CHAPTER

13 Simple Harmonic Motion

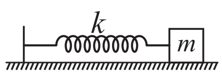
Simple Harmonic Motion

$F = -kx$

General equation of S.H.M. is $x = A\sin(\omega t + \phi)$; $(\omega t + \phi)$ is phase of the motion and ϕ is initial phase of the motion.

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Time period (T): $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$



Speed: $v = \omega\sqrt{A^2 - x^2}$

Acceleration: $a = -\omega^2 x$

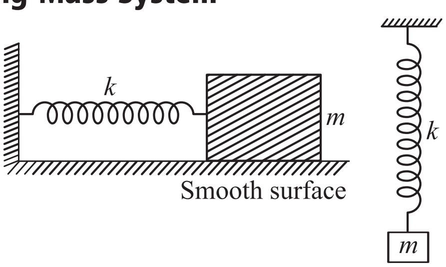
Kinetic Energy (KE): $\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}k\left(A^2 - x^2\right)$

Potential Energy (PE): $\frac{1}{2}kx^2$

Total Mechanical Energy (TME)

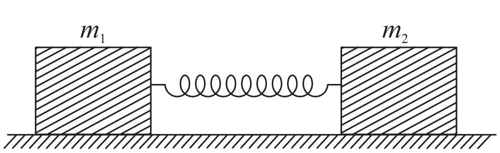
$= \text{K.E.} + \text{P.E.} = \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}Kx^2 = \frac{1}{2}KA^2$ = constant

Spring-Mass System

1. 

$T = 2\pi\sqrt{\frac{m}{k}}$

2. $T = 2\pi\sqrt{\frac{\mu}{k}}$, where $\mu = \frac{m_1 m_2}{(m_1 + m_2)}$ is known as "reduced mass".



Combination of Springs

Series Combination: $1/k_S = 1/k_1 + 1/k_2$

Parallel combination: $k_P = k_1 + k_2$

Simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$

$T = 2\pi\sqrt{\frac{l}{g_{eff.}}}$ (in accelerating Reference Frame); g_{eff} is net acceleration due to pseudo force and gravitational force.

Time period of simple pendulum in Accelerating lift.

(i) It velocity of lift in constant,

$g_{eff} = g$

$\therefore \quad T = 2\pi\sqrt{\frac{l}{g}} \qquad (\because a = 0)$

(ii) It lift is moving upwards with acceleration a $g_{eff} = g + a$

$\therefore \quad T = 2\pi\sqrt{\frac{l}{g + a}}$

(iii) It lift is moving downwards with acceleration a,

$g_{eff} = g - a$

$T = 2\pi\sqrt{\frac{l}{g - a}}$

(iv) It lift talls downwards treely,

$g_{eff} = 0$

$T = \infty$

Compound Pendulum/Physical Pendulum

Time Period (T): $T = 2\pi\sqrt{\frac{I}{mgl}}$

where, $I = I_{cm} + ml^2$; l is distance between point of suspension and centre of mass.

Torsional Pendulum

Time period (T): $T = 2\pi\sqrt{\frac{I}{C}}$ where, C = Torsional constant

Superposition of two SHM s along the same direction

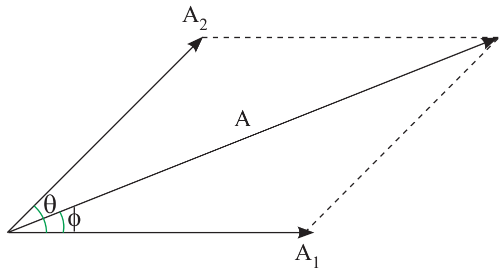
$x_1 = A_1 \sin \omega t$

and $x_2 = A_2 \sin(\omega t + \theta)$

If equation of resultant SHM is taken as $x = A \sin(\omega t + \phi)$

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$$

and $$\tan\phi = \frac{A_2\sin\theta}{A_1 + A_2\cos\theta}$$



Damped Harmonic Oscillations

If the damping force is given by $\vec{F}_d = -b\vec{v}$, where $\vec{v}$ is the velocity of the oscillator and b is a damping constant, then the displacement of the oscillator is given by

$$x(t) = A_0 e^{-bt/2m}\cos(\omega' t + \phi),$$

where ω' is the angular frequency of the damped oscillator, is given as $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

If the damping constant is small $\left(b \ll \sqrt{km}\right)$, then $\omega' \approx \omega$, where ω is the angular frequency of the undamped oscillator.

For small b, the mechanical energy E of the oscillator is given by

$$E(t) = \frac{1}{2}kA_0{}^2 e^{-bt/m}.$$

Forced Oscillations and Resonance

If an external driving force with angular frequency ω_d acts on an oscillating system with natural angular frequency ω_0, the system oscillates with angular frequency ω_d. The velocity amplitude v_m of the system is greatest when

$\omega_d = \omega$,

a condition called **resonance**. The amplitude A_0 of the system is (approximately) greatest under this condition.