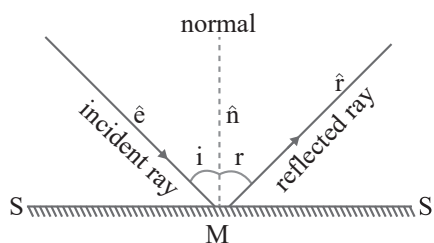# Ray Optics and Optical Instruments

REFLECTION

Laws of Reflection

❖ The incident ray, the reflected ray and normal to the surface of reflection at the point of incidence, all lie in the same plane.

❖ $\angle i = \angle r$

normal

In vector form $\boxed{\hat{r} = \hat{e} - 2(\hat{e}.\hat{n})\hat{n}}$

Object

Real: Point from which rays actually diverge.

Virtual: Point towards which rays appear to converge

Image: Image is decided by reflected or refracted rays only. The point of image for a mirror is that point at which the rays reflected from the mirror, actually converge (real image) or from which the reflected rays appear to diverge (virtual image).

Characteristics of Reflection By a Plane mirror

❖ The size of the image is the same as that of the object.

❖ For a real object the image is virtual and for a virtual object the image is real.

❖ For a fixed incident light ray, if the mirror be rotated through an angle θ the reflected ray turns through an angle 2θ in the same sense.

Number of images (n) in inclined mirror Find $\dfrac{360}{\theta} = m$

❖ If m is even, then n = m – 1, for all positions of object.

❖ If m is odd, then n = m, If object is not on bisector
    and n = m – 1, If object at bisector

❖ If m is fraction then n = nearest even number

SPHERICAL MIRRORS

principal axis

spherical surface   spherical mirror

concave mirror   convex mirror
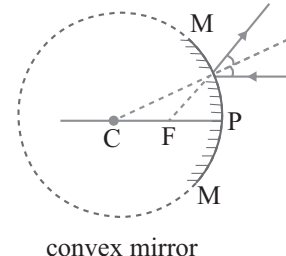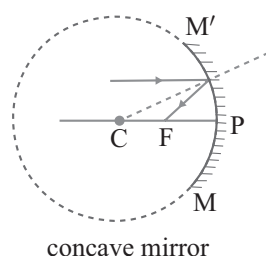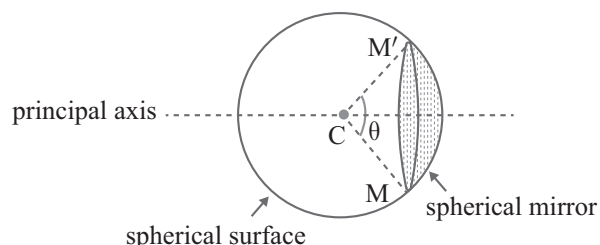
Mirror Formula: $\dfrac{1}{f} = \dfrac{1}{v} + \dfrac{1}{u}$.

f = Focal length of mirror

u = distance of object

v = distance of image

Note: Valid only for paraxial rays.

Transverse Magnification: $m_t = \dfrac{h_2}{h_1} = -\dfrac{v}{u}$

h_2 = height of image
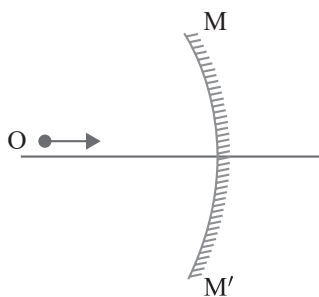
h_1 = height of the object

(both perpendicular to the principal axis of mirror)

Longitudinal magnification (m$_\ell$): $m_\ell = \dfrac{\text{Length of image}}{\text{Length of object}}$

for small object $m_\ell = -m_t^2$, m_t = transverse magnification.

Velocity of image of Moving Object (Spherical Mirror)

Velocity component along axis (Longitudinal velocity)

When an object is coming from infinite towards the focus of concave mirror

$$\because \frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\therefore -\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt} = 0$$

$$\Rightarrow \vec{v}_{IM} = -\frac{v^2}{u^2}\vec{v}_{OM} = -m^2\vec{v}_{OM}$$

❖ $V_{IM} = \dfrac{dv}{dt} =$ velocity of image with respect to mirror

❖ $V_{OM} = \dfrac{du}{dt} =$ velocity of object with respect to mirror.

Newton's Formula
Applicable to a pair of real object and real image position only. X_1, X_2 are the distance along the principal axis of the real object and real image respectively from the principal focus
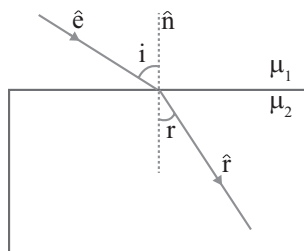
$$X_1 X_2 = f^2$$

Optical Power : Optical power of a mirror (in Diopters) $= -\dfrac{1}{f}$

where f = focal length (in meters) with sign.

REFRACTION

Laws of Refraction (At any Refracting Surface)
1. Incident ray, refracted ray and normal always lie in the same plane.
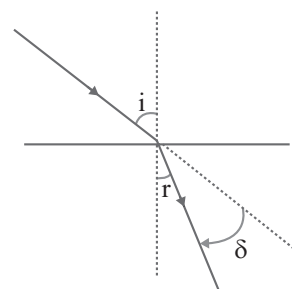

In vector form $(\hat{e} \times \hat{n}).\hat{r} = 0$

2. The product of refractive index and sine of angle of incidence at a point in a medium is constant. $\mu_1 \sin i = \mu_2 \sin r$ (Snell's law)

Snell's Law

$$\boxed{\frac{\sin i}{\sin r} = \mu_2 = \frac{\mu_2}{\mu_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}}$$ In vector form $\boxed{\mu_1 |\hat{e} \times \hat{n}| = \mu_2 |\hat{r} \times \hat{n}|}$
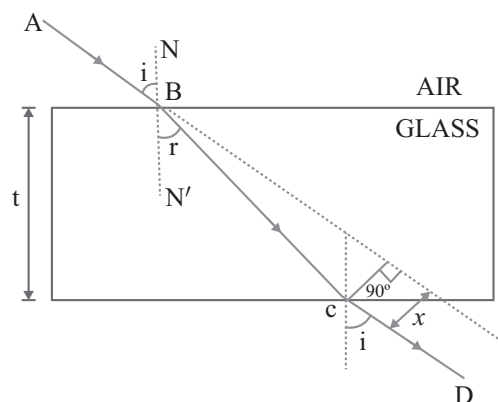
❖ Frequency of light does not change during refraction.

Deviation of a Ray due to refraction

$$\boxed{\text{angle of deviation}, \delta = i - r}$$ (clockwise)
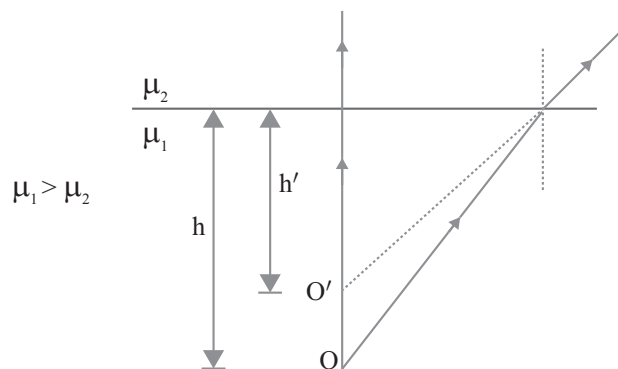
Refraction Through a Parallel Slab
Emergent ray is parallel to the incident ray, if medium is same on both sides.

❖ Lateral shift $x = \dfrac{t\sin(i-r)}{\cos r}$; t = thickness of slab

❖ Emergent ray will not be parallel to the incident ray if the medium on both the sides are different.

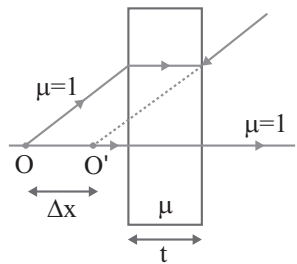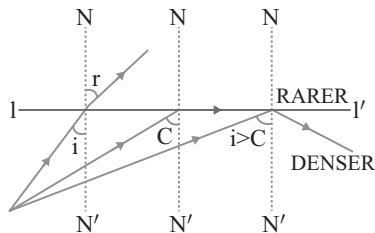## Apparent Depth of Submerged Object : (h' < h)

For near normal incidence $h' = \dfrac{\mu_2}{\mu_1} h$

h and h' are always measured from surface.

Δx = Apparent shift = $t\left(1 - \dfrac{1}{\mu}\right)$ always in direction of incident ray.
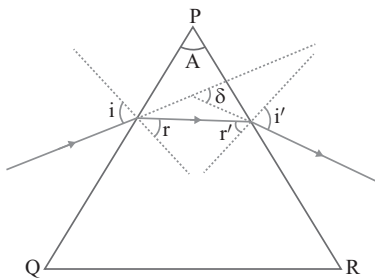

Critical Angle & Total Internal Reflection (TIR)
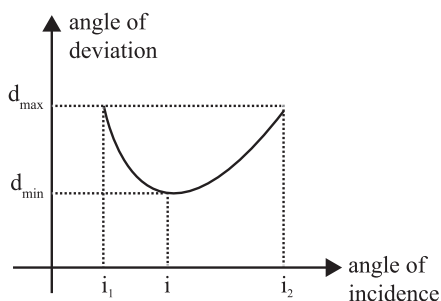

Conditions of TIR

❖ Ray is going from denser to rarer medium

❖ Angle of incidence should be greater than the critical angle $(i > C)$.

Critical angle $C = \sin^{-1}\dfrac{\mu_R}{\mu_D} = \sin^{-1}\dfrac{v_D}{v_R} = \sin^{-1}\dfrac{\lambda_D}{\lambda_R}$

Refraction Through Prism

❖ $\delta = (i + i') - (r + r')$

❖ $r + r' = A$

❖ $\delta = i + i' - A$

❖ Variation of δ versus i

❖ There is one and only one angle of incidence for which the angle of deviation is minimum. When $\delta = \delta_{min}$ then $i = i'$ and $r = r'$, the ray passes symetrically about the prism, and then

$$\mu = \dfrac{\sin\left[\dfrac{A + \delta m}{2}\right]}{\sin\left[\dfrac{A}{2}\right]}, \text{ where } \mu = \text{absolute R.I. of glass.}$$

❖ When the prism is dipped in a medium then μ = R.I. of glass w.r.t. medium.

❖ For a thin prism $(A \le 10^\circ)$; $\delta = (\mu - 1)A$

❖ **Dispersion of Light :** The angular splitting of a ray of white light into a number of components when it is refracted in a medium other than air is called **Dispersion of Light.**

❖ **Angle of Dispersion :** Angle between the rays of the extreme colours in the refracted (dispersed) light is called Angle of Dispersion. $\theta = \delta_v - \delta_r$

❖ Dispersive power (ω) of the medium of the material of prism.

$$\omega = \dfrac{\text{angular dispersion}}{\text{devitation of mean ray (yellow)}}$$
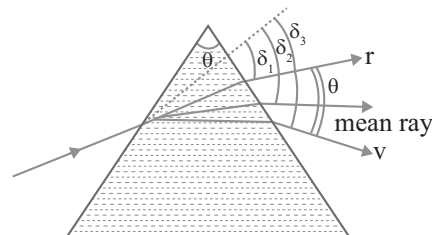
For small angled prism $(A \le 10^\circ)$;

$$\omega = \dfrac{\delta_v - \delta_R}{\delta_y} = \dfrac{\mu_v - \mu_R}{\mu_y - 1}; n = \dfrac{\mu_v + \mu_R}{2}$$

μ_y, μ_R and μ_y are R.I. of material for violet, red and yellow colours respectively.

REFRACTION AT SPHERICAL SURFACE

1. $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$
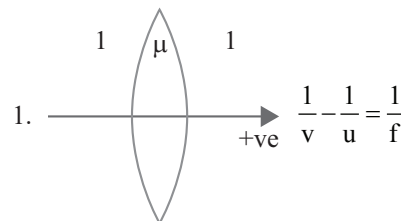
    v, u and R are to be kept with sign.

    v = PI
    u = –PO
    R = PC

2. $m = \dfrac{\mu_1 v}{\mu_2 u}$

Lens Formula

1. $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

2. $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

3. $m = \dfrac{v}{u}$

Power of Lenses

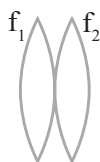Reciprocal of focal length in meter is known as power of lens.

SI unit : dioptre (D)

Power of lens : $P = \dfrac{1}{f\,(m)} = \dfrac{100}{f\,(cm)}$ dioptre

Combination of Lenses
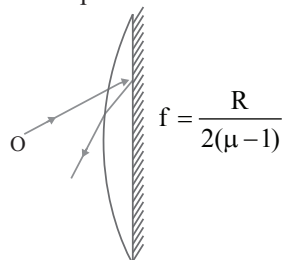
Two thin lenses are placed in contact to each other

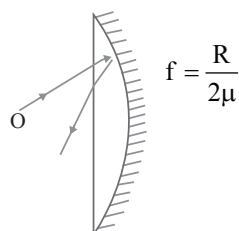Power of combination. $P = P_1 + P_2 \Rightarrow \dfrac{1}{F} = \dfrac{1}{f_1} + \dfrac{1}{f_2}$

Use sign convention while solving numericals.

Silvering of one surface of lens (use $P_{eq} = 2P_l + P_m$)

❖ When plane surface is silvered

$f = \dfrac{R}{2(\mu - 1)}$

❖ When convex surface is slivered

$f = \dfrac{R}{2\mu}$

OPTICAL INSTRUMENTS

For Simple microscope

❖ Magnifying power when image is formed at D : M = 1 + D/f

❖ When image is formed at infinity M = D/f

For Compound microscope

❖ $M = -\dfrac{v_0}{u_0}\left(\dfrac{D}{u_e}\right)$

❖ Magnifying power when final image is formed at D, M

$= -\dfrac{v_0}{u_0}\left(1 + \dfrac{D}{f_e}\right)$

❖ Tube length $L = v_0 + |u_e|$

❖ When final images is formed at infinity $M = -\dfrac{v_0}{u_0} \times \dfrac{D}{f_e}$ and

$L = v_0 + f_e$

Astronomical Telescope

$M = -\dfrac{f_0}{u_e}$

❖ Magnifying power when final image is formed at D:

$M = \dfrac{f_0}{f_e}\left(1 + \dfrac{f_e}{D}\right)$

❖ Tube length : $L = f_0 + |u_e|$

❖ When final image is formed at infinity: $M = \dfrac{f_0}{f_e}$ and

$L = f_0 + f_e$