Vectors

1 Vectors

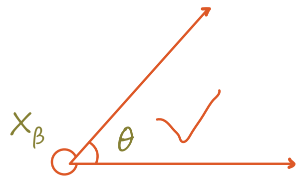
A vector is a quantity that has both magnitude and direction. Vectors are used to represent physical quantities that cannot be fully described by a single number alone. Examples of vector quantities include displacement, velocity, acceleration, and force.

Scalar Quantity	Vector Quantity
✦ Having Magnitude only	✦ Having Magnitude and direction
✦ Follow simple algebric addition	✦ follow triangle law of vector addition.
✦ Can be changed only by changing its value	✦ Can be changed by changing magnitude only, or changing dir^n only or changing both.
Ex-Speed, time, Mass, Volume, density current, etc.	Ex-Force, Velocity, current density, torque etc.

Properties of Vectors

- In a vector +ve sign and sign –ve indicate direction only.

  Ex: +5N and –5N, same magnitude of force in opposite direction.
- Angle between vector – When two vectors are placed head to head or tail to tail then smaller angle between vector is called angle between vector.

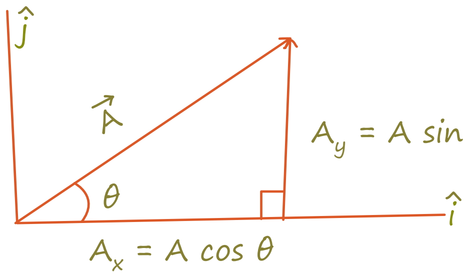


- Vector can be shifted parallel to itself by keeping magnitude and direction fixed.
- Rotation of vector not allowed it will change meaning of vector.
- If angle between $\vec{A}$ and $\vec{B}$ vector is θ then angle between $\vec{A}$ and $-\vec{B}$ is $(180° - \theta)$.
- All equal vectors are parallel but all parallels are not equal.
- All opposite (Negative) Vectors are Antiparallel but all antiparallel are not Opposite Vector

2 Type of Vectors

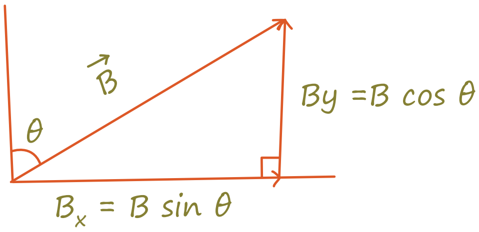
Type	Magnitude	Direction\Angle
Equal Vector	Same	Same ($\theta = 0$)
Parallel Vector	May or May not same	Same ($\theta = 0$)
Opposite Vector or Negative Vectors	Same	Opposite $\theta = 180°$
Anti-parallel Vector	May or May not same	$\theta = 180°$ opposite
Orthogonal	May same	$\theta = 90°$
Zero/Null Vector	Zero	any direction
Unit Vectors	One	$\hat{A} = \dfrac{\vec{A}}{A}$

Components of Vector in 2-D (effect of Vector)



✦ $\vec{A} = A_x\hat{i} + A_y\hat{j}$

$\vec{A} = A\cos\boldsymbol{\theta}\ \hat{i} + A\sin\boldsymbol{\theta}\,\hat{j}$



✦ $\vec{B} = B_x\hat{i} + B_y\hat{j}$

$= B\sin\boldsymbol{\theta}\,\hat{i} + B\cos\boldsymbol{\theta}\,\hat{j}$

Magnitude of resultant Vector

$A = \sqrt{A_x^2 + A_y^2}$ or $B = \sqrt{B_x^2 + B_y^2}$

Direction:

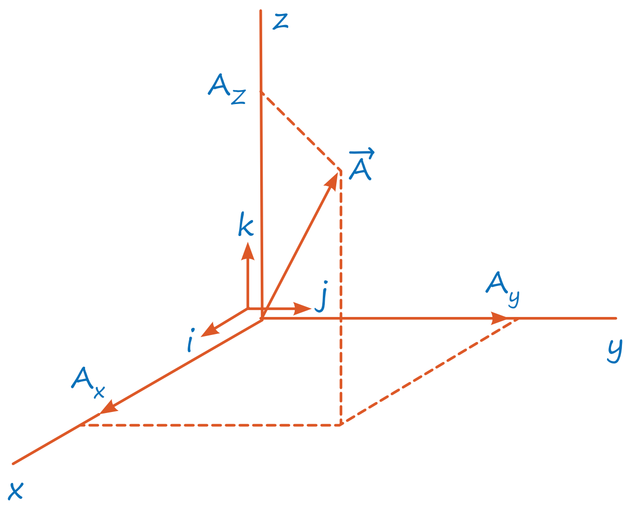
$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$ or $\theta = \tan^{-1}\left(\frac{B_x}{B_y}\right)$

Rectangular component of a vector in 3D

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

Magnitude

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$



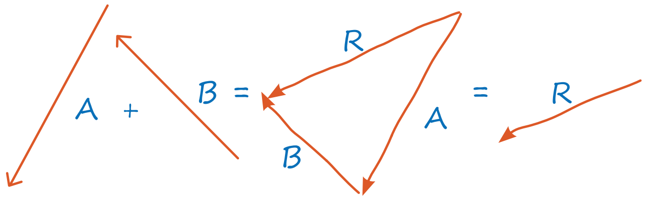
✦ If vector is making an angle α, β and γ from x, y and z-axis respectively then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$; $\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Direction Cosine

$\cos\alpha = \frac{Ax}{A}$ $\quad\cos\beta = \frac{Ay}{A}$ $\quad\cos\gamma = \frac{Az}{A}$

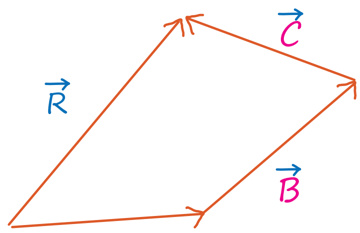
3 Vectors addition

✦ It is the process of combining two vectors by placing the tail of one vector at the head of the other. i.e.,

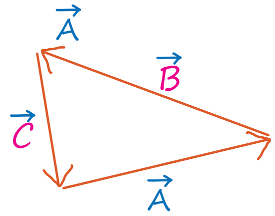


Polygon Law of vector addition

✦ Start tail of next vector from head of previous vector and so on.

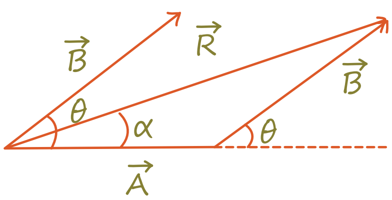


$\vec{A} + \vec{B} + \vec{C} = \vec{R}$



$\vec{A} + \vec{B} + \vec{C} = O$

Triangle Law of Vector addition



resultant must be in the plane of $\vec{A}$ and $\vec{B}$.

+ $|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

+ $\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$

+ If $\theta = 0°$, $R_{max} = A + B$ | $\theta = 90°$, $R = \sqrt{A^2 + B^2}$ | $\theta = 180°$, $R_{min} = A - B$

+ $A - B \le R \le A + B$

+ If $|\vec{A}| = |\vec{B}| = A$ and Angle b/w them θ

  $|\vec{R}| = 2A\cos(\theta/2)$ $\quad |\vec{D}| = 2A\sin(\theta/2)$

Vector Subtraction

Angle B/w $\vec{A}$ & $\vec{B}$ is θ then $\vec{D} = \vec{A} - \vec{B}$

Magnitude

$|\vec{D}| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

+ If $\theta = 0°$, $D_{min} = A - B$ | $\theta = 90°$, $D = \sqrt{A^2 + B^2}$ | $\theta = 180°$, $D = A + B$

+ $A - B \le D \le A + B$

$\theta = 0°$	$\theta = 60°$	$\theta = 90°$	$\theta = 120°$	$\theta = 180°$
$R = 2A$	$R = \sqrt{3}A$	$R = \sqrt{2}A$	$R = A$	$R = 0$
$D = 0$	$D = A$	$D = \sqrt{2}A$	$D = \sqrt{3}A$	$D = 2A$

Properties of vector addtion and substraction:

+ Magnitude of Vector addition and subtraction are same at 90°.

+ $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ Commutative

+ $n(\vec{A} + \vec{B}) = n\vec{A} + n\vec{B}$ distributive

+ $\vec{A} - \vec{B} \ne \vec{B} - \vec{A}$

+ $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$ then $\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j} + (A_z + B_z)\hat{k}$

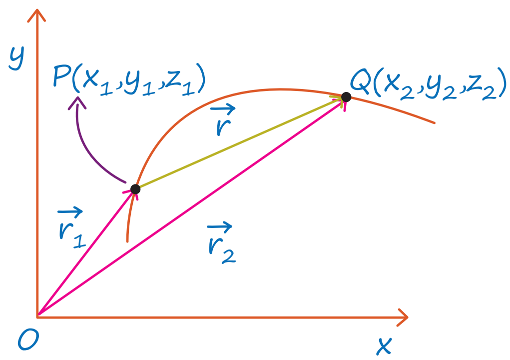
+ If $|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}|$ then angle between $\vec{A}$ and $\vec{B}$ is 120°

+ If $|\vec{A}| + |\vec{B}| = |\vec{A} + \vec{B}|$ then angle between $\vec{A}$ and $\vec{B}$ is zero.

+ If $\vec{A} + \vec{B} = \sqrt{A^2 + B^2}$ then angle between $\vec{A}$ and $\vec{B}$ is 90°.

+ If $|\vec{A} + \vec{B}| = |\vec{B} - \vec{A}|$ then angle between $\vec{A}$ and $\vec{B}$ is 90°.

4 Displacement vector



+ If a particle moves from the initial position (x_1, y_1, z_1) to the final position (x_2, y_2, z_2), the displacement vector r is given by:

$r = (x_2 - x_1)i + (y_2 - y_1)j + (z_2 - z_1)k$

or $\quad r = \Delta xi + \Delta yj + \Delta zk$

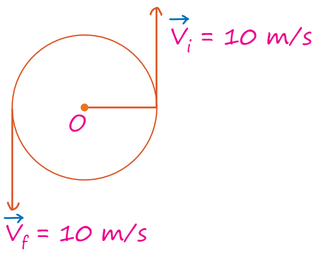
Magnitude of displacement vector

+ $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Q.1. Ramlal is moving with velocity 6m/s along east and pinky with 6 m/s at 30° east of north then relative velocity of pinky w.r.t Ramlal.

Sol. $\vec{V}_{PR} = \vec{V}_P - \vec{V}_R$ same vector ka subtraction at 60° $|\vec{V}_{PR}| = 6$ m/s

Q.2. Find change in Speed and velocity



Sol. Change in speed = 0 magnitude of change in velocity = 20 m/s

Q.3. If $\vec{A} = 0.6\hat{i} + \beta\hat{j}$ is a unit vector then find value of β.

Sol. $|\vec{A}| = 1$ if A is unit vector

$$\sqrt{(0.6)^2 + \beta^2} = 1$$

$$\beta^2 + 0.36 = 1$$

$$\beta = \sqrt{0.64} = 0.8$$

Q.4. Two force 10N and 6N acting then find resultant of these two force.

Sol. $10 - 6 \leq R \leq 10 + 6$

R will between 4N to 16N

Q.5. Find the angle which a vector $\hat{i} + \hat{j} + \sqrt{2}\hat{k}$ makes with x, y and z-axis

$$A = \sqrt{1^2 + 1^2 + (\sqrt{2})^2} = 2$$

$A_x = 1$

$A_y = 1$

$A_z = \sqrt{2}$

$\cos\alpha = \frac{A_x}{A}$ $\cos\beta = \frac{A_y}{A}$ $\cos\gamma = \frac{A_z}{A}$

$\alpha = 60°$ $\beta = 60°$ $\gamma = 45°$

Q.6. In which of the following combination of three force resultant will be zero.

(a) 3N, 7N, 8N

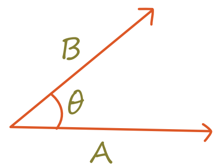
(b) 2N, 5N, 1N

(c) 3N, 12N, 7N

(d) 4N, 5N, 10N

Sol. (a) Sum of two smaller must be greater or equal to (3rd).

5 Scalar Product (Dot Product)



✦ $\vec{A}\cdot\vec{B} = A(B\cos\theta) = A$(Component of B along A)

$= (A\cos\theta)\,B = B$(Component of A along B)

Component of B along A $= \frac{\vec{A}\cdot\vec{B}}{A}$

Component of A along B $= \frac{\vec{A}\cdot\vec{B}}{B}$

✦ Result of dot product is always scalar.

$\hat{i}\cdot\hat{i} = 1$ $\hat{j}\cdot\hat{j} = 1$ $\hat{k}\cdot\hat{k} = 1$

$\hat{i}\cdot\hat{k} = 0$ $\hat{j}\cdot\hat{i} = 0$ $\hat{k}\cdot\hat{j} = 0$

$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_xB_x + A_yB_y + A_zB_z$

Application of dot Product

✦ To Find Angle B/W vectors

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB}$

+ To check unit vector

If $\vec{A}$ is a unit vector then $\vec{A}\cdot\vec{A} = 1$

+ To check perpendicular vector (orthogonal)

If $\vec{A}\cdot\vec{B} = AB\cos 90° = 0$

$\vec{A}\cdot\vec{B} = 0 \quad (\vec{A} \perp r\ \vec{B})$

+ To find component of one vector along other.

$\vec{A}\cdot\vec{B} = A(B\cos\theta)$

$B\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{A}$ = Comp^n of B along A

Q.7. If $\vec{A} = 4\hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{B} = \hat{i} + 3\hat{j} - 2\hat{k}$ then find $\vec{A}.\vec{B}$

(1) 10

(2) 16

(3) 3

(4) 14

Sol. (2) $\vec{A}.\vec{B} = (4\hat{i} + 2\hat{j} - 3\hat{k})\cdot(\hat{i} + 3\hat{j} - 2\hat{k})$

$= 4 + 6 + 6$

$= 16$

Q.8. A force $\vec{F} = 4\hat{i} + 5\hat{j}$ newton displaces a particle through $\vec{S} = 3\hat{i} + 6\hat{j}$ meter. Then find work done.

(1) 24 Joule

(2) 12 Joule

(3) 28 Joule

(4) 42 Joule

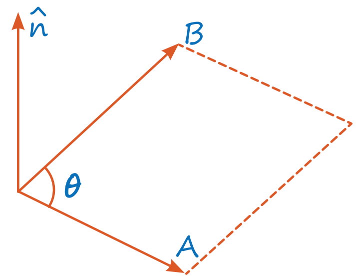
Sol. (1) $W = \vec{F}.\vec{S} = (4\hat{i} + 5\hat{j})\cdot(3\hat{i} + 6\hat{j})$

$= 12 + 30$

$= 42$ J

6 Cross-Product : [Vector Product]

+ $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$



+ $\hat{n}$ is direction of $\vec{A}\times\vec{B}$ which is perpendicular to $\vec{A}$ & $\vec{B}$.

+ $(\vec{A}\times\vec{B})\cdot\vec{A} = 0 \qquad (\vec{A}\times\vec{B})\cdot\vec{B} = 0$

+ $B\sin\theta = \dfrac{\vec{A}\times\vec{B}}{A}$ = component of B perpendicular of A

+ $\vec{R} = \vec{A}\times\vec{B}$

Place your finger of right hand along $\vec{A}$ and slap $\vec{B}$ then thumb will represent $\vec{R}$.

$\hat{i}\times\hat{i} = 0 = \hat{j}\times\hat{j} = \hat{k}\times\hat{k}$

$\hat{i}\times\hat{j} = \hat{k} \qquad \hat{j}\times\hat{i} = -\hat{k}$

$\hat{j}\times\hat{k} = \hat{i} \qquad \hat{k}\times\hat{j} = -\hat{i}$

$\hat{k}\times\hat{i} = \hat{j} \qquad \hat{i}\times\hat{k} = -\hat{j}$

+ $\vec{A}\times\vec{B} = \begin{vmatrix} i & j & k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$= i(A_yB_z - A_zB_y)$

$- J(A_xB_z - A_zB_x)$

$+ K(A_xB_y - A_yB_x)$

+ Unit vector does not have any unit only have direction and magnitude one.

- Minimum no. of vectors whose resultant can be zero is '2'.
- Minimum no of unequal vectors whose resultant can be zero is 3.
- The resultant of 3 Non- coplaner vectors can't be zero.
- Minimum no of Non-coplaner, vectors whose resultant can be zero is 4.
- Angle between $(\vec{A} \times \vec{B})$ and $(\vec{A} + \vec{B})$ is $90°$
- Division of vector with vector is not possible
- Division of magnitude of vector is possible
- Vector can be divided by scalar.
- If vector multiplied by positive scalar then magnitude change and direction remains same.
- If vector multiplied by negative scalar then magnitude change and direction becomes opposite.

Scalar triple Product

$R = (\vec{A} \times \vec{B}) \cdot \vec{C}$ Result R will be scalar and R will be zero if any of these two vector becomes parallel.

Area of parallelogram

$$\text{Area} = |A \times B|$$

Area of triangle:

$$\text{Area} = \frac{1}{2}|A \times B|$$

The condition for coplanarity is

$$A \cdot (B \times C) = 0$$

Q.9. Force acting on object $\vec{F} = 5\hat{i} + 3\hat{j} - 7\hat{k}$ position vector $\vec{r} = 2i + 2j - k$ then find torque ?? (NEET 2022)

Sol. $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ 5 & 3 & -7 \end{vmatrix}$

$\hat{i}(-14 - (-3)) - \hat{j}(-14 - (-5)) + \hat{k}(6 - 10)$

$-11\hat{i} + 9\hat{j} - 4\hat{k}$

Q.10. If $|\vec{A} \times \vec{B}| = \sqrt{3}\ \vec{A} \cdot \vec{B}$ then angle between $\vec{A}$ and $\vec{B}$ is?

Sol. $AB \sin\theta = \sqrt{3}\, AB \cos\theta$

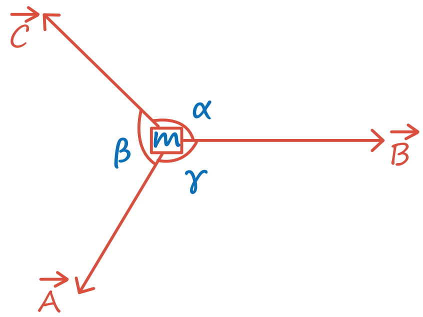
$\tan\theta = \sqrt{3} \Rightarrow \theta = 60°$

7 Lami's Theorem (Laws of motion) for 3 Vectors

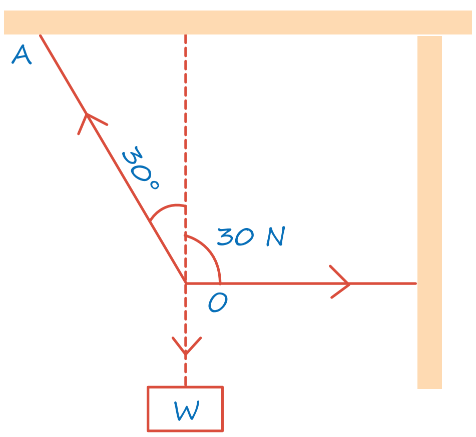
When,

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

Then, $\vec{A} + \vec{B} + \vec{C} = 0$



Q. 11. As shown in figure tension in the horizontal cord is 30 N. The weight W and tension in the string OA in Newton are



(a) $30\sqrt{3}, 30$ (b) $30\sqrt{3}, 60$

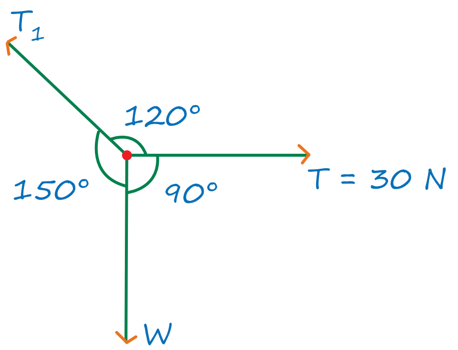
(c) $60\sqrt{3}, 30$ (d) None of these

Ans. $\dfrac{30}{\sin 150^\circ} = \dfrac{T_1}{\sin 90^\circ}$

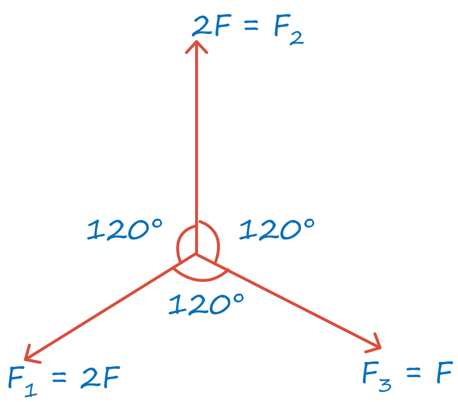
$\dfrac{T_1}{1} = \dfrac{30}{1/2}, \; T_1 = 60 \text{ N}$

$\dfrac{W}{\sin 120^\circ} = \dfrac{30}{\sin 150^\circ}$

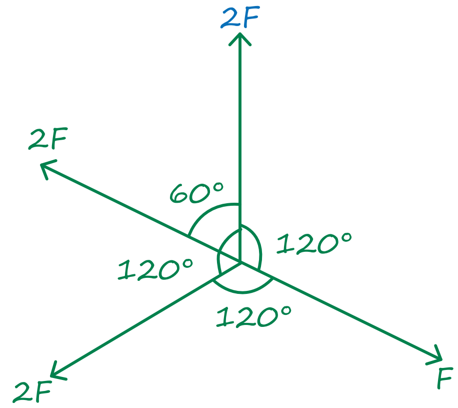
$\dfrac{W}{\sqrt{3}/2} = \dfrac{30}{1/2} = 30\sqrt{3}$



Q. 12. If three force acting on the object as shown in figure. Then find net force on object.



Sol. Net force on this object (point)



$F_{net} = 2F - F = F$

Q. 13. For two vector $\vec{A}$ and $\vec{B}$ vector addition $\vec{R} = \vec{A} + \vec{B}$ and vector subtraction $\vec{D} = \vec{A} - \vec{B}$ then angle between $\vec{A}$ and $\vec{B}$.

Condition	Angle
(i) $\lvert\vec{A} + \vec{B}\rvert = \lvert\vec{R}\rvert = \lvert\vec{A}\rvert = \lvert\vec{B}\rvert$	$\rightarrow \theta = 120^\circ$
(ii) $\lvert\vec{A}\rvert = \lvert\vec{B}\rvert = \lvert\vec{A} - \vec{B}\rvert$	$\rightarrow \theta = 60^\circ$
(iii) $A^2 + B^2 = \lvert\vec{A} - \vec{B}\rvert^2 = D^2$	$\rightarrow \theta = 90^\circ$
(iv) $A^2 + B^2 = R^2$	$\rightarrow \theta = 90^\circ$
(v) $\lvert\vec{A} + \vec{B}\rvert = \lvert\vec{A} - \vec{B}\rvert$	$\rightarrow \theta = 90^\circ$
(vi) $\lvert\vec{A} + \vec{B}\rvert = \lvert\vec{A}\rvert + \lvert\vec{B}\rvert$; $\lvert\vec{R}\rvert = A + B$	$\rightarrow \theta = 0^\circ$
(vii) $\lvert\vec{A} - \vec{B}\rvert = \lvert\vec{A}\rvert + \lvert\vec{B}\rvert$; $\lvert\vec{D}\rvert = A + B$	$\rightarrow \theta = 180^\circ$
(viii) $\lvert\vec{A} + \vec{B}\rvert = \lvert\vec{A}\rvert - \lvert\vec{B}\rvert$; $\lvert R\rvert = A - B$	$\rightarrow \theta = 180^\circ$
(ix) $\lvert\vec{A} - \vec{B}\rvert = \lvert\vec{A}\rvert - \lvert\vec{B}\rvert$	$\rightarrow \theta = 0^\circ$

MR*

"अपनी पढाई छोड़ जो तेरे पीछे चला आयेगा।
वो खुद का ना हो सका, तेरा क्या हो जायेगा।।"