Position and Velocity of SHM

$x = A\sin(\omega t + \Phi_0)$

$v = A\omega\cos(\omega t + \Phi_0)$

$v = \omega\sqrt{A^2 - x^2}$

a) $x = 0 \longrightarrow v_{max} = A\omega$

b) $x = \frac{A}{2} \longrightarrow v = \frac{\sqrt{3}A\omega}{2}$

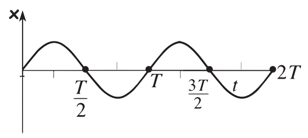
c) $x = \frac{A}{\sqrt{2}} \longrightarrow v = \frac{A\omega}{\sqrt{2}}$

d) $x = A \longrightarrow v = 0$

Graphical Representation of Position and Velocity

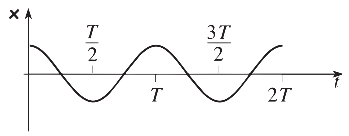
Start from mean position, $\Phi_0 = 0$

$x = A\sin(\omega t)$

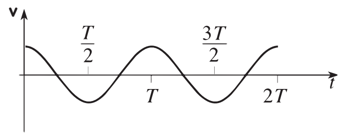


Start from extreme position $\Phi_0 = \frac{\pi}{2}$

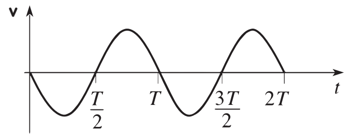
$x = A\cos(\omega t)$

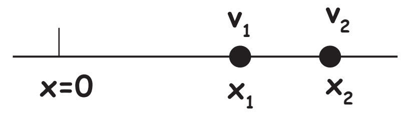


$v = A\omega\cos(\omega t)$



$v = -A\omega\sin(\omega t)$





$$A = \sqrt{\frac{v_1^2 x_2^2 - v_2^2 x_1^2}{v_1^2 - v_2^2}} \qquad \omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

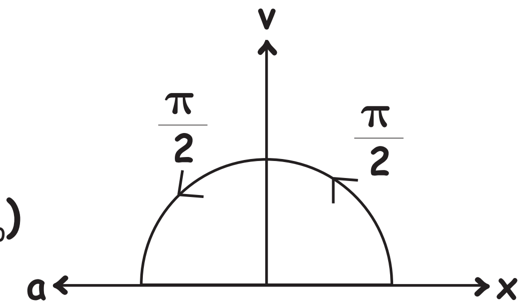
Acceleration of SHM

$x = A\sin(\omega t + \Phi_0)$

$v = A\omega\cos(\omega t + \Phi_0)$

$a = -A\omega^2\sin(\omega t + \Phi_0)$

$a_{max} = -A\omega^2$



Phase difference between x and v= $\frac{\pi}{2}$

Phase difference between v and a= $\frac{\pi}{2}$

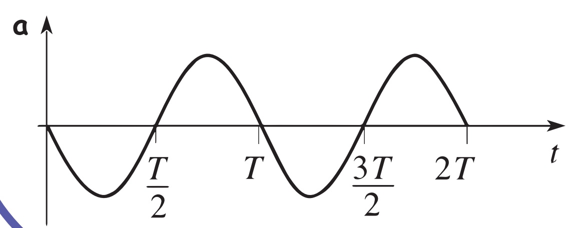
Phase difference between x and a= π

Simple Harmonic Motion 01

Graphical Representation of Acceleration

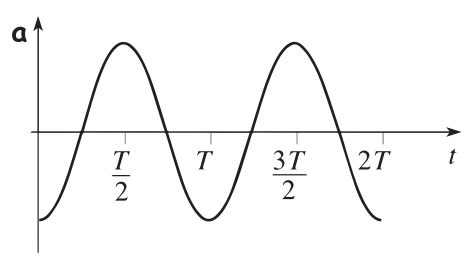
Start from mean position, $\Phi_0 = 0$

$a = -A\omega^2\sin(\omega t)$



Start from extreme position, $\Phi_0 = \frac{\pi}{2}$

$a = -A\omega^2\cos(\omega t)$



Calculation of Time period and amplitude

$v_{max} = A\omega \qquad a_{max} = A\omega^2$

$\omega = \frac{a_{max}}{v_{max}} \qquad A = \frac{v_{max}^2}{a_{max}}$

$\frac{2\pi}{T} = \frac{a_{max}}{v_{max}}$

$T = 2\pi\frac{v_{max}}{a_{max}}$

PHYSICS WALLAH

Simple Harmonic Motion 02

Energy of SHM

$$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$$

KE=0 | $KE_{max} = \frac{1}{2}m\omega^2A^2$ | KE=0

x=−A | x=0 | x=A

$$P.E = \frac{1}{2}m\omega^2x^2$$

$PE_{max} = \frac{1}{2}m\omega^2A^2$ | PE=0 | $PE_{max} = \frac{1}{2}m\omega^2A^2$

x=−A | x=0 | x=A

Total mechanical energy, $E = \frac{1}{2}m\omega^2A^2$ = Constant

1) $x = \frac{A}{\sqrt{2}}$

$K.E = P.E = \frac{E}{2}$

2) $x = \frac{A}{2}$

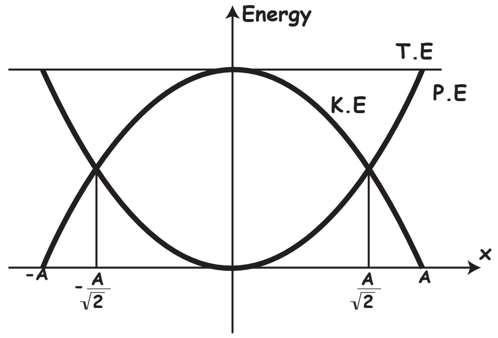
$K.E = \frac{3E}{4}$, $P.E = \frac{E}{4}$

3) x=0

K.E=E P.E=0

4) x=A

K.E=0 P.E=E

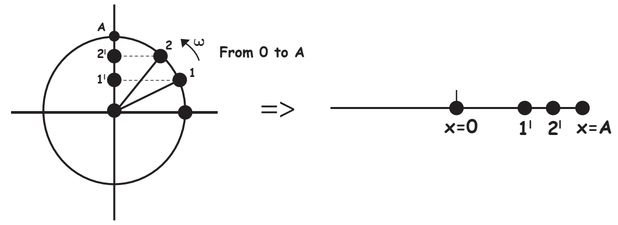


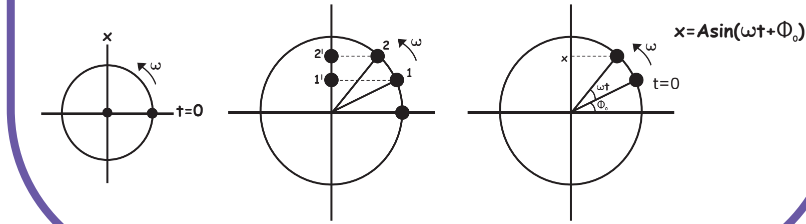
Total mechanical energy(E) $= \frac{1}{2}m\omega^2A^2$

Note: In SHM, if particle oscillates with frequency ω, then the K.E & P.E oscillate with 2 ω

PROJECTION OF CIRCULAR MOTION

Projection/shadow of uniform circular motion on y axis is SHM



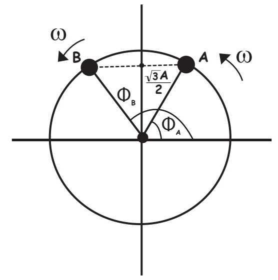


Two particles executing SHM meet at $x = \frac{\sqrt{3}A}{2}$

for particle A

$X = \frac{\sqrt{3}A}{2}$, t=0 $\Rightarrow \frac{\sqrt{3}A}{2} = A\sin\Phi_A$

$\Phi_A = 60^\circ = \frac{\pi}{3}$

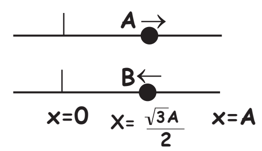


for particle B

$X = \frac{\sqrt{3}A}{2}$, t=0 $\Rightarrow \frac{\sqrt{3}A}{2} = A\sin\Phi_B$

$\Phi_B = 90^\circ + 30^\circ$

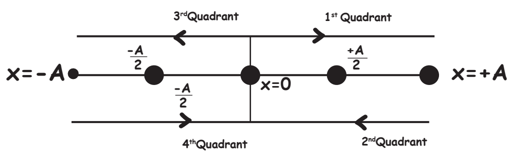
$\frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$



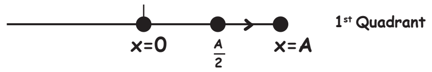
Phase difference between particles

$\Phi = \frac{2\pi}{3} - \frac{\pi}{3} = \frac{\pi}{3} = 60^\circ$

INITIAL PHASE FROM POSITION & DIRECTION



Eg: Particle is at $x = \frac{A}{2}$ [t=0] and moves towards A



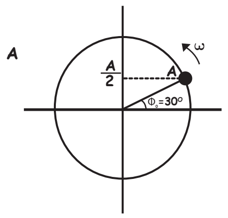
1st Quadrant

1) Particle is at $x = \frac{A}{2}$ [t=0] and moves towards A

$X = \frac{A}{2}$, t=0 $\Rightarrow \frac{A}{2} = A\sin\Phi_0$

$\Phi_0 = 30^\circ = \frac{\pi}{6}$

$x = A\sin(\omega t + \frac{\pi}{6})$

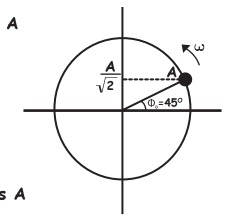


2) Particle is at $x = \frac{A}{\sqrt{2}}$ [t=0] and moves towards A

$X = \frac{A}{\sqrt{2}}$, t=0 $\Rightarrow \frac{A}{\sqrt{2}} = A\sin\Phi_0$

$\Phi_0 = 45^\circ = \frac{\pi}{4}$

$x = A\sin(\omega t + \frac{\pi}{4})$

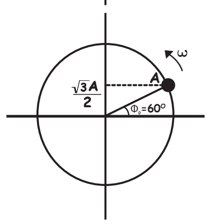


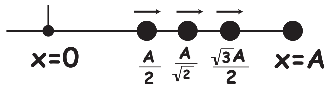
3) Particle is at $x = \frac{\sqrt{3}A}{2}$ [t=0] and moves towards A

$X = \frac{\sqrt{3}A}{2}$, t=0 $\Rightarrow \frac{\sqrt{3}A}{2} = A\sin\Phi_0$

$\Phi_0 = 60^\circ = \frac{\pi}{3}$

$x = A\sin(\omega t + \frac{\pi}{3})$





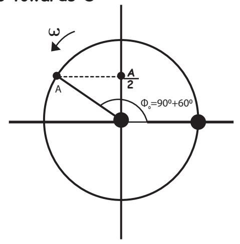
2nd Quadrant

1) Particle is at $x = \frac{A}{2}$ [t=0] and moves towards O

$X = \frac{A}{2}$, t=0 $\Rightarrow \frac{A}{2} = A\cos\Phi_0$

$\Phi_0 = 90^\circ + 60^\circ = \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

$x = A\cos(\omega t + \frac{5\pi}{6})$

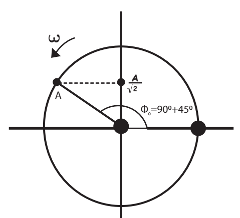


2) Particle is at $x = \frac{A}{\sqrt{2}}$ [t=0] and moves towards O

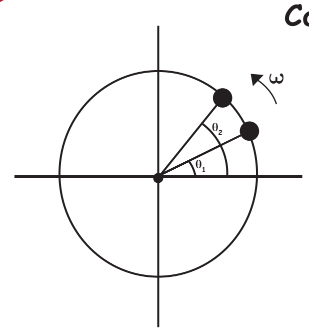
$X = \frac{A}{\sqrt{2}}$, t=0 $\Rightarrow \frac{A}{\sqrt{2}} = A\cos\Phi_0$

$\Phi_0 = 90^\circ + 45^\circ = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

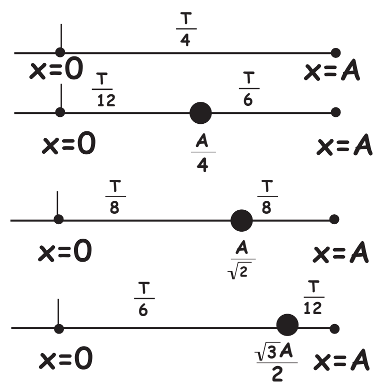
$x = A\cos(\omega t + \frac{3\pi}{4})$



Calculation of time

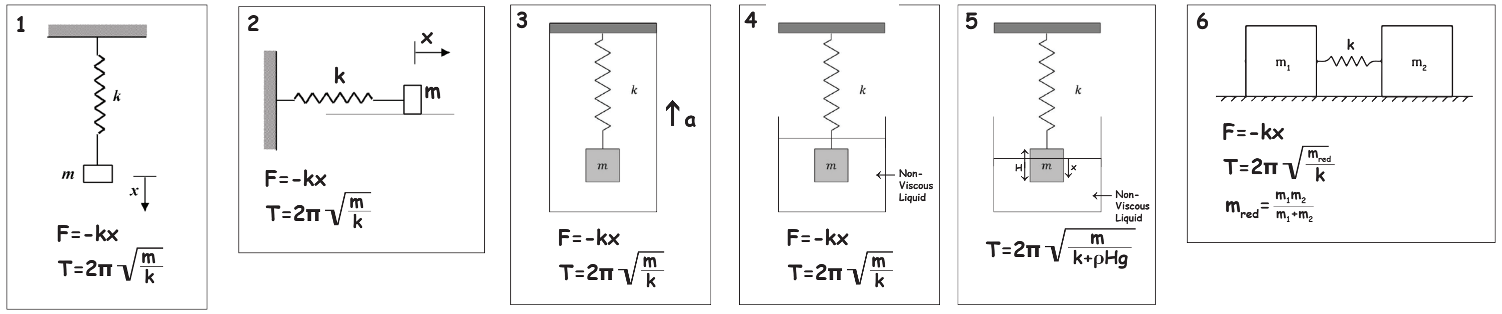


$$\Delta t = \frac{\Delta\theta}{\omega} = \left(\frac{\theta_1 - \theta_2}{2\pi}\right)T$$



Simple Harmonic Motion 03

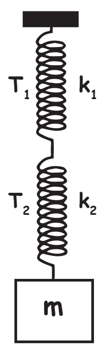
TIME PERIOD OF S.H.M



1. $F=-kx$, $T=2\pi\sqrt{\frac{m}{k}}$
2. $F=-kx$, $T=2\pi\sqrt{\frac{m}{k}}$
3. $F=-kx$, $T=2\pi\sqrt{\frac{m}{k}}$
4. $F=-kx$, $T=2\pi\sqrt{\frac{m}{k}}$
5. $T=2\pi\sqrt{\frac{m}{k+\rho Hg}}$
6. $F=-kx$, $T=2\pi\sqrt{\frac{m_{red}}{k}}$, $m_{red}=\frac{m_1m_2}{m_1+m_2}$

COMBINATIONS OF SPRINGS

1) Series



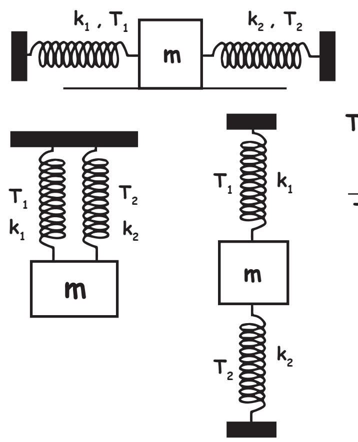
$k_{eq}=\frac{k_1k_2}{k_1+k_2}$

$T_{series}=2\pi\sqrt{\frac{m}{k_{eq}}}$

$T^2_{series}=T_1^2+T_2^2$

$T_{series}=\sqrt{T_1^2+T_2^2}$

2) Parallel

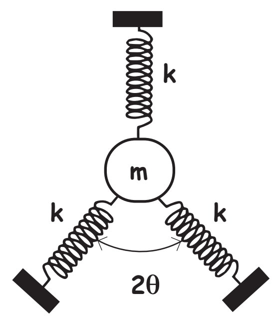


$k_{eq}=k_1+k_2$

$T_{parallel}=2\pi\sqrt{\frac{m}{k_{eq}}}$

$\frac{1}{T^2_{parallel}}=\frac{1}{T_1^2}+\frac{1}{T_2^2}$

SPECIAL CASE



$k_{eq}=k+2k\cos^2\theta$

$T=2\pi\sqrt{\frac{m}{k_{eq}}}$

eg:

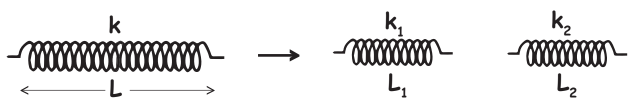
if $\theta=120°$ or $\theta=60°$

$k_{eq}=k+\frac{2k}{4}=\frac{3k}{2}$

$T=2\pi\sqrt{\frac{2m}{3k}}$

CUTTING OF SPRINGS

$k \propto \frac{1}{L}$ $\qquad k_1L_1=k_2L_2=kL$

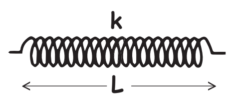


$L=L_1+L_2$

$k_1=\frac{kL}{L_1}$ $\qquad k_2=\frac{kL}{L_2}$

$k_1=\frac{k(L_1+L_2)}{L_1}$ $\qquad k_2=\frac{k(L_1+L_2)}{L_2}$

eg:



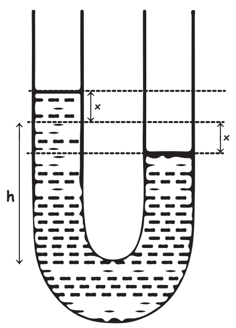
$L_1:L_2:L_3=1:2:3$

$k_1=\frac{k(L_1+L_2+L_3)}{L_1}=\frac{6k}{1}=6k$

$k_2=\frac{6k}{2}=3k$

$k_3=\frac{6k}{3}=2k$

OSCILLATION OF LIQUID COLUMN



$T=2\pi\sqrt{\frac{m}{2\rho gA}}$

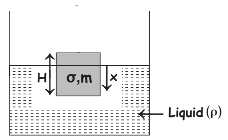
$T=2\pi\sqrt{\frac{h}{g}}$

m=mass of liquid

ρ=Density of liquid

A=Area of U-tube

OSCILLATION OF FLOATING BODY



$T=2\pi\sqrt{\frac{m}{\rho gA}}$

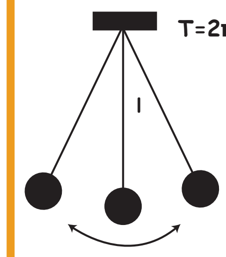
$T=2\pi\sqrt{\frac{\sigma H}{\rho g}}$

m=mass of body

σ=Density of Body

ρ=Density of liquid

SIMPLE PENDULUM



$T=2\pi\sqrt{\frac{l}{g}}$

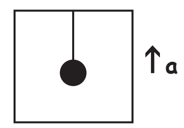
- Second's pendulum
  - T=2 second
  - l=1 meter

Concept of $g_{effective}$

$T=2\pi\sqrt{\frac{l}{g_{eff}}}$

Pendulum in lift

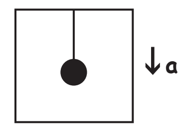
Case 1-Moving with constant upward acceleration 'a'



$g_{eff}=g+a$

$T=2\pi\sqrt{\frac{l}{g+a}}$

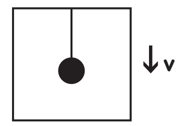
Case 2-Moving with constant downward acceleration 'a'



$g_{eff}=g-a$

$T=2\pi\sqrt{\frac{l}{g-a}}$

Case 3-Moving with constant velocity



a=0

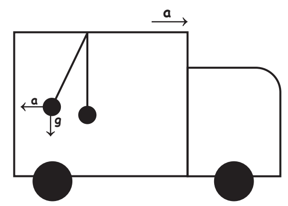
$T=2\pi\sqrt{\frac{l}{g}}$

Case 4-Free fall

a=g

$g_{eff}=g-a=g-g=0$ $\quad T \to \infty$

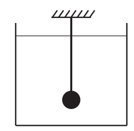
Pendulum in a truck moving with constant acceleration



$g_{eff}=\sqrt{g^2+a^2}$

$T=2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$

Pendulum in Water



σ=Density of Bob

ρ=Density of liquid

$g_{eff}=g\left(1-\frac{\rho}{\sigma}\right)$

$T=2\pi\sqrt{\frac{l}{g\left(1-\frac{\rho}{\sigma}\right)}}$

DIFFERENTIAL EQUATION OF S.H.M

$ma=-kx$

$m\frac{d^2x}{dt^2}=-kx$

$\frac{d^2x}{dt^2}=\frac{-k}{m}x$

$\frac{d^2x}{dt^2}+\frac{k}{m}x=0$

$\frac{d^2x}{dt^2}+\omega^2x=0$

where, $\omega^2=\frac{k}{m}$ $\qquad \omega=\sqrt{\frac{k}{m}}$

Solving,

$x=A\sin(\omega t+\Phi_0)$

A=Amplitude of SHM

Φ_0=Initial phase angle

Eg: $\left[\begin{array}{l} m=4\text{ Kg} \\ K=320\text{ N/m} \end{array}\right]$

$4\frac{d^2x}{dt^2}+320x=0$

$\frac{d^2x}{dt^2}+80x=0$

$\omega^2=80$ $\qquad \omega=\sqrt{80}$

$\frac{2\pi}{T}=\sqrt{80}$

$T=\frac{2\pi}{\sqrt{80}}$

PHYSICS WALLAH