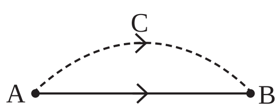
CHAPTER 2

Motion in a Straight Line

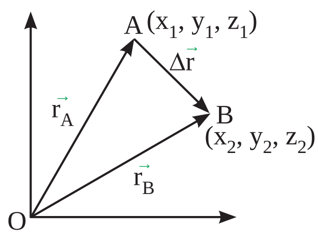
DISTANCE VERSUS DISPLACEMENT

Total length of path (ACB) covered by the particle is called distance. Displacement vector or displacement is the straught line distance (AB) and directed from initial position to final position.



DISPLACEMENT IS CHANGE OF POSITION VECTOR

From $\Delta OAB \quad \Delta\vec{r} = \vec{r}_B - \vec{r}_A$



$$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$$

and $\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

Average velocity $= \dfrac{\text{Displacement}}{\text{Time interval}} \Rightarrow \vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$

Average speed $= \dfrac{\text{Distance travelled}}{\text{Time interval}}$

For uniform motion

Average speed = | average velocity | = | instantaneous velocity

Velocity $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}(x\hat{i} + y\hat{j} + z\hat{k})$

$$= \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$$

Average Acceleration $= \dfrac{\text{Total change in velocity}}{\text{Total time taken}} = \vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$

Acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(v_x\hat{i} + v_y\hat{j} + v_z\hat{k})$$

$$= \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Important Points About 1D Motion

- Distance ≥ | displacement | and Average speed ≥ | average velocity |
- If distance > | displacement | it implies that atleast at one point in path velocity is zero.

Displa-cement	Differentiation → / ← Integration	Velocity	Differentiation → / ← Integration	Accele-ration

Motion with Constant Acceleration: Equations of Motion

- *In vector form:*

$$\vec{v} = \vec{u} + \vec{a}t \text{ and } \Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{s} = \left(\frac{\vec{u} + \vec{v}}{2}\right)t = \vec{u}t + \frac{1}{2}\vec{a}t^2 = \vec{v}t - \frac{1}{2}\vec{a}t^2$$

$$v^2 = u^2 + 2\vec{a}.\vec{s} \text{ and } \vec{s}_{n^{th}} = \vec{u} + \frac{\vec{a}}{2}(2n - 1)$$

($S_n^{th} \rightarrow$ displacement in n^{th} second)

- For one dimensional motion

$$v = u + at \qquad s = \left(\frac{u + v}{2}\right)t = ut + \frac{1}{2}at^2 = vt - \frac{1}{2}at^2$$

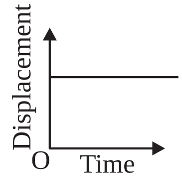
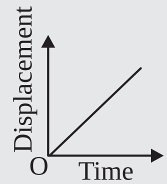
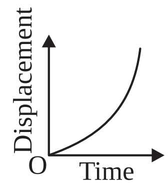
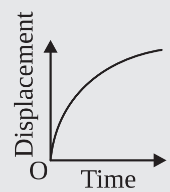
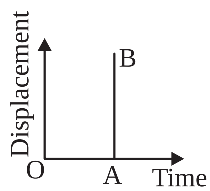
$$v^2 = u^2 + 2as \qquad s_n = u + \frac{a}{2}(2n - 1)$$

UNIFORM MOTION

If an object moving along the straight line covers equal distance in equal interval of time, it is said to be in uniform motion along a straight line.

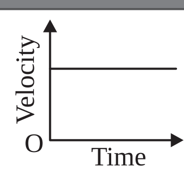
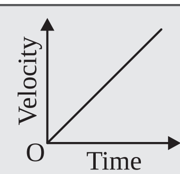
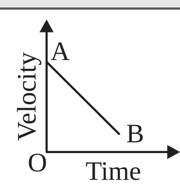
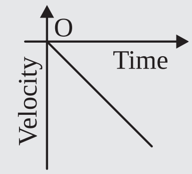
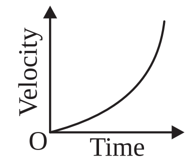
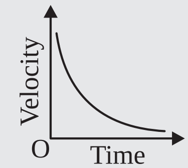
Velocity of the ofset is constant.

DIFFERENT GRAPHS OF MOTION DISPLACEMENT-TIME GRAPH

S.No.	Condition	Graph
1.	For a stationary body	
2.	Body moving with a constant velocity	
3.	Body moving with a constant acceleration	
4.	Body moving with a constant retardation	
5.	Body moving with infinite velocity. But such motion of body is never possible	

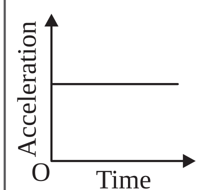
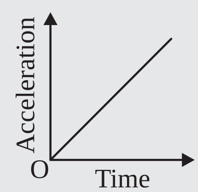
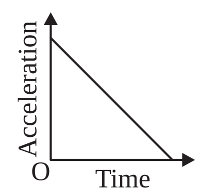
Note: Slooe of displacement time graph given elocity

VELOCITY-TIME GRAPH

S.No.	Condition	Graph
1.	Moving with a constant velocity	
2.	Moving with a constant acceleration having zero initial velocity	
3.	Body moving with a constant retardation and its initial velocity is not zero	
4.	Moving with a constant retardation with zero initial velocity	
5.	Moving with increasing acceleration	
6.	Moving with decreasing acceleration	

Note: Slope of velocity-time graph gives acceleration.

ACCELERATION-TIME GRAPH

S.No.	Condition	Graph
1.	When object is moving with constant acceleration	
2.	When object is moving with constantly increasing acceleration	
3.	When object is moving with constantly decreasing acceleration	

MOTION UNDER GRAVITY (NO AIR RESISTANCE)

If an object is falling freely under gravity, then equations of motion become

(*i*) $v = u + gt$

(*ii*) $h = ut + \frac{1}{2}gt^2$

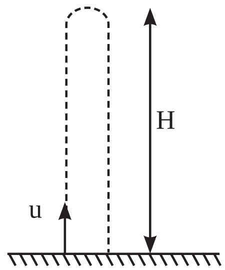
(*iii*) $v^2 = u^2 + 2gh$

Notes: If an object is thrown upward then g is replaced by –g in above three equations.

If a body is thrown vertically up with a velocity u in the uniform gravitational field then (*i*) Maximum height attained $H = \frac{u^2}{2g}$

(*ii*) Time of ascent = time of descent = $\frac{u}{g}$

(*iii*) Total time of flight = $\frac{2u}{g}$

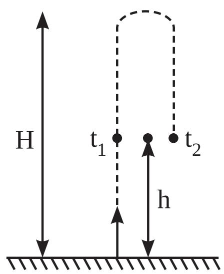


(*iv*) Velocity of fall at the point of projection = u (downwards)

(*v*) **Gallileo's law of odd numbers:** For a freely falling body dropped from rest, the ratio of successive distance covered in equal time interval, is

$S_1 : S_2 : S_3 : \ldots : S_n = 1 : 3 : 5 : \ldots : 2n - 1$

At any point on its path the body will have same speed for upward journey and downward journey.



If a body thrown upwards crosses a point in time t_1 and t_2 respectively then height of point $h = ½\, gt_1t_2$. Maximum height $H = \frac{1}{8}\, g(t_1 + t_2)^2$.

A body thrown upwards, downwards and horizontally with same speed takes time t_1, t_2 and t_3 respectively to reach the ground then $t_3 = \sqrt{t_1t_2}$ and height from where the particle was thrown is $H = \frac{1}{2}\, gt_1t_2$.