YAKEEN NEET 2.0

2026

Basic Maths and Calculus (Mathematical Tools)

PHYSICS | Lecture - 15

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Todays Goal

- Application of Differentiation
- Intigration

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$\frac{dy}{dx}$ ⟶ Rate of change of y wrt x

$\frac{dT}{dx}$ ⟶ Rate of change of temp wrt x

$\frac{dT}{dt}$ ⟶ Rate of change of temp wrt time.

$\frac{d(\text{कद्दू})}{dt}$ ⟶ Rate of change of कद्दू wrt time

$\frac{d(\text{Area})}{dt}$ ⟶ Rate of change of Area wrt time

Q

Area $= \pi r^2$

If radius of the ring is increasing at const rate 10 m/sec find rate of change of area of ring wrt time at $r = 5$ m

Sol^n

$$A = \pi r^2$$

$$\frac{dA}{dt} = \pi 2r \frac{dr}{dt}$$

$$\frac{dA}{dt} = \pi \times 2 \times 5 \times 10$$

$$\boxed{\frac{dA}{dt} = 100\pi}$$

$A = \pi r^2$

$\frac{dA}{dr} = \pi 2r$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

Q If radius of a solid sphere is increasing at const rate 5 m/sec. find rate of change of surface area wrt time and rate of change of Volume wrt time when $r = 2m$.

Soln $A = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

$\frac{dA}{dt} = 4\pi\, 2r \frac{dr}{dt} = 4\pi \times 2 \times 2 \times 5 = 80\pi$

$\frac{d\,vol^n}{dt} = \frac{4}{3}\pi\, 3r^2 \cdot \frac{dr}{dt} = 4\pi(2)^2 \times 5 = 80\pi$

Q Area of a ring is changing as

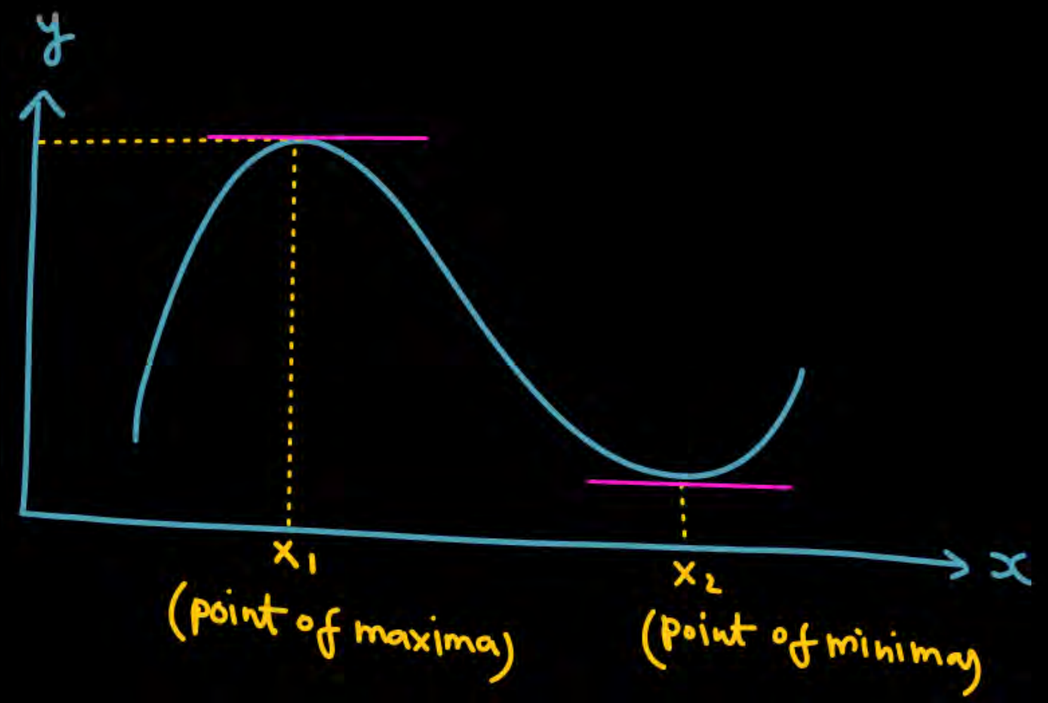
$A = 3t^2 - 4t + 6$

find rate of change of area wrt time at $t = 1$ sec

Soln $\frac{dA}{dt} = 6t - 4$ at $t = 1 \Rightarrow \frac{dA}{dt} = 6 - 4 = 2$

Maxima-minima

maxima	minima
$\frac{dy}{dx} = 0$	$\frac{dy}{dx} = 0$
$\frac{d^2y}{dx^2} < 0$	$\frac{d^2y}{dx^2} > 0$



Q $y = x^2 - 4x + 10$

$y_{min} = ?$

Sol^n $\frac{dy}{dx} = 2x - 4$

$\frac{dy}{dx} = 0$

$2x - 4 = 0$

$x = 2$

Verify $\frac{d^2y}{dx^2} = 2 > 0$ (minima)

$y_{min} \Rightarrow$ put $x = 2$

$y_{min} = 4 - 8 + 10 = 6$

Q $y = 5t^2 - 5t + 9$

Find where y is min and y_{min}

Sol $\frac{dy}{dt} = 10t - 5 = 0$

$t = \frac{5}{10} = \frac{1}{2}$.

$y_{min} = 5\left(\frac{1}{4}\right) - \frac{5}{2} + 9 = \checkmark$

$= \frac{31}{4}$ (check)

Q $y = x^3 - 3x^2 + 6$

find y_{min} y_{max}

Soln $\frac{dy}{dx} = 3x^2 - 6x$

$\frac{d^2y}{dx^2} = 6x - 6$

$\frac{dy}{dx} = 0$

$3x^2 - 6x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$(x = 0, 2 \equiv \text{slope} = 0)$

$x = 0$ पर क्या ?

$\frac{d^2y}{dx^2} = 6x - 6 = 6 \times 0 - 6$

$\frac{d^2y}{dx^2} = -6$ (Negative) (Maxima)

$y_{max} = 0 - 0 + 6 = 6$

(at $x = 0$)

$x = 2$ पर क्या

$\frac{d^2y}{dx^2} = 6x - 6 = 6 \times 2 - 6 = 6$

$\frac{d^2y}{dx^2} = 6$ (positive) (Minima)

$y_{min} = 2^3 - 3 \times 2^2 + 6 = 2$

Q $y = \sin\theta + \sqrt{3}\cos\theta$

find y_{max}

$$y_{max} = \sqrt{1^2 + (\sqrt{3})^2} = 2$$

$$y_{max} = \sqrt{a^2+b^2}$$

$$y_{min} = -\sqrt{a^2+b^2}$$

$$\frac{dy}{d\theta} = \cos\theta + \sqrt{3}(-\sin\theta) = 0$$

$$\cos\theta = \sqrt{3}\sin\theta$$

$$\tan\theta = \frac{1}{\sqrt{3}} \quad \theta = 30^\circ$$

at $\theta = 30^\circ$, $y = \sin 30^\circ + \sqrt{3}\cos 30^\circ$

$$= \frac{1}{2} + \sqrt{3}\,\frac{\sqrt{3}}{2}$$

$$= \frac{1}{2} + \frac{3}{2} = 2$$

Integration

→ Indefinite integration $\equiv \int f'(x)\,dx = f(x) + C$

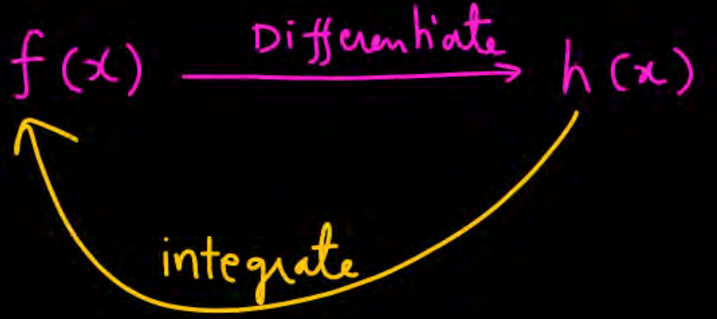
→ Definite integration

$$\int_{x=x_1}^{x=x_2} f'(x)\,dx = f(x)\Big|_{x=x_1}^{x=x_2} = f(x_2) - f(x_1)$$

Faltu Style

$$\int_{x_1}^{x_2} y\,dx = \text{Area}$$

Integration



- Reverse process of differentiation.

$$\frac{d}{dx} f(x) = f'(x)$$

If $\frac{d}{dx} f(x) = g(x)$

$$\int g(x)\, dx = f(x) + c$$

$$\int f'(x)\, dx = f(x) + c$$ (Indefinite Integration)

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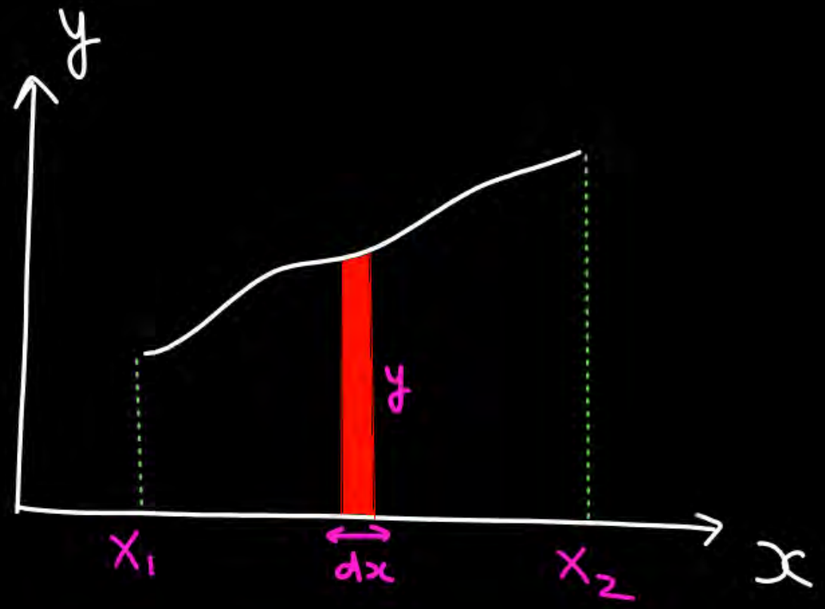


Area of small strip
$= y\,dx$

Hame aap $x = x_1$ se
$x = x_2$ tak ka poora area Bata o

upper limit of x ← $x = x_2$

$$\int y\,dx = \text{total area}$$

$x = x_1$ → lower limit of x

यहाँ तक
$\int y\,dx \Rightarrow$ Area Under curve
यहाँ से

* जिसके Resp. में integration कर रहे हो limit उसी की put करती है

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad \text{(Indefinite Integration)}$$

① $$\int x^3 dx = \frac{x^{3+1}}{3+1} + c = \frac{x^4}{4} + c$$

② $$\int x^5 dx = \frac{x^6}{6} + c$$

③ $$\int x^9 dx = \frac{x^{10}}{10} + c$$

④ $$\int x^2 dx = \frac{x^3}{3} + c$$

⑤ $$\int x^{-4} dx = \frac{x^{-4+1}}{-4+1} + c$$

$$= \frac{x^{-3}}{-3} + c$$

$$= -\frac{1}{3x^3} + c$$

$$\int 6x^4\,dx = 6\frac{x^5}{5} + c$$

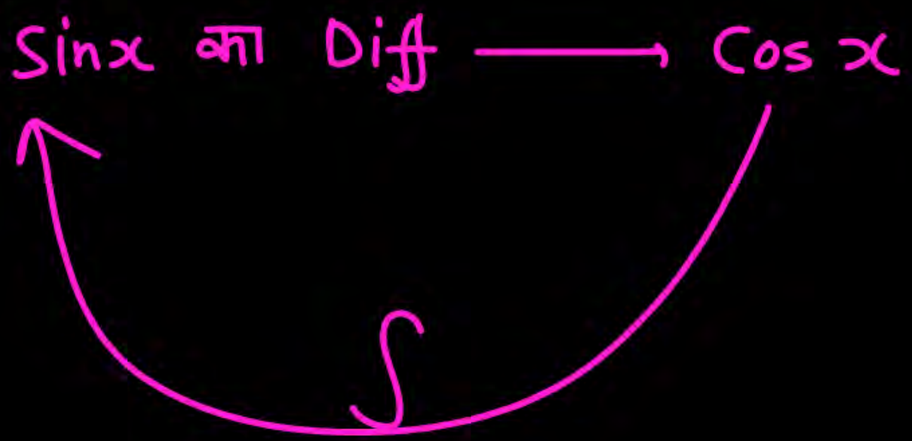
- $\int \cos x \cdot dx = \sin x + c$
- $\int \sin x\,dx = -\cos x + c$
- $\int e^x\,dx = e^x + c$
- $\int \sec^2 x \cdot dx = \tan x + c$
- $\int \frac{1}{x}\,dx = \ln x + c$

- $\int x^n dx = \frac{x^{n+1}}{n+1} + c, \quad (n \neq -1)$

① $y = e^x + \sin x + x^4$

$$\int y\,dx = \int (e^x + \sin x + x^4)\,dx$$

$$\int y\,dx = e^x + (-\cos x) + \frac{x^5}{5}$$



$\cos x \xrightarrow{\text{diff}} -\sin x$

$-\sin x$ का $\int$ ⟶ $\cos x$

$$\int \sin x \, dx = -\cos x + C$$

$$* \quad \int (y_1 + y_2)\,dx = \int y_1\,dx + \int y_2\,dx$$

$$\# \quad \int \underset{\text{Const}}{k}\, y\,dx = k\int y\,dx =$$

Q $\int 5\cos x\,dx = 5\int \cos x\,dx$

$= 5\sin x + c$

Q $\int (\cos x + x^6 + e^x)\,dx$

$= \sin x + \frac{x^7}{7} + e^x + c$

Homework

- KPP will uploaded today evening
- module →

THANK YOU