CHAPTER

4 Laws of Motion

NEWTON'S LAW AND LINEAR MOMENTUM

1. **Newton's First Law of Motion:** An object continues to be in the state of rest or of uniform motion unless a force is applied on it.
2. **Newton's Second Law of Motion:** Newton's second law of motion states that the rate of change of linear momentum of a body is directly proportional to the applied force and the change in momentum takes place in the direction of the applied force.

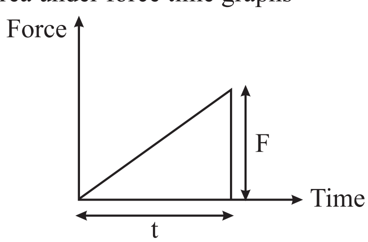
   $$\vec{F} = \frac{d\vec{p}}{dt}$$

   $$\vec{F} = \frac{dm\vec{v}}{dt} = m\vec{a} \qquad \text{(Here m = constant)}$$

3. **Newton's third law of motion:** It states that to every action there is an equal and opposite reaction.
4. **Linear momentum**, $\vec{p} = m\vec{v}$

IMPULSE

1. Let I is the impulse and F is the force acting in a very short time dt then,
2. Impulse = area under force time graphs



   $$I = \text{Area of triangle} = \frac{1}{2} \times \text{Base} \times \text{Height} = \frac{1}{2}\text{Ft}$$

3. Impulse in terms of momentum

   $I = \vec{P}_2 - \vec{P}_1 = \Delta\vec{P}$, [$\vec{P}$ is the momentum of the particle]

   $$\Rightarrow I = \int_{\vec{P}_1}^{\vec{P}_2} d\vec{P}$$

CONSERVATION OF LINEAR MOMENTUM

In the absence of any external force

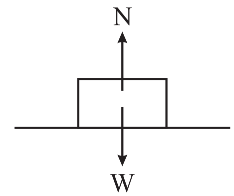
Total initial momentum = Total final momentum

$\Rightarrow P_i = P_f$

DIFFERENT TYPES OF FORCES

Normal Reaction

It is a contact force. It is the force between two surfaces in contact, which is always perpendicular to the surfaces in contact.



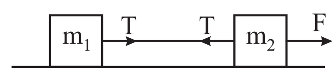
Tension

Tension force always pulls a body.

Tension is a reactive force. It is not an active force.

Tension across massless pulley or frictionless pulley remains constant.

Rope becomes slack when tension force becomes zero.



Spring Force

F = –kx

x is displacement of the free end from its natural length or deformation of the spring where K = spring constant.

K × l = constant

where l = Natural length of spring

Note: If spring is cut into two in the ratio m : n then spring constant is given by

$$\ell_1 = \frac{m\ell}{m+n}; \ell_2 = \frac{n\ell}{m+n}$$

$$k\ell = k_1\ \ell_1 = k_2\ell_2$$

For series combination of springs.

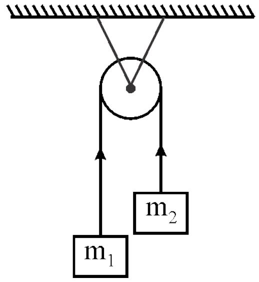
$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots$$

For parallel combination of spring.

$$k_{eq} = k_1 + k_2 + k_3 \ldots$$

MOTION OF CONNECTED BODIES

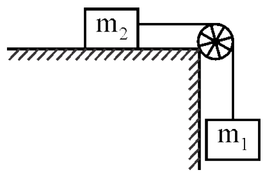
1. When two bodies of masses m_1 and m_2 are tied at the ends of an inextensible string that passes over a light frictionless pulley.



Acceleration of the blocks (a) = $\left(\frac{m_2 - m_1}{m_1 + m_2}\right)g$

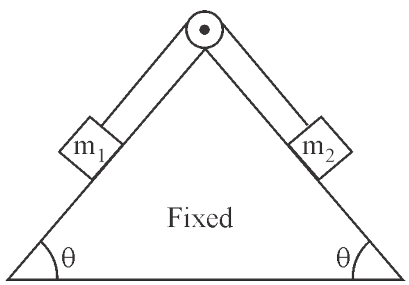
Tension in the string (T) = $\left(\frac{2m_1m_2}{m_1 + m_2}\right)g$

2. Acceleration of the system when blocks connected by a string which passes over a light frictionless pulley.



$$a = \left(\frac{m_1 g}{m_1 + m_2}\right)$$

3. In the system shown below if $m_1 > m_2$, then acceleration of the blocks is given by (All surfaces are smooth)



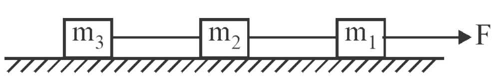
$$a = \frac{(m_1 - m_2)g\sin\theta}{m_1 + m_2}$$

[a is the acceleration of the block

tension in the string is given by

$$\frac{2m_1m_2 g\sin\theta}{m_1 + m_2}$$

4.



Equation of motion of first body is $F - T_1 = m_1a$

Equation of motion of second body is $T_1 - T_2 = m_2a$

Equation of motion of 3rd body is $T_2 = m_3a$

Common acceleration, (a is the comman acceleration)

$$a = \frac{F}{m_1 + m_2 + m_3}$$

$$T_1 = \left(\frac{m_2 + m_3}{m_1 + m_2 + m_3}\right)F$$

$$T_2 = \frac{m_3F}{m_1 + m_2 + m_3}$$

Frame of Reference

Inertial frames of reference: A reference frame which is either at rest or in uniform motion along the straight line. A non-accelerating frame reference is called an inertial frame of reference.

All the fundamental laws of physics have been formulates in respect of inertial frame of reference.

Non-inertial frame of reference: An accelerating frame of reference is called a non-inertial frame of reference. Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force.

Pseudo force: The force on a body due to acceleration of non-inertial frame is called fictitious or apparent or pseudo force and is given by $-m\vec{a}_0$ where $\vec{a}$ is acceleration of non-inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pseudo force must be opposite to the direction of acceleration of the non-inertial frame.

FRICTION

Friction is a force which resist the relative motion of two contact bodies.

Friction force is of two types– (1) Static, (2) Kinetic.

Static Friction

It exists between the two surfaces when there is tendency of relative motion but no relative motion along the two contact surfaces.

This means static friction is a variable and self adjusting force. However it has a maximum value called limiting friction.

$f_{max} = \mu_s \times N$

$0 < f_s < f_{smax}$

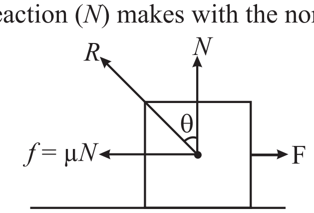
Kinetic Friction

$f_k = \mu_k \times N$

The proportionality constant u_k is called the coefficient of kinetic friction and its value depends on the nature of the two surfaces in contact.

Angle of Friction

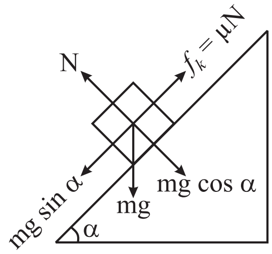
It is the angle which the resultant of the force of limiting friction and the normal reaction (N) makes with the normal reaction.



$\mu_1 = \tan\theta$

Angle of Repose or Angle of Sliding

It is the minimum angle of inclination of a plane with the horizontal, such that a body placed on it, just begins to slide down.



If angle of repose is α and coefficient of limiting friction is μ_l, then $\mu = \tan\alpha$

Note: Pulling (Fig. I) is easier than pushing (fig. II) on a rough horizontal surface beacuse normal reaction is less in pulling than in pushing.

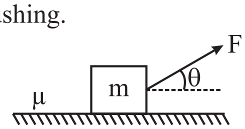


Fig.-I

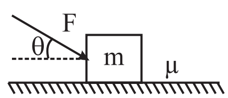


Fig.-II

DYNAMICS OF CIRCULAR MOTION

Centripetal Force: (F_C): It is a force which required to move a body in circular path. Direction of centripetal force is always perpendicular to direction of linear velocity.

Centripetal Force (F_C) = $\dfrac{mv^2}{r}$

Centrifugal Force: It is a pseudo force experienced radially outward by an object of mass m relative to the object, moving in a circular path of radius r, relative to an inertial frame. the centrifugal force is given by $\dfrac{mv^2}{r}$.

(v = speed of object relative to inertial frame)

Circular turning of roads:

(*i*) **Friction only:** Friction provides the necessary centripetal force for turning of vehicle.

$$v_{\max} = \sqrt{\mu_s rg}$$

$v_{\max}$ = Maximum safe velocity for turning.

(*ii*) **Banking of roads only:** Centripetal force provides by banking of road only:

$$v_{\max} = \sqrt{\tan\theta\, rg}$$

(*iii*) **Both friction and banking of roads:** To avoid the risk factor and reduce the wear of tyres, both friction and banking of rods are used to provide the necessary centripetal force.

Maximum safe speed without skidding:

$$v_{\max} = \frac{rg\left(\tan\theta + \mu_s\right)}{1 - \mu_s \tan\theta}$$

Note: In condition of both banking of rod and friction, the direction of frictional force is determined by the velocity of vehicle v for a given banking angle θ:

(*a*) It $v = 0$ then frictional force is outward.

(*b*) It $v > \sqrt{gr\tan\theta}$, then frictional force is inwards.

(*c*) It $v = \sqrt{gr\tan\theta}$, then frictional force $f = 0$.

(*d*) It $v < \sqrt{gr\tan\theta}$, then frictional force f is outwards.