# Mechanical Properties of Solids

## STRAIN

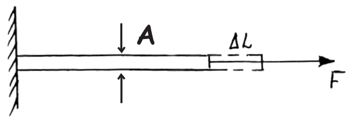
01 Longitudinal strain= $\frac{\Delta L}{L}$ $\frac{\text{Change in length}}{\text{Original length}}$

02 Volumetric strain= $\frac{\Delta V}{V}$ $\frac{\text{Change in volume}}{\text{Original volume}}$

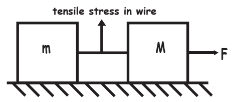
03 Shearing strain= $\phi = \frac{\Delta x}{L}$

## LONGITUDINAL STRESS

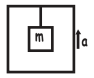
Longitudinal/Tensile stress causes increase in length



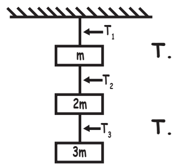

Longitudinal stress = $\frac{F}{A}$ unit N/m$^2$




$T.S = \frac{Fm}{(M+m)/A}$




$T.S = T = \frac{m(g+a)}{A}$

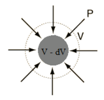



$T.S_1 = \frac{T_1}{A} = \frac{6\ mg}{A}$

$T.S_2 = \frac{T_2}{A} = \frac{5\ mg}{A}$

$T.S_3 = \frac{T_3}{A} = \frac{3\ mg}{A}$

## VOLUME STRESS



- Same as pressure
- Causes change in volume

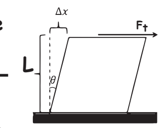
volume stress = $\frac{F}{A}$ = pressure

here, F = normal force (or thrust)

## SHEARING STRESS

- Causes change in shape

shearing stress = $\frac{F_t}{A}$



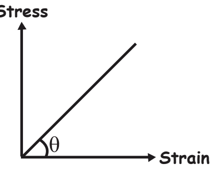

$F_t$ = tangential force

## HOOKE' S LAW

$E = \frac{\text{Stress}}{\text{Strain}}$

(E= modulus of elasticity)

$\tan\theta$ = modulus of elasticity




1. For rigid body E = infinity
2. Steel is more elastic than rubber
3. Depends on :-
   (a) Nature of metal
   (b) Temperature
4. Independent of dimensions

risid body

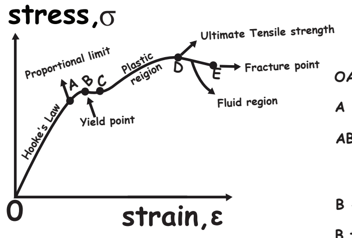
Young's modulus

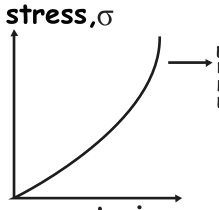
E

Bulk Modulus

Modulus of rigidity

## STRESS STRAIN CURVE




Elastomer Doesn't obey Hooke's Law Eg:-Rubber

OA - Hooke's law obeyed

A - Proportional limit

AB - stress is not proportional to strain but body regains it's original shape and size regains load is removed

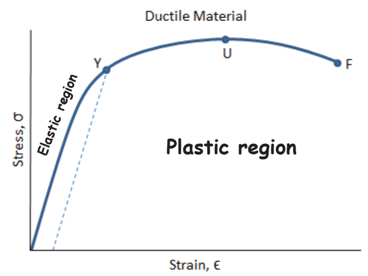
B - Yield point

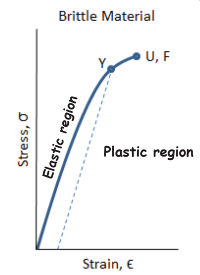
B to D - Body doesn't regain it's original dimension. Beyond B is plastic region.

D - Ultimate stress point

Beyond D - large strain is produced even for a small applied force.

E - Fracture occurs

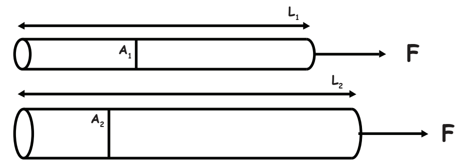



Plastic region is large for ductile materials and smaller for brittle materials

## BREAKING STRESS

Breaking Force= breaking stress x area, B.F $\propto$ A



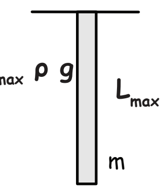

$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow (B.S)_1 = (B.S)_2$

### BREAKING OF WIRE UNDER IT'S OWN WEIGHT

B.S X A =T= mg

B.S X A = Vx ρxg = A $L_{max}$ ρ g

$L_{max} = \frac{B.S}{\rho g}$




## RATIO OF DENSITY OF BODY TO THAT OF LIQUID IN WHICH BODY IS IMMERSED

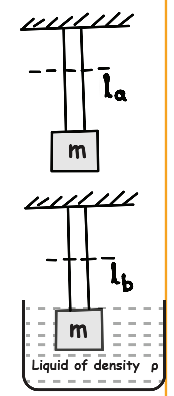
$mg = Kl_a$

$mg\left(1-\frac{\rho}{\sigma}\right) = Kl_b$

$\left(1-\frac{\rho}{\sigma}\right) = \frac{l_b}{l_a}$

$\frac{\rho}{\sigma} = \frac{l_a - l_b}{l_a}$

$\frac{\sigma}{\rho} = \frac{l_a}{l_a - l_b}$




here, $l_a$ and $l_b$ are extensions in the rod in the two cases.

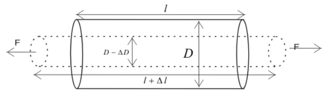
## YOUNG'S MODULUS

$Y = \frac{\text{Longitudnal stress}}{\text{Longitudnal strain}} = \frac{FL}{A\Delta L}$

Comparing with a spring of force constant K

$Y = \frac{FL}{Ax}$ $F = \frac{AYx}{L} = kx$ $k = \frac{AY}{L}$

## POISSON'S RATIO

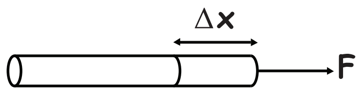



$\sigma = \frac{\text{Lateral strain}}{\text{Longitudinal strain}}$ $\sigma = \frac{(-dr/r)}{\left(\frac{dL}{L}\right)} = \frac{-\frac{\Delta r}{r}}{\frac{\Delta l}{l}} = \frac{-\frac{\Delta D}{D}}{\frac{\Delta l}{l}}$

Theoretical value :- $-1 < \sigma < 0.5$

Practical value :- $0 < \sigma < 0.5$

## ELASTIC POTENTIAL ENERGY AND ENERGY DENSITY




$E.PE = \frac{1}{2}kx^2 = \frac{F^2}{2k} = \frac{1}{2}Fx$

$= \frac{1}{2}\left(\frac{YA}{L}\right)x^2$

$= \frac{1}{2} \times \frac{\text{Stress}}{\text{Strain}} \times \text{volume} \times \text{strain}^2$

$= \frac{1}{2} \times \text{Stress} \times \text{strain} \times \text{volume}$

Energy Density= $\frac{\text{EPE}}{\text{volume}}$

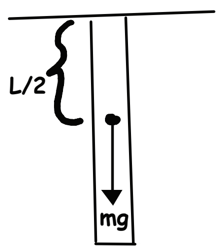
$\Rightarrow e = \frac{1}{2} \times \text{Stress} \times \text{strain}$

## INCREASE IN LENGTH DUE TO IT'S OWN WEIGHT

$F = kx$

$mg = \frac{YA}{\frac{L}{2}}x$

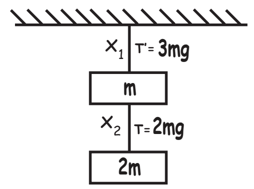
$x = \frac{mgL}{2YA}$




$P.E = \frac{F^2}{2k}$

$= \frac{m^2g^2}{2\frac{YA}{\frac{L}{2}}} = \frac{1}{4}\frac{m^2g^2L}{YA}$

## RATIO OF EXTENSION




$\frac{x_1}{x_2} = \frac{\frac{F_1}{k_1}}{\frac{F_2}{k_2}} = \frac{\frac{3mg \times L_1}{Y_1 \times A_1}}{\frac{2mg \times L_2}{Y_2 \times A_2}} = \frac{3l}{2yd^2}$

where $l = \frac{L_1}{L_2}$ $y = \frac{Y_1}{Y_2}$ $d = \frac{D_1}{D_2}$

## BULK MODULUS

Bulk modulus, B = $\frac{\text{Volumetric stress}}{\text{Volumetric strain}} = \frac{\Delta P}{-\frac{\Delta V}{V}}$

$K = \frac{1}{B}$ =compressibility

$B_{isothermal} = P$, $B_{adibatic} = \gamma P$, $B_{isotropic} = xP$

## MODULUS OF RIGIDITY

$\eta = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F}{A\phi} = \frac{Fl}{Ax}$