Semiconductors 101

"Oops! I did it again"

Hartree-Fock approximation

The Hamiltonian contains all the possible interactions between electrons an nuclei. We can approximate that to a central potential $V(\mathbf{r}_i)$ for each electron:

$$\mathscr{H} \simeq \sum_i \frac{-\hbar^2}{2m_i}\nabla_i^2 + V(\mathbf{r}_i)$$

This is factorizable, so for each electron

$$\left[\frac{-\hbar^2}{2m_i}\nabla_i^2 + V(\mathbf{r}_i)\right]\Psi_i = E_i\Psi_i$$

We use periodic boundary conditions (Born-Karman conditions). Tight-binding assumption lets us consider that levels are those of the base atom but with extra shifted levels.

Schrödinger, effective mass

If $E \sim E_c + \frac{\hbar^2 k^2}{2m_n^*}$ we can neglect $V(r)$ and write

$$\frac{-\hbar^2}{2m_n^*}\nabla\Psi = E'\Psi'$$

If $E_D \sim E_c + E''$, with small $|E''|$, we can neglect the $V(r)$ in $\mathcal{V}(r) = V(r) - \frac{q^2}{4\pi\varepsilon r}$, and write

$$\left(\frac{-\hbar^2}{2m_n^*}\nabla - \frac{q^2}{4\pi\varepsilon r}\right)\Psi'' = E''\Psi''$$

For the valence band ($E_A \sim E_V + E''$),

$$\left(\frac{-\hbar^2}{2m_p^*}\nabla - \frac{q^2}{4\pi\varepsilon r}\right)\Psi'' = -E''\Psi''$$

using $m_p^* = -m_n^*$ and the fact that the potential changes sign.

The effective mass, $m*_n = \frac{\hbar}{\frac{\partial^2 E}{\partial k^2}}$, is positive in the border of E_C, E_V next to the Fermi level and negative in the opposite one TODO WHY?.

Fermi level

Let N_i be the effective density of states and n_0, p_0 the total volume concentrations of charge carriers.

No doping

We obtain

$$n_0 = N_C e^{-\Delta E/kT} = N_C e^{-(E_C - E_F)/kT}$$
$$p_0 = N_V e^{-\Delta E/kT} = N_V e^{-(E_F - E_V)/kT}$$
$$n_i = \sqrt{n_0 p_0} \propto T^{3/2} e^{-E_G/2kT}$$

since $n_0 = p_0$,

$$E_F = \frac{E_V + E_C}{2} + \cancelto{0}{\frac{kT}{2}\log\left(\frac{N_V}{N_C}\right)^{3/2}}$$

The term cancels at $T \sim 300$ K.

N doping

E_F near E_C:

$$n_0 = N_D = N_C e^{-(E_C - E_F)/kT}$$
$$E_F = E_C - kT\log\frac{N_C}{N_D}$$

P doping

E_F near E_V:

$$p_0 = N_A = N_V e^{-(E_F - E_V)/kT}$$
$$E_F = E_V + kT\log\frac{N_V}{N_A}$$

Extrinsic semiconductors

Suppose $\rho = 0 = q(N_D^+ - N_A^- + p_0 - n_0)$.

N type ($N_A^- = 0$)	P type ($N_D^+ = 0$)
$n_0 = p_0 + N_D^+ = \frac{n_i^2}{n_0} + N_D^+$	$p_0 = n_0 + N_A^- = \frac{n_i^2}{p_0} + N_A^-$

Solve 2nd degree equation, obtaining in the N case:

$$n_0 = \frac{N_D^+ \pm \sqrt{(N_D^+)^2 + 4n_i^2}}{2}$$

for p_0, the relation $n_0 = p_0 + N_D^+$ is used.

Temperature ranges

Considering a N semiconductor, we analyze three regions:

Low temperatures The number of non-ionized impurities (N_D) is greater than the number N_D^+ of ionized ones , $N_D > N_D^+ \gg 2n_i$. From this inequality follows $n_0 \simeq N_D^+$ and $p_0 \simeq n_i^2/N_D^+$.

Extrinsic range The same reasoning as in the low temperature region, but this time *all* the impurities are ionized, so $N_D \simeq N_D^+$. About 150 K to 400 K.

High temperatures $n_i \gg N_D = N_D^+$, and the expression for n_0 simplifies to $n_0 \simeq p_0 \simeq n_i$.

Approximations

Low level injection Carrier variations being very small compared to n_0, p_0 implies that $R - G \propto n', p'$.

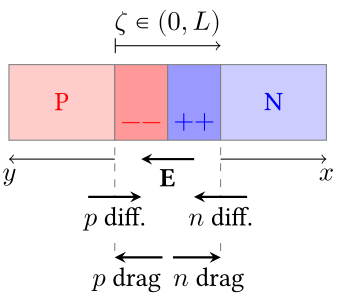
Quasi-neutrality condition In low level injection conditions, the homogeneous parts of a extrinsic semiconductor can be considered neutral, much like a metal.

Negligible minority carrier drag When determining the total current density, the minority carrier contribution to the drag current is negligible. They move mainly through diffusion (drift).

Diodes

"When everything else fails"

Forces



Zero bias In equilibrium, both forces (drag and drift) counterbalance. There is no current flow except for the thermal generation of carriers.

Forward bias When a positive voltage bias between the P and the N side is applied, holes and electrons recombine and L gets smaller. E decreases and even changes sign. If the applied V is big enough, E can not counteract the diffusion of carriers and there is a current flow.

Also, a smaller L increases $\nabla n, \nabla p$ in the transition zone and the diffusion current, further increasing the conductivity.

The majority of the current is through drag, so each region contributes with its majority carriers, of which it has a constant supply from the battery.

Backward bias If the bias is negative, L increases and there is even less current flow. The only carriers that flow are the minoritary ones through diffusion, the ones that the regions aren't receiving from the battery. The only source of them is the thermal generation, so the current is fixed to a value $I_S \neq f(V)$.

Analysis

Charged currents:

$$\mathbf{J}_{\text{drag}} = qn\mu_n\mathbf{E},\ qp\mu_p\mathbf{E}$$
$$\mathbf{J}_{\text{drift}} = qD_n\nabla n,\ -qD_p\nabla p$$
$$\nabla\mathbf{J} = q(G - R)$$

Continuity equation:

$$\dot{n}_p = \frac{1}{q}\frac{\partial J_n}{\partial x} + G_n - \frac{n_p'}{\tau_n}$$
$$\dot{p}_n = \frac{-1}{q}\frac{\partial J_p}{\partial x} + G_p - \frac{p_n'}{\tau_p}$$

Contact potential

The transition zone is characterized by a constant charge given by ionized impurities. $\mathbf{J}_n\big|_{\text{diff}} \simeq -\mathbf{J}_n\big|_{\text{drag}}$ implies

$$-qn\mu_n\nabla V \simeq -qD_n\nabla n$$
$$\frac{q}{\mathrm{k}T}\mathrm{d}V \simeq \frac{\mathrm{d}n}{n}$$
$$n(\zeta) \simeq n(0)\mathrm{e}^{q\Phi(\zeta)/\mathrm{k}T}$$

We have $n(\zeta = L) = N_D = \frac{n_i^2}{N_A}$ and $n(0) = n_{p0} = \frac{n_i^2}{N_A}$, so

$$\Phi_B = \frac{\mathrm{k}T}{q}\log\frac{N_A N_D}{n_i^2}$$

Circuit models

Analytical $I = I_S(\mathrm{e}^{qV/n\mathrm{k}T} - 1)$

DC If $V > V_u$ is 0.7 V, else is a open circuit. If a Zener diode is employed, below V_z it can be modeled by a battery of $V = V_z$. In both batteries the diode current flows from $+$ to $-$.

AC The notation is

$$\underbrace{v_D}_{\text{Total}} = \underbrace{V_D}_{\text{DC}} + \underbrace{v_d}_{\text{AC}}$$

The diode (if not in open circuit) is a resistor r_d, of value $r_d = \frac{n\mathrm{k}T}{q(I_D+I_S)}$.

Second order effects

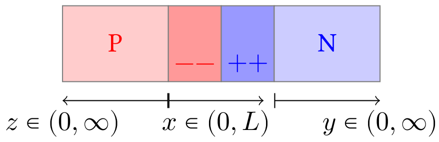
Resistive effects P,N zones show resistive effects, most notable at high I.

Avalanche breakdown For high reverse bias, the big $\mathbf{E}$ can accelerate electrons enough to ionize other electrons in a chain reaction, generating huge currents.

Generation-recombination In inverse bias, generation effects (which are predominant) make I be higher than expected. In forward bias, recombination effects (dominant, $R \sim n_i \sim \mathrm{e}^{qV/\mathrm{k}T}$) make current be lower than expected. This effects are modeled with n, the *ideality factor*:

$$I \sim I_0\left(\mathrm{e}^{qV/n\mathrm{k}T} - 1\right)$$

High level of injection Low level of injection is no longer the case for high I, where $I \sim e^{qV/2\mathrm{k}T}$.



Solving a Diode

"Because I didn't had anything better to do"

Transition zone

Drift must counteract drag. Using e.g. electrons:

$$
\begin{aligned}
\mathbf{J}_n\Big|_{\text{drag}} = qn\mu_n E_x = -qD_n \nabla n = -\mathbf{J}_n\Big|_{\text{drift}} \\
-qn\left(\frac{q}{\mathrm{k}T}\right)\frac{\mathrm{d}}{\mathrm{d}x}V = -qD_n\frac{\mathrm{d}}{\mathrm{d}x}n \\
\frac{q}{\mathrm{k}T}\mathrm{d}V \simeq \frac{\mathrm{d}n}{n} \\
n(x_2) \simeq n(x_1)\mathrm{e}^{\frac{q}{\mathrm{k}T}(V_2-V_1)}
\end{aligned}
$$

With $x_2 = x, x_1 = 0$ we obtain

$$
\begin{aligned}
n(x) &\simeq n(0)\mathrm{e}^{q\Phi/\mathrm{k}T} \\
p(x) &\simeq p(0)\mathrm{e}^{-q\Phi/\mathrm{k}T}
\end{aligned}
$$

With an external potential V, $\underbrace{n(L)}_{\simeq N_D} = n(0)\mathrm{e}^{\frac{q}{\mathrm{k}T}(\Phi_B - V)}$ and we end with

$$
\begin{aligned}
n(0) &= N_D\mathrm{e}^{\frac{q}{\mathrm{k}T}\Phi_B}\mathrm{e}^{\frac{q}{\mathrm{k}T}V} = n_{p0}\cdot\mathrm{e}^{qV/\mathrm{k}T} \\
p(L) &= N_A\mathrm{e}^{-\frac{q}{\mathrm{k}T}\Phi_B}\mathrm{e}^{\frac{q}{\mathrm{k}T}V} = p_{n0}\cdot\mathrm{e}^{qV/\mathrm{k}T}
\end{aligned}
$$

Φ_B, the *junction potencial*, can be found using $V = 0, n(0) = \frac{n_i^2}{N_A}$ in $\underbrace{n(L)}_{N_D} = n(0)\mathrm{e}^{q\Phi/\mathrm{k}T}$ as

$$
\Phi_B = \frac{\mathrm{k}T}{q}\log\frac{N_A N_D}{n_i^2}
$$

One can solve $\Phi(x)$ via Poisson's equation, using that the charges in the region are $-qN_A$ and qN_D. The boundary conditions are $\Phi(0) = 0$ and $\Phi(L) = \Phi_B$.

$$
\Phi(x) = \begin{cases} \frac{qN_A}{2\varepsilon}x^2 & , x \in (0, \ell_p) \\ \frac{qN_D}{2\varepsilon}(x-L)^2 & , x \in (\ell_p, L) \end{cases}
$$

Continuity of the potential gives the value of L as a function of Φ_B, continuity of $\mathbf{E}$ gives $N_D\ell_n = N_A\ell_p$.

P side

The minoritary carriers are electrons. The minoritary carriers move mainly through diffusion:

$$
\mathbf{J}_p \simeq \mathbf{J}_p\Big|_{\text{drift}} = qD_n\nabla_n
$$

Use the continuity equation in stationary state:

$$
\begin{aligned}
\cancelto{0}{\dot{n}'_p} &= \frac{1}{q}\frac{\partial}{\partial z}\mathbf{J}_p + \cancelto{0}{G_n} - \frac{n'_p}{\tau_n} \\
&= D_n\frac{\partial^2}{\partial z^2}n'_p - \frac{n'_p}{\tau_n}
\end{aligned}
$$

So $\frac{\partial^2 n'_p}{\partial z^2} = \frac{n'_p}{L_n^2}$ with $L_n^2 = \tau_n D_n$ the *diffusion length*, with solution $A\mathrm{e}^{-z/L_n} + B\mathrm{e}^{z/L_n}$.

Boundary conditions are:

- $n'_p(\infty) = 0 \ \rightarrow \ B = 0$
- $n'_p(0) = n_{p0}\mathrm{e}^{qV/\mathrm{k}T} - n_{p0}$

We obtain:

$$
\boxed{n'_p(z) = n_{p0}\left(\mathrm{e}^{qV/\mathrm{k}T} - 1\right)\mathrm{e}^{-z/L_n}}
$$

N side

Identical. We obtain:

$$
\boxed{p'_n(y) = p_{n0}\left(\mathrm{e}^{qV/\mathrm{k}T} - 1\right)\mathrm{e}^{-y/L_p}}
$$

Current

One can show with Gauss' law that $\mathbf{J} \neq f(x)$. Assuming no generation-recombination effects in the transition zone (very thin) one can write for the transition zone $\mathbf{J}_p(x) = \mathbf{J}_p(L)$ and $\mathbf{J}_n(x) = \mathbf{J}_n(0)$.

We sum minority carriers drift currents in the transition zone since both are known. Care must be taken because $\hat{z}$ points in the reverse direction from the other axis.

$$
\begin{aligned}
\mathbf{J}\hat{x} &= \mathbf{J}_p(y)\hat{x} - \mathbf{J}_p(z)\hat{x} \\
&= \left(-qD_p\frac{\partial}{\partial y}p'_n(y)\right)\Big|_0 - \left(qD_n\frac{\partial}{\partial z}n'_p(z)\right)\Big|_0 \\
&= q\left(\mathrm{e}^{qV/\mathrm{k}T} - 1\right)\left[\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right]
\end{aligned}
$$

Define I as going from P to N:

$$
I = \int \mathbf{J}\,\mathrm{d}\mathbf{S} = \underbrace{qS\left[\frac{D_p p_{n0}}{L_p} + \frac{D_n n_{p0}}{L_n}\right]}_{I_S}\left(\mathrm{e}^{\frac{qV}{\mathrm{k}T}} - 1\right)
$$

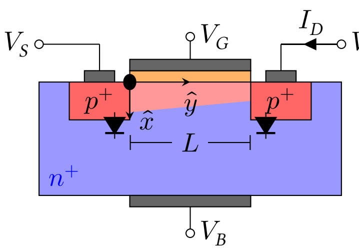
If a non zero rate of generation-recombination is assumed in the transition zone, one can approximate

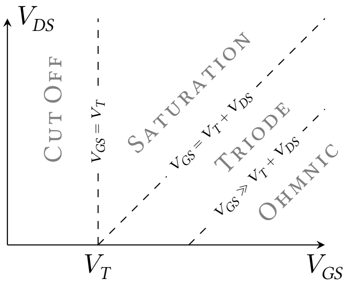
$$
\boxed{I = I_S\left[\exp\left(\frac{qV}{n\mathrm{k}T}\right) - 1\right]}
$$

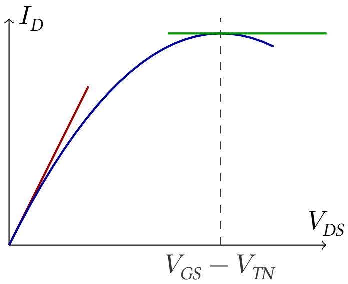
for $n \in (1, 2)$.

MOS TRANSISTOR

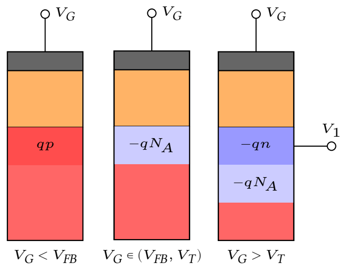
"Twice the diodes, twice as better"







MOS capacitor (P)



- Accumulation in reverse bias.
- Depletion, $N_A > p > n$.
- Weak inversion , $N_A > n > p$.
- Strong inversion, $n > N_A > p$.

Charge in the strong inversion channel:

$$\int qn \, \mathrm{d}x = -Q = C_0(V_G - V_1 - V_{TN})$$

With $V_B = 0$,

$$V_{TN0} = 2\Phi_0 + V_{FB} + \gamma\sqrt{2\Phi_0}$$

Each y of the MOS channel is a MOS capacitor!

Regions

Cut off $V_{GS} < V_{TN}$ implies no channel at all.

Ohmnic Homogeneous channel. $V_{GS} > V_{TN}$ and $V_{GS} - V_{TN} \gg V_{DS}$, so

$$\int_{\text{channel}} qn \, \mathrm{d}x \simeq C_0[V_{GS} - V_{TN} + \underbrace{\cancel{V(y)}}_{\sim 0}]$$

Triode The channel is not homogeneous. Impose that there is still channel $\forall y$:

$$\left.\int qn \, \mathrm{d}x\right|_{y=0} \geq 0 \;\rightarrow\; V_{GS} \geq V_{TN}$$

$$\left.\int qn \, \mathrm{d}x\right|_{y=L} \geq 0 \;\rightarrow\; V_{GS} - V_{TN} \geq V_{DS}$$

Saturation / active There are values of y without channel.

$$\left.\int qn \, \mathrm{d}x\right|_{y=0} \geq 0 \;\rightarrow\; V_{GS} \geq V_{TN}$$

$$\left.\int qn \, \mathrm{d}x\right|_{y=L} \leq 0 \;\rightarrow\; V_{GS} - V_{TN} \leq V_{DS}$$

Starts when $V(L) = V_{GS} - V_{TN} := V_{\text{sat}}$. Despite not being channel in some y, the electric field is strong and conduction continues, the depleted region conduces without problem.

For a PMOS they are the same, but with the inequalities reversed ($C_0 \rightarrow -C_0$).

Characteristic equation

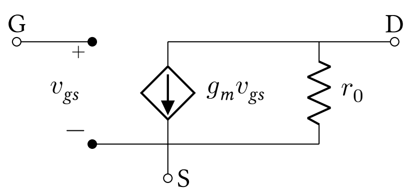
$$I_D = \beta(V_{GS} - V_{TN})V_{DS} \qquad \text{(Ohmnic)}$$

$$I_D = \beta\left[(V_{GS} - V_{TN})V_{DS} - \frac{V_{DS}^2}{2}\right] \qquad \text{(Triode)}$$

$$I_D = \frac{\beta}{2}(V_{GS} - V_{TN})^2 \qquad \text{(Saturation)}$$

with $\beta = C_0\mu_n\frac{W}{L}$. Everything at 1^{st} order of aproximation. The region depends on the *operation point*, given by (I_{DS}, V_{DS}). Valid for both PMOS and NMOS.

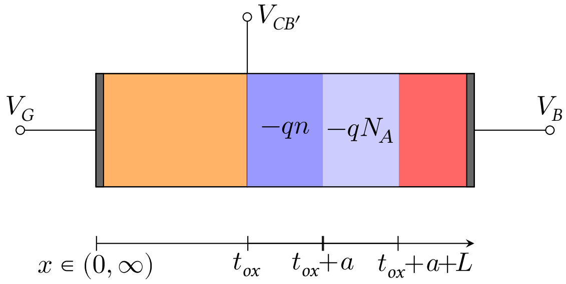
AC model (active zone)



where PMOS have ⬦ instead (or, alternatively, changes $v_{gs} \rightarrow v_{sg}$) and

$$g_m = \frac{2I_{DS}}{|V_{GS} - V_T|} \qquad r_0 = \left(\frac{\lambda I_{DS}}{\cancel{1 + \lambda V_{DS}}}\right)^{-1}$$

Also, $g_m = \beta(V_{GS} - V_T) = \sqrt{2\beta I_{DS}}$ in the saturation region.



Solving a MOS capacitor

"Fun AF!"

Voltages

We do KVL from $x = 0$ to $x = \infty$:

$$V_G = \underbrace{[V(0) - V(t_{ox})]}_{V_{ox}} + \underbrace{[V(t_{ox}) - V(\infty)]}_{V_{CB'}} - \Phi_{CMP} + V_B$$

where Φ_{CMP} is the voltage across the metal contact in V_B. Reordering terms, we obtain the voltage in the insulator:

$$\boxed{V_{ox} = V_G - V_B + \Phi_{CMP} - V_{CB'}}$$

Channel charge

In the insulator, $\nabla \mathbf{D} = 0$ so the displacement is constant and $\mathbf{E} = -\nabla V = \frac{D_0}{\varepsilon}$. We obtain a voltage $V_{ox} = \frac{D_0}{\varepsilon} t_{ox}$ after integrating in $x \in (0, t_{ox})$. The displacement can be writen in terms of this voltage $D_0 = \frac{\varepsilon}{t_{ox}} V_{ox} = C_0 V_{ox}$.

In the semiconductor, we have $\nabla \mathbf{D} = \rho$, so $\mathrm{d}D\,\hat{x} = \rho\,\mathrm{d}x\,\hat{x}$; let's integrate (why not) that in $x \in (t_{ox}, \infty)$:

$$\int_{t_{ox}}^{\infty} \hat{x}\mathbf{D}\,\mathrm{d}D = \int_0^{\infty} \rho\,\mathrm{d}x$$

$$\cancelto{0}{D(\infty)} - D(t_{ox}) = \int_{t_{ox}}^{t_{ox}+a} \rho\,\mathrm{d}x + \int_{t_{ox}+a}^{t_{ox}+a+L} \rho\,\mathrm{d}x + \cancelto{0}{\int_{t_{ox}+a+L}^{\infty} \rho\,\mathrm{d}x}$$

$$-D(t_{ox}) = \underbrace{\int_{t_{ox}}^{t_{ox}+a} -qn\,\mathrm{d}x}_{\text{Str. Inv.}} + \underbrace{\int_{t_{ox}+a}^{t_{ox}+a+L} -qN_A\,\mathrm{d}x}_{\text{Weak Inv.}} + \underbrace{0}_{\text{P region}}$$

So we obtain

$$\begin{aligned}
\int_{t_{ox}}^{t_{ox}+a} qn\,\mathrm{d}x &= D(t_{ox}) - qN_A L \\
&= C_0 V(t_{ox}) - qN_A L \\
&= C_0[V_G - V_B + \Phi_{CMP} - V_{CB'}] - qN_A L \\
&= C_0\left[V_G - V_B + \Phi_{CMP} - V_{CB'} - \frac{qN_A}{C_0} L\right] \\
&= C_0\left[V_G - V_B - V_{CB'} - \left(-\Phi_{CMP} + \frac{qN_A}{C_0} L\right)\right]
\end{aligned}$$

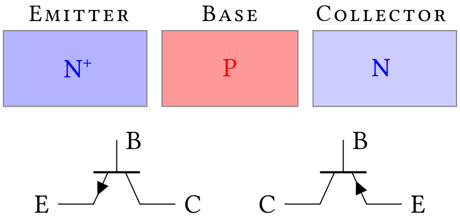
where $V(t_{ox})$ has been substituted by the monster found in the Voltages section. Because we added a strong inversion layer in the analysis, $V_{CB'}$ must be at least $2\Phi_0$ so it forms.

We are interested in an expresion that is a function of $V_1 = V_{CB'} + V_B + \Phi_{cmp}$, which is basically the voltage $V_{CB'}$ refered to V_B taking care of the metal contact voltage difference. Grouping everything else into a constant V_{TN},

$$\boxed{\int_{t_{ox}}^{t_{ox}+a} qn\,\mathrm{d}x = C_0[V_G - V_1 - V_{TN}]}$$

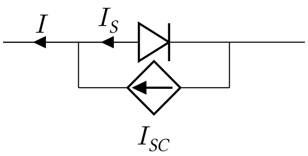
BJT transistor

"I love the smell of a PN junction in the morning"



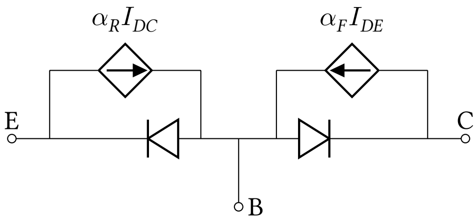
Inverse biased diode

The main idea is to control the saturation current of a diode. We model this as a controled generator of $I = I_{SC}$ in series with the diode, of current $I = I_S$ (saturation current).



Two diode model

General model for all work regions.



$$I_E = I_{ES}\left(e^{qV_{BE}/kT} - 1\right) - \alpha_R I_{CS}\left(e^{qV_{BC}/kT} - 1\right)$$
$$I_C = \alpha_F I_{ES}\left(e^{qV_{BE}/kT} - 1\right) - I_{CS}\left(e^{qV_{BC}/kT} - 1\right)$$
$$I_B = I_E - I_C$$

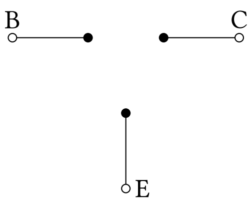
with $\alpha_R I_{CS} = \alpha_F I_{ES} = I_S = qAD_b \frac{n_{b0}}{W}$. This equations (Elbers-Moll equations) are too complicated for direct use, and simplifications are made depending on the work region.

The relationship of I_S with W shows that a small base region increases the diode interdependency.

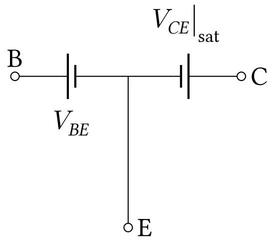
Region models (NPN)

The region depends on the *operation point*, (I_C, V_{CE}).

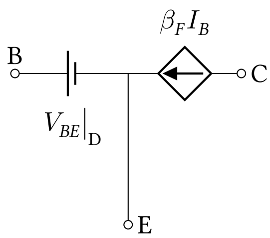
Cut Both junctions are inversely polarized, so $e^{qV/kT} \sim 0$. The only current is the diode saturation current, which can be neglected.



Saturation Both junctions are forward polarized; the circuit is equivalent to a short circuit with the PN junctions voltage drops.



Active Base-emitter junction forward biased, but base-collector junction reverse biased. This is the main region of the transistor, in which it behaves like an amplifier.

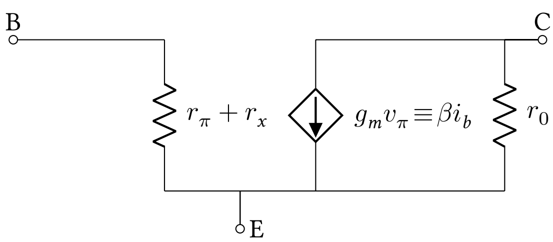


where $\beta_F = \frac{\alpha_F}{\alpha_F - 1}$.

Active inverse Is identical to the *active* zone, but the emitter and the collector switch roles. Is not very used because of the BJT being optimized for forward operation. The emitter is more doped than the collector for this exact thing.

PNP transistors are identical but defining I_E, I_C in the opposite sense (from emitter to collector) and reversing the current generator.

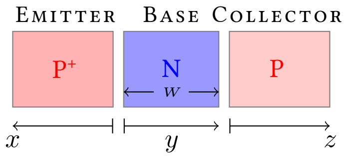
AC model



where r_x is the resistance of the base terminal. For a PNP, the model is exactly the same but I_B, I_E and I_C are defined in the opposite sense (and the current generator is reversed). The parameters are given by:

$$\frac{q}{kT} I_C = g_m \qquad \frac{I_C}{V_A + V_{CE}} = \frac{1}{r_0} \qquad \frac{g_m}{\beta_0} = \frac{1}{r_\pi}$$

Sometimes, the *hybrid model* is used, and $h_{fe} \equiv \beta$ and $h_{ie} \equiv r_\pi + r_x$ are given.



Solving a BJT

"You never have enough diodes"

Carrier concentrations

The continuity equation is the same for all the regions, with solution of the style $A\mathrm{e}^{\zeta/L} + B\mathrm{e}^{\zeta/L}$.

Emitter Solve $\frac{\mathrm{d}}{\mathrm{d}x^2}p'_e = p'_e/L_e^2$ with boundary conditions: $p'_e(0) = p_{e0}\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big)$ and $p'_e(\infty) = 0$. Solution:

$$p'_e(x) = p_{e0}\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big)\mathrm{e}^{-x/L_e}$$

Collector Exactly the same.

$$p'_c(z) = p_{c0}\big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)\mathrm{e}^{-z/L_e}$$

Base The boundary conditions are a bit more difficult, so the ending expresion is more complicated.

$$n'_b(0) = n_{b0}\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big)$$
$$n'_b(W) = n_{b0}\big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)$$

At the end of the day, we obtain

$$n'_b(y) = \frac{n_{b0}}{\sinh(W/L_b)}\Big[\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big)\sinh\frac{W-y}{L_b} + \big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)\sinh\frac{y}{L_b}\Big]$$

We can assume than generation-recombination rate is small; $n'_b/L_b^2 \sim 0$ in the differential equation or $W \ll L_b$ in the full expression. With this approximation, we obtain:

$$n'_b(y) = n_{b_0}\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big) - n_{b0}\left(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - \mathrm{e}^{qV_{BC}/\mathrm{k}T}\right)\frac{y}{W}$$

in a first order aproximation, $\sinh x = x + \mathcal{O}(x^2)$. We obtain something proportional to y.

Current densities

For the minoritary carriers, they are just the drift currents ($\sim qD\nabla n$). We evaluate the emitter and collector currents in the boundary of the transition zone:

$$\mathbf{J}_{pe}(0) = q\frac{D_e p_{e0}}{L_e}\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big)\hat{x}$$
$$\mathbf{J}_{pc}(0) = -q\frac{D_c p_{c0}}{L_c}\big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)\hat{y}$$

For the base current we derivate the full hiperbolic expression, and substitute $\cosh\varepsilon \sim 1$:

$$\mathbf{J}_{nb}(y) = -q\frac{D_b n_{b0}}{W}\Big[\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big) - \big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)\Big]\hat{z}$$

Intensities

They are independent of the coordinate, so we do the same trick as in the diode and compute them in the transition zone to be able to sum both carriers.

Beware the sign changes due to $\hat{x}$ being in the opposite direction from $\hat{y}$, $\hat{z}$.

Emitter

$$\begin{aligned}\mathbf{J}_E &\simeq \mathbf{J}_n(y=0) + \mathbf{J}_p(x=0)\\ &= -qD_b\frac{n_{b0}}{W}\left[\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big) - \big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)\right](-\hat{x})\\ &+ qD_e\frac{p_{e0}}{L_e}\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big)\hat{x}\end{aligned}$$

We define I_E as going from base to emitter ($\rightarrow \hat{x}$), and S as the cross section surface:

$$\begin{aligned}I_E &= \int \mathbf{J}\,\mathrm{d}\mathbf{S} = \big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big)\underbrace{\left[AqD_e\frac{p_{e0}}{L_e} + AqD_b\frac{n_{b0}}{W}\right]}_{I_{ES}}\\ &- \big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)\underbrace{\left[Aq\frac{D_b n_{b0}}{W}\right]}_{\alpha_R I_{CS}}\\ &= I_{ES}\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big) - \alpha_R I_{CS}\big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)\end{aligned}$$

Collector

$$\begin{aligned}\mathbf{J}_C &\simeq \mathbf{J}_n(y=W) + \mathbf{J}_p(z=0)\\ &= -qD_b\frac{n_{b0}}{W}\left[\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big) - \big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)\right](+\hat{z})\\ &+ qD_c\frac{p_{c0}}{L_c}\big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)\hat{z}\end{aligned}$$

We define I_C as going from collector to base ($\rightarrow -\hat{z}$):

$$\begin{aligned}I_C &= \int \mathbf{J}\,\mathrm{d}\mathbf{S} = -\big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)\underbrace{\left[AqD_e\frac{p_{e0}}{L_e} + AqD_b\frac{n_{b0}}{W}\right]}_{I_{CS}}\\ &+ \big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big)\underbrace{\left[Aq\frac{D_b n_{b0}}{W}\right]}_{\alpha_F I_{ES} = \alpha_R I_{CS}}\\ &= \alpha_F I_{ES}\big(\mathrm{e}^{qV_{BE}/\mathrm{k}T} - 1\big) - I_{CS}\big(\mathrm{e}^{qV_{BC}/\mathrm{k}T} - 1\big)\end{aligned}$$

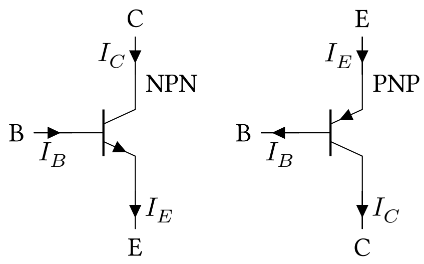
Base After defining I_B as entering the transistor we can write $I_B = I_E - I_C$, which are already calculated.

Doin' circuits

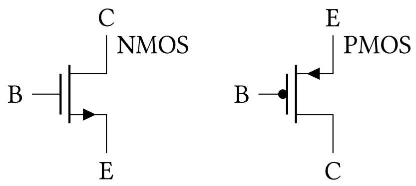
"Cheesesheet"

Symbols

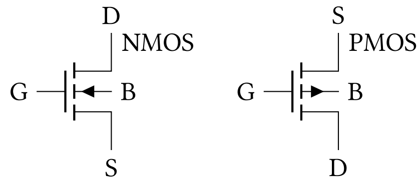
BJT's



MOSFET's



MESFET's are drawn whithout the arrow. If there is no connection between the body and the source, the following symbols can be used:



If it is an depletion device, a solid channel is drawn below the big line, from source to drain.

Lastly, the JFET's: