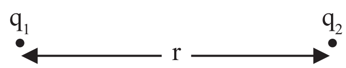
CHAPTER 1

Electric Charges and Fields

Coulomb's Law

Force between two charges $\vec{F} = \frac{1}{4\pi \epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2} \hat{r}$, ϵ_r = dielectric constant



Principle of Superposition

Force on a point charge due to many charges is given by

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ...$$

Notes: The force due to one charge is not affected by the presence of other charges.

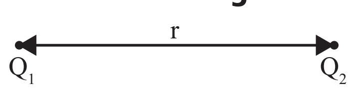
Electric Field or Electric Field Intensity (Vector Quantity)

$$\vec{E} = \frac{\vec{F}}{q}, \text{ unit is N/C or V/m.}$$

Electric Field Due to Charge Q

$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{1}{4\pi \epsilon_0} \frac{Q}{r^2} \hat{r}$$

Null Point for Two Charges



If $|Q_1| > |Q_2|$

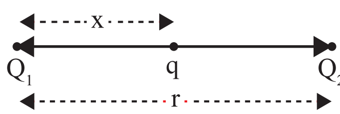
$\Rightarrow$ Null point near Q_2

$$x = \frac{\sqrt{Q_1}\, r}{\sqrt{Q_1} \pm \sqrt{Q_2}}; \; x \to \text{distance of null point from } Q_1 \text{ charge}$$

(+) for like charges

(–) for unlike charges

Equilibrium of Three Point Charges



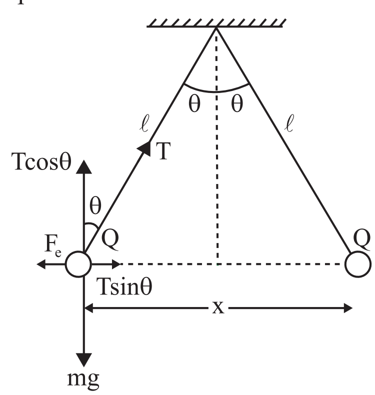
(i) Two charges must be of like nature.

(ii) Third charge should be of unlike nature.

$$x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}} r \text{ and } q = \frac{-Q_1 Q_2}{\left(\sqrt{Q_1} + \sqrt{Q_2}\right)^2}$$

Equilibrium of Suspended Point Charge System

For equilibrium position

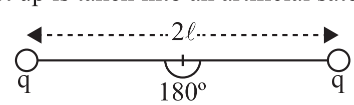


$T \cos\theta = mg$ & $T \sin\theta = F_e$

$$\Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

$$T = \sqrt{(F_e)^2 + (mg)^2}$$

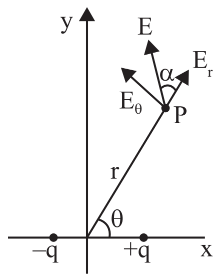
If whole set up is taken into an artificial satellite ($g_{eff} \simeq 0$)



$$\Rightarrow T = F_e = \frac{kq^2}{4\ell^2}$$

Electric Dipole

- Electric dipole moment p = qd
- Torque on dipole placed in uniform electric field $\vec{\tau} = \vec{p} \times \vec{E}$
- At a point which is at a distance r from dipole midpoint and making angle θ with dipole axis.



Electric field $E = \frac{1}{4\pi \in_0} \frac{p\sqrt{1+3\cos^2\theta}}{r^3}$

Direction $\tan\alpha = \frac{E_\theta}{E_r} = \frac{1}{2}\tan\theta$

- Electric field at axial point (or End-on) $\vec{E} = \frac{1}{4\pi \in_0} \frac{2\vec{p}}{r^3}$ of dipole
- Electric field at equatorial position (Broad-on) of dipole $\vec{E} = \frac{1}{4\pi \in_0} \frac{(-\vec{p})}{r^3}$

Electric flux: $\phi = \int \vec{E}.d\vec{s}$

Gauss's Law: $\oint \vec{E}.d\vec{s} = \frac{\sum}{\in}$ (Applicable only on closed surface)

Net flux emerging out of a closed surface is $\frac{q_{en}}{\varepsilon_0}$

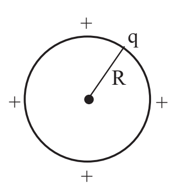
$\phi = \oint \vec{E}.d\vec{A} = \frac{q_{en}}{\varepsilon_0}$ where q_{en} = net charge enclosed by the closed surface.

φ does not depend on the

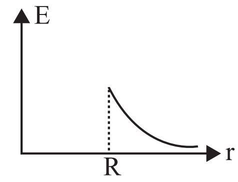
(i) Shape and size of the closed surface

(ii) The charges located outside the closed surface.

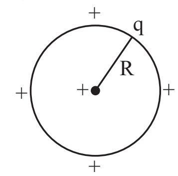
For a Conducting Sphere



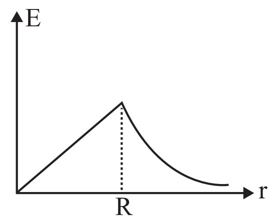
For $r \geq R : E = \frac{1}{4\pi \in_0} \frac{q}{r^2}$ and For r < R : E = 0



For a Non-conducting Sphere



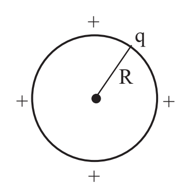
For $r \geq R : E = \frac{1}{4\pi \in_0} \frac{q}{r^2}$



For r < R : E $= \frac{1}{4\pi \in_0} \frac{qr}{R^3}$

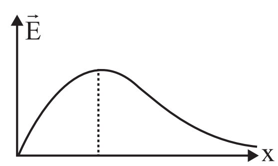
For a Conducting/Non-conducting Spherical Shell

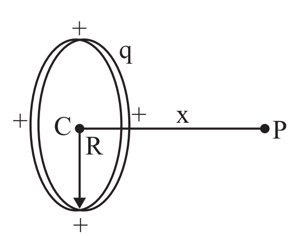
For $r \geq R : E = \frac{1}{4\pi \in_0} \frac{q}{r^2}$



For r < R : E = 0

For a Charged Circular Ring





$$E_P = \frac{1}{4\pi \epsilon_0} \frac{qx}{(x^2 + R^2)^{3/2}}$$

Electric field will be maximum at $x = \pm \frac{R}{\sqrt{2}}$

For a Charged Long Conducting Cylinder

- For $r \geq R : E = \frac{q}{2\pi \epsilon_0 r}$
- For $r < R : E = 0$

Electric Field Intensity at a Point near a Charged Conductor

$$E = \frac{\sigma}{\epsilon_0}$$

Mechanical Pressure on a Charged Conductor

$$P = \frac{\sigma^2}{2 \epsilon_0}$$

Electric Field for Non-conducting Infinite Sheet of Surface Charged Density σ

$$E = \frac{\sigma}{2 \epsilon_0}$$

Electric Field for Conducting Infinite Sheet of Surface Charge Density σ

$$E = \frac{\sigma}{\epsilon_0}$$