1. Translation preserves angles and parallel lines. Scaling preserves angles and parallel lines. Reflection preserves parallel lines and angles. Shearing preserves neither parallel lines nor angles. Rotation will also preserve parallel lines and angles.

2. (a) Given the point $p = (x, y)$, then $Mp = (x, 1.5x + y)$ as per the given answer. This is a *shearing* transformation. (b) Given the point $p = (x, y)$, then $Mp = (x, y - 1.5x)$. This is a *shearing* transformation in the opposite direction of the last. (c) Given the point $p = (x, y)$, then $Mp = (x, -y)$. This is a *reflection* transformation across the x-axis. (d) Given the point $p = (x, y)$, then $Mp = (-2x, -2y)$. This is a *scaling* by a factor of -2 in each coordinate. (it also has the effect of reflection across the origin, then scaling by 2.) (e) Given the point $p = (x, y)$, then $Mp = (x\cos(\theta) - y\sin(\theta), x\sin(\theta) + y\cos(\theta))$. This is a reflection in the plane by θ degrees counter clockwise. (f) Given the point $p = (x, y)$, then $Mp = (x\alpha - y\sqrt{1-\alpha^2}, x\sqrt{1-\alpha^2}, \alpha)$. For $\alpha \in [0, 1]$ this has the effect of rotation by $\pi(1-\alpha)/2$ degrees. (g) i. p is rotated by θ degrees clockwise. ii. p is then reflected across the x axis. iii. p is then rotated θ degrees counter-clockwise.

3. (a) The point $p = (x, y, 1) \to (2x, 2y, 1)$. This is scaling by a factor of 2. (b) The point $p = (x, y, 1) \to (2x, 2y, 0.5) \to (4x, 4y, 1)$. This is scaling by a factor of 4. (c) The point $p = (x, y, 1) \to (x, 0.5x + y, 1)$. This is shearing. (d) The point $p = (x, y, 1) \to (x\cos(\theta) - y\sin(\theta), x\sin(\theta) + y\cos(\theta), 1)$. This is still rotation.

4. If $M(\theta)$ represents rotation counter clockwise by θ, then $M^{-1}(\theta)$ must be rotation clockwise by θ. We can achieve this by seeing that $M^{-1}(\theta)$ must then be $M(-\theta)$.

5. Instead of viewing a rotation matrix as rotating our vector by θ degrees counter clockwise, we can instead imagine taking the standard basis vectors $(1, 0)$ and $(0, 1)$ and rotating *those* counter clockwise by theta. Then, the new coordinate of p will be the same in this new basis.

$(1, 0) \to (\cos(-\theta), \sin(-\theta))(0, 1) \to (\cos(\pi/2 - \theta), \sin(\pi/2 - \theta))$. Then converting this into a change of basis matrix we get

$$M = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \cos(\pi/2 - \theta) & \sin(\pi/2 - \theta)) \end{bmatrix}$$

But using the rules of trigonometry, $\cos(\pi/2 - \theta) = \sin(\theta)$, and $\sin(\pi/2 - \theta) = \cos(\theta)$. Therefore we see that

$$M = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$$