentum than the boat of mass m (but with the same kinetic energy). Can you show that the boat of mass $2m$ has $\sqrt{2}$ times as much momentum at the finish line as the boat of mass m?

KEYCONCEPT The momentum of an object equals the product of its mass and its velocity $\vec{v}$, and also equals the impulse (net external force multiplied by time) needed to accelerate it from rest to $\vec{v}$.

EXAMPLE 8.2 A ball hits a wall

You throw a ball with a mass of 0.40 kg against a brick wall. It is moving horizontally to the left at 30 m/s when it hits the wall; it rebounds horizontally to the right at 20 m/s. (a) Find the impulse of the net external force on the ball during its collision with the wall. (b) If the ball is in contact with the wall for 0.010 s, find the average horizontal force that the wall exerts on the ball during the impact.

IDENTIFY and SET UP We're given enough information to determine the initial and final values of the ball's momentum, so we can use the impulse–momentum theorem to find the impulse. We'll then use the definition of impulse to determine the average force. **Figure 8.6** shows our sketch. We need only a single axis because the motion is purely horizontal. We'll take the positive x-direction to be to the right. In part (a) our target variable is the x-component of impulse, J_x, which we'll find by using Eqs. (8.9). In part (b), our target variable is the average x-component of force $(F_{av})_x$; once we know J_x, we can also find this force by using Eqs. (8.9).

EXECUTE (a) With our choice of x-axis, the initial and final x-components of momentum of the ball are

$$p_{1x} = mv_{1x} = (0.40\ \text{kg})(-30\ \text{m/s}) = -12\ \text{kg}\cdot\text{m/s}$$
$$p_{2x} = mv_{2x} = (0.40\ \text{kg})(+20\ \text{m/s}) = +8.0\ \text{kg}\cdot\text{m/s}$$

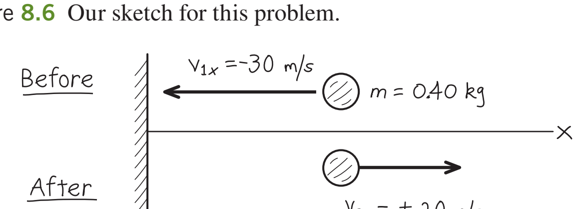
From the x-equation in Eqs. (8.9), the x-component of impulse equals the *change* in the x-momentum:

$$\begin{aligned} J_x &= p_{2x} - p_{1x} \\ &= 8.0\ \text{kg}\cdot\text{m/s} - (-12\ \text{kg}\cdot\text{m/s}) = 20\ \text{kg}\cdot\text{m/s} = 20\ \text{N}\cdot\text{s} \end{aligned}$$

(b) The collision time is $t_2 - t_1 = \Delta t = 0.010$ s. From the x-equation in Eqs. (8.9), $J_x = (F_{av})_x(t_2 - t_1) = (F_{av})_x\Delta t$, so

$$(F_{av})_x = \frac{J_x}{\Delta t} = \frac{20\ \text{N}\cdot\text{s}}{0.010\ \text{s}} = 2000\ \text{N}$$

Figure 8.6 Our sketch for this problem.



EVALUATE The x-component of impulse J_x is positive—that is, to the right in Fig. 8.6. The impulse represents the "kick" that the wall imparts to the ball, and this "kick" is certainly to the right.

CAUTION Momentum is a vector Because momentum is a vector, we had to include the negative sign in writing $p_{1x} = -12\ \text{kg}\cdot\text{m/s}$. Had we omitted it, we would have calculated the impulse to be $8.0\ \text{kg}\cdot\text{m/s} - (12\ \text{kg}\cdot\text{m/s}) = -4\ \text{kg}\cdot\text{m/s}$. This would say that the wall had somehow given the ball a kick to the *left*! Remember the *direction* of momentum in your calculations.

The force that the wall exerts on the ball must have such a large magnitude (2000 N, equal to the weight of a 200 kg object) to change the ball's momentum in such a short time. Other forces that act on the ball during the collision are comparatively weak; for instance, the gravitational force is only 3.9 N. Thus, during the short time that the collision lasts, we can ignore all other forces on the ball. **Figure 8.7** shows the impact of a tennis ball and racket.

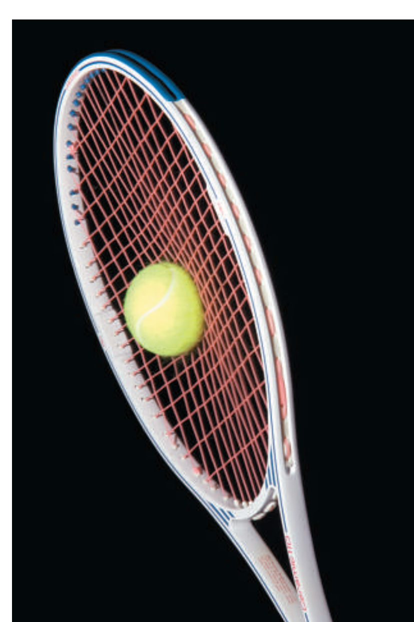


Figure 8.7 Typically, a tennis ball is in contact with the racket for approximately 0.01 s. The ball flattens noticeably due to the tremendous force exerted by the racket.

Continued

Note that the 2000 N value we calculated is the *average* horizontal force that the wall exerts on the ball during the impact. It corresponds to the horizontal line $(F_{av})_x$ in Fig. 8.3a. The horizontal force is zero before impact, rises to a maximum, and then decreases to zero when the ball loses contact with the wall. If the ball is relatively rigid, like a baseball or golf ball, the collision lasts a short time and the maximum force is large, as in the blue curve in Fig. 8.3b. If the ball is softer, like a tennis ball, the collision time is longer and the maximum force is less, as in the orange curve in Fig. 8.3b.

KEYCONCEPT The impulse-momentum theorem states that when a net external force acts on an object, the object's momentum changes by an amount equal to the impulse of the net external force. Both momentum and impulse are vector quantities.

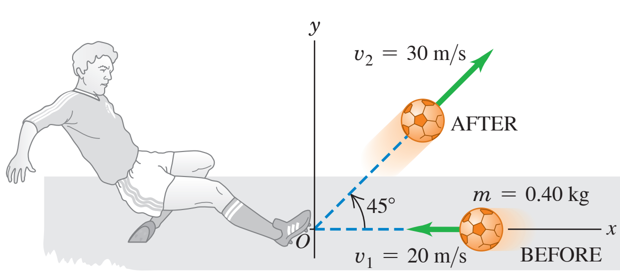
EXAMPLE 8.3 Kicking a soccer ball

A soccer ball has a mass of 0.40 kg. Initially it is moving to the left at 20 m/s, but then it is kicked. After the kick it is moving at 45° upward and to the right with speed 30 m/s (**Fig. 8.8a**). Find the impulse of the net external force and the average net external force, assuming a collision time $\Delta t = 0.010$ s.

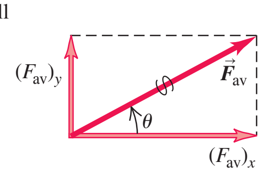
IDENTIFY and SET UP The ball moves in two dimensions, so we must treat momentum and impulse as vector quantities. We take the x-axis to be horizontally to the right and the y-axis to be vertically upward. Our target variables are the components of the net impulse on the ball, J_x and J_y, and the components of the average net external force on the ball, $(F_{av})_x$ and $(F_{av})_y$. We'll find them by using the impulse–momentum theorem in its component form, Eqs. (8.9).

Figure **8.8** (a) Kicking a soccer ball. (b) Finding the average force on the ball from its components.

(a) Before-and-after diagram



(b) Average force on the ball



EXECUTE Using $\cos 45° = \sin 45° = 0.707$, we find the ball's velocity components before and after the kick:

$$v_{1x} = -20 \text{ m/s} \qquad v_{1y} = 0$$
$$v_{2x} = v_{2y} = (30 \text{ m/s})(0.707) = 21.2 \text{ m/s}$$

From Eqs. (8.9), the impulse components are

$$\begin{aligned} J_x &= p_{2x} - p_{1x} = m(v_{2x} - v_{1x}) \\ &= (0.40 \text{ kg})[21.2 \text{ m/s} - (-20 \text{ m/s})] = 16.5 \text{ kg}\cdot\text{m/s} \end{aligned}$$

$$\begin{aligned} J_y &= p_{2y} - p_{1y} = m(v_{2y} - v_{1y}) \\ &= (0.40 \text{ kg})(21.2 \text{ m/s} - 0) = 8.5 \text{ kg}\cdot\text{m/s} \end{aligned}$$

From Eq. (8.8), the average net external force components are

$$(F_{av})_x = \frac{J_x}{\Delta t} = 1650 \text{ N} \qquad (F_{av})_y = \frac{J_y}{\Delta t} = 850 \text{ N}$$

The magnitude and direction of the $\vec{F}_{av}$ vector (Fig. 8.8b) are

$$F_{av} = \sqrt{(1650 \text{ N})^2 + (850 \text{ N})^2} = 1.9 \times 10^3 \text{ N}$$
$$\theta = \arctan\frac{850 \text{ N}}{1650 \text{ N}} = 27°$$

The ball was not initially at rest, so its final velocity does *not* have the same direction as the average force that acted on it.

EVALUATE $\vec{F}_{av}$ includes the force of gravity, which is very small; the weight of the ball is only 3.9 N. As in Example 8.2, the average force acting during the collision is exerted almost entirely by the object that the ball hit (in this case, the soccer player's foot).

KEYCONCEPT In two-dimensional problems involving impulse and momentum, you must apply the impulse–momentum theorem separately to the x-components and the y-components.

TEST YOUR UNDERSTANDING OF SECTION 8.1 Rank the following situations according to the magnitude of the impulse of the net external force, from largest value to smallest value. In each situation a 1000 kg automobile is moving along a straight east–west road. The automobile is initially (i) moving east at 25 m/s and comes to a stop in 10 s; (ii) moving east at 25 m/s and comes to a stop in 5 s; (iii) at rest, and a 2000 N net external force toward the east is applied to it for 10 s; (iv) moving east at 25 m/s, and a 2000 N net external force toward the west is applied to it for 10 s; (v) moving east at 25 m/s; over a 30 s period, the automobile reverses direction and ends up moving west at 25 m/s.

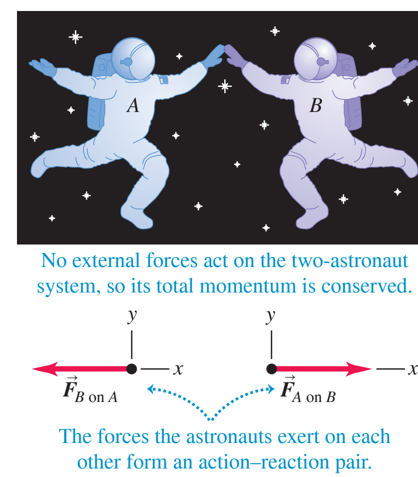
ANSWER

❙ **(v), (i) and (ii) (tied for second place), (iii) and (iv) (tied for third place)** We use two interpretations of the impulse of the net external force: (1) the net external force multiplied by the time that the net external force acts, and (2) the change in momentum of the particle on which the net external force acts. Which interpretation we use depends on what information we are given. We take the positive x-direction to be to the east. (i) The force is not given, so we use interpretation 2: $J_x = mv_{2x} - mv_{1x} = (1000\text{ kg})(0) - (1000\text{ kg})(25\text{ m/s}) = -25{,}000\text{ kg}\cdot\text{m/s}$, so the magnitude of the impulse is $25{,}000\text{ kg}\cdot\text{m/s} = 25{,}000\text{ N}\cdot\text{s}$. (ii) For the same reason and values as in (i), we use interpretation 2, and the magnitude of the impulse is again $25{,}000\text{ N}\cdot\text{s}$. (iii) The final velocity is not given, so we use interpretation 1: $J_x = (\sum F_x)^{\text{av}}(t_2 - t_1) = (2000\text{ N})(10\text{ s}) = 20{,}000\text{ N}\cdot\text{s}$, so the magnitude of the impulse is $20{,}000\text{ N}\cdot\text{s}$. (iv) For the same reason as in (iii), we use interpretation 1: $J_x = (\sum F_x)^{\text{av}}(t_2 - t_1) = (-2000\text{ N})(10\text{ s}) = -20{,}000\text{ N}\cdot\text{s}$, so the magnitude of the impulse is $20{,}000\text{ N}\cdot\text{s}$. (v) The force is not given, so we use interpretation 2: $J_x = mv_{2x} - mv_{1x} = (1000\text{ kg})(-25\text{ m/s}) - (1000\text{ kg})(25\text{ m/s}) = -50{,}000\text{ kg}\cdot\text{m/s}$, so the magnitude of the impulse is $50{,}000\text{ kg}\cdot\text{m/s} = 50{,}000\text{ N}\cdot\text{s}$.

8.2 CONSERVATION OF MOMENTUM

The concept of momentum is particularly important in situations in which we have two or more objects that *interact*. To see why, let's consider first an idealized system of two objects that interact with each other but not with anything else—for example, two astronauts who touch each other as they float freely in the zero-gravity environment of outer space (**Fig. 8.9**). Think of the astronauts as particles. Each particle exerts a force on the other; according to Newton's third law, the two forces are always equal in magnitude and opposite in direction. Hence, the *impulses* that act on the two particles are equal in magnitude and opposite in direction, as are the changes in momentum of the two particles.

Figure **8.9** Two astronauts push each other as they float freely in the zero-gravity environment of space.



Let's go over that again with some new terminology. For any system, the forces that the particles of the system exert on each other are called **internal forces.** Forces exerted on any part of the system by some object outside it are called **external forces.** For the system shown in Fig. 8.9, the internal forces are $\vec{F}_{B \text{ on } A}$, exerted by particle B on particle A, and $\vec{F}_{A \text{ on } B}$, exerted by particle A on particle B. There are *no* external forces; when this is the case, we have an **isolated system.**

The net external force on particle A is $\vec{F}_{B \text{ on } A}$ and the net external force on particle B is $\vec{F}_{A \text{ on } B}$, so from Eq. (8.4) the rates of change of the momenta of the two particles are

$$\vec{F}_{B \text{ on } A} = \frac{d\vec{p}_A}{dt} \qquad \vec{F}_{A \text{ on } B} = \frac{d\vec{p}_B}{dt} \tag{8.10}$$

The momentum of each particle changes, but these changes are related to each other by Newton's third law: Forces $\vec{F}_{B \text{ on } A}$ and $\vec{F}_{A \text{ on } B}$ are always equal in magnitude and opposite in direction. That is, $\vec{F}_{B \text{ on } A} = -\vec{F}_{A \text{ on } B}$, so $\vec{F}_{B \text{ on } A} + \vec{F}_{A \text{ on } B} = \mathbf{0}$. Adding together the two equations in Eq. (8.10), we have

$$\vec{F}_{B \text{ on } A} + \vec{F}_{A \text{ on } B} = \frac{d\vec{p}_A}{dt} + \frac{d\vec{p}_B}{dt} = \frac{d(\vec{p}_A + \vec{p}_B)}{dt} = \mathbf{0} \tag{8.11}$$

The rates of change of the two momenta are equal and opposite, so the rate of change of the vector sum $\vec{p}_A + \vec{p}_B$ is zero. We define the **total momentum** $\vec{P}$ of the system of two particles as the vector sum of the momenta of the individual particles; that is,

$$\vec{P} = \vec{p}_A + \vec{p}_B \tag{8.12}$$

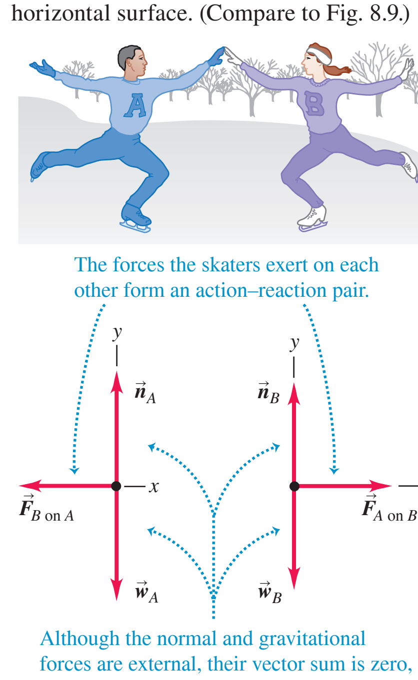
Then Eq. (8.11) becomes

$$\vec{F}_{B \text{ on } A} + \vec{F}_{A \text{ on } B} = \frac{d\vec{P}}{dt} = \mathbf{0} \tag{8.13}$$

The time rate of change of the *total* momentum $\vec{P}$ is zero. Hence the total momentum of the system is constant, even though the individual momenta of the particles that make up the system can change.

If external forces are also present, they must be included on the left side of Eq. (8.13) along with the internal forces. Then the total momentum is, in general, not constant. But if the vector sum of the external forces is zero, as in **Fig. 8.10**, these forces have no

Figure **8.10** Two ice skaters push each other as they skate on a frictionless, horizontal surface. (Compare to Fig. 8.9.)



effect on the left side of Eq. (8.13), and $d\vec{P}/dt$ is again zero. Thus we have the following general result:

CONSERVATION OF MOMENTUM **If the vector sum of the external forces on a system is zero, the total momentum of the system is constant.**

This is the simplest form of the **principle of conservation of momentum.** This principle is a direct consequence of Newton's third law. What makes this principle useful is that it doesn't depend on the detailed nature of the internal forces that act between members of the system. This means that we can apply conservation of momentum even if (as is often the case) we know very little about the internal forces. We have used Newton's second law to derive this principle, so we have to be careful to use it only in inertial frames of reference.

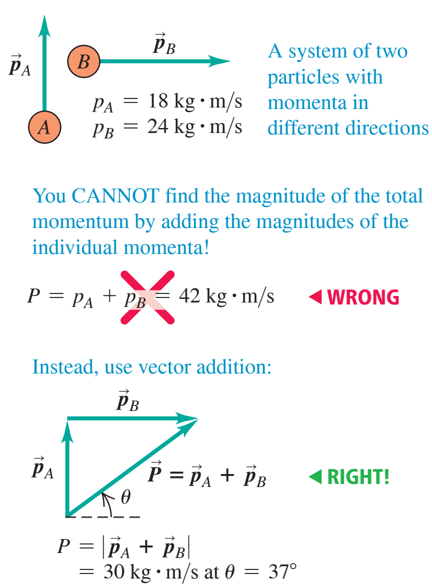
We can generalize this principle for a system that contains any number of particles A, B, C, … interacting only with one another, with total momentum

Total momentum of a system of particles $A, B, C, \ldots$

$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots = m_A\vec{v}_A + m_B\vec{v}_B + \cdots \tag{8.14}$$

... equals vector sum of momenta of all particles in the system.

Figure **8.11** When applying conservation of momentum, remember that momentum is a vector quantity!



We make the same argument as before: The total rate of change of momentum of the system due to each action–reaction pair of internal forces is zero. Thus the total rate of change of momentum of the entire system is zero whenever the vector sum of the external forces acting on it is zero. The internal forces can change the momenta of individual particles but not the *total* momentum of the system.

CAUTION **Conservation of momentum means conservation of its components** When you apply the conservation of momentum to a system, remember that momentum is a *vector* quantity. Hence you must use vector addition to compute the total momentum of a system (**Fig. 8.11**). Using components is usually the simplest method. If p_{Ax}, p_{Ay}, and p_{Az} are the components of momentum of particle A, and similarly for the other particles, then Eq. (8.14) is equivalent to the component equations

$$P_x = p_{Ax} + p_{Bx} + \cdots, \quad P_y = p_{Ay} + p_{By} + \cdots, \quad P_z = p_{Az} + p_{Bz} + \cdots \tag{8.15}$$

If the vector sum of the external forces on the system is zero, then P_x, P_y, and P_z are all constant.

In some ways the principle of conservation of momentum is more general than the principle of conservation of total mechanical energy. For example, total mechanical energy is conserved only when the internal forces are *conservative*—that is, when the forces allow two-way conversion between kinetic and potential energies. But conservation of momentum is valid even when the internal forces are *not* conservative. In this chapter we'll analyze situations in which both momentum and total mechanical energy are conserved, and others in which only momentum is conserved. These two principles play a fundamental role in all areas of physics, and we'll encounter them throughout our study of physics.

PROBLEM-SOLVING STRATEGY 8.1 Conservation of Momentum

IDENTIFY *the relevant concepts:* Confirm that the vector sum of the external forces acting on the system of particles is zero. If it isn't zero, you can't use conservation of momentum.

SET UP *the problem* using the following steps:

1. Treat each object as a particle. Draw "before" and "after" sketches, including velocity vectors. Assign algebraic symbols to each magnitude, angle, and component. Use letters to label each particle and subscripts 1 and 2 for "before" and "after" quantities. Include any given values.
2. Define a coordinate system and show it in your sketches; define the positive direction for each axis.
3. Identify the target variables.

EXECUTE *the solution:*

4. Write an equation in symbols equating the total initial and final x-components of momentum, using $p_x = mv_x$ for each particle. Write a corresponding equation for the y-components. Components can be positive or negative, so be careful with signs!
5. In some problems, energy considerations (discussed in Section 8.4) give additional equations relating the velocities.
6. Solve your equations to find the target variables.

EVALUATE *your answer:* Does your answer make physical sense? If your target variable is a certain object's momentum, check that the direction of the momentum is reasonable.

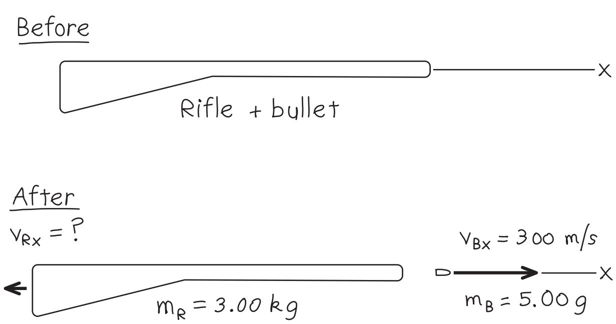
EXAMPLE 8.4 Recoil of a rifle

WITH VARIATION PROBLEMS

A marksman holds a rifle of mass $m_R = 3.00$ kg loosely, so it can recoil freely. He fires a bullet of mass $m_B = 5.00$ g horizontally with a velocity relative to the ground of $v_{Bx} = 300$ m/s. What is the recoil velocity v_{Rx} of the rifle? What are the final momentum and kinetic energy of the bullet and rifle?

IDENTIFY and SET UP If the marksman exerts negligible horizontal forces on the rifle, then there is no net horizontal force on the system (the bullet and rifle) during the firing, and the total horizontal momentum of the system is conserved. **Figure 8.12** shows our sketch. We take the positive x-axis in the direction of aim. The rifle and the bullet are initially at rest, so the initial x-component of total momentum is zero. After the shot is fired, the bullet's x-momentum is $p_{Bx} = m_B v_{Bx}$ and the rifle's x-momentum is $p_{Rx} = m_R v_{Rx}$. Our target variables are v_{Rx}, p_{Bx}, p_{Rx}, and the final kinetic energies $K_B = \frac{1}{2}m_B v_{Bx}^2$ and $K_R = \frac{1}{2}m_R v_{Rx}^2$.

Figure 8.12 Our sketch for this problem.



EXECUTE Conservation of the x-component of total momentum gives

$$P_x = 0 = m_B v_{Bx} + m_R v_{Rx}$$

$$v_{Rx} = -\frac{m_B}{m_R}v_{Bx} = -\left(\frac{0.00500\ \text{kg}}{3.00\ \text{kg}}\right)(300\ \text{m/s}) = -0.500\ \text{m/s}$$

The negative sign means that the recoil is in the direction opposite to that of the bullet.

The final momenta and kinetic energies are

$$p_{Bx} = m_B v_{Bx} = (0.00500\ \text{kg})(300\ \text{m/s}) = 1.50\ \text{kg}\cdot\text{m/s}$$

$$K_B = \tfrac{1}{2}m_B v_{Bx}^2 = \tfrac{1}{2}(0.00500\ \text{kg})(300\ \text{m/s})^2 = 225\ \text{J}$$

$$p_{Rx} = m_R v_{Rx} = (3.00\ \text{kg})(-0.500\ \text{m/s}) = -1.50\ \text{kg}\cdot\text{m/s}$$

$$K_R = \tfrac{1}{2}m_R v_{Rx}^2 = \tfrac{1}{2}(3.00\ \text{kg})(-0.500\ \text{m/s})^2 = 0.375\ \text{J}$$

EVALUATE The bullet and rifle have equal and opposite final *momenta* thanks to Newton's third law: They experience equal and opposite interaction forces that act for the same *time,* so the impulses are equal and opposite. But the bullet travels a much greater *distance* than the rifle during the interaction. Hence the force on the bullet does more work than the force on the rifle, giving the bullet much greater *kinetic energy* than the rifle. The 600:1 ratio of the two kinetic energies is the inverse of the ratio of the masses; in fact, you can show that this always happens in recoil situations. (See Exercise 8.26 for an application of these ideas to the recoil of atomic nuclei from a fission reaction.)

KEYCONCEPT The momentum of a system is conserved if no net external force acts on the system. If the momentum of one part of the system changes by $\Delta\vec{p}$, the momentum of the other parts of the system changes by $-\Delta\vec{p}$ so that the total momentum remains the same.

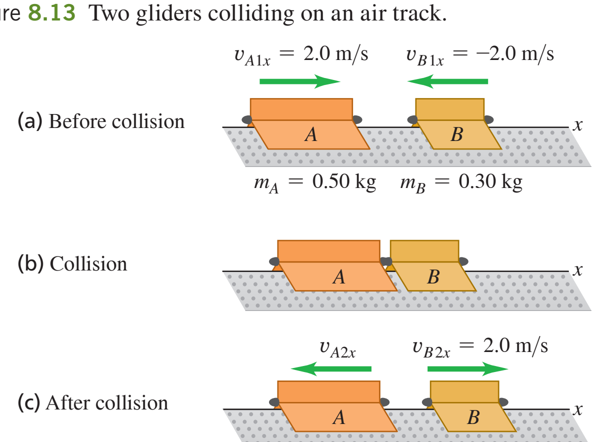
EXAMPLE 8.5 Collision along a straight line

WITH VARIATION PROBLEMS

Two gliders with different masses move toward each other on a frictionless air track (**Fig. 8.13a**). After they collide (Fig. 8.13b), glider B has a final velocity of $+2.0$ m/s (Fig. 8.13c). What is the final velocity of glider A? How do the changes in momentum and in velocity compare?

IDENTIFY and SET UP As for the skaters in Fig. 8.10, the total vertical force on each glider is zero, and the net external force on each individual glider is the horizontal force exerted on it by the other glider. The net external force on the *system* of two gliders is zero, so their total momentum is conserved. We take the positive x-axis to be to the right. We are given the masses and initial velocities of both gliders and the final velocity of glider B. Our target variables are v_{A2x} (the final x-component of velocity of glider A) and the changes in momentum and in velocity of the two gliders (the value *after* the collision minus the value *before* the collision).

Figure 8.13 Two gliders colliding on an air track.



EXECUTE The x-component of total momentum before the collision is

$$P_x = m_A v_{A1x} + m_B v_{B1x}$$
$$= (0.50\ \text{kg})(2.0\ \text{m/s}) + (0.30\ \text{kg})(-2.0\ \text{m/s}) = 0.40\ \text{kg}\cdot\text{m/s}$$

This is positive (to the right in Fig. 8.13) because A has a greater magnitude of momentum than B. The x-component of total momentum has the same value after the collision, so

$$P_x = m_A v_{A2x} + m_B v_{B2x}$$

We solve for v_{A2x}:

$$v_{A2x} = \frac{P_x - m_B v_{B2x}}{m_A} = \frac{0.40\ \text{kg}\cdot\text{m/s} - (0.30\ \text{kg})(2.0\ \text{m/s})}{0.50\ \text{kg}}$$
$$= -0.40\ \text{m/s}$$

Continued

The changes in the x-momenta are

$$m_A v_{A2x} - m_A v_{A1x} = (0.50\text{ kg})(-0.40\text{ m/s}) - (0.50\text{ kg})(2.0\text{ m/s}) = -1.2\text{ kg}\cdot\text{m/s}$$

$$m_B v_{B2x} - m_B v_{B1x} = (0.30\text{ kg})(2.0\text{ m/s}) - (0.30\text{ kg})(-2.0\text{ m/s}) = +1.2\text{ kg}\cdot\text{m/s}$$

The changes in x-velocities are

$$v_{A2x} - v_{A1x} = (-0.40\text{ m/s}) - 2.0\text{ m/s} = -2.4\text{ m/s}$$
$$v_{B2x} - v_{B1x} = 2.0\text{ m/s} - (-2.0\text{ m/s}) = +4.0\text{ m/s}$$

EVALUATE The gliders were subjected to equal and opposite interaction forces for the same time during their collision. By the impulse–momentum theorem, they experienced equal and opposite impulses and therefore equal and opposite changes in momentum. But by Newton's second law, the less massive glider (B) had a greater magnitude of acceleration and hence a greater velocity change.

KEYCONCEPT In any collision, momentum is conserved: The total momentum of the colliding objects has the same value just after the collision as just before the collision.

EXAMPLE 8.6 Collision in a horizontal plane

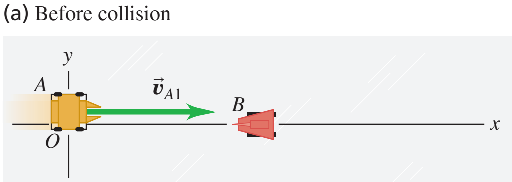
WITH VARIATION PROBLEMS

Figure 8.14a shows two battling robots on a frictionless surface. Robot A, with mass 20 kg, initially moves at 2.0 m/s parallel to the x-axis. It collides with robot B, which has mass 12 kg and is initially at rest. After the collision, robot A moves at 1.0 m/s in a direction that makes an angle $\alpha = 30°$ with its initial direction (Figure 8.14b). What is the final velocity of robot B?

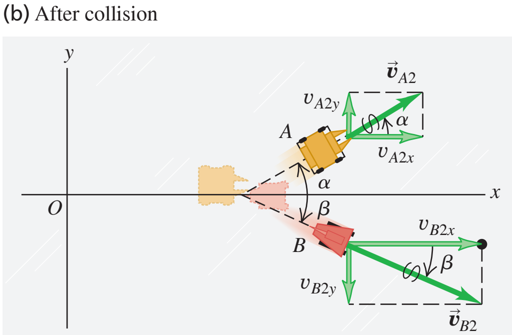
IDENTIFY and SET UP There are no horizontal external forces, so the x- and y-components of the total momentum of the system are conserved. Hence the sum of the x-components of momentum *before* the collision (subscript 1) must equal the sum *after* the collision (subscript 2), and similarly for the sums of the y-components. Our target variable is $\vec{v}_{B2}$, the final velocity of robot B.

Figure 8.14 Views from above of the robot velocities.

(a) Before collision



(b) After collision



EXECUTE The momentum-conservation equations and their solutions for v_{B2x} and v_{B2y} are

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$v_{B2x} = \frac{m_A v_{A1x} + m_B v_{B1x} - m_A v_{A2x}}{m_B}$$

$$= \frac{\begin{bmatrix}(20\text{ kg})(2.0\text{ m/s}) + (12\text{ kg})(0) \\ -(20\text{ kg})(1.0\text{ m/s})(\cos 30°)\end{bmatrix}}{12\text{ kg}} = 1.89\text{ m/s}$$

$$m_A v_{A1y} + m_B v_{B1y} = m_A v_{A2y} + m_B v_{B2y}$$

$$v_{B2y} = \frac{m_A v_{A1y} + m_B v_{B1y} - m_A v_{A2y}}{m_B}$$

$$= \frac{\begin{bmatrix}(20\text{ kg})(0) + (12\text{ kg})(0) \\ -(20\text{ kg})(1.0\text{ m/s})(\sin 30°)\end{bmatrix}}{12\text{ kg}} = -0.83\text{ m/s}$$

Figure 8.14b shows the motion of robot B after the collision. The magnitude of $\vec{v}_{B2}$ is

$$v_{B2} = \sqrt{(1.89\text{ m/s})^2 + (-0.83\text{ m/s})^2} = 2.1\text{ m/s}$$

and the angle of its direction from the positive x-axis is

$$\beta = \arctan\frac{-0.83\text{ m/s}}{1.89\text{ m/s}} = -24°$$

EVALUATE Let's confirm that the components of total momentum before and after the collision are equal. Initially robot A has x-momentum $m_A v_{A1x} = (20\text{ kg})(2.0\text{ m/s}) = 40\text{ kg}\cdot\text{m/s}$ and zero y-momentum; robot B has zero momentum. Afterward, the momentum components are $m_A v_{A2x} = (20\text{ kg})(1.0\text{ m/s})(\cos 30°) = 17\text{ kg}\cdot\text{m/s}$ and $m_B v_{B2x} = (12\text{ kg})(1.89\text{ m/s}) = 23\text{ kg}\cdot\text{m/s}$; the total x-momentum is $40\text{ kg}\cdot\text{m/s}$, the same as before the collision. The final y-components are $m_A v_{A2y} = (20\text{ kg})(1.0\text{ m/s})(\sin 30°) = 10\text{ kg}\cdot\text{m/s}$ and $m_B v_{B2y} = (12\text{ kg})(-0.83\text{ m/s}) = -10\text{ kg}\cdot\text{m/s}$; the total y-component of momentum is zero, as before the collision.

KEYCONCEPT In problems that involve a two-dimensional collision, write separate conservation equations for the x-component and y-component of total momentum.

TEST YOUR UNDERSTANDING OF SECTION 8.2 A spring-loaded toy sits at rest on a horizontal, frictionless surface. When the spring releases, the toy breaks into equal-mass pieces A, B, and C, which slide along the surface. Piece A moves off in the negative x-direction, while piece B moves off in the negative y-direction. (a) What are the signs of the velocity components of piece C? (b) Which of the three pieces is moving the fastest?

ANSWER

(a) $v_{C2x} > 0$, $v_{C2y} > 0$, (b) **piece C** There are no external horizontal forces, so the x- and y-components of the total momentum of the system are conserved. Both components of the total momentum are zero before the spring releases, so they must be zero after the spring releases. Hence,

$$P_x = 0 = m_A v_{A2x} + m_B v_{B2x} + m_C v_{C2x}$$
$$P_y = 0 = m_A v_{A2y} + m_B v_{B2y} + m_C v_{C2y}$$

We are given that $m_A = m_B = m_C$, $v_{A2x} < 0$, $v_{A2y} = 0$, $v_{B2x} = 0$, and $v_{B2y} < 0$. You can solve the above equations to show that $v_{C2x} = -v_{A2x} > 0$ and $v_{C2y} = -v_{B2y} > 0$, so both velocity components of piece C are positive. Piece C has speed $\sqrt{v_{C2x}^2 + v_{C2y}^2} = \sqrt{v_{A2x}^2 + v_{B2y}^2}$, which is greater than the speed of either piece A or piece B.

8.3 MOMENTUM CONSERVATION AND COLLISIONS

To most people the term "collision" is likely to mean some sort of automotive disaster. We'll broaden the meaning to include any strong interaction between objects that lasts a relatively short time. So we include not only car accidents but also balls colliding on a billiard table, neutrons hitting atomic nuclei in a nuclear reactor, and a close encounter of a spacecraft with the planet Saturn.

If the forces between the colliding objects are much larger than any external forces, as is the case in most collisions, we can ignore the external forces and treat the objects as an *isolated* system. Then momentum is conserved and the total momentum of the system has the same value before and after the collision. Two cars colliding at an icy intersection provide a good example. Even two cars colliding on dry pavement can be treated as an isolated system during the collision if the forces between the cars are much larger than the friction forces of pavement against tires.

Elastic and Inelastic Collisions

If the forces between the objects are also *conservative,* so no mechanical energy is lost or gained in the collision, the total *kinetic* energy of the system is the same after the collision as before. Such a collision is called an **elastic collision.** A collision between two marbles or two billiard balls is almost completely elastic. **Figure 8.15** shows a model for an elastic collision. When the gliders collide, their springs are momentarily compressed and some of the original kinetic energy is momentarily converted to elastic potential energy. Then the gliders bounce apart, the springs expand, and this potential energy is converted back to kinetic energy.

A collision in which the total kinetic energy after the collision is *less* than before the collision is called an **inelastic collision.** A meatball landing on a plate of spaghetti and a bullet embedding itself in a block of wood are examples of inelastic collisions. An inelastic collision in which the colliding objects stick together and move as one object after the collision is called a **completely inelastic collision. Figure 8.16** shows an example; we have replaced the spring bumpers in Fig. 8.15 with Velcro®, which sticks the two objects together.

Figure 8.15 Two gliders undergoing an elastic collision on a frictionless surface. Each glider has a steel spring bumper that exerts a conservative force on the other glider.

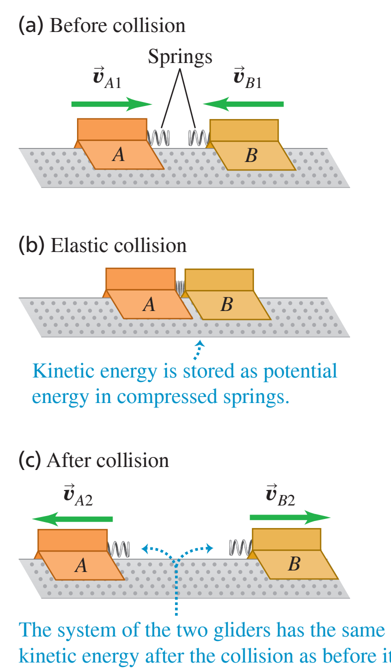
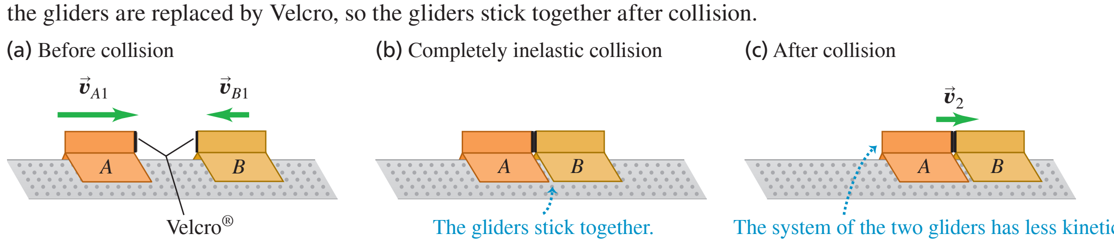


Figure 8.16 Two gliders undergoing a completely inelastic collision. The spring bumpers on the gliders are replaced by Velcro, so the gliders stick together after collision.



CAUTION An inelastic collision doesn't have to be completely inelastic Inelastic collisions include many situations in which the objects do *not* stick. If two cars bounce off each other in a "fender bender," the work done to deform the fenders cannot be recovered as kinetic energy of the cars, so the collision is inelastic (**Fig. 8.17**).

Figure 8.17 Cars are designed so that collisions are inelastic—the structure of the car absorbs as much of the energy of the collision as possible. This absorbed energy cannot be recovered, since it goes into a permanent deformation of the car.

Remember this rule: **In any collision in which external forces can be ignored, momentum is conserved and the total momentum before equals the total momentum after; in elastic collisions *only*, the total kinetic energy before equals the total kinetic energy after.**

Completely Inelastic Collisions

Let's look at what happens to momentum and kinetic energy in a *completely* inelastic collision of two objects (A and B), as in Fig. 8.16. Because the two objects stick together after the collision, they have the same final velocity $\vec{v}_2$:

$$\vec{v}_{A2} = \vec{v}_{B2} = \vec{v}_2$$

Conservation of momentum gives the relationship

$$m_A\vec{v}_{A1} + m_B\vec{v}_{B1} = (m_A + m_B)\vec{v}_2 \quad \text{(completely inelastic collision)} \tag{8.16}$$

If we know the masses and initial velocities, we can compute the common final velocity $\vec{v}_2$.

Suppose, for example, that an object with mass m_A and initial x-component of velocity v_{A1x} collides inelastically with an object with mass m_B that is initially at rest ($v_{B1x} = 0$). From Eq. (8.16) the common x-component of velocity v_{2x} of both objects after the collision is

$$v_{2x} = \frac{m_A}{m_A + m_B}v_{A1x} \quad \begin{array}{l}\text{(completely inelastic collision,}\\ B \text{ initially at rest)}\end{array} \tag{8.17}$$

Let's verify that the total kinetic energy after this completely inelastic collision is less than before the collision. The motion is purely along the x-axis, so the kinetic energies K_1 and K_2 before and after the collision, respectively, are

$$K_1 = \tfrac{1}{2}m_A v_{A1x}^2$$

$$K_2 = \tfrac{1}{2}(m_A + m_B)v_{2x}^2 = \tfrac{1}{2}(m_A + m_B)\left(\frac{m_A}{m_A + m_B}\right)^2 v_{A1x}^2$$

The ratio of final to initial kinetic energy is

$$\frac{K_2}{K_1} = \frac{m_A}{m_A + m_B} \quad \begin{array}{l}\text{(completely inelastic collision,}\\ B \text{ initially at rest)}\end{array} \tag{8.18}$$

The right side is always less than unity because the denominator is always greater than the numerator. Even when the initial velocity of m_B is not zero, the kinetic energy after a completely inelastic collision is always less than before.

Please note: Don't memorize Eq. (8.17) or (8.18)! We derived them only to prove that kinetic energy is always lost in a completely inelastic collision.

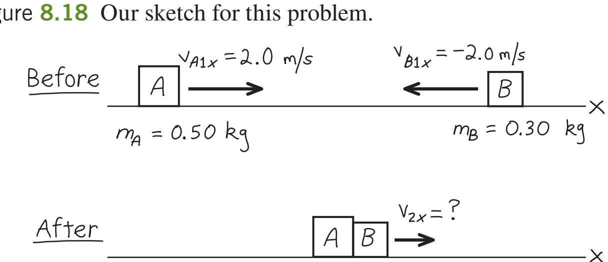
EXAMPLE 8.7 A completely inelastic collision

WITH VARIATION PROBLEMS

We repeat the collision described in Example 8.5 (Section 8.2), but this time equip the gliders so that they stick together when they collide. Find the common final x-velocity, and compare the initial and final kinetic energies of the system.

IDENTIFY and SET UP There are no external forces in the x-direction, so the x-component of momentum is conserved. **Figure 8.18** shows our sketch. Our target variables are the final x-velocity, v_{2x}, and the initial and final kinetic energies, K_1 and K_2.

Figure 8.18 Our sketch for this problem.



EXECUTE From conservation of momentum,

$$m_A v_{A1x} + m_B v_{B1x} = (m_A + m_B)v_{2x}$$

$$v_{2x} = \frac{m_A v_{A1x} + m_B v_{B1x}}{m_A + m_B}$$

$$= \frac{(0.50\ \text{kg})(2.0\ \text{m/s}) + (0.30\ \text{kg})(-2.0\ \text{m/s})}{0.50\ \text{kg} + 0.30\ \text{kg}}$$

$$= 0.50\ \text{m/s}$$

Because v_{2x} is positive, the gliders move together to the right after the collision. Before the collision, the kinetic energies are

$$K_A = \tfrac{1}{2}m_A v_{A1x}{}^2 = \tfrac{1}{2}(0.50\ \text{kg})(2.0\ \text{m/s})^2 = 1.0\ \text{J}$$

$$K_B = \tfrac{1}{2}m_B v_{B1x}{}^2 = \tfrac{1}{2}(0.30\ \text{kg})(-2.0\ \text{m/s})^2 = 0.60\ \text{J}$$

The total kinetic energy before the collision is $K_1 = K_A + K_B = 1.6\ \text{J}$. The kinetic energy after the collision is

$$K_2 = \tfrac{1}{2}(m_A + m_B)v_{2x}{}^2 = \tfrac{1}{2}(0.50\ \text{kg} + 0.30\ \text{kg})(0.50\ \text{m/s})^2$$
$$= 0.10\ \text{J}$$

EVALUATE The final kinetic energy is only $\frac{1}{16}$ of the original; $\frac{15}{16}$ of the total mechanical energy is converted to other forms. If there is a wad of chewing gum between the gliders, it squashes and becomes warmer. If there is a spring between the gliders that is compressed as they lock together, the energy is stored as potential energy of the spring. In both cases the *total* energy of the system is conserved, although *kinetic* energy is not. In an isolated system, however, momentum is *always* conserved whether the collision is elastic or not.

KEYCONCEPT In a completely inelastic collision, the colliding objects come together and stick. Momentum is conserved, but kinetic energy is not.

EXAMPLE 8.8 The ballistic pendulum

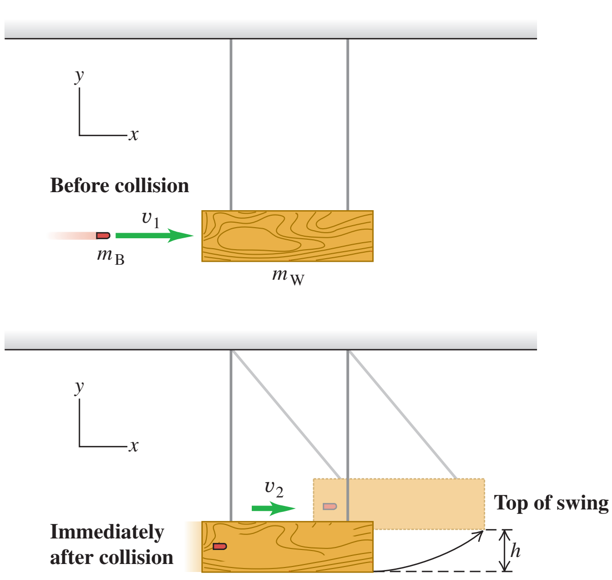
WITH VARIATION PROBLEMS

Figure 8.19 shows a ballistic pendulum, a simple system for measuring the speed of a bullet. A bullet of mass m_B makes a completely inelastic collision with a block of wood of mass m_W, which is suspended like a pendulum. After the impact, the block swings up to a maximum height h. In terms of h, m_B, and m_W, what is the initial speed v_1 of the bullet?

IDENTIFY We'll analyze this event in two stages: (1) the bullet embeds itself in the block, and (2) the block swings upward. The first stage happens so quickly that the block does not move appreciably. The supporting strings remain nearly vertical, so negligible external horizontal force acts on the bullet–block system, and the horizontal component of momentum is conserved. Total mechanical energy is *not* conserved during this stage, however, because a nonconservative force does work (the force of friction between bullet and block).

In the second stage, the block and bullet move together. The only forces acting on this system are gravity (a conservative force) and the string tensions (which do no work). Thus, as the block swings, *total mechanical energy* is conserved. Momentum is *not* conserved during this stage, however, because there is a net external force (the forces of gravity and string tension don't cancel when the strings are inclined).

Figure 8.19 A ballistic pendulum.



SET UP We take the positive x-axis to the right and the positive y-axis upward. Our target variable is v_1. Another unknown quantity is the speed v_2 of the system just after the collision. We'll use momentum conservation in the first stage to relate v_1 to v_2, and we'll use energy conservation in the second stage to relate v_2 to h.

EXECUTE In the first stage, all velocities are in the $+x$-direction. Momentum conservation gives

$$m_B v_1 = (m_B + m_W)v_2$$

$$v_1 = \frac{m_B + m_W}{m_B}v_2$$

At the beginning of the second stage, the system has kinetic energy $K = \frac{1}{2}(m_B + m_W)v_2{}^2$. The system swings up and comes to rest for an instant at a height h, where its kinetic energy is zero and the potential energy is $(m_B + m_W)gh$; it then swings back down. Energy conservation gives

$$\tfrac{1}{2}(m_B + m_W)v_2{}^2 = (m_B + m_W)gh$$

$$v_2 = \sqrt{2gh}$$

We substitute this expression for v_2 into the momentum equation:

$$v_1 = \frac{m_B + m_W}{m_B}\sqrt{2gh}$$

EVALUATE Let's plug in realistic numbers: $m_B = 5.00\ \text{g} = 0.00500\ \text{kg}$, $m_W = 2.00\ \text{kg}$, and $h = 3.00\ \text{cm} = 0.0300\ \text{m}$:

$$v_1 = \frac{0.00500\ \text{kg} + 2.00\ \text{kg}}{0.00500\ \text{kg}}\sqrt{2(9.80\ \text{m/s}^2)(0.0300\ \text{m})} = 307\ \text{m/s}$$

$$v_2 = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(0.0300\ \text{m})} = 0.767\ \text{m/s}$$

The speed v_2 of the block after impact is *much* lower than the initial speed v_1 of the bullet. The kinetic energy of the bullet before impact is $\frac{1}{2}(0.00500\ \text{kg})(307\ \text{m/s})^2 = 236\ \text{J}$. Just after impact the kinetic energy of the system is $\frac{1}{2}(2.005\ \text{kg})(0.767\ \text{m/s})^2 = 0.589\ \text{J}$. Nearly all the kinetic energy disappears as the wood splinters and the bullet and block become warmer.

KEYCONCEPT Conservation of momentum holds true only when the net external force is zero. In some situations momentum is conserved during part of the motion (such as during a collision) but not during other parts.

EXAMPLE 8.9 An automobile collision

WITH VARIATION PROBLEMS

A 1000 kg car traveling north at 15 m/s collides with a 2000 kg truck traveling east at 10 m/s. The occupants, wearing seat belts, are uninjured, but the two vehicles move away from the impact point as one. The insurance adjustor asks you to find the velocity of the wreckage just after impact. What is your answer?

IDENTIFY and SET UP Any horizontal external forces (such as friction) on the vehicles during the collision are very small compared with the forces that the colliding vehicles exert on each other. (We'll verify this below.) So we can treat the cars as an isolated system, and the momentum of the system is conserved. **Figure 8.20** shows our sketch and the x- and y-axes. We can use Eqs. (8.15) to find the total momentum $\vec{P}$ before the collision. The momentum has the same value just after the collision; hence we can find the velocity $\vec{V}$ just after the collision (our target variable) by using $\vec{P} = M\vec{V}$, where $M = m_C + m_T = 3000$ kg is the mass of the wreckage.

EXECUTE From Eqs. (8.15), the components of $\vec{P}$ are

$$P_x = p_{Cx} + p_{Tx} = m_C v_{Cx} + m_T v_{Tx}$$
$$= (1000\ \text{kg})(0) + (2000\ \text{kg})(10\ \text{m/s}) = 2.0 \times 10^4\ \text{kg}\cdot\text{m/s}$$
$$P_y = p_{Cy} + p_{Ty} = m_C v_{Cy} + m_T v_{Ty}$$
$$= (1000\ \text{kg})(15\ \text{m/s}) + (2000\ \text{kg})(0) = 1.5 \times 10^4\ \text{kg}\cdot\text{m/s}$$

The magnitude of $\vec{P}$ is

$$P = \sqrt{(2.0 \times 10^4\ \text{kg}\cdot\text{m/s})^2 + (1.5 \times 10^4\ \text{kg}\cdot\text{m/s})^2}$$
$$= 2.5 \times 10^4\ \text{kg}\cdot\text{m/s}$$

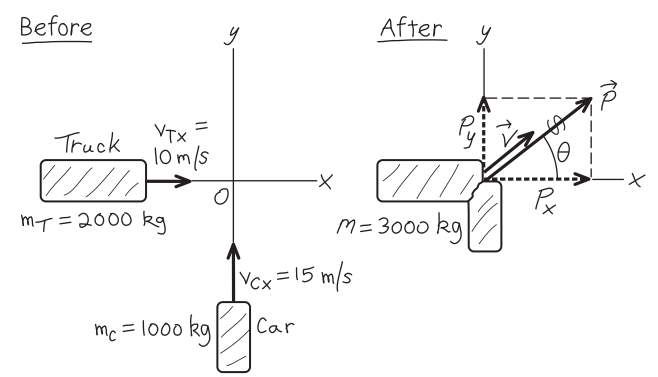
and its direction is given by the angle θ shown in Fig. 8.20:

$$\tan\theta = \frac{P_y}{P_x} = \frac{1.5 \times 10^4\ \text{kg}\cdot\text{m/s}}{2.0 \times 10^4\ \text{kg}\cdot\text{m/s}} = 0.75 \qquad \theta = 37^\circ$$

From $\vec{P} = M\vec{V}$, the direction of the velocity $\vec{V}$ just after the collision is also $\theta = 37^\circ$. The velocity magnitude is

$$V = \frac{P}{M} = \frac{2.5 \times 10^4\ \text{kg}\cdot\text{m/s}}{3000\ \text{kg}} = 8.3\ \text{m/s}$$

Figure **8.20** Our sketch for this problem.



EVALUATE As you can show, the initial kinetic energy is 2.1×10^5 J and the final value is 1.0×10^5 J. In this inelastic collision, the total kinetic energy is less after the collision than before.

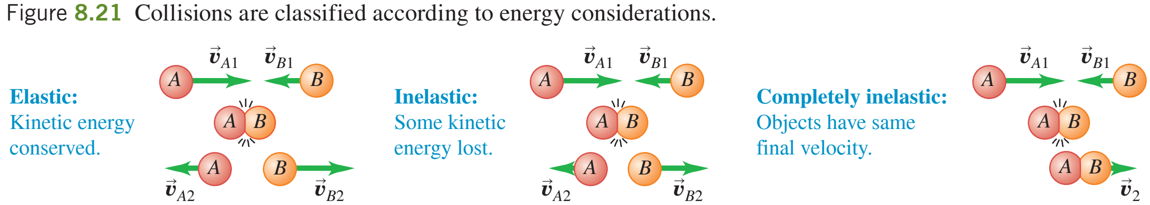
We can now justify our neglect of the external forces on the vehicles during the collision. The car's weight is about 10,000 N; if the coefficient of kinetic friction is 0.5, the friction force on the car during the impact is about 5000 N. The car's initial kinetic energy is $\frac{1}{2}(1000\ \text{kg})(15\ \text{m/s})^2 = 1.1 \times 10^5$ J, so -1.1×10^5 J of work must be done to stop it. If the car crumples by 0.20 m in stopping, a force of magnitude $(1.1 \times 10^5\ \text{J})/(0.20\ \text{m}) = 5.5 \times 10^5$ N would be needed; that's 110 times the friction force. So it's reasonable to treat the external force of friction as negligible compared with the internal forces the vehicles exert on each other.

KEYCONCEPT You can use conservation of momentum in collision problems even though external forces act on the system. That's because the external forces are typically small compared to the internal forces that the colliding objects exert on each other.

Classifying Collisions

It's important to remember that we can classify collisions according to energy considerations (**Fig. 8.21**). A collision in which kinetic energy is conserved is called *elastic*. (We'll explore this type in more depth in the next section.) A collision in which the total kinetic energy decreases is called *inelastic*. When the two objects have a common final velocity, we say that the collision is *completely inelastic*. There are also cases in which the final kinetic energy is *greater* than the initial value. Rifle recoil, discussed in Example 8.4 (Section 8.2), is an example.

Figure **8.21** Collisions are classified according to energy considerations.



Finally, we emphasize again that we can typically use momentum conservation for collisions even when external forces are acting on the system. That's because the net external force acting on the colliding objects is typically small in comparison with the internal forces during the collision (as in Example 8.9).

TEST YOUR UNDERSTANDING OF SECTION 8.3 For each situation, state whether the collision is elastic or inelastic. If it is inelastic, state whether it is completely inelastic. (a) You drop a ball from your hand. It collides with the floor and bounces back up so that it just reaches your hand. (b) You drop a different ball from your hand and let it collide with the ground. This ball bounces back up to half the height from which it was dropped. (c) You drop a ball of clay from your hand. When it collides with the ground, it stops.

ANSWER

(a) **elastic**, (b) **inelastic**, (c) **completely inelastic** In each case gravitational potential energy is converted to kinetic energy as the ball falls, and the collision is between the ball and the ground. In (a) all of the initial energy is converted back to gravitational potential energy, so no kinetic energy is lost in the bounce and the collision is elastic. In (b) there is less gravitational potential energy at the end than at the beginning, so some kinetic energy is lost in the bounce. Hence the collision is inelastic. In (c) the ball loses all of its kinetic energy, the ball and the ground stick together, and the collision is completely inelastic.

8.4 ELASTIC COLLISIONS

We saw in Section 8.3 that an *elastic collision* in an isolated system is one in which kinetic energy (as well as momentum) is conserved. Elastic collisions occur when the forces between the colliding objects are *conservative*. When two billiard balls collide, they squash a little near the surface of contact, but then they spring back. Some of the kinetic energy is stored temporarily as elastic potential energy, but at the end it is reconverted to kinetic energy (**Fig. 8.22**).

Figure 8.22 Billiard balls deform very little when they collide, and they quickly spring back from any deformation they do undergo. Hence the force of interaction between the balls is almost perfectly conservative, and the collision is almost perfectly elastic.

Let's look at a *one-dimensional* elastic collision between two objects A and B, in which all the velocities lie along the same line. We call this line the x-axis, so each momentum and velocity has only an x-component. We call the x-velocities before the collision v_{A1x} and v_{B1x}, and those after the collision v_{A2x} and v_{B2x}. From conservation of kinetic energy we have

$$\tfrac{1}{2}m_A v_{A1x}{}^2 + \tfrac{1}{2}m_B v_{B1x}{}^2 = \tfrac{1}{2}m_A v_{A2x}{}^2 + \tfrac{1}{2}m_B v_{B2x}{}^2$$

and conservation of momentum gives

$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$

If the masses m_A and m_B and the initial velocities v_{A1x} and v_{B1x} are known, we can solve these two equations to find the two final velocities v_{A2x} and v_{B2x}.

Elastic Collisions, One Object Initially at Rest

The general solution to the above equations is a little complicated, so we'll concentrate on the particular case in which object B is at rest before the collision (so $v_{B1x} = 0$). Think of object B as a target for object A to hit. Then the kinetic energy and momentum conservation equations are, respectively,

$$\tfrac{1}{2}m_A v_{A1x}{}^2 = \tfrac{1}{2}m_A v_{A2x}{}^2 + \tfrac{1}{2}m_B v_{B2x}{}^2 \qquad (8.19)$$

$$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x} \qquad (8.20)$$

We can solve for v_{A2x} and v_{B2x} in terms of the masses and the initial velocity v_{A1x}. This involves some fairly strenuous algebra, but it's worth it. No pain, no gain! The simplest approach is somewhat indirect, but along the way it uncovers an additional interesting feature of elastic collisions.

Figure **8.23** One-dimensional elastic collisions between objects with different masses.

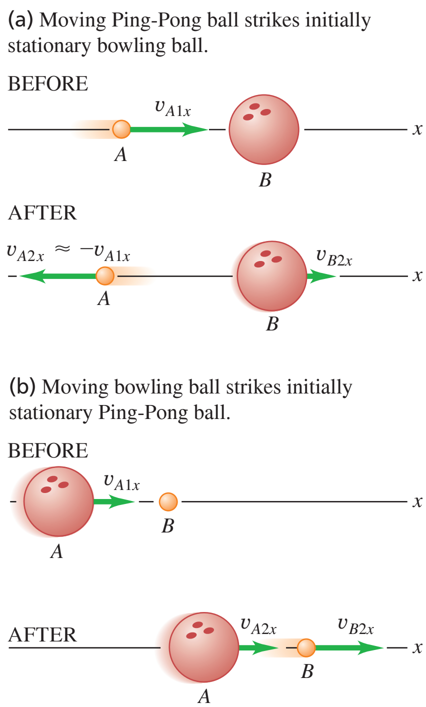
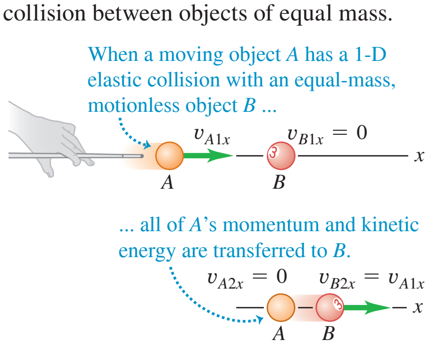


Figure **8.24** A one-dimensional elastic collision between objects of equal mass.



First we rearrange Eqs. (8.19) and (8.20) as follows:

$$m_B v_{B2x}{}^2 = m_A(v_{A1x}{}^2 - v_{A2x}{}^2) = m_A(v_{A1x} - v_{A2x})(v_{A1x} + v_{A2x}) \tag{8.21}$$

$$m_B v_{B2x} = m_A(v_{A1x} - v_{A2x}) \tag{8.22}$$

Now we divide Eq. (8.21) by Eq. (8.22) to obtain

$$v_{B2x} = v_{A1x} + v_{A2x} \tag{8.23}$$

We substitute this expression back into Eq. (8.22) to eliminate v_{B2x} and then solve for v_{A2x}:

$$m_B(v_{A1x} + v_{A2x}) = m_A(v_{A1x} - v_{A2x})$$

$$v_{A2x} = \frac{m_A - m_B}{m_A + m_B} v_{A1x} \tag{8.24}$$

Finally, we substitute this result back into Eq. (8.23) to obtain

$$v_{B2x} = \frac{2m_A}{m_A + m_B} v_{A1x} \tag{8.25}$$

Now we can interpret the results. Suppose A is a Ping-Pong ball and B is a bowling ball. Then we expect A to bounce off after the collision with a velocity nearly equal to its original value but in the opposite direction (**Fig. 8.23a**), and we expect B's velocity to be much less. That's just what the equations predict. When m_A is much smaller than m_B, the fraction in Eq. (8.24) is approximately equal to (-1), so v_{A2x} is approximately equal to $-v_{A1x}$. The fraction in Eq. (8.25) is much smaller than unity, so v_{B2x} is much less than v_{A1x}. Figure 8.23b shows the opposite case, in which A is the bowling ball and B the Ping-Pong ball and m_A is much larger than m_B. What do you expect to happen then? Check your predictions against Eqs. (8.24) and (8.25).

Another interesting case occurs when the masses are equal (**Fig. 8.24**). If $m_A = m_B$, then Eqs. (8.24) and (8.25) give $v_{A2x} = 0$ and $v_{B2x} = v_{A1x}$. That is, the object that was moving stops dead; it gives all its momentum and kinetic energy to the object that was at rest. This behavior is familiar to all pool players.

Elastic Collisions and Relative Velocity

Let's return to the more general case in which A and B have different masses. Equation (8.23) can be rewritten as

$$v_{A1x} = v_{B2x} - v_{A2x} \tag{8.26}$$

Here $v_{B2x} - v_{A2x}$ is the velocity of B relative to A *after* the collision; from Eq. (8.26), this equals v_{A1x}, which is the *negative* of the velocity of B relative to A *before* the collision. (We discussed relative velocity in Section 3.5.) The relative velocity has the same magnitude, but opposite sign, before and after the collision. The sign changes because A and B are approaching each other before the collision but moving apart after the collision. If we view this collision from a second coordinate system moving with constant velocity relative to the first, the velocities of the objects are different but the *relative* velocities are the same. Hence our statement about relative velocities holds for *any* straight-line elastic collision, even when neither object is at rest initially. *In a straight-line elastic collision of two objects, the relative velocities before and after the collision have the same magnitude but opposite sign.* This means that if B is moving before the collision, Eq. (8.26) becomes

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x}) \tag{8.27}$$

It turns out that a *vector* relationship similar to Eq. (8.27) is a general property of *all* elastic collisions, even when both objects are moving initially and the velocities do not all lie along the same line. This result provides an alternative and equivalent definition of an elastic collision: *In an elastic collision, the relative velocity of the two objects has the same magnitude before and after the collision.* Whenever this condition is satisfied, the total kinetic energy is also conserved.

When an elastic two-object collision isn't head-on, the velocities don't all lie along a single line. If they all lie in a plane, then each final velocity has two unknown components, and there are four unknowns in all. Conservation of energy and conservation of the x- and y-components of momentum give only three equations. To determine the final velocities uniquely, we need additional information, such as the direction or magnitude of one of the final velocities.

EXAMPLE 8.10 An elastic straight-line collision

We repeat the air-track collision of Example 8.5 (Section 8.2), but now we add ideal spring bumpers to the gliders so that the collision is elastic. What are the final velocities of the gliders?

IDENTIFY and SET UP The net external force on the system is zero, so the momentum of the system is conserved. **Figure 8.25** shows our sketch. We'll find our target variables, v_{A2x} and v_{B2x}, by using Eq. (8.27), the relative-velocity relationship for an elastic collision, and the momentum-conservation equation.

EXECUTE From Eq. (8.27),

$$v_{B2x} - v_{A2x} = -(v_{B1x} - v_{A1x})$$
$$= -(-2.0\text{ m/s} - 2.0\text{ m/s}) = 4.0\text{ m/s}$$

From conservation of momentum,

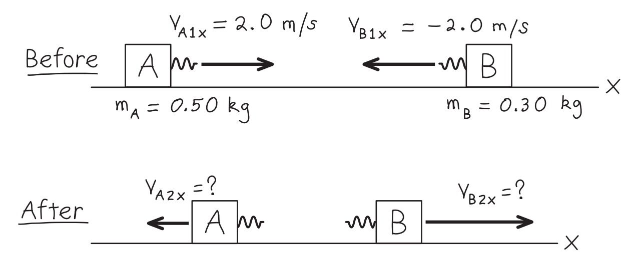
$$m_A v_{A1x} + m_B v_{B1x} = m_A v_{A2x} + m_B v_{B2x}$$
$$(0.50\text{ kg})(2.0\text{ m/s}) + (0.30\text{ kg})(-2.0\text{ m/s})$$
$$= (0.50\text{ kg})v_{A2x} + (0.30\text{ kg})v_{B2x}$$
$$0.50\,v_{A2x} + 0.30\,v_{B2x} = 0.40\text{ m/s}$$

(To get the last equation we divided both sides of the equation just above it by 1 kg. This makes the units the same as in the first equation.) Solving these equations simultaneously, we find

$$v_{A2x} = -1.0\text{ m/s} \qquad v_{B2x} = 3.0\text{ m/s}$$

EVALUATE Both objects reverse their direction of motion; A moves to the left at 1.0 m/s and B moves to the right at 3.0 m/s. This is unlike the result of Example 8.5 because that collision was *not* elastic. The more massive glider A slows down in the collision and so loses kinetic energy.

Figure 8.25 Our sketch for this problem.



The less massive glider B speeds up and gains kinetic energy. The total kinetic energy before the collision (which we calculated in Example 8.7) is 1.6 J. The total kinetic energy after the collision is

$$\tfrac{1}{2}(0.50\text{ kg})(-1.0\text{ m/s})^2 + \tfrac{1}{2}(0.30\text{ kg})(3.0\text{ m/s})^2 = 1.6\text{ J}$$

The kinetic energies before and after this elastic collision are equal. Kinetic energy is transferred from A to B, but none of it is lost.

CAUTION Be careful with the elastic collision equations You could *not* have solved this problem by using Eqs. (8.24) and (8.25), which apply only if object B is initially *at rest*. Always be sure that you use equations that are applicable!

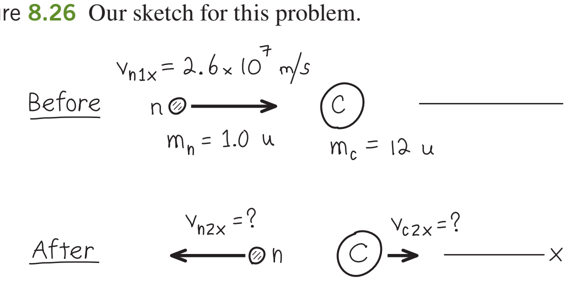
KEYCONCEPT In an elastic collision both total momentum and total kinetic energy are conserved. The relative velocity of the two colliding objects has the same magnitude after the collision as before, but in the opposite direction.

EXAMPLE 8.11 Moderating fission neutrons in a nuclear reactor

The fission of uranium nuclei in a nuclear reactor produces high-speed neutrons. Before such neutrons can efficiently cause additional fissions, they must be slowed down by collisions with nuclei in the *moderator* of the reactor. The first nuclear reactor (built in 1942 at the University of Chicago) used carbon (graphite) as the moderator. Suppose a neutron (mass 1.0 u) traveling at 2.6×10^7 m/s undergoes a head-on elastic collision with a carbon nucleus (mass 12 u) initially at rest. Neglecting external forces during the collision, find the velocities after the collision. (1 u is the *atomic mass unit,* equal to 1.66×10^{-27} kg.)

IDENTIFY and SET UP We ignore external forces, so momentum is conserved in the collision. The collision is elastic, so kinetic energy is also conserved. **Figure 8.26** shows our sketch. We take the x-axis to be in the direction in which the neutron is moving initially. The collision is head-on, so both particles move along this same axis after the collision. The carbon nucleus is initially at rest, so we can use Eqs. (8.24) and (8.25); we replace A by n (for the neutron) and B by C (for the carbon nucleus). We have $m_n = 1.0$ u, $m_C = 12$ u, and $v_{n1x} = 2.6 \times 10^7$ m/s. The target variables are the final velocities v_{n2x} and v_{C2x}.

Figure 8.26 Our sketch for this problem.



EXECUTE You can do the arithmetic. (*Hint:* There's no reason to convert atomic mass units to kilograms.) The results are

$$v_{n2x} = -2.2 \times 10^7\text{ m/s} \qquad v_{C2x} = 0.4 \times 10^7\text{ m/s}$$

Continued

EVALUATE The neutron ends up with $|(m_n - m_C)/(m_n + m_C)| = \frac{11}{13}$ of its initial speed, and the speed of the recoiling carbon nucleus is $|2m_n/(m_n + m_C)| = \frac{2}{13}$ of the neutron's initial speed. Kinetic energy is proportional to speed squared, so the neutron's final kinetic energy is $\left(\frac{11}{13}\right)^2 \approx 0.72$ of its original value. After a second head-on collision, its kinetic energy is $(0.72)^2$, or about half its original value, and so on. After a few dozen collisions (few of which are head-on), the neutron speed will be low enough that it can efficiently cause a fission reaction in a uranium nucleus.

KEYCONCEPT When a particle undergoes a head-on elastic collision with a more massive, initially stationary target, the particle bounces back in the opposite direction. The recoiling target carries away some of the particle's initial kinetic energy.

EXAMPLE 8.12 A two-dimensional elastic collision

Figure 8.27 shows an elastic collision of two pucks (masses $m_A = 0.500$ kg and $m_B = 0.300$ kg) on a frictionless air-hockey table. Puck A has an initial velocity of 4.00 m/s in the positive x-direction and a final velocity of 2.00 m/s in an unknown direction α. Puck B is initially at rest. Find the final speed v_{B2} of puck B and the angles α and β.

IDENTIFY and SET UP We'll use the equations for conservation of energy and conservation of x- and y-momentum. These three equations should be enough to solve for the three target variables.

EXECUTE The collision is elastic, so the initial and final kinetic energies of the system are equal:

$$\frac{1}{2}m_A v_{A1}^2 = \frac{1}{2}m_A v_{A2}^2 + \frac{1}{2}m_B v_{B2}^2$$

$$v_{B2}^2 = \frac{m_A v_{A1}^2 - m_A v_{A2}^2}{m_B}$$

$$= \frac{(0.500\text{ kg})(4.00\text{ m/s})^2 - (0.500\text{ kg})(2.00\text{ m/s})^2}{0.300\text{ kg}}$$

$$v_{B2} = 4.47\text{ m/s}$$

Conservation of the x- and y-components of total momentum gives

$$m_A v_{A1x} = m_A v_{A2x} + m_B v_{B2x}$$

$$(0.500\text{ kg})(4.00\text{ m/s}) = (0.500\text{ kg})(2.00\text{ m/s})(\cos\alpha) + (0.300\text{ kg})(4.47\text{ m/s})(\cos\beta)$$

$$0 = m_A v_{A2y} + m_B v_{B2y}$$

$$0 = (0.500\text{ kg})(2.00\text{ m/s})(\sin\alpha) - (0.300\text{ kg})(4.47\text{ m/s})(\sin\beta)$$

These are two simultaneous equations for α and β. You can supply the details of the solution. (*Hint:* Solve the first equation for $\cos\beta$ and the second for $\sin\beta$; square each equation and add. Since $\sin^2\beta + \cos^2\beta = 1$, this eliminates β and leaves an equation that you can solve for $\cos\alpha$ and hence for α. Substitute this value into either of the two equations and solve for β.) The results are

$$\alpha = 36.9^\circ \qquad \beta = 26.6^\circ$$

EVALUATE To check the answers we confirm that the y-momentum, which was zero before the collision, is in fact zero after the collision. The y-momenta are

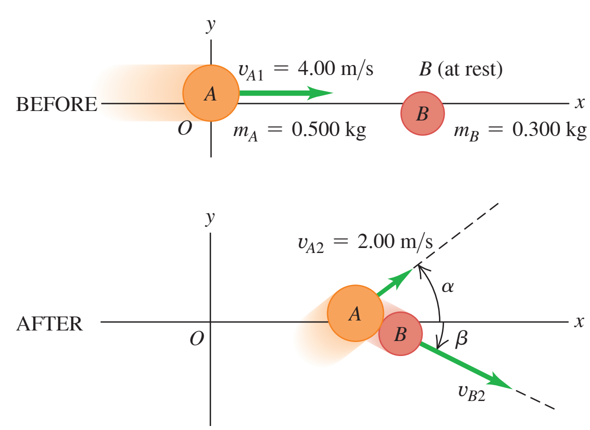
$$p_{A2y} = (0.500\text{ kg})(2.00\text{ m/s})(\sin 36.9^\circ) = +0.600\text{ kg}\cdot\text{m/s}$$

$$p_{B2y} = -(0.300\text{ kg})(4.47\text{ m/s})(\sin 26.6^\circ) = -0.600\text{ kg}\cdot\text{m/s}$$

and their sum is indeed zero.

KEYCONCEPT There are three conservation equations for a problem that involves a two-dimensional elastic collision: one for kinetic energy, one for the x-component of momentum, and one for the y-component of momentum.

Figure **8.27** An elastic collision that isn't head-on.



TEST YOUR UNDERSTANDING OF SECTION 8.4 Most present-day nuclear reactors use water as a moderator (see Example 8.11). Are water molecules (mass $m_w = 18.0$ u) a better or worse moderator than carbon atoms? (One advantage of water is that it also acts as a coolant for the reactor's radioactive core.)

ANSWER

worse After colliding with a water molecule initially at rest, the neutron has speed $|(m_n - m_w)/(m_n + m_w)| = |(1.0\text{ u} - 18\text{ u})/(1.0\text{ u} + 18\text{ u})| = \frac{17}{19}$ of its initial speed, and its kinetic energy is $\left(\frac{17}{19}\right)^2 = 0.80$ of the initial value. Hence a water molecule is a worse moderator than a carbon atom, for which the corresponding numbers are $\frac{11}{13}$ and $\left(\frac{11}{13}\right)^2 = 0.72$.

8.5 CENTER OF MASS

We can restate the principle of conservation of momentum in a useful way by using the concept of **center of mass.** Suppose we have several particles with masses m_1, m_2, and so on. Let the coordinates of m_1 be (x_1, y_1), those of m_2 be (x_2, y_2), and so on. We define the center of mass of the system as the point that has coordinates $(x_{\text{cm}}, y_{\text{cm}})$ given by

$$x_{\text{cm}} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_ix_i}{\sum_i m_i}$$
$$y_{\text{cm}} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_iy_i}{\sum_i m_i} \quad \text{(center of mass)} \tag{8.28}$$

We can express the position of the center of mass as a vector $\vec{r}_{\text{cm}}$:

Position vectors of individual particles

Position vector of **center of mass** of a system of particles

$$\vec{r}_{\text{cm}} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i} \tag{8.29}$$

Masses of individual particles

We say that the center of mass is a *mass-weighted average* position of the particles.

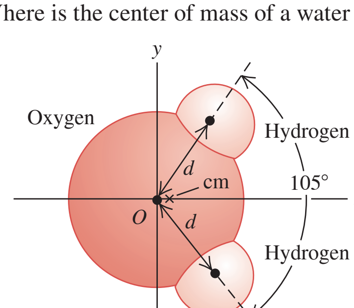
EXAMPLE 8.13 Center of mass of a water molecule

WITH VARIATION PROBLEMS

Figure 8.28 shows a simple model of a water molecule. The oxygen–hydrogen separation is $d = 9.57 \times 10^{-11}$ m. Each hydrogen atom has mass 1.0 u, and the oxygen atom has mass 16.0 u. Find the position of the center of mass.

IDENTIFY and SET UP Nearly all the mass of each atom is concentrated in its nucleus, whose radius is only about 10^{-5} times the overall radius of the atom. Hence we can safely represent each atom as a point particle. Figure 8.28 shows our coordinate system, with the x-axis chosen to lie along the molecule's symmetry axis. We'll use Eqs. (8.28) to find x_{cm} and y_{cm}.

Figure **8.28** Where is the center of mass of a water molecule?



EXECUTE The oxygen atom is at $x = 0$, $y = 0$. The x-coordinate of each hydrogen atom is $d\cos(105°/2)$; the y-coordinates are $\pm d\sin(105°/2)$. From Eqs. (8.28),

$$x_{\text{cm}} = \frac{\left[\begin{array}{c}(1.0\text{ u})(d\cos 52.5°) + (1.0\text{ u})(d\cos 52.5°) \\ + (16.0\text{ u})(0)\end{array}\right]}{1.0\text{ u} + 1.0\text{ u} + 16.0\text{ u}} = 0.068d$$

$$y_{\text{cm}} = \frac{\left[\begin{array}{c}(1.0\text{ u})(d\sin 52.5°) + (1.0\text{ u})(-d\sin 52.5°) \\ + (16.0\text{ u})(0)\end{array}\right]}{1.0\text{ u} + 1.0\text{ u} + 16.0\text{ u}} = 0$$

Substituting $d = 9.57 \times 10^{-11}$ m, we find

$$x_{\text{cm}} = (0.068)(9.57 \times 10^{-11}\text{ m}) = 6.5 \times 10^{-12}\text{ m}$$

EVALUATE The center of mass is much closer to the oxygen atom (located at the origin) than to either hydrogen atom because the oxygen atom is much more massive. The center of mass lies along the molecule's *axis of symmetry*. If the molecule is rotated 180° around this axis, it looks exactly the same as before. The position of the center of mass can't be affected by this rotation, so it *must* lie on the axis of symmetry.

KEYCONCEPT The x-coordinate of the center of mass of a collection of particles is a weighted sum of the x-coordinates of the individual particles, and similarly for the y-coordinate.

Figure **8.29** Locating the center of mass of a symmetric object.

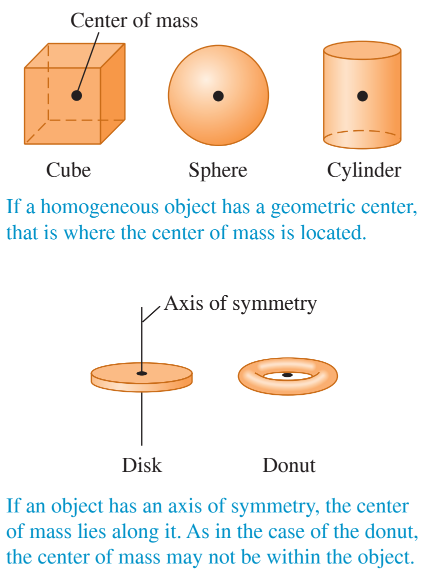
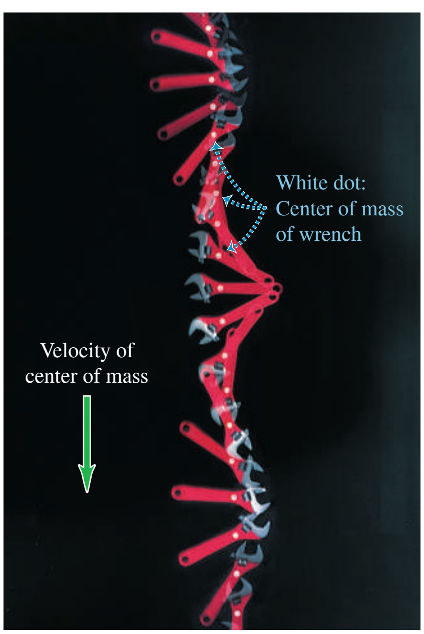


Figure **8.30** The net external force on this wrench is almost zero as it spins on a smooth, horizontal surface (seen from above). Hence the center of mass moves in a straight line with nearly constant velocity.



For solid objects, in which we have (at least on a macroscopic level) a continuous distribution of matter, the sums in Eqs. (8.28) have to be replaced by integrals. The calculations can get quite involved, but we can say three general things about such problems (**Fig. 8.29**). First, whenever a homogeneous object has a geometric center, such as a billiard ball, a sugar cube, or a can of frozen orange juice, the center of mass is at the geometric center. Second, whenever an object has an axis of symmetry, such as a wheel or a pulley, the center of mass always lies on that axis. Third, there is no law that says the center of mass has to be within the object. For example, the center of mass of a donut is in the middle of the hole.

Motion of the Center of Mass

To see the significance of the center of mass of a collection of particles, we must ask what happens to the center of mass when the particles move. The x- and y-components of velocity of the center of mass, $v_{\text{cm-}x}$ and $v_{\text{cm-}y}$, are the time derivatives of x_{cm} and y_{cm}. Also, dx_1/dt is the x-component of velocity of particle 1, so $dx_1/dt = v_{1x}$, and so on. Taking time derivatives of Eqs. (8.28), we get

$$v_{\text{cm-}x} = \frac{m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} + \cdots}{m_1 + m_2 + m_3 + \cdots}$$
$$v_{\text{cm-}y} = \frac{m_1 v_{1y} + m_2 v_{2y} + m_3 v_{3y} + \cdots}{m_1 + m_2 + m_3 + \cdots} \tag{8.30}$$

These equations are equivalent to the single vector equation obtained by taking the time derivative of Eq. (8.29):

$$\vec{\boldsymbol{v}}_{\text{cm}} = \frac{m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2 + m_3\vec{\boldsymbol{v}}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \tag{8.31}$$

We denote the *total* mass $m_1 + m_2 + \cdots$ by M. We can then rewrite Eq. (8.31) as

Total mass of a system of particles

Momenta of individual particles

$$M\vec{\boldsymbol{v}}_{\text{cm}} = m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2 + m_3\vec{\boldsymbol{v}}_3 + \cdots = \vec{\boldsymbol{P}} \tag{8.32}$$

Velocity of center of mass

Total momentum of system

So *the total momentum* $\vec{\boldsymbol{P}}$ *of a system equals the total mass times the velocity of the center of mass.* When you catch a baseball, you are really catching a collection of a very large number of molecules of masses $m_1, m_2, m_3, \ldots$. The impulse you feel is due to the total momentum of this entire collection. But this impulse is the same as if you were catching a single particle of mass $M = m_1 + m_2 + m_3 + \cdots$ moving with $\vec{\boldsymbol{v}}_{\text{cm}}$, the velocity of the collection's center of mass. So Eq. (8.32) helps us justify representing an extended object as a particle.

For a system of particles on which the net external force is zero, so that the total momentum $\vec{\boldsymbol{P}}$ is constant, the velocity of the center of mass $\vec{\boldsymbol{v}}_{\text{cm}} = \vec{\boldsymbol{P}}/M$ is also constant. **Figure 8.30** shows an example. The overall motion of the wrench appears complicated, but the center of mass follows a straight line, as though all the mass were concentrated at that point.

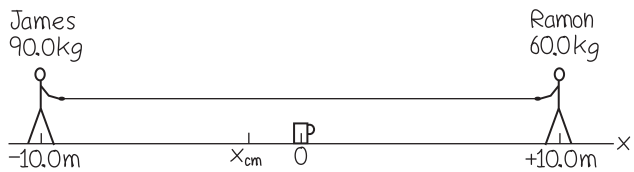
EXAMPLE 8.14 A tug-of-war on the ice

WITH VARIATION PROBLEMS

James (mass 90.0 kg) and Ramon (mass 60.0 kg) are 20.0 m apart on a frozen pond. Midway between them is a mug of their favorite beverage. They pull on the ends of a light rope stretched between them. When James has moved 6.0 m toward the mug, how far and in what direction has Ramon moved?

IDENTIFY and SET UP The surface is horizontal and (we assume) frictionless, so the net external force on the system of James, Ramon, and the rope is zero; their total momentum is conserved. Initially there is no motion, so the total momentum is zero. The velocity of the center of mass is therefore zero, and it remains at rest. Let's take the origin at the

Figure 8.31 Our sketch for this problem.



position of the mug and let the $+x$-axis extend from the mug toward Ramon. **Figure 8.31** shows our sketch. We use the first of Eqs. (8.28) to calculate the position of the center of mass; we ignore the mass of the light rope.

EXECUTE The initial x-coordinates of James and Ramon are -10.0 m and $+10.0$ m, respectively, so the x-coordinate of the center of mass is

$$x_{cm} = \frac{(90.0\text{ kg})(-10.0\text{ m}) + (60.0\text{ kg})(10.0\text{ m})}{90.0\text{ kg} + 60.0\text{ kg}} = -2.0\text{ m}$$

When James moves 6.0 m toward the mug, his new x-coordinate is -4.0 m; we'll call Ramon's new x-coordinate x_2. The center of mass doesn't move, so

$$x_{cm} = \frac{(90.0\text{ kg})(-4.0\text{ m}) + (60.0\text{ kg})x_2}{90.0\text{ kg} + 60.0\text{ kg}} = -2.0\text{ m}$$

$$x_2 = 1.0\text{ m}$$

James has moved 6.0 m and is still 4.0 m from the mug, but Ramon has moved 9.0 m and is only 1.0 m from it.

EVALUATE The ratio of the distances moved, $(6.0\text{ m})/(9.0\text{ m}) = \frac{2}{3}$, is the *inverse* ratio of the masses. Can you see why? Because the surface is frictionless, the two men will keep moving and collide at the center of mass; Ramon will reach the mug first. This is independent of how hard either person pulls; pulling harder just makes them move faster.

KEYCONCEPT If there is no net external force on a system of particles, the center of mass of the system maintains the same velocity. As a special case, if the center of mass is at rest, it remains at rest.

External Forces and Center-of-Mass Motion

If the net external force on a system of particles is not zero, then total momentum is not conserved and the velocity of the center of mass changes. Let's look at this situation in more detail.

Equations (8.31) and (8.32) give the *velocity* of the center of mass in terms of the velocities of the individual particles. We take the time derivatives of these equations to show that the *accelerations* are related in the same way. Let $\vec{a}_{cm} = d\vec{v}_{cm}/dt$ be the acceleration of the center of mass; then

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots \tag{8.33}$$

Now $m_1\vec{a}_1$ is equal to the vector sum of forces on the first particle, and so on, so the right side of Eq. (8.33) is equal to the vector sum $\sum\vec{F}$ of *all* the forces on *all* the particles. Just as we did in Section 8.2, we can classify each force as *external* or *internal*. The sum of all forces on all the particles is then

$$\sum\vec{F} = \sum\vec{F}_{ext} + \sum\vec{F}_{int} = M\vec{a}_{cm}$$

Because of Newton's third law, all of the internal forces cancel in pairs, and $\sum\vec{F}_{int} = \mathbf{0}$. What survives on the left side is the sum of only the *external* forces:

$$\underbrace{\sum\vec{F}_{ext}}_{\text{Net external force on an object or a collection of particles}} = \overbrace{M}^{\text{Total mass of object or collection of particles}}\underbrace{\vec{a}_{cm}}_{\text{Acceleration of center of mass}} \tag{8.34}$$

When an object or a collection of particles is acted on by external forces, the center of mass moves as though all the mass were concentrated at that point and it were acted on by a net external force equal to the sum of the external forces on the system.

This result is central to the whole subject of mechanics. In fact, we've been using this result all along; without it, we would not be able to represent an extended object as a point particle when we apply Newton's laws. It explains why only *external* forces can affect the motion of an extended object. If you pull upward on your belt, your belt exerts an equal downward force on your hands; these are *internal* forces that cancel and have no effect on the overall motion of your body.

As an example, suppose that a cannon shell traveling in a parabolic trajectory (ignoring air resistance) explodes in flight, splitting into two fragments with equal mass (**Fig. 8.32**, next page). The fragments follow new parabolic paths, but the center of mass continues on the original parabolic trajectory, as though all the mass were still concentrated at that point.

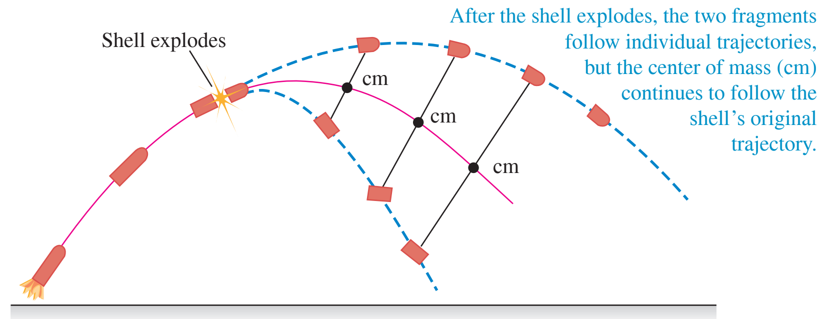
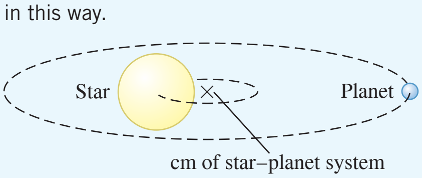


Figure **8.32** A shell explodes into two fragments in flight. If air resistance is ignored, the center of mass continues on the same trajectory as the shell's path before the explosion.

APPLICATION Finding Planets Beyond Our Solar System Planets orbiting distant stars are so faint that they cannot be seen with even the best telescopes. But they can be detected by using the idea that a planet and its parent star orbit around their common center of mass (cm). If we observe a star "wobbling" around a point, we can infer that there is an unseen companion planet and can actually determine the planet's mass. Hundreds of planets around distant stars have been discovered in this way.



This property of the center of mass is important when we analyze the motion of rigid objects. In Chapter 10 we'll describe the motion of an extended object as a combination of translational motion of the center of mass and rotational motion about an axis through the center of mass. This property also plays an important role in the motion of astronomical objects. It's not correct to say that the moon orbits the earth; rather, both the earth and the moon move in orbits around their common center of mass.

There's one more useful way to describe the motion of a system of particles. Using $\vec{a}_{cm} = d\vec{v}_{cm}/dt$, we can rewrite Eq. (8.33) as

$$M\vec{a}_{cm} = M\frac{d\vec{v}_{cm}}{dt} = \frac{d(M\vec{v}_{cm})}{dt} = \frac{d\vec{P}}{dt} \qquad (8.35)$$

The total system mass M is constant, so we're allowed to move it inside the derivative. Substituting Eq. (8.35) into Eq. (8.34), we find

$$\sum\vec{F}_{ext} = \frac{d\vec{P}}{dt} \quad \text{(extended object or system of particles)} \qquad (8.36)$$

This equation looks like Eq. (8.4). The difference is that Eq. (8.36) describes a *system* of particles, such as an extended object, while Eq. (8.4) describes a single particle. The interactions between the particles that make up the system can change the individual momenta of the particles, but the *total* momentum $\vec{P}$ of the system can be changed only by external forces acting from outside the system.

If the net external force is zero, Eqs. (8.34) and (8.36) show that the center-of-mass acceleration $\vec{a}_{cm}$ is zero (so the center-of-mass velocity $\vec{v}_{cm}$ is constant) and the total momentum $\vec{P}$ is constant. This is just our statement from Section 8.3: If the net external force on a system is zero, momentum is conserved.

TEST YOUR UNDERSTANDING OF SECTION 8.5 Will the center of mass in Fig. 8.32 continue on the same parabolic trajectory even after one of the fragments hits the ground? Why or why not?

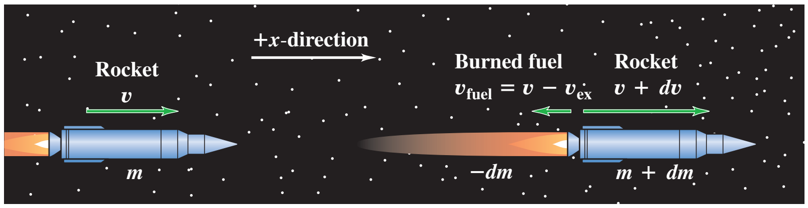
ANSWER

no If gravity is the only force acting on the system of two fragments, the center of mass will follow the parabolic trajectory of a freely falling object. Once a fragment lands, however, the ground exerts a normal force on that fragment. Hence the net external force on the system has changed, and the trajectory of the center of mass changes in response.

8.6 ROCKET PROPULSION

Momentum considerations are particularly useful for analyzing a system in which the masses of parts of the system change with time. In such cases we can't use Newton's second law $\sum\vec{F} = m\vec{a}$ directly because m changes. Rocket propulsion is an important example of this situation. A rocket is propelled forward by rearward ejection of burned fuel that initially was in the rocket (which is why rocket fuel is also called *propellant*). The forward force on the rocket is the reaction to the backward force on the ejected material. The total mass of the system is constant, but the mass of the rocket itself decreases as material is ejected.

(a) (b)



At time t, the rocket has mass m and x-component of velocity v.

At time $t + dt$, the rocket has mass $m + dm$ (where dm is inherently *negative*) and x-component of velocity $v + dv$. The burned fuel has x-component of velocity $v_{\text{fuel}} = v - v_{\text{ex}}$ and mass $-dm$. (The minus sign is needed to make $-dm$ *positive* because dm is negative.)

Figure **8.33** A rocket moving in gravity-free outer space at (a) time t and (b) time $t + dt$.

For simplicity, let's consider a rocket in outer space, where there is no gravitational force and no air resistance. Let m denote the mass of the rocket, which will change as it expends fuel. We choose our x-axis to be along the rocket's direction of motion. **Figure 8.33a** shows the rocket at a time t, when its mass is m and its x-velocity relative to our coordinate system is v. (To simplify, we'll drop the subscript x in this discussion.) The x-component of total momentum at this instant is $P_1 = mv$. In a short time interval dt, the mass of the rocket changes by an amount dm. This is an inherently negative quantity because the rocket's mass m *decreases* with time. During dt, a *positive* mass $-dm$ of burned fuel is ejected from the rocket. Let v_{ex} be the exhaust *speed* of this material *relative to the rocket;* the burned fuel is ejected opposite the direction of motion, so its x-component of *velocity* relative to the rocket is $-v_{\text{ex}}$. The x-velocity v_{fuel} of the burned fuel relative to our coordinate system is then

$$v_{\text{fuel}} = v + (-v_{\text{ex}}) = v - v_{\text{ex}}$$

and the x-component of momentum of the ejected mass $(-dm)$ is

$$(-dm)v_{\text{fuel}} = (-dm)(v - v_{\text{ex}})$$

Figure 8.33b shows that at the end of the time interval dt, the x-velocity of the rocket and unburned fuel has increased to $v + dv$, and its mass has decreased to $m + dm$ (remember that dm is negative). The rocket's momentum at this time is

$$(m + dm)(v + dv)$$

Thus the *total* x-component of momentum P_2 of the rocket plus ejected fuel at time $t + dt$ is

$$P_2 = (m + dm)(v + dv) + (-dm)(v - v_{\text{ex}})$$

According to our initial assumption, the rocket and fuel are an isolated system. Thus momentum is conserved, and the total x-component of momentum of the system must be the same at time t and at time $t + dt$: $P_1 = P_2$. Hence

$$mv = (m + dm)(v + dv) + (-dm)(v - v_{\text{ex}})$$

This can be simplified to

$$m\,dv = -dm\,v_{\text{ex}} - dm\,dv$$

We can ignore the term $(-dm\,dv)$ because it is a product of two small quantities and thus is much smaller than the other terms. Dropping this term, dividing by dt, and rearranging, we find

$$m\frac{dv}{dt} = -v_{\text{ex}}\frac{dm}{dt} \qquad (8.37)$$

Now dv/dt is the acceleration of the rocket, so the left side of Eq. (8.37) (mass times acceleration) equals the net external force F, or *thrust,* on the rocket:

$$F = -v_{\text{ex}}\frac{dm}{dt} \qquad (8.38)$$

BIO APPLICATION Jet Propulsion in Squids Both a jet engine and a squid use variations in their mass to provide propulsion: They increase their mass by taking in fluid (air for a jet engine, water for a squid) at low speed, then decrease their mass by ejecting that fluid at high speed. The Caribbean reef squid (*Sepioteuthis sepioidea*), shown here, can use jet propulsion to vault to a height of 2 m above the water and fly a total distance of 10 m—about 50 times its body length!

Figure 8.34 To provide enough thrust to lift its payload into space, this *Atlas V* launch vehicle ejects more than 1000 kg of burned fuel per second at speeds of nearly 4000 m/s.

The thrust is proportional both to the relative speed v_{ex} of the ejected fuel and to the mass of fuel ejected per unit time, $-dm/dt$. (Remember that dm/dt is negative because it is the rate of change of the rocket's mass, so F is positive.)

The x-component of acceleration of the rocket is

$$a = \frac{dv}{dt} = -\frac{v_{ex}}{m}\frac{dm}{dt} \tag{8.39}$$

This is positive because v_{ex} is positive (remember, it's the exhaust *speed*) and dm/dt is negative. The rocket's mass m decreases continuously while the fuel is being consumed. If v_{ex} and dm/dt are constant, the acceleration increases until all the fuel is gone.

Equation (8.38) tells us that an effective rocket burns fuel at a rapid rate (large $-dm/dt$) and ejects the burned fuel at a high relative speed (large v_{ex}), as in **Fig. 8.34**. In the early days of rocket propulsion, people who didn't understand conservation of momentum thought that a rocket couldn't function in outer space because "it doesn't have anything to push against." In fact, rockets work *best* in outer space, where there is no air resistance! The launch vehicle in Fig. 8.34 is *not* "pushing against the ground" to ascend.

If the exhaust speed v_{ex} is constant, we can integrate Eq. (8.39) to relate the velocity v at any time to the remaining mass m. At time $t = 0$, let the mass be m_0 and the velocity be v_0. Then we rewrite Eq. (8.39) as

$$dv = -v_{ex}\frac{dm}{m}$$

We change the integration variables to v' and m', so we can use v and m as the upper limits (the final speed and mass). Then we integrate both sides, using limits v_0 to v and m_0 to m, and take the constant v_{ex} outside the integral:

$$\int_{v_0}^{v} dv' = -\int_{m_0}^{m} v_{ex}\frac{dm'}{m'} = -v_{ex}\int_{m_0}^{m}\frac{dm'}{m'}$$

$$v - v_0 = -v_{ex}\ln\frac{m}{m_0} = v_{ex}\ln\frac{m_0}{m} \tag{8.40}$$

The ratio m_0/m is the original mass divided by the mass after the fuel has been exhausted. In practical spacecraft this ratio is made as large as possible to maximize the speed gain, which means that the initial mass of the rocket is almost all fuel. The final velocity of the rocket will be greater in magnitude (and is often *much* greater) than the relative speed v_{ex} if $\ln(m_0/m) > 1$—that is, if $m_0/m > e = 2.71828\ldots$

We've assumed throughout this analysis that the rocket is in gravity-free outer space. However, gravity must be taken into account when a rocket is launched from the surface of a planet, as in Fig. 8.34.

EXAMPLE 8.15 Acceleration of a rocket

The engine of a rocket in outer space, far from any planet, is turned on. The rocket ejects burned fuel at a constant rate; in the first second of firing, it ejects $\frac{1}{120}$ of its initial mass m_0 at a relative speed of 2400 m/s. What is the rocket's initial acceleration?

IDENTIFY and SET UP We are given the rocket's exhaust speed v_{ex} and the fraction of the initial mass lost during the first second of firing, from which we can find dm/dt. We'll use Eq. (8.39) to find the acceleration of the rocket.

EXECUTE The initial rate of change of mass is

$$\frac{dm}{dt} = -\frac{m_0/120}{1\text{ s}} = -\frac{m_0}{120\text{ s}}$$

From Eq. (8.39),

$$a = -\frac{v_{ex}}{m_0}\frac{dm}{dt} = -\frac{2400\text{ m/s}}{m_0}\left(-\frac{m_0}{120\text{ s}}\right) = 20\text{ m/s}^2$$

EVALUATE The answer doesn't depend on m_0. If v_{ex} is the same, the initial acceleration is the same for a 120,000 kg spacecraft that ejects 1000 kg/s as for a 60 kg astronaut equipped with a small rocket that ejects 0.5 kg/s.

KEYCONCEPT The thrust provided by a rocket equals the exhaust speed multiplied by the mass of fuel ejected per unit time. The resulting acceleration equals the thrust divided by the rocket's mass.

EXAMPLE 8.16 Speed of a rocket

Suppose that $\frac{3}{4}$ of the initial mass of the rocket in Example 8.15 is fuel, so the fuel is completely consumed at a constant rate in 90 s. The final mass of the rocket is $m = m_0/4$. If the rocket starts from rest in our coordinate system, find its speed at the end of this time.

IDENTIFY, SET UP, and EXECUTE We are given the initial velocity $v_0 = 0$, the exhaust speed $v_{ex} = 2400\text{ m/s}$, and the final mass m as a fraction of the initial mass m_0. We'll use Eq. (8.40) to find the final speed v:

$$v = v_0 + v_{ex} \ln \frac{m_0}{m} = 0 + (2400\text{ m/s})(\ln 4) = 3327\text{ m/s}$$

EVALUATE Let's examine what happens as the rocket gains speed. (To illustrate our point, we use more figures than are significant.) At the start of the flight, when the velocity of the rocket is zero, the ejected fuel is moving backward at 2400 m/s relative to our frame of reference. As the rocket moves forward and speeds up, the fuel's speed relative to our system decreases; when the rocket speed reaches 2400 m/s, this relative speed is *zero*. [Knowing the rate of fuel consumption, you can solve Eq. (8.40) to show that this occurs at about $t = 75.6$ s.] After this time the ejected burned fuel moves *forward,* not backward, in our system. Relative to our frame of reference, the last bit of ejected fuel has a forward velocity of $3327\text{ m/s} - 2400\text{ m/s} = 927\text{ m/s}$.

KEYCONCEPT Because the mass of a rocket decreases as it ejects fuel, its acceleration is not constant even if the thrust is constant.

TEST YOUR UNDERSTANDING OF SECTION 8.6 (a) If a rocket in gravity-free outer space has the same thrust at all times, is its acceleration constant, increasing, or decreasing? (b) If the rocket has the same acceleration at all times, is the thrust constant, increasing, or decreasing?

ANSWER

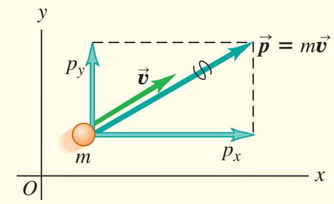
(a) **increasing,** (b) **decreasing** From Eqs. (8.37) and (8.38), the thrust F is equal to $m(dv/dt)$, where m is the rocket's mass and dv/dt is its acceleration. Because m decreases with time, if the thrust F is constant, then the acceleration must increase with time (the same force acts on a smaller mass); if the acceleration dv/dt is constant, then the thrust must decrease with time (a smaller force is all that's needed to accelerate a smaller mass).

CHAPTER 8 SUMMARY

Momentum of a particle: The momentum $\vec{p}$ of a particle is a vector quantity equal to the product of the particle's mass m and velocity $\vec{v}$. Newton's second law says that the net external force on a particle is equal to the rate of change of the particle's momentum.

$$\vec{p} = m\vec{v} \qquad (8.2)$$

$$\sum\vec{F} = \frac{d\vec{p}}{dt} \qquad (8.4)$$

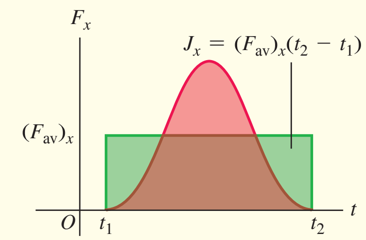


Impulse and momentum: If a constant net external force $\sum\vec{F}$ acts on a particle for a time interval Δt from t_1 to t_2, the impulse $\vec{J}$ of the net external force is the product of the net external force and the time interval. If $\sum\vec{F}$ varies with time, $\vec{J}$ is the integral of the net external force over the time interval. In any case, the change in a particle's momentum during a time interval equals the impulse of the net external force that acted on the particle during that interval. The momentum of a particle equals the impulse that accelerated it from rest to its present speed. (See Examples 8.1–8.3.)

$$\vec{J} = \sum\vec{F}(t_2 - t_1) = \sum\vec{F}\,\Delta t \qquad (8.5)$$

$$\vec{J} = \int_{t_1}^{t_2} \sum\vec{F}\,dt \qquad (8.7)$$

$$\vec{J} = \vec{p}_2 - \vec{p}_1 \qquad (8.6)$$

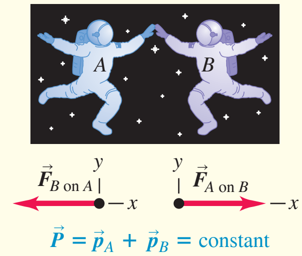


Conservation of momentum: An internal force is a force exerted by one part of a system on another. An external force is a force exerted on any part of a system by something outside the system. If the net external force on a system is zero, the total momentum of the system $\vec{P}$ (the vector sum of the momenta of the individual particles that make up the system) is constant, or conserved. Each component of total momentum is separately conserved. (See Examples 8.4–8.6.)

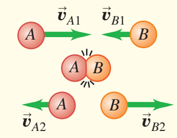
$$\vec{P} = \vec{p}_A + \vec{p}_B + \cdots$$

$$= m_A\vec{v}_A + m_B\vec{v}_B + \cdots \qquad (8.14)$$

If $\sum\vec{F} = \mathbf{0}$, then $\vec{P}$ = constant.



Collisions: In typical collisions, the initial and final total momenta are equal. In an elastic collision between two objects, the initial and final total kinetic energies are also equal, and the initial and final relative velocities have the same magnitude. In an inelastic two-object collision, the total kinetic energy is less after the collision than before. If the two objects have the same final velocity, the collision is completely inelastic. (See Examples 8.7–8.12.)

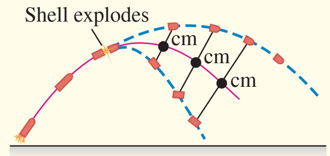


Center of mass: The position vector of the center of mass of a system of particles, $\vec{r}_{cm}$, is a weighted average of the positions $\vec{r}_1$, $\vec{r}_2$, . . . of the individual particles. The total momentum $\vec{P}$ of a system equals the system's total mass M multiplied by the velocity of its center of mass, $\vec{v}_{cm}$. The center of mass moves as though all the mass M were concentrated at that point. If the net external force on the system is zero, the center-of-mass velocity $\vec{v}_{cm}$ is constant. If the net external force is not zero, the center of mass accelerates as though it were a particle of mass M being acted on by the same net external force. (See Examples 8.13 and 8.14.)

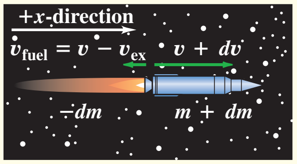
$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i} \qquad (8.29)$$

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots = M\vec{v}_{cm} \qquad (8.32)$$

$$\sum\vec{F}_{ext} = M\vec{a}_{cm} \qquad (8.34)$$



Rocket propulsion: In rocket propulsion, the mass of a rocket changes as the fuel is used up and ejected from the rocket. Analysis of the motion of the rocket must include the momentum carried away by the spent fuel as well as the momentum of the rocket itself. (See Examples 8.15 and 8.16.)



Chapter 8 Media Assets

GUIDED PRACTICE

For assigned homework and other learning materials, go to Mastering Physics.

KEY EXAMPLE VARIATION PROBLEMS

Be sure to review EXAMPLES 8.4, 8.5, and 8.6 (Section 8.2) before attempting these problems.

VP8.6.1 You hold glider A of mass 0.125 kg and glider B of mass 0.375 kg at rest on an air track with a compressed spring of negligible mass between them. When you release the gliders, the spring pushes them apart. (a) Once the gliders are no longer in contact with the spring, glider A is moving to the right at 0.600 m/s. What is the velocity (magnitude and direction) of glider B at this time? (b) Glider A moving to the right at 0.600 m/s then collides head-on with a third glider C of mass 0.750 kg that is moving to the left at 0.400 m/s. After this collision, glider C is moving to the left at 0.150 m/s. What is the velocity (magnitude and direction) of glider A after this collision?

VP8.6.2 Two hockey players skating on essentially frictionless ice collide head-on. Madeleine, of mass 65.0 kg, is moving at 6.00 m/s to the east just before the collision and at 3.00 m/s to the west just after the collision. Buffy, of mass 55.0 kg, is moving at 3.50 m/s to the east just after the collision. (a) Find Buffy's velocity (magnitude and direction) just before the collision. (b) What are the changes in the velocities of the two hockey players during the collision? Take east to be the positive direction. Who has the greater magnitude of velocity change: more massive Madeleine or less massive Buffy?

VP8.6.3 A 2.40 kg stone is sliding in the $+x$-direction on a horizontal, frictionless surface. It collides with a 4.00 kg stone at rest. After the collision the 2.40 kg stone is moving at 3.60 m/s at an angle of 30.0° measured from the $+x$-direction toward the $+y$-direction, and the 4.00 kg stone is moving at an angle of 45.0° measured from the $+x$-direction toward the $-y$-direction. (a) What is the y-component of momentum of the 2.40 kg stone after the collision? What must be the y-component of momentum of the 4.00 kg stone after the collision? (b) What is the speed of the 4.00 kg stone after the collision? (c) What is the x-component of the total momentum of the two stones after the collision? (d) What is the speed of the 2.40 kg stone before the collision?

VP8.6.4 A hockey puck of mass m is moving in the $+x$-direction at speed v_{P1} on a frictionless, horizontal surface. It collides with a stone of mass $2m$ that is initially at rest. After the collision the hockey puck is moving at an angle θ measured from the $+x$-direction toward the $+y$-direction, and the stone is moving at the same angle θ but measured from the $+x$-direction toward the $-y$-direction. (a) In order for the y-component of momentum to be conserved, what must be the ratio of the final speed v_{S2} of the stone to the final speed v_{P2} of the hockey puck? (b) Use conservation of the x-component of momentum to find v_{S2} and v_{P2} in terms of v_{P1} and θ.

Be sure to review EXAMPLES 8.7, 8.8, and 8.9 (Section 8.3) before attempting these problems.

VP8.9.1 Two blocks of clay, one of mass 1.00 kg and one of mass 4.00 kg, undergo a completely inelastic collision. Before the collision one of the blocks is at rest and the other block is moving with kinetic energy 32.0 J. (a) If the 4.00 kg block is initially at rest and the 1.00 kg block is moving, what is the initial speed of the 1.00 kg block? What is the common final speed of the two blocks? How much kinetic energy is lost in the collision? (b) If the 1.00 kg block is initially at rest and the 4.00 kg block is moving, what is the initial speed of the 4.00 kg block? What is the common final speed of the two blocks? How much kinetic energy is lost in the collision? (c) In which case is more of the initial kinetic energy lost in a completely inelastic collision: a moving object collides (i) with a heavier object at rest or (ii) with a lighter object at rest?

VP8.9.2 A 0.500 kg block of cheese sliding on a frictionless tabletop collides with and sticks to a 0.200 kg apple. Before the collision the cheese was moving at 1.40 m/s and the apple was at rest. The cheese and apple then slide together off the edge of the table and fall to the floor 0.600 m below. (a) Find the speed of the cheese and apple just after the collision. In this collision, what is conserved: momentum, total mechanical energy, both, or neither? (b) What is the speed of the cheese and apple just before they hit the floor? During the fall from the tabletop to the floor, what is conserved: momentum, total mechanical energy, both, or neither?

VP8.9.3 A 2.40 kg can of coffee moving at 1.50 m/s in the $+x$-direction on a kitchen counter collides head-on with a 1.20 kg box of macaroni that is initially at rest. After the collision the can of coffee is moving at 0.825 m/s in the $+x$-direction. (a) What is the velocity (magnitude and direction) of the box of macaroni after the collision? (b) What are the kinetic energies of the can before and after the collision, and of the box after the collision? (c) Is this collision elastic, inelastic, or completely inelastic? How can you tell?

VP8.9.4 A block of mass m moving due east at speed v collides with and sticks to a block of mass $2m$ that is moving at the same speed v but in a direction 45.0° north of east. Find the direction in which the two blocks move after the collision.

Be sure to review EXAMPLES 8.13 and 8.14 (Section 8.5) before attempting these problems.

VP8.14.1 Find the x- and y-coordinates of the center of mass of a system composed of the following particles: a 0.500 kg particle at the origin; a 1.25 kg particle at $x = 0.150$ m, $y = 0.200$ m; and a 0.750 kg particle at $x = 0.200$ m, $y = -0.800$ m.

VP8.14.2 Three objects lie along the x-axis. A 3.00 kg object is at the origin, a 2.00 kg object is at $x = 1.50$ m, and a 1.20 kg object is at an unknown position. The center of mass of the system of three objects is at $x = -0.200$ m. What is the position of the 1.20 kg object?

VP8.14.3 You hold a 0.125 kg glider A and a 0.500 kg glider B at rest on an air track with a compressed spring of negligible mass between them. When you release the gliders, the spring pushes them apart so that they move in opposite directions. When glider A has moved 0.960 m to the left from its starting position, how far to the right from its starting position has glider B moved?

VP8.14.4 Three objects each have mass m. Each object feels a force from the other two, but not from any other object. Initially the first object is at $x = -L$, $y = 0$; the second object is at $x = +L$, $y = 0$; and the third object is at $x = 0$, $y = L$. At a later time the first object is at $x = -L/3$, $y = +L/4$; and the second object is at $x = +L/2$, $y = -L$. At this later time, where is the third object?

BRIDGING PROBLEM One Collision After Another

Sphere A, of mass 0.600 kg, is initially moving to the right at 4.00 m/s. Sphere B, of mass 1.80 kg, is initially to the right of sphere A and moving to the right at 2.00 m/s. After the two spheres collide, sphere B is moving at 3.00 m/s in the same direction as before. (a) What is the velocity (magnitude and direction) of sphere A after this collision? (b) Is this collision elastic or inelastic? (c) Sphere B then has an off-center collision with sphere C, which has mass 1.20 kg and is initially at rest. After this collision, sphere B is moving at 19.0° to its initial direction at 2.00 m/s. What is the velocity (magnitude and direction) of sphere C after this collision? (d) What is the impulse (magnitude and direction) imparted to sphere B by sphere C when they collide? (e) Is this second collision elastic or inelastic? (f) What is the velocity (magnitude and direction) of the center of mass of the system of three spheres (A, B, and C) after the second collision? No external forces act on any of the spheres in this problem.

SOLUTION GUIDE

IDENTIFY and SET UP

1. Momentum is conserved in these collisions. Can you explain why?
2. Choose the x- and y-axes, and use your choice of axes to draw three figures that show the spheres (i) before the first collision, (ii) after the first collision but before the second collision, and (iii) after the second collision. Assign subscripts to values in each of situations (i), (ii), and (iii).
3. Make a list of the target variables, and choose the equations that you'll use to solve for these.

EXECUTE

4. Solve for the velocity of sphere A after the first collision. Does A slow down or speed up in the collision? Does this make sense?
5. Now that you know the velocities of both A and B after the first collision, decide whether or not this collision is elastic. (How will you do this?)
6. The second collision is two-dimensional, so you'll have to demand that *both* components of momentum are conserved. Use this to find the speed and direction of sphere C after the second collision. (*Hint:* After the first collision, sphere B maintains the same velocity until it hits sphere C.)
7. Use the definition of impulse to find the impulse imparted to sphere B by sphere C. Remember that impulse is a vector.
8. Use the same technique that you employed in step 5 to decide whether the second collision is elastic.
9. Find the velocity of the center of mass after the second collision.

EVALUATE

10. Compare the directions of the vectors you found in steps 6 and 7. Is this a coincidence? Why or why not?
11. Find the velocity of the center of mass before and after the first collision. Compare to your result from step 9. Again, is this a coincidence? Why or why not?

PROBLEMS

•, ••, •••: Difficulty levels. **CP**: Cumulative problems incorporating material from earlier chapters. **CALC**: Problems requiring calculus. **DATA**: Problems involving real data, scientific evidence, experimental design, and/or statistical reasoning. **BIO**: Biosciences problems.

DISCUSSION QUESTIONS

Q8.1 In splitting logs with a hammer and wedge, is a heavy hammer more effective than a lighter hammer? Why?

Q8.2 Suppose you catch a baseball and then someone invites you to catch a bowling ball with either the same momentum or the same kinetic energy as the baseball. Which would you choose? Explain.

Q8.3 When rain falls from the sky, what happens to its momentum as it hits the ground? Is your answer also valid for Newton's famous apple?

Q8.4 A car has the same kinetic energy when it is traveling south at 30 m/s as when it is traveling northwest at 30 m/s. Is the momentum of the car the same in both cases? Explain.

Q8.5 A truck is accelerating as it speeds down the highway. One inertial frame of reference is attached to the ground with its origin at a fence post. A second frame of reference is attached to a police car that is traveling down the highway at constant velocity. Is the momentum of the truck the same in these two reference frames? Explain. Is the rate of change of the truck's momentum the same in these two frames? Explain.

Q8.6 (a) If the momentum of a *single* point object is equal to zero, must the object's kinetic energy also be zero? (b) If the momentum of a *pair* of point objects is equal to zero, must the kinetic energy of those objects also be zero? (c) If the kinetic energy of a pair of point objects is equal to zero, must the momentum of those objects also be zero? Explain your reasoning in each case.

Q8.7 A woman holding a large rock stands on a frictionless, horizontal sheet of ice. She throws the rock with speed v_0 at an angle α above the horizontal. Consider the system consisting of the woman plus the rock. Is the momentum of the system conserved? Why or why not? Is any component of the momentum of the system conserved? Again, why or why not?

Q8.8 In Example 8.7 (Section 8.3), where the two gliders of Fig. 8.18 stick together after the collision, the collision is inelastic because $K_2 < K_1$. In Example 8.5 (Section 8.2), is the collision inelastic? Explain.

Q8.9 In a completely inelastic collision between two objects, where the objects stick together after the collision, is it possible for the final kinetic energy of the system to be zero? If so, give an example in which this would occur. If the final kinetic energy is zero, what must the initial momentum of the system be? Is the initial kinetic energy of the system zero? Explain.

Q8.10 Since for a particle the kinetic energy is given by $K = \frac{1}{2}mv^2$ and the momentum by $\vec{P} = m\vec{v}$, it is easy to show that $K = p^2/2m$. How, then, is it possible to have an event during which the total momentum of the system is constant but the total kinetic energy changes?

Q8.11 In each of Examples 8.10, 8.11, and 8.12 (Section 8.4), verify that the relative velocity vector of the two objects has the same magnitude before and after the collision. In each case, what happens to the *direction* of the relative velocity vector?

Q8.12 A glass dropped on the floor is more likely to break if the floor is concrete than if it is wood. Why? (Refer to Fig. 8.3b.)

Q8.13 In Fig. 8.23b, the kinetic energy of the Ping-Pong ball is larger after its interaction with the bowling ball than before. From where does the extra energy come? Describe the event in terms of conservation of energy.

Q8.14 A machine gun is fired at a steel plate. Is the average force on the plate from the bullet impact greater if the bullets bounce off or if they are squashed and stick to the plate? Explain.

Q8.15 A net external force of 4 N acts on an object initially at rest for 0.25 s and gives it a final speed of 5 m/s. How could a net external force of 2 N produce the same final speed?

Q8.16 A net external force with x-component $\sum F_x$ acts on an object from time t_1 to time t_2. The x-component of the momentum of the object is the same at t_1 as it is at t_2, but $\sum F_x$ is not zero at all times between t_1 and t_2. What can you say about the graph of $\sum F_x$ versus t?

Q8.17 A tennis player hits a tennis ball with a racket. Consider the system made up of the ball and the racket. Is the total momentum of the system the same just before and just after the hit? Is the total momentum just after the hit the same as 2 s later, when the ball is in midair at the high point of its trajectory? Explain any differences between the two cases.

Q8.18 In Example 8.4 (Section 8.2), consider the system consisting of the rifle plus the bullet. What is the speed of the system's center of mass after the rifle is fired? Explain.

Q8.19 An egg is released from rest from the roof of a building and falls to the ground. As the egg falls, what happens to the momentum of the system of the egg plus the earth?

Q8.20 A woman stands in the middle of a perfectly smooth, frictionless, frozen lake. She can set herself in motion by throwing things, but suppose she has nothing to throw. Can she propel herself to shore *without* throwing anything?

Q8.21 At the highest point in its parabolic trajectory, a shell explodes into two fragments. Is it possible for *both* fragments to fall straight down after the explosion? Why or why not?

Q8.22 When an object breaks into two pieces (explosion, radioactive decay, recoil, etc.), the lighter fragment gets more kinetic energy than the heavier one. This is a consequence of momentum conservation, but can you also explain it by using Newton's laws of motion?

Q8.23 An apple falls from a tree and feels no air resistance. As it is falling, which of these statements about it are true? (a) Only its momentum is conserved; (b) only its total mechanical energy is conserved; (c) both its momentum and its total mechanical energy are conserved; (d) its kinetic energy is conserved.

Q8.24 Two pieces of clay collide and stick together. During the collision, which of these statements are true? (a) Only the momentum of the clay is conserved; (b) only the total mechanical energy of the clay is conserved; (c) both the momentum and the total mechanical energy of the clay are conserved; (d) the kinetic energy of the clay is conserved.

Q8.25 Two objects of mass M and $5M$ are at rest on a horizontal, frictionless table with a compressed spring of negligible mass between them. When the spring is released, which of the following statements are true? (a) The two objects receive equal magnitudes of momentum; (b) the two objects receive equal amounts of kinetic energy from the spring; (c) the heavier object gains more kinetic energy than the lighter object; (d) the lighter object gains more kinetic energy than the heavier object. Explain your reasoning in each case.

Q8.26 A very heavy SUV collides head-on with a very light compact car. Which of these statements about the collision are correct? (a) The amount of kinetic energy lost by the SUV is equal to the amount of kinetic energy gained by the compact; (b) the amount of momentum lost by the SUV is equal to the amount of momentum gained by the compact; (c) the compact feels a considerably greater force during the collision than the SUV does; (d) both cars lose the same amount of kinetic energy.

EXERCISES

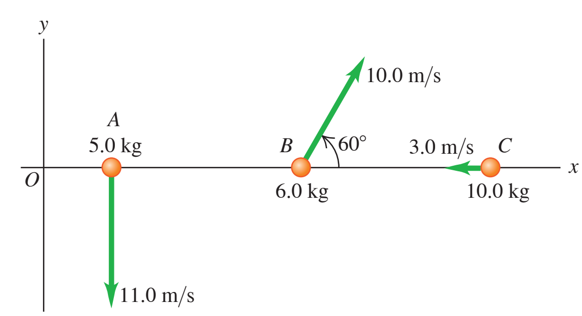
Section 8.1 Momentum and Impulse

8.1 • (a) What is the magnitude of the momentum of a 10,000 kg truck whose speed is 12.0 m/s? (b) What speed would a 2000 kg SUV have to attain in order to have (i) the same momentum? (ii) the same kinetic energy?

8.2 • In a certain track and field event, the shotput has a mass of 7.30 kg and is released with a speed of 15.0 m/s at 40.0° above the horizontal over a competitor's straight left leg. What are the initial horizontal and vertical components of the momentum of this shotput?

8.3 • Objects A, B, and C are moving as shown in **Fig. E8.3**. Find the x- and y-components of the net momentum of the particles if we define the system to consist of (a) A and C, (b) B and C, (c) all three objects.

Figure **E8.3**



8.4 • Two vehicles are approaching an intersection. One is a 2500 kg pickup traveling at 14.0 m/s from east to west (the $-x$-direction), and the other is a 1500 kg sedan going from south to north (the $+y$-direction) at 23.0 m/s. (a) Find the x- and y-components of the net momentum of this system. (b) What are the magnitude and direction of the net momentum?

8.5 • One 110 kg football lineman is running to the right at 2.75 m/s while another 125 kg lineman is running directly toward him at 2.60 m/s. What are (a) the magnitude and direction of the net momentum of these two athletes, and (b) their total kinetic energy?

8.6 •• **BIO** **Biomechanics.** The mass of a regulation tennis ball is 57 g (although it can vary slightly), and tests have shown that the ball is in contact with the tennis racket for 30 ms. (This number can also vary, depending on the racket and swing.) We shall assume a 30.0 ms contact time. One of the fastest-known served tennis balls was served by "Big Bill" Tilden in 1931, and its speed was measured to be 73 m/s. (a) What impulse and what total force did Big Bill exert on the tennis ball in his record serve? (b) If Big Bill's opponent returned his serve with a speed of 55 m/s, what total force and what impulse did he exert on the ball, assuming only horizontal motion?

8.7 • **Force of a Golf Swing.** A 0.0450 kg golf ball initially at rest is given a speed of 25.0 m/s when a club strikes it. If the club and ball are in contact for 2.00 ms, what average force acts on the ball? Is the effect of the ball's weight during the time of contact significant? Why or why not?

8.8 • **Force of a Baseball Swing.** A baseball has mass 0.145 kg. (a) If the velocity of a pitched ball has a magnitude of 45.0 m/s and the batted ball's velocity is 55.0 m/s in the opposite direction, find the magnitude of the change in momentum of the ball and of the impulse applied to it by the bat. (b) If the ball remains in contact with the bat for 2.00 ms, find the magnitude of the average force applied by the bat.

8.9 • A 0.160 kg hockey puck is moving on an icy, frictionless, horizontal surface. At $t = 0$, the puck is moving to the right at 3.00 m/s. (a) Calculate the velocity of the puck (magnitude and direction) after a force of 25.0 N directed to the right has been applied for 0.050 s. (b) If, instead, a force of 12.0 N directed to the left is applied from $t = 0$ to $t = 0.050$ s, what is the final velocity of the puck?

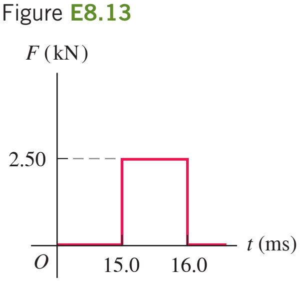
8.10 •• A bat strikes a 0.145 kg baseball. Just before impact, the ball is traveling horizontally to the right at 40.0 m/s; when it leaves the bat, the ball is traveling to the left at an angle of 30° above horizontal with a speed of 52.0 m/s. If the ball and bat are in contact for 1.75 ms, find the horizontal and vertical components of the average force on the ball.

8.11 • CALC At time $t = 0$ a 2150 kg rocket in outer space fires an engine that exerts an increasing force on it in the $+x$-direction. This force obeys the equation $F_x = At^2$, where t is time, and has a magnitude of 781.25 N when $t = 1.25$ s. (a) Find the SI value of the constant A, including its units. (b) What impulse does the engine exert on the rocket during the 1.50 s interval starting 2.00 s after the engine is fired? (c) By how much does the rocket's velocity change during this interval? Assume constant mass.

8.12 •• A packing crate with mass 80.0 kg is at rest on a horizontal, frictionless surface. At $t = 0$ a net horizontal force in the $+x$-direction is applied to the crate. The force has a constant value of 80.0 N for 12.0 s and then decreases linearly with time so it becomes zero after an additional 6.00 s. What is the final speed of the crate, 18.0 s after the force was first applied?

8.13 • A 2.00 kg stone is sliding to the right on a frictionless, horizontal surface at 5.00 m/s when it is suddenly struck by an object that exerts a large horizontal force on it for a short period of time. The graph in **Fig. E8.13** shows the magnitude of this force as a function of time. (a) What impulse does this force exert on the stone? (b) Just after the force stops acting, find the magnitude and direction of the stone's velocity if the force acts (i) to the right or (ii) to the left.

Figure E8.13



8.14 •• CALC Starting at $t = 0$, a horizontal net external force $\vec{F} = (0.280\ \text{N/s})t\hat{\imath} + (-0.450\ \text{N/s}^2)t^2\hat{\jmath}$ is applied to a box that has an initial momentum $\vec{p} = (-3.00\ \text{kg}\cdot\text{m/s})\hat{\imath} + (4.00\ \text{kg}\cdot\text{m/s})\hat{\jmath}$. What is the momentum of the box at $t = 2.00$ s?

8.15 • A young ice skater with mass 40.0 kg has fallen and is sliding on the frictionless ice of a skating rink with a speed of 20.0 m/s. (a) What is the magnitude of her linear momentum when she has this speed? (b) What is her kinetic energy? (c) What constant net horizontal force must be applied to the skater to bring her to rest in 5.00 s?

Section 8.2 Conservation of Momentum

8.16 • A 68.5 kg astronaut is doing a repair in space on the orbiting space station. She throws a 2.25 kg tool away from her at 3.20 m/s relative to the space station. What will be the change in her speed as a result of this throw?

8.17 •• The expanding gases that leave the muzzle of a rifle also contribute to the recoil. A .30 caliber bullet has mass 0.00720 kg and a speed of 601 m/s relative to the muzzle when fired from a rifle that has mass 2.80 kg. The loosely held rifle recoils at a speed of 1.85 m/s relative to the earth. Find the momentum of the propellant gases in a coordinate system attached to the earth as they leave the muzzle of the rifle.

8.18 • Two figure skaters, one weighing 625 N and the other 725 N, push off against each other on frictionless ice. (a) If the heavier skater travels at 1.50 m/s, how fast will the lighter one travel? (b) How much kinetic energy is "created" during the skaters' maneuver, and where does this energy come from?

8.19 • BIO **Animal Propulsion.** Squids and octopuses propel themselves by expelling water. They do this by keeping water in a cavity and then suddenly contracting the cavity to force out the water through an opening. A 6.50 kg squid (including the water in the cavity) at rest suddenly sees a dangerous predator. (a) If the squid has 1.75 kg of water in its cavity, at what speed must it expel this water suddenly to achieve a speed of 2.50 m/s to escape the predator? Ignore any drag effects of the surrounding water. (b) How much kinetic energy does the squid create by this maneuver?

8.20 •• You are standing on a sheet of ice that covers the football stadium parking lot in Buffalo; there is negligible friction between your feet and the ice. A friend throws you a 0.600 kg ball that is traveling horizontally at 10.0 m/s. Your mass is 70.0 kg. (a) If you catch the ball, with what speed do you and the ball move afterward? (b) If the ball hits you and bounces off your chest, so afterward it is moving horizontally at 8.0 m/s in the opposite direction, what is your speed after the collision?

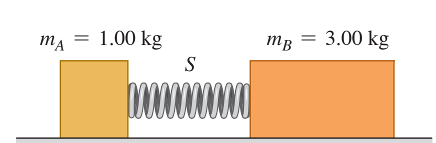
8.21 •• On a frictionless, horizontal air table, puck A (with mass 0.250 kg) is moving toward puck B (with mass 0.350 kg), which is initially at rest. After the collision, puck A has a velocity of 0.120 m/s to the left, and puck B has a velocity of 0.650 m/s to the right. (a) What was the speed of puck A before the collision? (b) Calculate the change in the total kinetic energy of the system that occurs during the collision.

8.22 •• When cars are equipped with flexible bumpers, they will bounce off each other during low-speed collisions, thus causing less damage. In one such accident, a 1750 kg car traveling to the right at 1.50 m/s collides with a 1450 kg car going to the left at 1.10 m/s. Measurements show that the heavier car's speed just after the collision was 0.250 m/s in its original direction. Ignore any road friction during the collision. (a) What was the speed of the lighter car just after the collision? (b) Calculate the change in the combined kinetic energy of the two-car system during this collision.

8.23 •• Two identical 0.900 kg masses are pressed against opposite ends of a light spring of force constant 1.75 N/cm, compressing the spring by 20.0 cm from its normal length. Find the speed of each mass when it has moved free of the spring on a frictionless, horizontal table.

8.24 • Block A in **Fig. E8.24** has mass 1.00 kg, and block B has mass 3.00 kg. The blocks are forced together, compressing a spring S between them; then the system is released from rest on a level, frictionless surface. The spring, which has negligible mass, is not fastened to either block and drops to the surface after it has expanded. Block B acquires a speed of 1.20 m/s. (a) What is the final speed of block A? (b) How much potential energy was stored in the compressed spring?

Figure E8.24



8.25 •• A hunter on a frozen, essentially frictionless pond uses a rifle that shoots 4.20 g bullets at 965 m/s. The mass of the hunter (including his gun) is 72.5 kg, and the hunter holds tight to the gun after firing it. Find the recoil velocity of the hunter if he fires the rifle (a) horizontally and (b) at 56.0° above the horizontal.

8.26 • An atomic nucleus suddenly bursts apart (fissions) into two pieces. Piece A, of mass m_A, travels off to the left with speed v_A. Piece B, of mass m_B, travels off to the right with speed v_B. (a) Use conservation of momentum to solve for v_B in terms of m_A, m_B, and v_A. (b) Use the results of part (a) to show that $K_A/K_B = m_B/m_A$, where K_A and K_B are the kinetic energies of the two pieces.

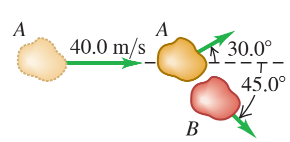
8.27 •• Two ice skaters, Daniel (mass 65.0 kg) and Rebecca (mass 45.0 kg), are practicing. Daniel stops to tie his shoelace and, while at rest, is struck by Rebecca, who is moving at 13.0 m/s before she collides with him. After the collision, Rebecca has a velocity of magnitude 8.00 m/s at an angle of 53.1° from her initial direction. Both skaters move on the frictionless, horizontal surface of the rink. (a) What are the magnitude and direction of Daniel's velocity after the collision? (b) What is the change in total kinetic energy of the two skaters as a result of the collision?

8.28 •• You are standing on a large sheet of frictionless ice and holding a large rock. In order to get off the ice, you throw the rock so it has velocity 12.0 m/s relative to the earth at an angle of 35.0° above the horizontal. If your mass is 70.0 kg and the rock's mass is 3.00 kg, what is your speed after you throw the rock? (See Discussion Question Q8.7.)

8.29 •• You (mass 55 kg) are riding a frictionless skateboard (mass 5.0 kg) in a straight line at a speed of 4.5 m/s. A friend standing on a balcony above you drops a 2.5 kg sack of flour straight down into your arms. (a) What is your new speed while you hold the sack? (b) Since the sack was dropped vertically, how can it affect your *horizontal* motion? Explain. (c) Now you try to rid yourself of the extra weight by throwing the sack straight up. What will be your speed while the sack is in the air? Explain.

8.30 •• **Asteroid Collision.** Two asteroids of equal mass in the asteroid belt between Mars and Jupiter collide with a glancing blow. Asteroid A, which was initially traveling at 40.0 m/s, is deflected 30.0° from its original direction, while asteroid B, which was initially at rest, travels at 45.0° to the original direction of A (**Fig. E8.30**). (a) Find the speed of each asteroid after the collision. (b) What fraction of the original kinetic energy of asteroid A dissipates during this collision?

Figure **E8.30**



Section 8.3 Momentum Conservation and Collisions

8.31 • An ice hockey forward with mass 70.0 kg is skating due north with a speed of 5.5 m/s. As the forward approaches the net for a slap shot, a defensive player (mass 110 kg) skates toward him in order to apply a body-check. The defensive player is traveling south at 4.0 m/s just before they collide. If the two players become intertwined and move together after they collide, in what direction and at what speed do they move after the collision? Friction between the two players and the ice can be neglected.

8.32 • Two skaters collide and grab on to each other on frictionless ice. One of them, of mass 70.0 kg, is moving to the right at 4.00 m/s, while the other, of mass 65.0 kg, is moving to the left at 2.50 m/s. What are the magnitude and direction of the velocity of these skaters just after they collide?

8.33 •• A 15.0 kg fish swimming at 1.10 m/s suddenly gobbles up a 4.50 kg fish that is initially stationary. Ignore any drag effects of the water. (a) Find the speed of the large fish just after it eats the small one. (b) How much total mechanical energy was dissipated during this meal?

8.34 •• CP An apple with mass M is hanging at rest from the lower end of a light vertical rope. A dart of mass $M/4$ is shot vertically upward, strikes the bottom of the apple, and remains embedded in it. If the speed of the dart is v_0 just before it strikes the apple, how high does the apple move upward because of its collision with the dart?

8.35 •• Two large blocks of wood are sliding toward each other on the frictionless surface of a frozen pond. Block A has mass 4.00 kg and is initially sliding east at 2.00 m/s. Block B has mass 6.00 kg and is initially sliding west at 2.50 m/s. The blocks collide head-on. After the collision block B is sliding east at 0.50 m/s. What is the decrease in the total kinetic energy of the two blocks as a result of the collision?

8.36 • A 1050 kg sports car is moving westbound at 15.0 m/s on a level road when it collides with a 6320 kg truck driving east on the same road at 10.0 m/s. The two vehicles remain locked together after the collision. (a) What is the velocity (magnitude and direction) of the two vehicles just after the collision? (b) At what speed should the truck have been moving so that both it and the car are stopped in the collision? (c) Find the change in kinetic energy of the system of two vehicles for the situations of parts (a) and (b). For which situation is the change in kinetic energy greater in magnitude?

8.37 •• On a very muddy football field, a 110 kg linebacker tackles an 85 kg halfback. Immediately before the collision, the linebacker is slipping with a velocity of 8.8 m/s north and the halfback is sliding with a velocity of 7.2 m/s east. What is the velocity (magnitude and direction) at which the two players move together immediately after the collision?

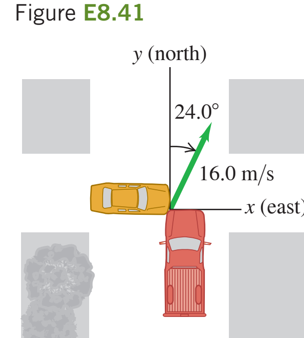
8.38 •• **Accident Analysis.** Two cars collide at an intersection. Car A, with a mass of 2000 kg, is going from west to east, while car B, of mass 1500 kg, is going from north to south at 15 m/s. As a result, the two cars become enmeshed and move as one. As an expert witness, you inspect the scene and determine that, after the collision, the enmeshed cars moved at an angle of 65° south of east from the point of impact. (a) How fast were the enmeshed cars moving just after the collision? (b) How fast was car A going just before the collision?

8.39 •• Jack (mass 55.0 kg) is sliding due east with speed 8.00 m/s on the surface of a frozen pond. He collides with Jill (mass 48.0 kg), who is initially at rest. After the collision, Jack is traveling at 5.00 m/s in a direction 34.0° north of east. What is Jill's velocity (magnitude and direction) after the collision? Ignore friction.

8.40 •• BIO **Bird Defense.** To protect their young in the nest, peregrine falcons will fly into birds of prey (such as ravens) at high speed. In one such episode, a 600 g falcon flying at 20.0 m/s hit a 1.50 kg raven flying at 9.0 m/s. The falcon hit the raven at right angles to the raven's original path and bounced back at 5.0 m/s. (These figures were estimated by the author as he watched this attack occur in northern New Mexico.) (a) By what angle did the falcon change the raven's direction of motion? (b) What was the raven's speed right after the collision?

8.41 • At the intersection of Texas Avenue and University Drive, a yellow subcompact car with mass 950 kg traveling east on University collides with a red pickup truck with mass 1900 kg that is traveling north on Texas and has run a red light (**Fig. E8.41**). The two vehicles stick together as a result of the collision, and the wreckage slides at 16.0 m/s in the direction 24.0° east of north. Calculate the speed of each vehicle before the collision. The collision occurs during a heavy rainstorm; ignore friction forces between the vehicles and the wet road.

Figure **E8.41**

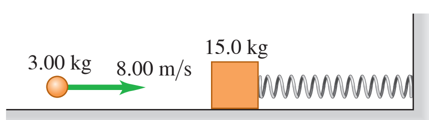


8.42 •• A 5.00 g bullet is fired horizontally into a 1.20 kg wooden block resting on a horizontal surface. The coefficient of kinetic friction between block and surface is 0.20. The bullet remains embedded in the block, which is observed to slide 0.310 m along the surface before stopping. What was the initial speed of the bullet?

8.43 •• **A Ballistic Pendulum.** A 12.0 g rifle bullet is fired with a speed of 380 m/s into a ballistic pendulum with mass 6.00 kg, suspended from a cord 70.0 cm long (see Example 8.8 in Section 8.3). Compute (a) the vertical height through which the pendulum rises, (b) the initial kinetic energy of the bullet, and (c) the kinetic energy of the bullet and pendulum immediately after the bullet becomes embedded in the wood.

8.44 •• **Combining Conservation Laws.** A 15.0 kg block is attached to a very light horizontal spring of force constant 500.0 N/m and is resting on a frictionless horizontal table (**Fig. E8.44**). Suddenly it is struck by a 3.00 kg stone traveling horizontally at 8.00 m/s to the right, whereupon the stone rebounds at 2.00 m/s horizontally to the left. Find the maximum distance that the block will compress the spring after the collision.

Figure **E8.44**



8.45 •• **CP** A 0.800 kg ornament is hanging by a 1.50 m wire when the ornament is suddenly hit by a 0.200 kg missile traveling horizontally at 12.0 m/s. The missile embeds itself in the ornament during the collision. What is the tension in the wire immediately after the collision?

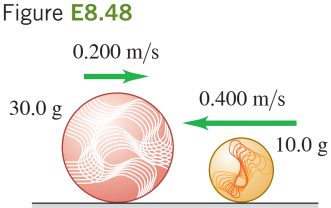
Section 8.4 Elastic Collisions

8.46 •• A 0.150 kg glider is moving to the right with a speed of 0.80 m/s on a frictionless, horizontal air track. The glider has a head-on collision with a 0.300 kg glider that is moving to the left with a speed of 2.20 m/s. Find the final velocity (magnitude and direction) of each glider if the collision is elastic.

8.47 •• Blocks *A* (mass 2.00 kg) and *B* (mass 6.00 kg) move on a frictionless, horizontal surface. Initially, block *B* is at rest and block *A* is moving toward it at 2.00 m/s. The blocks are equipped with ideal spring bumpers, as in Example 8.10 (Section 8.4). The collision is head-on, so all motion before and after the collision is along a straight line. (a) Find the maximum energy stored in the spring bumpers and the velocity of each block at that time. (b) Find the velocity of each block after they have moved apart.

8.48 • A 10.0 g marble slides to the left at a speed of 0.400 m/s on the frictionless, horizontal surface of an icy New York sidewalk and has a head-on, elastic collision with a larger 30.0 g marble sliding to the right at a speed of 0.200 m/s (**Fig. E8.48**). (a) Find the velocity of each marble (magnitude and direction) after the collision. (Since the collision is head-on, all motion is along a line.) (b) Calculate the *change in momentum* (the momentum after the collision minus the momentum before the collision) for each marble. Compare your values for each marble. (c) Calculate the *change in kinetic energy* (the kinetic energy after the collision minus the kinetic energy before the collision) for each marble. Compare your values for each marble.

Figure **E8.48**



8.49 •• **Moderators.** Canadian nuclear reactors use *heavy water* moderators in which elastic collisions occur between the neutrons and deuterons of mass 2.0 u (see Example 8.11 in Section 8.4). (a) What is the speed of a neutron, expressed as a fraction of its original speed, after a head-on, elastic collision with a deuteron that is initially at rest? (b) What is its kinetic energy, expressed as a fraction of its original kinetic energy? (c) How many such successive collisions will reduce the speed of a neutron to 1/59,000 of its original value?

8.50 •• You are at the controls of a particle accelerator, sending a beam of 1.50×10^7 m/s protons (mass *m*) at a gas target of an unknown element. Your detector tells you that some protons bounce straight back after a collision with one of the nuclei of the unknown element. All such protons rebound with a speed of 1.20×10^7 m/s. Assume that the initial speed of the target nucleus is negligible and the collision is elastic. (a) Find the mass of one nucleus of the unknown element. Express your answer in terms of the proton mass *m*. (b) What is the speed of the unknown nucleus immediately after such a collision?

8.51 •• Object *B* is at rest when object *A* collides with it. The collision is one-dimensional and elastic. After the collision object *B* has half the velocity that object *A* had before the collision. (a) Which object has the greater mass? (b) How much greater? (c) If the velocity of object *A* before the collision was 6.0 m/s to the right, what is its velocity after the collision?

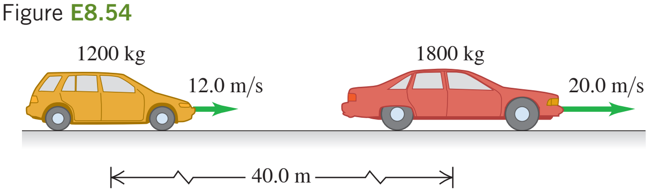
Section 8.5 Center of Mass

8.52 • Find the position of the center of mass of the system of the sun and Jupiter. (Since Jupiter is more massive than the rest of the solar planets combined, this is essentially the position of the center of mass of the solar system.) Does the center of mass lie inside or outside the sun? Use the data in Appendix F.

8.53 •• **Pluto and Charon.** Pluto's diameter is approximately 2370 km, and the diameter of its satellite Charon is 1250 km. Although the distance varies, they are often about 19,700 km apart, center to center. Assuming that both Pluto and Charon have the same composition and hence the same average density, find the location of the center of mass of this system relative to the center of Pluto.

8.54 • A 1200 kg SUV is moving along a straight highway at 12.0 m/s. Another car, with mass 1800 kg and speed 20.0 m/s, has its center of mass 40.0 m ahead of the center of mass of the SUV (**Fig. E8.54**). Find (a) the position of the center of mass of the system consisting of the two cars; (b) the magnitude of the system's total momentum, by using the given data; (c) the speed of the system's center of mass; (d) the system's total momentum, by using the speed of the center of mass. Compare your result with that of part (b).

Figure **E8.54**



8.55 • A uniform cube with mass 0.500 kg and volume 0.0270 m^3 is sitting on the floor. A uniform sphere with radius 0.400 m and mass 0.800 kg sits on top of the cube. How far is the center of mass of the two-object system above the floor?

8.56 • At one instant, the center of mass of a system of two particles is located on the x-axis at $x = 2.0$ m and has a velocity of $(5.0\text{ m/s})\hat{\imath}$. One of the particles is at the origin. The other particle has a mass of 0.10 kg and is at rest on the x-axis at $x = 8.0$ m. (a) What is the mass of the particle at the origin? (b) Calculate the total momentum of this system. (c) What is the velocity of the particle at the origin?

8.57 •• In Example 8.14 (Section 8.5), Ramon pulls on the rope to give himself a speed of 1.10 m/s. What is James's speed?

8.58 • **CALC** A system consists of two particles. At $t = 0$ one particle is at the origin; the other, which has a mass of 0.50 kg, is on the y-axis at $y = 6.0$ m. At $t = 0$ the center of mass of the system is on the y-axis at $y = 2.4$ m. The velocity of the center of mass is given by $(0.75\text{m/s}^3)t^2\hat{\imath}$. (a) Find the total mass of the system. (b) Find the acceleration of the center of mass at any time t. (c) Find the net external force acting on the system at $t = 3.0$ s.

8.59 • **CALC** A radio-controlled model airplane has a momentum given by $[(-0.75\text{ kg}\cdot\text{m/s}^3)t^2 + (3.0\text{ kg}\cdot\text{m/s})]\hat{\imath} + (0.25\text{ kg}\cdot\text{m/s}^2)t\hat{\jmath}$. What are the x-, y-, and z-components of the net external force on the airplane?

8.60 •• You have three identical, uniform, square pieces of wood, each with side length L. You stack the three pieces of wood at the edge of the horizontal top of a table. The first block extends a distance $L/4$ past the edge of the table. The next block extends a distance $L/4$ past the edge of the first block, so a distance $L/2$ past the edge of the table. The third block extends a distance $L/4$ past the edge of the block beneath it, so $3L/4$ past the edge of the table. The stack is unstable if the center of mass of the stack extends beyond the edge of the table. Calculate the horizontal location of the center of mass of the three-block stack.

Section 8.6 Rocket Propulsion

8.61 •• A 70 kg astronaut floating in space in a 110 kg MMU (manned maneuvering unit) experiences an acceleration of 0.029 m/s^2 when he fires one of the MMU's thrusters. (a) If the speed of the escaping N_2 gas relative to the astronaut is 490 m/s, how much gas is used by the thruster in 5.0 s? (b) What is the thrust of the thruster?

8.62 • A small rocket burns 0.0500 kg of fuel per second, ejecting it as a gas with a velocity relative to the rocket of magnitude 1600 m/s. (a) What is the thrust of the rocket? (b) Would the rocket operate in outer space where there is no atmosphere? If so, how would you steer it? Could you brake it?

PROBLEMS

8.63 •• **CP** A large shipping crate is at rest on the horizontal floor of a warehouse. The coefficient of static friction between the crate and the floor is $\mu_s = 0.500$; the coefficient of kinetic friction is $\mu_k = 0.300$. (a) Estimate the weight, in pounds, of the heaviest crate you could start sliding by pushing on it horizontally. Based on this estimate, what is the magnitude of the maximum force you can apply? (b) If you continue to push on the crate with a constant force equal to the force calculated in part (a), use the impulse–momentum theorem to calculate how long you must push on the crate to give it a speed of 8.0 m/s. Don't forget to take into account the kinetic friction force. (c) With the same force, how far would the crate travel before reaching a speed of 8.0 m/s? Use the work–energy theorem.

8.64 •• A steel ball with mass 40.0 g is dropped from a height of 2.00 m onto a horizontal steel slab. The ball rebounds to a height of 1.60 m. (a) Calculate the impulse delivered to the ball during impact. (b) If the ball is in contact with the slab for 2.00 ms, find the average force on the ball during impact.

8.65 •• Just before it is struck by a racket, a tennis ball weighing 0.560 N has a velocity of $(20.0\text{ m/s})\hat{\imath} - (4.0\text{ m/s})\hat{\jmath}$. During the 3.00 ms that the racket and ball are in contact, the net external force on the ball is constant and equal to $-(380\text{ N})\hat{\imath} + (110\text{ N})\hat{\jmath}$. What are the x- and y-components (a) of the impulse of the net external force applied to the ball; (b) of the final velocity of the ball?

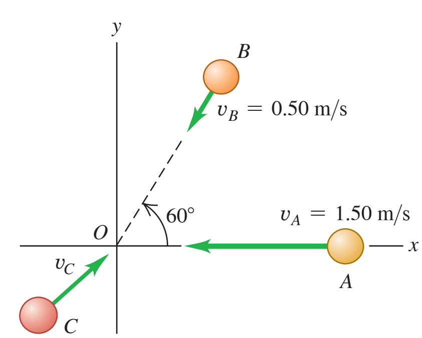
8.66 •• Starting at $t = 0$ a net external force in the $+x$-direction is applied to an object that has mass 2.00 kg. A graph of the force as a function of time is a straight line that passes through the origin and has slope 3.00 N/s. If the object is at rest at $t = 0$, what is the magnitude of the force when the object has reached a speed of 9.00 m/s?

8.67 •• Blocks A (mass 2.00 kg) and B (mass 10.00 kg, to the right of A) move on a frictionless, horizontal surface. Initially, block B is moving to the left at 0.500 m/s and block A is moving to the right at 2.00 m/s. The blocks are equipped with ideal spring bumpers, as in Example 8.10 (Section 8.4). The collision is head-on, so all motion before and after it is along a straight line. Find (a) the maximum energy stored in the spring bumpers and the velocity of each block at that time; (b) the velocity of each block after they have moved apart.

8.68 •• A railroad handcar is moving along straight, frictionless tracks with negligible air resistance. In the following cases, the car initially has a total mass (car and contents) of 200 kg and is traveling east with a velocity of magnitude 5.00 m/s. Find the *final velocity* of the car in each case, assuming that the handcar does not leave the tracks. (a) A 25.0 kg mass is thrown sideways out of the car with a velocity of magnitude 2.00 m/s relative to the car's initial velocity. (b) A 25.0 kg mass is thrown backward out of the car with a velocity of 5.00 m/s relative to the initial motion of the car. (c) A 25.0 kg mass is thrown into the car with a velocity of 6.00 m/s relative to the ground and opposite in direction to the initial velocity of the car.

8.69 • Spheres A (mass 0.020 kg), B (mass 0.030 kg), and C (mass 0.050 kg) are approaching the origin as they slide on a frictionless air table. The initial velocities of A and B are given in **Fig. P8.69**. All three spheres arrive at the origin at the same time and stick together. (a) What must the x- and y-components of the initial velocity of C be if all three objects are to end up moving at 0.50 m/s in the $+x$-direction after the collision? (b) If C has the velocity found in part (a), what is the change in the kinetic energy of the system of three spheres as a result of the collision?

Figure **P8.69**



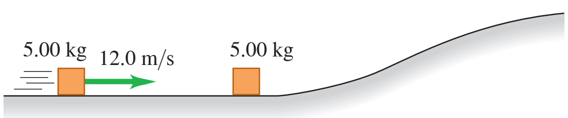
8.70 •• Starting at $t = 0$ a net external force $F(t)$ in the $+x$-direction is applied to an object that has mass 3.00 kg and is initially at rest. The force is zero at $t = 0$ and increases linearly to 5.00 N at $t = 4.00$ s. The force then decreases linearly until it becomes zero at $t = 10.0$ s. (a) Draw a graph of F versus t from $t = 0$ to $t = 10.0$ s. (b) What is the speed of the object at $t = 10.0$ s?

8.71 •• **CP** An 8.00 kg block of wood sits at the edge of a frictionless table, 2.20 m above the floor. A 0.500 kg blob of clay slides along the length of the table with a speed of 24.0 m/s, strikes the block of wood, and sticks to it. The combined object leaves the edge of the table and travels to the floor. What horizontal distance has the combined object traveled when it reaches the floor?

8.72 ••• CP A small wooden block with mass 0.800 kg is suspended from the lower end of a light cord that is 1.60 m long. The block is initially at rest. A bullet with mass 12.0 g is fired at the block with a horizontal velocity v_0. The bullet strikes the block and becomes embedded in it. After the collision the combined object swings on the end of the cord. When the block has risen a vertical height of 0.800 m, the tension in the cord is 4.80 N. What was the initial speed v_0 of the bullet?

8.73 •• Combining Conservation Laws. A 5.00 kg chunk of ice is sliding at 12.0 m/s on the floor of an ice-covered valley when it collides with and sticks to another 5.00 kg chunk of ice that is initially at rest (**Fig. P8.73**). Since the valley is icy, there is no friction. After the collision, how high above the valley floor will the combined chunks go?

Figure **P8.73**



8.74 •• CP Block *B* (mass 4.00 kg) is at rest at the edge of a smooth platform, 2.60 m above the floor. Block *A* (mass 2.00 kg) is sliding with a speed of 8.00 m/s along the platform toward block *B*. *A* strikes *B* and rebounds with a speed of 2.00 m/s. The collision projects *B* horizontally off the platform. What is the speed of *B* just before it strikes the floor?

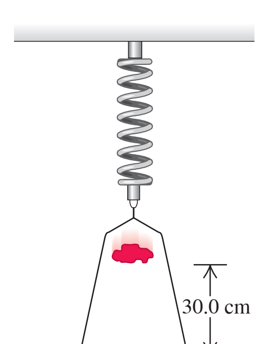
8.75 •• Two carts of equal mass are on a horizontal, frictionless air track. Initially cart *A* is moving toward stationary cart *B* with a speed of v_A. The carts undergo an inelastic collision, and after the collision the total kinetic energy of the two carts is one-half their initial total kinetic energy before the collision. What is the speed of each cart after the collision?

8.76 •• Automobile Accident Analysis. You are called as an expert witness to analyze the following auto accident: Car *B*, of mass 1900 kg, was stopped at a red light when it was hit from behind by car *A*, of mass 1500 kg. The cars locked bumpers during the collision and slid to a stop with brakes locked on all wheels. Measurements of the skid marks left by the tires showed them to be 7.15 m long. The coefficient of kinetic friction between the tires and the road was 0.65. (a) What was the speed of car *A* just before the collision? (b) If the speed limit was 35 mph, was car *A* speeding, and if so, by how many miles per hour was it *exceeding* the speed limit?

8.77 •• A 1500 kg sedan goes through a wide intersection traveling from north to south when it is hit by a 2200 kg SUV traveling from east to west. The two cars become enmeshed due to the impact and slide as one thereafter. On-the-scene measurements show that the coefficient of kinetic friction between the tires of these cars and the pavement is 0.75, and the cars slide to a halt at a point 5.39 m west and 6.43 m south of the impact point. How fast was each car traveling just before the collision?

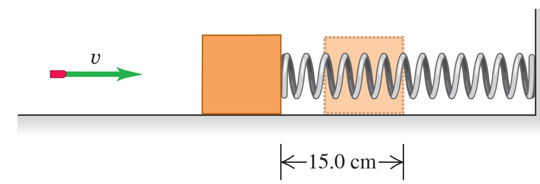
8.78 ••• CP A 0.150 kg frame, when suspended from a coil spring, stretches the spring 0.0400 m. A 0.200 kg lump of putty is dropped from rest onto the frame from a height of 30.0 cm (**Fig. P8.78**). Find the maximum distance the frame moves downward from its initial equilibrium position.

Figure **P8.78**



8.79 • A rifle bullet with mass 8.00 g strikes and embeds itself in a block with mass 0.992 kg that rests on a frictionless, horizontal surface and is attached to an ideal spring (**Fig. P8.79**). The impact compresses the spring 15.0 cm. Calibration of the spring shows that a force of 0.750 N is required to compress the spring 0.250 cm. (a) Find the magnitude of the block's velocity just after impact. (b) What was the initial speed of the bullet?

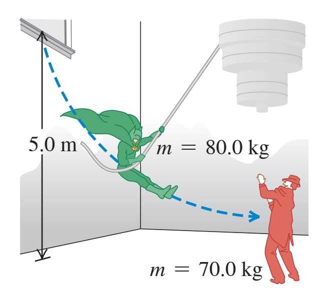
Figure **P8.79**



8.80 •• A Ricocheting Bullet. A 0.100 kg stone rests on a frictionless, horizontal surface. A bullet of mass 6.00 g, traveling horizontally at 350 m/s, strikes the stone and rebounds horizontally at right angles to its original direction with a speed of 250 m/s. (a) Compute the magnitude and direction of the velocity of the stone after it is struck. (b) Is the collision perfectly elastic?

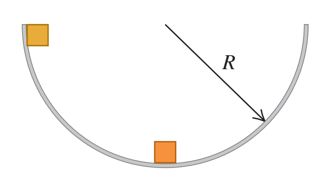
8.81 •• A movie stuntman (mass 80.0 kg) stands on a window ledge 5.0 m above the floor (**Fig. P8.81**). Grabbing a rope attached to a chandelier, he swings down to grapple with the movie's villain (mass 70.0 kg), who is standing directly under the chandelier. (Assume that the stuntman's center of mass moves downward 5.0 m. He releases the rope just as he reaches the villain.) (a) With what speed do the entwined foes start to slide across the floor? (b) If the coefficient of kinetic friction of their bodies with the floor is $\mu_k = 0.250$, how far do they slide?

Figure **P8.81**



8.82 •• CP Two identical masses are released from rest in a smooth hemispherical bowl of radius *R* from the positions shown in **Fig. P8.82**. Ignore friction between the masses and the surface of the bowl. If the masses stick together when they collide, how high above the bottom of the bowl will they go after colliding?

Figure **P8.82**



8.83 •• Objects *A* and *B* undergo a one-dimensional elastic collision. The initial speed of *A* is v_{Ai} and the initial speed of *B* is zero. Equations (8.24) and (8.25) give the final velocity components of the objects after the collision. Let $m_A = \alpha m_B$, where α is a constant. (a) What is the value of α if the final kinetic energy of *B* equals the initial kinetic energy of *A*? (b) What is α if the final kinetic energies of *A* and *B* are equal?

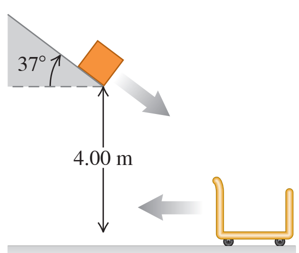
8.84 ••• CP A 20.00 kg lead sphere is hanging from a hook by a thin wire 2.80 m long and is free to swing in a complete circle. Suddenly it is struck horizontally by a 5.00 kg steel dart that embeds itself in the lead sphere. What must be the minimum initial speed of the dart so that the combination makes a complete circular loop after the collision?

8.85 •• A 4.00 g bullet, traveling horizontally with a velocity of magnitude 400 m/s, is fired into a wooden block with mass 0.800 kg, initially at rest on a level surface. The bullet passes through the block and emerges with its speed reduced to 190 m/s. The block slides a distance of 72.0 cm along the surface from its initial position. (a) What is the coefficient of kinetic friction between block and surface? (b) What is the decrease in kinetic energy of the bullet? (c) What is the kinetic energy of the block at the instant after the bullet passes through it?

8.86 •• A 5.00 g bullet is shot *through* a 1.00 kg wood block suspended on a string 2.00 m long. The center of mass of the block rises a distance of 0.38 cm. Find the speed of the bullet as it emerges from the block, along a horizontal, straight line, if its initial speed is 450 m/s.

8.87 •• CP In a shipping company distribution center, an open cart of mass 50.0 kg is rolling to the left at a speed of 5.00 m/s (**Fig. P8.87**). Ignore friction between the cart and the floor. A 15.0 kg package slides down a chute that is inclined at 37° from the horizontal and leaves the end of the chute with a speed of 3.00 m/s. The package lands in the cart and they roll together. If the lower end of the chute is a vertical distance of 4.00 m above the bottom of the cart, what are (a) the speed of the package just before it lands in the cart and (b) the final speed of the cart?

Figure P8.87



8.88 ••• **Neutron Decay**. A neutron at rest decays (breaks up) to a proton and an electron. Energy is released in the decay and appears as kinetic energy of the proton and electron. The mass of a proton is 1836 times the mass of an electron. What fraction of the total energy released goes into the kinetic energy of the proton?

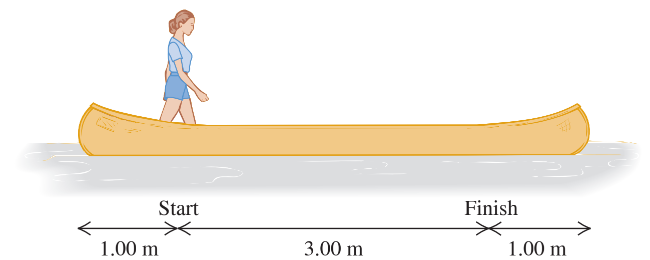
8.89 •• CP In a game of physics and skill, a rigid block (A) with mass m sits at rest at the edge of a frictionless air table, 1.20 m above the floor. You slide an identical block (B) with initial speed v_B toward A. The blocks have a head-on elastic collision, and block A leaves the table with a horizontal velocity. The goal of the game is to have block A land on a target on the floor. The target is a horizontal distance of 2.00 m from the edge of the table. What is the initial speed v_B that accomplishes this? Neglect air resistance.

8.90 •• Jonathan and Jane are sitting in a sleigh that is at rest on frictionless ice. Jonathan's weight is 800 N, Jane's weight is 600 N, and that of the sleigh is 1000 N. They see a poisonous spider on the floor of the sleigh and immediately jump off. Jonathan jumps to the left with a velocity of 5.00 m/s at 30.0° above the horizontal (relative to the ice), and Jane jumps to the right at 7.00 m/s at 36.9° above the horizontal (relative to the ice). Calculate the sleigh's horizontal velocity (magnitude and direction) after they jump out.

8.91 •• Friends Burt and Ernie stand at opposite ends of a uniform log that is floating in a lake. The log is 3.0 m long and has mass 20.0 kg. Burt has mass 30.0 kg; Ernie has mass 40.0 kg. Initially, the log and the two friends are at rest relative to the shore. Burt then offers Ernie a cookie, and Ernie walks to Burt's end of the log to get it. Relative to the shore, what distance has the log moved by the time Ernie reaches Burt? Ignore any horizontal force that the water exerts on the log, and assume that neither friend falls off the log.

8.92 •• A 45.0 kg woman stands up in a 60.0 kg canoe 5.00 m long. She walks from a point 1.00 m from one end to a point 1.00 m from the other end (**Fig. P8.92**). If you ignore resistance to motion of the canoe in the water, how far does the canoe move during this process?

Figure P8.92



8.93 •• CP Two sticky spheres are suspended from light ropes of length L that are attached to the ceiling at a common point. Sphere A has mass $2m$ and is hanging at rest with its rope vertical. Sphere B has mass m and is held so that its rope makes an angle with the vertical that puts B a vertical height H above A. Sphere B is released from rest and swings down, collides with sphere A, and sticks to it. In terms of H, what is the maximum height above the original position of A reached by the combined spheres after their collision?

8.94 •• CP In a fireworks display, a rocket is launched from the ground with a speed of 18.0 m/s and a direction of 51.0° above the horizontal. During the flight, the rocket explodes into two pieces of equal mass (see Fig. 8.32). (a) What horizontal distance from the launch point will the center of mass of the two pieces be after both have landed on the ground? (b) If one piece lands a horizontal distance of 26.0 m from the launch point, where does the other piece land?

8.95 •• A block with mass 0.500 kg sits at rest on a light but not long vertical spring that has spring constant 80.0 N/m and one end on the floor. (a) How much elastic potential energy is stored in the spring when the block is sitting at rest on it? (b) A second identical block is dropped onto the first from a height of 4.00 m above the first block and sticks to it. What is the maximum elastic potential energy stored in the spring during the motion of the blocks after the collision? (c) What is the maximum distance the first block moves down after the second block has landed on it?

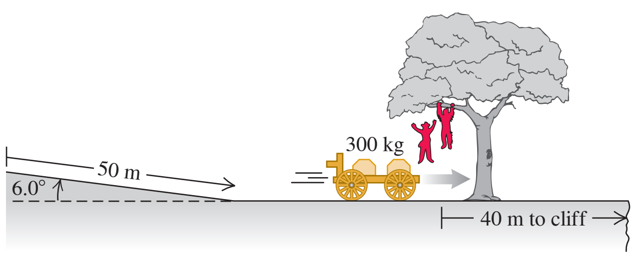
8.96 •• CP A 20.0 kg projectile is fired at an angle of 60.0° above the horizontal with a speed of 80.0 m/s. At the highest point of its trajectory, the projectile explodes into two fragments with equal mass, one of which falls vertically with zero initial speed. Ignore air resistance. (a) How far from the point of firing does the other fragment strike if the terrain is level? (b) How much energy is released during the explosion?

8.97 ••• CP A fireworks rocket is fired vertically upward. At its maximum height of 80.0 m, it explodes and breaks into two pieces: one with mass 1.40 kg and the other with mass 0.28 kg. In the explosion, 860 J of chemical energy is converted to kinetic energy of the two fragments. (a) What is the speed of each fragment just after the explosion? (b) It is observed that the two fragments hit the ground at the same time. What is the distance between the points on the ground where they land? Assume that the ground is level and air resistance can be ignored.

8.98 ••• A 12.0 kg shell is launched at an angle of 55.0° above the horizontal with an initial speed of 150 m/s. At its highest point, the shell explodes into two fragments, one three times heavier than the other. The two fragments reach the ground at the same time. Ignore air resistance. If the heavier fragment lands back at the point from which the shell was launched, where will the lighter fragment land, and how much energy was released in the explosion?

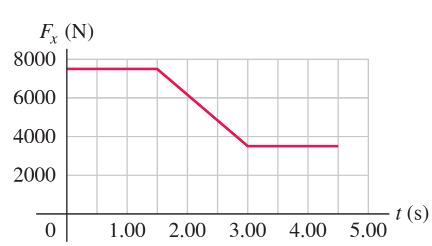
8.99 • **CP** An outlaw cuts loose a wagon with two boxes of gold, of total mass 300 kg, when the wagon is at rest 50 m up a 6.0° slope. The outlaw plans to have the wagon roll down the slope and across the level ground, and then fall into a canyon where his accomplices wait. But in a tree 40 m from the canyon's cliff wait the Lone Ranger (mass 75.0 kg) and Tonto (mass 60.0 kg). They drop vertically into the wagon as it passes beneath them (**Fig. P8.99**). (a) If they require 5.0 s to grab the gold and jump out, will they make it before the wagon goes over the cliff? The wagon rolls with negligible friction. (b) When the two heroes drop into the wagon, is the kinetic energy of the system of heroes plus wagon conserved? If not, does it increase or decrease, and by how much?

Figure **P8.99**



8.100 •• **DATA** A 2004 Prius with a 150 lb driver and no passengers weighs 3071 lb. The car is initially at rest. Starting at $t = 0$, a net horizontal force $F_x(t)$ in the $+x$-direction is applied to the car. The force as a function of time is given in **Fig. P8.100**. (a) For the time interval $t = 0$ to $t = 4.50$ s, what is the impulse applied to the car? (b) What is the speed of the car at $t = 4.50$ s? (c) At $t = 4.50$ s, the 3500 N net external force is replaced by a constant net braking force $B_x = -5200$ N. Once the braking force is first applied, how long does it take the car to stop? (d) How much work must be done on the car by the braking force to stop the car? (e) What distance does the car travel from the time the braking force is first applied until the car stops?

Figure **P8.100**



8.101 •• **DATA** In your job in a police lab, you must design an apparatus to measure the muzzle velocities of bullets fired from handguns. Your solution is to attach a 2.00 kg wood block that rests on a horizontal surface to a light horizontal spring. The other end of the spring is attached to a wall. Initially the spring is at its equilibrium length. A bullet is fired horizontally into the block and remains embedded in it. After the bullet strikes the block, the block compresses the spring a maximum distance d. You have measured that the coefficient of kinetic friction between the block and the horizontal surface is 0.38. The table lists some firearms that you'll test:

Bullet ID	Type	Bullet Mass (grains)	Muzzle Velocity (ft/s)
A	.38Spec Glaser Blue	80	1667
B	.38Spec Federal	125	945
C	.44Spec Remington	240	851
D	.44Spec Winchester	200	819
E	.45ACP Glaser Blue	140	1355

Source: www.chuckhawks.com

A grain is a unit of mass equal to 64.80 mg. (a) Of bullets A through E, which will produce the maximum compression of the spring? The minimum? (b) You want the maximum compression of the spring to be 0.25 m. What must be the force constant of the spring? (c) For the bullet that produces the minimum spring compression, what is the compression d if the spring has the force constant calculated in part (b)?

8.102 •• **DATA** For the Texas Department of Public Safety, you are investigating an accident that occurred early on a foggy morning in a remote section of the Texas Panhandle. A 2012 Prius traveling due north collided in a highway intersection with a 2013 Dodge Durango that was traveling due east. After the collision, the wreckage of the two vehicles was locked together and skidded across the level ground until it struck a tree. You measure that the tree is 35 ft from the point of impact. The line from the point of impact to the tree is in a direction 39° north of east. From experience, you estimate that the coefficient of kinetic friction between the ground and the wreckage is 0.45. Shortly before the collision, a highway patrolman with a radar gun measured the speed of the Prius to be 50 mph and, according to a witness, the Prius driver made no attempt to slow down. Four people with a total weight of 460 lb were in the Durango. The only person in the Prius was the 150 lb driver. The Durango with its passengers had a weight of 6500 lb, and the Prius with its driver had a weight of 3042 lb. (a) What was the Durango's speed just before the collision? (b) How fast was the wreckage traveling just before it struck the tree?

CHALLENGE PROBLEMS

8.103 •• **CALC** **A Variable-Mass Raindrop.** In a rocket-propulsion problem the mass is variable. Another such problem is a raindrop falling through a cloud of small water droplets. Some of these small droplets adhere to the raindrop, thereby *increasing* its mass as it falls. The force on the raindrop is

$$F_{\text{ext}} = \frac{dp}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

Suppose the mass of the raindrop depends on the distance x that it has fallen. Then $m = kx$, where k is a constant, and $dm/dt = kv$. This gives, since $F_{\text{ext}} = mg$,

$$mg = m\frac{dv}{dt} + v(kv)$$

Or, dividing by k,

$$xg = x\frac{dv}{dt} + v^2$$

This is a differential equation that has a solution of the form $v = at$, where a is the acceleration and is constant. Take the initial velocity of the raindrop to be zero. (a) Using the proposed solution for v, find the acceleration a. (b) Find the distance the raindrop has fallen in $t = 3.00$ s. (c) Given that $k = 2.00$ g/m, find the mass of the raindrop at $t = 3.00$ s. (For many more intriguing aspects of this problem, see K. S. Krane, *American Journal of Physics,* Vol. 49 (1981), pp. 113–117.)

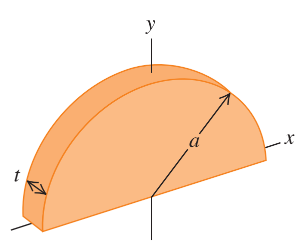
8.104 •• CALC In Section 8.5 we calculated the center of mass by considering objects composed of a *finite* number of point masses or objects that, by symmetry, could be represented by a finite number of point masses. For a solid object whose mass distribution does not allow for a simple determination of the center of mass by symmetry, the sums of Eqs. (8.28) must be generalized to integrals

$$x_{\mathrm{cm}} = \frac{1}{M}\int x\,dm \qquad y_{\mathrm{cm}} = \frac{1}{M}\int y\,dm$$

where x and y are the coordinates of the small piece of the object that has mass dm. The integration is over the whole of the object. Consider a thin rod of length L, mass M, and cross-sectional area A. Let the origin of the coordinates be at the left end of the rod and the positive x-axis lie along the rod. (a) If the density $\rho = M/V$ of the object is uniform, perform the integration described above to show that the x-coordinate of the center of mass of the rod is at its geometrical center. (b) If the density of the object varies linearly with x—that is, $\rho = \alpha x$, where α is a positive constant—calculate the x-coordinate of the rod's center of mass.

8.105 •• CALC Use the methods of Challenge Problem 8.104 to calculate the x- and y-coordinates of the center of mass of a semicircular metal plate with uniform density ρ and thickness t. Let the radius of the plate be a. The mass of the plate is thus $M = \frac{1}{2}\rho\pi a^2 t$. Use the coordinate system indicated in **Fig. P8.105**.

Figure P8.105



MCAT-STYLE PASSAGE PROBLEMS

BIO Momentum and the Archerfish. Archerfish are tropical fish that hunt by shooting drops of water from their mouths at insects above the water's surface to knock them into the water, where the fish can eat them. A 65 g fish at rest just at the surface of the water can expel a 0.30 g drop of water in a short burst of 5.0 ms. High-speed measurements show that the water has a speed of 2.5 m/s just after the archerfish expels it.

8.106 What is the momentum of one drop of water immediately after it leaves the fish's mouth? (a) 7.5×10^{-4} kg · m/s; (b) 1.5×10^{-4} kg · m/s; (c) 7.5×10^{-3} kg · m/s; (d) 1.5×10^{-3} kg · m/s.

8.107 What is the speed of the archerfish immediately after it expels the drop of water? (a) 0.0025 m/s; (b) 0.012 m/s; (c) 0.75 m/s; (d) 2.5 m/s.

8.108 What is the average force the fish exerts on the drop of water? (a) 0.00015 N; (b) 0.00075 N; (c) 0.075 N; (d) 0.15 N.

8.109 The fish shoots the drop of water at an insect that hovers on the water's surface, so just before colliding with the insect, the drop is still moving at the speed it had when it left the fish's mouth. In the collision, the drop sticks to the insect, and the speed of the insect and water just after the collision is measured to be 2.0 m/s. What is the insect's mass? (a) 0.038 g; (b) 0.075 g; (c) 0.24 g; (d) 0.38 g.

ANSWERS

Chapter Opening Question ?

(ii) Both hailstones have the same magnitude of momentum $p = mv$ (the product of mass and speed), but the faster, lighter hailstone has twice the kinetic energy $K = \frac{1}{2}mv^2$ of the slower, heavier one. Hence, the lightweight hailstone can do the most work on whatever it hits (and do the most damage) in the process of coming to a halt (see Section 8.1).

Key Example VARIATION Problems

VP8.6.1 (a) 0.200 m/s, to the left (b) 0.900 m/s, to the left
VP8.6.2 (a) 7.14 m/s, to the west (b) −9.00 m/s for Madeleine, +10.6 m/s for Buffy; Buffy
VP8.6.3 (a) +4.32 kg · m/s; −4.32 kg · m/s (b) 1.53 m/s (c) +11.8 kg · m/s (d) 4.92 m/s
VP8.6.4 (a) 1/2 (b) $v_{\mathrm{P2}} = v_{\mathrm{P1}}/2\cos\theta$, $v_{\mathrm{S2}} = v_{\mathrm{P1}}/4\cos\theta$
VP8.9.1 (a) 8.00 m/s before, 1.60 m/s after; 25.6 J lost (b) 4.00 m/s before, 3.20 m/s after; 6.4 J lost (c) case (i)
VP8.9.2 (a) 1.00 m/s; momentum (b) 3.57 m/s; total mechanical energy
VP8.9.3 (a) 1.35 m/s, in the $+x$-direction (b) can before, 2.70 J; can after, 0.817 J; box after, 1.09 J (c) inelastic; kinetic energy is lost, but the objects do not stick together after the collision
VP8.9.4 30.4°, north of east
VP8.14.1 $x_{\mathrm{cm}} = +0.135$ m, $y_{\mathrm{cm}} = -0.140$ m
VP8.14.2 $x = -3.53$ m
VP8.14.3 0.240 m
VP8.14.4 $x = -L/6$, $y = +7L/4$

Bridging Problem

(a) 1.00 m/s, to the right
(b) elastic
(c) 1.93 m/s, at −30.4°
(d) 2.31 kg · m/s, at 149.6°
(e) inelastic
(f) 1.67 m/s, in the $+x$-direction