Vectors

1. If $\vec{A}=2\vec{i}-3\vec{j}+7\vec{k}, \vec{B}=\vec{i}+2\vec{k}$ and $\vec{C}=\vec{j}-\vec{k}$ find $\vec{A}\cdot(\vec{B}\times\vec{C})$

2. Find the maximum or minimum values of the function $y=x+\frac{1}{x}$ for $x>0$.

3. Evaluate $\int_0^t A \sin \omega t \, dt$ where A and ω are constants.

4. The velocity v and displacement x of a particle executing simple harmonic motion are related as $v\frac{dv}{dx}=-\omega^2 x$.

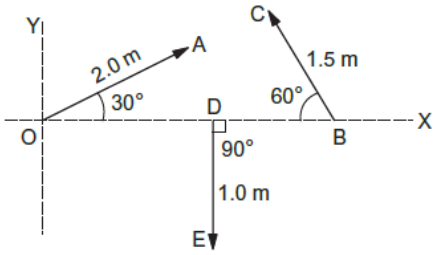
At x = 0, $v = v_0$. Find the velocity u when the displacement becomes x.

5. The charge flown through a circuit in the time interval between t and t + dt is given by dq = $e^{-t/\tau}$ dt, where τ is a constant. Find the total charge flown through the circuit between t = 0 to t = τ.

6. A vector $\overline{A}$ makes an angle of 20° and $\overline{B}$ makes an angle of 110° with the X-axis. The magnitudes of these vectors are 3 m and 4 m respectively. Find the resultant.

7. Add vectors $\overline{A}$, $\overline{B}$ and $\overline{C}$ each having magnitude of 100 unit and inclined to the X-axis at angles 45°, 135° and 315° respectively.

8. Refer to figure. Find (a) the magnitude, (b) x and y components and (c) the angle with the X-axis of the resultant of $\overrightarrow{OA}$, $\overrightarrow{BC}$ and $\overrightarrow{DE}$.



9. Two vectors have magnitudes 3 unit and 4 unit respectively. What should be the angle between them if the magnitude of the resultant is (a) 1 unit, (b) 5 unit and (c) 7 unit.

10. Two vectors have magnitudes 2 m and 3 m. The angle between them is 60°. Find (a) the scalar product of the two vectors, (b) the magnitude of their vector product.

11. Prove that $\overline{A}\cdot\left(\overline{A}\times\overline{B}\right)=0$.

12. A curve is represented by $y = \sin x$. If x is changed from $\frac{\pi}{3}$ to $\frac{\pi}{3}+\frac{\pi}{100}$, find approximately the change in y.

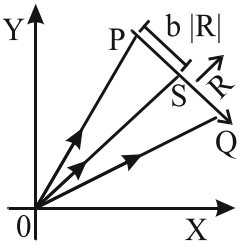
13. The electric current in a charging R–C circuit is given by $i = i_0 e^{-t/RC}$ where i_0, R and C are constant parameters of the circuit and t is time. Find the rate of change of current at (a) $t = 0$, (b) $t = RC$, (c) $t = 10\,RC$.

14. Find the area bounded under the curve $y = 3x^2 + 6x + 7$ and the X-axis with the ordinates at $x = 5$ and $x = 10$.

15. Find the area bounded by the curve $y = e^{-x}$, the X-axis and the Y-axis.

16. Two vectors $\overline{A}$ and $\overline{B}$ are defined as $\overline{A}=a\,\hat{i}$ and $\overline{B}=a(\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j})$, where a is a constant $\omega=\pi/6$ rad s^{-1}. If $|\overline{A}+\overline{B}|=\sqrt{3}\,|\overline{A}-\overline{B}|$ at time t = τ for the first time, the value of τ, in second is …….. . **[JEE ADV. 2018]**

17. Three vectors $\vec{P},\vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the point P and S is b $|\vec{R}|$ and $\vec{R}=\vec{Q}-\vec{P}$. The general relation among vectors $\vec{P},\vec{Q}$ and $\vec{S}$ is



[JEE ADV. 2017]

(A) $\vec{S}=(b-1)\vec{P}+b\vec{Q}$

(B) $\vec{S}=\left(1-b^2\right)\vec{P}+b\vec{Q}$

(C) $\vec{S}=(1-b)\vec{P}+b^2\vec{Q}$

(D) $\vec{S}=(1-b)\vec{P}+b\vec{Q}$

ANSWER KEY

1. 0

2. at x = 1, y is y = 2

3. $\frac{A}{\omega}(1 - \cos \omega t)$

4. $v = \sqrt{v_0^2 - \omega^2 x^2}$.

5. $\tau\left(1 - \frac{1}{e}\right)$

6. (5 m at 73° with *X*-axis)

7. (100 unit at 45° with *X*-axis)

8. (a) 1.6 m (b) 0.98 m and 1.3 m respectively
(c) tan− 1(1.32)

9. (a) 180° (b) 90° (c) 0

10. (a) 3 m 2 (b) 3√3 m2

11. **?**

12. **(0.0157)**

13. $(a)\ \frac{-i_0}{RC}\ (b)\ \frac{-i_0}{RCe}\ (c)\ \frac{-i_0}{RCe^{10}}$

14. (1135)

15. (1)

16. **?**

17. $\vec{S} = (1 - b)\vec{P} + b\vec{Q}$