UNITS & DIMENSIONS

MEASUREMENT OF MASS & TIME

MASS

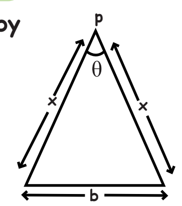
- Unified atomic mass unit(amu) is used to measure mass of atoms & molecules
- 1amu =(1/12)th mass of one C^{12} atom
- 1amu = 1.66×10^{-27} kg
- Electron mass- 10^{-30} kg
- Earth mass : 10^{25} kg
- Observable Universe 10^{55} kg

TIME

- SI unit is second (based on caesium clock with an uncertainity less than 1 part in 10^{-13} ie, 3μs loss every year)
- Timespan of unstable particle: 10^{-24} s
- Age of universe: 10^{17} s

MEASUREMENT OF LENGTH

- Large distance is measured by parallax method
- Parallax angle= $\frac{\text{BASIS}}{\text{DISTANCE}} = \frac{b}{x}$



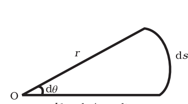
- $1^\circ = 1.745 \times 10^{-2}$ rad
- $1' = 2.91\times10^{4}$ rad.
- $1'' = 4.85\times10^{6}$ rad.
- For very small sizes, optical microscope, tunneling microscope, electron microscope are used.
- 1 AU = 1.496×10^{11} m
- 1 ly = 9.46×10^{15} m
- 1parsec= 3.08×10^{16} m
- Size of proton: 10^{-15} m
- Radius Of Earth: 10^{7}m
- Distance to Boundary Of Observable Universe : 10^{26} m

SI SYSTEM

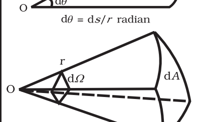
7 Base units and 2 supplementary units

NO.	Base Units: Quantity	Unit	Symbol
1	Length	meter	m
2	Mass	kilogram	kg
3	Time	second	s
4	Temperature	kelvin	K
5	Electric current	ampere	A
6	Luminous intensity	candela	cd
7	Amount of substance	mole	mol

NO.	Supplementary Units: Quantity	Unit	Symbol
1	Plane angle	radian	rad
2	Solid angle	steradian	sr



$d\theta = ds/r$ radian



$d\Omega = dA/r^2$ steradian

What is the unit of permittivity of free space ε_0?

(a) coloumb/newton-metre
(b) newton-metre^2 /coloumb^2
(c) coloumb^2/newton-metre^2
(d) coloumb^2/(newton-metre)^2

SIGNIFICANT FIGURES

The digits in a measured quantity which are reliable and confidence in our measurement + the digit which is uncertain.

RULES FOR SIGNIFICANT FIGURES

1. All non-zero digits are significant. For example, 42.3 has three significant figures; 243.4 has four significant figures; and 24.123 has five significant figures.

2. A zero becomes significant figure if it appears between two non-zero digits. For example, 5.03 has three significant figures; 5.604 has four significant figures; and 4.004 has four significant figures.

3. Leading zeros or the zeros placed to the left of the number are never significant. For example,0.543 has three significant figures; 0.045 has two significant figures; and 0.006 has one significant figure.

4. Trailing zeros or the zeros placed to the right of the number are significant. For example, 4.330 has four significant figures; 433.00 has five significant figures; and 343.000 has six significant figures.

5. In exponential notation, the numerical portion gives the number of significant figures. For example,1.32 x 10^{-2} has three significant figures and 1.32 x 10^{4} has three significant figures.

RULES FOR ROUNDING OF A MEASUREMENT

1. If the digit to be dropped is less than 5, then the preceding digit is left unchanged. For example,x = 7.82 is rounded off to 7.8 and x = 3.94 is rounded off to 3.9.

2. If the digit to be dropped is more than 5, then the preceding digit is raised by one. For example, x = 6.87 is rounded off to 6.9 and x = 12.78 is rounded off to 12.8.

3. If the digit to be dropped is 5 followed by digits other than zero, then the preceding digit is raised by one. For example, x = 16.351 is rounded off to 16.4 and x = 6.758 is rounded off to 6.8.

4. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit, if it is even, is left unchanged. For example, x = 3.250 becomes 3.2 on rounding off and x = 12.650 becomes 12.6 on rounding off.

5. If the digit to be dropped is 5 or 5 followed by zeros, then the preceding digit, if it is odd, is raised by one. For example, x = 3.750 is rounded off to 3.8, again x = 16.150 is rounded off to 16.2.

RULES FOR ROUNDING OF A MEASUREMENT

ADDITION & SUBTRACTION

In addition or subtraction, the final result should be reported to the same number of decimal places as that of the original number with minimum number of decimal places

```
  3.1421
  0.241
 +0.09    ⟵ (has two decimal places)
  3.4731  ⟵ (Answer should be reported to two decimal
             places after rounding off)
```

Answer = 3.47

MULTIPLICATION & DIVISION

When numbers are multiplied or divided, the number of significant figures in the answer equals the smallest number of significant figures in any of the original numbers

```
   51.028
  x 1.31    ⟵ (Three significant figures)
  66.84668  ⟵ (Answer should have three significant figures
               after rounding off)
```

Answer = 66.8

If L=2.331cm, B= 2.1cm,then L+B = ?

(a) 4.431 cm (b) 4.43 cm
(c) 4.4 cm (d) 4 cm

Dimensional Analysis

Dimensions of a physical quantity are the powers to which units of base quantity are raised. Eg: $[M]^a [L]^b [T]^c [A]^d [K]^e$

APPLICATIONS

- checking the correctness of various formulae
  Eg: If Z=A+B,[Z]=[A]=[B]
- conversion of one system of unit into another
  $n_1u_1 = n_2u_2$
  Eg: $n_1[M_1^A L_1^B T_1^C] = n_2[M_2^A L_2^B T_2^C]$
  $n_1 = n_2 \left[\frac{M_2}{M_1}\right]^A \left[\frac{L_2}{L_1}\right]^B \left[\frac{T_2}{T_1}\right]^C$
- Deducing relation among physical quantity

DIMENSIONAL FORMULA

1) Pressure=stress=Young's modulus=$ML^{-1}T^{-2}$
2) Work=Energy=Torque=ML^2T^{-2}
3) Power P=ML^2T^{-3}
4) Gravitational constant G=$M^{-1}L^3T^{-2}$
5) Force constant=Spring constant=MT^{-2}
6) Coefficient of viscosity=$ML^{-1}T^{-1}$
7) Latent heat = L^2T^{-2}
8) Electric potential= $\frac{P}{I}$ = $ML^2T^{-3}A^{-1}$
9) Resistance=$\sqrt{\frac{\mu_0}{\varepsilon_0}}$ =$ML^2T^{-3}A^{-2}$
10) Capacitance=$M^{-1}L^{-2}T^4A^2$
11) Permittivity ε_0=$M^{-1}L^{-3}T^4A^2$
12) Angular momentum = planck's constant =$M^1L^2T^{-1}$
13) $M = k\sqrt{\frac{hc}{G}}$ $L = k\sqrt{\frac{hG}{c^2}}$ $T = k\sqrt{\frac{hG}{c^5}}$

Time period $T \propto \sqrt{\frac{l}{g}} \propto \sqrt{\frac{m}{k}} \propto \sqrt{\frac{R}{g}}$
$\frac{L}{R} = RC = \sqrt{LC}$

DIMENSIONLESS QUANTITIES

1) Strain
2) Refractive index
3) Relative density
4) Plane angle
5) Solid angle

In SI Units, the dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is:

a)$A^{-1}TML^3$ b)$AT^2M^{-1}L^{-1}$
c)$AT^{-3}ML^{3/2}$ d)$A^2T^3M^{-1}L^{-2}$

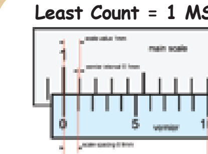
INSTRUMENTS

Least Count:
Smallest quantity an instrument can measure

mm scale	vernier scale	screw gauge
↓ 1mm	↓ 0.1mm	↓ 0.01mm

VERNIER CALIPERS

Least Count = 1 MSD - 1VSD



If n VSD Coincides with (n-1) MSD,
then (n-1) MSD= n VSD
∴ 1VSD = $\frac{n-1}{n}$ MSD

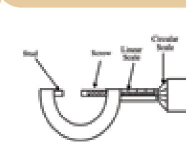
Least Count = 1MSD - $\frac{n-1}{n}$ MSD = $\frac{1MSD}{n}$

Total Reading = Main Scale Reading + (coinciding Vernier Scale division x least count)

In a vernier calipers, one main scale division is x cm & n division of vernier scale coincide with n-1 divisions of the main scle. The least count (in cm) of the calipers is;

a) $\left(\frac{n-1}{n}\right)x$ b) $\frac{nx}{(n-1)}$ c) $\frac{x}{(n-1)}$ d) $\frac{x}{n}$

SCREW GAUGE



Pitch = $\frac{\text{Main Scale Reading}}{\text{No.of rotations}}$

Least Count = $\frac{\text{pitch}}{\text{Total no.of divisions on circular scale}}$

Total Reading = Linear Scale Reading + circular scale reading x least count

The least count of the main scale of a screw gauge is 1mm. The minimum no.of divisions on its circular scale required to measure 5μm diameter of wire is;

a) 200 b) 50 c) 400 d) 100

ERRORS IN MEASUREMENT

Difference between true value & measured value of a quantity

- Systematic Errors: Errors which tend to occur only in one direction, either positive or negative
  - Instrumental: Due to inbuilt defect of measuring instrument
  - Experimental: Limitations in experimental technique
  - Personal: Due to individual bias,Lack of proper setting of apparatus
- Random Errors: Irregular and random in magnitude & direction

- Least count error is the smallest value that can be measured by instrument (occurs with random & systematic errors)
- Absolute Error :- $\Delta a = a_i - a_{mean}$, $a_{mean} = \frac{a_1+a_2+a_3+ \ldots +a_n}{n}$
- Relative Error:- $\frac{\Delta a_{mean}}{a_{mean}}$ $\Delta a_{mean} = \frac{|\Delta a_1|+|\Delta a_2|+|\Delta a_3|+ \ldots +|\Delta a_n|}{n}$
- Percentage Error:- $\frac{\Delta a_{mean}}{a_{mean}} \times 100$

COMBINATION OF ERRORS

Operations	Formula Z	Absolute error ΔZ	Relative error ΔZ/Z	Percentage error 100 x ΔZ/Z
Sum	$A+B$	$\Delta A+\Delta B$	$\frac{\Delta A+\Delta B}{A+B}$	$\frac{\Delta A+\Delta B}{A+B}\times 100$
Difference	$A-B$	$\Delta A+\Delta B$	$\frac{\Delta A+\Delta B}{A-B}$	$\frac{\Delta A+\Delta B}{A-B}\times 100$
Multiplication	$A\times B$	$A\Delta B+B\Delta A$	$\frac{\Delta A}{A}+\frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right)\times 100$
Division	$\frac{A}{B}$	$\frac{B\Delta A+A\Delta B}{B^2}$	$\frac{\Delta A}{A}+\frac{\Delta B}{B}$	$\left(\frac{\Delta A}{A}+\frac{\Delta B}{B}\right)\times 100$
Power	A^n	$nA^{n-1}\Delta A$	$n\frac{\Delta A}{A}$	$n\frac{\Delta A}{A}\times 100$
Root	$A^{\frac{1}{n}}$	$\frac{1}{n}A^{\frac{1}{n}-1}\Delta A$	$\frac{1}{n}\frac{\Delta A}{A}$	$\frac{1}{n}\frac{\Delta A}{A}\times 100$

General rule:

If $Z = \frac{A^pB^q}{C^r}$,Then the maximum fractional relative error in Z will be:

$$\frac{\Delta Z}{Z} = p\frac{\Delta A}{A} + q\frac{\Delta B}{B} + r\frac{\Delta C}{C}$$

In an expirement four quantities a,b,c and d are measured with percentage error1%, 2%, 3% and 4% respectively. Quantity P is calculated as shown below. What is the percentage error in P?

$$P = \frac{a^2b^2}{cd}$$

(a) 14% (b) 10%
(c) 7% (d) 4%