AADS: Assignment Week 2

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Question 1

1. $T(n) = 4T(n/2) + n^2$

   As we can see in the function above, $a = 4$ and $b = 2$, we know that because the formula is

   $$T(n) = aT(n/b) + f(n)$$

   Also, we can know that $f(n) = n^2$. Now, we have think in which case we are, and to do this, we have to know the value of $n^{log_b a}$, in this case, the value is n^2, so we can deduce that $f(n) = \ominus(g_{a,b}()n)$

   With all this information we decide to take the second case. Likewise, we can say that $T(n) = \ominus(n^2 log_2 n)$

2. $T(n) = 2T(n/2) + \sqrt{n}$ In this case, $a = 2$, $b = 2$ and $f(n) = \sqrt{n}$. Knowing this, we have to calculate the value, as well as before, of $n^{log_b a}$. Now, the value is n, so we can say that $f(n) = O(g_{a,b}(n) * n^{-\epsilon})$.

   At this moment, we can deduce that we have to apply the first case, then, we say that $T(n) = \ominus(n)$

Question 2

To proof that n is an exact power of 2, the recurrence in the function is $T(n) = nlog_2 n$, we are going to do five steps:

1. First, we have to establish a function on which we are going to perform the induction. As we know that it has to be a power of 2, we decide to take: $F(K) = T(2^K)$

2. We also know that the base is $F(1) = T(2) = 2 = 2lg2 = 2^1 lg2^1$

3. Then, we want to prove that $T(n) = nlog_2 n$, so we deduce $F(K) = 2^K lg2^K$.

4. Now, we havve to prove with K+1:

   $$F(K+1) = T(2^{K+1}) =$$
   $$2 * T(\frac{2^{K+1}}{2}) + 2^{K+1} =$$
   $$2 * T(2^K) + 2^{K+1} =$$
   $$2 * 2^K lg2^K + 2^{2+1} =$$
   $$2^{K+1}(lg2^{K+1}) =$$
   $$2^{K+1}(lg2^K + lg2) =$$
   $$2^{K+1} * lg2^{K+1}$$

5. Finally, if $n = 2^{K+1}$ then, $n * lg(n) = T(n)$

Question 3

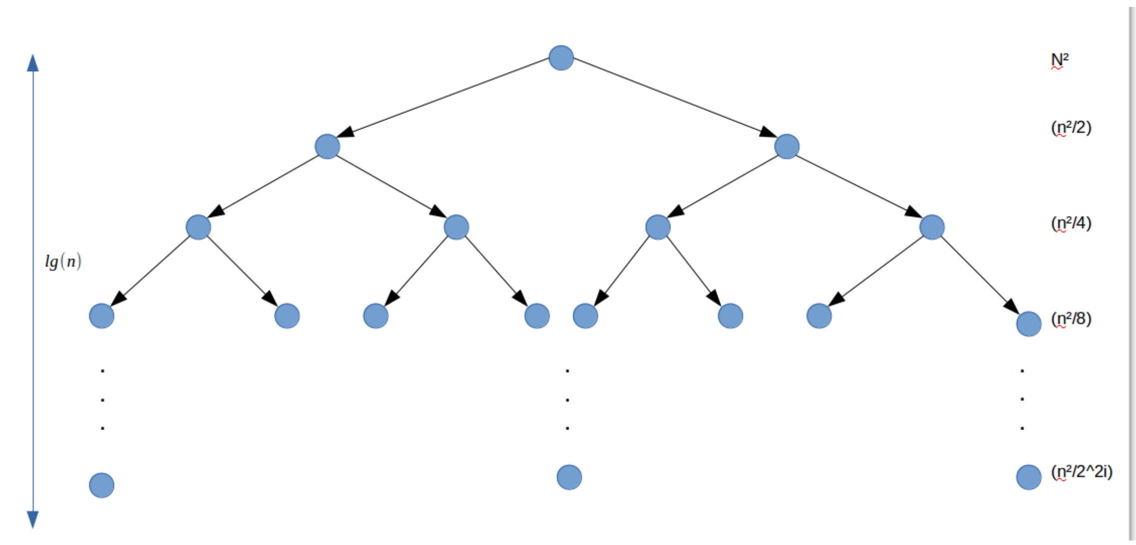

Figure 1: Recursive tree

As we can see in the figure above, at depth i, it's weight is $(n/2^i)^2$. Also, since the number of levels of the tree is $lg(n)$, we guess that the solution is $O(n^2)$. For the next step, we use the substitution method, so we assume that $T(n) \leq cn^2$.
Knowing this, we just have to apply it to the original equation:

$$T(n) = T(n/2) + n^2$$

$$T(n) \leq c(\frac{n}{2})^2 + n^2$$

$$T(n) = n^2 + (\frac{c}{4} + 1) \leq cn^2$$

With that, we have proved that the best asymptotic bound for the recurrence $T(n) = T(\frac{n}{2}) + n^2$ is $O(n^2)$