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Putting you to work !

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Wed, Aug 4, 2021 at 3:10 PM

C_1 = First Material Const
C_2 = Second Material Cont

Chen,

Below are some Mooney-Rivlin parameters for four different soft, stretchable, photo-curable materials we have examined. I believe c1 and c2 correspond to A and B in the M-R strain energy function, per the SolidWorks user's manual. All parameters have units of MPa.

Material	AFRL Elastomer	SUV Elastomer	SilOHflex	ELAST-BLK 10
Parameter Values	$c_1 = 0.0083$ $c_2 = 2.35e-14$	$c_1 = 0.2767$ $c_2 = 0.0308$	$c_1 = 0.095$ $c_2 = 5.75e-07$	$c_1 = 0.7757$ $c_2 = 2.2514e-07$
NRMSE	0.01789	0.12661	0.06684	0.29564

M-R

Of these four materials, the only one currently printable at UDRI is ELAST-BLK 10, a commercially available material from 3D Systems. Frankly, it is a lousy material and has effectively been phased out. The other three are from literature. We (Braeden and Joseph) are trying to get a similar material to the "AFRL elastomer" up and running on the Figure 4 printers at UDRI, but we don't have formal test data or calibrated models ... yet. For Poisson's ratio, it is generally recommended to use a value of 0.49 to 0.499 so that the material is weakly compressible. This preserves the mathematical structure of the governing equations and keeps the numerical method happy, while preserving near-incompressibility characteristic of most rubbery and elastomeric materials.

Ogden parameters are shown below. mu_i are in MPa, alpha_i are dimensionless. **I do not recommend using this model.** As it is "higher-order," It typically does not predict other deformation modes (e.g., compression, biaxial tension, shear, torsion) well in FEA analyses when only calibrated to uniaxial tension data.

Material	AFRL Elastomer	SUV Elastomer	SilOHflex	ELAST-BLK 10	
Parameter Values	$\mu_1 = -0.0111$ $\alpha_1 = 1.4292$ $\mu_2 = 6.03e-06$ $\alpha_2 = 5.8233$ $\mu_3 = 0.0241$ $\alpha_3 = 16.8645$	$\mu_1 = 2.1198$/MPa $\alpha_1 = 0.8211$ $\mu_2 = 0.0117$ $\alpha_2 = 3.5776$ $\mu_3 = 0.4076$ $\alpha_3 = 21.9032$	$\mu_1 = 0.2671$ $\alpha_1 = 0.2671$ $\mu_2 = 8.05e-05$ $\alpha_2 = 6.8677$ $\mu_3 = -0.0066$ $\alpha_3 = 343.4955$	$\mu_1 = 0.0562$ $\alpha_1 = 3.7848$ $\mu_2 = 0.0047$ $\alpha_2 = 9.0703$ $\mu_3 = 0.4093$ $\alpha_3 = 10.6912$	MPa N/D MPa N/D MPa N/D
NRMSE	0.00035	0.00751	0.00230	0.04109	

Ogden

Blatz-Ko is a compressible hyper-elastic model. As such, it is most appropriate for compressible materials like foams or porous elastomers. We do not have a calibrated version of this model.

Hope that helps. If I can assist further, please don't hesitate to reach out.

Best Regards,
Bob

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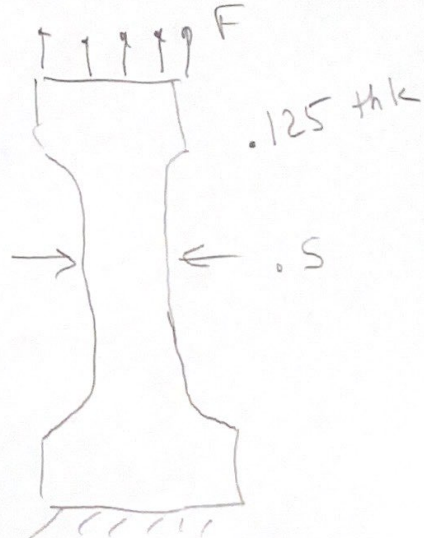

$\sigma = \frac{F}{A} = P_1$

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$$\Psi = C_1(I_1 - 3) + C_2(I_2 - 3)$$

M-R

3.1.1 Uniaxial tension (UT)

In uniaxial tension, the specimen is elongated in only one direction, for example, $\lambda_1 = \lambda$. From incompressibility, that is, $I_3 = \lambda_1^2\lambda_2^2\lambda_3^2 = 1$, and the assumption of isotropy, the complementary principal stretches follow as $\lambda_2 = \lambda_3 = \lambda^{-1/2}$. Thus, the corresponding deformation gradient and the invariants, cf. Eq. (11), are

$$\boldsymbol{F}^{\mathrm{UT}} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda^{-1/2} & 0 \\ 0 & 0 & \lambda^{-1/2} \end{bmatrix}, \quad I_1^{\mathrm{UT}} = 2\lambda^{-1} + \lambda^2, \quad I_2^{\mathrm{UT}} = \lambda^{-2} + 2\lambda. \tag{18}$$

Since the contraction is unhindered in the transversal directions, both $P_{2,3}^{\mathrm{UT}}$ are zero and only $P_1^{\mathrm{UT}}(\lambda)$ has to be determined. By setting Eq. (17) to zero, for example, for $i = 2$, and calculating the derivatives $\partial I_{1,2}/\partial\lambda_2$ from Eq. (11), the resultant pressure is determined as

$$p^{\mathrm{UT}} = \frac{2}{\lambda}\frac{\partial\Psi}{\partial I_1} + 2\left[\lambda + \frac{1}{\lambda^2}\right]\frac{\partial\Psi}{\partial I_2}. \tag{19}$$

Inserting this into Eq. (17) for $i = 1$, we obtain the analytical formulation for the first principal stress:

P_1 = axial stress

$$P_1^{\mathrm{UT}} = 2\left[\underbrace{\frac{\partial\Psi}{\partial I_1}}_{C_1} + \frac{1}{\lambda}\underbrace{\frac{\partial\Psi}{\partial I_2}}_{C_2}\right]\left[\lambda - \frac{1}{\lambda^2}\right]. \tag{20}$$

$\lambda = 1 + \varepsilon$ axial strain

3.1.2 Equibiaxial tension (ET)

The specimen is equally stretched in two orthogonal directions, that is, $\lambda_1 = \lambda_2 = \lambda$. Again due to incompressibility, the remaining principal stretch reads $\lambda_3 = \lambda^{-2}$, and the corresponding deformation gradient and invariants follow as

$$\boldsymbol{F}^{\mathrm{ET}} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^{-2} \end{bmatrix}, \quad I_1^{\mathrm{ET}} = \lambda^{-4} + 2\lambda^2, \quad I_2^{\mathrm{ET}} = 2\lambda^{-2} + \lambda^4. \tag{21}$$

The stresses in load directions are equal, while the third direction is stress-free due to unhindered contraction, that is, $P_1^{\mathrm{ET}} = P_2^{\mathrm{ET}}$ and $P_3^{\mathrm{ET}} = 0$, respectively. The pressure is determined by setting Eq. (17) to zero for $i = 3$:

$$p^{\mathrm{ET}} = \frac{2}{\lambda^4}\frac{\partial\Psi}{\partial I_1} + \frac{4}{\lambda^2}\frac{\partial\Psi}{\partial I_2}, \tag{22}$$

and by reinserting this into (17) for $i = 1$, we obtain the first and second principal stresses:

$$P_1^{\mathrm{ET}} = P_2^{\mathrm{ET}} = 2\left[\frac{\partial\Psi}{\partial I_1} + \lambda^2\frac{\partial\Psi}{\partial I_2}\right]\left[\lambda - \frac{1}{\lambda^5}\right]. \tag{23}$$

3.1.3 Pure shear (PS)

The pure shear set-up of Treloar [52] utilises rectangular sheets having a much larger width than length to realise a zero deformation perpendicular to the loading direction $\lambda_1 = \lambda$, that is, $\lambda_2 = 1$ holds almost everywhere except for the vicinity of the free edges. From incompressibility, the third principal stretch $\lambda_3 = \lambda^{-1}$ and the corresponding deformation gradient and the invariants read

$$\boldsymbol{F}^{\mathrm{PS}} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda^{-1} \end{bmatrix}, \quad I^{\mathrm{PS}} = I_1^{\mathrm{PS}} = I_2^{\mathrm{PS}} = \lambda^2 + \lambda^{-2} + 1. \tag{24}$$

From unhindered contraction in the third direction, the pressure follows from setting (17) to zero for $i = 3$:

$$p^{\mathrm{PS}} = \frac{2}{\lambda^2}\frac{\partial\Psi}{\partial I_1} + 2\left[1 + \frac{1}{\lambda^2}\right]\frac{\partial\Psi}{\partial I_2}. \tag{25}$$

Insertion of (25) into (17) yields the principal stresses in load direction and perpendicular to this:

$$P_1^{\mathrm{PS}} = 2\left[\frac{\partial\Psi}{\partial I_1} + \frac{\partial\Psi}{\partial I_2}\right]\left[\lambda - \frac{1}{\lambda^3}\right], \quad P_2^{\mathrm{PS}} = 2\left[\frac{\partial\Psi}{\partial I_1} + \lambda^2\frac{\partial\Psi}{\partial I_2}\right]\left[1 - \frac{1}{\lambda^2}\right]. \tag{26}$$