YAKEEN NEET 2.0

2026

Vectors

Physics Assignment Solution 04

By- Manish Raj (MR Sir)

16. Two vectors $\overline{A}$ and $\overline{B}$ are defined as $\vec{A} = a\hat{i}$ and $\overline{B} = a(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$, where a is a constant $\boxed{\omega = \pi/6}$ rad s^{-1}. If $|\overline{A}+\overline{B}| = \sqrt{3}\,|\overline{A}-\overline{B}|$ at time t = τ for the first time, the value of τ, in second is .... **[JEE ADV. 2018]**

$\omega t = \frac{\pi}{3}$

$\frac{\pi}{6} t = \frac{\pi}{3}$

$t = 2$ sec

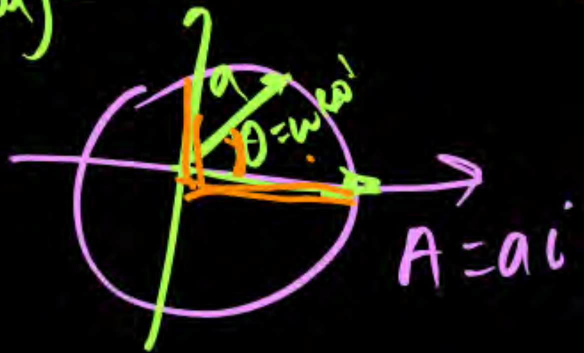
$\vec{A} = a\hat{i}$

$\vec{B} = a(\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j})$

$|\vec{B}| = \sqrt{a^2[\cos^2\omega t + \sin^2\omega t]^2}$

$|\vec{B}| = a$

Same magnitude



$|\vec{A}+\vec{B}| = \sqrt{A^2+B^2+2AB\cos\theta} = \sqrt{3}a$

$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos\theta} = a$

$\sqrt{2a^2+2a^2\cos\theta} = \sqrt{3}\sqrt{2a^2-2a^2\cos\theta}$

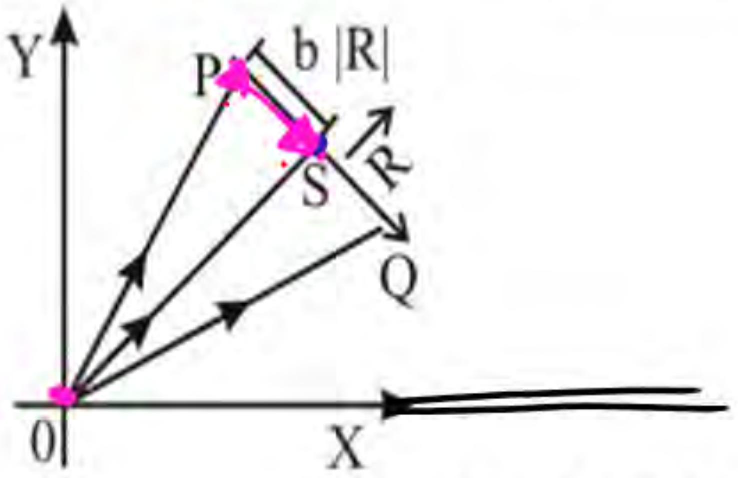
$2\cancel{a^2}+2\cancel{a^2}\cos\theta = 3(2\cancel{a^2}-2\cancel{a^2}\cos\theta)$

$2+2\cos\theta = 6-6\cos\theta$

$8\cos\theta = 4$

$\cos\theta = \frac{\cancel{4}}{\cancel{8}\,2}$

17. Three vectors $\vec{P}, \vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the point P and S is $b\,|\vec{R}|$ and $\vec{R} = \vec{Q} - \vec{P}$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is



[JEE ADV. 2017]

(A) $\vec{S} = (b-1)\vec{P} + b\vec{Q}$

(B) $\vec{S} = \left(1-b^2\right)\vec{P} + b\vec{Q}$

(C) $\vec{S} = (1-b)\vec{P} + b^2\vec{Q}$

✓(D) $\vec{S} = (1-b)\vec{P} + b\vec{Q}$

$(PS)_{\text{dist}} = b|\vec{R}|$

$\vec{R} = \vec{Q} - \vec{P}$ — (1)

$\hat{R} = \dfrac{\vec{R}}{|\vec{R}|}$

$\vec{S} = \vec{P} + b|\vec{R}|\,\hat{R}$

$\vec{S} = \vec{P} + b\cancel{|\vec{R}|}\dfrac{\vec{R}}{\cancel{|\vec{R}|}}$

$\vec{S} = \vec{P} + b\vec{R}$

$= \vec{P} + b(\vec{Q} - \vec{P})$

$= \vec{P} + b\vec{Q} - b\vec{P} = \vec{P}(1-b) + b\vec{Q}$

P

1. If $\vec{A} = 2\vec{i} - 3\vec{j} + 7\vec{k}$, $\vec{B} = \vec{i} + 2\vec{k}$ and $\vec{C} = \vec{j} - \vec{k}$ find $\vec{A} \cdot (\vec{B} \times \vec{C})$ ← scalar Triple product

$\frac{d\frac{1}{x}}{dx} = -\frac{1}{x^2}$

2. Find the maximum or minimum values of the function $y = x + \frac{1}{x}$ for $x > 0$.

$y_{min} = 1 + \frac{1}{1} = 1+1 = 2$ at $x = 1$

$y = x + \frac{1}{x}$

$\frac{dy}{dx} = 1 - \frac{1}{x^2} = 0$

$1 = \frac{1}{x^2}$

$x^2 = 1$

$x = \pm 1$

$\boxed{\text{Consider only } x = +1}$

$\frac{d^2y}{dx^2} = 0 - \left(-\frac{2}{x^3}\right)$

$\left(\frac{d^2y}{dx^2}\right) = +\frac{2}{x^3}$ ↪ minimum.

3. Evaluate $\int_0^t A \sin \omega t \; dt$ where A and ω are constants.

$$\int_0^t A \sin(\omega t)\, dt = A \int_0^t \sin(\omega t)\, dt = -A \left[\frac{\cos(\omega t)}{\omega}\right]_0^t$$

$$= -\frac{A}{\omega}\left[\cos(\omega t) - 1\right]$$

4. The velocity v and displacement x of a particle executing simple harmonic motion are related as

$$v\frac{dv}{dx} = -\omega^2 x.$$

At x = 0, $v = v_0$. Find the velocity u when the displacement becomes x.

5. The charge flown through a circuit in the time interval between t and t + dt is given by dq = $e^{-t/\tau}$ dt, where τ is a constant. Find the total charge flown through the circuit between t = 0 to t = τ .

6. A vector $\vec{A}$ makes an angle of 20° and $\vec{B}$ makes an angle of 110° with the X-axis. The magnitudes of these vectors are 3 m and 4 m respectively. Find the resultant.

$$v\frac{dv}{dx} = -\omega^2 x$$

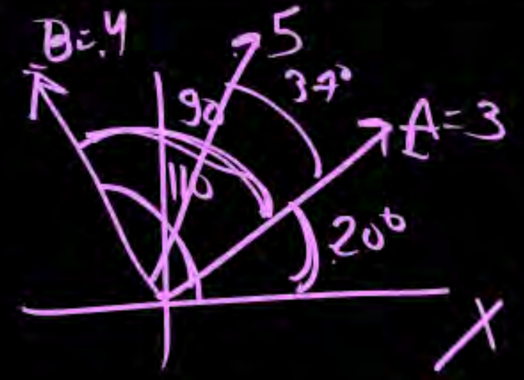
$$\int_{v_0}^{v} v\,dv = -\int_{x=0}^{x} \omega^2 x\,dx$$

$$\left[\frac{v^2}{\cancel{2}}\right]_{v_0}^{v} = -\omega^2\left(\frac{x^2}{\cancel{2}}\right]$$

$$v^2 - v_0^2 = -\omega^2 x^2$$

$$v^2 = v_0^2 - \omega^2 x^2$$

$$v = \sqrt{v_0^2 - \omega^2 x^2}$$

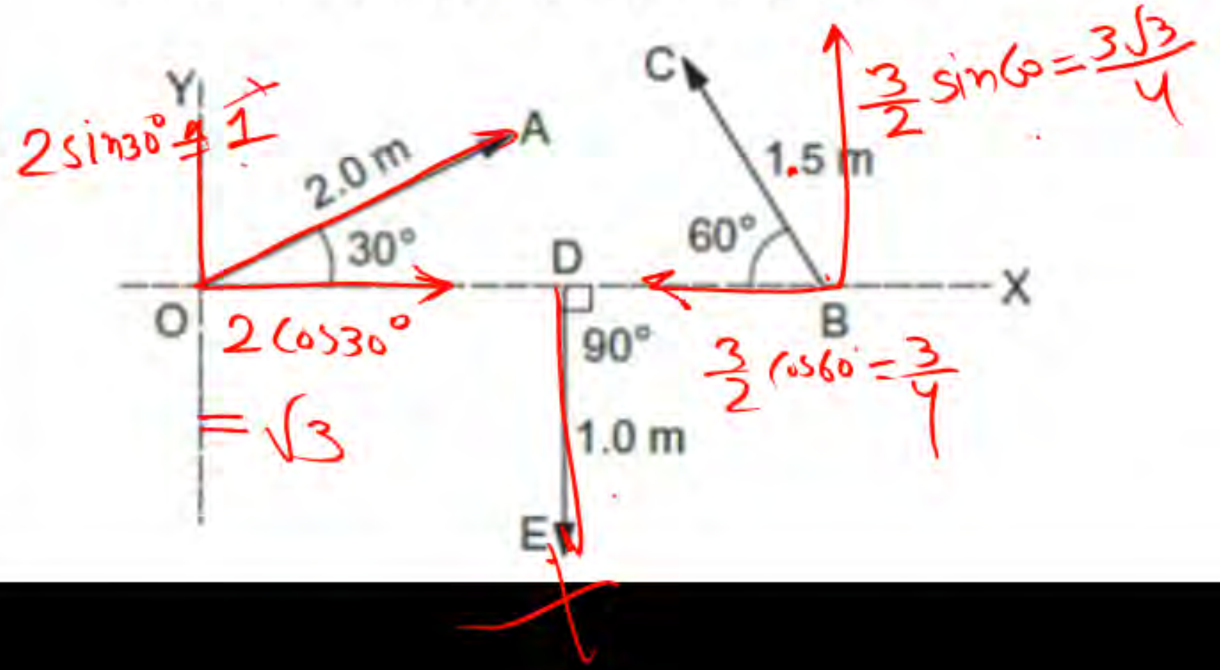


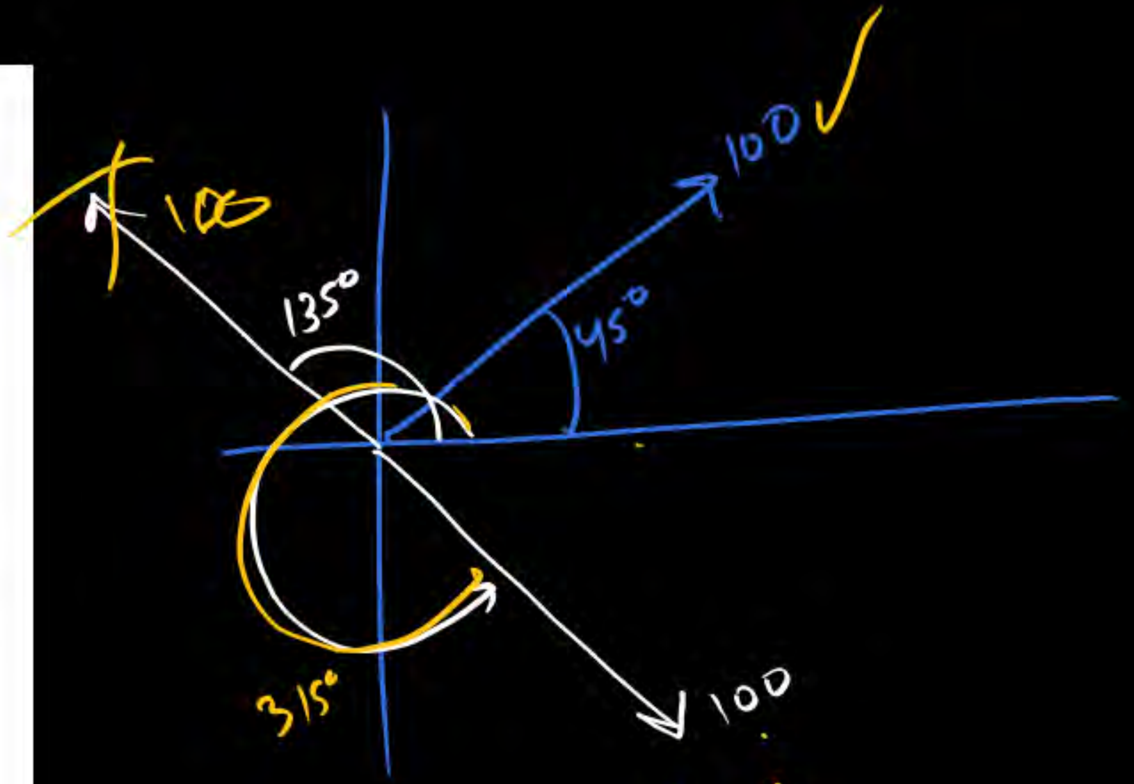
$$\int dq = \int_0^{\tau} e^{-t/\tau}\,dt$$

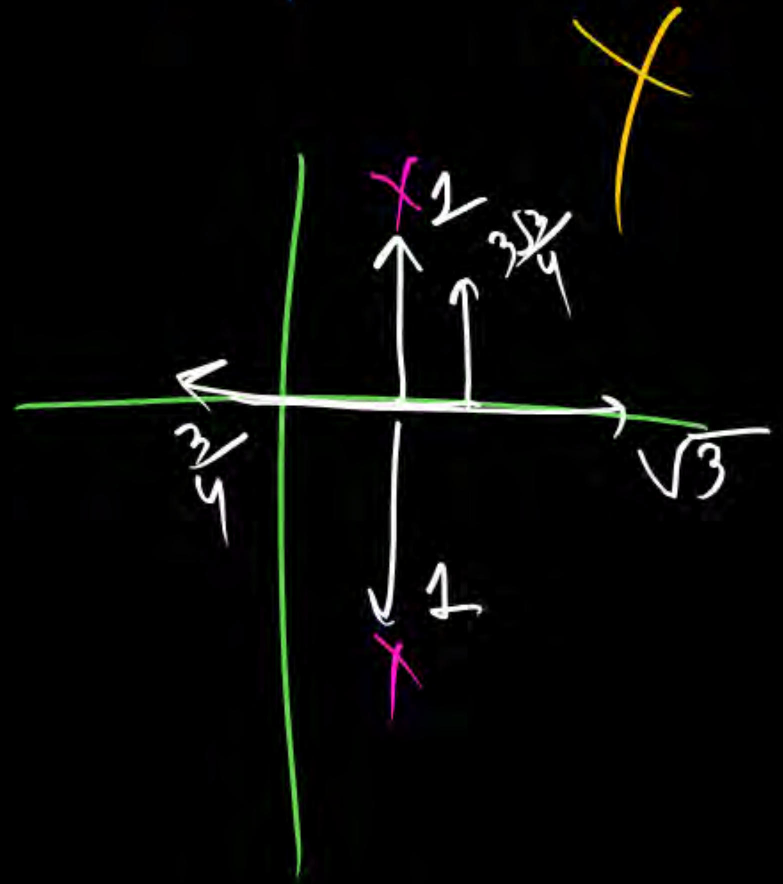
$$q = \frac{e^{-t/\tau}}{-\frac{1}{\tau}} = -\tau\left[e^{-\tau/\tau} - e^0\right] = -\tau\left[e^{-1} - 1\right] = \tau\left(1 - e^{-1}\right) = \tau\left(1 - \frac{1}{e}\right)q$$

7. Add vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ each having magnitude of 100 unit and inclined to the X-axis at angles 45°, 135° and 315° respectively.

8. Refer to figure. Find (a) the magnitude, (b) x and y components and (c) the angle with the X-axis of the resultant of $\overrightarrow{OA}$, $\overrightarrow{BC}$ and $\overrightarrow{DE}$.







$\frac{3\sqrt{3}}{4}\hat{j} + \left(\sqrt{3} - \frac{3}{4}\right)\hat{i}$

$\frac{3\sqrt{3}}{4}\hat{j} + 1\hat{i}$

$= \sqrt{\frac{27}{16} + 1}$

$= \sqrt{\frac{43}{16}}$

9. Two vectors have magnitudes $\boxed{3 \text{ unit and } 4}$ unit respectively. What should be the angle between them if the magnitude of the resultant is (a) 1 unit, (b) 5 unit and (c) 7 unit.

180°

90° 0°

10. Two vectors have magnitudes 2 m and 3 m. The angle between them is 60°. Find (a) the scalar

$AB\cos 60 = \cancel{2} \times 3 \times \frac{1}{\cancel{2}} = 3$

$|AB\sin 60| = \cancel{2} \times 3 \times \frac{\sqrt{3}}{\cancel{2}} = 3\sqrt{3}$

11. Prove that $\vec{A}\cdot(\vec{A}\times\vec{B})=0$.

$\hat{i}\cdot\hat{k}=0$

12. A curve is represented by $y=\sin x$. If x is changed from $\underset{x_i=}{\frac{\pi}{3}}$ to $\underbrace{\frac{\pi}{3}+\frac{\pi}{100}}_{x_f}$, find approximately the change in y.

$dx=\frac{\pi}{100}$

13. The electric current in a charging R–C circuit is given by $i=i_0\,e^{-t/RC}$ where i_0, R and C are constant parameters of the circuit and t is time. Find the rate of change of current at (a) $t=0$, (b) $t=RC$, (c) $t=10\,RC$.

Let $\vec{A}=\hat{i}$

$\vec{B}=\hat{j}$

$(\vec{A}\times\vec{B})=\hat{k}$

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$i=i_0\,e^{-t/RC}$

$\frac{di}{dt}=i_0\,e^{-t/RC}\left(-\frac{1}{RC}\right)$

$=\frac{-i_0}{RC}e^{-t/RC}$

$y=\sin x$

$\frac{dy}{dx}=\cos x$

$dy=\cos x\,dx$

$dy=\cos\frac{\pi}{3}\times\frac{\pi}{100}=\frac{1}{\cancel{2}}\left(\frac{\overset{1.57}{\cancel{3.14}}}{100}\right)=\underline{0.0157}$

14. Find the area bounded under the curve $y = 3x^2 + 6x + 7$ and the X-axis with the ordinates at $x = 5$ and $x = 10$.

15. Find the area bounded by the curve $y = e^{-x}$, the X-axis and the Y-axis.

$-x = \log_e y$

$\int x\,dy = -\int \log_e y\,dy$ No need in this

$y = e^{-x}$

$\int e^{-x}\,dx = -e^{-x}$

$y = 3x^2 + 6x + 7$

$\int_5^{10} y\,dx = \int (3x^2 + 6x + 7)\,dx = \left[\cancel{3}\frac{x^3}{\cancel{3}} + \cancel{6}^{3}\frac{x^2}{\cancel{2}} + 7x\right]_5^{10}$

THANK YOU