CHAPTER 3

Motion in a Plane

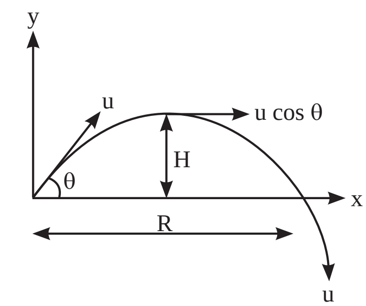
PROJECTION MOTION

Horizontal Motion of Projectile

$u \cos \theta = u_x$

$a_x = 0$

$x = u_x t = (u \cos \theta)t$



Vertical Motion of Projectile

$v_y = u_y - gt$ where $u_y = u \sin \theta$; $y = u_y t - \frac{1}{2}gt^2 = u \sin \theta t - \frac{1}{2}gt^2$

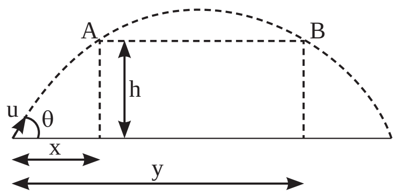
Net acceleration = $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$

At any Instant

$v_x = u \cos \theta, \quad v_y = u \sin \theta - gt$

For Projectile Motion

- A body crosses two points at same height in time t_1 and t_2 the points are at distance x and y from starting point then



  - $x + y = R$
  - $t_1 + t_2 = T$
  - $h = ½\, g t_1 t_2$
  - Average velocity from A to B is $u \cos \theta$
- If a person can throw a ball to a maximum distance 'x' then the maximum height to which he can throw the ball will be (x/2).

Velocity of Particle at Time t

- $\vec{v} = v_x\hat{i} + v_y\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$
- If angle of velocity $\vec{v}$ with horizontal is α, then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$

At highest point: $v_y = 0$, $v_x = u \cos \theta$

Time of flight: $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$

Horizontal range: $R = (u \cos \theta)T$

$$= \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g}$$

It is same for θ and (90° – θ) and maximum for θ = 45°.

Maximum height: $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = \frac{1}{8}gT^2$

$$\frac{H}{R} = \frac{1}{4}\tan\theta$$

Equation of trajectory:

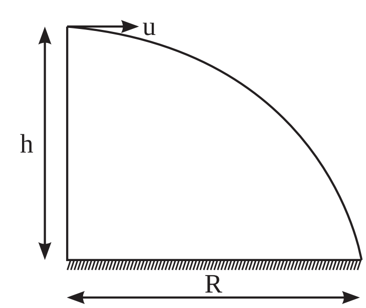
$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$$

Horizontal Projection from a Height h

Time of flight: $T = \sqrt{\frac{2h}{g}}$

Horizontal range: $R = uT = u\sqrt{\frac{2h}{g}}$

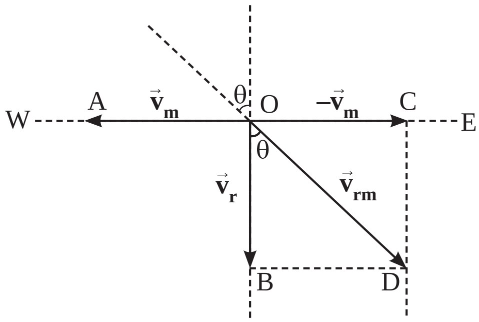
Angle of velocity at any instant with horizontal: $\theta = \tan^{-1}\left(\frac{gt}{u}\right)$



RELATIVE MOTION

Relative Velocity of Rain w.r.t. the Moving Man

A man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\overrightarrow{OB}$ as shown in figure.



The relative velocity of rain w.r.t. man $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$ will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

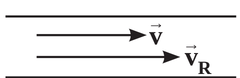
$$\therefore \quad v_{rm} = \sqrt{v_r^2 + v_m^2 - 2v_r v_m \cos 90^\circ} = \sqrt{v_r^2 + v_m^2}$$

If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction then

$$\tan\theta = \frac{BD}{OB} = \frac{v_m}{v_r} \Rightarrow \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$$

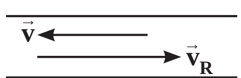
Swimming into the River

- A man can swim with velocity $\vec{v}$, i.e., it is the velocity of man w.r.t. still water. If water is also flowing with velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{\mathbf{v}}_\mathbf{m} = \vec{\mathbf{v}} + \vec{\mathbf{v}}_\mathbf{R}$.
  - If the swimming is in the direction of flow of water or along the downstream then



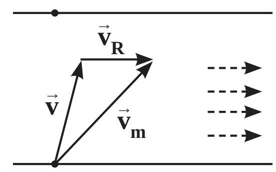
$v_m = v + v_R$

  - If the swimming is in the direction opposite to the flow of water or along the upstream then

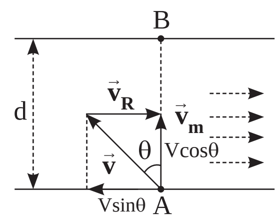


$v_m = v - v_R$

- If man is crossing the river as shown in the figure i.e. $\vec{v}$ and $\vec{v}_R$ are non-collinear then use the vector algebra $\vec{v}_m = \vec{v} + \vec{v}_R$ **(assuming v > v_R)**

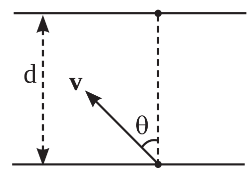


For shortest path



(For minimum displacement)

- To reach at B, v sin θ = v_R ⇒ sin θ = $\frac{v_R}{v}$

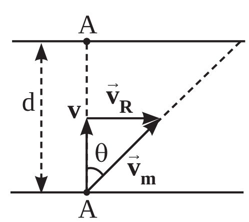


t = (d/v cos θ)

- Time of crossing

Notes: If v_R > v then for minimum drifting sin θ = $\frac{v}{v_R}$.

For minimum time



(For minimum time)

then, Drift = $t_{min}\ V_R = \frac{dV_R}{V}$