THEORETICAL QUESTIONS

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Theoretical Questions:

1 Rotation, Reflection, Scaling, Shearing, and Translations all preserve parallel lines. Rotation, Reflection, Translation, and Scaling preserve angles

2 .

(a) if $p = (x, y)$ then $M \cdot p = (x, y + 1.5x)$ This is a shearing transformation

(b) if $p = (x, y)$ then $M \cdot p = (x, y - 1.5x)$ This is a shearing transformation

(c) if $p = (x, y)$, then $M \cdot p = (x, -y)$ This is a reflection transformation

(d) if $p = (x, y)$, then $M \cdot p = (-2x, -2y)$ This is a reflection and scaling transformation

(e) if $p = (x, y)$, then $M \cdot p = (x\cos(\theta) - y\sin(\theta), x\sin(\theta) + y\cos(\theta))$ This is a rotation transformation clockwise by θ

(f) if $p = (x, y)$, then $M \cdot p = (x\alpha - y\sqrt{1-\alpha^2}, x\sqrt{1-\alpha^2} + y\alpha)$ this is a counter clockwise rotation by $\arccos(\alpha)$

(g) if $p = (x, y)$ then $R(-\theta)p = (x\cos(-\theta) - y\sin(-\theta), x\sin(-\theta) + y\cos(-\theta))$. Then $M_3(R(-\theta)p) = (x\cos(-\theta) - y\sin(-\theta), -x\sin(-\theta) - y\cos(-\theta))$. Then $R(\theta)(M_3(R(-\theta)p)) = (\cos(\theta)(x\cos(-\theta) - y\sin(-\theta)) - \sin(\theta)(-x\sin(-\theta) - y\cos(-\theta)), \sin(\theta)(x\cos(-\theta) - y\sin(-\theta)) + \cos(\theta)(-x\sin(-\theta) - y\cos(-\theta)))$

3 .

(a) Let $p = (x, y, 1)_h$ then $M \cdot p = (2x, 2y, 1)_h$ this is a scale transformation.

(b) Let $p = (x, y, 1)_h$ then $M \cdot p = (2x, 2y, \frac{1}{2}) = (4x, 4y, 1)_h$ this is a scale transformation

(c) Let $p = (x, y, 1)_h$ then $M \cdot p = (x, y + 0.5x, 1)_h$ this is a shear transformation

(d) Let $p = (x, y, 1)_h$ then $M \cdot p = (x\cos(\theta) - y\sin(\theta), x\sin(\theta) + y\cos(\theta), 1)$ this is a rotation transformation

(4) $M = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$. Now we want to find the inverse of M. M is the rotation matrix that rotates counterclockwise by θ. The applying the inverse to the rotation should lead to the initial state (identity matrix). This transformation is the rotation matrix that rotates clockwise by theta. so $M^{-1} = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}$