CHAPTER

5 Work Energy and Power

WORK DONE BY CONSTANT FORCE

$$W = \vec{F}\cdot\vec{S}$$

Work Done by Multiple Forces

$$W = [\Sigma\vec{F}]\cdot\vec{S}$$

$$W = \vec{F}_1\cdot\vec{S} + \vec{F}_2\cdot\vec{S} + \vec{F}_3\cdot\vec{S} + \cdots \; (\because \; \Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots)$$

or $W = W_1 + W_2 + W_3 + \ldots$

Work Done by Variable Force

$$W = \int dW = \int \vec{F}\cdot d\vec{s}$$

- Area under the force and displacement curve gives work done.

RELATION BETWEEN MOMENTUM AND KINETIC ENERGY

$K = \dfrac{p^2}{2m}$ and $P = \sqrt{2mK}$; P = linear momentum

Potential Energy

$$\int_{U_1}^{U_2} dU = -\int_{r_1}^{r_2} \vec{F}\cdot d\vec{r}$$

i.e., $U_f - U_i = -\int_r \vec{F}\cdot d\vec{r} = -W_{Conservative}$

Conservative Force

$$\vec{F} = -\frac{dU}{dr}\hat{r}, \quad \vec{F} = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

EQUILIBRIUM CONDITIONS

- Stable Equilibrium

$$F(r) = -\frac{\partial U}{\partial r} = 0; \quad \text{and} \quad \frac{\partial F}{\partial r} < 0; \quad \text{and} \quad \frac{\partial^2 U}{\partial r^2} > 0$$

- Unstable Equilibrium

$$F(r) = -\frac{\partial U}{\partial r} = 0; \quad \text{therefore} \quad \frac{\partial F}{\partial r} > 0; \quad \text{and} \quad \frac{\partial^2 U}{\partial r^2} < 0$$

- Neutral Equilibrium

$$F(r) = -\frac{\partial U}{\partial r} = 0; \quad \text{therefore} \quad \frac{\partial F}{\partial r} = 0; \quad \text{and} \quad \frac{\partial^2 U}{\partial r^2} = 0$$

WORK-ENERGY THEOREM

$$W_{All} = \Delta K$$

$$\Rightarrow \quad W_C + W_{NC} + W_{Ext} + W_{Pseudo} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Law of Conservation of Mechanical Energy

If the net external force acting on a system is zero, then the mechanical energy is conserved.

$$K_f + U_f = K_i + U_i$$

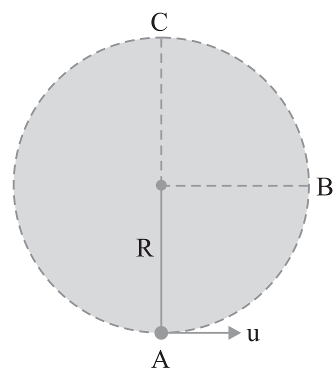
POWER

The average poiwer delivered by an agent is given by

$$P_{avg} = \frac{W}{t}$$

$$P = \frac{d\vec{F}\cdot\vec{S}}{dt} = \vec{F}\cdot\frac{d\vec{S}}{dt} = \vec{F}\cdot\vec{v}$$

Circular Motion in Vertical Plane



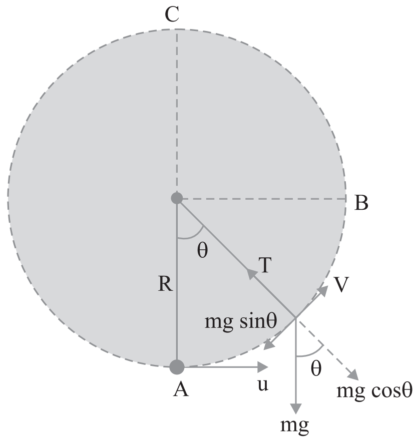
A. Condition to completge vertical circle $u \geq \sqrt{5gR}$

If $u = \sqrt{5gR}$ then Tension at C is equal to 0 and tension at A is equal to 6 mg.

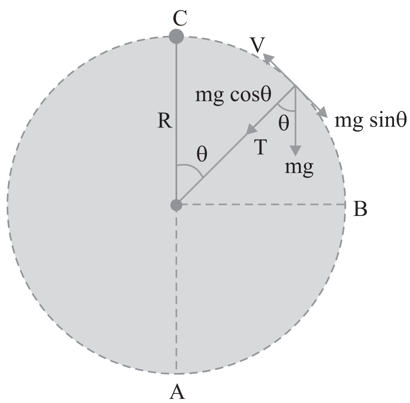
Velocity at B : $v_B = \sqrt{3gR}$

Velocity at C : $v_C = \sqrt{gR}$

From A to B : $T = mg\cos\theta + \dfrac{mv^2}{R}$



From B to C : $T = \frac{mv^2}{R} - mg\cos\theta$



B. Condition for pendulum motion (oscillating condition)

$u \le \sqrt{2gR}$ (in between A to B)

Velocity can be zero but T can never be zero between A and B.

Because T is given by $T = mg\cos\theta + \frac{mv^2}{R}$.

Collisions

(*a*) Collision is the interaction between two (or) more particles where exchange of momentum takes place.

(*b*) In case of collisions as the impulsive force acting during collision is internal, the total momentum of system always remains conserved.

(*c*) If the velocities of the colliding particles are along the same line before and after the collision then the collision is said to be one dimensional collision.

(*d*) In a collision, if the motion of colliding particles before and after the collision are not along the initial line of motion, then the collision is said to be oblique collision.

(*e*) In an oblique elastic collision, if $m_1 = m_2$ and m_2 is initially at rest, then after the collision the two masses will move in directions inclined at 90° to each other.

Coefficient of Restitution

(*a*) $e = \frac{\text{relative velocity of separation}}{\text{relative velocity of approach}}$

$\therefore\ e = \frac{v_2 - v_1}{u_1 - u_2}$

(*b*) The value of coefficient of restitution (e) is independent of masses and velocities of the colliding bodies. It depends on their materials.

(*c*) For a perfectly elastic collision, $e = 1$

For a perfectly inelastic collision, $e = 0$

For other collisions, e lies between 0 and 1

(*d*) If a body falls from a height 'h' and strikes the level ground with velocity V in time seconds and rebounds with velocity V_1 upto height h_1 in time t_1 seconds.

The coefficient of restitution is given by

$$e = \frac{V_1}{V} \quad \text{(or)} \quad e = \sqrt{\frac{H_1}{H}} \quad \text{(or)} \quad e = \frac{t_1}{t}$$

For a perfectly elastic collision, $H_1 = H$

For a perfectly inelastic collision, $H_2 = 0$

For other collisions, $H_1 < H$

For any collision, H_1 cannot be greater than h

(*e*) A small metal sphere falls freely from a height 'H' upon a fixed horizontal plane. If e is the coefficient of restitution, then

(i) The height to which it rebounds after n collision is $H_n = e^{2n} H$

(ii) The velocity with which it rebounds from the ground after nth collision is $v_n = e^n v$, where v is the velocity of the sphere just before first collision.

(iii) The total distance travelled by it before it stops rebounding is $d = H\left(\frac{1+e^2}{1-e^2}\right)$

(iv) The total time taken by it to come to rest is $T = \sqrt{\frac{2H}{g}}\left(\frac{1+e}{1-e}\right)$

(*f*) In one dimensional semi elastic (or) inelastic collisions, linear momentum is conserved but kinetic energy of the system is not conserved.

(*g*) In above collisions, there is loss of kinetic energy of the system in the form of heat, sound, light etc.

(*h*) In one dimensional semi elastic collision relative velocity of seperation = e × realtive velocity of approach i.e., $v_2 - v_1 = e(u_1 - u_2)$

Inelastic Collisions

(*a*) Formulas for final velocities in case of one dimensional semi elastic collision are

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_2 + \left(\frac{1+e}{m_1 + m_2}\right)m_2u_2$$

$$v_2 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 + \left(\frac{1+e}{m_1 + m_2}\right)m_1u_1$$

(*b*) Loss in Kinetic energy of the system in one dimensional semi elastic collision is $\Delta E_{\text{k}} = \frac{1}{2}\left[\frac{m_1m_2}{m_1 + m_2}\right][1-e^2][u_2 - u_2]^2$

(*c*) In one dimensional perfectly inelastic collision the two particles stick together after the collision and move with common velocity.

(*d*) The formula for common velocity of compound body after perfectly inelastic collision is $\vec{v} = \frac{m_1\overrightarrow{u_1} + m_2\overrightarrow{u_2}}{m_1 + m_2}$

(*e*) The loss of kinetic energy in a perfectly inelastic collision ($e = 0$) is given by $\Delta E_{\text{k}} = \frac{1}{2}\left[\frac{m_1m_2}{m_1 + m_2}\right][u_1 - u_2]^2$

(*f*) In a perfectly inelastic collision, the ratio of loss of energy of the system and its initial energy, if u_2 is zero, is given by

$$\frac{\Delta E_{\text{k}}}{E_1} = \frac{m_2}{m_1 + m_2}$$

(*g*) In a perfectly inelastic collision, the ratio of final energy to initial energy, of the system, if u_2 is zero, is given by

$$\frac{E_{\text{k}}}{E_1} = \frac{m_1}{m_1 + m_2} \Rightarrow E_k < E_1$$