

# YAKEEN NEET 2.0

2026

(One Shot)

Motion in a plane

Physics

Summary Lecture

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# Summary Lecture

Motion in Plane

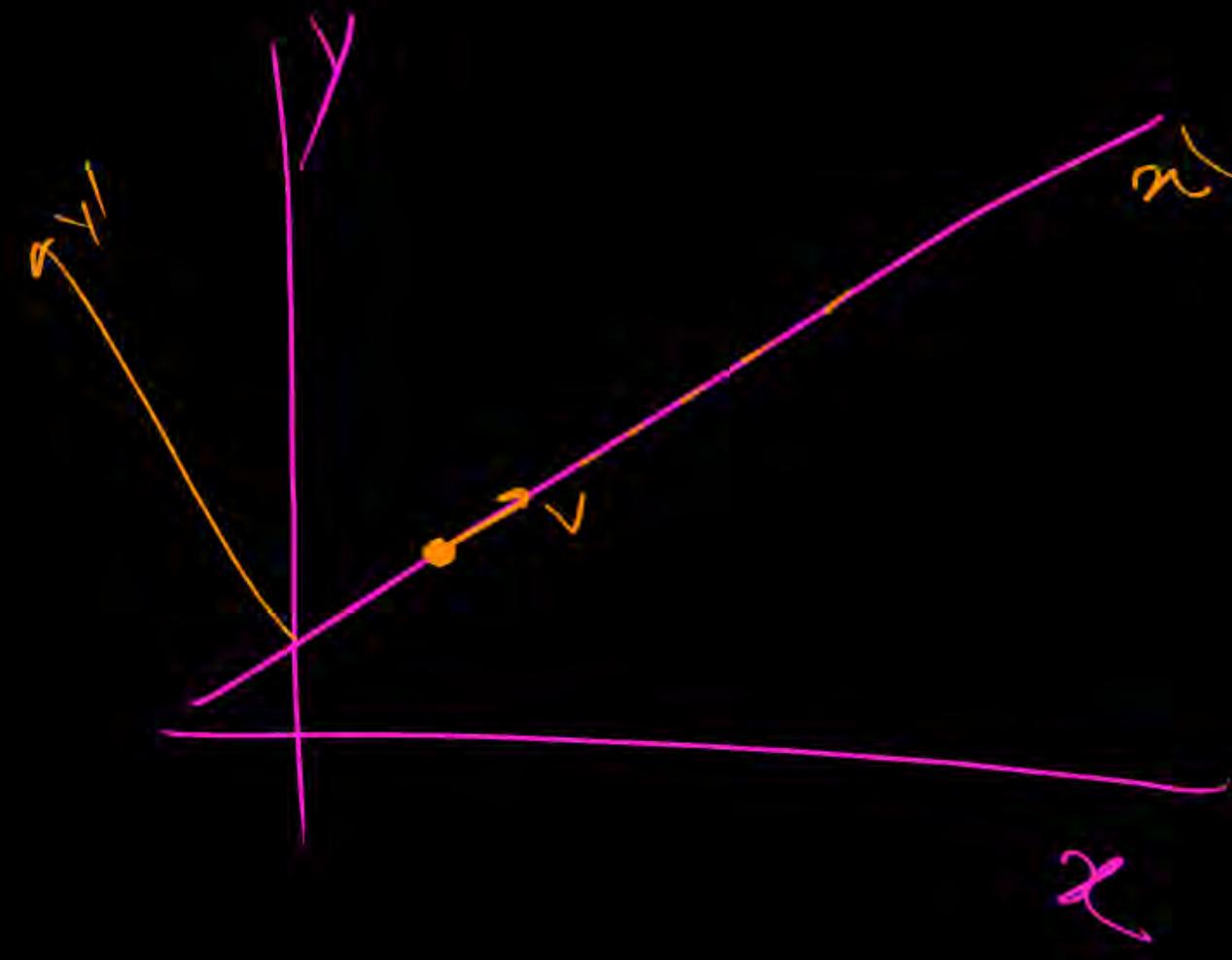


## To days Goal

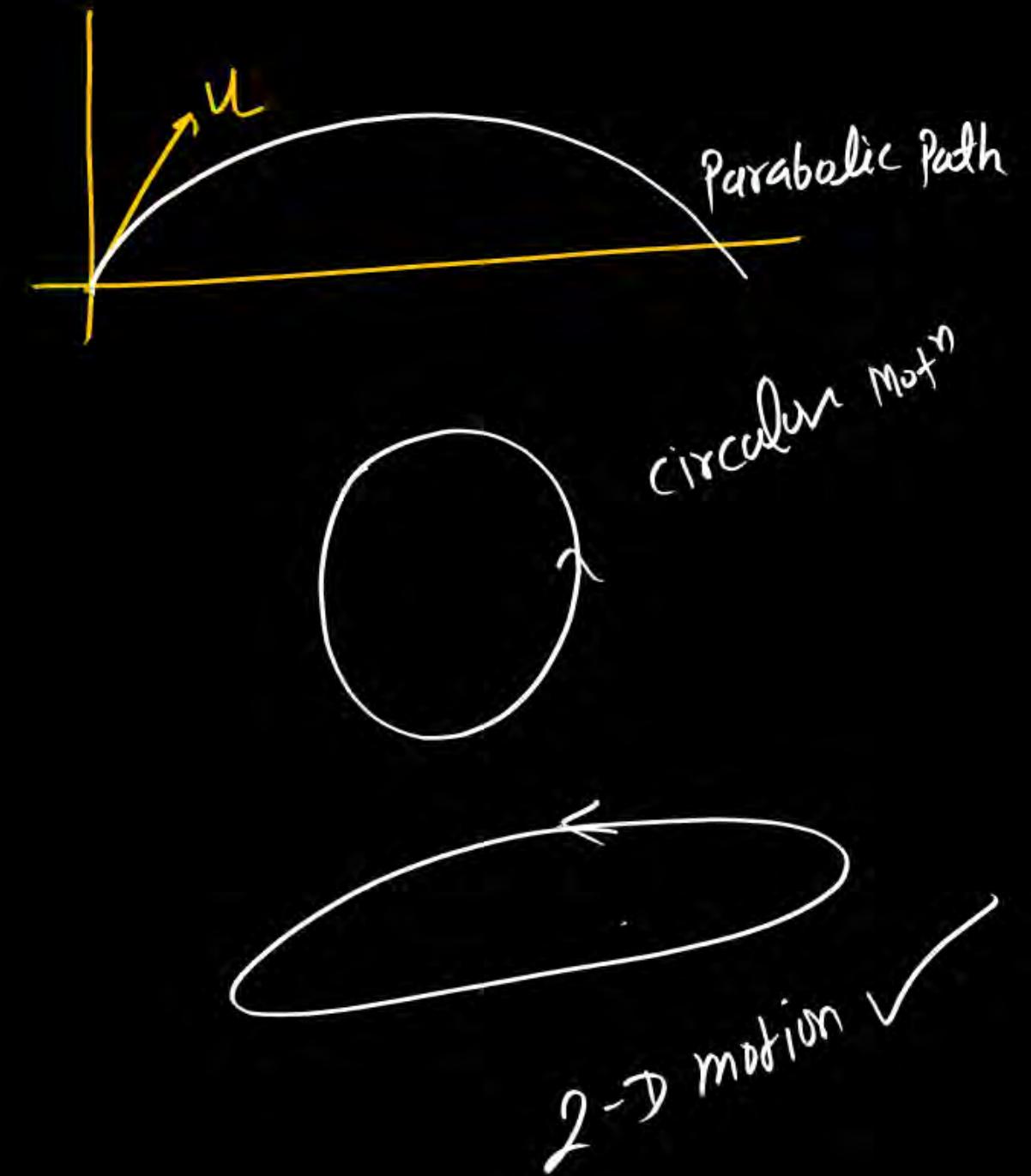
1. Equation of Trajectory ✓
2. General 2-D motion ✓
3. Equation of motion in 2-D. ✓
4. Projectile motion
5. Horizontal Projectile motion

6. Projectile on Inclined\*

7. Relative velocity in 1-D ✓
8. Relative velocity in 2-D ✓
9. River - man Prob<sup>m</sup>
10. Condition of collision .
11. Rain man Prob<sup>m</sup>

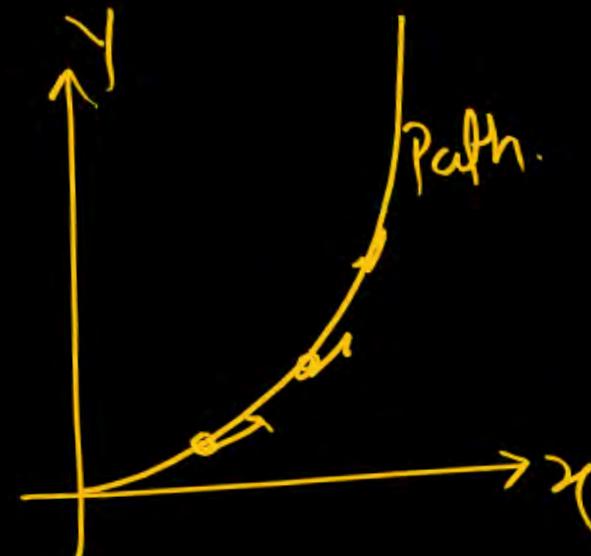
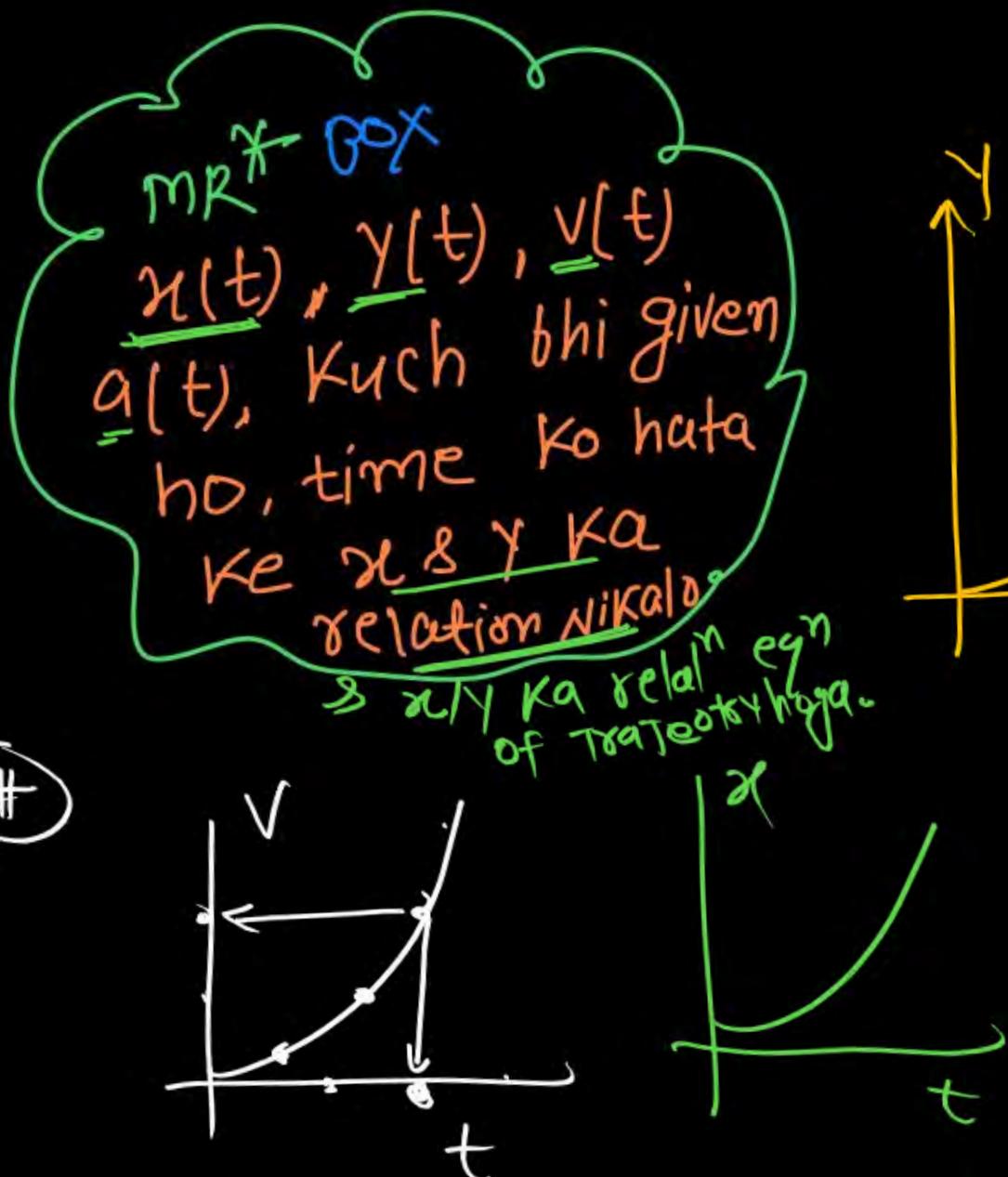


Projectile



# Equation of Trajectory $\rightarrow$ Relation between $x$ - and $y$ -co-ordinate

of motion ✓



MR\* Box  $\rightarrow$  Graph b/w  $x$  and  $y$  represent Path, but  $v(t)$ ,  $x(t)$ ,  $a(t)$  graph does not represent Path of motion.

MR\* Box  
equation  $\rightarrow$  Path  
 $y = mx + c$   
 $y = x^2$   
 $x^2 + y^2 = R^2$   
 $x^2/a^2 + y^2/b^2 = 1$

straight line ✓  
 parabola ✓  
 circular ✓  
 elliptical path

$$\textcircled{1} \quad x = 8 \sin(t) \quad \textcircled{10}$$

$$y = 4 \sin(t) \quad \textcircled{11}$$

$$\frac{x}{y} = \frac{8 \sin(t)}{4 \sin(t)}$$

$$\frac{x}{y} = 2$$

$$y = \frac{x}{2}$$
 straight line

$$\textcircled{5} \quad x = 4 \sin(t)$$

$$y = 5 \cos(t)$$

$$\frac{x}{4} = \sin(t) \quad \textcircled{1}$$

$$\frac{y}{5} = \cos(t) \quad \textcircled{11}$$

$$\frac{x^2}{16} + \frac{y^2}{25} = 1$$

$$\textcircled{2} \quad x = 3t$$

$$y = 9t$$

$$\frac{x}{y} = \frac{3t}{9t} = \frac{1}{3}$$

1-D

$$\textcircled{3} \quad y = 3t^2 \quad \textcircled{10}$$

$$x = 2t \quad \textcircled{11}$$

Putting value of 't'  
from eqn \textcircled{11} to \textcircled{1}

$$t = \frac{x}{2}$$

$$y = 3\left(\frac{x}{2}\right)^2 = \frac{3x^2}{4}$$

Parabola

$$\textcircled{4} \quad x = 3 \sin(t) \quad \textcircled{10}$$

$$y = 3 \cos(t) \quad \textcircled{11}$$

$$\textcircled{10}^2 + \textcircled{11}^2$$

$$x^2 + y^2 = 9 \left[ \sin^2 t + \cos^2 t \right]$$

$$x^2 + y^2 = 9$$

circle

$$\textcircled{6} \quad \vec{r} = 2t \hat{i} + 4t^2 \hat{j}$$

$$\vec{r} = x \hat{i} + y \hat{j}$$

$$x = 2t$$

$$y = 4t^2$$

Parabola

$$\textcircled{7} \quad \vec{v} = 2 \hat{i} + x \hat{j}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$v_x = 2 \quad v_y = x$$

$$\frac{dx}{dt} = 2 \quad \frac{dy}{dt} = x$$

$$\textcircled{10} \quad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{x}{2}$$

$$\frac{dy}{dx} = \frac{x}{2}$$

$$\int 2dy = \int x dx$$

$$2y = \frac{x^2}{2} + C$$

## General 2-D Motion

$$\vec{r} = x \hat{i} + y \hat{j}$$

Position of object

Diff w.r.t. time

$$\frac{d\vec{r}}{dt} = \left( \frac{dx}{dt} \right) \hat{i} + \left( \frac{dy}{dt} \right) \hat{j}$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

Magnitude of velocity

$$\text{speed } V = \sqrt{v_x^2 + v_y^2}$$

Direction of motion

$$\tan \alpha = \frac{v_y}{v_x}$$

velocity in vector form

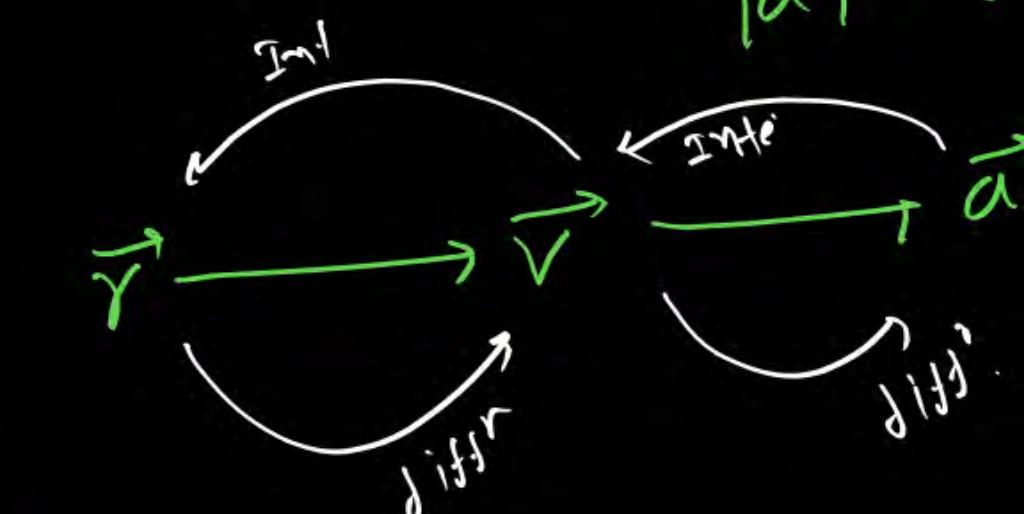
$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

Diff^n w.r.t. time

$$\frac{d\vec{v}}{dt} = \frac{dv_x}{dt} \hat{i} + \frac{dv_y}{dt} \hat{j}$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2}$$



$$2-D = [1-D]_x + [1-D]_y = 1-D \text{ motion with concept of } \underline{\text{vector}}$$

$$\vec{a} = a_x \hat{i} + a_y \hat{j}$$

$$($$

$$a_x = \frac{dv_x}{dt}$$

$$v_x$$

$$\int dv_x = \int a_x dt$$

$$v_x - v_n = \int a_n dt$$

$$a_y = \frac{dv_y}{dt}$$

$$dv_y = a_y dt$$

$$v_y - v_n = \int a_y dt$$

$$\vec{v} = v_x \hat{i} + v_y \hat{j}$$

$$v_x = \frac{dx}{dt}$$

$$v_y = \frac{dy}{dt}$$

Motion in plane with constant accn. {Equation of motion}

x-axis

y-axis

$$\vec{v}_x = \vec{u}_x + \vec{a}_x t$$

$$\vec{s}_x = \vec{u}_x t + \frac{1}{2} \vec{a}_x t^2$$

$$v_x^2 - u_x^2 = 2 a_x s_x$$

$$\vec{v}_y = \vec{u}_y + \vec{a}_y t$$

$$\vec{s}_y = \vec{u}_y t + \frac{1}{2} \vec{a}_y t^2$$

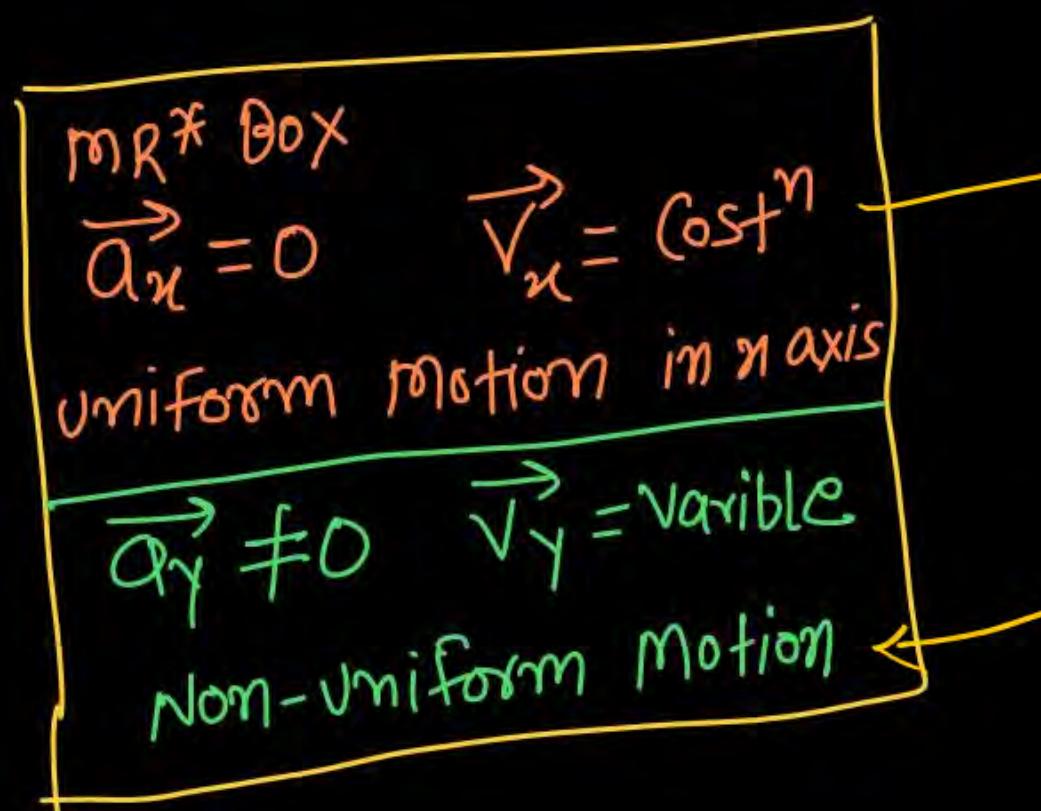
$$v_y^2 - u_y^2 = 2 a_y s_y$$



MR\* BOX for 2-d motion

- question se sare physical quantity KO x and y axis me separate karao (component)
- x and y me separate equation of motion lagao
- vector ke concept se VMKO add karo
- time dono axis me same hi hoga

dis<sup>n</sup> in  $\vec{x}$  =  $s_x$



$$\vec{s}_x = x_f - x_i = u_x t$$

$$\vec{s}_y = y_f - y_i = u_y t + \frac{1}{2} a_y t^2$$

$x$  ka kisi P.Q. Ka effect  
 $y$  me nahi hoga

$y$  ke kisi P.Q. Ka  
effect  $x$  me nahi hoga

(Q) Position  $\vec{r} = 3t^2 \hat{i} + 2t \hat{f}$  find velocity and acceleration  
 at  $t = 2$  sec

(Q) initial velocity of object  $\vec{V} = 3\hat{i} + 4\hat{j}$  and  $\vec{a} = 0.4\hat{i} + 0.3\hat{j}$   
 then find velocity after  $t = 10 \text{ sec}$

$$\vec{V}_x = U_x + a_x t$$

$$= 3 + 0.4 \times 10$$

#  $\vec{V}_x = (3+4)\hat{i} = 7\hat{i}$

①  $\vec{u}_{t=0} = 2\hat{i}$  &  $\vec{a} = 3\hat{j}$  find disp in 2-sec.

$$u_x = 2 \text{ m/s } \hat{i}$$

$$a_x = 0$$

$$\Delta x = u_x t \\ = 2 \times 2$$

$$\Delta x = 4 \text{ m}$$

$$\Delta x = 4 \text{ m}$$

$$v_y = 0$$

$$a_y = 3$$

$$\Delta y = v_y t + \frac{1}{2} a_y t^2 \\ = 0 + \frac{1}{2} 3 (2)^2$$

$$y = \frac{1}{2} 3 \times 4^2$$

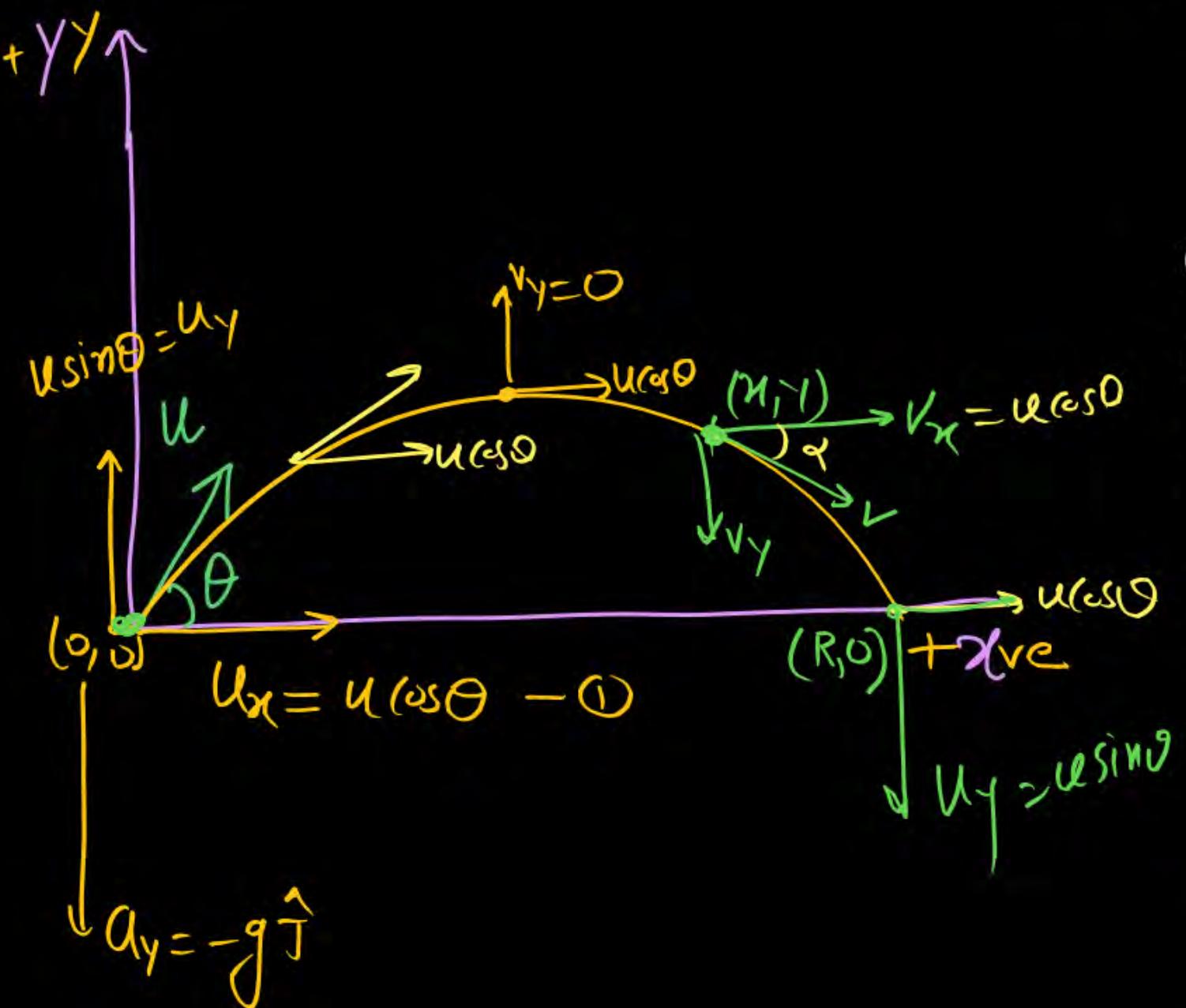
$$\Delta y = 6\hat{j}$$

$$\boxed{\text{dis} = 4\hat{i} + 6\hat{j}}$$



# Projectile motion :

2D Non Uniform Motion [variable velocity]  
 with Uniform acceleration. ( $a_y = g$  down)



Physical Quantity	x-axis	y-axis	Net
initial velocity	$\vec{u}_x = u \cos \theta \hat{i}$	$\vec{u}_y = u \sin \theta \hat{j}$	$\vec{v} = u_x \hat{i} + u_y \hat{j}$
accn	$\vec{a}_x = 0$	$\vec{a}_y = -g \hat{j}$	$\vec{a} = -g \hat{j}$
velocity at time $t$	$\vec{v}_x = u \cos \theta \hat{i}$	$\vec{v}_y = \vec{u}_y - g t \hat{j}$	$\vec{v} = u_x \hat{i} + (u_y - g t) \hat{j}$
Disp <sup>m</sup> of $t$	$x = u_x t$	$y = \vec{u}_y t - \frac{1}{2} g t^2$	$\vec{r} = x \hat{i} + y \hat{j}$
Velocity at max <sup>m</sup> height	$u_x = u \cos \theta$	$v_y = 0$	$\vec{v}_{\text{at max}} = u_x \hat{i}$
Velocity of collision at ground	$\vec{u}_x = u \cos \theta$	$\vec{v}_y = -u_y \hat{j}$	$\vec{v} = u_x \hat{i} - u_y \hat{j}$
Eqn of Traject(+)	$y = u_y t - \frac{1}{2} g t^2 =$	$y = \frac{u_y x}{u_x} - \frac{1}{2} g \left(\frac{x}{u_x}\right)^2$	Purabit

- ① Velocity of maximum height :-  $U_a = U \cos \theta \hat{i} \checkmark$
- ② Velocity of collision :-  $\vec{V}_c = U \cos \theta \hat{i} - U \sin \theta \hat{j} \checkmark$
- ③ Change in velocity in Compt motion :-  $\vec{V}_f - \vec{V}_i = (U \cos \theta \hat{i} - U \sin \theta \hat{j}) - (U \cos \theta \hat{i} + U \sin \theta \hat{j}) = -2 U \sin \theta \hat{j} \checkmark$
- ④ Average velocity in Compt Motion  $\rightarrow \boxed{\vec{V}_{Av} = \frac{\vec{V}_i + \vec{V}_f}{2}} = \frac{U \cos \theta \hat{i} + U \cos \theta \hat{i} - 2 U \sin \theta \hat{j}}{2} = \frac{2 U \cos \theta \hat{i} - 2 U \sin \theta \hat{j}}{2} = U \cos \theta \hat{i} - U \sin \theta \hat{j} = \boxed{\frac{R}{T_f}}$
- ⑤ Velocity at time  $t' \quad \vec{V} = U_x \hat{i} + (U_y - gt) \hat{j}$
- ⑥ Angle of projection  $\tan \theta = \frac{U_y - gt}{U_x}$   
 $\text{at } t=0 \quad \tan \theta = \frac{U_y}{U_x}$  Angle of proj

⑦ Direction of motion at time 't'

Consider motn in y-axis

$$y = u_y t - \frac{1}{2} a_y t^2$$

$$0 = u_y t - \frac{1}{2} g t^2$$

$$u_y t = \frac{1}{2} g t^2$$

$$t = \frac{2u_y}{g}$$

Time of flight.

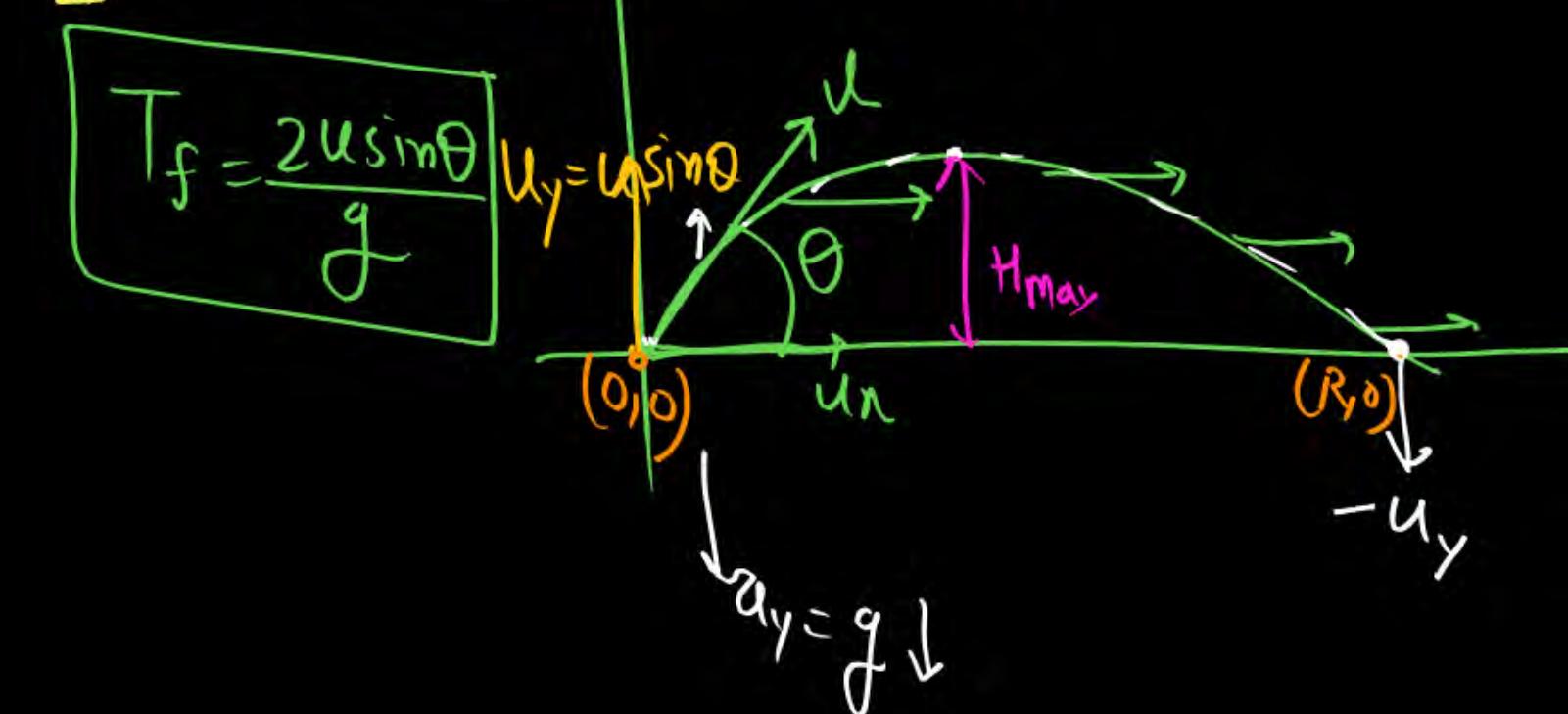
⑧ Time of flight

$$T_f = \frac{2u_y}{g} = \frac{2u \sin \theta}{g}$$

feel of motion  
under gravity

⑨ Maximum Height

$$H_{\text{Max}} = \frac{u_y^2}{2g} = \frac{u^2 \sin^2 \theta}{2g}$$



(10) Horizontal Range  $R_{\text{Ray}} = u_x \bar{t}_f = u_x \times \left[ \frac{2u_y}{g} \right] = \frac{2u_x u_y}{g}$

$$R = \frac{u^2 \sin 2\theta}{g}$$

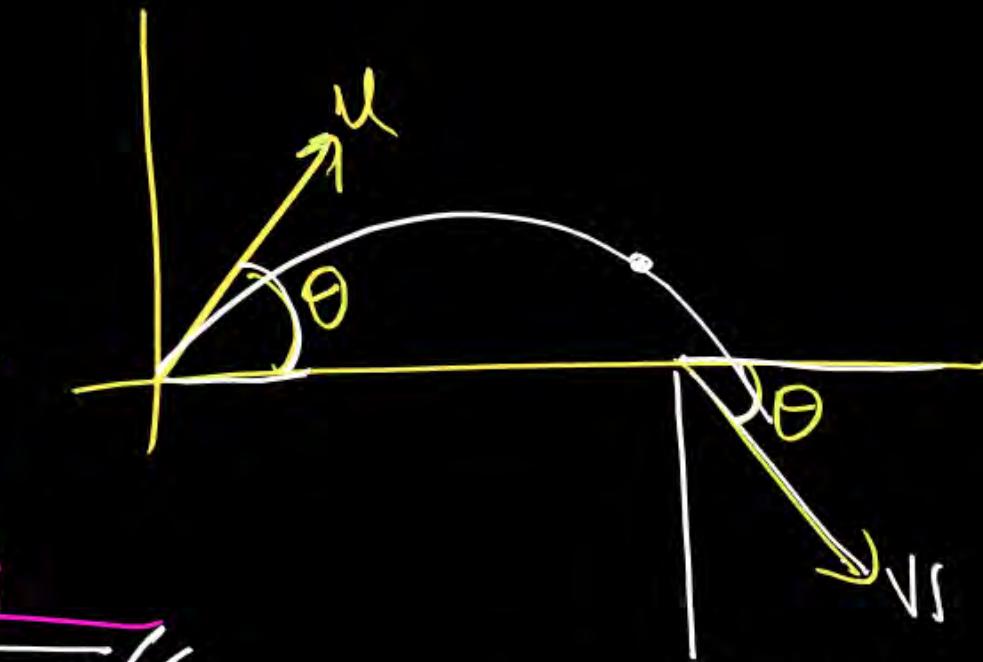
(11) Time when velocity is  $\perp \hat{y}$  to initial velocity of projection

only possible ( $\theta \geq 45^\circ$ )  
Angle of projection.

$$\vec{u} = u_x \hat{i} + u_y \hat{j}$$

$$\vec{v} = u_x \hat{i} + (u_y - gt) \hat{j}$$

$$\vec{v} \perp \vec{u} \quad \boxed{\vec{v} \cdot \vec{u} = 0}$$



(12) equation of trajectory

$$x = u_x t$$

$$y = u_y t - \frac{1}{2} g t^2$$

$$y = u_y \frac{x}{u_x} - \frac{1}{2} g \left( \frac{x}{u_x} \right)^2$$

$$\boxed{y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u_x^2 \cos^2 \theta}}$$

$$t = \frac{u}{g \sin \theta}$$

$$\boxed{y = x \tan \theta \left( 1 - \frac{x}{R} \right)}$$

$$(Q) \quad y = \sqrt{3}x - 4x^2$$

$$y = x \tan \theta - \frac{g x^2}{2 u^2 \cos^2 \theta}$$

$$\tan \theta = \sqrt{3}(\theta - b)$$

(13)

Relation between  $H$  and  $R$ 

$$\left[ \begin{array}{l} H = \frac{u_y^2 u_x}{2g} \\ R = \frac{2u_x u_y}{g} \end{array} \right]$$

$$\frac{H}{R} = \frac{\cancel{u_y^2 u_x}}{2g \cancel{2u_x u_y}} = \frac{u_y}{g u_x} = \frac{\tan \theta}{4}$$

$$H = \frac{R \tan \theta}{4} *$$

(14)

Condition of maximum Range :-

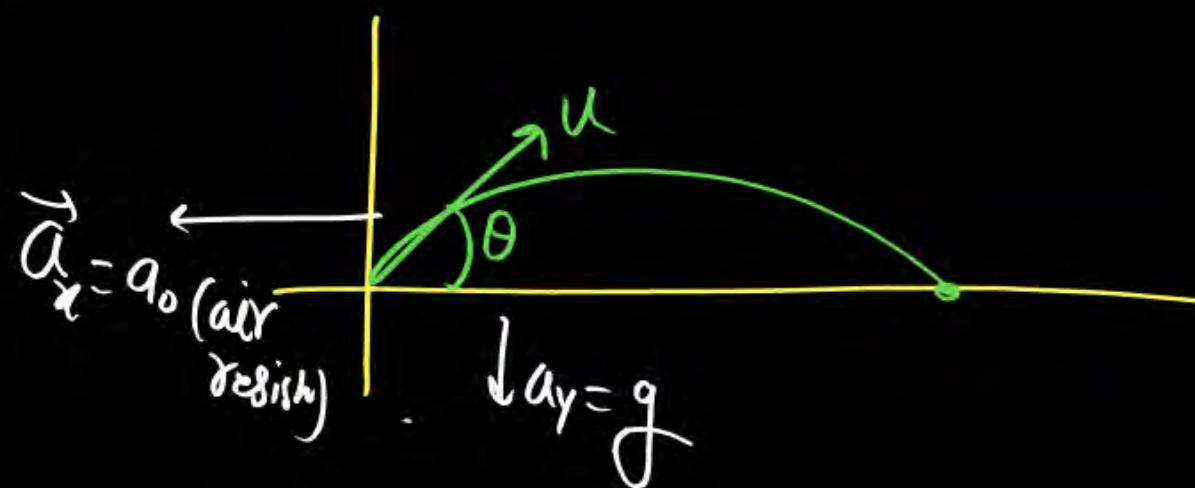
$$R_{\max} = \frac{u^2 \sin(2\theta)_{\max}}{g} = \frac{u^2}{g}$$

$$\sin(2\theta)_{\max} = 1$$

$$2\theta = 90^\circ \quad \boxed{\theta = 45^\circ}$$

(15)

if air

Resistance in  $x$ -axis only

$$T_f = \frac{2u_y}{g} \text{ (same)}$$

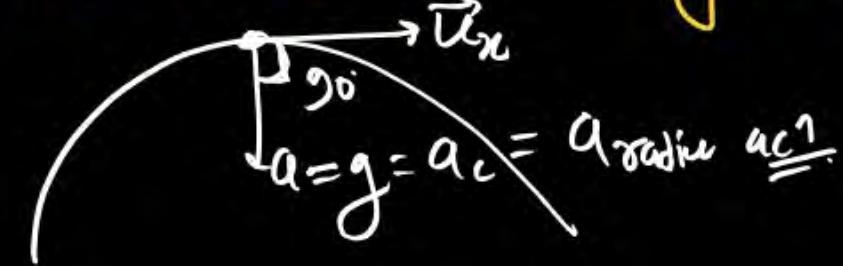
$$H_{\max} = \frac{u_y^2}{2g} \text{ (same)}$$

$$R \rightarrow \infty$$

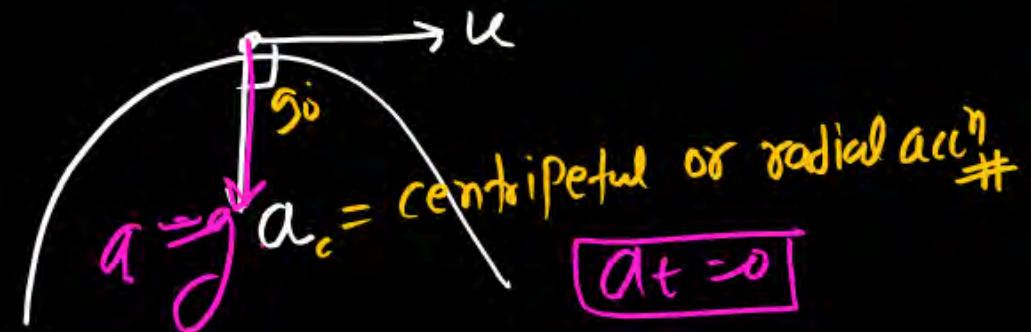
$$R = u_x t + \frac{1}{2} a_{xx} t^2$$

⑯ Angle b/w arcn & velocity at maximum Height

$$\theta = 90^\circ *$$



⑰ Tangential and centripetal accn at maximum Height



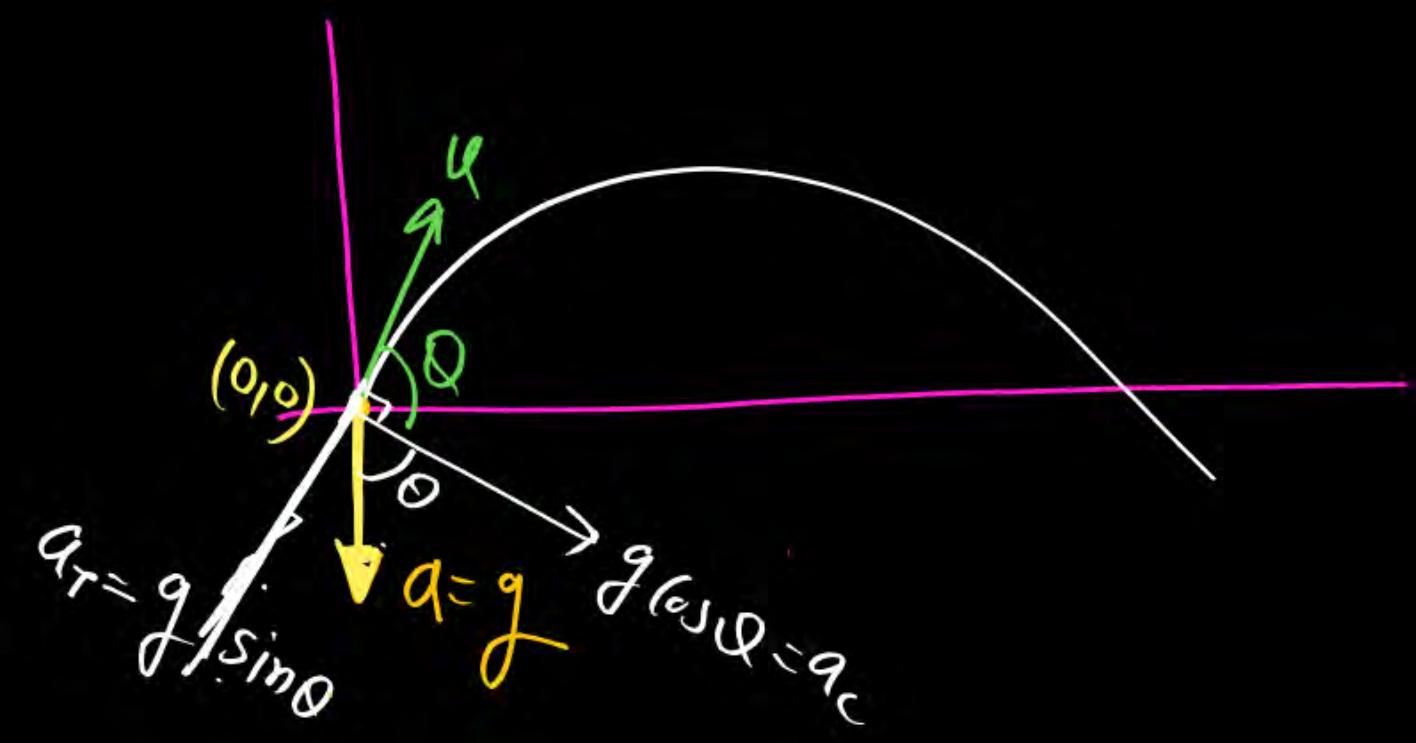
⑱ complementary Angle :-

Object is projected with same speed at two diffn angle  $\alpha$  &  $\beta$  but

Range are same.

$$\alpha + \beta = 90^\circ$$

$$\alpha = 30^\circ / \beta = 60^\circ$$



⑯ Ball is projected with same speed at  $\alpha$  and  $\beta$  such that Range are same then :-

$$\frac{H_1}{H_2} = \frac{\frac{u^2 \sin^2 \alpha}{2g}}{\frac{u^2 \sin^2 \beta}{2g}}$$

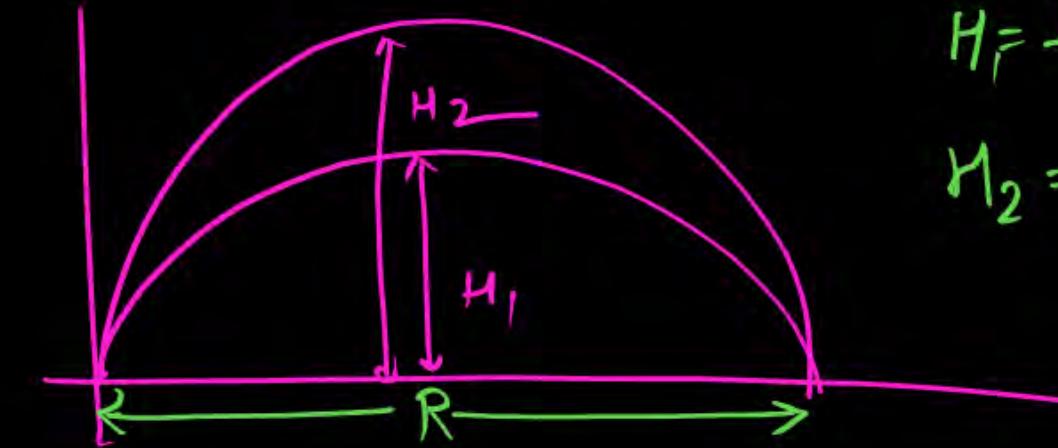
$$\frac{H_1}{H_2} = \frac{\sin^2 \alpha}{\sin^2 \beta} \quad [\alpha + \beta = 90]$$

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$$\frac{H_1}{H_2} = \tan^2 \alpha$$

$$\frac{T_1}{T_2} = \frac{\frac{2u \sin \alpha}{g}}{\frac{2u \sin \beta}{g}} = \frac{\sin \alpha}{\sin (\alpha - \beta)} = \frac{\sin \alpha}{\sin \beta}$$

(P.G. 2)



$$H_1 = \frac{R \tan \alpha}{4}$$

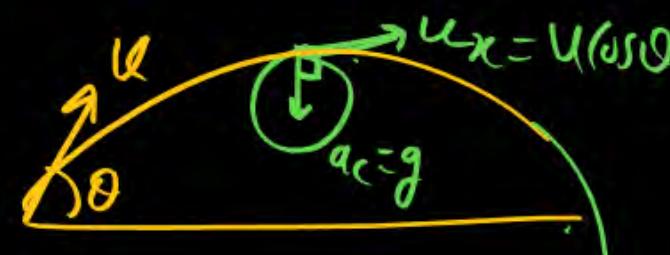
$$H_2 = \frac{R \tan \beta}{4}$$

$$H_1 H_2 = \frac{R^2 \tan \alpha \tan \beta}{16}$$

$$\frac{R^2}{16} = H_1 H_2$$

proved

⑰ Radius of curvature at maximum Height :-



$$a_c = \frac{v^2}{R}$$

$$R_{max} = \frac{u^2}{a_c} = \frac{u^2 \cos^2 \theta}{g}$$

② Rate of change in velocity  $V$  at time  $t$

$$\vec{a} = g \text{ (downward)}$$

③ Rate of Change in Moment

$$\overrightarrow{\text{Force}} = \frac{d\vec{P}}{dt} = mg \text{ (downward)}$$

Object is projected at  $\vec{U} = 30\hat{i} + 40\hat{j}$  then find :-

① Angle of projection

$$\tan \theta = \frac{u_y}{u_x} = \frac{40}{30} = \frac{4}{3}$$

$\boxed{\theta = 53^\circ}$

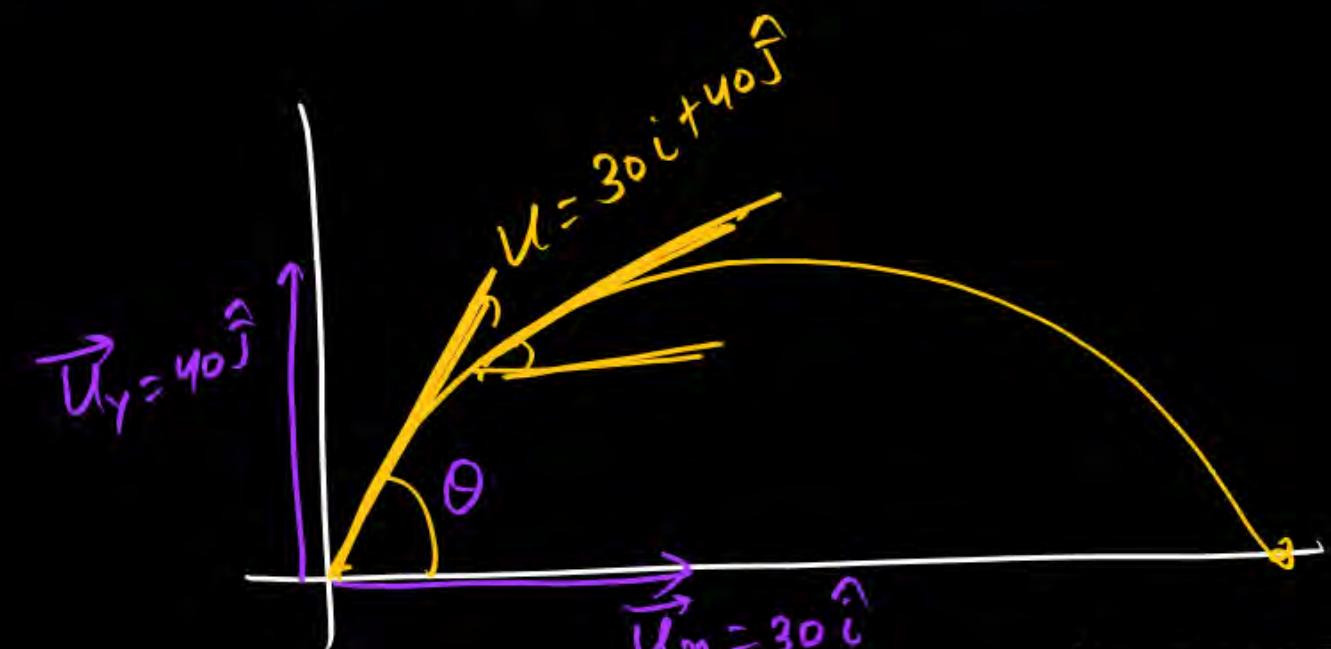
②  $T_f = \frac{2u_y}{g} = \frac{2 \times 40}{10} = 8 \text{ sec}$

③  $H_{\max} = \frac{u_y^2}{2g} = \frac{(40)^2}{2 \times 10} = \frac{40 \times 16}{2 \times 10} = 80 \text{ m}$

④  $R_{\max} = u_x T_f = 30 \times 8 = 240 \text{ m}$

⑤ Velocity at Max Height  $= u_x = 30 \text{ m/s}$

⑥  $t = \frac{u_y}{g \sin \theta} \Rightarrow \text{time when velo} \perp g$



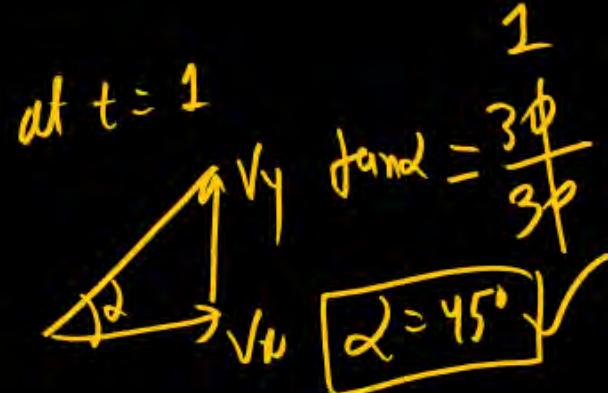
⑥ Velocity at time  $[t = 1 \text{ sec}]$

$$\vec{U}_x = 30 \text{ m/s} \hat{i}$$

$$\vec{U}_y = 40 - 10 \times 1 = 40 - 10 = 30 \hat{j}$$

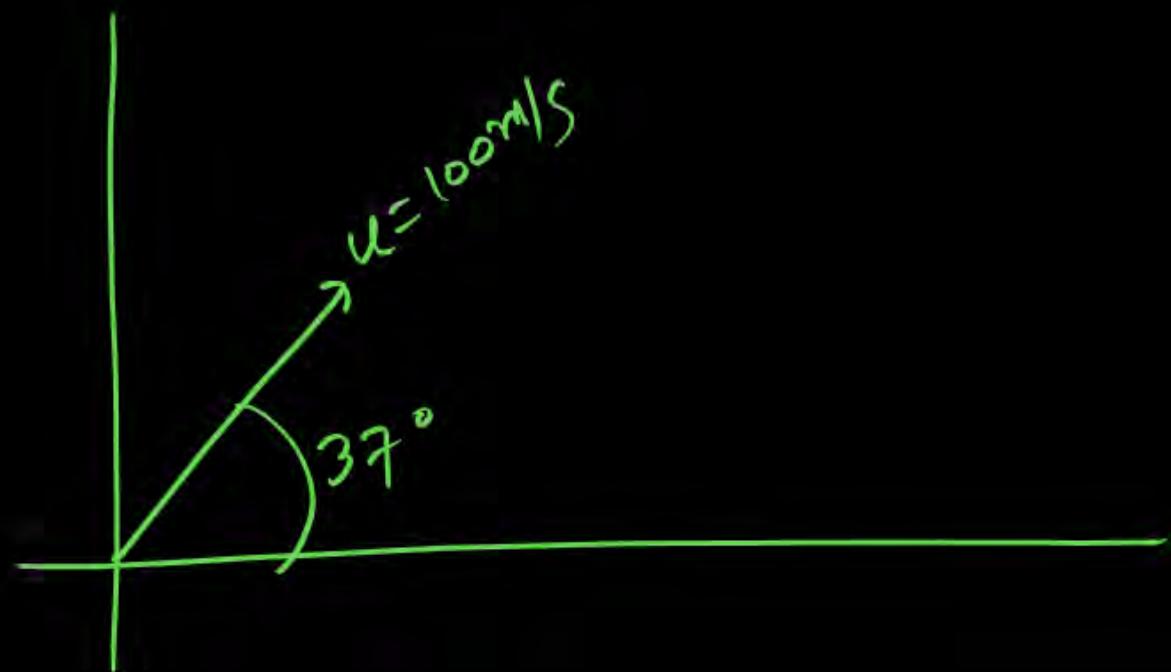
$$\vec{V} = 30\hat{i} + 30\hat{j}$$

⑦ direction of motion at  $t = 1$

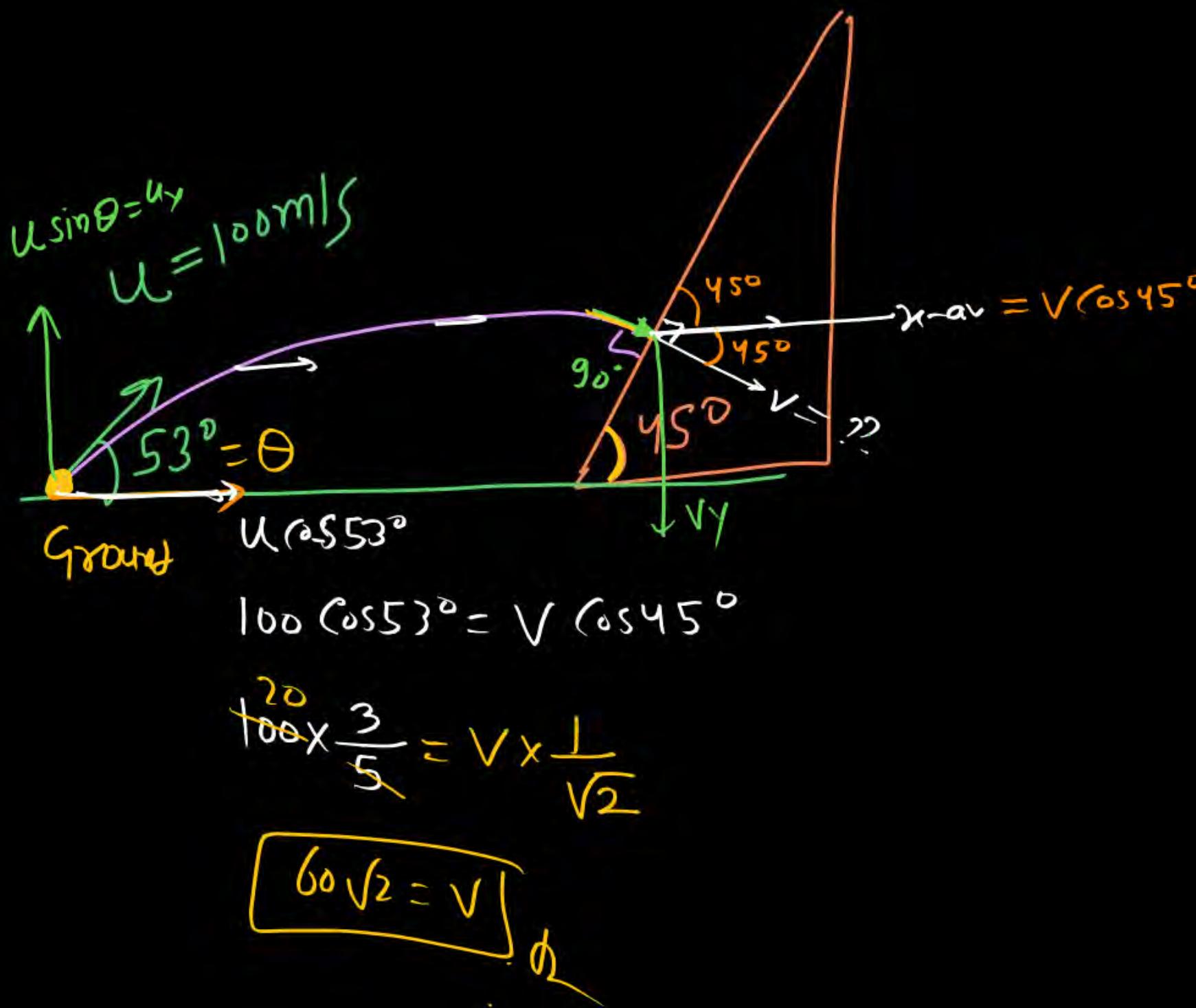


$H/\omega$

- ①  $T_f$
  - ②  $R_r$
  - ③  $H_{Ma}$
  - ④ Velocity at  $t = 1 \text{ sec}$ .
  - ⑤ Velocity at  $t = 4 \text{ sec}$
- 



find velocity of collision & Time of collision ?? ✓



$$\cos 53^\circ = \frac{3}{5}$$

$$\text{at } t=0 \quad U_y = U \sin 53^\circ = \frac{20}{100} \times \frac{4}{5} = 80 \text{ m/s}$$

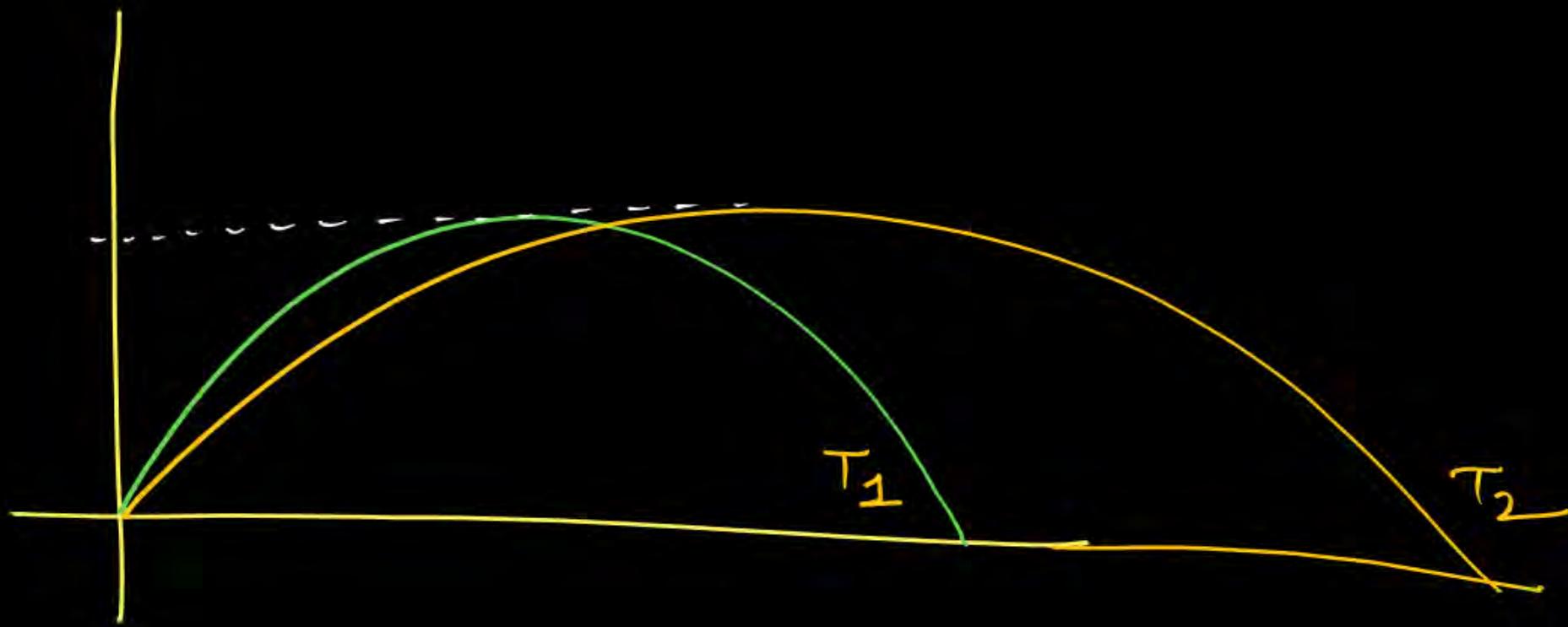
$$V_y = V \sin 45^\circ = 60\sqrt{2} \times \frac{1}{\sqrt{2}} = 60$$

$$\vec{V}_f = \vec{u} + \vec{a}t$$

$$-60 = 80 - 10t$$

$$10t = 140$$

$$t = 14 \text{ sec}$$

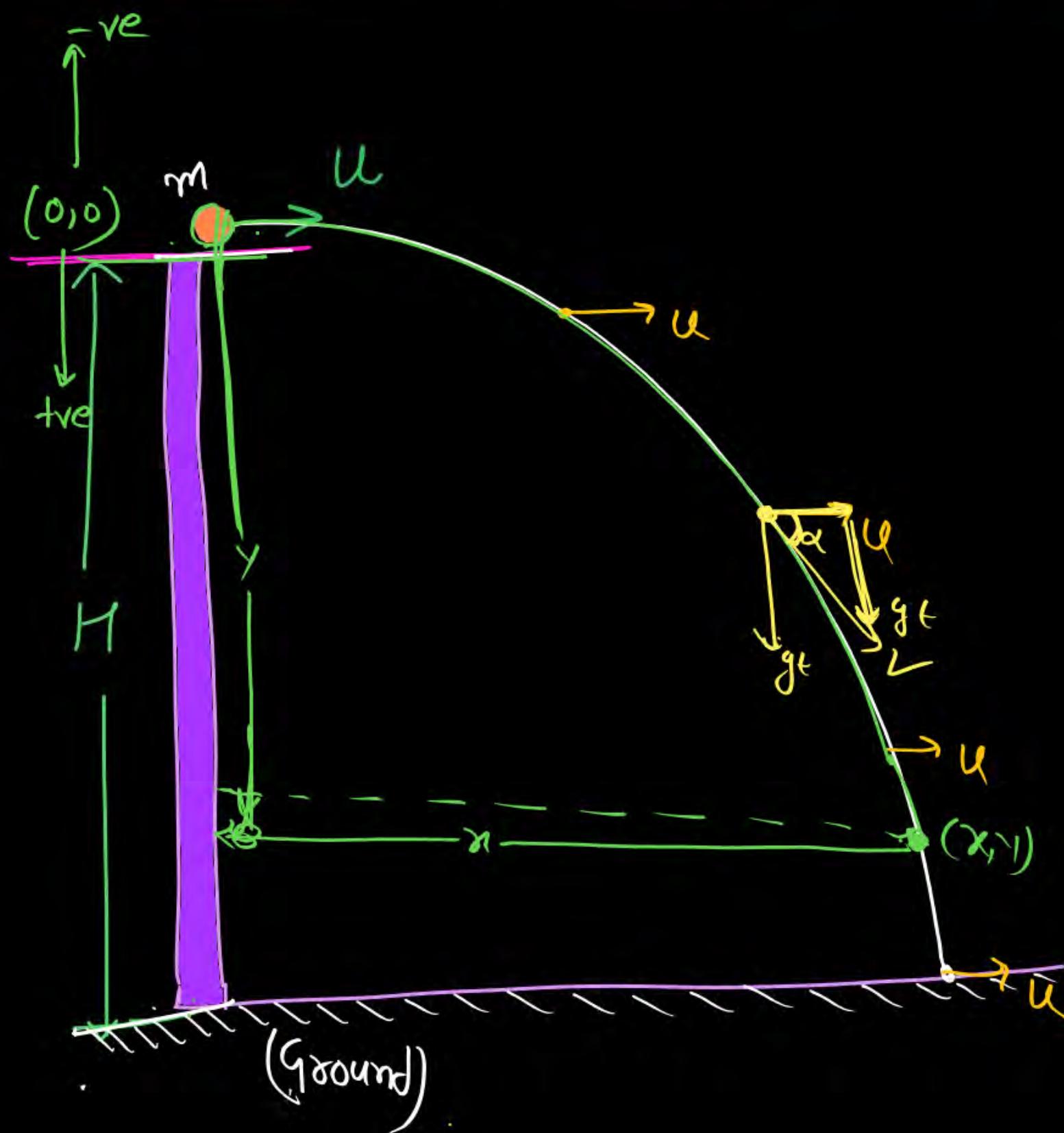


dono Path me  $H_{\max}$  hai :—

$\Rightarrow T_1 = T_2$  Same time of flight hoga ✓

# Horizontal Projectile Motion

→ 2-D Motion → Non-uniform motion!  
 accn due to gravity =  $g$  (const) Uniform accn



X-axis

$$\vec{u}_x = u$$

$$a_x = 0$$

$$\vec{v}_x = u$$

uniform motion in X

$$x = u_x t$$

Y-axis

$$u_y = 0$$

$$a_y = g$$

$$v_y = gt \hat{j}$$

$$y = \frac{1}{2}gt^2$$

$$\vec{v}_{t=0} = u\hat{i} + 0\hat{j}$$

$$\vec{a} = 0 + g\hat{j}$$

$$\vec{v} = u\hat{i} + gt\hat{j}$$

$$\tan \alpha = \frac{gt}{u}$$

$$T_f = \sqrt{\frac{2H}{g}}$$

$$R = u \sqrt{\frac{2H}{g}}$$

$$y = \frac{1}{2}g(t)^2$$

$$y = \frac{1}{2}g\left(\frac{x}{u}\right)^2$$



$$\rightarrow u = 30 \text{ m/s}$$

$$u_y = 0$$

$$a = g$$

$$H = 80 \text{ m}$$

① Time of Flight

$$T_f = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 80}{10}} = \sqrt{16} = 4 \text{ sec}$$

② Range

$$R = u_x T_f = 30 \times 4 = 120 \text{ m}$$

③ Velocity at ground

$$\vec{V} = 30\hat{i} + 40\hat{j}$$

④ Velocity at  $t=2 \text{ sec}$

$$\vec{V} = 30\hat{i} + 20\hat{j}$$

3rd eqn of motion  
in y-axis

$$v_y = ?$$

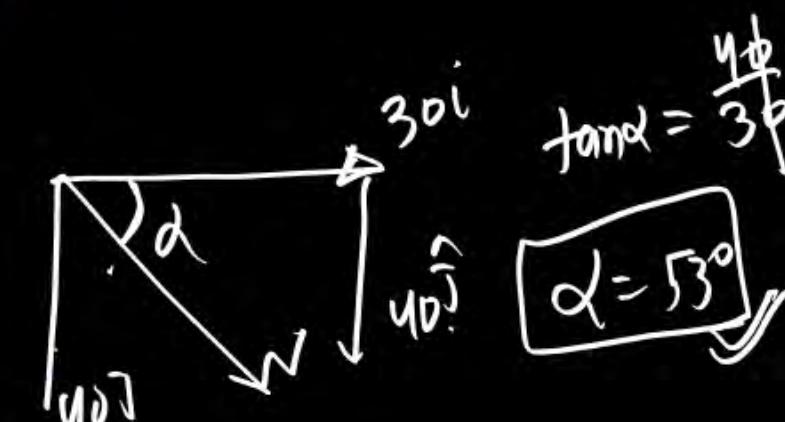
$$v_y = \sqrt{2gh}$$

$$v_y = \sqrt{2 \times 10 \times 80} = 40 \text{ m/s}$$

⑤

Direction of motion at  $t=4 \text{ sec}$

$$\vec{V} = 30\hat{i} + 40\hat{j}$$



⑥ Position of object at  $t = 3\text{ sec}$

$$x = u_x t = 30 \times 3 = 90 \text{ m i}$$

$$y = u_y t + \frac{1}{2} g t^2 = 10(3)^2 = 5 \times 9 = 45 \text{ J}$$

$$\vec{r} = 90 \text{ m i} + 45 \text{ m J}$$

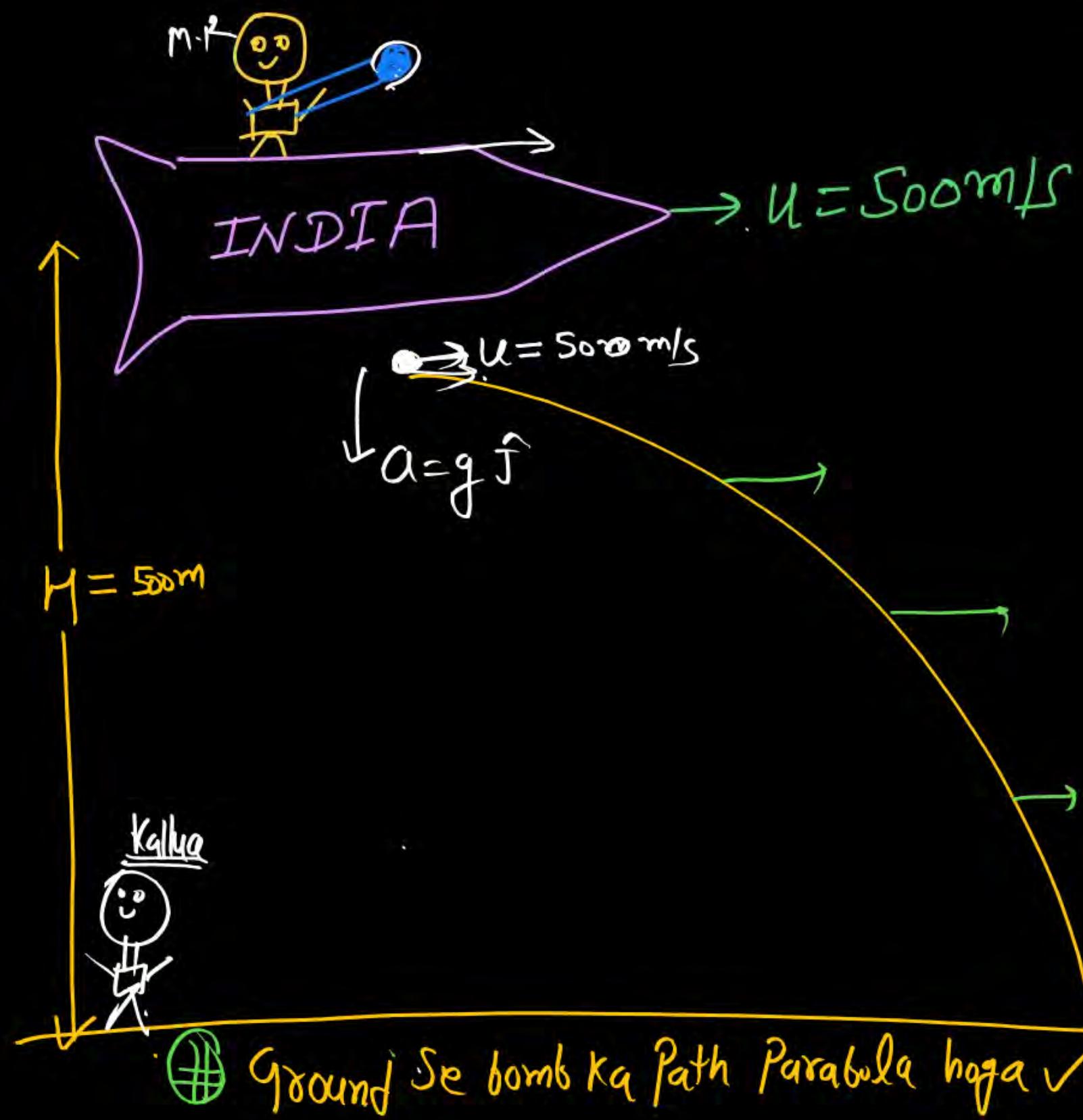
⑦ Position vector at  $t$

$$\vec{r} = ut \hat{i} + \frac{1}{2} g t^2 \hat{j}$$

⑧ Equation of Trajectory

$$y = \frac{1}{2} g t^2$$

$$x = ut$$



bomb is dropped from 500m  
 Height then Range of bomb

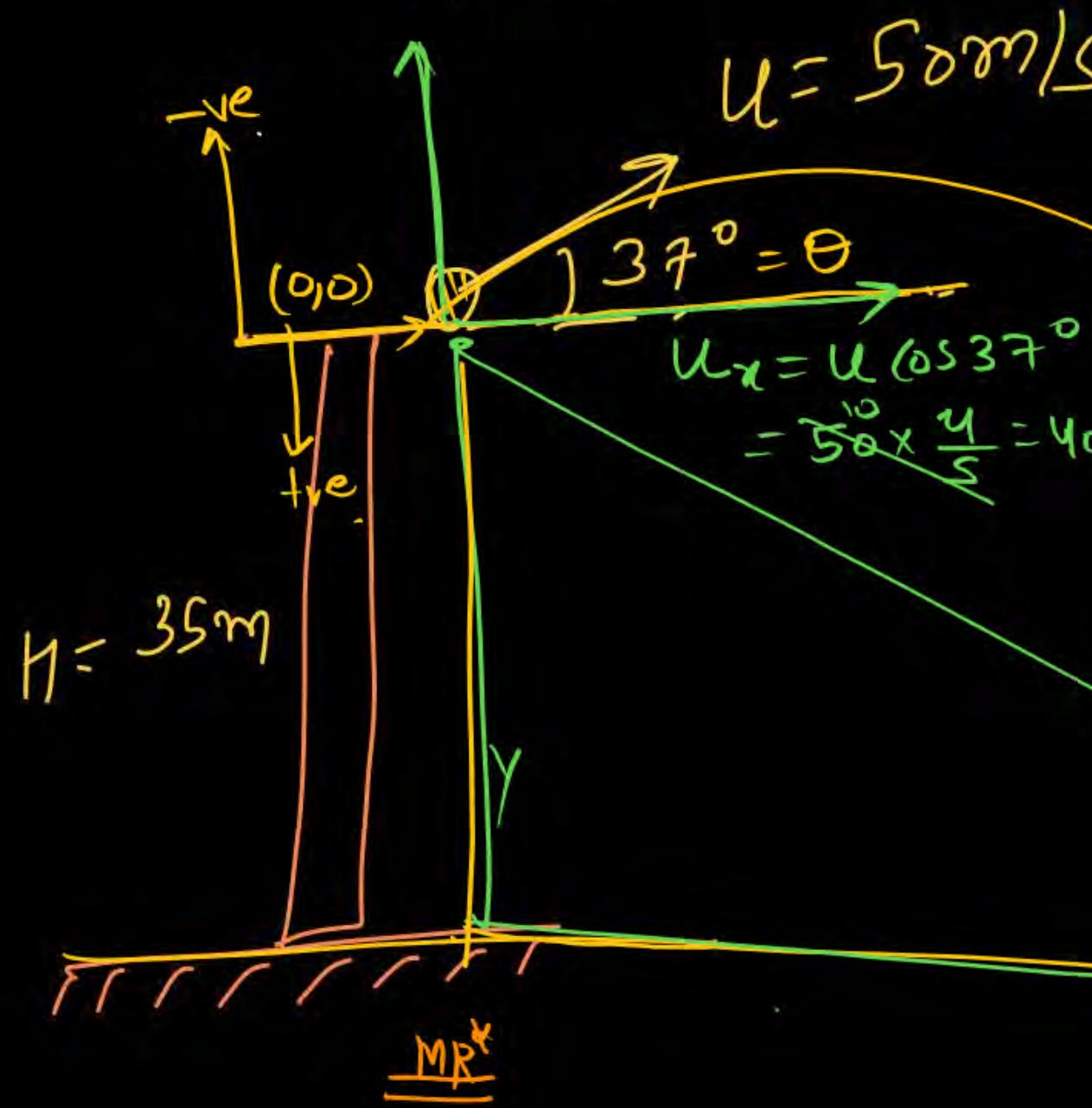
# Path of bomb is Parabolic

$$\# T_f = \sqrt{\frac{2H}{g}} = \sqrt{\frac{2 \times 500}{10}} = \sqrt{100} = 10\text{sec}$$

$$\begin{aligned}
 R &= u_n T_f \\
 &= 500 \times 10 \\
 &= 5000\text{m} \quad \# \text{ Answer}
 \end{aligned}$$

(Q) find Time of Flight and Range :-

$$u_y = 50 \sin 37^\circ = 50 \times \frac{3}{5} = 30 \hat{j}$$



Consider motion in y-axis

$$u_y = -30$$

$$a_y = +10$$

$$s_y = +35 \text{ m}$$

$$y = ut + \frac{1}{2} at^2$$

$$35 = -30t + \frac{1}{2} 10t^2$$

$$\Rightarrow 7t^2 - 60t - 7 = 0$$

$$t^2 - 6t - 1 = 0$$

$$t^2 - 7t + t - 1 = 0$$

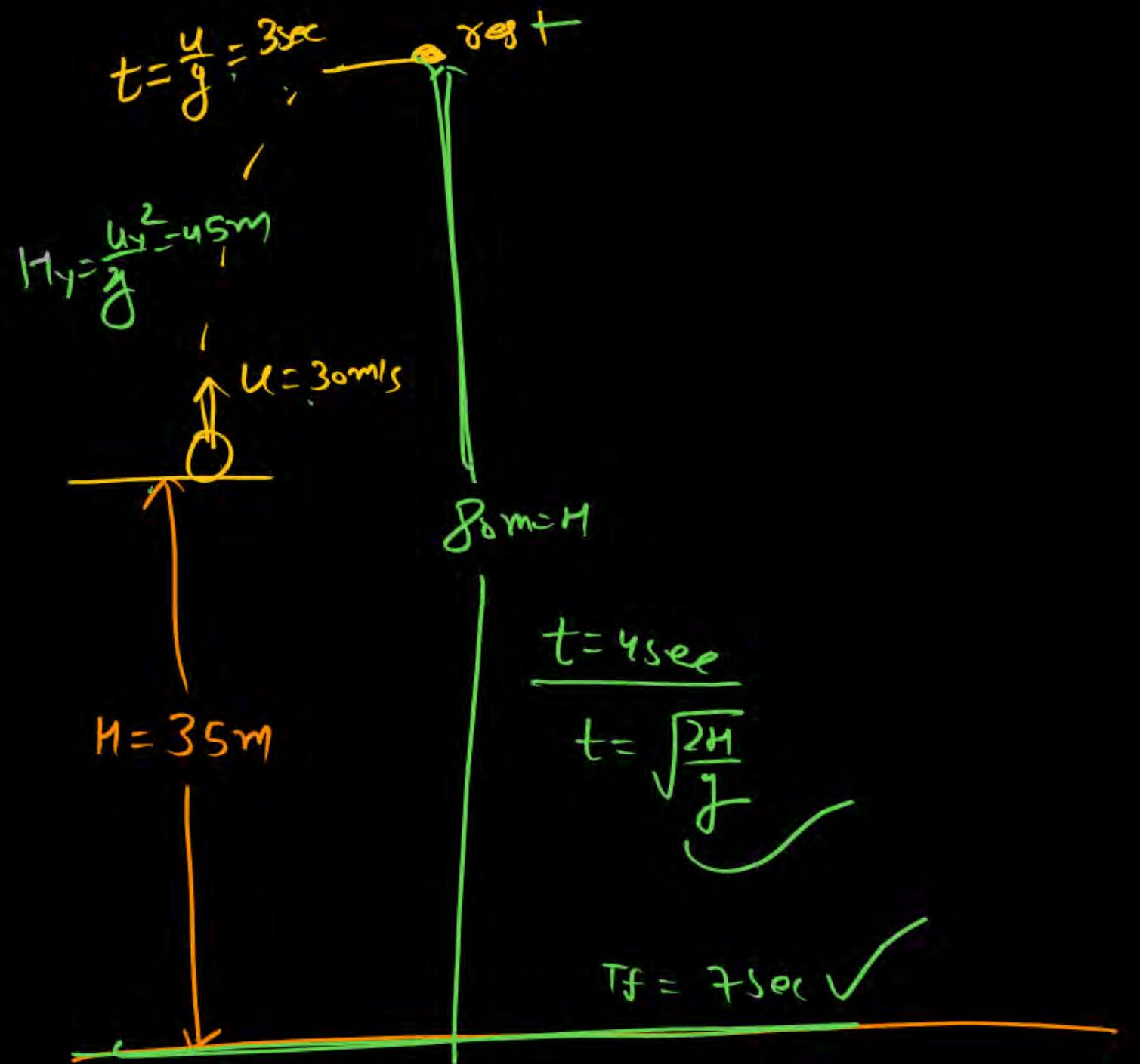
$$t(t-1) + 1(t-1) = 0$$

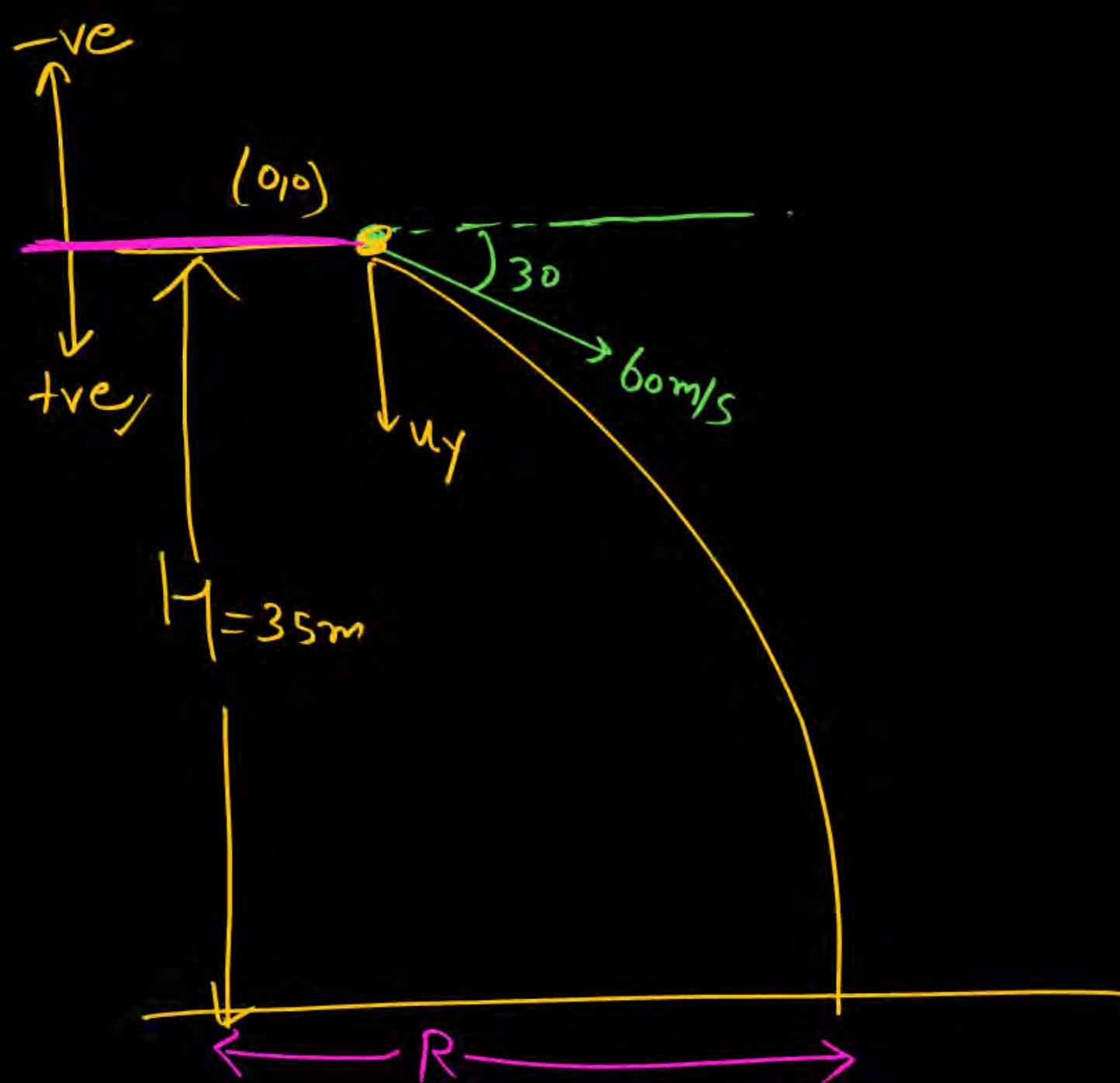
$$(t-1)(t+1) = 0$$

$$t=1 \quad \checkmark$$

$$(x_1, y) = (R, +35)$$

$$\text{Range} = u_x T_f = 40 \times 7 = 280 \text{ m} \quad \checkmark$$



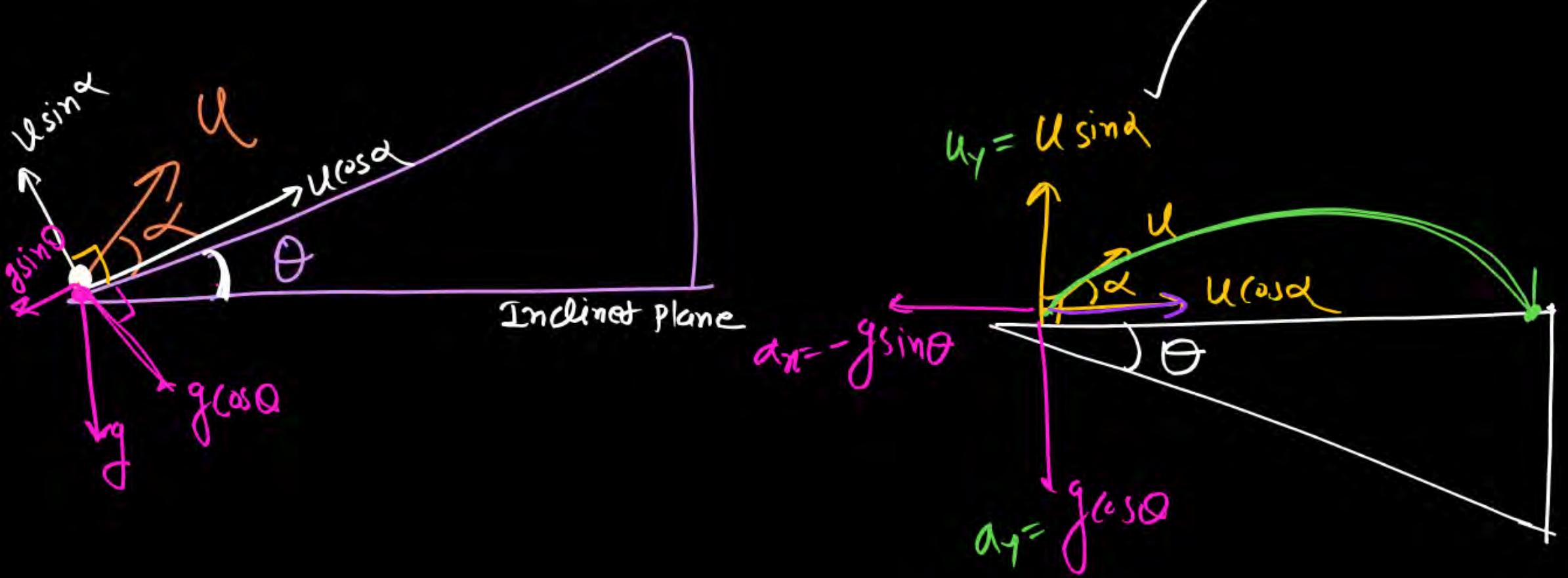


$$\vec{U}_y = 60 \sin 30 = 60 \times \frac{1}{2} = 30 \hat{j}$$

$$y = 35 \text{ m}$$

$$a = 10 \text{ m/s}^2$$

# Projectile on Inclined:-



Time of flight:-  
consider motn in y-axis!

$$T = \frac{2u_y}{a_y} = \frac{2u \sin \alpha}{g \cos \theta}$$

along Inclined Plane:-

$$u_n = u \cos \alpha \quad a_n = -g \sin \theta$$

$$x = u_n t_f + \frac{1}{2} a_n t_f^2$$

$$R = u \cos \alpha t_f - \frac{1}{2} g \sin \theta t_f^2$$

Relative velocity :— Velocity ka vector subtraction.

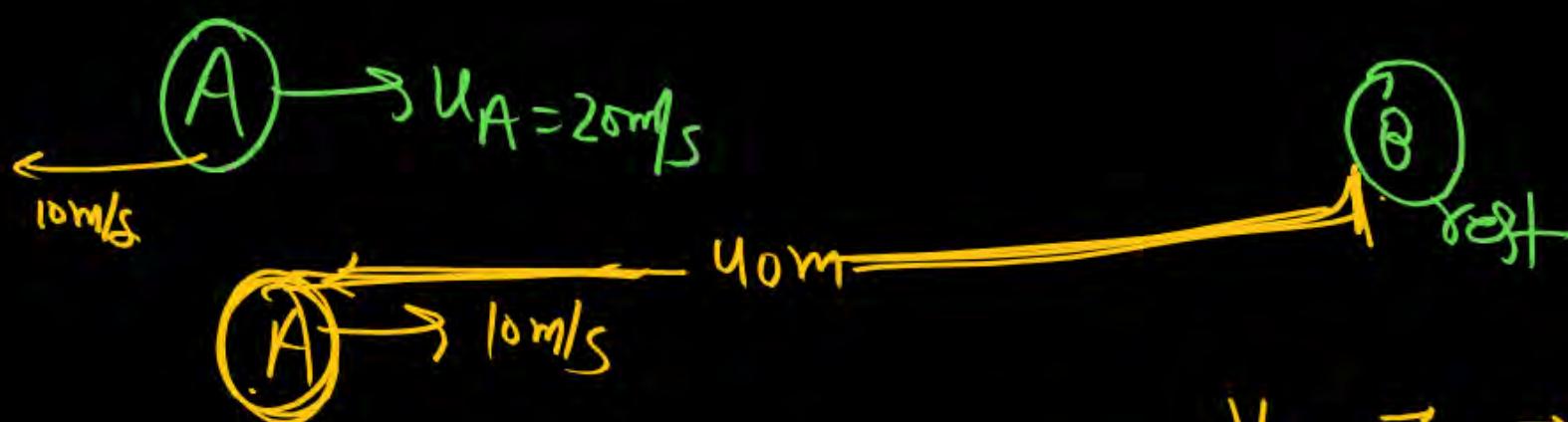
$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

"velocity of A w.r.t. B"

(A)  $\rightarrow u_A = 20 \text{ m/s}$

$d = 40 \text{ m}$

time of collision



$$S = u t$$

$$y_B = 10t$$

$$t = 4 \text{ sec}$$

$$S_{AB} = 40 \text{ m}$$

$$a_{AB} = 0$$

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B = 20 - 10 = 10 \text{ m/s}$$

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

observes (Velocity of B w.r.t. A)

Object B moves to the right at  $10 \text{ m/s}$ .

$$\vec{V}_{AB} = -\vec{V}_{BA}$$

✓

$$\vec{r}_{AB} = \vec{r}_A - \vec{r}_B$$

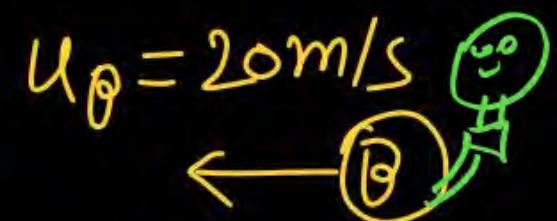
$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

$$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$$

MRX Box:—  
aage wale  
par obseb2 ban kar baith  
jao, usko rest me man  
lo, Pichhe wale ka distn, relative motion mein yeh lafz  
Nikalo & eqn of motion se solve karo.

$$\textcircled{A} \rightarrow u_A = 10 \text{ m/s}$$

$$40 \text{ m} \longrightarrow u_{OB} \longleftarrow$$



Time of collision -

$$\textcircled{A} \rightarrow \begin{cases} 10 \text{ m/s} \\ 20 \text{ m/s} \end{cases} \sum v_{AB} = 30 \text{ m/s}$$

$$40 \text{ m} = S_{AB} \rightarrow \text{rest}$$

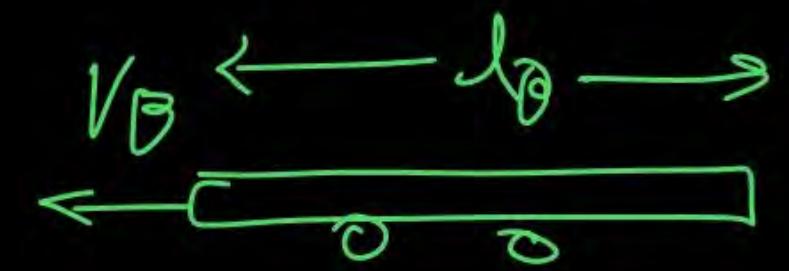
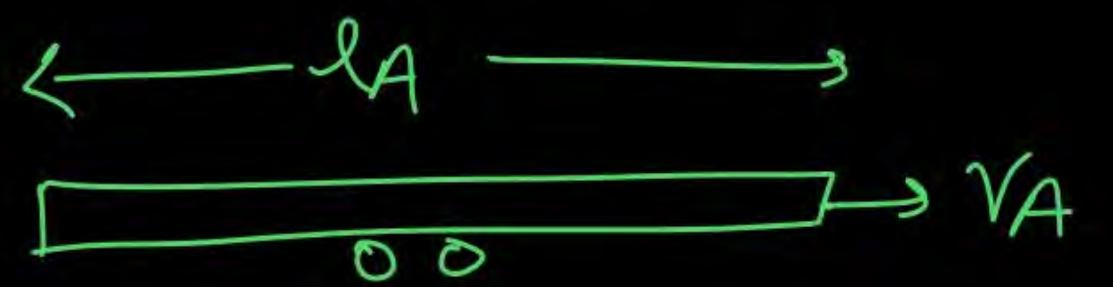
$$S_{AB} = 40 = 30 t$$

$$t = \frac{4}{3} \text{ sec} \quad \checkmark$$

Q If  $v_A > v_B$ ; time when train A will overtake:-



$$t_{\text{overtake}} = \frac{l_A + l_B}{v_A - v_B}$$

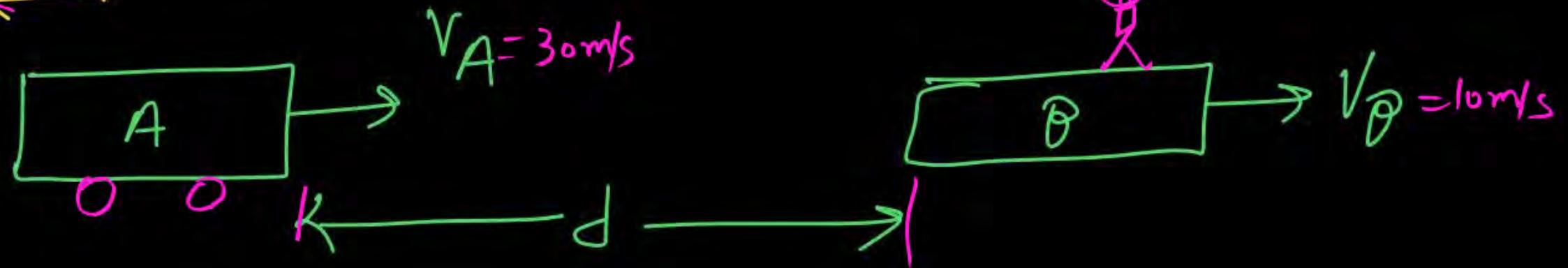


$$t_{\text{cross}} = \frac{l_A + l_B}{v_A + v_B}$$

✓

(Q)

$$v_{\text{initial}} = u = 2 \text{ m/s}^2$$

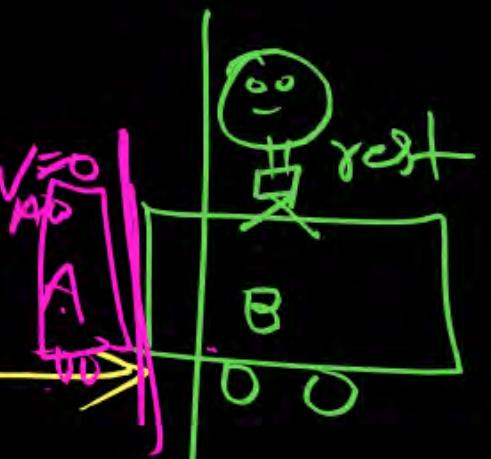


Condition for No Collision :-

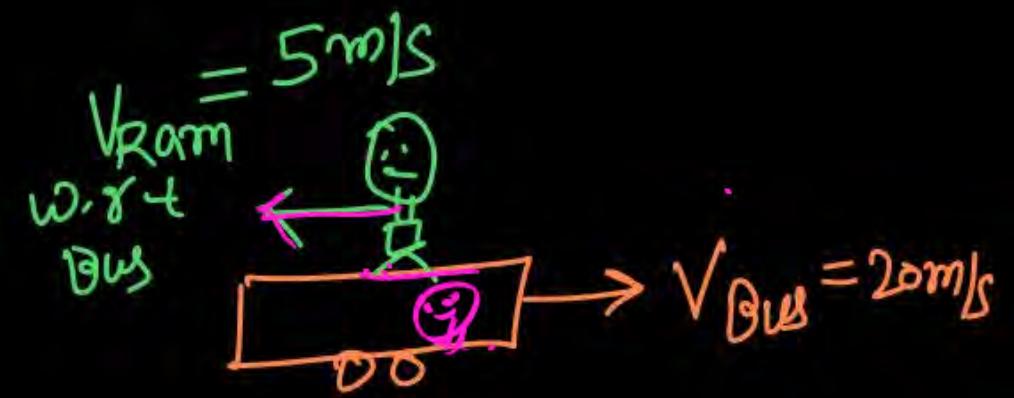
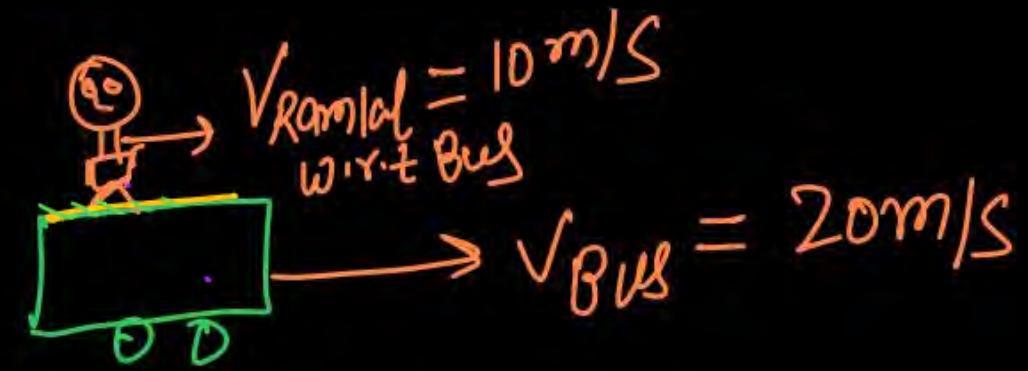
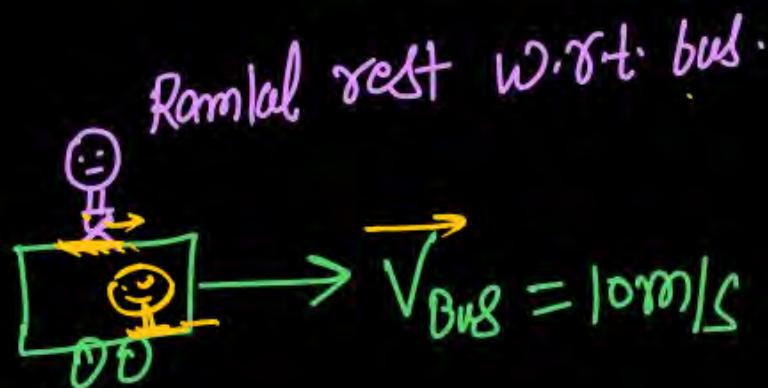
$$A = 2 \text{ m/s}^2$$
$$(30 - 10) = 20 \text{ m/s}$$

$$V^2 - U^2 = 2as$$

$$0 - (20)^2 = 2 \times 2 \times d$$
$$d = \frac{20 \times 20}{4} = \underline{\underline{100 \text{ m}}}$$



# Relative Motion from moving frame :-



$$V_{Ram Bus} = -5 \text{ m/s}$$

$$V_{Ram Grav} = 15 \text{ m/s}$$

$$V_{Ram Lal, Bus} = 0 \quad \checkmark$$

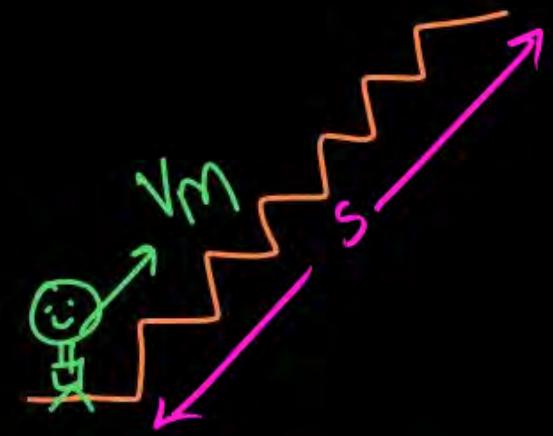
$$V_{Ram Bus} = 10 \text{ m/s}$$

$$V_{Ram Grav} = 30 \text{ m/s}$$

$$V_{Ram Group} = 10 \text{ m/s} \quad \checkmark$$

~~MR\* Box:~~ —  
Jab bhi Koi Object  
moving journe me hai  
to wo object journe  
ki velocity as it is copy  
kar lega but  
acn copy nahi karega

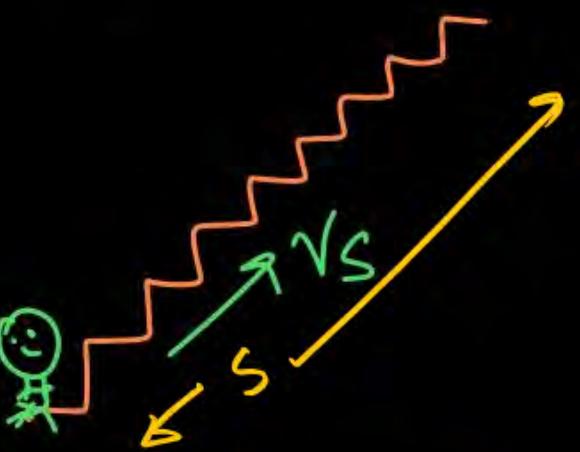
escalator at rest



Man is moving

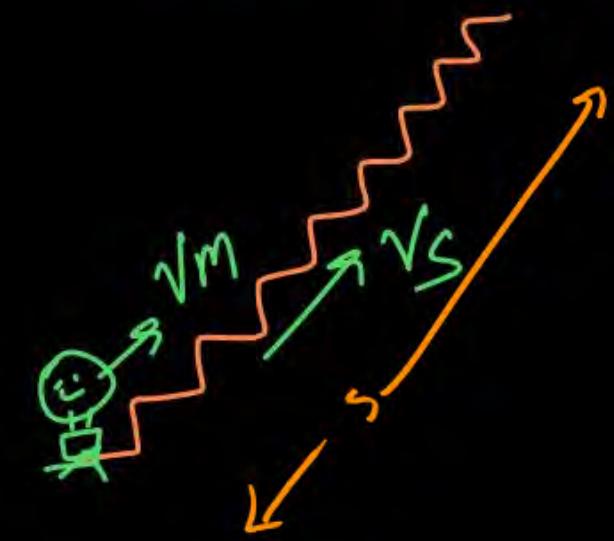
$$t_1 = \frac{s}{v_m}$$

escalator moving  
(man at rest)



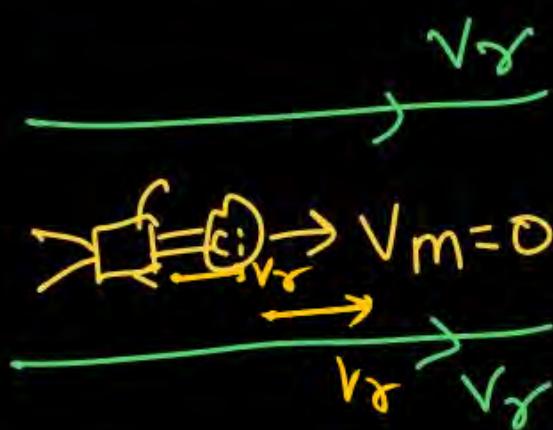
$$t_2 = \frac{s}{v_s} - ①$$

escalator & man  
both are moving



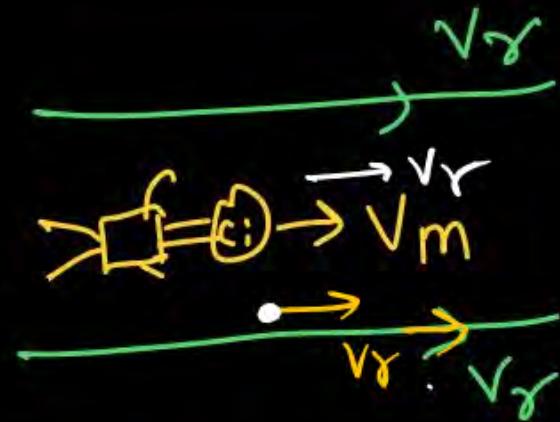
$$t = \frac{s}{(v_m + v_s)} - ①$$

## River man in 1-D

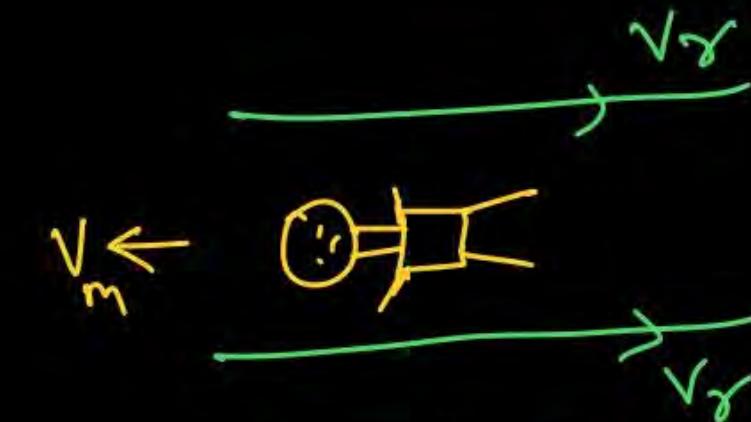


(man is not swimming)

$$V_{\text{man river}} = 0 \quad \checkmark$$



Man is swimming with  $V_m$



Man is swimming backward.

$$V_m > V_r \quad \checkmark$$

$$\vec{V}_{\text{man river}} = -\vec{V}_m \quad \text{Back}$$

$$V_{\text{man river}} = V_m \quad \checkmark$$

Velocity by which Man swims =  $V_m$

$$V_{\text{man ground}} = V_r - \textcircled{1}$$

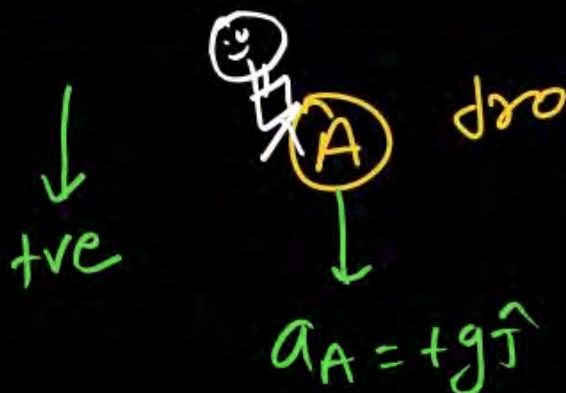
$$V_{\text{man ground}} = V_m + V_r - \textcircled{1}$$

$$\vec{V}_{\text{man ground}} = \vec{V}_m - \vec{V}_r \quad (\text{Baln})$$

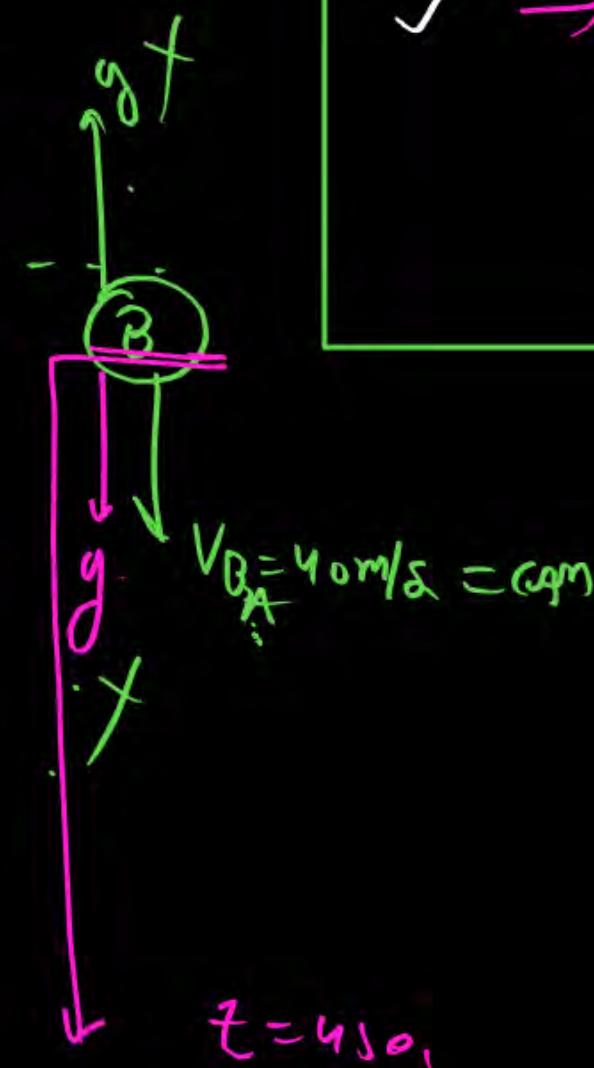
$$\vec{V}_{\text{man ground}} = \vec{V}_r - \vec{V}_m \quad \checkmark$$

$V_r > V_m$

## Relative motion in motion under gravity :-



A diagram showing a circular object labeled 'B' falling vertically downwards. An arrow points downwards from the center of the circle, labeled 'G'. Below it, the equation  $a_B = g\hat{j}$  is shown. To the right, the initial velocity  $v_{B0} = 40 \text{ m/s}$  is given.



$\Rightarrow$

$$\begin{aligned}\vec{a}_{BA} &= \vec{a}_B - \vec{a}_A \\ &= g\hat{j} - g\hat{j} \\ &= 0\end{aligned}$$

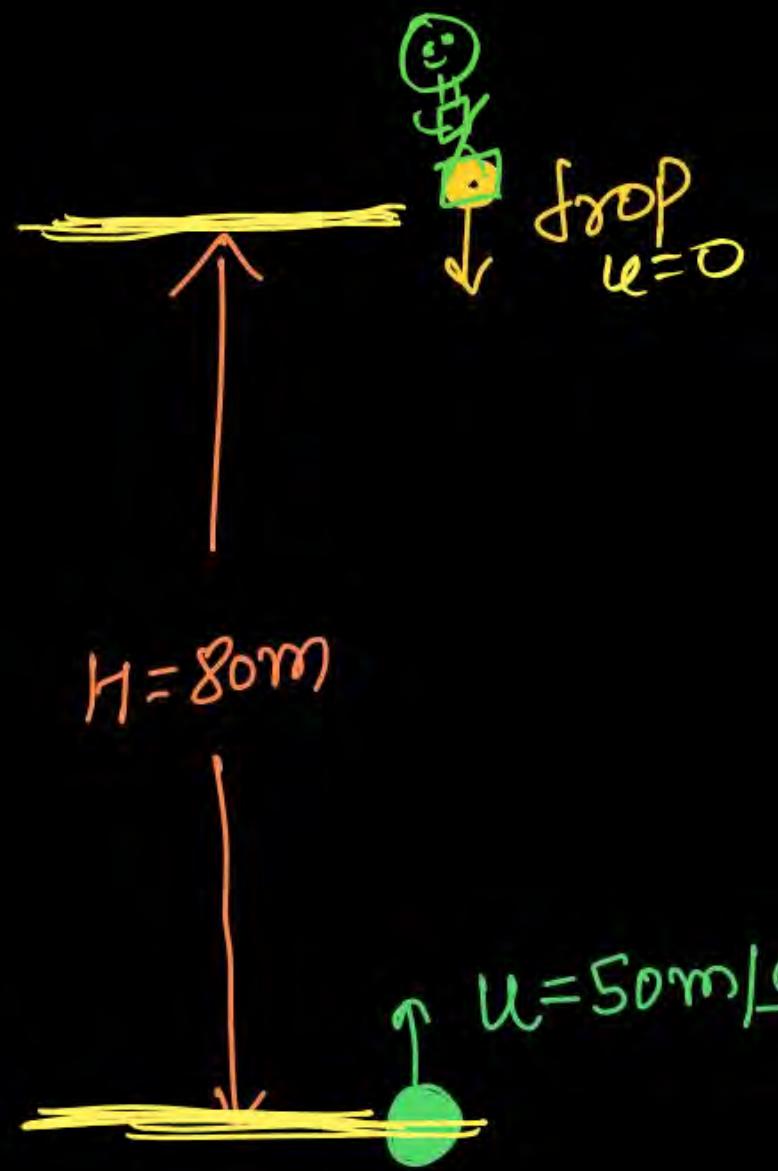
- ✓ → motion under gravity of a object is Non-uniform motion
- ✓ → Motion under gravity of one object w.r.t other object who also in motion under gravity is uniform motion ✓

$$v_{B0} = 40 \text{ m/s} = 40 \text{ m/s}$$

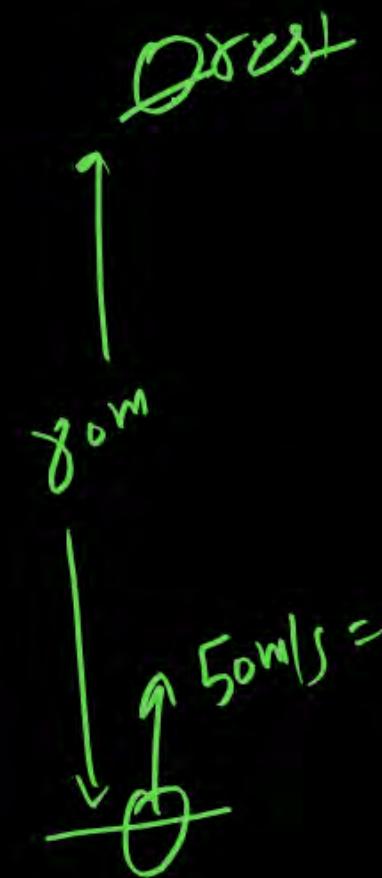
$$t = 4 \text{ s}$$

$$Y = 40 \times 4 = 160 \text{ m}$$

If relative accn is zero, then  
velocity constant  
hence relative motion uniform



find time of collision.



$$t = \frac{80}{50} \text{ sec} \checkmark$$

$$t = \frac{8}{5} \text{ sec} \checkmark$$



Same in projectile Mot<sup>n</sup> relative accn one object in projectile motion w.r.t. to other object in Projectile motion is zero, then relative Velocity is uniform, hence , relative Motion straight Line

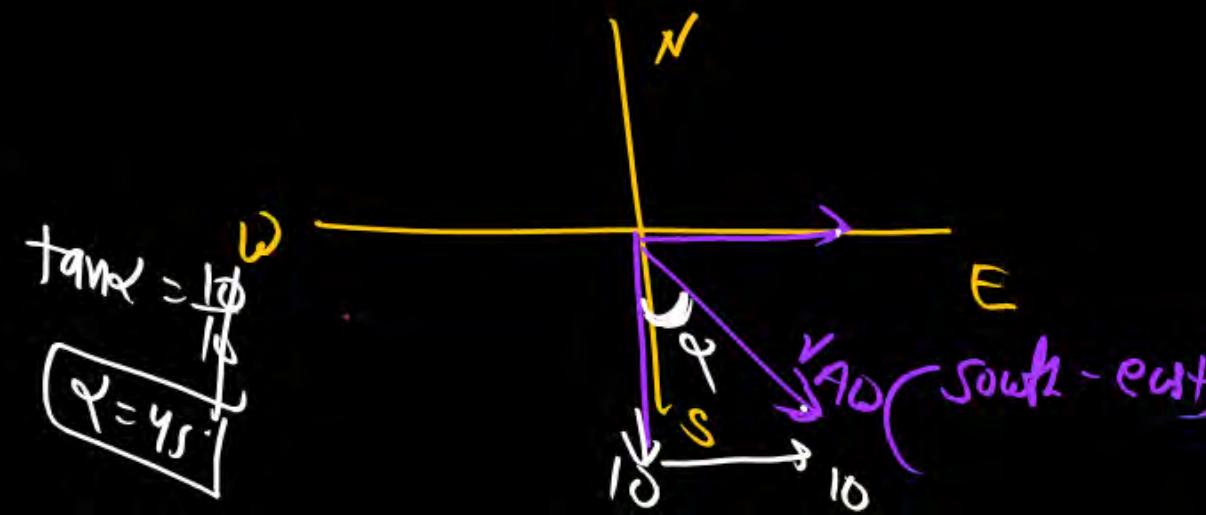
Relative motion in 2-D :- Same as 1-D bas vector ka fcel  
dalma hei :-

$$(A) \rightarrow V_A = 10 \text{ m/s} \hat{i} (\text{east})$$

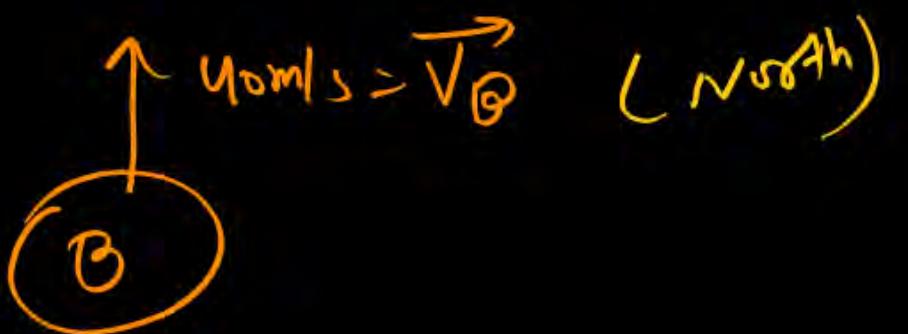
(B)  $V_B = 10 \text{ m/s} \hat{j} (\text{North})$

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

$$\underline{\vec{V}_{AB}} = 10 \hat{i} - 10 \hat{j}$$



$$V_A = 30 \text{ m/s} \text{ west}$$



$$\vec{V}_{BA} = ??$$

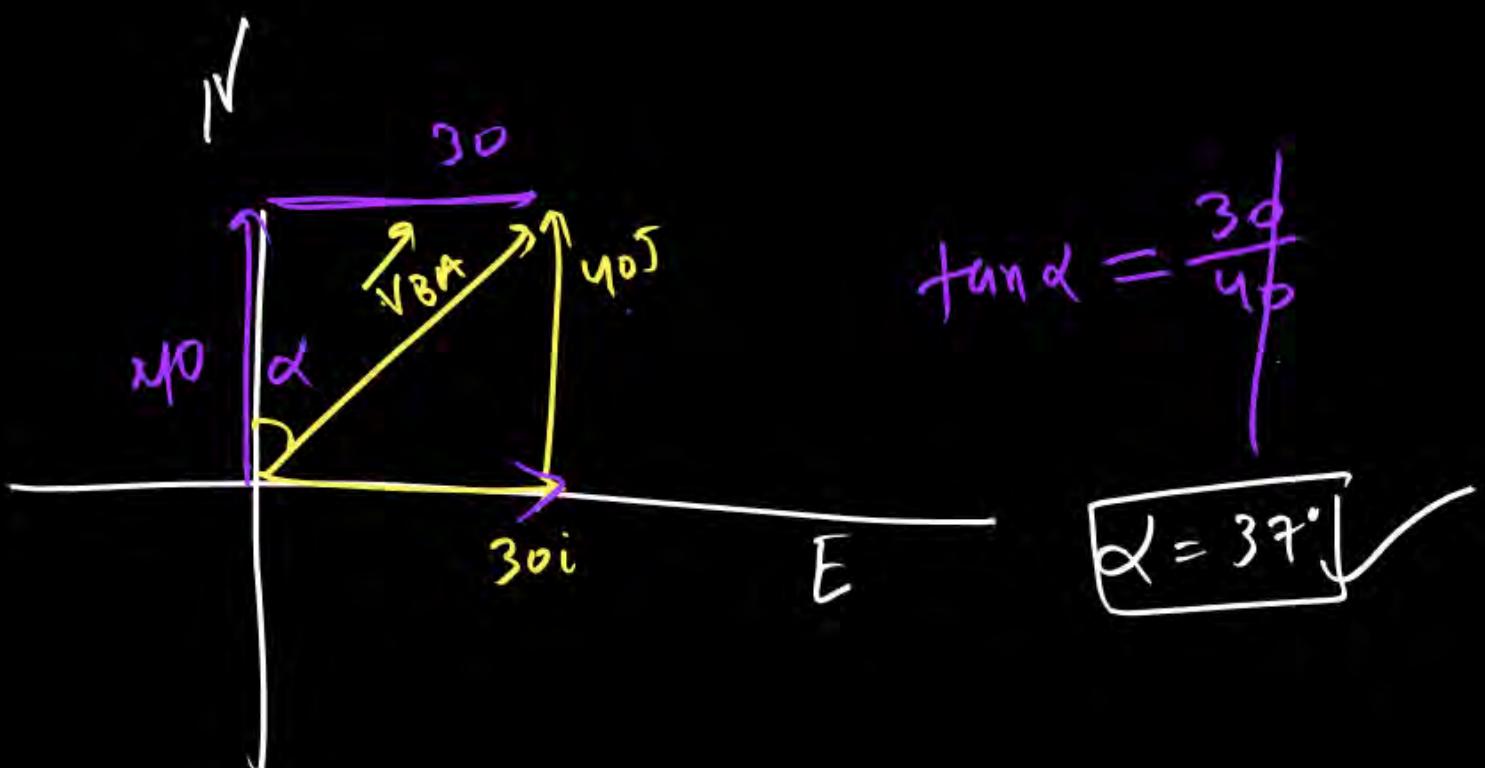
So

$$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$$

$$= 40\hat{j} - (-30\hat{i})$$

$$= 40\hat{j} + 30\hat{i}$$

$$|\vec{V}_{BA}| = \sqrt{(40)^2 + (30)^2} = 50 \text{ m/s}$$





Bird

$\vec{V}_{0y} = 3\hat{j}$

$V_B = 5 \text{ m/s}$

$37^\circ$

$\vec{V}_{0x} = V_B \cos 37^\circ = 5 \times \frac{4}{5} = 4\hat{i}$

Pinky

$\vec{V}_p = 3 \text{ m/s } \hat{j}$

$$\vec{V}_{B \text{ Ramlal}} = \vec{V}_B - \vec{V}_R$$

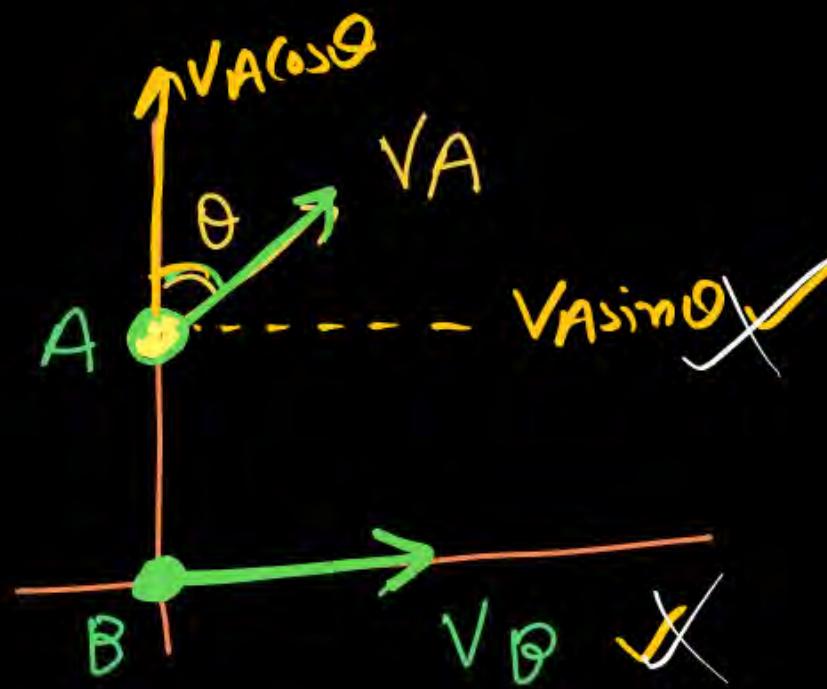
$$= 4\hat{i} + 3\hat{j} - 4\hat{i}$$

$\boxed{\vec{V}_{B \text{ Ramlal}} = 3\hat{j}}$

$$\vec{V}_{\text{Ramlal Org}} = -3\hat{j}$$

$$\vec{V}_{B \text{ Pinky}} = \vec{V}_B - \vec{V}_p = 4\hat{i} + 3\hat{j} - 3\hat{j} = 4\hat{i}$$

$$\vec{V}_{\text{Pinky Bir}} = -4\hat{i}$$



(Q) find  $\frac{V_A}{V_B}$  so that A appears to move vertically upward w.r.t. B

$$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$$

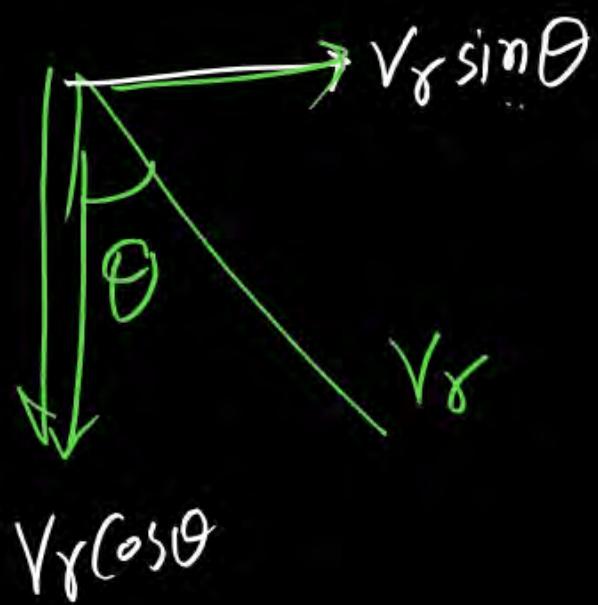
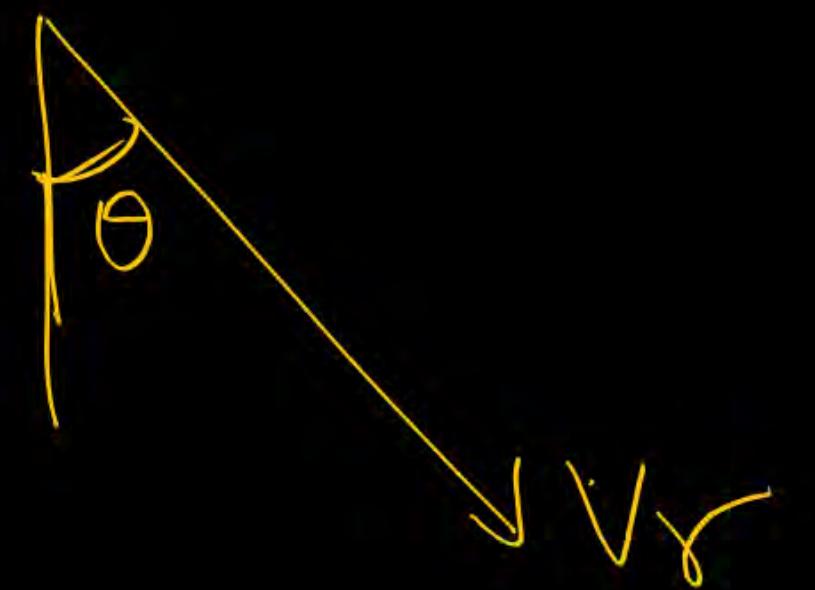
$$\vec{V}_{AB} = V_A \sin \theta \hat{i} + V_B \cos \theta \hat{j} - V_B \hat{i}$$

$$= V_B \cos \theta \hat{j} + (V_A \sin \theta - V_B) \hat{i}$$

$$V_A \sin \theta - V_B = 0$$

$$V_A \sin \theta = V_B$$

$$\frac{V_A}{V_B} = \sin \theta = \frac{1}{\cos \theta} = \sec \theta$$



$V_m = V_r \sin \theta$

for this cond<sup>n</sup>  
 $\gamma$  will appr to  
full verti<sup>l</sup>  
downward

## Minimum Separation :-

**MR\* POX:-** Kisi ek par Ram Lal ko bitha ke dusre object ka relative velocity likho, dusra object relative velocity ke direction me Jayga. relative velocity ke direction se A par perpendicular line hi minimum separation hoga.

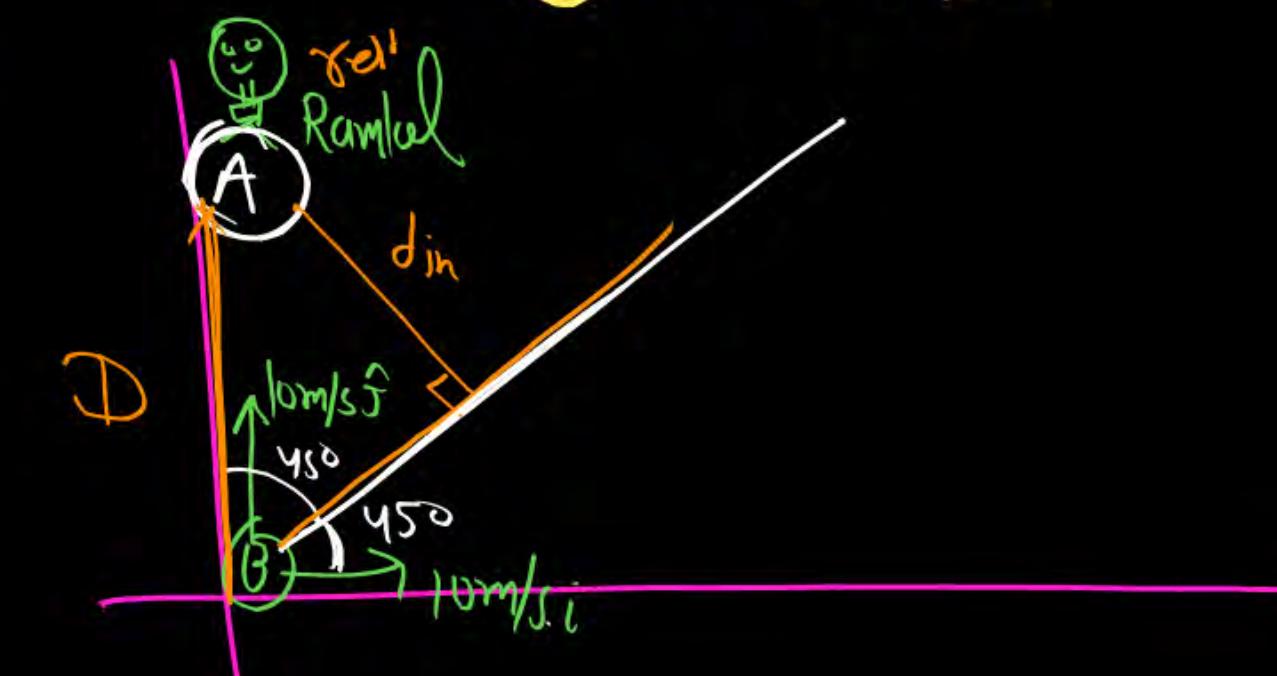
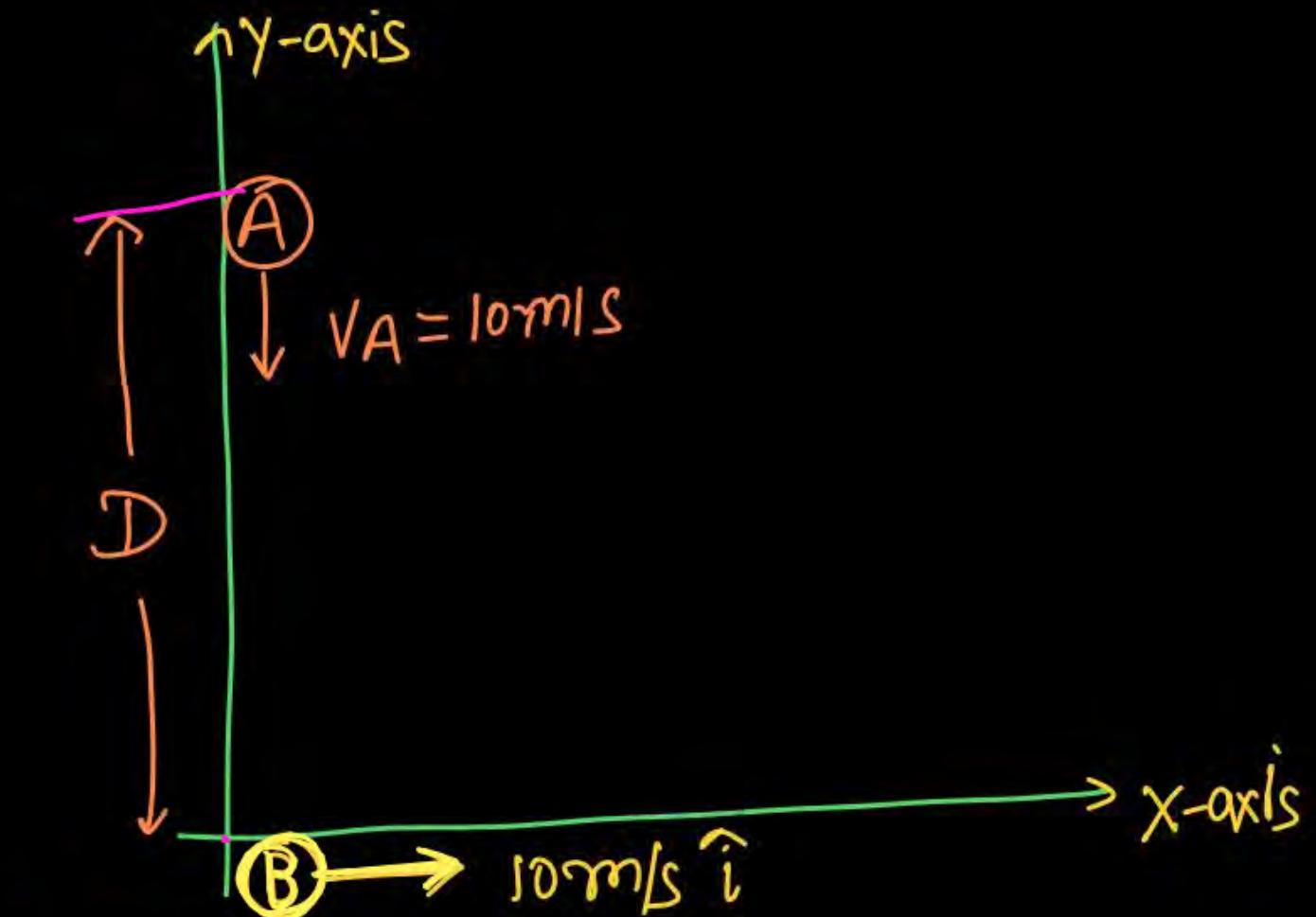
$$t = \frac{D}{\sqrt{2} (10\sqrt{2})} = \frac{D}{20}$$

at which they are at min<sup>m</sup> Sepn

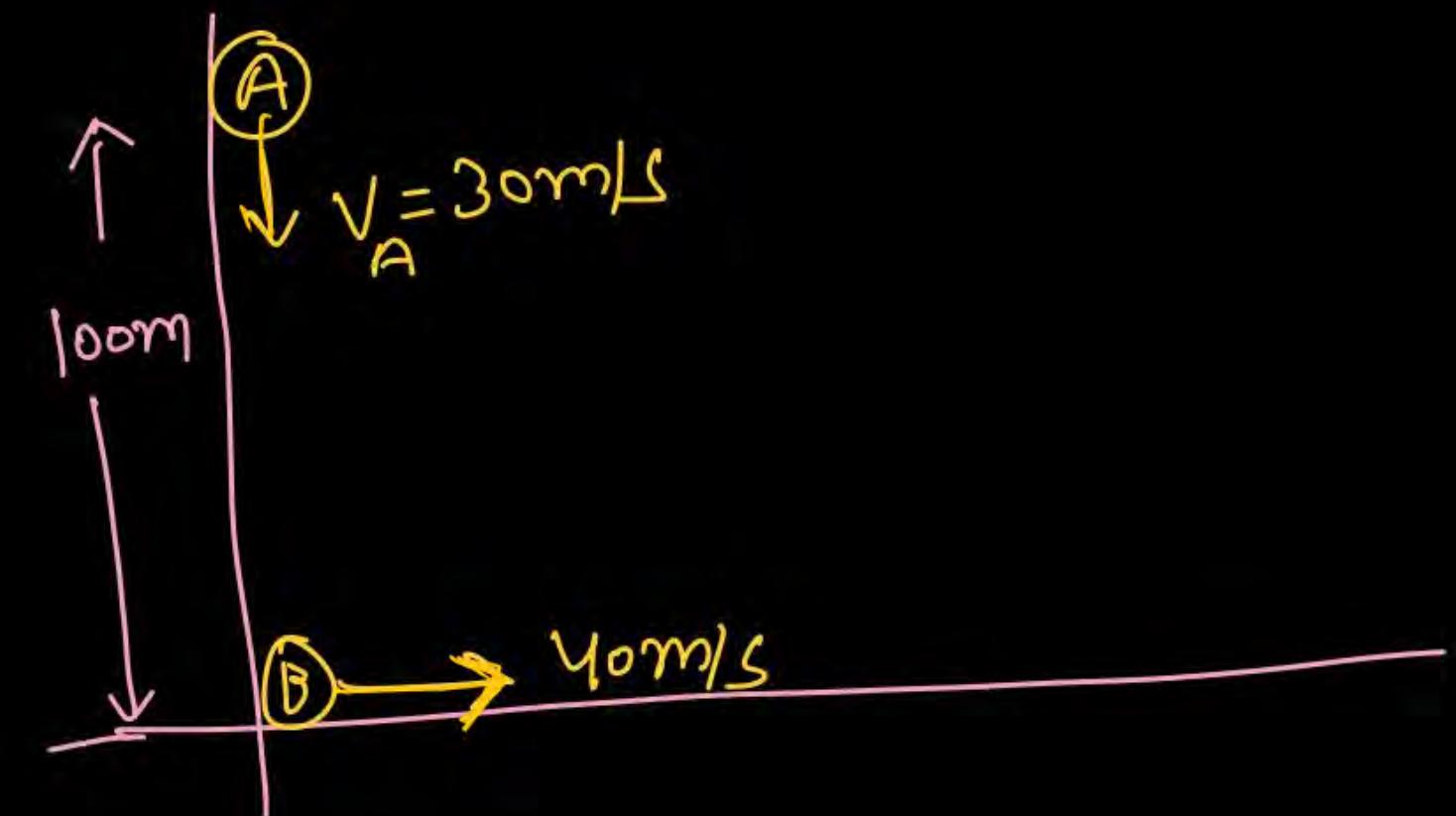
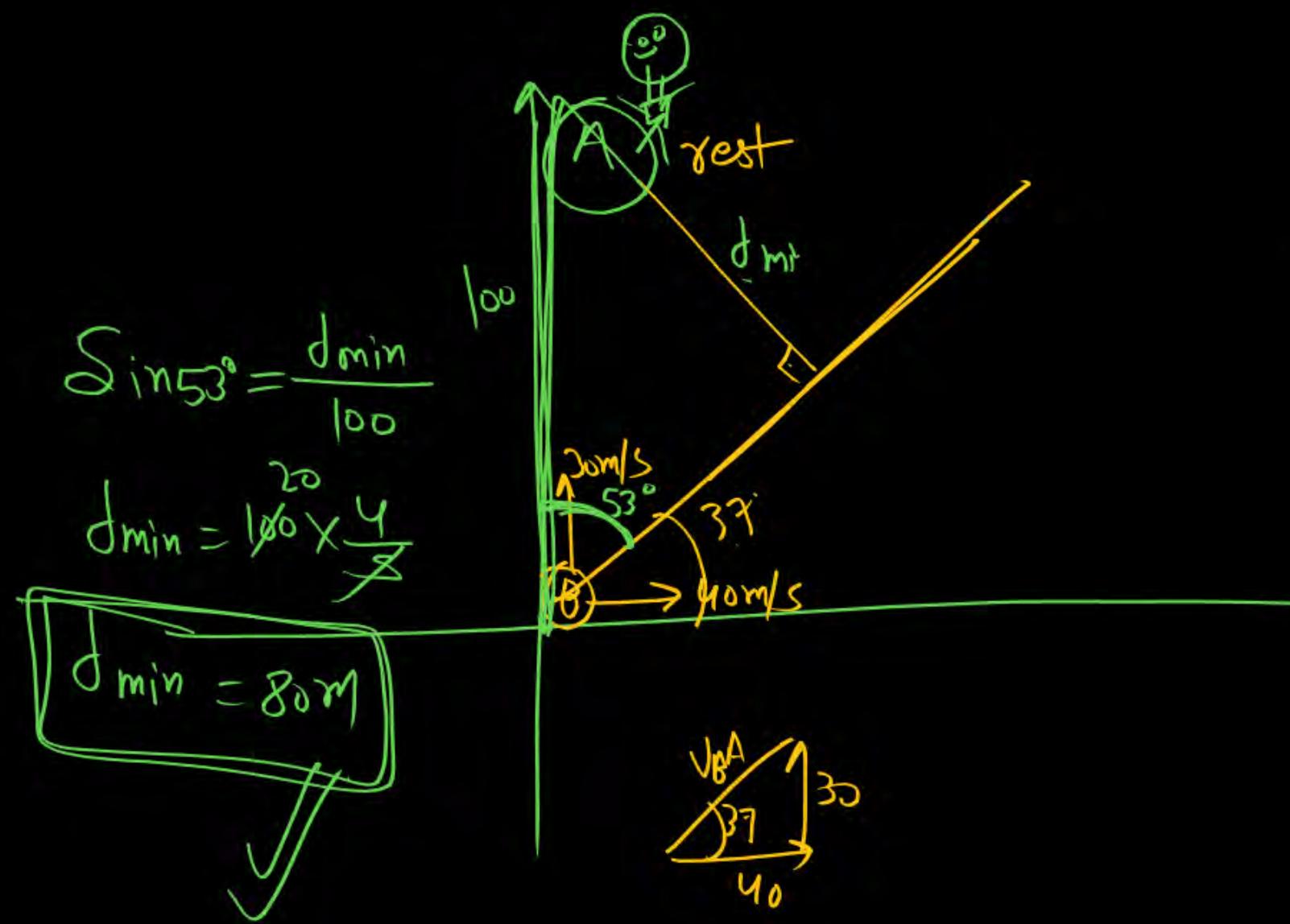
$$\sin 45^\circ = \frac{d_{min}}{D}$$

$$d_{min} = D \times \frac{1}{\sqrt{2}}$$

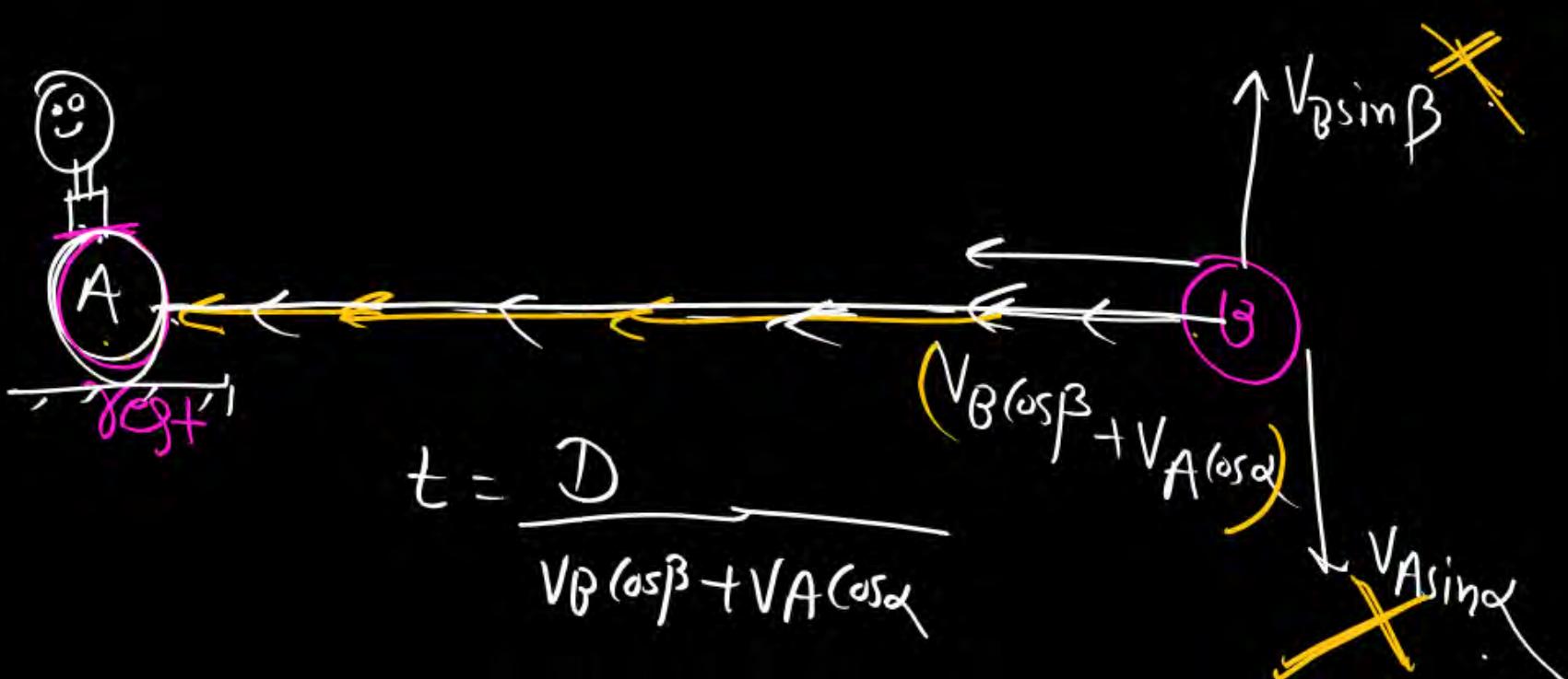
$$V_{BA} = 10\sqrt{2}$$



find minimum separation :-



# Condition of collision



MR\* Box:-

① relative velocity must be along line of joining.

② Relative velocity must be zero Perpendicular to the line Joining of object

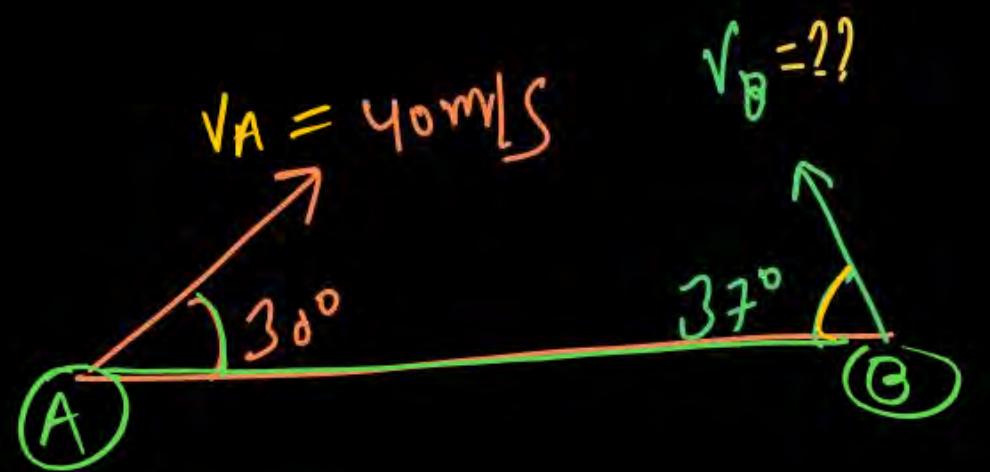
Cond<sup>n</sup> of collision (AI PMT -2015)

$$\left\{ \hat{V}_{BA} = -\hat{V}_{AB} \right\}$$

Comp<sup>n</sup> of their velocity must be same wrt to line joining ✓

Projectile motion:-

$$\vec{a}_{AB} = 0$$



Cond<sup>n</sup> of Collision

$$40 \sin 30^\circ = v_B \sin 37^\circ$$

$$\cancel{40 \times \frac{1}{2}} = v_B \times \frac{3}{5}$$

$$\boxed{\frac{100}{3} = v_B} \checkmark$$



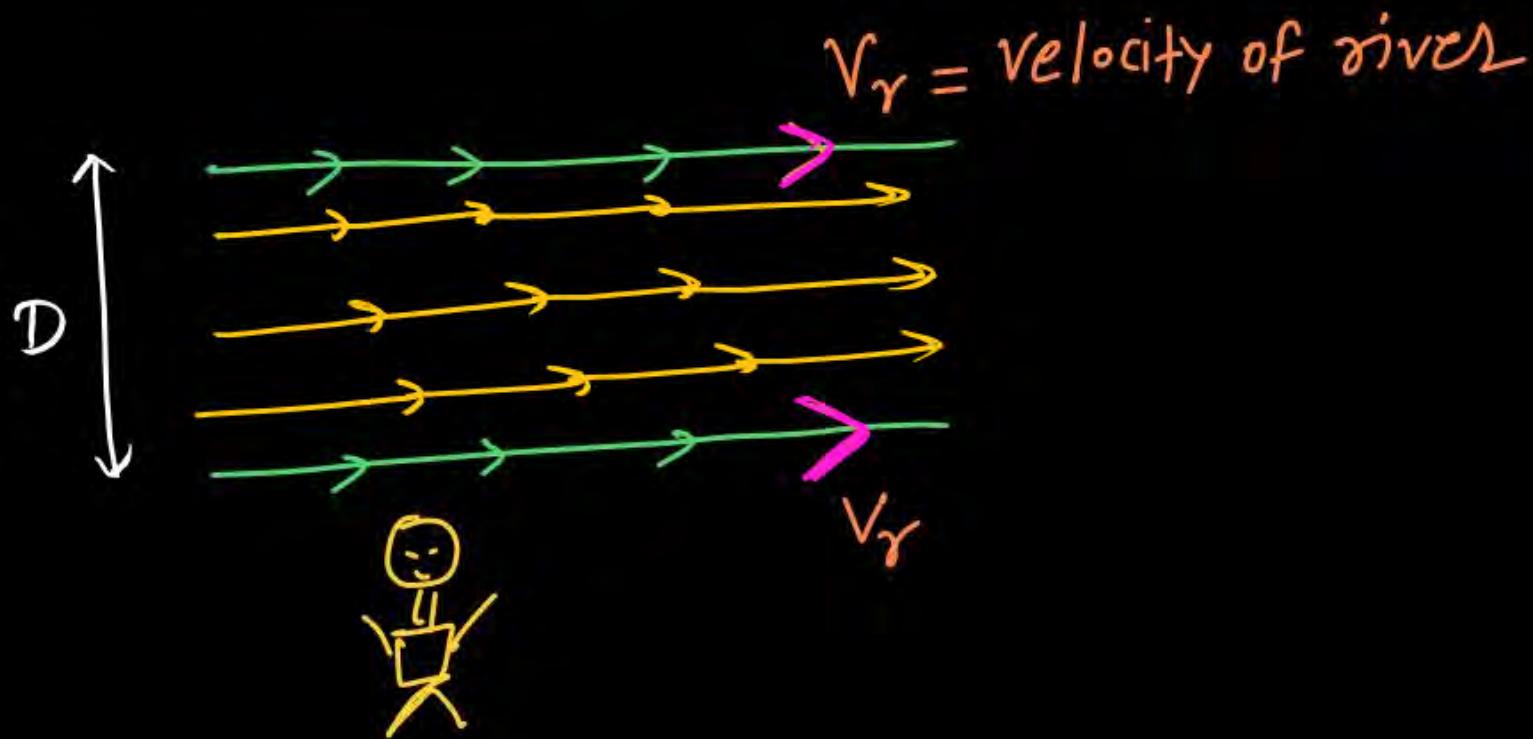
$\rightarrow u_A \sin \alpha = u_B \sin \beta$      $\# [R_1 + R_2 \geq x]$  ✓  
 $\rightarrow$  Cond'n of collision

$\Rightarrow x > R_1 + R_2 \rightarrow$  No Colls.

## River Man Problem :-

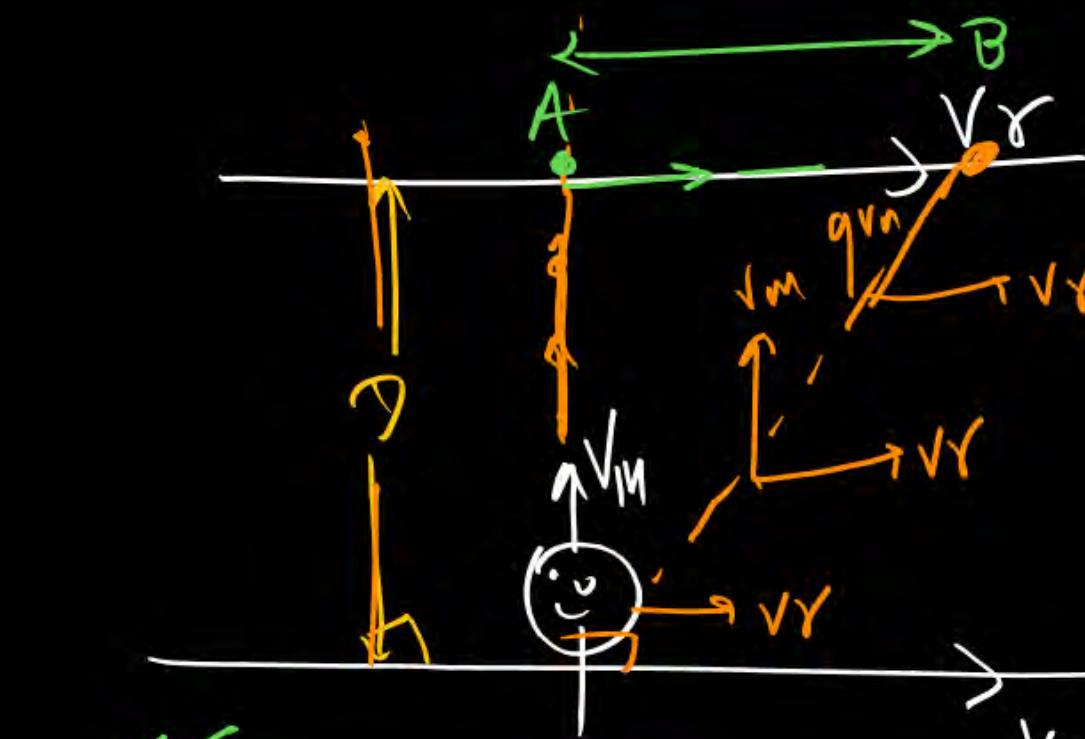
River ka flow Na to river ke across support karega Na oppose. River ka flow river ke along velocity deg.

NEET



MP\*  
River ko apne  
dam par cross kiya  
Jata hai, river ka flow  
Na to support karega Na  
oppose karega.

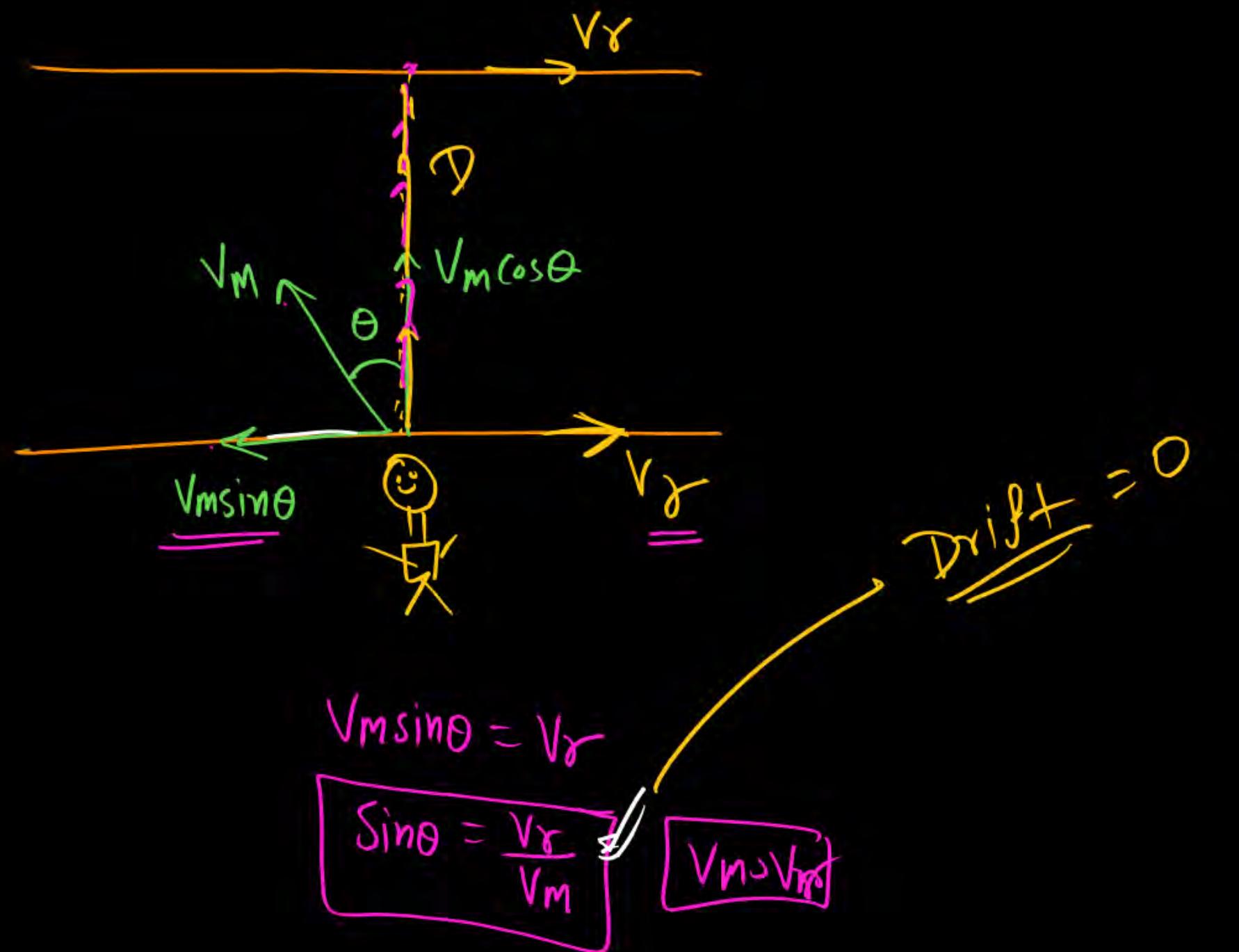
① Minimum time:-



Drift =  $v_r t = v_r \frac{D}{v_m}$

$t = \frac{D}{v_m}$

③ Man is swimming such that want to reach just opposite end of river  $\Leftrightarrow (V_m > V_r)$  {minimum Path}.



$$t = \frac{D}{V_m \cos \theta}$$

$$\sin \theta = \frac{V_r}{V_m}$$

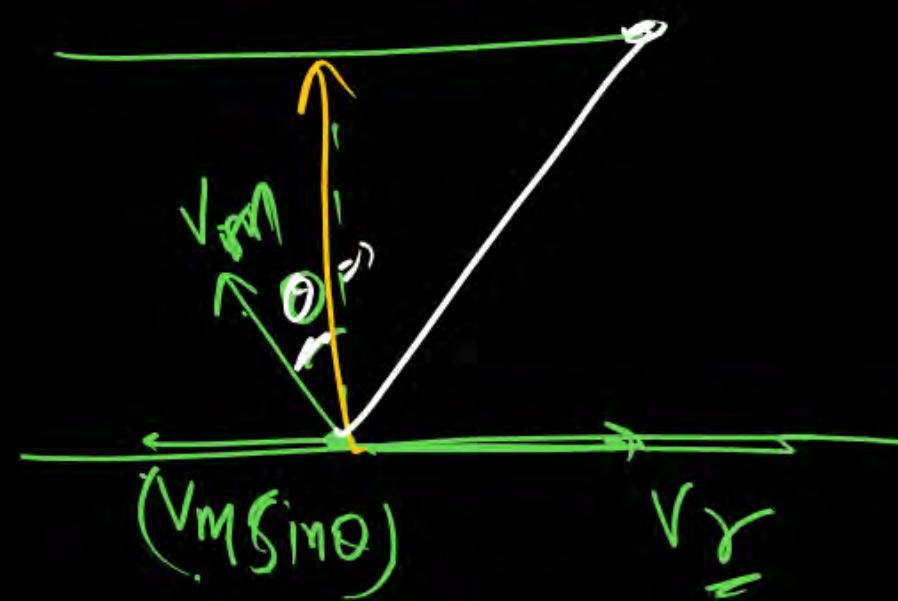
$$\sin \theta = \frac{10}{20}$$

$\theta = 30^\circ$



$$\alpha = 120^\circ \text{ from flow direction}$$

(4) If  $\sqrt{v_y} > \sqrt{v_m}$  then find direction of swimming, for minimum drift:-



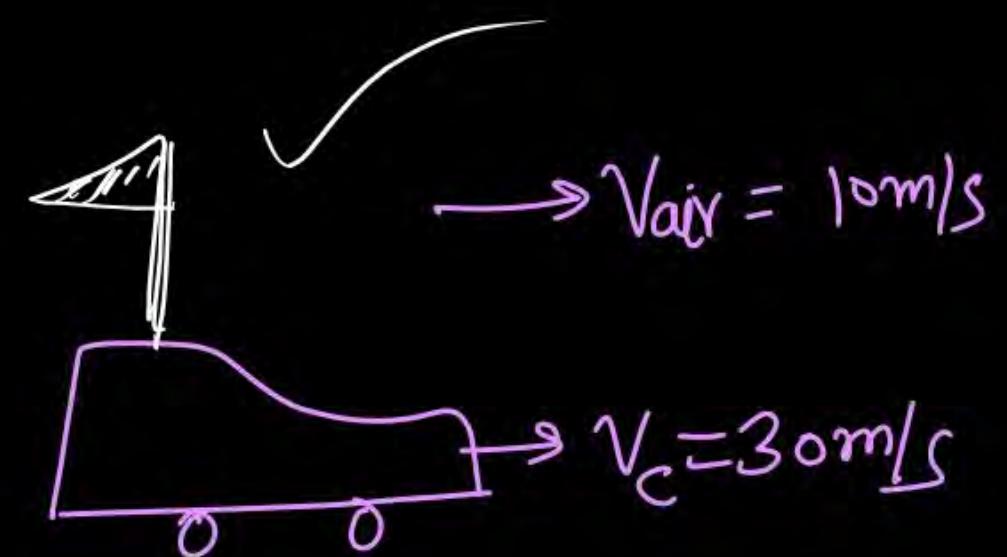
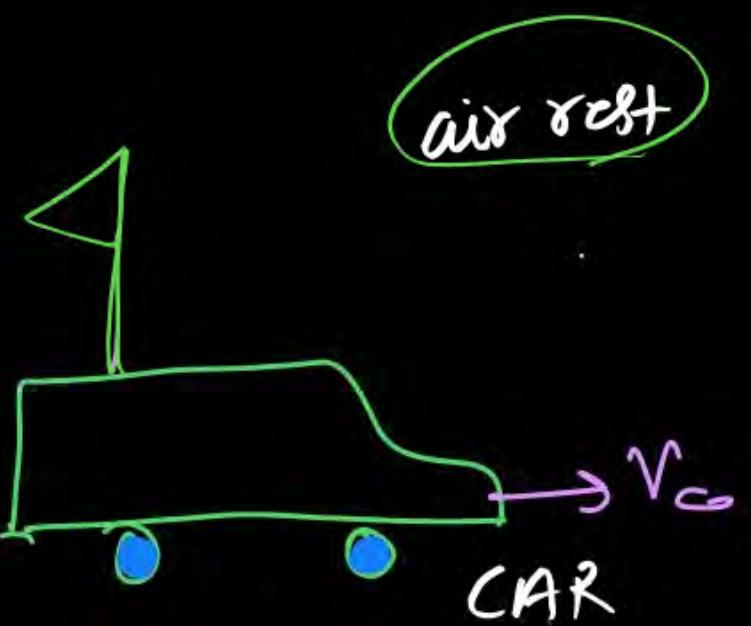
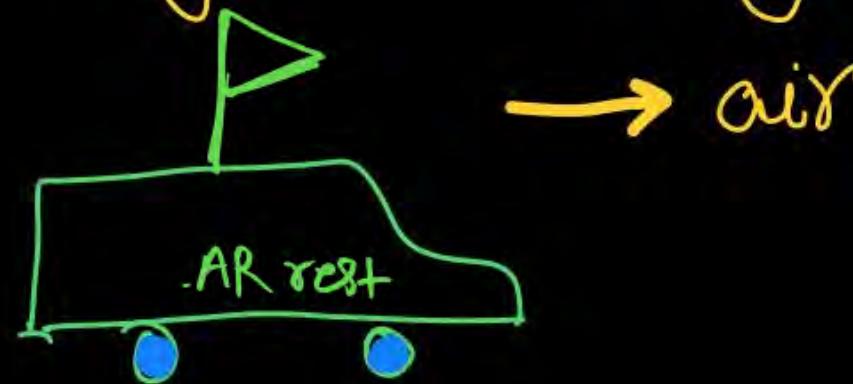
Zero drift is not possible

$$\sin \theta = \frac{\sqrt{v_m} (\text{small})}{\sqrt{v_y} (\text{large})}$$

from across  $v_m$

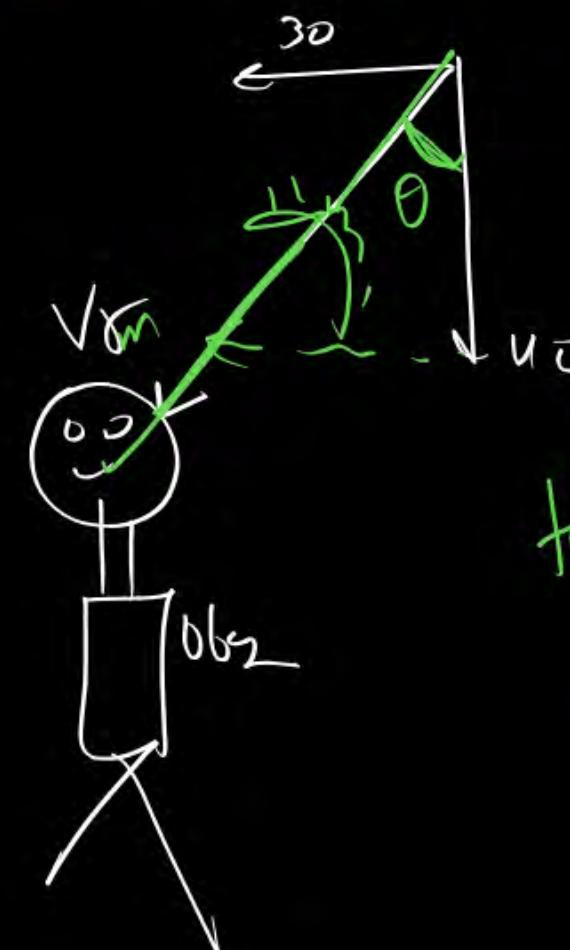
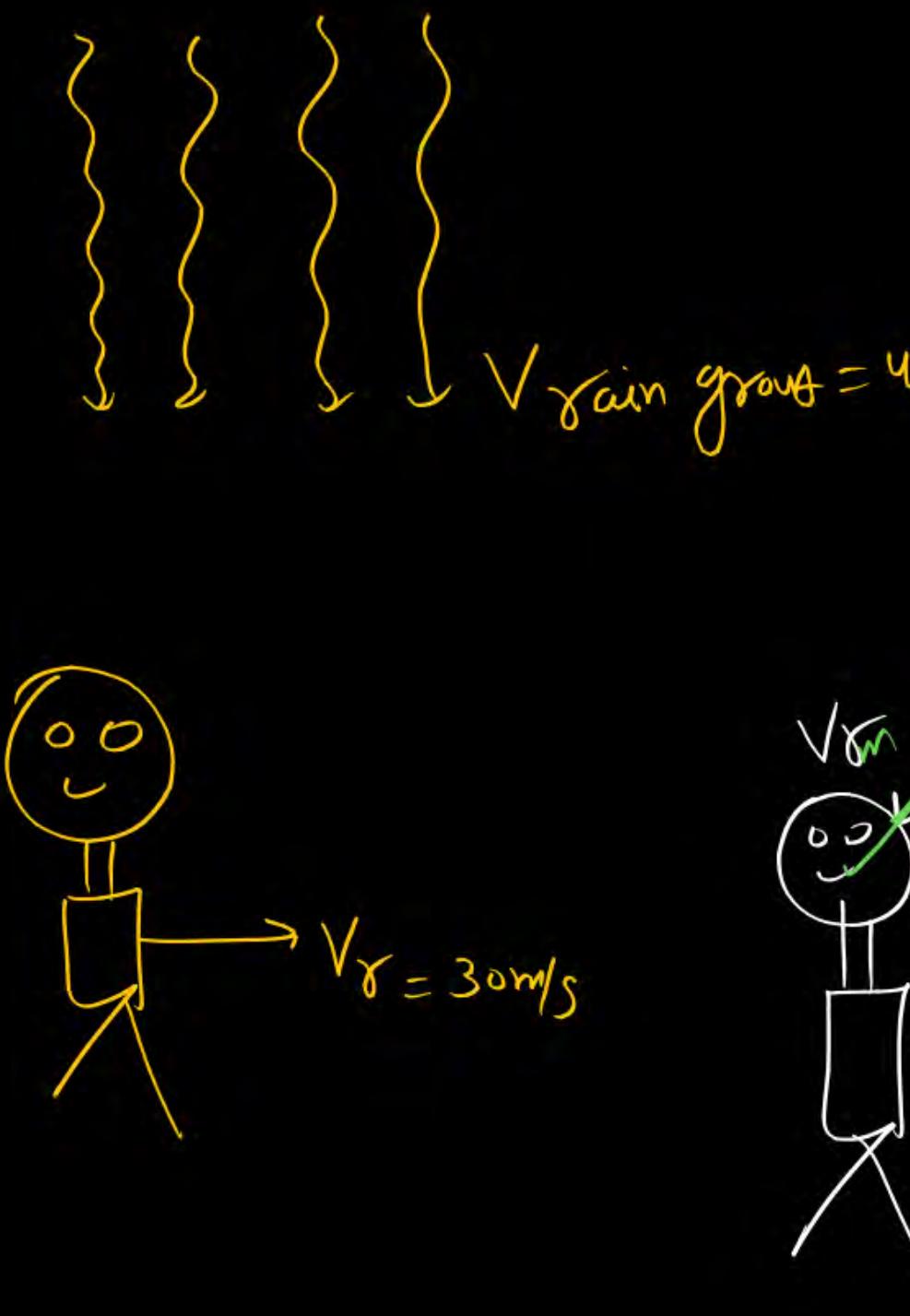
$$\boxed{\sqrt{v_m} < \sqrt{v_y}}$$

flag fluttering :-



- flag flutters in the direction flow of air.
- flag flutters in the opposite dirn of motion of CAR

# Rain-Man Problem :-



$$\tan \theta = \frac{30}{40}$$

$$\tan \theta = \frac{3}{4}$$

$$\theta = 37^\circ$$

Answer

$$\vec{V}_{RM} = -30i - 40j$$

$$|V_{RM}| = 50 \text{ m/s} \quad \text{Answer}$$

MR\* Box :-

$$(1) \text{ find } \vec{V}_{RM}$$

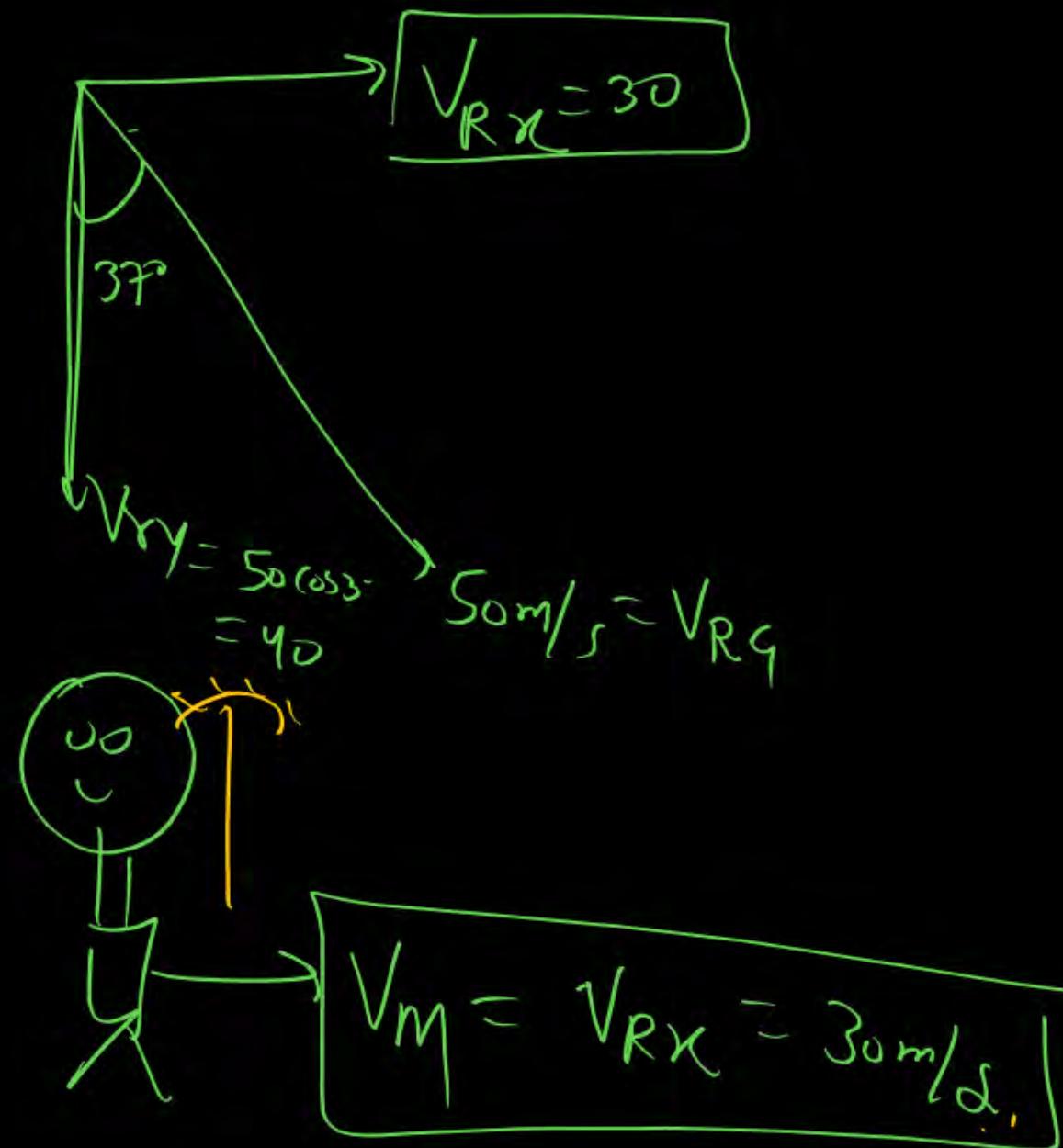
$$\vec{V}_{RM} = \vec{V}_R - \vec{V}_m$$

(2) Draw  $\vec{V}_{RM}$  in vector form  $\Rightarrow$  find Angle using  $\tan \theta$

(3) use Umbrella in direction of  $\vec{V}_{RM}$

(4) given velocity of ask kiske respect me hai, ye dhyani se dekhna. ✓

Rain is falling with  $50\text{m/s}$  at angle  $37^\circ$  from vertical towards east  
• Man starts running with  $v_m$ , then rain appears to fall vertically  
on his head then find  $v_m$  ??



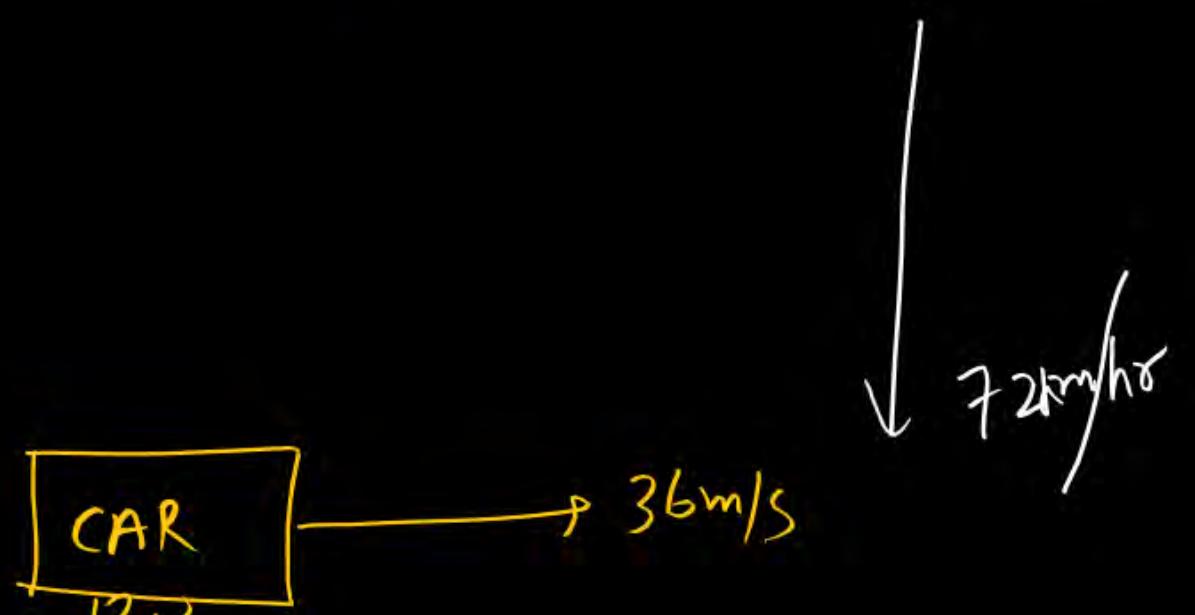
$$\boxed{\sqrt{v_{ym}^2 + v_{Ry}^2} = 40 \text{ m/s}}$$

CAR is moving with speed 36 m/s in east & rain is falling at 72 km/hr vertically downward. find Angle at which rain strike to the vertical mirror of CAR.

$$V_{rg} = 72 \text{ km/hr}$$

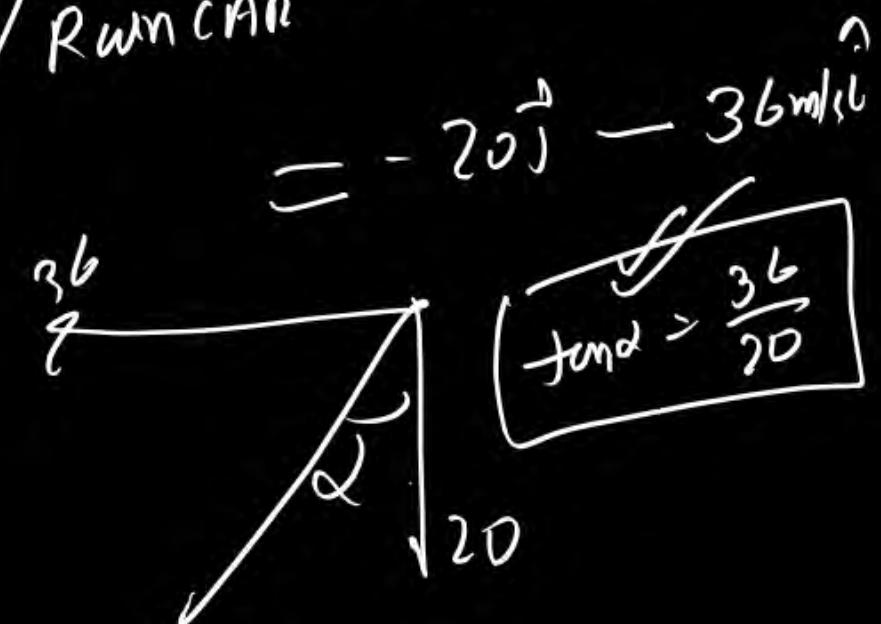
$$= 72 \times \frac{5}{18}$$

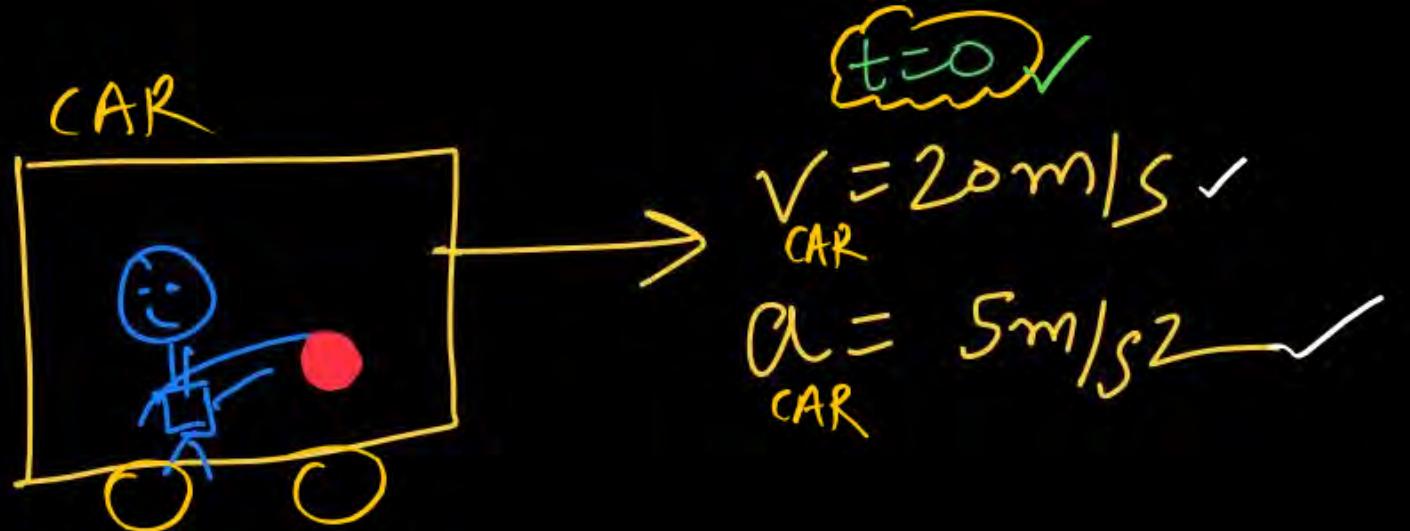
$$= 20 \text{ m/s} \hat{j}$$



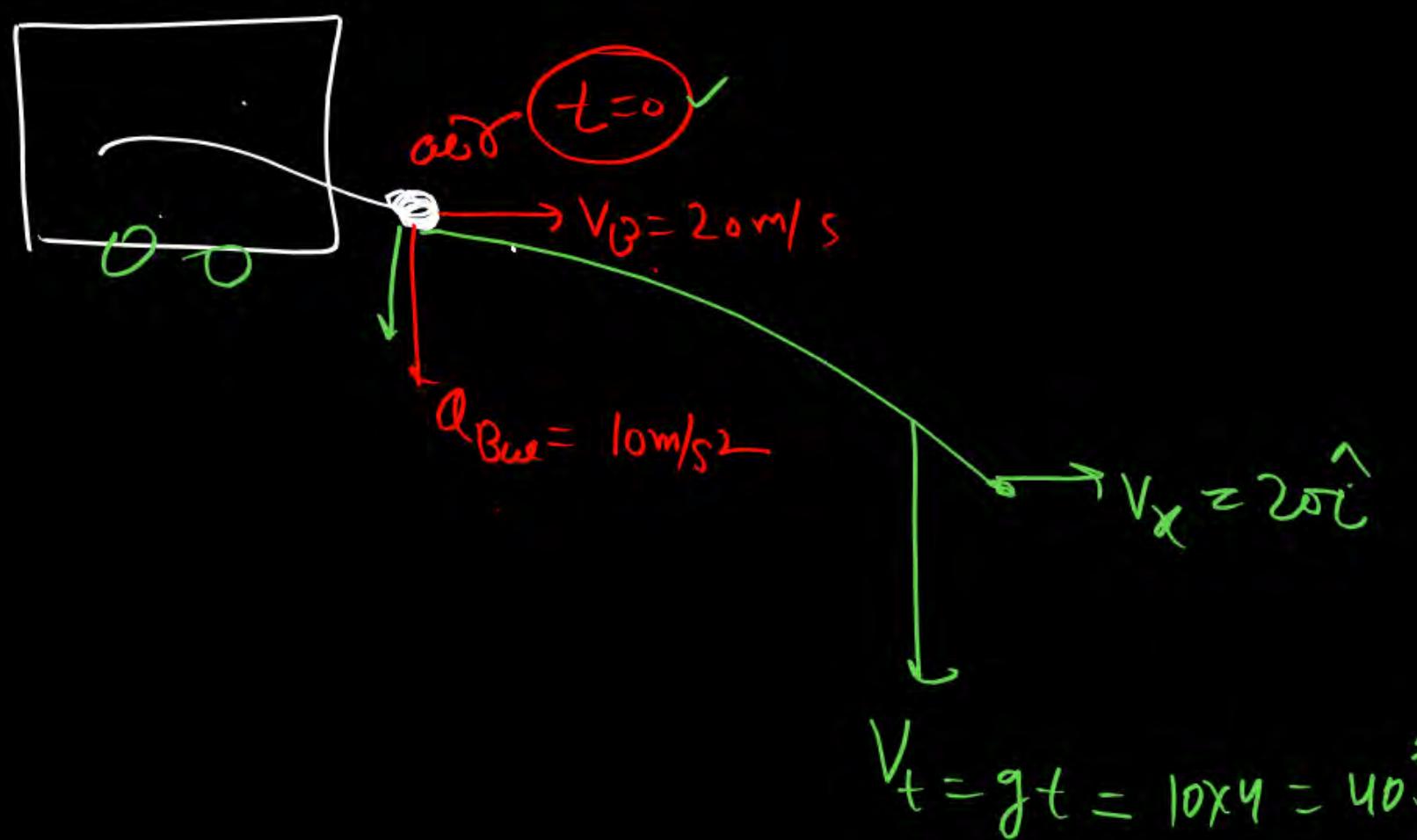
$$V_{Rain\,CAR} = V_R - V_C$$

$$= -20\hat{j} - 36\hat{i}$$





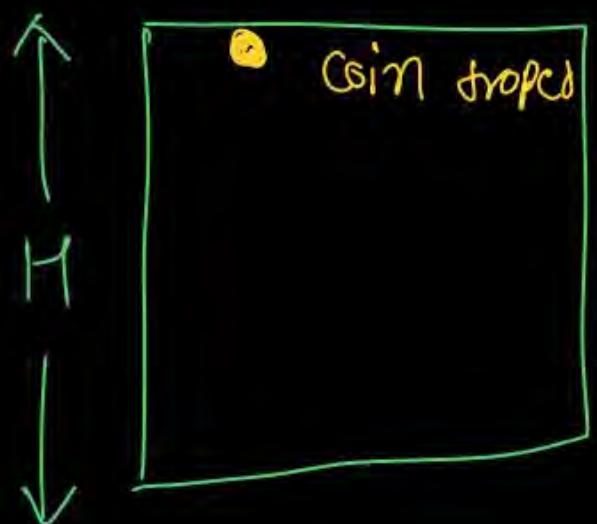
Ball is dropped; then find its velocity and acceleration after  $t = 4 \text{ sec}$ .



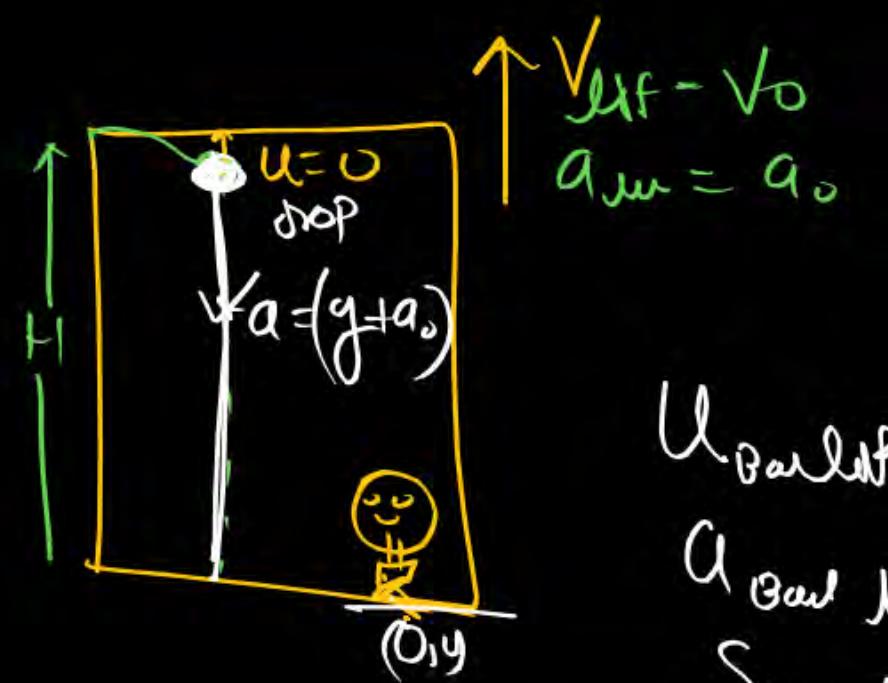
$$\vec{v}_{t=4} = 20\hat{i} + 40\hat{j}$$

$$\vec{a}_{t=4} = g \text{ down}$$

Find time when coin reach to the base of lift.



$$\begin{aligned} \uparrow & \\ a_{\text{lift}} &= a_0 \\ \text{velocity}_{\text{lift}} &= v_0 \end{aligned}$$



$$\begin{aligned} \uparrow & \\ v_{\text{f}} &= v_0 \\ a_{\text{air}} &= a_0 \end{aligned}$$

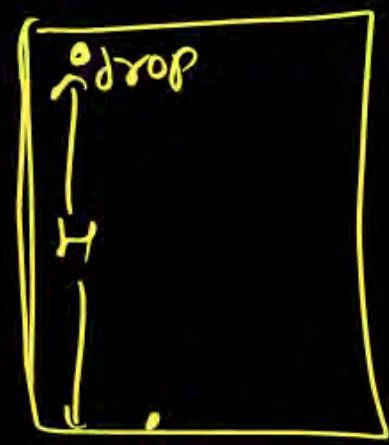
$$u_{\text{Ball lift}} = 0$$

$$a_{\text{Ball lift}} = g + a$$

$$s_{\text{Ball lift}} = H$$

$$s = ut + \frac{1}{2}at^2$$

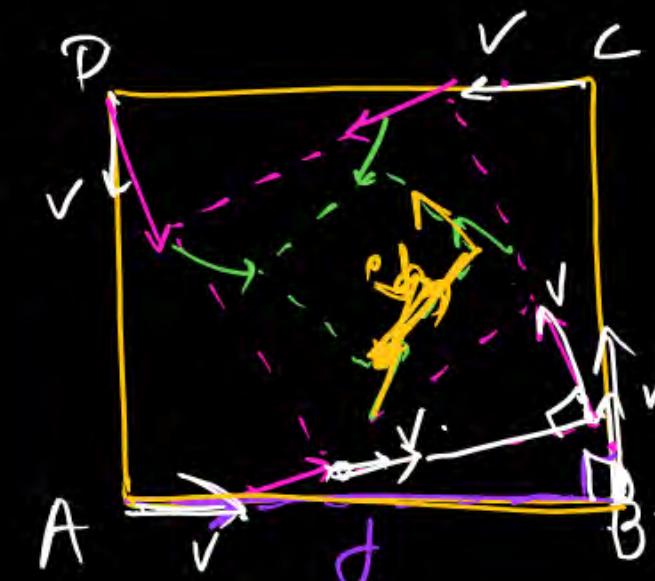
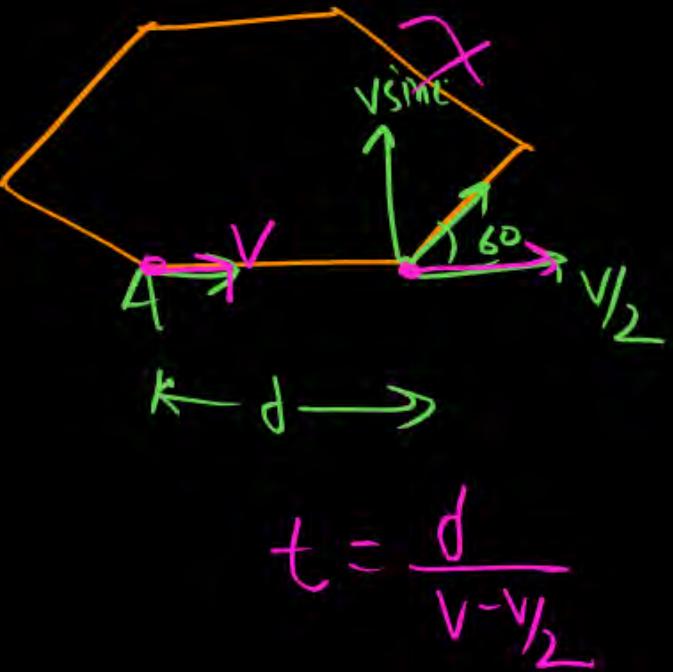
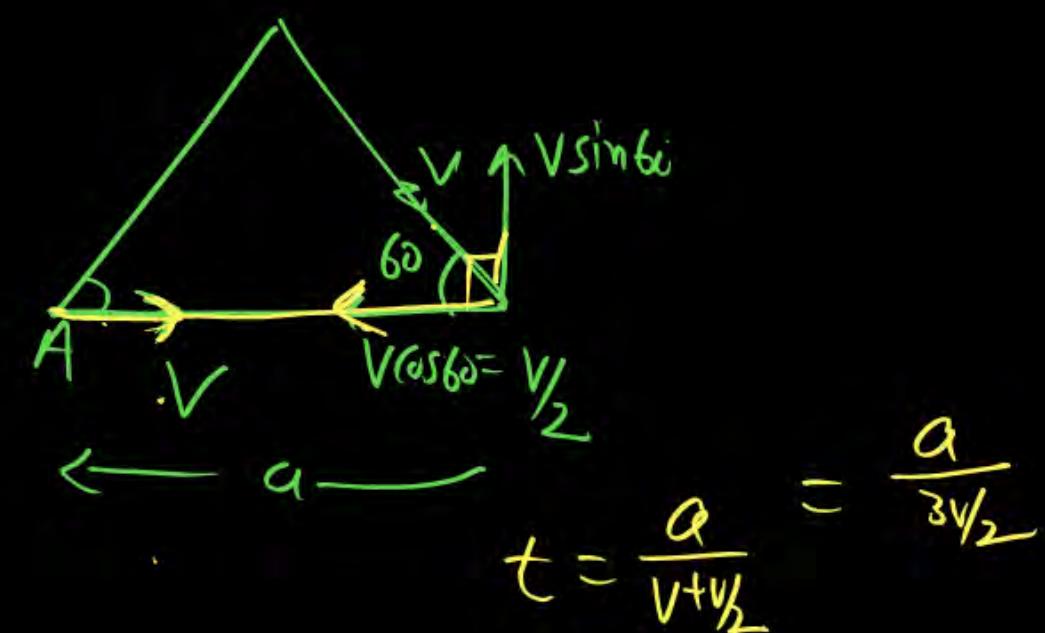
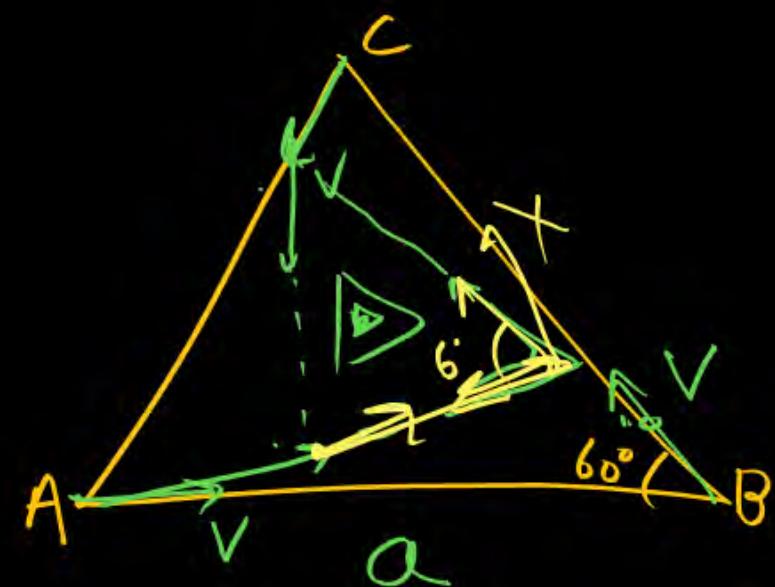
$$\begin{aligned} H &= \frac{1}{2} (g+a) t^2 \\ t &= \sqrt{\frac{2H}{g+a}} \quad \text{Ans} \end{aligned}$$



$$a_{\text{lift}} = g$$

$$t = \sqrt{\frac{2H}{(g-a)}}$$

(Q) Three persons standing on corners of equilateral triangle start moving towards each other with speed  $v$  then find time when they will meet.



mot'n along line joining

$$t = \frac{d}{v}$$



# THANK YOU

