ψ

$$\psi_{j\ell m} = \begin{pmatrix} \frac{iG(r)}{r}\Omega_{j\ell m}(\theta,\phi) \\ \frac{-F(r)}{r}\Omega_{j\ell' m}(\theta,\phi) \end{pmatrix} \tag{1}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}r}+\frac{\kappa}{r}\right)G=\left(\frac{E+mc^2}{\hbar c}+\frac{Z\alpha}{r}\right)F \tag{2}$$

$$\left(\frac{\mathrm{d}}{\mathrm{d}r} - \frac{\kappa}{r}\right) F = \left(\frac{E - mc^2}{\hbar c} + \frac{Z\alpha}{r}\right) G \tag{3}$$

$r \to 0$

$F, G \sim r^{\pm\gamma}$

$\gamma = \sqrt{R^2 - (Z\alpha)^2}$

$$\int \psi^\dagger \psi r^2 \, \mathrm{d}r < \infty \;\to\; \int (|F|^2 + |G|^2) \, \mathrm{d}r < \infty \tag{4}$$

$\int_0^\infty \frac{1}{r^{2\gamma}}\,\mathrm{d}r$

$\gamma > 1/2$

$\gamma \in [0, 1/2]$

$\rho = 2\lambda r$

$\lambda = \frac{1}{\hbar c}\sqrt{m^2c^4 - E^2}$

$$\frac{\mathrm{d}G}{\mathrm{d}\rho} = -\frac{\kappa}{\rho}G + \left(\frac{E+mc^2}{2\hbar c\lambda} + \frac{Z\alpha}{\rho}\right)F \tag{5}$$

$$\frac{\mathrm{d}F}{\mathrm{d}\rho} = +\frac{\kappa}{\rho}F - \left(\frac{E-mc^2}{2\hbar c\lambda} + \frac{Z\alpha}{\rho}\right)G \tag{6}$$

$r \to \infty$

$$\frac{\mathrm{d}G}{\mathrm{d}\rho} = \frac{E + mc^2}{2\hbar c\lambda} F \tag{7}$$

$$\frac{\mathrm{d}F}{\mathrm{d}\rho} = \frac{E - mc^2}{2\hbar c\lambda} G \tag{8}$$

$F \sim e^{-1/2\rho}$

$G \sim e^{--1/2\rho}$