Centre of Mass and System of Particles

SHORT NOTES

Centre of Mass of a System of 'N' Discrete Particles

$$r_{cm} = \frac{m_1r_1 + m_2r_2 + \cdots + m_nr_n}{m_1 + m_2 + \cdots + m_n};\ r_{cm} = \frac{\sum_{i=1}^{n} m_ir_i}{\sum_{i=1}^{n} m_i},\ r_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_ir_i$$

Centre of Mass of a Continuous Mass Distribution

$$x_{cm} = \frac{\int x\,dm}{\int dm},\ y_{cm} = \frac{\int y\,dm}{\int dm},\ z_{cm} = \frac{\int z\,dm}{\int dm}$$

$$\int dm = M(\text{mass of the body})$$

Collisions

(*a*) Collision is the interaction between two (or) more particles where exchange of momentum takes place.

(*b*) In case of collisions as the impulsive force acting during collision is internal, the total momentum of system always remains conserved.

(*c*) If the velocities of the colliding particles are along the same line before and after the collision then the collision is said to be one dimensional collision.

(*d*) In a collision, if the motion of colliding particles before and after the collision are not along the initial line of motion, then the collision is said to be oblique collision.

(*e*) In an oblique elastic collision, if $m_1 = m_2$ and m_2 is initially at rest, then after the collision the two masses will move in directions inclined at 90° to each other.

Coefficient of Restitution

(*a*) $e = \dfrac{\text{relative velocity of separation}}{\text{relative velocity of approach}}$

$\therefore\ e = \dfrac{v_2 - v_1}{u_1 - u_2}$

(*b*) The value of coefficient of restitution (e) is independent of masses and velocities of the colliding bodies. It depends on their materials.

(*c*) For a perfectly elastic collision, $e = 1$

For a perfectly inelastic collision, $e = 0$

For other collisions, e lies between 0 and 1

(*d*) If a body falls from a height 'h' and strikes the level ground with velocity V in time seconds and rebounds with velocity V_1 upto height h_1 in time t_1 seconds.

The coefficient of restitution is given by

$$e = \frac{V_1}{V} \quad (\text{or})\ e = \sqrt{\frac{H_1}{H}} \quad (\text{or})\ e = \frac{t_1}{t}$$

For a perfectly elastic collision, $H_1 = H$

For a perfectly inelastic collision, $H_2 = 0$

For other collisions, $H_1 < H$

For any collision, H_1 cannot be greater than h

(*e*) A small metal sphere falls freely from a height 'H' upon a fixed horizontal plane. If e is the coefficient of restitution, then

(i) The height to which it rebounds after n collision is $H_n = e^{2n} H$

(ii) The velocity with which it rebounds from the ground after nth collision is $v_n = e^n v$. Where v is the velocity of the sphere just before first collision.

(iii) The total distance travelled by it before it stops rebounding is $d = H\left(\dfrac{1+e^2}{1-e^2}\right)$

(iv) The total time taken by it to come to rest is $T = \sqrt{\dfrac{2H}{g}}\left(\dfrac{1+e}{1-e}\right)$

(*f*) In one dimensional semi elastic (or) inelastic collisions, Linear momentum is conserved but kinetic energy of the system is not conserved.

(*g*) In above collisions, there is loss of kinetic energy from the system in the form of heat, sound, light etc.

(*h*) In one dimensional semi elastic collision relative velocity of seperation = e × realtive velocity of approach i.e., $v_2 - v_1 = e(u_1 - u_2)$

Inelastic Collisions

(*a*) Formulas for final velocities in case of one dimensional semi elastic collision are

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_2 + \left(\frac{1+e}{m_1 + m_2}\right)m_2u_2$$

$$v_2 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 + \left(\frac{1+e}{m_1 + m_2}\right)m_1u_1$$

(*b*) Loss in Kinetic energy of the system in one dimensional semi elastic collision is $\Delta E_k = \dfrac{1}{2}\left[\dfrac{m_1m_2}{m_1 + m_2}\right][1-e^3][u_2 - u_2]^2$

(*c*) In one dimensional perfectly inelastic collision the two particles stick together after the collision and move with common velocity.

(*d*) The formula for common velocity of compound body after perfectly inelastic collision is $\vec{v} = \dfrac{m_1\vec{u_1} + m_2\vec{u_2}}{m_1 + m_2}$

(*e*) The loss of kinetic energy in a perfectly inelastic collision ($e = 0$) is given by $\Delta E_k = \dfrac{1}{2}\left[\dfrac{m_1m_2}{m_1 + m_2}\right][u_1 - u_2]^2$

(*f*) In a perfectly inelastic collision, the ratio of loss of energy of the system and its initial energy, if u_2 is zero, is given by

$$\frac{\Delta E_k}{E_1} = \frac{m_2}{m_1 + m_2}$$

(*g*) In a perfectly inelastic collision, the ratio of final energy to initial energy, of the system, if u_2 is zero, is given by

$$\frac{E_k}{E_1} = \frac{m_1}{m_1 + m_2} \Rightarrow E_k < E_1$$