Wave Motion 1

Waves

Wave is a disturbance which carries energy and momentum from one place to another without the transport of medium.

⟹ The medium should have elasticity and inertia

Characteristics of Wave

⟹ The particles of the medium are executing simple harmonic motion.

⟹ The phase of vibration of the particle keeps on changing.

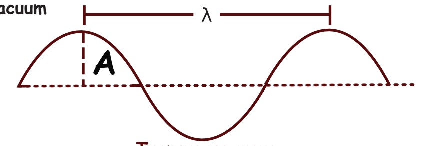
⟹ Wave carries energy and momentum.

⟹ The velocity of the particle is not equal to velocity of wave.

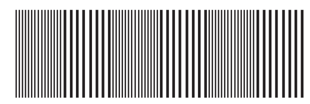
Classification of waves

According to

- **Necessity of medium**
  - Mechanical waves: can not travel in vaccum
  - Non-mechanical waves(EMW):z can travel in vacuum
- **Energy propagation**
  - Progressive waves
  - Stationary waves
- **Dimension**
  - One Dimensional
  - Two Dimensional
  - Three Dimensional
- **Vibration of particles**
  - Transverse waves
  - Longitudinal waves

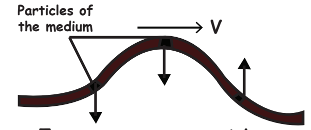
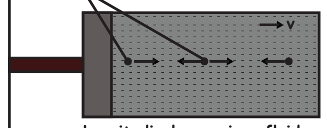
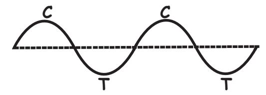
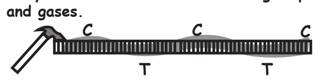


Transverse wave



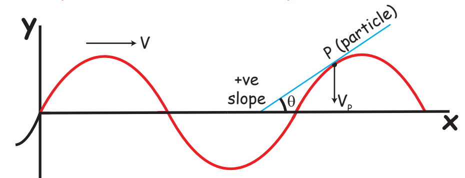
Longitudinal wave

Classification of waves based on vibration of particles

Transverse waves	Longitudinal waves
Particles of the medium vibrate in a direction perpendicular to the direction of propagation of wave.  Transverse wave on a string	Particles of the medium vibrate in the direction of wave motion. Particles of the medium.  Longitudinal wave in a fluid
It travels in the form of crests (C) and troughs (T). 	It travels in the form of compression (C) and rarefaction (R). Maximum pressure and density  Minimum Pressure and density
Transverse waves can be transmitted through solids. They can be setup on the surface of liquids but not inside the liquids. They can not be transmitted through liquids and gases.  Transverse-wave in a rod	These waves can be transmitted through solids, liquids and gases because for propagation of these waves, Only volume elasticity is necessary.  Longitudinal wave in a rod
Medium should possess the property of rigidity	Medium should possess the property of volume elasticity
Transverse waves can be polarised	Longitudinal waves can not be polarised
Movement of string of a sitar or violin, movement of the membrane of a tabla or Dholak, movement of kink on a rope, waves set-up on the surface of water.	Sound waves travelling through air, vibration of air column in organ pipes, vibration of air column above the surface of water in the tube of resonance apparatus.

Wave equation

All travelling waves satisfy a differential equation called wave equation.



$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{V^2}\frac{\partial^2 y}{\partial t^2}$$

$$\boxed{\frac{\partial y}{\partial t} = -\frac{\omega}{k}\times\frac{\partial y}{\partial x}}$$

Velocity of particle = -Velocity of wave × Slope of the waveform

Velocity of particle / Acceleration of particle

velocity of particle

$V=\omega\sqrt{A^2-y^2}$

$V_{max}=\omega A$

Acceleration of particle

$a=\omega^2 y$

$a_{max}=\omega^2 A$

If v_{max} of the particle $= n \times v_{wave}$

$\omega A = n \times f\lambda$

$2\pi f A = n \times f\lambda$

$n=\frac{2\pi A}{\lambda}$

$\lambda=\frac{2\pi A}{n}$

$A=\frac{n\lambda}{2\pi}$

Intensity of wave

$$\boxed{I = \frac{\text{Energy}}{\text{time}\times\text{Area}} = 2\pi^2 f^2 A^2 \rho v}$$

$$\boxed{I=\frac{P}{4\pi r^2}} \quad I \propto f^2 A^2$$

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Equation of progressive wave

(i) $y=A\sin(\omega t-kx)$

(ii) $y=A\sin(\omega t-\frac{2\pi}{\lambda}x)$

(iii) $y=A\sin 2\pi\left[\frac{t}{T}-\frac{x}{\lambda}\right]$

(iv) $y=A\sin\frac{2\pi}{\lambda}(vt-x)$

(v) $y=A\sin\omega\left(t-\frac{x}{v}\right)$

The general equation of a plane progressive wave with initial phase is

$$y(x,t)=A\sin(\omega t \pm kx + \phi_0)$$

Displacement, Amplitude, Oscillating term phase, Angular frquency, Initial phase, Direction, wave number

Important Terms

A= Amplitude

λ= wave length

v= wave velocity

ω= Angular frequency

f= Frequency

T= Time period

K=Wave number

$\omega=\frac{2\pi}{T}$ or $2\pi f$

$K=\frac{2\pi}{\lambda}$

Rate of Energy Transmission

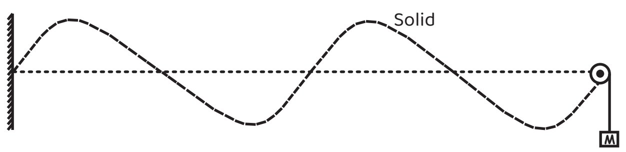
$$\boxed{\frac{dK}{dt}=\frac{dU}{dt}=\frac{1}{4}\mu V\omega^2 A^2}$$

Power transmitted by a wave

$$\boxed{\frac{1}{2}\mu V\omega^2 A^2}$$

μ=mass per unit length of string

Velocity of Transmission of wave in a string



$$\boxed{V=\sqrt{\frac{T}{\mu}}}$$

$$\boxed{V=\sqrt{\frac{T}{\rho\pi r^2}}}$$

$$\boxed{\frac{V_1}{V_2}=\sqrt{\frac{T_1}{T_2}}\times\frac{r_2}{r_1}}$$

T=Mg

$\mu=\frac{m}{l}$

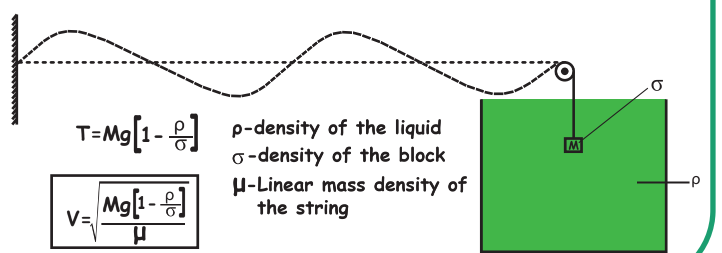
$\mu=\rho A$

M-Mass of block

m-mass of string

ρ-density of material of string

Case 1



$$T=Mg\left[1-\frac{\rho}{\sigma}\right]$$

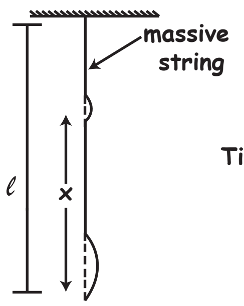
$$\boxed{V=\sqrt{\frac{Mg\left[1-\frac{\rho}{\sigma}\right]}{\mu}}}$$

ρ-density of the liquid

σ-density of the block

μ-Linear mass density of the string

Case 2



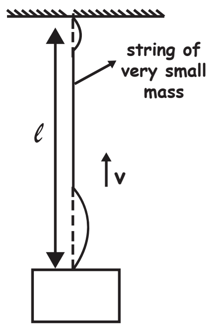
Velocity at any Point

$$\boxed{V=\sqrt{gx}}$$

Time taken by the pulse to reach the top

$$\boxed{t=2\sqrt{\frac{\ell}{g}}}$$

Case 3



Velocity, $v=\sqrt{\frac{T}{\mu}}$

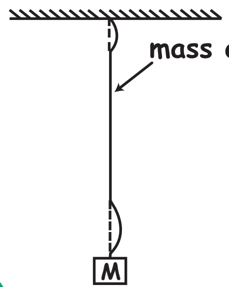
Time taken by the pulse to reach the top

$$t=\frac{\ell}{\sqrt{\frac{T}{\mu}}} = \ell\sqrt{\frac{\mu}{T}}$$

T= Tension in the string

μ= Linear mass density of the rope

Case 4



velocity at bottom

$$\boxed{V_1=\sqrt{\frac{Mg}{\mu}}}$$

velocity at Top

$$\boxed{V_2=\sqrt{\frac{(M+m)g}{\mu}}}$$

$$\frac{V_1}{V_2}=\frac{\lambda_1}{\lambda_2}=\sqrt{\frac{M}{M+m}}$$

Velocity of Longitudiual Wave

$V=\sqrt{\frac{E}{\rho}}$ (E=Elasticity of the medium; ρ=density of the medium)

(1) As solids are most elastic while gases are least, i.e. $E_S>E_L>E_G$; so the velocity of sound is maximum in solids and minimum in gases

Velocity Sound (air)

$\boxed{V=\sqrt{\frac{P}{\rho}}}$ — Newton

$\boxed{V=\sqrt{\frac{\gamma P}{\rho}}}$ — Laplace

$\gamma=\frac{C_P}{C_V}$

Mono atomic	γ= 5/3
diatomic	γ= 7/5

Factors affecting velocity of sound

→ Pressure

- Velocity of sound in air is independent of pressure

→ Density

$V\propto\frac{1}{\sqrt{\rho}}$

$\frac{V_1}{V_2}=\sqrt{\frac{\rho_2}{\rho_1}}$

→ Temp

$V=\sqrt{\frac{\gamma RT}{M}}$

$\frac{V_1}{V_2}=\sqrt{\frac{T_1}{T_2}}$

Temp Coefficient(α)

Increase in velocity of sound for 1°C or 1K rise in temperature of gas

Value of $\alpha = 0.608 \frac{m/s}{°C}$

$=0.61$

Humidity

Humidity ↑

Speed of sound ↑

Sound travels faster in moist air than in dry air

Relation between Δx and ΔΦ

$$\Delta\Phi=\frac{2\pi}{\lambda}\Delta x$$

Wave Motion 2

Reflection of Mechanical waves

Transverse wave
- Reflection from rigid end/denser medium → Phase change by π
- Reflection from free end/rarer medium → No phase change

Principle of superposition

The displacement at any time due to number of waves meeting simulatoneously at a point in a medium is the vector sum of individual displacements due each one of the waves at that point at same time

Superposition

- Interference
  - Constructive
  - Destructive
- Beats
- Standing wave
  - In a string
  - In an open pipe
  - In a closed pipe

Interference of sound wave

Condition:-

- Two waves of same frequency, same wavelength, same velocity
- Resultant intensities will be different from the sum of intensities of each wave seperately
- This is due to the interference of waves

$y_1 = a_1 \sin \omega t,\ y_2 = a_2 \sin(\omega t + \phi)$

ϕ-Phase difference between two waves

$\vec{y} = \vec{y}_1 + \vec{y}_2 \Rightarrow y = A \sin(\omega t + \theta)$

$A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

$\tan\theta = \dfrac{a_2\sin\phi}{a_1 + a_2\cos\phi}$

Intensity $\propto A^2$

$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\phi$

$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$\dfrac{I_1}{I_2} = \left(\dfrac{a_1}{a_2}\right)^2;\ \dfrac{I_{max}}{I_{min}} = \dfrac{(a_1 + a_2)^2}{(a_1 - a_2)^2} = \dfrac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$

i) For Constructive interference:-

$\phi = 0, 2\pi, 4\pi, ---$, OR $\phi = 2n\pi$; where $n = 0, 1, 2, ---$

$\Delta x = 0, \lambda, 2\lambda, ---$, OR $\Delta x = n\lambda$; where $n = 0, 1, 2, ---$

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

$= (\sqrt{I_1} + \sqrt{I_2})^2 \propto (A_1 + A_2)^2$

ii) For Destructive interference:-

$\phi = \pi, 3\pi, 5\pi, ---$ OR $\phi = (2n-1)\pi$; where $n = 1, 2, 3, ---$

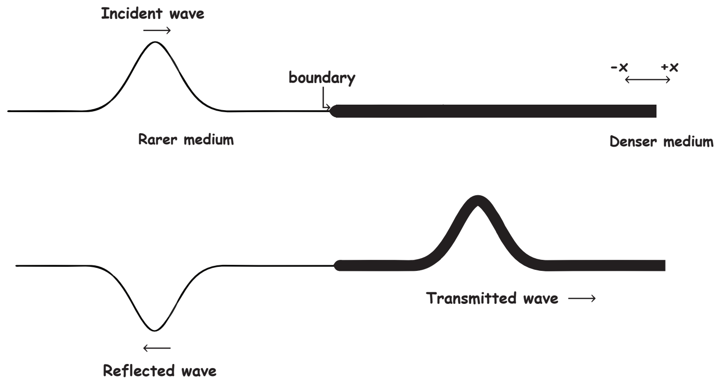
$\Delta x = \dfrac{\lambda}{2}, \dfrac{3\lambda}{2}, ---$ OR $\Delta x = (2n-1)\dfrac{\lambda}{2}$; where, $n = 1, 2, 3, -----$

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$\Longrightarrow I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2 \propto (A_1 - A_2)^2$

Waves on combination of strings

1) From rarer to denser medium



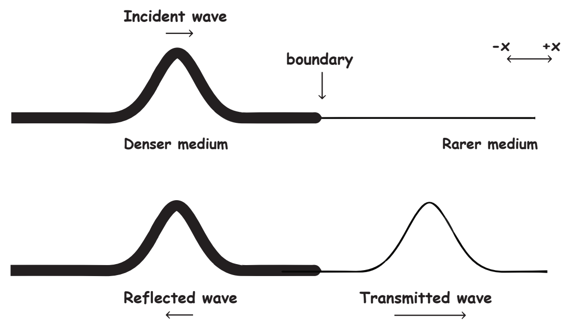
Incident wave $y_i = a_i \sin(\omega t - k_1 x)$

Reflected wave $y_r = a_r \sin(\omega t - k_1(-x) + \pi)$

$= -a_r \sin(\omega t + k_1 x)$

Transmitted wave $y_t = a_t \sin(\omega t - k_2 x)$

2) From rarer to denser medium



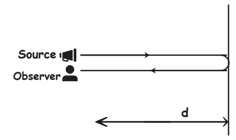
Incident wave $y_i = a_i \sin(\omega t - k_1 x)$

Reflected wave $y_r = a_r \sin(\omega t - k_1(-x) + o)$

$= a_r \sin(\omega t + k_1 x)$

Transmitted wave $y_t = a_t \sin(\omega t - k_2 x)$

Echo



Source at distance "d" from screen

$t = \dfrac{d}{v} + \dfrac{d}{v} = \dfrac{2d}{v}$

Persistence of hearing for human ear is 0.1 sec

Conditions for echo:

if $t > 0.1 \Rightarrow \dfrac{2d}{V} > 0.1 \Rightarrow d > \dfrac{V}{20}$

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Beats:-

- sound waves travelling in same medium with slightly different frequencies superimpose on each other.
- The intensity of resultant sound at particular position rises and falls regularly with time.
- The phenomenon of variation of intensity of sound with time at a particular position is called beats.

Point to remember:-

1) One beat:-

Maximum intensity (at t=0)
↓
Intensity becomes minimum
↓
Again acheives maximum

} One beat is formed

Beat period:-

Time interval between two sucessive beats (ie.two sucessive maximum of sound) is called beat period.

Beat frequency:-

No.of beats produced per second

Beat frequency:- $n = |n_1 - n_2|$

Beat period:- $T = \dfrac{1}{\text{Beat frequency}} = \dfrac{1}{|n_1 - n_2|}$

Determination of Unknown Frequency

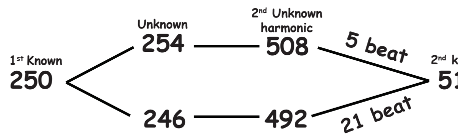
Let n_2 is the unknown frequency of tuning fork B, and this tuning fork B produce x beats per second with another tuning fork of known frequency n_1.
As number of beat/sec is equal to the difference in frequencies of two sources, therefore $n2 = n1 \pm x$

By waxing	By filing
If B is loaded with wax, its frequency decreases	If B is filed, its frequency increases

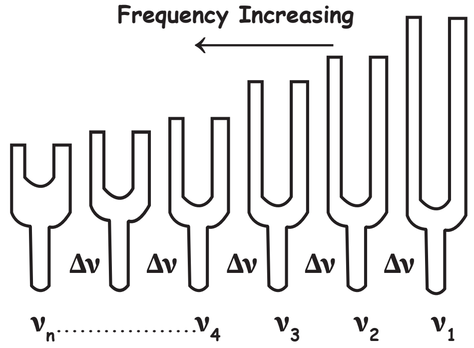
Q) A source of unknown frequency produces 4 beats/s when sounded with a source of known frequency 250 Hz. The second harmonic of the source of unknown frequency gives 5 beat/s when sounded with a source of frequency 513 Hz. The unknown frequency is?

a) 254 Hz b) 246 Hz c) 240 Hz d) 260 Hz

Solution:



Hence unknown frequency is 254 Hz



$\nu_n^{th} = \nu_1 + (n-1)\,\Delta\nu$

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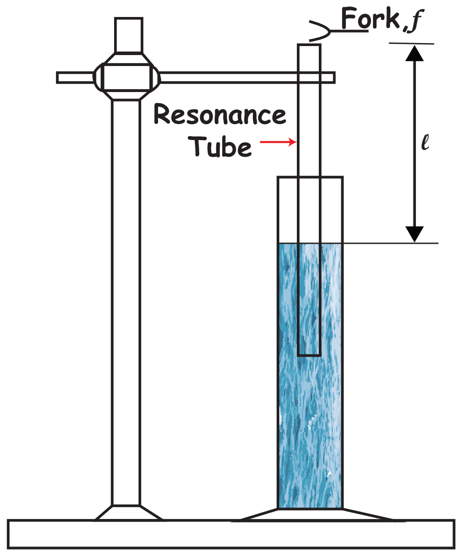
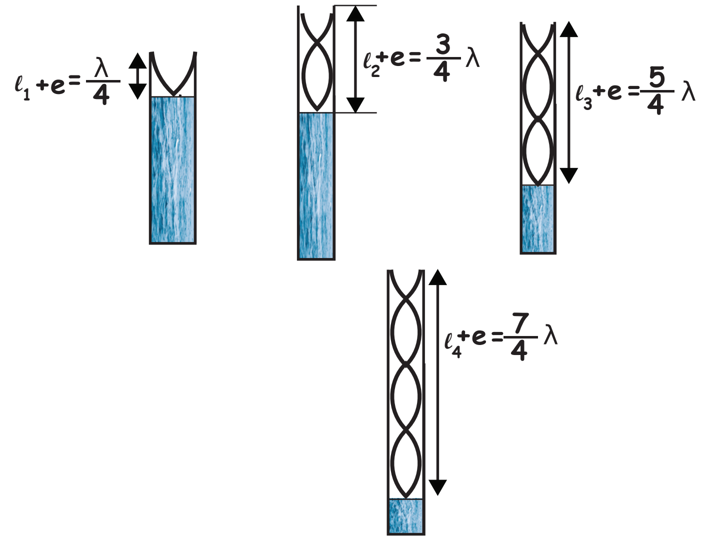
Resonance tube experiment



Speed of sound, $v = 2f(l_2 - l_1)$

End correction:-

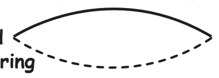
$e = \frac{1}{2}(\ell_2 - 3\ell_1)$



Standing Waves:

• When two progressive waves (both longitudinal and transverse) having same amplitude, time period, frequency moving along a straight line in opposite direction superpose, a new wave is formed. It is called stationary Or standing wave.

Both end fixed String



$\nu_1:\nu_2:\nu_3 = 1:2:3$

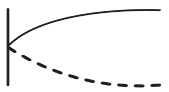
$y = 2a\sin(kx)\cos(\omega t)$

$\nu = \frac{n}{2\ell}\sqrt{\frac{T}{\mu}}, \ \ell = n \times \frac{\lambda}{2}$

$n = 1, 2, 3....$

No. of nodes = n+1
No. of Antinodes = n

One end fixed String



$\nu_1:\nu_2:\nu_3 = 1:3:5$

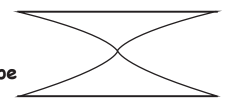
$y = 2a\sin(kx)\cos(\omega t)$

$\nu = \frac{n}{4\ell}\sqrt{\frac{T}{\mu}}, \ \ell = \frac{n\lambda}{4}$

$n = 1, 3, 5....$

• ℓ = length of string or air column

Open pipe



$\nu_1:\nu_2:\nu_3 = 1:2:3$

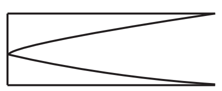
$y = 2a\cos(kx)\sin(\omega t)$

$\nu = \frac{n}{2\ell}\sqrt{\frac{\gamma P}{\rho}}, \ \ell = n \times \frac{\lambda}{2}$

$n = 1, 2, 3....$

No. of nodes = n+1
No. of Antinodes = n

Closed pipe



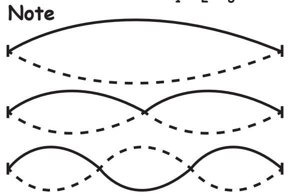
$\nu_1:\nu_2:\nu_3 = 1:3:5$

$y = 2a\sin(kx)\cos(\omega t)$

$\nu = \frac{n}{4\ell}\sqrt{\frac{\gamma P}{\rho}}, \ \ell = n \times \frac{\lambda}{4}$

$n = 1, 3, 5....$

Note



Distance between an adjacent node & antinode is $\frac{\lambda}{4}$

Phase difference between 2 particles at the nodes is 180° or π

Strain and pressure is maximum at node and minimum at antinode

Both end fixed string/open pipe	One end fixed string/closed pipe
n=1⇒ Fundamental frequency / 1st harmonic n=2⇒ First overtone/ 2nd harmonic n=3⇒ Second overtone / 3rd harmonic ⋮	n=1⇒ Fundamental frequency / 1st harmonic n=3⇒ First overtone/ 2nd harmonic n=5⇒ Second overtone / 3rd harmonic ⋮

Octave: The tone whose frequency is double the fundamental frequency is called as Octave.

(i) If $n_2 = 2n_1$ it means n_2 is an octave higher than n_1 or n_1 is an octave lower than n_2.

(ii) If $n_2 = 2^3 n_1$ it means n_1 is 3-octave higher than n_1 or n_1 is 3-octave lower than n_2.

(iii) Similarly if $n_2 = 2^n n_1$, it means n_2 is n-octave higher than n_1 or n_1 is n octave lower.

Unison: If the two frequencies are equal then vibrating bodies are said to be in unison.

Resonance: The phenomenon of making a body vibrate with it's natural frequency under the influence of another vibrating body having same frequency is called resonance.

Comparative Study of Stretched Strings, Open Organ Pipe and Closed Organ Pipe

S . NO	Parameter	Stretched string		Open organ pipe	Closed organ pipe
		Both ends fixed	one ends fixed		
1	Fundamental frequency or 1st harmonic	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{4l}$	$n_1 = \frac{v}{2l}$	$n_1 = \frac{v}{4l}$
2	Frequency of or 2nd harmonic	$n_2 = 2n_1$ 1st overtone	$n_2 = 3n_1$ 1st overtone	$n_2 = 2n_1$ 1st overtone	$n_2 = 3n_1$ 1st overtone
3	Frequency of or 3rd harmonic	$n_3 = 3n_1$ 2nd overtone	$n_3 = 5n_1$ 2nd overtone	$n_3 = 3n_1$ 2nd overtone	$n_3 = 5n_1$ 2nd overtone
4	Frequency ratio of overtones	2:3:4.....	3:5:7.....	2:3:4.....	3:5:7.....
5	Frequency ratio of harmonics	1:2:3:4.....	1:3:5:7.....	1:2:3:4.....	1:3:5:7.....
6	Nature of waves	Transverse stationary	Transverse stationary	Longitudinal stationary	Longitudinal stationary

Wave Motion 3

Relation between loudness and intensity

$L \propto \log_{10}$ (Intensity)

unit(dB) unit W/m²

$I_0 = 10^{-12}\,W/m^2$

$dB = 10 \times \log_{10}\frac{I}{I_0}$

I_0 = Threshold intensity

$L_1 = 10 \times \log_{10}\frac{I_1}{I_0}$ $I_1 \longrightarrow L_1$

$L_2 = 10 \times \log_{10}\frac{I_2}{I_0}$ $I_2 \longrightarrow L_2$

$L_2 - L_1 = 10\left[\log\left(\frac{I_2}{I_0}\right) - \log\left(\frac{I_1}{I_0}\right)\right]$ ΔL = change in loudness

$\Delta L = 10\,\log\left(\frac{I_2}{I_1}\right)$

Doppler Effect

Whenever there is a relative motion between a source of sound and the listener, the apparent frequency of sound heard by the listener is different from the actual frequency of sound emitted by the source.

General equation
(when both source & listener are moving)

$$\nu' = \left(\frac{V \pm V_L}{V \mp V_s}\right)\nu$$

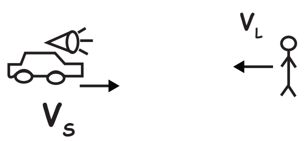
Case 1
(listener is stationary & source is approaching the listener)

$$\nu' = \left(\frac{V}{V - V_s}\right)\nu$$

Case 2
(The source is stationary & listener is approaching the source)

$$\nu' = \left(\frac{V + V_L}{V}\right)\nu$$

Case 3
(source & listener are approaching each other)



$$\nu' = \left(\frac{V + V_L}{V - V_s}\right)\nu$$

Case 4
(source is stationary, listener is moving away from the source)

$$\nu' = \left(\frac{V - V_L}{V}\right)\nu$$

Case 5
(source is moving away from the listener, listener is stationary)

$$\nu' = \left(\frac{V}{V + V_s}\right)\nu$$

Case 6
(source and listener moving in same direction)

$$\nu' = \nu\,\frac{V + V_L}{V + V_s}$$

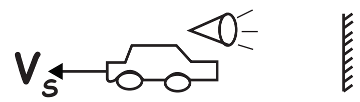
Case 7
(source and listener moving away from each other)

$$\nu' = \left(\frac{V - V_L}{V + V_s}\right)\nu$$

Case 8
(source approaching a stationary wall)

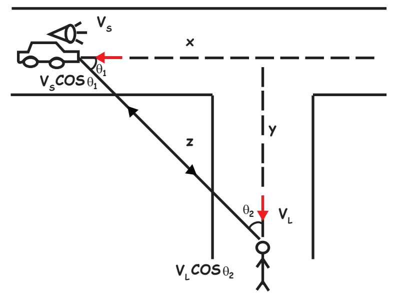
$$\nu' = \frac{V + V_s}{V - V_s}\,\nu$$

Case 9
(source is moving away from stationary wall)



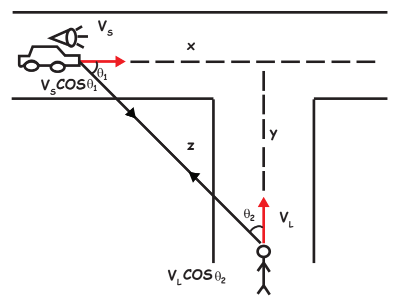
$$\nu' = \left(\frac{V - V_s}{V + V_s}\right)\nu$$

Case 10



$$\nu' = \left(\frac{V - V_L\cos\theta_2}{V + V_s\cos\theta_1}\right)\nu$$

Case 11

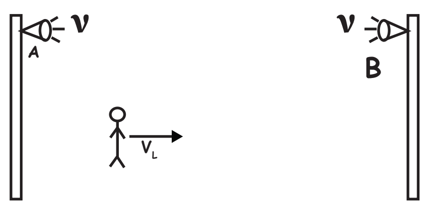


$$\nu' = \left(\frac{V + V_L\cos\theta_2}{V - V_s\cos\theta_1}\right)\nu$$

$$\cos\theta_1 = \frac{x}{\sqrt{x^2+y^2}}$$

$$\cos\theta_2 = \frac{y}{\sqrt{x^2+y^2}}$$

Note:



$$\nu'_A = \left(\frac{V - V_L}{V}\right)\nu \qquad \nu'_B = \left(\frac{V + V_L}{V}\right)\nu$$

Beat frequency $(\Delta\nu) = \nu'_B - \nu'_A$

$$= \frac{\nu}{V}[V + V_L - V + V_L]$$

$$\Delta\nu = \frac{2V_L\nu}{V}$$