Topics to be covered

1. Graph → P.h.D on straight line, Parabola, Rectangular hyperbola, circle, ellipse
2. differential, application of diff^n, Quadratic eq^n, max^m/Minima.
3. Integration
4. Application of Integration

graph $\longrightarrow$ always b/w two physical quantity

dependent P.Q (depends on x)

$y = y(x) = kx$
$= kx^2$
$= \frac{k}{x}$

s (depen) $\quad s = \frac{1}{2}at^2 \quad t = \sqrt{\frac{2s}{a}}$

t (Independent)

x = Independent P.Q

$F = ma$
const

$C = \frac{\varepsilon_0 A}{d}$ → const

d = c

$f = \frac{1}{T}$

$K.E = \frac{1}{2}mv^2$

$V = \frac{\Delta s}{\Delta t}$

$s = ut$

$p = mv$

$F = ma$

$F = \frac{kq}{r^2}$

$F = \frac{Gmm}{r^2}$

सरल रेखा

Straight line graph (linear relation)

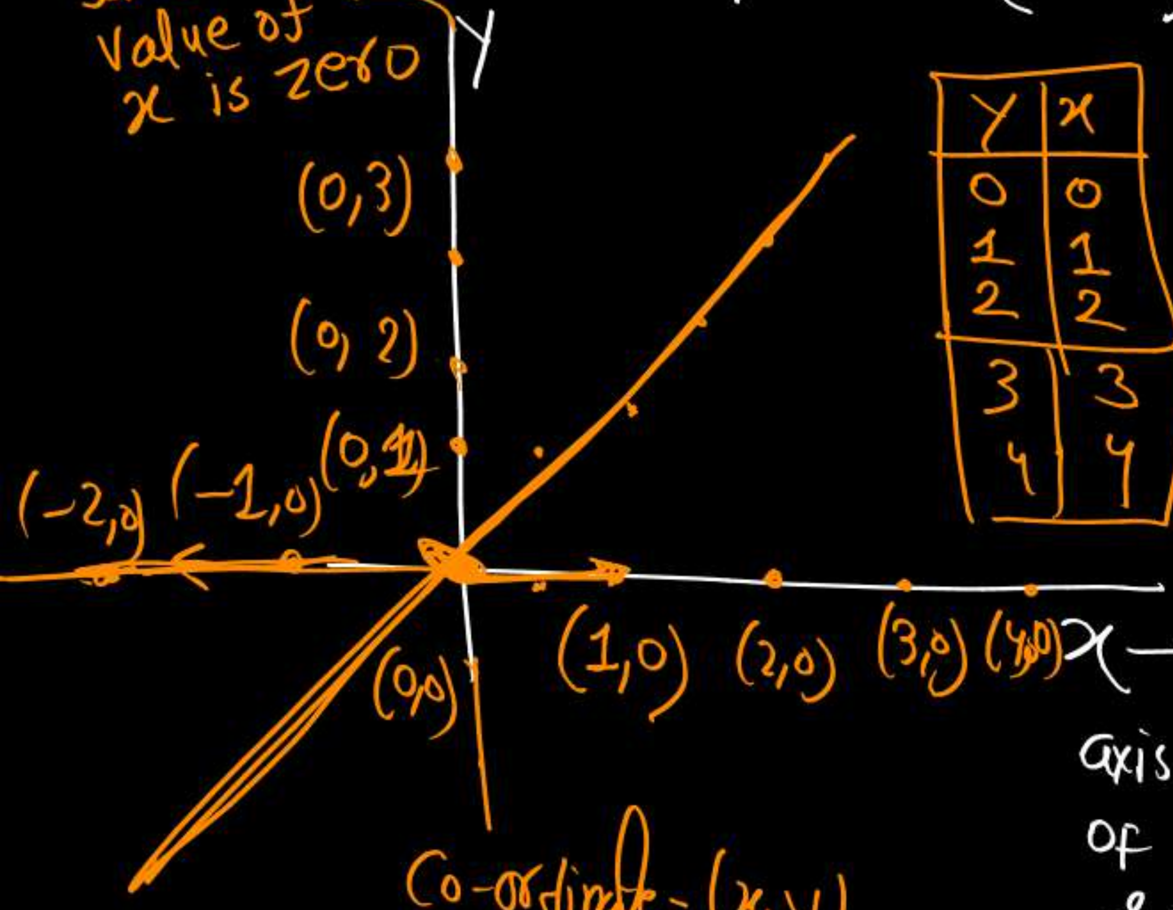
$y \propto x$

$y = cx$

$y = x$ (if $c = 1$)

y	x
0	0
1	1
2	2
3	3
4	4

• A vertical line on which value of x is zero



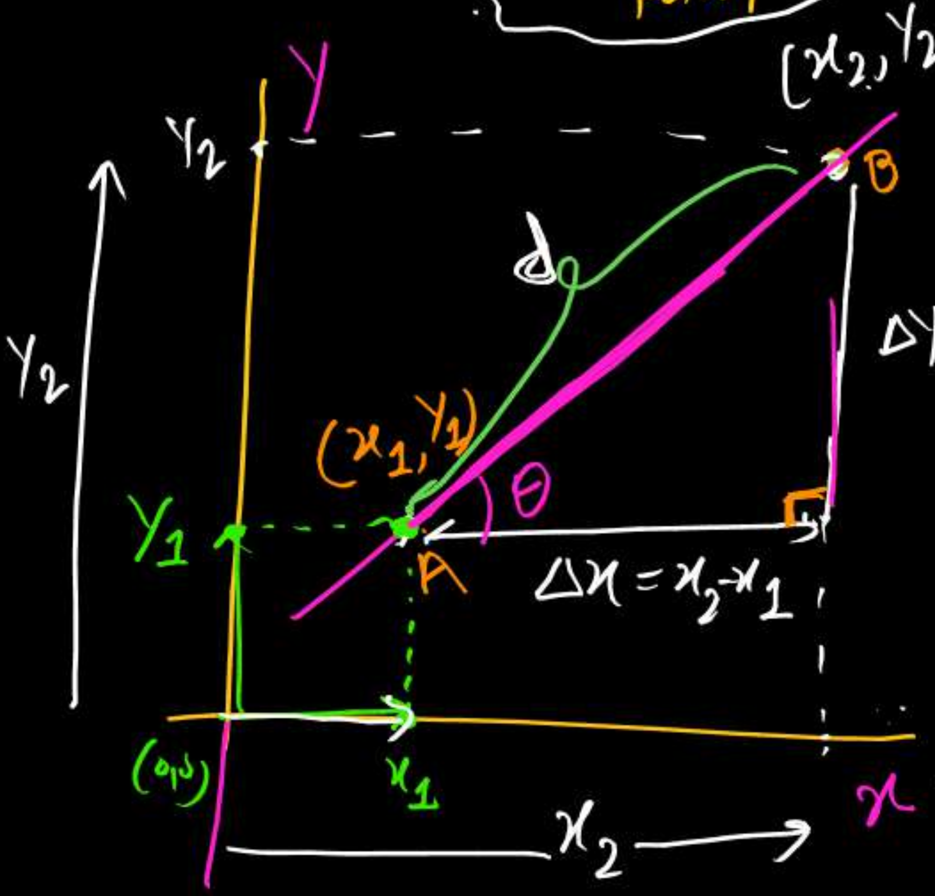
$x \rightarrow$ A Horizontal axis on which value of y is zero & x will vary.

Co-ordinate = (x, y)

Straight dist^n $AB = \sqrt{\Delta x^2 + (\Delta y)^2}$

distance formula b/w Two point

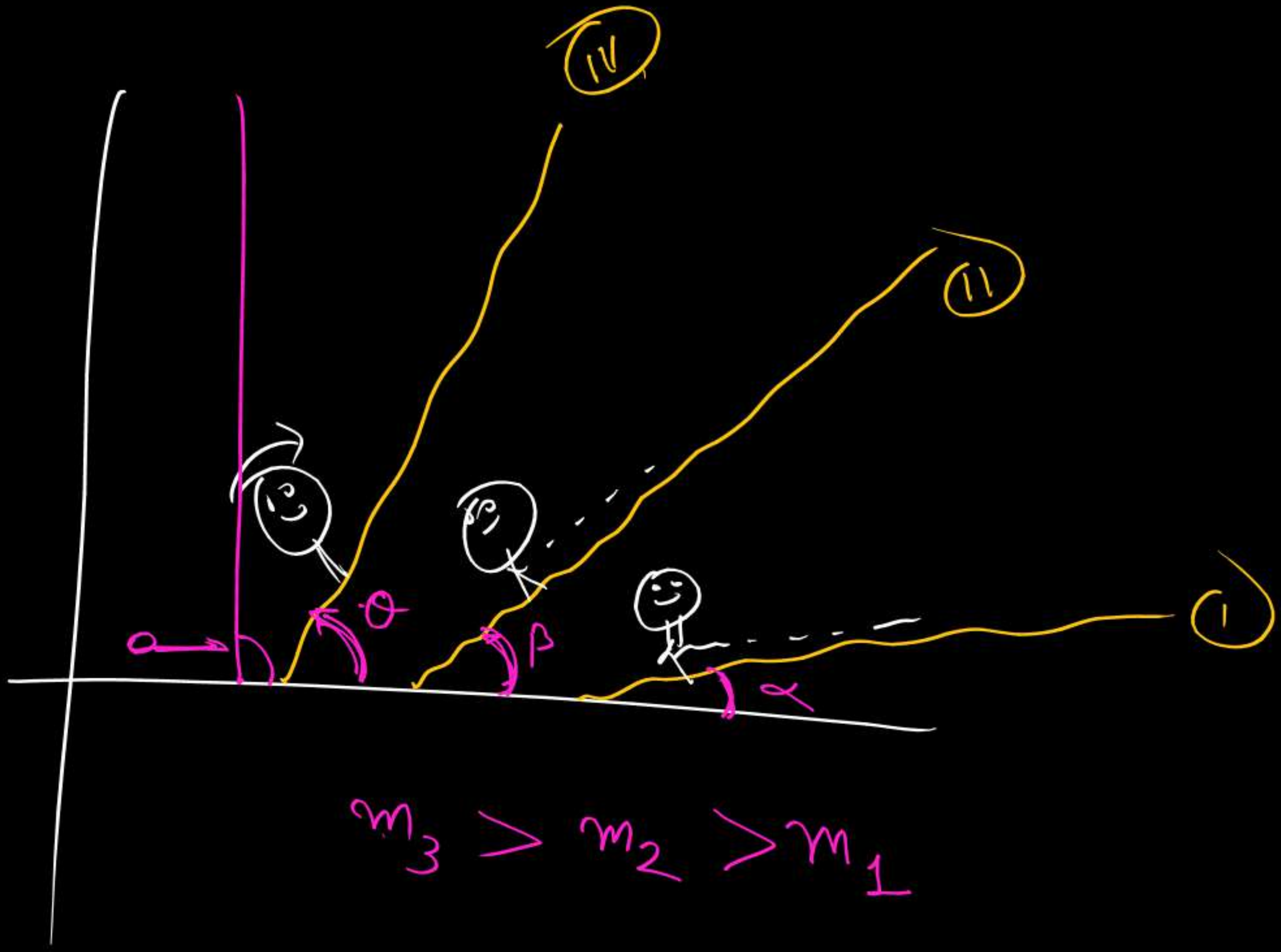
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



$(\tan\theta) = \frac{\Delta y}{\Delta x}$

m [Slope (Inclination)] = $\tan\theta$

$$\boxed{m = \tan\theta = \frac{(\Delta y)}{(\Delta x)}}$$

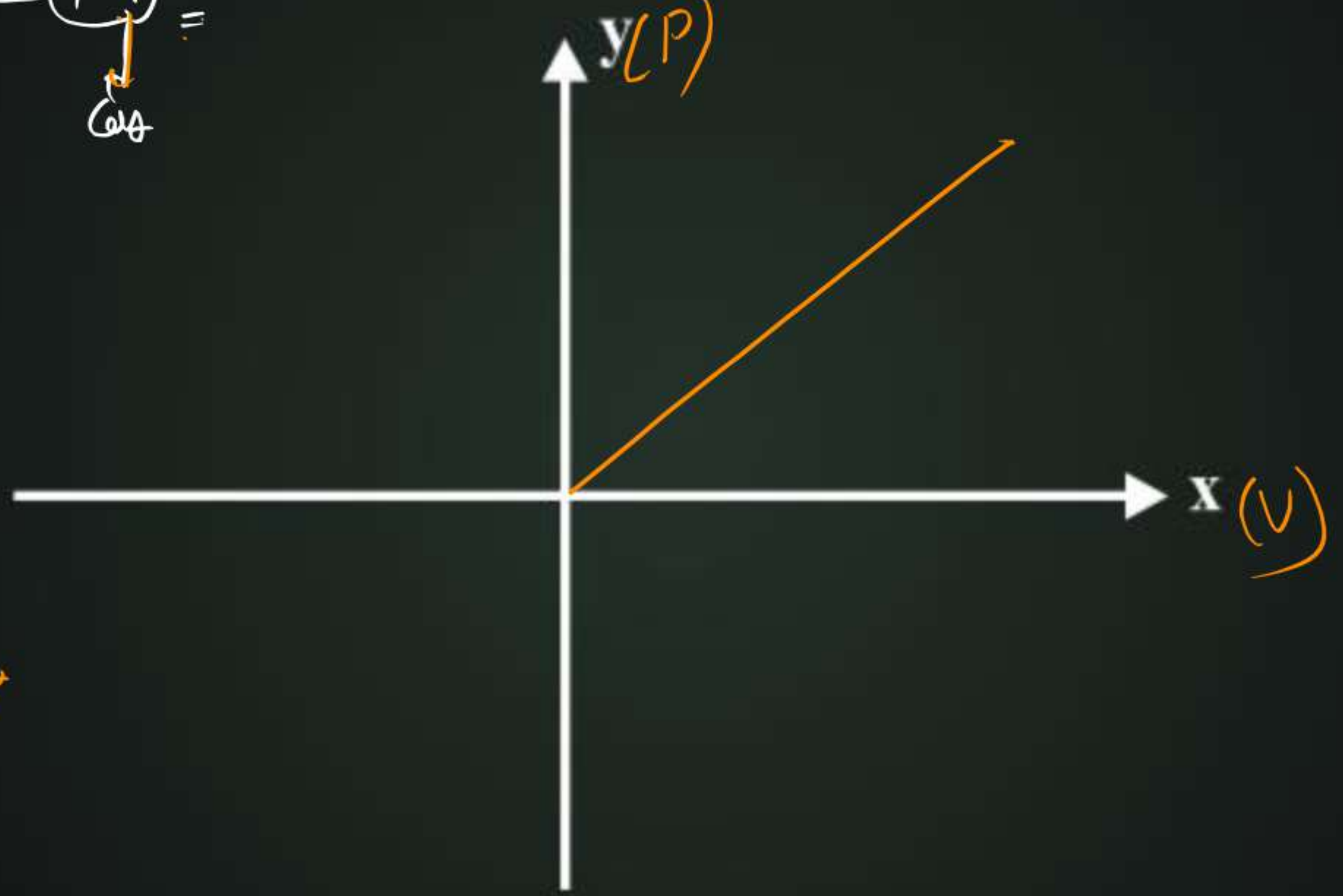


$$m_3 > m_2 > m_1$$

Graph of straight line [linear relation between two physical quantity]:

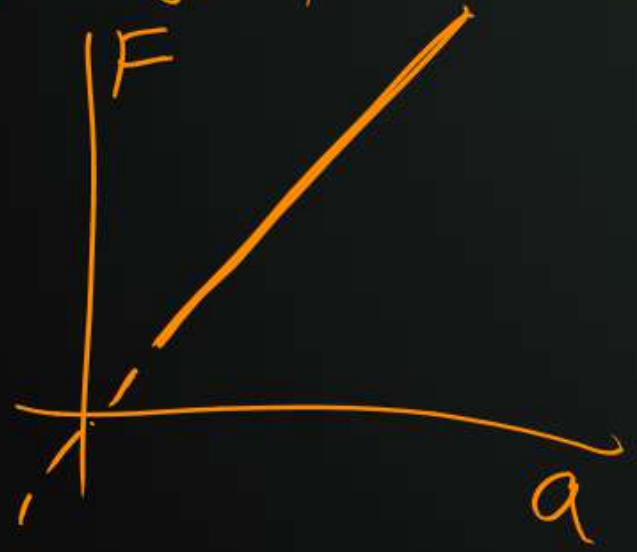
$P = m V$

Const



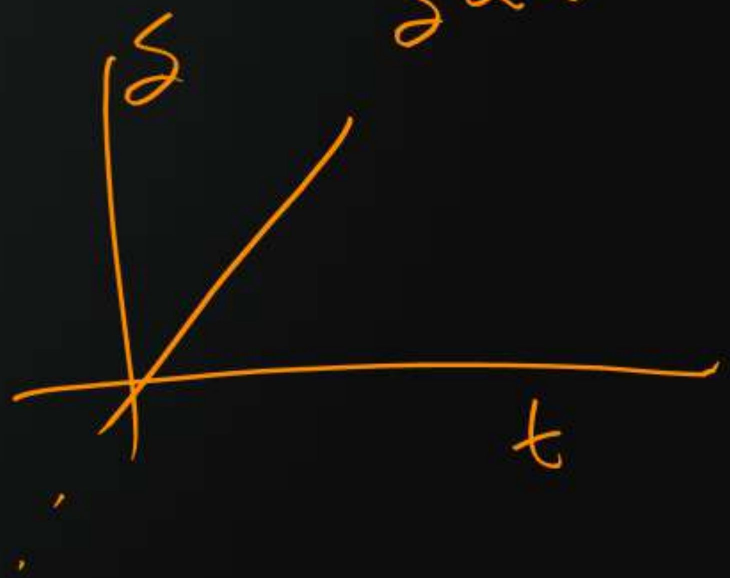
Const^n

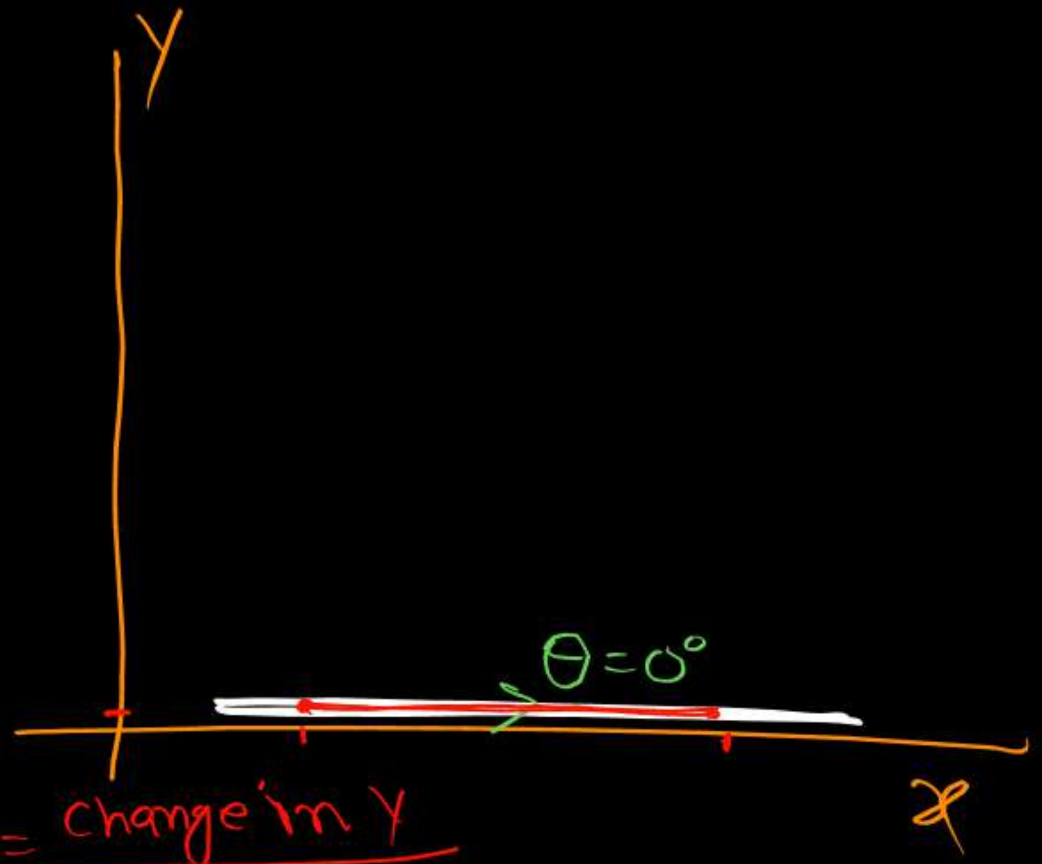
$F = m a$



$s = u t$

$s \propto t$





$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

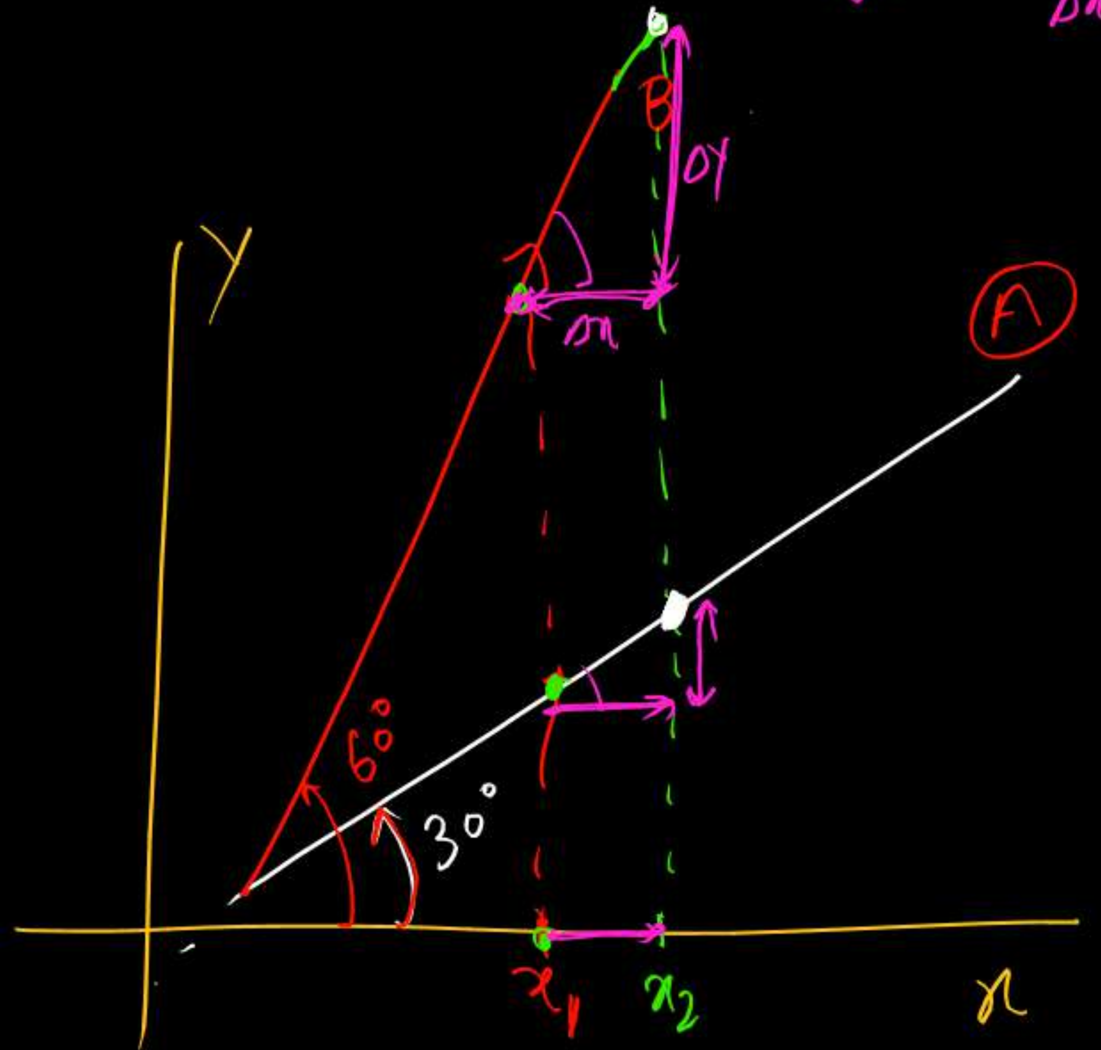
$\text{Slope}(m) = \tan\theta = \frac{\Delta y}{\Delta x} = \frac{0}{\Delta x} = 0$

$= \tan 0^\circ$

$$\boxed{m = 0}$$

A straight line Parallel to x-axis have zero slope.

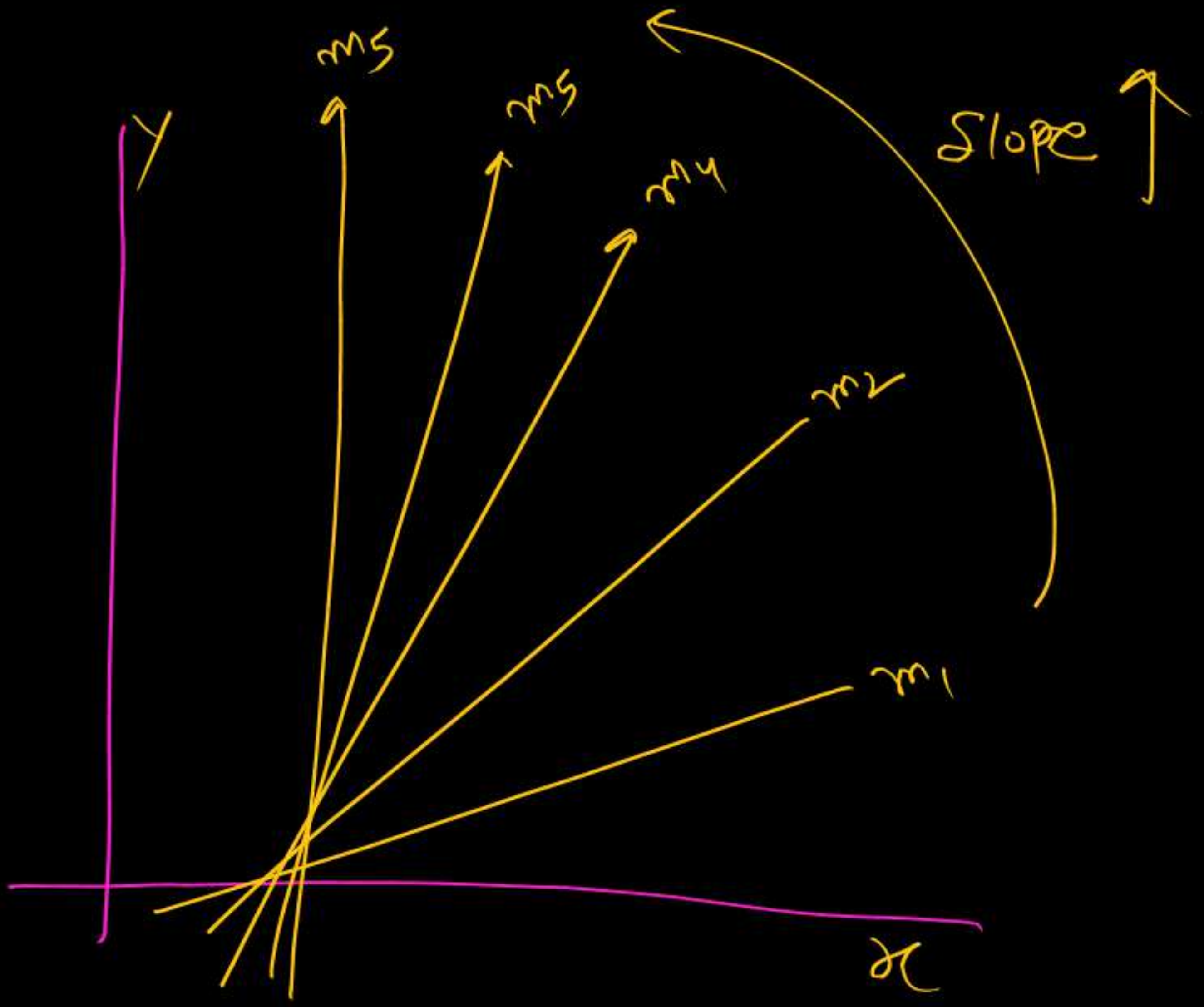
$\tan\theta = \frac{\Delta y}{\Delta x}$

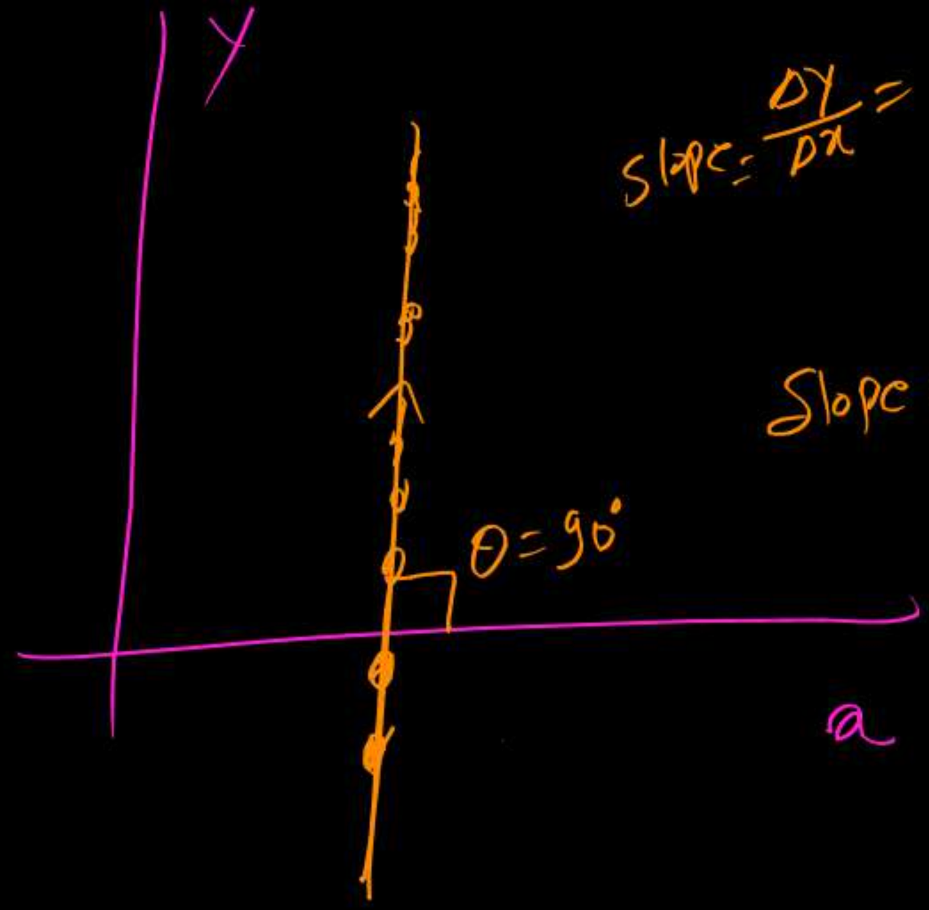


$m_B = \tan 60^\circ = \sqrt{3}$

$M_A = \tan 30^\circ = \frac{1}{\sqrt{3}}$

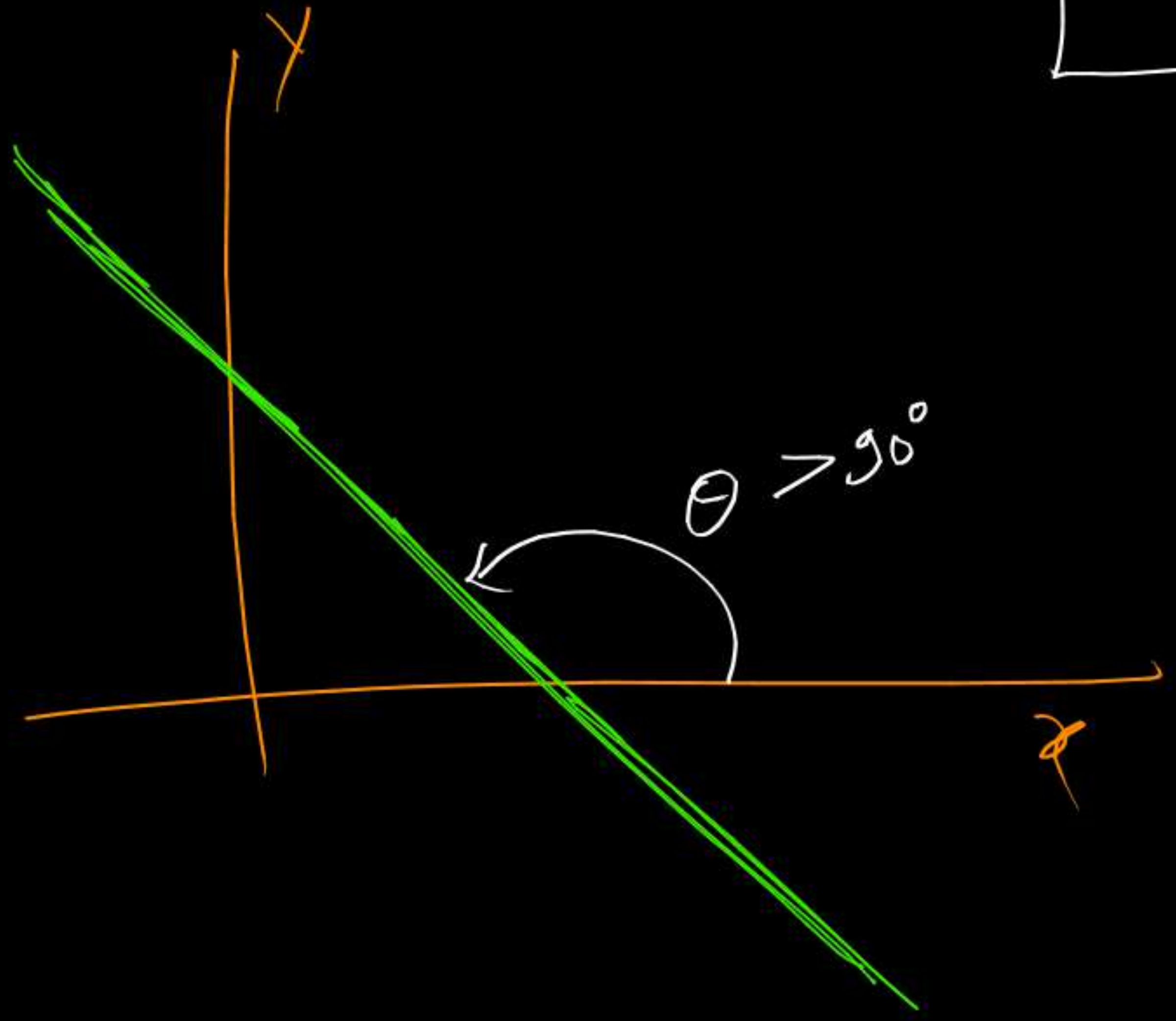
$m_B > m_A$





$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\Delta y}{0} = \text{infinite}$$

$$\text{Slope} = \tan 90^\circ$$
$$= \infty$$



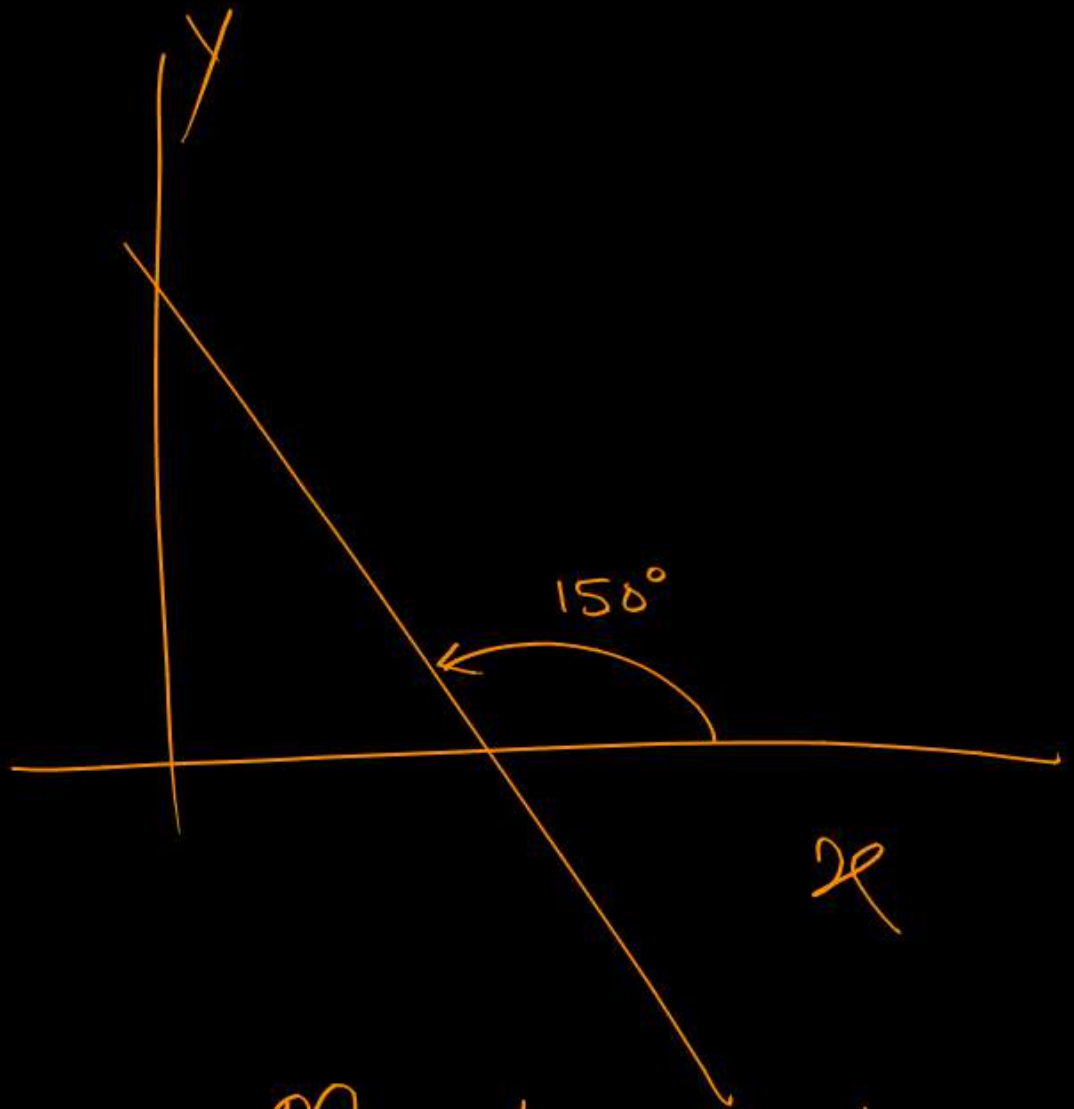
$$\boxed{m = \tan\theta = -ve}$$

$\underline{\theta > 90^\circ}$

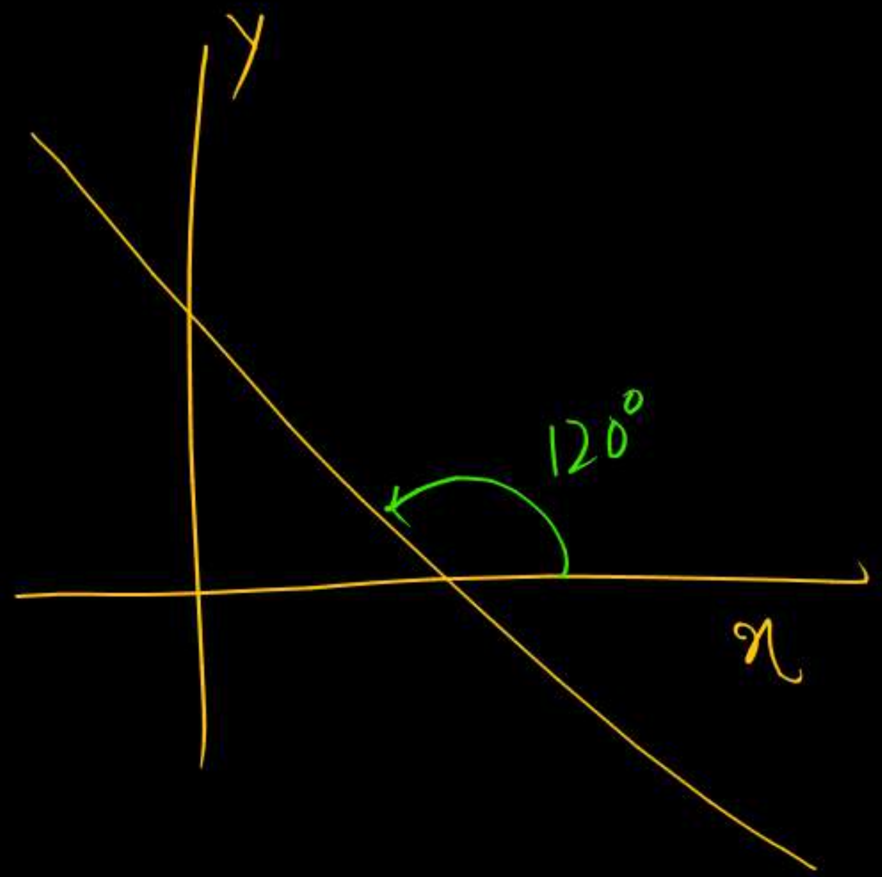
always taken from +ve x-axis in Anticlock wise direction

(Obtuse Angle)

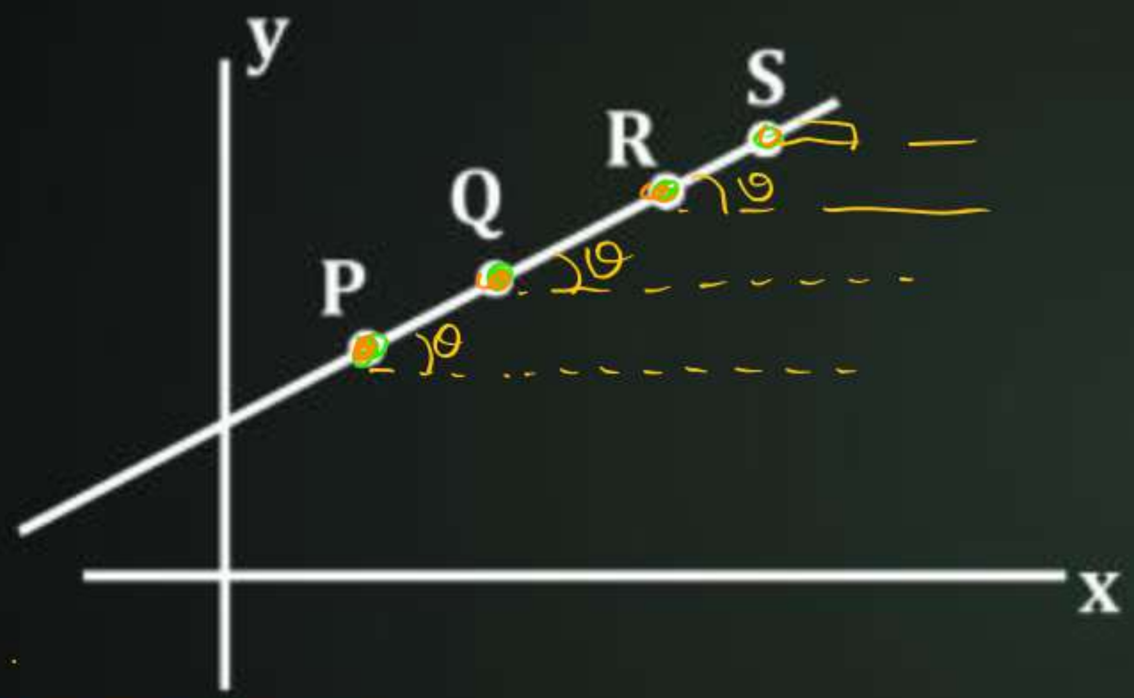
$\underline{\theta > 90}$



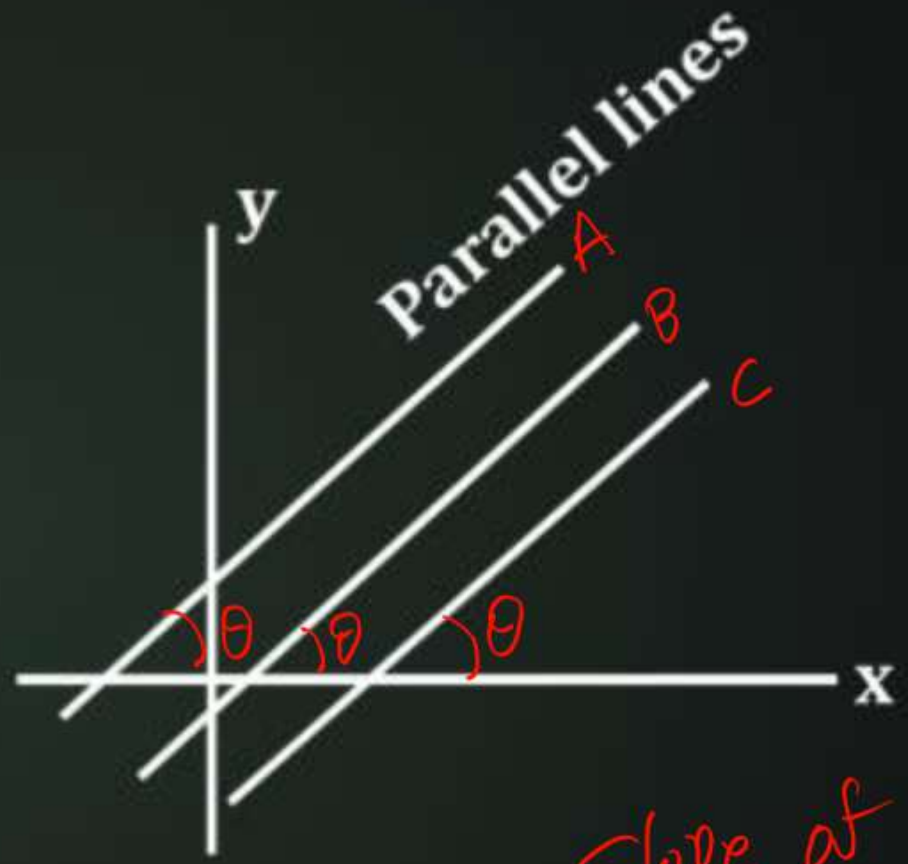
$$m = \tan\theta = \tan 150^\circ = -\frac{1}{\sqrt{3}}$$



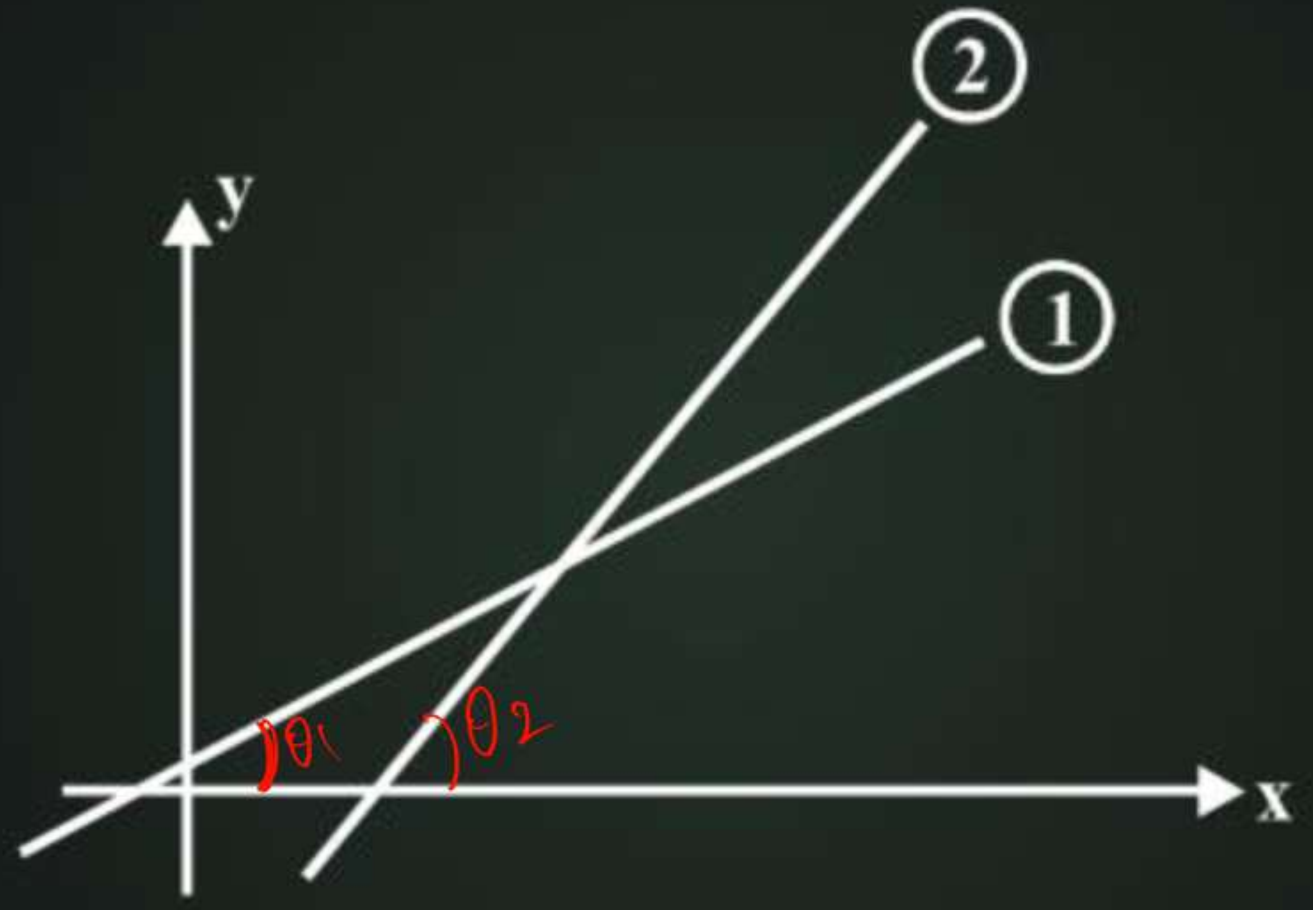
$$m = \tan(120^\circ)$$
$$= -\sqrt{3}$$



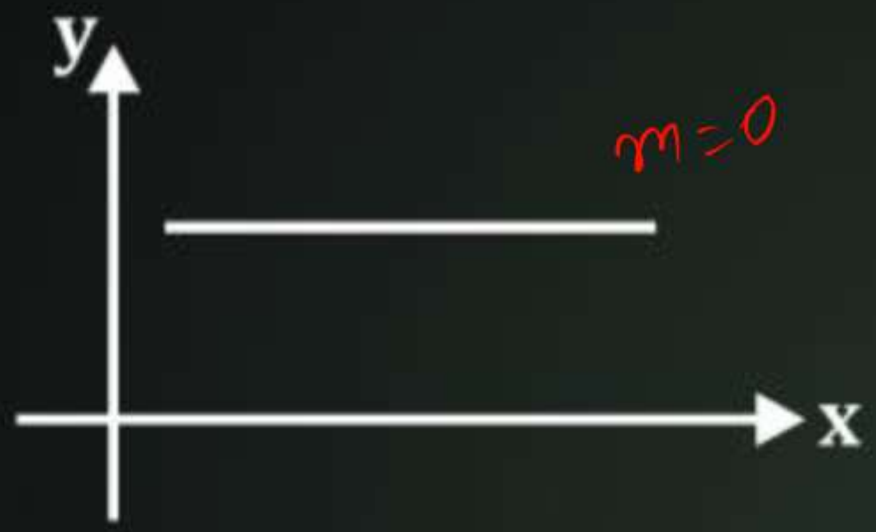
$$\boxed{m_P = m_Q = m_R = m_S}$$

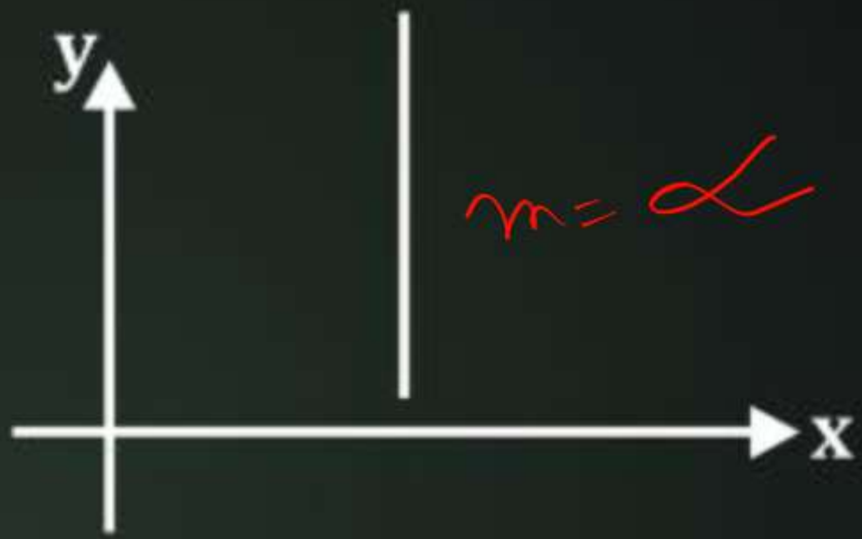


Slope of Parallel line will be same



y
x
1
2
θ1
θ2

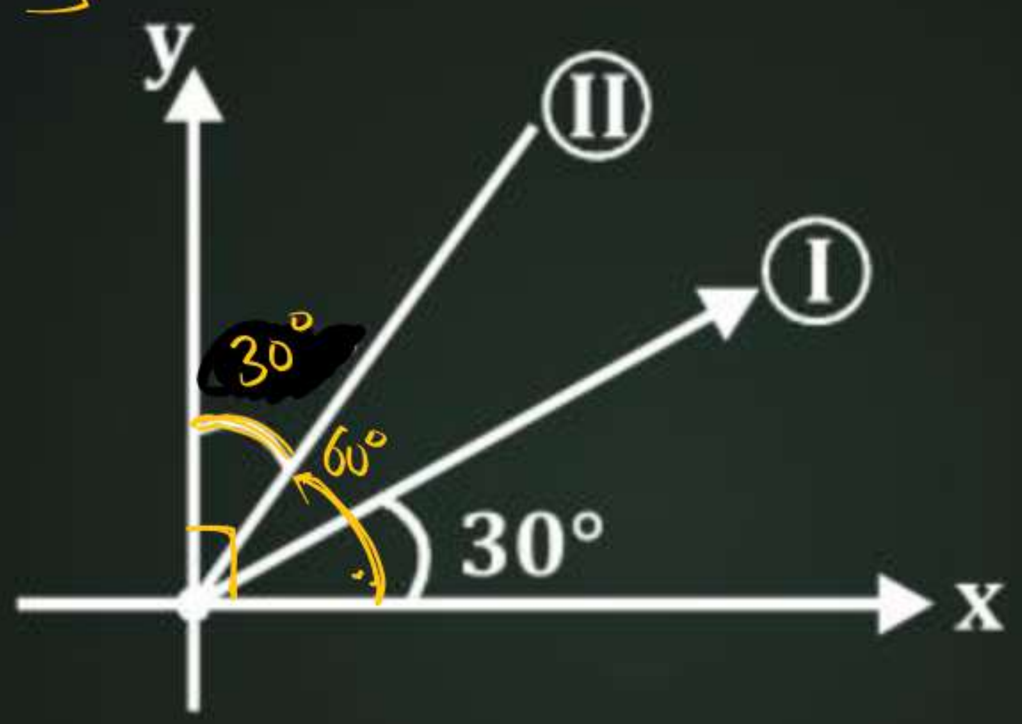

y
x
m=0


y
x
m= ∞


y
x
m= -ve
θ

QUESTION

Find (slope)$_{\mathrm{I}}$/(slope)$_{\mathrm{II}}$ = $\frac{1}{3}$

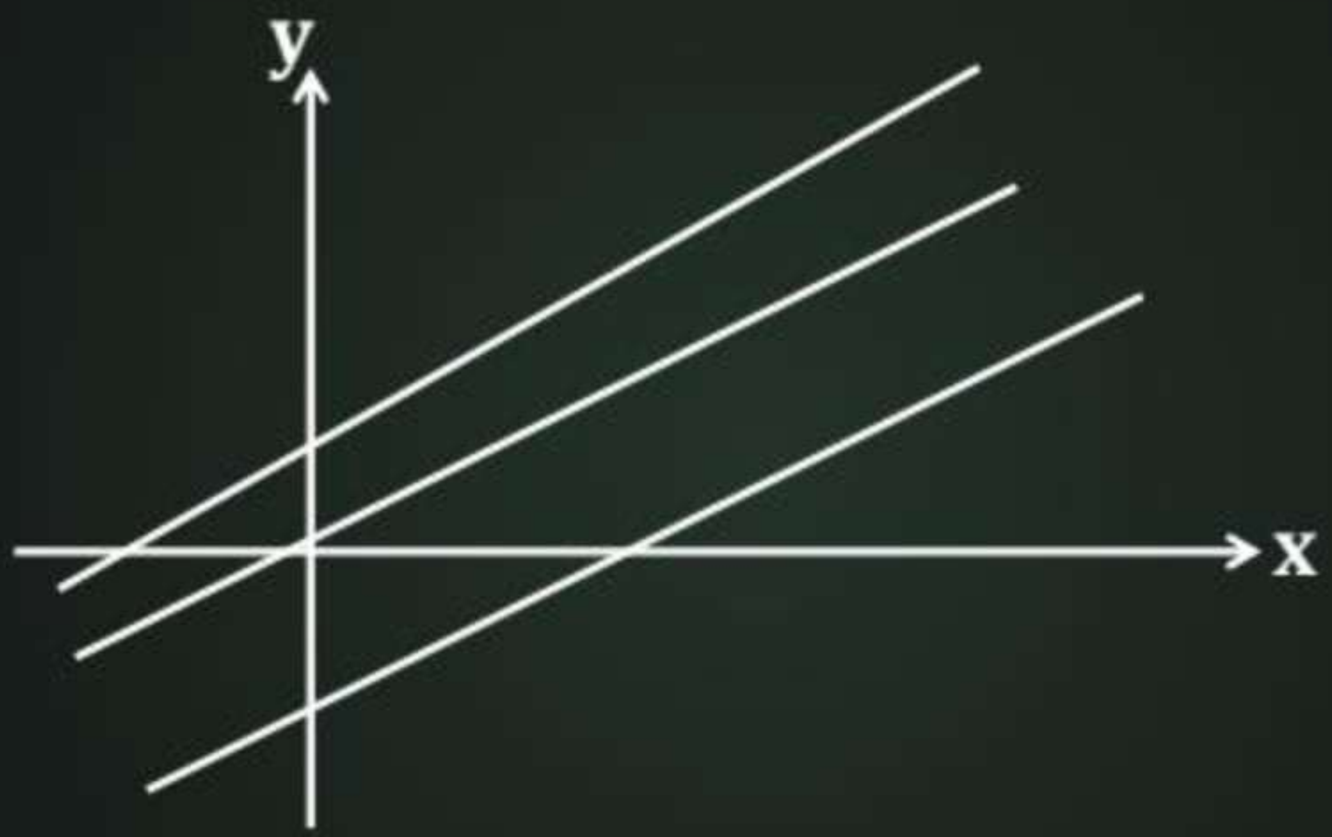


$$\frac{(\text{slope})_{II}}{(\text{slope})_{I}} = \frac{\tan 60^\circ}{\tan 30^\circ} = \frac{\sqrt{3}}{1/\sqrt{3}} = \frac{3}{1}$$

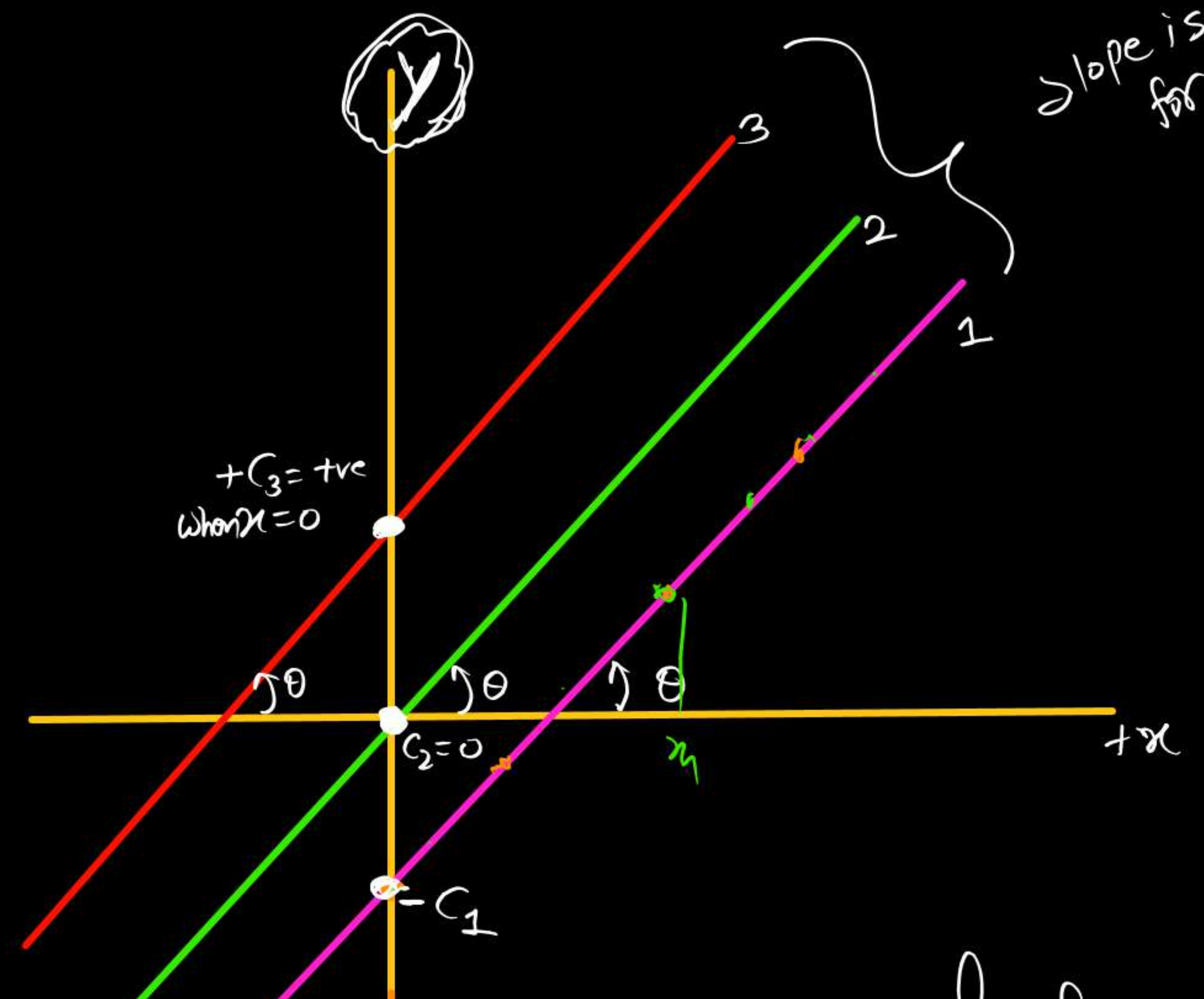
QUESTION

$c = y$ intercept

$= \text{value of } y \text{ at } x = 0$



Slope is same for all.



$C = (\text{Y-Intercept}) = \underbrace{\text{Value of Y when } x \text{ is zero.}}$

Kisi bhi straight line ka slope & y-intercept fixed hota hai.

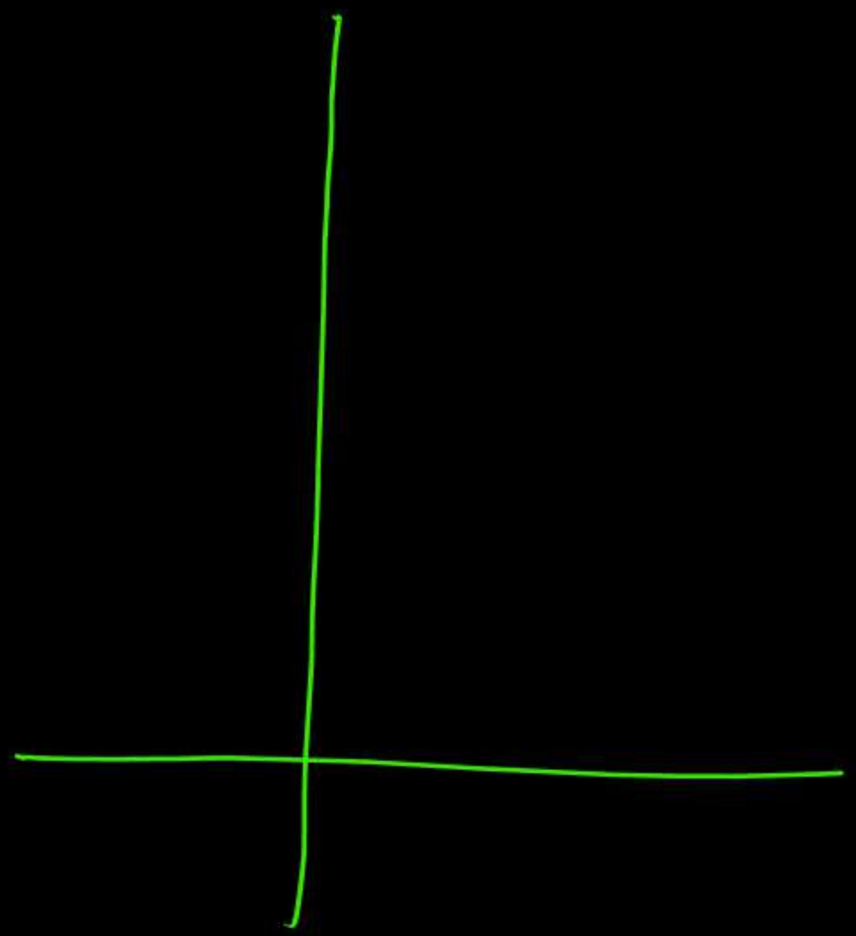
$y = mx + c$

slope ↑ (m), y-Intercept ↑ (c)

$$\boxed{y = mx + c}$$

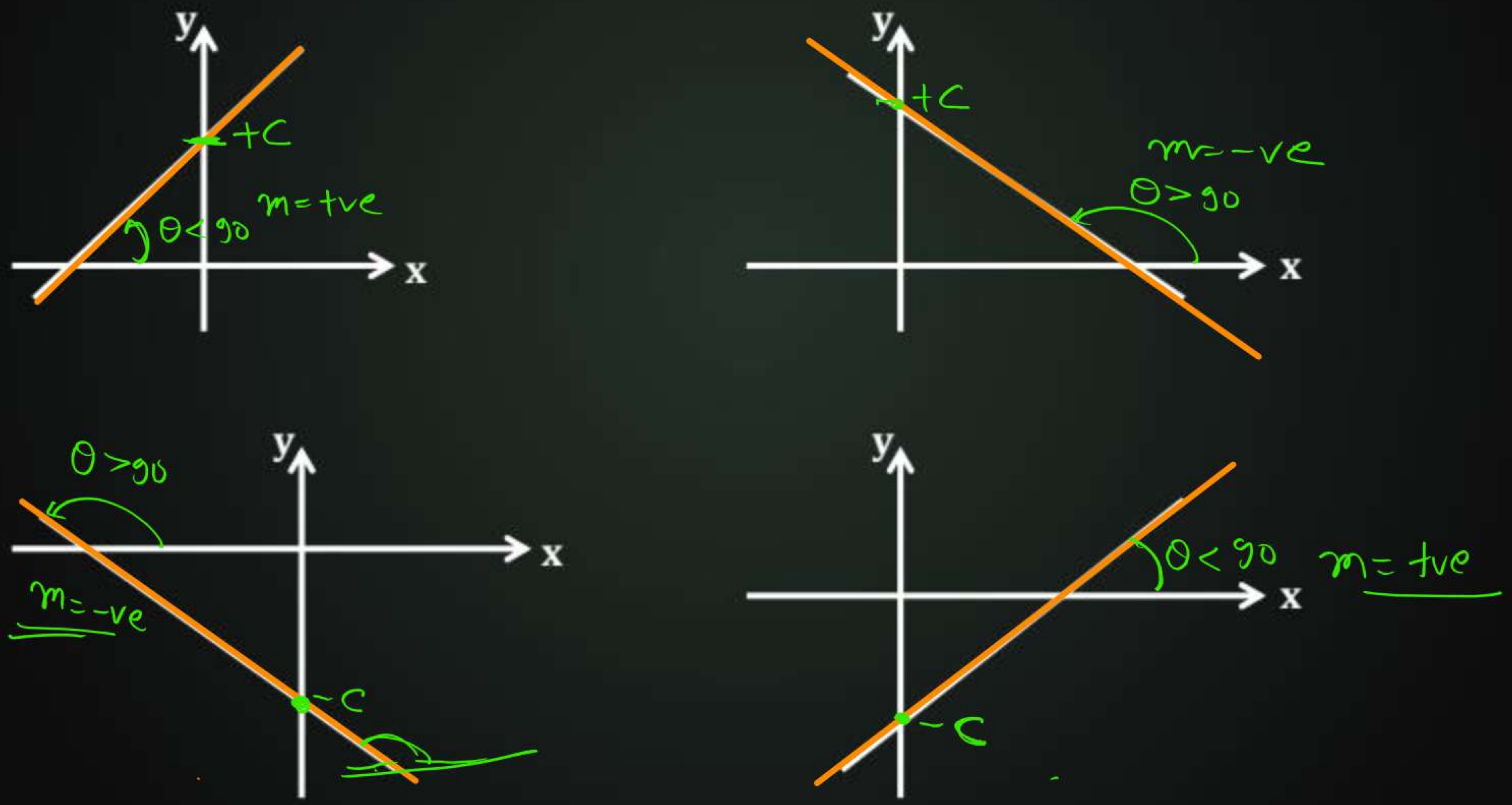
Slope = $\tan\theta$ fixed

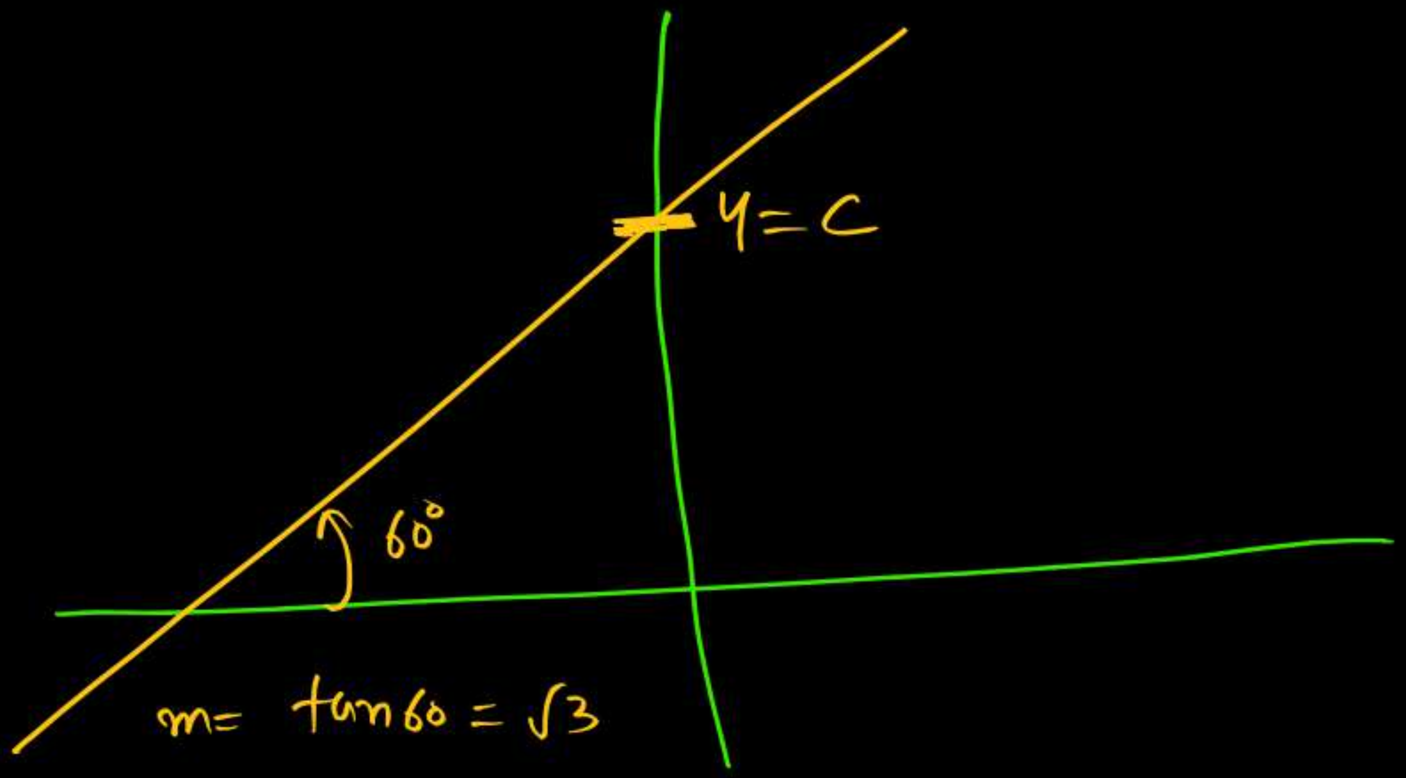
y-intercept
Value of y at x = 0



QUESTION

Comment on slope and intercept.

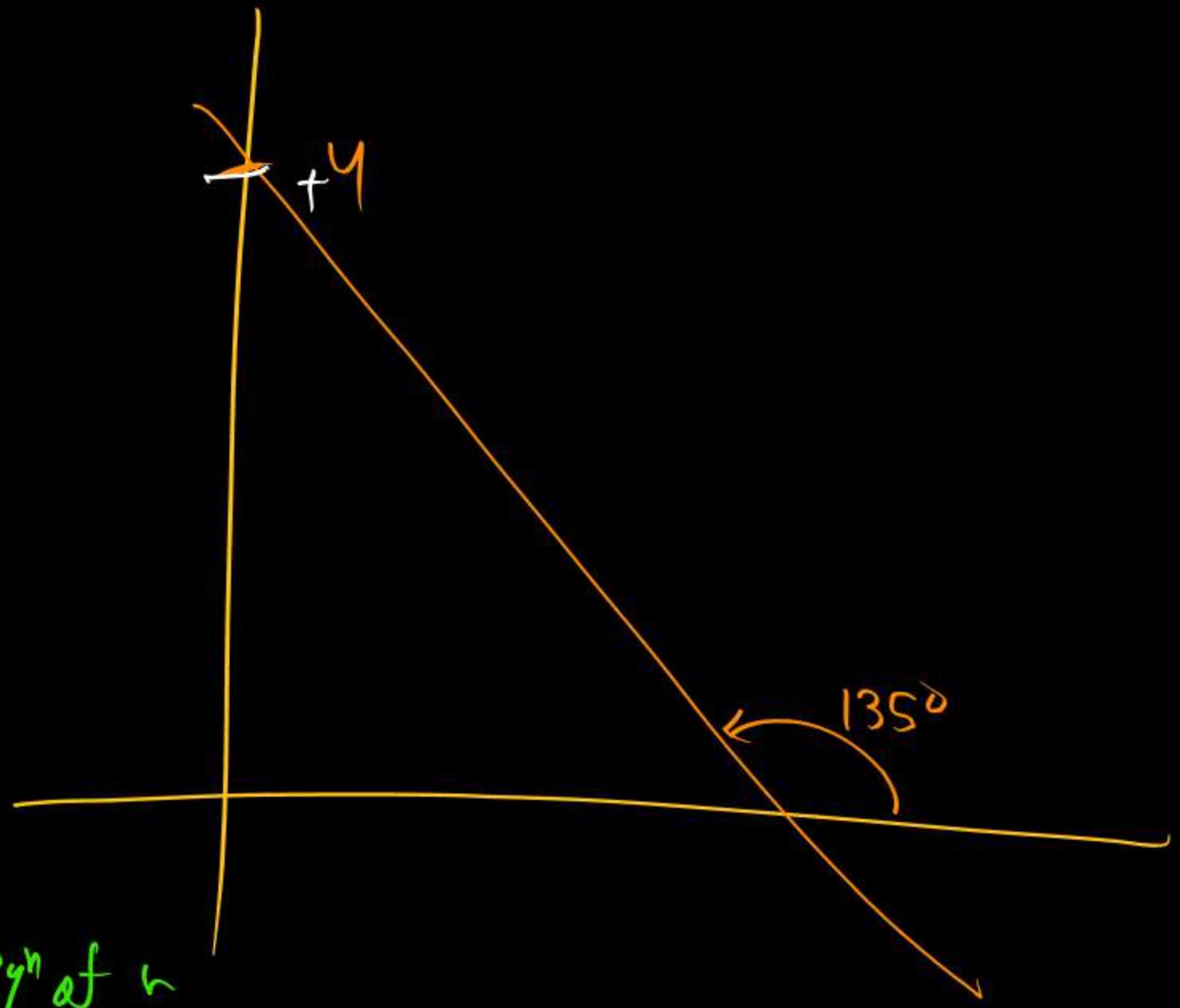




write eqn of straigh-lin

$$y = mx + c$$

✓ $$\boxed{y = \sqrt{3}x + 4}$$

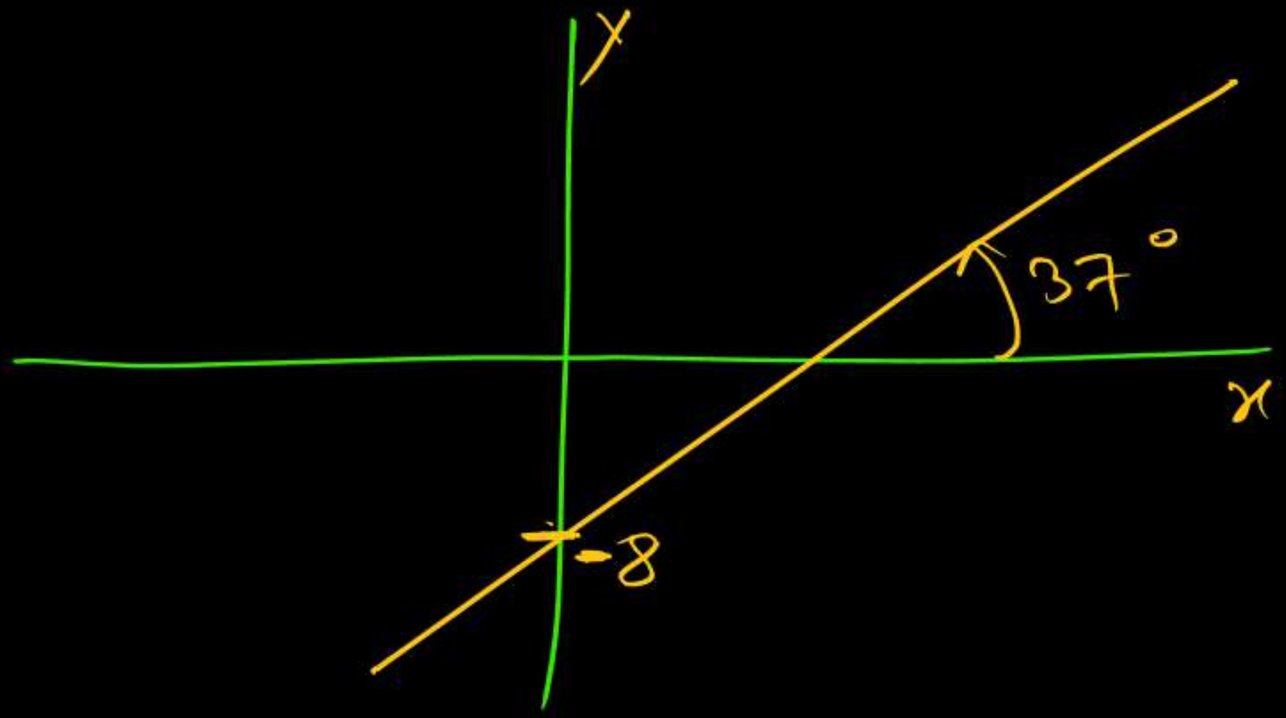


eqⁿ of n

$$y = mx + c$$

$$y = \tan 135^\circ x + 4$$

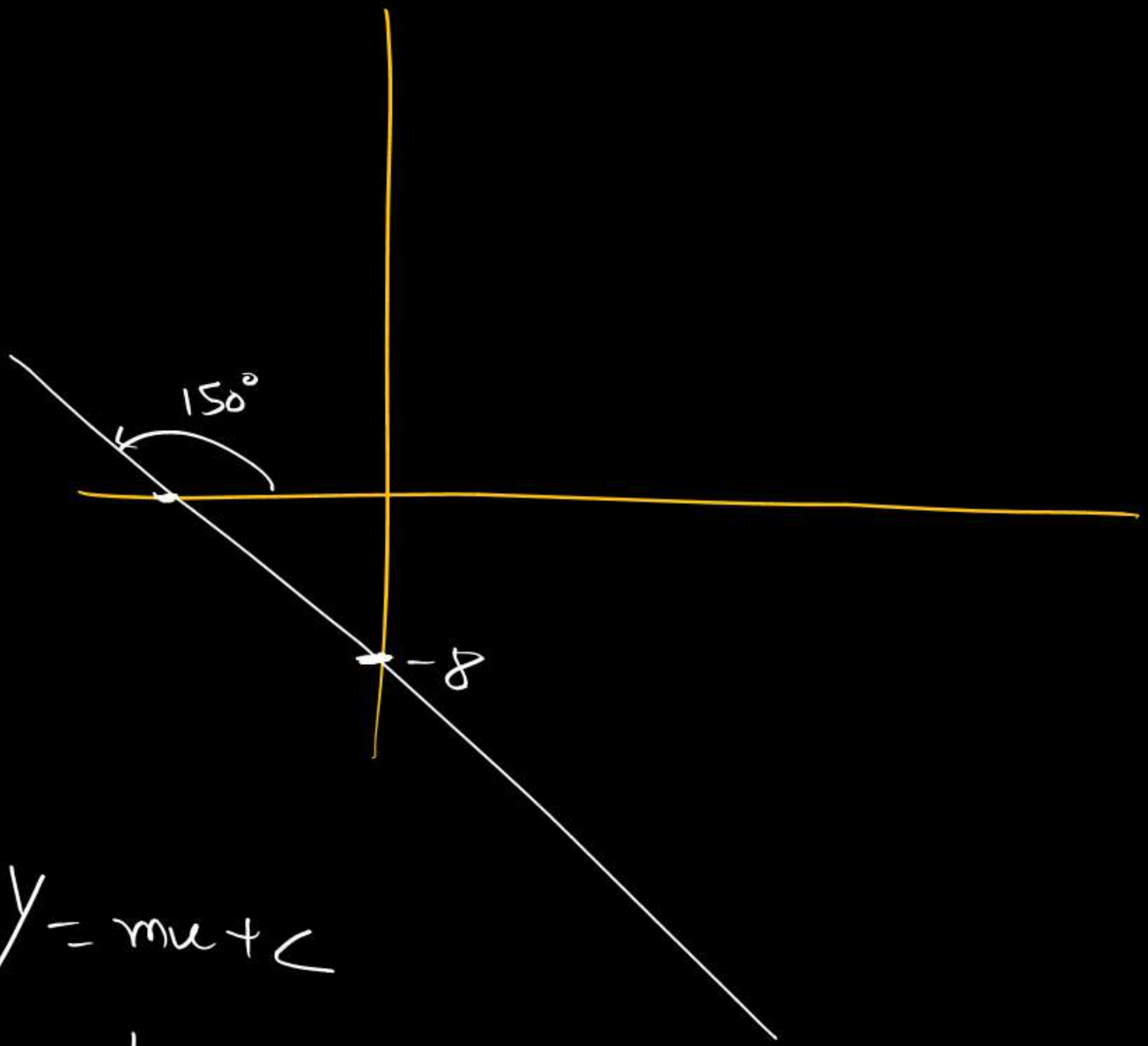
$$\boxed{y = -x + 4}$$



$$y = mx + c$$

$$y = \tan 37^\circ x - 8$$

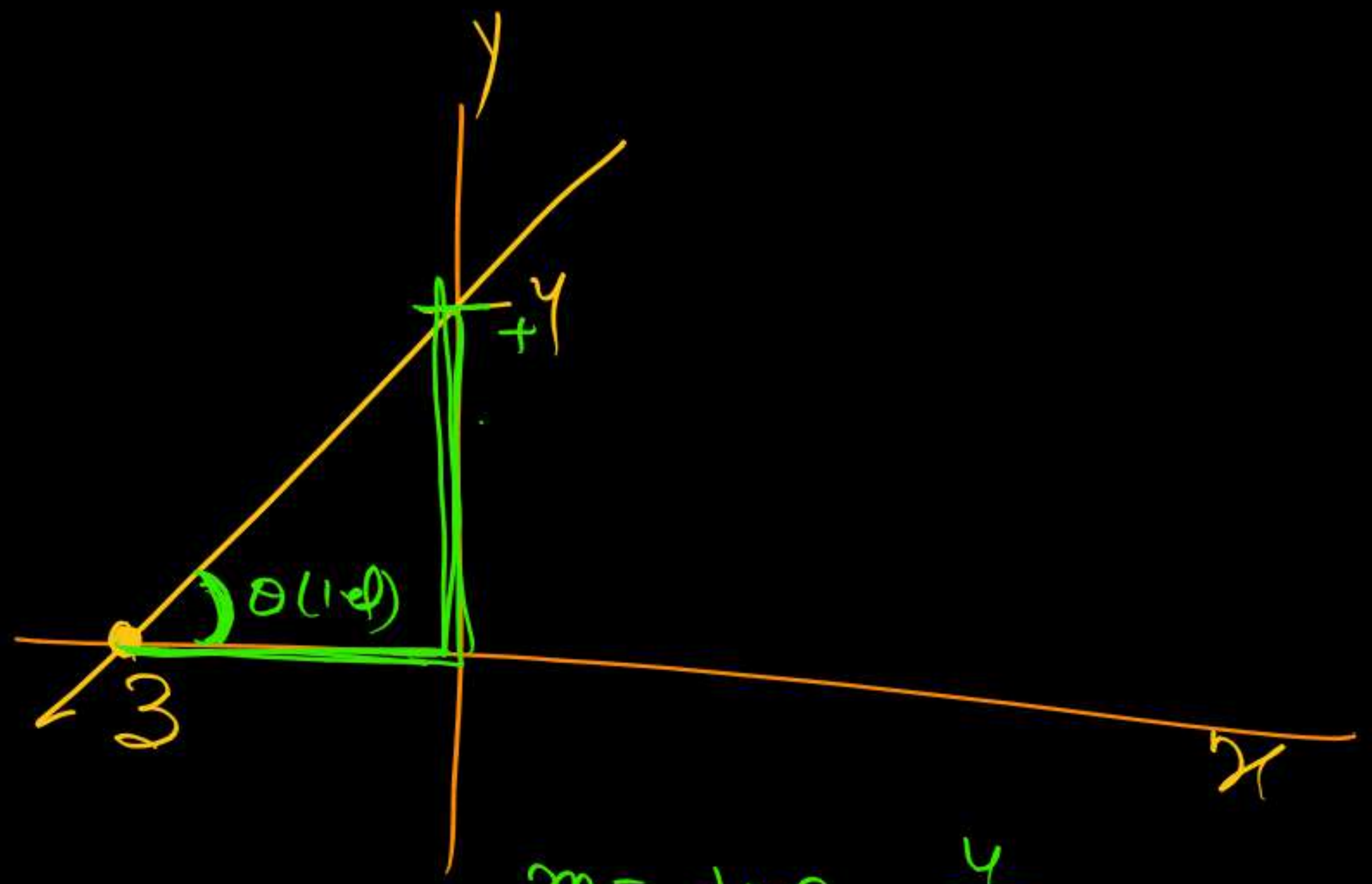
$$y = \frac{3x}{4} - 8$$



$y = mx + c$

$= \tan 150° x - 8$

$$\boxed{y = -\frac{1}{\sqrt{3}}x - 8}$$



$$\underline{m} = \underline{\tan\theta} = \frac{4}{3}$$

$\theta = 53^\circ$

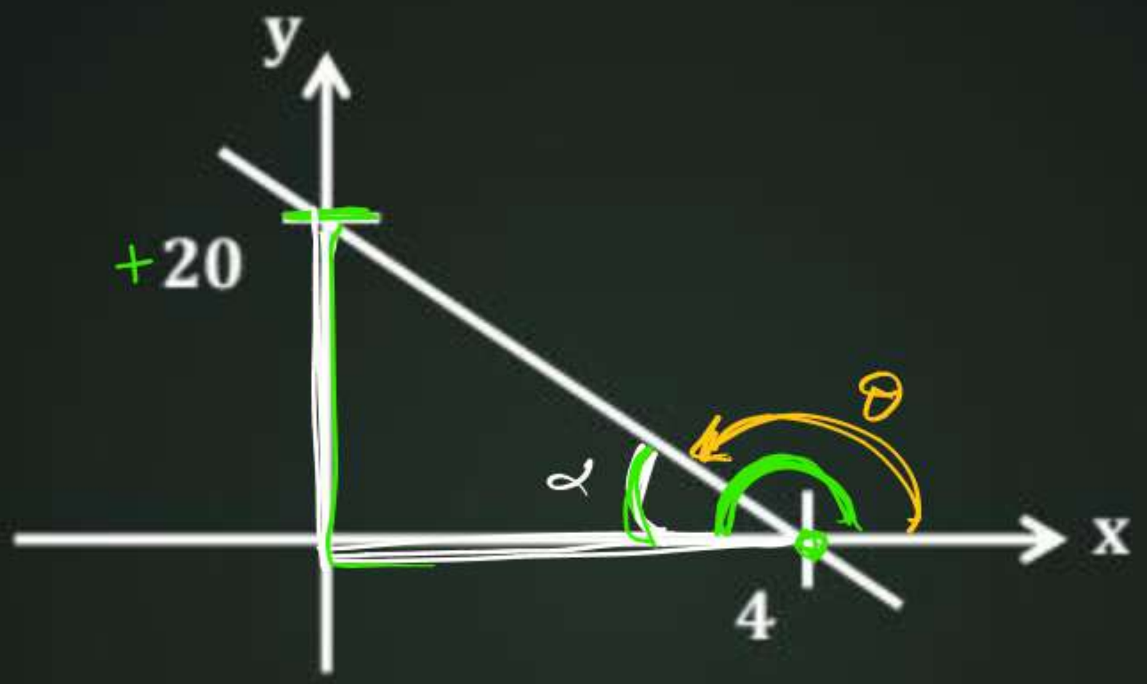
$$y = mx + c$$

$$y = mx + 4$$

$$y = \frac{4}{3}x + 4$$

QUESTION

Find value of y at $x = 8$



$c = 20$

$m = \tan\theta = -5$ $\qquad \tan\alpha = \frac{20}{4} = 5$

MR Ratta

$y = mx + c$

$\boxed{y = -5x + 20}$

$y_{at\, x=8} = -5 \times 8 + 20$

$= -40 + 20$

$= -20$

Slope $= \tan\theta$

$= \tan(180 - \alpha)$

$= -\tan\alpha$

$\boxed{\text{Slope} = -5}$

QUESTION

Draw graph between y and x for given equation :

(i) $y = \sqrt{3}x + 4$

(ii) $y = -x$

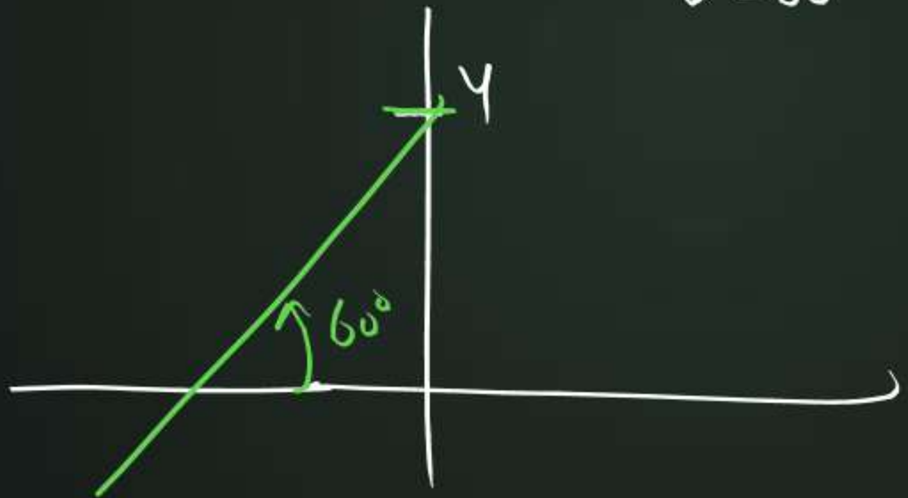
(iii) $y = |x|$

(iv) $\frac{x}{3} + \frac{y}{4} + 1 = 0$

$\underline{\text{Sol}^n}$ (i) $y = \sqrt{3}x + 4$

$c = 4$

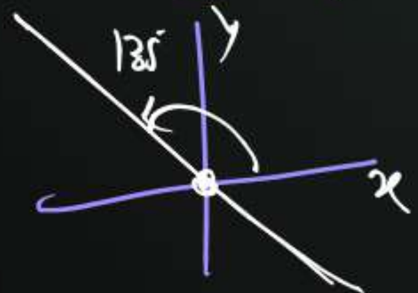
$m = \underbrace{\sqrt{3} = \tan\theta}_{\theta = 60^\circ}$



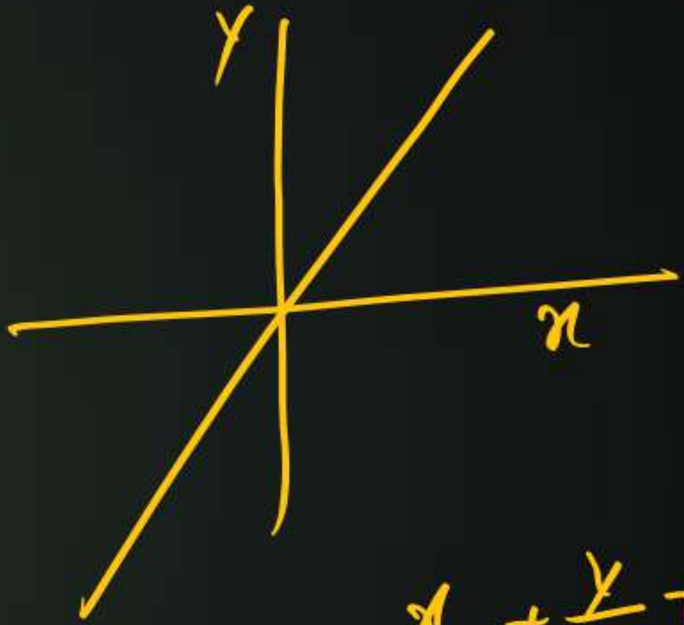
(ii) $y = -x$ $(y = mx + c)$

$* \; c = 0$

$m = -1 = \tan\theta \; (\theta = 135^\circ)$



$y = x$



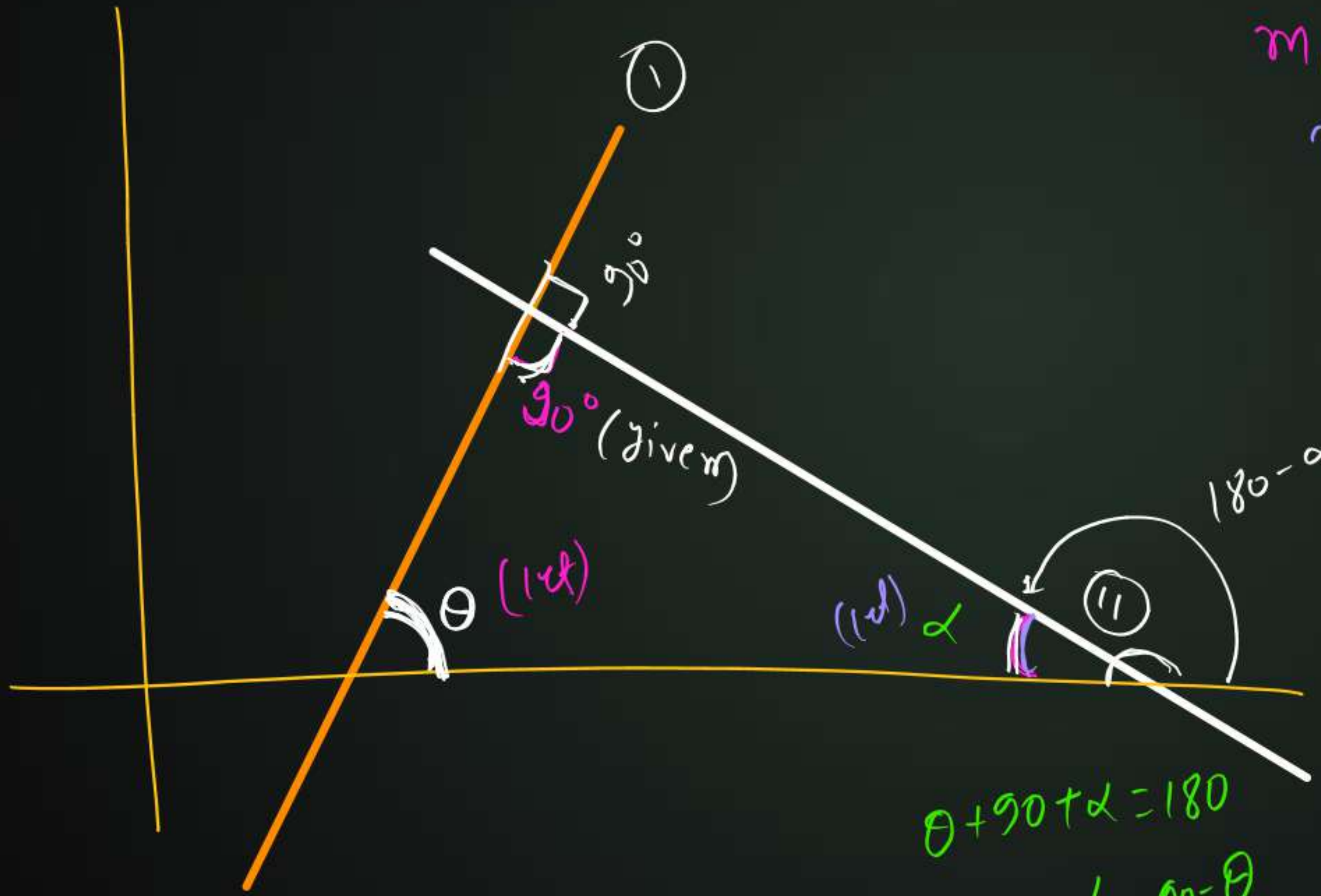
(iv) $\frac{x}{3} + \frac{y}{4} + 1 = 0$

$\frac{y}{4} = -\frac{x}{3} - 1$

$y = -\frac{4}{3}x - 4$

QUESTION

If two straight line perpendicular to each other then prove that product of their slope is –1.



$m_1 = \tan\theta$ — (1)

$m_2 = \tan(90+\theta) = -\cot\theta$ — (11)

$180-\alpha = 180-(90-\theta)$

$= 90+\theta$

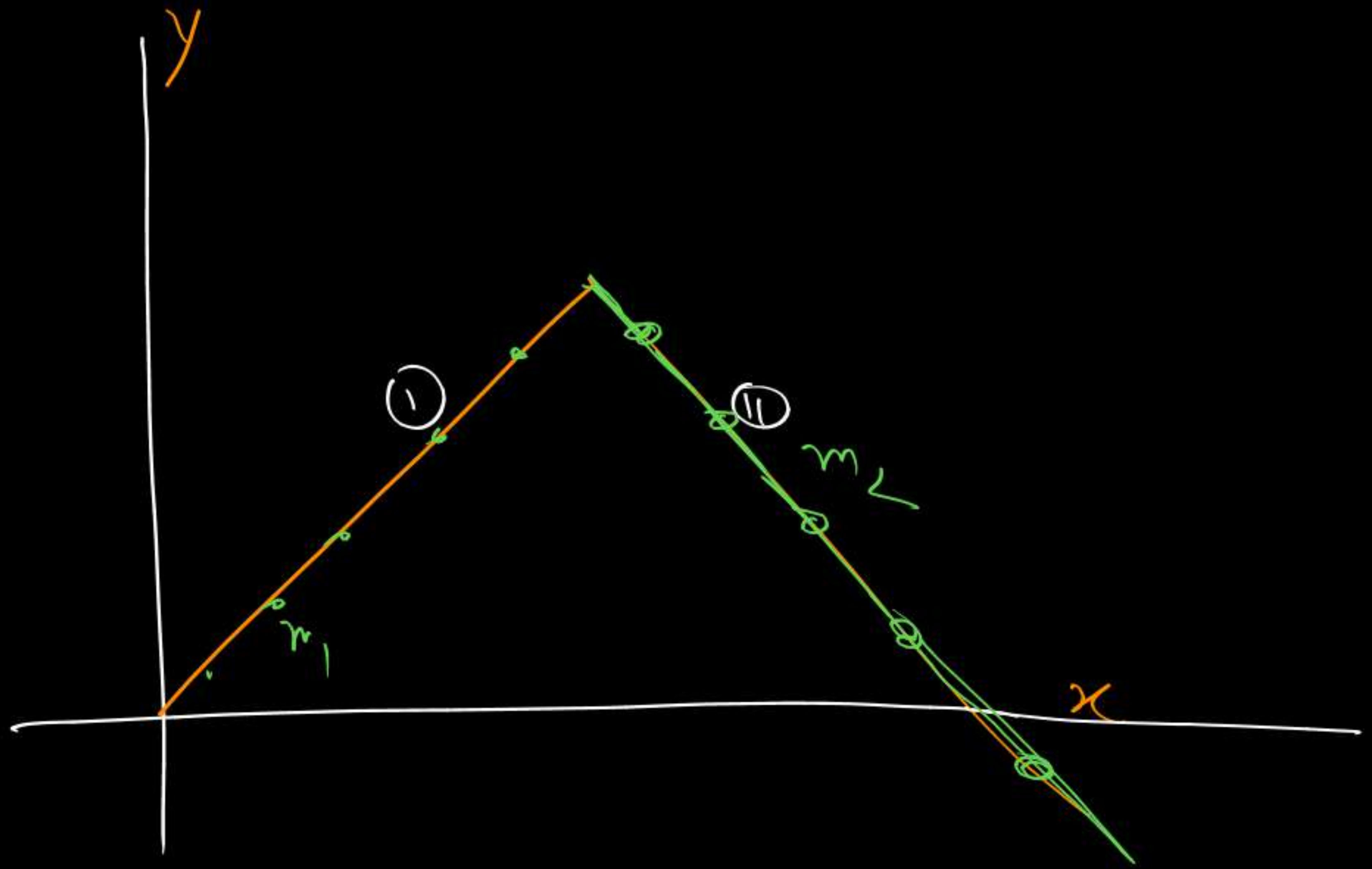
$\theta + 90 + \alpha = 180$

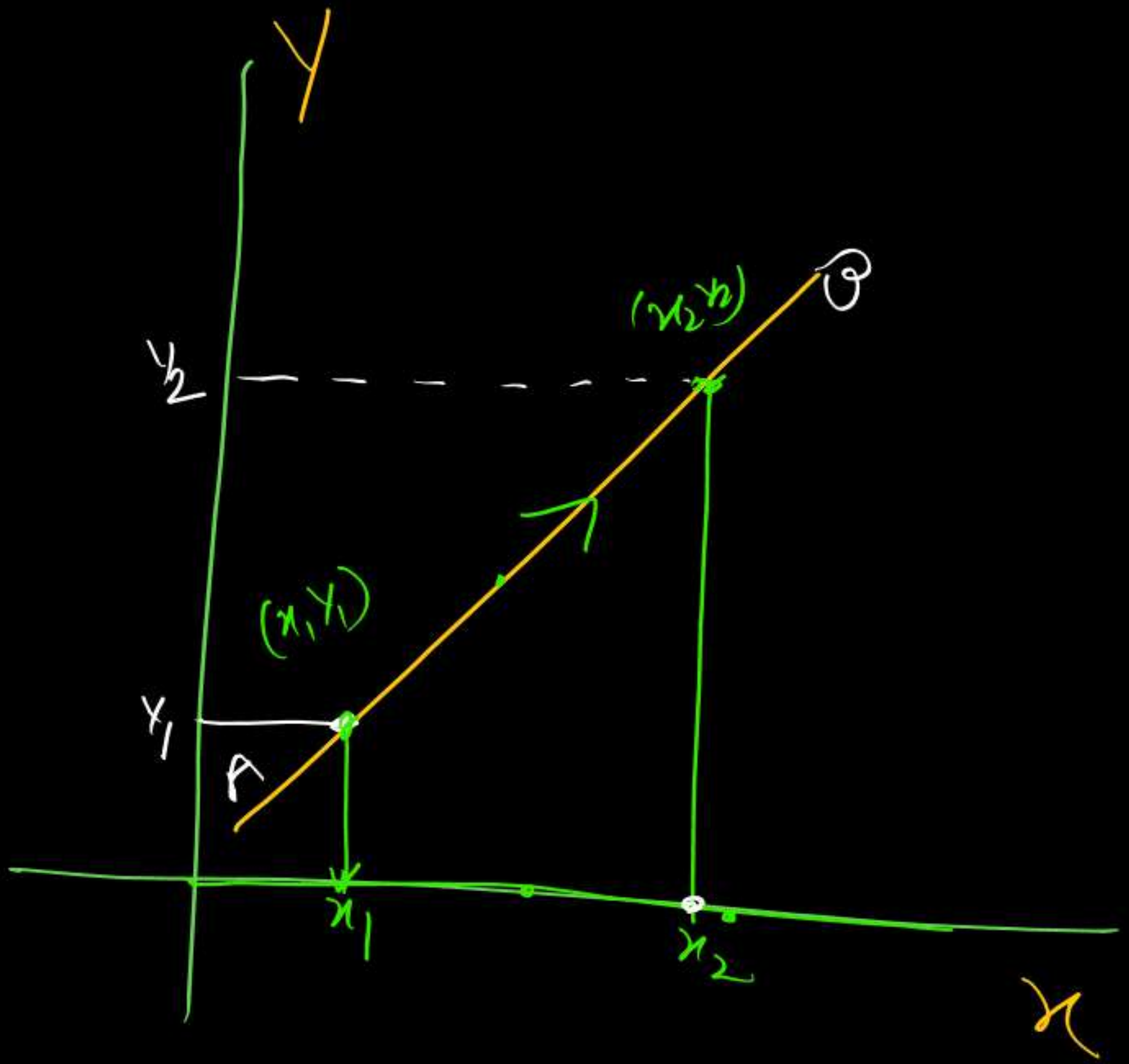
$\alpha = 90 - \theta$

$m_1 m_2 = -\cancel{\tan\theta}\cdot\cancel{\cot\theta}$

$$\boxed{m_1 m_2 = -1}$$

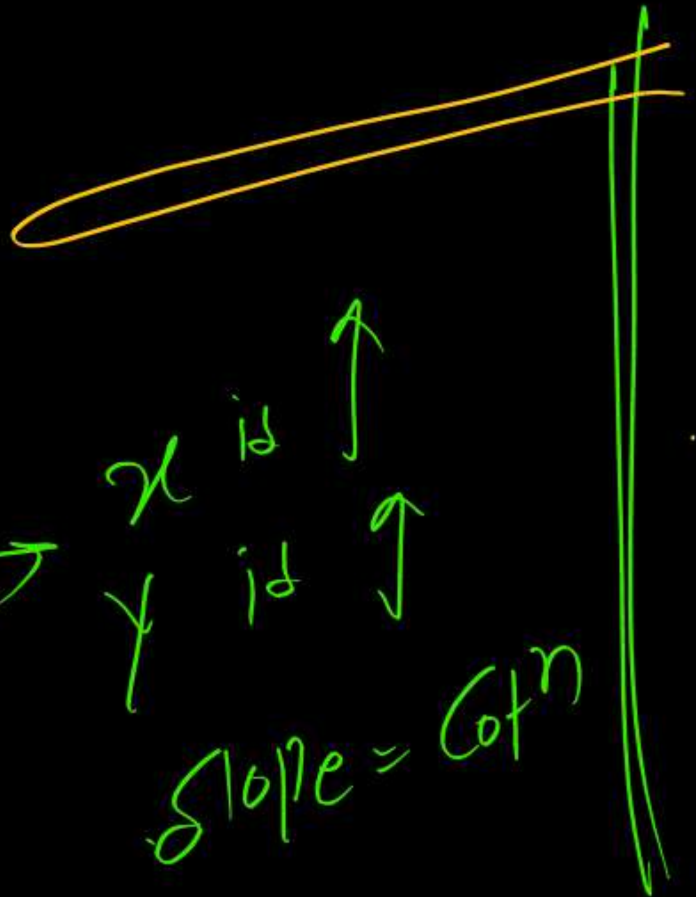
Proved


y
x
1
11
m1
m2

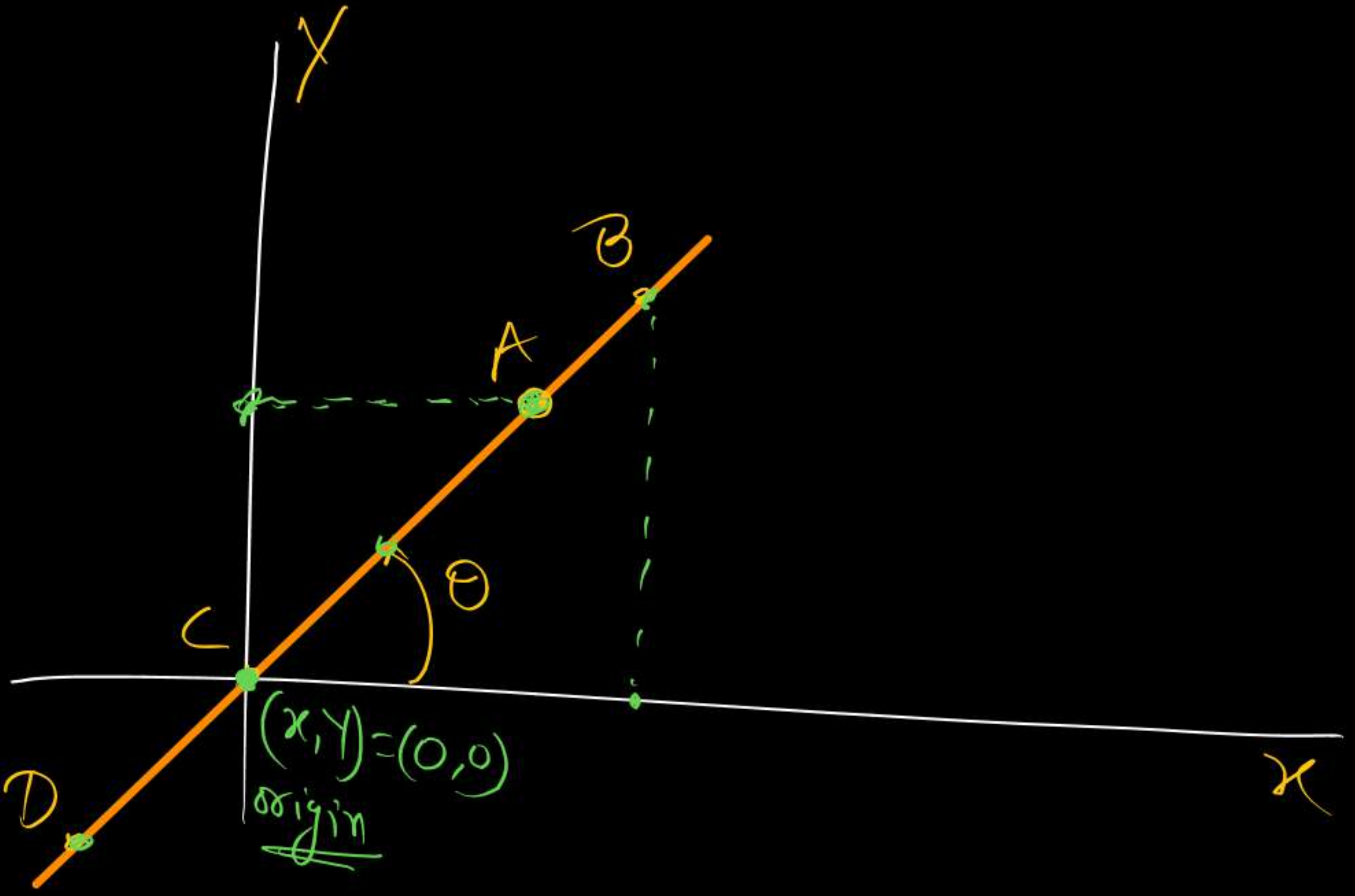


Slope = $\tan\theta$

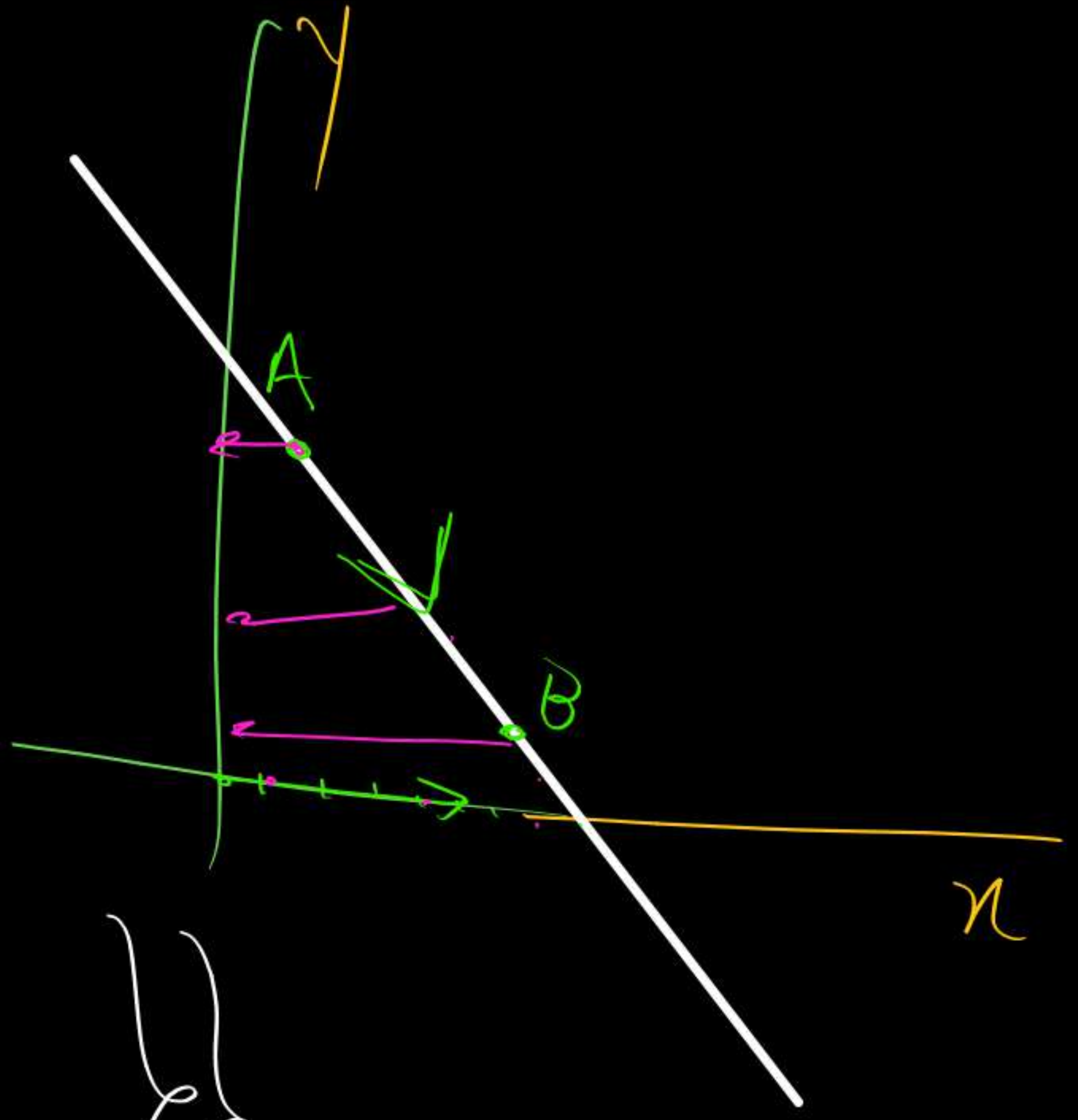
When we move from A to B → x is ↑, y is ↑



Slope = Cot n



$$m_A = m_B = m_C = m_D = \text{+ve} \quad \text{const}^n$$



$A \longrightarrow B$

$x \longrightarrow y$

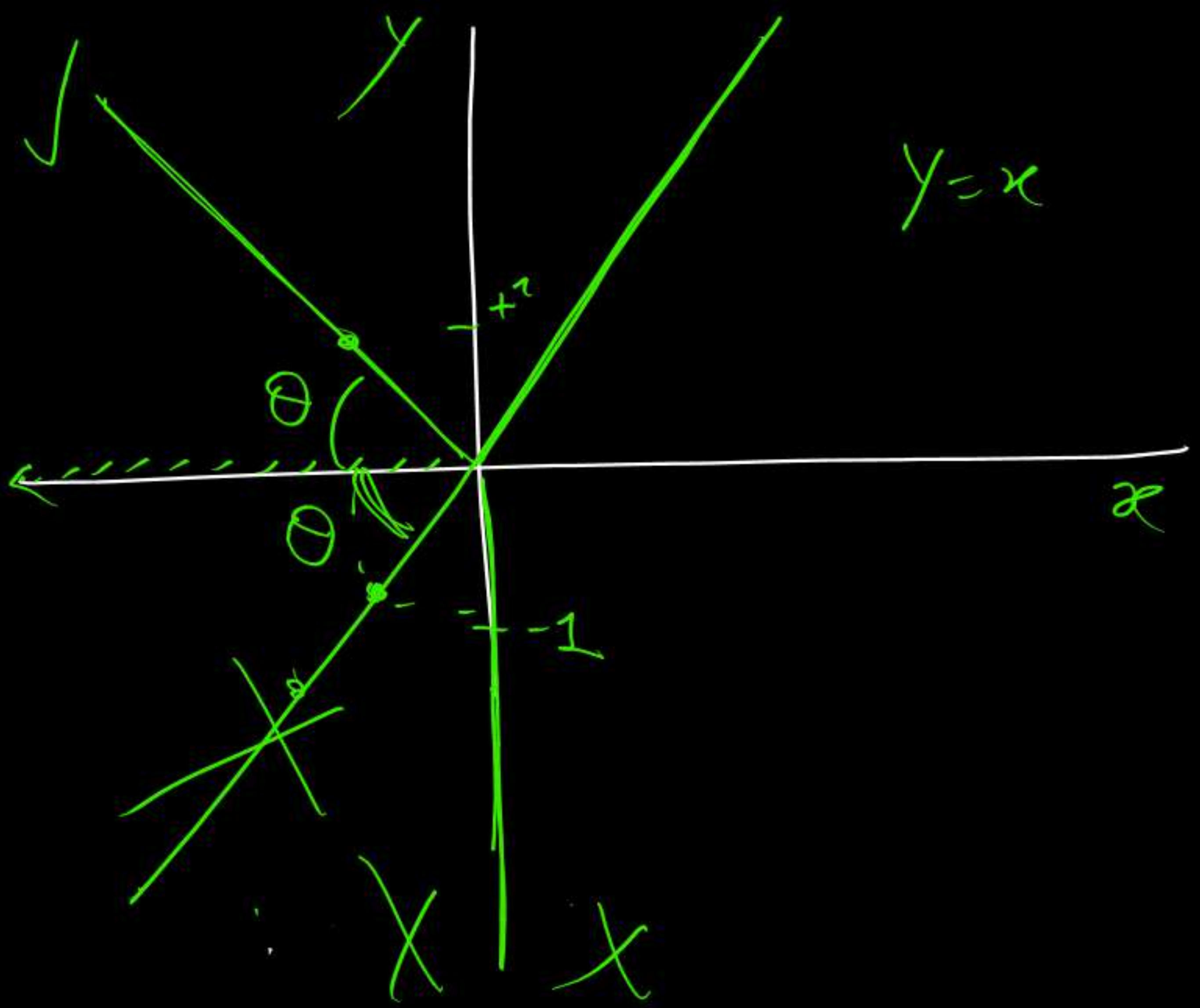
$X \longrightarrow \downarrow$

Slope = Cot η

$y = |x|$ ← mod

MR^{x} ⟶ (I) mode हटा दो, simple graph बनाओ

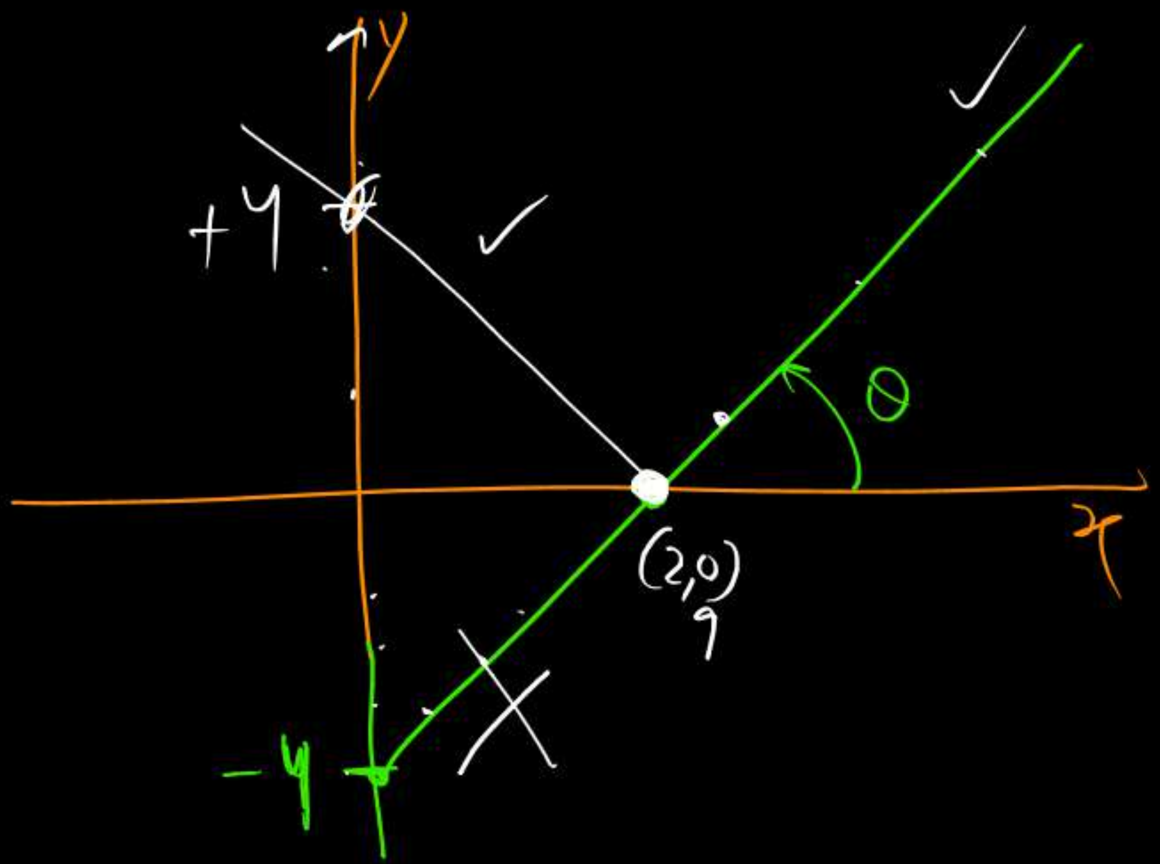
(II) जहाँ y Negative वहा पर Y का +ve mirror Image ले लो।



$$y = |2x - 4|$$

Sol^n step-1 mod हटा दो

$$y = 2x - 4$$

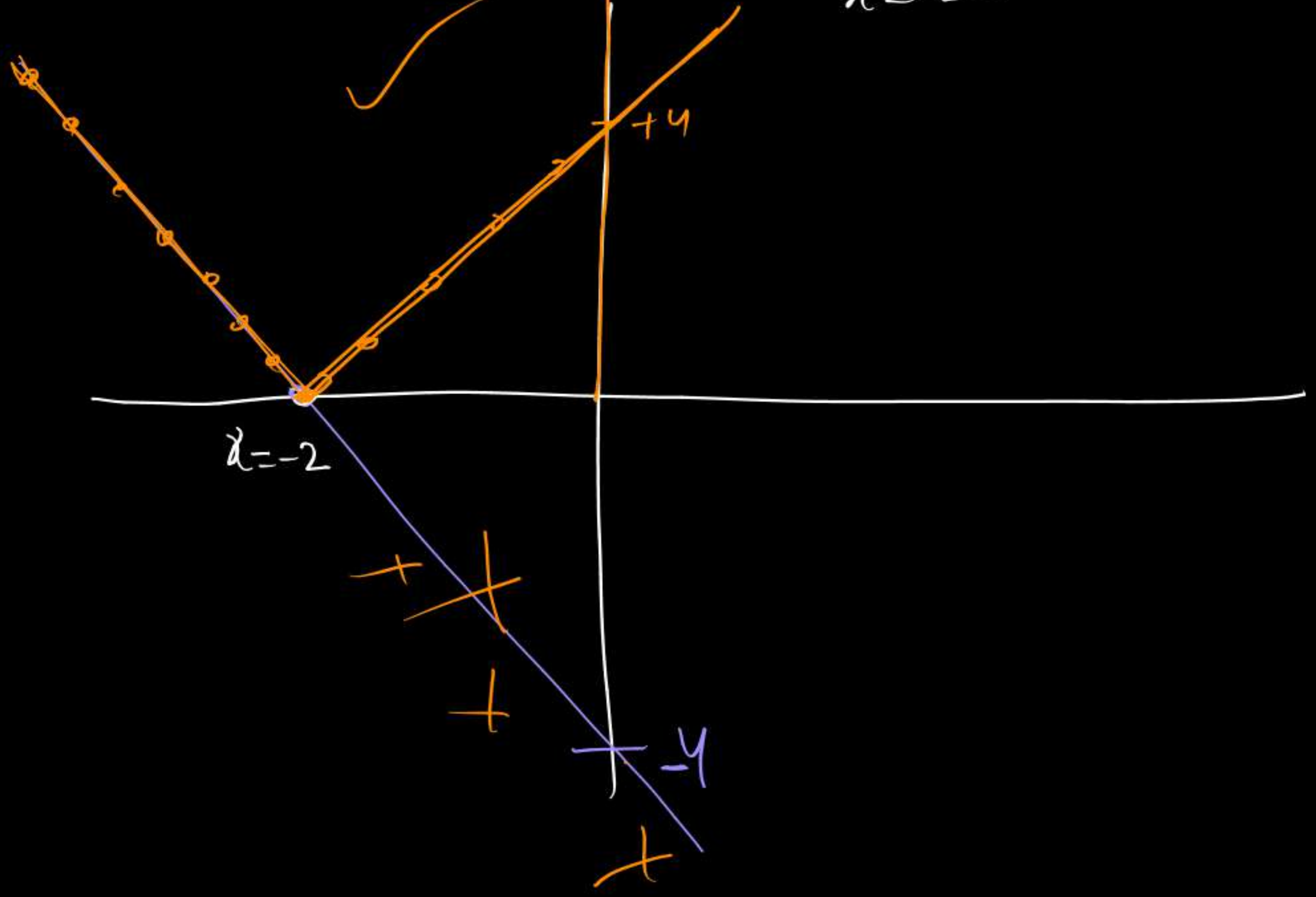


$$y = -2x - 4$$

$$0 = -2x - 4$$

$$4 = -2x$$

$$x = -2$$



QUESTION

Draw graph having Y-intercept 4 and passing through (2, 6).

(x, y)

Sol^n

$c = 4$

$y = mx + c$

$y = mx + 4$

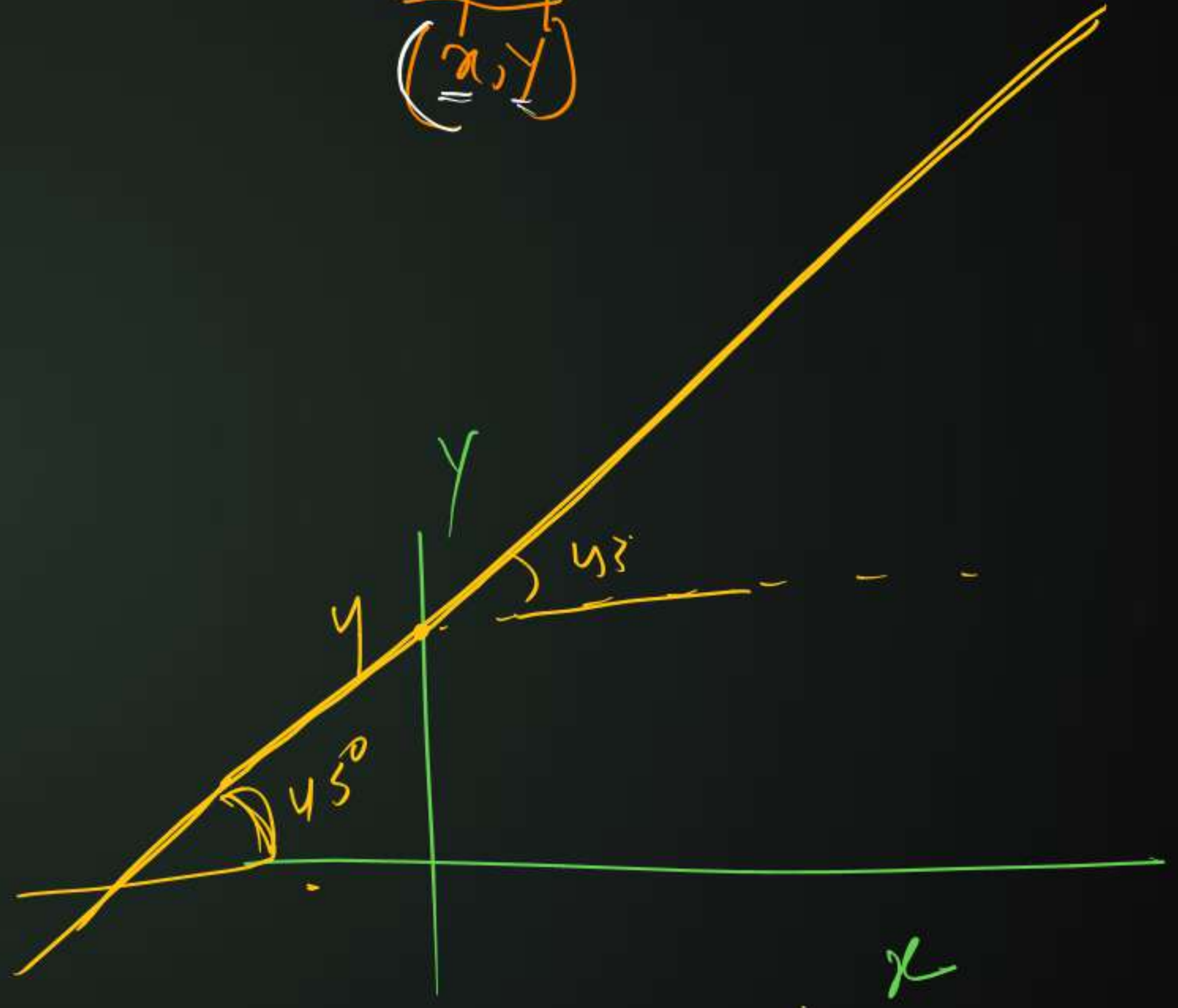
$6 = m2 + 4$

$6 - 4 = 2m$

$\cancel{2}m = \cancel{2}$

$m = 1$ (slope) $= \tan\theta$

$\theta = 45$



QUESTION

Find equation of a straight line passing through point (3, 4) and (2, 6).

$$y = mx + c \quad -(1)$$

$$\Rightarrow 4 = 3m + c \quad -(\text{I})$$

$$\Rightarrow 6 = 2m + c \quad -(\text{II})$$

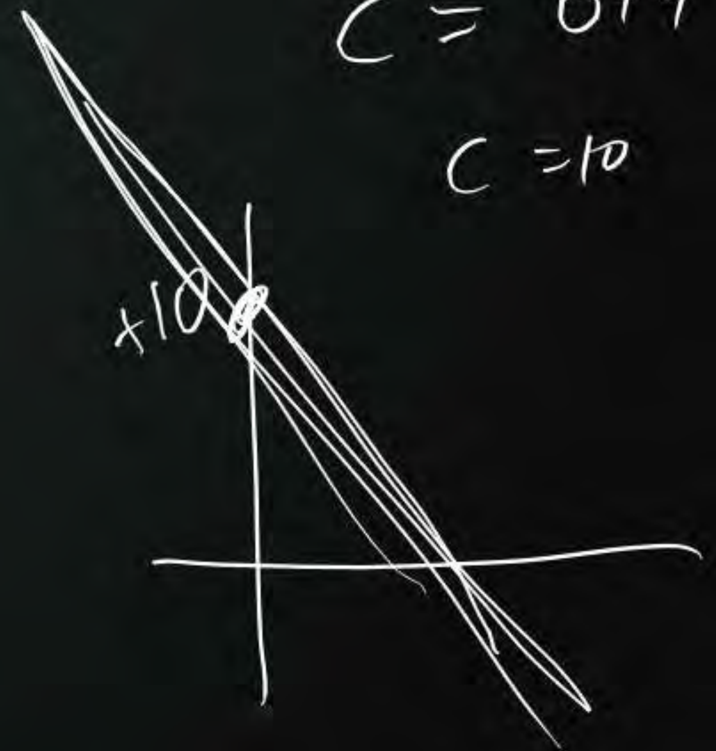
$$-2 = m$$

use eqn (1)

$$y = 3\times(-2) + c$$

$$c = 6+4 = 10$$

$$c = 10$$



QUESTION

Draw graph passing through (2, 3) and slope 1.

(x, y)

$m = 1$

$\tan\theta = 1$

$\theta = 1?$

QUESTION

Find equation of the line which makes intercept +4 and +5 on the x and y-axis.

(A) $5x + 4y + 20 = 0$

(B) $4y + 5\cancel{y} - 20 = 0$

(C) $4y - 5x = -20$

(D) $4x + 5y + 20 = 0$

$4y + 5x = 20$

If $(x = 0)$ $\cancel{4}y = \cancel{20}^{5}$

$y = 5$

If $y = 0$ $5x = 20$

$x = 4$

x - intercept (value of x when y is zero)

$$\# \quad \frac{x}{4} + \frac{y}{3} + 6 = 0$$

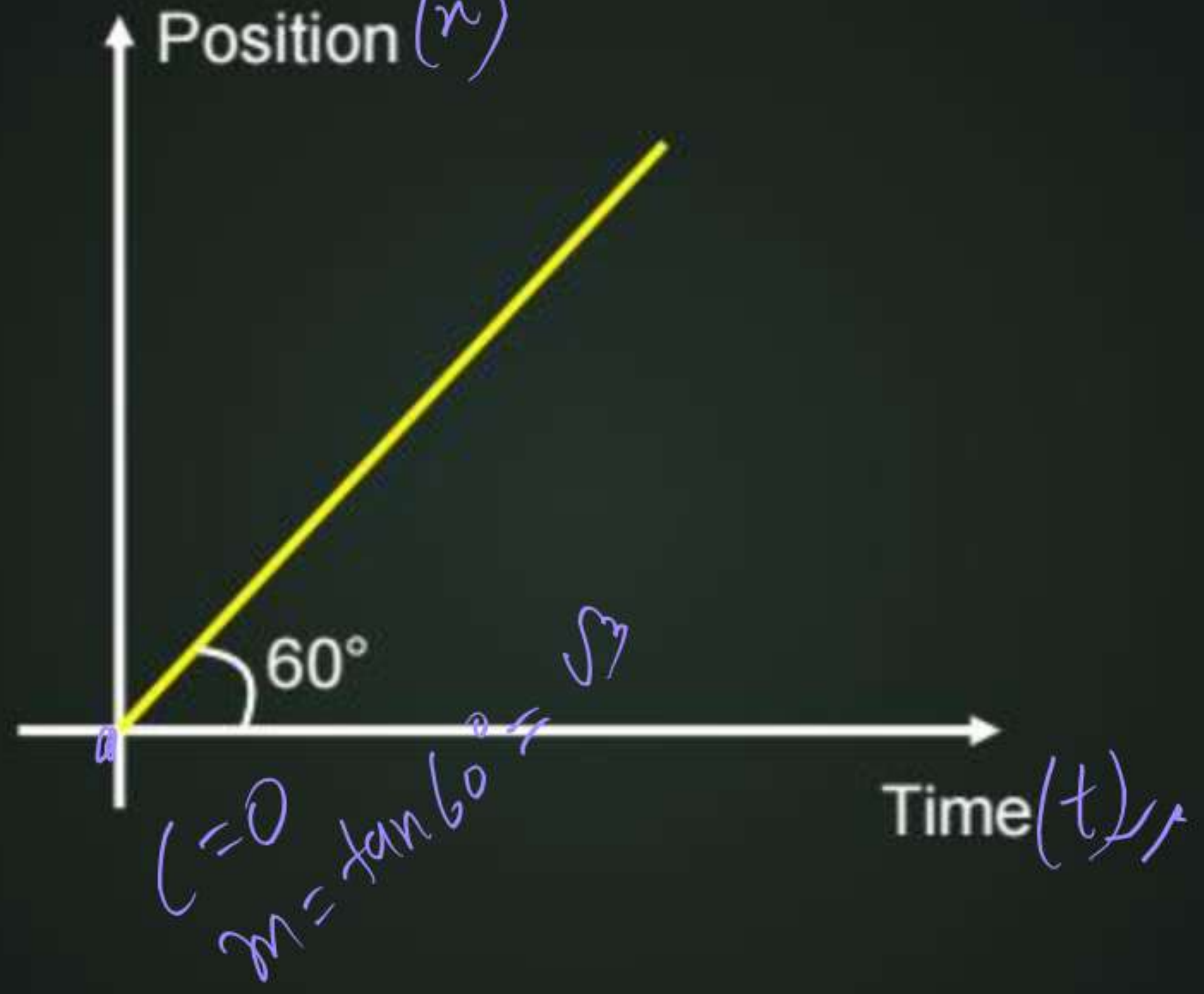
$$\frac{y}{3} = -\frac{x}{4} - 6$$

$$y = -\frac{3x}{4} - 18$$

$$\underbrace{(y-5) + (x-y) - 3 = 0}$$

QUESTION

Find position of object at $t = 2$ sec.



$$y = mx + c$$

$$y = \sqrt{3}\,t + 0$$

$$y = \sqrt{3}\,t$$

$$y = 2\sqrt{3}$$

Parabolic graph (Non-linear variation)

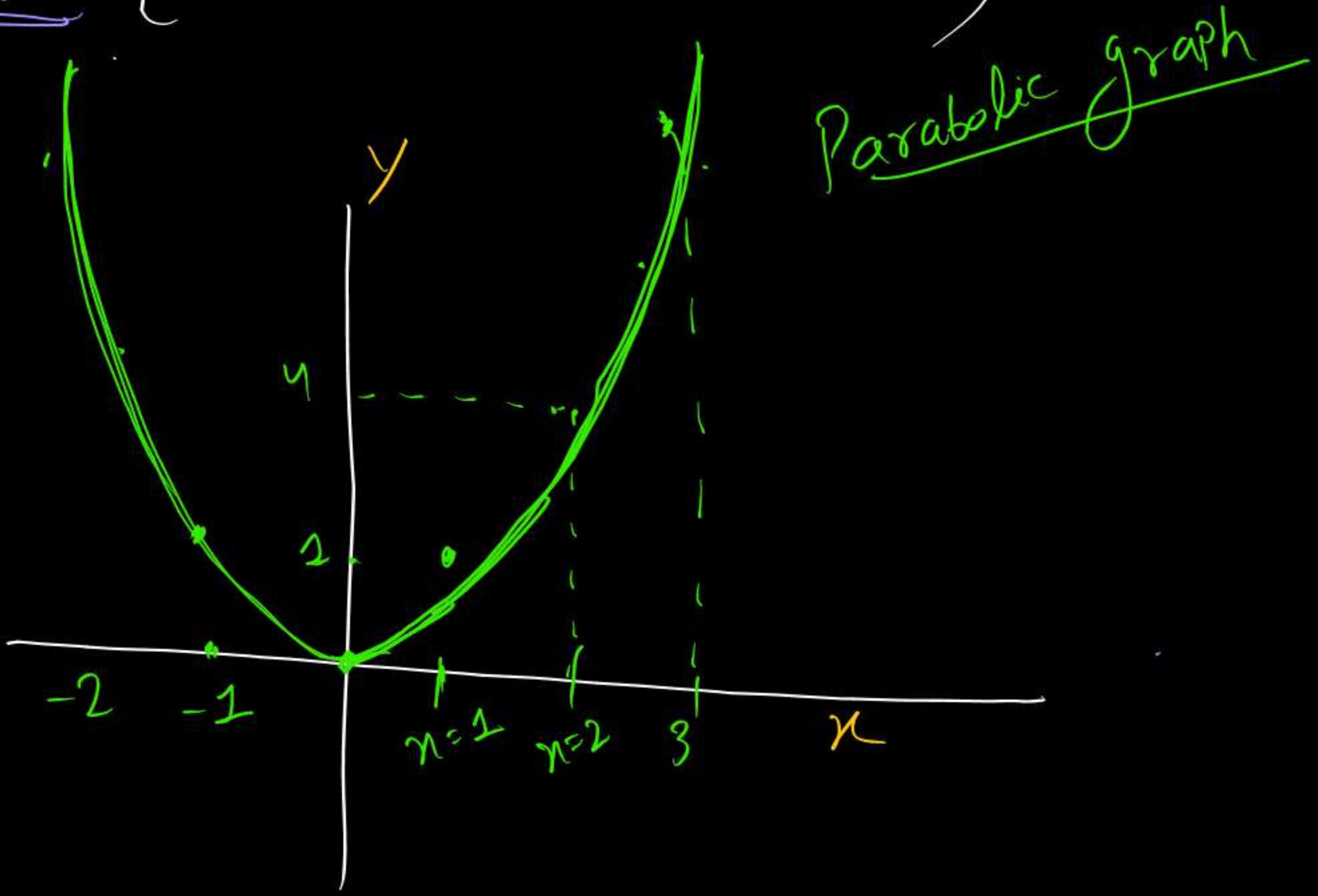
$y \propto x^2$

$y = cx^2$

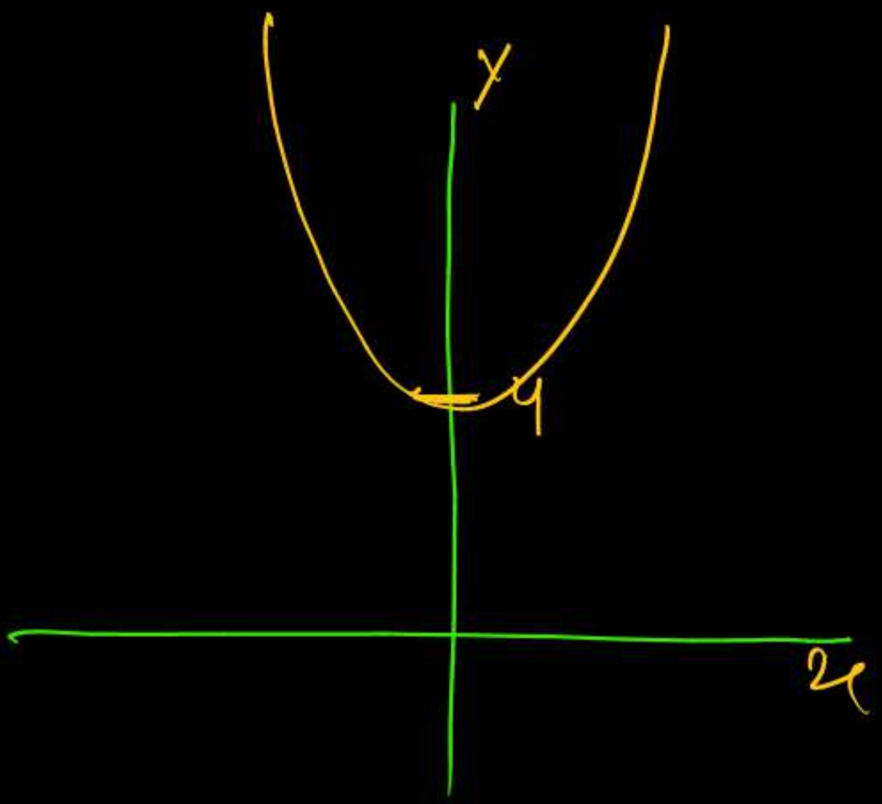
$\boxed{\text{If } c = 1}$

$y = x^2$

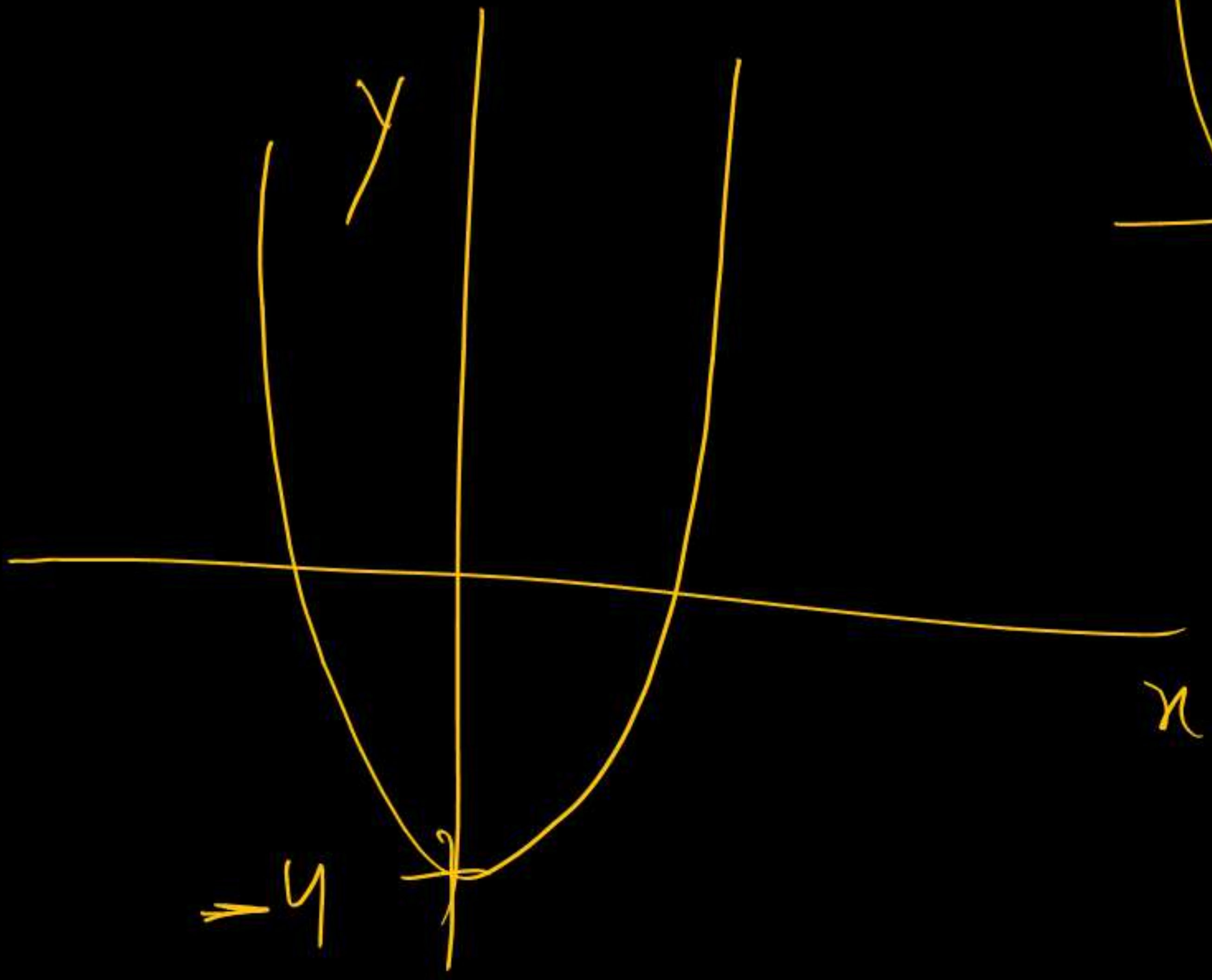
y	x
0	0
+1	±1
+4	±2
9	±3
16	±4



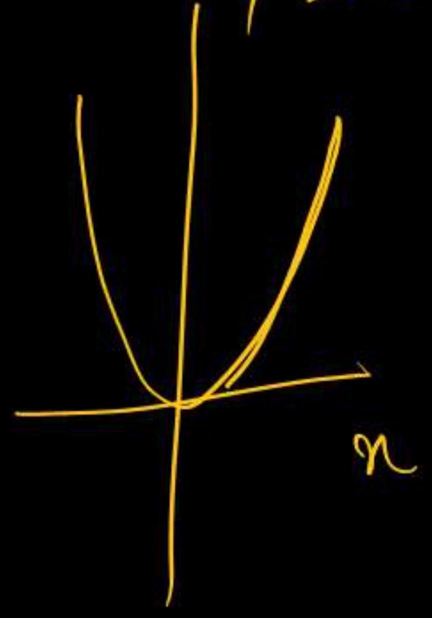
$$y = x^2 + 4$$



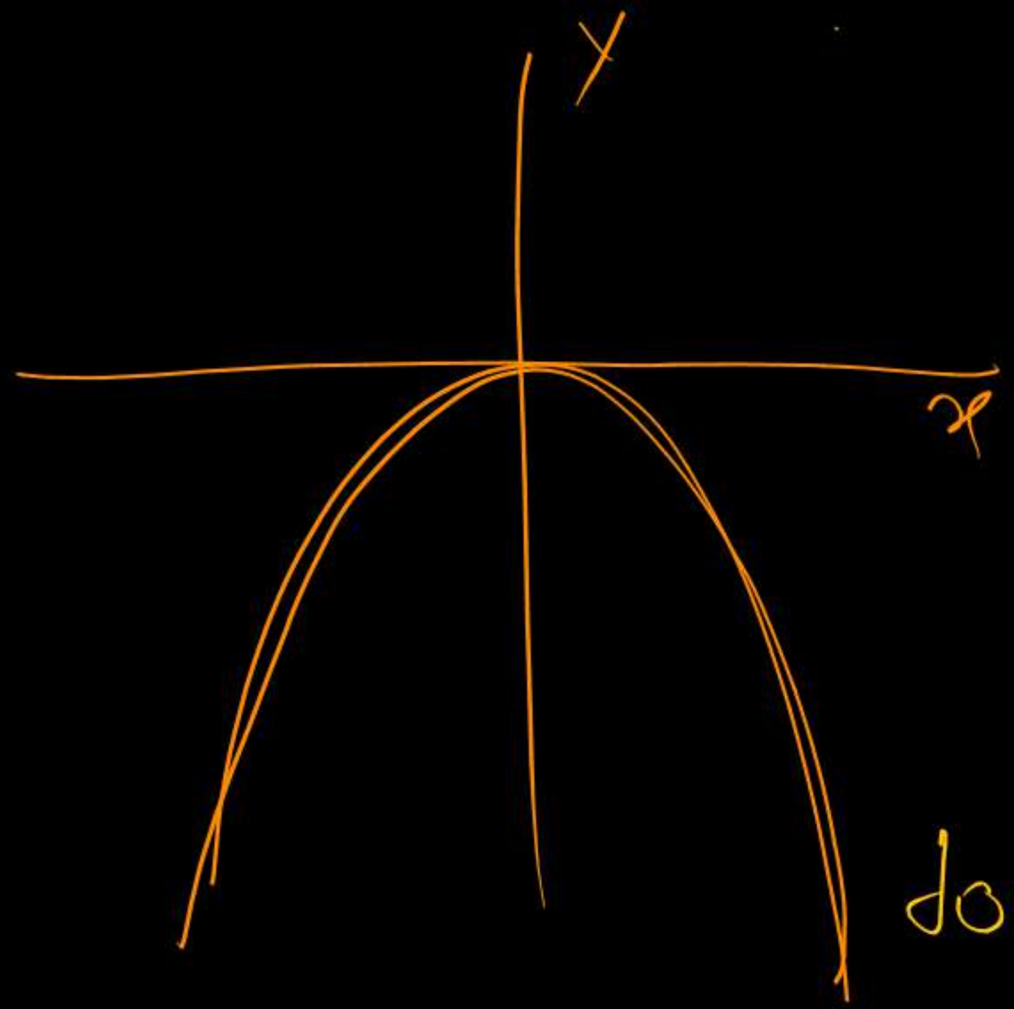
$$y = x^2 - 4$$



$$y = x^2$$

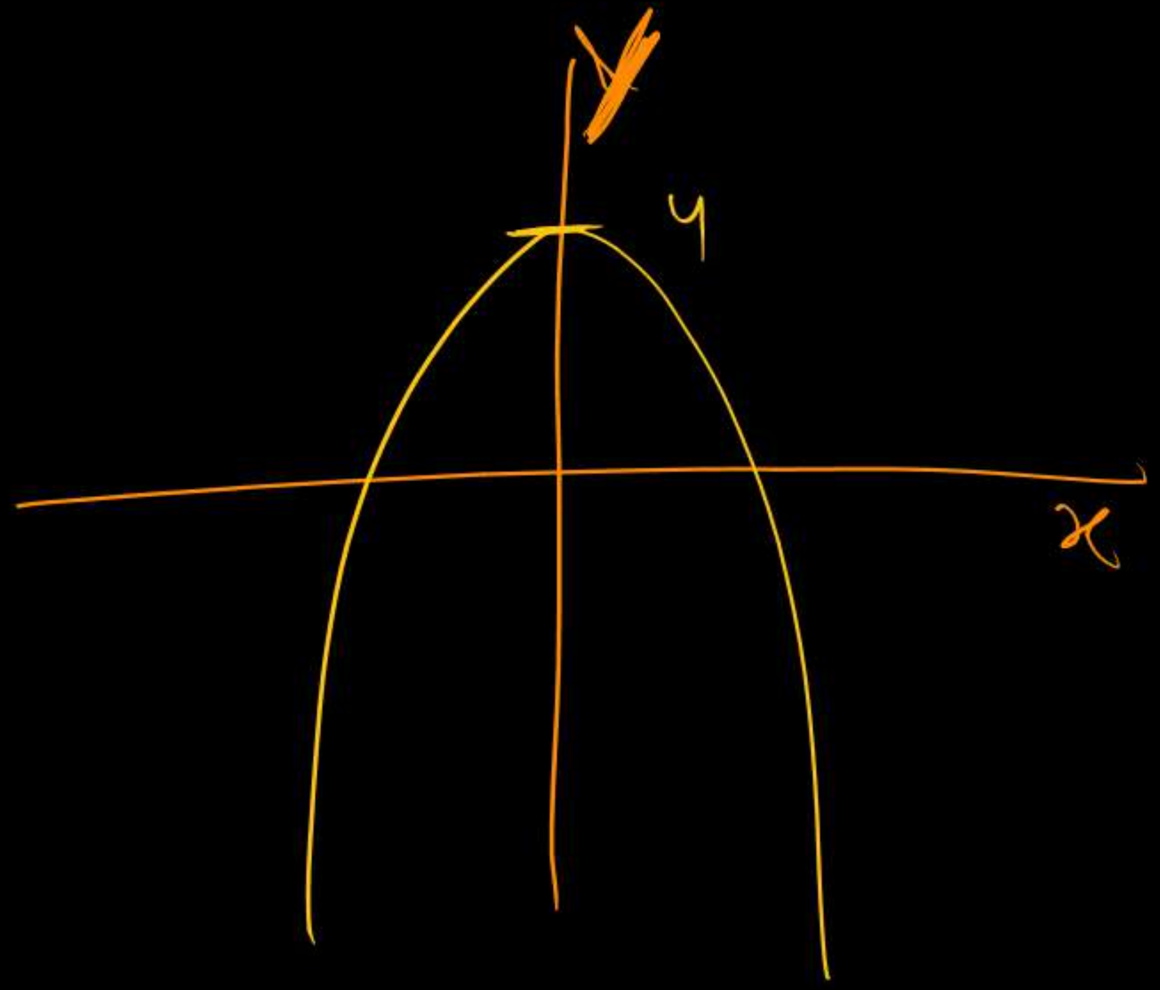


$$y = -(x)^2$$

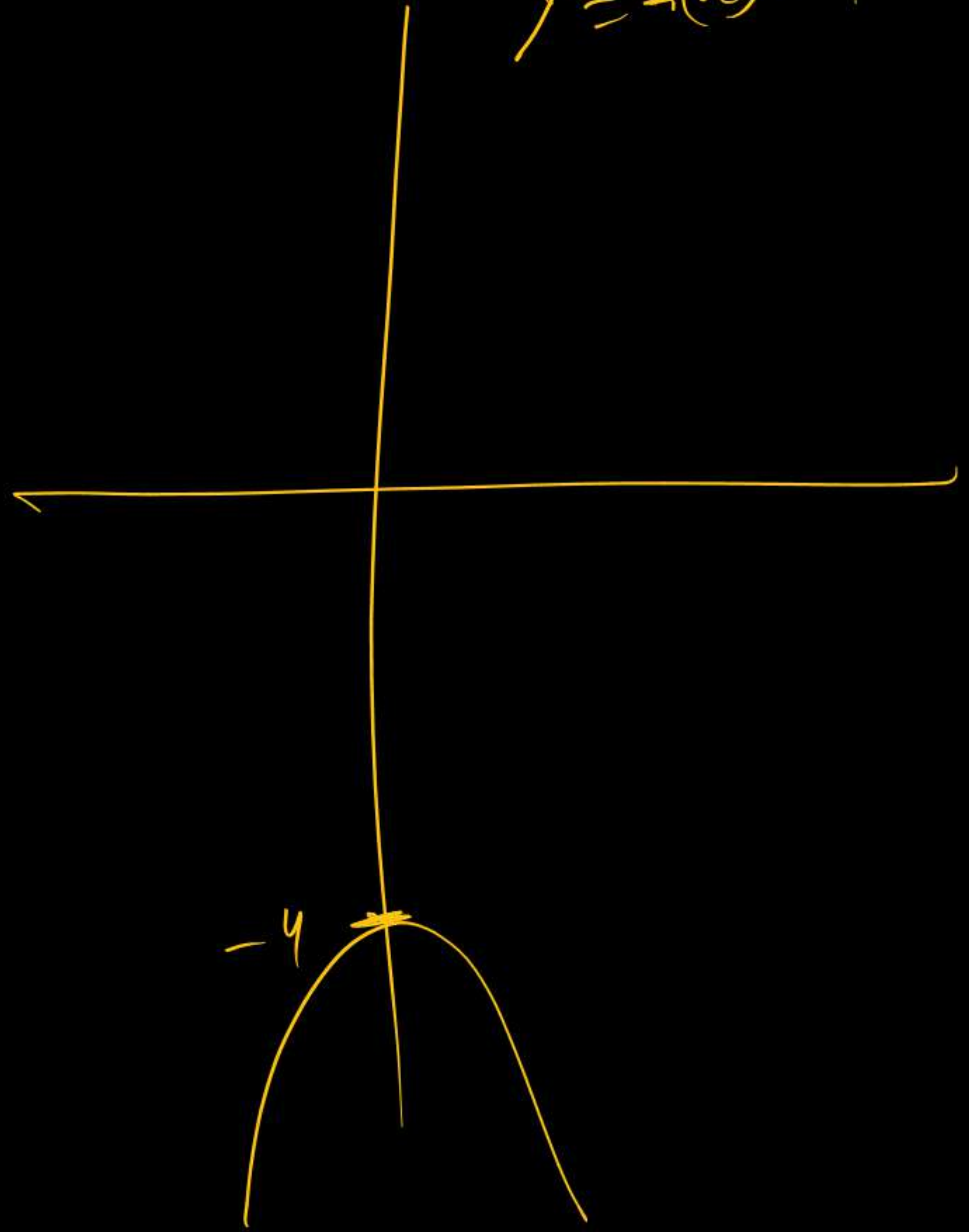


downward opening Parabola.

$$y = -x^2 + 4$$

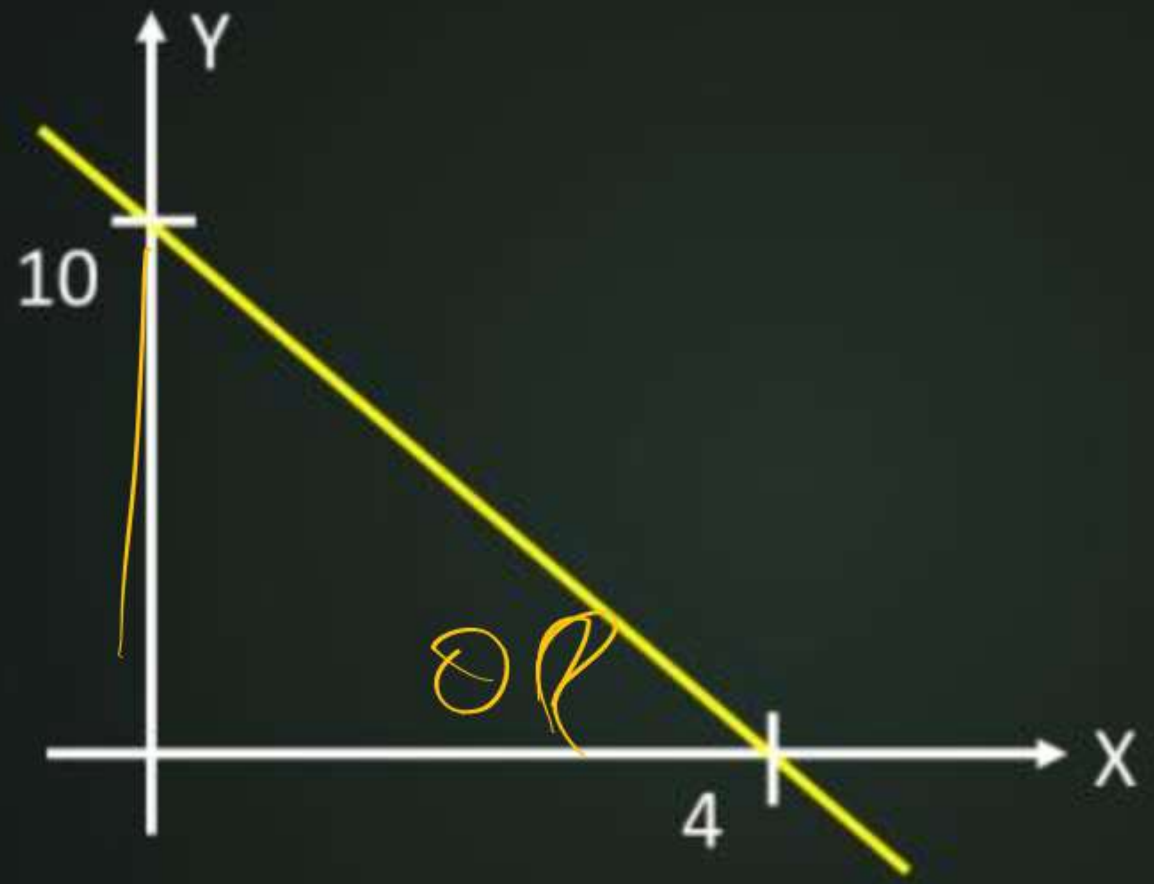


$$y = -(x)^2 - 4$$



QUESTION

Find slope of graph and value of y at $x = 2$.



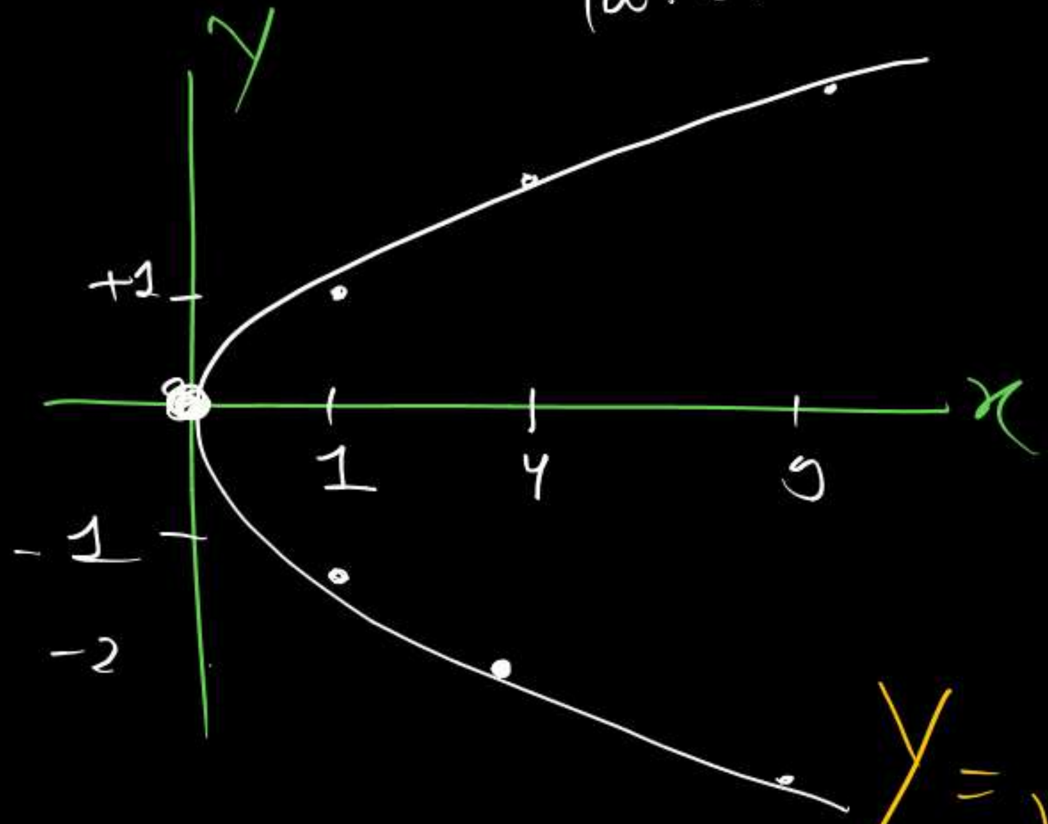
Slope $= \frac{10}{4} = -2.5$

$$y = 4x^2 + 2$$

$y = \sqrt{x}$ → x can't be -ve

y	x
± 1	1
± 2	4
± 3	9
± 4	16
0	0

lateral Parapol



$\sqrt{4} = \pm 2$

$\sqrt{9} = \pm 3$

$\sqrt{-4}$ = Not defined

$\sqrt{-10}$ = "

$y = \sqrt{-x}$

Possible

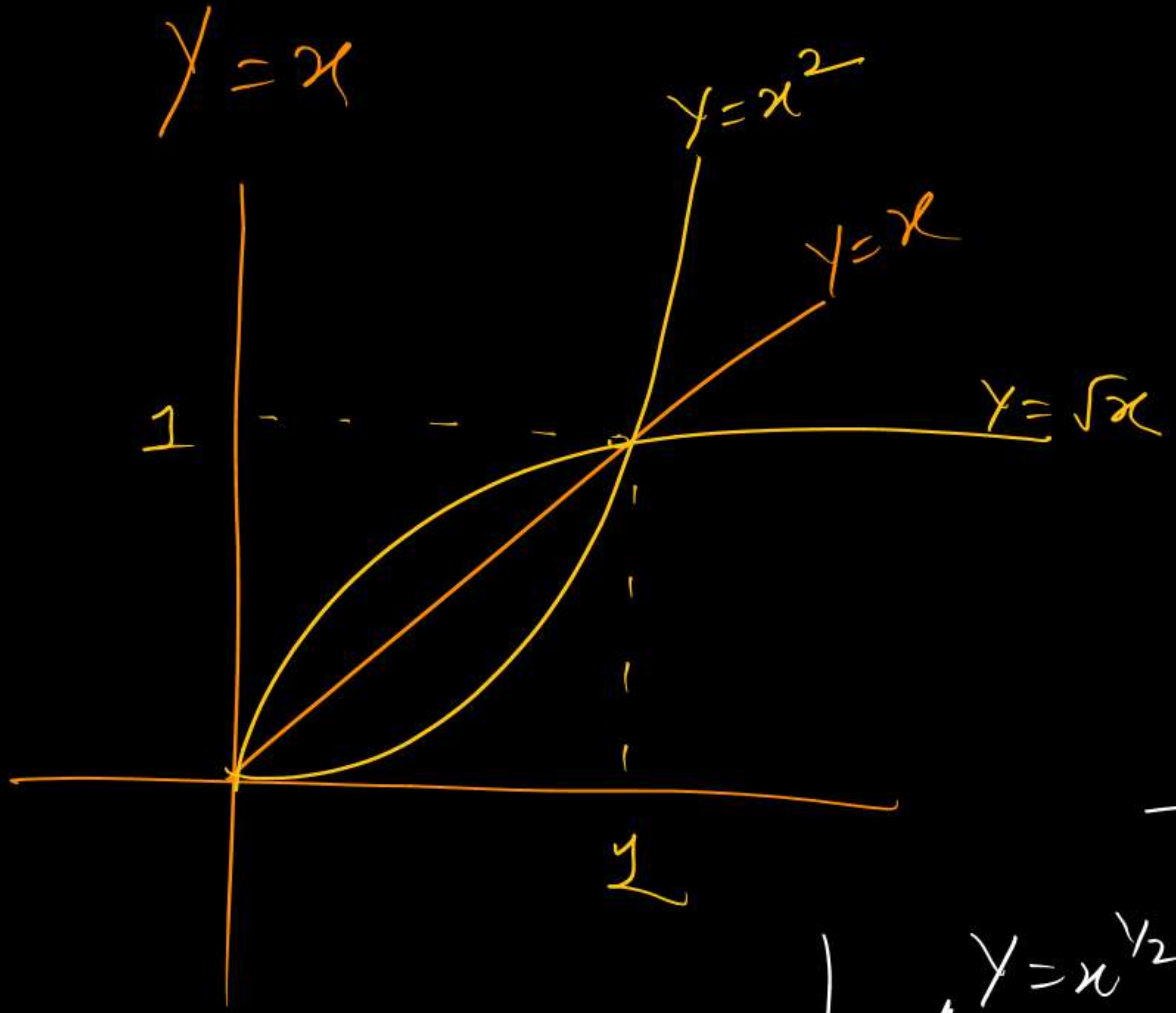
x → Can't be +ve

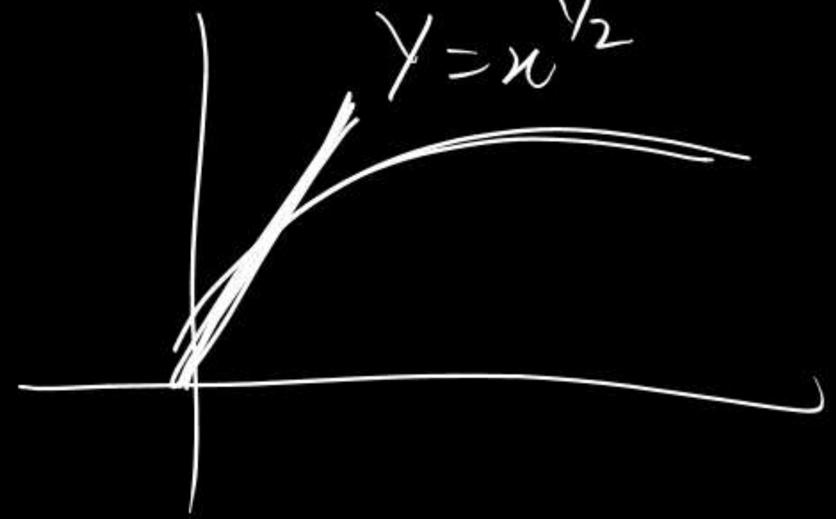
x → Can be -ve here

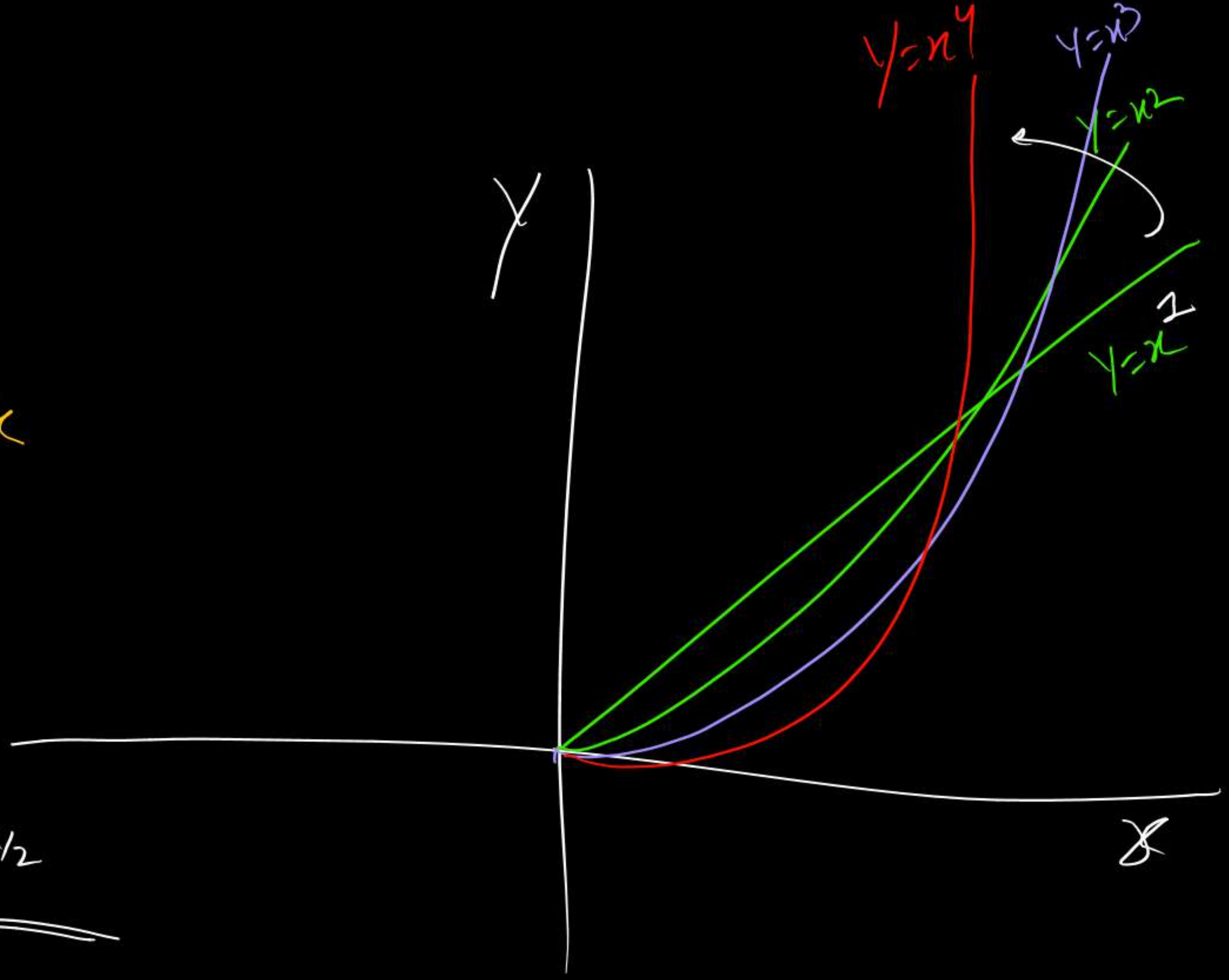
$\boxed{y = \sqrt{-x}}$ → Ha, Na

→ Possible ✓

$y = \sqrt{-x}$


y = x
y = x^2
y = x
y = √x
1
1


y = x^1/2


y = x^4
y = x^3
y = x^2
y = x^1
y
x

Rectangular hyperbola

$$y \propto \frac{1}{x}$$

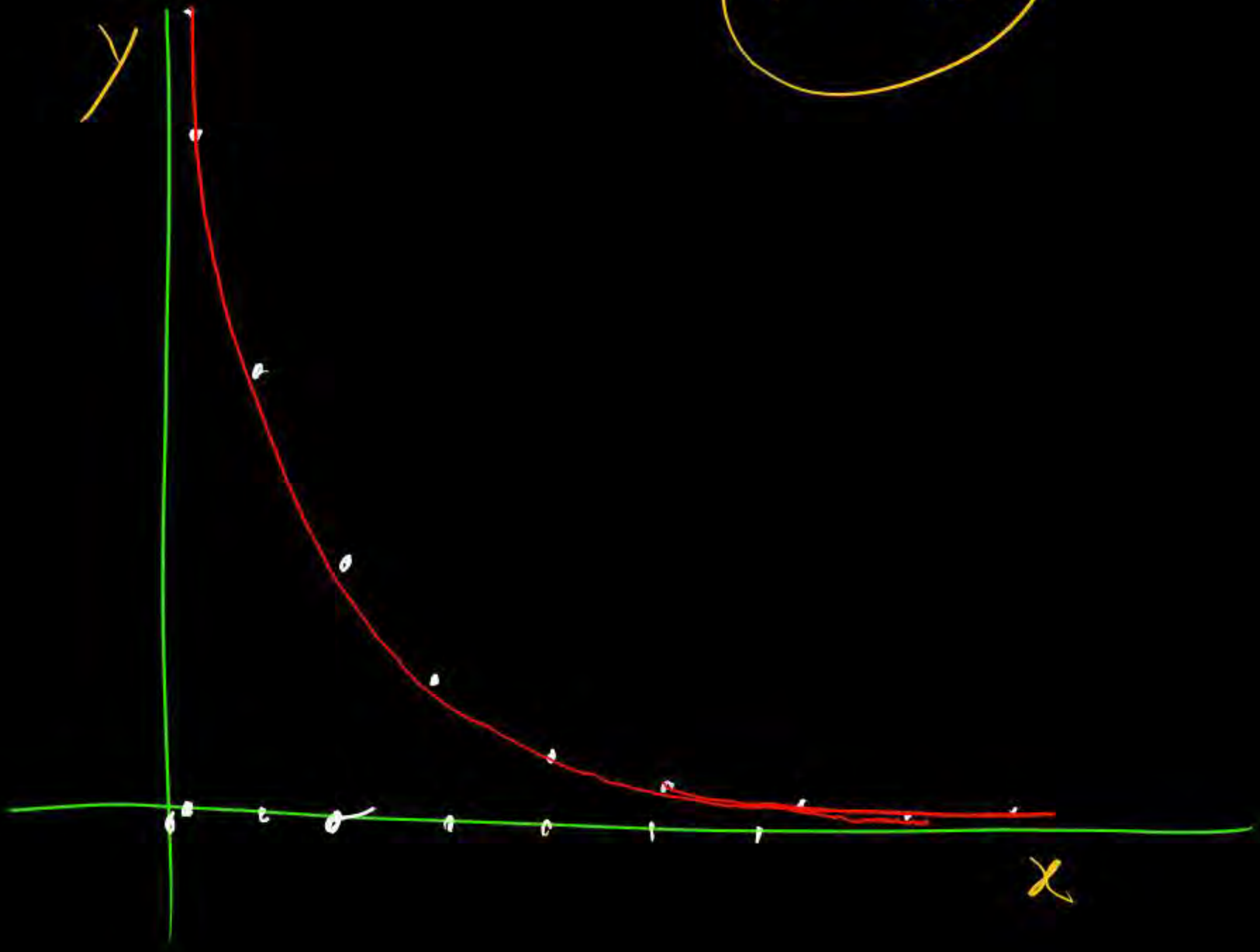
$$\boxed{y = \frac{1}{x}}$$

$$y = \frac{1}{x}$$

$$= \frac{1}{0.1} = 10$$

y	x
∞	0
10	0.1
4	0.25
2	0.5 ✓
1	1
0.5	2
0.25	4
0.1	10
0.01	100

$$y = \frac{1}{x}$$



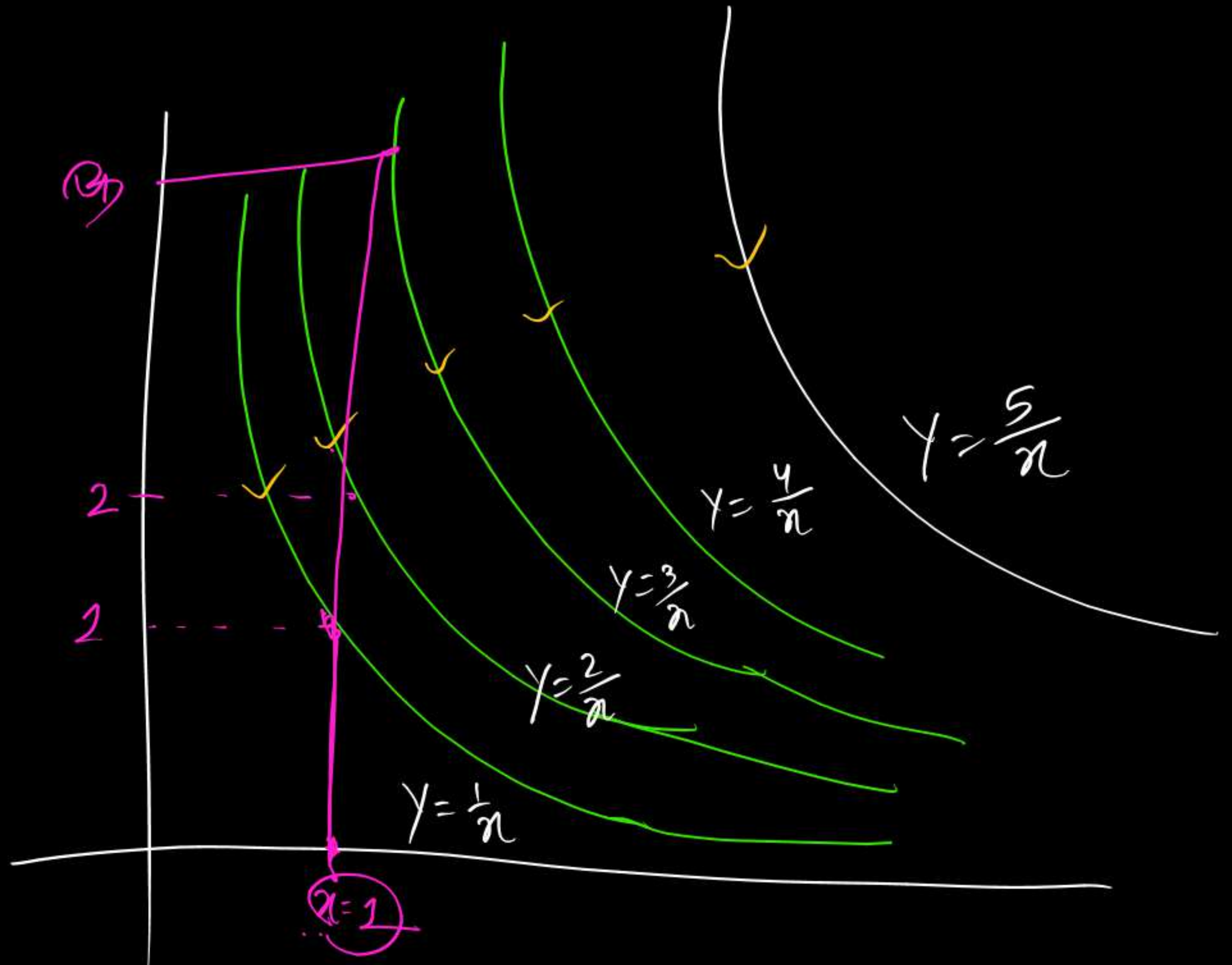
$$y = \frac{1}{x}$$

$$y = \frac{2}{x}$$

$$\downarrow y = \frac{3}{x}\uparrow$$

$$\downarrow y = \frac{4}{x}\uparrow$$

$$\downarrow y = \frac{5}{x}\uparrow$$



$$PV = \boxed{nRT}$$

Tempr = const
Isothermal

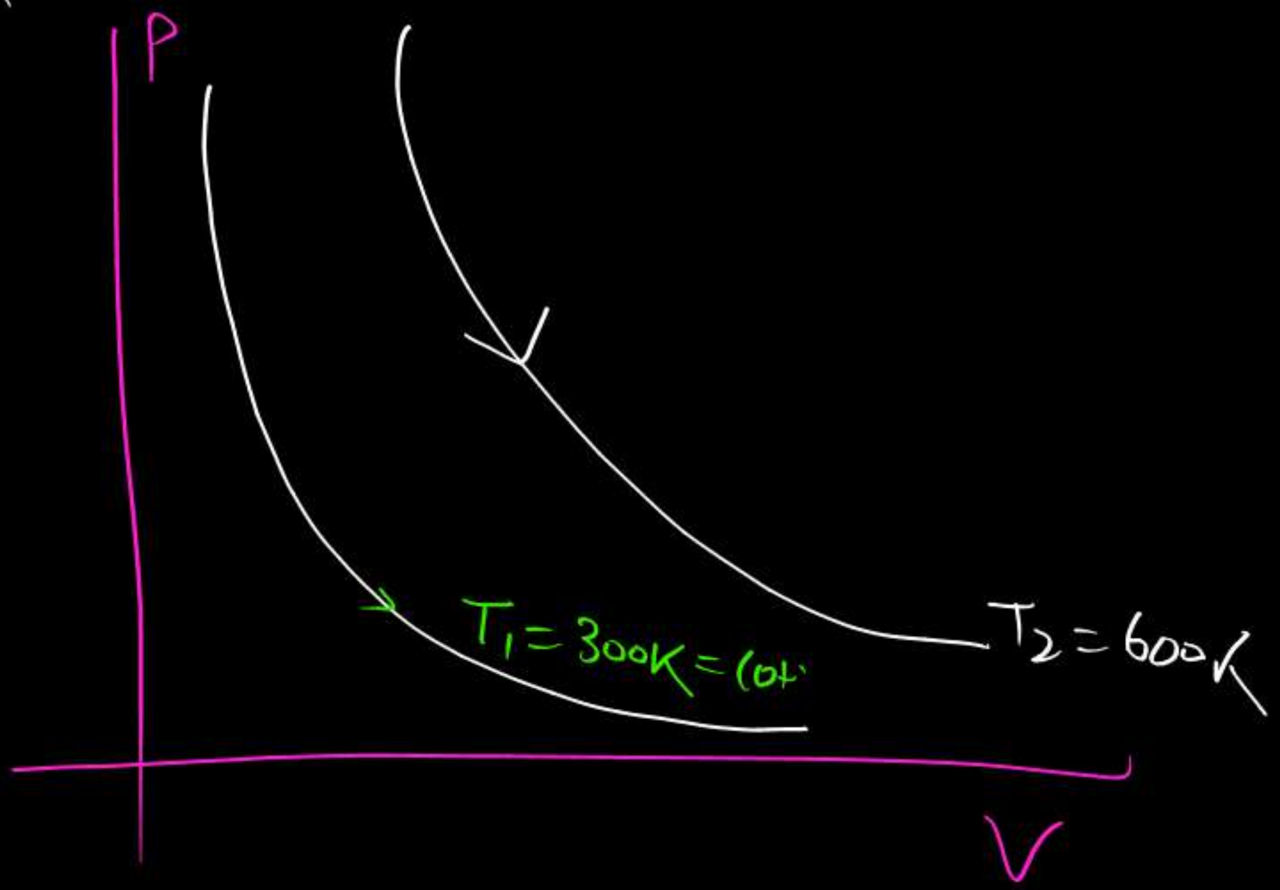
$$PV = nRT$$

at $T_1 = 300K$ fo

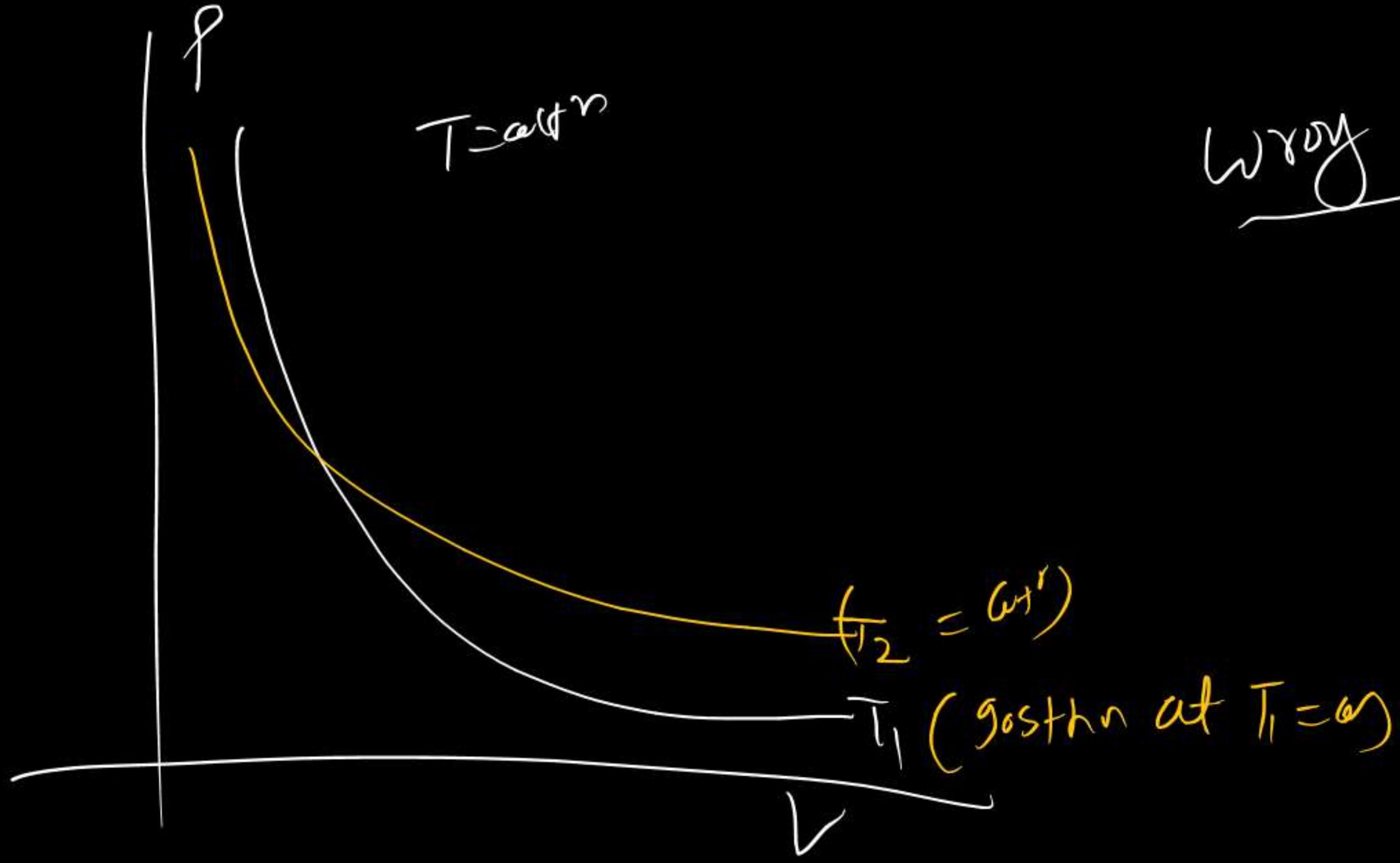
$$\downarrow P = \frac{nRT}{V\uparrow}$$ → const

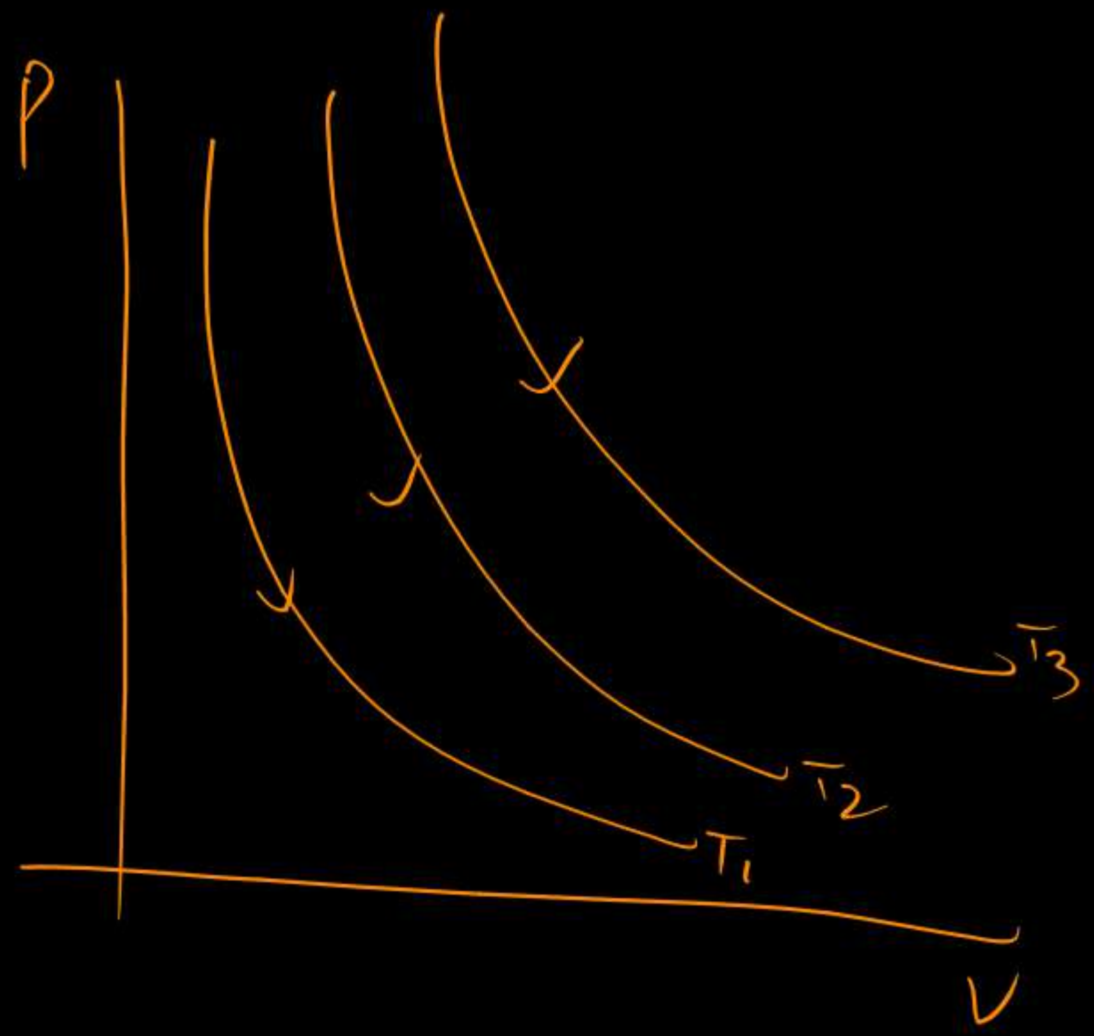
for same gas

$$P = \frac{nRT_2}{V}$$ → 600K

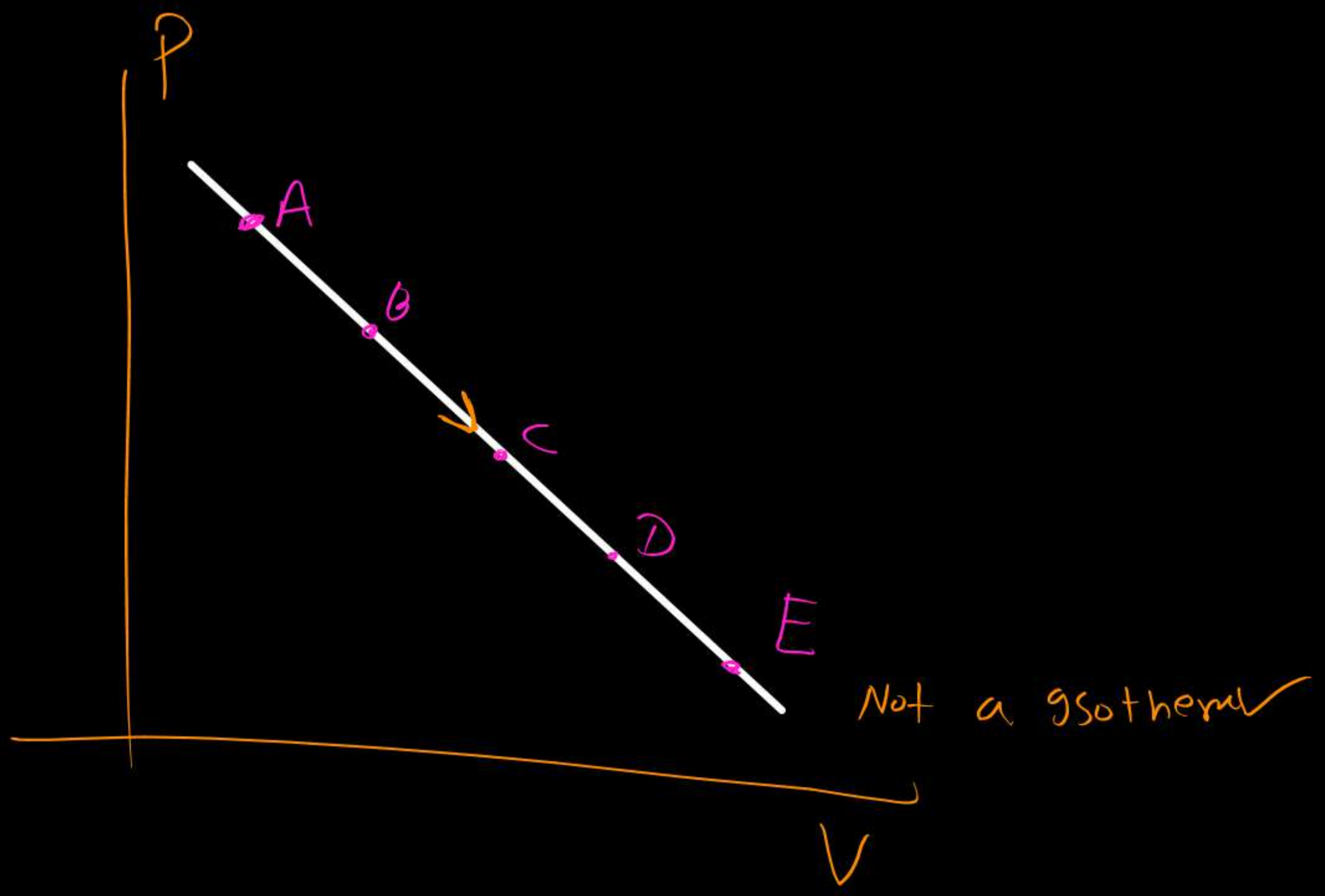


Wrong graph





$$T_3 > T_2 > T_1$$



$E \to D \to C \to B \to A$

Tempr $\to$??

Tempr 1st ↑ then decs

g will tell you in cls

$A \to B \to C \to D \to E$

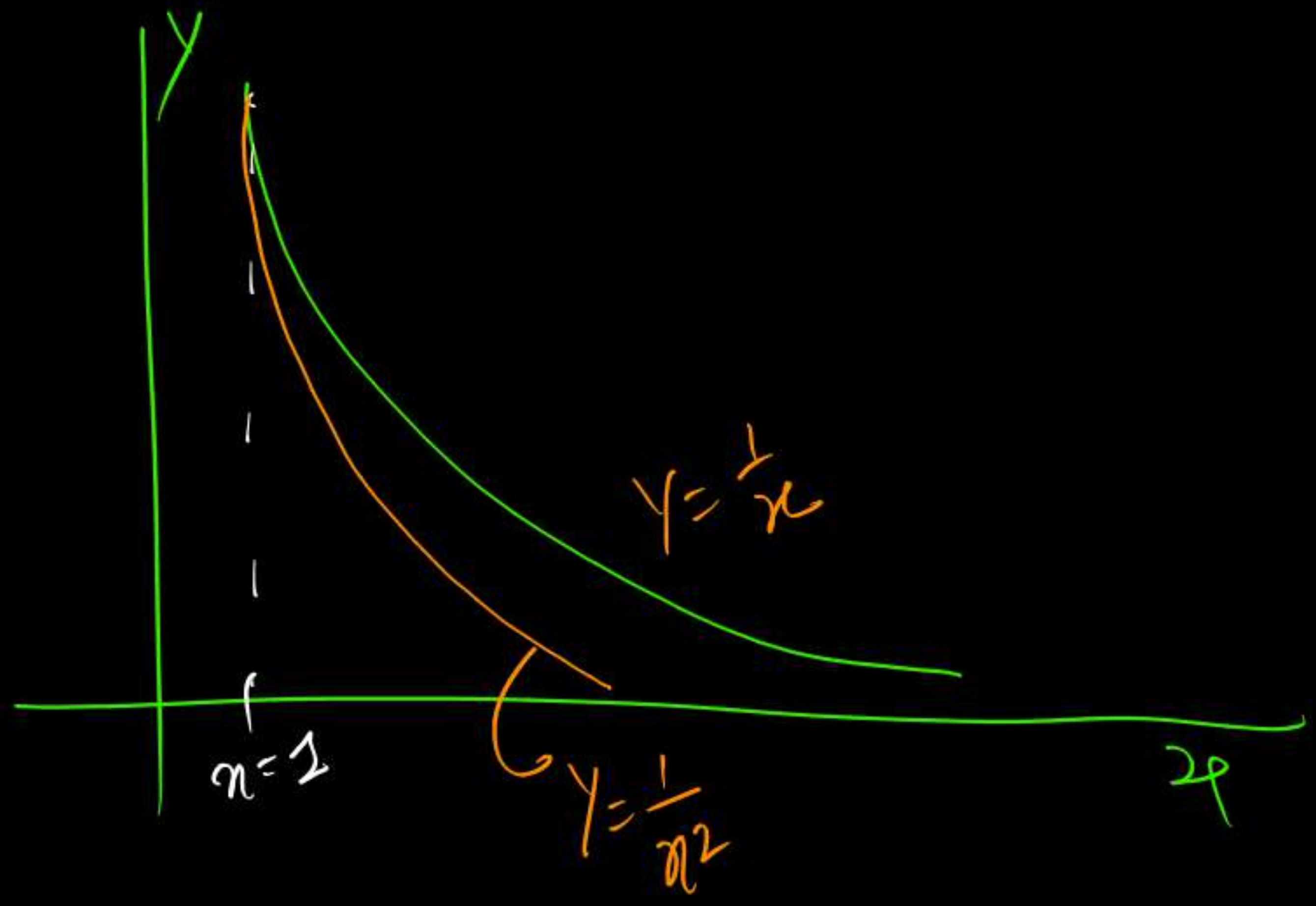
Tempr ↑ 1st then decrs

$y \propto \frac{1}{x}$ (y is Inversly Propotional to x)

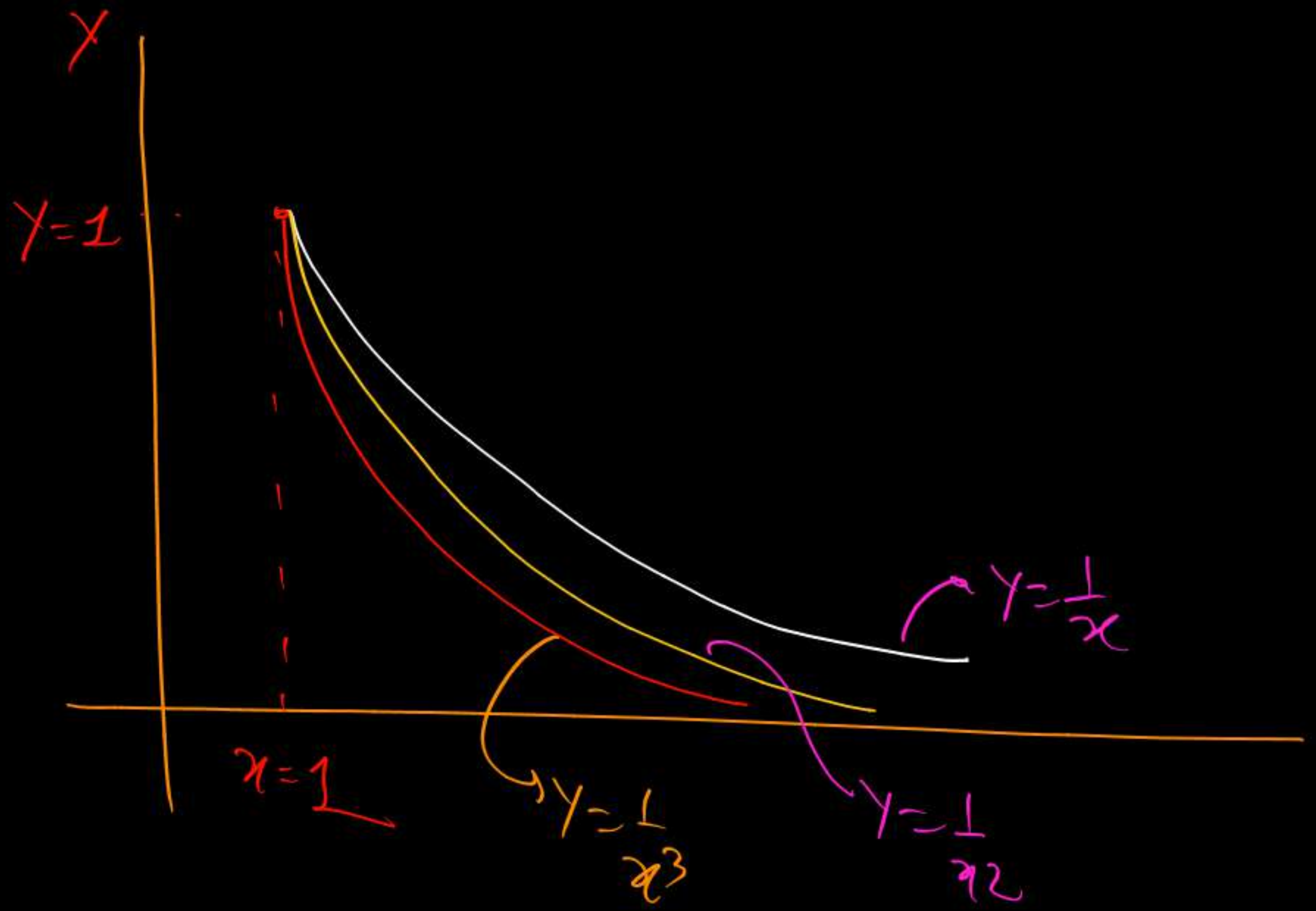
$y \propto \frac{1}{x^2}$ (y is Inversly Propotional to x^2)

y	x
∞	0
100	0.1
1	1
0.25	2
0.01	10
0.0001	100

$$y = \frac{1}{(0.1)^2} = \frac{1}{\frac{1}{100}} = 100$$



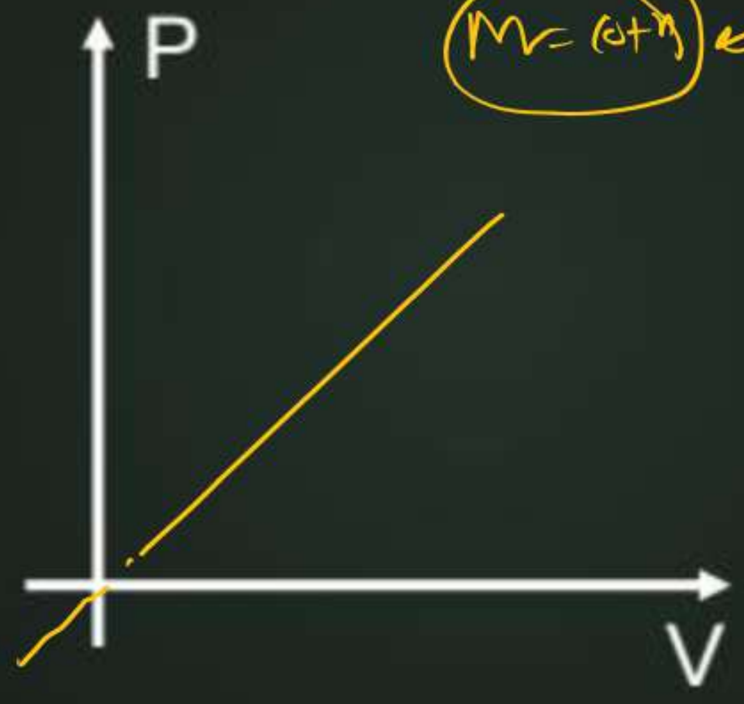
$\downarrow y = \frac{1}{x\uparrow}$ $\quad$ $\downarrow\downarrow y = \frac{1}{x^2\uparrow}$ $\quad$ $\downarrow\downarrow\downarrow y = \frac{1}{x^3\uparrow}$



GRAPH

Draw graph between momentum and velocity P = mV.

m = const

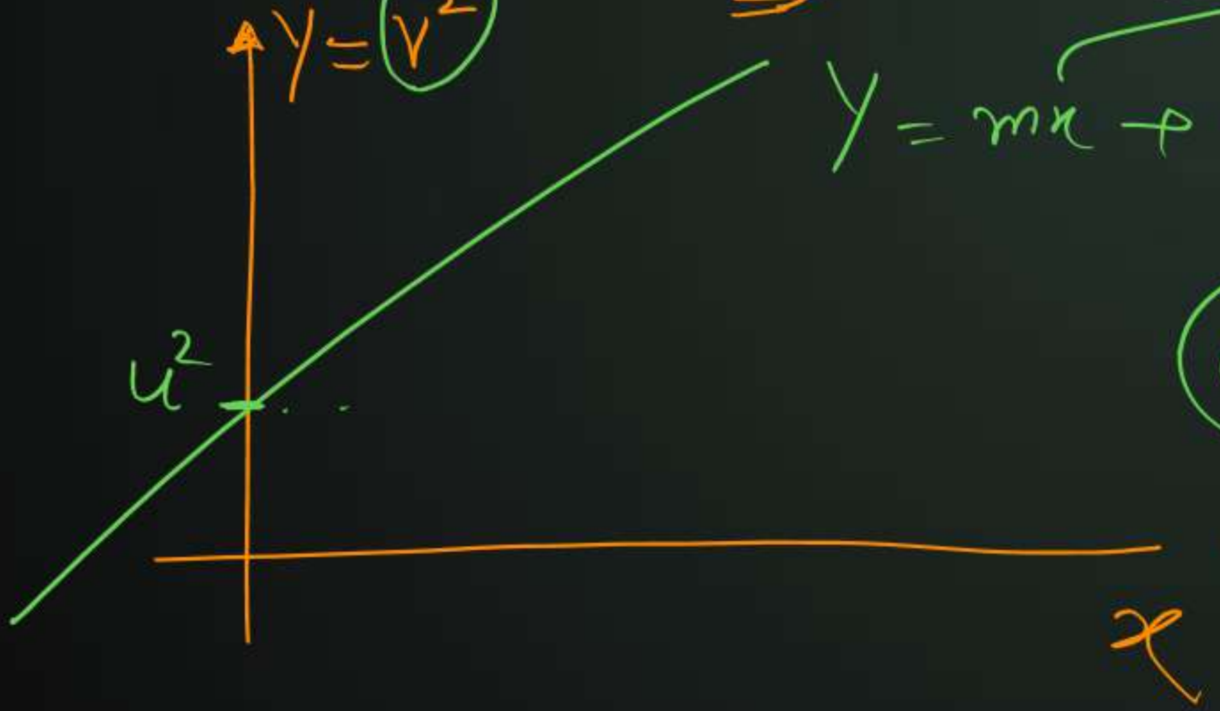


QUESTION

Plot graph between V^2 and position x according to equation $v^2 = u^2 + 2ax$; u is contant.

$$\boxed{v^2 = u^2 + 2ax}$$

$$y = u^2 + \underline{2a}\,x$$

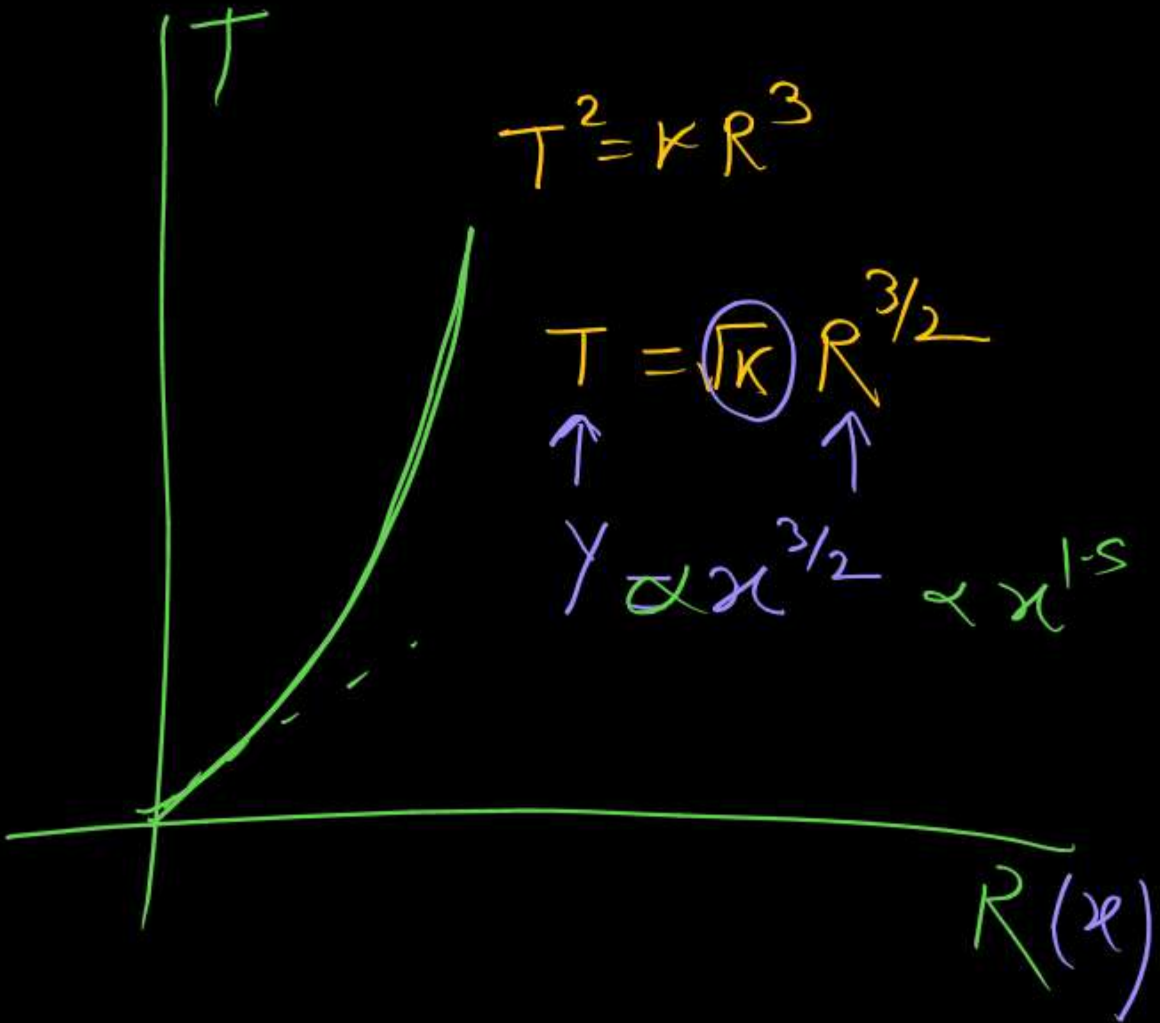


$$y = mx + c$$

$$c = u^2$$

$$m = 2a$$

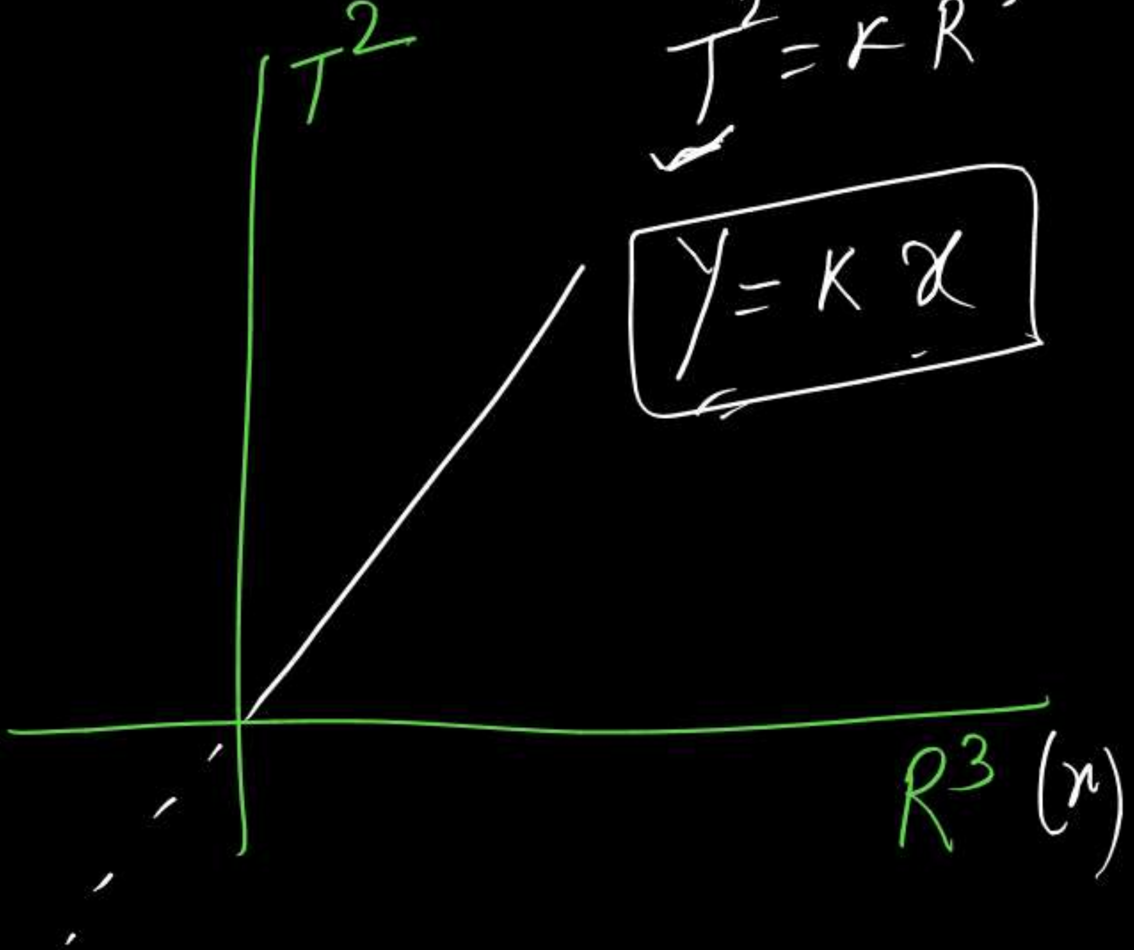
$T^2 = K R^3$ → for this eqn Plot graph B/w $\underline{\underline{T^2}}$ & $\underline{\underline{R^3}}$



$T^2 = K R^3$

$T = \sqrt{K}\, R^{3/2}$

$Y \propto x^{3/2} \propto x^{1.5}$

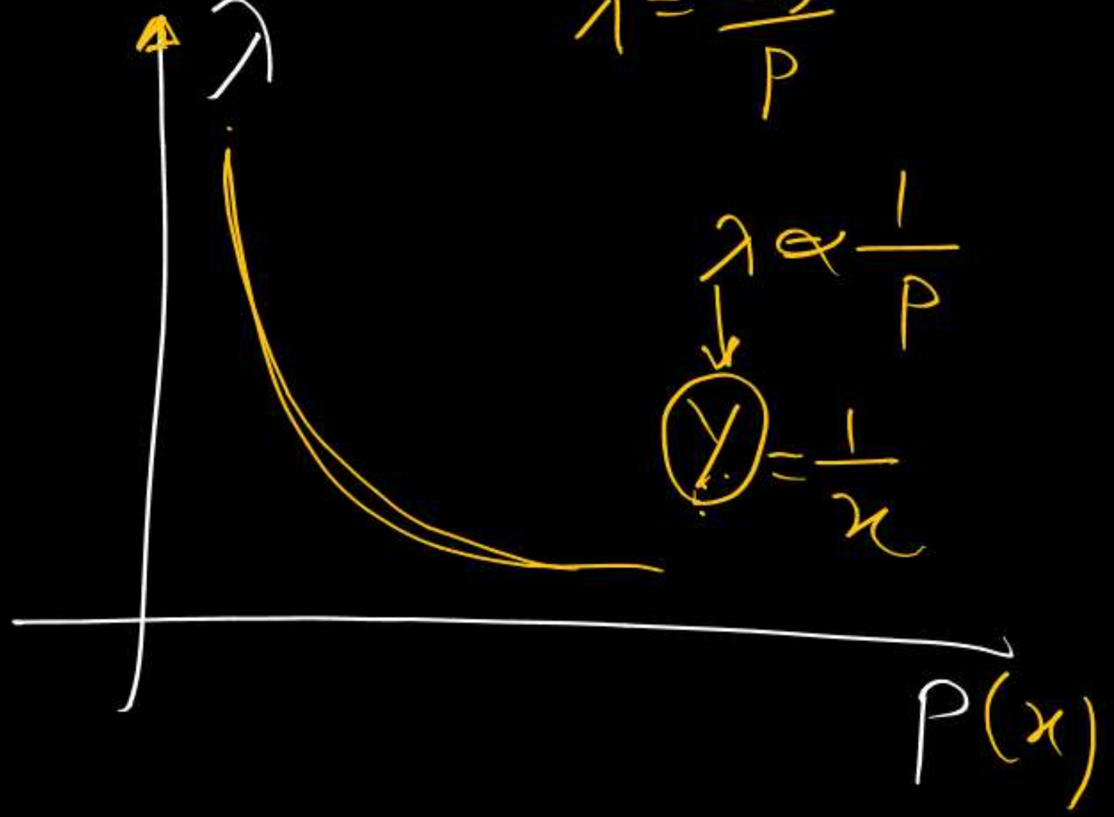


$T^2 = K R^3$

$\boxed{Y = K x}$

$$\lambda = \frac{h}{p}$$

NEET-2014



$$\lambda = \frac{h}{p}$$

$\lambda \propto \frac{1}{p}$

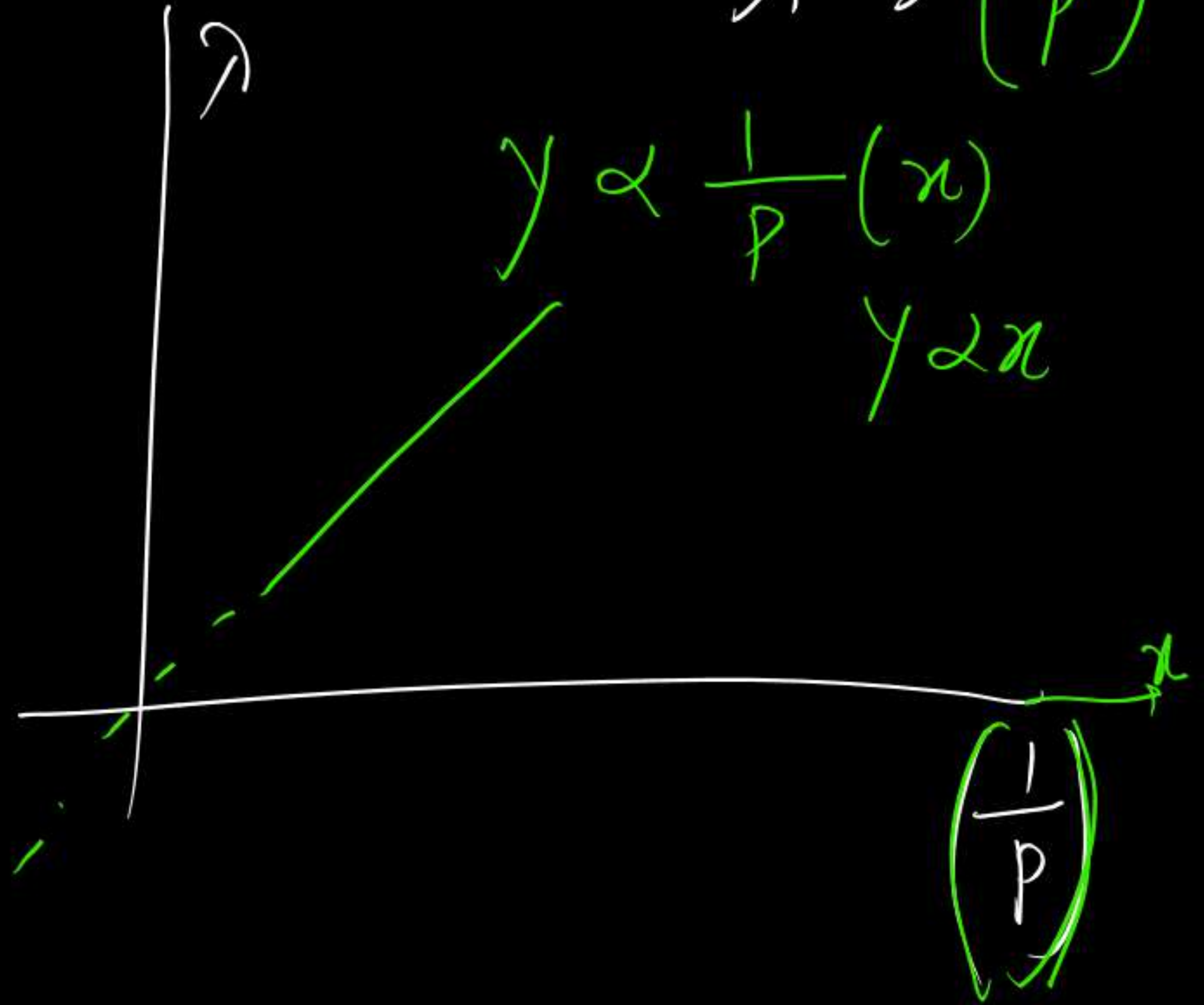
$y = \frac{1}{x}$

$$\lambda = \frac{h}{p}$$

graph B/W 'λ' & $\left(\frac{1}{p}\right)$

$y \propto \frac{1}{p} (x)$

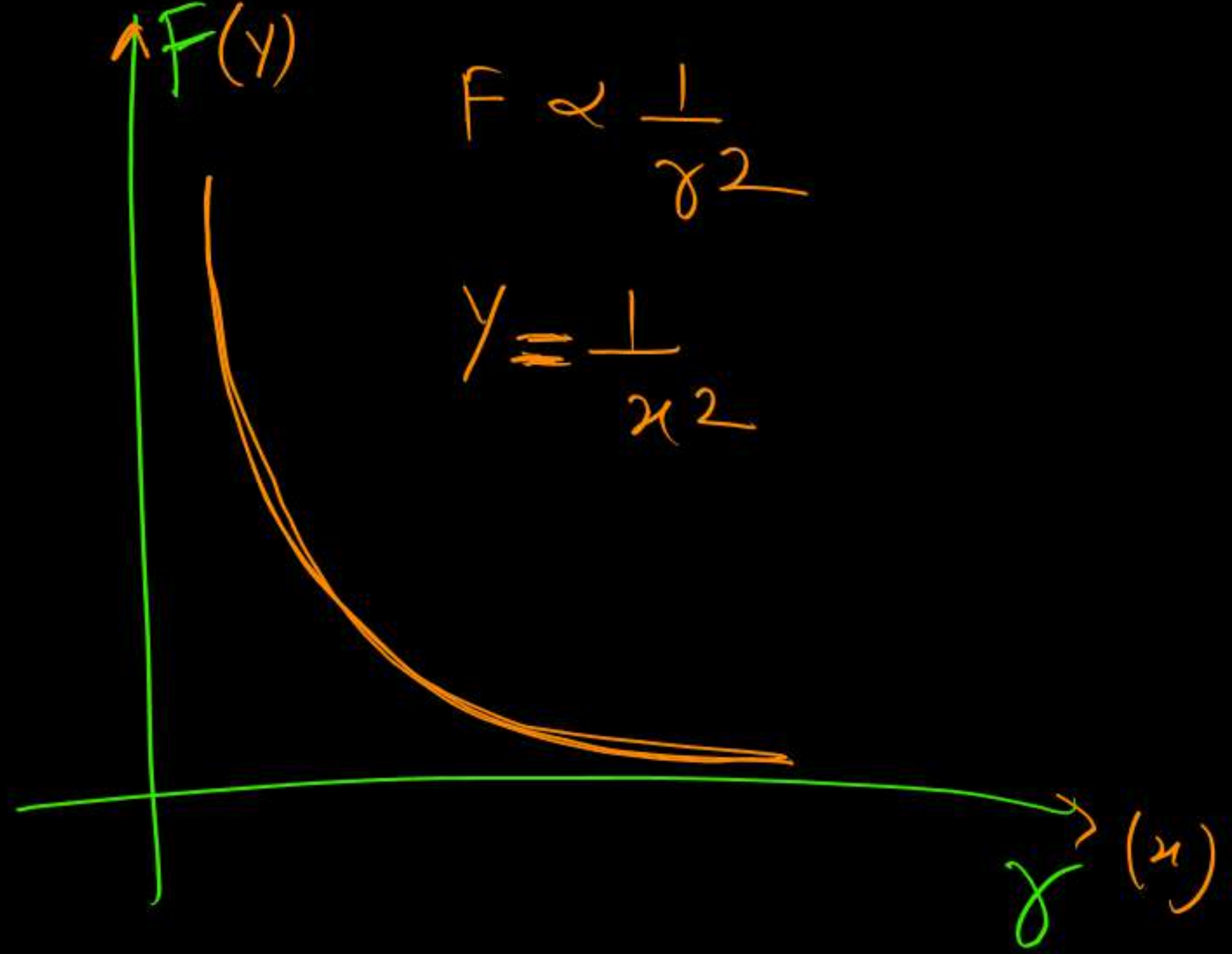
$y \propto x$



$$F = \frac{K Q_1 Q_2}{r^2}$$

$$F \propto \frac{1}{r^2}$$

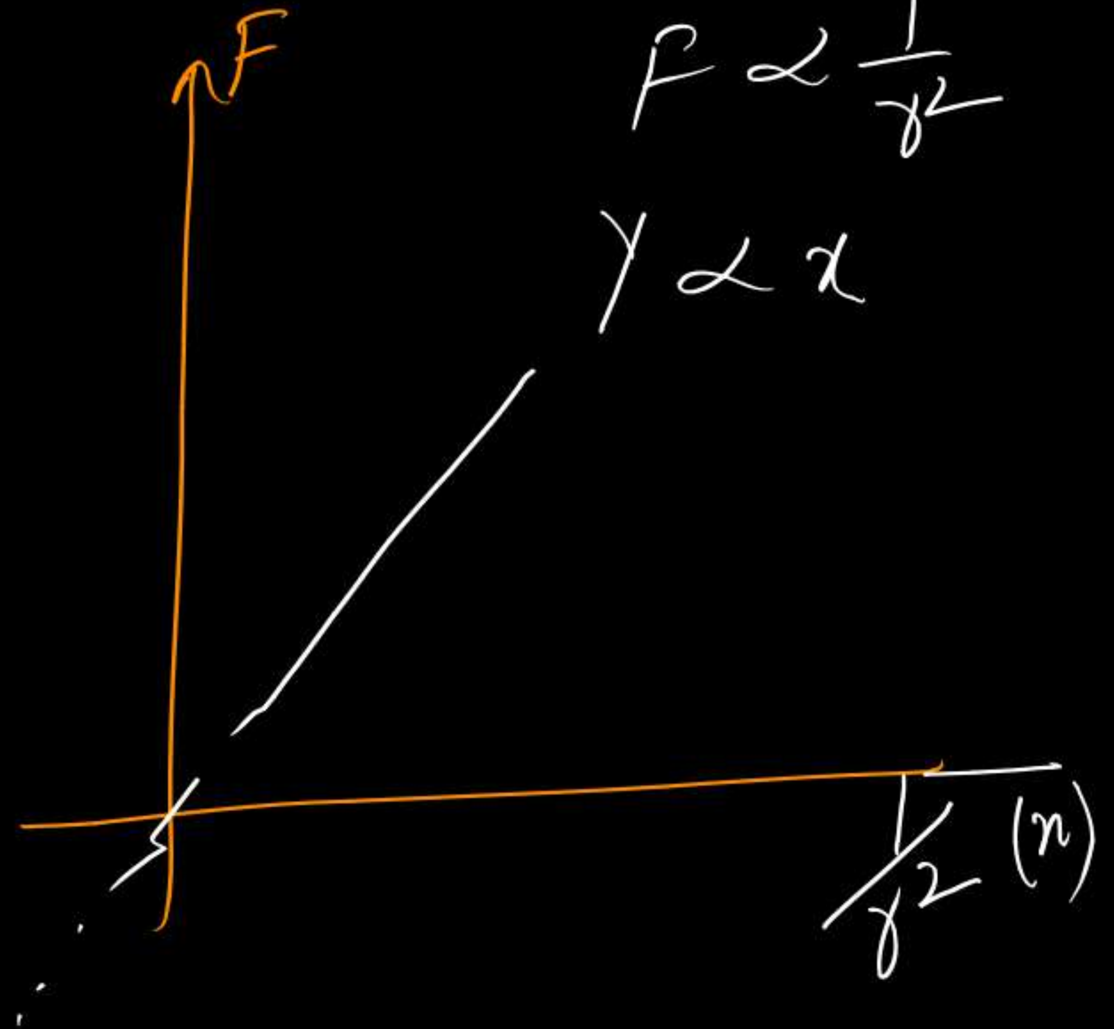
$$y = \frac{1}{x^2}$$



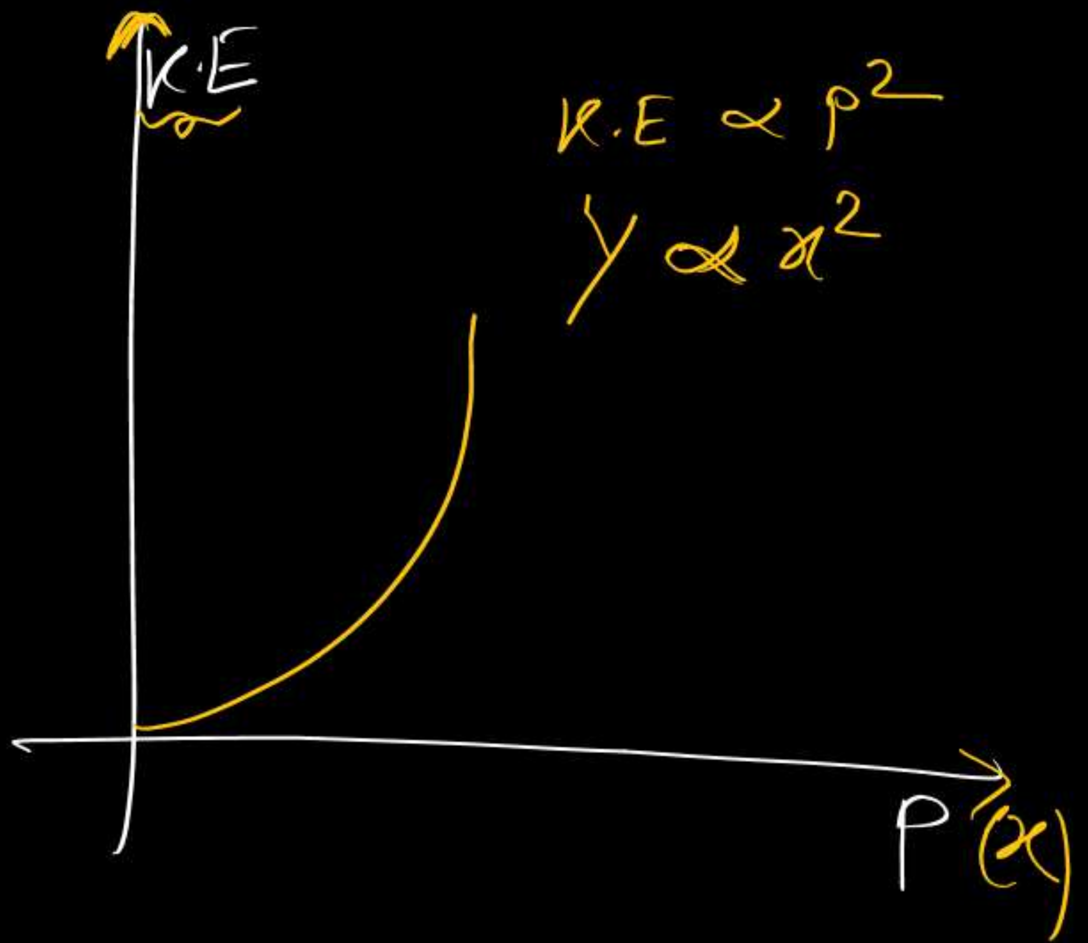
graph b/w F & $\left(\frac{1}{r^2}\right)$

$$F \propto \frac{1}{r^2}$$

$$y \propto x$$



$K \cdot E = \frac{p^2}{2m} \rightarrow \text{cnt}$ (given)



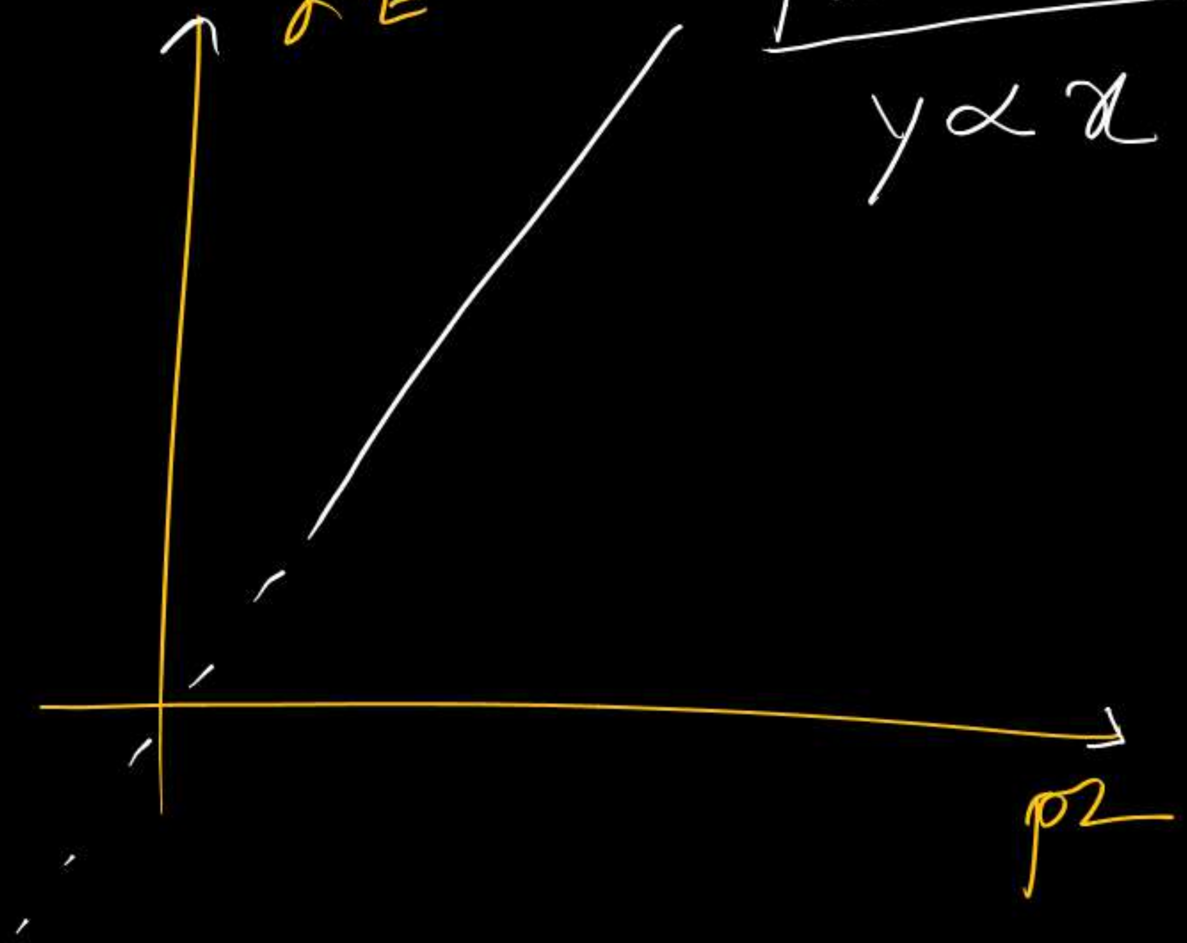
$K \cdot E \propto p^2$

$y \propto x^2$

This is graph B/w K.E & Momentum

$\boxed{K \cdot E \propto p^2}$

$y \propto x$



This is the graph B/w K.E & p^2

QUESTION

$$PV = nRT$$

Draw graph between pressure temptation for constant volume.

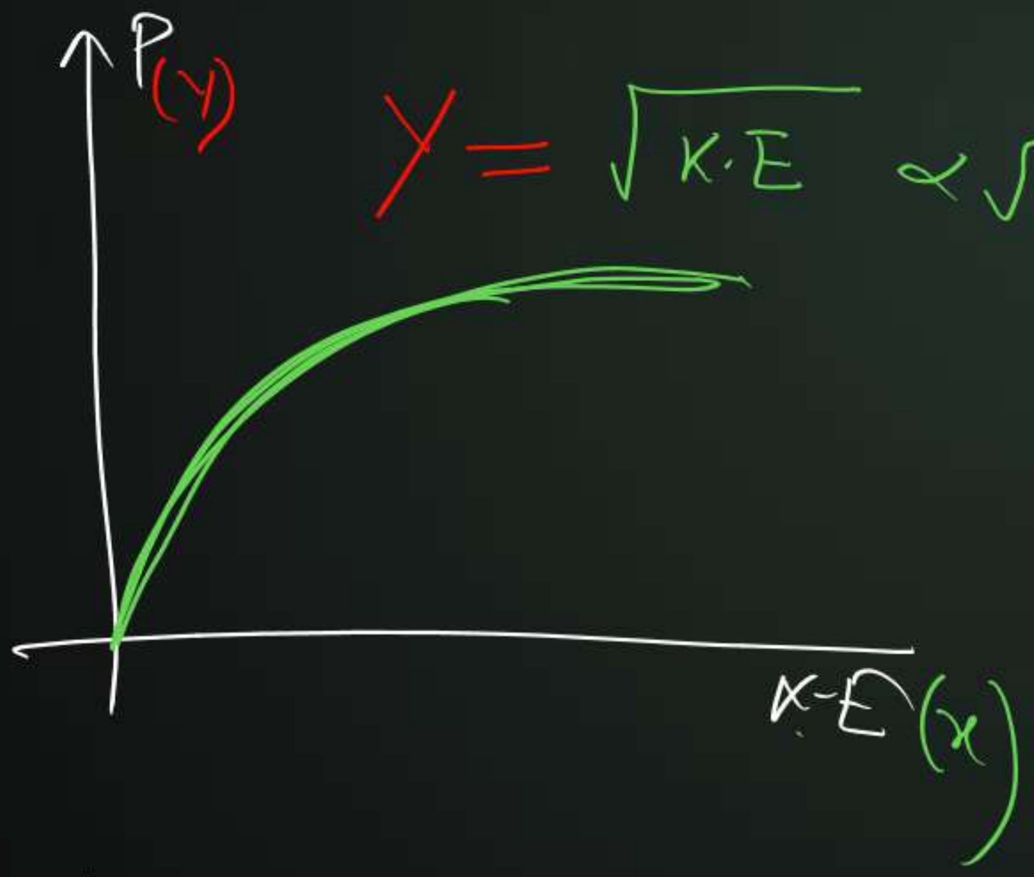
We know electric field due to point charge $E = \frac{k\theta}{r^2}$ then, draw electric field distance graph.

De-Broglie wavelength $\lambda = \frac{h}{P}$. Draw graph between wavelength and momentum.

We know K.E. and momentum relation $P = \sqrt{2m\,K.E.}$ Draw graph between P and K.E.

$$P = \sqrt{\underbrace{2m}_{\text{const}}\ K.E}$$

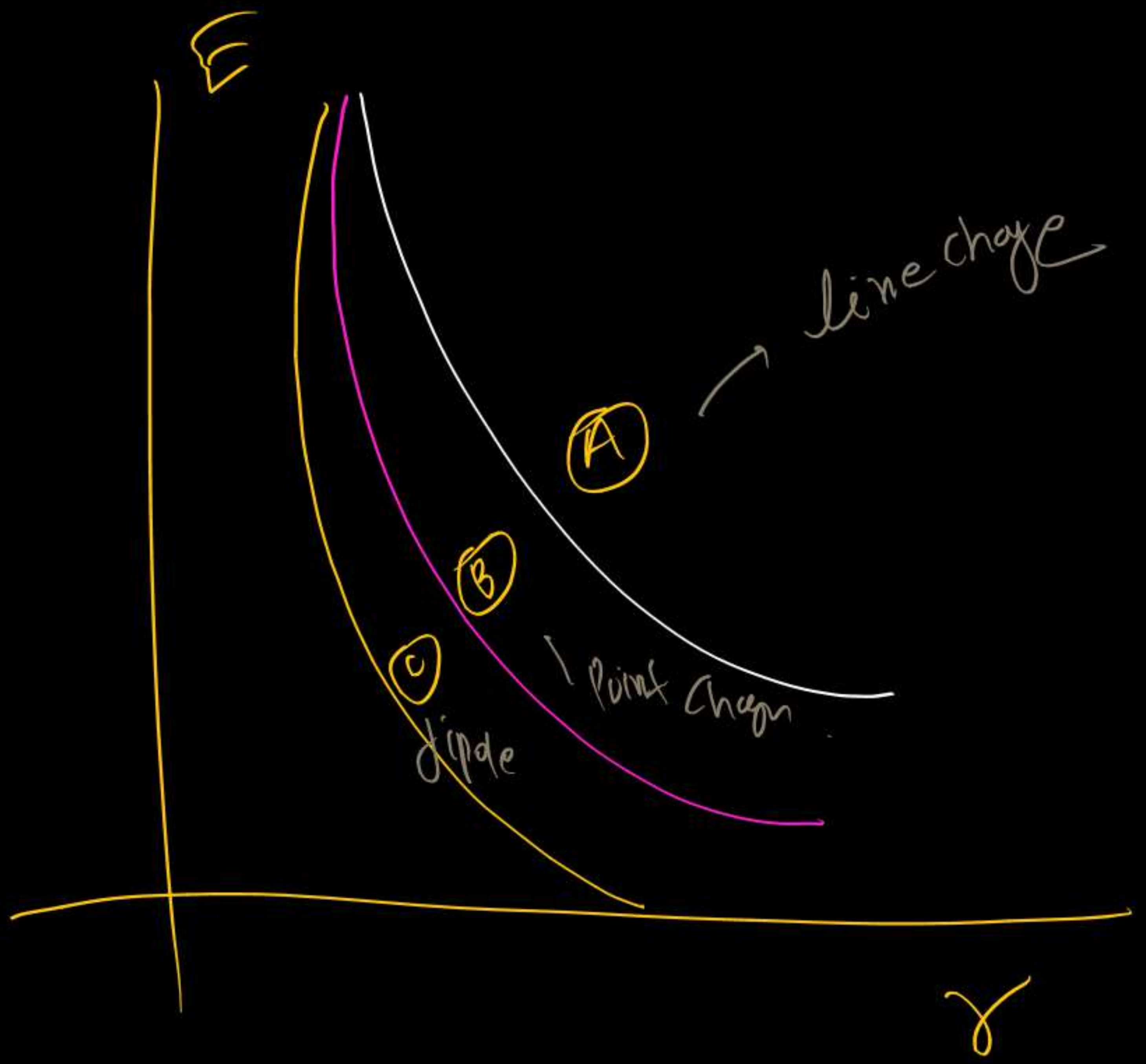
$$Y = \sqrt{K.E} \propto \sqrt{x}$$



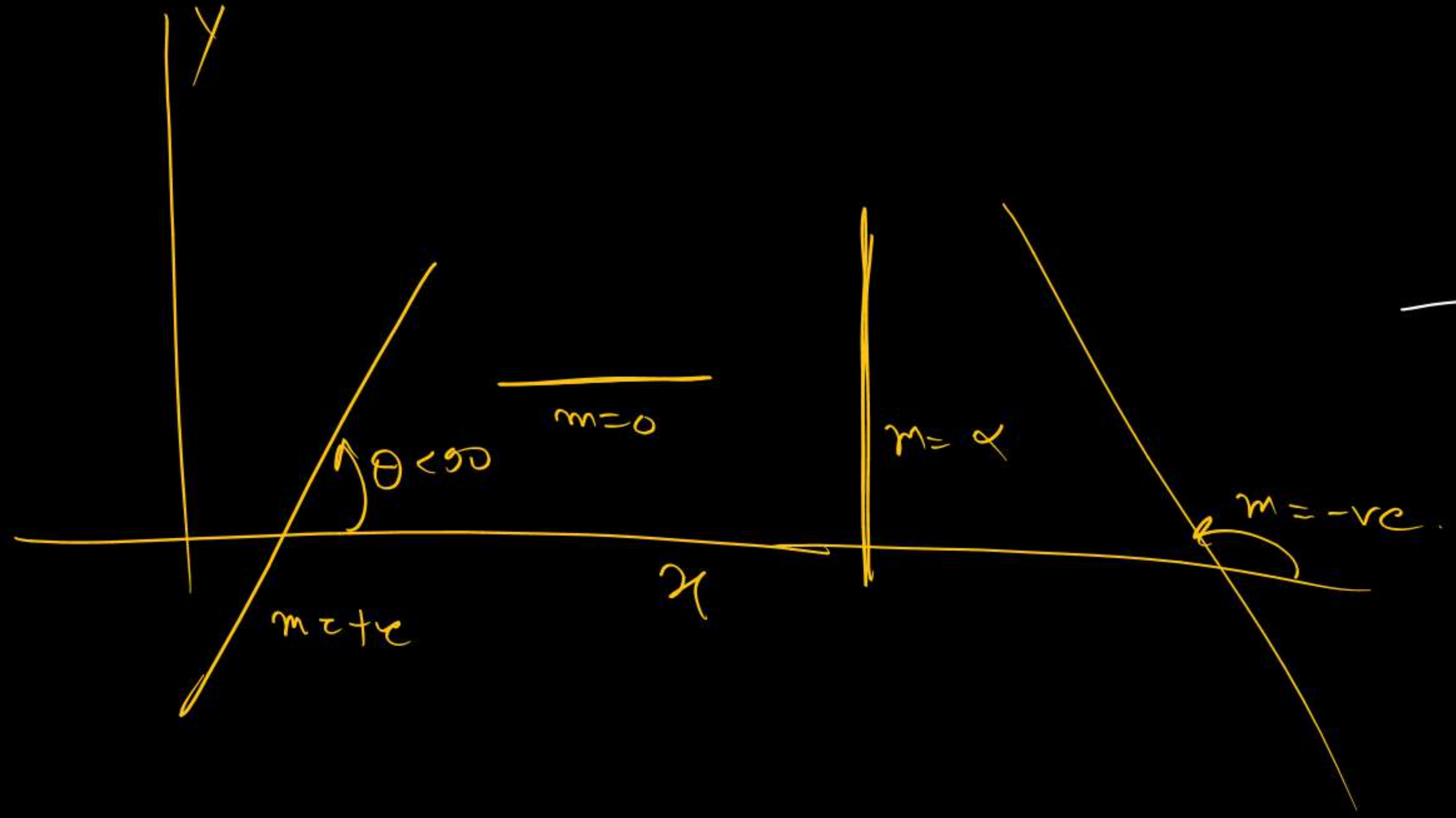
$$E_{\text{line charge}} = \frac{2K\lambda}{r}$$

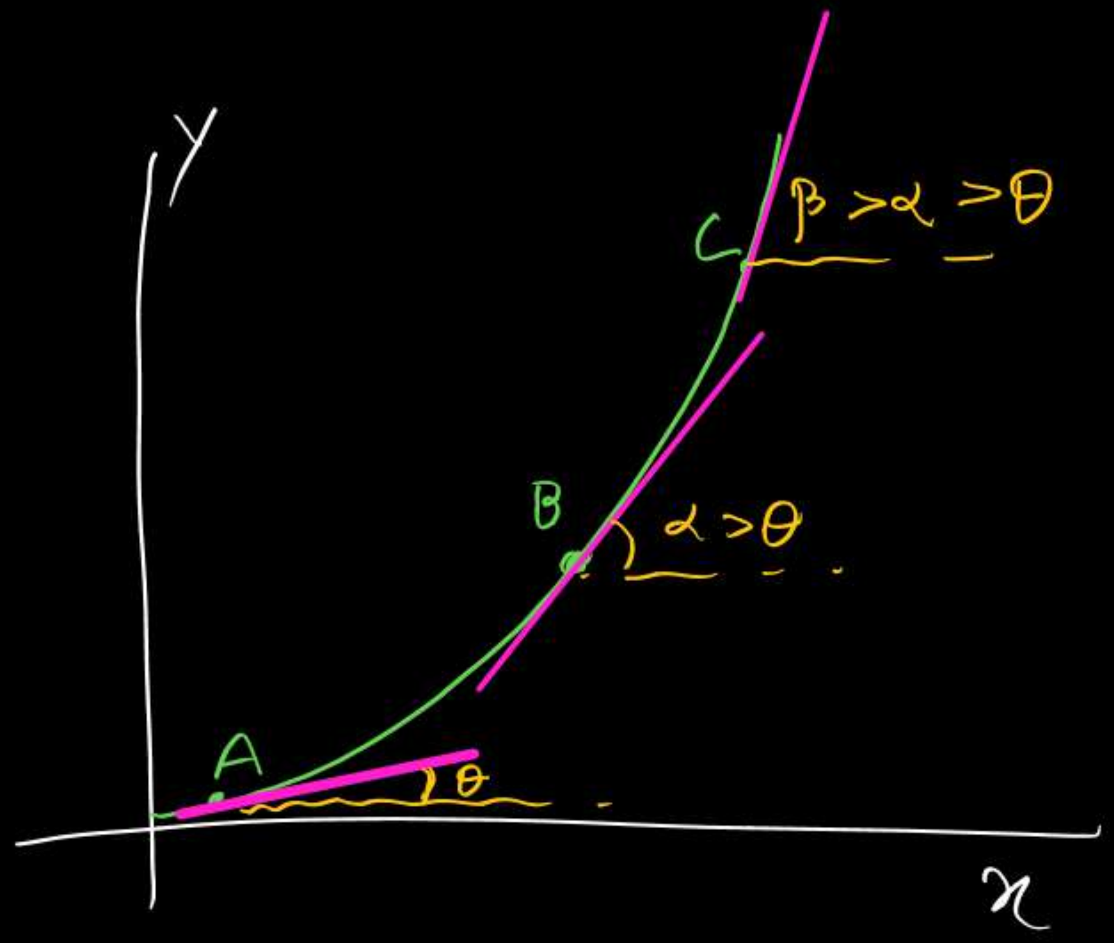
$$E_{\text{dipole}} = \frac{2Kp}{r^3}$$

$$E_{\text{Point ch}} = \frac{KQ}{r^2}$$



Slope - variation





$m_A = +ve$

$m_B = +ve$

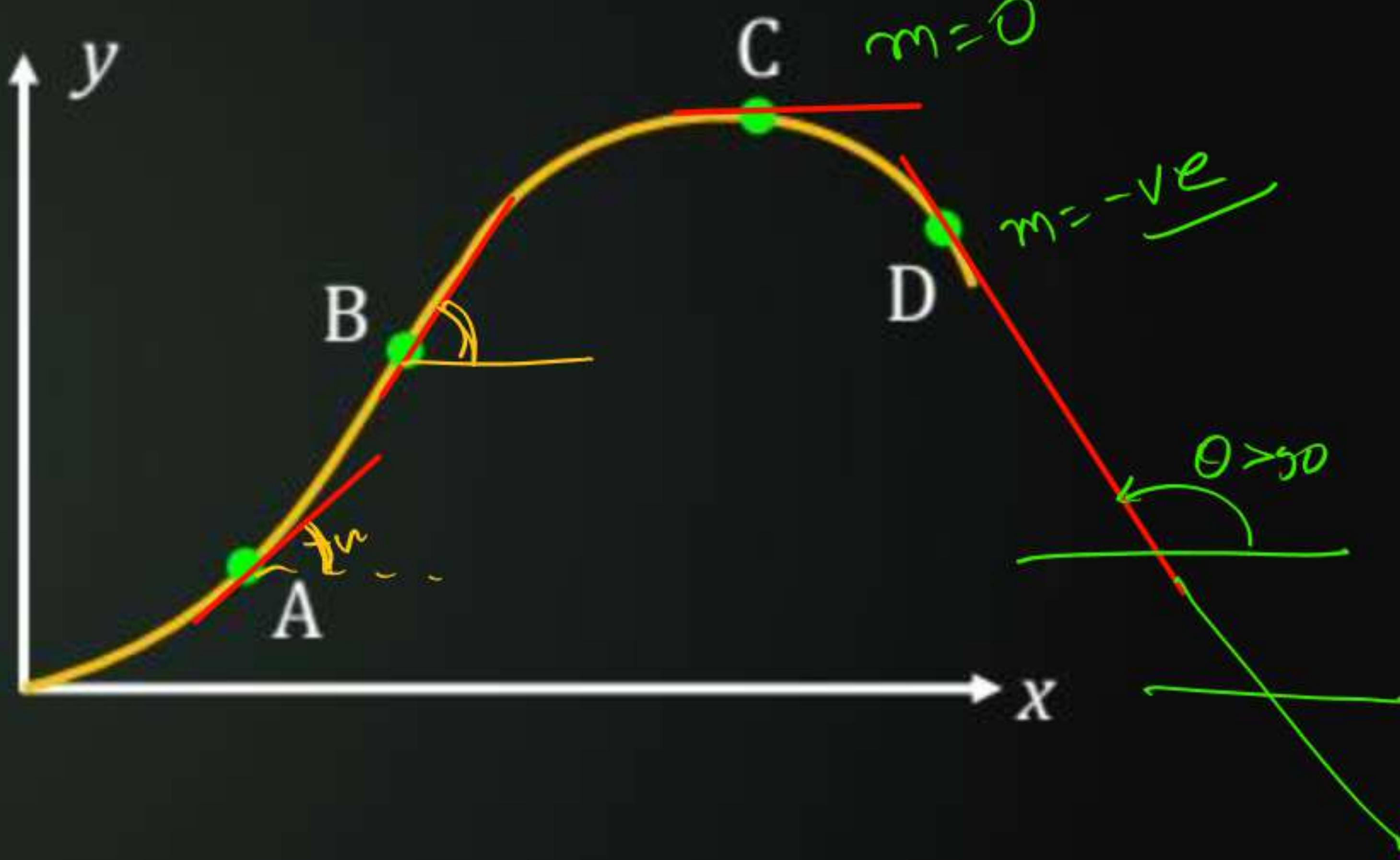
$m_C = +ve$

+ve (Incrsiy)

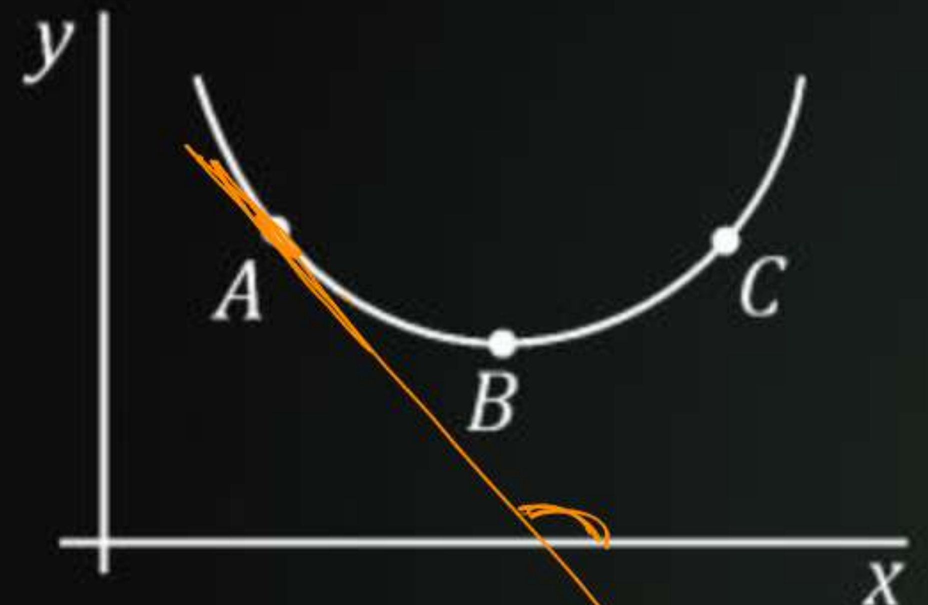
QUESTION

Match the matrix :

Point	Slope
A	Zero
B	negative
C	~~Maximum~~
D	Positive



Comment on x-slope and Variation of Slope



$-4 \quad -3 \quad -2 \quad 0 \quad 2 \quad 3 \quad 4$

$m_A = -ve$

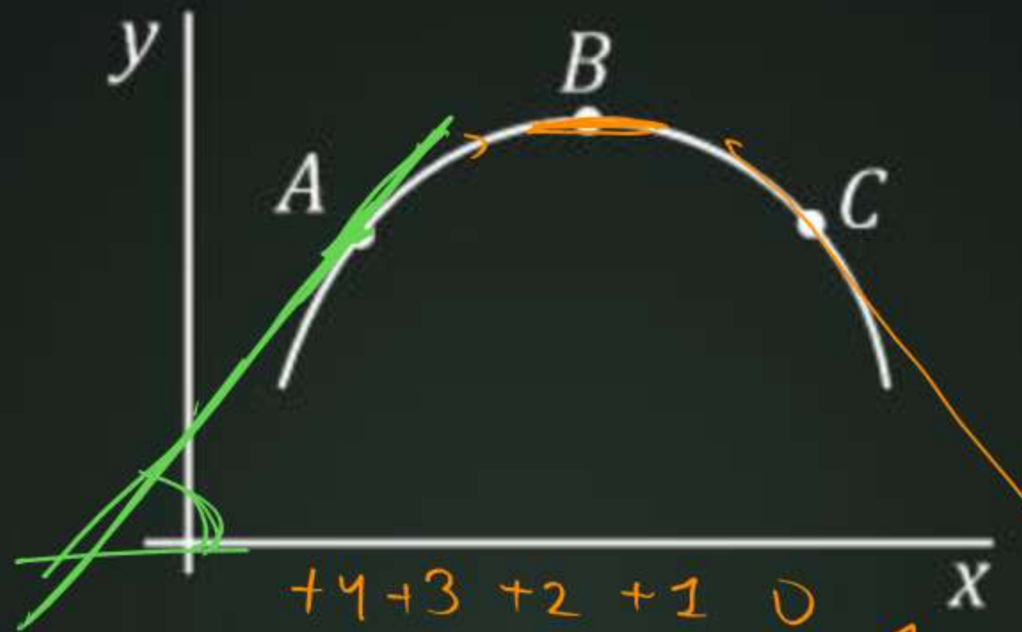
$m_B = 0$

$m_C = +ve$

Slope ↑

Magnitude of Slope =

1st decrea then Incre



$+4 \quad +3 \quad +2 \quad +1 \quad 0 \quad -1 \quad -2 \quad -3 \quad -4$

$m_A = +ve$

$m_B = 0$

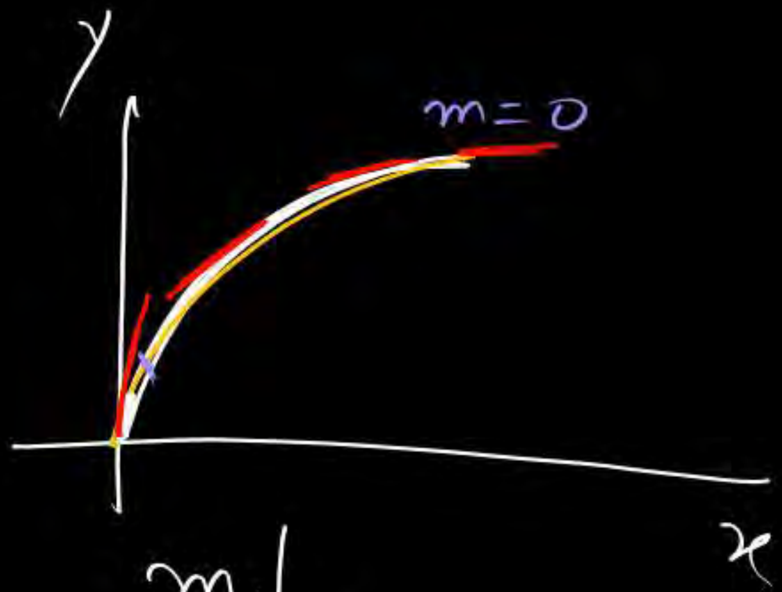
$m_C = -ve$

Slope → decresing

magnitude of slope | 1st decresing then incresing

Slope ↑

Slope ↓

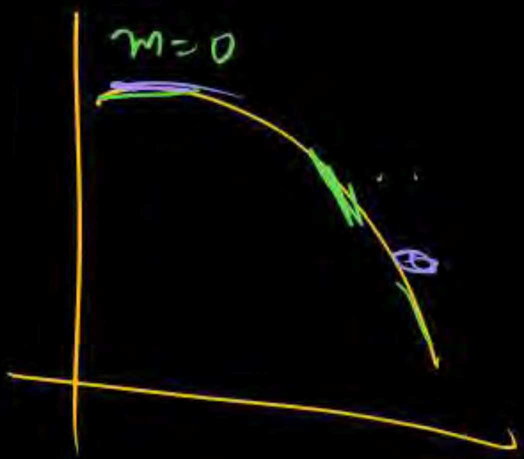


$m \downarrow$

Magnitude of Slope → ↓

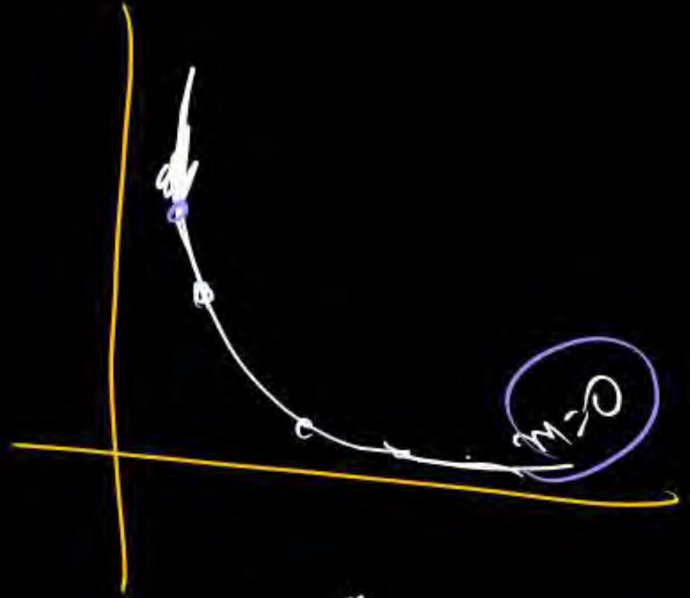
$m \uparrow$ +ve

Magnitude of Slope → ↑



$m \downarrow$ (-ve)

Magnitude of Slope → ↑



$m \uparrow$

Magnitude of Slope → ↓

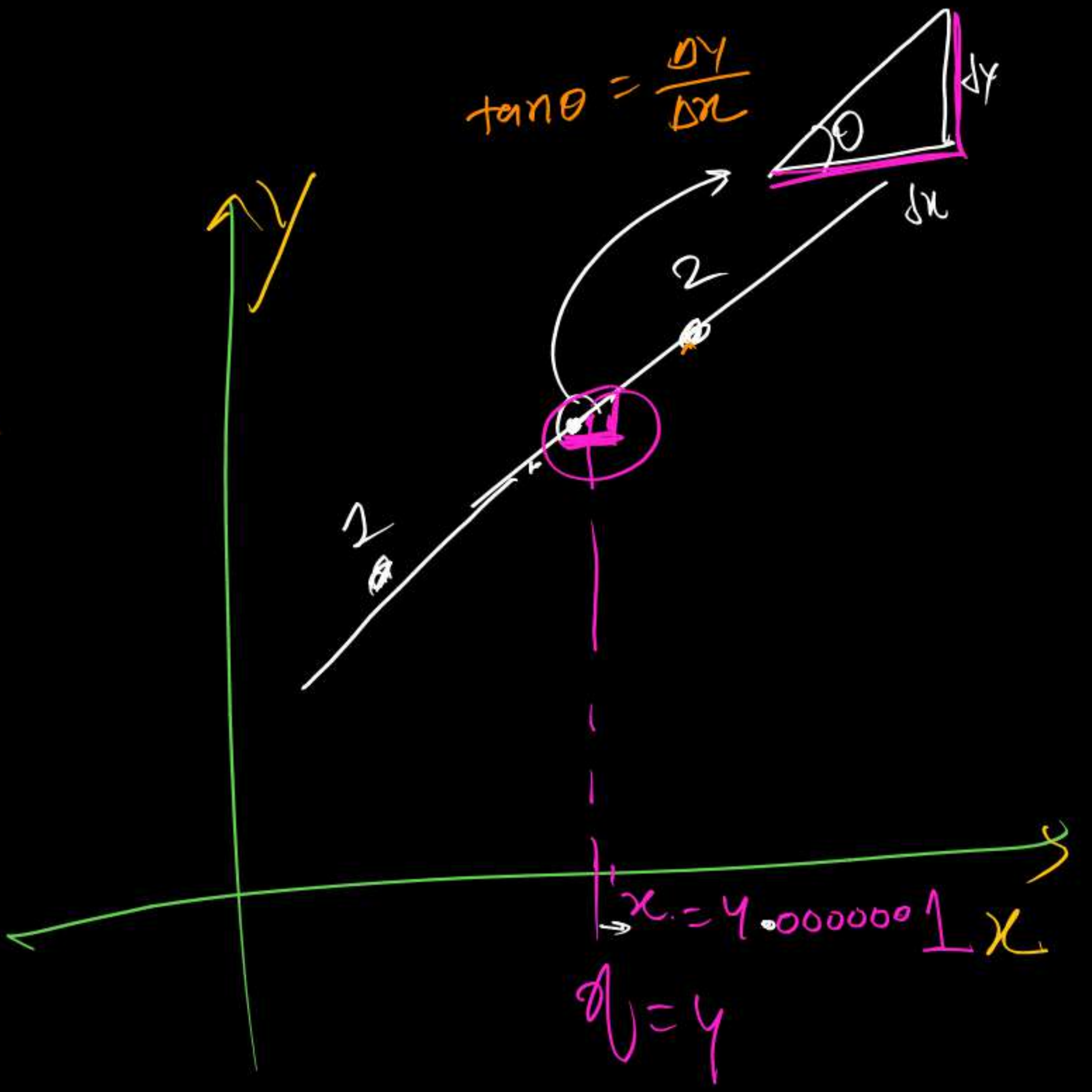
$$\text{Slope}_{\Delta y}\ (m) = \left(\frac{\Delta y}{\Delta x}\right)$$

$$\text{Slope of graph} = (\tan\theta) = \underbrace{\frac{dy}{dx}}_{\text{Inst. diff}^n} = \frac{d(y)}{dx}$$

$$\frac{d[y]}{dx} = \text{diff}^n \text{ of } y \text{ w.r.t. } x$$

$$\frac{dy}{dx} = \text{Rate of change 'y' w.r.t. 'x'}$$

$\cancel{\frac{dy}{dx}}$ $\cancel{\frac{dy}{dx}}$ $\frac{d[y]}{dx} = \text{diff}^n \text{ of } y$



$\frac{d}{dx}$ = differential operator.

$\frac{d \boxed{y}}{dt}$ = differentiation of $\boxed{y}$ w.r.t t

$\frac{d\boxed{}}{dp}$ =

$\frac{d\boxed{}}{dm}$ =

$\frac{d}{d(M.R)}$ =

Rule of diff^n $\longrightarrow$ 1

diff^n of const value

$$\frac{dy}{dx} = 0$$

cst

$$\frac{d(5)}{dt} = 0$$

$$\frac{d(6)}{dt} = 0 \qquad \frac{d7}{dt} = 0$$

$$\frac{d\pi}{dt} = 0$$

$$\frac{dG}{dt} = 0$$

$$\frac{dR}{dt} = 0$$

If $\frac{dy}{dx} = 0$

then y must be const^n

$y = $ cst^n

$$\frac{d(\sin 90^\circ)}{d\theta} = 0$$

$$\frac{d(\cos\theta \cdot \sec\theta)}{d\theta} = 0$$

$$\frac{d(\sin^2\theta + \cos^2\theta)}{d\theta} = 0$$

$$\frac{d\boxed{\square}}{dx} = \text{Differential operator}$$

$$= \text{Rate of change in } \square \text{ w.r.t. '}x\text{'.}$$

$$\frac{dy}{dt} = \text{Rate of charge in } y \text{ w. r. t. time}$$

Rule: (1)

Differentiation of any constant is zero. $Y = C$

$$\frac{d5}{dx} = 0$$

$$\frac{dG}{dt} = 0$$

$$\frac{d\pi}{dt} = 0$$

$$\frac{d(\sin 90°)}{dt} = 0$$

$$\frac{d(x^{0})}{dx} = \frac{d1}{dx} = 0$$

QUESTION

If $\frac{dy}{dx} = 0$ then y is?

(A) Zero

(B) Variable

(C) Constant

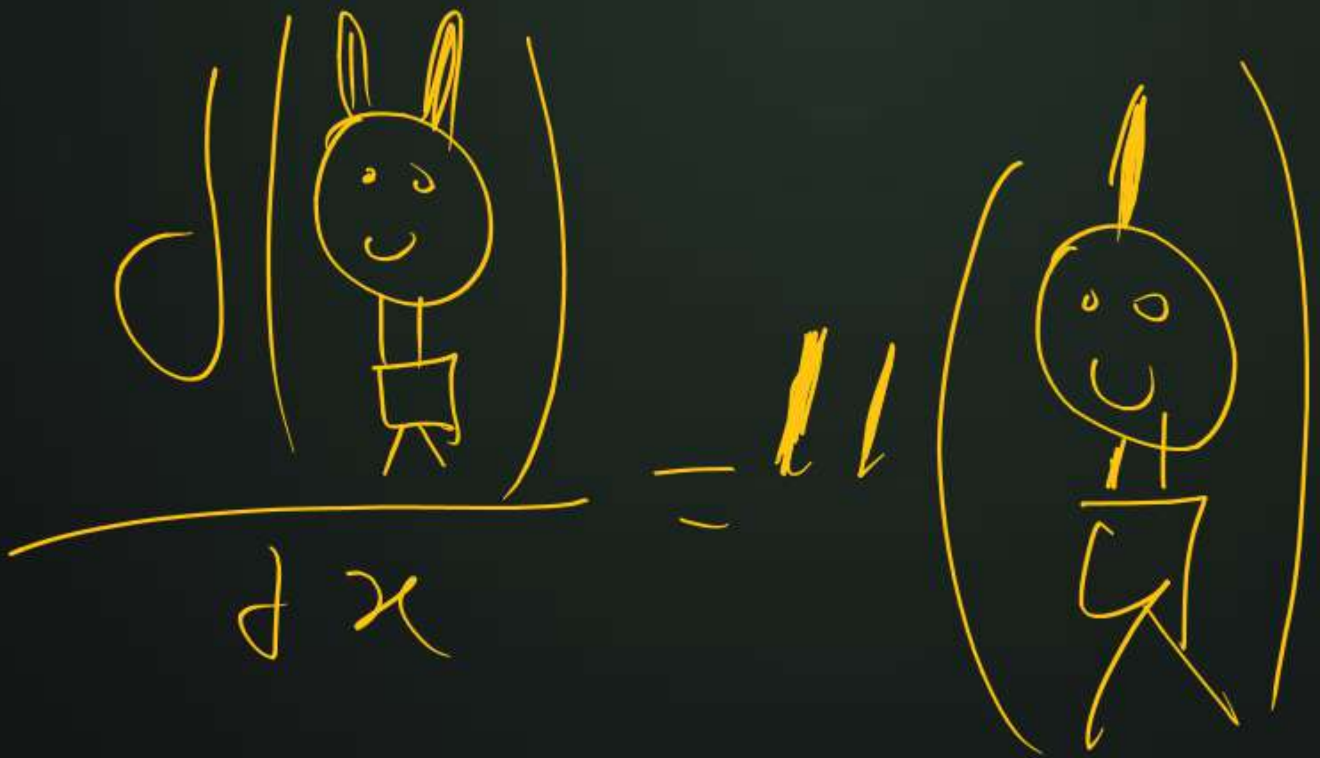
(D) Non-zero number

Rule: (2)

Differentiation of algebraic function $y = \boxed{f(x) = x^n}$

n = constant

$$\frac{d(x^n)}{dx} = nx^{n-1}$$



$$\frac{dx^2}{dx} = 2x^{2-1}$$

$$= 2x$$

$$\frac{dx^3}{dx} = 3x^{3-1} = 3x^2$$

$$\# \quad \frac{d\sqrt{x}}{dx} = \frac{dx^{1/2}}{dx} = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-1/2} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$$

Example:

$$\frac{dx^5}{dx} = 5x^4$$

$$\frac{dx^{+9}}{dx} = 9x^8$$

$$\frac{dx^y}{dt} =$$

$$\frac{dx^{3/2}}{dx} = \frac{3}{2}x^{\frac{3}{2}-1} = \frac{3}{2}\sqrt{x}$$

$$\frac{d\sqrt{x}}{dx} = \frac{1}{2\sqrt{x}}$$

$$\frac{d\left(\frac{1}{x}\right)}{dx} = \frac{dx^{-1}}{dx} = -1x^{-1-1} = -1x^{-2} = \frac{-1}{x^2}$$

$$\frac{d\left(\frac{1}{x^4}\right)}{dx} = \frac{-4}{x^5} \qquad \frac{d\left(\frac{1}{x^3}\right)}{dx} = \frac{-3}{x^4}$$

$$\frac{d\,\frac{1}{x^2}}{dx} = \frac{-2}{x^3}$$

$=$

Example:

$\sqrt{x} = x^{1/2}$ $\quad \frac{1}{\sqrt{x}} = \frac{1}{x^{1/2}} = x^{-1/2}$

$$\frac{d\left(\frac{1}{\sqrt{x}}\right)}{dx} = \frac{d x^{-1/2}}{dx} = -\frac{1}{2} x^{-\frac{1}{2}-1}$$

$$= \frac{-1 x^{-3/2}}{2}$$

$$\frac{dx^{-\frac{1}{3}}}{dx} = -\frac{1}{3} x^{-\frac{1}{3}-1}$$

$$\frac{d\left(\frac{1}{x^2}\right)}{dx} = -\frac{2}{x^3}$$

$$\frac{dx^{-7}}{dx} = -7 x^{-7-1} = -7x^{-8}$$

Rule: (3)

If any constant number is multiplied with variable then it comes out from differentiation.

$y = c\,f(x)$

$y = 3x^2$

Example:

$y = 3x^2$

$\frac{dy}{dx} = \frac{d\,3x^2}{dx} = 3\frac{dx^2}{dx} = 3(2x) = 6x$

$y = 4x^2 \longrightarrow \frac{dy}{dx} = 4\frac{dx^2}{dx} = 4(2x)$

$= 8x$

$y = 4\sqrt{x}$

$\frac{dy}{dx} = 4\frac{d\sqrt{x}}{dx} = 4 \times \frac{1}{2} x^{\frac{1}{2}-1}$

$y = \frac{2}{x^3} = 2x^{-3}$

$\frac{dy}{dx} = 2(-3x^{-3-1})$

$= -6x^{-4}$

$y = A\sin x$

$y = 4e^{x}$

$$\frac{d(\sin\theta)}{d\theta} = \cos\theta$$

$$\frac{d\sin(x)}{dx} = \cos(x)$$

$$\frac{d\cos(\theta)}{d\theta} = -\sin\theta$$

$$\frac{d\tan(x)}{dx} = \sec^2\theta$$

$$* \quad \boxed{\frac{d(e^x)}{dx} = e^x}$$

$$\frac{d(\cot\theta)}{d\theta} = -\operatorname{cosec}^2\theta$$

$$\frac{d\sec\theta}{dt} = \sec\theta\cdot\tan\theta$$

$$\frac{d(\log_e x)}{dx} = \frac{d\ln x}{dx} = \frac{1}{x}$$

Addition Rule	Subtraction Rule	Multiplication Rule	Division Rule
$Y = A + B$ "diff^n w.r.t. x" $\frac{dy}{dx} = \frac{d(A+B)}{dx}$ $= \frac{dA}{dx} + \frac{dB}{dx}$	$Y = A - B$ $\frac{dy}{dx} = \frac{d(A-B)}{dx}$ $= \frac{dA}{dx} - \frac{dB}{dx}$	$Y = \frac{A}{B}$ $\frac{dy}{dx} = \frac{B\frac{dA}{dx} - A\frac{dB}{dx}}{B^2}$	$Y = A \cdot B$ $\frac{dy}{dx} = \frac{d(A \cdot B)}{dx}$ $= \left(\frac{dA}{dx}\right)B + \left(\frac{dB}{dx}\right)A$

$= \frac{dA}{dx} \cdot \frac{dB}{dx}$

QUESTION

$y = \tan x \cdot \log x$

$$\frac{dy}{dx} = \frac{d\tan x}{dx}\log x + \tan x\,\frac{d\log x}{dx}$$

$$= \sec^2 x \log x + \tan x\,\frac{1}{x}$$

QUESTION

$y = x^2 - 4x + 3$, then find value of $\frac{dy}{dx}$ at $x = 2$

$$\frac{dy}{dx} = 2x - 4\times 1 + 0$$

$$\left(\frac{dy}{dx}\right) = 2x - 4$$

$$\left.\frac{dy}{dx}\right|_{x=2} = 2\times 2 - 4 = 0$$

QUESTION

$y = e^x(\sin(x))$

$y = e^x \cdot \sin x$

$\frac{dy}{dx} = \frac{de^x}{dx}\sin x + \frac{d\sin x}{dx} e^x$

$= \underbrace{e^x \sin x + \cos x\, e^x}$

$y = e^x + \sin x$

$\frac{dy}{dx} = e^x + \cos x$

$y = \underbrace{e^x - \sin x}$

$\frac{dy}{dx} = e^x - \cos x$ ✓

QUESTION

$$y = \cos x + \frac{2}{x^3}$$

$$\frac{dy}{dx} = \frac{d\cos x}{dx} + 2\frac{dx^{-3}}{dx}$$

QUESTION

$\frac{dx}{dx} = 1$

Double Differentiation

$y = 1x^5 + 2x^4 + 3x^3 + 4x^2 + 5x^1 + 6x^0$

find $\frac{dy}{dx}$ (diffⁿ of Rule addition)

$$\frac{d(x^5 + 2x^4 + 3x^3 + 4x^2 + 5x^1 + 6x^0)}{dx} = 5x^4 + 8x^3 + 9x^2 + 8x + 5 \times 1$$

$$\left(\frac{dy}{dx}\right) = 5x^4 + 8x^3 + 9x^2 + 8x + 5$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \left(\frac{d^2y}{dx^2}\right) = 20x^3 + 24x^2 + 18x + 8$$

$$\frac{d^3y}{dx^3} = 60x^2 + 48x + 18$$

$$\frac{d^4y}{dx^4} = 120x + 48$$

$$\frac{d^5y}{dx^5} = 120$$

$$\frac{d^6y}{dx^6} = 0$$

$$y = A \sin x$$

$$\frac{dy}{dx} = A \frac{d \sin x}{dx}$$

$$\frac{dy}{dx} = A \cos\theta$$

$$\frac{d^2y}{du^2} = -A \sin\theta$$

$$\frac{d^3y}{du^3} = -A \cos\theta \qquad \frac{d^4y}{du^4} = +A \sin\theta$$

$$y = e^t$$

$$\frac{dy}{dt} = e^t$$

$$\frac{d^2y}{dt^2} = e^t$$

$$\frac{d^3y}{dt^3} = e^t \checkmark$$

QUESTION

Find 5th order differentiation of given function:

$y = e^x$

$y = \sin x$

Function of a Function

* $y = \sin x \cdot e^x$

Product Rule of diffⁿ.

$y = \log x . (x^5)$

Produce Rule

$y = \sin x + \cos(x)$

addⁿ Rule of diff

$y = x^5 - \tan x$

Subtra Rule

(*) $y = \sin(e^x)$

$Y = \sin(e^x)$

$y = e^{(4x)}$ $\quad Y = e^{(x^2)}$ $\quad Y = e^{(\sin x)}$

$y = \sin(x^2 + 2x)$

$y = (x^2 + 4)^4$

अल्जीत ~~Chain Rule~~

outside Inside Rule

Out-side - Inside Rule of diffⁿ

$y = \sin(x^2)$

Inside

Chain Rule

(Let) $x^2 = t$

diffⁿ w.r.t x

$$\frac{dx^2}{dx} = \frac{dt}{dx}$$

$$2x = \frac{dt}{dx}$$

$$2x\,dx = dt \quad \text{—(1)}$$

$y = \sin(t)$

diffⁿ w.r.t.

$$\frac{dy}{dt} = \cos t$$

$$\frac{dy}{2x\,dx} = \cos t$$

$$\frac{dy}{dx} = \cos t \times 2x$$

$$\boxed{\frac{dy}{dx} = \cos(x^2) \times 2x}$$

out-sinside

$$\frac{dy}{dx} = \left(\begin{array}{c}\text{diff}^n \text{ of outer function} \\ \text{keep inside} \\ \text{as it is}\end{array}\right) \times \left(\begin{array}{c}\text{diff}^n \text{ of Inner function} \\ \text{w.r.t. } x\end{array}\right)$$

$$y = \underset{\text{out}}{\sin}(x^2)$$

$$\frac{dy}{dx} = \cos(x^2) \times (2x)$$

$$y = \underset{\text{out}}{e}^{(4x)} \longrightarrow \frac{dy}{dx} = e^{(4x)} \times \frac{d(4x)}{dx} = e^{4x} \times 4$$

QUESTION

$y = 4 \sin(\beta t)$ find $\frac{dy}{dt}$ where α and β are constant.

$$y = 4 \sin \beta(t)$$

$$\frac{dy}{dt} = 4 \frac{d \sin(\beta t)}{dt}$$

$$= 4 \left[\cos(\beta t) \times \beta\right)$$

$$= 4\beta \cos(\beta t)$$

$$\# \quad \boxed{y = A \times \sin(\omega t)}$$

$$\frac{dy}{dt} = \frac{d(A\sin(\omega t))}{dt}$$

$$= A\,\frac{d(\sin(\underline{\omega t}))}{dt}$$

$$= A\cos(\omega t) \times \omega$$

$$\frac{dy}{dt} = A\omega\cos(\omega t)$$

QUESTION

$y = 4e^{3t}$ find $\frac{dy}{dt}$

$$\frac{dy}{dt} = 4\ e^{3t} \times 3$$

$$= 12\,e^{3t}$$

QUESTION

$y = 5\sin(4+3t)$ find $\frac{dy}{dt}$

$y = 5 \sin(4+3t)$

outer inner

$$\frac{dy}{dt} = \frac{5\, d \sin(4+3t)}{dt}$$

$$= 5\cos(4+3t) \times \frac{d(4+3t)}{dt}$$

$\frac{dy}{dt} = (5\times3)\cos(4+3t)$

QUESTION

$y = (x^2 - 4x)^3$ find $\frac{dy}{dx}$.

$y = (x^2 - 4x)^3$ find $\frac{dy}{dx}$.

$\frac{d(x^2-4x)^3}{d \cdot x} = 3(x^2-4x)^{3-1} \times (2x-4)$

$= 3(x^2-4x)^2 \times (2x-4)$

Ap

$\frac{dx^3}{dx} = 3x^{3-2}$

$= 3x^2$

QUESTION

$y = e^{-\alpha^2 t}$ find $\frac{dy}{dt}$ α is constant.

$$y = e^{-\alpha^2 t}$$

$$\frac{dy}{dx} = e^{-\alpha^2 t}\left(-\alpha^2\right)$$

$$= -\alpha^2 e^{-\alpha^2 t}$$

$$y = e^{(\alpha t^2)}$$

$$\frac{dy}{dx} = e^{\alpha t^2}\left[\alpha 2t\right)$$

$$= \alpha 2t\, e^{\alpha t^2}$$

QUESTION

$y = 4e^{3t}$ find $\frac{dy}{dt}$

$$\frac{dy}{dt} = 4\, e^{3t} (3)$$

$$= 12\, e^{3t}$$

$$y = \sqrt{x^2+2x} = (\underbrace{x^2+2x}_{\text{Inside}})^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}\left(x^2+2x\right)^{\frac{1}{2}-1} \times (2x+2)$$

$$= \frac{1}{2}\left(x^2+2x\right)^{-\frac{1}{2}} \times (2x+2)$$

$$y = (x^2+4)^{-2}$$

$$\frac{dy}{dx} = -2\,(x^2+4)^{-3} \times (2x+0)$$

$$\frac{dy}{dx} = -4x\,(x^2+4)^{-3}$$

QUESTION

$y = \underline{\alpha \sin(\beta t)}$ find $\dfrac{dy}{dt}$ where α and β are constant.

$$\frac{dy}{dt} = \alpha\left(\cos(\beta t) \times \beta\right.$$

QUESTION

$y = 5\sin(4 + 3t)$ find $\dfrac{dy}{dt}$

QUESTION

$y = (x^2 - 4x)^3$ find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = 3(x^2-4x)^2(2x-4)$$

QUESTION

$y = e^{-\alpha^2 t}$ find $\frac{dy}{dt}$ α is constant.

$$y = x^2$$

$$\frac{dy}{dx} = \frac{dx^2}{dx} = 2x$$

$$y = x^2 \quad \text{find } \frac{dy}{dt} = ??$$

$$\frac{dy}{dx} = \frac{dx^2}{dx}.$$

$$\frac{dy}{dt} = 2x \times \frac{dx}{dt}$$

$$y = t^3$$

find $\dfrac{dy}{dx}$

$$\frac{dy}{dx} = 3t^2 \frac{dt}{dx}$$

QUESTION

If $V = \frac{4}{3}\pi R^3$; find rate of change in volume w. r. t. time $\left(\frac{dV}{dt}\right)$ = ??

$$V = \frac{4}{3}\pi R^3$$

$$\frac{dV}{dR} = \frac{4}{3}\pi \frac{dR^3}{dR}$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cancel{3} R^2 \frac{dR}{dt}$$

$$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$$

R

QUESTION

If radius of circle is increasing $\frac{1}{\pi}$ m/s then find rate of change in area when radius is 4m.

$\rightarrow \quad \frac{dR}{dt} = \frac{1}{\pi}\ m/s$

$A = \pi R^2$

$\frac{dA}{dR} = \pi 2R$

$\frac{dA}{dt} = \pi 2R \frac{dR}{dt} = \cancel{\pi} 2 \times 4 \frac{1}{\cancel{\pi}} = 8$

Partial diffⁿ

$$V = f(x, y, z)$$

Partial diffⁿ of V w.r.t x $= \left(\frac{\partial v}{\partial x}\right)_{y, \& z \text{ const}}$

$$+ \left(\frac{\partial v}{\partial y}\right)_{x, z \text{ const}} + \left(\frac{\partial v}{\partial z}\right)_{x \& y \text{ const}}$$

$$\frac{\partial v}{\partial r} = \left(\frac{\partial r}{\partial x}\right)_{y, \& z \text{ const}} + \left(\frac{\partial r}{\partial y}\right)_{x, z \text{ const}} + \left(\frac{\partial r}{\partial z}\right)_{x, y \text{ const}}$$

QUESTION

If $V = x^2y + y^2z + z^2x$, find $\dfrac{\partial v}{\partial x}$

$$\left(\frac{\partial v}{\partial x}\right) = \frac{\partial(x^2y + y^2z + z^2x)}{\partial x} = y\frac{\partial x^2}{\partial x} + \cancel{\frac{\partial(y^2z)}{\partial x}}^{\,0} + z^2\frac{\partial x}{\partial x}$$

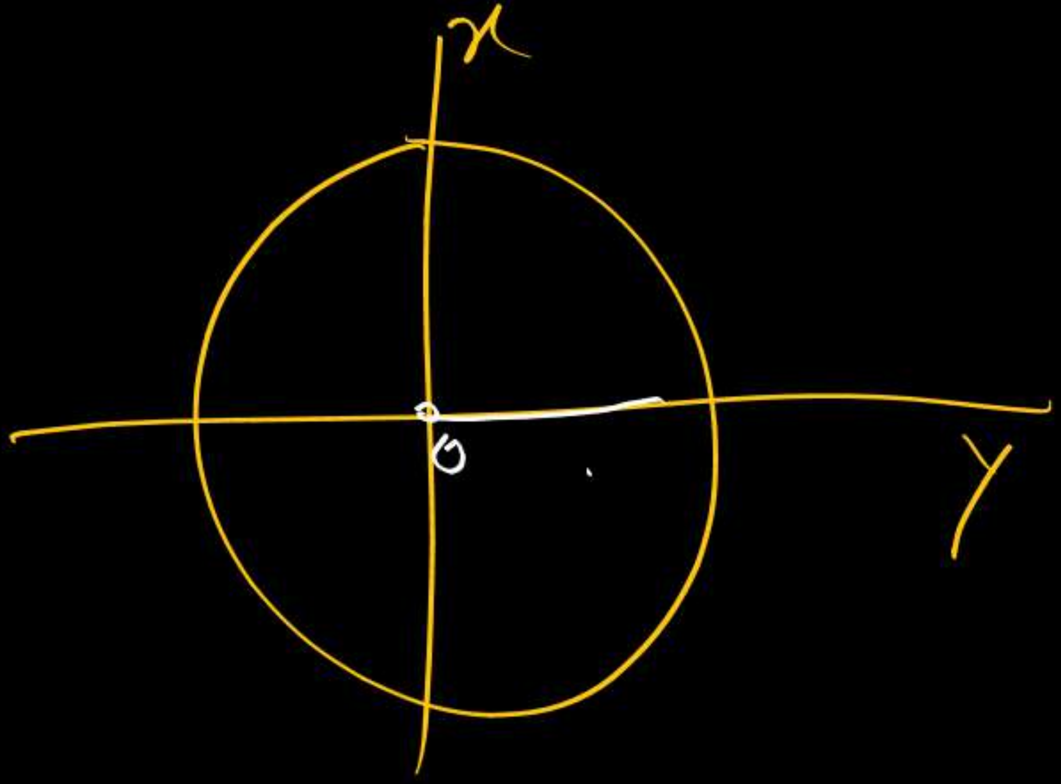
$$= y(2x) + 0 + z^2$$

$$\# \left(\frac{\partial v}{\partial y}\right)_{x,z\ \text{const}} = \frac{\partial(x^2y + y^2z + z^2x)}{\partial y} = x^2\frac{\partial y}{\partial y} + z\frac{\partial y^2}{\partial y} + \cancel{\frac{\partial z^2x}{\partial y}}^{\,0} = 2xy + z^2$$

$$= x^2 + z2y$$

graph of circle

$$x^2+y^2=R^2$$



$$(x-x_0)^2+(y-y_0)^2=R^2$$

$(x_0,y_0)\to$ co-or of centre

Radius

$$(x_0,y_0)=(0,0)$$

centre is at origin

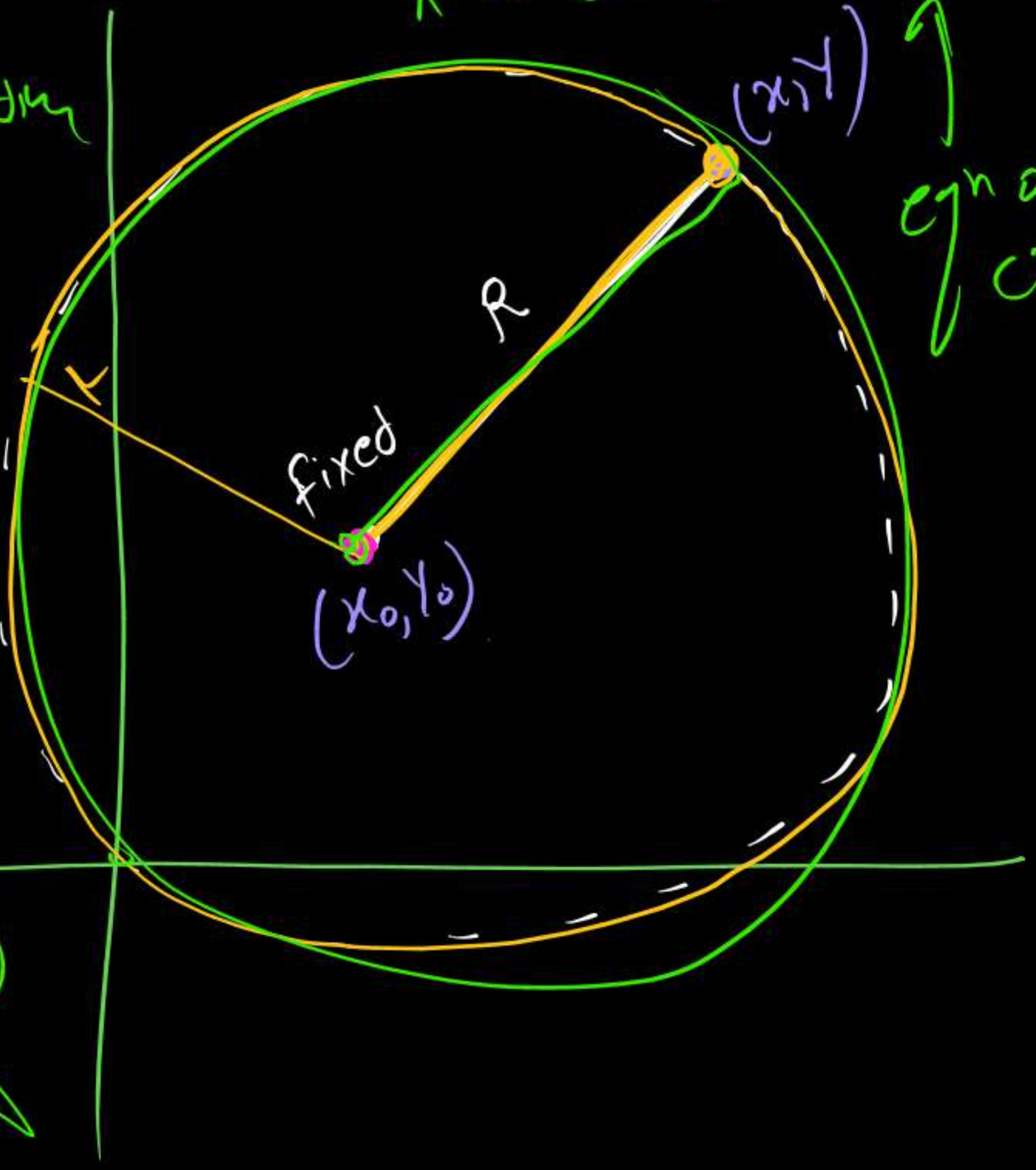
$$x^2+y^2=R^2$$

circle ka equation

$$R=\sqrt{(x-x_0)^2+(y-y_0)^2}$$

$$R^2=(x-x_0)^2+(y-y_0)^2$$

eqn of circle

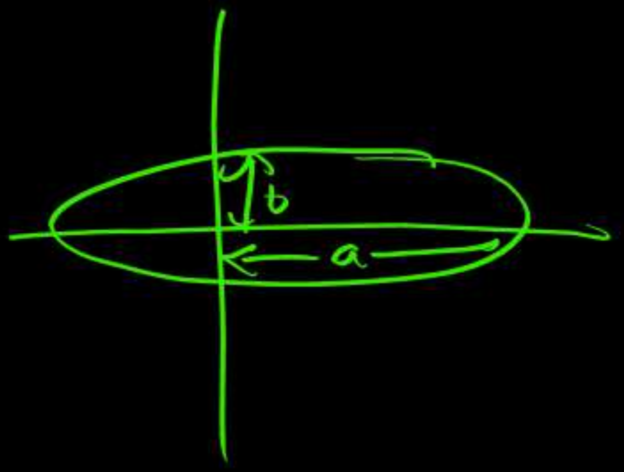


$$\boxed{(y-4)^2+(x-3)^2=25}$$

$$\text{Centre} = (3,4)$$

$$R=5$$

ellipse $\longrightarrow$ खाराब circle

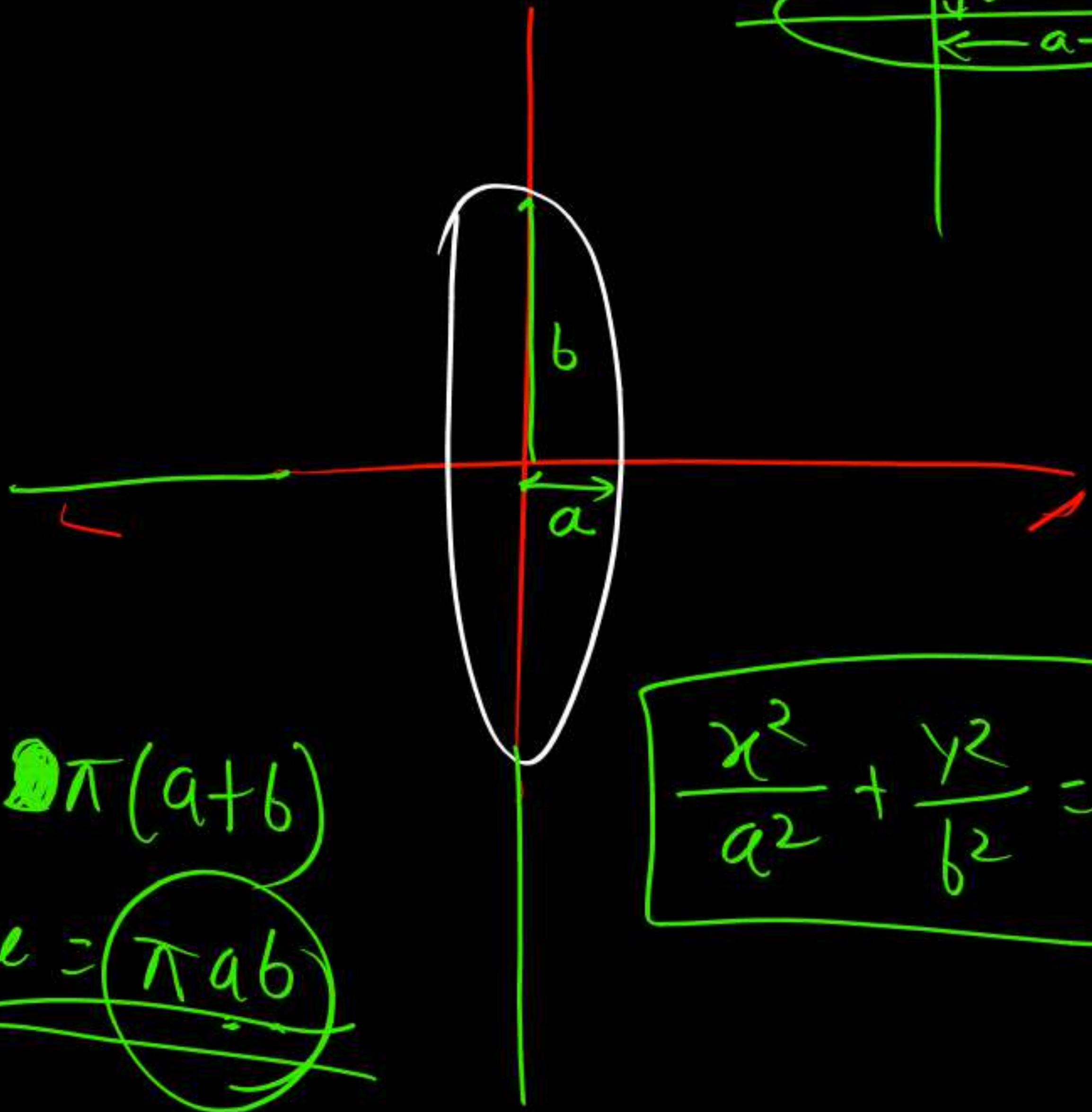


$$x^2 + y^2 = R^2$$

&

$$\boxed{\frac{x^2}{R^2} + \frac{y^2}{R^2} = 1}$$

eqn of circle

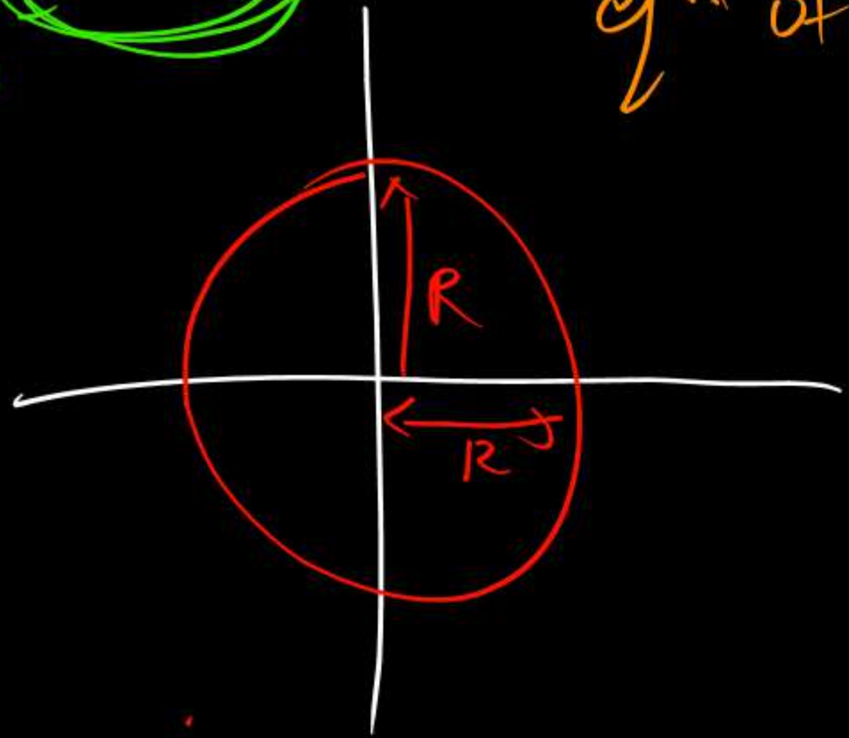


$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

$a = b = R$ circle

Circ $= \pi(a+b)$

Are $= \pi ab$



QUESTION

$y = x^3 - 3x^2 + 4$, find maximum and minimum value.

QUESTION

If $y = e^x$ then find $\frac{dy}{dx}$ at $x = 2$.

QUESTION

$$y = \sin(2x)$$

$$y = \ln(3x + 4)$$

Outside-Inside Rule For Function of a Function

$$\frac{dy}{dx} = \text{(different of outer function keep inside as it is)}$$
$$\times \text{ (different of inner function)}$$

(i) $y = \sin(3x + 4)$

(ii) $y = \cos(e^x)$

QUESTION

Find differentiation of function $y = \sin(2x^2 - 6x)$

QUESTION

Find differentiation of $y = (x^4 - 1)^{50}$.

(iii) $y = e^{(\sin x)}$ $\longrightarrow$ $\frac{dy}{dx} = e^{\sin x} \times \cos x$

(iv) $y = \cos(x^2 + 4x)$

(v) $y = 4\sin(8x)$

(vi) $y = (x^2 + 4x)^2$

QUESTION

Find differentiation of

$$y = A \sin(\omega t - kx)$$

QUESTION

Find differentiation of

(i) $y = \log(3x + 4)$

(ii) $y = (4x + 3)^2$

QUESTION

Find differentiation of

$$y = \frac{x^2}{x^2+1}$$

QUESTION

Find differentiation of

(i) $y = \sin^2 x = (\sin x)^2$

(ii) $y = \sin(x^2)$

QUESTION

Find differentiation of

(i) $y = e^{-\alpha x}$

(ii) $y = e^{(4x-3)}$

(iii) $y = e^{(x^2+2)d}$

QUESTION

If $y = x^2 - 4x$ then find y when rate of change in y w.r.t. x is zero.

QUESTION

Find maxima and minima of

(i) $y = x^2 + 5$

(ii) $y = x^2 - 9x$

(iii) $y = 4x - x^2$

QUESTION

Find double differentiation

$$y = e^{\alpha x}$$

QUESTION

Find double differentiation

$$y = A \sin(\omega t)$$


THANK YOU