# Homework | Theoretical Questions

1. which geometric transformations (from the ones covered in class so far), transfer parallel lines to parallel lines? which transformation maintains angles?

- translation: parallel lines remain parallel, angles are preserved
- Rotation: Parallel lines remain parallel, angles are also preserved
- Scaling: Parallel lines remain parallel, angles are preserved
- Reflection: parallel lines remain parallel, angles are preserved
- Shear: parallel lines remain parallel (but not oriented the same), and angles <u>are not</u> preserved

2. what is effect of the following matrices?:

(a)

$M = \begin{bmatrix} 1 & 0 \\ 1.5 & 1 \end{bmatrix}$ let $P=(x,y)$ then $M \cdot P = \begin{bmatrix} 1 & 0 \\ 1.5 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 1.5x+y \end{bmatrix} = (x, 1.5x+y)$

this is a sheering transformation

(b)

$M = \begin{bmatrix} 1 & 0 \\ -1.5 & 1 \end{bmatrix}$, if $P=(x,y)$ then $M \cdot P = \begin{bmatrix} 1 & 0 \\ -1.5 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} x \\ -1.5x+y \end{bmatrix} = M \cdot P = (x, -1.5x+y)$

this is also sheering, but in the oposite direction as (a).

2. (c)

$M_3 = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$ if $P = (x,y)$, then $M_3 \cdot P = \begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}x\\ -y\end{bmatrix} = M_3 \cdot P = (x, -y)$

this is a reflection over the X axis.

(d)

$M = \begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}$ ; if $P = (x,y)$ $M \cdot P = \begin{bmatrix}-2 & 0\\ 0 & -2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}-2x\\ -2y\end{bmatrix} = (-2x, -2y)$

this is a reflection over the X and Y axis, and a scaling of 2

(e)

$M = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$ if $P = (x,y)$ $M \cdot P = \begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}x\cos\theta - y\sin\theta\\ x\sin\theta + y\cos\theta\end{bmatrix}$

$M \cdot P = (x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta)$

this is a rotation by $\theta$ (angle) around the origin counter clockwise.

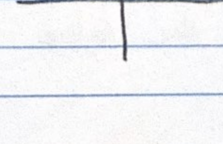
(f)

$M = \begin{bmatrix}\alpha & -\sqrt{1-\alpha^2}\\ \sqrt{1-\alpha^2} & \alpha\end{bmatrix}$ where $0 \leq \alpha \leq 1$ if $(x,y) = P$ so $P \cdot M = \begin{bmatrix}\alpha & -\sqrt{1-\alpha^2}\\ \sqrt{1-\alpha^2} & \alpha\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}x\alpha - \sqrt{1-\alpha^2}\\ \alpha y + \sqrt{1-\alpha^2}\end{bmatrix}$

$PM = (\alpha x - \sqrt{1-\alpha^2},\ \alpha y + \sqrt{1-\alpha^2})$

this is a rotation sheer, its a transformation that keeps the same size of the object, and same angles. it rotates the object by simultaneously sheering both points along the x & Y axis.

(very rough sketch):



(g) $M = R(\theta) \cdot \begin{bmatrix}1 & 0\\0 & -1\end{bmatrix} \cdot R(-\theta)$  $R(\theta)$ is Rotation Matrix

1) $R(\theta)$ is a rotation Matrix, that rotates Point P Counter Clockwise around orgin.

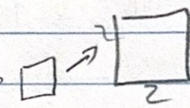
2) $\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$ is a Matrix that flips Point P over the X-AXiS.

3) $R(-\theta)$ is a rotation Matrix that rotates P clockwise around orgin

this combination of matries will Flip an image while maintaining its Position or orientation.

3. what is the effect of the homogeneous coord Matrices:

(a) $M = \begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 1\end{bmatrix}$ this will Scale by 2: $\begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\1\end{bmatrix}$ $\begin{bmatrix}2x\\2y\\1\end{bmatrix}$ →




(b) $M = \begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 0.5\end{bmatrix}$ this Scales x, y up by 2, however it then Scales the w down by 0.5, So the end result is the same as the starting State

$$\begin{bmatrix}2 & 0 & 0\\0 & 2 & 0\\0 & 0 & 0.5\end{bmatrix} \cdot \begin{bmatrix}x\\y\\w\end{bmatrix} = \begin{bmatrix}2x\\2y\\0.5w\end{bmatrix} \left( = \begin{bmatrix}1x\\1y\\1w\end{bmatrix} \right)$$

(c) $M = \begin{bmatrix}1 & 0 & 0\\0.5 & 1 & 0\\0 & 0 & 1\end{bmatrix}$ let $P = \begin{bmatrix}x\\y\\w\end{bmatrix}$ $PM = \begin{bmatrix}1 & 0 & 0\\0.5 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x\\y\\w\end{bmatrix}$ $\begin{bmatrix}x\\0.5x+y\\1\end{bmatrix}$

this Yeilds a shear effect where the Y coord is Shifted depending on the value of the X coord.

(2) $M = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ let $P = \begin{pmatrix} x \\ y \\ w \end{pmatrix}$ $M \cdot P = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix} \begin{bmatrix} \cos\theta x - y\sin\theta \\ x\sin\theta + y\cos\theta \\ w \end{bmatrix}$

we can see $p = (x, y, w) \to (\cos\theta x - y\sin\theta,\ x\sin\theta + y\cos\theta, w)$ is the formula for a rotation around the origin counterclockwise by an angle $\theta$.

4. Let $M = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, Find $M^{-1}$ such that $M^{-1} \cdot M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

to Find $M'$, we can think about this as a rotation in the oposite direction, aka $-\theta$

So $m^{-1} = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ ✓