# H-principle solution

Ethan Phillip Riley

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## 1 proof

1. Assume in h-principle

$$\Phi(U_m, Y_n) = \Phi(U_m, {J_f}^k) = 0$$

2. given arbitrary transform:

$$T : Y_n \longleftrightarrow {J_f}^k$$

3. given 1. we use to define our limits of such transform

$$\lim_{f(U_m)\to 0} T(Y_n) = 0$$

$$\lim_{f(U_m)\to 0} T({J_f}^k) = 0$$

4. As they both converge to the limit we can rearrange to:

$$\Phi(U_m, Y_n) - \Phi(U_m, {J_f}^k) = 0$$

Reflecting this in the limits:

$$\lim_{f(U_m)\to 0} T(Y_n) - T({J_f}^k) = 0$$

5. Set this is as our boundary of our manifold to integrate over transform T:

$$U = [{J_f}^k, Y_n]$$

6. we can express this transform as the integral with the limits as basis:

$$\int_U T(Y_n) - T(J_f^k) dT = 0$$

7. using generalized stokes theorem (second fundamental theorem of calculus):

$$\int_U \int_U T(f) dT dT = 0$$

8. then using the stokes theorem of manifolds:

$$\int_{\partial U} \int_{U} T(T(f))dT = 0$$

$$\int_{\partial U} \int_{\partial U} T(T(T(f))) = 0$$

9. remembering the property of bidirectional transform, it cancels out to:

$$\int_{\partial U} \int_{\partial U} T(f) = 0$$

10. this shows that this integral is closed and exact using Poincare's lemma ■