YAKEEN NEET 2.0

2026

Motion in a Plane

Physics | Lecture - 01

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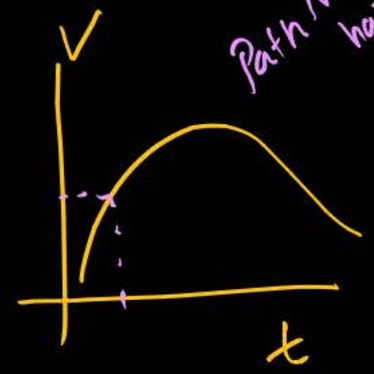
Topics to be covered

1. # equation of path :—
2. Projectile Motion
3.
4.

(Q) If Position of object $x = 4\sin(t)$ & $y = 8\sin(t)$ then Path of object is"

(a) 1-D [Straight line] NR scam.

(b) 2-D [motion Dn curve]

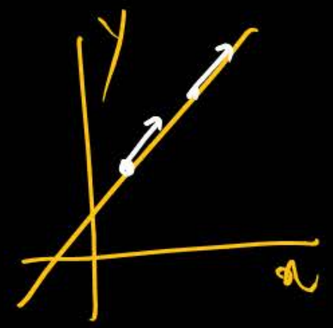


Path Nahi hai.

MR^{+} BOX eq^{n} of x/t or v/t or a/t Object ke Path ko represent Nahi karta hai, only x & y ka rel^{n} or x & y ka graph Path ko reprent Karta hai.

equation of Path → Rel^{n} b/w y & x;

case 1 $y = mx + c$ ← straight line



$x^2 + y^2 = 4^2$ circular Path.

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

↳ elliptical Path.

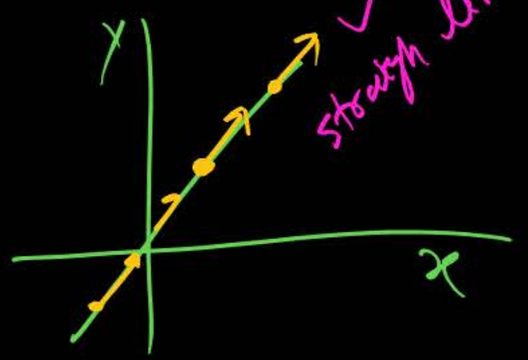
① $y = 8\sin(t)$ — (i)

$x = 4\sin(t)$ — (ii)

(i) / (ii)

$$\frac{y}{x} = \frac{8\sin t}{4\sin t}$$

$$\boxed{y = 2x}$$ 1-D



② $x = 3t$ — (i)

$y = 9t$ — (ii)

$$\frac{y}{x} = \frac{9t}{3t}$$

$$\boxed{y = 3x}$$

straight line

1-D

③ $y = 3t^2$ — (i)

$x = 2t$ — (ii)

* Puting value of t from eqn (ii) to (i)

$$t = x/2$$

$$y = 3(t)^2$$

$$= 3\left(\frac{x}{2}\right)^2$$

$$\boxed{y = 3\frac{x^2}{4}}$$

Parabolic [2-D]

④ $x = 3\sin(t)$ — (i)

$y = 3\cos(t)$ — (ii)

Soln $\frac{x}{y} = \frac{3\sin t}{3\cos t}$

$\frac{x}{y} = \tan t$

$(i)^2 + (ii)^2$

$x^2 + y^2 = [3\sin(t)]^2 + [3\cos(t)]^2$

$x^2 + y^2 = 9[\sin^2 t + \cos^2 t]$

$$\boxed{x^2 + y^2 = 9}$$

circular path [2-D]

⑤ $x = 4\sin(t)$

$y = 5\cos(t)$

Soln

$$\frac{x}{4} = \sin t \quad - (i)$$

$$\frac{y}{5} = \cos(t) \quad - (ii)$$

$$(i)^2 + (ii)^2$$

$$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{5}\right)^2 = \sin^2 t + \cos^2 t$$

$$\boxed{\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1} \rightarrow \text{elliptical Path.}$$

$$[\sin\theta]^2 = \sin^2\theta$$

$$[\cos\theta]^2 = \cos^2\theta$$

Question

Position of object at time 't' $\vec{r} = 2t\,\hat{i} + 4t^2\,\hat{j}$, then find equation of trajectory.

$\vec{r} = 2t\,\hat{i} + 4t^2\,\hat{j}$

$x = 2t \longrightarrow t = x/2$

$y = 4t^2$

$y = 4\left(\frac{x}{2}\right)^2 = \frac{4x^2}{4}$

$y = x^2$ Parabol

Question (111)

Velocity of object $\vec{V} = 2\,\hat{i} + x\,\hat{j}$ then equation of trajectory.

$\vec{V} = 2\hat{i} + x\hat{j} = V_x\hat{i} + V_y\hat{j}$

velocity

$\Rightarrow V_x = 2 \quad \Rightarrow V_y = x$

$\hookrightarrow \frac{dx}{dt} = 2 \quad -(1) \qquad \frac{dy}{dt} = x \quad -(11)$

(a)

Straight line

$\int dx = \int 2\,dt$

$x = 2t \quad -(111)$

$dy = x\,dt$

$\int dy = \int 2t\,dt$

$y = 2\frac{t^2}{2} = t^2$

$y = t^2 = \left(\frac{x}{2}\right)^2$

$y = \frac{x^2}{4}$

(11)/(1)

$\frac{dy}{dx} = \frac{x}{2}$

$dy = \int \frac{x}{2}\,dx = \frac{1}{2}\left(\frac{x^2}{2}\right) = \frac{x^2}{4}$

$m\vec{a}^*$

Question

H/W MR^2

A particle moving with velocity $V = y\,\hat{i} + x\,\hat{j}$ then find equation of trajectory.

[IIT-2020]

Motion in a Plane [2-D motion] $\Longrightarrow$ [1-D] + [1-D]

[1-D] → x-axis, [1-D] → y-axis

$[2\text{-}D]_{motion}$ = 1-D motion + concept of vector

MR* → Jab bhi 2-D motion ka question ho Vector ke component method se usko Break karo, then $[1\text{-}D]_x$ & $[1\text{-}D]_y$ me seprate solve karo then Vector ke magnitude concept se final Answer likho.

MR*

x ka P.Q sirf x ke P.Q se solve hoga

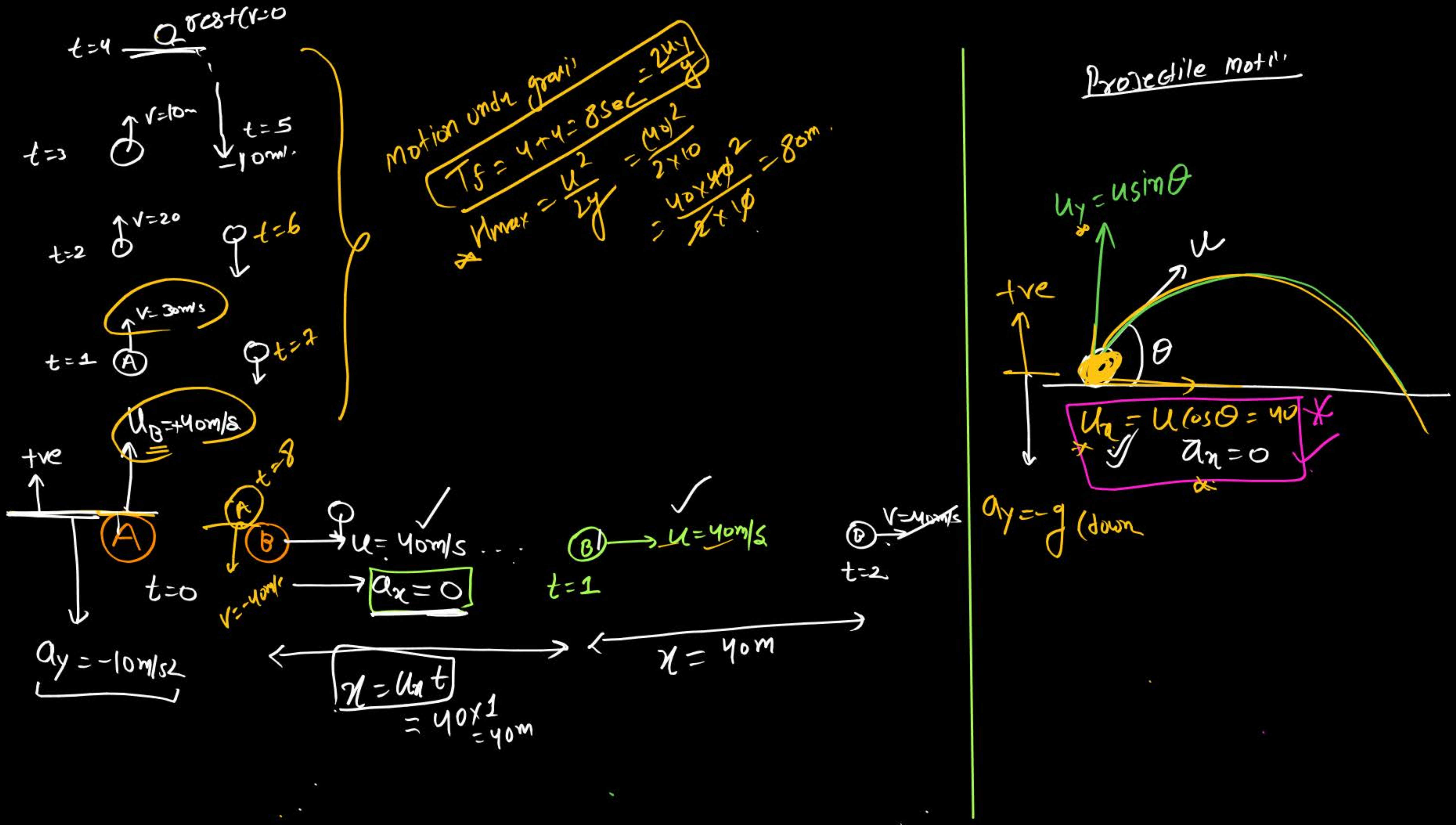
y ka P.Q. sirf y ke P.Q. se solve hoga

MR* Box

* $a_x = 0$; $\vec{V}_x$ = Cost^n — uniform motion in x

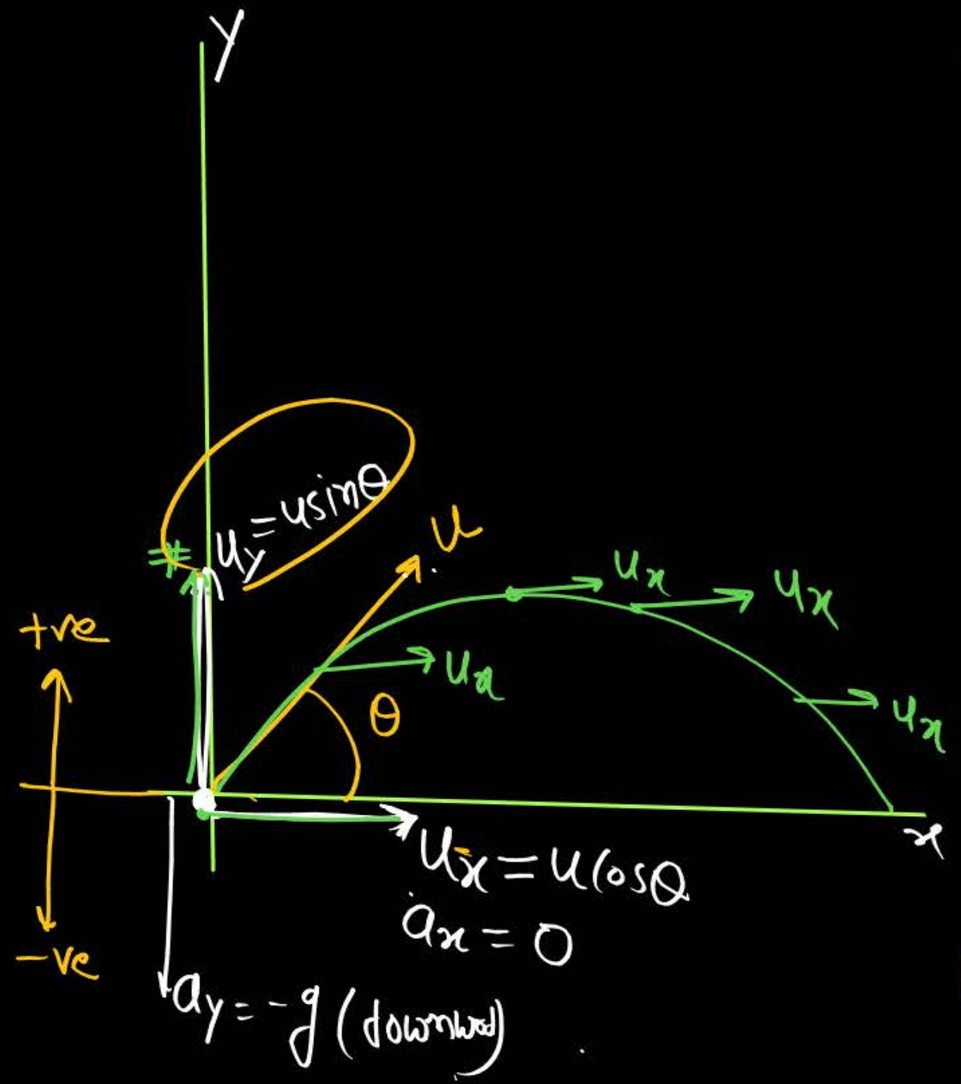
* $a_y \neq 0$; $\vec{V}_y$ = varible — Non-uniform in y

MR* Koin hai wo Jo dono axis ka majale rahai → time



rest (v=0)
t=4
t=3
v=10m
t=5
-10m/s
t=2
v=20
t=6
v=30m/s
t=1
A
t=7
+ve
$u_B = +40$ m/s
t=0
t=8
v=-40m/s
$a_y = -10$ m/s²
Motion under gravity
$T_f = 4+4 = 8$ sec $= \frac{2u_y}{g}$
$H_{max} = \frac{u^2}{2g} = \frac{(40)^2}{2\times10} = \frac{40\times40}{2\times10} = 80$ m
u = 40m/s
$a_x = 0$
$x = u_x t$
$= 40\times1$
$= 40$m
B
u = 40m/s
t=1
x = 40m
v=40m/s
t=2
Projectile motion
$u_y = u\sin\theta$
u
+ve
θ
$u_x = u\cos\theta = 40$
$a_x = 0$
$a_y = -g$ (down)

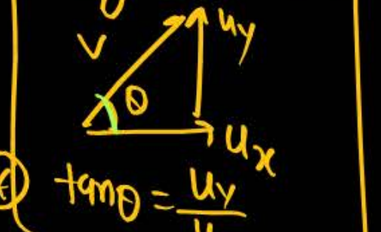
Projectile motion → 2-D Non uniform motion [variable velocity] with Uniform acc^n.



P.Q.	x-axis	y-axis	Net
initial velocity	u_x	u_y	$\vec{v} = u_x\hat{i} + u_y\hat{j}$ ✓
			$\lvert\vec{v}\rvert = \sqrt{u_x^2 + u_y^2}$
acc^n	$a_x = 0$	$a_y = -g\hat{j}$	$\vec{a} = -g\hat{j} = -10\,m/s^2$
velocity at time	$v_t = u_x$ ↳ Const	$v_y = (u_y - gt)\hat{j}$	$\vec{v} = u_x\hat{i} + (u_y - gt)\hat{j}$ ✓
displ^m	$x = u_x t$	$y = u_y t - \frac{1}{2}gt^2$	$\vec{s} = x\hat{i} + y\hat{j}$

$t = \frac{x}{u_x}$

Angle of Projection



(*) $\tan\theta = \frac{u_y}{u_x}$

(#) $y = u_y t - \frac{1}{2}gt^2 = u_y\left[\frac{x}{u_x}\right] - \frac{1}{2}g\left(\frac{x}{u_x}\right)^2$

No Ratta

$$y = \frac{u_y}{u_x}x - \frac{1}{2}g\frac{x^2}{u_x^2}$$

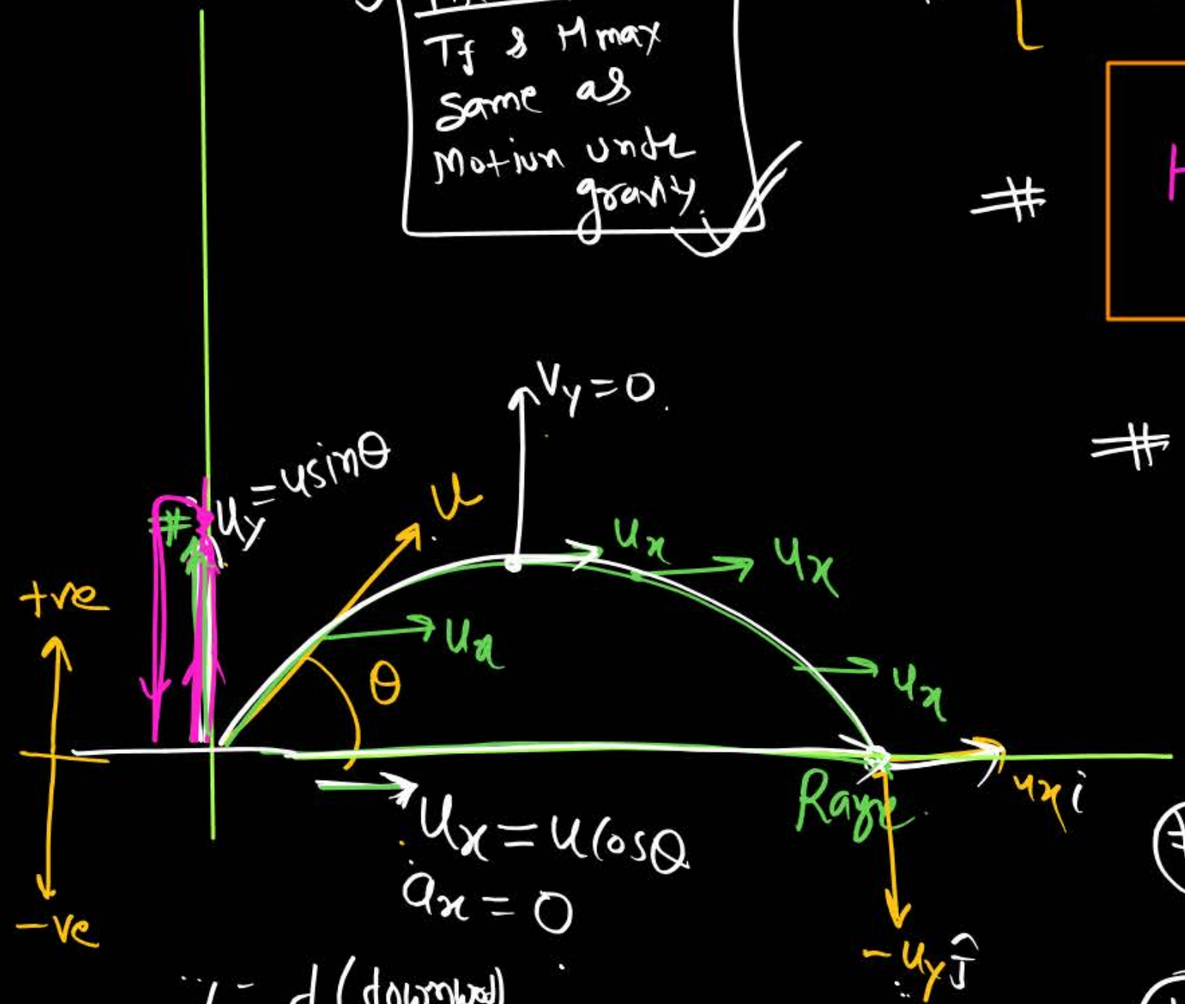
eq^n of Trajectory

Projectile motion → 2-D Non uniform motion [variable velocity] with Uniform acc^{n}.

MR* Box
Tf & Hmax
Same as
Motion under
gravity

$$\# \left[T(\text{Time of flight}) = \frac{2u_y}{g} = \frac{2u\sin\theta}{g} \right]$$ consider motion in y-axis.

$$\# \quad H_{max} = \frac{u_y^2}{2g} = \frac{(u\sin\theta)^2}{2g} = \frac{u^2\sin^2\theta}{2g}$$



$$\# \quad R = u_x t_f = u_x \frac{2u_y}{g}$$ } consider in x-axis

$$R = \frac{2u_x u_y}{g}$$

∴ - d (downward)

(#) Velocity at max^{m} hight $= u_x$

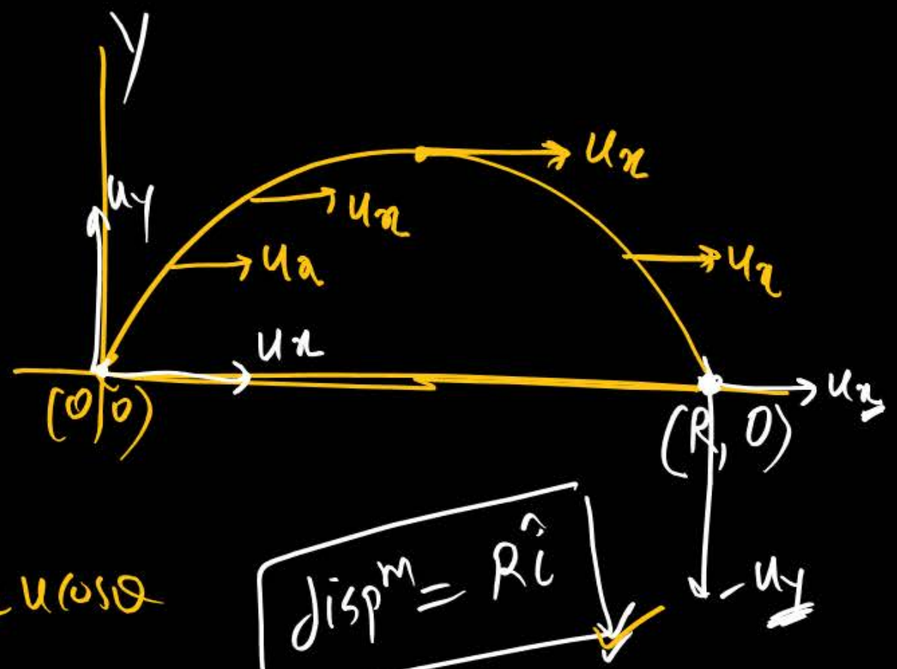
(#) Velocity of collsion $= u_x\hat{i} - u_y\hat{j}$ }

(#)(+) Change in velocity $= \vec{v}_f - \vec{v}_i$

$= (u_x\hat{i} - u_y\hat{j}) - [u_x\hat{i} + u_y\hat{j}]$

$= u_x\hat{i} - u_y\hat{j} - u_x\hat{i} - u_y\hat{j}$

(+) Change in velocity $= -2u_y\hat{j}$

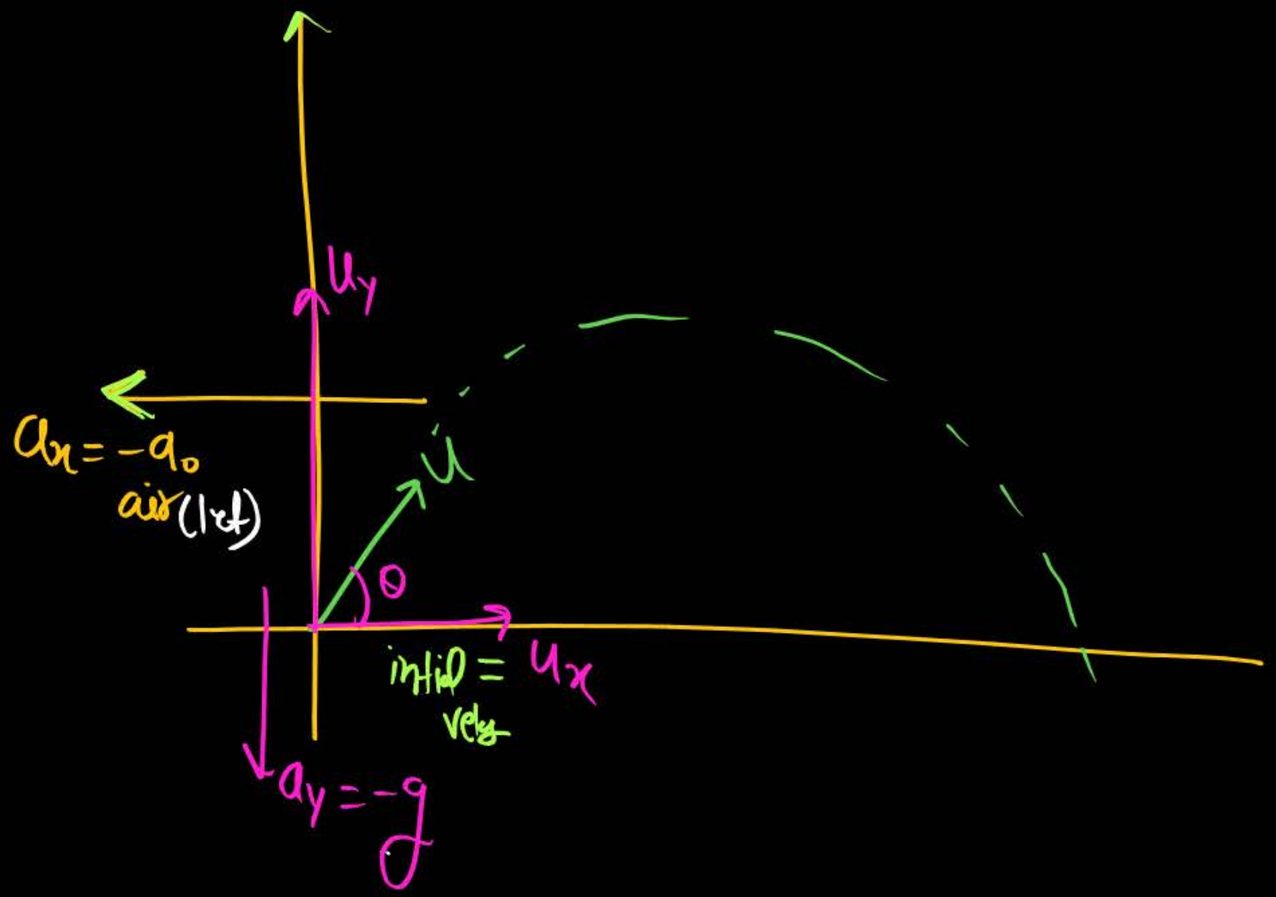


(+) Avg velocity $= \dfrac{\text{Total disp}^m}{\text{time}} = \dfrac{R}{T_f} = \dfrac{u_x T_f}{T_f} = u_x = u\cos\theta$

$\text{disp}^m = R\hat{i}$

(#) Avg velocity $= \dfrac{\vec{u}_i + \vec{v}_f}{2} = \dfrac{u_x\hat{i} + u_y\hat{j} + u_x\hat{i} - u_y\hat{j}}{2} = \dfrac{2u_x\hat{i}}{2} = u_x\hat{i}$

(#) If air resistance consider in x-axis only (air resistance opposite to motion) then what will be effect of H_{max} / T_f / R.



$$T_f = \frac{2u_y}{g_{y\text{-axis}}} \longrightarrow \text{same}$$

$$H_{max} = \frac{u_y^2}{2g} \rightarrow \text{Same.}$$

$$R = u_x T_f \times$$

$$R = u_x T_f - \frac{1}{2} a_0 T_f^2$$ decrease

Important

Rel^n b/w H_{max} & R.

(#) $$\frac{H_{max}}{R} = \frac{\cancel{u_y^2}\ 2u_y}{2g\ u_x \frac{2u_y}{g}}$$

$$\frac{H}{R} = \frac{u_y}{4u_x}$$

$$\tan\theta = \frac{u_y}{u_x}$$

$$\frac{H}{R} = \frac{\tan\theta}{4}$$

(*) $$H = \frac{R\tan\theta}{4}$$

$$\text{Range} = u_x T_f$$ (*)

(Q) If H_{max} is equal to Range then find Angle of Projection

NEET-2014

Sol^n $$H = \frac{R\tan\theta}{4}$$

$$\tan\theta = 4$$

$$\theta = \tan^{-1}(4)$$

Object is Projected with $\vec{u} = u_x\hat{i} + u_y\hat{j}$ then find t after which it is moving $\perp^r$ to initial velocity ✓

Soln $\quad \vec{u}_{t=0} = u_x\hat{i} + u_y\hat{j}$

$$\vec{v} = u_x\hat{i} + (u_y - gt)\hat{j}$$

When Two P.Q is $\perp^r$ then their dot product will be zero.

$$\vec{u} \cdot \vec{v} = 0$$

$$u_x^2 + u_y^2 - u_y gt = 0$$

$$u_x^2 + u_y^2 = u_y gt$$

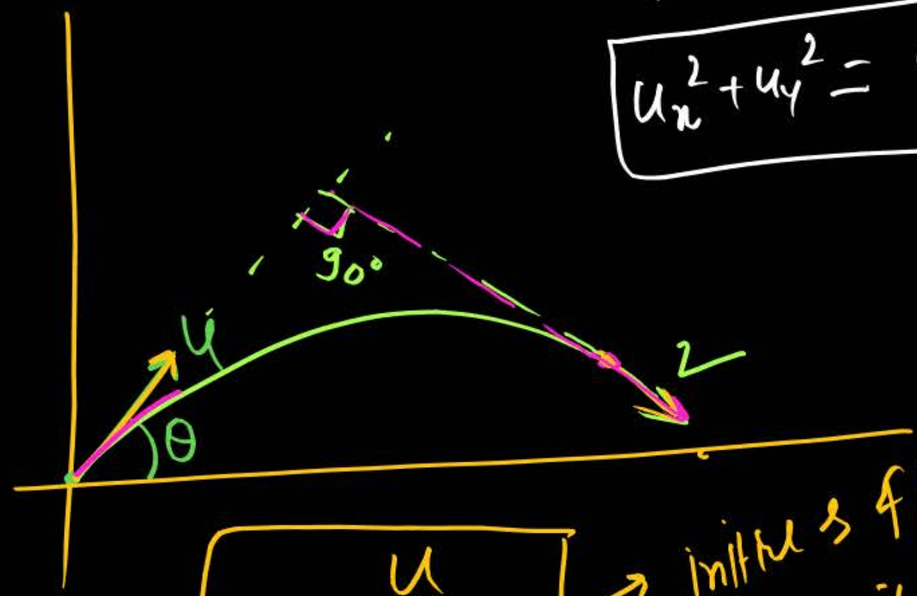
$$u \cancel{u^2} = \cancel{u}\sin\theta\, gt$$

$$\boxed{t = \frac{u}{g\sin\theta}}$$

$u_x = u\cos\theta$

$u_y = u\sin\theta$

$$\boxed{u_x^2 + u_y^2 = u^2}$$



$$\boxed{t = \frac{u}{g\sin\theta}}$$

→ initial & final velocity is $\perp^r$ to each other

(2) object of mass 1kg projected with velocity $\vec{u} = 40\hat{i} + 40\hat{j}$ then find.

(1) Speed of Projection

$$\vec{v} = 40\hat{i} + 40\hat{j}$$

$$|v| = \sqrt{(40)^2 + (40)^2} = 40\sqrt{2}$$

(2) Angle of Projection

$$\tan\theta = \frac{u_y}{u_x}$$

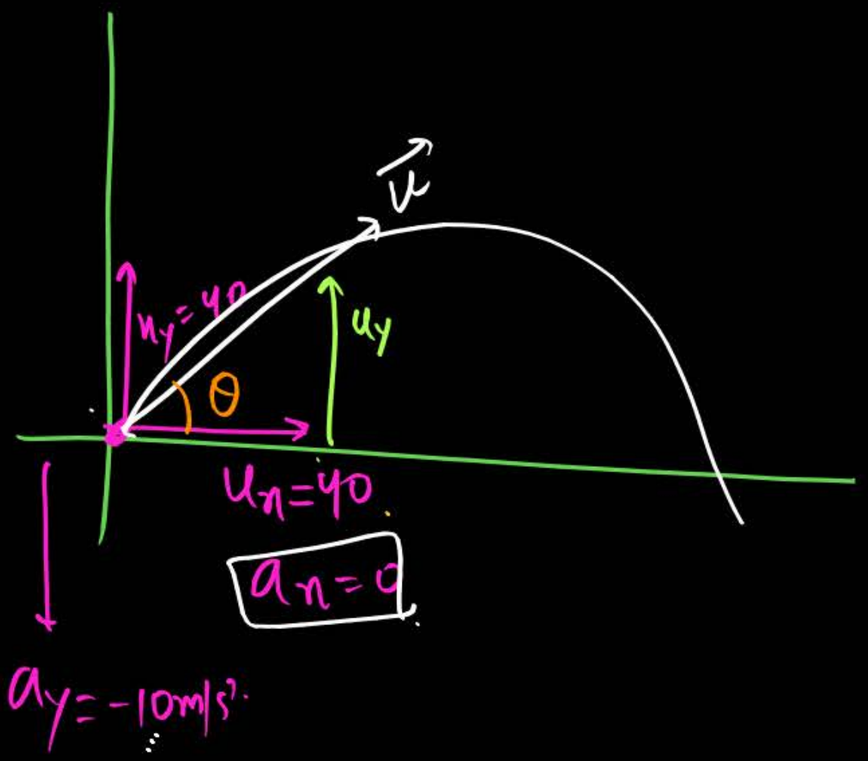
$$\tan\theta = \frac{40}{40} = 1$$

$$\theta = 45^\circ$$

(3) Velocity after time '2'-sec.

$$\vec{v} = 40\hat{i} + 20\hat{j}$$

$$|\vec{v}| = \sqrt{(40)^2 + (20)^2}$$



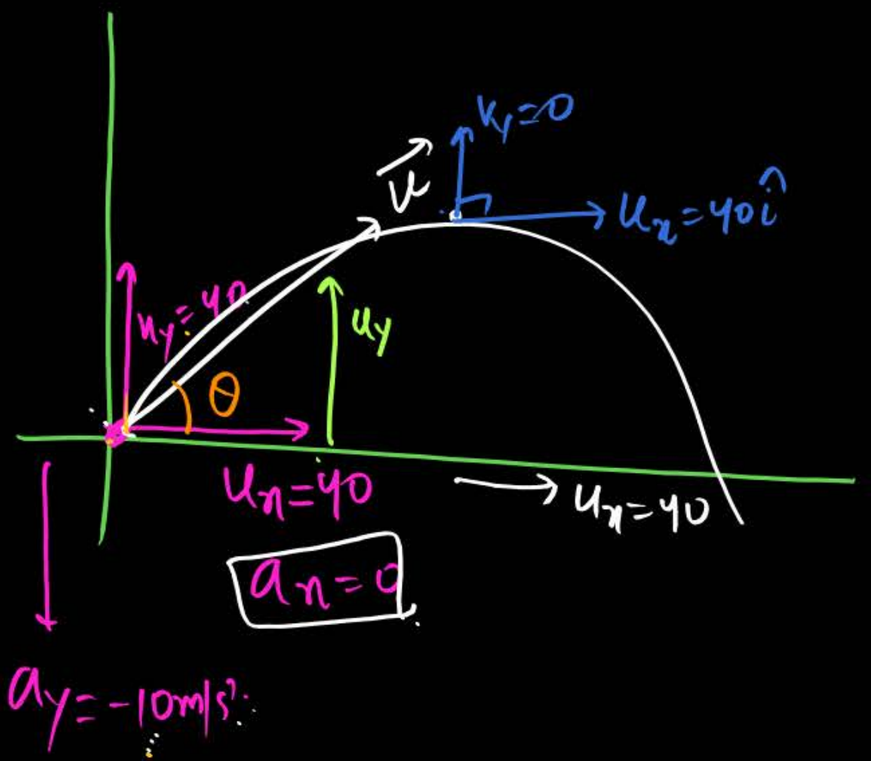
(8) object of mass 1kg projected with velocity $\vec{u} = 40\hat{i} + 40\hat{j}$ then find.

(4) velocity at $t = 4sec$.

$$\vec{V} = 40\hat{i} + (40 - gt)\hat{j}$$

$$= 40\hat{i} + (40 - 10\times 4)\hat{j}$$

$$\boxed{V = 40\hat{i} + 0\hat{j}}$$



(5) Time of flight (Same as Motn under gravity)

$T_f = T_{up} + T_{down} = 4 + 4 = 8 sec$

$t = 4sec$ पर Maxm Height पर होगा

$T_f = \frac{2u_y}{g} = \frac{2\times 40}{10} = 8 sec$ ⇒

(8) object of mass 1kg projected with velocity $\vec{u} = 40\hat{i} + 40\hat{j}$ then find.

(6) Max^m Height

$$H_{max} = \frac{u_y^2}{2g}$$

$$= \frac{(40)^2}{2\times 10}$$

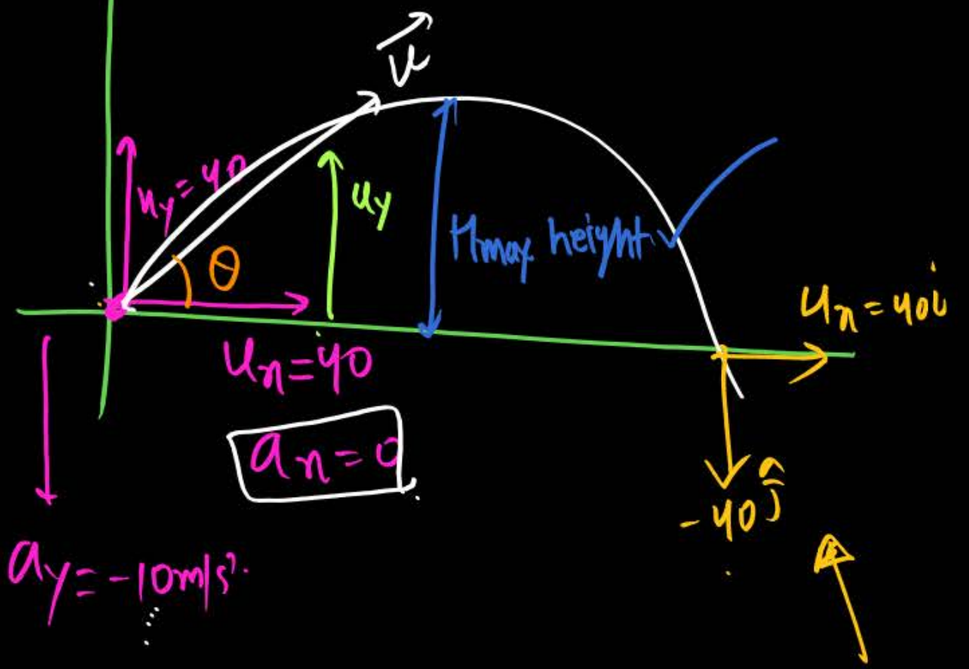
$$= \frac{40\times \cancel{40}^{2}}{\cancel{2}\times \cancel{10}} = 80m$$

(7) Velocity of collision:—

$\vec{V}_{collision} = 40\hat{i} - 40\hat{j}$ at ground.

(8) velocity at time 't'

$\vec{V} = u_x\hat{i} + (v_y - gt)\hat{j}$ ✓



(8) object of mass 1kg projected with velocity $\vec{u} = 40\hat{i} + 40\hat{j}$ then find.

(9) Range ?

↳ consider motion in x- only

$R = u_x T_f$

$= 40 \times 8 = 320\,m$.

(10) Position at time 't'

$t = x/40$

$x = u_x t = 40t$

$y = u_y t - \frac{1}{2} g t^2 = 40t - \frac{1}{2}(10)t^2 = 40t - 5t^2$.

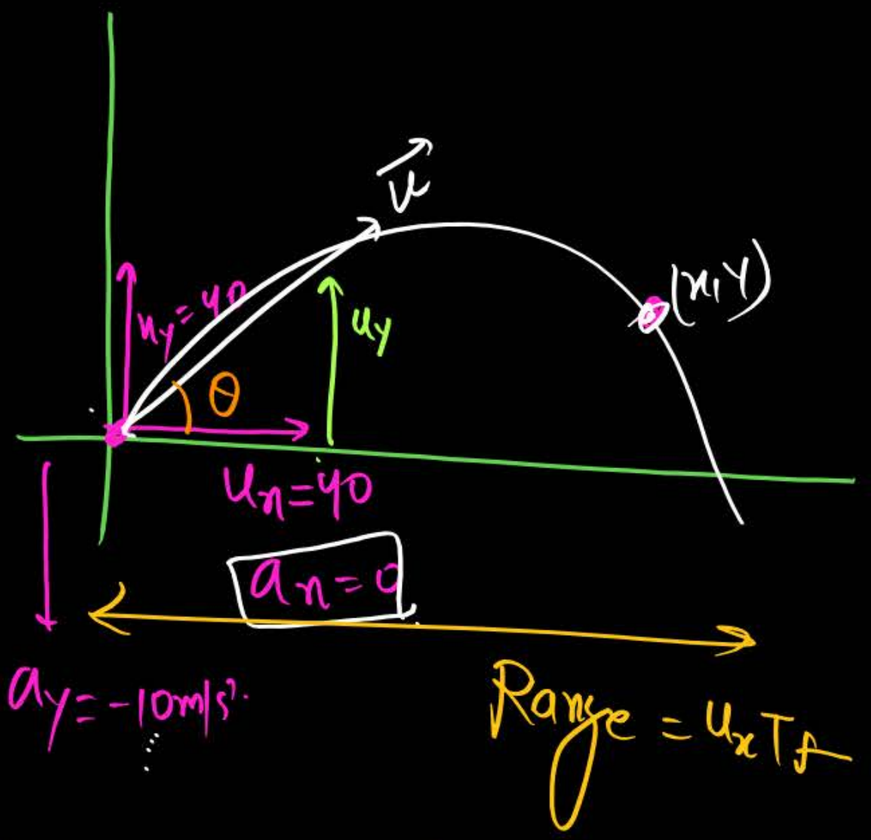
(11) Equation of Trajectory

$y = 40t - 5t^2$

Put value of (1)

$= 40 \times \frac{x}{40} - 5\left(\frac{x}{40}\right)^2$

$y = x - \frac{5x^2}{1600}$ Parabola



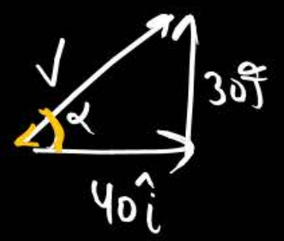
(8) object of mass 1kg projected with velocity $\vec{u} = 40\hat{i} + 40\hat{j}$ then find.

(12) direction of motn at $t = 1$ sec.

→ dirn of velocity.

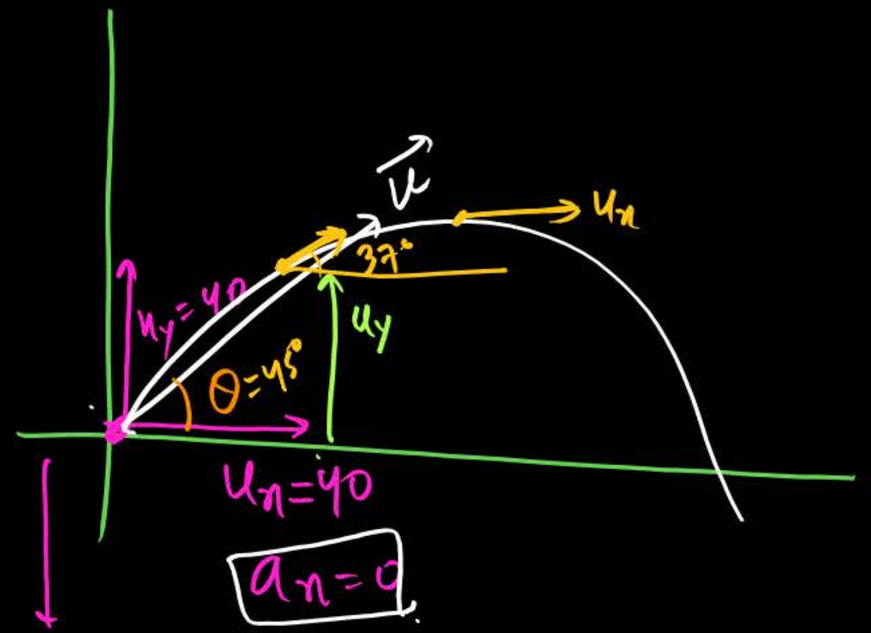
$$\vec{V}_{t=1} = 40\hat{i} + (40 - 10\times 1)\hat{j}$$

$$= 40\hat{i} + 30\hat{j}$$



$$\tan\alpha = \frac{30}{40} = \frac{3}{4}$$

$\alpha = 37°$



(8) object of mass 1kg projected with velocity $\vec{u} = 40\hat{i} + 40\hat{j}$ then find.

(13) find Position at $t = 4\,sec$

$$x = u_x t = 40 \times 4 = 160m$$

$$y = u_y t - \frac{1}{2} g t^2 = 40 \times 4 - \frac{1}{2} 10 (4)^2$$

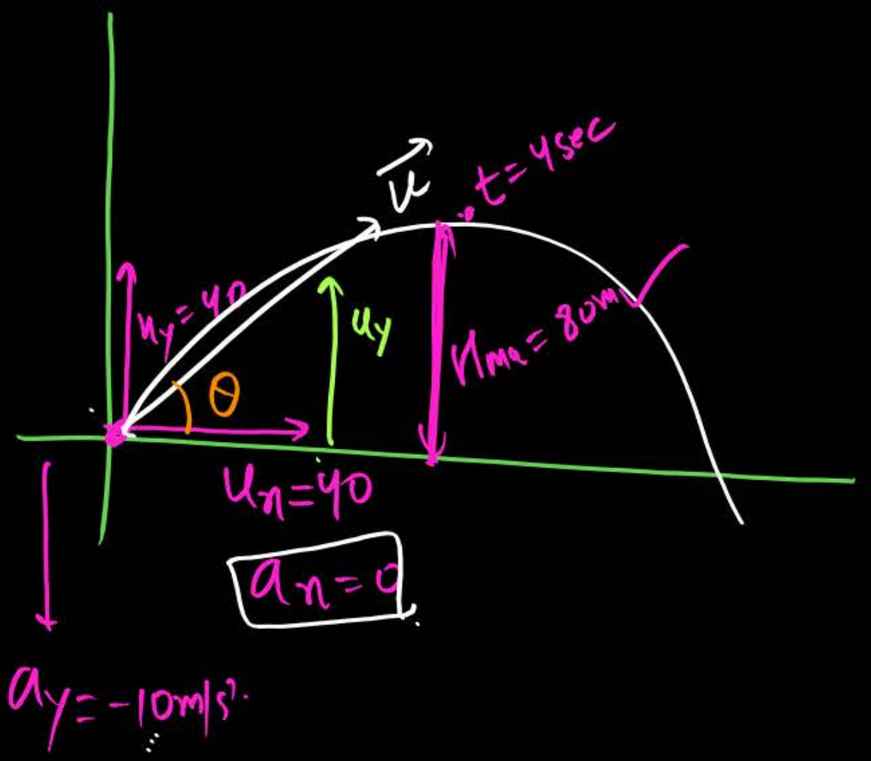
$$= 160 - 5 \times 16$$

$$= 160 - 80$$

$$= 80m$$

$T_f = 8\,sec$

at $t = 4$ sec Max^{m} height



(8) object of mass 1kg projected with velocity $\vec{u} = 40\hat{i} + 40\hat{j}$ then find.

(14) find time 't' at which velocity is $\perp^r$ to intial dir^n of project

$\vec{u_i} = 40\hat{i} + 40\hat{j}$

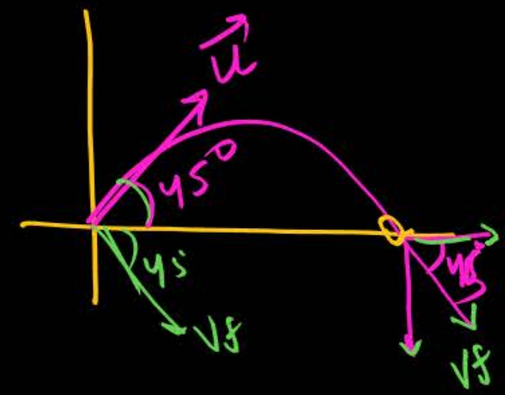
$\vec{V_t} = 40\hat{i} + (40 - 10t)\hat{j}$

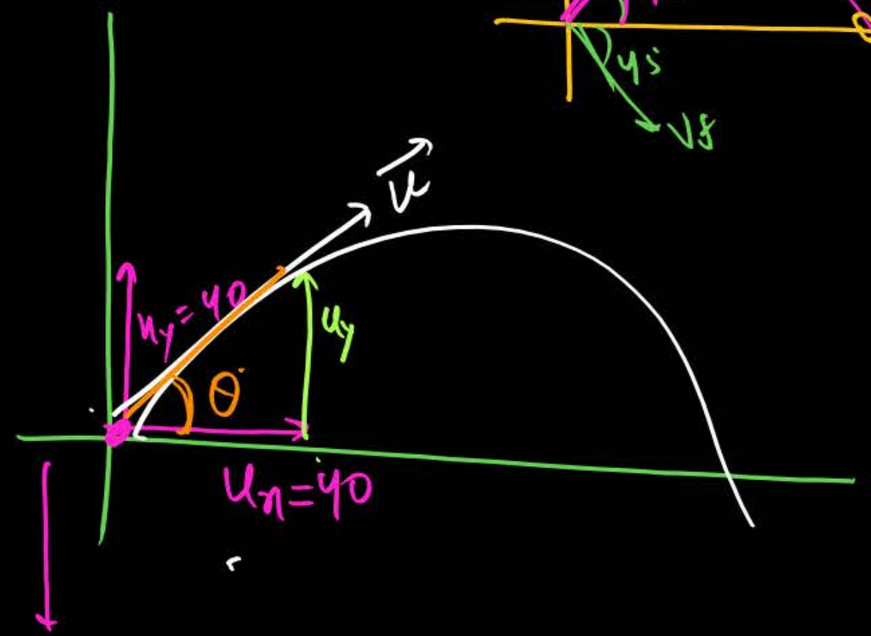
$\vec{u_i} \cdot \vec{V_t} = 0$

$$t = \frac{u}{g\sin\theta} = \frac{40\sqrt{2}}{10\sin(45^\circ)} = \frac{4\sqrt{2}}{1/\sqrt{2}} = 4\sqrt{2}\times\sqrt{2}$$

$= 4 \times 2$

$= 8$ sec





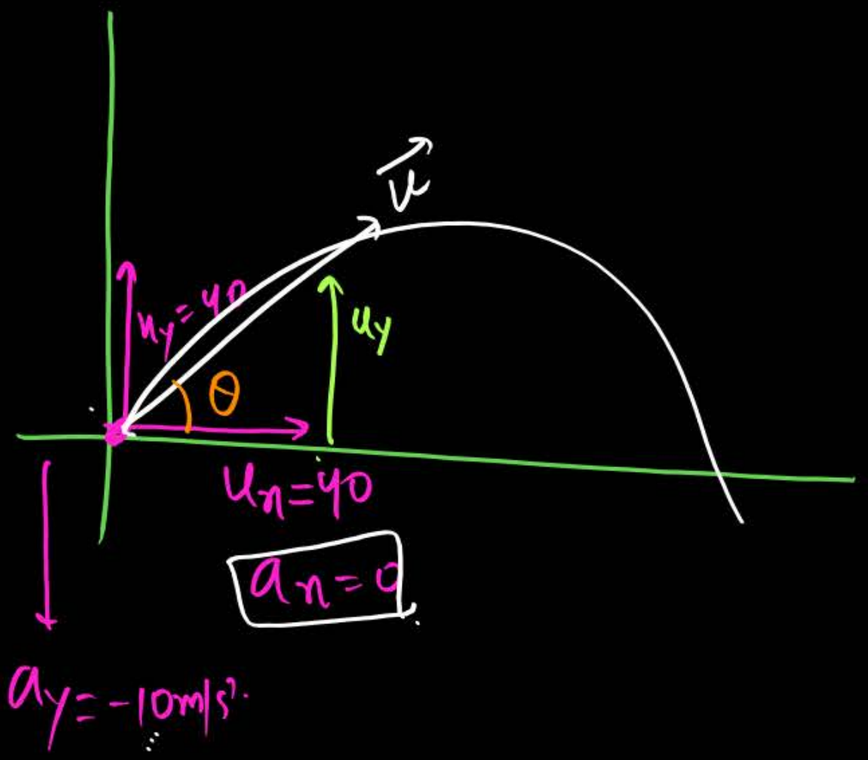
(8) object of mass 1kg projected with velocity $\vec{u} = 40\hat{i} + 40\hat{j}$ then find.

(15) find change in velocity in compt motion:-

$$\Delta\vec{V} = \vec{V}_f - \vec{V}_i = (40\hat{i} - 40\hat{j}) - (40\hat{i} + 40\hat{j})$$

$$= 40\hat{i} - 40\hat{j} - 40\hat{i} - 40\hat{j}$$

$$= -80\hat{j}$$



(16) Change in momentum in comt motn.

$$\Delta P = m(\vec{V}_f - \vec{V}_i)$$

$$= 1[-80)\hat{j} = -80\hat{j} \text{ kgm/s}$$

(17) Avg velocity in Compt Journey :

$$\vec{u}_{Avg} = u_x = 40\,\hat{i}$$

$$\vec{V}_{Avg} = \frac{\vec{u_i} + \vec{v_f}}{2}$$

(18) Rate of change in momentum at $t = 1$ sec.

Rate of change in momentum = force

$= -mg = -1 \times 10$

$= -10$ J New.

(19) accn at max^m height ✓

$\vec{a} = \text{Cat}^n = -g\,\hat{j}$

(20) accn at $t = 4$ sec

$\vec{a}_{Avg} = -g\,\hat{j}$

· Cat^n ka Avg instantaneous ke equal hota hai

(8) object of mass 1kg projected with velocity $\vec{u} = 40\hat{i} + 40\hat{j}$ then find.

(1) Speed of Projection.

$$\vec{v} = 40\hat{i} + 40\hat{j}$$

$$|v| = \sqrt{(40)^2 + (40)^2} = 40\sqrt{2}.$$

(2) Angle of Projection :-

$$\tan\theta = \frac{u_y}{u_x}$$

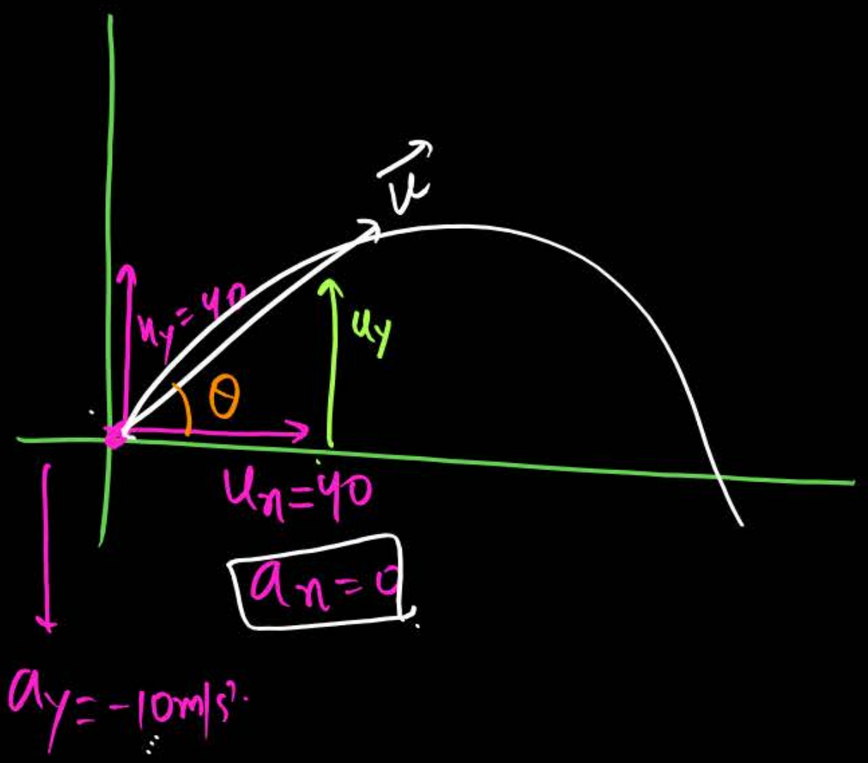
$$\tan\theta = \frac{40}{40} = 1$$

$$\theta = 45^\circ.$$

(3) Velocity aft 2 time '2'-sec.

$$\vec{v} = 40\hat{i} + 20\hat{j}$$

$$|\vec{v}| = \sqrt{(40)^2 + (20)^2}$$



(8) object of mass 1kg projected with velocity $\vec{u} = 40\hat{i} + 40\hat{j}$ then find.

(1) Speed of Projection.

$$\vec{v} = 40\hat{i} + 40\hat{j}$$

$$|v| = \sqrt{(40)^2 + (40)^2} = 40\sqrt{2}.$$

(2) Angle of Projection :-

$$\tan\theta = \frac{u_y}{u_x}$$

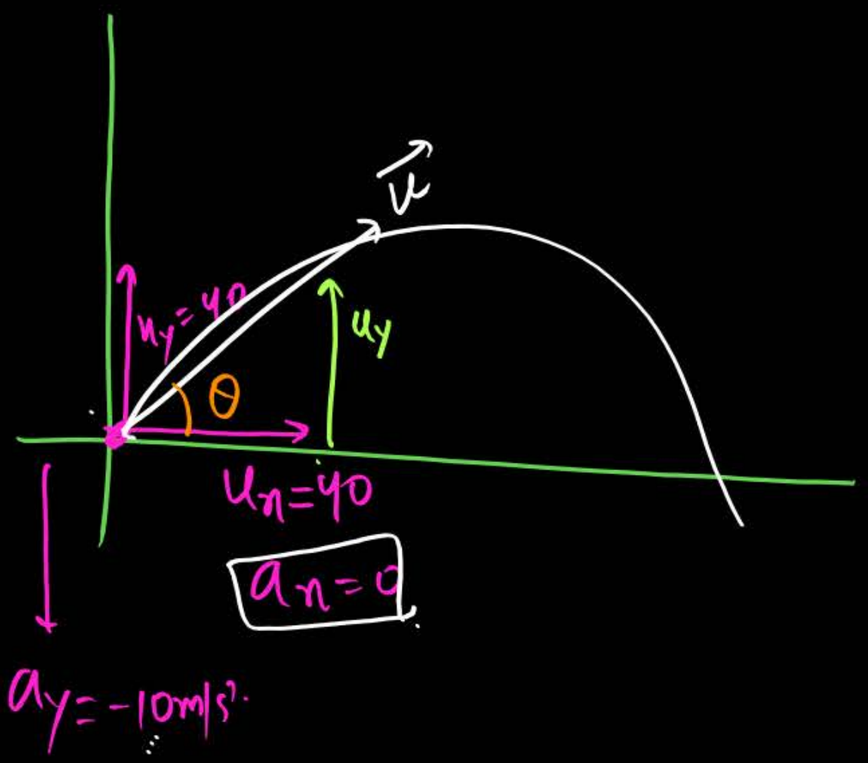
$$\tan\theta = \frac{40}{40} = 1$$

$$\theta = 45^\circ.$$

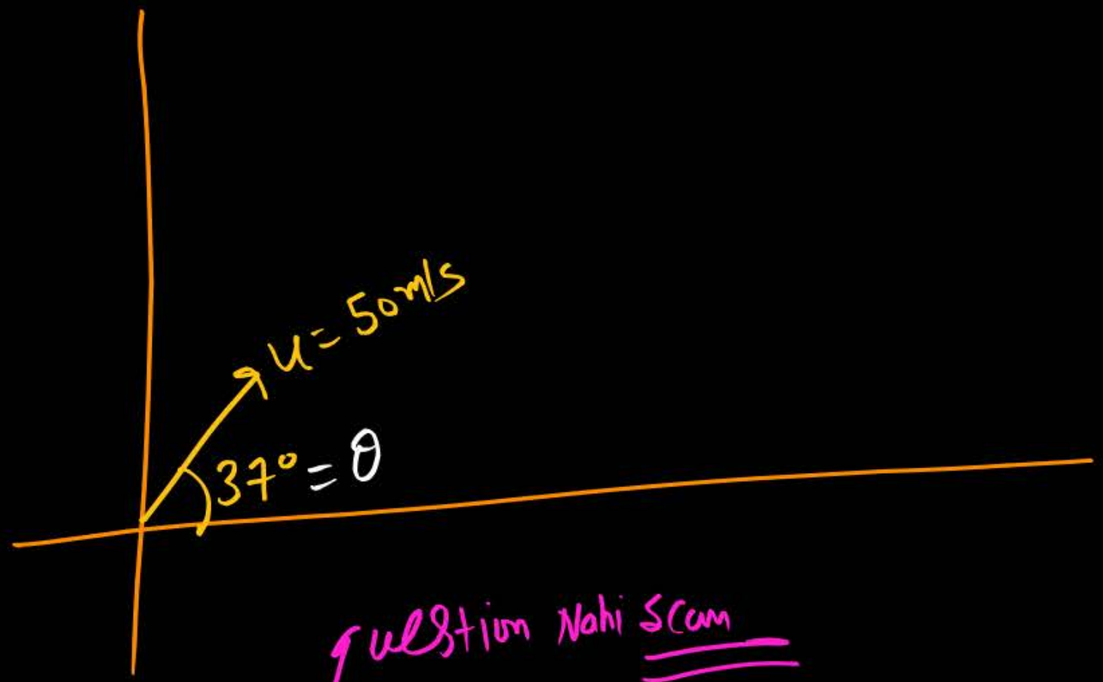
(3) Velocity aft 2 time '2'-sec.

$$\vec{v} = 40\hat{i} + 20\hat{j}$$

$$|\vec{v}| = \sqrt{(40)^2 + (20)^2}$$



Find time when final velocity is ⊥r to intial velocity.



Question Nahi Scam

Why ??

$$t = \frac{u}{g\sin\theta}$$

$$= \frac{50}{10 \times \frac{3}{5}}$$

$$= \frac{25}{3} = 8.1 \text{ sec}$$

Question

A bullet is fired from a gun at the speed of 280 ms^{-1} in the direction 30° above the horizontal. The maximum height attained by the bullet is _____.

($g = 9.8\ ms^{-2}$, $\sin 30° = 0.5$) **[NEET-2023]**

1. 80 m
2. 100 m
3. 60 m
4. 40 m

Question

Velocity of projection $\vec{u} = 40\hat{\imath} + 30\hat{\jmath}$ then find H, T, R and angle of project.

Question

The horizontal range and the maximum height of a projectile are equal. The angle of projection of the projectile is: **[AIPMT Pre. 2012]**

1. $\theta = \tan^{-1}$
2. $\theta = 45°$
3. $\theta = \tan^{-1}\left(\frac{1}{4}\right)$
4. $\theta = \tan^{-1}(4)$

Question

The horizontal range of a projectile is $4\sqrt{3}$ times its maximum height. Its angle of projectile will be:

1. $45°$
2. $60°$
3. $90°$
4. $30°$

THANK YOU