CHAPTER 7

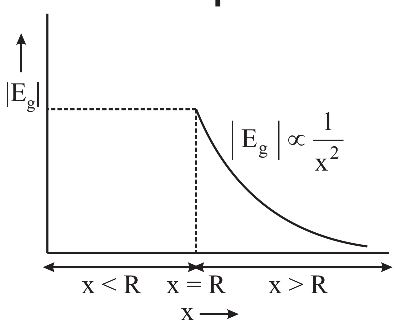
Gravitation

Newton's Universal Law of Gravitation

$$m_1 \longleftrightarrow m_2$$

- Force of attraction between two point masses $F = \frac{Gm_1m_2}{r^2}$, where $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$
- Directed along the line joining the point masses.
- It is a conservative force $\Rightarrow$ mechanical energy will be conserved.
- It is a central force $\Rightarrow$ angular momentum will be conserved.

Gravitational Field due to Spherical Shell



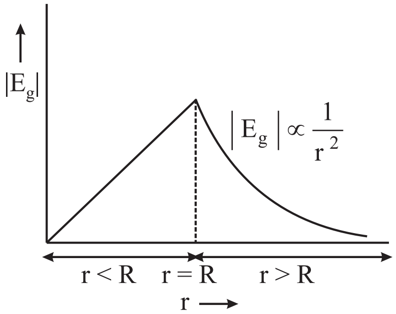
Outside Region $E_g = \frac{GM}{r^2}$, where x > R

On the surface $E_g = \frac{GM}{R^2}$, where x = R

Inside Region $E_g = 0$, where x < R

Note: Direction always towards the centre of the sphere, radially inwards.

Gravitational Field Due to Solid Sphere



Outside Region $E_g = \frac{GM}{r^2}$, where r > R

On the surface $E_g = \frac{GM}{R^2}$, where r = R

Inside Region $E_g = \frac{GMr}{R^3}$, where r < R

Gravity 'g'

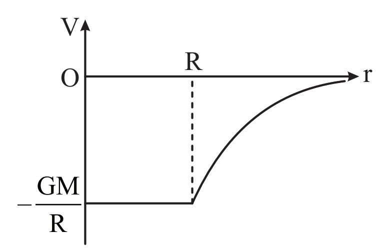
- Acceleration due to gravity $g_S = \frac{GM}{R^2}$ (on the surface of earth)
- At height h, $g_h = \frac{GM}{(R+h)^2}$
- If h << R ; $g_h \approx g_s\left(1-\frac{2h}{R}\right)$
- At depth d, $g_d = \frac{GM(R-d)}{R^3} = g_s\left(1-\frac{d}{R}\right)$
- Effect of rotation on g : $g' = g - \omega^2 R\cos^2\lambda$
  (where λ is angle of latitude.)

Gravitational Potential

- Due to a point mass at a distance r
  $$V = \frac{GM}{r}$$
- Gravitational potential due to spherical shell

  Outside the shell $V = \frac{GM}{r}$, $r > R$

  Inside/on the surface of the shell $V = \frac{GM}{R}$, $r < R$

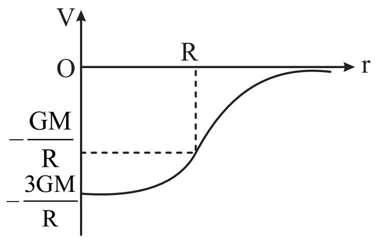


- Potential due to a solid sphere

Outside Region $V = -\dfrac{GM}{r},\ r > R$

On the surface $V = -\dfrac{GM}{R},\ r = R$

Inside Region $V = -\dfrac{GM(3R^2 - r^2)}{2R^3},\ r < R$



- Potential on the axis of a thin ring at a distance r from the centre

$$V = -\frac{GM}{\sqrt{R^2 + r^2}}$$

Motion of a Satellite

- Escape velocity from a planet of mass M and radius R

$$V_e = \sqrt{\frac{2GM}{R}}$$

- Orbital velocity of satellite (orbital radius r)

$$V_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+H)}}$$

- For nearby satellite

$$V_0 = \sqrt{\frac{GM}{R}} \quad \frac{V_e}{\sqrt{2}}$$

Here V_e = escape velocity on earth surface.

Time Period of Satellite

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

Energies of a Satellite

Potential energy $U = -\dfrac{GMm}{r}$

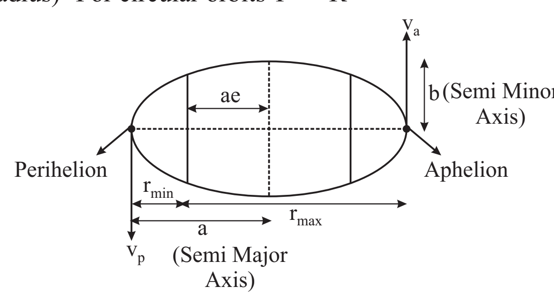
Kinetic energy $K = \dfrac{1}{2}mv^2 = \dfrac{GMm}{2r}$

Mechanical energy $E = U + K = -\dfrac{GMm}{2r}$

Binding energy $BE = -E = \dfrac{GMm}{2r}$

Kepler's Laws

- 1^{st} Law of orbitals: Path of a planet is elliptical with the sun at one of the focus.
- II^{nd} Law of areas: Areal velocity $\dfrac{d\vec{A}}{dt} = \text{constant} = \dfrac{\vec{L}}{2m}$
- III^{rd} Law of periods: $T^2 \propto a^3$ or $T^2 \propto \left(\dfrac{r_{max} + r_{min}}{2}\right)^3 \propto$ (mean radius)^3 For circular orbits $T^2 \propto R^3$



- $r_{max} = a(1 + e)$

  $r_{min} = a(1 - e)$
- $\dfrac{v_p}{v_a} = \dfrac{v_{max}}{v_{min}} = \dfrac{1+e}{1-e}$

  (where, e is eccentricity.)