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Thermodynamics

- First law of themodynamics $\Delta Q = \Delta U + \Delta W$
- * Work done.

$$\Delta W = P\Delta V$$

$$\therefore \Delta Q = \Delta U + P\Delta V$$

Relation between specific heats for a gas

$$C_{\rm p} - C_{\rm v} = R$$

Polytropic Process

It is a thermodynamic process that can be expressed as follows:

$$PV^x =$$
Constant

x (Polytropic exponent)	Type of standard process	Expression
0	Isobaric (dP = 0)	P = Constant
1	Isothermal (dT = 0)	PV = Constant
γ	Adiabatic (dQ = 0)	$PV^{\gamma} = Constant$
∞	Isochoric (dV = 0)	V = Constant

Process	Definition	P-V graph	P-T graph	V-T graph
Isothermal	Temperature constant	P ₁ , V ₁ Compression P ₂ , V ₂ Expansion	$\begin{array}{c} P_{1}, T \\ \downarrow \\ \downarrow \\ P_{2}, T \end{array}$	V_2 , T V_1 , T
Isobaric	Pressure constant	$P_0 = P_0 = P_0 = P_0 = P_0 = P_0$	$P \\ P_0 \\ P_1 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_4 \\ P_5 \\ P_6 \\ P_6 \\ P_6 \\ P_6 \\ P_6 \\ P_6 \\ P_7 \\ P_8 \\ $	V ₂ , T ₂ V ₁ , T ₁
Isochoric	Volume constant	$\begin{array}{c} P \\ \hline \\ P_2, T \\ \hline \\ P_1, T \\ \hline \\ V \end{array}$	P P ₂ , T ₂ P ₁ , T ₁ T	V, T ₁ V, T ₂

For any General Process

$$\Delta Q = \Delta U + W$$

$$\Rightarrow nC\Delta T = \frac{f}{2}nR\Delta T + \frac{nR\Delta T}{1-x}$$

[: Work done in a general polytropic process = $[nR\Delta T/(1-x)]$

$$\Rightarrow$$
 $C = \frac{f}{2}R + \frac{R}{1-x}$

For infinitesimal changes in Q, U, and W, we ca write,

$$dQ = dU + dW$$

$$\Rightarrow \ nCdT = \frac{f}{2}nRdT + PdV$$

$$\Rightarrow C = \frac{f}{2}R + \frac{P}{n}\frac{dV}{dT}$$

Process	Equation of State	W	$\Delta \mathbf{U}$
Isobaric (dP = 0)	$\frac{V}{T} = c$	$P(V_f - V_i) = nR(T_f - T_i)$	$\frac{f}{2}nR(T_f - T_i) = \frac{f}{2}P(V_f - V_i)$
Isochoric (dV = 0)	$\frac{P}{T} = c$	0	$\frac{f}{2}nR(T_f - T_i) = \frac{f}{2}V(P_f - P_i)$
Isothermal (dT = 0)	PV = c	$nRT \ln \left(\frac{V_f}{V_i}\right) = nRT \ln \left(\frac{P_i}{P_f}\right)$	0
Adiabatic (dQ = 0)	$PV^{\gamma} = c$	$\frac{f}{2}nR(T_i - T_f)$	$\frac{f}{2}nR(T_f - T_i)$
		$=\frac{\mathrm{f}}{2}(\mathrm{P_{i}}\mathrm{V_{i}}-\mathrm{P_{f}}\mathrm{V_{f}})$	

Process	$\Delta \mathbf{Q}$	
Isobaric (dP = 0)	$\left(\frac{f}{2}+1\right)nR\Delta T = \left(\frac{f}{2}+1\right)P(V_f - V_i)$	
Isochoric (dV = 0)	$\frac{f}{2}nR(T_f - T_i) = \frac{f}{2}V(P_f - P_i)$	
Isothermal (dT = 0)	$nRT \ln \left(\frac{V_f}{V_i}\right) = nRT \ln \left(\frac{P_i}{P_f}\right)$	
Adiabatic $(dQ = 0)$	0	

- Slope of adiabatic = γ (slope of isotherm)
- * Carnot engine

$$\eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$W = Q_1 - Q_2$$
(efficiency) $\eta = \frac{W}{Q_1}$

* Refrigerator

Coefficient of performance is β

$$\beta = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2}{W}$$
$$\beta = \frac{1 - \eta}{\eta}$$

Heat pump

$$\alpha = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2} = \frac{1}{\eta}$$