YAKEEN NEET 2.0

2026

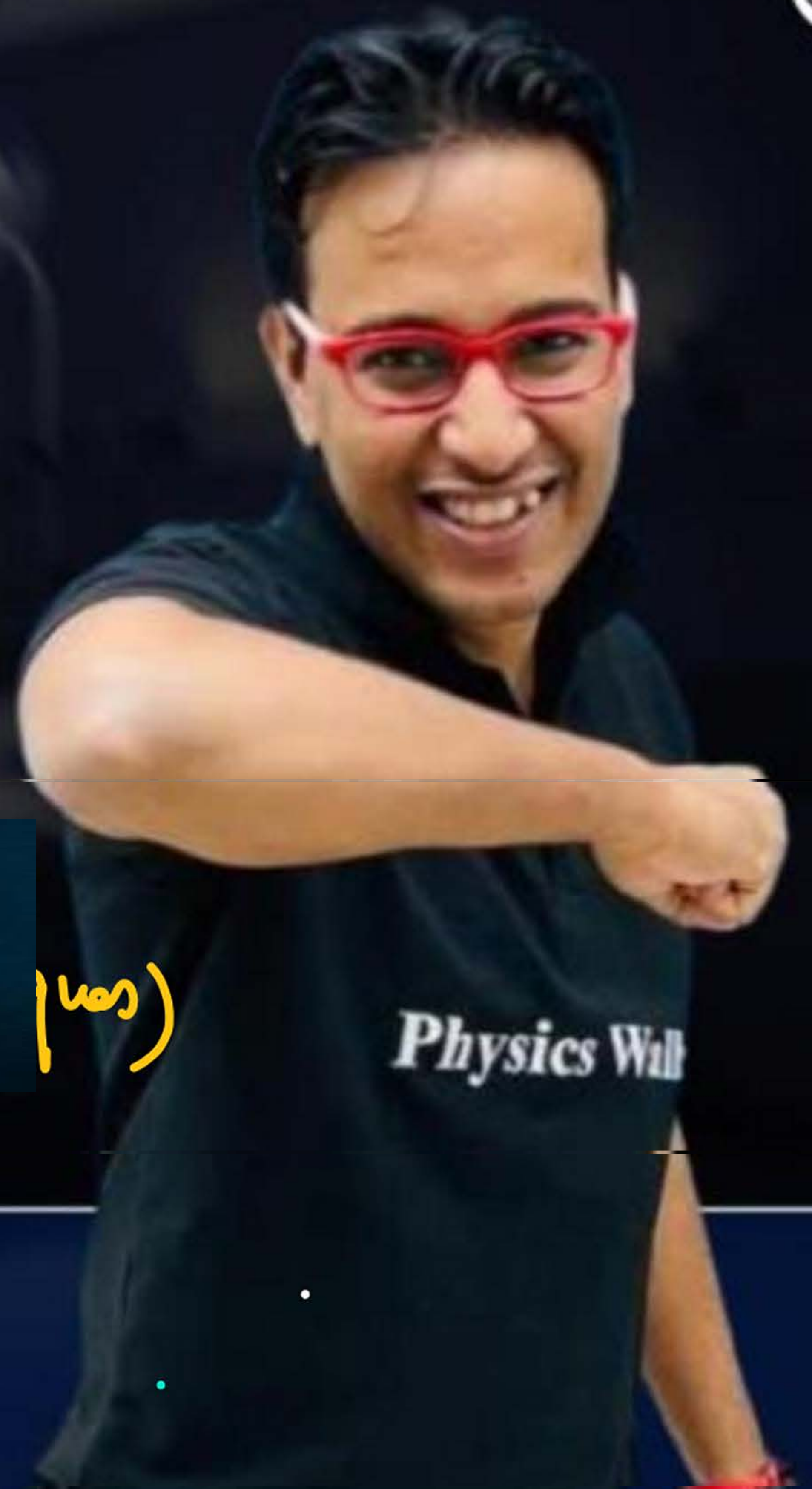
Microlecture

Motion in a plane

PHYSICS

Lecture -04

By - Saleem Ahmed Sir



Diff & Integration based ques.

position vector.

Q $\vec{r} = 3t^2\hat{i} - t^3\hat{j} + 2t^2\hat{k}$

① find $\vec{v}$

② Find $\vec{a}$

③ $t=2$, $\vec{v}$, $\vec{a}$

④ Find $\langle\vec{v}\rangle$ & $\langle\vec{a}\rangle$ from $t=1 \longrightarrow t=2$

$t=1$, $\vec{r}_i = (3,-1,2)$

$t=2$, $\vec{r}_f = (12,-8,8)$

Solⁿ ① $\vec{v} = \dfrac{d\vec{r}}{dt} = 6t\hat{i} - 3t^2\hat{j} + 4t\hat{k}$

② $\vec{a} = \dfrac{d\vec{v}}{dt} = 6\hat{i} - 6t\hat{j} + 4\hat{k}$

③ $t=2$, $\vec{v} = 12\hat{i} - 12\hat{j} + 8\hat{k}$

$\vec{a} = 6\hat{i} - 12\hat{j} + 4\hat{k}$

④ $\langle\vec{v}\rangle = \dfrac{\vec{r}_f - \vec{r}_i}{t_2 - t_1} = \dfrac{9\hat{i} - 7\hat{j} + 6\hat{k}}{2-1}$

$\langle\vec{a}\rangle = \dfrac{\vec{V}_2 - \vec{V}_1}{t_2 - t_1} = \dfrac{(12,-12,8)-(6,-3,4)}{2-1} = \dfrac{6\hat{i} - 9\hat{j} + 4\hat{k}}{1}$

Q $x = 6t \longrightarrow \frac{dx}{dt} = v_x = 6 \longrightarrow a_x = 0$

$y = 3t - 5t^2 \longrightarrow v_y = 3 - 10t \longrightarrow a_y = -10$

$\vec{r} = 6t\hat{i} + (3t - 5t^2)\hat{j}$ $\quad\quad \vec{v} = 6\hat{i} + (3 - 10t)\hat{j}$ $\quad\quad \vec{a} = -10\hat{j}$

$\vec{v} = 6\hat{i} + (3 - 10t)\hat{j}$

$\vec{a} = 0 + -10\hat{j}$

$$\begin{tikzcd} x \arrow[r] & v \arrow[r] \arrow[l, bend left] & a \arrow[l, bend left] \end{tikzcd}$$

Q A particle start motion from rest at $t=0$ from origin $(x=0)$, on the x-Axis

$V = 3t^2$

find location of particle at $t = 3$ sec.

Sol^n (1D)

$V = 3t^2$

$\frac{dx}{dt} = 3t^2$

$dx = 3t^2 dt$

$\int_0^x dx = \int_0^3 3t^2 dt$

$x \Big|_0^x = 3\frac{t^3}{3} \Big|_0^3$

$x - 0 = 3^3 = 27$

Definite

Indefinite

$\int dx = \int 3t^2 dt$

$x = t^3 + c$

At $t=0, x=0$

$0 = 0 + c$

$c = 0$

$x = t^3$

At $t=3$, $x = 3^3 = 27$

$\therefore$

Q A particle start motion from $t=0$ from $x=-10$ with initial velocity 5 m/s from origin s.t.

$V = 3t^2+5$

find location of particle at $t = 3$ sec.

Solⁿ

$$\frac{dx}{dt} = 3t^2+5$$

$$\int dx = \int (3t^2+5)\,dt$$

$$x = t^3+5t+c$$

At $t=0$, $x=-10$

$$-10 = 0+0+c$$

$$\boxed{c=-10}$$

$$x = t^3+5t-10$$

At $t=3$

$$x = 27+15-10$$

$$x = 32$$

$$\frac{dx}{dt} = 3t^2+5$$

$$\int_{-10}^{x} dx = \int_0^3 (3t^2+5)\,dt$$

$$x\Big|_{-10}^{x} = (t^3+5t)\Big|_0^3$$

$$x-(-10) = 3^3+5\times 3 - 0$$

$$x+10 = 27+15$$

$$\boxed{x=32}$$

Q A particle start motion from origin ($x=0$) from rest at $t=0$, s.t. $a_x = 6t$. find

(1) $v = f(t)$

(2) $x = f(t)$

(3) v & x at $t = 2$ sec.

Solⁿ

$$a = 6t$$

$$\frac{dv}{dt} = 6t$$

$$\int_0^v dv = \int_0^t 6t\,dt$$

$$v - 0 = 6\frac{t^2}{2}\Big|_0^t = 3t^2$$

$$\boxed{v = 3t^2}$$

$$v = 3t^2$$

$$\frac{dx}{dt} = 3t^2$$

$$\int_0^x dx = \int_0^t 3t^2\,dt$$

$$x - 0 = t^3 - 0$$

$$\boxed{x = t^3}$$

At $t = 2$, $v = 12$

$x = 8$

M-2

$$\Delta v = \int a\,dt$$

$$v_f - v_i = \int_0^t 6t\,dt$$

$$v - 0 = 3t^2$$

$$v = 3t^2$$

$$x_f - x_i = \int 3t^2\,dt$$

$$\boxed{x_f - 0 = t^3}$$

$$\frac{dv}{dt} = a$$

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt$$

$$v_2 - v_1 = \int_{t_1}^{t_2} a\,dt$$

Q Particle start motion at $t=0$ from $x=+10$ s.t.

$$V = 4t^3 + 3t^2 + 2t$$

Find location of particle at $t = 1$ sec.

Sol^n

$$\frac{dx}{dt} = 4t^3 + 3t^2 + 2t$$

$$\int_{10}^{x} dx = \int_{0}^{1} (4t^3 + 3t^2 + 2t)\,dt$$

$$x - 10 = t^4 + t^3 + t^2 \Big|_0^1$$

$$x - 10 = 3 - 0$$

$$\boxed{x = 13}$$

or

$$x_f - x_i = \int_0^1 v\,dt$$

$$x_f - 10 = t^4 + t^3 + t^2 \Big|_0^1$$

$$x_f - 10 = 3 - 0$$

$$\underline{x_f = 13}$$

Question

A particle start motion at $t=0$ having initial velocity $v=+10$ and initial location at $x=20$ such that $a=24t^2$

find (1) v, x (location) at $t = 2$ sec

(2) $v = f(t)$

$x = f(t)$

Ans : ($v = 74$, $x = 72$)

Question

A particle start motion at $t=0$ having initial velocity $v=+10$ and initial location at $x=20$ such that $a=24t^2$

find v, x (location) at $t=2$ sec

Sol^n

$$a = 24t^2$$

$$\frac{dv}{dt} = 24t^2$$

$$\int_{10}^{v} dv = \int_0^t 24t^2\,dt$$

$$v-10 = 24\frac{t^3}{3}\Big|_0^t$$

$$\boxed{v = 8t^3+10}$$

$$v = 8t^3+10$$

$$\frac{dx}{dt} = 8t^3+10$$

$$\int_{20}^{x} dx = \int_0^t (8t^3+10)\,dt$$

$$x-20 = \left(8\frac{t^4}{4}+10t\right)\Big|_0^t$$

$$\boxed{x = 20+2t^4+10t}$$

Ans : ($v = 74$, $x = 72$)

$$v = \frac{dx}{dt}$$

$$a = \frac{dv}{dt}$$

$$\boxed{a = v\frac{dv}{dx}}$$

$$a = v\frac{dv}{dx}$$

Q $V = x^2 + 8x$

find acc. at $x = 2$

Solⁿ $\frac{dv}{dx} = 2x + 8$

$$a = v\frac{dv}{dx} = (x^2 + 8x)\cdot(2x+8)$$

At $x = 2$, $a = (4+16)(4+8)$

$a =$

$$\left\{\begin{aligned} v &= \frac{dx}{dt} \\ a &= \frac{dv}{dt} \\ a &= v\frac{dv}{dx} \end{aligned}\right\}$$

Question

A particle is moving on x-axis such that its acc is given by $a = \frac{3}{v}$. At $t = 0$ its velocity is 1 m/s. Find velocity at $t = 40$ sec.

v, t

Sol^n

$$a = \frac{3}{v}$$

$$\frac{dv}{dt} = \frac{3}{v}$$

$$\int_1^v v\,dv = \int_0^{40} 3\,dt$$

$$\left.\frac{v^2}{2}\right|_1^v = \left.3t\right|_0^{40}$$

$$\frac{v^2}{2} - \frac{1}{2} = 3 \times (40 - 0)$$

$$\frac{v^2 - 1}{2} = 120$$

$$v = \sqrt{241}$$

Ans : $(v = \sqrt{241})$

SKC

ye dekhen ki
Diya kya hai
or pucha kya hai

Question

Acceleration of a particle moving on x-axis having initial speed v_0 with distance from origin is given by $a = \sqrt{x}$ Distance covered by particle where its speed become thrice that of initial speed.

$$a = \sqrt{x}$$

$$v\frac{dv}{dx} = \sqrt{x}$$

$$\int_{V_0}^{3V_0} v\,dv = \int_0^x x^{\frac{1}{2}}\,dx$$

$$\left.\frac{v^2}{2}\right|_{V_0}^{3V_0} = \frac{x^{3/2}}{3/2}$$

$$\frac{1}{2}\left[(3V_0)^2 - V_0^2\right] = \frac{2}{3}x^{3/2}$$

$$4V_0^2 = \frac{2}{3}x^{3/2}$$

$$x = (6V_0^2)^{2/3}$$

Ans : $x = \cancel{\left(\frac{2}{3}c\right)^{\frac{1}{3}}}$.

Question

v, x

A particle is projected with velocity $v_0 = 4$ m/s along +x-axis from origin and acc. is $a = -3x^2$. Find where particle will comes to at rest

$a = -3x^2$

$v\frac{dv}{dx} = -3x^2$

$\int_4^0 v\,dv = \int_0^{x_0} -3x^2\,dx$

$\frac{v^2}{2}\Big|_4^0 = -3.\frac{x^3}{3}\Big|_0^{x_0}$

$0 - \frac{4^2}{2} = -x_0^3$

$8 = x_0^3$

$\boxed{x_0 = 2}$

$a = -3x^2$

When rest $x = 2$

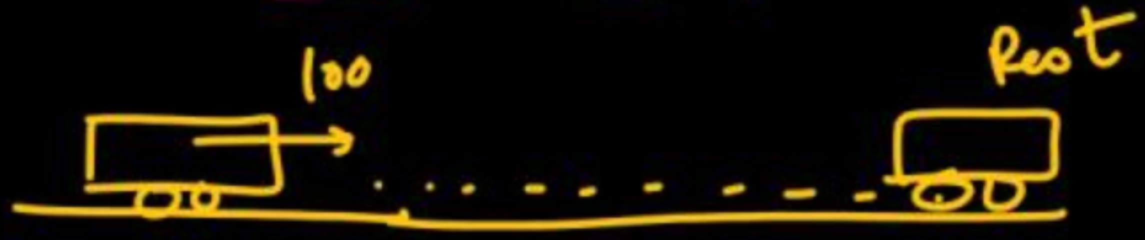
$a = -3\times 2^2 = -12$

Ans : $x_f = 2$, $a = -12$

Question

The retardation of a car when its engine is shut off depends on its velocity as $a = -\alpha v$ where α is positive constant. Find the total distance travelled by the car if its initial velocity is 20 m/s and $\alpha = 0.5/s$.

(Rest where)



Sol^n

$$a = -\alpha v$$

$$v\frac{dv}{dx} = -\alpha v$$

$$\int_{20}^{0} dv = \int_{0}^{x_0} -\alpha\, dx$$

$$0 - 20 = -\alpha x_0$$

$$20 = \frac{1}{2} x_0$$

$$\boxed{x_0 = 40}$$

Ans : (d = 40 m)

find

Question

Acceleration of particle moving rectilinearly is $a = 4 - 2x$ (where x is position in metre and a in m/s^2). It is at rest at $x = 0$. At what position x (in metre) will the particle again come to instantaneous rest?

v, x

$$a = 4 - 2x$$

$$v\frac{dv}{dx} = 4 - 2x$$

$$\int_0^0 v\,dv = \int_0^{x_0} (4 - 2x)\,dx$$

$$0 = \left(4x - x^2\right)\Big|_0^{x_0}$$

$$\left(4x_0 - x_0^2\right) - 0 = 0$$

$$4x_0 - x_0^2 = 0$$

$$x_0(4 - x_0) = 0$$

$$x_0 = 0$$
$$x_0 = 4$$

Ans : (x = 0, 4)

Question

copy ×

A block of mass m is fired horizontally along a level surface that is lubricated with oil. The oil provides a viscous (force) resistance that varies as the 3/2 power of the speed. If the initial speed of the block is v_0 at $x = 0$, find the maximum distance reached by the block. Assume no resistance to motion other than that provided by the oil.

$$F = -v^{3/2}$$

$$ma = -v^{3/2}$$

$$a = -\frac{v^{3/2}}{m}$$

$$v\frac{dv}{dx} = -\frac{v^{3/2}}{m}$$

$$\int_{v_0}^{0} v^{-\frac{1}{2}}\,dv = -\frac{1}{m}\int_0^{x_0} dx$$

$$\frac{v_0^{\frac{1}{2}}}{1/2} = \frac{1}{m}(x_0 - 0)$$

$$2\sqrt{v_0} = \frac{x_0}{m}$$

$$\boxed{x_0 = 2m\sqrt{v_0}}$$

Ans : (*)

Question

The deceleration experienced by a moving motor boat, after its engine is cut-off is given by $dv/dt = -kv^3$, where k is constant. If v_0 is the magnitude of the velocity at cut-off, the magnitude of the velocity at a time t after the cut-off is

(1) $v_0/2$

(2) v

(3) $v_0 e^{-kt}$

(4) $\frac{v_0}{\sqrt{2v_0^2 kt + 1}}$

$$\frac{dv}{dt} = -kv^3$$

$$\int_{v_0}^{v} \frac{dv}{v^3} = \int_{0}^{t} -k\,dt$$

Ans : (4)

An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be :-

6.25 m/s की चाल से गतिशील एक वस्तु के मन्दन की दर इससे दी जाती है।

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

$$\int_{6.25}^{0}\frac{dv}{\sqrt{v}} = -\int_{0}^{t}2.5\,dt$$

जहाँ v तात्क्षणिक चाल है। वस्तु को विराम अवस्था में आने में लगा समय है :- [AIEEE-2011]

(1) 4 s (2) 8 s (3) 1 s (4) 2 s

Ans. (4)

A particle is projected with velocity v_0 along x–axis. The deceleration on the particle is proportional to the square of the distance from the origin i.e., $a = -\alpha x^2$. The distance at which the particle stops is:–

एक कण x अक्ष के अनुदिश v_0 वेग से प्रक्षेपित किया जाता है। कण का मंदन, मूल बिन्दु से इसकी दूरी के वर्ग के समानुपाती है अर्थात् $a = -\alpha x^2$ है। किस दूरी पर कण रूक जायेगा ?

(A) $\sqrt{\dfrac{3v_0}{2\alpha}}$ (B) $\left(\dfrac{3v_0}{2\alpha}\right)^{\frac{1}{3}}$ (C) $\sqrt{\dfrac{3v_0^2}{2\alpha}}$ (D) $\left(\dfrac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$

Ans. (D)

Sol:

$$v\frac{dv}{dx} = -\alpha x^2$$

$$\int_{v_0}^{0} v\,dv = -\int_{0}^{x_0} \alpha x^2\,dx$$

The acceleration vector along x–axis of a particle having initial speed v_0 changes with distance as $a = \sqrt{x}$. The distance covered by the particle, when its speed becomes twice that of initial speed is:–

प्रारम्भिक चाल v_0 वाले एक कण का x- अक्ष के अनुदिश त्वरण सदिश, दूरी के साथ $a = \sqrt{x}$ के अनुसार परिवर्तित होता है। जब कण की चाल प्रारम्भिक चाल की दुगुनी हो जाये उस समय कण द्वारा तय की गई दूरी होगी:-

(A) $\left(\frac{9}{4}v_0\right)^{\frac{4}{3}}$ (B) $\left(\frac{3}{2}v_0\right)^{\frac{4}{3}}$ (C) $\left(\frac{2}{3}v_0\right)^{\frac{4}{3}}$ (D) $2v_0$

Ans. (B)

$$v\frac{dv}{dx} = x^{\frac{1}{2}}$$

copy

Q A particle has an initial velocity of $3\hat{i}+4\hat{j}$ and an acceleration of $\underbrace{0.4\hat{i}+0.3\hat{j}}_{\text{const}}$. Its speed after 10s is:-

[AIEEE-2009]

(1) 7 units (2) 8.5 units (3) 10 units (4) $7\sqrt{2}$ units

$$\vec{v}=\vec{u}+\vec{a}t$$

$$\vec{v}=(3\hat{i}+4\hat{j})+(.4\hat{i}+.3\hat{j})\times 10$$

magnitude

Question

A particle is moving along the x-axis whose instantaneous speed is given by $v^2 = 108 - 9x^2$. The acceleration of the particle is

(1) $-9x \text{ m s}^{-2}$ (2) $-18x \text{ m s}^{-2}$

(3) $\frac{-9x}{2} \text{ m s}^{-2}$ (4) None of these

$V^2 = 108 - 9x^2$

Diff wrt 'x'

$2V \frac{dv}{dx} = 0 - 9 \times 2x$

$2a = -18x$

$a = -9x$

m2

$2V \frac{dv}{dt} = 0 - 9 \times 2x \cdot \frac{dx}{dt}$

$2\cancel{v} a = -18x \cdot \cancel{v}$

$a = -9x$

Ans : (1)

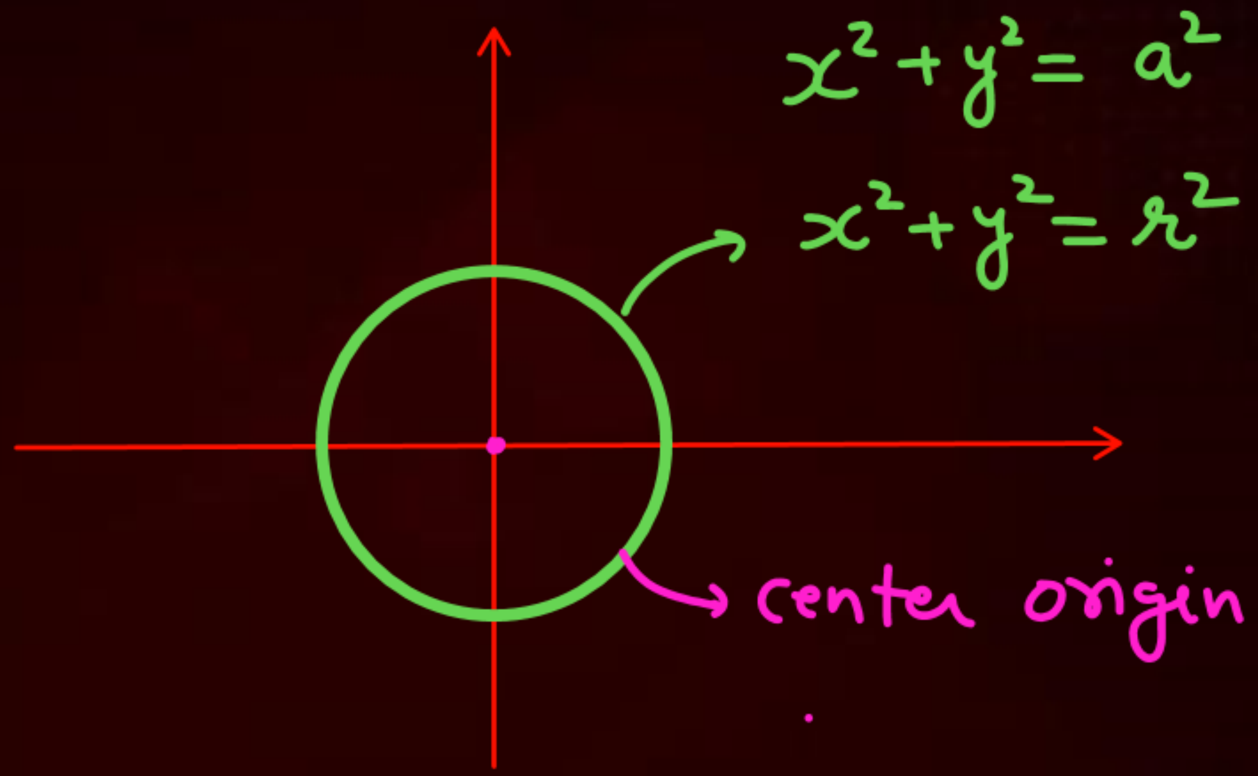
(1) St. line $\longrightarrow$ $y = mx + c$

(2) parabola $y^2 = x$

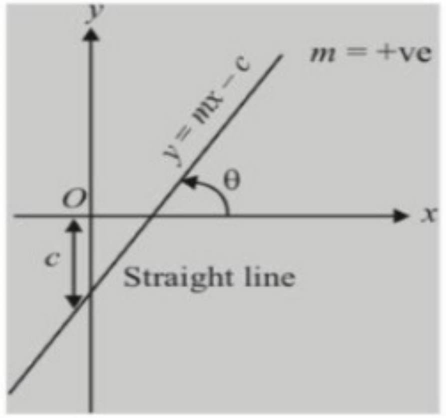
$y = x^2$

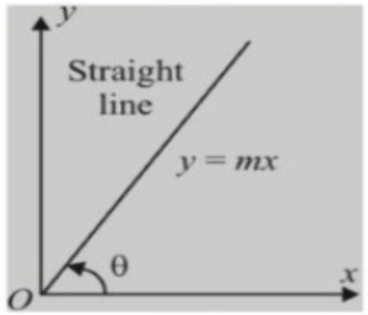
(3) Circle

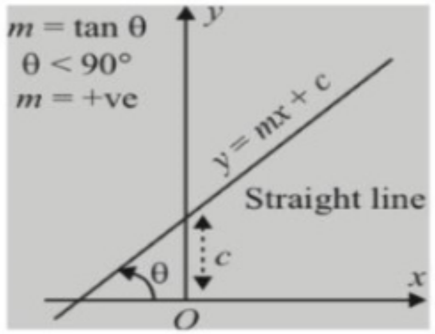
$x^2 + y^2 = r^2$

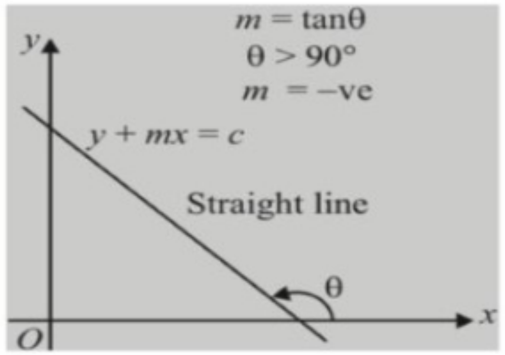


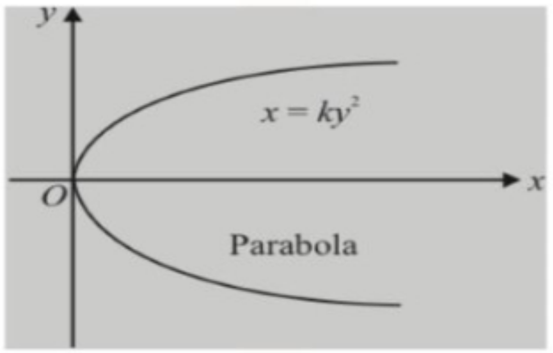
$$\boxed{\begin{array}{l} (x - x_1)^2 + (y - y_1)^2 = r^2 \\ \text{Center } (x_1, y_1) \end{array}}$$

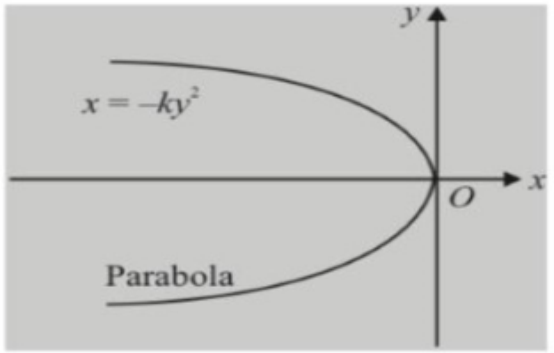

y
$m = +ve$
$y = mx - c$
θ
O
x
c
Straight line

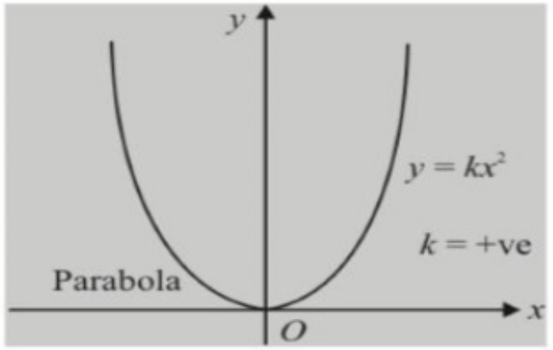

y
Straight line
$y = mx$
θ
O
x

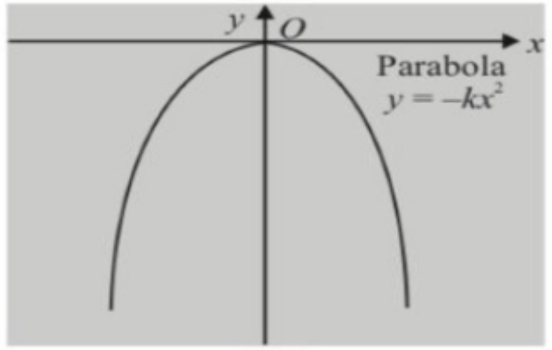

$m = \tan\theta$
$\theta < 90°$
$m = +ve$
y
$y = mx + c$
Straight line
θ
c
O
x

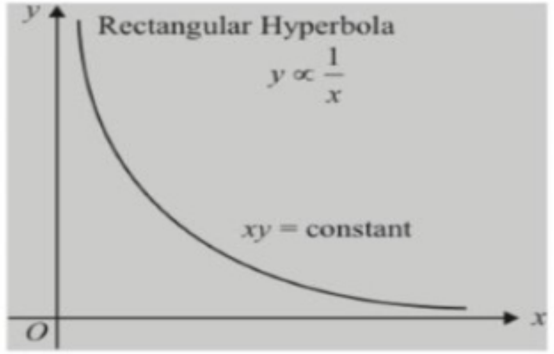

$m = \tan\theta$
$\theta > 90°$
$m = -ve$
y
$y + mx = c$
Straight line
θ
O
x

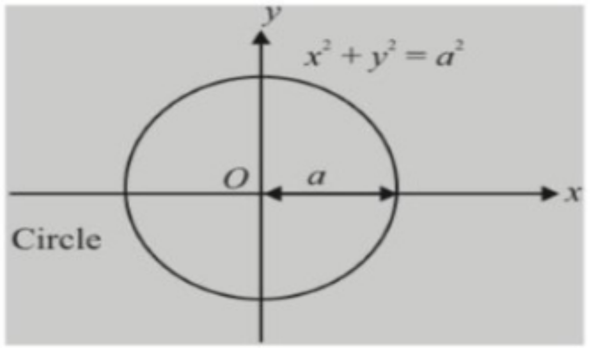

y
$x = ky^2$
O
x
Parabola

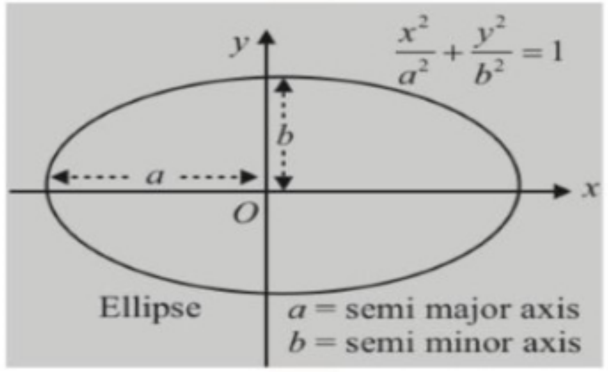

y
$x = -ky^2$
O
x
Parabola

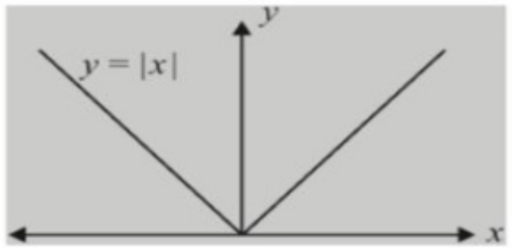

y
$y = kx^2$
$k = +ve$
Parabola
O
x


y
O
x
Parabola
$y = -kx^2$


y
Rectangular Hyperbola
$y \propto \frac{1}{x}$
xy = constant
O
x


y
$x^2 + y^2 = a^2$
O
a
x
Circle


$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
y
b
a
O
x
Ellipse
a = semi major axis
b = semi minor axis


y
$y = |x|$
x

$y = mx + c$

$\boxed{\begin{array}{c}\text{parabola} \\ y = x^2 \\ y^2 = x\end{array}}$

$x^2 + y^2 = r^2$ Equation of circle of center $(0,0)$ & radius 'r'

or

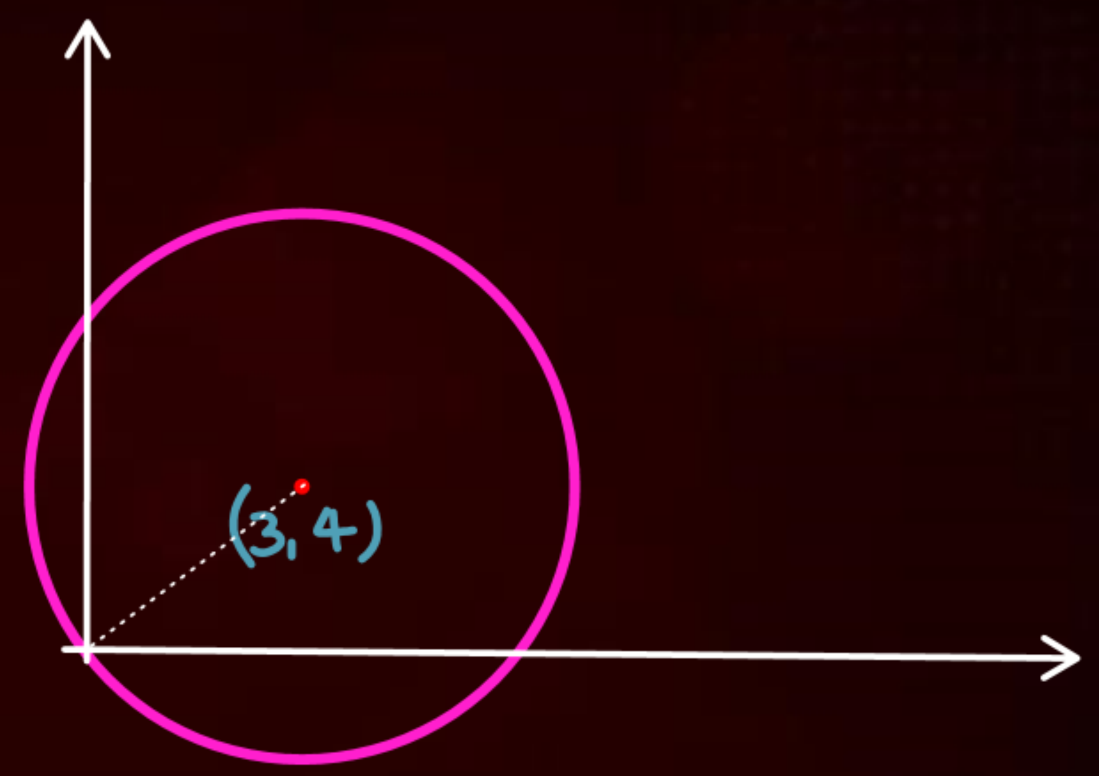
$x^2 + y^2 = a^2$

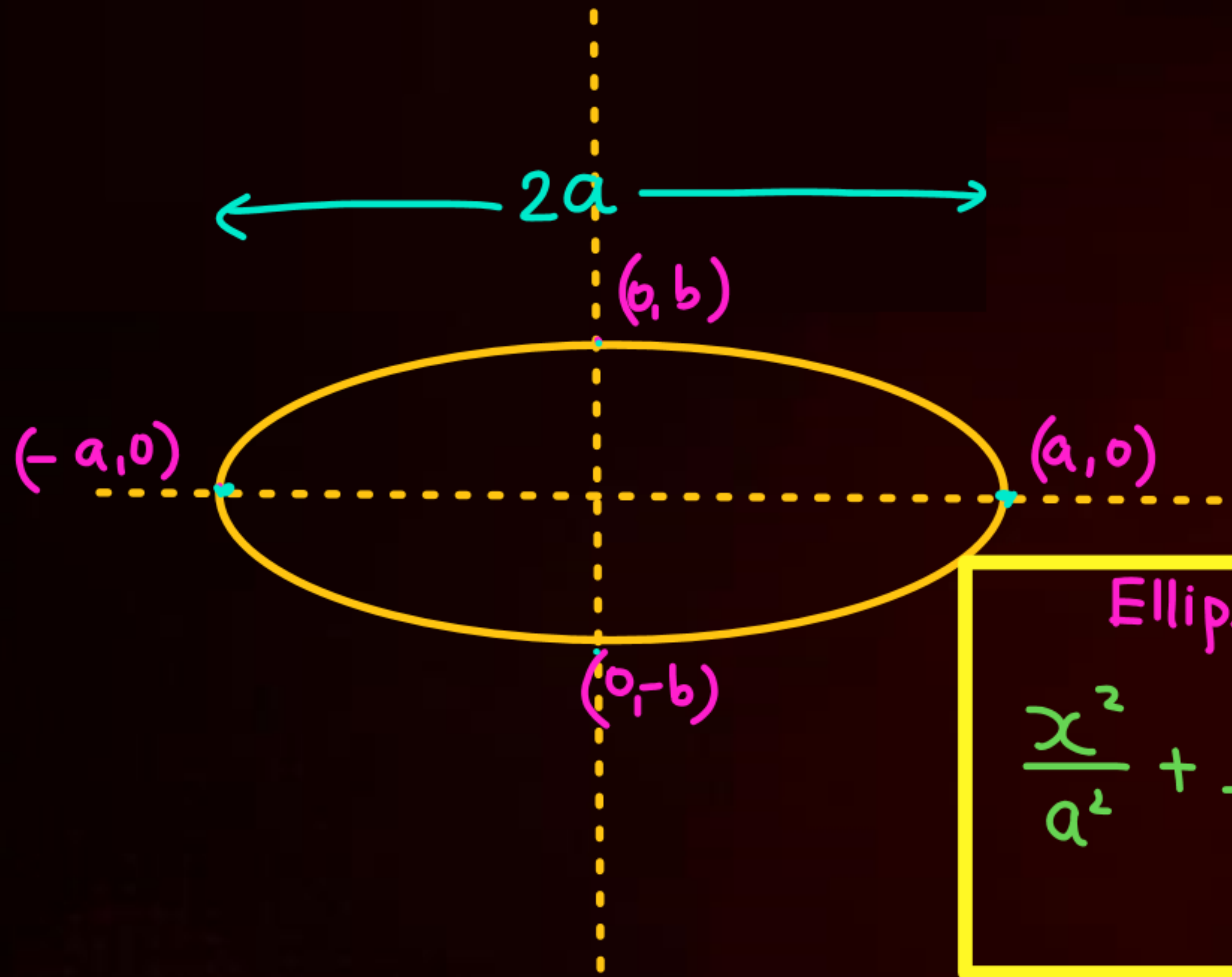
Eq^n $(x - x_1)^2 + (y - y_1)^2 = r^2$

Q $(x-3)^2+(y-4)^2=25$

center $\longrightarrow (3,4)$

radius $\longrightarrow \sqrt{25}=5$





Ellipse.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

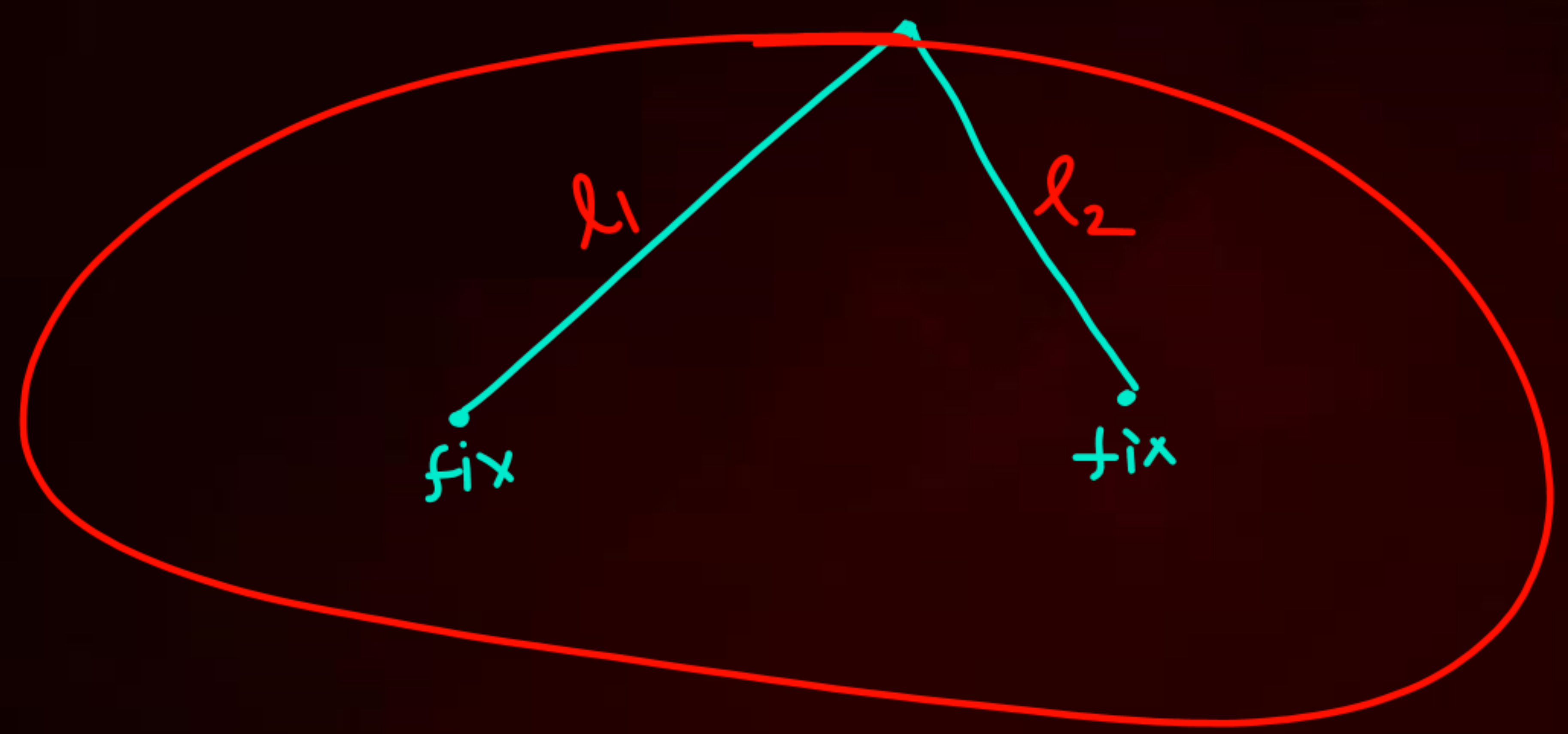
If $a=b \Rightarrow$ circle

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

put, $y=0$

$x^2=a^2$

$x=\pm a$



l1
l2
fix
fix

Find eqⁿ of trajectory

(1) $x = 2t$

$y = 4t$

Solⁿ $t = \frac{x}{2}$

$y = 4 \cdot \frac{x}{2}$

$\boxed{y = 2x}$

St. line, 1D

(2) $x = 2t$

$y = 4t^2$

Solⁿ $t = \frac{x}{2}$

$y = 4\left(\frac{x}{2}\right)^2$

$y = 4\frac{x^2}{4} = x^2$

$y = x^2$

(parabola), 2D

(3) $x = A\sin\omega t$

$y = A\cos\omega t$

Solⁿ

$\sin^2\omega t + \cos^2\omega t = 1$

$\left(\frac{x}{A}\right)^2 + \left(\frac{y}{A}\right)^2 = 1$

$x^2 + y^2 = A^2$

(circular), 2D


@SALEEMSIR_PW

THANK YOU