



SK



PHYSICS CRUSH

Class Notes in Handwritten Format

A beautiful journey From basic to JEE advanced via Mains/ NEET







Saleemians Khopcha Concept

PHYSICS CRUSH

Class Notes in Handwritten Format

A beautiful journey From basic to JEE advanced via Mains/ NEET

By: Saleem Bhaiya



Contents

1.	Units and Measurements	1-20
2.	Vector	21-37
3.	Kinematics	38-72
4.	Newton's Laws of Motion & Friction	73-115
5.	Work, Power and Energy	116-144
6.	Circular Motion	145-159
7.	Centre of Mass	160-193
8.	Rotation	194-247
9.	Gravitation	248-266
10.	Mechanical Properties of Solid	267-276
11.	Fluid Mechanics and Dynamics	277-310
12.	Gas Laws	311-319
13.	KTG and Thermodynamics	320-344
14.	Simple Harmonic Motion	345-363
15.	Wave	364-397
16.	Heat Transfer	398-430
17.	Error and Measurement	431-449

Share your video review on Instagram and tag me! I'd love to see it!

Join my Telegram channel & Insta.





Saleem.nitt *





Newton's Laws of Motion & Friction

देख भाई इस chapter में mains or advance weightage ढूंढने की गलती मत करना Mains में हर साल इससे question पूछे जाते है but advance में बहुत ही कम question पूछे गए है but पूरी mechanics का ये fundamental chapter है जो पूरी physics मे use होगा, so इस chapter को हलके मे ना ले।



INERTIA

- * It is the tendency to resist the change.
- Tt is the property due to which a body wants to be in state of rest or in uniform motion in a straight line.
- * Mass is the measurement of inertia.

Momentum

It is the net motion contain in the system.

$$\overrightarrow{P} = \overrightarrow{m}\overrightarrow{V}$$

देख भाई बाद में हम यह देखेंगे kissi body या system का momentum निकालने का तरीका होता है।

Force ----- Push or Pull

Types of Force

- 1. Gravitational force \longrightarrow mg, $\frac{Gm_1 m_2}{m^2}$
- 2. Electromagnetic force

Tension force, Normal force, Friction, Electrostatic force, magnetic force etc.

- 3. Strong nuclear force
- 4. Weak nuclear force

यह हम 12th class मे पढेंगे।

Newton 1st Law (Law of inertia)

* A body continue to be in state of rest or in state of uniform velocity (st. line) until or unless net external force acts on it.

Newton Second Law

$$(\vec{F}_{net})_{ext} = \frac{d\vec{p}}{dt}$$
 $\vec{P} = m\vec{v}$

If
$$m \rightarrow Const$$

$$\overrightarrow{F}_{net} = ma$$

$$\overrightarrow{F}_{net} = \frac{d(m\overrightarrow{v})}{dt} = m \frac{d\overrightarrow{v}}{dt} + \overline{v} \frac{dm}{dtv}$$

If mass of system is constant then

$$\overrightarrow{F}_{net} = \frac{md\overrightarrow{v}}{dt} = m\overrightarrow{a}$$

अबे इसका मतलब है हर जगह F = ma मत लगा देना जैसे rocket propultion मे mass variable होता है तो वहाँ F = ma नही लगा सकते।



Newton Third law

* For every action there is equal and opposite reaction at the same time, simultaneously of the same nature.

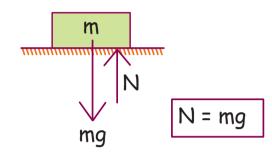
- * Action and reaction simultaneously लगते है इसलिए we can't say को कौन action है और कौन reaction.
- * Two different body event.



According to this law अगर A ने B पर force लगाएगा F = तो B भी A पर वापस force लगाएगा F, of same nature at same time



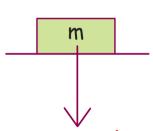




यहाँ normal और mg action reaction pair नहीं है।

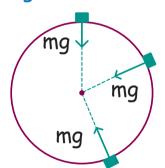
चलो पहले mg, normal, tension force पढ़ लेते है।

GRAVITATIONAL FORCE



mg (ये earth ने mass पर लगाया)

वैसे तो mg की direction towards the centre of earth है। जिसे हम gravitation में detail मे पढ़ेंगे



Block चाहे earth पर हो या हवा मे हो mg force तो लगेगा ही।

NORMAL FORCE

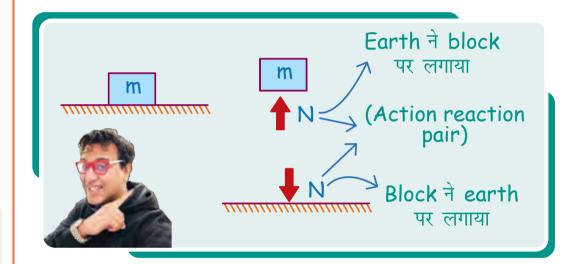
- ★ It act towards the body perpendicular to the surface.
- Pushing nature.

अगर कोई body kissi के साथ छुअन-छुआई कर रही है मतलब किसी surface के साथ contact/touch कर रही है तो उस body पर body की तरफ normal force लगेगा अब ये मत कहना mg क्यों नही लगाया Bcz अभी हम only normal force की बात कर रहे है।

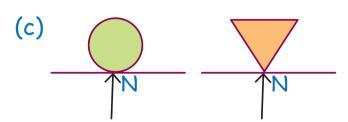




Block, earth के contact में है तो earth भी तो block के contact में है Hence, earth block को touch कर रही है तो earth पर भी normal लगेगा जो block लगाएगा।







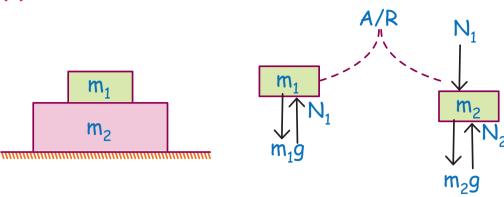
Draw the FBD of Block



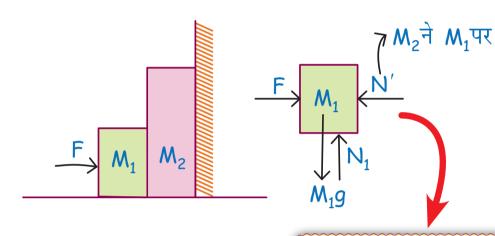


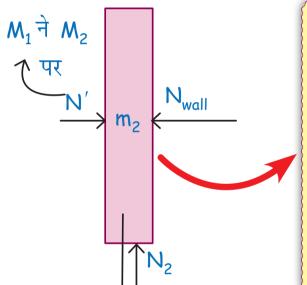
Q. Draw FBD of all the block in following cases.





(b)



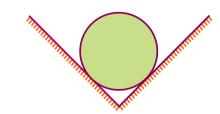


Since, both block are in equillibrium so ऊपर वाले force = नीचे वाले force and दाएँ वाले force = बाएँ वाले

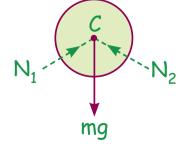
force

$$F = N'$$
 $N_1 = m_1g$
 $N' = N_{wall}$ $N_2 = m_2g$
 $F = N' = N_{wall}$

Q. A cylinder of weight W is resting on a V-groove as shown in figure. Draw its free body diagram.

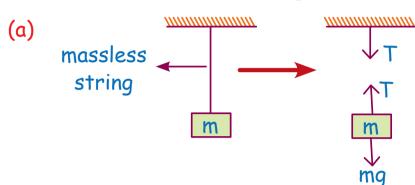


Sol.

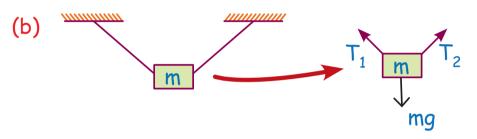


TENSION FORCE

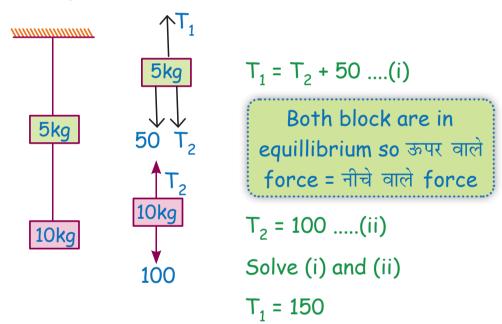
- * It act away from the body towards string.
- * It is a pulling force.
 - Q. Draw FBD of mass in following case.



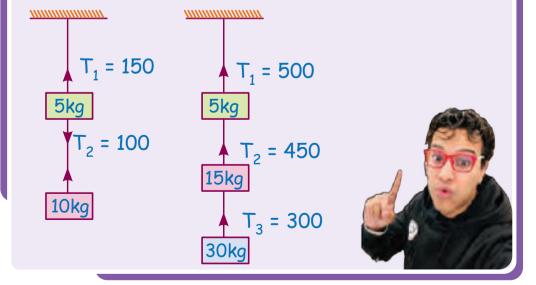
Here if string is massless then tension at every point is massless



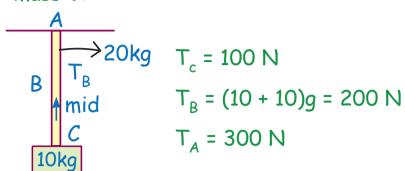
Q. Find tension in each string if all string are mass less.

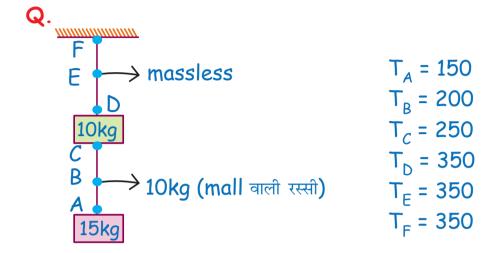


अबे ये detail मे तो मुझे मजबूरी मे लिखना पड़ रहा है भाई तू तो इसे direct कर, ऊपर वाली रस्सी को पकड़ देख नीचे कितना mass लटका है (15 kg so, T_1 = 150) similarly नीचे वाली रस्सी को पकड़ कर tension निकालो।



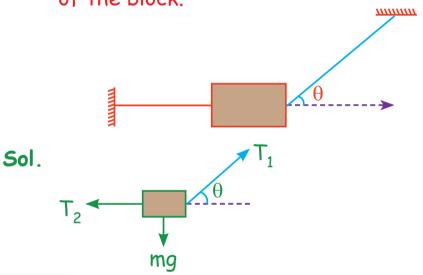
- **Q**. 10 kg block is suspended through a string of mass 20 kg as shown in figure. Find T_A , T_B , T_C .
- Sol. जिस जगह tension पूछी है देखो उसके नीचे total कितना mass है।





Mall वाली रस्सी अबे मतलब mass वाली रस्सी

Q. A block of mass m is attached with two strings as shown in figure. Draw the free body diagram of the block.



EQUILIBRIUM वाले सवाल

If body is in equil

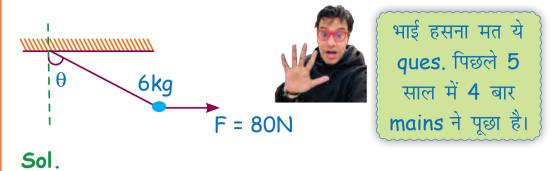
force = right वाले force

$$\Rightarrow \overrightarrow{F}_{net} = 0$$
 (Translational equil.)

$$\Sigma \overrightarrow{F}_x = 0$$
, $\Sigma \overrightarrow{F}_y = 0$, $\Sigma \overrightarrow{F}_y = 0$



Q. Body is in equilibrium find $\theta = ?$



Tsin θ Tcos θ Tsin θ Tsin θ Tsin θ Tsin θ

mg = 60

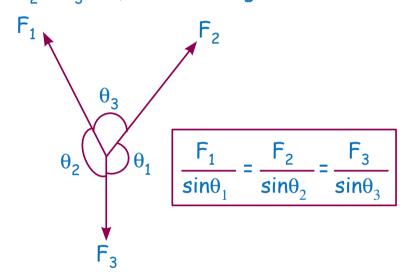
 $Tsin\theta = 80$ $Tcos\theta = 60$ $tan\theta = \frac{80}{60} = \frac{4}{3}$ $\theta = 53$

$$\theta = 53$$
Tsin53° = 80
T = 100

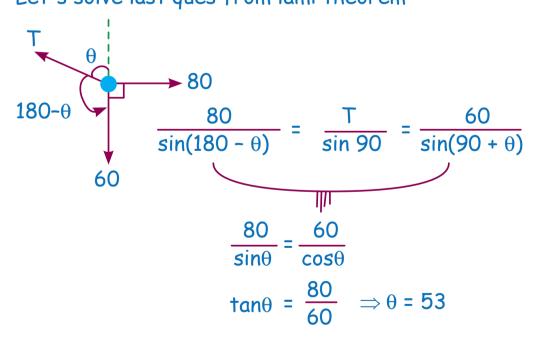
or after solving we got $T = \sqrt{F^2 + (mg)^2}$

M-2 Lami theorem

If $\overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} = 0$, in following case.

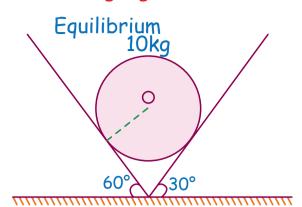


Let's solve last ques from lami theorem

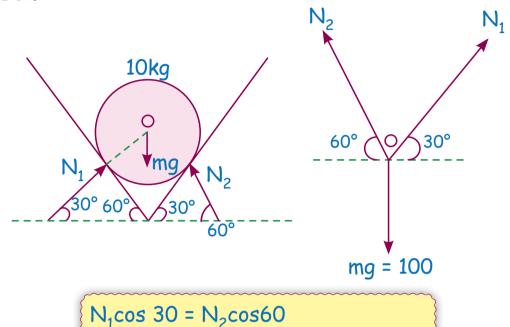


भाई मुझे तो ये lami theorem घटिया method लगता है bcz ना तो इससे feel आती है और time उतना ही लगेगा मेने यहाँ बता दिया है कहीं तू बाद मे बोले बताया नही। So, i will suggest की component लेके, force equate करके solve करो।

Q. Sphere is in equil find force applied by inclined plane in following figure.

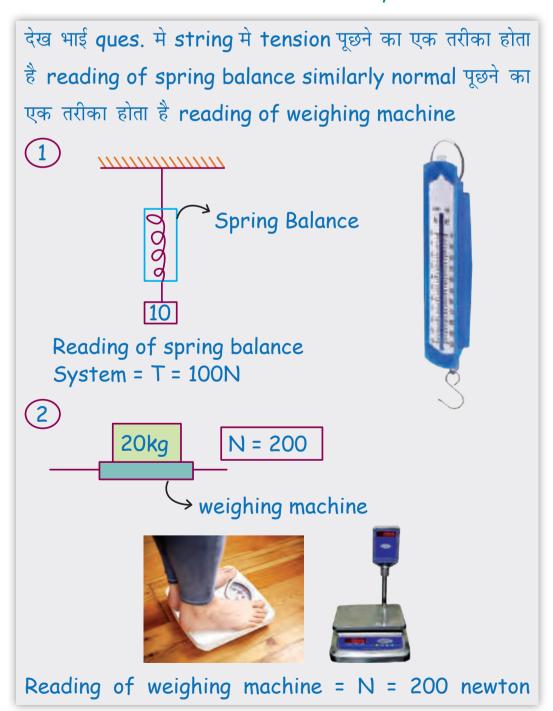


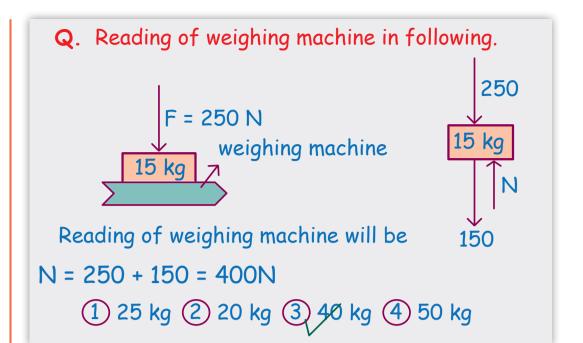
Sol.



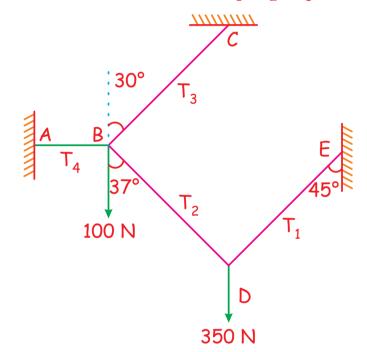
 $N_2 \sin 60 + N_1 \sin 30 = mg = 100$

Now you can solve and get N_1 and N_2 और मन करे तो lami theorem से answer verify कर लेना।

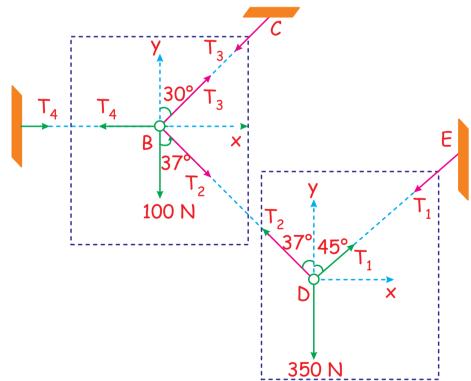




Q. Two particles of masses 10 kg and 35 kg are connected with four strings at points B and D as shown in figure. Find T_1 , T_2 , T_3



Sol.



Analysing the equilibrium of point D:

$$\sum F_x = 0 \text{ or } T_1 \sin 45^\circ - T_2 \sin 37^\circ = 0 \qquad ... (i)$$
 and
$$\sum F_y = 0 \text{ or } T_1 \cos 45^\circ + T_2 \cos 37^\circ = 350$$
 ... (ii) Solve both eqn. and get
$$\Rightarrow T_1 = 150\sqrt{2} \, \text{N} \text{ and } T_2 = 250 \, \text{N}$$

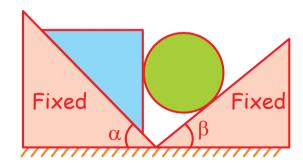
Similarly for equil of point B $T_3 \cos 30^\circ = 100 + T_2 \cos 37^\circ$ $T_3 \sin 30^\circ + T_2 \sin 37^\circ = T_4$ Put value of $T_2 = 250$, & solve both eqn get the ans.



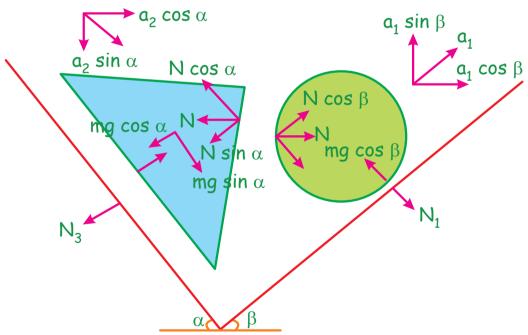
अबे क्यों रो रहा है मुझे पता calculation मुश्किल है लेकिन advance के point of view से आदत डाल ले। ये ले दो सवाल और ले।



Q. A cylinder and a wedge of same masses with a vertical face, touching each other, move along two smooth inclined planes forming the same angle α and β respectively with the horizontal. Determine the force of normal N exerted by the wedge on the cylinder, neglecting the friction between them.



Sol.



It is obvious that acceleration of cylinder is parallel to the wedge I and acceleration of triangular block is parallel to the wedge 2.

$$a_2 \cos \alpha = a_1 \cos \beta$$
 ... (i)

[constrained relation between the contact surface of block and cylinder]

N cos
$$\beta$$
 - m₁ g sin β = m₁a₁ ... (ii)

[Newton's II law for cylinder along the direction parallel to the wedge1]

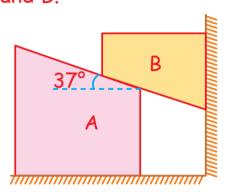
$$m_2 g \sin \alpha - N \cos \alpha = m_2 a_2$$
 ... (iii)

[Newton's II law for block along the direction parallel to the wedge 2]

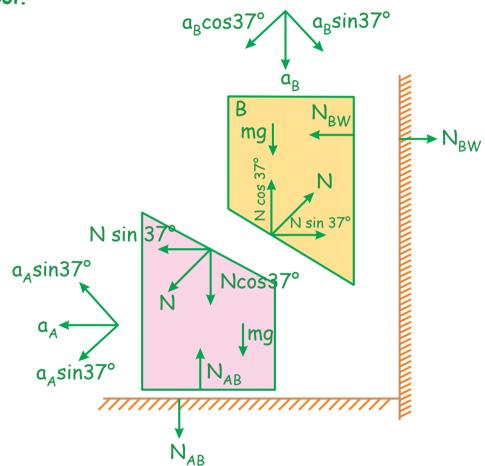
By solving equation (i), (ii) and (iii) we get

$$N = mg \left(\frac{\sin \alpha \cos \alpha + \sin \beta \cos \beta}{\cos^2 \alpha + \cos^2 \beta} \right)$$

Q. The masses of blocks A and B are same and equal to m. Friction is absent everywhere. Find the magnitude of normal force with which block B presses on the wall and accelerations of the blocks A and B.



Sol.



 $mg - N \cos 37 = ma_R$

[Newton's II law for block B in vertical direction]

 $N \sin 37 = ma_A$

[Newton's II law for block A in horizontal direction]

$$a_B \cos 37 = a_A \sin 37$$

[constrained relation for contact surface between block A and B]

By solving above three equations we get

$$a_A = \frac{12g}{25} a_B = \frac{9g}{25} N = \frac{4mg}{5}$$

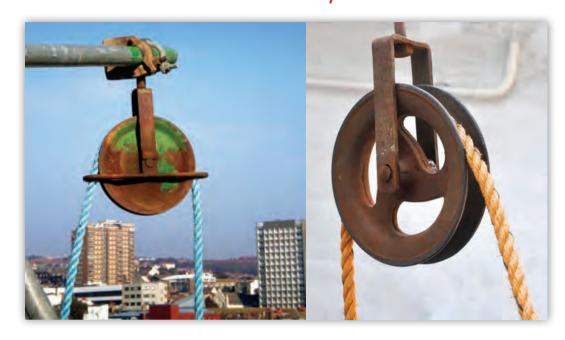
 $N_{BW} = N \sin 37$

[Equilibrium of block B in horizontal direction]

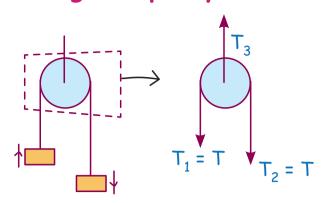
$$\Rightarrow$$
 N_{BW} = $\frac{12 \, \text{mg}}{25}$

PULLEY SYSTEM / ATWOOD MACHINE

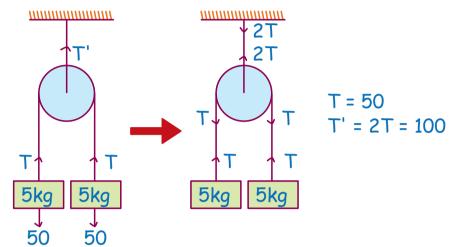
Real Pulley

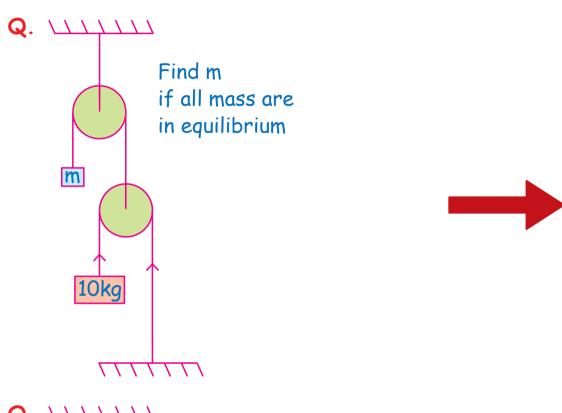


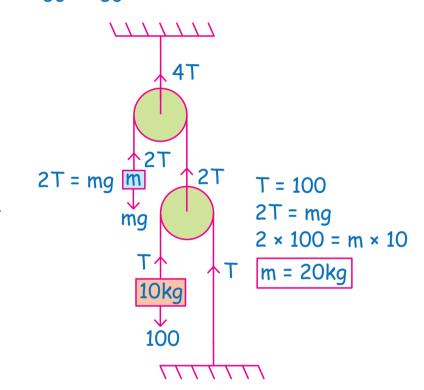
Ideal Pulley \rightarrow massless, no friction between string and pulley.

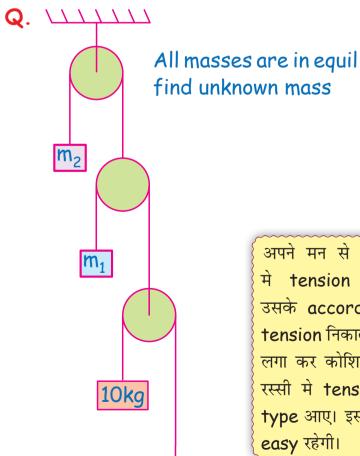


If pulley is ideal then $T_3 = T_1 + T_2 = 2T$

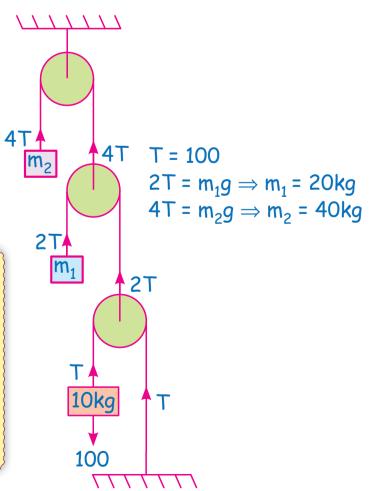


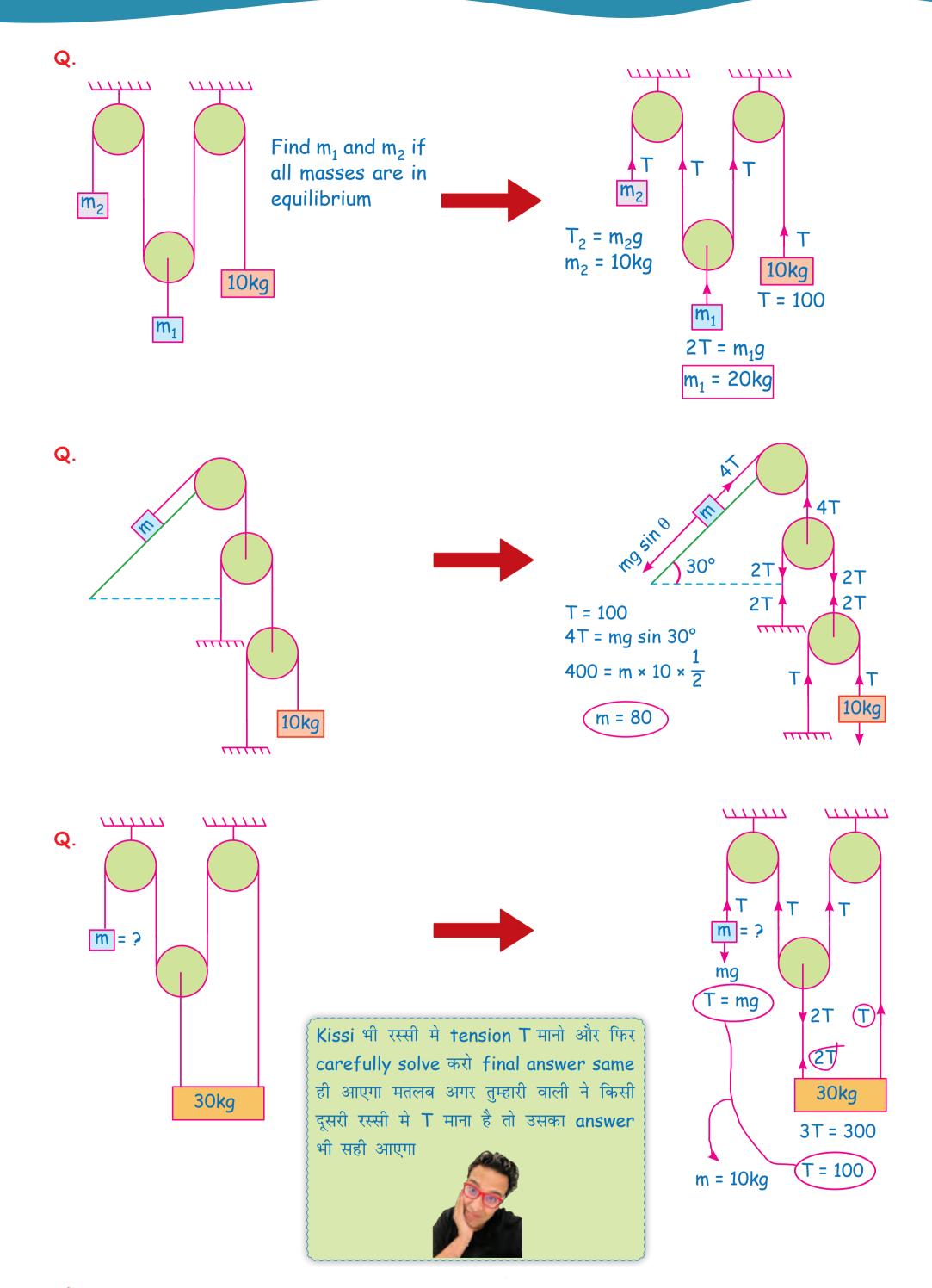


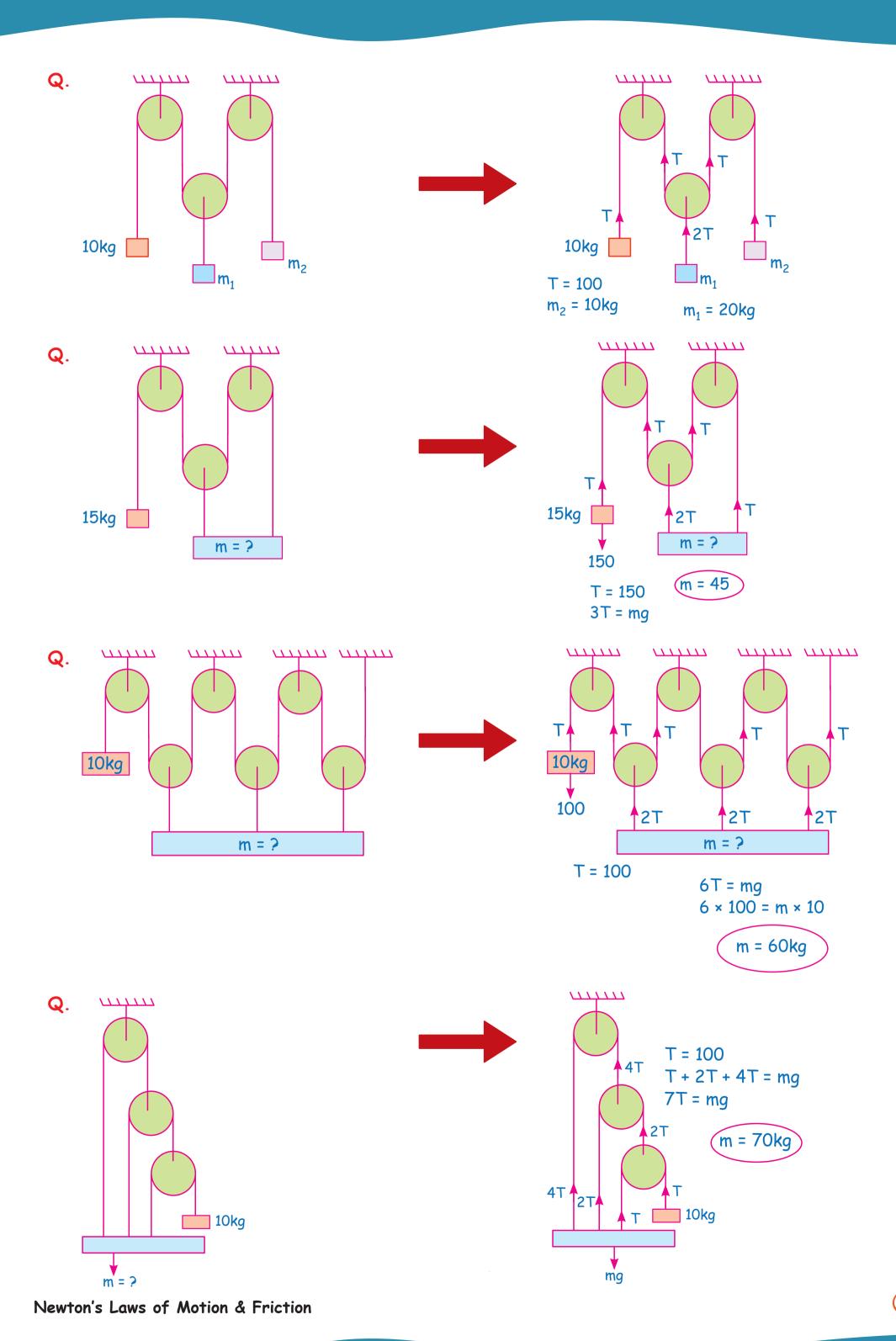




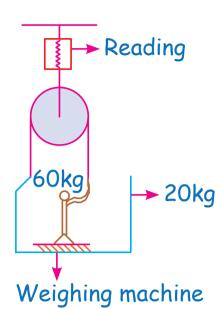
अपने मन से kissi एक रस्सी
मे tension T मानलो और
उसके according हर रस्सी मे
tension निकालो थोड़ा सा घुटना
लगा कर कोशिश करो की बाकी
रस्सी मे tension T, 2T, 4T
type आए। इससे calculation
easy रहेगी।



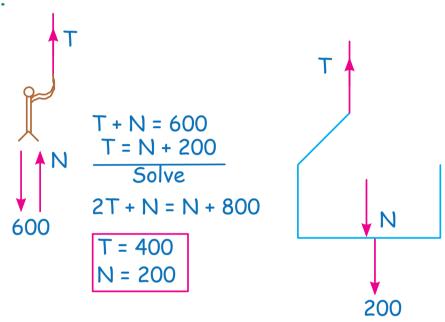




Q. Man is pulling the string by applying force F such that whole system is in air and equili.
Find T, Normal contact force between man and platform

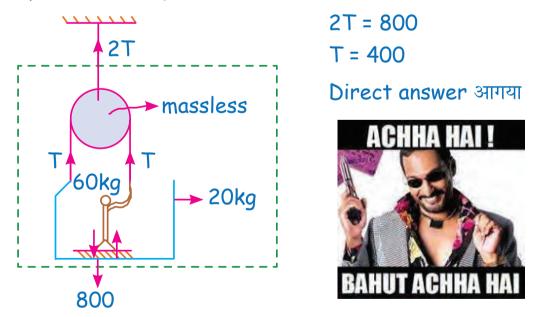


Sol.

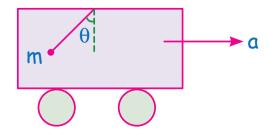


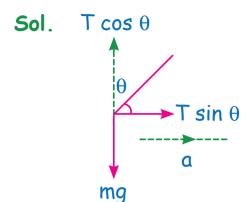
- * Reading of spring balance = 2T = 800 N
- * Reading of weighing machine = N = 200N

Method-2: अगर मै pulley, man, cage को system मान लूं तो system is in equilibrium.



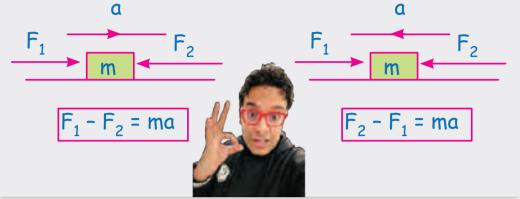
Q. A car is moving with constant acc a, as shown in fig. if the string is making constant angle θ . Find θ .

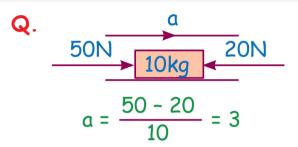




 $T \cos \theta = mg$ $T \sin \theta = ma$ $\tan \theta = \frac{a}{g}$

Very very simple but very very very important इसे lightly बिलकुल मत लेना bcz एक करोड़ बार ये use होगा a

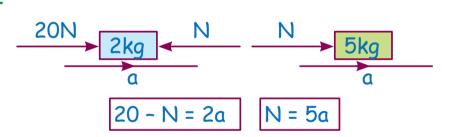




Q. Find acc of block and normal force between them.



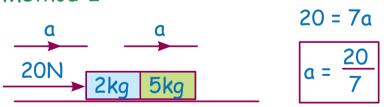
Sol.



Solve and get

$$a = \frac{20}{7}$$
, $N = \frac{100}{7}$

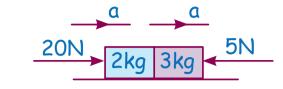
Method-2:



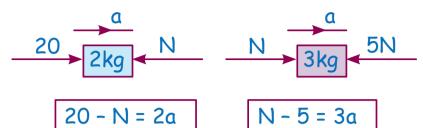
$$N = 5a = \frac{5 \times 20}{7} = \frac{100}{7}$$



Q. Find acc and normal between blocks.



Sol.



Solve and get a = 3, N = 14

Method-2:

$$= 20 - 5 = (2 + 3) a$$

$$= 20 - 5 = (2 + 3) a$$

$$= 3$$

$$a = 3$$

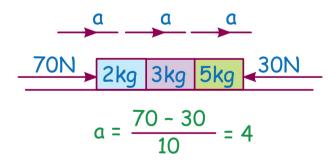
$$20 - N = 2 \times 3$$



भाई जो बच्चे पहली बार NLM पढ़ रहे हैं उनके लिए एक बहुत ही important बात बता रहा हुँ जब भी कभी normal, tension या spring force पूछे ये याद रखो की FBD बनाकर eqn लिखनी है (VVI)



Q. Find acc and normal between blocks.

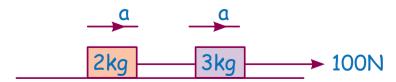


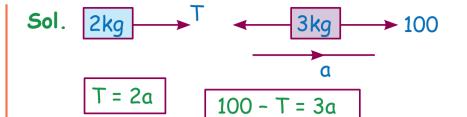
force applied by 3kg on 5kg

$$N - 30 = 5a$$

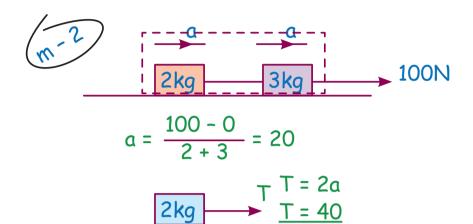
 $N = 30 + 20 = 50$

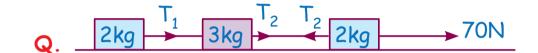
Q. Find acc of each block and tension in string.



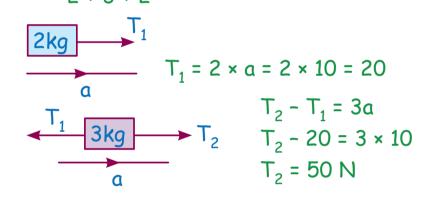


Solve and get a = 20 and T = 40





Sol.
$$a = \frac{70 - 0}{2 + 3 + 2} = 10$$



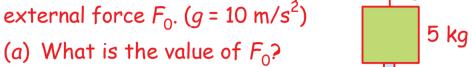
Sol.
$$a = \frac{100 - 40}{2 + 3} = 12$$

$$T - 40 = 2 \times 12$$
 $a = 12$
 $T = 64$



Sol.
$$a = \frac{100 - 20}{2 + 3 + 5} = 8$$
 $100 - T_2 = 5 \times 8$ $T_1 - 20 = 2 \times 8$ $T_2 = 60$

Q. A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at an acceleration of 2 m/s^2 by an external force F_0 . $(g = 10 \text{ m/s}^2)$



2 kg

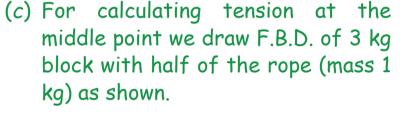
↓ 4g

- (b) What is the net force on the rope?
- 3 kg (c) What is the tension at middle point of the rope?
- **Sol**. For calculating the value of F_0 , consider two blocks with the rope as a system.

F.B.D. of whole system

(a)
$$F_0 - 100 = 10 \times 2$$

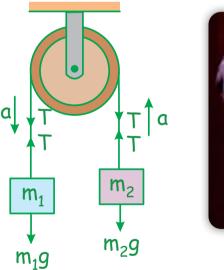
 $F_0 = 120 \text{ N}$... (i) 10 kg
(b) According to Newton's 2m/s²



$$T-4g=4\times(2)\Rightarrow T=48N$$

 \mathbf{Q} . When two bodies of mass \mathbf{m}_1 and \mathbf{m}_2 are attached at the ends of a string passing over a pulley (neglecting the mass of pulley).

Sol.

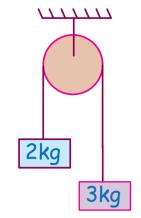


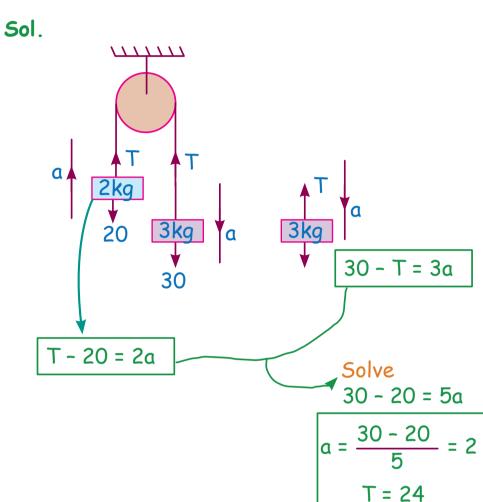


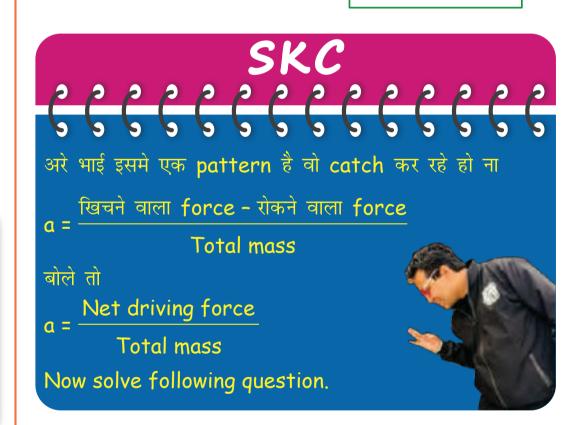
 $m_1 a = m_1 g - T (for m_1)$ $m_2 a = T - m_2 g \text{ (for } m_2 \text{)}$ Solve and get (i) and (ii)

$$a = \frac{(m_1 - m_2)g}{(m_1 + m_2)}$$
, $T = \left(\frac{2m_1m_2}{m_1 + m_2}\right)g$

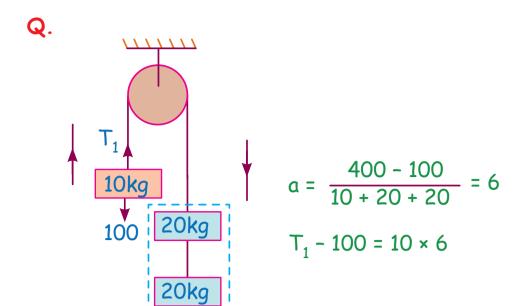
Q. Find acc of each mass if pulley is ideal, string is massless and their is no friction between pulley and rope.





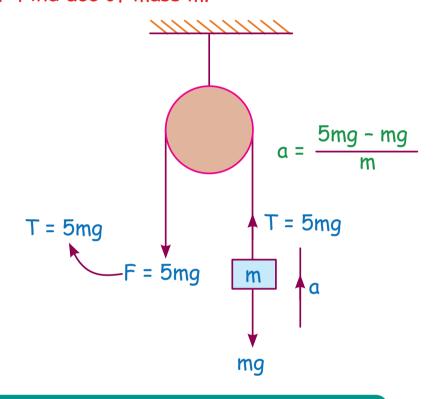


Q. $a = \frac{60 - 40}{6 + 4}$ a = 260 - T = 6aT = 48



Q. Find acc of mass m.

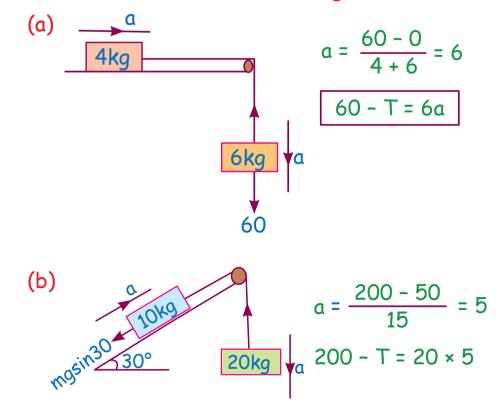
400

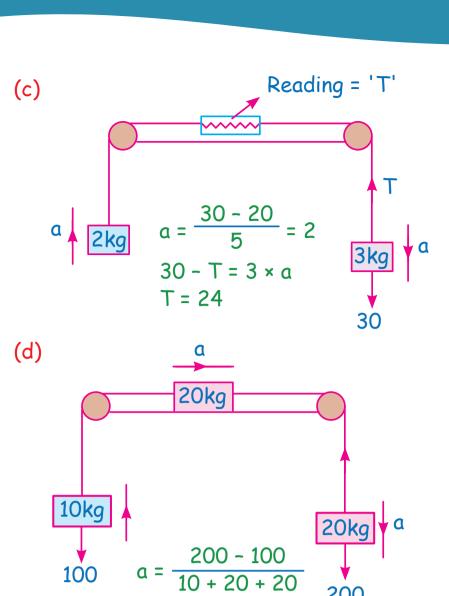


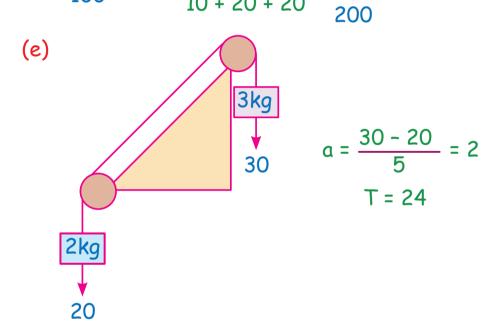
देख भाई इस बात को ध्यान में रखते हुए की ये 11th class की book है और many of you are 10th pass so, बहुत patience के साथ मैंने इस profile के basic fundamental ques करवा दिए है अब नीचे वाले एक-एक ques आपको solve करना है।

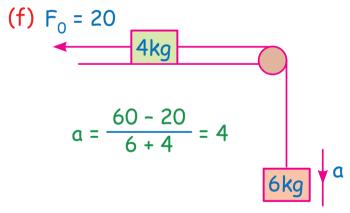


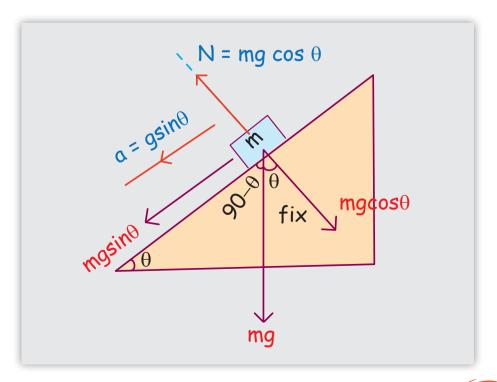
Q. Find acc and tension in following case

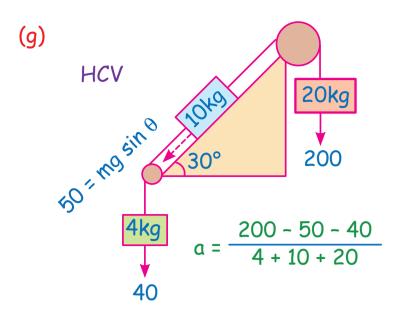


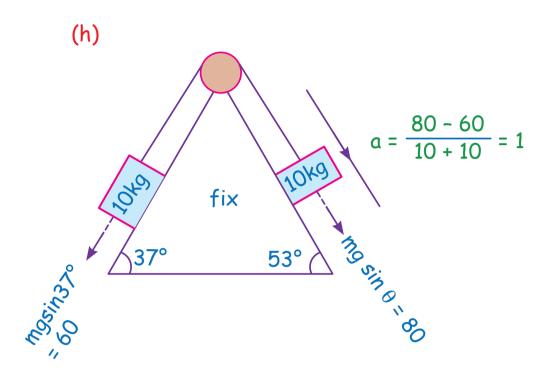


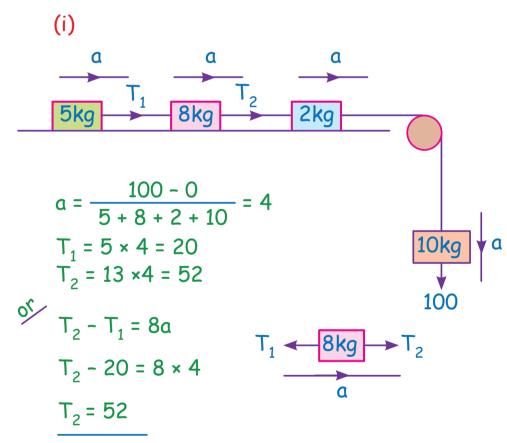


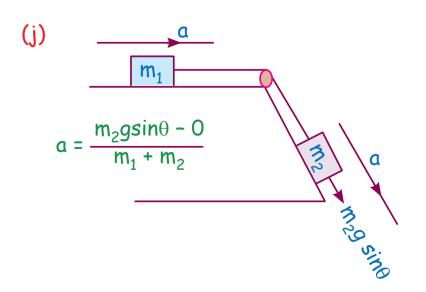


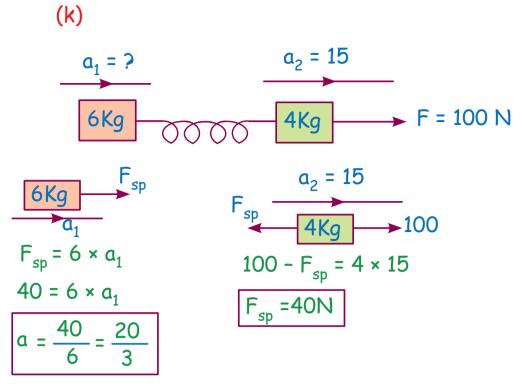




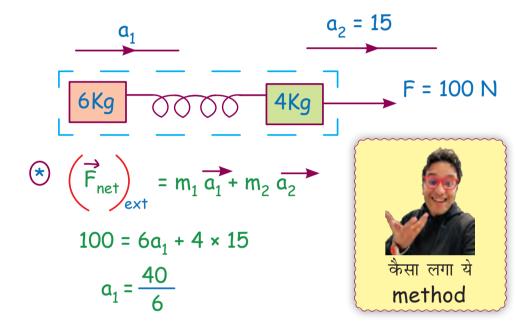




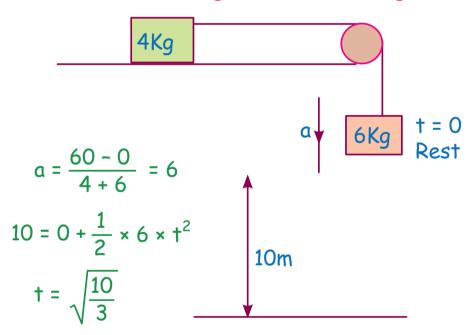




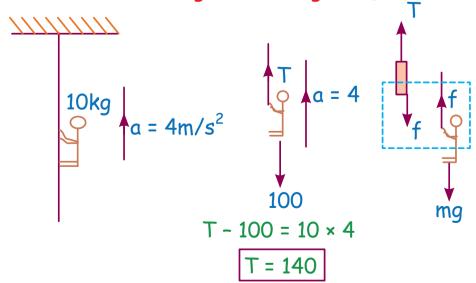
(m) Method-2:



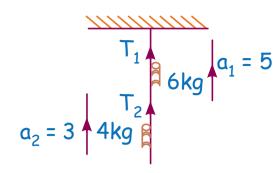
Q. Find time when 6kg block strike the ground.



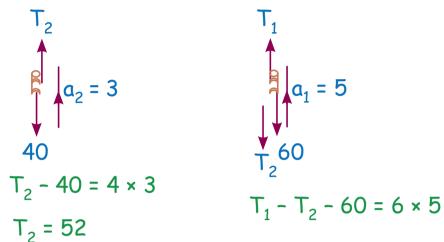
Q. Find force applied by the monkey on string (i.e tension in the string in following case)



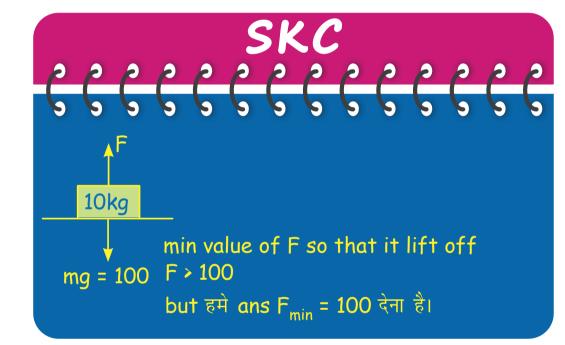
Q. Find tension in upper part of string and in middle part of string in the following figure.



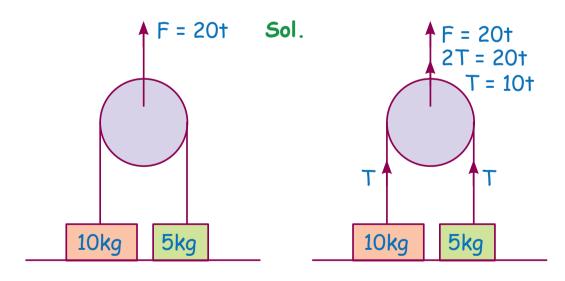
Sol.



Solve and get T_2 = 52 and T_1 = 142



Q. (a) When 5kg block leave the floor



5kg कब उडेगा जब T = 50

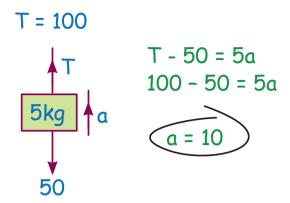
10 t = $50 \Rightarrow$ t = 5 seconds

(b) When 10kg block leave the floor

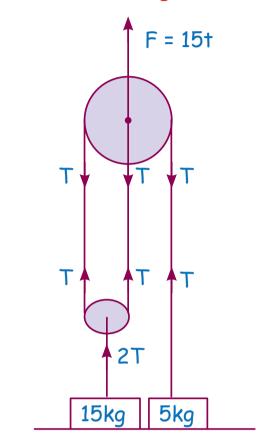
$$T = 100$$

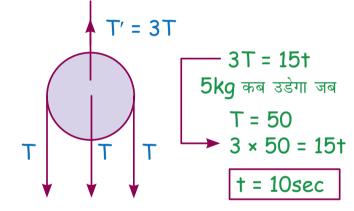
$$10 \dagger = 100 \Rightarrow \dagger = 10 \text{sec}$$

(c) find acceleration of 5kg block when 10kg block will leave the floor



Q. Find the time when 5kg block lift off



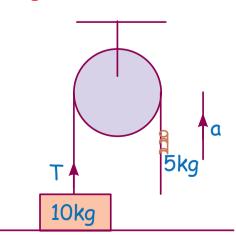


(b) when 15 kg block will lift off

$$2T = 150 \Rightarrow T = 75$$

F = 15t = 3T = 3 × 75
t = 5 sec

Q. What should be min ace of monkey so that block lift off the ground

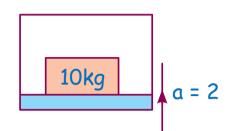


Sol.
$$T = 100$$

 $T = 50 = 100$
 $100 - 50 = 100$

$$T - 50 = 5a$$
 $100 - 50 = 5a$
 $a = 10$

Q. Find normal contact force in following figure.



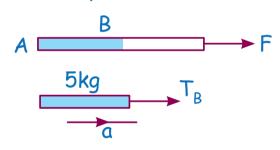
Q. Find tension at the mid point of rod

rod or string
$$(10kg, 20m)$$

$$A \longrightarrow F = 100N$$

Sol.
$$a = \frac{100}{10} = 10$$

Tat point 'B'



$$T_B = 5 \times a = 5 \times 10 = 50$$

Q. In above question find T = f(x), where x is distance from end A.



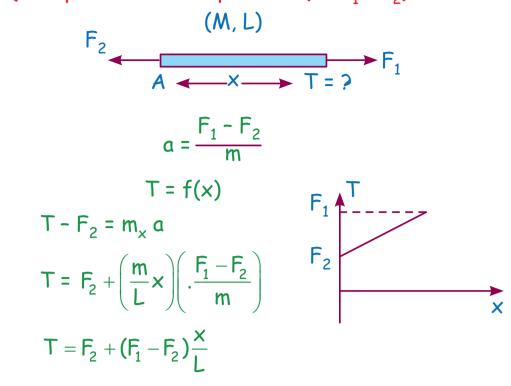
$$a = \frac{F}{M}$$

$$T = m_x a$$
 $L \longrightarrow m$

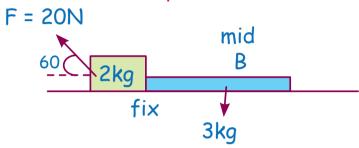
$$T = \frac{m}{L} \times \cdot \frac{F}{m} \qquad i \longrightarrow \frac{m}{L}$$

$$T = \frac{Fx}{L}$$

Q. Repeat the above problem (If $F_1 > F_2$).



Q. Find tension at midpoint of the rod.

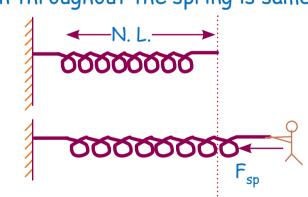


Sol.
$$a = \frac{F\cos 60}{2+3} = \frac{20 \times (0.5)}{5} = 2$$

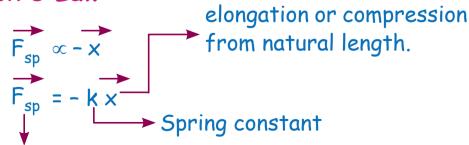
$$T_B = \frac{3}{2} \times a = \frac{3}{2} \times 2 = 3$$

IDEAL SPRING

- * Massless
- * Pitch same
- N.L
 ⇒ Natural length = released length
 = original length
- * Tension throughout the spring is same.



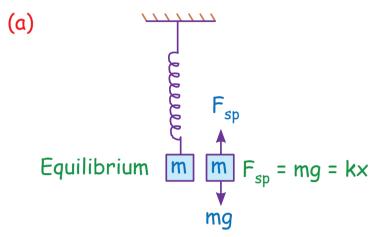
Hook's Law



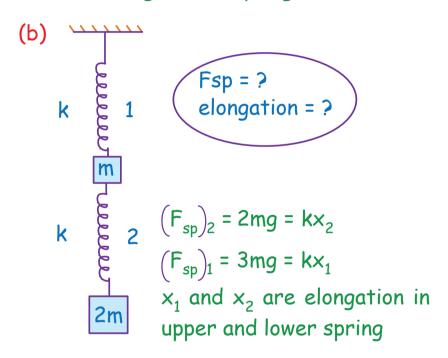
Towards the natural length

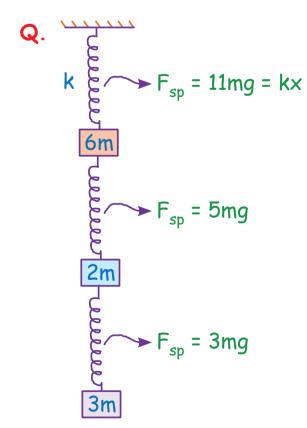


Q. Block is in equilibrium find spring force and elongation in the spring in following cases.



elongation in spring = x





बस spring force
निकाल कर उसे kx
के बराबर कर दो तो
elongation in the
spring (x) पता चल
जाएगा

Spring Cutting

- Q. A spring of length I and spring constant k is cut into two spring of length l_1 and l_2 find spring constant of both spring after cutting.

बस इतना याद रखो अगर material same है तो k × l का product same होगा



$$k \downarrow k_{1} \Rightarrow k_{2} \downarrow k_{2} \downarrow k_{1} \Rightarrow k_{2} \downarrow k_{2} \downarrow k_{2} \downarrow k_{2} \downarrow k_{1} \Rightarrow k_{1} \downarrow k_{2} \downarrow$$

$$k_1 = 2K$$

 $k_2 = 2K$

- \bigstar 2 equal parts \Rightarrow 2 mm mm mm 3k 3k 3k
 - Q. (a) Find tension and acc of each block.

$$a_1 = 2$$

$$T_A = m_A a_A = 1 \times 2 = 2N$$

(b) Find spring force & elongation in the spring

$$\begin{array}{c}
T \\
2kg \\
F_{sp} - T = ma \\
F_{sp} - 2 = 2 \times 2
\end{array}$$

$$F_{sp} = 6N$$

$$F_{sp}$$
 = kx \Rightarrow 6 = 100x \Rightarrow x = .06m

(c) Find acc of 3kg block

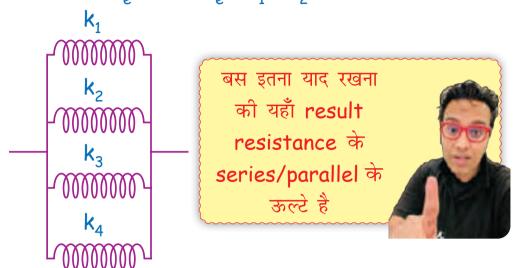
$$a_c = 4$$

$$F_{sp} = 6N$$

$$3kg \rightarrow 18N$$

EQUIVALENT SPRING CONSTANT

When two springs are connected in parallel then we can replace them by single spring of spring constant k_e where $k_e = k_1 + k_2$.



For more than two spring $k = k_1 + k_2 + k_3 + \dots$

When two springs are connected in series then we can replace them by single spring of spring constant $k_{\rm e}$

Where $1/k_e = 1/k_1 + 1/k_2$. $k_1 k_2 k_3 k_4$ $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2} + \dots$

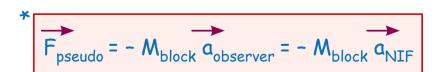
INERTIAL FRAME / NON INERTIAL FRAME

Inertial Frame

- वो frome where NLM are valid.
- Rest wrt ground or moving with const velocity. wrt ground.

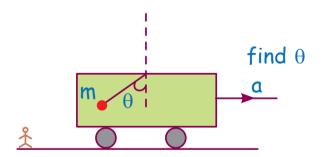
Non Inertial Frame (NIF)

- वो frame where NLM fails & directly not applicable.
- * all the accelerated frame wrt ground are NIF
- ★ अगर a है \Rightarrow Non inertial.
- ★ In order to validate NLM eqn in a non inertial frame, we must apply a correction factor called pseudo force.



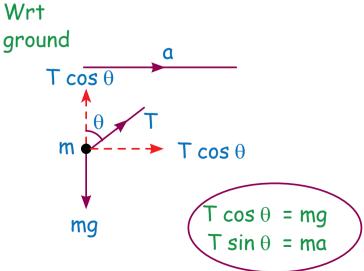
देख भाई मुद्दे की बात ये है की

- 1. Ground के respect में अगर किसी frame के पास acc है तो वो non-inertial frame है, अगर उस frame में जाके हम directly NLM की eqn लिखे तो eqn गलत आई। तो newton law की इज्जत बचाने के लिए हमने एक जबरदस्ती का force लगाया जो की सच्ची मुच्ची में नहीं लग रहा है (false force, pseudo force) ताकी हम NIF में बिंदास NLM eqn लिख सके।
- 2. Pseudo force = (Mass of block) × (Acc of NIF) और इसकी direction NIF के acc की direction के opposite
- 3. ये समझलो कि ये NSP (ताड़ने) का licsence/किराया है मतलब जब भी कभी accelerated lift, wedge, car, balloon पर जाकर मजे लेने हो तो सबसे पहले block पर pseudo force लगा दो और फिर मजा लो...... physics का
- 4. अगर आप ground पर खड़े होकर ques solve कर रहे हैं तो pseudo force की जरूरत नहीं है Bcz in this case pseudo force = 0
- 5. Ques चाहे ground पर खड़े होकर solve करो या NIF/ acc wedge, car मे बैठ कर ans same आएगा मतलब सारे ques बिना pseudo force के ground frame भी solve किए जा सकते है।
- **Q**. In following question find θ if block is at rest with respect to car. Analyse the situation in ground frame and also in car frame.



Sol.

Method-1: In ground frame



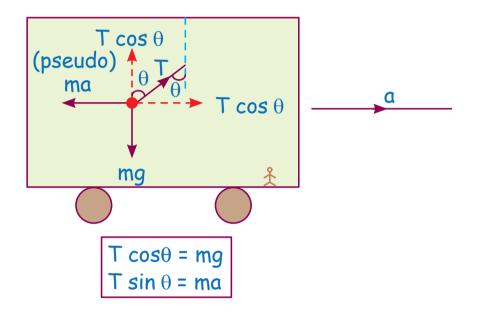
You \rightarrow sir हमेशा ये confusion रहता है कब pseudo force लगाना है और कब नहीं ?



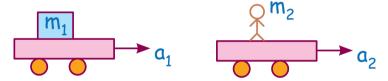


अरे जब ground पर खड़े होकर question solve कर रहे हो तो pseudo force मत लगाओ और जब car $(a \neq 0)$ के अंदर बैठ कर question solve कर रहे हो तो pseudo force लगा दो।

Method-2: In car frame



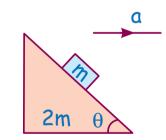
SSSQ.



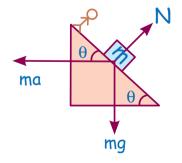
What will be the Pseudo force on 'm₁' wrt man?

Ans.
$$\overrightarrow{F}_{Pseudo} = m_1 a_2 (-\hat{i})$$

Q. What should be the accleration of wedge 'a' so that the block remains at rest wrt wedge.



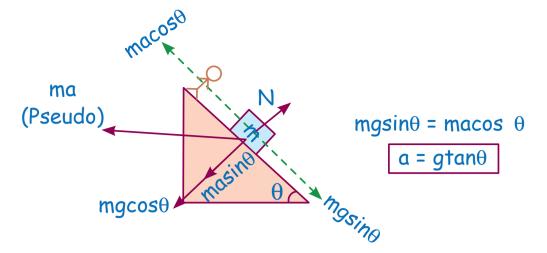
Sol.



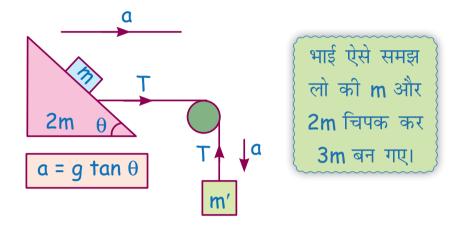
Now resolve mg and ma along the incline and perpendicular to incline.



अगर मै wedge के ऊपर जाकर बैठ जाऊँ तो मुझे block रूका हुआ दिखाई देगा, बस बैठने से पहले pseudo force देना जरूरी है।



Q. Find the value of m' so that m is at rest with respect to 2m.



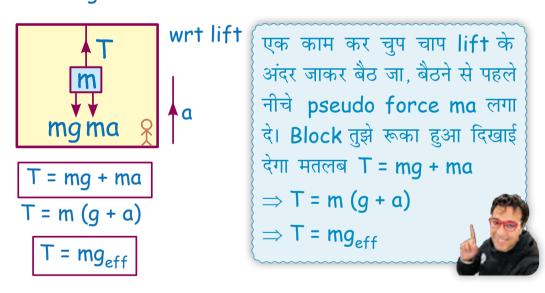
Sol. $a = g \tan \theta$

$$m'g - T = m'(g \tan \theta) \qquad ...(i)$$

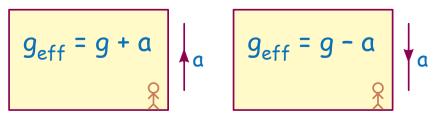
$$T = (3m) g \tan \theta$$
 ...(ii)

Solving above equations we get m'.

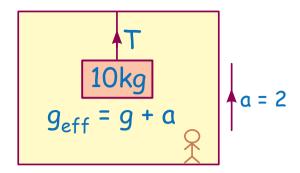
★ Consider a lift moving upward with acc a, and a mass m is hanging inside lift by a string as shown in fig. Find tension.







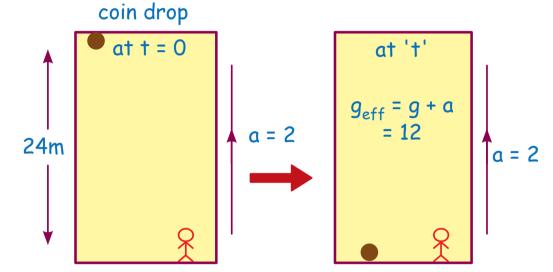
Q. Find tension is string.



Sol.
$$T = m g_{eff} = m (g + a)$$

 $T = 10 \times 12 = 120$

Q. A coin is dropped at t = 0 from ceiling of a lift as shown in fig. Find time when it will hit the floor of lift.



Sol.
$$g_{eff} = 12 \text{ m/s}^2$$

 $24 = 0 + \frac{1}{2} \times 12 \times t^2$
 $t = 2$
 $t = \frac{2h}{g_{eff}}$

पहले Kinematics
$$U_{coin/lift} = 0$$
, $a_{coin/lift} = -10 - 2$
 $S_{coin/lift} = -24 = -12$
 $-24 = 0 + \frac{1}{2}(-12) + \frac{1}{2}$
 $+ = 2 \sec$

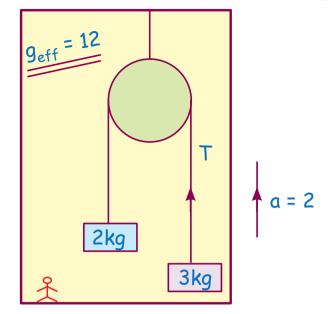
Q. Find time, when the block will reach at B.

$$g_{eff}^1 = g + a$$
 drop

Sol.
$$I = 0 + \frac{1}{2} \times (g + a)\sin \theta t^2$$

$$\Rightarrow t = \sqrt{\frac{2I}{(g+a)\sin \theta}}$$

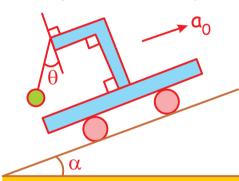
Q. Find tension in the string in following case.



$$T = \frac{2m_1m_2}{m_1 + m_2}g_{eff} = \frac{2 \times 2 \times 3}{2 + 3} \times 12$$



Q. A pendulum of mass m hangs from a support fixed to a trolley. The direction of the string when the trolley rolls up a plane of inclination α with acceleration a_0 is (String and bob remain fixed with respect to trolley)

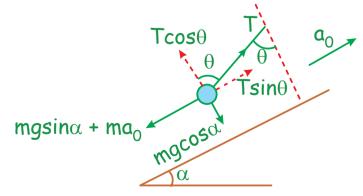


Sol. Balancing forces in the frame of trolley (non-inertial frame)

 $T \sin \theta = m (g \sin a + a_0)$

 $T\cos\theta = mg\cos\alpha$

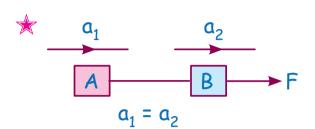
$$\Rightarrow \tan \theta = \left(\frac{g\sin \alpha + a_0}{g\cos \alpha}\right)$$

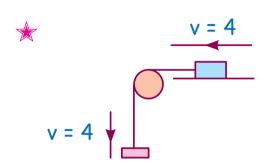


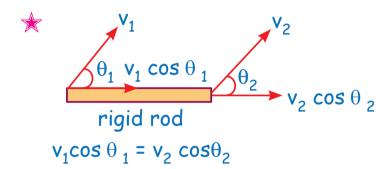
$$\theta = \tan^{-1} \left(\frac{g \sin \alpha + a_0}{g \cos \alpha} \right)$$

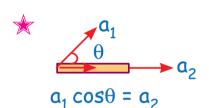
CONSTRAINT MOTION

Along the rigid rod or taut string; Components of magnitude of velocity & acc. should be same.

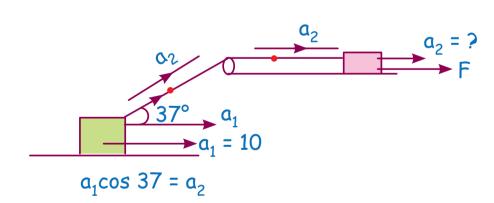






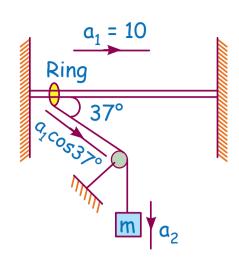


Q. Relation between a_1 and a_2 .



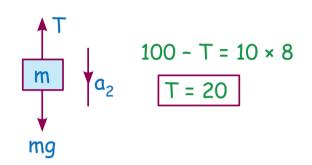


Q. Ring is constant to move along the fix rod as shown in fig. If at a instant acc of ring is 10 m/ s^2 along +x-axis. Find acceleration of block.



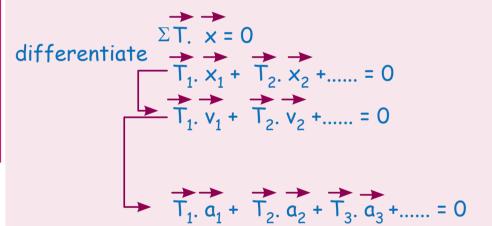
$$a_2 = a_1 \cos 37^\circ$$
 $a_2 = 10 \times \frac{4}{5} = 8 \text{ m/s}^2$

(b) If m = 10 kg find T at this instant

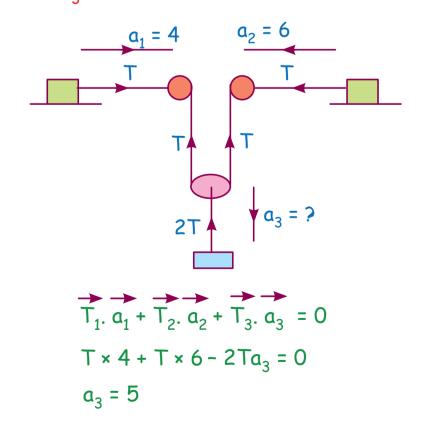


Virtual Work Method

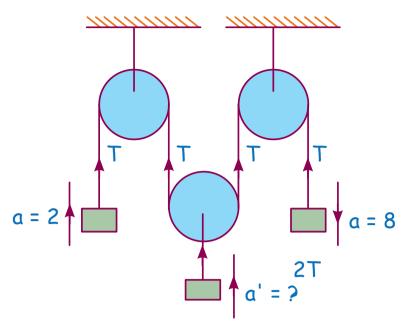
ये कैसे आया इसका reason WPE में पढ़ेंगे (net WD by internal tension = 0) अभी ये सिखो इसको apply कैसे करना है।



 \mathbf{Q} . Find \mathbf{a}_3 .



Q. Find a' in following figure.



Sol.
$$T \times 2 + 2Ta' - T \times 8 = 0$$
 $a' = 3$

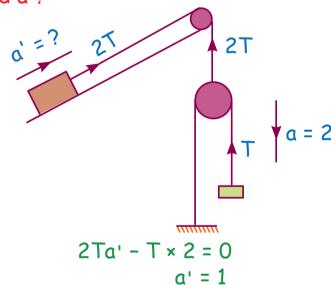
कुछ बच्चो के दिमाग मे ये doubt आएगा कि sir यहाँ mass तो given नही है।



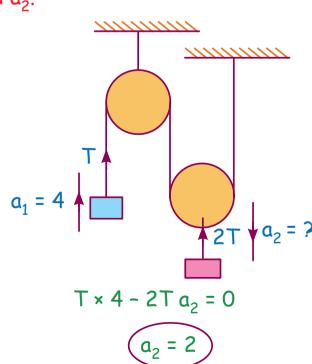


बेटा यहाँ mass की जरूरत नहीं है यही तो सीखना है, अगर रस्सी मे tension पूछता तो mass की जरूरत होती।

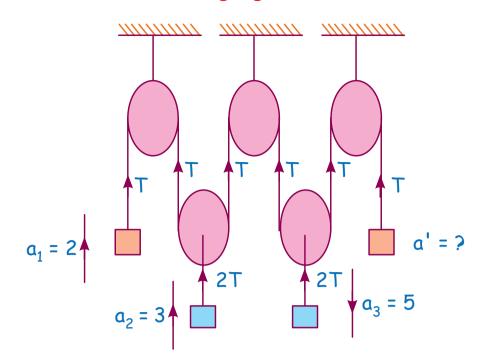
Q. Find a'?



\mathbf{Q} . Find \mathbf{a}_2 .



Q. Find a' in following figure.



$$T \times 2 + 2T \times 3 - 2T \times 5 + Ta' = 0$$

 $2 + 6 - 10 + a' = 0$
 $a' = 2$

देख भाई किसी एक रस्सी में tension T मान और उसके बाद सभी रस्सी में tension निकाल दे जिसका acc नहीं पता उसका acc ऊपर की तरफ मान लो और solve करदों अगर answer negative आया तो समझ जाना acc नीचे था।

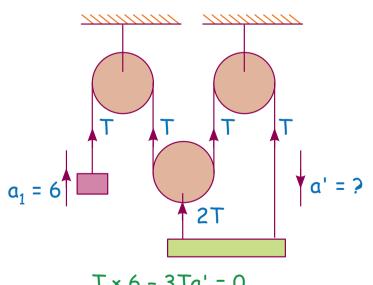




अबे जा कहाँ रहा है पहले आगे वाले ques और solve कर



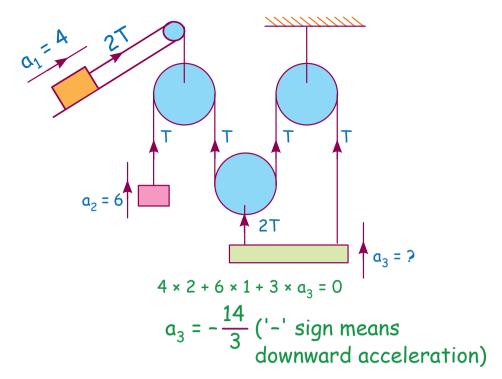
Q. Find a' in following figure.



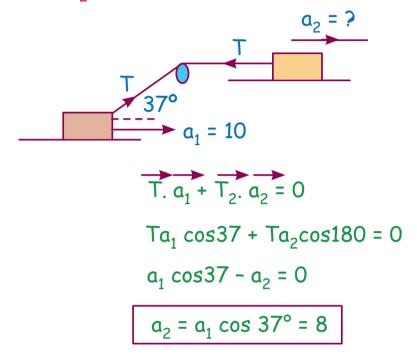
$$T \times 6 - 3Ta' = 0$$

$$a' = 2$$

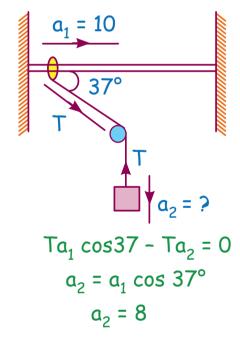
Q. Find a' in following figure.



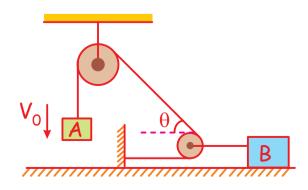
Q. Find a_2 ?



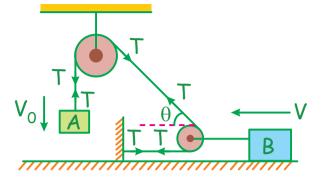
Q. Find a_2 ?



Q. In the figure, find the velocity of block B, if velocity of A is V_0 in downward direction?

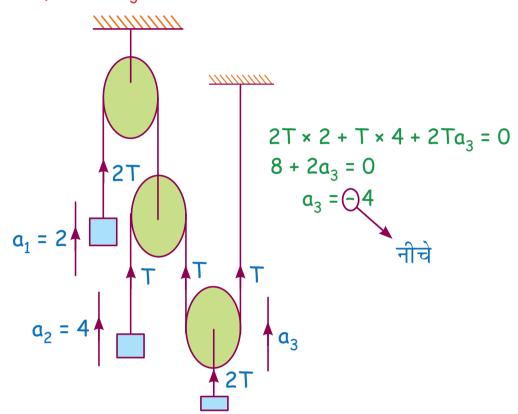


Sol.

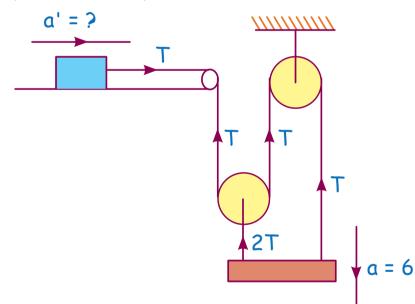


$$-TV_0 + TV\cos\theta + TV = 0 \Rightarrow V = \frac{V_0}{1 + \cos\theta}$$

 \mathbf{Q} . Find \mathbf{a}_3 .



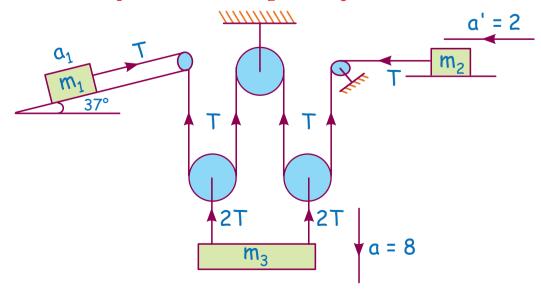
Q. Find a' and T.



$$Ta' - 3T \times 6 = 0$$

 $a' = 18$

Q. If $m_1 = 10$ kg find m_2 and m_3 in following fig.



$$Ta_1 - 32T + 2T = 0$$

 $a_1 = 30$

$$T - 100 \sin 37 = 10 \times 30 \text{ (For } m_1)$$

$$T = 360$$

$$m_3 g - 4T = m_3 \times a$$

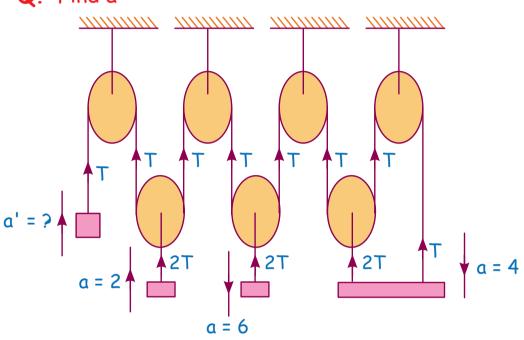
$$m_3 \times 10 - 4 \times 360 = m_3 \times 8$$

$$m_3 = 720 \text{ kg}$$

$$T = m_2 a' \Rightarrow 360 = m_2 \times 2$$

$$m_2 = 180$$

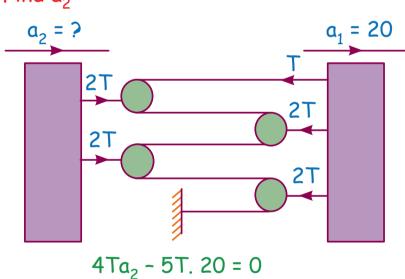
Q. Find a



$$Ta' + 4T - 12T - 12T = 0$$

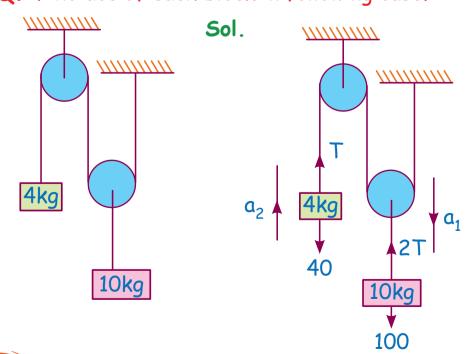
 $a' = 20$

Q. Find a₂



Q. Find acc of each block in following case.

 $a_2 = 25$



Method-1:

$$T - 40 = 4a_2$$
 ...(1)

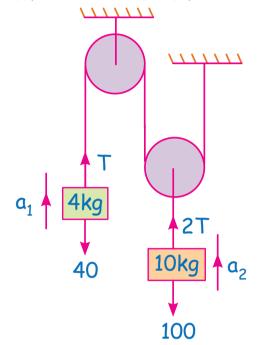
$$100 - 2T = 10a_1$$
 ...(2)

$$Ta_2 - 2Ta_1 = 0$$

$$a_2 = 2a_1$$
 (constraint relation) ...(3)

Solve & get

Method-2: SKC Method

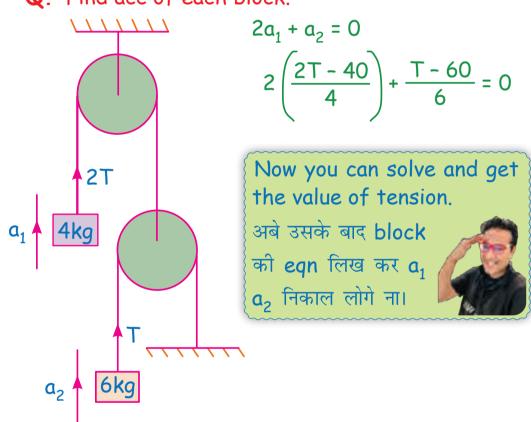


$$Ta_1 + 2Ta_2 = 0$$

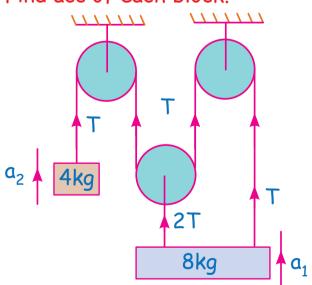
 $a_1 + 2a_2 = 0$
 $\frac{T - 40}{4} + 2\left(\frac{2T - 100}{10}\right) = 0$

Solve and get value of T and then \mathbf{a}_1 and \mathbf{a}_2

Q. Find acc of each block.



Q. Find acc of each block.

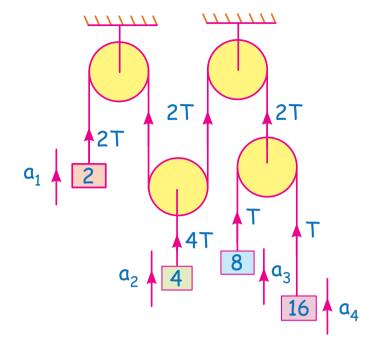


$$Ta_2 + 3Ta_1 = 0$$

$$a_2 + 3a_1 = 0$$

$$\frac{T - 40}{4} + 3\left(\frac{3T - 80}{8}\right) = 0$$

Q. Find acc of each block.

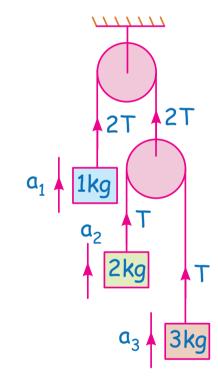


$$2a_1 + 4a_2 + a_3 + a_4 = 0$$

$$2\left(\frac{2T - 20}{2}\right) + 4\left(\frac{4T - 40}{4}\right) + \left(\frac{T - 80}{8}\right) + \frac{T - 160}{16} = 0$$

ये जरूर check कर लेना कि जितने mass उतने terms आएंगें

Q. Find acc of each block.



$$2Ta_1 + Ta_2 + Ta_3 = 0$$

$$2a_1 + a_2 + a_3 = 0$$

$$2\left(\frac{2T-10}{1}\right)+\frac{T-20}{2}+\frac{T-30}{3}=0$$

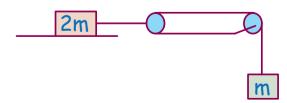
Solve and get T

$$a_1 = (2T - 10)/1$$

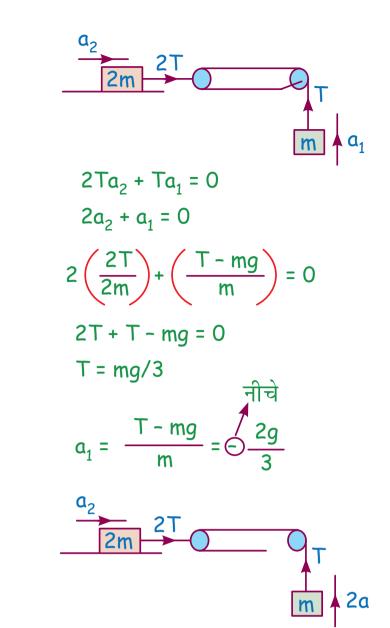
$$a_2 = (T - 20)/2$$

$$a_3 = (T - 30)/3$$

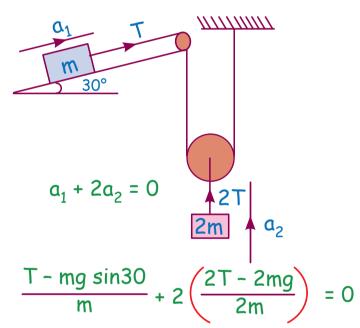
Q. Find acc of both the block.



Sol.

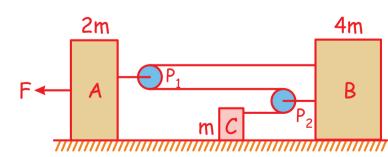


Q. Find acc of both the block.

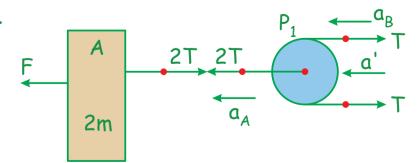


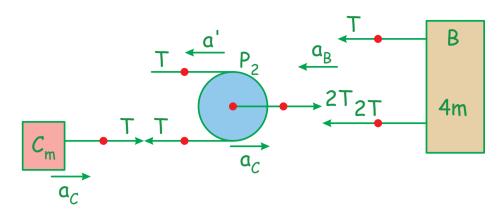
Now solve by yourself.

Q. In the shown figure, find the acceleration of block B. All surfaces are frictionless.



Sol.





$$-2Ta_A + 3Ta_B + Ta_C = 0 \Rightarrow 3a_B + a_C = 2a_A$$

From above equation

 $3a_B + a_c = 2a_A$...(i) [Constraint relation]

 $F - 2T = 2ma_A...(ii)$ [Newton's II law for block A]

 $3T = 4m a_B \dots (iii)$ [Newton's II law for block B]

 $T = ma_C$ (iv) [Newton's II law for block C]

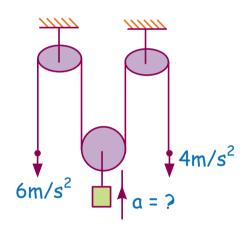
From (i), (ii), (iii) and (iv)

$$3 \times \frac{3T}{4m} + \frac{T}{m} = 2 \left(\frac{F-2T}{2m} \right)$$

$$\frac{9T}{4} + T = F - 2T \Rightarrow T = \frac{4F}{21}$$

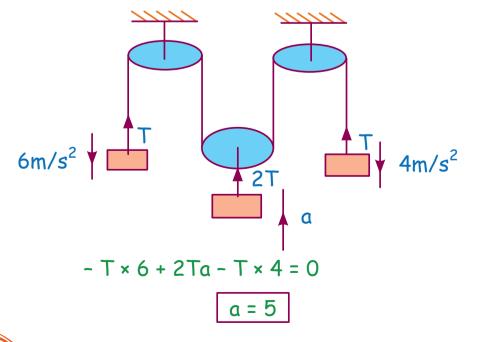
$$a_B = \frac{3T}{4m} \Rightarrow a_B = \frac{F}{7m}$$
.

Q. Find a?

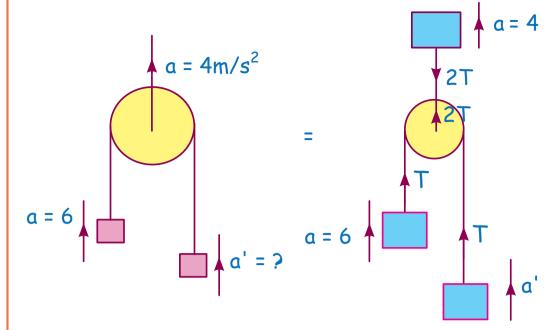


इस type के ques में अगर block missing है तो जरूरत के हिसाब से बना लो।

Sol.



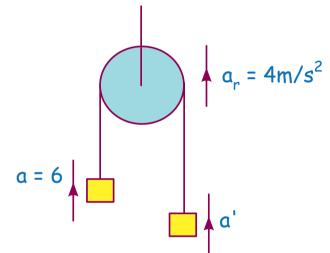
Q. Find a'?



$$+T \times 6 + Ta' - 2T_4 = 0$$

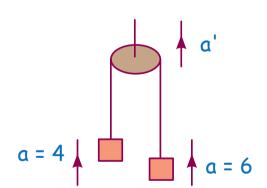
 $+6 + a' - 8 = 0$

Q. Find a'



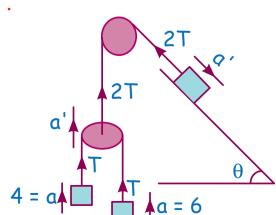
 $4 = \frac{6 + a'}{2}$ (ये एक मस्त वाला result है कैसे आया वो छोड़ो इसका pattern याद करलो)

Q. Find a'



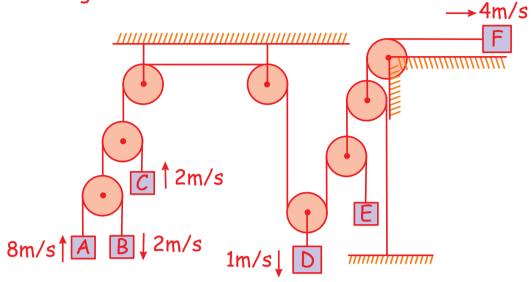
Sol.
$$a' = \frac{4+6}{2} = 5$$

Q. Find a'.



Method-2:
$$a' = \frac{4+6}{2} = 5$$

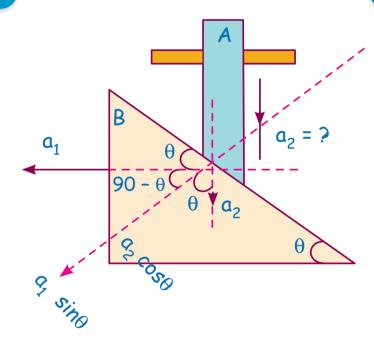
Q. Find out the velocity of block E as shown in figure.



Sol.
$$8T - 2T + (2T \times 2) - (8T \times 1) + 4TV_E - (16T \times 4) = 0$$

 $4V_E = -8 + 2 - 4 + 8 + 64 = 62$
 $V_E = \frac{62}{4} = \frac{31}{2} \text{ms}^{-1}$

WEDGE CONSTRAINT



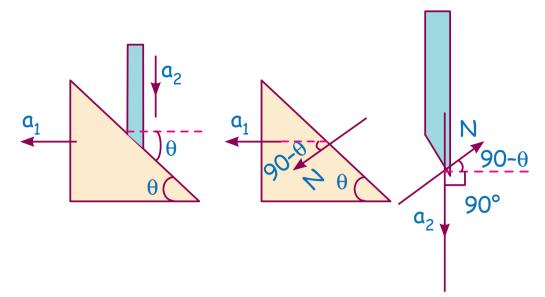
Method-1:

- * There no relative motion along common normal.
- * Component of acc of A & B along the common normal same

$$a_1 \sin\theta = a_2 \cos\theta$$

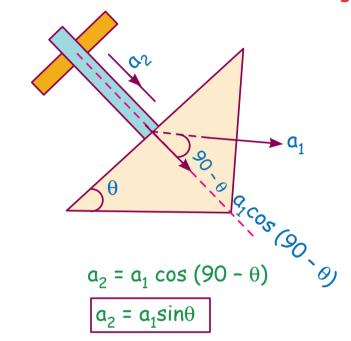


Method-2:

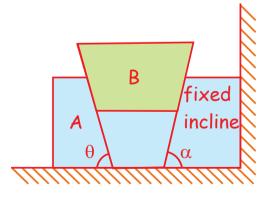


$$N_1$$
. $a_1 + N_2$. $a_2 = 0$
 $Na_1 \cos (90 - \theta) + Na_2 \cos (90 + 90 - \theta) = 0$
 $a_1 \sin \theta - a_2 \cos \theta = 0$
 $a_1 \sin \theta = a_2 \cos \theta$

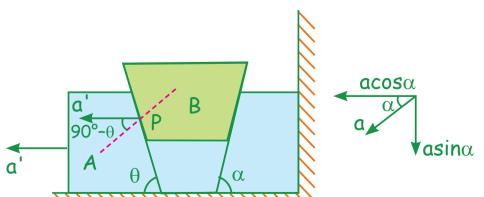
Q. Find constraint relation for following fig.



 \mathbf{Q} . In the arrangement shown in figure, if the acceleration of B is a, then find the acceleration of A.



Sol. Let a' be the acceleration of block A using wedge constraint at point P



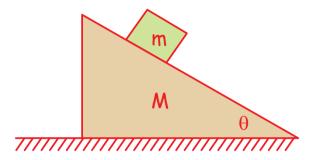
$$a\sin\alpha \cos\theta + a\cos\alpha \sin\theta = a'\cos(90^\circ - \theta)$$

$$a\sin(\alpha + \theta) = a'\sin\theta$$

$$a' = \frac{a\sin(\theta + \alpha)}{a}$$

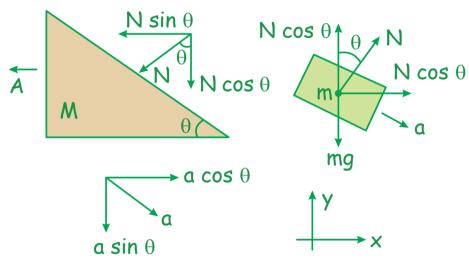
$$a' = \frac{a\sin(\theta + \alpha)}{\sin\theta}$$

Q. A block of mass m is placed on the inclined surface of a wedge as shown. Calculate the acceleration of the wedge and the block when the block is released. Assume all surfaces are frictionless.



Sol. Let the acceleration of wedge be A and that of block be a (w.r.t. wedge). Then acceleration of m w.r.t. ground is

$$(a \cos \theta - A)\hat{i} - a \sin \theta \hat{j}$$



For M: N sin
$$\theta$$
 = MA ... (i)

For m:
$$mg - N \cos \theta = ma \sin \theta$$
 ... (ii)

N
$$\sin \theta = m(a \cos \theta - A)$$
 ... (iii)

Solved (i), (ii) and (iii) to get

$$A = \frac{\text{mgsin}\theta\cos\theta}{\text{M} + \text{msin}^2\theta} \text{ and } \alpha = \frac{(\text{M} + \text{m})\text{gsin}\theta}{\text{M} + \text{msin}^2\theta}$$

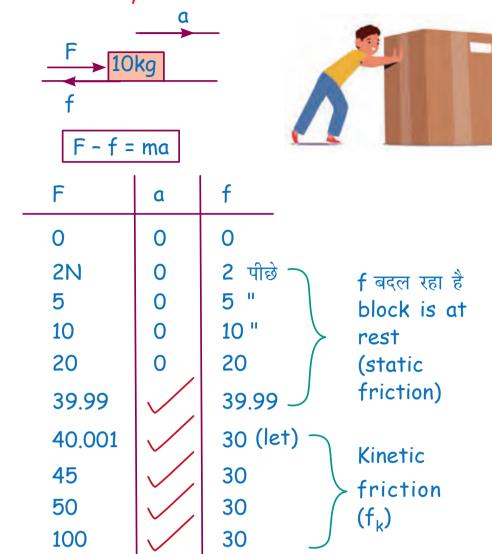
Now acceleration of block can be found by putting the values of A and a in (i).

FRICTION

देख भाई ये कोई नया chapter नहीं है, NLM ही है इसमे एक force पढ़ेंगे जिसका नाम है friction और उसे पुराने forces के साथ mix करेंगे, बस इस बात का ध्यान रखना की हर question में friction कितना लग रहा है और उसका nature क्या है।



Q. A block of mass 10 kg is at rest on rough surface. A boy is applying force F on the block such that value of F is increasing slowly from zero analyse the situation



Static Friction

It is variable force and self adjusting force.

Experimentally
$$\Rightarrow$$
 $(f_s)_{max} \propto N$ $(f_s)_{max} = \mu_s N$ coeff of static friction

- * It oppose relative motion b/w contact surface
- \star (f_s)_{max} = limiting friction
- * Independent on area of contact.

Kinetic Friction

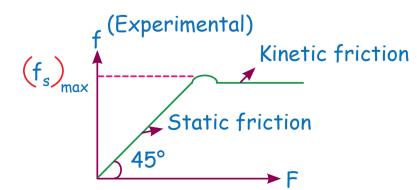
- It opposes the relative motion b/w contact surface

$$f_k \propto N$$

$$f_k = \mu_k N = Const$$

$$Coeff of kinetic friction$$

* It's value is constant and independent of area of contact surface.



Q. In given fig. Find value of acc and friction if F = 10 N, 20 N, 35 N, 39.99 N, 40.01 N, 50 N.

F
$$\mu_s = 0.4$$

 $\mu_k = 0.3$

Sol.
$$(f_s)_{max} = \mu_s N = 0.4 \times mg = 40 N$$

$$f_k = \mu_k N = 0.3 \times 100 = 30N$$



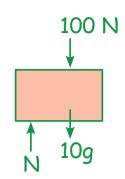
देख भाई सबसे पहले carefully $(f_s)_{max}$ और f_k निकालो यहाँ $(f_s)_{max} = 40$ N आया मतलब रूके हुए block पर अगर 40 N या उससे कम force लगाओंगे तो block रूका रहेगा और friction static लगेगा जिसकी value जितना force लगाया उसके बराबर होगी। अगर 40 N से ज्यादा force लगाया तो block move कर जाएगा।

 $f_k = 30 N$ आया means अगर block move कर रहा है तो friction kinetic लगेगा जिसकी value constant 30 N होगी।

- (a) Find a & f if F = 10N a = 0, f = 10N, (Static)
- (b) Find a & f if F = $20N \Rightarrow a = 0$, $f_s = 20$
- (c) 11 11 11 F = 35N \Rightarrow a = 0, f_s = 35
- (d) II II II F = 39.99 \Rightarrow a = 0, f_s = 39.99
- (f) F = 50N, $a = \frac{50 30}{10} = 2$
 - Q. Find frictional force acting on the block in the following cases given that $\mu = 0.1$ [Take $g = 10 \text{ m/s}^2$]

 Case-I: $F_1 = 0$ Case-II: $F_1 = 20 \text{ N}$

Sol. F.B.D. of the block



$$f_{s.max} = \mu_s N = 200 \times 0.1 = 20 N$$

If $|F_2 - F_1| < \mu_s N$ block will not move and self adjusting static friction will act on its block and will be equal to $|F_2 - F_1|$.

If
$$|F_2 - F_1| > \mu_s N$$

then,
$$f_k = \mu_k N$$

Case-I:
$$F_1 = 0$$

$$F_2 = 5N$$
 $f = 5 N \text{ (towards left)}$

$$F_2 = 15N f = 15 N \text{ (towards left)}$$

$$F_2 = 25N f = 20 N$$
 (towards left)

Case-II:
$$F_1 = 20 N$$

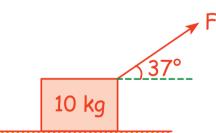
$$F_2$$
 = 10 N f = 10 N (towards right)

$$F_2 = 20 N f = 0$$

$$F_2$$
 = 42 N f = 20 N (towards left)

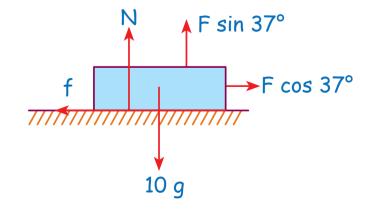
$$F_2$$
 = 50 N f = 20 N (towards left)

Q. A force F is applied on the block as shown in the figure. If the coefficient of static friction μ_s = 0.3 and coefficient of kinetic friction is μ_k = 0.2.



Find frictional force and acceleration of the block if

- (i) F = 10 N
- (ii) F = 50 N
- Sol. F.B.D. of the block



(i)
$$F = 10 \text{ N}$$

Balancing forces normal to the surface

$$N + 10\sin 37^{\circ} - 10g = 0 \implies N = 94 N$$

$$f_{s. max} = \mu_s N = 0.3 \times 94 = 28.2 N$$

Fcos37° =
$$10 \times \frac{4}{5} = 8 \text{ N}$$

Since, Fcos37° $< \mu_s N$, block will not move and static friction $f_s = F\cos 37^\circ = 8N$.

(ii)
$$F = 50 \text{ N}$$

$$N + 50\sin 37^{\circ} - 10 g = 0 \implies N = 70N$$

$$f_{s, \text{max}} = \mu_s N = 0.3 \times 70 = 21N$$

$$F\cos 37^{\circ} = 50 \times \frac{4}{5} = 40 \text{ N}$$

Since $F\cos 37^{\circ} > \mu_s N$, friction will not be able to prevent slipping.

Block will slide and kinetic friction will act on the block

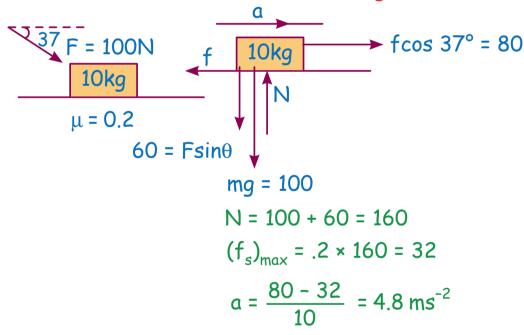
$$f_k = 0.2 \text{ N} = 0.2 \times 70 = 14 \text{ N}$$

To find acceleration, we apply Newton's law along horizontal direction

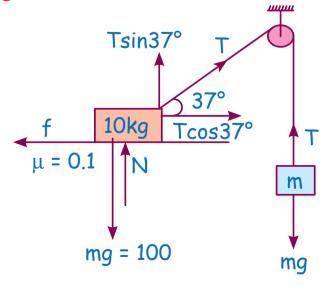
$$F\cos 37^{\circ} - f_k = ma$$

$$\Rightarrow a = \frac{40-14}{10} = 2.6 \text{ m/s}^2$$

Q. Find the acc of block in following case.



Q. Find min value of m so that 10 kg block just move



$$Tsin37 + N = 100$$

कायदे में सही
$$\Rightarrow$$
 Tcos37° \rightarrow (f_s)_{max}

$$T = mg$$

Tcos37° =
$$(f_s)_{max}$$
 = μN

$$mg \frac{4}{5} = 1 (100 - Tsin37)$$

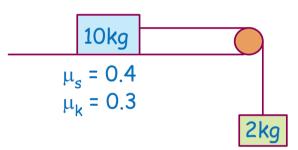
$$mg \frac{4}{5} = 10 - .1 \times mg \times \frac{3}{5}$$

$$mg\left(\frac{4}{5} \times \frac{.3}{5}\right) = 10$$

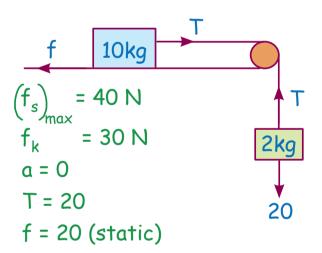
$$m \times 10 \times \frac{4.3}{5} = 10$$

$$m = \frac{5}{43} kg$$

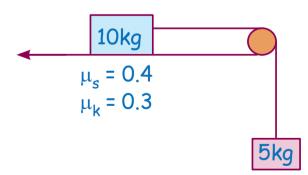
Q. Find acc. of block and value of friction.



Sol.



Q. Find acc. of block and value of friction.



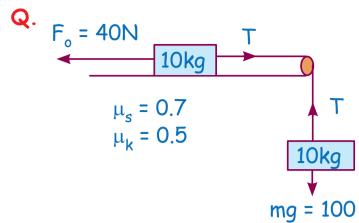
Sol. Since
$$(f_s)_{max} = 40$$
 and $f_k = 30$ N

$$mg = 50 > (f_s)_{max}$$
 (Block will move)

$$a = \frac{50 - f_k}{10 + 5} = \frac{50 - 30}{15} = \frac{4}{3}$$

$$50 - T = 5 \times a$$

$$T = 50 - 5 \times \frac{4}{3} = \frac{130}{3}$$



Sol.



देख भाई 10 kg के block को move कराने के लिए effectively 70 N का force चाहिए और यहाँ ध्यान से देखो तो 100 - 40 = 60 है means block move नहीं करेगा।

$$a = 0$$

$$T = 100$$

$$f_{o}$$

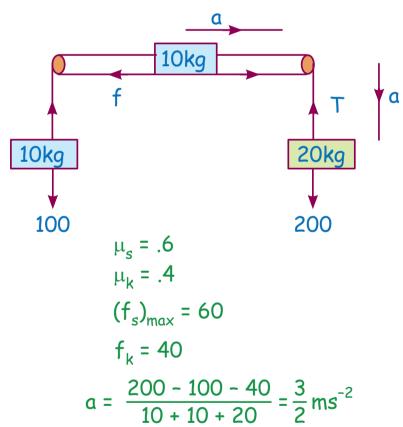
$$10kg \rightarrow T$$

$$f_{s}$$

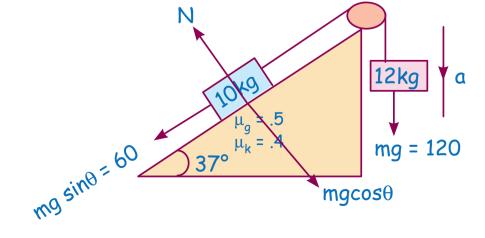
$$T = F_{o} + f_{s}$$

$$100 = 40 + f_{s}$$

Q. Find the acc. of each block.

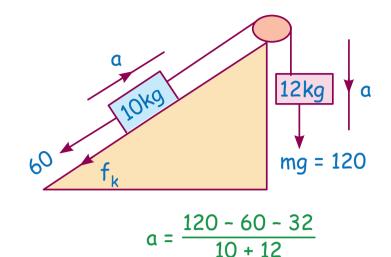


Q. Find acc. and friction.

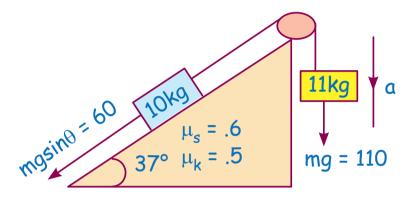


Newton's Laws of Motion & Friction

N = mg
$$cos\theta$$
 = 80
 $(f_s)_{max}$ = .5 × 80 = 40
 f_k = .4 × 80 = 32



Q. Find a?



$$(f_s)_{max} = \mu_s N = \mu_s mgcos\theta$$

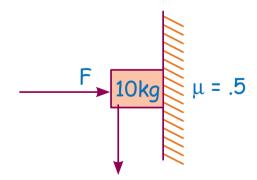
$$(f_s)_{max} = .6 \times 80 = 48$$

$$(f_k) = \mu_k N = .5 \times 80 = 40$$

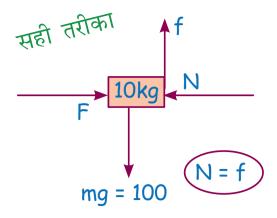
$$a = \frac{110 - 60 - 40}{10 + 11} = \frac{10}{21} \text{ ms}^{-2}$$

Friction की understanding को बढ़ाने के लिए नीचे ques attach कर रहा हूँ जिससे no. of page बढ़ेंगे, publisher वाले भईया गुस्सा होंगे क्योंकि price तो हम same रखेंगे लेकिन तुम्हे हर ques solve करना है। Bcz exam मे इन ques के आने कि probability बहुत ज्यादा है।

Q.



(a) Find min value of F so that block does not fall.



Block does not fall

$$mg \leq (f_s)_{max}$$

$$mg \leq \mu_s N$$

Shortcut

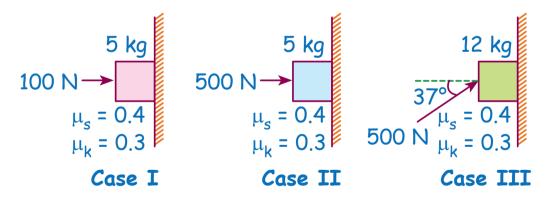
$$mg = (f_s)_{max} = \mu_s N$$
 $100 = 5 \times F_{min}$
 $F = 200 = F_{min}$

(b) Find max value of F so that block fall

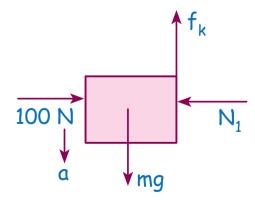
200 से कम लगाओंगे तो गिरेगा 199.99999......

Ans. 200

Q. Determine the magnitude of frictional force and acceleration of the block in each of the following cases: [Take $g = 10 \text{ m/s}^2$]



Sol. Case-I: $N_1 = 100 \text{ N}$, mg = 50 N

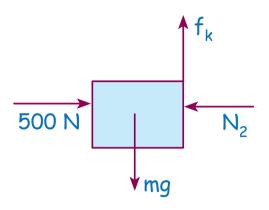


$$f_1 = \mu_s N_1 = 0.4 \times 100 = 40 \text{ N}, f_k = \mu_k N_1$$

= 0.3 × 100 = 30 N

Here mg (driving force) is greater than maximum friction f_1 = 40 N. Hence the block will not be able to stay at rest. It will accelerate downwards. But when it starts slipping, then kinetic friction will come into play.

Case-II: $N_2 = 500 \text{ N}$, mg = 50 N

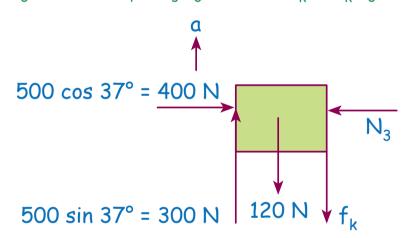


$$f_1 = \mu_s N_2 = 200 \text{ N}, f_k = \mu_k N_2 = 150 \text{ N}$$

Here f_1 is greater than mg (driving force), Hence, block will not move. So in this case a=0, $f=mg=50\ N$

Case-III:

$$N_3 = 400 \text{ N, } f_1 = \mu_s N_3 = 160 \text{ N, } f_k = \mu_k N_3 = 120 \text{ N}$$



Here driving force = $300 - 120 = 180 \, \text{N}$ in upward direction; hence, friction will act downwards. Driving force is more than f_{\parallel} . So the block will accelerate upwards.

$$a = \frac{180 - f_k}{m} = \frac{180 - 120}{12} = 5 \text{ ms}^{-2} \text{ (upwards)}$$

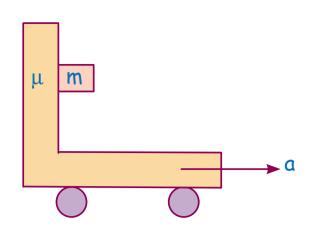
So in this case $f = f_k = 120 \text{ N}$ and $a = 5 \text{ ms}^{-2}$ (upwards)

Now

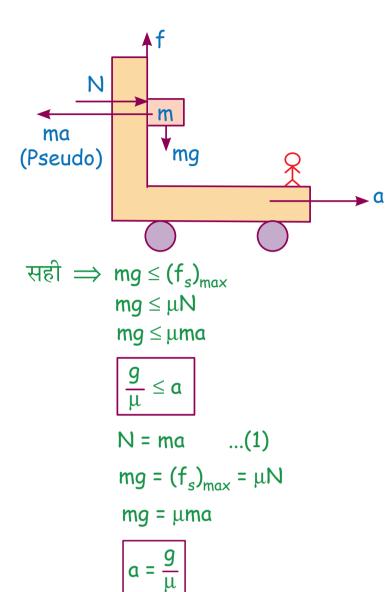
$$a = \frac{mg - f_k}{m} = \frac{50 - 30}{5} = 4 \text{ ms}^{-2}$$

So in this case $f = f_k = 30 \text{ N}$ and $a = 4 \text{ ms}^{-2}$ (downwards)

- Q. (a) Find min acc 'a' so that block does not fall
 - (b) Find max acc 'a' so that block does fall

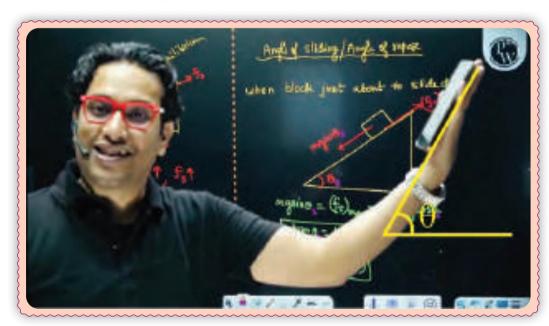


Sol.



Answer for both the (a) and (b) is same and equal to $\frac{g}{\mu}$

Angle of Repose/Angle of Sliding

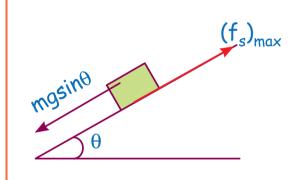


अगर मै ऊपर वाली image मे अपने हाथ को ऊपर उठाता जाऊँ मतलब θ increase करता जाऊँ तो जिस angle पर ये mobile just नीचे फिसलना start हो जाएगा उस angle को हम फिसलने वाला angle कहते है बोले तो angle of sliding.



A block of mass m is at rest on a rough incline plane making angle θ with horizontal. The angle θ at wedge block is just about to slide down is called angle of repose.

When block is just about to slide





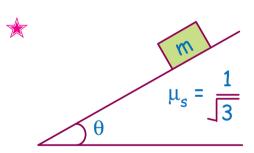
$$mgsin\theta = (f_s)_{max} = \mu_s mgcos\theta$$

$$\mu_s = \tan\theta$$

$$\theta = \tan^{-1}(\mu_s) = \text{angle of repose}$$

If
$$\theta \le \theta_s = \text{Rest}$$

$$\theta > \theta_s = Slide down$$



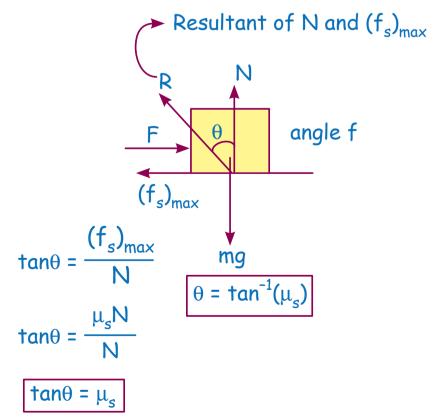
$$tan\theta = \mu_s = \frac{1}{\sqrt{3}}$$

$$\theta_s = 30$$

$$\theta \leq 30 \Rightarrow Block Rest$$

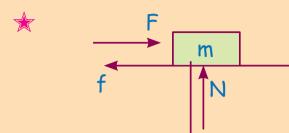
$$\theta > 30 \Rightarrow Block Move$$

Angle of Friction

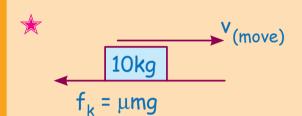




काम का डब्बा



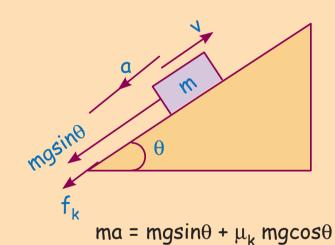
Netcontact force = $\sqrt{N^2 + f^2}$



$$a = \frac{f_k}{m} = \frac{\mu mg}{m} = \mu g$$
 (पीछे)

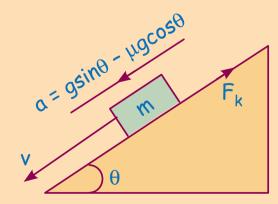
Stopping distance निकालने के लिए 3rd eqn of motion लगाओ $0^2 = v^2 - 2(\mu g)x$

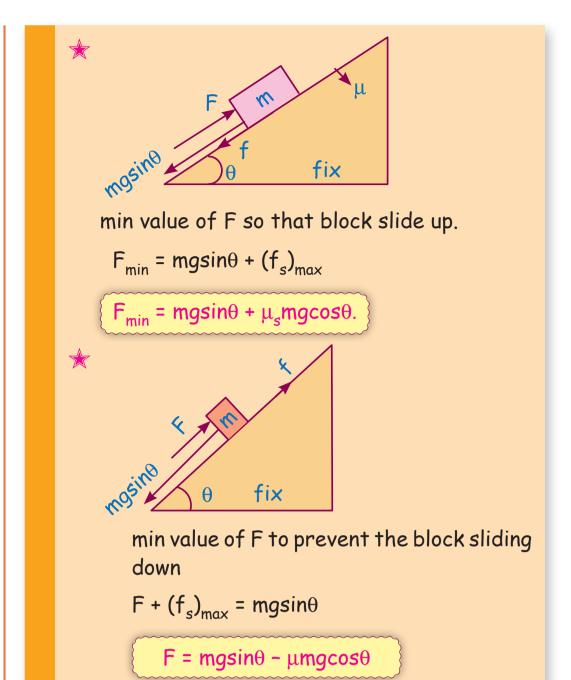
* When block is moving up along the inclined



$$a = a\sin\theta + \mu a\cos\theta$$

* When block is moving down along the inclined

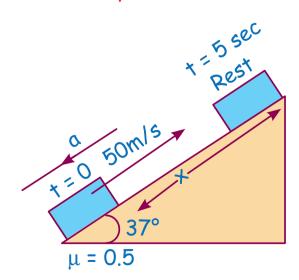




Q. In following fig. Find distance travel by block before coming to rest.

Sol. 20m/s $V_f = 0$ $\mu = .4$ \times $\alpha = \mu g = 4 (418) = cost.$ $O^2 = (20)^2 - 2 \times 4 \times x$ $\times = 50 \text{ m}$

Q. In the given fig. block is projected along the rough incline (μ = 0.5) with speed 50 m/s. Find distance travel by block before coming to rest.



$$a = gsin\theta + \mu gcos\theta = 6 + .5 \times 8 = 10 \text{ m/s}^2$$

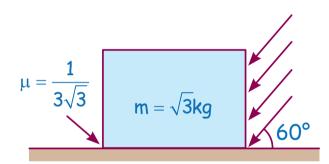
$$O^2 = (50)^2 - 2 \times 10 \times x$$

$$x = \frac{2500}{20} = 125 \text{ m}$$

Advance में directly friction से सवाल नहीं पूछे गए हैं लेकिन mains में हर साल repetative concept पर question पूछे जा रहे हैं। So, मैं कुछ important PYQ attach कर रहा हुँ इन्हें अच्छे से solve करे देखना यहाँ से सवाल जरूर आएगा।



Q. As shown in the figure, a block of mass $\sqrt{3}$ kg is kept on a horizontal rough surface of coefficient of friction $\frac{1}{3\sqrt{3}}$. The critical force to be applied on the vertical surface as shown at an angle 60° with horizontal such that it does not move. The value of force will be?



$$g = 10 \,\text{m/s}^2; \sin 60^\circ = \frac{\sqrt{3}}{2}; \cos 60^\circ = \frac{1}{2}$$

Sol. Net force in the Vertical direction is zero so.

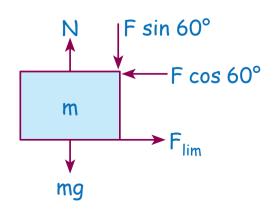
$$N = mg + F \sin 60^{\circ}$$

$$f_{\ell im}$$
 = Fcos 60° $\Rightarrow \mu N$ = Fcos60°

$$\Rightarrow \mu$$
(mg + Fsin 60°) = Fcos 60°

$$\Rightarrow \mu$$
mg = F[cos 60° - μ sin 60°]

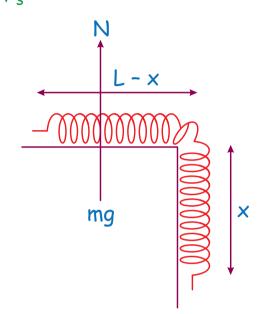
$$F = \frac{\mu mg}{\cos 60^{\circ} - \mu \sin 60^{\circ}} = \frac{\frac{1}{3\sqrt{3}} \times \sqrt{3} \times 10}{\frac{1}{2} - \frac{1}{3\sqrt{3}} \times \frac{\sqrt{3}}{2}} = 10$$



Q. A uniform chain of 6 m length is placed on a table such that a part of its length is hanging over the edge of the table. The system is at rest. The co-efficient of static friction between the chain and the surface of the table is 0.5, the maximum length of the chain hanging from the table is _____m.

$$N = mg = \lambda(L - x)g$$

$$f_{s,\text{max}} = \mu_s N$$



$$f_{s,max} = (0.5) (\lambda) (L - x)g$$

And also
$$f_{s,max} = m_x g$$

$$0.5\lambda (L - x)g = \lambda xg$$

$$\frac{L-x}{2}=x$$

$$\frac{L}{2} = \frac{3x}{2} \Rightarrow x = \frac{L}{3} = \frac{6}{3} = 2 \text{ m}$$

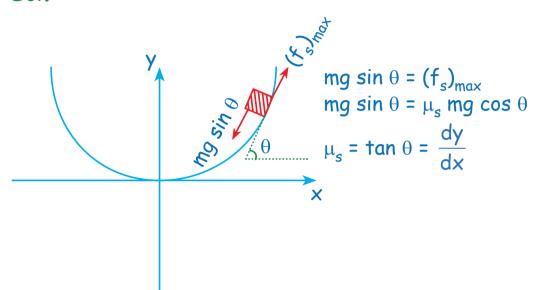
- Q. A body of mass 1 kg rests on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F (in N). The value of F will be _____ (Round off to the Nearest Integer) [Take $g = 10 \text{ ms}^{-2}$]
- Sol. Minimum possible force,

$$F = \frac{\mu mg}{\sqrt{1 + \mu^2}}$$

$$F_{min} = \frac{\frac{1}{\sqrt{3}} \times 1 \times 10}{\sqrt{1 + \frac{1}{3}}} = 5N$$

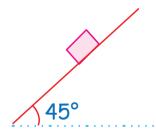
Q. A block of mass m is placed on a surface having vertical cross section given by $y = x^2/4$. If coefficient of friction is 0.5, the maximum height above the ground at which block can be placed without slipping is:

Sol.

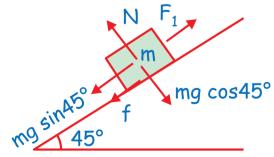


$$y = x^2/4$$
, $\mu = 0.5$
 $\frac{dy}{dx} = \tan \theta = \frac{x}{2} = \mu = \frac{1}{2}$
 $x = 1$, $y = 1/4$

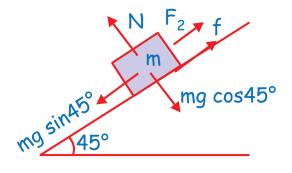
Q. Consider a block kept on an inclined plane (inclined at 45°) as shown in the figure. If the force required to just push it up the incline is 2 times the force required to just prevent it from sliding down, the coefficient of friction between the block and inclined plane (μ) is equal to:



Sol. (a) Force F_1 required to just push the block up the incline



$$F_1 = mgsin 45^{\circ} + f = \frac{mg}{\sqrt{2}} + \mu mg \cos 45^{\circ} = \frac{mg}{\sqrt{2}} (1 + \mu)$$



Force F₂ required to prevent the block from sliding

$$F_2$$
 = mgsin 45° - f
= mgsin 45° - $\mu N = \frac{mg}{\sqrt{2}} (1 - \mu)$

Given,
$$F_1 = 2F_2$$

$$\Rightarrow \frac{mg}{\sqrt{2}}\Big(1+\mu\Big) = 2\frac{mg}{\sqrt{2}}\Big(1-\mu\Big)$$

$$\Rightarrow$$
 μ = 1/3 = 0.33

- Q. The time taken by an object to slide down a 45° rough inclined plane is n times as it takes to slide down a perfectly smooth 45° inclined plane. The coefficient of kinetic friction between the object and the incline plane is
- Sol. For a perfectly smooth inclined plane

$$a_1 = g \sin \theta = g / \sqrt{2}$$

For the rough inclined plane

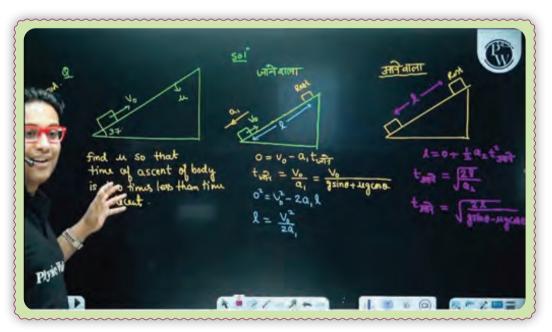
$$a_2 = g \sin \theta - \mu_k g \cos \theta = \frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}$$

Given,
$$t_2 = nt_1$$

and
$$a_1 t_1^2 = a_2 t_2^2$$

$$\Rightarrow \frac{g}{\sqrt{2}} t_1^2 = \left(\frac{g}{\sqrt{2}} - \frac{kg}{\sqrt{2}} \right) n^2 t_1^2$$

$$\Rightarrow \mu_K = 1 - \frac{1}{n^2}$$

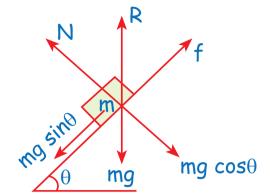


- Q. A block of mass M slides down on a rough inclined plane with constant velocity. The angle made by the incline plane with horizontal is θ . The magnitude of the contact force will be:
- Sol. From F. B. D

$$N = Mg \cos \theta$$

$$f = Mg \sin \theta$$

$$\therefore R = \sqrt{N^2 + f^2} = \sqrt{(Mg\cos\theta)^2 + (Mg\sin\theta)^2} = Mg$$



- Q. A body of mass 'm' is launched up on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$ if the time of ascent is half of the time of descent. The value of x is
- Sol. For Ascent:

$$t_a = \sqrt{\frac{2\ell}{a_{ascent}}} = \sqrt{\frac{2\ell}{g(\sin\theta + \mu\cos\theta)}} \qquad ...(i)$$

For Descent:

$$t_d = \sqrt{\frac{2\ell}{a_{descent}}} = \sqrt{\frac{2\ell}{g(\sin\theta - \mu\cos\theta)}}$$
 ...(ii)

According to question, $t_a = \frac{1}{2}t_d$

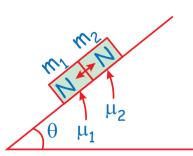
$$\Rightarrow \frac{1}{\sin\theta + \mu\cos\theta} = \left(\frac{1}{4}\right)\left(\frac{1}{\sin\theta - \mu\cos\theta}\right)$$

[From (i) and (ii)]

$$\Rightarrow \mu = \frac{3}{5} \tan \theta = \frac{3}{5} \tan 30^{\circ} = \frac{\sqrt{3}}{5}$$

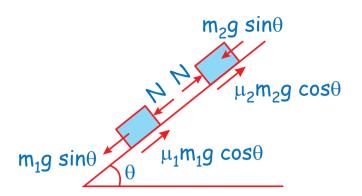
TWO BLOCKS ON AN INCLINED PLANE

Consider two blocks having masses m_1 and m_2 placed on a rough inclined plane. μ_1 and μ_2 are the friction coefficient for m_1 and m_2 respectively. If N is the normal force between the contact surface of m_1 and m_2 .



Now two condition arises.

(i) If $\mu_1 > \mu_2$ both will move with same acc



$$a = \frac{(m_1 + m_2)g\sin\theta - (\mu_1m_1 + \mu_2m_2)g\cos\theta}{m_1 + m_2}$$

We can also find normal force between block just by writting NLM eqn of any block.

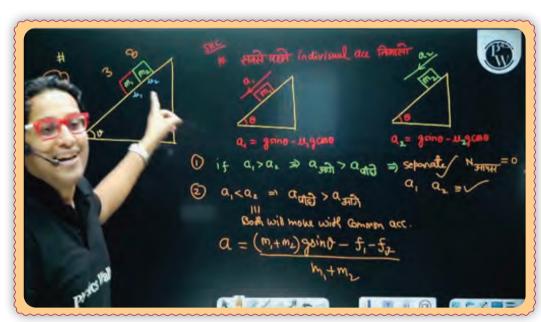
(ii) If $\mu_1 < \mu_2$ then, blocks will move with different acceleration.

N = 0 because, there is no contact between the blocks.

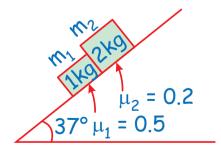
$$a_1 = g \sin \theta - \mu_1 g \cos \theta$$

$$a_2 = g \sin \theta - \mu_2 g \cos \theta$$

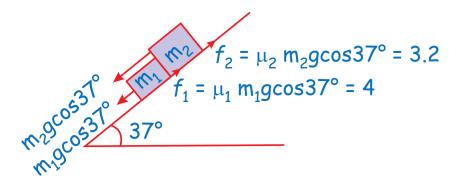
$$\Rightarrow$$
 $a_1 > a_2$



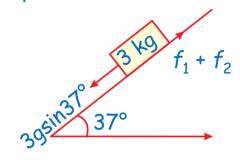
 ${\bf Q}.$ Mass ${\bf m}_1$ and ${\bf m}_2$ are placed on a rough inclined plane as shown in figure. Find out the acceleration of the blocks and contact force in between these surface.



- Sol. As we know if $\mu_1 > \mu_2$ both will travel together so $a_1 = a_2 = a$
 - F.B.D

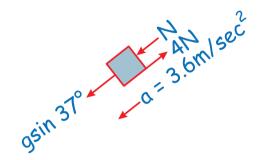


which is equivalent to



$$a = \frac{3g \sin 37^{\circ} - (f_1 + f_2)}{3} \implies a = \frac{18 - 7.2}{3} = 3.6 \,\text{m/sec}^2$$

Now F.B.D of 1 kg block is

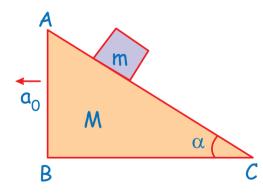


$$g \sin 37^{\circ} + N - 4 = (1) a$$

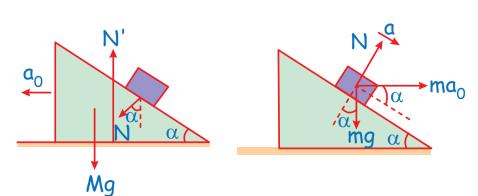
$$N = 3.6 + 4 - 6 = 1.6$$
 Newton

Now practice following question for Adv.

Q. All the surfaces shown in figure are assumed to be frictionless. The block of mass m slides on the prism which in turn slides backward on the horizontal surface. Find the acceleration of the smaller block with respect to the prism.



Sol. Let the acceleration of the prism be a_0 in the backward direction. Consider the motion of the smaller block from the frame of the prism The forces on the block are (figure)



$$ma_0 \cos \alpha + mg \sin \alpha = ma$$
 (for block)

or;
$$a = a_0 \cos \alpha + g \sin \alpha$$
 ...(i)

N + ma₀ sin
$$\alpha$$
 = mg cos α (for block) ...(ii)

N sin α = Ma₀ (for prism)

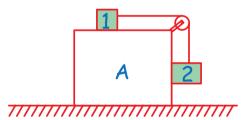
$$N = Ma_0/\sin \alpha$$
 ... (iii)

Putting in (ii)

$$\frac{\text{Ma}_0}{\text{sin}\alpha} + \text{ma}_0 \text{sin}\alpha = \text{mgcos}\alpha \ \text{or} \ \text{a}_0 = \frac{\text{mgsin}\alpha \text{cos}\alpha}{\text{M} + \text{msin}^2\alpha}$$

From (i),
$$a = \frac{\text{mgsin}\alpha\cos^2\alpha}{\text{M} + \text{msin}^2\alpha} + \text{gsin}\alpha = \frac{(\text{M} + \text{m})\text{gsin}\alpha}{\text{M} + \text{msin}^2\alpha}$$

Q. What is the minimum acceleration with which bar A should be shifted horizontally to keep the bodies 1 and 2 stationary relative to the bar? The masses of the bodies are equal and the coefficient of friction between the bar and the bodies equal to μ . The masses of the pulley and the threads are negligible while the friction in the pulley is absent.



Sol. Let us place the observer on A.

Since we have non-inertial frame we have pseudo forces.

For body '1' we have,

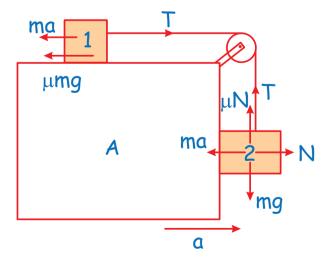
$$T = ma + \mu mq \qquad ... (i)$$

For body '2' we have,

N = ma

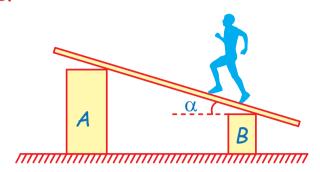
$$mg - T - \mu ma = 0$$

∴
$$mg = T + \mu ma$$
 ... (ii)

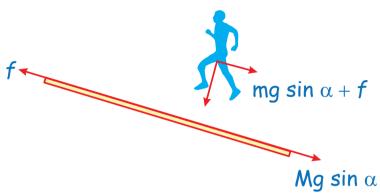


From (i) and (ii)
$$a_{min} = g \left(\frac{1-\mu}{1+\mu} \right)$$

Q. A plank is held at an angle α to the horizontal (Fig.) on two fixed supports A and B. The plank can slide against the supports (without friction) because of its weight Mg. With what acceleration and in what direction, a man of mass m should move so that the plank does not move.



Sol. F.B.D. of man and plank are



For plank be at rest, applying Newton's second law to plank along the incline

Mg sin
$$\alpha$$
 = f ... (i)

and applying Newton's second law to man along the incline.

mg sin
$$\alpha$$
 + f = ma ... (ii)
$$a = g \sin \alpha \left(1 + \frac{M}{m}\right) \text{ down the incline}$$

Alternate Solution: If the friction force is taken up the incline on man, then application of Newton's second law to man and plank along incline yields.

$$f + Mg \sin \alpha = 0$$
 ... (i)

$$mg \sin \alpha - f = ma$$
 ... (ii)

Solving (i) and (ii)

$$a = g \sin \alpha \left(1 + \frac{M}{m}\right)$$
 down the incline

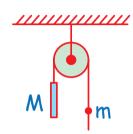
Alternate Solution: Application of Newton's seconds law to system of man + plank along the incline yields

 $mg \sin \alpha + Mg \sin \alpha = ma$

$$a = g \sin \alpha \left(1 + \frac{M}{m}\right)$$
 down the incline

Therefore, option (b) is the correct answer.

Q. In the arrangement shown in figure the mass of the rod M exceeds the mass m of the ball. The ball has an opening permitting it to slide along the thread with some friction. The mass of the pulley, mass of the string and the friction in its axle are negligible. At the initial moment the ball was located opposite the lower end of the rod. When set free, both bodies began moving with constant accelerations. Find the friction force between the ball and the thread if t seconds after the beginning of motion the ball got opposite the upper end of the rod. The rod length equals ℓ .



Sol. Step-I: Draw force diagram separately: In fig B, P is a point on the string

From fig. A,

$$mg - f = ma_1$$

$$\therefore a_1 = \frac{mg - f}{m} \qquad \dots (i)$$

From figure B,

$$T = f$$
 ... (ii)

P rod a₂
mg
Fig. A Fig. B Fig. C

From fig. C

$$Mg - T = Ma_2$$

$$Mg - f = Ma_2$$

$$\therefore a_2 = \frac{Mg - f}{M} \qquad ... (iii)$$

Step-II: Apply kinematic relation:

$$s_{\text{rel}} = u_{\text{rel}} t + \frac{1}{2} a_{\text{n}} t^2$$
 (Shown in fig. D)

Here
$$s_{rel} = \ell$$
, $u_{rel} = 0$

$$a_{rel} = a_2 - a_1$$

$$\therefore \ell = \frac{1}{2} (a_2 - a_1) t^2$$

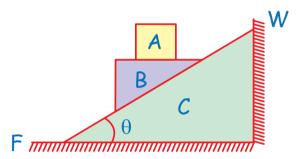
$$\therefore t = \sqrt{\frac{2\ell}{(a_2 - a_1)}}$$

$$rod \qquad \ell \qquad mg \qquad ball$$
Fig. D

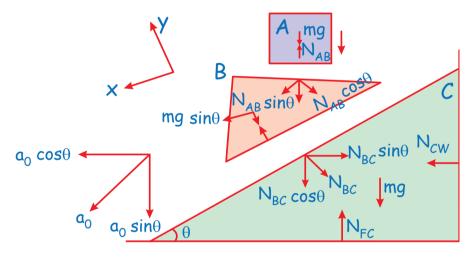
Putting the value of a_1 and a_2 , a_1

$$f = \frac{2\ell \text{ Mm}}{(\text{M} - \text{m})t^2}$$

Q. In the figure shown all the surfaces are smooth. The blocks A, B and C have the same mass m. F is floor and W is the wall. Find the magnitude of the contact forces at all the surfaces after the system is released from rest. The angle of inclination of the inclined plane with the horizontal is θ .



Sol.



$$mg - N_{AB} = ma_A$$

[Newton's II law for block A in vertical direction]

$$mg \sin \theta + N_{AB} \sin \theta = ma_{B}$$

[Newton's II law for block B in x direction]

$$a_A = a_B \sin \theta$$

[Constrained relation for contact surface between block A and B]

Solving above three equations we get

$$N_{AB} = \frac{mg \cos^2 \theta}{1 + \sin^2 \theta}$$

 $mg \cos \theta + N_{AB} \cos \theta - N_{BC} = 0$

[Equilibrium of block B in y direction]

$$\Rightarrow N_{BC} = mg \cos \theta + \frac{mg \cos^2 \theta \cos \theta}{1 + \sin^2 \theta}$$

$$\Rightarrow N_{BC} = \frac{2 mg \cos \theta}{1 + \sin^2 \theta}$$

$$N_{BC} \sin \theta - N_{WC} = 0$$

[Equilibrium of block C in horizontal direction]

$$N_{WC} = \frac{2mg \sin \theta \cos \theta}{1 + \sin^2 \theta}$$

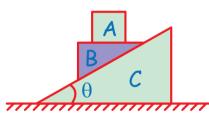
$$N_{BC} \cos \theta + mg - N_{FC} = 0$$

[Equilibrium of block C in vertical direction]

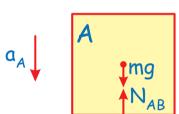
$$\Rightarrow N_{FC} = \frac{2 \operatorname{mg} \cos^2 \theta}{1 + \sin^2 \theta} + \operatorname{mg}$$

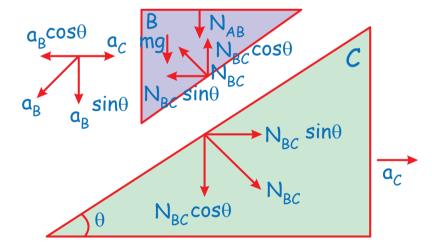
$$\Rightarrow N_{FC} = \frac{\text{mg} (2 + \cos^2 \theta)}{1 + \sin^2 \theta}$$

Q. In the figure shown all blocks are of equal mass 'm'. All surfaces are smooth. Find the acceleration of all the blocks.



Sol.





 ${\bf a}_{A}$ and ${\bf a}_{C}$ are acceleration of block A and C w.r.t ground and ${\bf a}_{\rm B}$ is the acceleration of block B w.r.t. block C

$$N_{BC} \sin \theta = ma_C$$

[Newton's II law for block C in horizontal direction in ground frame]

$$N_{AB} + mg - N_{BC} \cos \theta = ma_B \sin \theta$$

[Newton's II law for block B in vertical direction in ground frame]

$$N_{BC}\sin\theta = m (a_{B}\cos\theta - a_{c})$$

[Newton's II law for block B in horizontal direction in ground frame]

$$mg - N_{AB} = ma_A$$

[Newton's II law for block A in vertical direction in ground frame]

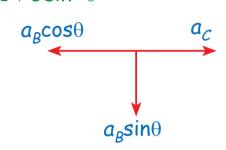
$$a_A = a_B \sin \theta$$

[constrained relation for block A and B in vertical direction]

Now we have 5 equations and 5 unknowns.

$$\Rightarrow a_A = \frac{4g\sin^2\theta}{1 + 3\sin^2\theta} \Rightarrow a_B = \frac{4g\sin\theta}{1 + 3\sin^2\theta}$$

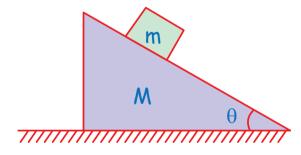
$$\Rightarrow a_C = \frac{2g\sin\theta\cos\theta}{1 + 3\sin^2\theta}$$



⇒ Acceleration of B w.r.t ground

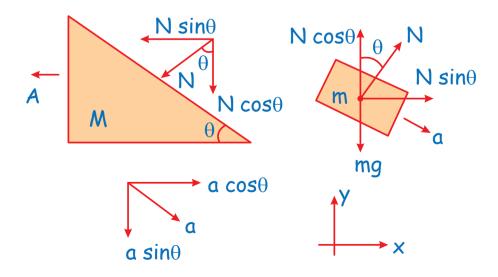
$$= \sqrt{(a_B \cos \theta - a_C)^2 + (a_B \sin \theta)^2} = \frac{2g \sin \theta}{\sqrt{1 + 3\sin^2 \theta}}$$

Q. A block of mass m is placed on the inclined surface of a wedge as shown. Calculate the acceleration of the wedge and the block when the block is released. Assume all surfaces are frictionless.



Sol. Let the acceleration of wedge be A and that of block be a (w.r.t. wedge). Then acceleration of m w.r.t. ground is

$$(a \cos \theta - A)\hat{i} - a \sin \theta \hat{j}$$



For M:
$$N \sin \theta = MA$$
 ... (i)

For m:
$$mg - N \cos \theta = ma \sin \theta$$
 ... (ii)

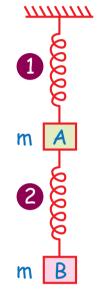
$$N \sin \theta = m(a \cos \theta - A) \qquad ... (iii)$$

Solved (i), (ii) and (iii) to get

$$A = \frac{\text{mgsin}\,\theta\cos\theta}{\text{M} + \text{msin}^2\theta} \text{ and } a = \frac{(\text{M} + \text{m})\text{gsin}\,\theta}{\text{M} + \text{msin}^2\theta}$$

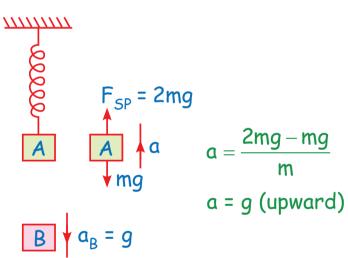
Now acceleration of block can be found by putting the values of A and a in (i).

Q. Find acc of each block just after spring cutting if

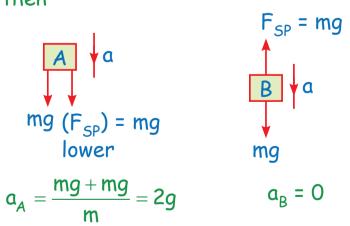


(a) Lower spring is cut

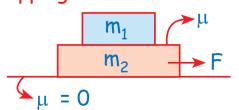
$$(F_{SP})_1 = 2 \text{ mg} \rightarrow \text{will not change suddenly}$$
 hence



(b) If upper spring cut at t = 0 then



 \mathbf{Q} . Find \mathbf{F}_{\max} so that both block move together without slipping between them



Sol. (a)_{upper} \rightarrow given by friction only

$$(a_{upper})_{max} = \frac{f_{max}}{m_1} = \frac{\mu m_1 g}{m_1}$$

$$(a_{upper})_{max} = \mu g$$

 $(a_{lower})_{max}$ so that both will move with same acc = μg

$$\Rightarrow$$
 F_{max} = (m₁ + m₂)a = (m₁ + m₂)µg

 \mathbf{Q} . \mathbf{F}_{max} so that both move with same acc.

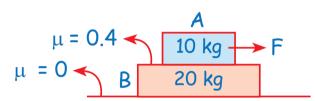


Sol.
$$(a_A)_{max} = \frac{f_{max}}{m_A} = \frac{0.4 \times 100}{10} = 4$$

$$F_{max} = (10 + 20) \times 4 = 120 \text{ N}$$

If $F \le 120$ both move with same acc.

 \mathbf{Q} . Find \mathbf{F}_{max} so that both move with same acc.



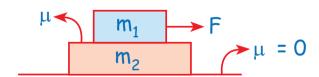
Sol.
$$(a_B)_{max} = \frac{f_{max}}{m_B} = \frac{0.4 \times 100}{20} = 2$$

$$F_{max} = (10 + 20) \times 2 = 60 \text{ N}$$

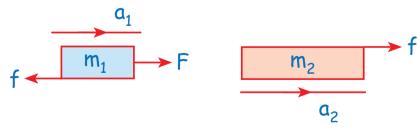
If F > 60 N Both move with difference acc

If $F \le 60$ Both move with same acc.

 \mathbf{Q} . Find \mathbf{F}_{\max} so that both block move with same



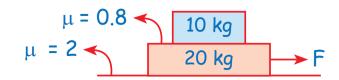
Sol.



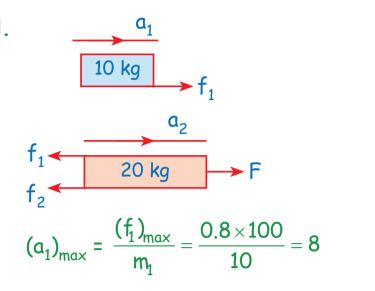
$$(a_2)_{max} = \frac{f_{max}}{m_2} = \frac{\mu m_1 g}{m_2}$$

$$\Rightarrow F_{\text{max}} = (m_1 + m_2) \frac{\mu m_1 g}{m_2}$$

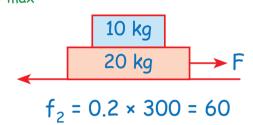
 \mathbf{Q} . Find \mathbf{F}_{max} so that both move with same acc.



Sol.



$$(a_{common})_{max} = 8$$



$$F - f_2 = (10 + 20) a_{common}$$

$$F - 60 = 30 \times 8$$

$$F = 300 = F_{max}$$

F > 300 both block will move with diff acc.

 $60 < F \le 300$ both block will move with same acc.

$$F \le 60 \ a_1 = a_2 = 0$$

Q. In the above question find the acceleration of both the blocks when

(i)
$$F = 18 N$$

(ii)
$$F = 36 N$$

Sol. (i) Since F < 30 both the blocks will move together

F.B.D

$$a = \frac{18}{6} = 3 \text{ m/s}^2$$

(ii)
$$F = 36 N$$

When F > 30 both the blocks will move separately so we treat each block independently

F.B.D of 2 kg block

A
$$2 \text{ kg} \rightarrow \text{F} = 10 \text{ N}$$
(Friction force)

$$a_A = 5 \text{ m/s}^2$$

F.B.D of 4 kg block

$$F = 10 \text{ N} + \frac{4 \text{ kg}}{B} + \frac{36 \text{ N}}{B}$$

$$a_B = \frac{36-10}{4} = \frac{26}{4} \text{m/s}^2$$

Q. Friction coefficient between the blocks is 0.5 and ground is smooth. If force of 120 N is applied on the upper block as shown in figure find acceleration of the blocks.



Sol. $f_{\text{max}} = 0.5 \times 10g = 50 \text{ N}$

Let's assume they have same acceleration. The value of common acceleration is $\frac{120}{30} = 4 \text{ m/s}^2$. Making free-body diagram

Newton's second law given

$$f = 20 \times 4 = 80 \text{ N}$$

Solving we get, $a = 4 \text{ m/s}^2$ and f = 80 N while limiting value of friction is 50 N. hence our assumption is wrong.

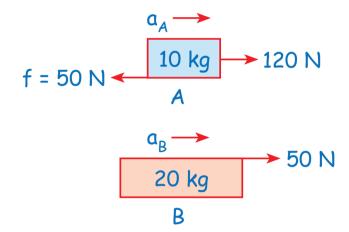
Let's say they are not moving together. Then $f_k = 50 \text{ N}$.

By Newton's second law.

$$120 - 50 = 10 a_A$$

$$50 = 20 a_B$$

$$a_A = 7 \text{ m/s}^2$$
, $a_B = 2.5 \text{ m/s}^2$



This gives us the correct direction of friction.

· Achche se in notes ko sevise karen...

Vese to bahut sari cheeze aur hai

But pahle inhe complete karen.....

If you found skc helpful

ye flipcart/Amazon/pw. store

par Avail. hai hard copy..... cut

very affordable price ---
वैसे पदन का असली मजा consistency

क साथ तो Hard copy से ही आता है