YAKEEN NEET 2.0

2026

Basic Maths and Calculus (Mathematical Tools)

PHYSICS

Lecture - 14

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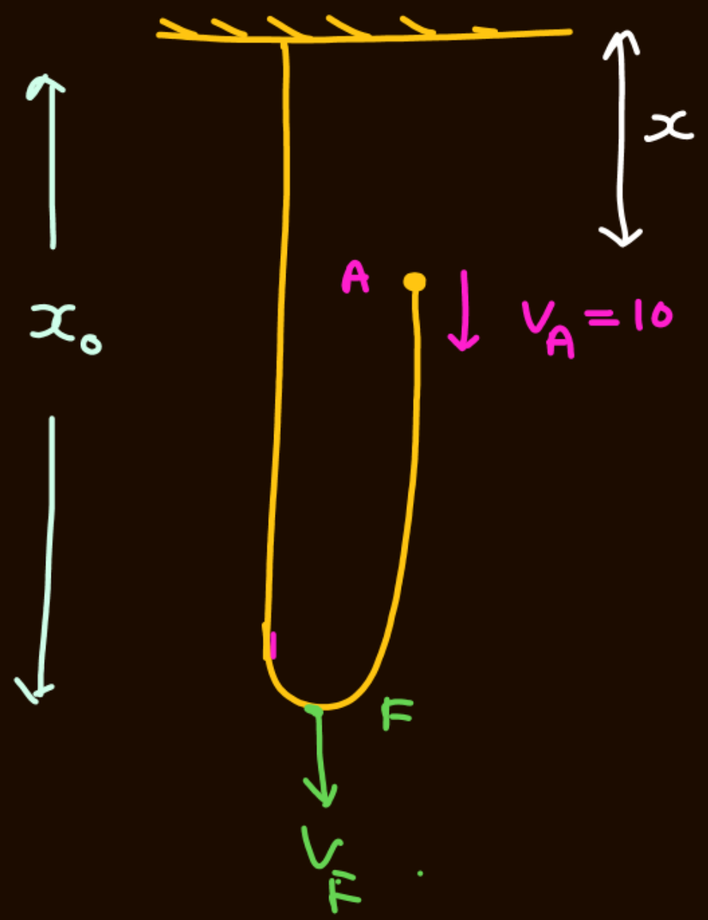
Topics to be covered

1. Chain rule
2.
3.
4.



Length of string $l = x_0 + x_0 - x$

Const $\leftarrow$ $l = 2x_0 - x$

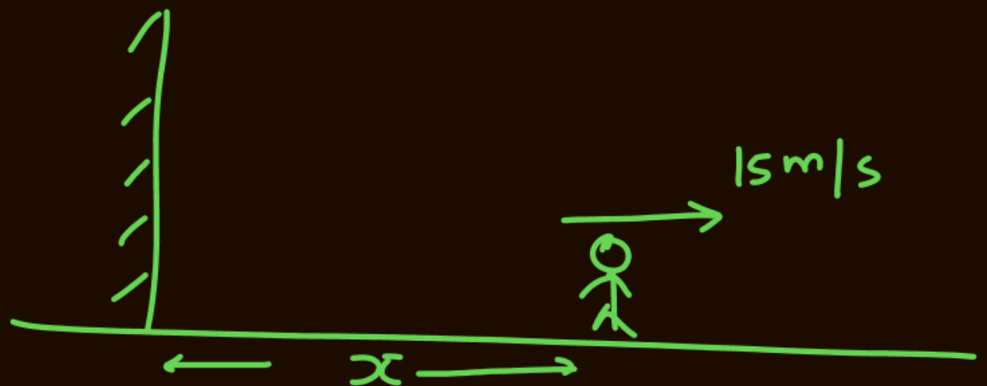
$$\frac{dl}{dt} = 2\frac{dx_0}{dt} - \frac{dx}{dt}$$

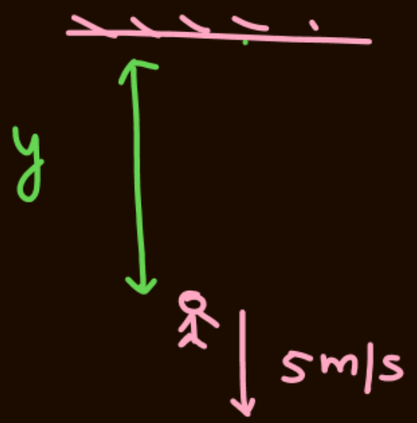
0

$$0 = 2V_F - 10$$

$$\boxed{V_F = 5}$$

Copy





$\frac{dx}{dt} = 15$

$\frac{dy}{dt} = 5$

Q $y = \sin(\ln x)$

$$\frac{dy}{dx} = \cos(\ln x) \times \frac{1}{x}$$

Q $y = \sin \ln(3x^2 + e^x)$

$$\frac{dy}{dx} = \cos\left(\ln(x^2+e^x)\right) \frac{1}{x^2+e^x} (2x+e^x)$$

Chain rule (Power ki Nautanki)

Sabse pahle power ko Lapeto

① $y = (\sin x)^4$

$\frac{dy}{dx} = 4(\sin x)^3 \times \cos x$

② $y = (\sin x)^6 = \sin^6 x$

$\frac{dy}{dx} = 6(\sin x)^5 \times \cos x$

③ $y = (\ln x)^5$

$\frac{dy}{dx} = 5(\ln x)^4 \times \frac{1}{x}$

④ $y = (\tan x)^7$

$\frac{dy}{dx} = 7(\tan x)^6 \cdot \sec^2 x$

∴

* ⑤ $y = [\sin(3x^2+4x)]^7$

$$\frac{dy}{dx} = 7[\sin(3x^2+4x)]^6 \times \cos(3x^2+4x) \times (6x+4)$$

⑥ $y = [\ln(x^2+4x)]^3$

$$y' = 3[\ln(x^2+4x)]^2 \times \frac{2x+4}{x^2+4x}$$

Q $y = \sin^2 x$

$y' = 2\sin x \cdot \cos x = \sin 2x$

Q $y = \sin^2(2x+3)$

$y' = 2\sin(2x+3) \times \cos(2x+3)\ (2+0)$

Q $y = \sin x^3 = \sin(x^3)$

$y' = \cos(x^3) \times 3x^2$

Q $y = \cos x^3$

$y' = -\sin x^3 \times 3x^2$

Q $y = 10 \sin(20\pi x + \pi/2)$

$y' = 10 \cos(20\pi x + \pi/2) \times (20\pi + 0)$

Q $y = e^{kx}$

$y' = e^{kx} \times k$

Q $y = e^{-kx}$

$y' = e^{-kx} \times (-k)$

Q $y = 10\, e^{-5x}$

$y' = 10x\, e^{-5x} \times -5$

Q $y = 10\, e^{-\frac{x}{5}}$

$y' = 10x\, e^{-x/5} \times \left(-\frac{1}{5}\right)$

$$y = \frac{x}{5}$$

$$y' = \frac{1}{5}$$

$$\Rightarrow \quad y = -\frac{x}{2}$$

$$y' = -\frac{1}{2}$$

$$y = -\frac{x}{2} + 10$$

$$y' = -\frac{1}{2} + 0$$

Q $y = \tan x$

$$\frac{dy}{dx} = \sec^2 x = (\sec x)^2$$

$$\frac{d^2y}{dx^2} = 2(\sec x) \times \sec x \cdot \tan x$$

physics में काम आने वाला chain rule

4 month

① $y = A\sin(\omega t + \phi)$, $(A, \omega, \phi \rightarrow \text{const})$

$$\frac{dy}{dt} = A\omega\cos(\omega t + \phi)$$

$$\frac{d^2y}{dt^2} = -A\omega\sin(\omega t + \phi)\,\omega$$

$$= -A\omega^2\sin(\omega t + \phi)$$

② $q = q_0 e^{-t/\tau}$ $(q_0, \tau \text{ const})$

$$\frac{dq}{dt} = q_0 \cdot e^{-t/\tau} \times -\frac{1}{\tau}$$

③ $q = q_0(1 - e^{-t/\tau})$

$$q = q_0 - q_0 e^{-t/\tau}, \quad (q_0, \tau \text{ const})$$

$$\frac{dq}{dt} = 0 - q_0 e^{-t/\tau} \times \left(\frac{-1}{\tau}\right)$$

$$= \frac{q_0}{\tau} e^{-t/\tau}$$

वर्ग Note

Q $y = 10\sin(\pi t + \pi/2)$

$$\frac{dy}{dt} = 10\cos(\pi t + \pi/2) \times (\pi + 0)$$

$$\frac{dy}{dt} = 10\pi\cos(\pi t + \pi/2)$$

$$\frac{d^2y}{dt^2} = 10\pi \times \left[-\sin(\pi t + \pi/2)\right](\pi + 0)$$

$$\frac{d^2y}{dt^2} = -10\pi^2 \sin(\pi t + \pi/2)$$

Q $q = 50\, e^{-5t}$

$$\frac{dq}{dt} = 50\, e^{-5t} \times (-5)$$

$$\frac{dq}{dt} = -250\, e^{-5t}$$

Q $q = 50\, e^{-t/5}$

$$\frac{dq}{dt} = 50\, e^{-t/5} \times \left(-\frac{1}{5}\right)$$

$$= -10\, e^{-t/5}$$

Q $y = \frac{-x}{5}$

$$\frac{dy}{dx} = -\frac{1}{5}$$

\# $y = e^{-5x}$

$$\frac{dy}{dx} = e^{-5x} \times (-5)$$

Q $y = e^{-x/5}$

$$\frac{dy}{dx} = e^{-x/5} \times \left(\frac{-1}{5}\right)$$

Q $y = \sin(2x+3)$

$$\frac{dy}{dx} = \cos(2x+3)\,(2+0)$$
$$= 2\cos(2x+3)$$

Q $y = \sin(2t+3)$

$$\frac{dy}{dt} = 2\cos(2t+3)$$

Q $y = 10\sin(2t+3)$

$$\frac{dy}{dt} = 20\cos(2t+3)$$

SKC

$\Rightarrow \quad y = \sin(\text{कदद्द})$

$\frac{dy}{dx} = \cos(\text{कदद्द}) \times \frac{d}{dx}(\text{कदद्द})$

$\Rightarrow \quad y = e^{\text{कदद्द}}$

$\frac{dy}{dx} = e^{\text{कदद्द}} \frac{d}{dx}(\text{कदद्द})$

$\# \quad y = (\text{कदद्द})^4$

$\frac{dy}{dx} = 4(\text{कदद्द})^3 \times \frac{d}{dx}(\text{कदद्द})$

परेशान करेगा

Chain rule

① $y = x^3$

$$\frac{dy}{dx} = 3x^2$$

##

② $y = x^3$

$$\frac{dy}{dx} = 3x^2$$

$$dy = 3x^2\,dx$$

$$\frac{dy}{dt} = 3x^2\,\frac{dx}{dt}$$

Shortcut (SKC)

$y = x^3$

$\frac{dy}{dt} = 3x^2$ (x का t के सापेक्ष differentiation wrt t)

$$\frac{dy}{dt} = 3x^2\,\frac{dx}{dt}$$

Q $y = x^5$

$\frac{dy}{dx} = 5x^4$

$\frac{dy}{dt} = 5x^4 \cdot \frac{dx}{dt}$

Q $y = x^7$

$\frac{dy}{dt} = 7x^6 \left(\frac{dx}{dt}\right)$

Q $y = \sin x$

$\frac{dy}{dt} = \cos x \times \frac{dx}{dt}$

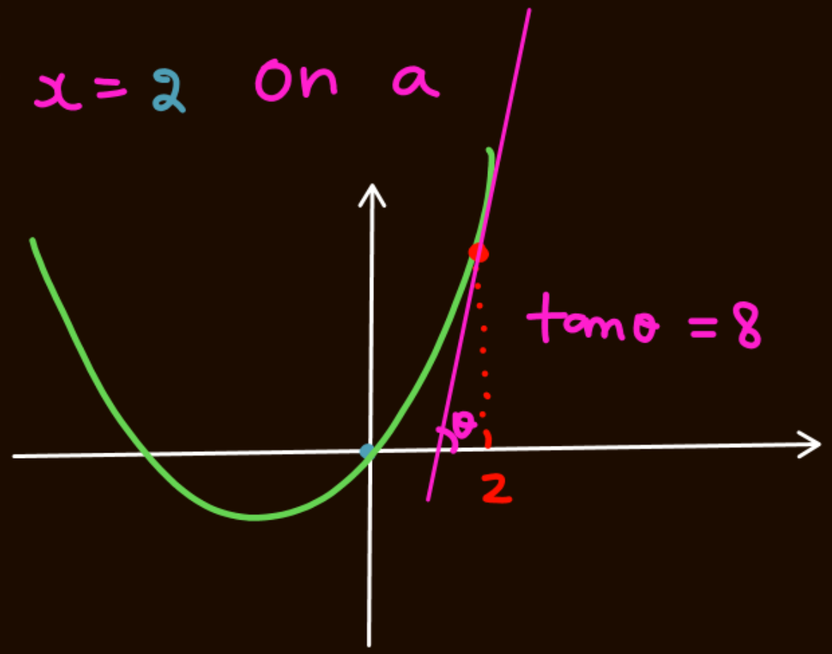
Q $y = e^x$

$\frac{dy}{dt} = e^x \cdot \frac{dx}{dt}$

H.W. दूजा वाली Qus isi slide पर Last me.

$\frac{dy}{dx}$ ⟶ slope of the tangent at that point.

Q find the slope of the tangent at $x = 2$ on a curve $y = x^2 + 4x$

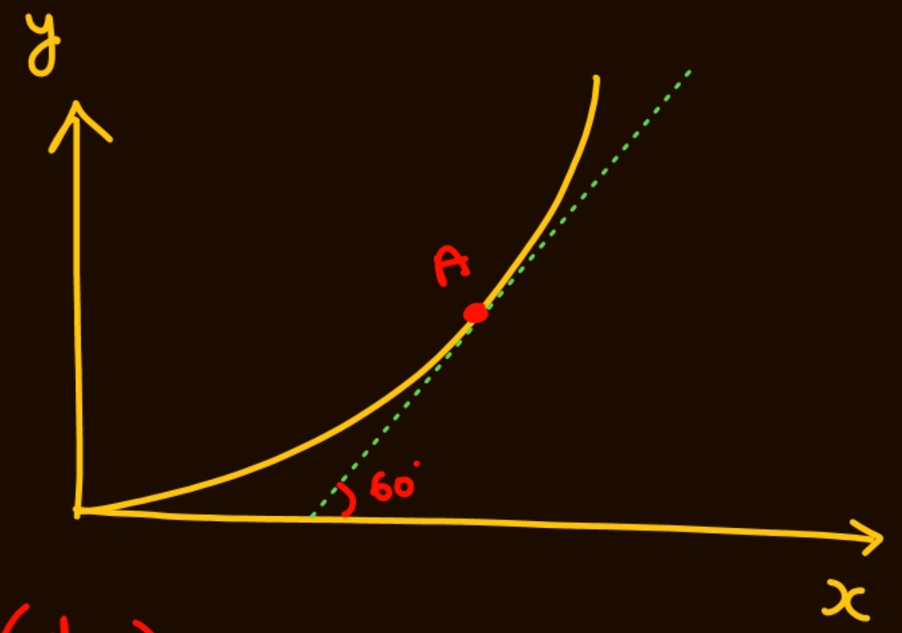


Soln

$$y = x^2 + 4x$$

$$\frac{dy}{dx} = 2x + 4 = \text{Slope}$$

At $x = 2$ पर $\frac{dy}{dx} = \text{slope} = 2 \times 2 + 4 = 8$ ∴

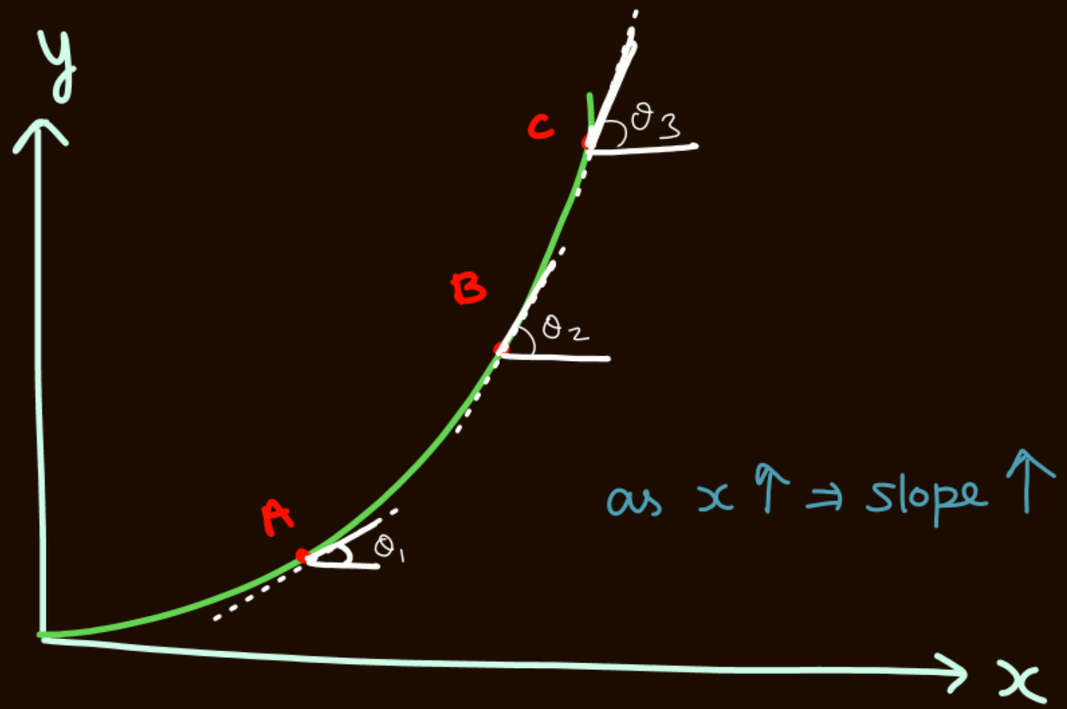


SKC

किसी भी curve के किसी भी point पर $\frac{dy}{dx}$ का मतलब है उस point पर tangent का slope

$\left(\frac{dy}{dx}\right)_{\text{at 'A'}}$ = Slope of the tangent at A

$= \tan 60 = \sqrt{3}$

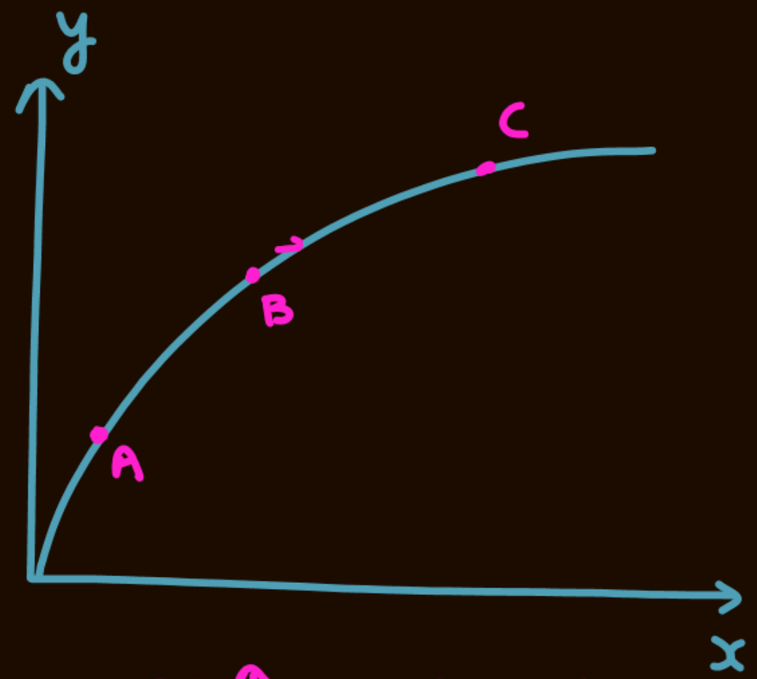


as $x \uparrow \Rightarrow$ slope $\uparrow$

$m_A < m_B < m_C$

(All slope are positive)

($m \rightarrow$ slope)



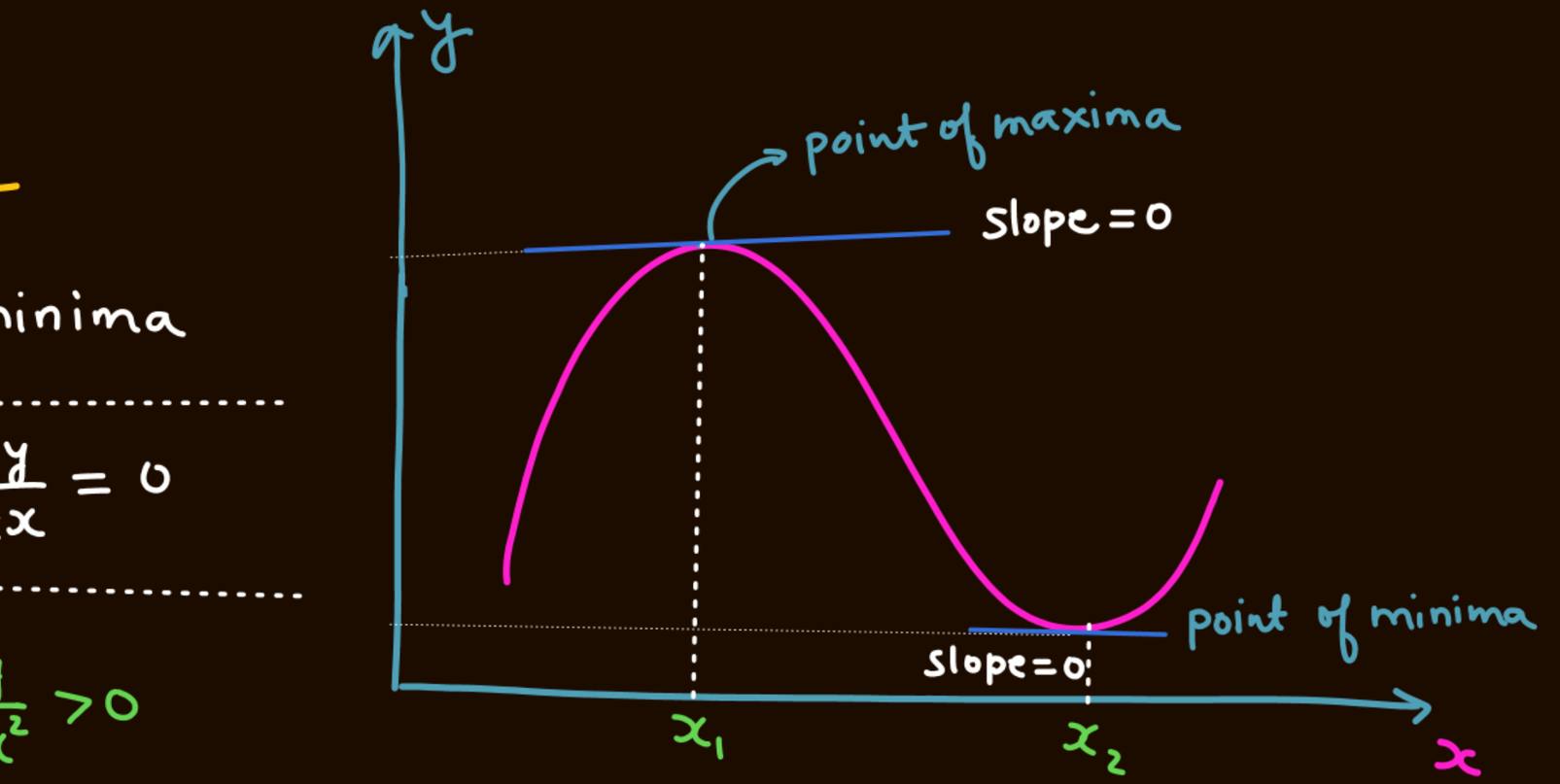
$x \uparrow \Rightarrow$ slope $\downarrow$

$m_A > m_B > m_C$

(slope +ve)

Maxima minima

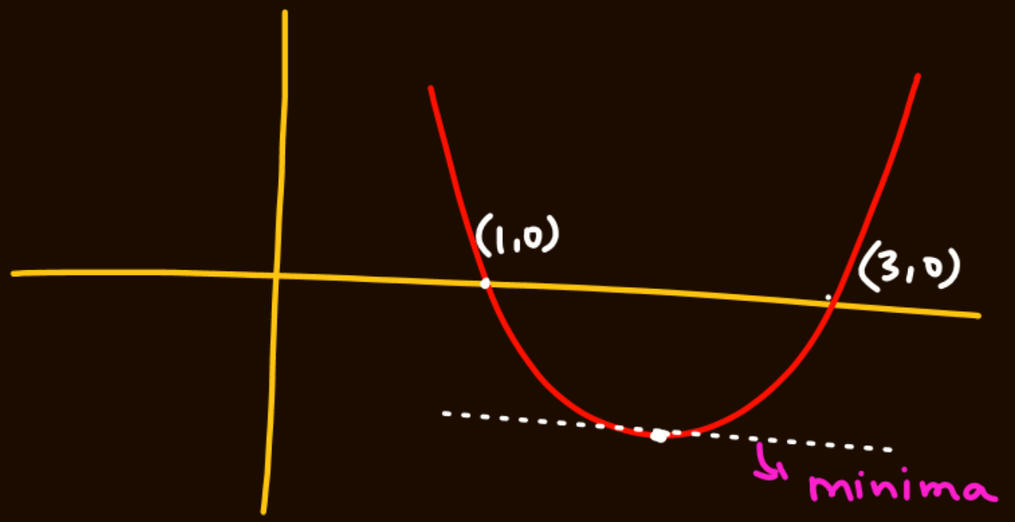
maxima	minima
$\frac{dy}{dx} = 0$	$\frac{dy}{dx} = 0$
$\frac{d^2y}{dx^2} < 0$	$\frac{d^2y}{dx^2} > 0$



Q $y = x^2 - 4x + 3$

$$\frac{dy}{dx} = 2x - 4 = 0$$

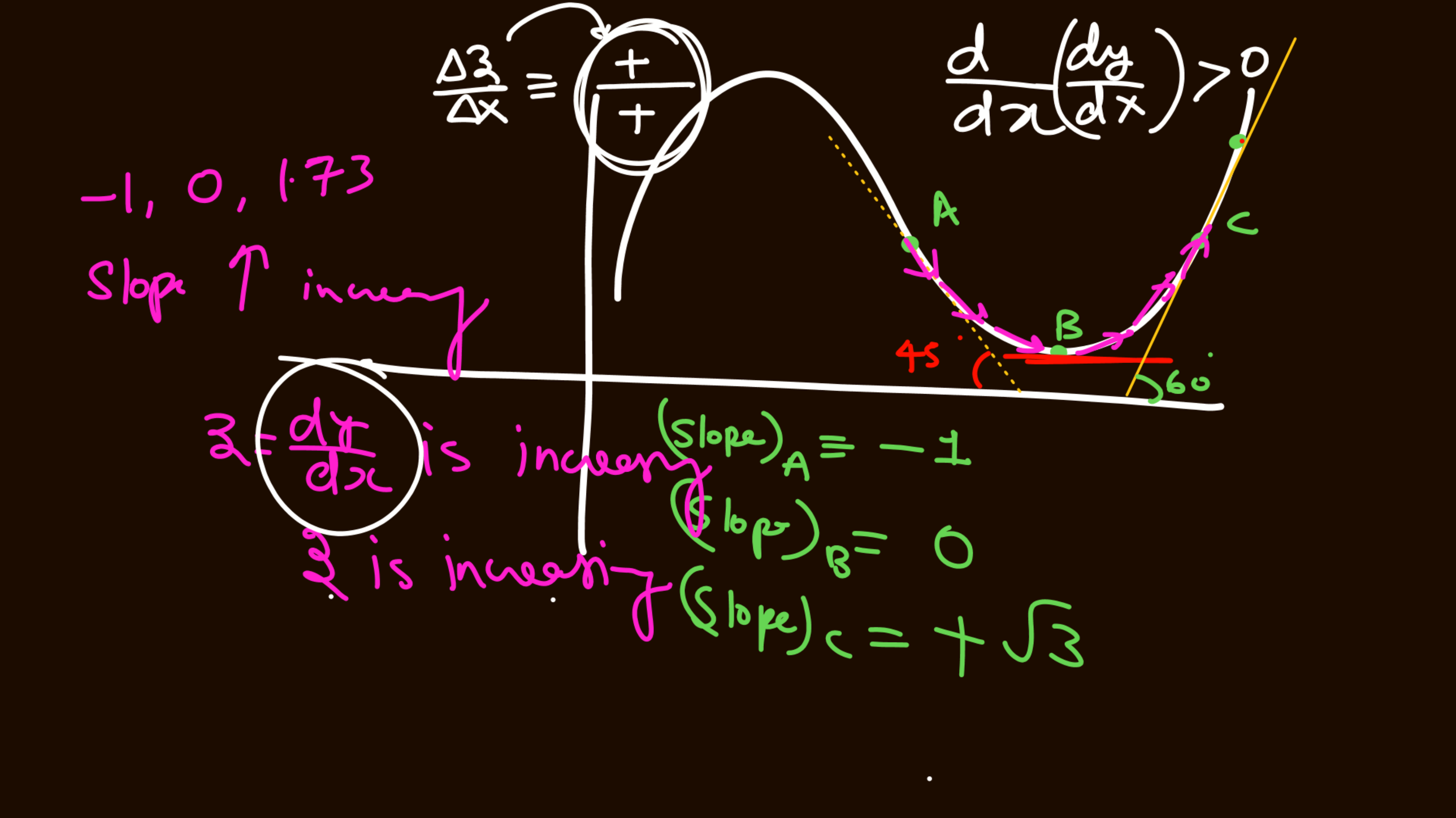
$$x = 2$$



$$\frac{dy}{dx} = 2x - 4$$

$$\frac{d^2y}{dx^2} = 2 > 0$$

$$\frac{d^2y}{dx^2} > 0 \equiv x = 2 \text{ minima}$$



$-1, 0, 1.73$

Slope ↑ increasing

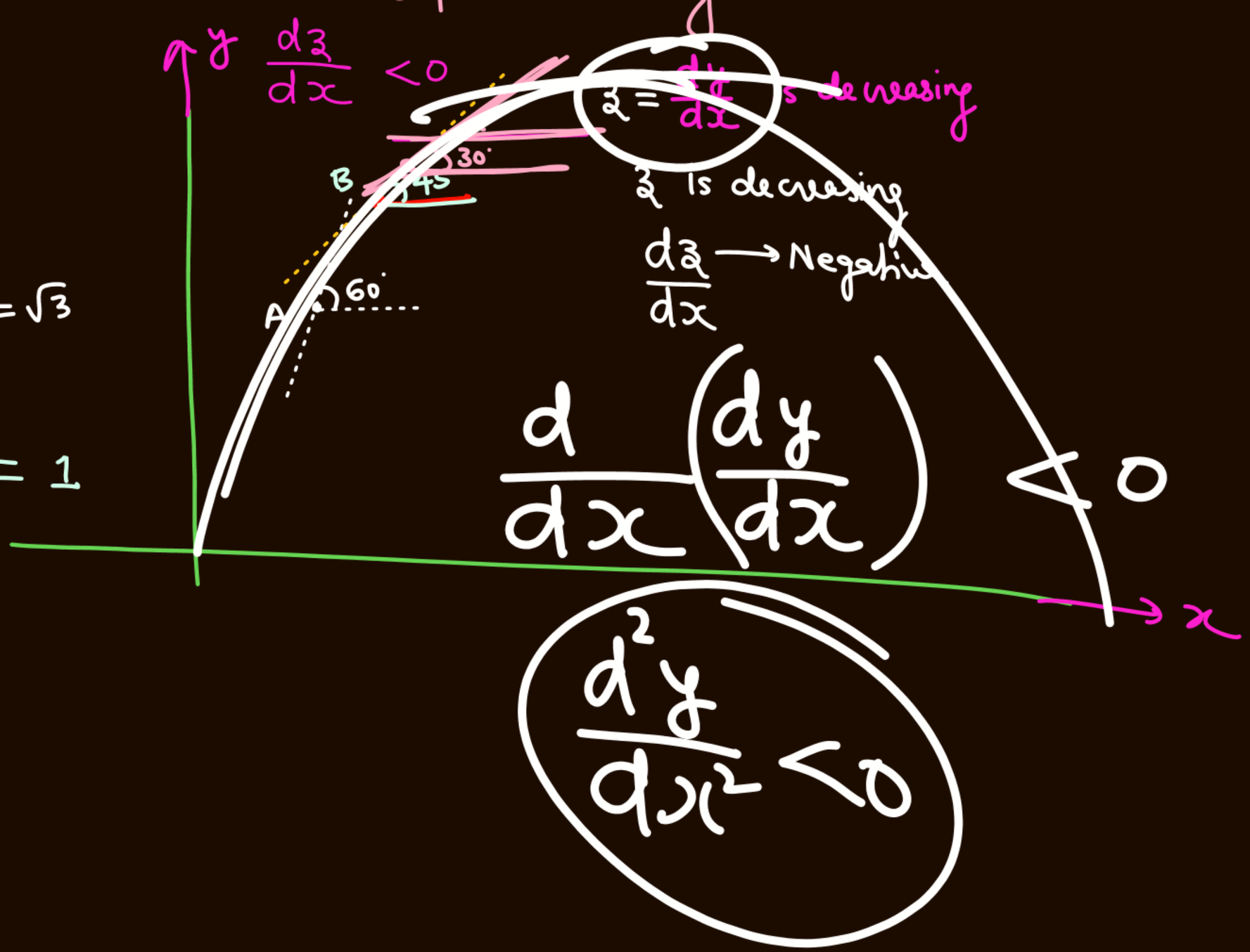
$3 = \frac{dy}{dx}$ is increasing

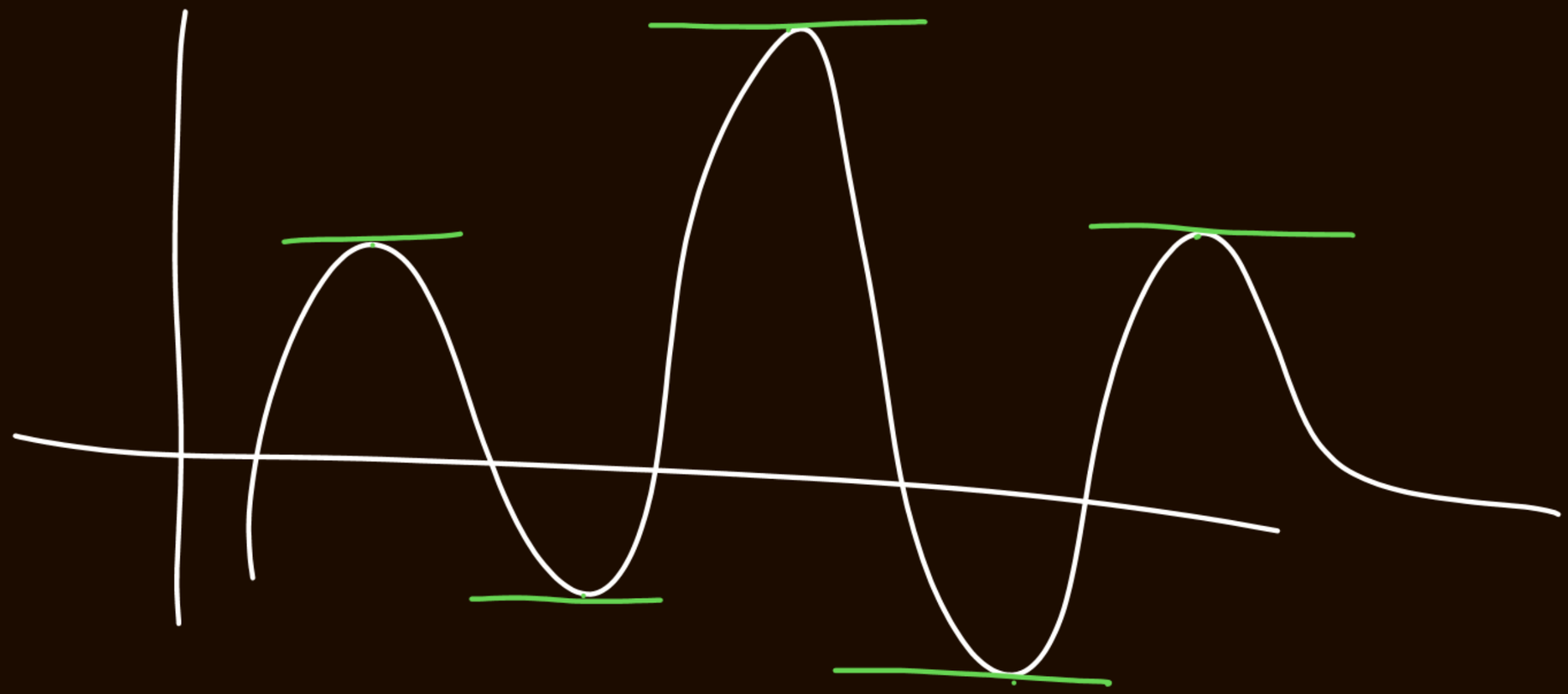
3 is increasing

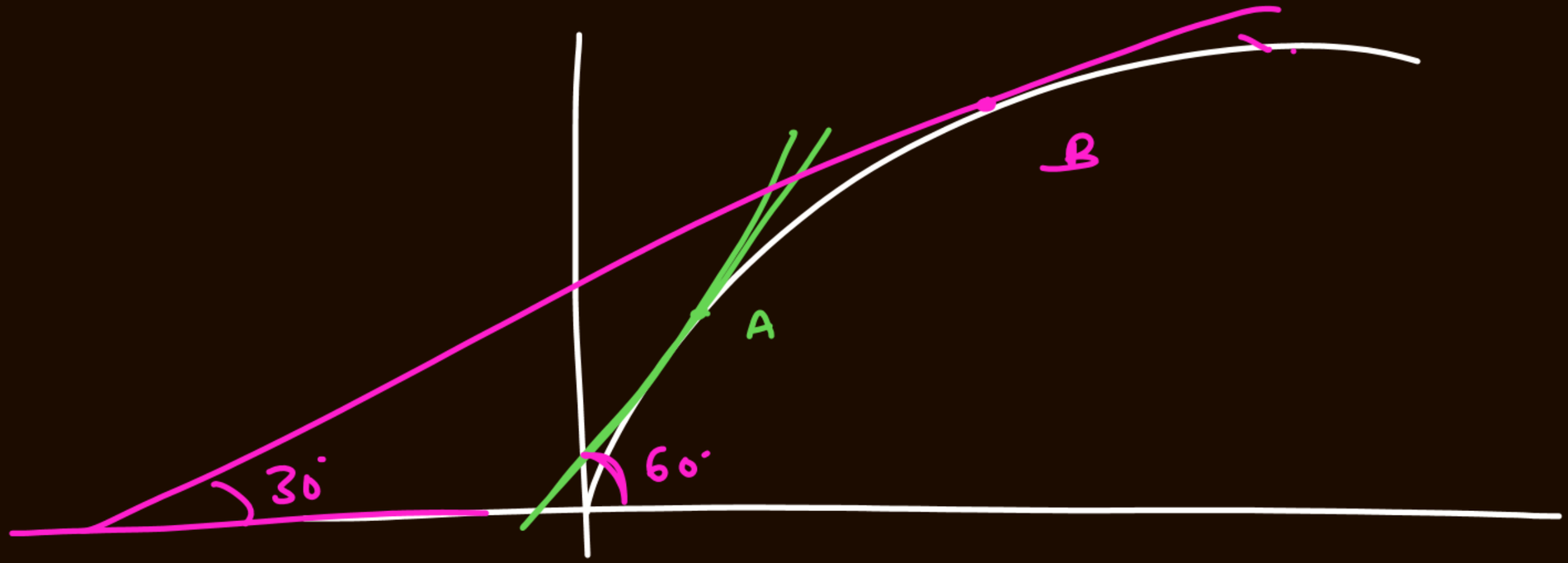
$(\text{Slope})_A \equiv -1$

$(\text{Slope})_B = 0$

$(\text{Slope})_C = +\sqrt{3}$


y
x
$\frac{dy}{dx} < 0$
$z = \frac{dy}{dx}$ is decreasing
z is decreasing
$\frac{dz}{dx}$ → Negative
30°
45°
60°
A
B
$\left(\frac{dy}{dx}\right)_A = \tan 60 = \sqrt{3}$
$\left(\frac{dy}{dx}\right)_B = \tan 45 = 1$
$\frac{d}{dx}\left(\frac{dy}{dx}\right) < 0$
$\frac{d^2y}{dx^2} < 0$




B
A
30°
60°

Maxima minima

maxima - minima

Q $y = x^3 - 3x^2 + 6$

Find y_{max} & y_{min}

$\frac{dy}{dx} = 3x^2 - 6x$

$\frac{dy}{dx} = 0, \quad x = 0, 2$

$\frac{d^2y}{dx^2} = 6x - 6$

Check $x = 0$

$\frac{d^2y}{dx^2} = 6 \times 0 - 6 = -6$

$\frac{d^2y}{dx^2} < 0$ ($x = 0$ maxima)

$y_{max} = 6$

$x = 2, \quad \frac{d^2y}{dx^2} = 6 \times 2 - 6 = 6 > 0$

$\frac{d^2y}{dx^2} > 0 \Rightarrow$ minima

$y_{min} = 8 - 12 + 6$

Q $x = t^3 - 3t^2 + 6$

9 $v = 6t^2 - 6t^3$

find v_{max}

$t = 0 \equiv$ min

minim

$v = 6t^2 - 6t^3$

$\frac{dv}{dt} = 12t - 18t^2$

$\frac{dv}{dt} = 0$, $6t(2 - 3t) = 0$

$t = 0$, $t = 3/2$

$\frac{d^2v}{dt^2} = 12 - 36t$

$12 - 36 \times \frac{3}{2}$

$12 - 54$

< 0

At $t = 0$, $\frac{d^2v}{dt^2} = 12 - 0 = 12 > 0$

Q $x = 5t^2 - 9t + 3$

find x_{max}, also plot graph

Q $y = \sin\theta + \sqrt{3}\cos\theta$

$y_{max} = ?$

Q $y = x^2 - 4x + 10$

$y_{min} = ?$

THANK YOU