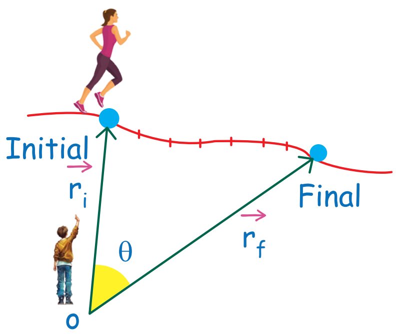
5 Circular Motion

पूरी mechanics मे circular motion सबसे ज्यादा important chapter है जिसका use WPE, COM, Rotation, Electrostatic, Magnetism etc. मे होगा...... बहुत ही आराम से सुकुन से basic fundamental and definition अच्छे से पढ़ना क्योंकि ये आपका base ready करेगा..... are you ready

Let us feel a beautiful girl is running infront of you and you are watching her.... अबे feel कर ना.... बोले तो

Let a particle is moving in a random curved path as shown in fig. and we are observing it from O.



SKC

Bhai teri mundi kitna angle ghumi is called angular displacement kis rate se ghumi is called angular velocity of radius vector.

★ Angular position is the angle (θ) made by the position vector of a point (measured from a fixed origin) with respect to a chosen reference line or direction.

★ $\theta \rightarrow$ angular displacement $\Rightarrow$ angle rotated by radius vector or (position vector).

★ Difference between the two angular positions of the particle moving along any arbitrary path w.r.t. some fixed point is called angular displacement.

or

★ Angular velocity $(\omega) = \frac{d}{d}$ = Rate of change of angular position (for radius vector).

★ Angular acc. 'α' = Rate of change of angular velocity $\frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$

★ Angular variables: θ, ω, α depend on observer.

Average Angular Speed

It is the ratio of total angular displacement and the time taken to do so.

$$\omega_{av} = \frac{\text{Total angular displacement}}{\text{Total time taken}} = \frac{\Delta\theta}{\Delta t}$$

Instantaneous Angular Velocity

It is the rate of change of angular position at some particular instant.

Instantaneous angular velocity

$$\vec{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\theta}}{\Delta t} = \vec{\omega} = \frac{d\vec{\theta}}{dt}$$

★ Instantaneous angular velocity can also be called simply as angular velocity.

Q. If angular displacement is given as $\theta = 2t^2 + 3$ where 't' is time then

(i) Find out average angular speed upto 3 sec.

(ii) Angular velocity, angular acceleration at 3 sec.

Sol. (i) $\omega_{avg} = \frac{\text{Total angular displacement}}{\text{total time}} = \frac{\theta_f - \theta_i}{t_2 - t_1}$

$\theta_f = 2\,(3)^2 + 3 = 21$ rad (at t = 3 sec)

$\theta_i = 2\,(0) + 3 = 3$ rad (at t = 0 sec)

So, $\omega_{avg} = \frac{21-3}{3} = 6$ rad/sec

(ii) $\omega_{instantaneous} = \frac{d\theta}{dt} = 4t$

$\omega_{at\ t = 3\ sec} = 4 \times 3 = 12$ rad/sec

$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = 4$ rad/sec^2

भाई θ, ω, α के बीच बिलकुल वही relation है जो kinematics मे x, v, a के बीच हुआ करता था.....

$x \longrightarrow \theta$

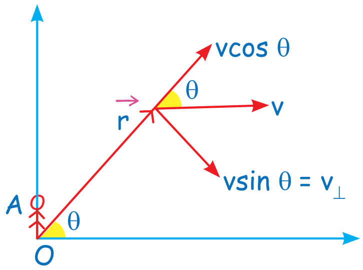
$v = \frac{dx}{dt} \longrightarrow \omega = \frac{d\theta}{dt}$

$a = \frac{dv}{dt} \longrightarrow \alpha = \frac{d\omega}{dt}$

सर मै तो circular motion पढ़ने आया था वहाँ पर θ, α, ω होता था लेकिन आपने तो यहाँ पर किसी random path के लिए चाहे वो straight line हो उसके लिए θ, α, ω define कर दिए

बेटा बिलकुल कर सकते है ये समझ ले कि circular motion इन सबका special case ही तो है

★ Suppose a particle is moving parallel to x-axis on xy plane with constant velocity $\vec{v}$ if observer is at origin as shown in fig. below. Then angular velocity of radius vector w.r.t origin is given by



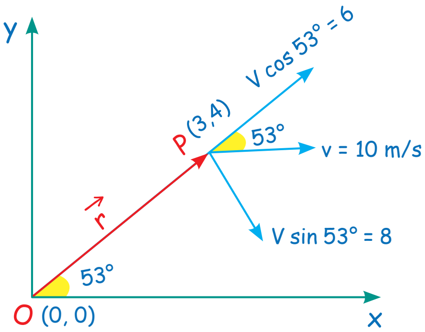
$\frac{d\theta}{dt} = \omega$ = ang.vel.of Position vector (P.V)

$$\omega = \frac{v\sin\theta}{r} = \frac{v_\perp}{r}$$

Magnitude

Q. A particle is moving with constant velocity 10 m/s along positive x-axis. Find ω of radius vector about origin when particle is at (3, 4).

Sol.

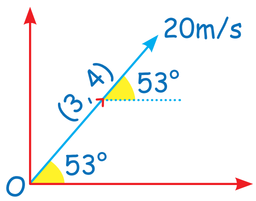


$(\omega_p)_{abt\ 'O'} = \frac{V\sin\theta}{r} = \frac{8}{5}$ rad/s

$$\omega = \frac{\vec{r} \text{ के } \perp^r \text{ वाली vel.}}{r}$$

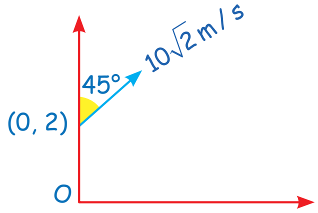
Q. Find ω of radius vector about 'O' in following question.

(a)



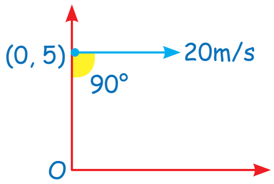
Sol. $\omega = 0$

(b)



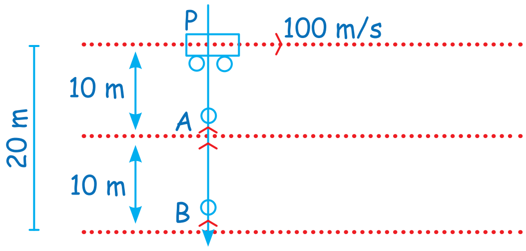
Sol. $\omega = \frac{10}{2} = 5 \text{ rad/s}$

(c)



Sol. $\omega = \frac{20}{5} = 4$ rad/s

(d)

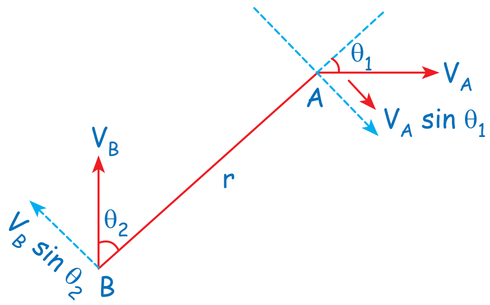


Sol. $(\omega_P)_{wrt}$ 'A' $= \frac{100}{10} = 10 \text{ rad/s}$

$(\omega_P)_{wrt}$ 'B' $= \frac{100}{20} = 5 \text{ rad/s}$

Calculation of $\omega_{A/B}$ When A and B Both are Moving

Relative angular velocity of a particle 'A' with respect to the other particle 'B' is the angular velocity of the position vector of 'A' with respect to 'B'. Which means that it is the rate at which position vector of 'A' with respect to 'B' rotates at that instant.



$$\omega_{AB} = \frac{(V_{AB})_\perp}{r_{AB}}$$

Here $(V_{AB})_\perp$ = Relative velocity $\perp$ to position vector AB

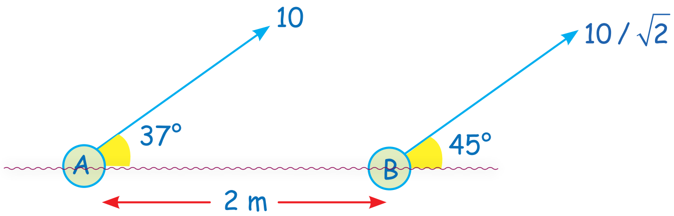
r_{AB} = separation between A and B.

$$(V_{AB})_\perp = V_A \sin\theta_1 + V_B \sin\theta_2$$

$$r_{AB} = r$$

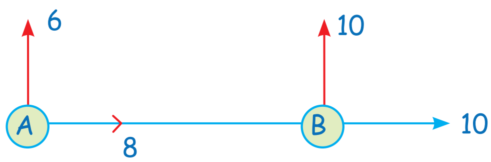
$$\omega_{AB} = \frac{V_A \sin\theta_1 + V_B \sin\theta_2}{r}$$

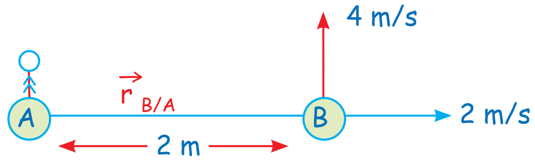
Q. Calculate angular velocity of B wrt A.



$\omega_{B/A}$ = ?

Sol. Breaking in components





$\vec{\omega}_{B/A} = \frac{4}{2} = 2(ACW) = 2\hat{k}$

SKC

* for $\omega_{B/A}$

→ Sabse pehle dono particle Ko ek line se jor do.

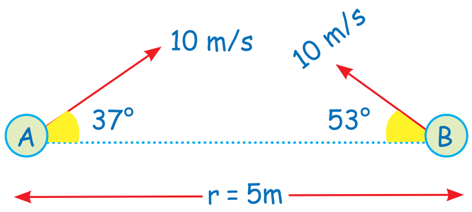
→ Dono vel. Ko, line Ki taraf aur line K $\perp^{ar}$ tod lo.

→ Kisi ek particle ke uper jake beth jao [relative]

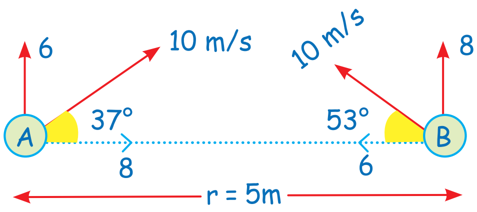
→ Ghumane wali vel. / r [divide] kro.

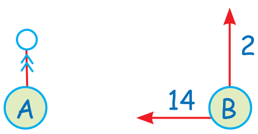
Q. Find $\omega_{B/A}$

(a)



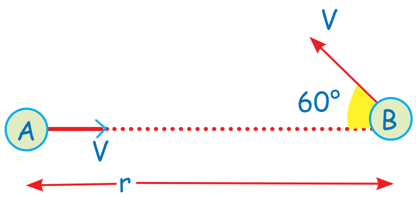
Sol.



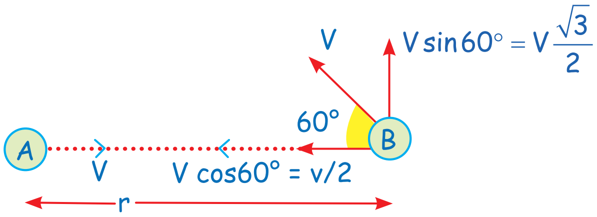


$\omega_{B/A}$ = 2/5 (ACW)

(b) Find $\omega_{B/A}$



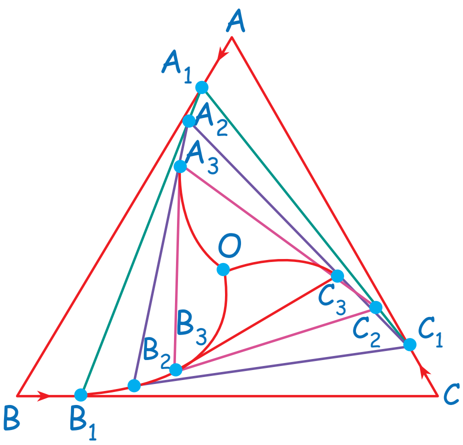
Sol.



$$\omega_{B/A} = \frac{V\sqrt{3}/2}{r}$$

Q. Three particles A, B and C are situated at the vertices of an equilateral triangle ABC of side ℓ at t = 0. Each of the particles moves with constant speed v. A always has its velocity along AB, B along BC and C along CA. At what time will the particles meet each other?

Sol. Roughly motion of the particles will be like as shown in fig. and by symmetry they will meet at the centroid.



Velocity of A is v along AB. The velocity of B is along BC. Hence rate of decrease of separation

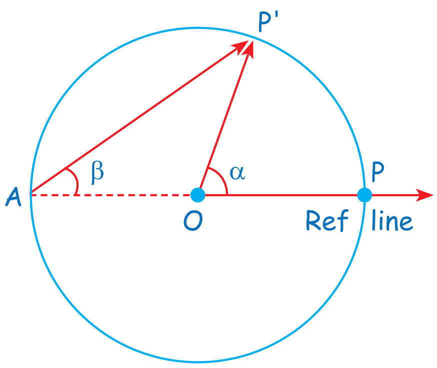
$= v + v\cos 60^\circ = \frac{3v}{2}$ = constant

Time taken to reduce the separation from ℓ to zero.

$$t = \frac{\ell}{\frac{3v}{2}} = \frac{2\ell}{3v}$$

Relative Angular Velocity

Angular velocity is defined with respect to the point from which the position vector of the moving particle is drawn. Here angular velocity of the particle w.r.t. 'O' and 'A' will be different.



$$\omega_{PO} = \frac{d\alpha}{dt} = \omega \text{ (say);}$$

$$\omega_{PA} = \frac{d\beta}{dt} = \frac{\omega}{2} \quad \left[\because \beta = \frac{\alpha}{2}\right]$$

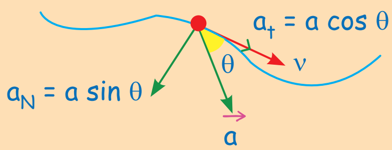
NORMAL ACCELERATION AND TANGENTIAL ACCELERATION

Suppose a particle is moving in a random curved path with variable speed then

Component of acceleration towards the velocity is called tangential acceleration and component acceleration perpendicular to the velocity is called normal acceleration.

काम का डब्बा

Tangential Acceleration



★ Component of $\vec{a}$ along vel. = acosθ

= a_t = Tangential Acceleration

★ $a_t = a\cos\theta = \dfrac{\vec{a}\cdot\vec{v}}{v}$ (Magnitude)

★ In vector form

$\vec{a}_t = a\cos\theta\,\hat{v} = \dfrac{\vec{a}\cdot\vec{v}}{v}\hat{v}$

★ It is responsible to change the magnitude of $\vec{v}$

$\dfrac{d(\text{speed})}{dt} = a_t$	Rate of change of speed is tangential acceleration.

★ If $\vec{a}_t$ is in the direction x[n] of $\vec{v}$ ⇒ speed up

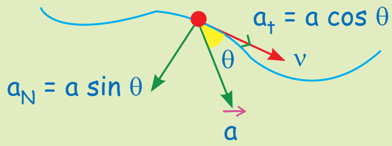
★ If $\vec{a}_t$ is in the opp. direction of $\vec{v}$ ⇒ speed down.

★ If speed is constant ⇒ $a_t = 0$

Normal Acceleration

★ Component of acceleration perpendicular to the velocity.

★ It is responsible to change the direction of velocity.



★ Normal acceleration = $a_N = a\sin\theta$

★ $\vec{a}_N = \vec{a} - \vec{a}_t$ (very important)

SKC

★ Agar particle kisi curve par chal rha ho to, uski vel. Ki dir[n] nikalne ke liye us curve per tangent Khicho.

★ Agar speed const. hai to tangential acc. O hoga, chaye Kesa bhi motion ho [1D, 2D, 3D]

★ Agar speed badli to tangential acc hai. Agar dir[n] badli matlb normal acc. hai

Q. If a particle is moving such that it's acceleration velocity is given by

$\vec{a} = 4\hat{i} - 3\hat{j}$

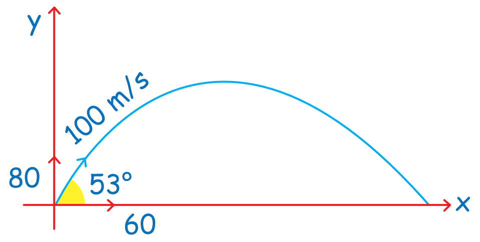
$\vec{v} = \hat{i} + \hat{j}$ Find a_t and a_N

Sol. $\vec{a}_t = \dfrac{\vec{a}\cdot\vec{v}}{v}\hat{v} = \left(\dfrac{(4\hat{i}-3\hat{j})\hat{i}+\hat{j}}{\sqrt{2}}\right)\cdot\dfrac{(\hat{i}+\hat{j})}{\sqrt{2}} = \dfrac{\hat{i}+\hat{j}}{2}$

$\vec{a}_N = \vec{a} - \vec{a}_t = (4\hat{i} - 3\hat{j}) - \dfrac{\hat{i}+\hat{j}}{2} = \dfrac{7\hat{i}-7\hat{j}}{2}$

भाई आगे बढ़ने से पहले तुझे यह promise करना पड़ेगा की अगर तुम्हे velocity, acc vector form मे दिए गए तो formula मे value put करके a_t और a_N निकाल लोगे। अगर यहाँ अच्छे से practice नही की तो पता है ना exam मे क्या होगा।

Q. A particle is projected with velocity 100 m/s at an angle 53° with horizontal as shown in fig. Find $\vec{a}_t$ & $\vec{a}_N$ at t = 2 sec.



Sol. t = 2, $\vec{v} = 60\hat{i} + 60\hat{j}$

$\vec{a} = -10\hat{j}$

$\vec{a}_t = \frac{\vec{a}.\vec{v}}{v}.\hat{v}$

$= \frac{-600}{60\sqrt{2}} \cdot \frac{60\hat{i}+60\hat{j}}{60\sqrt{2}}$

$\vec{a}_N = \vec{a} - \vec{a}_t$

Now you can solve.

Q. If $\vec{V} = 3t^2\hat{i} + 4t\hat{j}$ at t = 1 sec find a_t & a_N.

Sol. $\vec{v} = 3t^2\hat{i} + 4t\hat{j}$, t = 1, $\vec{v} = 3\hat{i} + 4\hat{j}$

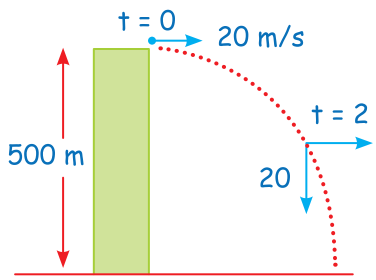
$\vec{a} = 6t\hat{i} + 4\hat{j}$, t = 1, $\vec{a} = 6\hat{i} + 4\hat{j}$

$\vec{a}_t = \frac{(18+16)\hat{v}}{5} = \frac{34}{5}\left(\frac{3\hat{i}+4\hat{j}}{5}\right)$

$\vec{a}_N = \vec{a} - \vec{a}_t = 6\hat{i} + 4\hat{j} - 34/5 \cdot \left(\frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}\right)$

Now you can solve

Q. Find tangential acc. at t = 2 sec.

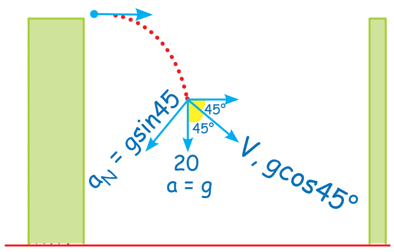


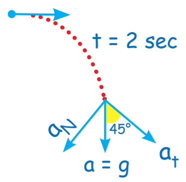
Sol. $\vec{v} = 20\hat{i} - 20\hat{j}$

$\vec{a} = -10\hat{j}$

$\vec{a}_t = \frac{\vec{a}.\vec{v}}{v}\hat{v} = \frac{0+200}{20\sqrt{2}} \times \left(\frac{20\hat{i}-20\hat{j}}{20\sqrt{2}}\right) = 5\hat{i} - 5\hat{j}$

OR





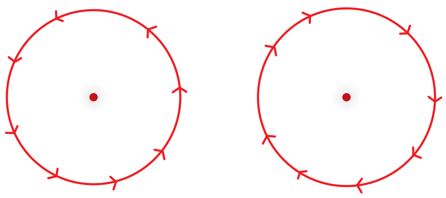
$a_t = g \cos 45° = \frac{10}{\sqrt{2}} = m/s^2$

तो चलो अब शुरू करते है Circular motion.... यही है ना... जिस के लिए पढ़ रहे थे। हम पहले ही बोले दे रहे है कि ये सबसे important chapter है जो WPE, COM, Rotation, Fluid, Electrostatic, Magnetism etc. सभी जगह use होगा।

CIRCULAR MOTION

★ When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point. That fixed point is called centre and the distance is called radius of circular path.

★ Direction of $\vec{\omega}$ is given by right hand thumb rule.

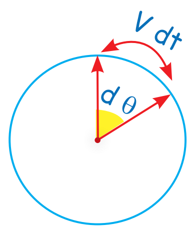


ACW (Sense) — direction of $\vec{\omega}$ = ⊙$\hat{k}$, $\perp^r$ outward

CW (Sense) — direction of ω = ⊗, $-\hat{k}$, $\perp^r$ inwards

★ $\theta \rightarrow$ angular displacement For a complete revolution angular disp. is 2π but disp. = 0

★ $\omega = \frac{d\theta}{dt}$, $\alpha = \frac{d\omega}{dt}$



Uniform Circular Motion

- ★ Motion of a particle along the circumference of a circle with a constant speed is called uniform circular motion.
- ★ AngularSpeed, (ω), constant
- ★ $a_t = 0$, $\alpha = 0$
- ★ It is an accelerated motion because direction of velocity vector keeps on changing.
- ★ The position vector and velocity vector keep on changing, though their magnitudes remain constant.
- ★ The acceleration of the particle points always towards center at each point on its path. Which is normal acc. or centripetal acc. Whose value is v^2/r, or $r\omega^2$
- ★ $\vec{r}\cdot\vec{v} = 0$ ★ $\vec{v}\cdot\vec{a} = 0$
- ★ $\omega = \dfrac{v}{R}$ (magnitude)
- ★ $\boxed{v = R\omega}$ $\boxed{\vec{v} = \vec{\omega} \times \vec{r}}$
- ★ For 1 complete revolution

  Speed = Dist./time = $2\pi R/T = R\omega$

  $\omega = 2\pi/T$

Non-Uniform Circular Motion

Non-uniform circular motion

If the speed of the particle moving in a circle is not constant, the acceleration has both the radial and the tangential components.

Examples of non uniform circular motion

A merry-go-round spinning up from rest to full speed, or a ball whirling around in a vertical circle.

In non-uniform circular motion

Speed $|\vec{v}| \neq$ constant

angular velocity $\omega \neq$ constant

If at any instant

v = magnitude of velocity of particle

ω = angular velocity of particle,

then, at that instant $v = r\omega$

where r = radius of circular path

भाई θ, ω, α के बीच बिलकुल वही relation है जो kinematics मे x, v, a के बीच हुआ करता था.....

$$x \longrightarrow \theta$$

$$v = \frac{dx}{dt} \longrightarrow \omega = \frac{d\theta}{dt}$$

$$a = \frac{dv}{dt} \longrightarrow \alpha = \frac{d\omega}{dt}$$

- ★ $\vec{v}, \vec{a} \Rightarrow$ Same sign (speed up)
- ★ $\vec{\omega}, \vec{\alpha} \Rightarrow$ Same sign (speed up)

भाई रूक पहले इन्हे detail मे अच्छे से digest कर

Linear Kinematics	Circular Motion Kinematics
$v = \dfrac{dx}{dt}$	$\omega = \dfrac{d\theta}{dt}$
$\vec{a} = \dfrac{d\vec{v}}{dt}$	$\vec{\alpha} = \dfrac{d\vec{\omega}}{dt}$
(x-t) $\Rightarrow$ slope $\Rightarrow$ vel.	(θ-t) $\Rightarrow$ slope $\Rightarrow$ ω
(v-t) $\Rightarrow$ slope $\Rightarrow$ acc.	(ω-t) $\Rightarrow$ slope $\Rightarrow$ α
(a-t) $\Rightarrow$ area $\Rightarrow$ change in velocity	(α-t) $\Rightarrow$ area $\Rightarrow$ ω
(v-t) $\Rightarrow$ area $\Rightarrow$ disp.	(ω-t) $\Rightarrow$ area $\Rightarrow$ $d\theta$

$$x \xrightarrow{\text{diff}} v \xrightarrow{\text{diff}} a$$

Integrate Integrate

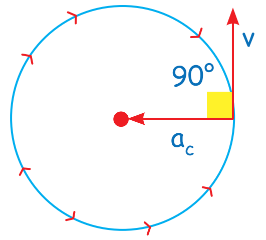
If $a \to$ const.		If $\alpha \to$ const
$v = u + at$	$s \to \theta$	$\omega = \omega_i + \alpha t$
$S = ut + 1/2\, at^2$	$a \to \alpha$	$\theta = \omega_i t + 1/2\, \alpha t^2$
$v^2 = u^2 + 2as$	$v \to \omega$	$\omega^2 = \omega_i^2 + 2\alpha\theta$

CENTRIPETAL ACCELERATION (NORMAL ACC.)

Component of acc. $\perp^r$ to the vel. If at any instant speed of particle performing circular motion is v then

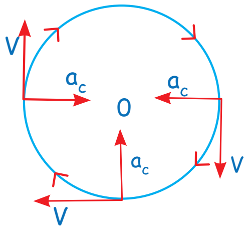
Centripetal acc. (a_c)

$= \dfrac{v^2}{R}$ (toward centre)



Q. A particle is moving in a circular path of radius 4 m with speed 10 m/s in clockwise sense in x-y plane. Find ω, a_t, a_c

Sol. $\omega = \dfrac{V}{R}$



$\omega = 10/4$ or $\vec{\omega} = -2.5\,\hat{k}$,

$a_t = 0$

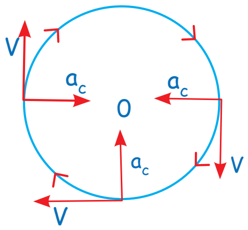
$a_t = R\alpha \Rightarrow \alpha = 0$

$a_c = \dfrac{V^2}{R} = \dfrac{100}{4} = 25$ (towards centre)

Next ques. is very important ques.
अच्छे से एक-एक चीज निकालना सीख लेना।

Q. A particle is moving in a circular path of radius 2 m such that is speed changes as $v = 2t^2$. Find ω, α, a_t, a_c at t = 2s and angle made by a_{net} with velocity at t = 2s.

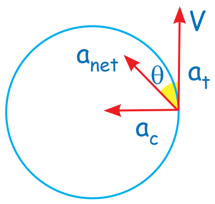
Sol. $a_t = \dfrac{d}{dt}(\text{speed}) = 4t$



at t = 2 $\Rightarrow$ v = 2 × 4 = 8 m/s

$\omega = \dfrac{V}{R} = \dfrac{8}{2} = 4\ \text{rad/s}$

$a_c = \dfrac{v^2}{R} = \dfrac{8^2}{2} = 32\ \ m/s^2$



a_{net} = resultant of a_t & a_c

$= \sqrt{a_t^2 + a_c^2} = \sqrt{8^2 + 32^2}$

$\tan\theta = \dfrac{a_c}{a_t} = \dfrac{32}{8} = 4$

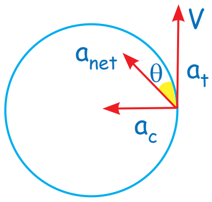
SKC

Agar particle circular motion Kar raha hai to a_c hoga hi hoga. Aur uski speed badal rehi hae to a_t hoga hi hoga. If speed up $\Rightarrow a_t \Rightarrow \vec{v}$ की side If speed down $\Rightarrow a_t \Rightarrow \vec{v}$ ke opposite side. Jab bhi Kabhi circular motion me acceleration puchega matlab net acc. puch rha h.

Q. A particle is rotating in a circular path of radius 2 m such that angle rotated by (radius vector) is given as $\theta = t^3/6$.

Find $\omega, \alpha, a_t, a_c, a_{net}$, v at t = 2 sec.

Sol. $\theta = \dfrac{t^3}{6}$



$\omega = \dfrac{d\theta}{dt} = \dfrac{3t^2}{6} = \dfrac{t^2}{2}$

$\alpha = \dfrac{2t}{2} = t$

$\tan\theta = \dfrac{a_c}{a_t}$

$t = 2, \omega = 2, \alpha = 2$

$a_t = R\alpha$

$a_t = 4$

$a_c = R\omega^2 = 2 \times 4 = 8\ m/s^2$

$a_{net} = \sqrt{4^2 + 8^2}$

$v = R\omega$

$v = R\dfrac{t^2}{2} = 2 \times \dfrac{4}{2} = 4$

Q. A particle travels in a circle of radius 20 cm at a speed that uniformly increases. If the speed changes from 5 m/s to 6 m/s in 2 s, find the angular acceleration.

Sol. The tangential acceleration is given by

$a_t = \dfrac{dv}{dt} = \dfrac{v_2 - v_1}{t_2 - t_1}$

$= \dfrac{6.0 - 5.0}{2}\ m/s^2 = 0.5\ m/s^2$

The angular acceleration is $\alpha = a_t / r$

$= \dfrac{0.5\ m/s^2}{20\ cm} = 2.5\ rad/s^2$

Q. A particle is moving in a circular path with velocity varying with time as $v = 1.5t^2 + 2t$. If the radius of circular path is 2 cm, the angular acceleration at t = 2 sec will be

Sol. Given $v = 1.5\,t^2 + 2t$

Tangential acceleration $a_t = \dfrac{dv}{dt} = 3t + 2$

Now angular acceleration at time t:

$\alpha = \dfrac{a_t}{r} \Rightarrow \alpha = \dfrac{3t + 2}{2 \times 10^{-2}}$

Angular acceleration at t = 2 sec

$(\alpha)_{at\ t\ =\ 2sec} = \dfrac{3 \times 2 + 2}{2 \times 10^{-2}} = \dfrac{8}{2} \times 10^2 = 4 \times 10^2$

= 400 rad/sec^2

Q. The shaft of an electric motor starts from rest and on the application of a torque, it gains an angular acceleration given by $\alpha = 3t - t^2$ during the first 2 seconds after it starts after which $\alpha = 0$. The angular velocity after 6 sec will be

Sol. Given $\alpha = 3t - t^2$

$$\Rightarrow \frac{d\omega}{dt} = 3t - t^2 \Rightarrow d\omega = (3t - t^2)dt$$

$$\Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} + c$$

at $t = 0$, $\omega = 0$

$$\therefore c = 0 \Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3}$$

Angular velocity at $t = 2$ sec,

$$(\omega)_{t=2\text{ sec}} = \frac{3}{2}(4) - \frac{8}{3} = \frac{10}{3} \text{ rad/sec}$$

Since there is no angular acceleration after 2 sec

$\therefore$ The angular velocity after 2 sec remains the same.

Q. Two particles *A* and *B* move anticlockwise with the same speed v in a circle of radius *R* and are diametrically opposite to each other. At $t = 0$, *A* is imparted a tangential acceleration of constant magnitude $a_t = \frac{72v^2}{25\pi R}$. Calculate the time in which *A* collides with *B*, the angle traced by *A* during this time, its angular velocity and radial acceleration of *A* at the time of collision.

Sol. $\omega_{0(rel)} = 0, \theta_{rel} = \pi, \alpha_{rel} = \frac{72v^2}{25\pi R^2}$

$$\theta_{rel} = \omega_{0(rel)}t + \frac{1}{2}\alpha_{rel}t^2$$

$$\pi = 0 + \frac{1}{2}\frac{72v^2}{25\pi R^2}t^2 \Rightarrow t = \frac{5\pi R}{6v}$$

Angle traced by *A*, $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$$\theta = \frac{v}{R}\cdot\frac{5\pi R}{6v} + \frac{1}{2}\frac{72v^2}{25\pi R^2}\cdot\left(\frac{5\pi R}{6v}\right)^2$$

$$= \frac{5\pi}{6} + \pi = \frac{11}{6}\pi$$

Angular velocity $\omega = \omega_0 + \alpha t = \frac{v}{R} + \frac{72v^2}{25\pi R^2}\cdot\left(\frac{5\pi R}{6v}\right)$

$$= \frac{v}{R} + \frac{12v}{5R} = \frac{17v}{5R}$$

$$a_c = \omega^2 R = \left(\frac{17v}{5R}\right)^2 R = \frac{289v^2}{25R}$$

UNIFORM AND NON-UNIFORM CIRCULAR MOTION

Uniform Circular Motion	Non-uniform Circular Motion
Speed $\rightarrow$ const.	Speed $\rightarrow$ change up (speed or down)
$\omega \rightarrow$ const.	$\omega \rightarrow$ change
$a_{net} = \sqrt{a_t^2 + a_c^2}$	$a_t \neq 0, \alpha \neq 0$
$a_{net} = a_c = \frac{V^2}{R} = R\omega^2$	$a_{net} = \sqrt{a_t^2 + a_c^2}$

DYNAMICS OF CIRCULAR MOTION

SKC

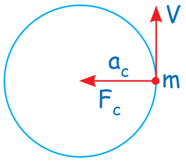
अब जो article हम पढ़ने जा रहे है बहुत ही interesting है अब तक हमने देखा कोई particle अगर circular motion कर रहा है तो उसके पास a_c होता है तो भाई ये a_c ऐसे थोड़े ही अपने आप पैदा हो जाएगा..... अबे कोई तो force particle पर लग रहे होंगे जिन्होंने a_c centripetal acc. दिया है some of all these forces towards the centre is called centripetal force.

Centripetal Force

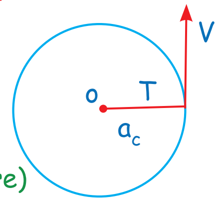
$a_c = \frac{V^2}{R} = R\omega^2$ (toward the centre)

Centripetal force $= ma_c = \frac{mv^2}{R} = mR\omega^2$

Centripetal force लगने वाला force नही है So, block पर centripetal force centre की तरफ लगता है जैसे गधो वाले sentence use मत करना...... just say centripetal force is the vector sum of all the forces towards the centre acting on the particle performing circular motion.



Q. A particle is moving on a smooth horizontal floor in a circular path with const. speed v, as shown in diagram find Tension

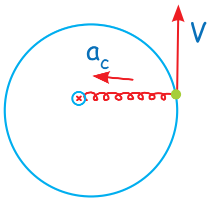


Sol. mg = $\perp^r$ inside

N = $\perp^r$ outside

$T = ma_c = \dfrac{mv^2}{R}$ (Towards centre)

Q. A particle is moving on a smooth horizontal floor in a circular path with const. speed v, as shown in digram. If natural length of spring is l_o find elongation in spring.

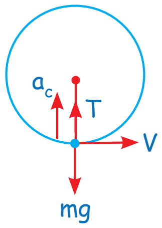


$R = l_o + x$

Sol. $Kx = \dfrac{mv^2}{R} = \dfrac{mv^2}{l_o + x}$

x : elongation

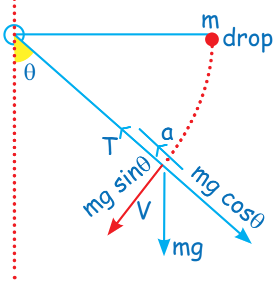
Q. A particle is performing circular motion in vertical plane such that its vel. at lowest point is v. Find tension at lowest point.



Sol. $T - mg = m \times a_c = \dfrac{mv^2}{r}$

$a_t = 0$

Q. A particle is released when string is horizontal as shown in fig. Find tension when string makes angle θ with verticle and having speed V.

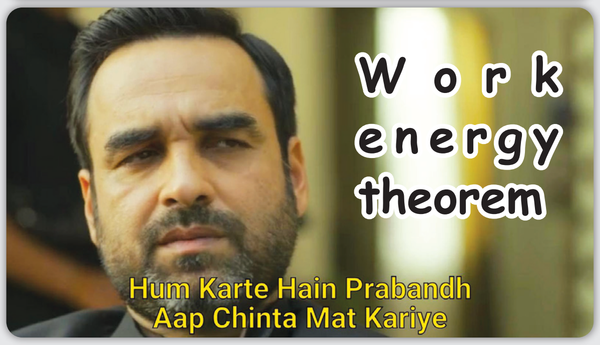


Sol. $T - mg\cos\theta = \dfrac{mv^2}{R}$

or $T = mg\cos\theta + \dfrac{mv^2}{R}$

mg sinθ = ma_t

a_t = g sinθ



SKC

भाई circular motion 12th तक तेरे बहुत काम आएगा इसलिए नीचे 4 Saleemians Khopcha Concept लिख रहा हुँ इनसे तुम्हारी सारी problem solve हो जाएगी।

1. सबसे पहले यह देखो की centre कहाँ है।
2. Partical की FBD बनाओ।
3. सारे forces को centre की तरफ और उसके perpendicular तोड़लो।
4. Centre की तरफ वाले force का sum mv^2/r या $mr\omega^2$ के बराबर करदो

बस इन्हे अच्छे से apply करना सीख लेना इसके लिए बहुत सारे ques. add कर रहा हुँ।

Conical Pendulum

Q. Particle is performing circular motion in horizontal circle with constant angular speed ω as shown in fig.

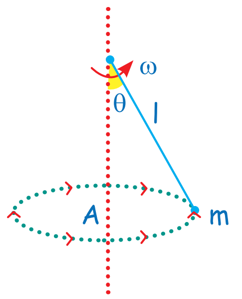
$\theta \to$ Const., length of string $\to l$

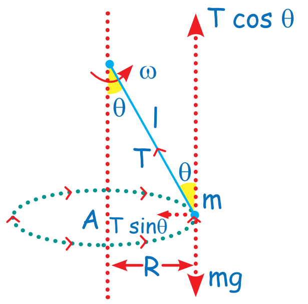
Sol. T cosθ = mg

$T\sin\theta = \dfrac{mv^2}{R} = mR\omega^2$

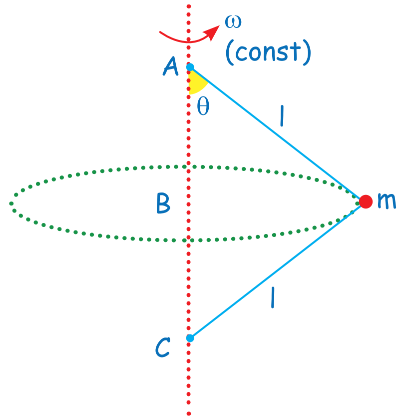
$l \sin\theta = R$

यह बहुत बड़ा बेशर्म ques है JEE MAINS मे तो यह बहुत बार आ चुका है।

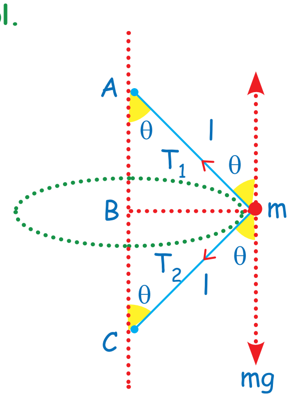


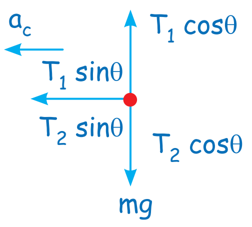


Q. In following fig. particle m is moving in circular motion. Find tension in both the string (massless).



Sol.





$T_1 \cos\theta = mg + T_2 \cos\theta$

$T_1 \sin\theta + T_2 \sin\theta = mr\omega^2$

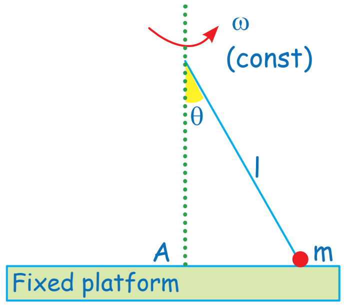
$r = l \sin\theta$

Now you can solve above eqn.

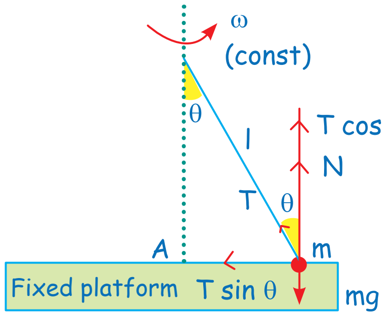
> आप ये सिखो की circular motion की eqn कैसे लिख रहे है ques वाला कोई भी physical quantity पूछ सकता है It's just maths

Q. In following fig. a circular platform is rotating with constant ω on smooth horizontal surface, on the platform a ball is attached to the axis of platform by a thin rod of length l.

(a) Find normal force exerted by ball on the platform. (θ = 60°, ω = 10 rad/s, l = 10 cm)



Sol.



$T \cos\theta + N = mg$

$T \sin\theta = mr\omega^2$

$r = l \sin\theta$

Solve and get N = 5 newton

(b) In above ques. find ω_{max} at which particle does not loose contact with platform. Find T in string at that condition.

Sol. $T \cos\theta + N = mg$

$T \cos\theta = mg \quad [N = 0]$

$T \sin\theta = mr\omega^2$

Solve and get

$$\omega = \sqrt{g \tan\theta / r}$$

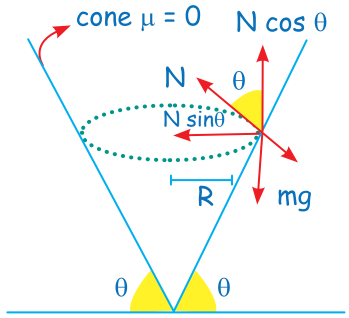
> अबे कुछ नही करना बस circular motion की eqn N = 0 put करदो

Q. Particle is moving with const. speed on the smooth frictionless surface (cone) as shown in fig in a circular path of radius R. find Normal force on particle?

Sol. $N \cos\theta = mg$

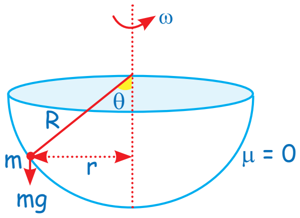
$$N \sin\theta = \frac{mv^2}{R}$$

Now you can solve



देख रहा है Binod.... कैसे एक conical pendulum के concept का use 5-6 सवालो मे किया जा रहा है

Q. Hemisphere bowl is rotating with const ω as shown in figure such that particle is at rest wrt bowl. (चिपकर धूम रहा है)
Find normal (N) force on particle.



Sol. Replica of conical Pendulum.

$N \cos\theta = mg$

$N \sin\theta = mr\omega^2$

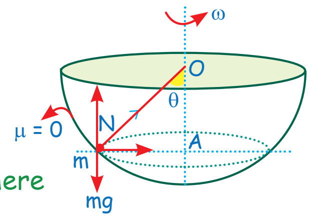
$R \sin\theta = r$

$R \rightarrow$ Radius of hemisphere

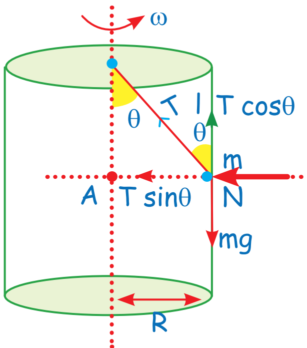
$r \rightarrow$ Radius of circle

Solve and get

$N = mR\omega^2$



Q. Cylinder is rotating with const ω. Particle is at rest wrt cylinder. Write the releavent circular motion eqn



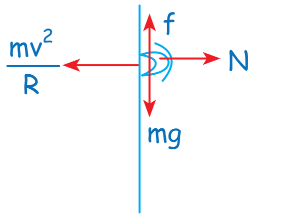
Sol. $T \cos\theta = mg$

$T \sin\theta + N = mR\omega^2$

$= m(l \sin\theta)\,\omega^2$ (l: length of string)

Q. A motor cyclist is driving in a horizontal circle on the inner surface of vertical cylinder of radius R. Find out the minimum velocity for which the motorcyclist can do this. (Death Well)

Sol. $N = \dfrac{mv^2}{R}$



$f = mg$

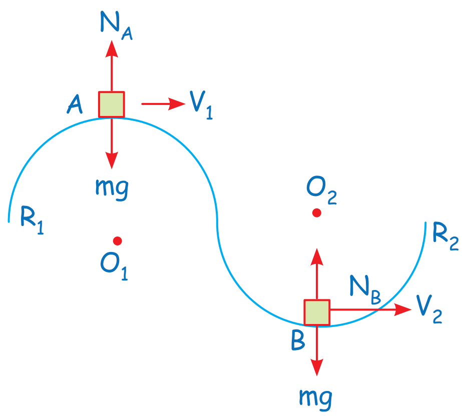
$f_{max} = \dfrac{\mu mv^2}{R}$

Cyclist does not drop down when

$f_{max} \geq mg \Rightarrow \dfrac{\mu mv^2}{R} \geq mg$

$v \geq \sqrt{\dfrac{gR}{\mu}}$

Q. Particle moving an a curved path an shown in vertical plane. Assume particle do not lose contact. Write the circular motion eqn when particle is at point A and B.

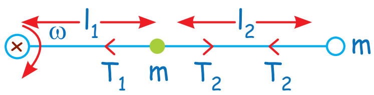


Sol.

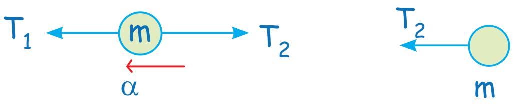
$mg - N_A = \dfrac{mV_1^2}{R_1}$; $N_B - mg = \dfrac{mV_2^2}{R_2}$

$N_B > N_A$

Q. System is on the frictionless Horizontal floor, both masses are moving in a circular path with same ω. Find Tension in String.



Sol.

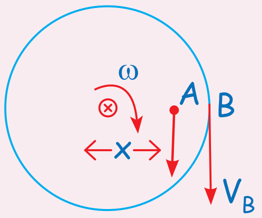


$T_1 - T_2 = ml_1\omega^2$ (for first mass)

$T_2 = m(l_1 + l_2)\,\omega^2$ (for second mass)

Suppose a disk is rotating on a horizontal floor with ω then velocity of different points in following diagram will be

★

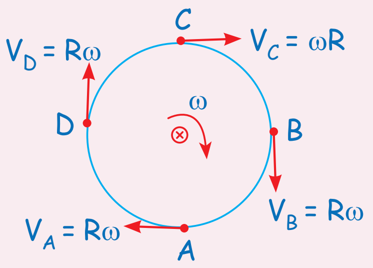


$\omega = \frac{2\pi}{T}$

$V_A = x\omega$

$V_B = R\omega$

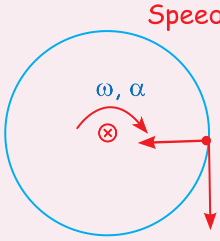
★



★

Speed up

$[\omega, \alpha \to CW]$



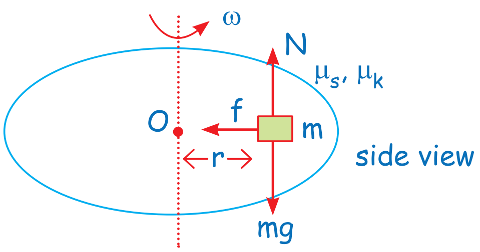
$R\omega^2 = V^2/R$

$a_t = R\alpha$

$V = R\omega$

Q. Table of radius r_0 is rotating about an axis $\perp^r$ to the plane of table and passing through centre of table. If angular vel of table is ω const, A mass 'm' is placed at a distance 'r' from centre of table such that mass remains at rest wrt table, find friction force acting on block.

Sol.

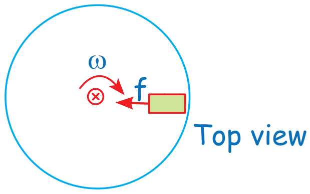


$a_t = 0$

$mg \to$ Inside

$N \to$ outside

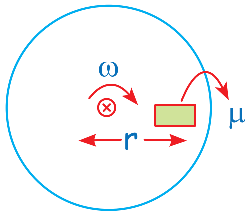
$f_s = mr\omega^2$



Q. Find ω_{max} so that block does not slip

OR

Find ω_{max} so there block remains at rest wrt disc



Sol. $mg \to$ Inside

$N \to$ Outside

$a_t = 0$

$f_s = mr\omega^2$

$\downarrow$

$(f_s)_{max} = \mu_s mg$

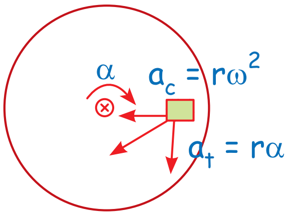
$$\omega_{max} = \sqrt{\frac{\mu_s g}{r}}$$

Q. Suppose in above ques, disk start rotating from rest having constant angular acc α find when block will slip if cofficient of friction is μ.

Sol. $f_{net} = ma_{net}$

$$f_{net} = m\sqrt{a_c^2 + a_t^2}$$

$a_t = r\alpha$ $\quad a_c = r\omega^2$



$$a_{net} = \sqrt{a_c^2 + a_t^2}$$

When block is just about to slip. At this time

$\omega = 0 + \alpha t = \alpha t$

$f = f_{max} = \mu_s mg$

$$\mu_s mg = m\sqrt{(r\omega^2)^2 + (r\alpha)^2}$$

$$\mu_s mg = m\sqrt{(r\alpha^2 t^2)^2 + (r\alpha)^2}$$

Now you can find t by puting value of μ_s, α, r, t

अबे value put करके इतना तो solve कर लोगे ना यहाँ ये important है की friction का एक component a_t दे रहा है और दुसरा component required centripetal force.

Q. Find the max speed that a vehicle (bicycle) can possess moving in a circle with const speed v such that surface is horizontal & coff. of friction is μ.

Sol.

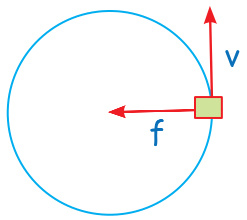
$$f = \frac{mv^2}{R}$$

max max

$$\mu N = \frac{mV_{max}^2}{R}$$

$$\mu mg = \frac{mV_{max}^2}{R}$$

$$V_{max} = \sqrt{\mu g R}$$



mg → अंदर

N → बाहर

Banking of Road

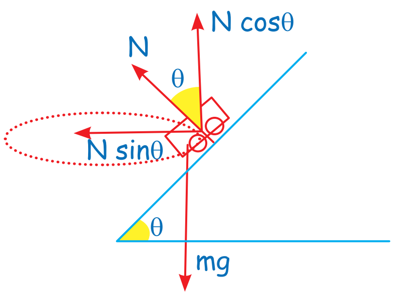
(i) Frictionless

N cosθ = mg

$$N\sin\theta = \frac{mv^2}{R}$$

$$\tan\theta = \frac{V^2}{Rg}$$

$$V = \sqrt{Rg\tan\theta}$$

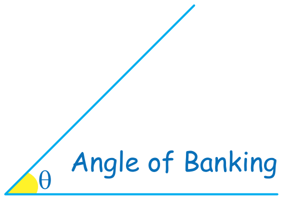


(ii) If friction is present

$$V_{max} = \sqrt{Rg\tan(\theta+\phi)}$$

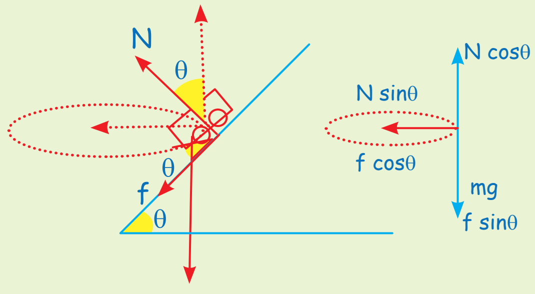
$$V_{min} = \sqrt{Rg\tan(\theta-\phi)}$$

where $\mu_s = \tan\phi$ (ϕ: Angle of friction)



Calculation for V_{max}:

N cosθ = mg + fsinθ



$$N\sin\theta + f\cos\theta = \frac{mv^2}{R}$$

for V_{max} = $(f_s)_{max} = \mu_s N$

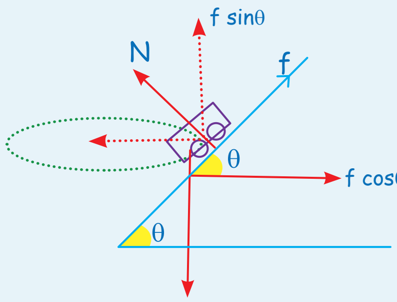
Solve and get

$$V_{max} = \sqrt{Rg\tan(\theta+\phi)}$$

Calculation for V_{min}:

Just Replace f to -f in V_{max}.

$$V_{min} = \sqrt{Rg\tan(\theta-\phi)}$$



Q. Suppose in above article we have R = 20m, $\mu_s = 1/\sqrt{3}$, θ = 45°.

(a) Find max/min possible speed for which car turn safely

Sol. $V_{max} = \sqrt{Rg\tan(\theta+\phi)} = \sqrt{20\times10\tan(45+30)}$

$V_{min} = \sqrt{Rg\tan(\theta-\phi)} = \sqrt{200\tan(45^\circ-30^\circ)}$

(b) At what velocity friction is not required to turns safely.

Sol. $V = \sqrt{Rg\tan\theta} = \sqrt{200\tan 45^\circ} = \sqrt{200}$

काम का डब्बा

देख भाई JEE MAINS के लिए ये 4 formula बहुत काम के है exam से पहले इन्हे जरूर पढ़के जाना।

★ $V_{max} = \sqrt{\mu gR}$ (Max speed on horizontal plane for safely turn of a bicycle)

★ $V = \sqrt{Rg\tan\theta}$ (banking of road when μ = 0)

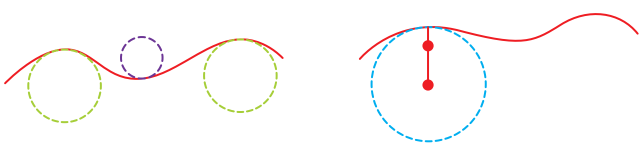
★ Banking of road when friction is present

$V_{max} = \sqrt{Rg\tan(\theta+\phi)}$

$V_{min} = \sqrt{Rg\tan(\theta-\phi)}$ $\tan\theta = \mu_s = \tan\phi$

Radius of Curvature (ROC)

Any curved path can be assume to be made of infinite circular arcs. ROC at a point is defined as the radius of that circular arc which fits at that particular point on the curve.



(a) (b)

★ $a_n = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_n}$ = ROC

★ ROC = $R = \frac{v^2}{a_\perp}$

भाई कुछ नही करना बस सबसे पहले normal acc निकाल लो और v^2/a_n करदो यही ROC है और अगर y = f(x) given है means particle moves in a trajectory given by y = f(x) then ROC at any point (x, y) of the trajectory is given by

$$R = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$

Q. A stone is projected with speed u and angle of projection is θ.

(a) Find radius of curvature at t = 0.

(b) Find ROC at highest point.

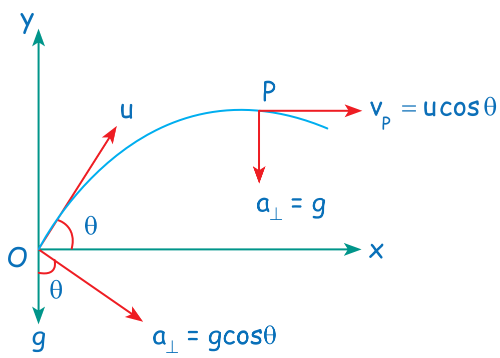
Sol. (a) At t = 0

$a_\perp = g\cos\theta$,

$R = \frac{v^2}{a_\perp} = \frac{u^2}{g\cos\theta}$

(b) At highest point $v_p = u\cos\theta$ and $a_\perp$ or $a_n = g$

Hence ROC at 'P' $= \frac{u^2\cos^2\theta}{g}$



6 Work, Power and Energy

ये एक बहुत ही जोरदार, easy and scoring chapter है जिससे Mains or Advance दोनो मे सवाल पुछे जाते है इसको बहुत अच्छे से करना bcz COM, rotation, electrostatic etc मे यह बहुत use होगा।

WORK

Work done (dW) by force $\vec{F}$ is given by

★ $dW = \vec{F} \cdot \vec{ds}$

$dW = \vec{F} \cdot \vec{ds}$

displacement of point of application of force

Chota sa WD jo $\vec{F}$ Ne kia

Vo force hai jiska WD hum nikal rahe hai

★ $(WD)_{\text{by Force F}} = W_{net} = \int dw = \int \vec{F} \cdot \vec{ds}$

★ If F is const $\Rightarrow (WD)_{\text{by F}} = \vec{F} \cdot \int \vec{ds} = \vec{F} \cdot \vec{S}$

$WD = \vec{F} \cdot \vec{S}$ or $WD = \vec{F} \cdot \vec{d}$

Here $\vec{S}$ or $\vec{d}$ are displacement.

Constant force $\vec{F}$ का WD $\vec{F} \cdot \vec{d}$ होता है ये कभी मत भूलना bcz WD by gravity, any other external constant force etc मे यह बहुत use होगा। Constant force का मतलब है magnitude के साथ direction भी constant होना ध्यान रखना इस बात को।

WORK DONE BY CONSTANT FORCE $\vec{F}$

$WD = \vec{F} \cdot \vec{d}$

$WD = F\ d\cos\theta$

angle between $\vec{F}$ & $\vec{d}$

magnitude of $\vec{d}$

WD by force $\vec{F}$

Magnitude of $\vec{F}$

$WD = (F\cos\theta) \cdot d$

WD = (component of force along displacement) × displacement (magnitude)

$WD = F\,(d\cos\theta)$

WD = Force × Component of displacement (magnitude) walong force

1. If $\theta = 0° \Rightarrow (WD)_F = Fd$
2. If $\theta = 90° \Rightarrow (WD)_F = 0$
3. If $\theta = 180 \Rightarrow (WD)_F = -Fd$
4. If $\theta < 90$, $(WD)_F > 0$
5. If $\theta > 90$, $(WD)_F < 0$

Q. If a force $\vec{F} = 3\hat{i} + 4\hat{j} + 6\hat{k}$ is acting on a particle such that particle moves from (1, 2, 3) to (7, 8, 5). Find WD by force $\vec{F}$.

Sol. $\vec{F} = 3\hat{i} + 4\hat{j} + 6\hat{k}$

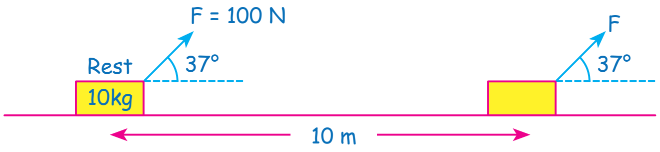
Displacement = change in position = $6\hat{i} + 6\hat{j} + 2\hat{k}$

$(WD)_F = \vec{F} \cdot \vec{d}$

$= 18 + 24 + 12 = 54$

Q. A constant force $\vec{F}$ of magnitude 100 N is applied at an angle 37° with horizontal on a block of mass 10 kg placed on ground. Find work done by force F and gravity when block move by 10 m.

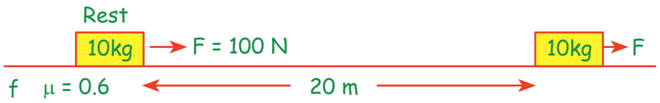
Sol.



Sol. $\vec{F} = 80\hat{i} + 60\hat{j}$ $\qquad \vec{d} = 10\hat{i}$

$(WD) = \vec{F} \cdot \vec{d} = 800 + 0 = 800 \text{ J}$

Q. Find the work done by forces F, f, mg and N if block move 20 m along + x axis



1. (WD) by F = 100 × 20 = 2000 J
2. (WD) $\text{gravity}_{(mg)} = 0 \cdot (\because \vec{F} \perp \vec{d})$
3. (WD) by $\text{normal}_{(N)} = 0$
4. (WD) by $\text{normal}_{(f)} = -f_k d = -60 \times 20 = -1200$

(WD) by all force $= W_g + W_N + W_f + W_F$

$= 0 + 0 - 1200 + 2000$

$= + 800 \text{ J}$

(WD) by net force = + (F – f) × d

↓

const

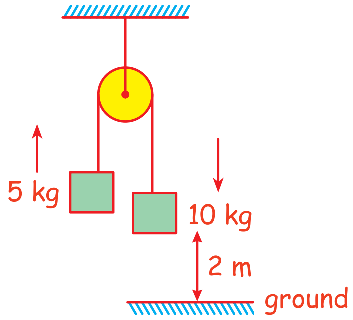
$= (100 - 60) \times 20 = + 800 \text{ J}$

SKC

यहाँ हमने देखा (WD) by all the force निकालो या (WD) by net force निकालो दोनो same है।

देख भाई जब किसी force का work done लिख रहे है तो NSP नही करनी है, NSP बोले तो Nayan sukh prapti मतलब ताड़ा ताड़ी नही करनी है अरे मेरा मतलब है जिसका force का WD निकाल रहे हो उसी force पर focus करो, दुसरी की तरफ देखना भी नही है।

Q. In given fig. system is released from rest. When 10 kg block reaches ground then find



(i) Work done by gravity on 10 kg.
(ii) Work done by gravity on 5 kg.
(iii) Work done by tension on 10 kg.
(iv) Work done by tension on 5 kg.

Sol. (i) $(W_g)_{10\,kg} = 10g \times 2 \cos 0° = 200 \text{ J}$

(ii) $(W_g)_{5\,kg} = 5g \times 2 \times \cos 180° = -100 \text{ J}$

(iii) $T = \dfrac{2m_1m_2}{m_1 + m_2} g = \dfrac{200}{3} \text{N}$

$(W_T)_{10kg} = \dfrac{200}{3} \times 2 \times \cos 180° = \dfrac{-400}{3} \text{ J}$

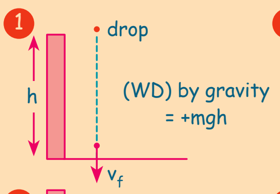
(iv) $(W_T)_{5\,kg} = \dfrac{200}{3} \times 2 \times \cos 0° = \dfrac{400}{3} \text{J}$

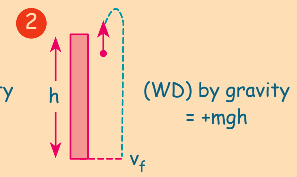
Net work done by tension is zero. Work done by internal tension i.e. (tension acting within system) on the system is always zero if the length of string remains constant.

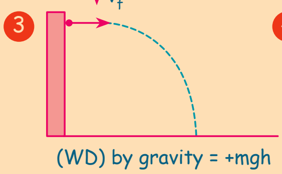
काम का डब्बा

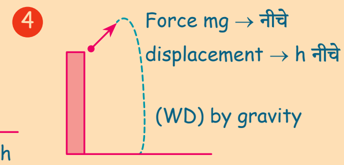
Agar koi particle kaise bhi neche 'h' aya to work done by gravity = +mgh hoga

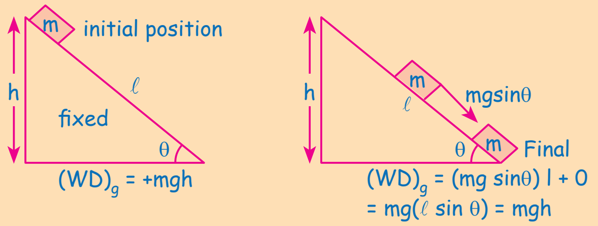
Agar uper 'h' gaya hai to WD by gravity = –mgh hoga

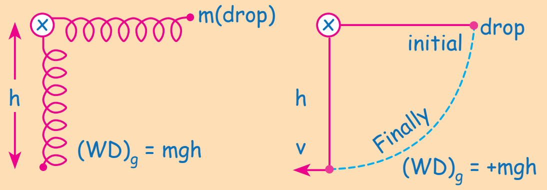


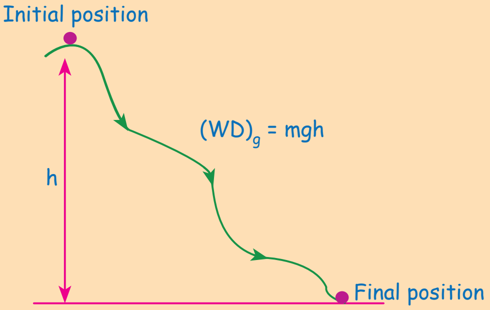


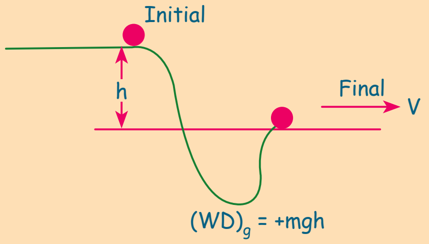












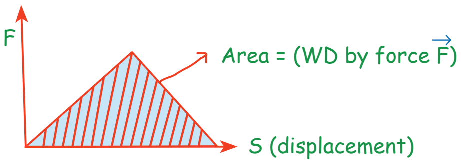
WORK DONE BY VARIABLE FORCE

यहाँ आपको दो प्रकार के ques देखने को मिलेंगे एक Area calculation and दूसरा integration based since,

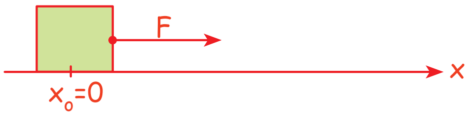
WD = $\int \vec{F}\cdot d\vec{s}$ so, F vs displacement (s) के graph का area under curve, WD by force F देगा।

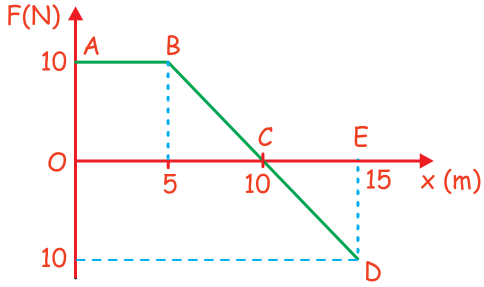
WD = $\int \vec{F}\cdot d\vec{s}$

Area = $\int F ds$



Q. A horizontal force F is used to pull a box placed on floor. Variation in the force with position coordinate *x* measured along the floor as shown in the graph.





(*a*) Calculate work done by the force in moving the box from x = 0 m to x = 10 m.

(*b*) Calculate work done by the force in moving the box from x = 0 m to x = 15 m.

Sol. In rectilinear motion work done by a force equals to area under the force-position graph.

(a) $W_{0\to10}$ = Area of trapazium OABC = 75 J

(b) $W_{0\to15}$ = Area of trapazium OABC – Area of triangle CDE = 50 J

अब आपको only mathematics देखने को मिलेगी so, maths का दिमाग on करदो और sachin sir का नाम लेकर नागिन को याद करो बोले तो....... EX को याद करने को नही बोल रहा हुँ i mean integration

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$$

$$(WD) \text{ by } F = \int \vec{F}\cdot d\vec{s} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

Q. If particle move from x = 0 to x = 2. Find (WD) by F = $3x^2$.

Sol. $\int \vec{F}\cdot d\vec{x} = \int_0^2 3x^2 dx = 3\left.\dfrac{x^3}{3}\right|_0^2 = 2^3 - 0^3 = 8J$

Q. $\vec{F} = 2x\hat{i} + 2y\hat{j} + 4z^3\hat{k}$

If particle move from (0, 0, 0) to (1, 2, 2), WD=?

Sol. $(WD)_F = \int_0^1 2x dx + \int_0^2 2y dy + \int_0^2 4z^3 dz$

$$= \cancel{2}\left.\frac{x^2}{\cancel{2}}\right|^1 + \cancel{2}\left.\frac{y^2}{\cancel{2}}\right|_0^2 + \cancel{4}\left.\frac{z^4}{\cancel{4}}\right|_0^2 = 1 + 4 + 16 = 21\text{ J}$$

Q. $\vec{F} = 2x\hat{i} + 3x^2y\hat{j} = F_1\hat{i} + F_2\hat{j}$

If a particle moves parallel to x axis from (0, 5) to (10, 5) find WD by $\vec{F}$.

F_2 का WD zero है क्योंकि F_2 along y-axis है और displacement along x-axis

Sol. $(WD)_{F_1} = \int_0^{10} 2x dx = 100$

$(WD)_{F_2} = 0$

(No displacement in y- direction)

Q. If a particle moves from (0, 0) to (3, 4) on a straight along line $y = \frac{4x}{3}$ then find the WD by force by force $\vec{F} = 2x\hat{i} + x^2y\hat{j}$

Sol. $(WD) = \int_0^3 2xdx + \int_0^4 x^2ydy$

अब इसे directly integrate नही कर सकते और ना ही x^2 को बाहर निकाल सकते है so, अब क्या करे

अरे कुछ नही करना x की value y के term मे put करदो

$$= \left(2 \times \frac{9}{2}\right) + \int_0^4 \frac{9}{16} y^3 dy \quad \left[x = \frac{3}{4}y\right]$$

$$= 9 + \frac{9}{16}\frac{4^4}{4} = 9 + (9 \times 4) = 45$$

Q. Find (WD) by force $F = 4x\hat{i} + y^2x\hat{j}$ if a particle move from (0, 0) to (2, 5) on the curve $y^2 = x$.

Sol. $WD = \int_0^2 4xdx + \int_0^5 y^2y^2dy = 633$

Q. Find the work done by force $\vec{F} = (xy\hat{i} + y\hat{j})N$ on particle, when it moves from origin to point (1, 1) along the path $y = x^2$, where x and y are in meters.

Sol. $W = \int F_x dx + \int F_y dy$

$W = \int xy\,dx + \int y\,dy$

$\Rightarrow y = x^2 \Rightarrow \frac{dy}{dx} = 2x \text{ or } dy = 2x\,dx$

$\Rightarrow W = \int_0^1 x^3dx + \int_0^1 x^2 2x\,dx$

$$= \frac{x^4}{4}\Big|_0^1 + \frac{2x^4}{4}\Big|_0^1 = \frac{1}{4} + \frac{2}{4} = \frac{3}{4}J$$

$WD = \int \vec{F}.d\vec{s}$

We know: $\vec{v} = \frac{d\vec{s}}{dt}$

$d\vec{s} = \vec{v}\,dt$

If $\vec{F}$ is $\perp^{ar}$ $\vec{V}$

$$\boxed{WD = \int \vec{F}\cdot\vec{V}\,dt}$$

$\Rightarrow (WD)_F = 0$

Q. A force $\vec{F} = (3t\hat{i} + 5\hat{j})N$ acts on a body due to which its position varies as $\vec{S} = (2t^2\hat{i} - 5\hat{j})$. Work done by this force in first two seconds is:

Sol. $\frac{d\vec{S}}{dt} = 4t\hat{i}$

$d\vec{S} = 4t\,dt\,\hat{i}$

$W = \int \vec{F}.\vec{ds}$

$= \int (3t\hat{i} + 5\hat{j}) . (4t\,dt\,\hat{i})$

$$= \int_0^2 12t^2\,dt = \frac{12\left[t^3\right]_0^2}{3} = 32\text{ J}$$

Work Done by Spring Force

$\because \vec{F}_{sp} = -k\vec{x}$

$(WD)_{\text{spring force}} = \int \vec{F} d\vec{x} = \int_{x_i}^{x_f} -kx\,dx$

$(WD)_{\text{spring force}} = -\frac{1}{2}k\left(x_f^2 - x_i^2\right) = \frac{1}{2}k\left(x_i^2 - x_f^2\right)$

Where $x_i \rightarrow$ Initial compression/elongation in spring from natural length

Where $x_f \rightarrow$ Final compression/elongation in spring from natural length

ये एक बहुत ही important result है जो आपको याद रखना है की $(WD)_{\text{spring force}} = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$ इसकी calculation मे spring compress है या elongated है कोई फर्क नही पड़ता

Q. If initial compression in spring is 2m. Find (WD) by spring force. If final compression in spring is 5m (k = 1000 N/m).

Sol. $(WD)_{sp} = -1/2k\,(x_f^2 - x_i^2)$

$= -1/2 \times 1000 \times (5^2 - 2^2)$

$= -500 \times 21$

$= -10500$ J

Q. Find (WD) by spring force if spring is compressed by x_o from natural length.

Sol. $(WD)_{sp} = -1/2k\,(x_o^2 - 0^2)$

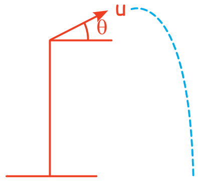
$= -1/2\,(kx_0^2)$

WORK ENERGY THEOREM (WET)

WD by all force = change in K.E= $\Delta KE = (K.E)_{final} - (K.E)_{Initial}$

यह बहुत ही ज्यादा important theorem है जो एक करोड़ बार use होगी

Q. Find final speed of particle just before it hits the ground

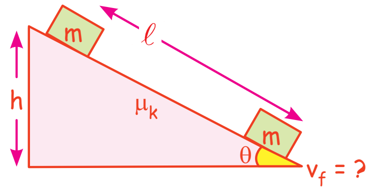


Sol. $(WD)_g = \Delta kE = k_f - k_i$

$mgh = \frac{1}{2}mv_f^2 - \frac{1}{2}mu^2$

$v_F^2 = u^2 + 2gh$

Q. Find final velocity V_f if block starts from rest

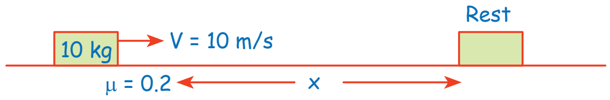


Sol. $W_g + W_N + W_f = \Delta kE$

$+mgh + 0 - (\mu mg\cos\theta\,\ell) = \frac{1}{2}mv_f^2 - 0$

Solve for V_f.

Q. Calculate distance covered before block stops.



Sol. M-1

$W_g + W_N + W_f = \Delta KE$

$0 + 0 - \mu mgx = 0 - \frac{1}{2}mV^2$

$0.2 \times 10 \times x = \frac{1}{\cancel{2}} \times \cancel{10}^{5} \times 10$

$x = \frac{50}{2} \quad \therefore \quad x = 25$ m

M-2

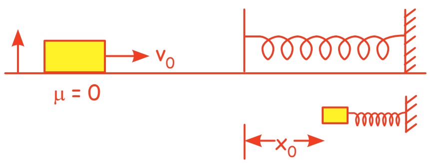
$v^2 = u^2 + 2as$

$0 = 10^2 - 2 \times \mu gx$

$2\mu gx = 100$

$x = \frac{100}{2 \times 0.2 \times 10} = 25$

Q. Find maximum compression in spring



Sol. $W_g + W_N + W_{SP} = \Delta KE$

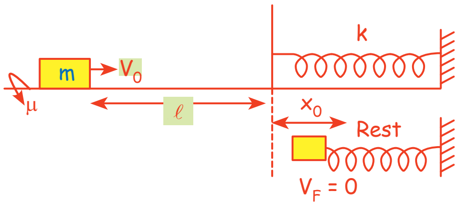
$$0 + 0 + - 1/2\ k\ (x_0^2 - 0^2) = 0 - 1/2 mv_0^2$$

$$x_0 = v_0\sqrt{\frac{m}{k}}$$

Maximum compression in spring तब होगा जब block rest पर आजाएगा। मतलब $v_f = 0$ और यह concept करोड़ो बार use होगा, 10-12 question तो मै ही add कर दे रहा हुँ।

मान लो in above ques friction भी दिया होता..... तो कर लोगे ना...... only friction का WD ही तो लिखना है।

Q. Surface in rough. Find max compression x_0

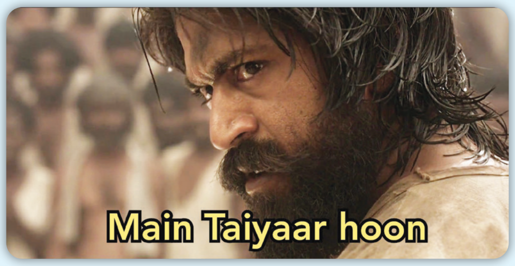


Sol. $W_g + W_N + W_f + W_{sp} = \Delta KE$

$$0 + 0 - \mu mg(l + x_0) - 1/2\ k\ (x_0^2 - 0^2)$$

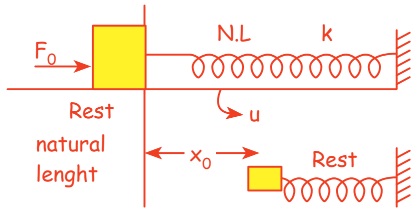
$$= 0 - 1/2\ mV_0^2$$

अब ध्यान से सुनो.... we have limited no. of pages but हम question practice मे कोई कमी नही छोड़ेंगे.... ये लो pen और जितने question मै attach कर रहा हुँ that's sufficient to build your concept so rough copy पर खुद से इन्हे जरूर solve करे are you ready....



मुझे insta (Saleem.nitt) पर update करें only if you have a account.

Q. Contant force F_0 acts on block. Find maximum compression.

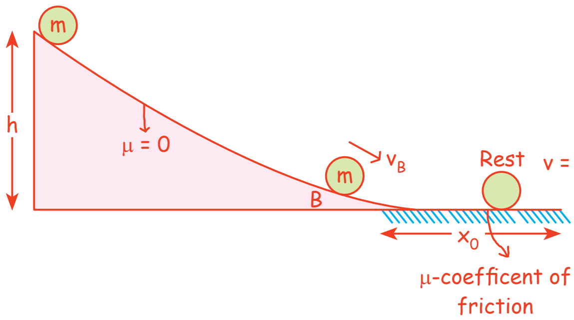


Sol. $W_g + W_N + W_f + W_{sP} + W_F = \Delta KE$

$$0 + 0 - \mu mgx_0 - 1/2k\ (x_0^2 - 0^2)$$

$$+ F_0\ x_0 = 0 - 0$$

Q. Horizontal surface is rough. Find velocity at B & x_0.



For velocity at B

$Wg + w_N = \Delta KE$

$mgh + 0 = 1/2\ mV_B^2 - 0$

$v_B = \sqrt{2gh}$

To find x_0

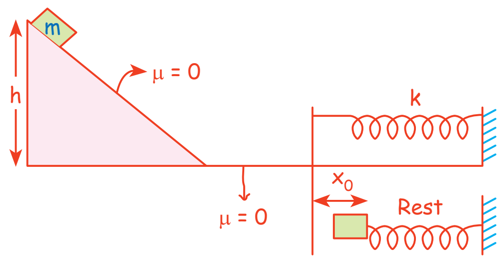
$W_g + W_N + W_f = \Delta KE$

$mgh\ + (0 - \mu mgx_0) = 0 - 0$

$\cancel{m}gh = \mu\ \cancel{m}gx_0$

$x_0 = h/\mu$

Q. Find max compression in spring

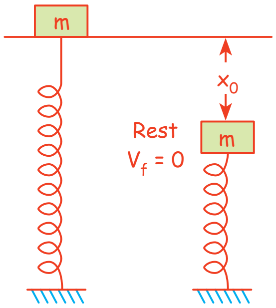


Sol. $W_g + W_N + W_f + W_{SP} = \Delta KE$

$$mgh + 0 + 0 - 1/2k\ (x_0^2 - 0^2) = 0 - 0$$

$$x_0 = \sqrt{\frac{2mgh}{k}}$$

Q. Find maximum compression in spring.
Spring is initially in its natural length.

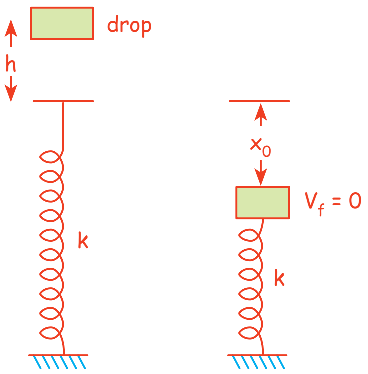


Sol. $W_g + W_{SP} = \Delta KE$

$mgx_0 - 1/2k(x_0^2 - 0) = 0 - 0$

$$x = \frac{2mg}{k}$$

Q. Find maximum compression. At max elongation v = 0

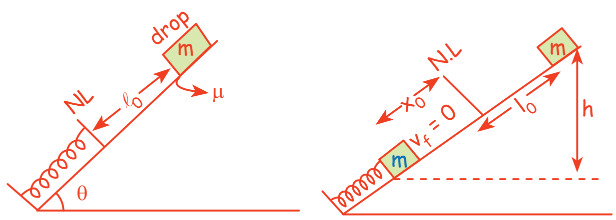


Sol. $W_g + W_{SP} = \Delta KE$

$mg(h + x_0) - 1/2k(x_0^2 - 0^2) = 0 - 0$

$\Rightarrow$ Solve for x_0.

Q. Find maximum compression in spring.



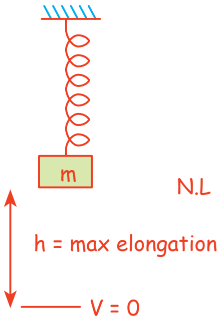
Sol. $W_g + W_N + W_{SP} + W_f = \Delta KE$

$mg(l_0 + x_0)\sin\theta + 0 - \frac{-1}{2}k(x_0^2 - 0)$

$-\mu mg\cos\theta\,(l_0 + x_0)$

$= 0 - 0$

Q. Find maximum elongation. When a block of mass m which is attached to a spring, is released from its Natural length.

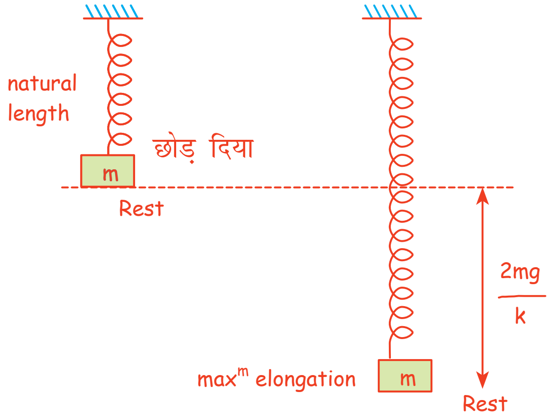


Sol. $W_g + W_{SP} + W_N = \Delta KE$

$mgh - 1/2\,k(h^2 - 0^2) + 0 = 0 - 0$

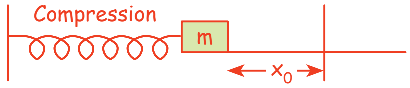
$mgh = 1/2\,kh^2$

$$h = \frac{2mg}{k}$$

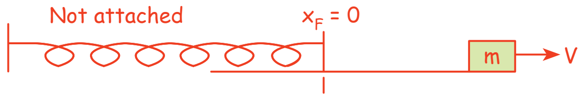


याद रखना यहाँ natural length से equilibrium position, mg/k दूरी पर है जहाँ velocity max. होगी

Q. Find K.E when block is released from rest.



Sol.

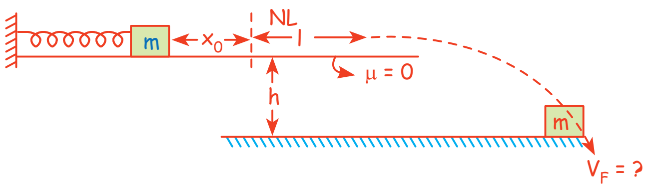


$W_g + W_N + W_F + W_{SP} = \Delta KE$

$0 + 0 + 0 - 1/2k\,(0^2 - x_0^2) = 1/2\,mV^2$

$$V = \sqrt{\frac{kx_0^2}{m}}$$

Q. Find V_f, initial compression is x_0.



Sol. $W_g + W_N + W_f + W_{SP} = \Delta KE$

$+ mgh + 0 + 0 - 1/2k\,(0^2 - x_0^2) = 1/2\ mV^2_F - 0$

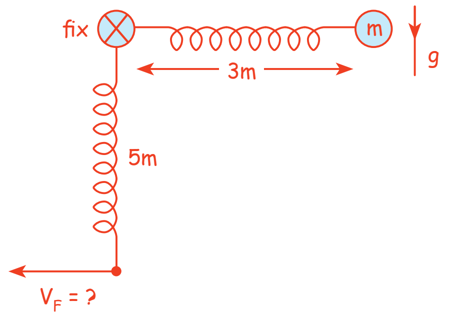
If friction (μ) is present

$W_g + W_N + W_f + W_{SP} = \Delta KE$

$+mgh + 0 - \mu mg\,(x_0 + l) - 1/2\ K\,(0^2 - x_0^2)$

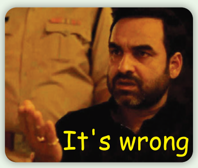
$= 1/2\ mV_f^2 - 0$

Q. A particle is released from rest in following diagram find velocity V_F if natural length of the spring is 2m.



Sol.

$(mg)5 - 1/2\ K\,[5^2 - 3^2] = 1/2\ mV_f^2$...........(wrong)



$mg5 - 1/2k\,[5^2 - 1^2] = 1/2\ mV_f^2$

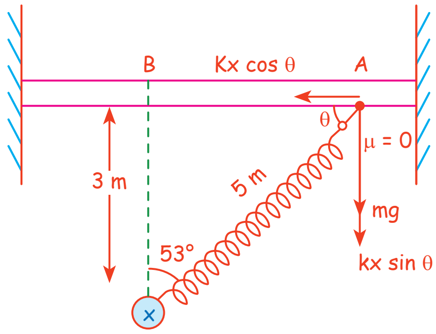
(It's also wrong)

x_f and x_i are elongation from natural length

$mg5 - 1/2k\,[3^2 - 1^2] = 1/2\ mV_f^2$

(Now it's correct)

Q. Find velocity of ring when it reaches at B, natural length = 2m



Sol. $x_i = 5 - 2 = 3$ m

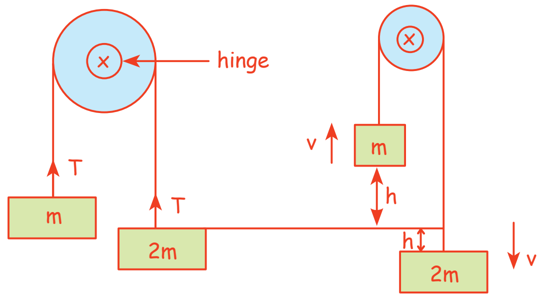
$x_f = 3 - 2 = 1$ m

$A \rightarrow B$

$w_g + w_N + w_{sp} + w_f = \Delta KE$

$0 + 0 - 1/2k\,(1^2 - 3^2) + 0 = 1/2\ mV_B^2 - 0$

Q. Blocks are released from rest, find speed of each block when 2m block goes down by h.



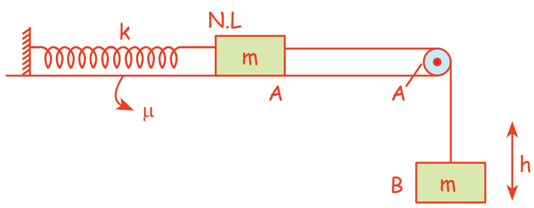
Sol. $W_g + W_T + W_{hinge} = \Delta KE$

$(+2mgh - mgh) + (-Th + Th) + 0$

$= \left(\frac{1}{2}\ 2mv^2 + \frac{1}{2}\ mv^2\right) - 0$

$mgh = \frac{3mv^2}{2}$ $\boxed{v = \sqrt{\frac{2gh}{3}}}$

Q. Find speed of both block if B goes down by 'h'

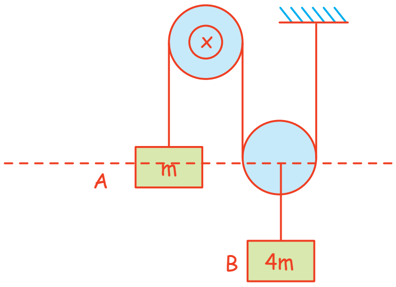


Sol. $W_{SP} + W_f + W_T + W_g + W_{hinge} + W_N = \Delta KE$

$-\frac{1}{2}k\,(h^2 - 0^2) - \mu mgh + 0 + (mgh + 0) + 0$

$= \left(\frac{1}{2}\ mv^2 + \frac{1}{2}\ mv^2\right) - 0$

Q. Find speed of both blocks when block B reaches distance h in downward direction



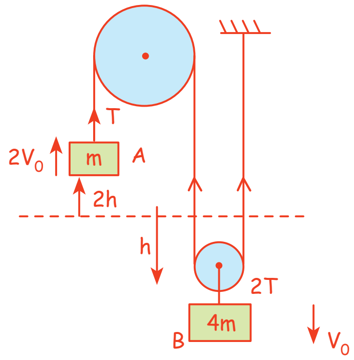
Sol. $w_g + w_T + w_{hinge} = \Delta KE$

$(+4mgh - mg2h) = \frac{1}{2}4mV_0^2 + \frac{1}{2}m^2V_0^2 - 0$

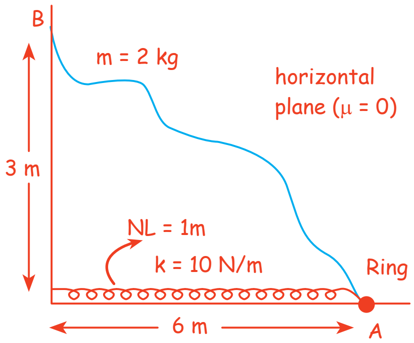
$2mgh = 4mv_0^2 \Rightarrow V_0 = \sqrt{\frac{gh}{2}}$

$V_A = 2v_0 = \sqrt{2gh}$

$V_B = v_0 = \sqrt{\frac{gh}{2}}$.



Q. When an unknown variable force 'F' is applied on ring lying on horizonatal plane such that when ring reaches at point B it aquire speed 5 m/s. Natural length of spring as 1 m and k = 10 N/m, Find $(WD)_F$ = ?



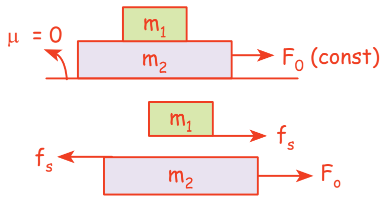
Sol. $W_g + W_N + W_{sp} + W_F = \Delta KE$

$0 + 0 - \frac{1}{2} \times 10\,(2^2 - 5^2) + w_F = \left(\frac{1}{2} \times 2 \times 5^2 - 0\right)$

$\frac{1}{2} \times 10 \times 21 + w_F = \frac{50}{2}$

$w_F = -80$ J

Q. In the following problem if both block move together without slipping with same acceleration by distance x. Find net WD by friction force.



Sol. (WD) by friction on $m_1 = +f_s \cdot x$

(WD) by friction of $m_2 = -f_s x$

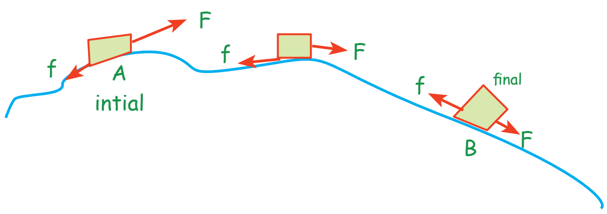
Net (WD) by friction = $f_s x + (-f_s \cdot x) = 0$

Work done by action reaction pair of constraint forces such as static friction, normal reaction and tension in the string is always zero when they are internal forces.

SKC

- WD by static friction can be positive / negative और zero but net WD by static friction always zero.
- WD by kinetic friction can be positive / negative और zero but net WD by static friction always negative.

WD BY FORCE IF F IS ALWAYS PARALLEL TO DISPLACEMENT OR F IS ALWAYS TANGENT TO THE PATH



Q. Find (WD) by force F from A to B if magnitude of force is constant and direction is parallel to displacement at each point on path.

Sol. F $\rightarrow$ always tangent/parallel has Displacement

$\theta = 0 = (WD)_F$ = F × distance = F × (path length)

F $\rightarrow$ always Anti parallel, $\theta = 180°$, displacement

$(WD)_F = -F \times$ (Distance)

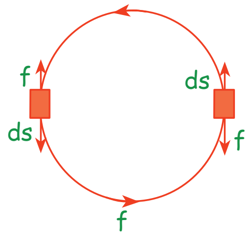
Q. A block move on horizontal rough surface in a circular path of radius R as shown in diagram

Find (WD) by friction in one complete revolution.

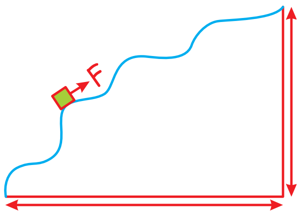
Sol. Force always antiparallel to displacement ($\theta = 180°$)

$w = -f \cdot (2\pi R)$ path length

$w = -\mu mg \cdot 2\pi R$



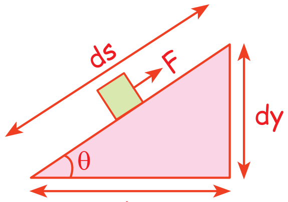
Q. A body of mass m is slowly hauled up the rough hill by a force F which at each point is directed along a tangent to the hill. Work done by the force.



x

Sol. $w_g + w_N + w_f + w_F = \Delta KE$

$-mgh + 0 + W_f + W_F = 0$



dx

$ds \cos\theta = dx$

$\int ds \cos\theta = \int dx = x$

and $|W_f| = \int (\mu mg \cos\theta) ds$

$= \mu mg \int ds \cos\theta = \mu mg \int dx = \mu mgx$

$-mgh + 0 - \mu mgx + W_F = 0$

$W_F = mgh + \mu mgx$

अब कुछ ऐसे सवाल solve करते है जिनमे maths और integration involve होती है ये question ज्यादातर Mains मे पूछे जाते है।

Q. If a particle of mass 2kg moving on x-axis such that $x = t^4$ under the action of several force. Find WD by all the force from t = 0 to t = 2 sec

Sol. m-1 $x = t^4$,

$v = 4t^3$

$a = 12t^2$

$F = ma = 2 \times 12t^2$

$F = 24t^2$

$WD = \int \vec{F} \cdot \vec{v}\, dt$

$= \int 24t^2 \cdot 4t^3\, dt$

$= 24 \times 4 \times \frac{2^6}{6} = 1024$

ये अच्छा method नही है सीधा WET लगाओ

m-2

$x = t^4$

$v = 4t^3$

$t = 0, v_i = 0$

$t = 2 = 4 \times 2^3 = 32 = v_f$

(WD) all force $= K_F - K_i$

$= 1/2 \times 2 \times 32^2 - 0 = 1024.$

Q. Find (WD) by all forces from t = 0 to t = 3 if m = 2kg, $v = 2t^2$.

Sol. $t = 0 \quad v = 0, \quad (KE)_i = 0$

$t = 2 \quad v = 2 \times 3^2 = 2 \times 3^2 = 18$

$(KE)_f = 1/2 \times 2 \times 18^2 = 324$

(WD) all force = 324 – 0

Q. A particle (2kg) start from rest from origin & start moving on x-axis its speed v/s x relation is find (WD) by all force from x = 0 to $v = 4x^{3/2}$, x = 4.

Sol. $x = 0, \quad v = 0 \Rightarrow (KE)_i = 0$

$x = 4, \quad v = 4 \times 4^{3/2} = 4 \times 2^3 = 32 \ (KE)_i = 0$

(WD) all force $= 1/2 \times 2 \times 32^2 - 0 \quad (KE)_f = (32)^2$

Q. A particle (1 kg) move on x-axis under the action of several force such that $x = \sqrt{t} - 2$.

Find (WD) by all force from $t = 1 \rightarrow t = 2$ sec.

Sol. $v = \frac{dx}{dt} = \frac{1}{2\sqrt{t}}$

$t = 1, v = 1/2 \qquad KE_i = 1/2 \times 1 \times 1/4$

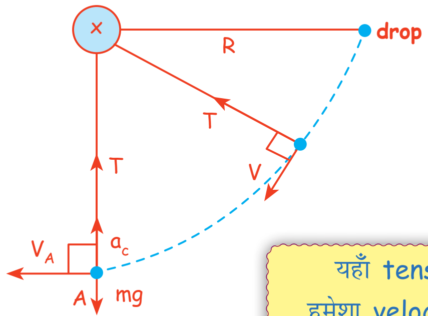
$t = 2, v = \frac{1}{2\sqrt{2}} \qquad KE_f = \frac{1}{2} \times 1 \times 1/8$

$W = \Delta KE = \frac{1}{16} - \frac{1}{8} = -\frac{1}{16} J$

अब हम WET + circular motion के mix question की practice करेंगे इन question मे बस ये याद रखना हमे mostly दो eqn के साथ खेलना है

1. Work energy theorem की eqn
2. Circular motion की centripital force की eqn

Q. The particle is released from rest as shown in figure. Find velocity of particle, tension, centripetal and tangential accelerations. (a_t, a_c) when it reaches at lowest point.



यहाँ tension हमेशा velocity के perpendicular है so, WD by tension = 0 (यहाँ अच्छे से समझ लो बहुत बार use होगा)

Sol. $w_g + w_T = \Delta KE$

$$+mgR + 0 = \frac{1}{2} mV_A^2 - 0$$

$$V_A = \sqrt{2gR}$$

Tension when reacheas at lowest pt

$$T_A - mg = ma_c$$

$$T_A - mg = \frac{mV^2}{R}$$

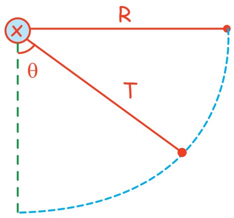
$$T_A - mg = \frac{m2g\cancel{R}}{\cancel{R}}$$

$$T_A = 3\ mg$$

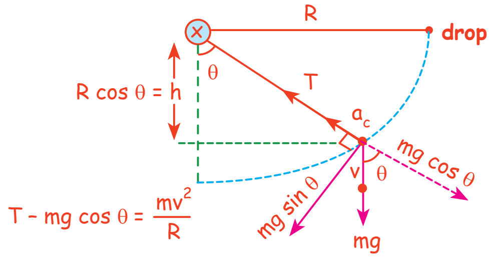
$$a_c = \frac{v^2}{R} = \frac{2gR}{R} = 2g$$

$$a_t = 0$$

Q. Find speed of ball and Tension in string T when string makes angle θ with vertical. Take θ = 60°, m = 2kg, r = 10 m.



Sol.



$$w_g + w_T = \Delta KE$$

$$mgh = \frac{1}{2} mv^2 - 0$$

$$V^2 = 2gh$$

$$V = \sqrt{2gR \cos\theta} = \sqrt{2 \times 10 \times 10 \cos 60} = 10\ m/s$$

$$mg \sin\theta = ma_t$$

$$a_t = g \sin\theta$$

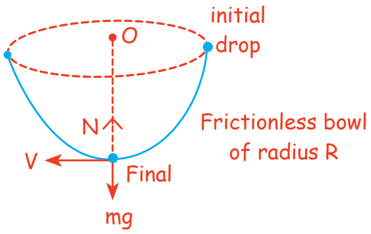
Now let's write circular motion eqn

$$T - mg \cos 60° = \frac{mV^2}{R}$$

$$T - 2 \times 10 \times \frac{1}{2} = \frac{2 \times 100}{10}$$

$$\therefore\ T = 30\ N$$

Q. A particle is dropped from a point inside a smooth fixed hemisphere as shown in fig. When particle reaches at lowest point find Speed (V) and Normal (N).



Sol. $W_f + W_g + W_N = \Delta KE$

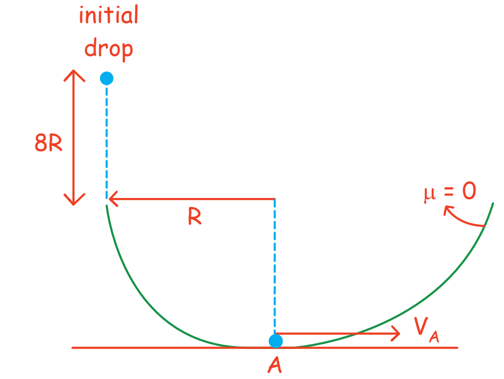
$$0 + mgR + 0 = \frac{1}{2} mV^2 - 0$$

$$V = \sqrt{2gR}$$

$$N - mg = \frac{mV^2}{R} = N = 3\ mg$$

यार last question की तरह यहाँ अगर किसी angle पर v, N पुछा जाए तो निकाल लोगे ना..... same as it is eqn बनेगी बस tension की जगह normal आजाएगा और WD by normal is 0.

Q. Find speed V_A of particle when it reaches at lowest point A.

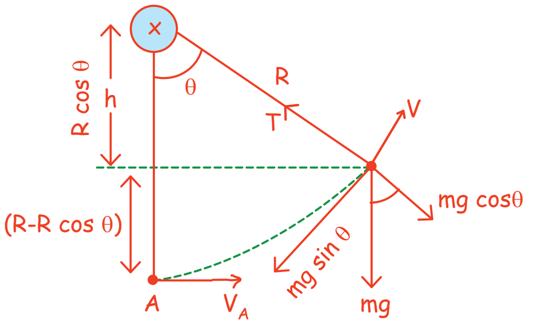


Sol. $W_g + W_N = \Delta KE$

$$mg9R + 0 = \frac{1}{2}mv^2 - 0$$

$$V_A = \sqrt{18gh}$$

Q. Bob of pendulum is given velocity V_A at lowest position. Find Tension when string makes angle θ (if $\theta < 90$). Find min velocity of particle to perform vertical circular motion at C.



Sol. $w_g + w_T = \Delta KE$ (WET)

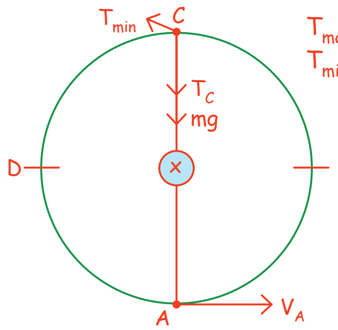
$$-mg(R - R\cos\theta) + 0 = \frac{1}{2}mv^2 - \frac{1}{2}mv_A^2$$

$$T - mg\cos\theta = \frac{mV^2}{R} \text{ (Centripetal force eqn)}$$

or

$$T = mg\cos\theta + \frac{mV^2}{R}$$

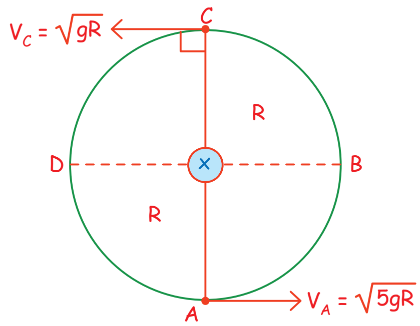
जैसे जैसे ball ऊपर जाएगी tension कम होती जाएगी, for minimum velocity at C, highest point पर tension zero करदो ऐसा करने पर mg ने इज्जत बचा ली बोले तो centripital force requirement fullfill करदी।



T_{max} = lower + point
T_{min} = highest point

at C $\Rightarrow mg + T_C = \dfrac{mV_C^2}{R}$

$T_C \rightarrow$ min = 0



$$mg = \frac{mV_C^2}{R}$$

$$V_C = \sqrt{gR}$$

$(C \rightarrow A)$

$w_g + w_T = \Delta KE$

$$mg2R + 0 = \frac{1}{2}mV_A^2 - \frac{1}{2}m(gR)$$

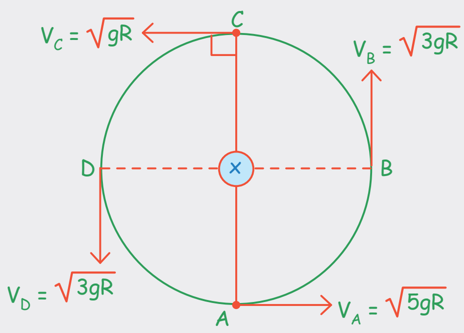
$$4mgR = mV_A^2 - mgR$$

then if $V_C = \sqrt{gR}$, $V_A = \sqrt{5gR}$

If $V_C = \sqrt{gR}$ then $V_A = \sqrt{5gR}$

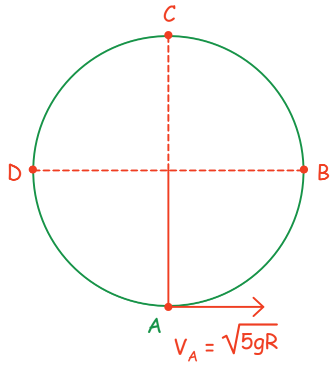
<u>This is Very Important Part</u>

Minimum velocity at A to complete vertical circular motion is $\sqrt{5gR}$, इस case मे remember हमने at highest point $T_c = 0$ किया, mg ने इज्जत बचाई, $V_b = \sqrt{3gR}$ और $V_c = \sqrt{gR}$



अब आपको हर point पर V, T, a_t, a_c निकालना आना चाहिए मै मस्त तरीके से हर एक-एक चीज को solve करके represent कर दे रहा हुँ plz इन्हे खुद से जरूर solve करें cost same को लेकर भाई publisher से मे लड़ लुँगा।

Q. Find T, v, a_t, a_c, a_{net} at A, B, C, D points in given case.



Sol.

At point 'A'

★ $T_A - mg = \dfrac{mV_A^2}{R}$

$T_A - mg = \dfrac{m5gR}{R}$

$T_A = 6\ mg$

★ $a_c = \dfrac{V^2}{R} = \dfrac{5gR}{R} = 5g$

★ $a_t = 0$

★ $a_{net} = \sqrt{a_t^2 + a_c^2} = 5g$

★ $V_A = \sqrt{5gR}$

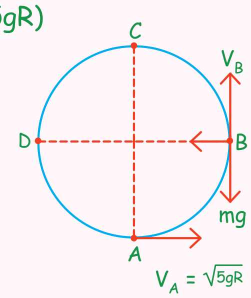
At point 'B' and 'D'

★ V_B : WET from $A \to B$

$w_g + w_T = \Delta KE$

$-mgR + 0 = \dfrac{1}{2} mV_B^2 - \dfrac{1}{2} m\ (5gR)$

$V_B = \sqrt{3gR}$



★ $a_C : \dfrac{V^2}{R} = \dfrac{\sqrt{(3gR)^2}}{R} = 3g$

★ $T_B = \dfrac{mV^2}{R} = \dfrac{m3gR}{R} = 3\ mg$

★ $mg = ma_t$

$a_t = g$ (downward)

★ $a_{net} = \sqrt{(3g)^2 + (g)^2} = g\sqrt{10}$

At point 'C'

★ V_C : apply WET from $A \to C$

$w_g + w_T = \Delta KE$

$-mg2R + 0 = \dfrac{1}{2} mV_C^2 - \dfrac{1}{2} m5gR;\ V_C = \sqrt{gR}$

★ $T_C : T + mg = \dfrac{mV^2}{R}$

$T + mg = \dfrac{mgR}{R} \Rightarrow T_C = 0$

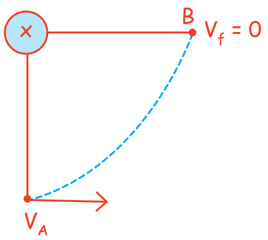
★ $a_C = \dfrac{V^2}{R} = \dfrac{gR}{R} = g$

★ $a_t = 0$

★ $a_{net} = \sqrt{(g)^2 + (0)^2} = g$

	A	B, D	C	P(general point)
Velocity	$\sqrt{5gl}$	$\sqrt{3gl}$	$\sqrt{gl}$	$\sqrt{gl(3+2\cos\theta)}$
Tension	6mg	3mg	0	$3mg(1 + \cos\theta)$
Potential Energy	0	mgl	2mgl	$mgl(1 - \cos\theta)$
Radial acceleration	5g	3g	g	$g(3 + 2\cos\theta)$
Tangential acceleration	0	g	0	$g \sin\theta$

Q. Find velocity at A (lowest point) so that, string become horizontal & particle comes to rest at B.

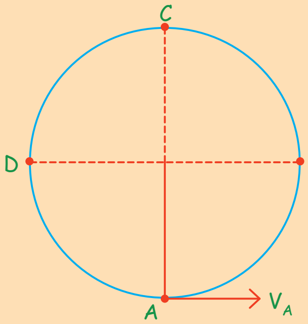


Sol. $w_g + w_T = \Delta KE$

$-mgR + 0 = 0 - \dfrac{1}{2} mV_A^2,\ V_A < \sqrt{2gR}$

काम का डब्बा

Particle given horizontal velocity at A

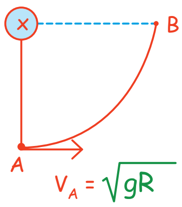


ये result जरूर याद कर लेना bcz इसके बाद आपको पहले से पता होगा की particle का motion क्या होगा।

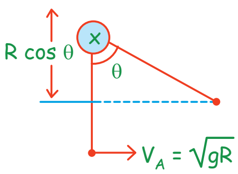
- ★ $V_A < \sqrt{2gR}$ = particle comes to rest before 'B' (v = 0 between A & B)
- ★ $V_A = \sqrt{2gR}$ = particle comes to rest at 'B'
- ★ $V_A = \sqrt{5gR}$ = vertical circular motion just completed ($T_C = 0$, $V_C = \sqrt{gR}$)
- ★ $V_A > \sqrt{5gR}$ ⇒ चमचमाता vertical circular motion चमचमाता $T_C \neq 0$
- ★ $\sqrt{2gR} < V_A < \sqrt{5gR}$ ⇒ somewhere between B & C, T will zero and subsequently motion will be projectile.

Q. 1. Find the 'maximum' height attained by particle.

2. Find maximum angle by string with vertical.



Sol. $w_g + w_T = \Delta KE$



$$-mg\,(R - R\cos\theta) + 0 = 0 - \frac{1}{2}m\,(\sqrt{gR})^2$$

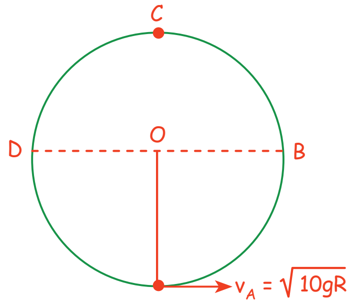
$$\cos\theta = \frac{1}{2} \quad \Rightarrow \theta = 60$$

$$h = R - R\cos 60° = R/2$$

Solve above question again if $V_A = \sqrt{1.5gR}$

Ans. $\theta = \cos^{-1}(1/4)$ (verify this on your own)

Q. Find T, v at A, B, C.



Sol.

★ at 'A'

$$T_A:\ T_A - mg = \frac{m10g\cancel{R}}{\cancel{R}}$$

$$T_A = 11mg$$

★ at 'B'

Applying WET from A → B

$$-mgR = \frac{1}{2}mv_B^2 - \frac{1}{2}m\times(\sqrt{10gR})^2$$

$$v_B = \sqrt{8gR}$$

$$T_B = \frac{mv_B^2}{R} \quad \therefore \frac{m8gR}{R} = 8mg$$

★ at 'C'

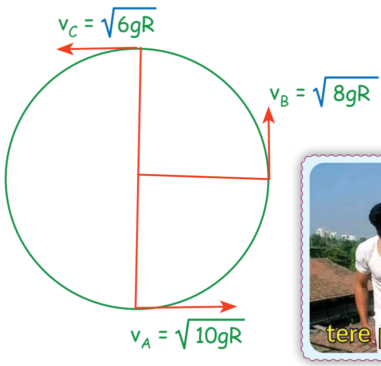
$$-mgh + 0 = \frac{1}{2}mv_c^2 + \frac{-1}{2}m10gR$$

$$v_C = \sqrt{6gR}$$

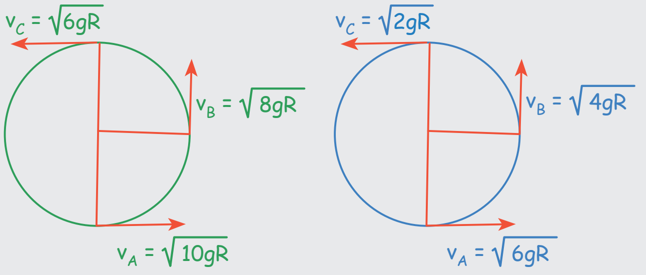
$$T_C + mg = \frac{mv_C^2}{R}$$

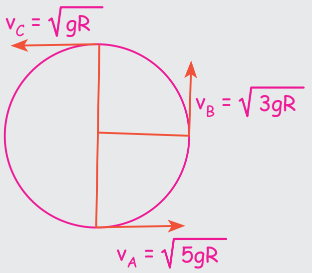
$$T_C = 5mg$$

Find $\frac{T_{max}}{T_{min}} = ? \Rightarrow \frac{T_{max}}{T_{min}} = \frac{T_A}{T_C}$



देख भाई ये pattern पहचान ने की कोशिश करो अगर A पर velocity $\sqrt{10gR}$ है तो B पर velocity $\sqrt{8gR}$ आई और C पर velocity $\sqrt{6gR}$ आई मतलब underroot के अंदर की digit दो कम होती जा रही है (minus two).... अबे समझ आया ले और example देख





Q. A small bob tied at one end of a thin string of length 1m is describing a vertical circle so that the maximum and minimum tension in the string are in the ratio 5 : 1. The velocity of the bob at the highest position is ________ m/s. (Take $g = 10\ m/s^2$)

Sol. Let v is velocity at highest position.

$T_{max} = 5T_{min}$

$$\Rightarrow \frac{mg + \frac{m(v^2+4gl)}{l}}{5\frac{mv^2}{l} - mg} \Rightarrow 4.\frac{v^2}{l} = 10g \Rightarrow \frac{T_{max}}{T_{min}} = 5$$

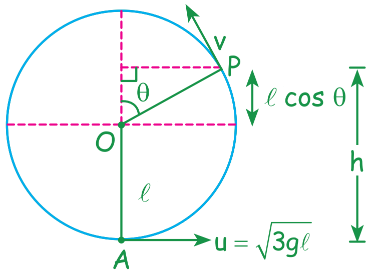
$$\Rightarrow v = \sqrt{\frac{5}{2}gl.} = \sqrt{\frac{5}{2}\times 10\times 1} = 5\ m/s$$

Q. A bob of a stationary pendulum is given a sharp hit to impart it a horizontal speed of $\sqrt{3gl}$. Find the angle rotated by the string before it becomes slack.

Sol. Suppose the string becomes slack at point P.

Let the bob rise to a height h.

$h = \ell + \ell\cos\theta$



From the work-energy theorem,

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = -mgh$$

$v^2 = u^2 - 2g(\ell + \ell\cos\theta)$ ...(i)

Again, $\frac{mv^2}{\ell} = mg\cos\theta$

$v^2 = \ell g\cos\theta$ ...(ii)

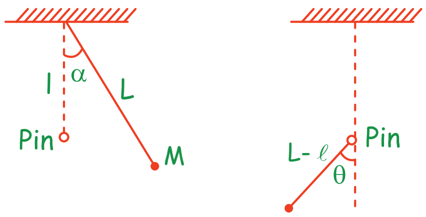
Using equation (i) and (ii) and the value of u, we get,

$g\ell\cos\theta = 3g\ell - 2g\ell - 2g\ell\cos\theta$

$3\cos\theta = 1$

$\cos\theta = \frac{1}{3}$

Q. A simple pendulum consisting of a mass M attached to a string of length L = 7m is released from rest at an angle $\alpha = 37°$. A pin is located at a distance ℓ = 4m below the pivot point. When the pendulum swings down, the string hits the pin as shown in figure. The maximum angle which the string makes with the vertical after hitting the pin is θ. Find the value of cosθ.



Sol. Since the pendulum started with no kinetic energy, conservation of energy implies that the potential energy at θ_{max} must be equal to the original potential energy, i.e., the vertical position will be same. Therefore,

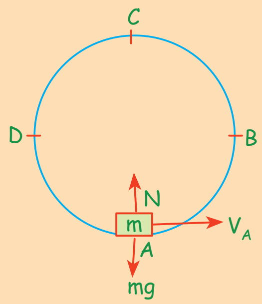
$mg\cos\alpha = mg[\ell + (L-\ell)\cos\theta]$

$$\Rightarrow \cos\theta = \frac{L\cos\alpha - \ell}{L-\ell}$$

$$\Rightarrow \theta = \cos^{-1}\left[\frac{L\cos\alpha - 1}{L-\ell}\right]$$

काम का डब्बा

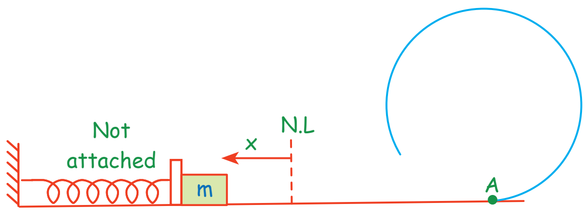
Particle given horizontal velocity at A



ये result बिल्कुल पहले जैसे है बस tension की जगह अब normal आगया और भाई ये VCM है horizontal नही be carefull

- ★ $V_A < \sqrt{2gR}$ = particle comes to rest before 'B' (v = 0 between A & B)
- ★ $V_A = \sqrt{2gR}$ = particle comes to rest at 'B'
- ★ $V_A = \sqrt{5gR}$ = vertical circular motion just completed ($N_C = 0$, $V_C = \sqrt{gR}$)
- ★ $V_A > \sqrt{5gR} \Rightarrow$ चमचमाता vertical circular motion चमचमाता $N_C \neq 0$
- ★ $\sqrt{2gR} < V_A < \sqrt{5gR} \Rightarrow$ somewhere between B & C, N will zero and subsequently motion will be projectile.

Q. Block is released from rest in following fig. Find minimum compression in spring so that block completes circle. Block not attached to spring.



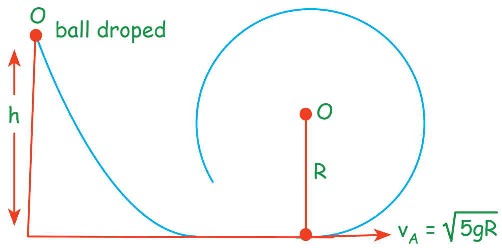
Sol. A पर minimum velocity $\sqrt{5gR}$ होनी चाहिए

$W_{SP} + W_g + W_N + W_f = \Delta KE$

$\frac{-1}{2}k(0 - x^2) + 0 + 0 + 0 = 1/2m(\sqrt{5gR})^2 - 0$

$\frac{1}{2}kx^2 = \frac{1}{2}m5gR \qquad x = \frac{\sqrt{5gRm}}{\sqrt{k}}$

Q. Find minimum value of h to complete circle.

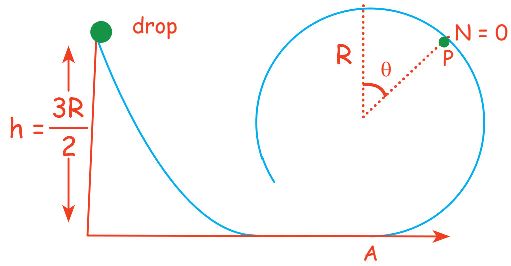


Sol. $W_g + W_N = \Delta KE$

$mgh + 0 = 1/2m(5gR) - 0$

$h = \frac{5R}{2} = h_{min}$

Q. If particle loose contact at point P find θ.

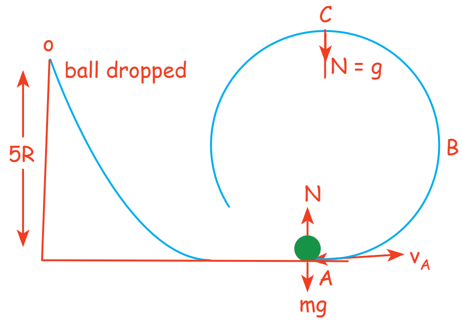


Sol. $mg\left(\frac{3R}{2}\right) + 0 = 1/2mv_A^2 - 0$

$v_A = \sqrt{3\,gR} \Rightarrow \cos\theta = \frac{1}{3}$

H.W. If in above ques. h = 7R/4. Find θ.... (ans. 60°)

Q. Find force applied by track when particle reaches at A & C



Sol. $mg(5R) + 0 = \frac{1}{2}mv_A^2 - 0$

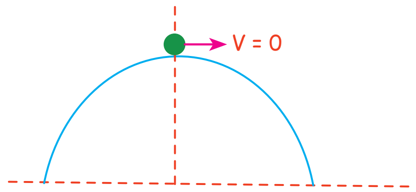
$v_A = \sqrt{2g(5R)} = \quad v_A = \sqrt{10gR}$

$N_A - mg = \frac{mv_A^2}{R} = 11mg$

$v_C = \sqrt{6gR}$

$N_C + mg = \frac{mv_C^2}{R}$ (Now solve and get)

Q. If a ball is slightly displace to right
Find the angle at which ball loses contact with the surface.

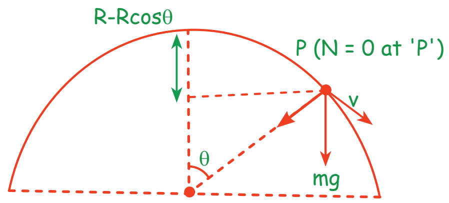


Sol. Suppose at point 'P' contact is lost (N = 0)

$W_g + W_N = \Delta KE$

$mg(R - R\cos\theta) + 0 = 1/2mv^2 - 0$ (i)

and $mg\cos\theta - N = \dfrac{mv^2}{R}$ (ii)

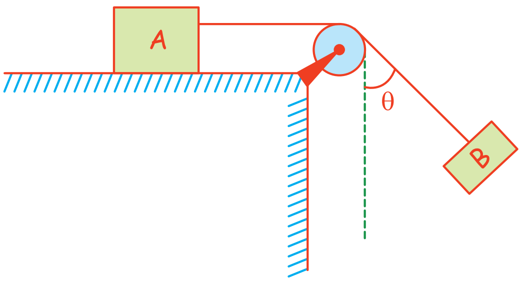


Solve (i) and (ii) and get

$\cos\theta = 2/3$

अबे क्या तुमने ये सोचा है की ये particle ground पर कहाँ जाके गिरेगा बोले तो distance of the point from centre where it will strike the ground...... चलो इसको solve करो मस्त सवाल है and its ans. is 1.46 R

Q. Two particles A and B each of mass m are connected by a massless string. A is placed on the rough table. The string passes over a small, smooth peg. B is left from a position making an angle θ with the vertical. Find the minimum coefficient of friction between A and the table so that A does not slip during the motion of mass B.



Sol. Block B rotates in vertical plane. Tension is maximum in string at lowest position. When block B is at lowest position and block A does not slide then it will not slide at any other position of B.

At lowest position

$T - mg = \dfrac{mv^2}{\ell} \Rightarrow T = mg + \dfrac{mv^2}{\ell}$ ... (*i*)

From energy conservation

$mg\,\ell\,(1 - \cos\theta) = \dfrac{1}{2}\,mv^2$ ... (*ii*)

from equation (i) and (ii)

$T = mg + 2mg\,(1 - \cos\theta) = 3mg - 2mg\cos\theta$

For no slipping.

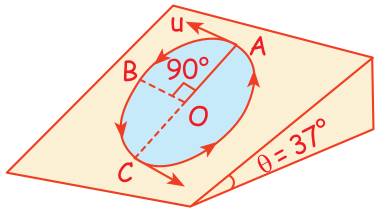
$T = \mu mg$

$\Rightarrow 3mg - 2mg\cos\theta \le \mu mg$

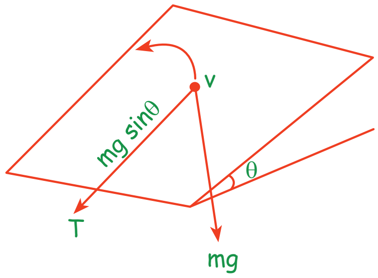
$\Rightarrow \mu \ge 3 - 2\cos\theta$

or $\mu_{min} = 3 - 2\cos\theta$

Q. A small sphere of mass m is connected by a string to a nail at O and moves in a circle of radius 'r' on the smooth plane inclined at an angle θ with the horizontal. If the sphere has a velocity u at the top position A. Mark the correct option(s).



Sol. (a)



$T + mg\sin 37° = \dfrac{mv^2}{r}$ $[\because T = 0]$

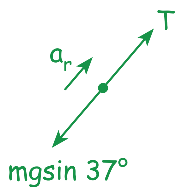
$\therefore v = \sqrt{\dfrac{3gr}{5}}$

(b) $mg\,(2r\sin 37°) = \dfrac{1}{2}mv^2 - \dfrac{1}{2}m\left(\dfrac{3gr}{5}\right)$

$\therefore \dfrac{mv^2}{r} = 3\,mg$

$T - mg\sin 37° = ma_r$

$\therefore T = \dfrac{18mg}{5}$



मानलो अगर inclined plane rough होता $\mu = \tan\theta$ तो पता करो tension कहाँ minimum होगी।

CONSERVATIVE FORCE

- ★ Those forces whose work done is independent of path taken i.e., their work depend on initial & final position only.
- ★ (WD) by such a force in a closed loop is always zero
- ★ Gravitation force, spring force, electrostatic force are the examples of conservative forces.
- ★ The concept of potential energy exists only in the case of conservative forces.

NON-CONSERVATIVE FORCE

- ★ A force is said to be non-conservative if work done by the force in moving a body depends upon the path taken between the initial and final positions.
- ★ Friction force, velocity-dependent forces such as air resistance, viscous force etc., are non-conservative forces.

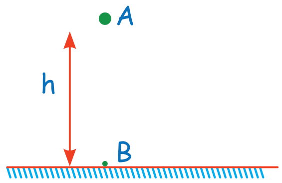
POTENTIAL ENERGY(U)

Potential energy of the system is defined as negative of WD by internal conservative forces

- ★ $\Delta U = -(WD)_{\text{internal conservative forces (icf)}}$
- ★ $\Delta U = -\int \vec{F}\cdot\vec{dr}$
- ★ $dU = -\vec{F}\cdot\vec{dr}$
- ★ $\Delta U = U_f - U_i = -WD = -\int_i^f \vec{F}\cdot\vec{dr}$
- ★ $\Delta U \rightarrow$ same for all frame
- ★ P.E only defined for conservative force
- ★ P.E is defined for system of particle
- ★ P.E at a point depends on reference line

  But ΔU ("change" in P.E) it doesn't depend on Reference line

Gravitational Potential Energy

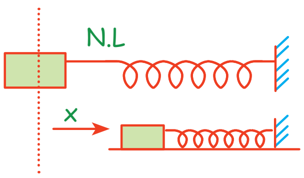


Reference
(U = 0)

$\Delta U = U_A - U_B = mgh$

If $U_B = 0 \Rightarrow U_A = mgh$

Potential Energy in Spring



$\Delta U = -(WD)_{icf} = -(WD)_{Sp}$

$\Delta U = -\left(-\left(\frac{1}{2}\right)k(x_f^2 - x_i^2)\right)$

$U_f - U_i = \left(\frac{1}{2}\right)kx_f^2$

Since at N.L, $U_i = 0$

so $U_f = \left(\frac{1}{2}\right)kx_f^2$

SKC

'h' उपर P.E mgh तभी होती जब जमीन पर P.E = 0 मानोगे अगर spring x दबा है या x खिचा तो उसके पेट में $1/2kx^2$ PE होगी।

अब अगले article मे maths maths and maths है so sachin sir का नाम लो और maths का mind on करलो

RELATION BETWEEN COMERVATIVE FORCE & P.E

जब F देकर U पूछे

$dU = -\vec{F}\cdot\vec{dr}$

Let $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$\vec{dr} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$\vec{F}\cdot\vec{dr} = F_x dx + F_y dy + F_z dz$

$\Delta U = U_f - U_i = -\int_i^f \vec{F}\cdot\vec{dr}$

$\Delta U = -[\int F_x dx + \int F_y dy + \int F_z dz]$

यहाँ definition का एक – (minus) आ रहा है इसे गलती से भी मत भूलना वरना पिटाई होगी।

Q. For a conservative force, $\vec{F} = -4x\hat{i} + 10\hat{j} + 20\hat{k}$ If P.E. at origin is 20J. Find P.E. at (1, 2, 3) will be?

Sol. $U_f - 20 = -\left[\int_0^1 -4x\,dx + \int_0^2 10\,dy + \int_0^3 20\,dz\right]$

जब U देकर $\vec{F}$ पूछे तब:

$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right]$

$F_x = \frac{-\partial U}{\partial x}\hat{i};\ F_y = \frac{-\partial U}{\partial y}\hat{j};\ F_z = \frac{-\partial U}{\partial Z}\hat{k}$

यहाँ भी definition का एक – (minus) आ रहा है इसे भी गलती से भी मत भूलना वरना.....

$\frac{\partial U}{\partial x}$ = differentiation of U wrt 'x' by keeping 'y' and 'z' constants. (Partial Differentiation)

$\frac{\partial U}{\partial y}$ = differentiation of U wrt 'y' by keeping 'x' and 'z' constant

$\frac{\partial U}{\partial z}$ = differentiation of U wrt 'z' by keeping 'x' and 'y' constant

Q. Given $U = x^2y^2$ Find $\vec{F}$ at (2, 3).

Sol. $F_x = \frac{-dU}{dx} = -2xy^2$

$\vec{F_y} = \frac{-dU}{dy} = -x^2 2y;\ F_z = \frac{-\partial U}{\partial z} = 0$

$\vec{F}_{net} = -2xy^2\hat{i} - x^2 2y\hat{j}$

at (2, 3) $\vec{F} = -36\hat{i} - 24\hat{j}$

Q. If U = 5xy Find $\vec{F}$ on particle at (2, 3)

Sol. $F_x = -[5y]$

$F_y = -[5x]$, $F_z = 0$

$\vec{F} = 15\hat{i} - 10\hat{j}$

Q. Given $U = ax^2 - bxy^2$ solve for $\vec{F}$

Sol. $F_x = -(a2x - by^2 1)$

$F_y = 0 - bx\,.\,2y \Rightarrow \vec{F} = (-2ax + by^2)\hat{i} + (2bxy)\hat{j}$

Q. Given $U = x^2 + y^2 + z^2$, find $\vec{F}$.

Sol. $F_x = -(2x + 0 + 0)$

$F_y = -(0 + 2y + 0)$

$F_z = -(0 + 0 + 2z) \Rightarrow \vec{F} = -2x\hat{i} - 2y\hat{j} - 2z\hat{k}$

Q. Given $U = x^2 - 8x$ Find equilibrium position.

Sol. at equilibrium $F = \frac{-\partial U}{\partial x} = 0$

or $\frac{dU}{dx} = 0$

$\Rightarrow 2x - 8 = 0$

$\boxed{x = 4}$

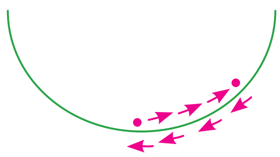
TYPES OF EQUILIBRIUM

★ Type of equilibrium are

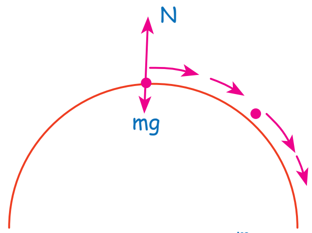
(1) stable equilibrium

(2) Unstable equilibrium

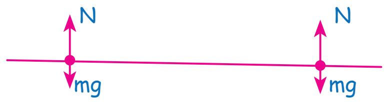
(3) Neutral equilibrium

1. 

Stable eqb^m

2. 

Unstable eqb^m

3. 

Neutral eqb^m

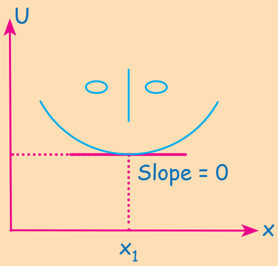
SKC

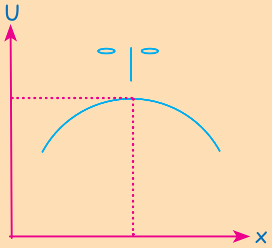
देख भाई result यह है कि, equillibrium position से थोड़ा सा displace कर अगर छोड़ कर गई तो बंदा unstable है, अगर वापस आई तो बंदा stable है और जब वापस आती है तो खुशी (happy) होती है और छोड़ कर जाती है तो दुख (sad) होता है।

काम का डब्बा

★ $F = -\dfrac{dU}{dx} = -(\text{slope})$

★ F = –(slope of U x graph)





At stable equil

slop = 0, $\dfrac{dU}{dx} = 0$

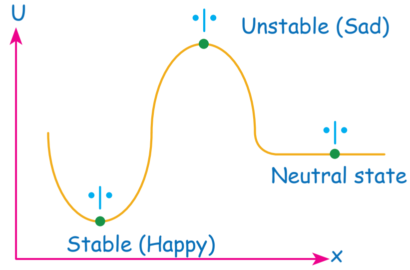
$\dfrac{d^2U}{dx^2} > 0$

At unstable equibm

slop = 0, $\dfrac{dU}{dx} = 0$

$\dfrac{d^2U}{dx^2} < 0$

Stable (Happy)	Unstable (Sad)	Neutral
$\dfrac{du}{dx} = 0$	$\dfrac{du}{dx} = 0$	$\dfrac{du}{dx} = 0$
$\dfrac{d^2u}{dx^2} > 0$	$\dfrac{d^2u}{dx^2} < 0$	$\dfrac{d^2u}{dx^2} = 0$



जब U का x की term मे function given हो और stable or unstable पता करना हो सबसे पहले $\dfrac{dU}{dx} = 0$ करके equillibrium position निकालो फिर x की उन value पर $\dfrac{d^2U}{dx^2}$ निकालो अगर $\dfrac{d^2U}{dx^2} > 0$ आया तो stable equil और $\dfrac{d^2U}{dx^2} < 0$ आया तो unstable equil और $\dfrac{d^2U}{dx^2} = 0$ आया तो neutral equil.

Q. For a given potential $U = x^2 - 8x$,. Find position of equilibrium and its type?

Sol. $\dfrac{du}{dx} = 2x - 8 = 0 \Rightarrow x = 4$

Now $\dfrac{d^2u}{dx^2} = 2$ (ये तो positive आगया) $\dfrac{d^2U}{dx^2} > 0$ Hence,

x = 4, is stable equilibrium point

Q. For a given potential $U = \dfrac{x^3}{3} - 9x$ find equilibrium and its types ?

Sol. $\dfrac{dU}{dx} = \dfrac{3x^2}{3} - 9 = x^2 - 9$

Put $\dfrac{dU}{dx} = 0 \Rightarrow x^2 - 9 = 0$

$\Rightarrow x = +3, -3$ (equilibrium position)

Now $\dfrac{d^2u}{dx^2} = 2x$

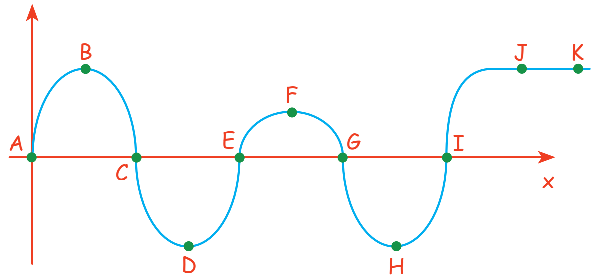
For x = 3 find $\dfrac{d^2u}{dx^2}$

$\dfrac{d^2u}{dx^2} = +6 > 0$ so, x = 3 is unstable equilibrium point)

For x = –3

$\dfrac{d^2U}{dx^2} = 2 \times -3 = -6$ so x = –3 is unstable equilibrium point)

Q. Identify stable & unstable equilibrium positions in following.



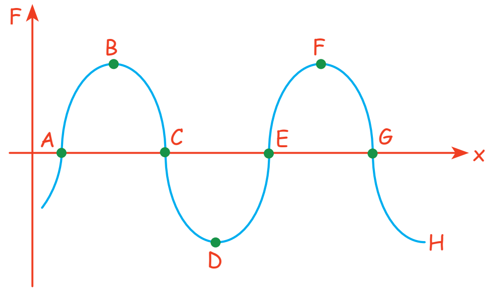
Sol. Slope = 0, $\frac{dU}{dx} = 0$ = (at equilibrium)

B, F $\rightarrow$ Unstable equilibrium.

D, H $\rightarrow$ stable equilibrium

J, K: Neutral equilibrium

Q. In following figure, force(F) vs. Position 'x' curve is shown, Identify stable, unstable and neutral points



Sol. eqbm. वहाँ होगी जहाँ $F = \frac{dU}{dx} = 0;$

So, A, C, E, G are equilibrium points.

A, E $\rightarrow$ stable $\left(\because \frac{dF}{dx} > 0\right)$

C, G $\rightarrow$ unstable $\left(\because \frac{dF}{dx} < 0\right)$

Q. The potential energy between two atoms in a molecule is given by, $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where '$a$' and b are positive constants and x is the distance between the atoms. The system is in stable equilibrium when

Sol. Given that, $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$

We, know $F = -\frac{dU}{dx} \quad -\left(\frac{-12a}{x^{13}} + \frac{6b}{x^7}\right) = 0$

or $\frac{6b}{x^7} = \frac{12a}{x^{13}}$ or $x^6 = 12a/6b = 2a/b$

or $x = \left(\frac{2a}{b}\right)^{1/6}$

Q. The potential energy of a conservative system is given by $U = ax^2 - bx$ where a and b are positive constants. Find the equilibrium position and discuss whether the equilibrium is stable, unstable or neutral.

Sol. In a conservative field, $F = -\frac{dU}{dx}$

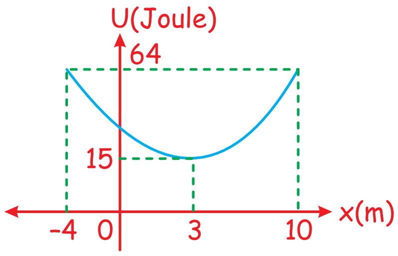
$\therefore F = -\frac{d}{dx}(ax^2 - bx) = b - 2ax$

For equilibrium, $F = 0$ or $b - 2ax = 0 \Rightarrow x = \frac{b}{2a}$

From the given equation we can see that $\frac{d^2U}{dx^2} = 2a$ (positive), i.e., U is minimum.

Therefore, $x = b/2a$ is the stable equilibrium position.

Q. A single conservative force $F(x)$ acts on a particle that moves along the x-axis. The graph of the potential energy with x is given. At $x = 5$ m, the particle has a kinetic energy of 50 J and its potential energy is related to position 'x' as $U = 15 + (x - 3)^2$ Joule, where x is in meter.



Sol. At $x = 5$m, $\quad KE = 50$J

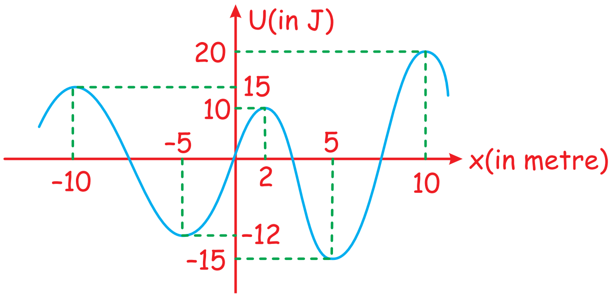
$PE = 15 + (5 - 3)^2 = 19$ J

Mechanical energy = 69 J

KE_{max} = Total Energy – PE_{mini} = 69 – 15 = 54 J

Comprehension questions:

In the figure the variation of potential energy of a particle of mass $m = 2$ kg is represented with respect to its x-coordinate. The particle moves under the effect of this conservative force along the x-axis.



(a) If it is relesed at the origin it will move in which direction?

Sol. It will move along the direction of force, as $\frac{dU}{dx} > 0;\ F = \frac{-dU}{dx}$ negative hence particle will move along negative x-axis towards origin.

(b) If the particle is released at $x = 2 + \Delta x$ where $\Delta x \to 0$ (it is positive) then its maximum speed in subsequent motion will be

Sol. When the particle is released at $x = 2 + \Delta x$ it will reach the point of least possible potential energy (–15 J) where it will have maximum kinetic energy.

$\therefore\ \frac{1}{2}mv_{max}^2 = 25 \quad \Rightarrow \quad v_{max} = 5$ m/s

(c) $x = -5$ m and $x = 10$ m positions represents which type of equilibrium?

Sol. Conditions for stable equilibrium is:

$\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} > 0$

Condition for unstable equilibrium is:

$\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} < 0$

hence x = -5 is stable and x = 10 m is unstable equilibrium position

MECHANICAL ENERGY CONSERVATION

w_{iCF} = work by internal conservative force

According to work energy theorem

$(w)_{\text{all the forces}} = \Delta KE$

$(w)_{ext} + (WD)_{iCF} + (WD)_{\text{internal non conservative}} = \Delta KE$

If $w_{ext} = 0$ and $(WD)_{\text{internal non conservative}} = 0$

$\Rightarrow 0 + (WD)_{iCF} + 0 = \Delta KE$

$-\Delta U = \Delta KE$

$-(U_f - U_i) = K_f - K_i$

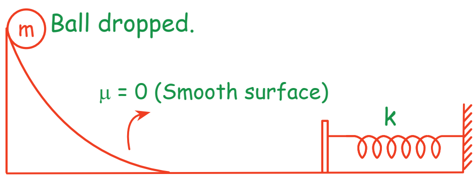
$\boxed{K_i + U_i = K_f + U_f}$

or $(ME)_i = (ME)_f$

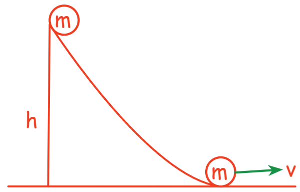
(mechanical energy conservation)

अगर हम ध्यान से देखे तो ये WET का एक special case है जहाँ अगर $(WD)_{ext} = 0$ and $(WD)_{inc} = 0$ हुआ तो mechanical energy will conserve

Q. Find maximum compression.



Sol.

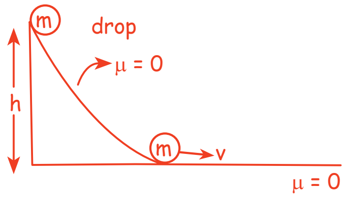


$K_i + U_i = k_f + U_f$

$0 + mgh = 0 + \left(0 + \frac{1}{2}kx^2_{max}\right)$

$x_{max} = \sqrt{\frac{2mgh}{k}}$

Q. Find final velocity of ball in following problem.



Sol. $W_g + W_N + W_f = \Delta KE$

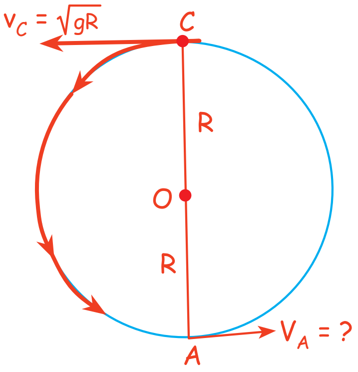
$mgh + 0 + 0 = \frac{1}{2}mv^2 - 0,\ V = \sqrt{2gh}$

By M.E. conservation

$K_i + U_i = K_f + U_f$

$0 + mgh = \frac{1}{2}mv^2 + 0,\quad V = \sqrt{2gh}$

Q. Find velocity at point 'A' (V_A) if a particle performing vertical circular motion and starts at point 'c' with velocity $V_c = \sqrt{gR}$



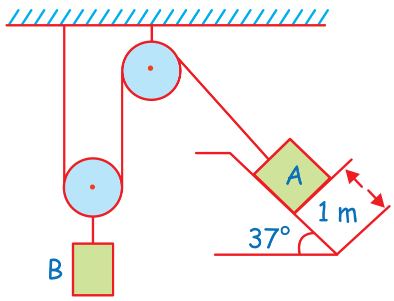
Sol. $U_C + K_C = U_A + K_A$

$$mg2R + \frac{1}{2}m(\sqrt{gh})^2 = 0 + \frac{1}{2}mV_A^2$$

$$2mgR + \frac{mgR}{2} = \frac{1}{2}mV_A^2$$

$$V_A = \sqrt{5gR}$$

Q. The blocks shown in figure have masses $M_A = 5$ kg and $M_B = 4$ kg. The system is released from rest. The speed of B after A has travelled a distance 1 m along the incline is



Sol. If A moves down the incline by 1 m, B shall move up by $\frac{1}{2}$ m. If the speed of B is v, then the speed of A will be 2v.

From conservation of energy,

Gain in KE = loss in PE

$$\frac{1}{2}m_A(2v)^2 + \frac{1}{2}m_B v^2 = m_A g \times \frac{3}{5} - m_B g \times \frac{1}{2}$$

Solving, we get

$$v = \frac{1}{2}\sqrt{\frac{g}{3}}$$

POWER

ये सारे point अच्छे से देख लेना।

1. Rate of work done
2. $P = \frac{dw}{dt} = \frac{\vec{F}\cdot\vec{dr}}{dt}$

Unit: Joule/sec. or Watt.

3. $P = \vec{F}\cdot\vec{v}$
4. $P = \frac{dw}{dt}$ or $\int dw = \int Pdt$
5. WD = $\int Pdt$

   means P vs t के graph का area W.D देगा
6. Average power = $\frac{\int Pdt}{\int dt}$

Q. If work as a function of 't' given as $w = 4t^2 + 3t$. Find 'P' at t = 2 sec.

Sol. $P = \frac{dw}{dt} = 8t + 3$

at t = 2, P = 19 watt

Q. Find power by friction & gravity at t = 5 sec.

Sol. f = 0.2 × 100 = 20 N

$a = \mu g = 0.2 \times 10 = 2$ (deceleration)

Velocity at t = 5 will be, v = 100 – 2 × 5 = 90 m/s

Power by friction at t = 5 sec

$= \vec{F}\cdot\vec{v} = f\,v\cos 180° = = -20 \times 90 = -1800$ watt

Power supply by gravity at t = 5 sec = $\vec{F_g}\cdot\vec{v} = (mg).v\cos 90° = 0$,

P = 0 (Bcz $\theta = 90°$)

Q. $P = 2t^2 + t$ Find power at t = 2 sec

Sol. $P_{inst} = 2 \times 2^2 + 2 = 10$

Find avg. power from $t = 0 \to t = 2$sec

Avg. power $= \frac{\int Pdt}{\int dt} = \int_0^2 \frac{(2t^2 + t)dt}{dt}$ (solve)

Q. A particle of mass 2 kg is projected with speed 100 m/s at an angle 37° with horizontal. Find power supply by gravity at t = 2sec.

Sol. At t = 2 $\quad \vec{v} = 80\hat{i} + 40\hat{j}$ and $\vec{F} = mg = -20\hat{j}$

$P = \vec{F}.\vec{v} = -20 \times 40 = -800$ watt

ये question हर साल JEE Mains मे पूछा जा रहा है इसका result याद करलो।

Q. A body is moving unidirectionally under the influence of a constant power source. Its displacement in time t is proportional to:

Sol. P = Fv = constant

$$\Rightarrow m\frac{dv}{dt}v = \text{constant}$$

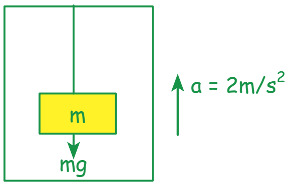
$$\int_0^v vdv = (C)\int_0^t dt \Rightarrow \left(\frac{v^2}{2}\right) = Ct$$

$$v \propto t^{1/2} \Rightarrow \frac{ds}{dt} \propto t^{1/2}$$

$$\int_0^S ds = K\int_0^t t^{1/2}dt \Rightarrow S = K \times \frac{2}{3}t^{3/2}$$

$$S \propto t^{3/2}$$

Q. Lift starts moving upward from rest having acceleration $2 m/s^2$ upward at t = 0. Find Power supply by tension & gravity to the block at t = 5 sec in ground frame (m = 2kg).



Sol. $P = \vec{F} \cdot \vec{v}$

v = u + at = 0 + 2 × 5 $\quad$ v = 10m/s

Power supplied by gravity = –mgv = –20 × 10

= –200 watt

Power supplied by tension force = +T.v

= 24 × 10 = 240 watt

$\Rightarrow$ T = 20 + 2 × 2 = 24 N $\quad (\because T = Tg + ma)$

काम का डब्बा

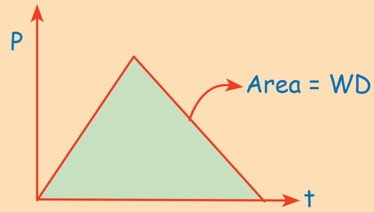
★ $P = \vec{F} \cdot \vec{v}$

★ $P = \frac{dw}{dt}$ (instantaneous power)

★ $WD = \int Pdt$

★ Avrg power $= \frac{\int pdt}{\int dt}$

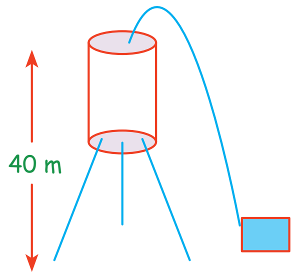
★ Area of power time graph given work.



$WD = \int Pdt$ = area under curve in P vs t graph

★ Efficiecncy = $\frac{\text{output power}}{\text{input power}}$

Q. A water pump rated 600 watt has efficiency 75%. If it is employ to raise water through a height of 40 meter. Find the mass of water drawn in 2 hour.



Sol. $\frac{dw}{dt} = \frac{d}{dt}(mgh)$

$$P = \frac{dw}{dt} = gh\left(\frac{dm}{dt}\right)$$

$$\boxed{\frac{dm}{dt} = \frac{p}{gh}}$$

Given $P = 600 \times \frac{75}{100} = 450$ watt

water drawn in 2 hour $= \frac{450}{10 \times 40} \times (3600 \times 2)$

= 8100 kg = 8.1 L

Q. Wind enters in a wind mill with a velocity of 20 m/sec. Facing area of the windmill is 10 m^2 and density of air is 1.2 kg/m^3. If wind energy is converted into electrical energy with 33.3% efficiency, then electrical power produced by the wind mill in kW is

Sol. Energy entering in the windmill = $\frac{1}{2}mv^2$

$$P_{in} = \frac{dE}{dt} = \left(\frac{1}{2}v^2\right)\left(\frac{dm}{dt}\right)$$

$$P_{in} = \left(\frac{1}{2}v^2\right)(\rho Av) = \frac{1}{2}\rho Av^3$$

Electrical power output

$$P_{out} = \frac{1}{3}\left(\frac{1}{2}\rho Av^3\right)$$

$$P_{out} = \frac{1}{6}\rho Av^3 = \frac{1}{6}\times 1.2\times 10\times (20)^3$$

$\Rightarrow P_{out}$ = 16 kW

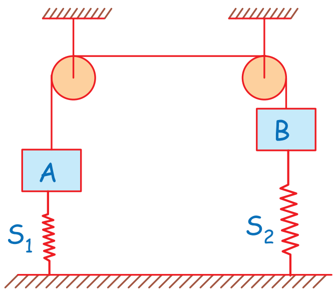
काम का डब्बा

Let us summarise the concepts developed so far in this chapter.

- ★ Work done on a particle is equal to the change in its kinetic energy.
- ★ Work done on a system by all the (external and internal) forces is equal to the change in its kinetic energy.
- ★ A force is called conservative if the work done by it during a round trip of a system is always zero. The force of gravitation, Coulomb force, force by a spring etc. are conservative. If the work done by it during a round trip is not zero, the force is nonconservative. Friction is an example of nonconservative force.
- ★ The change in the potential energy of a system corresponding to conservative internal forces is equal to negative of the work done by these forces.
- ★ If no external forces act (or the work done by them is zero) and the internal forces are conservative, the mechanical energy of the system remains constant. This is known as the principle of conservation of mechanical energy.
- ★ If some of the internal forces are nonconservative, the mechanical energy of the system is not constant.
- ★ If the internal forces are conservative, the work done by the external forces is equal to the change in mechanical energy.

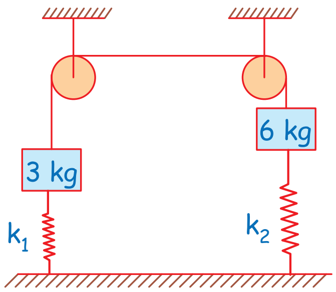
Now Practice following question for Advance

Q. In the figure shown below masses of blocks *A* and *B* are 3 kg and 6 kg respectively. The force constants of springs S_1 and S_2 are 160 N/m and 40 N/m respectively. Length of the light string connecting the blocks is 8 m. The system is released from rest with the springs at their natural lengths. The maximum elongation of spring S_1 will be:



Sol. $\Delta K = 0 \Rightarrow \Delta U_{gravity} = -6gx + 3gx$

$$\Delta U_{spring} = \frac{1}{2}k_2x^2 + \frac{1}{2}k_1x^2$$



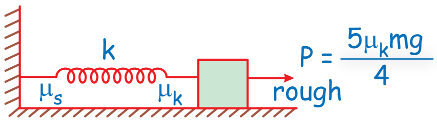
Applying, $\Delta K + \Delta U_{gravity} + \Delta U_{spring} = 0$

$$6g(x) - 3g(x) = \frac{1}{2}k_2x^2 + \frac{1}{2}k_1x^2 \quad [\because \Delta K = 0]$$

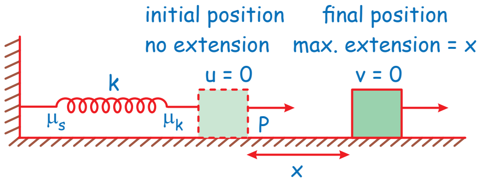
$$6g = (k_1 + k_2)x, x = \frac{6 \times 9.8}{200} = \frac{3 \times 9.8}{100} \Rightarrow x = 0.294 \text{ m}$$

Q. A block of mass m rests on a rough horizontal plane having coefficient of kinetic friction μ_k. The spring is in its natural length, when a constant force of magnitude $P = \frac{5\mu_k mg}{4}$ starts acting on the block. The spring force F is a function of extension x varies as $F = kx^3$. (where k is spring constant)

Find the maximum extension in the spring (Assume that the force P is sufficient to make the block move).



Sol. For motion to start $\frac{5\mu_k mg}{4} > \mu_s mg$ or $5\mu_k > 4\mu_s$



At the final position of the block extension in spring is maximum and the speed of the block is $v = 0$. Hence the net work done in taking the block from initial to final position

ΔW = (Work done by force P + work done by spring force F + work done by friction) = $\Delta K = 0$

$$= Px - \int_0^x Kx^3 . dx - \mu_k mgx$$

$$= \frac{5\mu_k mg}{4}.x - \frac{kx^4}{4} - \mu_k mgx$$

Solving we get, $x = \left(\frac{\mu_k mg}{k}\right)^{1/3}$

Q. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$ where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true?

Sol. $\frac{dK}{dt} = mv\frac{dv}{dt} = \gamma t$

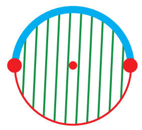
$v\,dv = \frac{\gamma}{m}\, t\, dt$

$\frac{v^2}{2} = \frac{\gamma}{m}\frac{t^2}{2} \Rightarrow v \propto t$

$\frac{dv}{dt}$ = constant $\Rightarrow$ F = constant

$\frac{dx}{dt} \propto t \Rightarrow x \propto t^2$

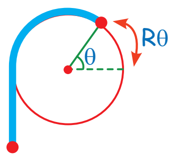
Q. Two identical particles are attached by an inextensible and massless string which passes over a smooth fixed cylinder of radius 'R' as shown in figure. Initially particles lie on same horizontal line and they are slightly displaced. The velocity of particles as a function of θ is given by



Sol. Loss in P.E. = gain in K.E.

$(mgR\theta - mgR\sin\theta) = \frac{1}{2} \times 2m \times v^2$

Solve for 'v'



Q. The angle θ at which the particle slipping on the cylinder leaves contact with the cylinder, satisfy the relation.

Sol. At the breaking point N = 0

$$\frac{mv^2}{R} = mg\cos\left(\theta - \frac{\pi}{2}\right)$$

$v^2 = gR\sin\theta$

$gR(\theta - \sin\theta) = gR\sin\theta$

$\sin\theta = \theta/2$

Q. The tangential acceleration of particle in contact with cylinder is

Sol. $T + mg\sin(\theta - \pi/2) = ma$ .....(i)

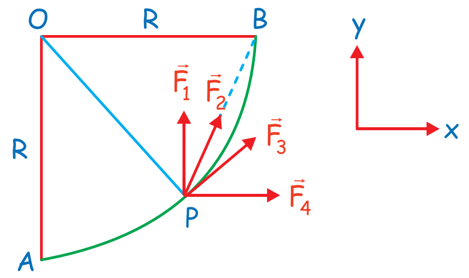
$mg - T = ma$ ...(ii)

on (i) + (ii)

$mg(1 - \cos\theta) = 2ma$

$a = g\sin^2(\theta/2)$

Q. AB is a quarter of a smooth horizontal circular track of radius R. A particle P of mass m moves along the track from A to B under the action of following forces:



$\vec{F}_1 = F$ (always towards y-axis)

$\vec{F}_2 = F$ (always towards point B)

$\vec{F}_3 = F$ (always along the tangent to path AB)

$\vec{F}_4 = F$ (always towards x-axis)

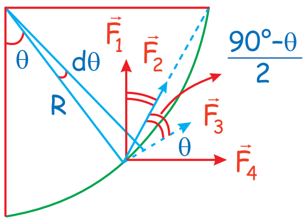
Column-I		Column-II	
A.	Work done by $\vec{F}_1$	p.	$\sqrt{2}FR$
B.	Work done by $\vec{F}_2$	q.	$\frac{1}{\sqrt{2}}FR$
C.	Work done by $\vec{F}_3$	r.	FR
D.	Work done by $\vec{F}_4$	s.	$\frac{\pi FR}{2}$

Sol. A-(r); B-(p); C-(s); D-(r)

For (A): Work done $\vec{F}_1 = FR$

For (B):

$dW = \vec{F}\cdot d\vec{s} = (FRd\theta)\cos$



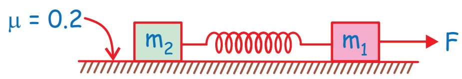
$$W = \int_0^{\pi/4} FR\cos\left(45 - \frac{\theta}{2}\right)$$

For (C) : $W = \int \vec{F}\cdot ds = F\left(\frac{\pi R}{2}\right) = \frac{\pi FR}{2}$

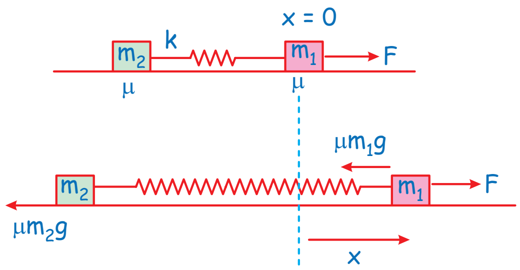
For (D) : $W = \int \vec{F}\cdot d\vec{s} = (F)(R) = FR$

Q. Two blocks of masses m_1 and m_2 are connected by a spring of stiffness k = 200 Nm^{-1}. The coefficient of friction between the blocks and the surface is μ. Find the minimum constant horizontal force F (in Newton) to be applied to m_1 in order to slide the mass m_2. (Initially spring is in its natural length).

(Take $m_1 = 3$ kg, $m_2 = 5$ kg, $g = 10$ m/s^2, $\mu = 0.2$)



Sol.



$W_F + W_{Sp} + W_{fric} = \Delta k$

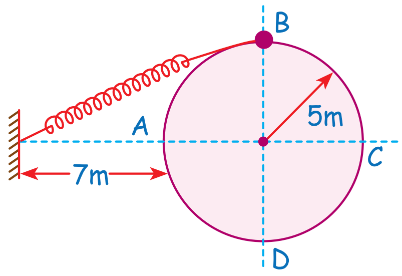
$\Rightarrow Fx - \frac{1}{2}kx^2 - \mu m_1 g x = 0$

and $kx = \mu m_2 g$

$\Rightarrow F - \frac{1}{2}\mu m_2 g - \mu m_1 g = 0 \Rightarrow F = \mu m_1 g + \frac{\mu m_2 g}{2}$

Q. A collar B of mass 2kg is constrained to move along a vertical smooth and fixed circular track of radius 5m as shown in figure. The spring is in plane of the track and has a spring constant of 200 N/m. It is underformed when collar is at A. It starts from rest at B. What is the normal force (in N) exerted by the track on the collar when it passes through A?

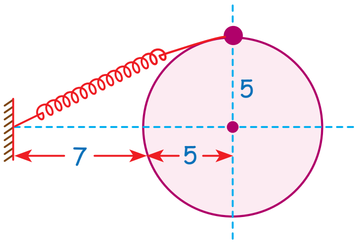
[g = 10 m/s^2]



Sol. $l_0 + x = \sqrt{5^2 + 12^2} = 13$

$x = 6$m

$mg \times 5 + \frac{1}{2}k(6^2 - 0^2) = \frac{1}{2}mv^2$

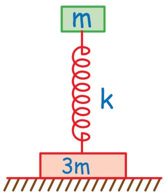


$$100 + \frac{1}{2} \times 200 \times 36 = \frac{1}{2} \times 2 \times v^2$$

$$v^2 = 3700$$

$$N = \frac{mv^2}{R} = \frac{2 \times 3700}{5} = 1480 \text{ N}$$

Q. In the figure shown the spring constant is k. The mass of the upper disc is m and that of the lower disc is 3m. The upper block is depressed down from its equilibrium position by a distance $\delta = 5mg/k$ and released at $t = 0$. Find the velocity of 'm' when normal reaction on 3m is mg.

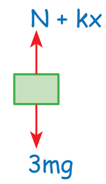


Sol. $N = mg$

$N + kx = 3mg$

$mg + kx = 3mg$

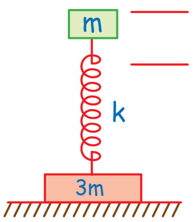
$x = \frac{2mg}{k}$

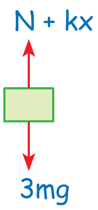


Applying C.O.E.

$$\frac{1}{2}k\left(\frac{6mg}{k}\right)^2 - \frac{1}{2}k\left(\frac{6mg}{k}\right)^2 - \frac{1}{2}k\left(\frac{2mg}{k}\right)^2$$

$$- \frac{8mg}{k}\, mg = \frac{1}{2}\, mv^2$$





$$\frac{36m^2g^2}{k} - \frac{4m^2g^2}{k} - \frac{16m^2g^2}{k} = mv^2$$

$$\frac{16m^2g^2}{k} = mv^2 \Rightarrow v = 4g\left[\frac{m}{k}\right]^{\frac{1}{2}}$$

Q. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration (a) is varying with time t as $a = k^2rt^2$, where k is a constant.

The power delivered to the particle by the force acting on it is given as

Sol. Centripetal acceleration, $a = k^2r\,t^2$

Since, centripetal acceleration is $\frac{v^2}{r}$

$$\therefore \frac{v^2}{r} = k^2r\,t^2$$

$v = krt$

Centripetal force is perpendicular to the tangential velocity, therefore power delivered by it will be zero.

Tangential acceleration,

$$\Rightarrow a_t = \frac{dv}{dt} = kr$$

$\Rightarrow$ Tangential force, $F_t = ma_t = mkr$

$\therefore$ Power delivered, $P = F_tv = (mkr)(krt) = mk^2r^2t$

Q. A particle moving in a circular track such that is gravitational potential energy $U = -\frac{k}{r}$. Find it's speed, kinetic energy, total energy.

Sol. $U = -\frac{k}{r}$

$$F = -\frac{\partial U}{\partial r} = -\frac{k}{r^2}$$

$$F = \frac{k}{r^2} = \frac{mv^2}{r} \text{ (magnitude)}$$

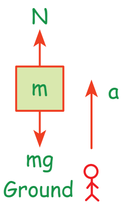
$$v = \sqrt{\frac{k}{mr}}$$

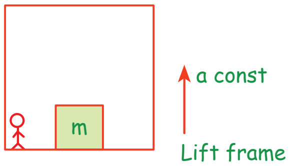
$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{k}{mr}\right) = \frac{k}{2r}$$

$$TE = -\frac{k}{r} + \frac{k}{2r} = -\frac{k}{2r}$$

Q. A block of mass M is placed inside the lift which starts moving from rest to upward direction with constant acc. 'a'. Find WD by all the forces on the block in ground frame and in lift frame.

Sol.





$N - mg = ma \Rightarrow N = m(g + a)$

displacement of block in 't_0' time.

$h = y = 0 + 1/2at_0^2$

- (WD) by gravity in ground frame = – mgh
- (WD) by normal force in ground frame

  $= Nh = + Ny = m(g + a)\ 1/2at_0^2$
- (WD) by pseudo force in ground frame = 0
- (WD) by gravity in lift frame

  $W = mg \times 0 = 0$
- (WD) by normal force in lift frame = 0
- (WD) by pseudo force in lift frame

  $= (ma) \times 0 = 0$
- That shows that WD by a force is also frame of reference dependent.