Basic Maths and Calculus (Mathematical Tools)

Physics

Revision -03

Find roots of equation

(i) $2x^2 - 2x - 24 = 0$

$$\boxed{ax^2 + bx + c = 0}$$

$a = 2 \quad b = -2 \quad c = -24$

$$x_1 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{4 - 4 \times 2(-24)}}{2 \times 2} = \frac{+2 \pm \sqrt{196}}{4} = \frac{2 \pm 14}{4}$$

$= \frac{16}{4} = 4$

$= \frac{-12}{4} = -3$

direct

$2x^2 - 2x - 24 = 0$

$2x^2 - 8x + 6x - 24 = 0$

$2x(x-4) + 6(x-4) = 0$

$(x-4)(2x+6) = 0$

$x = 4 \quad 2x = -6 \quad -3$

(ii) $x^2 - x - 6 = 0$

$x^2 - 3x + 2x - 6 = 0$

$x(x-3) + 2(x-3) = 0$

$(x-3)(x+2) = 0$

$x_1 = 3 \quad x_2 = -2$

$$x^2 - 2x - 15 = 0$$

Sol^n $x^2 - 5x + 3x - 15 = 0$

$$x(x-5) + 3(x-5) = 0$$

$$x = 5 \qquad x = -3$$

$$\Rightarrow \quad 8x^2 + 2x - 15 = 0$$

$$8x^2 + 2x - 15 = 0$$

$$8x^2 + 12x - 10x - 15 = 0 \qquad ac = -120$$

$$4x[2x+3] - 5(2x+3] = 0$$

12
10
170

$$(2x+3)(4x-5) = 0$$

$$x_1 = \frac{-3}{2} \qquad x_2 = \frac{5}{4}$$

$$x^2 + 12x + 32 = 0$$

$$x^2 + 8x + 4x + 32 = 0$$

$$x(x+8) + 4(x+8) = 0$$

$$x_1 = -8$$

$$x_2 = -4$$

maxima / minima of y w.r.t x

$*$ at max^m

$\# \boxed{\text{Slope}\left(\frac{dy}{dx}\right) = 0}$

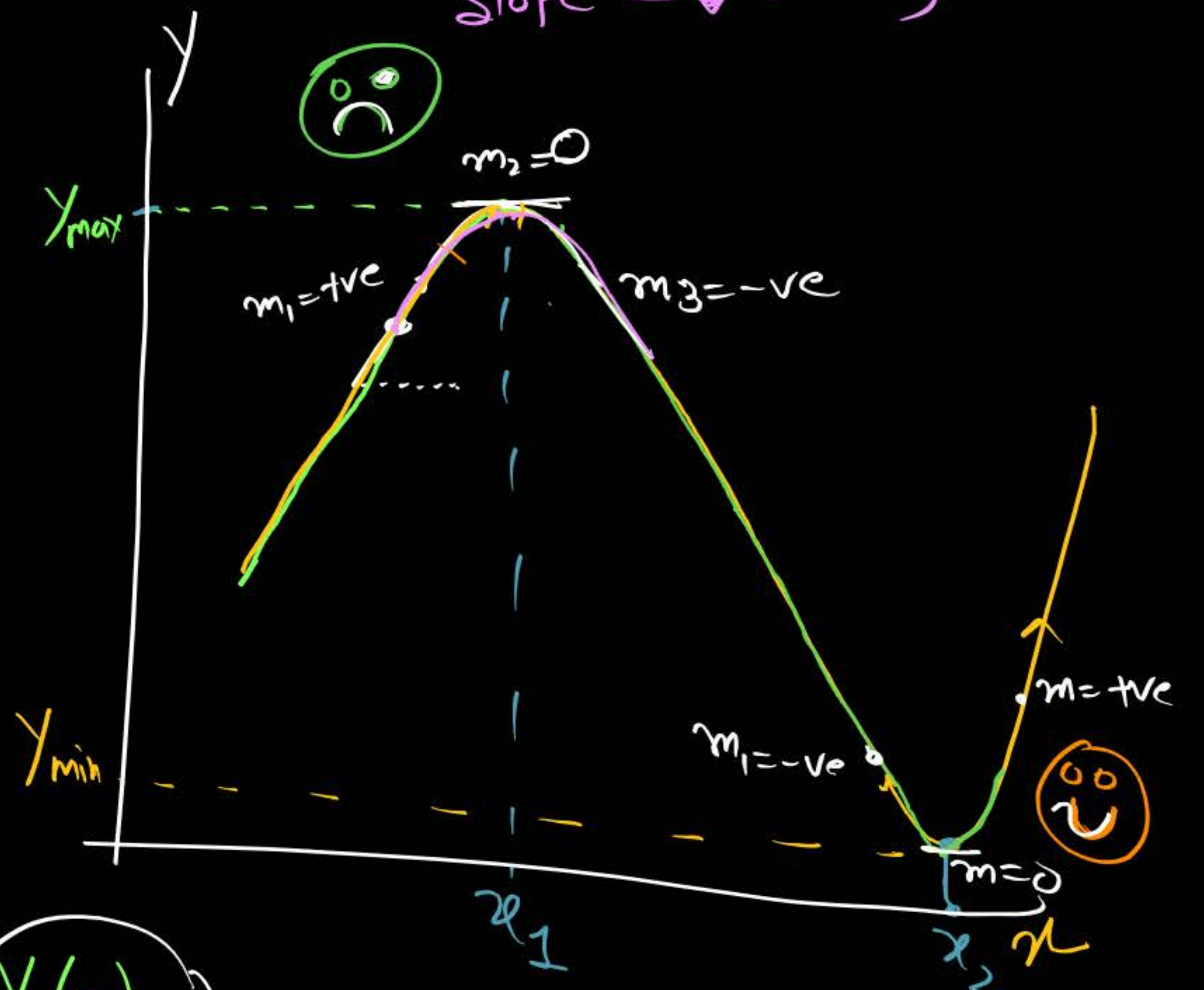
$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \left[\frac{d^2y}{dx^2}\right] = -ve$

$*$ at minima

$\boxed{\text{Slope}\left(\frac{dy}{dx}\right) = 0}$

$\boxed{\frac{d^2y}{dx^2} = +ve}$

slope = ↓ (decre)



MR* Extrem hapinees

$y \rightarrow$ find max^m value of y

$\boxed{\frac{dy}{dx} = 0} \rightarrow x_1$

$\boxed{y = x^3 - 2x^2 + 4}$

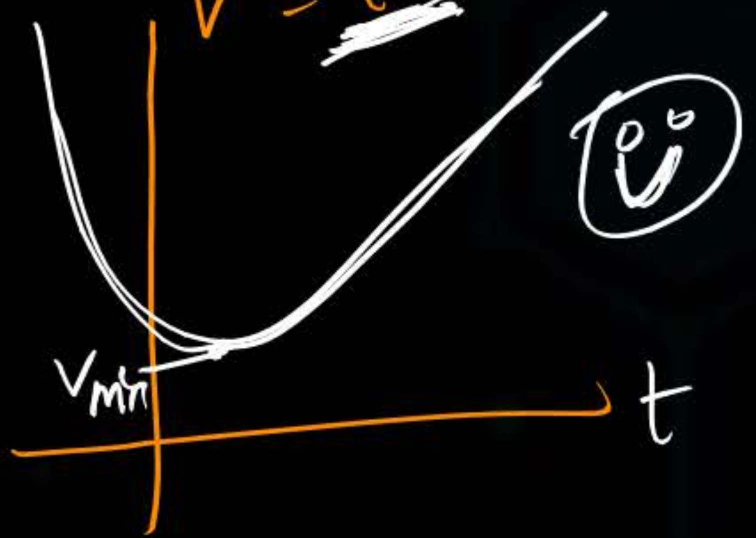
$\frac{dy}{dx} = 0 \rightarrow x_1, x_2$

$y_1(x_1) =$ max

$y_2(x_2)$ m

Question

If velocity of object $V = t^2 - 4t + 8$, find maximum or minimum velocity.



$$V = t^2 - 4t + 8$$

V will be max^m or minim

$$\frac{d^2V}{dt^2} = +2$$

minima

$$\left[\frac{dv}{dt} = 0\right]$$

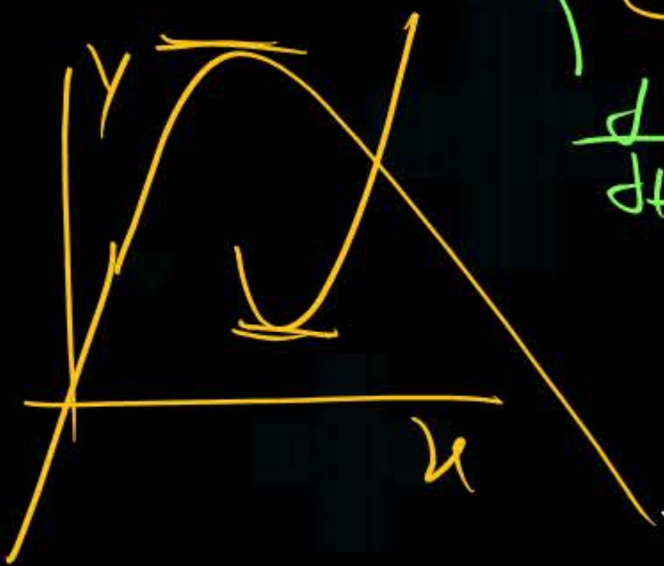
$$\frac{dv}{dt} = 2t - 4 = 0$$

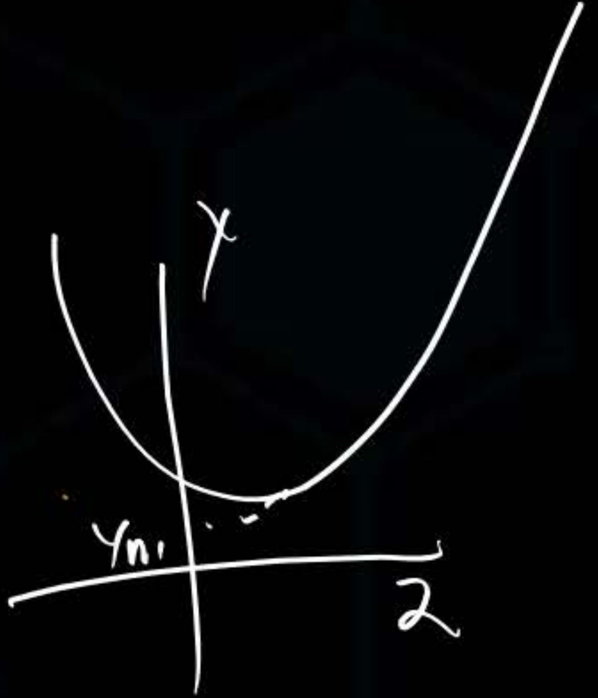
$$2t = 4$$

$$\boxed{t = 2\ \text{sec}}$$

$$V_{t=2} = 4 - 4 \times 2 + 8$$

$$V_{min} = 12 - 8 = 4\ \text{m/s}$$





Question

If acceleration of object $a = \frac{t^3}{3} - \frac{5t^2}{2} + 6t$ then find maximum and minimum acceleration.

$$\boxed{a = \frac{t^3}{3} - \frac{5t^2}{2} + 6t}$$

a will be max^m or min^m at $\frac{da}{dt} = 0$

$$\frac{da}{dt} = \frac{3t^2}{3} - 5\frac{(2t)}{2} + 6$$

$$\boxed{\frac{da}{dt} = t^2 - 5t + 6 = 0}$$

$$\Rightarrow t^2 - 5t + 6 = 0$$

$$t^2 - 3t - 2t + 6 = 0$$

$$t(t-3) - 2(t-3) = 0$$

$$(t-3)(t-2) = 0$$

$t_1 = 3$ | $t_2 = 2$

$(a)_{t=2 \text{ max}} = \frac{8}{3} - \frac{5 \times 4}{2} + 6 \times 2$

$= \frac{8}{3} - 10 + 12$

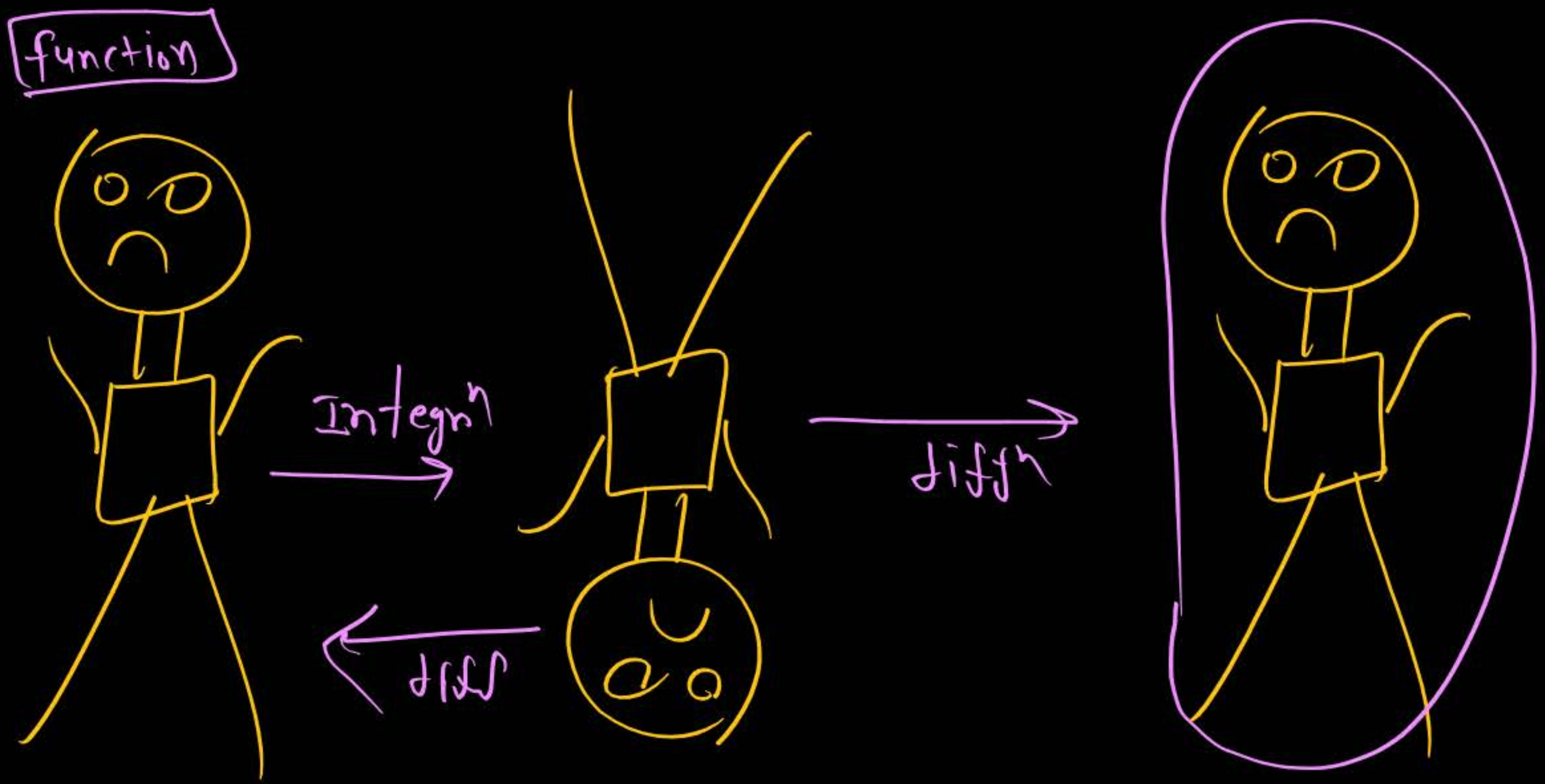
$= \frac{8}{3} + 2 = \frac{14}{3}$

$= 4.6$

$(a)_{t=3} = \frac{27}{3} - \frac{5 \times 9}{2} + 6 \times 3$

$= 27 - \frac{45}{2}$

$= \frac{54 - 45}{2} = \frac{9}{2} = 4.5$ min


function
Integnⁿ
diffⁿ
diffⁿ

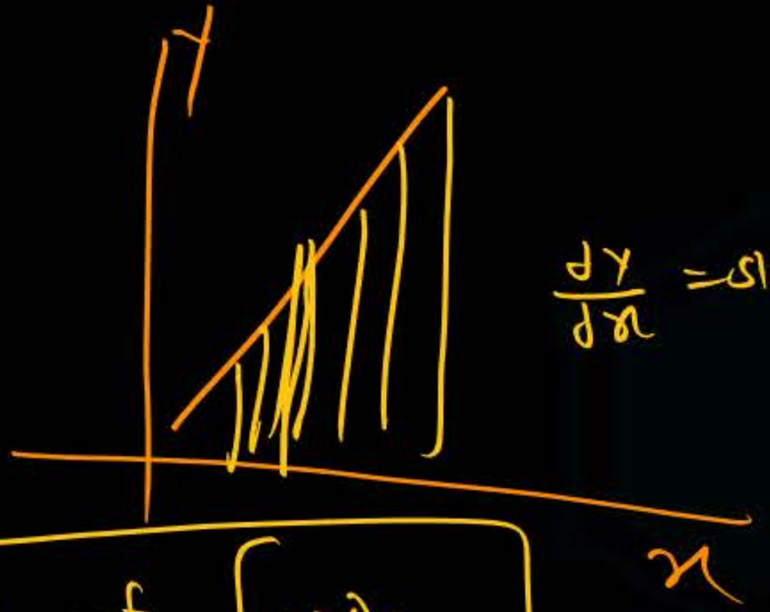
INTEGRATION

- Inverse of differentiation
- Addition of infinitely small term

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

Valid when $n \neq -1$

Not valid when $n = -1$



$\frac{dy}{dx}$ = slope

Area of curve $= \int (y)\, dx$

$$\int \boxed{x^7}\, dx = \frac{x^{7+1}}{7+1} + c = \underbrace{\frac{x^8}{8} + c}_{\text{diff}^n} \Rightarrow \frac{d\left(\frac{x^8}{8}+c\right)}{dx} = \frac{1}{8}\frac{dx^8}{dx} + \frac{dc}{dx}$$

$$= \frac{1}{\cancel{8}}\cancel{8}x^7 + 0$$

$$= x^7$$

$$\int 5\,dx = 5\int dx = 5x$$

$$\int 4x\,dx = 4\int x^1 dx = \cancel{4}\frac{x^2}{\cancel{2}} = 2x^2$$

$$\int 5x^3\,dx = 5\int x^3 dx = 5\frac{x^4}{4} + c$$

$$\int \sqrt{x}\,dx = \int x^{1/2} dx = \frac{x^{3/2}}{3/2} + c$$

$$\int x^{3/2}\,dx = \frac{x^{3/2+1}}{\frac{3}{2}+1}$$

$$\boxed{\int \frac{1}{x}\,dx = \int x^{-1} dx = \frac{x^{-1+1}}{-1+1} = \frac{x^0}{0} = \text{Not define.}}$$

$$\int \frac{1}{x^2}\,dx = \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} = \left(-\frac{1}{x}\right)$$

$$\boxed{\int \frac{1}{x}dx = \log x}$$

$$\int 1\,dx = \int x^0 dx = \frac{x^{0+1}}{0+1}$$

$$\boxed{\int dx = x}$$

$$\int dt = t$$

$$\int dP = P$$

$$\boxed{\int d(m.R) = m.R}$$

$$\int \sin x \, dx = \underbrace{-\cos x}_{} \xrightarrow{\text{diff}} -(-\sin x) = \underline{\sin x}$$

$$\int \cos x \, dx = \sin x$$

$$\int (x^2) + (\sin x) \, dx = \frac{x^3}{3} - \cos x$$

$$\int \sin(\cancel{5x+4}) \, dx =$$

$\underline{\text{MR}^{*}}$ $\frac{d \sin x}{dx} = \int \cos x = \underline{\sin x}$

$$\int e^x\,dx = e^x$$

$\downarrow$

diff

$$\frac{de^x}{dx} = e^x$$

$$\int (\sin x - 5 + 4x - e^x)\,dx$$

$$= \left(-\cos x - 5x + 4\frac{x^2}{2} - e^x\right)$$

As

outside Inside - function : outside Inside Rule of Integn.

$\Rightarrow$ $\sin(4x+5)$

$\Rightarrow$ $e^{(4x+3)}$

$(2x+4)^3$ ✓

$\cos(5x)$

$$\int y\,dx = \frac{\text{Integration of outer function keep Inside as it is}}{\text{coefficient of } x}$$

$\int \sin(4x+5)\,dx = \dfrac{-\cos(4x+5)}{4} + C$

outer Functi.

inner

$$\int \cos(2x)\,dx = \frac{\sin(2x)}{2} + C$$

$$\int (4x-6)^2\,dx = \frac{(4x-6)^3}{3\times 4} + C$$

$$\boxed{\int \frac{1}{(5x-3)}\,dx = \frac{\log(5x-3)}{5} + C}$$

$$\int e^{(5x+4)}\,dx = \frac{e^{(5x+4)}}{5}$$

$$\int e^{-3x}\,dx = \frac{e^{-3x}}{-3} \qquad \int e^{-x}\,dx = \frac{e^{-x}}{-1}$$

$$\cancel{\int \cos(2x^2)\,dx =}$$

$$\cancel{\int \sin(4x^2+3) =}$$

$$\cancel{\int \sin(4x^2-3x) =}$$

$\int(\sin x + 4\cos x + x^4)\,dx =$

$$\int\left(x - \frac{1}{x^2} + \frac{1}{x}\right)dx = \int x\,dx - \int\frac{1}{x^2}\,dx + \int\frac{1}{x}\,dx = \frac{x^2}{2} - \left(\frac{x^{-2+1}}{-2+1}\right) + \log x.$$

$$\int\cos(5x-4)\,dx = \frac{\sin(5x-4)}{5} + c$$

If velocity of object $V = t^2$ then find displacement in 2-sec

Sol^n

in Question

"$V = t^2$"

$$\frac{dx}{dt} = t^2$$

$$\int_{x_i}^{x_f} dx = \int_{t_i=0}^{t=2} t^2\, dt$$

$$(x)_{x_i}^{x_f} = \left(\frac{t^3}{3}\right)_0^2$$

$$x_f - x_i = \frac{1}{3}\left[(2)^3 - 0^3\right]$$

$$\Delta x = \frac{8}{3}\, m$$

$$a = t^2$$

$$\frac{dv}{dt} = t^2$$

$$\int_{u_i}^{v_f} dv = \int_{t_i}^{t_f} t^2 \, dt$$

(a) acc of object

$$\boxed{a = 3x^2}$$

if velocity at $x = 0$ is $2\,m/s$ then find velocity at $x = 4\,m$

$$a = 3x^2$$

$$\frac{dv}{dt} = 3x^2$$

$$\int dv = \int 3x^2 \, dt$$

Can't Solve

$$v\frac{dv}{dx} = 3x^2$$

$$\int_{u_i=2}^{v} v\,dv = \int_{x_i=0}^{x_f=4} 3x^2 \, dx$$

$$\left(\frac{v^2}{2}\right)_2^v = \cancel{3}\left[\frac{x^3}{\cancel{3}}\right]_0^4$$

$$\frac{1}{2}\left[v^2 - 2^2\right] = \left(x^3\right)_0^4 = 4^3 - 0^3 = 64$$

$$v^2 - 4 = 64 \times 2$$

$$v^2 = 128 + 4$$

$$v = \sqrt{132}$$

Definite Integratu → Integration with limit

$$\int_{x_1}^{x_2} y\,dx = \left[y'\right]_{x_1}^{x_2}$$

$$\int (t^2+2t)\,dx = \cancel{}$$

$y = x^2$ integm of y from $x_1 = 1$ $x_2 = 3$

$$\int y\,dx = \int_1^3 x^2\,dx = \left(\frac{x^3}{3}\right)_1^3 = \frac{1}{3}\left(x^3\right)_1^3 = \frac{1}{3}\left[3^3 - 2^3\right]$$

$$= \frac{1}{3}\left[27-1\right] = \frac{26}{3}$$

$\sin(-\theta) = -\sin\theta$

$$\int_0^{\pi/2 \,(90^\circ)} (\sin\theta)\, d\theta = \left[-\cos\theta\right]_0^{\pi/2} = -\left[\cos\frac{\pi}{2} - \cos 0^\circ\right]$$

$$= -\left[0 - 1\right] = +1 \checkmark$$

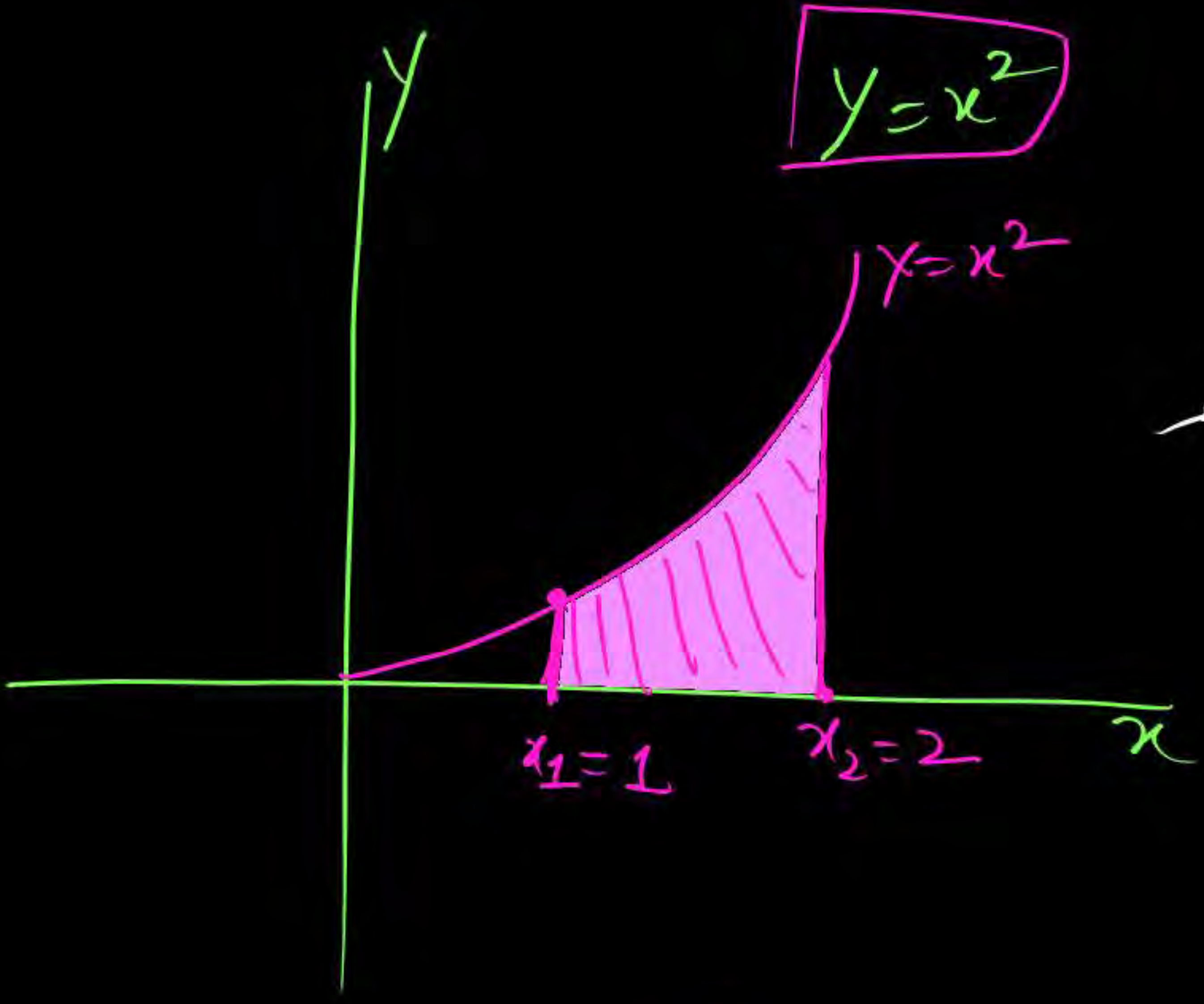
$$\int_{-\pi/2}^{3\pi/2} \cos\theta\, d\theta = \left(\sin\theta\right)_{-\pi/2}^{3\pi/2} = \sin\left(\frac{3\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right) = -1 - (-1) = -1+1$$

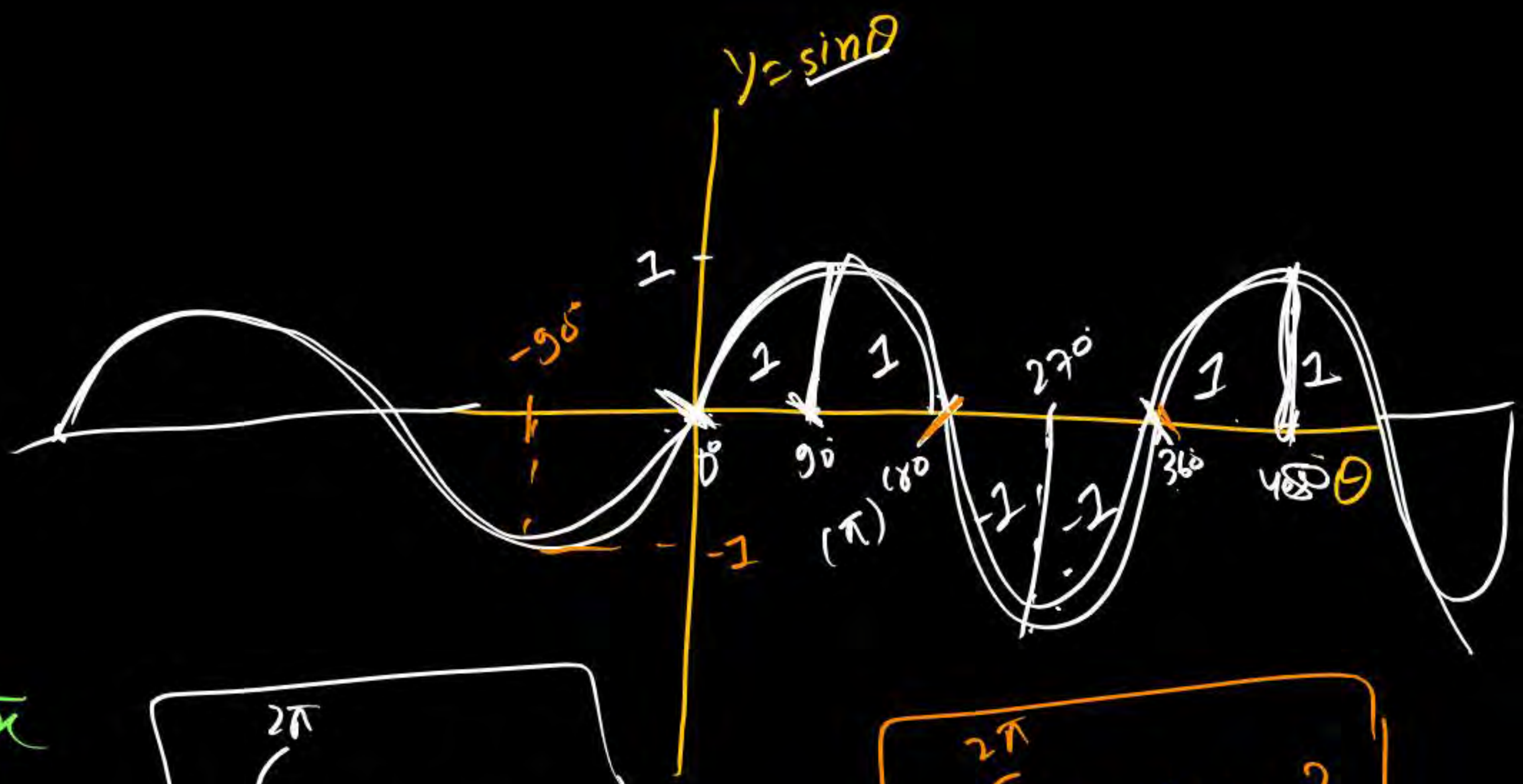
$$= 0$$

$$\int_0^{+\pi/2} (\sin\theta + \cos\theta)\, d\theta = \int_0^{\pi/2} \sin\theta\, d\theta + \int_0^{\pi/2} \cos\theta\, d\theta$$

$$= 2$$

$$\int_0^{90^\circ = 450^\circ} \sin\theta\, d\theta = 1$$



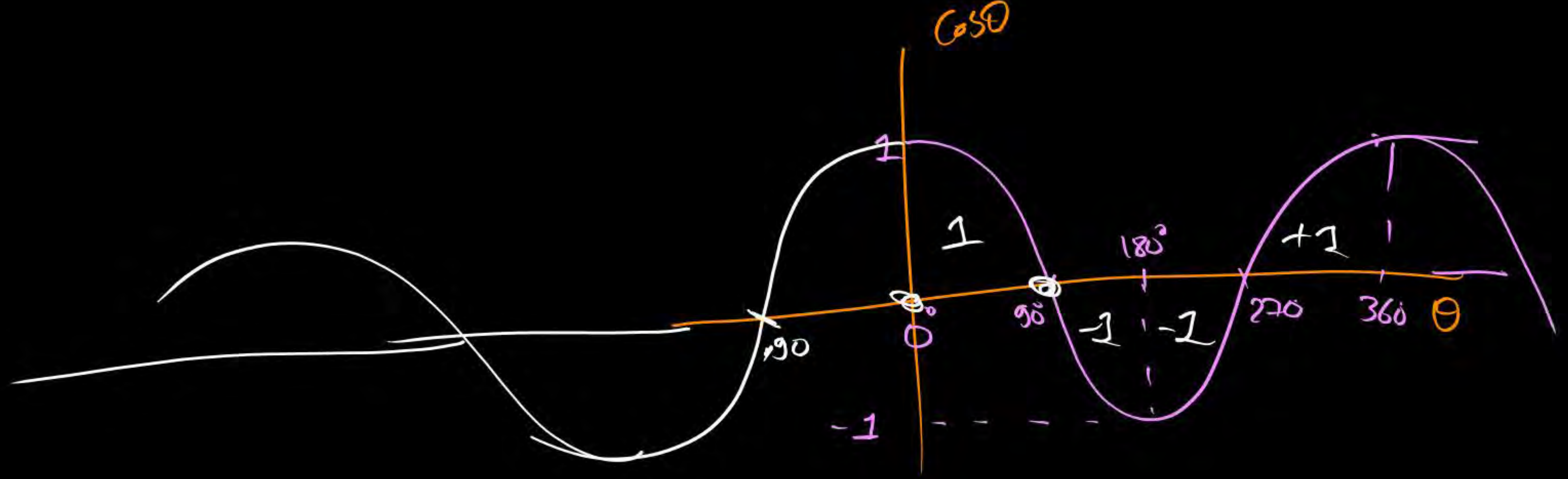


$$\boxed{\int_0^{2\pi} \sin\theta \, d\theta = 0}$$

$$\boxed{\int_\pi^{2\pi} \sin\theta \, d\theta = -2}$$

$$\text{Area} = \int y \, dx$$

$$= \int_1^2 x^2 \, dx = \left(\frac{x^3}{3}\right)_1^2 = \frac{1}{3}\left[2^3 - 1^3\right] = \frac{1}{3}\left[8 - 1\right] = \frac{7}{3} \text{ Ans.}$$



Cosθ
1
1
180°
+1
0°
90°
-90
-1
-1
270
360
θ
0
-1

$$\int_{90^\circ}^{360^\circ} \cos\theta \, d\theta =$$

$$\int_{0^\circ}^{270^\circ} \sin\theta =$$

$$\int_{90^\circ}^{180^\circ} \cos\theta =$$

π/ω


THANK YOU