Mechanical Properties of Fluids 01

- Density $(\rho) = \frac{\text{mass } (m)}{\text{volume } (v)}$
- unit= kg/m^3
- density of water=1000 $kg\ /\ m^3$=1g/cc
- for two bodies of same mass
  $\rho_1 v_1 = \rho_2 v_2$
  if $\rho_1 > \rho_2$
  $v_1 < v_2$

Mixing of liquid

Calculation of resultant/final density

1) If volumes of the liquids are equal

$$d = \frac{d_1 + d_2}{2}$$

2) If masses of two liquids are equal

For 2- liquids $\Rightarrow d = \frac{2d_1 d_2}{d_1 + d_2}$

For 3- liquids $\Rightarrow d = \frac{3d_1 d_2 d_3}{d_1 d_2 + d_2 d_3 + d_1 d_3}$

For n-liquids, $\frac{n}{d} = \frac{1}{d_1} + \frac{1}{d_2} + \frac{1}{d_3} + \ldots \frac{1}{d_n}$

3) If masses and volumes of two liquids are different

$$d = \frac{\text{Total mass}}{\text{Total volume}} = \frac{m_1 + m_2}{v_1 + v_2}$$

$m_1 = \rho_1 v_1$ & $m_2 = \rho_2 v_2$

$v_1 = \frac{m_1}{\rho_1}$ & $v_2 = \frac{m_2}{\rho_2}$

Relative density (R.D)/Specific gravity

1) Relative density of a body

$$(R.D)_{solid} = \frac{d_{solid}}{d_{water}} = \frac{w_a}{w_a - w_w}$$

2) Relative density of liquid

$$(R.D)_{liquid} = \frac{d_{liquid}}{d_{water}} = \frac{w_a - w_L}{w_a - w_w}$$

3) Relative density of a solid to that of liquid

$$\frac{(R.D)_{solid}}{(R.D)_{liquid}} = \frac{w_a}{w_a - w_L}$$

weight of object
where,
W_a = weight of object when in air
W_w = weight of object when dipped in water
W_L = weight of object when dipped in liquid

Pressure

Normal force or thrust exerted by liquid per unit area

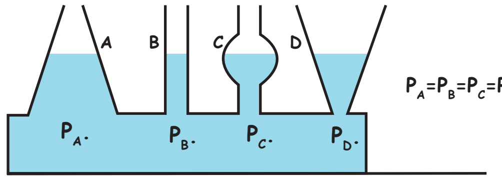
$$\boxed{P = \frac{F}{A}}$$

Pressure-depth relation

$$P = h \rho g$$

Hydrostatic paradox

Whatever the shape or width of vessel the pressure at any particular depth is same

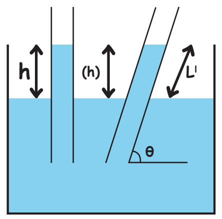


$P_A = P_B = P_C = P_D$

- Gauge pressure $= P - P_{atm} = h\rho g$
- $P_{atm} = 1.01325 \times 10^5$ Pa

Inclined barometer

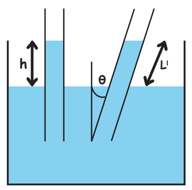
if θ=angle with horizontal



$\sin\theta = \frac{h}{L^I}$

$$\boxed{L^I = \frac{h}{\sin\theta}}$$

if θ=angle with vertical



$\cos\theta = \frac{h}{L^I}$

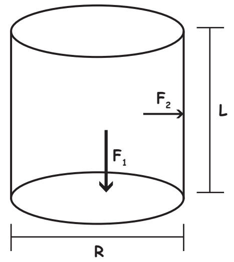
$L^I = \frac{h}{\cos\theta}$

Bubble rising up at constant temperature

radius 'r' becomes 'nr' when bubble rises in liquid from bottom to the surface

$$\rho g h = p_{atm}\,[n^3 - 1]$$

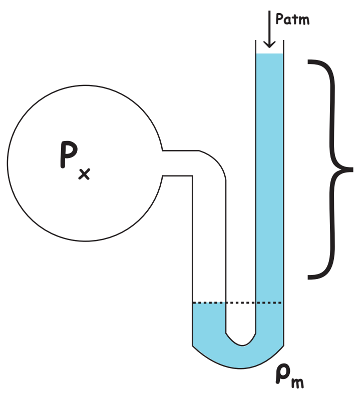
Conditions for equal forces on wall and bottom of a cylinder



If $\boxed{L = R}$

then, $F_1 = F_2$

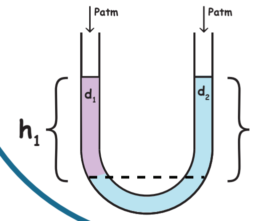
Manometer



$$P_x = P_{atm} + h\rho_m g$$

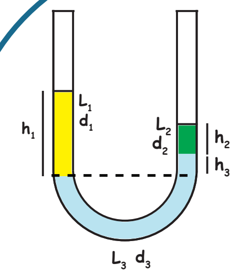
$$h\rho_m g = P_x - P_{atm}$$

1) U-Tube manometer



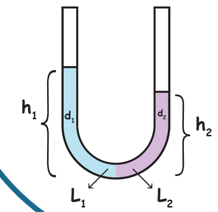
$$h_1 d_1 = h_2 d_2$$

4)



$$h_1 d_1 = h_2 d_2 + h_3 d_3$$

2) U-Tube type



$h_1 d_1 = h_2 d_2$

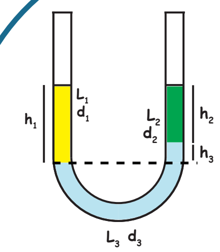
$\frac{d_1}{d_2} = \frac{h_2}{h_1}$

Note:
If one of the liquid water,

$(R.D) = \frac{d_L}{d_w} = \frac{h_w}{h_L}$

$(R.D) = \frac{h_w}{h_L}$

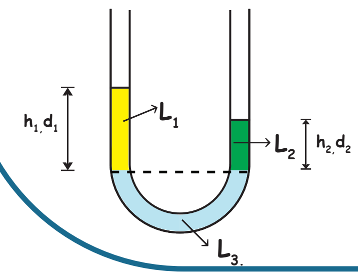
5)



$$h_1 = h_2 + h_3$$

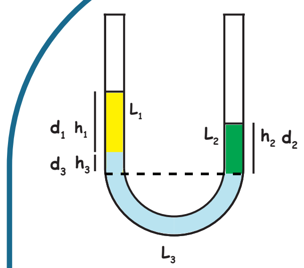
$$h_1 d_1 = h_2 d_2 + h_3 d_3$$

3) The third liquid is in level with other



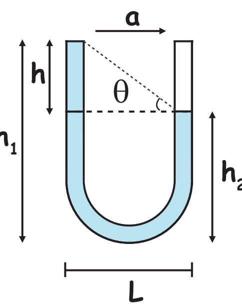
$$h_1 d_1 = h_2 d_2$$

6)



$$h_1 d_1 + h_3 d_3 = h_2 d_2$$

U - tube accelerating horizontally



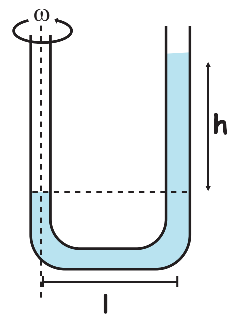
$h = h_1 - h_2$

$\tan\theta = \frac{h}{L} = \frac{a}{g}$

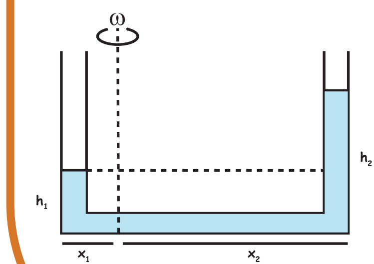
$\frac{h}{L} = \frac{a}{g}$

$h = \frac{La}{g}$

Special case: U – tube rotating



$$h = \frac{\omega^2 l^2}{2g}$$



$h_1 = \frac{\omega^2 x_1^2}{2g}$

$h_2 = \frac{\omega^2 x_2^2}{2g}$

$h = h_2 - h_1 = \frac{\omega^2 (x_2^2 - x_1^2)}{2g}$

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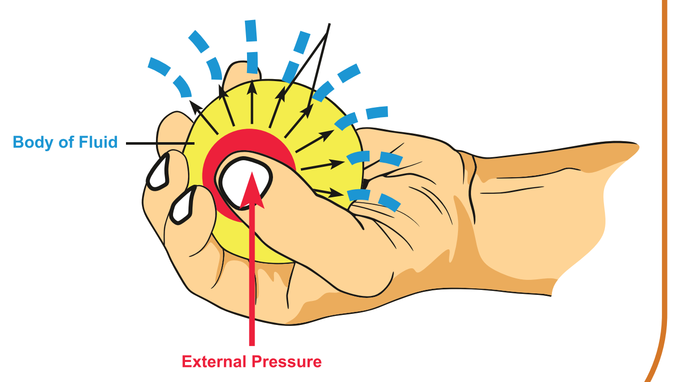
Mechanical Properties of Fluids 02

Pascal's Law

Any change in pressure at a point of an enclosed incompressible fluid is equally transmitted at all other points of fluid

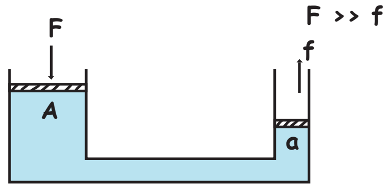
Pascal's Law



Pascal's Law States that Pressure Applied on One Point of Liquid Transmits Equally in All Direction

Application

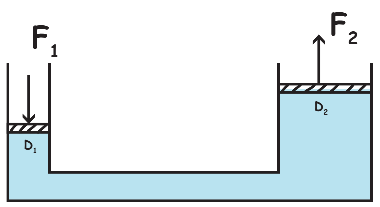
Hydraulic Lift



$$\boxed{\frac{F}{A} = \frac{f}{a}}$$

As A>> a therefore

If the cylinders are connected

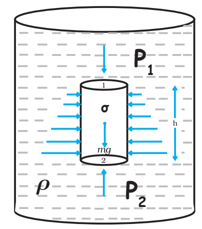


$$\frac{F_1}{\pi R_1^2} = \frac{F_2}{\pi R_2^2}$$

$$\frac{F_1}{R_1^2} = \frac{F_2}{R_2^2}$$

$$\boxed{\frac{F_1}{D_1^2} = \frac{F_2}{D_2^2}}$$

Archimede's principle



liquid applies net upward force on an immersed body, called as upthrust or buoyant force

upthrust=weight of the liquid displaced=$V\rho g$

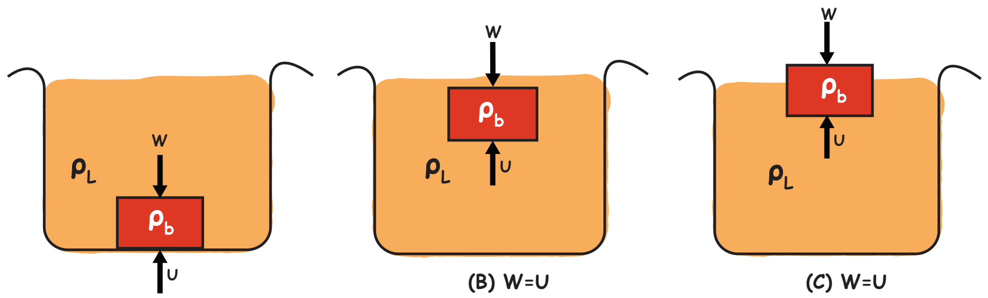
Apparent weight=Actual weight-upthrust

$W_{app}=W_{air}-U$

$=W_{air}\left[1-\frac{\sigma}{\rho}\right]$

$P_2 > P_1$, $P_2-P_1=$ Upthrust(U)

Law of floatation

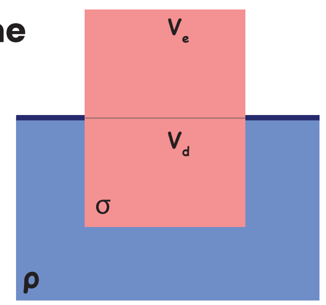


(A) W>U	(B) W=U	(C) W=U
$\rho_b > \rho_L$	$\rho_b = \rho_L$	$\rho_b < \rho_L$
Body sinks W ⇒ Weight of body U ⇒ Upthrust	Body floats just below surface	Body floats partially submerge

Fractional submerged volume

$$\frac{\text{Displaced volume}(V_d)}{\text{Total volume}(V)} = \frac{\sigma}{\rho} \text{ (submerged fraction)}$$

$$\frac{\text{Exposed volume}(V_e)}{\text{Total volume}(V)} = 1-\frac{\sigma}{\rho} \text{ (Exposed fraction)}$$



$$\text{Relative density of a solid} = \frac{\text{weight of solid in air}}{\text{Loss of weight in water}} = \frac{W_a}{W_a-W_w} = \frac{\rho_b}{\rho_W}$$

$$\text{Relative density of a liquid} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight of an object dipped in water}} = \frac{W_a-W_L}{W_a-W_w} = \frac{\rho_L}{\rho_W}$$

Newton's Law of viscosity

$$F \propto A\frac{dv}{dx} \Longrightarrow F = \eta A\frac{dv}{dx}$$

Where, Velocity gradient= $\frac{dv}{dx}$

$$\eta = \frac{F}{A dv/dx} \Longrightarrow \text{coefficent of viscosity} \Longrightarrow \eta = \frac{F/A}{dv/dx} = \frac{F/A}{v/l} = \frac{F/A}{\left(\frac{dx}{dt}\right)/l} = \frac{F/A}{\frac{d}{dt}\left(\frac{x}{l}\right)}$$

$$\Longrightarrow \eta = \frac{\text{shearing stress}}{\text{strain rate}}$$

Unit of Coefficient of viscosity

1) The CGS Unit of η is dyne s cm^{-2} and is called poise.

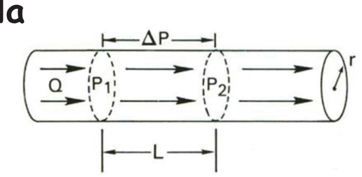
2) The SI unit of η is Nsm^{-2} called decapoise or poiseuille

1 poiseuille = 10 poise

Poiseuille's formula

$$Q = \frac{\pi \Delta P r^4}{8\eta L}$$

where, Q= rate of flow



Stoke's law

F=6ηπrv

F_{net}=Apparent weight-viscous force

Terminal velocity

$$V_t = \frac{2r^2}{9\eta}(\rho-\sigma)g$$

1) If $\rho>\sigma$, the body will attain terminal velocity in the downward direction.

2) If $\rho<\sigma$ the terminal velocity will be negative and the body will move in the upward direction.

3) $\rho=\sigma$, the body remain suspended in the fluid.

Critical velocity

Reynold's number

$$R_e = \frac{\rho v D}{\eta}$$

Significance of Reynold's number:

- If R_e lies between 0 and 2000 the flow is streamlined or laminar.
- If R_e>3000, the flow is turbulent.
- If R_e lies between 2000 & 3000 the flow of liquid is unstable. It may change from laminar to turbulent and vice versa.

Equation of continuity

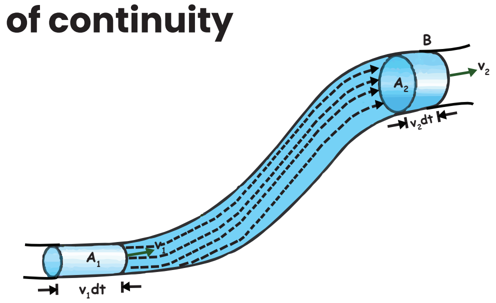
$v_1A_1\Delta t\rho_1 = v_2A_2\Delta t\rho_2$

since the liquid is incompressable $\rho_1=\rho_2$

$v_1A_1 = v_2A_2$

Av=constant.

$Av = \frac{dV}{dt} = Q \Longrightarrow$ Volume rate of flow



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Mechanical Properties of Fluids 03

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Energy of fluid in a study flow

kinetic Energy $= \frac{1}{2}mv^2$

kinetic energy per unit mass $= \frac{1}{2}v^2$

kinetic energy per unit volume $= \frac{1}{2}\rho v^2$

Potential Energy = mgh

Potential energy per unit mass = gh

Potential energy per unit volume = ρgh

Pressure energy = PV

Pressure energy per unit mass $= \frac{P}{\rho}$

Pressure energy per unit volume = P

BERNOULLI'S PRINCIPLE

$P_1V_1 - P_2V_2 = \frac{1}{2}m(v_2^2 - v_1^2) + mg(h_2 - h_1)$

$(P_1 - P_2)V = \frac{1}{2}m(v_2^2 - v_1^2) + mg(h_2 - h_1)$

$(P_1 - P_2)V = \frac{1}{2}\frac{m}{V}(v_2^2 - v_1^2) + \frac{mg}{V}(h_2 - h_1)$

$\Rightarrow P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}$

$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant}$, $\frac{P}{\rho g}$ = pressure head

$\frac{v^2}{2g}$ = velocity head h = Gravitational head

Conditions:

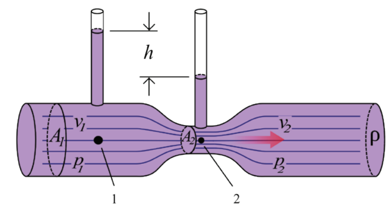
1. Flow should be streamlined.
2. Non-viscous and incompressible fluid.
3. Friction is absent everywhere.

Note: It is based on conservation of energy.

VENTURIMETER

Device to measure the flow of speed of incompressible fluid

$v_1 = \sqrt{\frac{2hg}{(A_1^2/A_2^2) - 1}}$

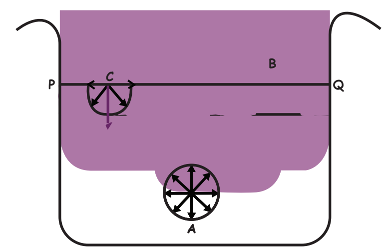


SURFACE TENSION

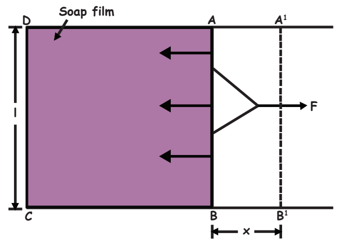
Surface tension T $= \frac{\text{Force}}{\text{Length}} = \frac{F}{l}$

Unit in SI system $= \frac{N}{m}$

Unit in CGS system = dyne / cm



SURFACE ENERGY



Work done W = F x x

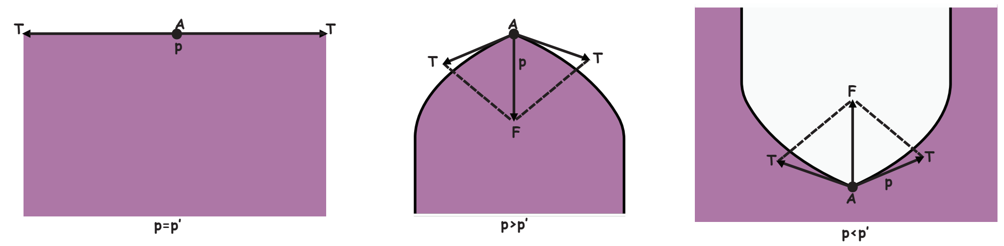
But F = 2TL

$\Rightarrow$ W = 2TL x x

Energy of the additional surface = W = 2TLx

$= T(2LX) = T\Delta A$

PRESSURE DIFFERENCE ACROSS A CURVED LIQUID SURFACE



Pressure on concave side> pressure on convex side

$P_{concave} - P_{convex} = \frac{2T}{R}$

$P_{inside} - P_{outside} = \frac{2T}{R}$ [Liquid drop or air bubble] One surface

$P_{inside} - P_{outside} = \frac{4T}{R}$ [Soap bubble] Two surfaces

APPLICATIONS OF BERNOULLI'S PRINCIPLE

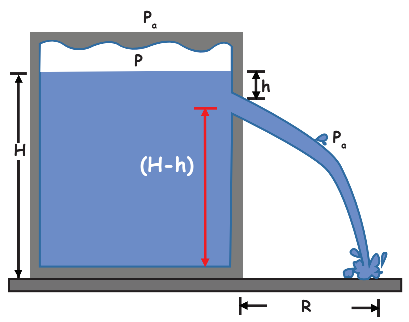
Torricelli's Law of Efflux

$v = \sqrt{\frac{2(P - P_o)}{\rho} + 2gh}$

- It is assumed that size of hole << size of top of tank

If tank is open, $P = P_a$

Then $v = \sqrt{2gh}$



Time of fall, $t = \sqrt{\frac{2(H-h)}{g}}$

Range R = vt

$= \sqrt{2gh} \times \sqrt{2(H-h)/g}$

$\Rightarrow R = 2\sqrt{h(H-h)}$

R is max, when $h = \frac{H}{2}$

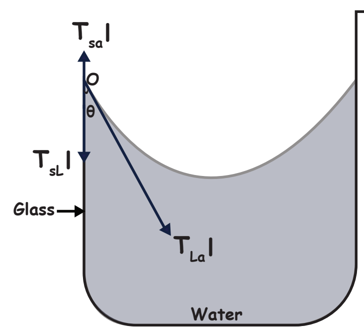
Excess pressure inside a liquid drop

$P_i - P_o = \frac{2T}{R}$

Excess pressure inside a soap bubble

$P_i - P_o = \frac{4T}{R}$

Shape of liquid meniscus



Consider the equilibrium along the surface at line of contact

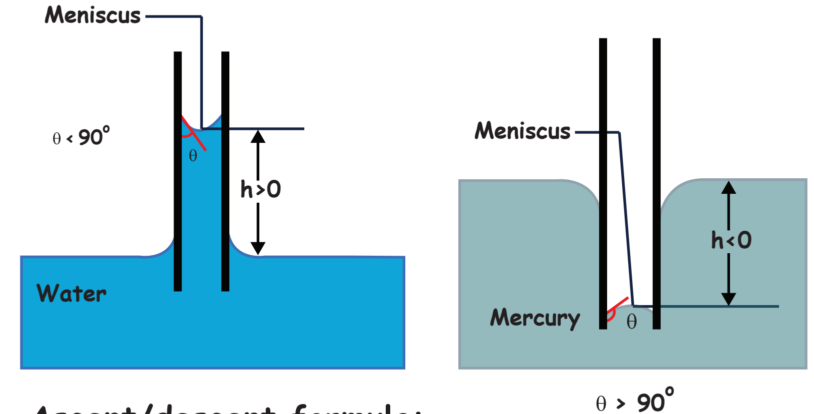
$T_{sa}l = T_{sL}l + T_{La}l\cos$

$T_{Sa} = T_{SL} + T_{La}\cos\theta$

$\cos\theta = \frac{T_{Sa} - T_{SL}}{T_{La}}$

θ = Angle of contact.

Capillarity



Ascent/descent formula:

$h = \frac{2T}{R\rho g}$, $h > 0\ (\theta < 90^\circ)$ $h = \frac{2T\cos\theta}{r\rho g}$, $h < 0\ (\theta > 90^\circ)$

where, R= radius of meniscus

r= radius of the tube