CHAPTER 4

Moving Charges and Magnetism

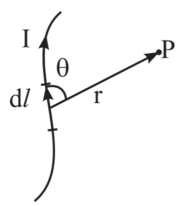
A static charge produces only electric field. A moving charge produces both electric field and magnetic field. A current carrying conductor produces only magnetic field.

MAGNETIC FIELD PRODUCED BY A CURRENT WIRE (BIOT-SAVART'S LAW)

The magnetic induction dB produced by an element $d\ell$ carrying a current I at a distance r is given by:

$$dB = \frac{\mu_0\mu_r}{4\pi}\frac{I\,dl\sin\theta}{r^2} \Rightarrow \vec{dB} = \frac{\mu_o\mu_r}{4\pi}\frac{I\left(\vec{d\ell}\times\vec{r}\right)}{r^3}$$

here the quantity $Id\ell$ is called as current element.



μ = permeability of the medium = $\mu_0\mu_r$

μ_0 = permeability of free space

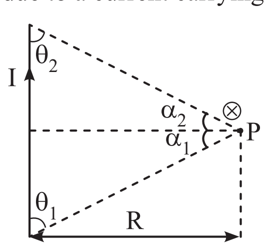
μ_r = relative permeability of the medium (Dimensionless quantity)

Unit of μ_0 & μ is NA^{-2} or Hm^{-1};

$\mu_0 = 4\pi \times 10^{-7}\ Hm^{-1}$

Magnetic Induction Due To a Straight Current Conductor

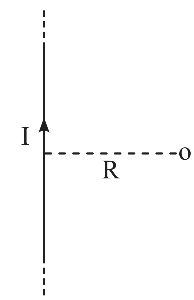
Magnetic induction due to a current carrying straight wire



$$B = \frac{\mu_0 I}{4\pi R}(\cos\theta_1 + \cos\theta_2) = \frac{\mu_0 I}{4\pi R}(\sin\alpha_1 + \sin\alpha_2)$$

Magnetic induction due to a infinitely long wire $B = \frac{\mu_0 I}{2\pi R}\otimes$

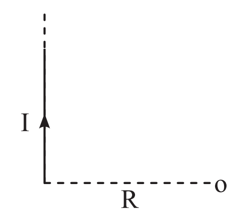
$\alpha_1 = 90^o$; $\alpha_2 = 90^o$



Magnetic induction due to semi infinite straight conductor

$B = \frac{\mu_0 I}{4\pi R}\otimes$

$\alpha_1 = 0^o$; $\alpha_2 = 90^o$



❖ Magnetic field due to a flat circular coil carrying a current:

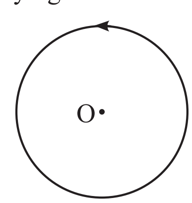
(*i*) At its centre $B = \frac{\mu_0 NI}{2R}$

where

N = total number of turns in the coil

I = current in the coil

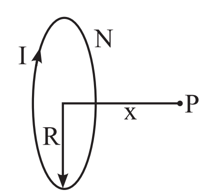
R = Radius of the coil



(*ii*) On the axis $B = \frac{\mu_0 NIR^2}{2\left(x^2+R^2\right)^{3/2}}$

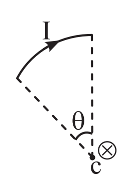
Where x = distance of the point from the centre.

It is maximum at the centre $B_C = \frac{\mu_0 NI}{2R}$



(*iii*) Magnetic field due to flat circular ARC :

$B = \frac{\mu_0 I\theta}{4\pi R}$



- Magnetic field due to infinite long solid cylindrical conductor of radius R
- For $r \geq R$: $B = \frac{\mu_0 I}{2\pi r}$
- For $r < R$: $B = \frac{\mu_0 I r}{2\pi R^2}$

Magnetic Induction Due to Solenoid

$B = \mu_0 nI$, direction along axis.

where $n \rightarrow$ number of turns per meter;

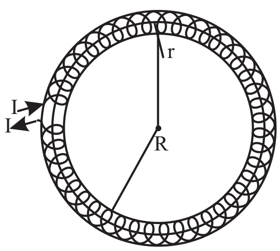
$I \rightarrow$ current

Magnetic Induction Due To Toroid

$B = \mu_0 nI$

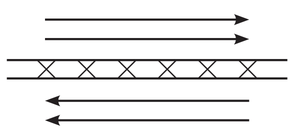
where $n = \frac{N}{2\pi R}$ (no. of turns per m)

N = total turns and $R \approx r$



Magnetic Induction Due To Current Carrying Sheet

$B = \frac{1}{2}\mu_0\lambda$, where λ = Linear current density (A/m)



Ampere's Circuital Law

$\oint \vec{B}.\vec{d}l = \mu\Sigma I$ where ΣI = algebraic sum of all the current.

Motion of A Charge In Uniform Magnetic Field

(a) When $\vec{V} \mid\mid \vec{B}$; Motion will be in a straight line and $\vec{F} = 0$

(b) When $\vec{V} \perp \vec{B}$: Motion will be in circular path with radius $R = \frac{mv}{qB}$ and angular velocity $\omega = \frac{qB}{m}$ and $F = qvB$.

(c) When $\vec{V}$ is at $\angle\theta$ to $\vec{B}$: Motion will be helical with radius $R_k = \frac{mv\sin\theta}{qB}$ and pitch $P_H = \frac{2\pi mv\cos\theta}{qB}$ and $F = qvB\sin\theta$.

LORENTZ FORCE

An electric charge 'q' moving with a velocity $\vec{V}$ through a magnetic field of magnetic induction $\vec{B}$ experiences a force $\vec{F}$, given by $\vec{F} = q\vec{v} \times \vec{B}$. Therefore, if the charge moves in a space where both electric and magnetic fields are superposed.

$\vec{F}$ = net electromagnetic force on the charge = $q\vec{E} + q\vec{v} \times \vec{B}$

This force is called the Lorentz Force.

Motion of Charge In Combined Electric Field & Magnetic Field

- When $\vec{v} \parallel \vec{B}$ & $\vec{v} \parallel \vec{E}$, Motion will be uniformly accelerated in a straight line as $F_{magnetic} = 0$ and $F_{electrostatic} = qE$

  So the particle will be either speeding up or speeding down
- When $\vec{v} \parallel \vec{B}$ & $\vec{v} \perp \vec{E}$, motion will be uniformly accelerated in a parabolic path
- When $\vec{v} \perp \vec{B}$ & $\vec{v} \perp \vec{E}$, the particle will move undeflected & undeivated with same uniform speed if $v = \frac{E}{B}$ (This is called as velocity selector condition)

Magnetic Force On A straight Current Carrying Wire : $\vec{F} = I\ (\vec{L} \times \vec{B})$

I = current in the straight conductor

$\vec{L}$ = displacement between the ends of the conductor in the direction of the current in it

$\vec{B}$ = magnetic induction. (Uniform throughout the length of conductor)

Note : In general, force is $\vec{F} = \int I\left(d\vec{l} \times \vec{B}\right)$

Magnetic Interaction Force Between Two Parallel Long Straight Currents

The interactive force between two parallel long straight wires is:

(i) Repulsive if the currents are anti-parallel.

(ii) Attractive if the currents are parallel.

This force per unit length on either conductor is given by $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$.

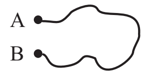
Where r = perpendicular distance between the parallel conductors

Magnetic Torque On a current loop

When a plane current loop of 'N' turns and of area 'A' per turn carrying a current I is placed in uniform magnetic field, it experiences zero net force, but experiences a torque given by $\vec{\tau} = NI\vec{A} \times \vec{B} = \vec{M} \times \vec{B} = BINA\sin\theta$ where $\vec{A}$ = area vector outward from the face of the circuit where the current is anticlockwise, $\vec{B}$ = magnetic induction of the uniform magnetic field.

$\vec{M}$ = magnetic moment of the current circuit = $NI\vec{A}$

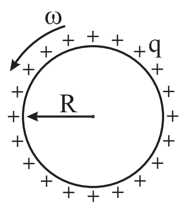
Force on A Random Shaped Conductor in A Uniform Magnetic Field



- Magnetic force on a closed loop in a uniform $\vec{B}$ is zero.
- Force experienced by a wire of any shape is equivalent to force on a wire joining points A & B in a uniform magnetic field.

Magnetic Moment of A Rotating Charge

If a charge q is rotating at an angular velocity ω, its equivalent current is given as $I = \frac{q\omega}{2\pi}$ & its magnetic moment is $M = I\pi R^2 = \frac{1}{2}q\omega R^2$.



Key Note

The ratio of magnetic moment to angular momentum of a uniform rotating object which is charged uniformly is always a constant, irrespective of the shape of conductor $M/L = q/2m$.

- Magnetic dipole
  - Magnetic moment $M = m \times 2l$ where m = pole strength of the magnet
  - Magnetic field at axial point (or End-on) of dipole $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{r^3}$
  - Magnetic field at equatorial position (Broad-on) of dipole $= \vec{B} = \frac{\mu_0}{4\pi}\frac{\left(-\vec{M}\right)}{r^3}$
  - At a point which is at a distance r from midpoint of dipole and making angle θ with dipole axis.

    Magnetic field $B = \frac{\mu_0}{4\pi}\frac{M\sqrt{1+3\cos^2\theta}}{r^3}$

- Torque on dipole placed in uniform magnetic field $\vec{\tau} = \vec{M}\times\vec{B}$
- Potential energy of dipole placed in an uniform field $U = -\vec{M}\cdot\vec{B}$
- Intensity of magnetisation $I = M/V$
- Magnetic induction $B = mH = m_0(H + I)$
- Magnetic permeability $\mu = \frac{B}{H}$
- Magnetic susceptibility $\chi_m = \frac{1}{H} = \mu - 1$
- Curies Law for paramagnetic materials $\chi_m \propto \frac{1}{T}$
- Curie-Wiess law for Ferromagnetic materials $\chi_m \propto \frac{1}{T - T_C}$

  Where T_C = Curie temperature