CLASS-11 SKC PHYSICS CRUSH Saleem Bhaiya

CLASS
11

# SKC
# PHYSICS CRUSH

## Class Notes in Handwritten Format

A beautiful journey From basic to JEE advanced via Mains/ NEET

By: Saleem Bhaiya



प्रयास है........................
Lakshya तक उड़ान भरने का
और Yakeen है................
Arjuna की तरह Focus करने का

Pen tod kar dikhao

# SKC

## Saleemians Khopcha Concept

# PHYSICS CRUSH

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# 2 Vector

सुनो भाई ये chapter वैसे तो mathematics का है जो maths वाले बच्चे 12th class मे extreme detail मे पढ़ोगे यहाँ हमे उतना ही पढ़ना है जितना physics मे use होना है जरूरत से ज्यादा PHD करने की कोशिश ना करे bcz वो हम 12th मे करेंगे कुछ बच्चे जो इसे पहली बार पढ़ रहे है उन्हे starting मे यह chapter मुश्किल लग सकता है उनसे मै यही कहुँगा की जितना content मै यहाँ दे रहा हुँ उसको 3-4 बार अच्छे से rough copy पर practice कर ले इतने से physics मे अच्छे से काम चल जाएगा

## Scalar Quantity

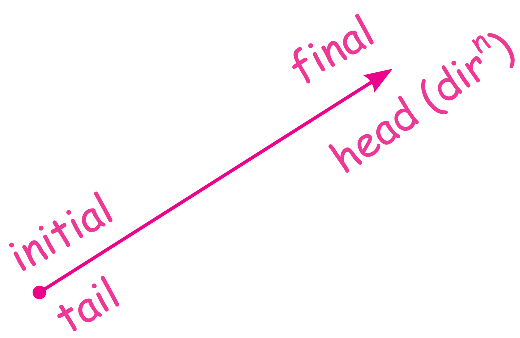
Those physical quantities which can be completely described by its magnitude only are called Scalar.

## Vector Quantity

Those physical quantity which have magnitude & direction and they follow law of vector algebra.
Eg. Force, Acceleration, Momentum.

## REPRESENTATION OF VECTOR

### Diagram वाला तरीका




Length of arrow represent magnitude of vector.

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कौन सी Physical quantity vector है या scalar है अभी हमे इस बात से कोई लेना देना नहीं है। क्योंकि अभी हमारी physics start नही हुई है so take it lightly Vector का सुरूर होले होले चढ़ेगा।

### Mathematically/Analytical

★ A vector P is represented by $\vec{P}$

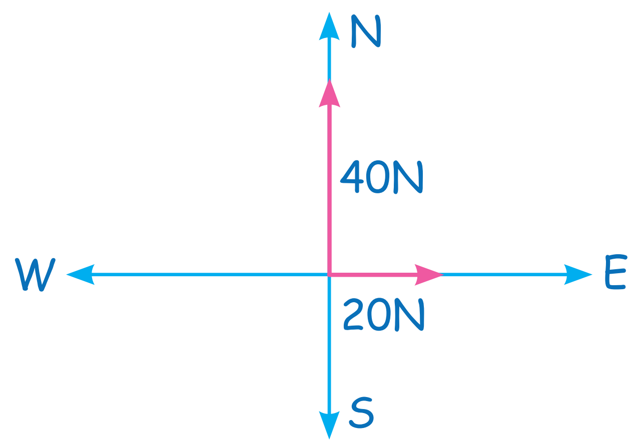
★ Force = $\vec{F}$

**Q.** Represent vector $\vec{P}$ & vector $\vec{Q}$

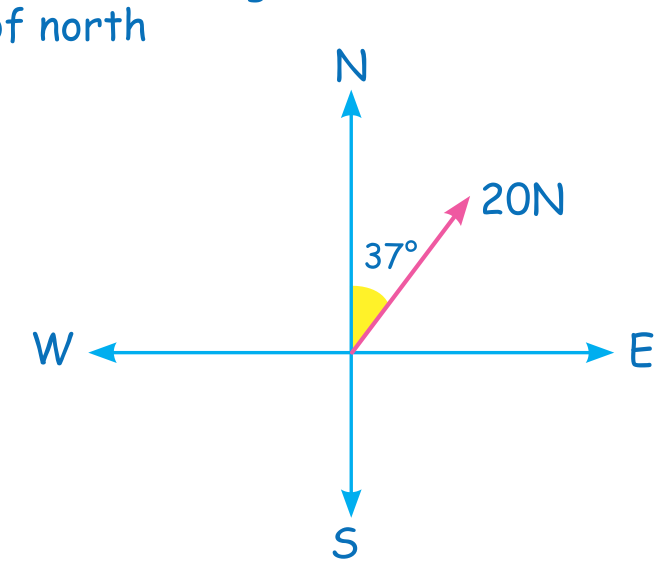
$\vec{P}$ = 20N along East

$\vec{Q}$ = 40N along north

**Sol.**




★ Represent $\vec{P}$ of magnitude 20N in direction of 37° east of north

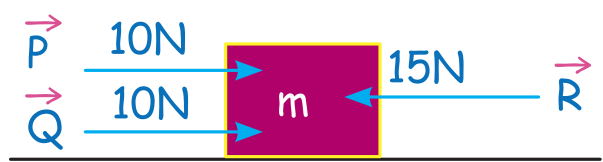



## TYPES OF VECTOR

अबे अच्छे से इनकी reading ले लेना।

### Equal Vector

Two vector are said to be equal vector, if they have same magnitude, same direction and same physical quantity.



$\vec{R}$ and $\vec{P}$ are equal vector : ✗

$\vec{P}$ and $\vec{Q}$ are equal vector : ✓

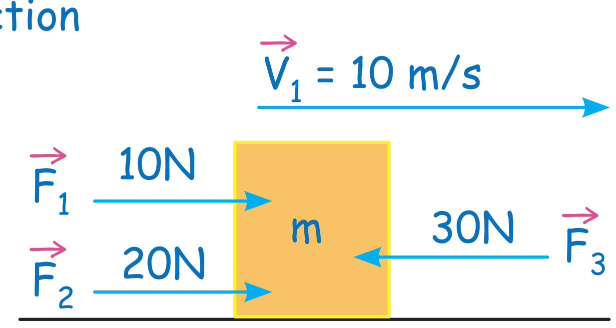
## Parallel Vector

Two vectors are said to be parallel if they have same direction.

Here, Physical quantity, अलग - अलग हो सकती है। All equal vectors are parallel vectors

## Anti-Parallel Vector = Direction Opposite

A Block is moving with constant velocity 10 m/s along east direction




$\vec{F}_1$ & $\vec{F}_2$ are parallel vector.

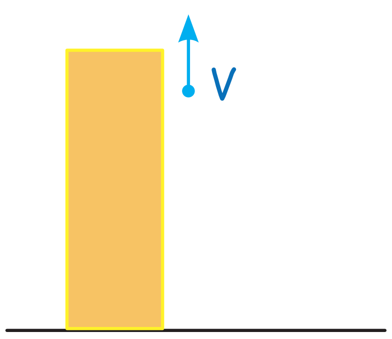
$\vec{F}_1$ & $\vec{F}_2$ are not equal vector.

$\vec{F}_1$ & $\vec{V}_1$ are parallel vector

$\vec{F}_2$ & $\vec{V}_1$ are parallel vector

$\vec{F}_1$ & $\vec{F}_3$ are anti-parallel vector

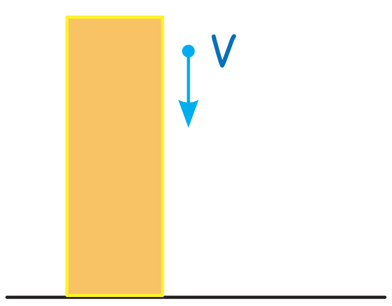
$\vec{V}_1$ & $\vec{F}_3$ are anti-parallel vector.




$\vec{V}$ → ऊपर

$\vec{a}$ → नीचे

$\vec{V}$, $\vec{a}$ are antiparallel.



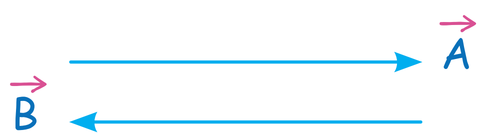

$\vec{V}$ → नीचे

$\vec{a}$ → नीचे

$\vec{V}$, $\vec{a}$ are parallel.

## Negative Vector

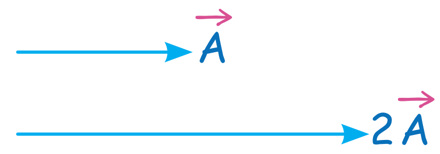
Two vectors $\vec{A}$, & $\vec{B}$, are said to be negative of each other if they have same magnitude & opposite direction.



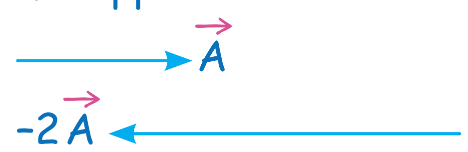

$\vec{A} = -\vec{B}$ are negative vector to each other.

## Multiplication of a Vector with a Number

★ If a vector $\vec{A}$ is multiply by a number n > 0. Then magnitude of vector becomes n times & direction remains same.




★ If a vector $\vec{A}$ is multiply by a number n < 0, then magnitude of vector becomes n times & direction become opposite.

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अगर हमने किसी vector को n से Multiply किया तो vector की length n times हो जाएगी / अगर n +ve है तो direction same रहेगी और अगर n-ve है तो direction opposite हो जाएगी।

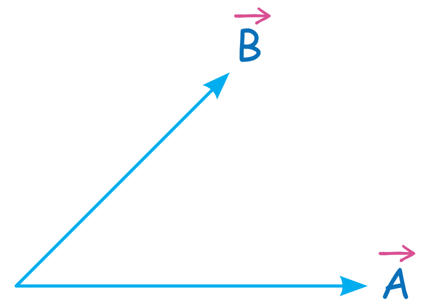
## Coplanar Vector

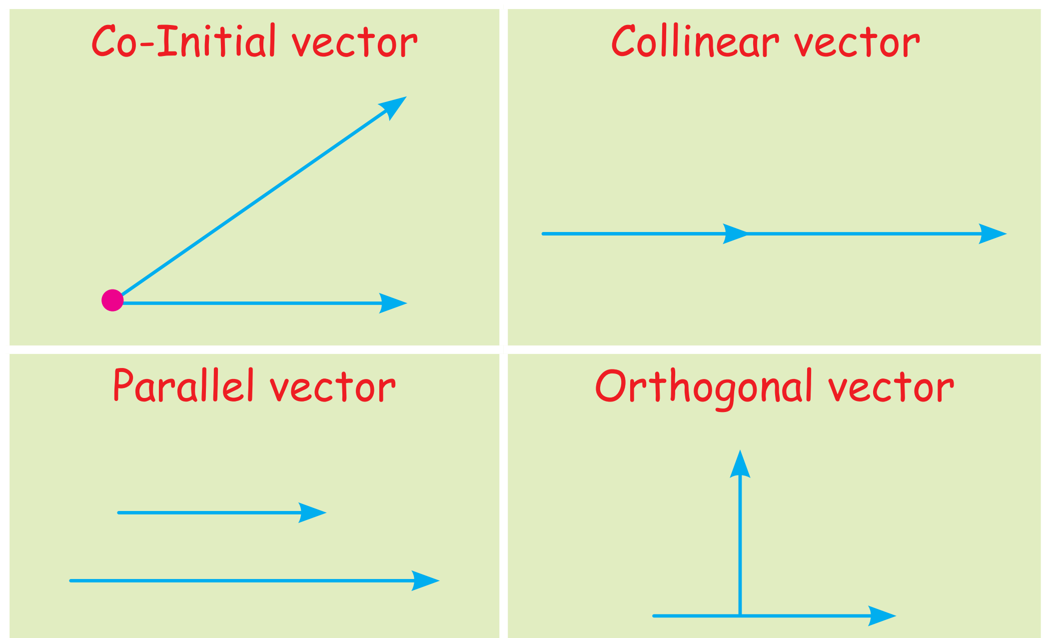
If all vectors lie in same plane then vectors are known as co-planar.

**NOTE:- TWO VECTORS are always COPLANAR**

## Coinitial Vector

[Tail to Tail ]: Two vectors are said to be coinitial if they have same initial point.



कौन-सी physical quantity scalar है कौन-सी vector है ये तू याद कर रहा है ????

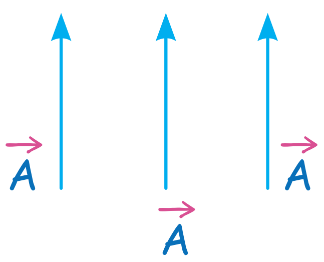
Haa humne RATT liye

Nhi RATTNA tha

पूरे दो साल यही पढ़ना है कि कौन-सी physical quantity क्या है So, इस पर time बिलकुल waste ना करें और मस्त रहे।

## Parallel Shifting of Vector

Whenever we need, we can parallely shift any vector without changing direction and magnitude we can can say that vector will not change.

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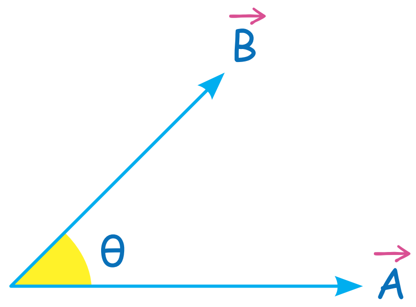
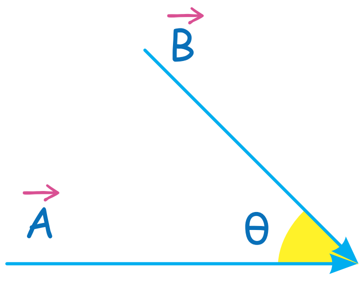
- देख भाई अभी हम अपनी जरूरत के अनुसार vector को parallelly shift कर सकते है जैसे computer के mouse का cursor sign होता है।
- अगर किसी vector की dirxn बदली मतलब vector बदल जाएगा।
- अगर किसी vector का Magnitude बदला मतलब vector बदला जाएगा।
- अगर Magnitude, direction same = vector same

## Angle Between Two Vectors

★ It is the angle b/w their tail.



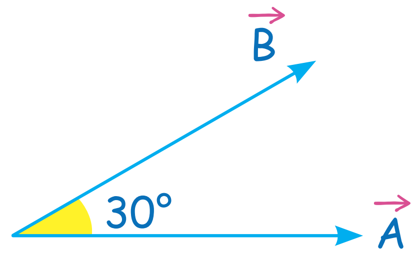

★ It is the angle b/w their head.




$[0 < \theta < 180]$ [$\theta \rightarrow$ angle b/w $\vec{A}$ & $\vec{B}$]

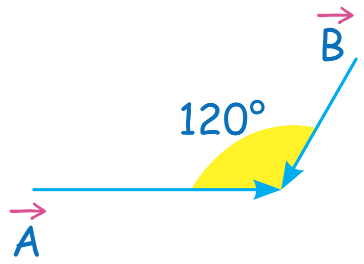
**Q.** Find angle b/w $\vec{A}$ & $\vec{B}$

(a)



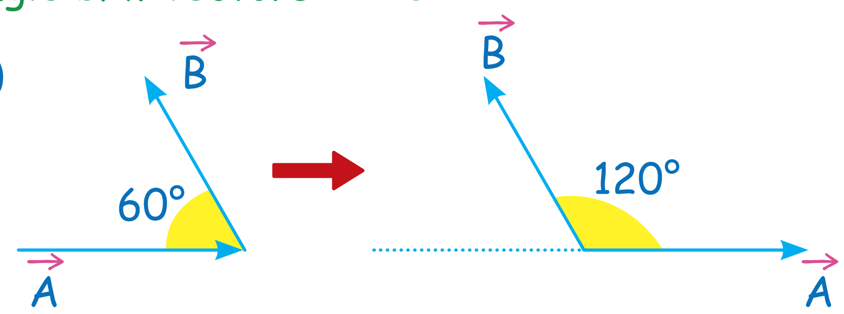

Angle b/w vectors = 30°

(b)




Angle b/w vectors = 120°

(c)

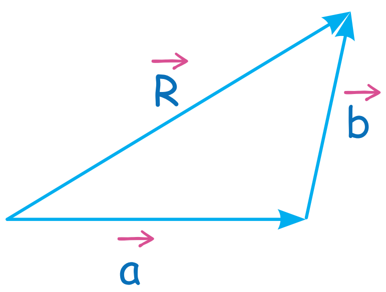



Angle b/w vectors = 180 – 60° = 120°

## ADDITION OF 2 VECTORS

### Triangle Rule of Vector Addition

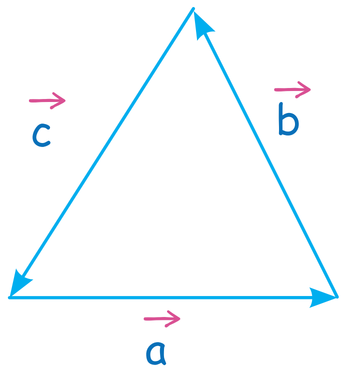
If 2 vectors represent two sides of a triangle in same sense (head of first vector coincide with tail of other vector) resultant is 3rd side of triangle in opposite sense.




$\vec{a} + \vec{b} = \vec{R}$

**Q.** Write the eqn for following fig. using triangle of vector addition.

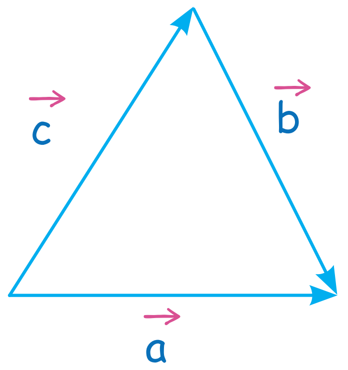
(a)




Closed loop same sense

$\vec{a} + \vec{c} + \vec{b} = 0$ इसका मतलब तीनो vector का sum/addition/resultant 0 आगया।

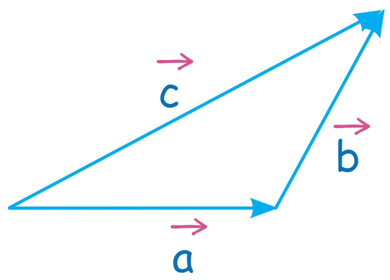
(b)




$\vec{a} = \vec{b} + \vec{c} \Rightarrow \vec{a} - \vec{c} = \vec{b}$

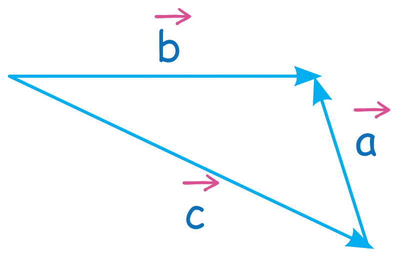
$\vec{a}$ is resultant of $\vec{c}$ and $\vec{b}$

(c)




$\vec{a} + \vec{b} = \vec{c}$

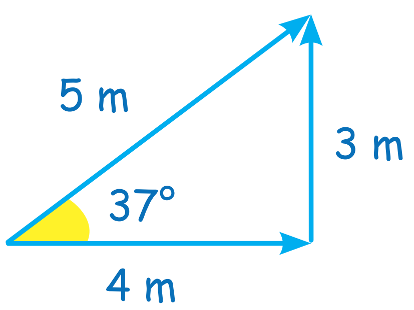
(d)




$\vec{c} + \vec{a} = \vec{b}$

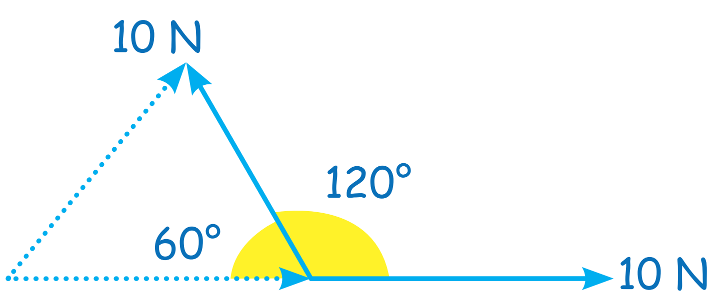
**Q.** A man moves 4m in east direction and then 3m in north direction. Find the resultant displacement of man.

**Sol.**



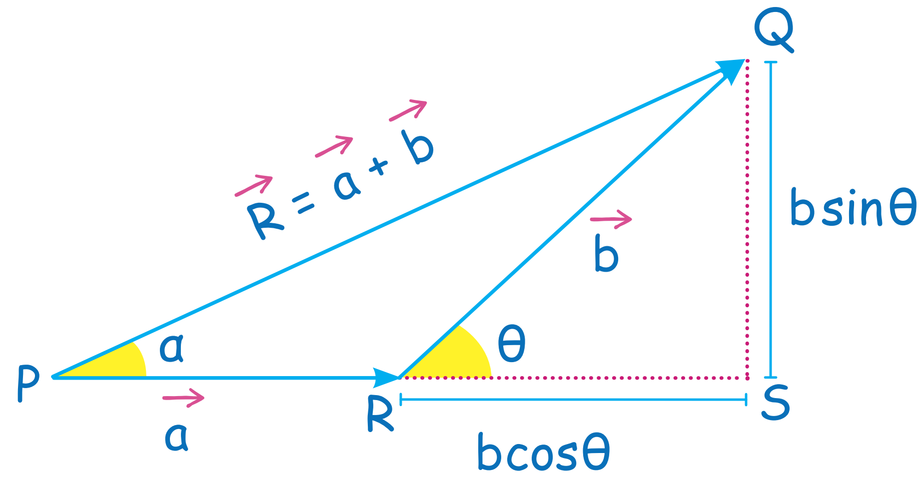

5 m in 37° north of east OR 5 m in 53° East of north.

**Q.**




**Sol.** Resultant 10 in 60° north of east.

## ANALYTICAL METHOD TO FIND RESULTANT AND ITS DIRECTION (OF 2 VECTORS)




$|\vec{a}| = a, |\vec{b}| = b, |\vec{R}| = |\vec{a} + \vec{b}| = R$

In ΔRQS

$\sin\theta = QS/b \Rightarrow QS = b\sin\theta$

$\cos\theta = RS/b \Rightarrow RS = b\cos\theta$

In ΔPQS

$(PQ)^2 = (PR + RS)^2 + QS^2$

$R^2 = (a + b\cos\theta)^2 + (b\sin\theta)^2$

$R^2 = a^2 + b^2(\cos^2\theta + \sin^2\theta)^2 + 2ab\cos\theta$

$$\boxed{R = \sqrt{a^2 + b^2 + 2ab\cos\theta}} \qquad \boxed{\tan\alpha = \frac{b\sin\theta}{a + b\cos\theta}}$$

R = Magnitude of resultant.

$\alpha$ = Angle b/w $\vec{R}$ & $\vec{a}$

देख भाई काम की बात यह है कि दो vector $\vec{a}$ and $\vec{b}$ के resultant का magnitude $R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$ होता है जो $\vec{a}$ के साथ angle $\alpha$ बनाता है

Where $\tan\alpha = \frac{b\sin\theta}{a + b\cos\theta}$ ये formula गलती से भी मत भूलना वरना पिटाई होगी अगर resultant R ने $\vec{b}$ के साथ angle $\beta$ बनाया तो $\tan\beta = \frac{a\sin\theta}{b + a\cos\theta}$

**Q.** Two vector of magnitude 10N each gives a resultant of magnitude $10\sqrt{3}$ N. Find angle b/w both vector.

**Sol.** $R = 10\sqrt{3}$ N

$R = \sqrt{10^2 + 10^2 + 200\cos\theta}$

$300 = 200 + 200\cos\theta \Rightarrow \theta = 60°$

**Q.** Two forces $\vec{A}$ and $\vec{B}$ of magnitude 10 N and 20 N are acting on a block having 60° angle between them. Find the magnitude of resultant of them

**Sol.** $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

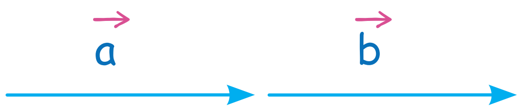
$= \sqrt{100 + 400 + 200} = \sqrt{700}$

Angle made by resultant of $\vec{A}$ & $\vec{B}$ with $\vec{A}$.

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} = \frac{20 \times \frac{\sqrt{3}}{2}}{10 + 20 \times \frac{1}{2}} = \frac{\sqrt{3}}{2}$$

## SPECIAL CASES

### 1 θ = 0° (Parallel Vectors)

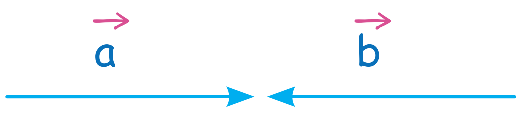



$\vec{R} = \vec{a} + \vec{b}$

$R = \sqrt{a^2 + b^2 + 2ab} = a + b$

$R_{max} = a + b = |\vec{a}| + |\vec{b}|$

### 2 θ = 180° (Antiparallel Vectors)




$\vec{R} = \vec{a} + \vec{b}$

$R = \sqrt{a^2 + b^2 - 2ab} = |a - b|$

$R_{min} = |a - b| = \left||\vec{a}| - |\vec{b}|\right|$

$\boxed{|a - b| \le R \le (a + b)}$

$R_{max} = a + b$ $\quad R_{min} = |a - b|$

### 3 θ = π/2 or 90°

$R = \sqrt{a^2 + b^2}$

**NOTE:** **Value of θ increases from 0° to 180° then magnitude of resultant will decrease.**

Vector की understanding बढ़ाने के लिए नीचे कुछ important ques. attach कर रहा हुँ भले ही आपको कुछ ques. easy लगे फिर भी आपको सारे ques. in the last solve करने है।


**Q.** Max and min magnitude of resultant of two forces are 7 N and 1 N respectively. Find the resultant of 2 forces when it act orthogonally.

**Sol.** $F_1 + F_2 = 7$ N;

$F_1 - F_2 = 1$ N

$\Rightarrow F_1 = 4$ N; $F_2 = 3$ N

When $F_1$ and $F_2$ are perpendicular to each other

then $R = \sqrt{3^2 + 4^2} = 5$

**Q.** Two vector A & B with same magnitude x. Find magnitude of resultant of A and B. θ = 60°.

**Sol.** $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{2x^2 + 2x^2\cos 60°}$

$= \sqrt{2x^2 + 2x^2 \frac{1}{2}} = \sqrt{3x^2} = x\sqrt{3}$

**Q.** In above question if A = x, B = x, θ = 120°, R = ?

**Sol.** $R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$= \sqrt{2x^2 + 2x^2\left(-\frac{1}{2}\right)}$

$= \sqrt{x^2} = x$

**Q.** If two vector $\vec{A}$ and $\vec{B}$ of magnitude 10 N and 6 N are at an angle 60°. If $\vec{B}$ become twice to its initial value & added to A. Find magnitude of resultant of A & B after $\vec{B}$ change.

**Sol.** B= 2 × 6 = 12 N

A= 10 N

अबे θ तो 60° ही रहेगा

$C = \sqrt{100 + 144 + 2 \times 10 \times 12 \times \cos 60} = \sqrt{364}$

**Q.** Magnitude of $\vec{A}$ is 8 N.

Magnitude of $\vec{B}$ is 6 N.

Which of the following can be magnitude of $\vec{A} + \vec{B}$

(a) 10 N (b) 22 N (c) 48 N

(d) 2 N (e) 2.001 N (f) 1.99 N

(g) 14.1 N (h) 13.999 N

**Sol.** $C_{max} = A + B = 14$

$C_{min} = A - B = 2$

$2 \le C \le 14$

**Ans.** (a), (d), (e), (h)

**Q.** Magnitude of resultant of $\vec{A}$ and $\vec{B}$ is 5 unit where magnitude of $\vec{A}$ is $5\sqrt{3}$ unit and magnitude of $\vec{B}$ = 5 unit. Find angle between A and B.

**Sol.** $\vec{C} = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$5^2 = 75 + 25 + 50\sqrt{3}\cos\theta$

$\frac{-\sqrt{3}}{2} = \cos\theta \Rightarrow \theta = 150°$

**Q.** Sum of the magnitude of $\vec{A}$ and $\vec{B}$ is 16 N. Magnitude of resultant of vector $\vec{A}$ and $\vec{B}$ is 8 N. When resultant is perpendicular to the $\vec{A}$. Find magnitude of $\vec{A}$ and $\vec{B}$.

**Sol.** $A + B = 16$ N

Magnitude of $\vec{R} = 8$ N

$\alpha = 90°$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$, $\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

$\tan 90° = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow A + B\cos\theta = 0$

$B\cos\theta = -A$

$64 = A^2 + B^2 + 2AB\cos\theta$

$64 = A^2 + B^2 - 2A^2$

$64 = B^2 - A^2 = (A + B)(B - A)$

$64 = 16(B - A)$

$4 = (B - A)$

$B - A = 4$

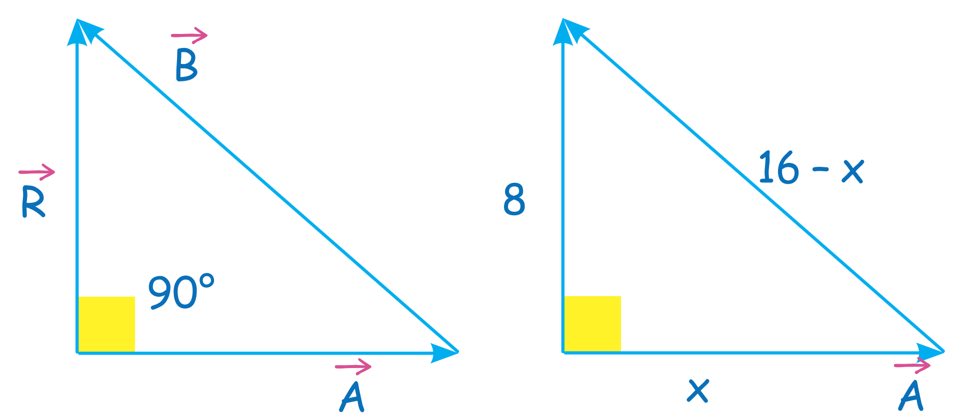
$B + A = 16$

$2B = 20$

$B = 10$

$A = 6$

**Method-2:**




By using Pythagoras theorem.

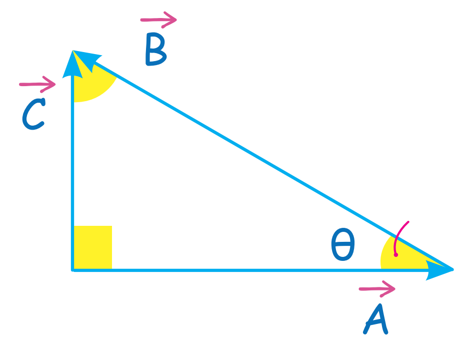
$(16 - x)^2 = 8^2 + x^2$

$256 + x^2 - 32x = 64 + x^2$

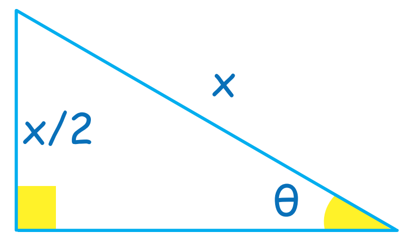
$256 - 32x = 64$

$\Rightarrow 6 = x$ & $16 - x = 10$

**Q.** Resultant of $\vec{A}$ and $\vec{B}$ is $\perp$ to $\vec{A}$ and its magnitude is equal to half of magnitude of $\vec{B}$. Find the angle between $\vec{A}$ and $\vec{B}$.




**Sol.** $\vec{A} + \vec{B} = \vec{C}$




$\sin\theta = \frac{\frac{x}{2}}{x} = \frac{1}{2}$

$\theta = 30°$

Angle between $\vec{A}$ & $\vec{B}$ = 180° – 30° = 150°

Angle between $\vec{B}$ and $\vec{C}$ = 60°

Angle between $\vec{A}$ and $\vec{C}$ = 90°

3. Two vector $\vec{A}$ and $\vec{B}$ have same magnitude 'a' and resultant has magnitude R. Now $\vec{B}$ is doubled and added to $\vec{A}$ and now new resultant become $a\sqrt{3}$. Find angle b/w $\vec{A}$ and $\vec{B}$.

**Sol.** $\vec{A} + \vec{B} = \vec{R}$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$R_{old} = \sqrt{2a^2 + 2a^2\cos\theta}$

$|2\vec{B} + \vec{A}| = a\sqrt{3}$

$R_{new} = \sqrt{a^2 + (2a)^2 + 2(a)(2a)\cos\theta}$

$a\sqrt{3} = \sqrt{a^2 + 4a^2 + 4a^2\cos\theta}$

$3a^2 = 5a^2 + 4a^2\cos\theta$

$-2\cancel{a^2} = 4\cancel{a^2}\cos\theta$

$\frac{-2}{4} = \frac{-1}{2} = \cos\theta$

$\theta = 120°$

## # काम का डब्बा

★ If $\vec{A} + \vec{B} = \vec{R}$

Magnitude of $\vec{R} = R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\theta$ = angle between A & B

★ $\tan\alpha = \dfrac{b\sin\theta}{a + b\cos\theta}$ (कैसे आया Not Important)

($\alpha$ is the angle made by $\vec{R}$ with $\vec{A}$)

★ $R = |\vec{A} + \vec{B}|$ = magnitude of resultant of $\vec{A}$ and $\vec{B}$.

★ $|A - B| \le R \le (A + B)$

★ दो vector का Resultant max तब होगा जब उनके बीच angle 0° होगा। और Min तब होगा जब उनके बीच angle 180° होगा।

If $\theta = 0 \Rightarrow R = A + B = R_{max}$

If $\theta = 180 \Rightarrow R = |A - B| = R_{min}$ = [बडा-छोटा]

If $\theta = 90 \Rightarrow R = \sqrt{A^2 + B^2}$

★ If $|\vec{A}| = |\vec{B}| = A$(let)

$\theta = 0 \longrightarrow R = 2A$

$\theta = 60° \longrightarrow R = A\sqrt{3}$

$\theta = 90° \longrightarrow R = A\sqrt{2}$

$\theta = 120° \longrightarrow R = A$

$\theta = 180° \longrightarrow R = 0$

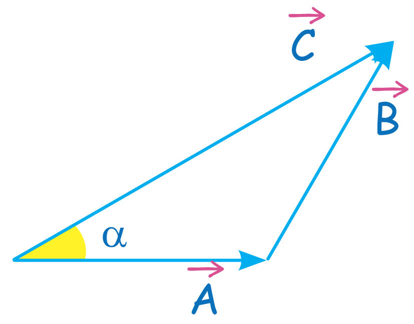
$R = 2A\cos\theta/2$

दो equal magnitude के vector अगर 60° पर है तो उनका resultant $\sqrt{3}$ times i.e $A\sqrt{3}$ होगा और वो 120° पर है तो उनका resultant A होगा।

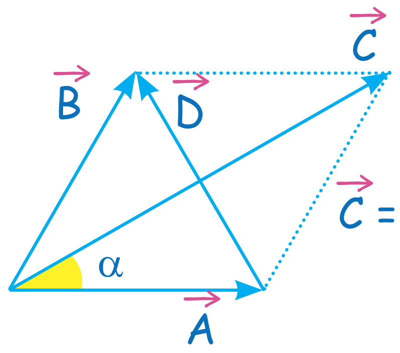
★ दो equal magnitude वाले vector का resultant उनके बीचो-बीच (along angle bi-sector) निकलेगा।

## PARALLELOGRAM LAW OF VECTOR ADDITION

If two coinitial vectors are given then resultant of these two vectors are given by diagonal of parallelogram (II gm) made from 2 given vector by shifting them parallel to their cointial vector & other diagonal of the II gm gives difference of vectors.




$\vec{C} = \vec{A} + \vec{B}$

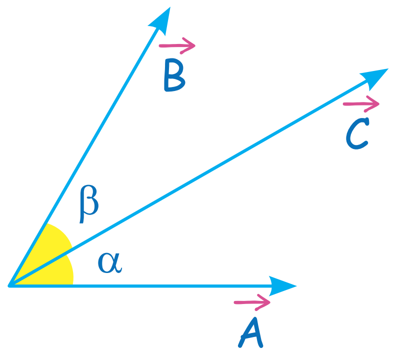



$\vec{C} = \vec{A} + \vec{B}$

Parallelogram

$\vec{A} + \vec{D} = \vec{B}$

$\vec{D} = \vec{B} - \vec{A}$




If $A > B, \alpha < \beta$

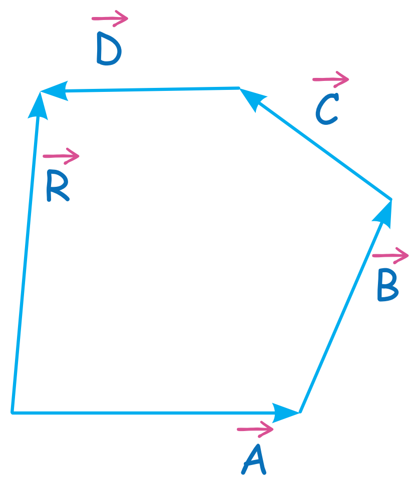
If $A < B, \alpha > \beta$

[जो Vector भारी होगा Resultant उस तरफ झुकेगा]

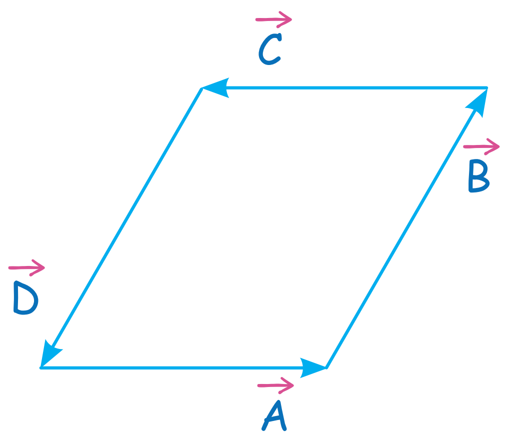
## POLYGON-LAW

### SKC

Resultant vector को draw करने के लिए पहले की Tail को Last के Head se Join Krdo




$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{R}$

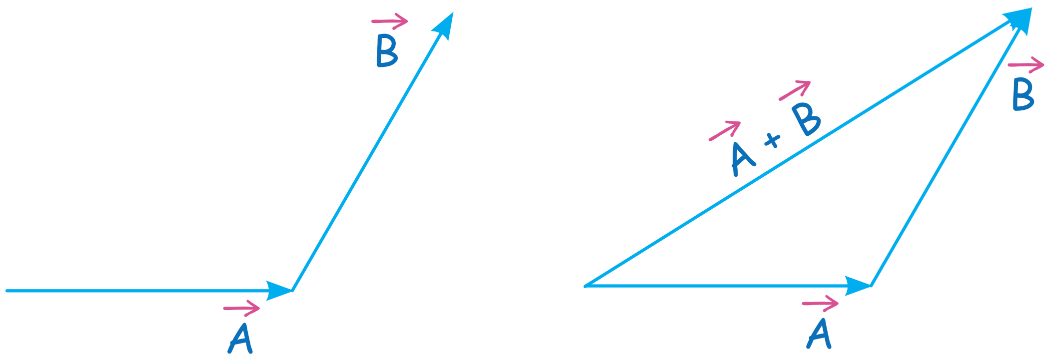



$\vec{A} + \vec{B} + \vec{C} + \vec{D} = \vec{O}$ [zero vector]

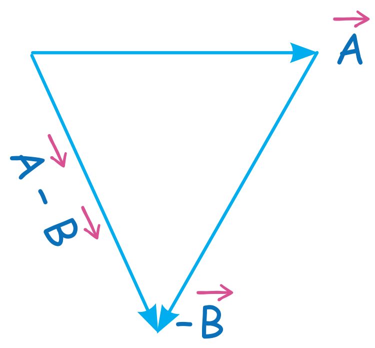
Dirxn of this zero vector is arbitary. [Jo Man Kre]

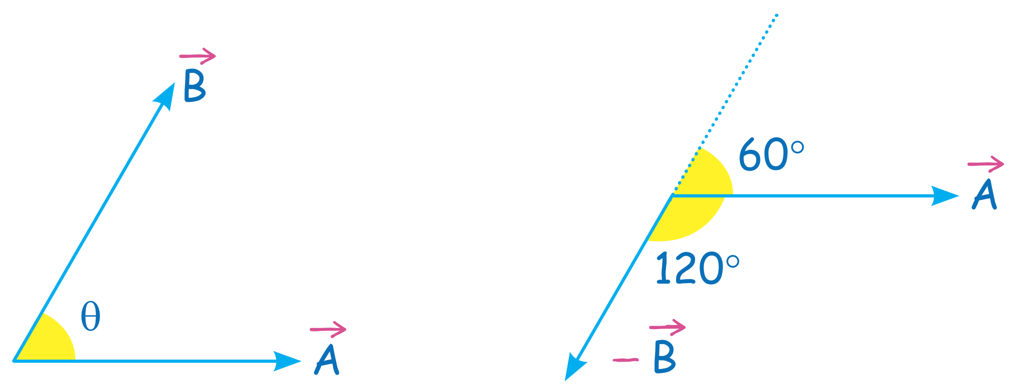
It is a vector where magnitude is zero.

## SUBTRACTION OF TWO VECTOR




$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$




$\vec{A}$ और $\vec{B}$ के बीच अगर angle θ है तो $\vec{A}$ में और $-\vec{B}$ के बीच 180-θ angle होगा।

**If angle between $\vec{A}$ & $\vec{B}$ is θ,**

Then, $\vec{C}=\vec{A}+\vec{B}$

$$C=\sqrt{A^2+B^2+2AB\cos\theta}$$

$$\tan\alpha=\frac{B\sin\theta}{A+B\cos\theta}$$

**SKC**

पूरी Physics में किसी vector की Physical quantity में (-) sign का मतलब ये होगा की जिधर आपने + ve dirx$^n$ मानी है। उसके opposite physical quantity की dirx$^n$ है।
[(-) मतलब dirx$^n$ से लेना देना]

& $\vec{D}=\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$

↳ Magnitude = ?

angle between $\vec{A}$ & $-\vec{B}$ will be 180 - θ.

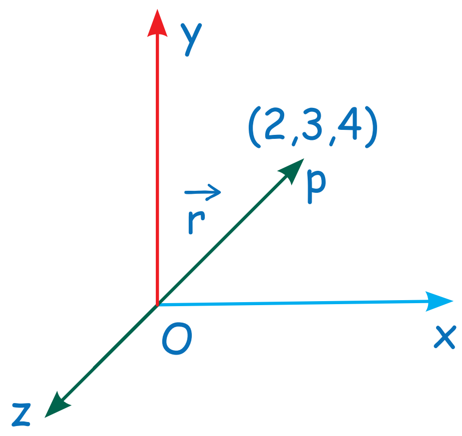
$$D=\sqrt{A^2+B^2+2AB\cos(180-\theta)}$$

$$=\sqrt{A^2+B^2-2AB\cos\theta}$$

$$\tan\alpha=\frac{B\sin(180-\theta)}{A+B\cos(180-\theta)}=\frac{B\sin\theta}{A-B\cos\theta}$$

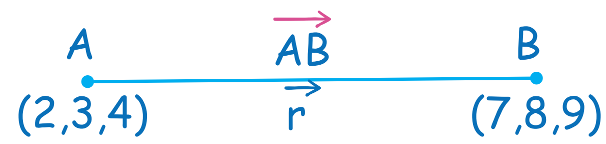
## POSITION VECTOR

A vector representing location or position of a point in space w.r.t origin is called position vector.




Position vector of P w.r.t origin = $2\hat{i}+3\hat{j}+4\hat{k}$

★

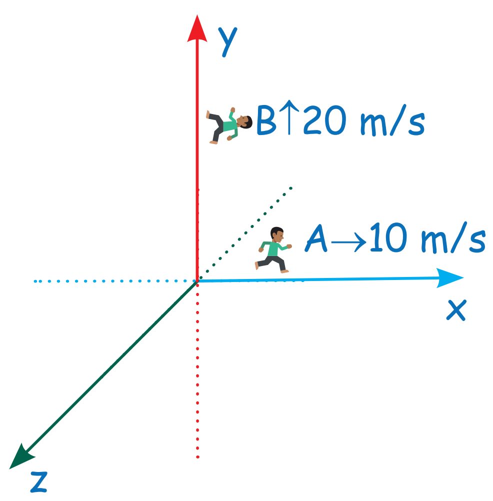



$\vec{AB}$ = P.V of B – P.V of A

$=(7-2)\hat{i}+(8-3)\hat{j}+(9-4)\hat{k}=5\hat{i}+5\hat{j}+5\hat{k}$

**Q.** A particle A is moving with speed 10 m/s along + x-axis and another particle B is moving along y-axis with speed 20 m/s. Find their velocity.

**Sol.**




→ Velocity of A = $10\hat{i}$

→ Velocity of B = $20\hat{j}$

## MAGNITUDE OF A VECTOR

**Q.** $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

**Sol.** Magnitude of $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

**Q.** $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $|\vec{A}| = ?$

**Sol.** Magnitude of A

$= \sqrt{2^2 + 3^2 + 4^2} = \sqrt{4+9+16} = \sqrt{29}$

**Q.** $\vec{A} = 2\hat{i} + 3\hat{j} - 4\hat{k}$, $|\vec{A}| = ?$

**Sol.** Magnitude of A

$= \sqrt{2^2 + 3^2 + (-4)^2} = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$

**Q.** $\vec{A} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$. Find magnitude of $\vec{A}$.

**Sol.** $|\vec{A}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$

**Q.** $\vec{A} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$, $|\vec{A}| = ?$

**Sol.** $|\vec{A}| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2} = 1$

## UNIT VECTOR

है तो vector ही $\hat{A}$ vector whose magnitude is 1. It represent direx$^n$.

अंधे की लाठी

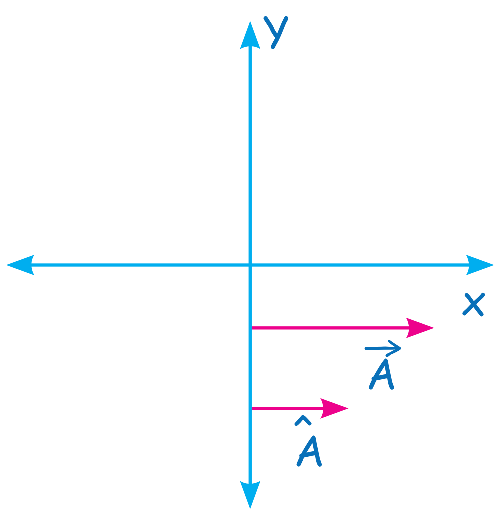
it is used to give direx$^n$ and is unitless & dimensionless

$\vec{A} = |\vec{A}|$ direx$^n$

$\vec{A} = |\vec{A}|\ \hat{A}$

└→ unit vector

$\hat{A} = \frac{\vec{A}}{|\vec{A}|}$





$\hat{A} \rightarrow$ unit vector

└→ ये ऐसा vector है जिसकी Magnitude '1' और dirx$^n$ $\vec{A}$ की तरफ है।

→ Unit vector along +x-axis = $\hat{i}$

→ Unit vector along +y-axis = $\hat{j}$

→ Unit vector along +z-axis = $\hat{k}$

**Q.** If $\vec{A}$ is unit vector find value of $\alpha$ where $A = 0.6\hat{i} + \alpha\hat{j}$

**Sol.** $\vec{A} = 0.6\hat{i} + \alpha\hat{j}$

$\vec{A} = \sqrt{0.36 + \alpha^2}$

$1 = 0.36 + \alpha^2$

$0.64 = \alpha^2$

$\pm 0.8 = \alpha$

**Q.** If a vector is given by $\vec{A} = 3\hat{i} + 4\hat{j}$. Find unit vector along $\vec{A}$.

**Sol.** Magnitude of $\vec{A} = \sqrt{3^2 + 4^2} = 5$

$\vec{A} = |\vec{A}| \cdot \hat{A}$

→ dirx$^n$

→ unit vector

→ Magnitude

किसी भी vector को उसके magnitude से divide करदो तो उस vector की तरफ का unit vector आजाएगा।

Unit vector along $\vec{A}$ or $\hat{A}$

$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{3\hat{i} + 4\hat{j}}{5} = \frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}$

$\vec{A}$ and $\hat{A}$ are parallel vector.

**Q.** Find the unit vector along $\vec{A}$ where $\vec{A} = 2\hat{i} + 6\hat{j} - 3\hat{k}$

**Sol.** $\hat{A} = \frac{2\hat{i} + 6\hat{j} - 3\hat{k}}{\sqrt{49}} = \frac{2}{7}\hat{i} + \frac{6}{7}\hat{j} - \frac{3}{7}\hat{k}$

**Q.** A particle has momentum of magnitude 20 kg m/s. If momentum is in the dirx$^n$ of $\vec{A}$. Find momentum in vector form, where $\vec{A} = \hat{i} + \hat{j}$

**Sol.** Momentum $= 20 \times \hat{A} = 20 \times \left(\frac{\hat{i} + \hat{j}}{\sqrt{2}}\right) = \sqrt{2}\left(10\hat{i} + 10\hat{j}\right)$

**Q.** A bird is flying with speed 10 m/s in the dirx$^n$ of a vector $\vec{A} = 3\hat{i} + 4\hat{j}$. Find velocity of bird.

**Sol.** $\vec{V}$ = Magnitude of dirx$^n$.

$\vec{V} = 10\hat{A} = 10\left[\frac{3\hat{i} + 4\hat{j}}{5}\right] = 2[3\hat{i} + 4\hat{j}] = 6\hat{i} + 8\hat{j}$

**Q.** Find force F in vector form if its magnitude is 21 N and dirx$^n$ is

(a) parallel to $6\hat{i} - 3\hat{j} + 2\hat{k}$

(b) opposite to $6\hat{i} - 3\hat{j} + 2\hat{k}$

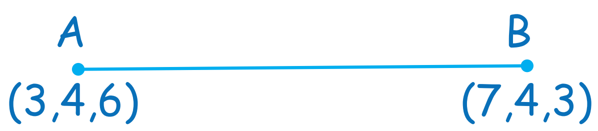
**Sol.** (a) [Parallel] $\vec{F}$

$$= 21\hat{A} = 21\left[\frac{6\hat{i} - 3\hat{j} + 2\hat{k}}{7}\right] = 18\hat{i} - 9\hat{j} + 6\hat{k}$$

(b) [Opposite] $\vec{F}$

$$= 21(-\hat{A}) = -21\hat{A} = -18\hat{i} + 9\hat{j} - 6\hat{k}$$

**Q.** A bird is flying with 10 m/s speed from point A to directly point $\vec{B}$. Find velocity of Bird.




**Sol.** $\overrightarrow{AB} = 4\hat{i} + 0\hat{j} - 3\hat{k}$

$$\vec{V} = 10 \times \hat{AB} = 10 \times \left(\frac{4\hat{i} - 3\hat{k}}{5}\right) = 8\hat{i} - 6\hat{k}$$

**Q.** $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$.

$\vec{B} = 2\hat{i} + 3\hat{j} - 2\hat{k}$. Find $\vec{A} + \vec{B}$

**Sol.** $\vec{A} + \vec{B} = 5\hat{i} + 7\hat{j} + 3\hat{k}$

**Q.** $\vec{A} = 3\hat{i} + 2\hat{j} + 5\hat{k}$.

$\vec{B} = 2\hat{i} + 3\hat{j} + 4\hat{k}$. Find following vectors.

**Sol.** $\vec{A} + \vec{B} = 5\hat{i} + 5\hat{j} + 9\hat{k}$

$\vec{A} - \vec{B} = \hat{i} - \hat{j} + \hat{k}$

$2\vec{A} + 3\vec{B} = 12\hat{i} + 13\hat{j} + 22\hat{k}$

$|\vec{A} + \vec{B}| = \sqrt{5^2 + 5^2 + 9^2}$

$|\vec{A} - \vec{B}| = \sqrt{3}$

$|2\vec{A} + 3\vec{B}| = \sqrt{12^2 + 13^2 + 22^2}$

$2\vec{A} - 3\vec{B} = 0\hat{i} + (4-9)\hat{j} + (10-12)\hat{k} = -5\hat{j} - 2\hat{k}$

**Q.** $\vec{A} = 2\hat{i} + \hat{j} + \hat{k}$.

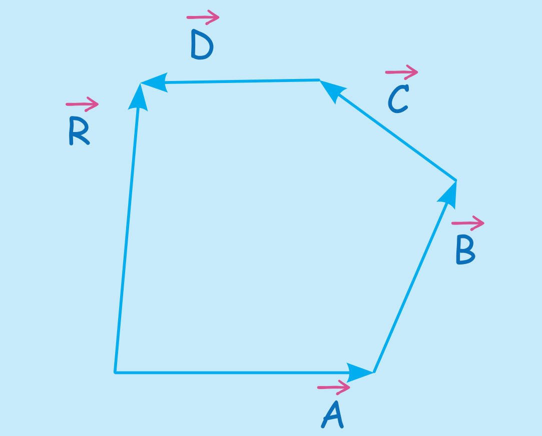
$\vec{B} = 4\hat{i} + \hat{j} + 4\hat{k}$.

Find a vector $\vec{C}$ whose magnitude is 20 and dirx$^n$ is opposite to $4\vec{A} + \vec{B}$.

**Sol.** $4\vec{A} + \vec{B} = 12\hat{i} + 5\hat{j} + 8\hat{k}$

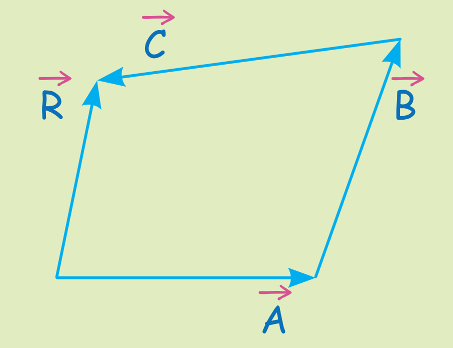
$$\vec{C} = -20 \times \left[\frac{12\hat{i} + 5\hat{j} + 8\hat{k}}{\sqrt{12^2 + 5^2 + 8^2}}\right]$$

## COMPONENT OF VECTOR




$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

Here, $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ are said to be four component of $\vec{R}$.




$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

Here, $\vec{A}$, $\vec{B}$, $\vec{C}$ are three component of $\vec{R}$.

**Q.** A man move 5 m along east, then turn left and move 10 m along north, then turn right move 20 m east and then turn right to south & move 15 m. Find net displacement and distance travel.

**Sol.** Distance = 5 + 10 + 20 + 15 = 50 m

$\vec{d}_1 = 5\hat{i}$

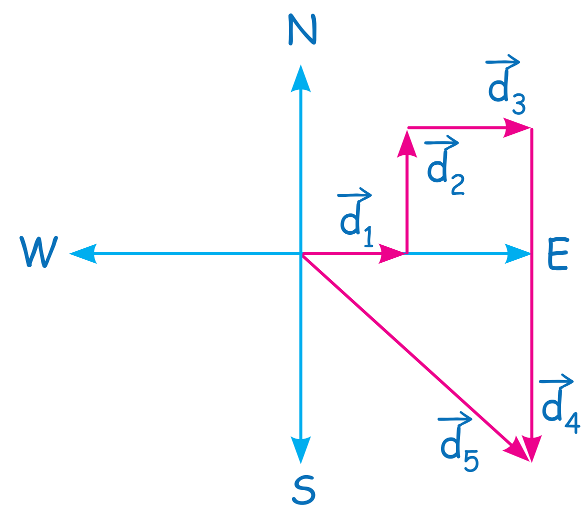
$\vec{d}_2 = 10\hat{j}$

$\vec{d}_3 = 20\hat{i}$

$\vec{d}_4 = -15\hat{j}$

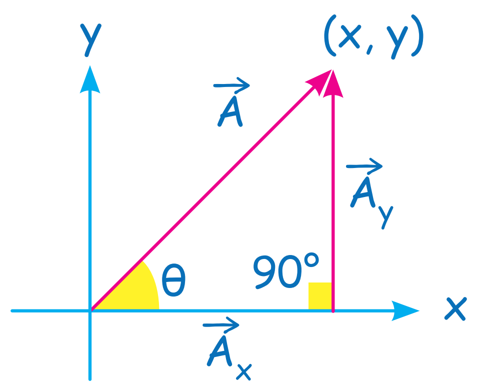
---

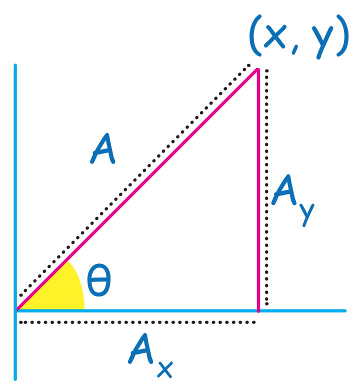
$d_{net} = 25\hat{i} - 5\hat{j}$




Magnitude = $\sqrt{25^2 + 5^2} = \sqrt{625 + 25} = \sqrt{650}$

## RECTANGULAR COMPONENT/ CARTESIAN COMPONENT OF A VECTOR (2D)




$\vec{A} = x\hat{i} + y\hat{j}$,

$\vec{A} = \vec{A}_x + \vec{A}_y$,

*$A_x$ & $A_y$ are two rectangular component of $\vec{A}$.

$\vec{A} = A_x\hat{i} + A_y\hat{j}$,

$\vec{A} = A\cos\theta\hat{i} + A\sin\theta\hat{j}$,

$\cos\theta = \frac{A_x}{A} \Rightarrow A_x = A\cos\theta$

$\sin\theta = \frac{A_y}{A} \Rightarrow A_y = A\sin\theta$

$\vec{A}_x$ = Component of $\vec{A}$ along x-axis

$\vec{A}_y$ = Component of $\vec{A}$ along y-axis

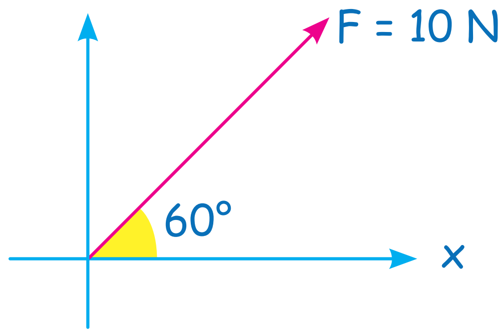
$\vec{A}_x = A\cos\theta\hat{i}$

$\vec{A}_y = A\sin\theta\hat{j}$

यह बहुत important है और कम से कम एक करोड़ बार use होगा so, अच्छे से समझ लेना कि कैसे एक vector के दो rectangular component ले रहे है या कैसे एक vector को तोड़ रहे है।

**Q.** Resolve the vector into $\hat{i}$ & $\hat{j}$

(a)

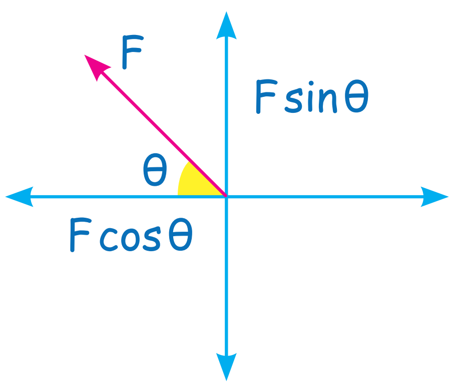



$\vec{F} = 10\cos 60\hat{i} + 10\sin 60\hat{j}$

$\vec{F} = 10 \times \frac{1}{2}\hat{i} + 10\frac{\sqrt{3}}{2}\hat{j}$

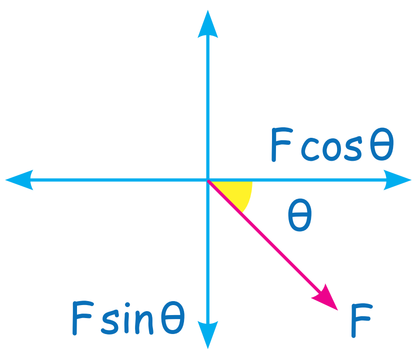
$= 5\hat{i} + 5\sqrt{3}\hat{j}$

(b)



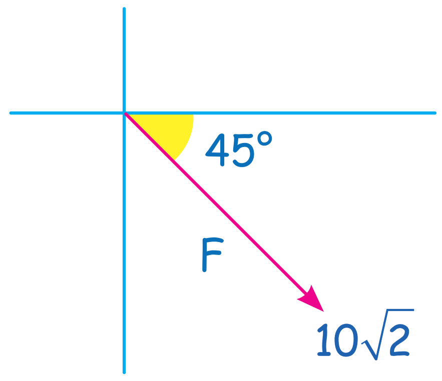

$\vec{F} = -F\cos\theta\hat{i} + F\sin\theta\hat{j}$

(c)



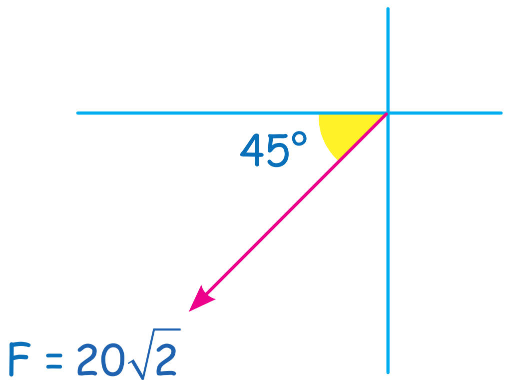

$\vec{F} = F\cos\theta\hat{i} - F\sin\theta\hat{j}$

(d)




$\vec{F} = 10\sqrt{2}\cos 45°\hat{i} - 10\sqrt{2}\sin 45°\hat{j} = 10\hat{i} - 10\hat{j}$

(e)

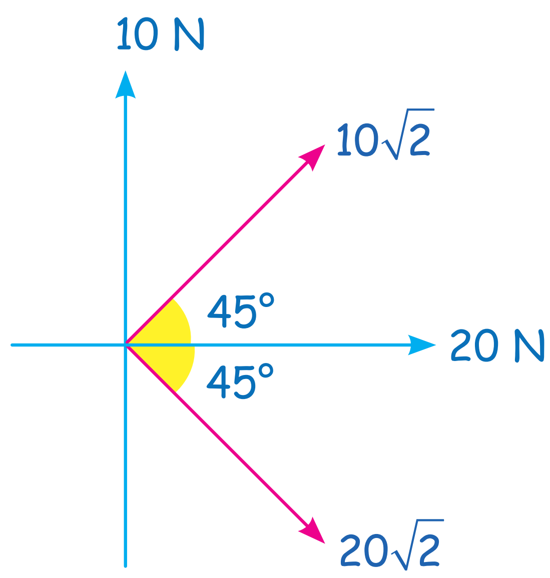



$\vec{F} = -20\hat{i} - 20\hat{j}$

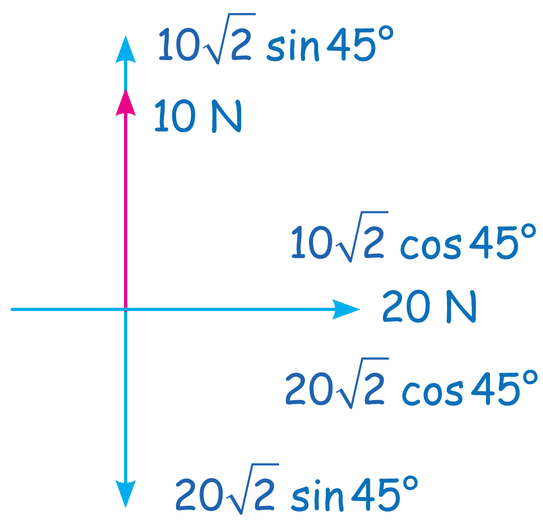
## SKC

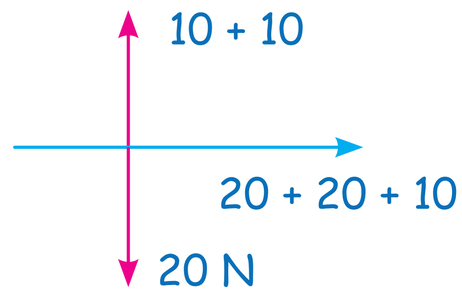
अब हम vector का सबसे ज्यादा important part पढ़ने जा रहे है जो electrostatic, mechanics मे बहुत use होगा यहाँ हमे बहुत सारे vector का net resultant निकालना होगा। कुछ नहीं करना बस सारे forces को x-y मे तोड़ लो और individually x और y मे collect करके answer लिख दो।

**Q.** Find net force.




**Sol.**

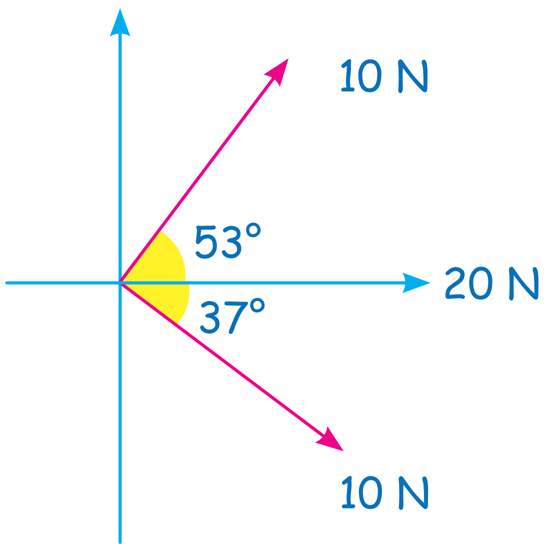


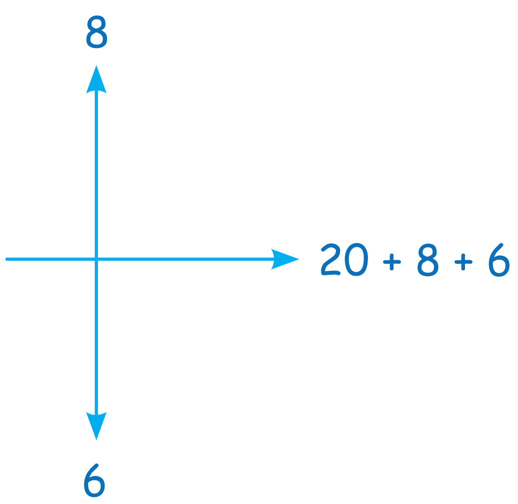



$F_{net} = 50\hat{i}$

**Q.** Find net force

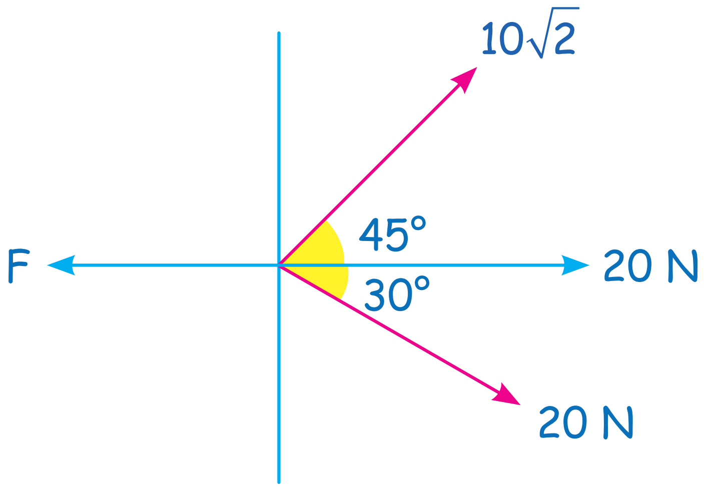
**Sol.**



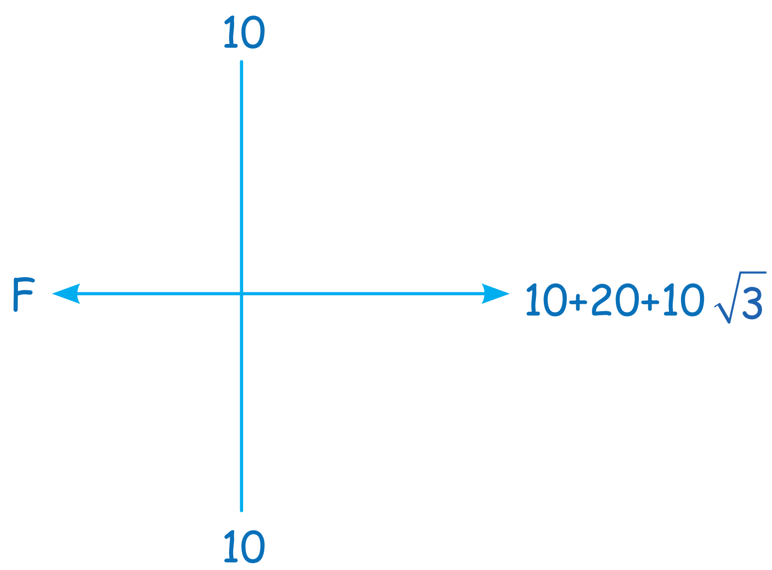




$F_{net} = 34\hat{i} + 2\hat{j}$

**Q.** Find the value of F so that the net force is zero

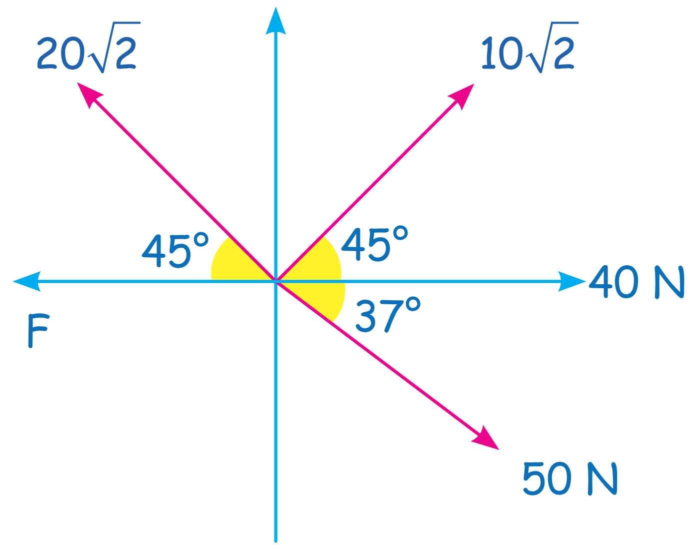



**Sol.**

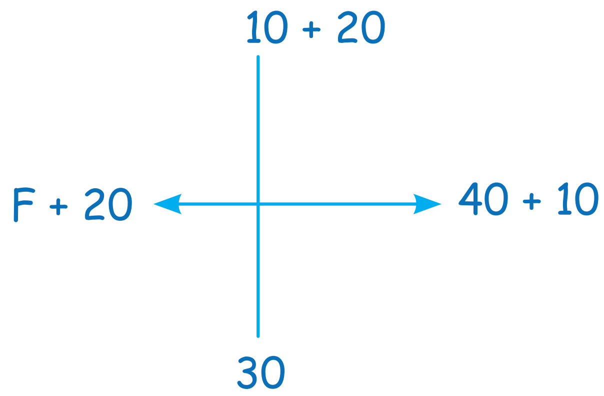



$30 + 10\sqrt{3} = F$

**Q.** Find value of F so that $\vec{F}_{net} = 0$.




**Sol.**




$F + 20 = 50$

$F = -30\hat{i}$

अब अगर मै 50 vector भी दे दूँ तो सभी को resolve करके कर लोगे ना, मानता हुँ maths/calculation लंबी होगी but physics तो आसान है अगर ये चीज समझ गए हो तो मुझे insta पर confirmation दो saleem.nitt only if you have account.

## VECTOR PRODUCT

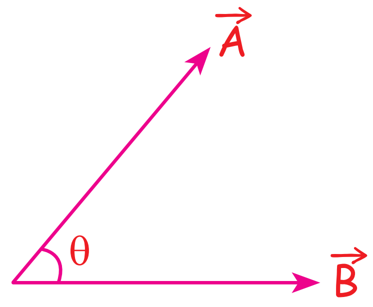
**1. Dot Product**

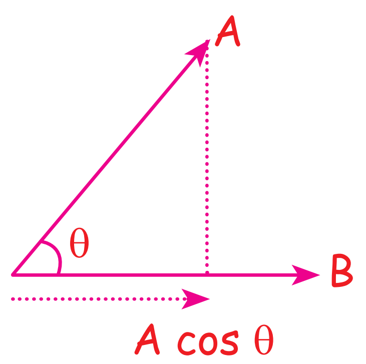
**2. Cross Product**


### Dot Product

$\vec{A} \cdot \vec{B} = A\ B\ \text{Cos}\theta$

Here, A and B are the magnitude of $\vec{A}$ and $\vec{B}$ and $\theta$ is the angel between $\vec{A}$ and $\vec{B}$.




$\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = A(B\cos\theta)$

= A (magnitude of component $\vec{B}$ along $\vec{A}$)

★ $\vec{A}\cdot\vec{B} = (A\cos\theta)B$

= (magnitude of componentof $\vec{A}$ along $\vec{B}$)B

★ If $\vec{A}\perp\vec{B} \Rightarrow \vec{A}\cdot\vec{B}\ A\cdot B\cos 90° = 0$

★ $\hat{i}\cdot\hat{j} = 1\times 1\times\cos 90° = 0$

★ $\hat{i}\cdot\hat{k} = 1\times 1\times\cos 90° = 0$

★ $\hat{j}\cdot\hat{k} = 1\times 1\times\cos 90° = 0$

★ $\vec{A}\cdot\vec{B}$ = number ayega scalar.

★ $\hat{i}\cdot\hat{i} = 1\times 1\times\cos 0° = 1$

★ $\hat{j}\cdot\hat{j} = 1\times 1\times\cos 0° = 1$

★ $\hat{k}\cdot\hat{k} = 1\times 1\times\cos 0° = 1$

★ $[\vec{A}\cdot\vec{A} = A^2]$

If $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

then $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

$\because\ \vec{A}\cdot\vec{B} = AB\cos\theta \Rightarrow \cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

★ Dot product of two vector is commutative

$\vec{A}.\vec{B} = \vec{B}.\vec{A}$

★ $\vec{A}.\vec{A} = A^2$

★ $(\vec{A}+\vec{B})\cdot(\vec{A}+\vec{B}) = A^2 + B^2 + \vec{A}.\vec{B} + \vec{B}.\vec{A}$

$= A^2 + B^2 + 2\vec{A}.\vec{B}$

**Q.** Find dot product of $\vec{A}$ and $\vec{B}$.

$\vec{A} = 2\hat{i} + 3\hat{j}$

$\vec{B} = 4\hat{i} + 5\hat{j}$

$\vec{A}\cdot\vec{B} = 8 + 15 = 23$

**Q.** $\vec{A} = 2\hat{i} - 3\hat{j} + 4\hat{k}$

$\vec{B} = \alpha\hat{i} + 2\hat{j} + 5\hat{k}$

if $\vec{A}\perp\vec{B}$ find α

VVVVVVVVVV
Important
अगर दो vector ⊥ है तो उनका Dot product zero hoga

**Sol.** $\vec{A}\cdot\vec{B} = 2\alpha - 6 + 20 = 0 \Rightarrow \alpha = -7$

**Q.** Find angle between $\vec{A}$ & $\vec{B}$ if $\vec{A} = \hat{i} + \hat{j}$, $\vec{B} = 3\hat{i} + 4\hat{j}$

**Sol.** $\vec{A}\cdot\vec{B} = AB\cos\theta$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$

Magnitude of B = 5

Mgnitude of A = $\sqrt{2}$

$\therefore\ \cos\theta = \dfrac{7}{5\sqrt{2}}$

**Q.** $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{B} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ Find angle between $\vec{A}$ & $\vec{B}$

**Sol.** $\vec{A}\cdot\vec{B} = 6 + 12 + 30 = 48$, $A = 5\sqrt{2}$, $B = 7$

$\vec{A}\cdot\vec{B} = AB\cos\theta$

$48 = 5\sqrt{2}\times 7\cos\theta$

$\dfrac{48}{35\sqrt{2}} = \cos\theta$

**Q.** Find angle between $\vec{A}$ & $\vec{B}$ if

$\vec{A} = \hat{i} + \hat{j} + \hat{k}$, $\vec{B} = \hat{i} - \hat{j} + \hat{k}$

**Sol.** $\vec{A}\cdot\vec{B} = AB\cos\theta$

$1 = \sqrt{3}\times\sqrt{3}\cos\theta$

$\dfrac{1}{3} = \cos\theta$

$\theta = \cos^{-1}\left(\dfrac{1}{3}\right)$

# $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

IF-

$\dfrac{A_x}{B_x} = \dfrac{A_y}{B_y} = \dfrac{A_z}{B_z} = n$

- $n > 0 \Rightarrow \vec{A}$ is parallel to $\vec{B}$
- $n < 0 \Rightarrow \vec{A}$ is anti-parallel to $\vec{B}$

**Q.** $\vec{A} = 3\hat{i} - 4\hat{j} + 5\hat{k}$

$\vec{B} = -9\hat{i} + 12\hat{j} - 15\hat{k}$. Are these vectors parallel or anti parallel?

**Sol.** $\dfrac{3}{-9} = \dfrac{-4}{12} = \dfrac{-5}{15}$

$\dfrac{-1}{3} = \dfrac{-1}{3} = \dfrac{-1}{3}$ [Antiparallel]

## SKC

$\vec{A}, \vec{B}$ $||^{rl}$ है या anti-$||^{rl}$ है या $\perp$ है या कुछ नहीं, ये पता करने के लिए

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{AB}$ निकाल लो। $\Rightarrow$ If $\theta = 0°$ = parallel

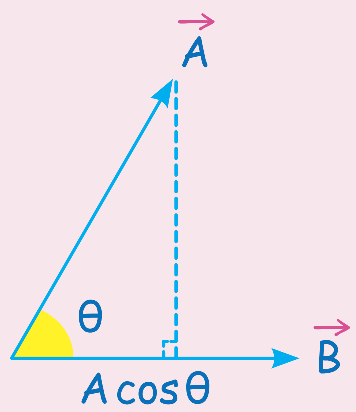
$\theta = 180°$ = Anti $||^{rl}$ If $\theta = 90°$ = $\perp$

**Q.** $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{B} = 4\hat{i} + 6\hat{j} - 8\hat{k}$. Are they parallel?

**Sol.** $\dfrac{2}{4} = \dfrac{3}{6} \neq \dfrac{-4}{8}$

$\dfrac{1}{2} = \dfrac{1}{2} \neq \dfrac{-1}{2}$ [Nothing]




Component of $\vec{A}$ along $\vec{B}$ = A cos θ (magnitude)

= A cos θ. $\hat{B}$ (In vector form) = $\vec{A}_{||}$

$\because A\cos\theta = \dfrac{\vec{A}.\vec{B}}{B}$

Component of $\vec{A}$ along $\vec{B} = \dfrac{\vec{A}.\vec{B}}{B} = \vec{A}.\hat{B}$

Component of $\vec{B}$ along $\vec{A} = \dfrac{\vec{A}.\vec{B}}{A} = \vec{B}.\hat{A}$

Component of $\vec{A}$ along $\vec{B}$ in vector form $(\vec{A}_{||}) = \left(\dfrac{\vec{A}.\vec{B}}{B}\right)\hat{B}$

Component of $\vec{A}$ perpendicular to $\vec{B} = \vec{A} - \vec{A}_{||}$

$\vec{A}_{||} + \vec{A}_{\perp} = \vec{A}$

इसका use आगे mechanics में होगा इसलिए rough copy पर पाँच बार लिख-लिख कर practice करे (for 11th students)

**Q.** Find component of $\vec{A}$ along $\vec{B}$

$\vec{A} = 3\hat{i} + 4\hat{j}$ $\vec{B} = \hat{i} + \hat{j}$

**Sol.** Component of $\vec{A}$ along $\vec{B}$ (scalar) = A cosθ

$= \dfrac{\vec{A}\cdot\vec{B}}{B} = \dfrac{3+4}{\sqrt{2}} = \dfrac{7}{\sqrt{2}}$ (magnitude)

Component of $\vec{A}$ along $\vec{B}$ (Vector) = $\dfrac{\vec{A}\cdot\vec{B}}{B} \times \hat{B}$

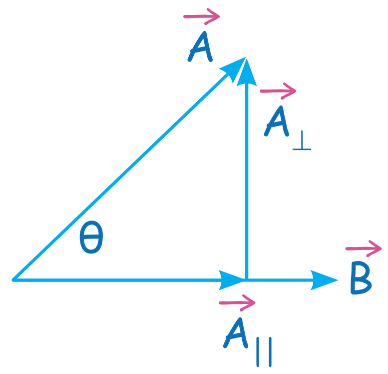
$= \dfrac{7}{\sqrt{2}}\left(\dfrac{\hat{i}+\hat{j}}{\sqrt{2}}\right) = \dfrac{7}{2}\left(\hat{i}+\hat{j}\right)$

**Q.** If $\vec{A} = \hat{i} + 3\hat{j}$ and $\vec{B} = 3\hat{i} + 4\hat{j}$,

Find component of $\vec{A}$ perpendicular to $\vec{B}$.

**Sol.** Component of $\vec{A}$ parallel (along) to $\vec{B} = \dfrac{3}{5}\left(3\hat{i} + 4\hat{j}\right)$

Component of $\vec{A}$ perpendicular to $\vec{B}$ = A sinθ




$\vec{A}_{||} + \vec{A}_{\perp} = \vec{A} \Rightarrow \vec{A}_{\perp} = \vec{A} - \vec{A}_{||}$

$A_{\perp} = \hat{i} + 3\hat{j} - \left[\dfrac{3}{5}\left(3\hat{i} + 4\hat{j}\right)\right] = \hat{i} + 3\hat{j} - \left(\dfrac{9\hat{i} + 12\hat{j}}{5}\right)$

$= \dfrac{-4\hat{i}}{5} + 3\hat{j}$

**Q.** $\vec{A} = 4\hat{i} - 2\hat{j}$, $\vec{B} = 3\hat{i} + 4\hat{j}$

Find component of $\vec{A}$ perpendicular to $\vec{B}$.

**Sol.** Component of $\vec{A}$ parallel to $\vec{B}$

$= \dfrac{\vec{A}\cdot\vec{B}}{B} = \dfrac{12-8}{5} = \dfrac{4}{5}$

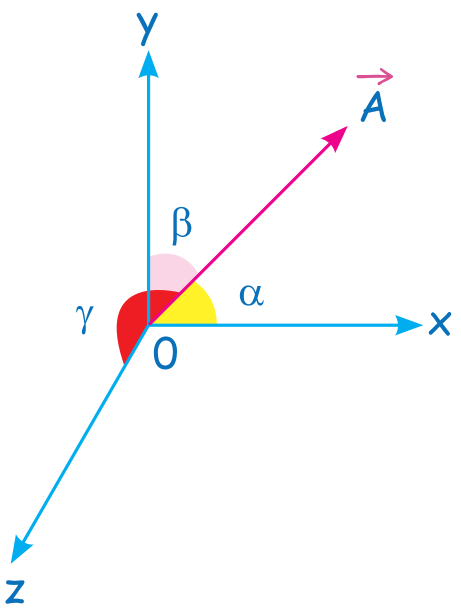
Vector form = $\dfrac{4}{5}\left(\dfrac{3\hat{i} + 4\hat{j}}{5}\right) = \dfrac{12\hat{i} + 16\hat{j}}{25}$

Component of $\vec{A}$ perpendicular to $\vec{B}$

$= A_{\perp} = \vec{A} - \vec{A}_{||}$

$= 4\hat{i} - 2\hat{j} - \left[\dfrac{12\hat{i} + 16\hat{j}}{25}\right] = \dfrac{88\hat{i} - 66\hat{j}}{25}$

## DIRECTION COSINE




$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

If $\vec{A}$ makes angle $\alpha$ with + x-axis

If $\vec{A}$ makes angle $\beta$ with + y-axis

If $\vec{A}$ makes angle $\gamma$ with + z-axis

Component of $\vec{A}$ along x-axis = $A\cos\alpha = A_x$

Component of $\vec{B}$ along y-axis = $A\cos\beta = A_y$

Component of $\vec{C}$ along z-axis = $A\cos\gamma = A_z$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

$\cos\alpha = \frac{A_x}{A}$ $\cos\beta = \frac{A_y}{A}$ $\cos\gamma = \frac{A_z}{A}$ here, $\cos\alpha$, $\cos\beta$, $\cos\gamma$ are the direction cosine of $\vec{A}$

**Q.** $\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$. Find direction cosine

**Sol.** $A = |\vec{A}| = \sqrt{2^2 + 3^2 + 6^2} = 7$

$$\cos\alpha = \frac{A_x}{A} = \frac{2}{7} \quad \cos\beta = \frac{A_y}{A} = \frac{3}{7} \quad \cos\gamma = \frac{A_z}{A} = \frac{6}{7}$$

**Q.** $\vec{A} = 3\hat{i} + 4\hat{j} + 5\hat{k}$, Find direction cosine

**Sol.** $A = \sqrt{9+16+25} = 5\sqrt{2}$

(a) Direction cosine

$$\cos\alpha = \frac{3}{5\sqrt{2}} \quad \cos\beta = \frac{4}{5\sqrt{2}} \quad \cos\gamma = \frac{5}{5\sqrt{2}}$$

(b) Component of $\vec{A}$ along x-axis = $3\hat{i}$

(c) Component of $\vec{A}$ along y-axis = $4\hat{j}$

(d) Component of $\vec{A}$ along z-axis = $5\hat{k}$

(e) Component of $\vec{A}$ on x-y plane = $3\hat{i} + 4\hat{j}$

(f) Component of $\vec{A}$ on y-z plane = $4\hat{j} + 5\hat{k}$

## Cross Product

भाई cross product की ज्यादा tension मत लेना बस ये सिख लेना कि कैसे cross product निकालते है rotation से पहले इसका बहुत ही कम use होगा, थोड़ा सा circular motion में use होगा।

If cross product of vector $\vec{A}$ and $\vec{B}$ is $\vec{C}$ means $\vec{A} \times \vec{B} = \vec{C}$ then $\vec{C}$ is a vector which is perpendicular to both $\vec{A}$ and $\vec{B}$ and magnitude of $\vec{C}$ is given by

$|\vec{C}| = AB\sin\theta$

★ $\vec{C} = (AB\sin\theta)\hat{C}$

★ $\vec{a} \times \vec{b} = \vec{c} = ab\sin\theta\cdot\hat{c}$

★ $\vec{c}\cdot\vec{a} = 0$, $\vec{c}\cdot\vec{b} = 0$

★ $\vec{c} \perp \vec{a}$, $\vec{c} \perp \vec{b}$

★ $(\vec{a} \times \vec{b})\cdot\vec{a} = 0$, $(\vec{a} \times \vec{b})\cdot\vec{b} = 0$

★ $\vec{a} \times \vec{a} = 0$

★ $\hat{i} \times \hat{i} = 0$, $\hat{j} \times \hat{j} = 0$, $\hat{k} \times \hat{k} = 0$

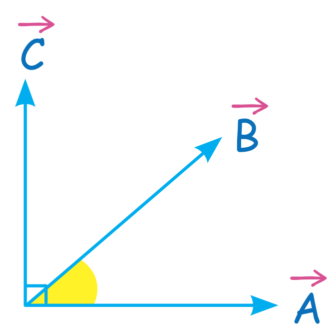
दो vector का cross product खुद मे एक vector होता है।

★ In terms of components $\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$= \hat{i}(A_yB_z - A_zB_y) + \hat{j}(A_zB_x - A_xB_z) + \hat{k}(A_xB_y - A_yB_x)$$

अगर आपने determinant नही पढ़ा तो tension लेने की कोई जरूरत नही है जब पढ़लो तब ये method apply करके cross product निकाल लेना।

★ $\vec{A} \times \vec{B} = \vec{C}$



# SKC

★ $\hat{i} \times \hat{j} = \hat{k}$

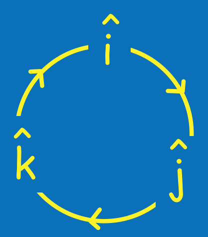
★ $\hat{j} \times \hat{k} = \hat{i}$

★ $\hat{k} \times \hat{i} = \hat{j}$

★ $\hat{j} \times \hat{i} = -\hat{k}$

★ $\hat{i} \times \hat{k} = -\hat{j}$

★ $\hat{k} \times \hat{j} = -\hat{i}$




अब याद करने का तरीका सुन, जबान से बोलो $\hat{i}\,\hat{j}\,k\hat{i}\,\hat{j}\,\hat{k}\hat{i}\,\hat{j}\,\hat{k}\hat{i}\,\hat{j}$...... और ये sequence catch करो जैसे i और j के बाद k आया तो $\hat{i} \times \hat{j} = \hat{k}$ उसी sequence में j और k के बाद i आया है तो $\hat{j} \times \hat{k} = \hat{i}$ अगर sequence गड़बड़/उल्टा हुआ तो minus लगा दो जैसे $\hat{j} \times \hat{i} = -\hat{k}$ अबे समझ गए ना।

**Q.** $\vec{a} = 3\hat{i} + 4\hat{j}$

$\vec{b} = 2\hat{i} + 5\hat{j}$  find $\vec{a} \times \vec{b}$

**Sol.** $\vec{a} \times \vec{b} = (3\hat{i} + 4\hat{j}) \times (2\hat{i} + 5\hat{j})$

$= 6\hat{i} \times \hat{i} + 15\hat{i} \times \hat{j} + 8\hat{j} \times \hat{i} + 20\hat{j} \times \hat{j}$

$= 0 + 15\hat{k} - 8\hat{k} + 0$

$= 7\hat{k} = \vec{C}$

$\vec{C} \perp \vec{A}$

$\vec{C} \perp \vec{B}$

**Q.** $\vec{a} = 4\hat{i} + 7\hat{j}$

$\vec{b} = 2\hat{i} + 3\hat{j}$  find $\vec{a} \times \vec{b}$

**Sol.** $\vec{a} \times \vec{b} = 8\hat{i} \times \hat{i} + 12\hat{i} \times \hat{j} + 14\hat{j} \times \hat{i} + 21\hat{j} \times \hat{j}$

$= 0 + 12\hat{k} - 14\hat{k} + 0 = -2\hat{k} = \vec{C}$

$\vec{C} \perp \vec{A},\ \vec{C} \perp \vec{B}$

$\vec{b} \times \vec{a} = (2\hat{i} + 3\hat{j}) \times (4\hat{i} + 4\hat{j})$

$= 0 + 14\hat{k} - 12\hat{k} + 0 = 2\hat{k}$

$\therefore\ \vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$

अब मै नीचे कुछ important question attach कर रहा हुँ इनको अच्छे से solve करो...... अबे करोगे ना......

## कुछ PRACTICE करलो

**Q.** If the sum of two unit vectors is a unit vector, then find the magnitude of their difference.

**Sol.** Let $\hat{n}_1$ and $\hat{n}_2$ are the two unit vectors, then the sum is $n_s = \hat{n}_1 + \hat{n}_2$ or $n_s^2 = n_1^2 + n_2^2 + 2n_1n_2\cos\theta$ $= 1 + 1 + 2\cos\theta$

Since it is given that $n_s$ is also a unit vector, therefore $1 = 1 + 1 + 2\cos\theta$

or $\cos\theta = -\frac{1}{2}$ or $\theta = 120°$

Now the difference of the vectors is $\vec{n}_d = \hat{n}_1 - \hat{n}_2$

or $n_d^2 = n_1^2 + n_2^2 - 2n_1n_2\cos\theta = 1 + 1 - 2\cos(120°)$

$\therefore\ n_d^2 = 2 - 2(-1/2) = 2 + 1 = 3 \Rightarrow n_d = \sqrt{3}$

**Q.** If vector $\vec{A} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{B} = -4\hat{i} - 6\hat{j} - \lambda\hat{k}$ are perpendicular to each other then value of $\lambda$ will be?

**Sol.** If $\vec{A}$ and $\vec{B}$ are perpendicular to each other then $\vec{A}.\vec{B} = 0$

So, $2(-4) + 3(-6) + (-1)(-\lambda) = 0 \Rightarrow \lambda = +26$

**Q.** If $\vec{a}_1$ and $\vec{a}_2$ are two non collinear unit vectors and if $|\vec{a}_1 + \vec{a}_2| = \sqrt{3}$, then find the value of $(\vec{a}_1 - \vec{a}_2).(2\vec{a}_1 + \vec{a}_2)$.

**Sol.** $a_1 = a_2 = 1$ and $a_1^2 + a_2^2 + 2a_1a_2\cos\theta = 3$

Or $1 + 1 + 2\cos\theta = 3$ or $\cos\theta = \frac{1}{2}$

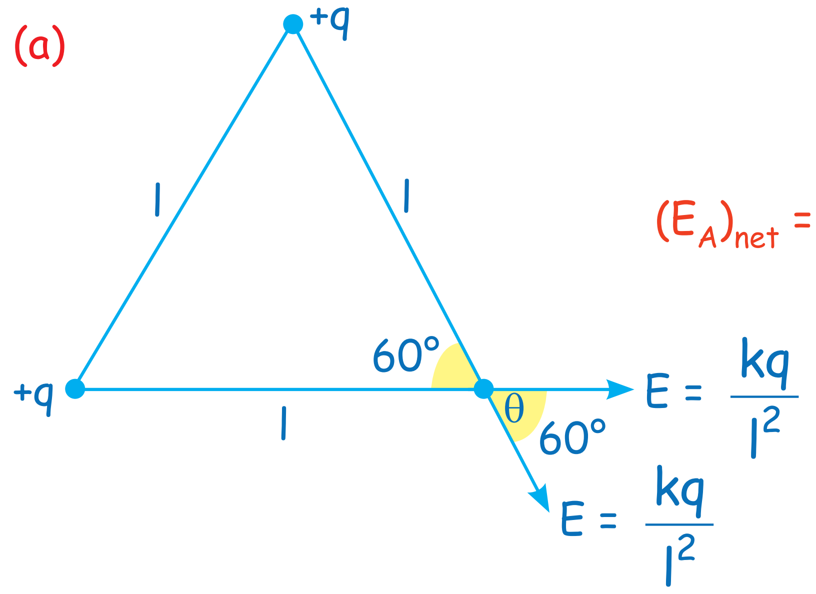
Now $(\vec{a}_1 - \vec{a}_2)\cdot(2\vec{a}_1 + \vec{a}_2) = 2a_1^2 - a_2^2 - a_1a_2\cos\theta$

$= 2 - 1 - \frac{1}{2} = \frac{1}{2}$

**Q.** In 12th class we will study in electrostatics that a point charge produce electric field which is a vector quantity and electric field due to a positive point charge at a distance r from charge has value $\frac{kq}{r^2}$ radially away from charge (along the line joining point to the charge) k is constant

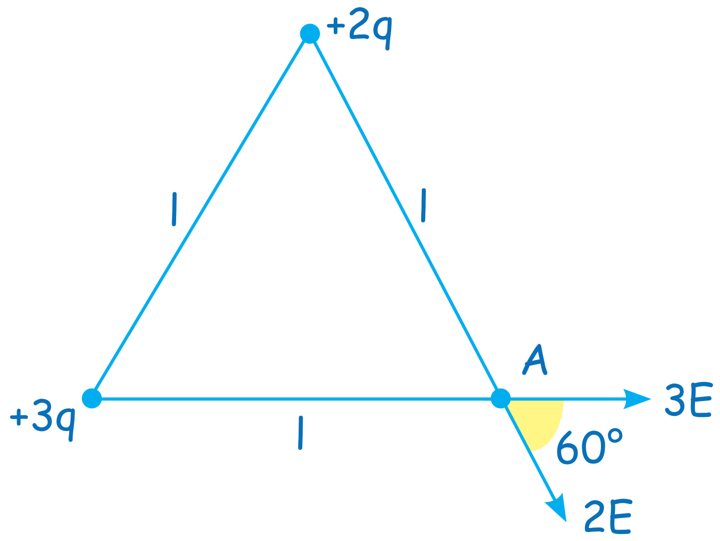
Using above data find the net electric field at point A in following case.

(a)




$(E_A)_{net} = E\sqrt{3}$

(b)

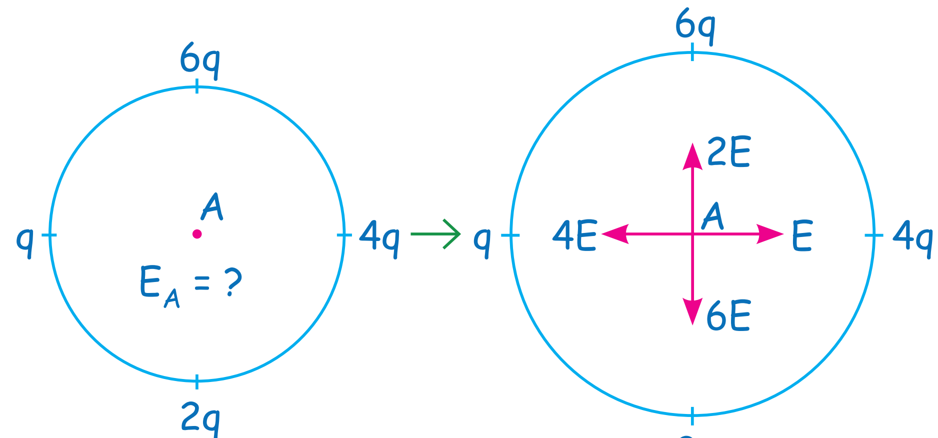


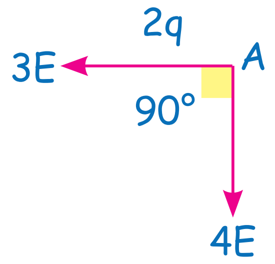

Let $E = \frac{kq}{l^2}$

$E_{net} = \sqrt{(2E)^2 + (3E)^2 + 2 \times 2E \times 3E \times \cos 60^\circ}$

$= E\sqrt{19}$

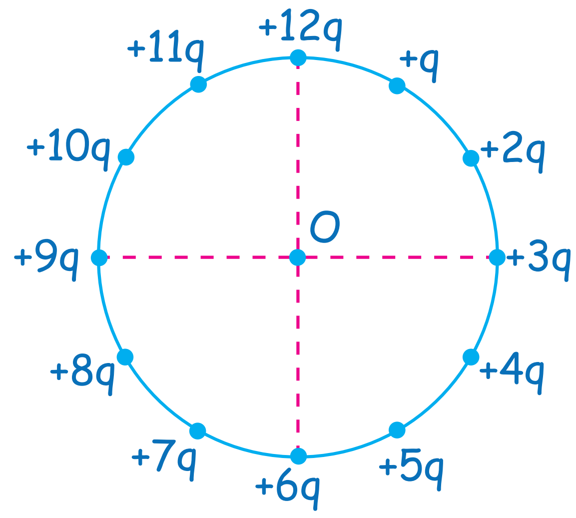
(c)

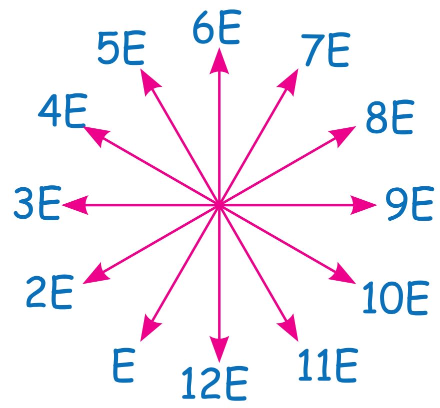


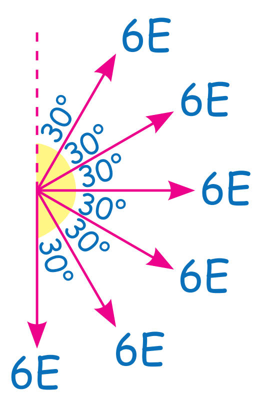




$(E_{net})_A = 5E = \frac{5kq}{r^2}$

(d) Find $E_{net}$ at centre O.




अब component लेकर solve कर लोगे ना..... नही तो कोई बात नही 12th मे कर लेना

$\vec{E}_{net} = (12E + 6E\sqrt{3})\hat{i} - 6E\hat{j}$

## IMPORTANT RESULT

If $|\vec{A}| = |\vec{B}| = x$

and angle between $\vec{A}$ and $\vec{B}$ is $\theta$

then $|\vec{A} + \vec{B}| = 2x \cos \theta/2$

$|\vec{A} - \vec{B}| = 2x \cos \theta/2$

**Q.** Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P} + \vec{Q}$ is n times the magnitude of $\vec{P} - \vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is:

**Ans.** $\cos^{-1}\left(\frac{n^2 - 1}{n^2 + 1}\right)$

# 3 Kinematics

## MOTION IN A STRAIGHT LINE

**Motion:** If a body changes its position with time body is called in motion

★ Rest and motion are relative terms. They depend on observer.

जैसे की यहाँ बाबू भईया आपके respect में motion मे है but ठेले के respect मे rest पर है।

**Distance:** Actual path travelled by the body -

★ Dependent upon path (we should know the path)

★ It can't be decreasing

★ It can't be negative

★ It can be zero or positive

## DISPLACEMENT

Change in position vector $\vec{r}$ or change in position

$\vec{d} = \overrightarrow{\Delta r} = \vec{r}_f - \vec{r}_i$

★ Displacement = final position - Initial position

★ It is the shortest distance between initial and final point.

★ Vector term

★ It can be positive, negative or zero.

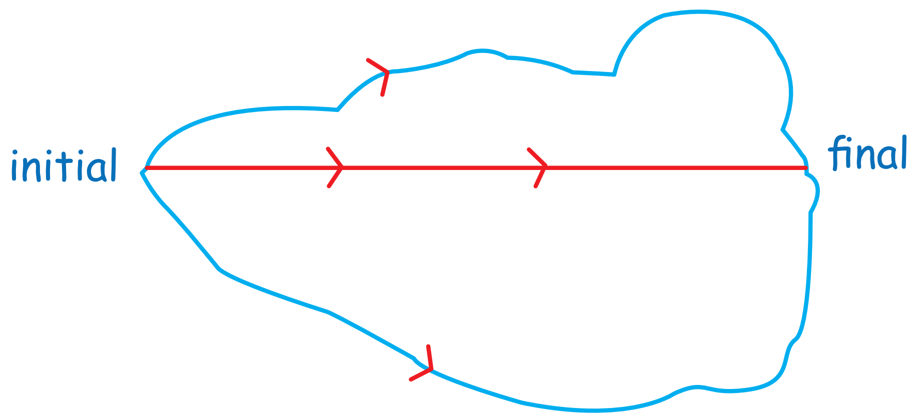
★ It can be increasing or decreasing.

★ Independent of path travelled.

**Q.** A particle move from point A to point B. Find its displacement in following case.


**Sol.** $\vec{d}$ = displacement $= 5\hat{i} + 2\hat{j} + 5\hat{j}$

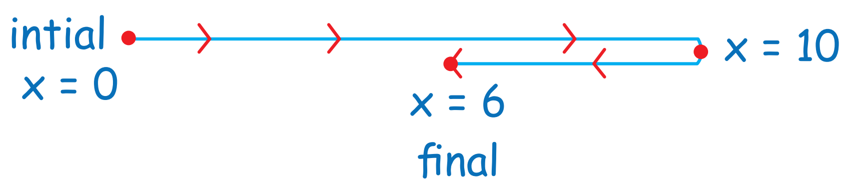
★




Displacement - same

Distance - different (depend upon path)

★

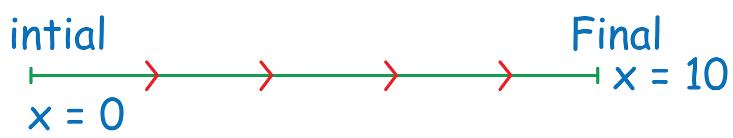



distance → 10 + 4 = 14

displacement → 6

(vector form) → $6\hat{i}$

★




Distance = 10

Displacement = $10\hat{i}$

Here, Distance = |Displacement|

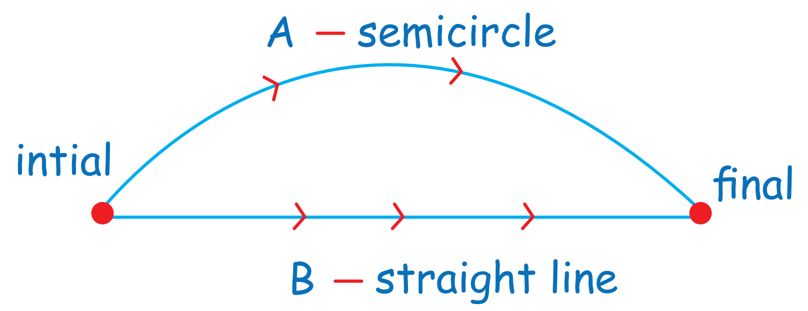
★ |Displacement| = magnitude of displacement

★ If a particle does not change direction then distance = |displacement|

★ Distance ≥ |displacement|

★ $\dfrac{\text{Distance}}{|\text{Displacement}|} \geq 1$

**Q.** A particle move from initial to final position in two different path A and B as shown in fig. Find distance and displacement for both.




**Sol.**

| | A | B |
|---|---|---|
| Distance | $\pi R$ | 2R |
| Displacement | 2R | 2R |

**Q.** A particle move 5m along east then 6m along north and 10m in upward direction. Find distance & displacement?

**Sol.** Distance 5m + 6m + 10m = 21

Displacement = $5\hat{i} + 6\hat{j} + 10\hat{k}$

Magnitude = $\sqrt{5^2 + 6^2 + 10^2}$

**Q.** A particle move

10m east = $10\hat{i}$

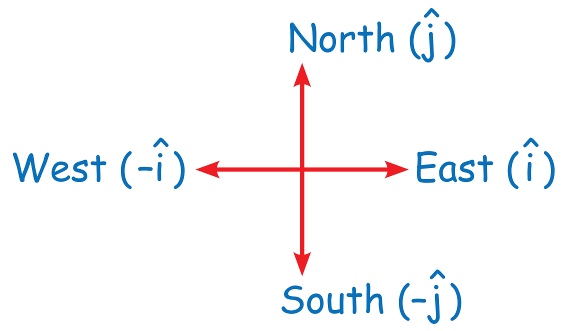
5m north = $5\hat{j}$

6m south = $-6\hat{j}$

8m west = $-8\hat{i}$

15m east = $15\hat{i}$

20m north = $20\hat{j}$

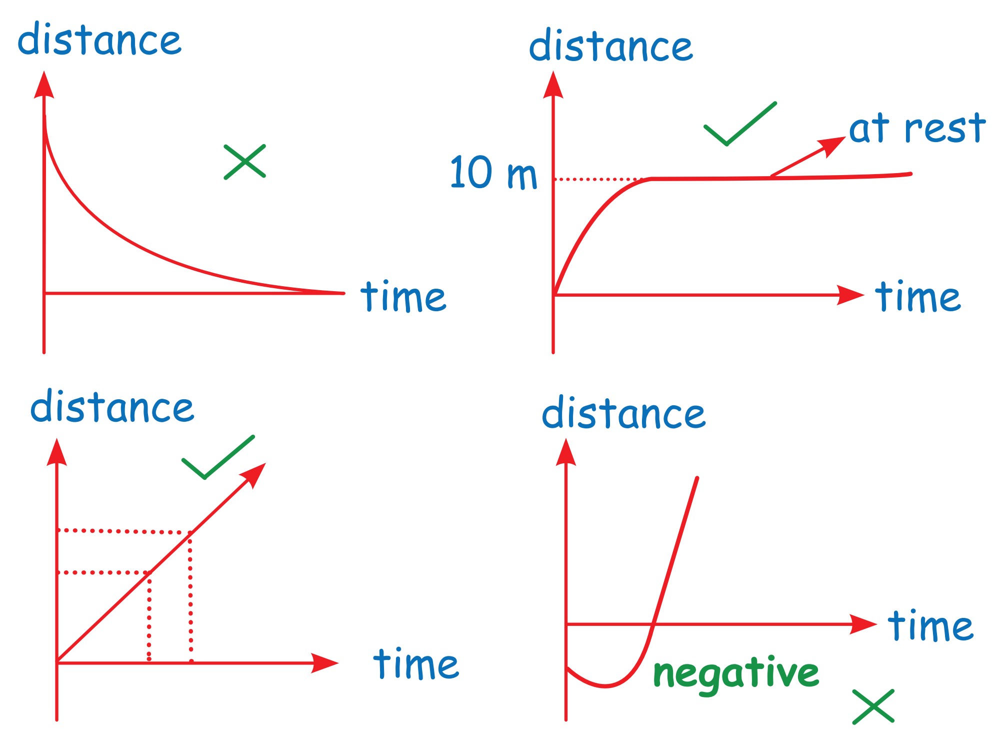


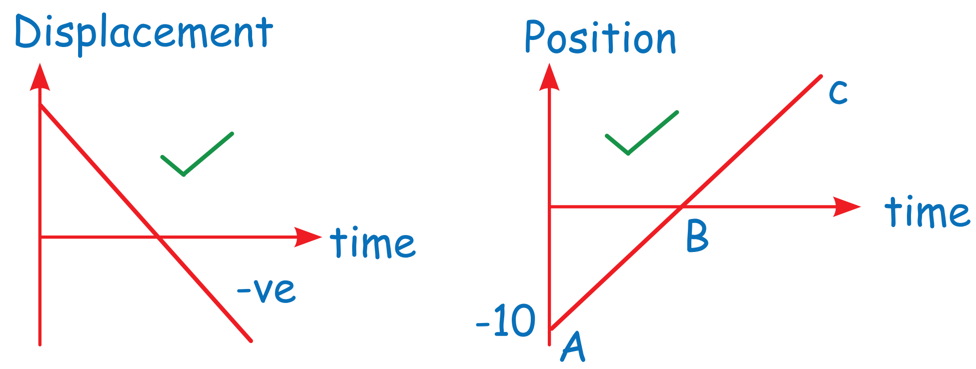

distance = 64 m

displacement = $(10 - 8 + 15)\hat{i} + (5 - 6 + 20)\hat{j}$

$= 17\hat{i} + 19\hat{j}$

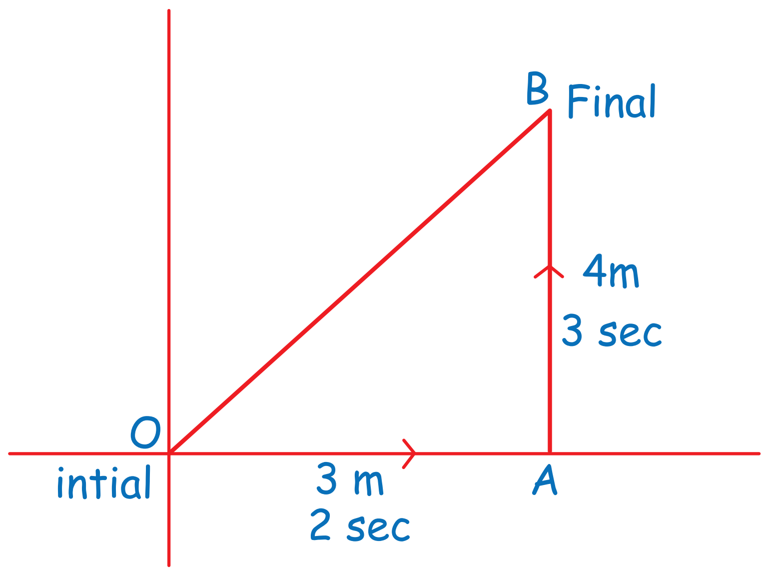
**Q.** Which of the graph is possible.




- displacement = change in position.
- Average velocity = $\dfrac{\text{total displacement}}{\text{total time}}$
- Instantaneous velocity = $\dfrac{d\bar{r}}{dt}$
- Average speed = $\dfrac{\text{total distance}}{\text{total time}}$
- Instantaneous speed = kisi inst. par speed
- |Instantaneous velocity| = Instantaneous speed

**Q.** A particle move from origin to point A and took 2 sec, then it move to point B and took 3 sec in following fig. Find displacement, average velocity and average speed.

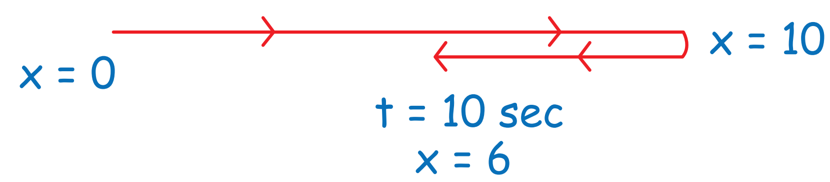



**Sol.** Displacement $\vec{d} = 3\hat{i} + 4\hat{j}$

Avrg. velocity = $\dfrac{3\hat{i} + 4\hat{j}}{5}$

Avrg. speed = $\dfrac{\text{Total distance}}{\text{Total time}} = \dfrac{3+4}{2+3} = \dfrac{7}{5}$

**Q.**



**Sol.** Displacement = 6 (mag)

Average velocity = $\frac{6}{10}$

Distance = 10 + 4 = 14

Average speed = $\frac{14}{10}$

**Q.** $x = 0$, $t = 0$ ———— $t = 5$ sec, $x = 10$

**Sol.** Distance = 10

Average speed = $\frac{10}{5}$

Displacement = 10 (Magnitude)

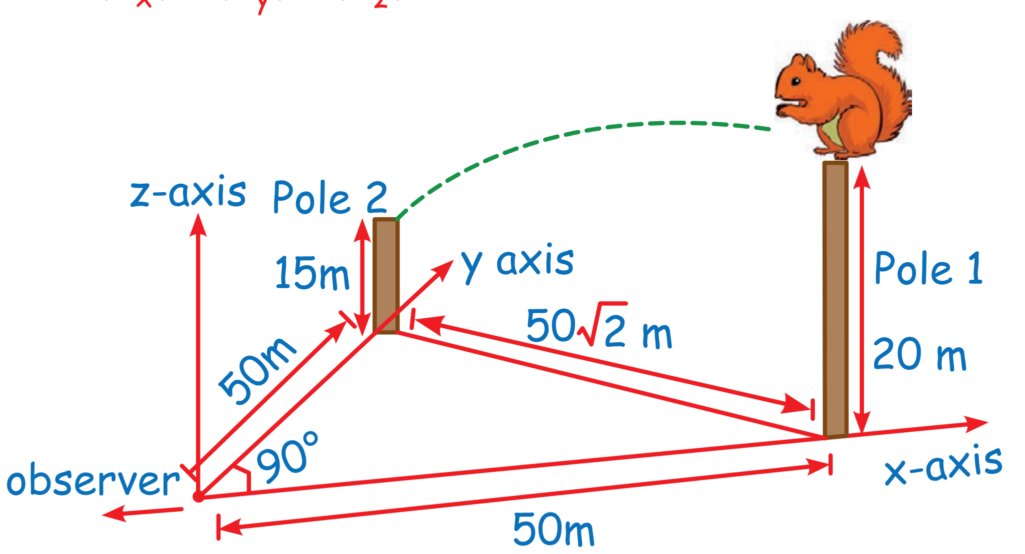
Average velocity = $\frac{10}{5}$ (Magnitude)

★ Agar particle ne apni direction nahi badli

★ distance = |displacement|

★ Avrg speed = |Avrg velocity|

**Q.** A small squirrel jumps from pole 1 to pole 2 and took 3 sec. What is average velocity vector of squirrel? If average velocity vector is expressed as $v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$, express your answer as sum of magnitudes of its components $|v_x| + |v_y| + |v_z|$ in unit m/s.




**Sol.** Intial coordinate is (50, 0, 20)

Final coordinate is (0, 50, 15)

Displacement = $-50\hat{i} + 50\hat{j} - 5\hat{k}$

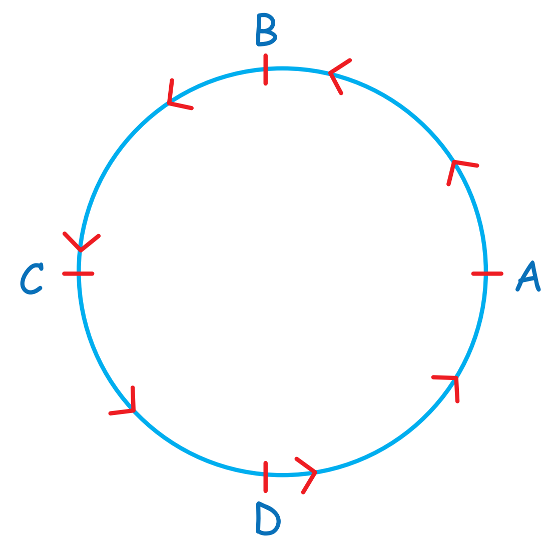
Average velocity = $\frac{-50\hat{i} + 50\hat{j} - 5\hat{k}}{3}$

$= -\frac{50}{3}\hat{i} + \frac{50}{3}\hat{j} - \frac{5}{3}\hat{k}$

$\frac{50}{3} + \frac{50}{3} + \frac{5}{3} = \frac{105}{3} = 35$ m/s

मुझे पता है कुछ google boys ने google पर search करके answer 105 निकाला होगा....... अबे सवाल ठीक से पढ़ लिया करो


**Q.** A particle is performing uniform circular motion with constant speed $v_1$ having time period T Anticlockwise. Find avg velocity and avg speed.




**Sol.**

| | Avrg Speed | Arvg Velocity |
|---|---|---|
| $A \to B$ | $\frac{2\pi R/4}{T/4} = \frac{2\pi R}{T}$ | $\frac{R\sqrt{2}}{T/4}$ |
| $A \to B \to C$ | $\frac{\pi R}{T/2} = \frac{2\pi R}{T}$ | $\frac{2R}{T/2}$ |
| $A \to B \to C \to D$ | $\frac{(3/4)2\pi R}{3T/4} = \frac{2\pi R}{T}$ | $\frac{R\sqrt{2}}{3T/4}$ |
| $A \to B \to C \to D \to A$ | $\frac{2\pi R}{T}$ | 0 |

**Q.** A car is moving along x-axis, in 1st four hour it travel with speed 50 km/hr, in next 2 hours it move with 70 km/hr and in last part of journey it travel for 5 hour with 80 km/hr. Find avg speed.

**Sol.** Avrg speed $= \frac{\text{total distance}}{\text{Time}}$

$= \frac{d_1 + d_2 + d_3}{t_1 + t_2 + t_3} = \frac{50 \times 4 + 70 \times 2 + 80 \times 5}{4 + 2 + 5}$

$$= \frac{200+140+400}{11}$$

$$\boxed{\text{Averg speed} = \frac{740}{11}}$$

note: 1D motion (on x-axis)

Average velocity $= \dfrac{\vec{x}_f - \vec{x}_i}{\text{time}} = \dfrac{\Delta \vec{x}}{\Delta t}$

मै average value को represent करने के कुछ symbol नीचे लिख रहा हुँ जो अक्सर use होते है और space बचाने के लिए हम भी use करेंगे

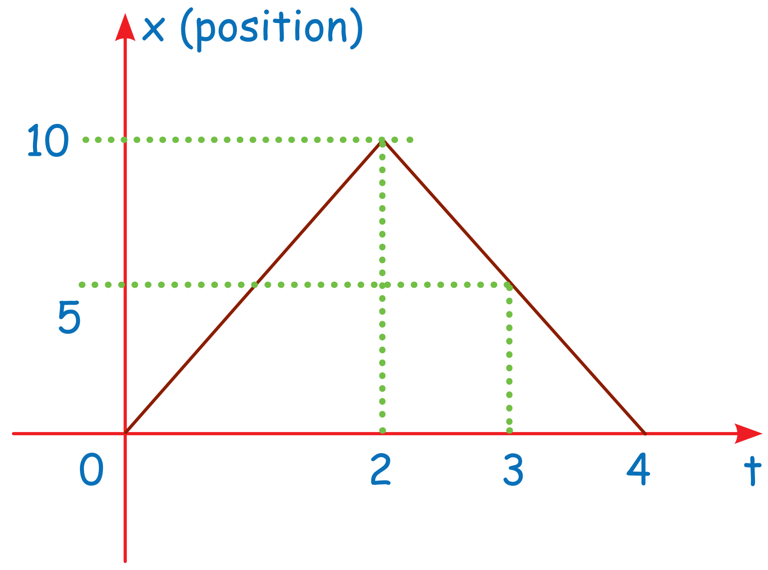
Average velocity = $< \vec{v} >$

Average speed = < speed > = $< |\vec{v}| >$

Average acc = $< \vec{a} >$

Average कद्दू = < कद्दू >

**Q.** A particle is moving on the x-axis as show its x-coordinate unit time. Find average velocity from t = 0 to t = 2 sec, from t = 0 to t = 3 sec.




**Sol.** 1 t = 0 → t = 2 sec

Average velocity $= \dfrac{x_f - x_i}{\text{time}} = \dfrac{10-0}{2} = 5$

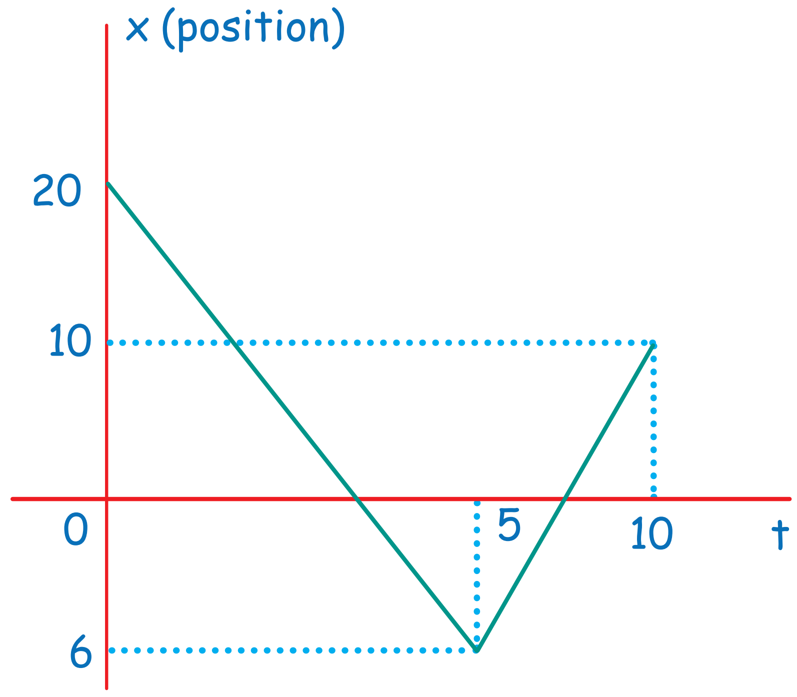
Average speed $= \dfrac{10}{2} = 5 \left| \dfrac{10_f - 0_i}{t} \right.$

2 t = 0 → t = 3

Average velocity $= \dfrac{x_f - x_i}{\text{time}} = \dfrac{5-0}{3}$

Average speed $= \dfrac{10+5}{3}$

**Q.** Find average velocity from t = 0 to t = 5s and t = 0 to t = 10s.




**Sol.** 1. t = 0 → t = 5

Average velocity $= \dfrac{x_f - x_i}{\text{time}} = \dfrac{(-6)-(+20)}{5}$

Average speed $= \dfrac{20+6}{5}$

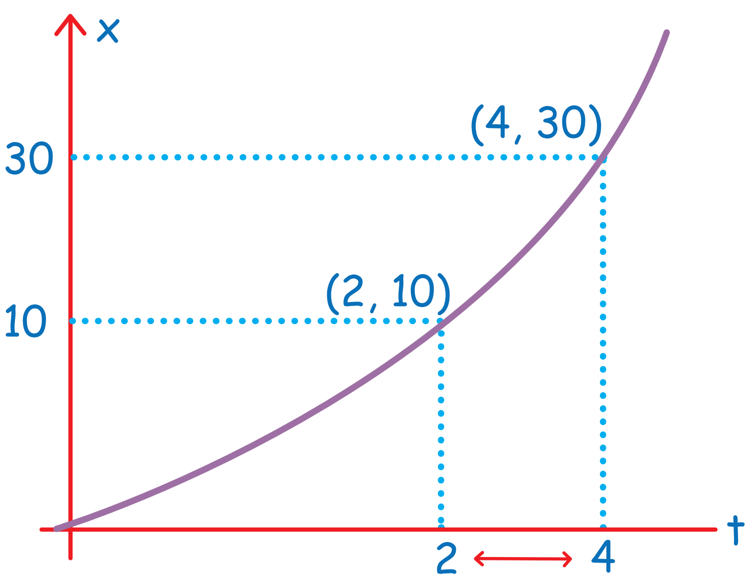
2. From t = 0 → t = 10

Average velocity $= \dfrac{x_f - x_i}{10} = \dfrac{10-20}{10} = \dfrac{-10}{10}$

Average speed $= \dfrac{20+6+6+10}{10} = 4.2$ m/s

**Q.** Find average velocity from

t = 2 → t = 4 sec




**Sol.** Average velocity (mag) $= \dfrac{x_f - x_i}{\Delta t}$

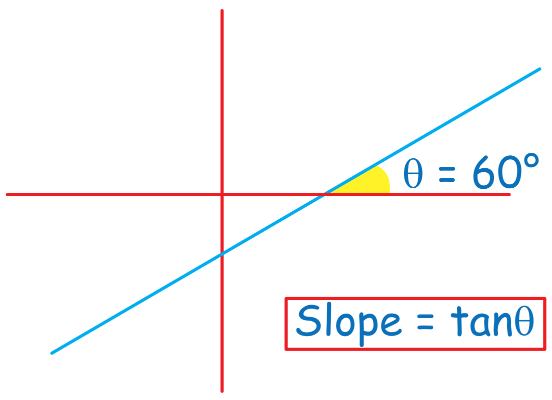
$$= \frac{20}{2} = 10$$

**Slope of line** $= \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{30-10}{4-2}$

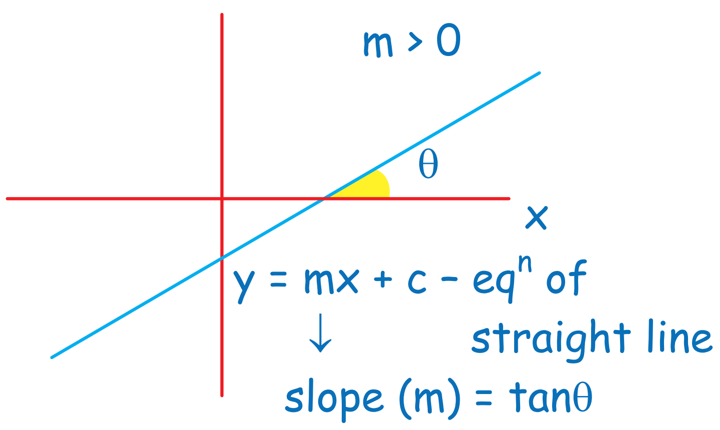
$$= \frac{20}{2} = 10$$

चलो अब थोड़ा सा काम का mathematics का revision हो जाए जिसकी हमे यहाँ बहुत जरूरत है।

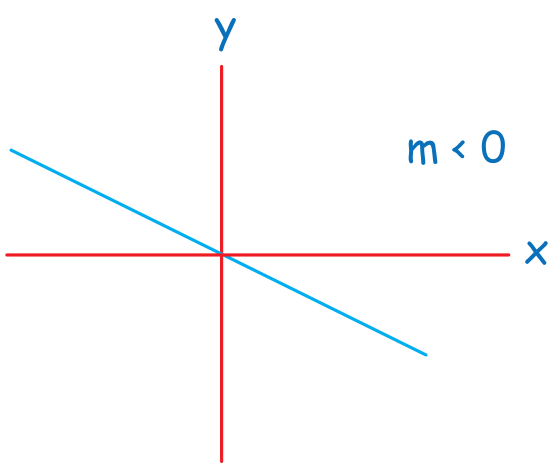
## SLOPE OF LINE

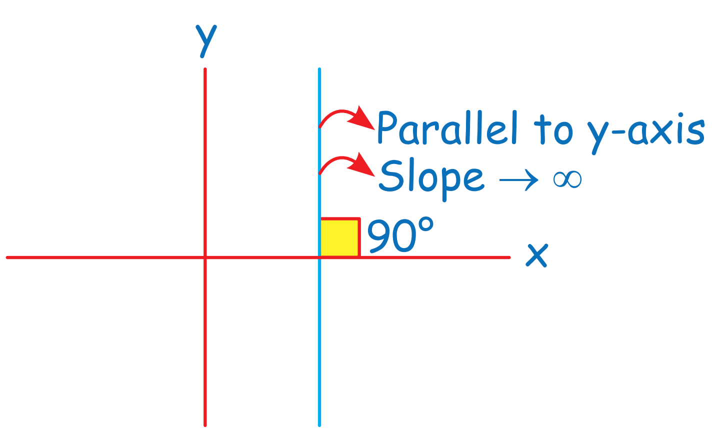


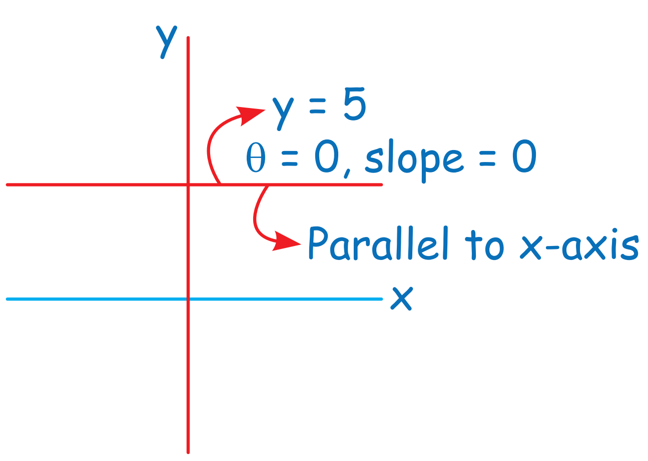

slope = $\sqrt{3} = \tan 60°$




slope = $\tan\theta$



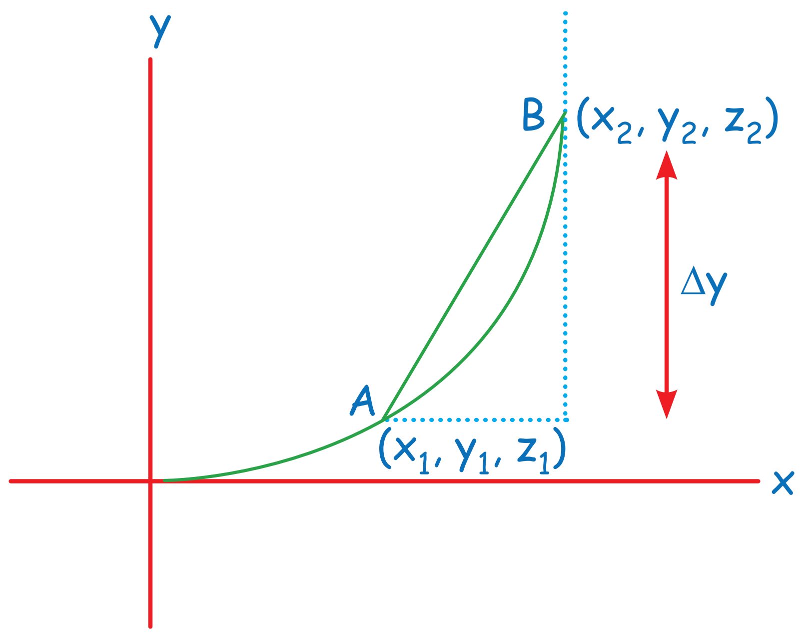




Slope of line join A to B

$$= \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x} = \tan\theta$$

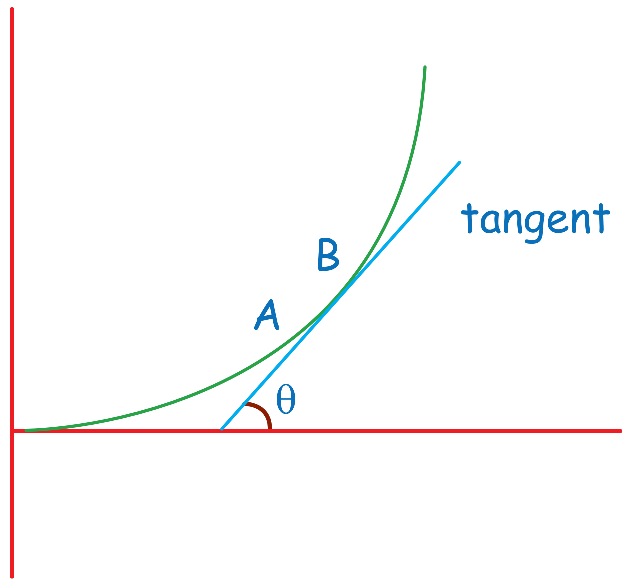
$\Delta y$ = change in y

$\Delta x$ = change in x




If $\Delta y$ is very small $\Delta y = \to 0$ $(\Delta y = dy)$

If $\Delta x$ is very small $\Delta x \to 0$ $(\Delta x = dx)$




$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

★ $\frac{dy}{dx}$ मतलब differentiation of y wrt x

→ और rate of change of y wrt x

→ Slope of tangent at that point

→ Ratio of very-2 small change in y to very - 2 small change in x

→ 1st derivation of y wrt x

## IMP FORMULA

| | |
|---|---|
| $\frac{d}{dx}(x^n) = nx^{n-1}$ | $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$ |
| $\frac{d}{dx}\sin x = \cos x$ | $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$ |
| $\frac{d}{dx}(\cos x) = -\sin x$ | $\frac{d}{dx}e^x = e^x$ |
| $\frac{d}{dx}(\tan x) = \sec^2 x$ | $\frac{d}{dx}(\ln x) = \frac{1}{x}$ |
| $\frac{d}{dx}(\sec x) = \sec x \tan x$ | |

**Q.** $y = x^3$

**Sol.** $\frac{dy}{dx} = 3x^{3-1} = 3x^2$

**Q.** $y = t^3$

**Sol.** $\frac{dy}{dt} = 3t^2$

**Q.** $y = 2x^5$

**Sol.** $\frac{dy}{dx} = 2x5x^4 = 10x^4$

**Q.** $y = \frac{1}{x^2} = x^{-2}$

**Sol.** $\frac{dy}{dx} = -2x^{-2-1} = \frac{-2}{x^3}$

**Q.** $y = x^2 + x^5$

**Sol.** $\frac{dy}{dx} = 2x + 5x^4$

**Q.** $y = x^9 + \frac{1}{x^5}$

**Sol.** $\frac{dy}{dx} = 9x^8 - \frac{5}{x^6}$

$\frac{dy}{dx} = 9x^8 - \frac{5}{x^6}$

differentiation of y wrt x

**Q.** $y = x$

**Sol.** $\frac{dy}{dx} = 1$

**Q.** $y = 5$ constant

**Sol.** $\frac{dy}{dx} = 0$

**Q.** $y = x^3 + \sin x$

**Sol.** $\frac{dy}{dx} = 3x^2 + \cos x$

**Q.** $y = x^7 + \tan x + 10$

**Sol.** $\frac{dy}{dx} = 7x^6 + \sec^2 x + 0$

**Q.** $y = 3x^3 + \sin x + \tan x$

**Sol.** $\frac{dy}{dx} = 9x^2 + \cos x + \sec^2 x$

**Q.** $y = 2x^2 + \cos x + 5$

**Sol.** $\frac{dy}{dx} = 4x - \sin x + 0$

**Q.** $y = 3x^2 + \cos x + e^x - \sin x + 10$

**Sol.** $\frac{dx}{dy} = 6x - \sin x + e^x - \cos x + 0$

**Q.** If $y = x^2 + 4x^{-1/2} - 3x^{-2}$ find $\frac{dy}{dx}$

**Sol.** $\frac{dy}{dx} = \frac{d}{dx}(x^2 + 4x^{-1/2} - 3x^{-2})$

$= \frac{d}{dx}(x^2) + \frac{d}{dx}(4x^{-1/2}) - \frac{d}{dx}(3x^{-2})$

$= \frac{d}{dx}(x^2) + 4\frac{d}{dx}(x^{-1/2}) - 3\frac{d}{dx}(x^{-2})$

$= 2x - 2x^{-3/2} + 6x^{-3}$

**Q.** If $3y = 4x^2 - 5$ find $\frac{dy}{dx}$

**Sol.** $y = \frac{4}{3}x^2 - \frac{5}{3} \Rightarrow \frac{dy}{dx} = \frac{8x}{3}$

**Q.** $\sqrt{y} = x - 1$, find $\frac{dy}{dx}$

**Sol.** $y = x^2 - 2x + 1 \Rightarrow \frac{dy}{dx} = 2x - 2$

$\frac{dy}{dx}$ को shortcut मे y' भी लिखते है इसका मतलब है y का एक बार differentiation, उसी प्रकार अगर y का दो बार differentiation करेंगे तो उसे y" या $\frac{d^2y}{dx^2}$ लिखते है जैसे

$y = x^3$

$y' = \frac{dy}{dx} = 3x^2$

$y'' = \frac{d^2y}{dx^2} = 6x$

## Multiply

**y = u . v**

**y′ = uv′ + vu′**

**Q.** $y = x^3 . \sin x$

**Sol.** $\frac{dy}{dx} = x^3\left(\frac{d}{dx}\sin x\right) + \sin x\left(\frac{d}{dx}x^3\right)$

**Q.** $y = x^5 \tan x$

**Sol.** $\frac{dy}{dx} = x^5 . \sec^2 x + [\tan x](5x^4)$

**Q.** $y = x^3 . e^x$

**Sol.** $\frac{dy}{dx} = x^3 e^x + e^x . 3x^2$

**Q.** $y = e^x . \sin x$

**Sol.** $\frac{dx}{dy} = e^x . \cos x + \sin x . e^x$

**Q.** $y = x^3 e^x$

**Sol.** $\frac{dy}{dx} = x^3 e^x + e^x . 3x^2$

**Q.** $y = x^4 \ln x$

**Sol.** $\frac{dy}{dx} = x^4 . \frac{1}{x} + (\ln x)(4x^3)$

## CHAIN RULE

**Q.** $y = \sin (x^2 + x^7)$

**Sol.** $\frac{dx}{dy} = \cos(x^2 + x^7) \times (2x + 7x^6)$

ऊपर वाले chain rule के question मे यह pattern पहचान ने की कोशिश करो कि पहले बाहर वाला function का differentiation करना है फिर multiply करते हुए अंदर वाले function का differentiation करते जाना है जैसे

$y = \sin(\text{कद्दू})$

$\frac{dy}{dx} = \text{Cos}(\text{कद्दू}) \times \frac{d}{dx}(\text{कद्दू})$

**Q.** $y = \ln x^5$

**Sol.** $\frac{dy}{dx} = \frac{1}{x^5} \times 5x^4$

**Q.** $y = \ln (\sin x)$

**Sol.** $\frac{dy}{dx} = \frac{1}{\sin x} \times \cos x$

**Q.** $y = \ln x^4$

**Sol.** $\frac{dy}{dx} = \frac{1}{x^4} \times 4x^3$

**Q.** $y = \tan^2 x$

**Sol.** $y = (2 \tan x).\sec^2 x$

**Q.** $y = \sin^3 x = (\sin x)^3$

**Sol.** $\frac{dy}{dx} = 3(\sin x)^2 . \cos x$

**Q.** $y = \sin^4 x = (\sin x)^4$

**Sol.** $\frac{dy}{dx} = 4(\sin x)^3 . \cos x$

**Q.** $y = \sin (x^2 + x^3)$

**Sol.** $\frac{dy}{dx} = \cos (x^2 + x^3)(2x + 3x^2)$

**Q.** $y = \ln (\sin x^3)$

**Sol.** $\frac{dy}{dx} = \frac{1}{\sin x^3} \times \cos(x^3) \times [3x^2]$

**Q.** $y = \sin (e^x)$

**Sol.** $\frac{dy}{dx} = \cos (e^x) . e^x$

★ $\frac{d}{dx}\sin(ax + b) = a \cos (ax + b)$

★ $\frac{d}{dx}\sin(2x + 3) = \cos (2x + 3) [2 + 0]$

★ $\frac{d}{dx}e^{2x+5} = e^{2x+5} (2 + 0)$

**Q.** $y = x^3$ find $\frac{dy}{dx}$ at $x = 2$

**Sol.** $y = x^3$

$\frac{dy}{dx} = 3x^2$

at $x = 2$, $\frac{dy}{dx} = 12$

**Q.** $x = t^2 - 4t + 20$ find value of $\frac{dx}{dt}, \frac{d^2x}{dt}$ at $t = 2$ sec

**Sol.** $\frac{dx}{dt} = 2t - 4 + 0 \quad \Rightarrow$ at $t = 2$ sec, $\frac{dx}{dt} = 0$

$\frac{d^2x}{dt^2} = 2 \quad \Rightarrow$ at $t = 2$ sec, $\frac{d^2x}{dt^2} = 2$

### Chain Rule Power वाले questions

**Q.** $y = \sin^2 x$

$\frac{dy}{dx} = 2 \sin x \frac{d}{dx}(\sin x) = 2 \sin x \cos x$

**Q.** $y = \sin^2(3x + 4)$

$\frac{dy}{dx} = 2 \sin (3x + 4) \cos (3x + 4) \times 3$

**Q.** $y = [\ln(3x^2)]^3$

$$\frac{dy}{dx} = 3\ln(3x^2)\times\frac{1}{3x^2}\times 6x$$

**Q.** $y = \left(x^3+4\right)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{1}{2}(x^2+4)^{-\frac{1}{2}}\times 2x$$

**Very Important**

**1.** $y = \pi x^3$

$$\frac{dy}{dx} = \pi\times 3x^2$$

**2.** $y = x^5$

$$\frac{dy}{dt} = 5x^4\frac{dx}{dt}$$

**3.** $y = \sin x$

$$\frac{dy}{dt} = \cos x\frac{dx}{dt}$$

**4.** $A = \pi r^2$

$$\frac{dA}{dt} = \pi 2r\frac{dr}{dt}$$

**5.** $y = 4/3\ \pi x^3$

$$\frac{dy}{dx} = \frac{4}{3}\pi 3x^2$$

**Q.** Radius of a circle changes wrt time with the rate of + 5 m/sec, find rate of change of area wrt time when radius is 10 m

**Sol.** $A = \pi r^2,\ \frac{dr}{dt} = 5$

$$\frac{dA}{dt} = \pi 2r\frac{dr}{dt} \Rightarrow \frac{dA}{dt} = \pi\times 2\times 10\times 5$$

$\frac{dy}{dx} \to$ Rate of change of y wrt x

$\frac{dA}{dt} \to$ Rate of change of Area with time

$\frac{dr}{dt} \to$ Rate of change of radius wrt time

**Q.** If radius of a sphere is increasing at the rate of 10 m/sec at what rate volume of sphere will change, when radius is 3 m

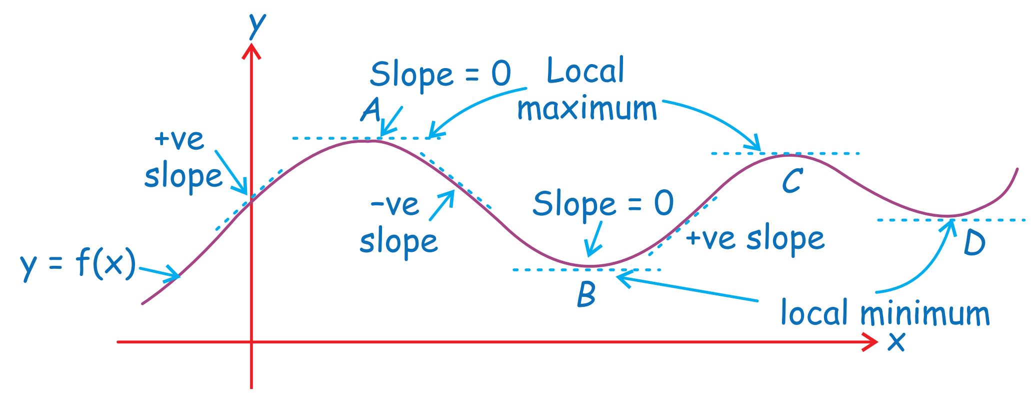
**Sol.** $V = 4/3\pi r^3 - \frac{dv}{dt} = \frac{4\pi}{3}3r^2\frac{dr}{dt}$

$$\frac{dv}{dt} = \frac{4}{3}\pi 3(3)^2\times 10 = 360\pi$$

## MAXIMA AND MINIMA

Finding maxima and minima of a function using derivatives:-

A maximum is a high point and minimum is low point of a function (see figure)




In a smoothly changing function a maximum or a minimum is always where function flattens out or where slope of tangent line is zero. We know slope = $\frac{dy}{dx}$. So a function reaches its maximum or minimum value when $\frac{dy}{dx} = 0$.

In the neighbourhood of maximum (point A), slope changes from positive to zero at point A and then becomes negative as x increases which means

$$\frac{d}{dx}(\text{slope}) < 0 \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} < 0$$

In the neigbourhood of minimum (point B), slope changes from negative to zero and then becomes positive as x increases which means

$$\frac{d}{dx}(\text{slope}) > 0 \Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2} > 0$$

## SECOND DERIVATIVE TEST

When a function's slope $\left(\frac{dy}{dx}\right) = 0$ at a point and its second derivative at that point is

(*i*) less than zero, it is a local maximum.

(*ii*) greater than zero, it is a local minimum.

★ Jab भी कभी किसी function की max या min value chahiye होगी हम उसे diff करके Zero कर देंगे.

★ For point of maxima $\frac{dy}{dx} = 0, \frac{d^2y}{dx^2} < 0$

★ For point of minima $\frac{dy}{dx} = 0, \frac{d^2y}{dx^2} > 0$

**Q.** $y = x^2 - 4x + 20$

Find Ymin = ?

**Sol.** $\frac{dy}{dx} = 2x - 4 = 0 \Rightarrow x = 2$

$\frac{d^2y}{dx^2} = 2 - 0 = 2 > 0$

$\frac{d^2y}{dx^2} > 0$ hence at $x = 2$, y is min

So, $y_{min} = 2^2 - 4 \times 2 + 20 = 16$

**Q.** $y = \sqrt{3}\sin\theta + \cos\theta$ Find y max

**Sol.** $\frac{dy}{d\theta} = \sqrt{3}\cos\theta - \sin\theta = 0$

$\sqrt{3}\cos\theta = \sin\theta$

$\theta° = 60°$

$\frac{d^2y}{d\theta^2} = -\sqrt{3}\sin\theta - \cos\theta$

at $\theta = 60°$

$\frac{d^2y}{d\theta^2} = \frac{-3}{2} - \frac{1}{2} < 0$

So y is max.

$y_{max} = \sqrt{3}\sin 60° + \cos 60°$

$\sqrt{3}(\sqrt{3}/2) + 1/2 = 2$

$y = a\sin\theta + b\cos\theta$

$y_{max} = \sqrt{a^2 + b^2}$

$y_{max} = \sqrt{3 + 1} = 2$

**Q.** What is the minimum value of y for the curve $y = -8x^2 + x^4$.

**Sol.** $y = -8x^2 + x^4$

$\frac{dy}{dx} = -16x + 4x^3 = -x(16 - 4x^2)$

The function will have a maximum or minimum value when $\frac{dy}{dx} = 0$

$\Rightarrow x(16 - 4x^2) = 0$

$\Rightarrow x = 0$ or $x = \pm 2$

Now

$\frac{d^2y}{dx^2} = -16 + 12x^2$

At $x = 0$, $\frac{d^2y}{dx^2} = -16$ (maximum)

At $x = \pm 2$, $\frac{d^2y}{dx^2} = -16 + 48 = +32$ (minimum)

So function has minimum value at $x = \pm 2$

$y_{min} = -8 \times 4 + 16 = -16$

**Q.** A ball is thrown vertically upward in the air. Its height y at any time t is given by $y = 10t - 5t^2$ where y is in meters and t is in seconds. What is the maximum height attained by the ball?

**Sol.** $y = 10t - 5t^2$

$\frac{dy}{dt} = 10 - 10t = 0$

$\Rightarrow t = 1$ sec

$\frac{d^2y}{dt^2} = -10$ (maximum)

ball attains maximum height at t = 1s

$y_{max} = 10 \times 1 - 5 \times 1^2 = 5$m.

★ Inst. velocity = $\vec{v} = \frac{d\vec{r}}{dt}$

★ If particle moving on x-axis

Inst. velocity = $\vec{v_x} = \frac{dx}{dt}\hat{i}$

सीधी बात x को time के respect मे differentiate करेंगे तो velocity आएगी।


**Q.** A particle is moving on x-axis such that its x-coordinate w.r.t time change as $x = t^2 - 6t + 10$

(a) Find velocity at t = 0, t = 3, t = 6 sec, = Inst velocity

**Sol.** $v = \frac{dx}{dt} = 2t - 6$

(now put the value of t)

$t = 0, v = -6$

$t = 3, v = 0$

$t = 6, v = +6$

★ किसी time पर v मतलब instantaneous velocity

★ किसी time पर velocity का magnitude ही उस time पर instantaneous speed है

(b) Find avrg velocity between $t = 0 \rightarrow t = 3$

**Sol.** Averge velocity = $\frac{x_f - x_i}{t_f - t_i}$

$t = 0, x_i = 0 - 0 + 10 = 10$

$t = 3, x_f = 3^2 - 6 \times 3 + 10 = 1$

Average velocity = $\frac{1 - 10}{3} = -3\hat{i}$

Definition carefully याद रखनी है, avg velocity के लिए x को differentiate मत कर देना।
Avg velocity निकालने के लिए बस ये देखो $x_i$ और $x_f$ क्या है।

**Q.** $x = t^2 - 2t + 10$. Find avg velocity and avg speed from t = 0 to t = 3 sec

**Sol.** At t = 0, x = 10

At t = 3, x = 9 - 6 + 10 = 13

Avg velocity = $\frac{13-10}{3} = 1$

Avg speed = 1 (क्यों भाई तूने यही किया ना)



अबे गलत है Avg speed हमेशा avg velocity के बराबर नही होती definition याद करो क्या थी।


## SKC

Avg speed के लिए हमे रास्ता पता होना जरूरी है। So, सबसे पहले initial और final position निकालो और velocity zero करके turning point निकालो, फिर रास्ता बना कर देखो particle कितना चला है।

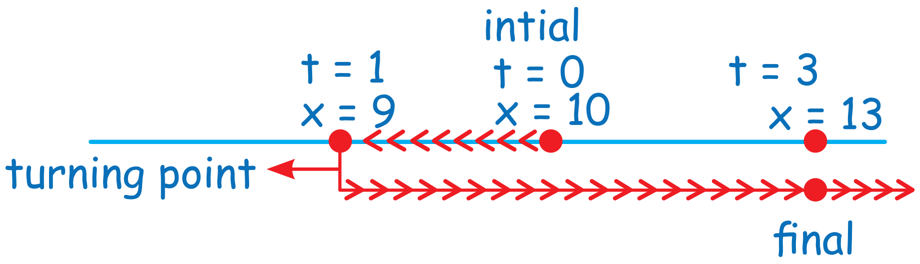
At t = 0, x = 10

At t = 3, x = 13 now find turning point

v = 2t - 2 = 0

t = 1 (देखो इस time पर particle मे अपनी direction change की है पता लगाओ t = 1 पर particle कहाँ है)

$t = 1, x = 1^2 - 2 \times 1 + 10 = 9$




Average speed $= \frac{1+4}{3} = \frac{5}{3}$

Better understanding के लिए नीचे वाले ques और solve करो

**Q.** $x = 2t^2 - 4t + 5$ from $t = 0 \rightarrow t = 2$ Find Average speed

**Sol.** $t = 0, x_i = 5$

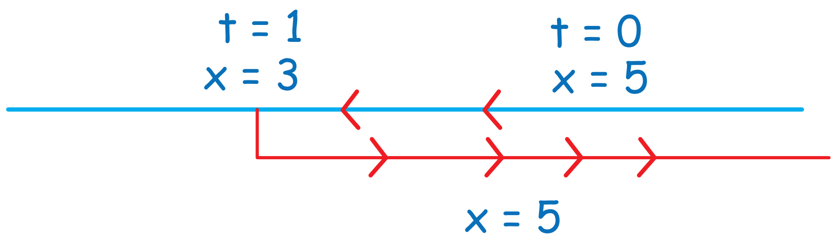
$t = 2, x_f = 5$

v = 4t - 4 = 0 (for turning point)

4t = 4

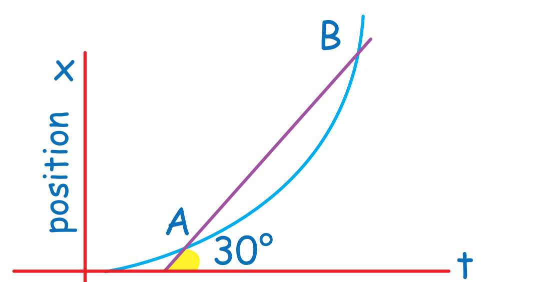
$\boxed{t = 1}$

At t = 1, x = 2 - 4 + 5 = 3

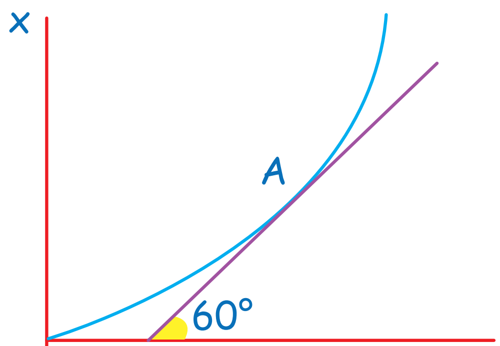



Average speed = $\frac{2+2}{4} = 1$

## GRAPHS

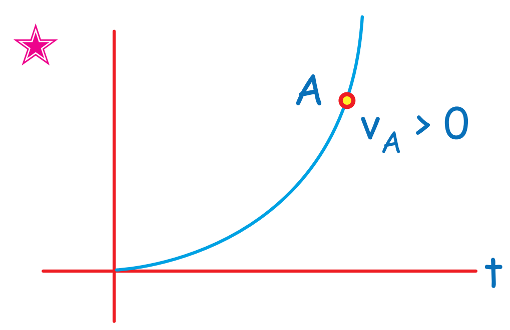


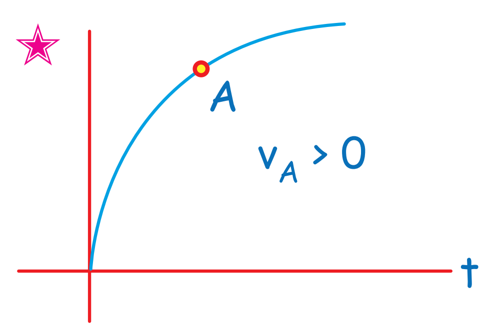

Average velocity = slope of line AB between A & B = tan 30°

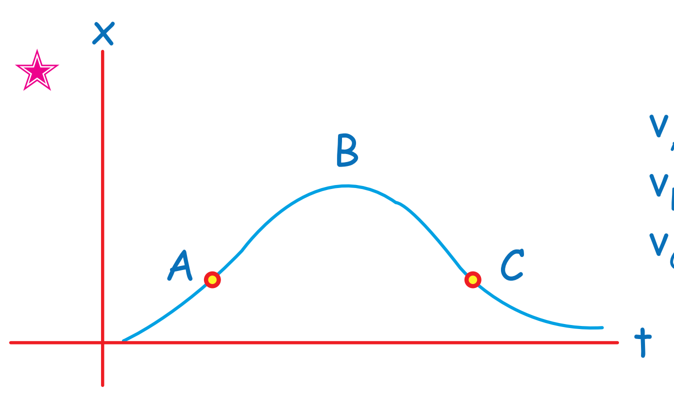



Instantaneous velocity at A = slope of tangent at A = tan 60°

x-t graph मे किसी point पर tangent का slope उस point पर velocity देगा...... इसे मत भूलना



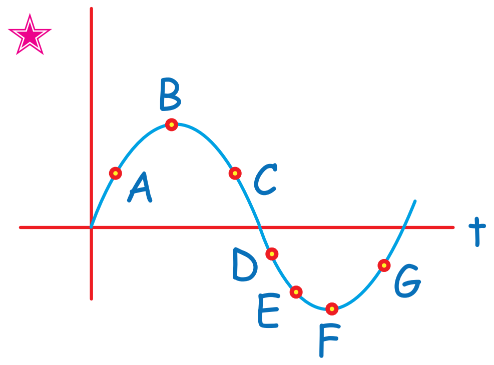




$v_A > 0$

$v_B = 0$ - turning point

$v_C < 0$




$v_A > 0$
$v_B = 0$
$v_C < 0$
$v_D < 0$
$v_E < 0$
$v_F = 0$
$v_G > 0$

## ACCELERATION

★ Inst acceln → Rate of change of velocity $\vec{a} = \frac{d\vec{v}}{dt}$

If $\vec{V}$ → constant

a = 0

Agar velocity badli to acceleration hai.

★ Average acceleration → $\frac{\text{change in velocity}}{\text{time}}$

$$= \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_F - \vec{v}_i}{t_f - t_i}$$

**Q.** A particle is moving on x-axis such that it's velocity is given at $V = t^2 + 4t + 10$.

Find acc at t = 3 sec

avg acc from t = 0 → t = 3

**Sol.** $\vec{a} = \frac{dv}{dt} = 2t + 4$

At t = 3  a = 2 × 3 + 4 = 10

$$\text{Average acc} = \frac{V_f - V_i}{\text{time}}$$

$t = 3,\ V_f = 9 + 12 + 10 = 31$

$t = 0,\ v_i = 10$

$$\text{Average acc} = \frac{31 - 10}{3 - 0} = 7$$

**Q.** $x = t^3 + 2t^2 + 5t$ Find velocity & acc at t = 2sec

**Sol.** $v = \frac{dx}{dt} = 3t^2 + 4t + 5$

$$a = \frac{dv}{at} = 6t + 4$$

t = 2, a = 12+ 4 = 16

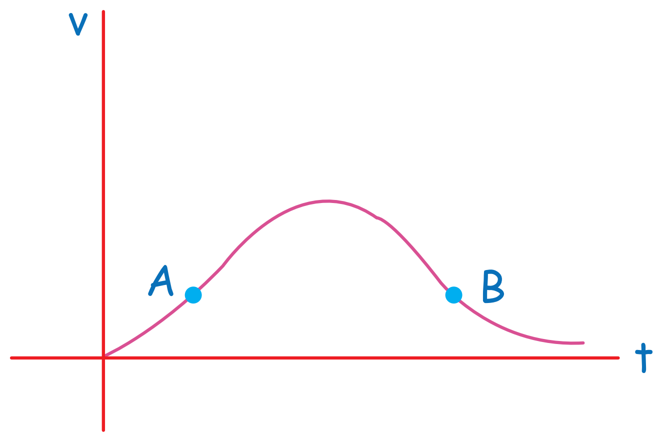
t = 2, v = 12 + 8 + 5 = 25

### # काम का डब्बा

अब तक का final result ये है की

- x-t graph मे किसी point पर tangent का slope उस point पर velocity देगा।
- v-t graph मे किसी point पर tangent का slope उस point पर acc देगा।
- $\vec{v}, \vec{a}$ ⇒ same sign ⇒ speed up
- $\vec{v}, \vec{a}$ ⇒ opposite sign ⇒ speed down.
- Average velocity = $\frac{\text{total displacement}}{\text{total time}}$
- Average speed = $\frac{\text{total distance}}{\text{total time}}$
- Average acc = $\frac{\text{change in velocity}}{\text{total time}} = \frac{\vec{v}_f - \vec{v}_i}{t}$
- (a - t) graph ka slope कददू देगा।

**Q.** Check if particle is slowing down or speed up at points A & B?

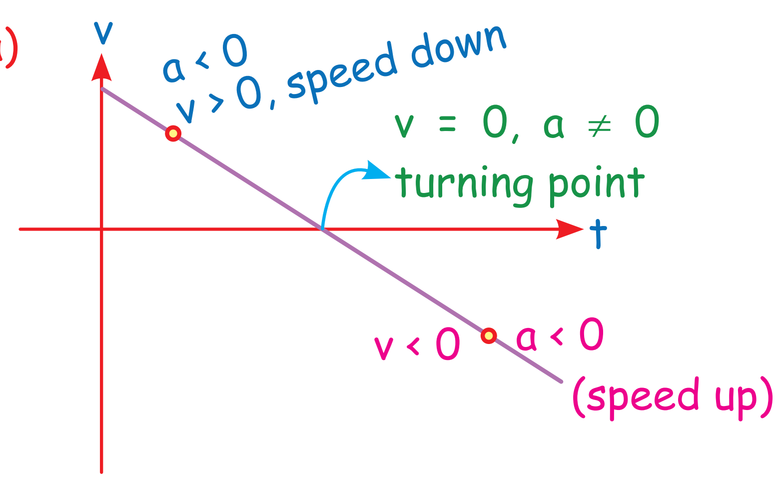



**Sol.** $v_A > 0,\ a_A > 0$ ⇒ speed up

$v_B > 0,\ a_B < 0$ ⇒ speed down

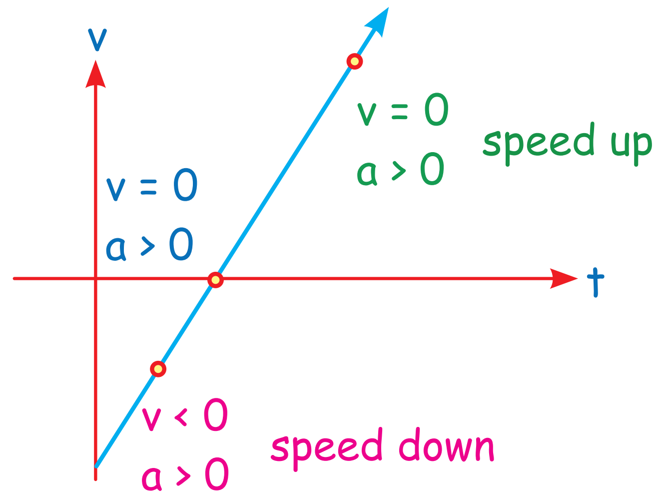
**Q.** Where is particle slowing down?

(a)

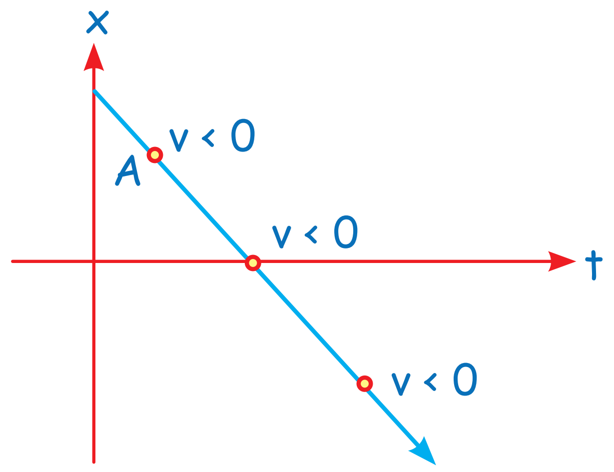


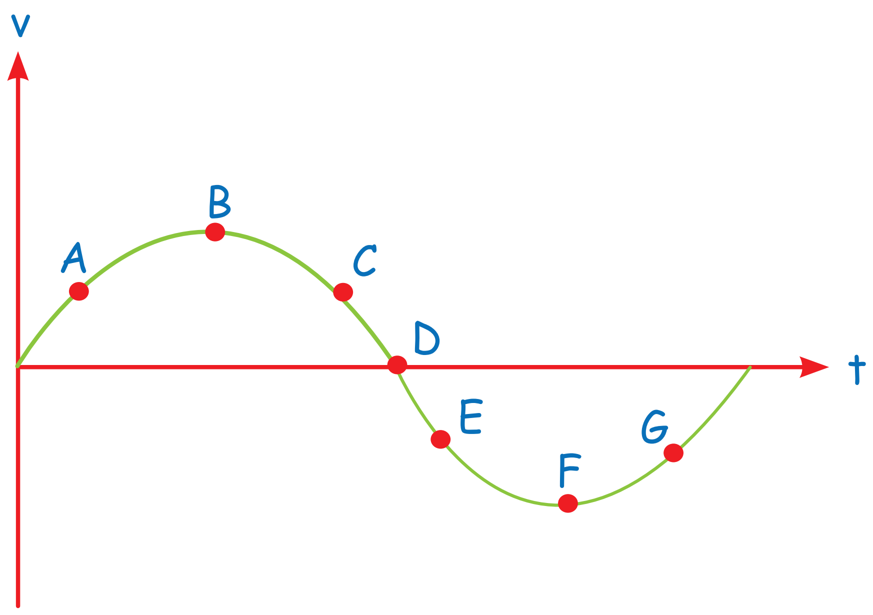

At turning point ⇒ v = 0 & v changes its sign at this point.

(b)




**Q.** Is velocity +ve or -ve in the x-t graph?




$A \Rightarrow v > 0, a > 0$, speed up

$B \Rightarrow v > 0, a = 0$

$C \Rightarrow v > 0, a < 0$, speed down

$D \Rightarrow v = 0, a < 0$

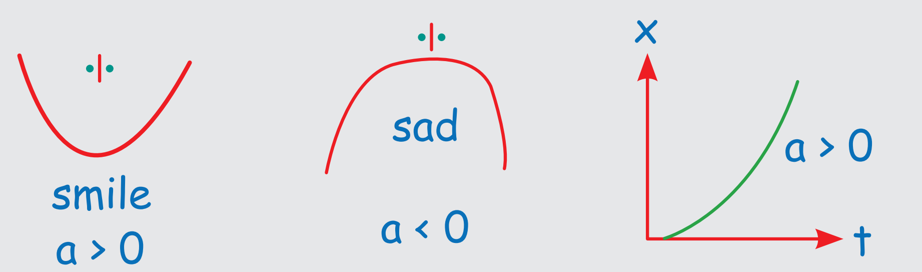
$E \Rightarrow v < 0, a < 0$, speed up

$F \Rightarrow v < 0, a = 0$

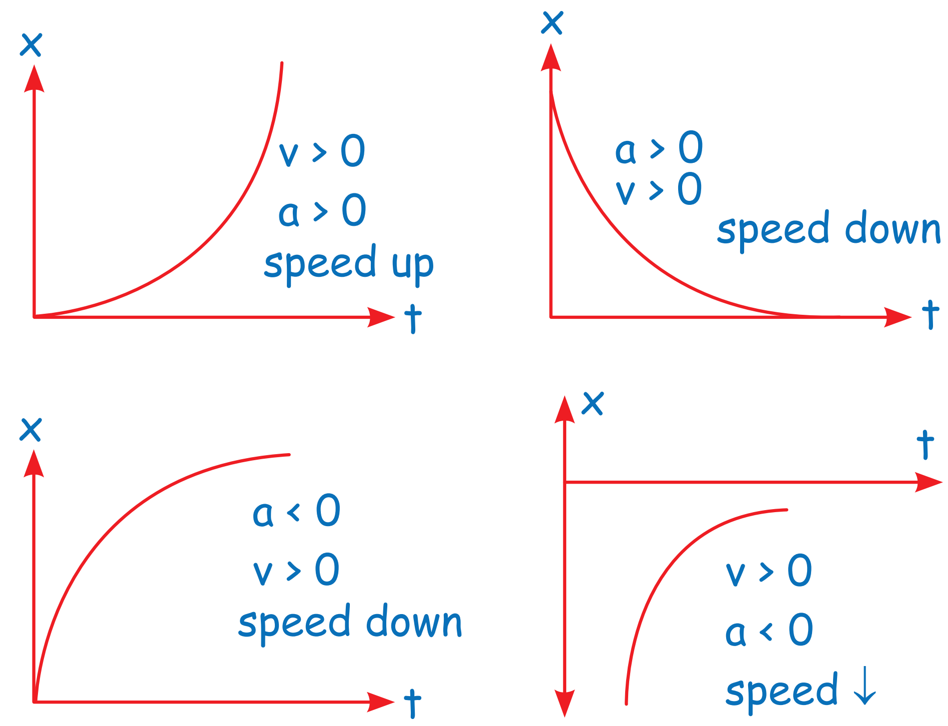
$G \Rightarrow v < 0, a > 0$, speed down

1. पहले ये देखो given क्या है पूछा क्या है ?
2. x - y axis mai kya given hai.

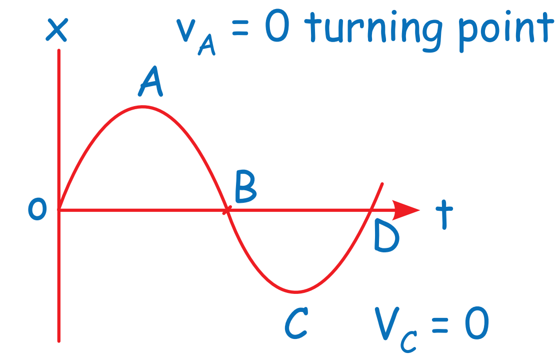
x - t graph se direct acc ka sign pata karna ho.




Only for x - t graph




**Q.**




| | v | a | Speed ↓↑ |
|---|---|---|---|
| $0 \to A$ | + | - | Speed ↓ |
| $A \to B$ | - | - | Speed up |
| $B \to C$ | - | + | Speed ↓ |
| $C \to D$ | + | + | Speed ↑ |

## INTEGRATION (MATHS)

Reverse of differentiation

$\frac{d}{dx} f(x) \Rightarrow$ differentiation of f(x) wrt x

$\frac{dy}{dx} \Rightarrow$ differentiation of y wrt x

$\int y dx \Rightarrow$ integration of y wrt x

$\frac{d}{dx} f(x) = f'(x) \Rightarrow \int f'(x) = f(x) + c$, where c is constant of integration.

$$f(x) \xrightarrow{\text{differentiation}} g(x)$$

$$g(x) \xrightarrow{\text{integration}} f(x) + c$$

## Indefinite Integration

$\int x^n\,dx$ = Integration of $x^n$ wrt x

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Physics मे सबसे ज्यादा यही formula use होगा

**Examples:**

1. $\int x^4 dx = \frac{x^{4+1}}{4+1} + C = \frac{x^5}{5} + C$

2. $\int x^7 dx = \frac{x^{7+1}}{7+1} + C = \frac{x^8}{8} + C$

3. $\int (x^2 + x^3) dx = \frac{x^3}{3} + \frac{x^4}{4} + C$

4. $\int 5x^4\, dx = 5\int x^4\, dx$

   $= \cancel{5}.\frac{x^5}{\cancel{5}} + C = x^5 + C$

5. $\int (3x^2 + 7x^6)\, dx$

   $\frac{3x^3}{3} + \frac{7x^7}{7} + C \Rightarrow = x^3 + x^7 + C$

6. $\int x^{-3} dx = \frac{x^{-3+1}}{-3+1} + C$

   $= \frac{-1}{2x^2} + C$

7. $\int \left(\frac{1}{x^9} + \frac{1}{x^{10}}\right) dx$

   $= \int (x^{-9} + x^{-10})\, dx$

   $= \frac{x^{-9+1}}{-9+1} + \frac{x^{-10+1}}{-10+1} + C = -\frac{x^{-8}}{8} - \frac{x^{-9}}{9} + C$

8. $\int dx = \int 1\, dx$

   $= \int x^0\, dx$

   $= \frac{x^{0+1}}{0+1} + C = x + C$

9. $\int 5\, dx = 5\int dx = 5x + C$

10. $\int (2t + 7t^6 + 10)\, dt = \frac{2t^2}{2} + \frac{7t^7}{7} + 10t + c$

    $= t^2 + t^7 + 10t + c$

### FORMULA

★ $f(x^n dx) = \frac{x^{n+1}}{n+1} + c$

★ $\int e^x dx = e^x + c$

★ $\int \frac{1}{x} dx = \ln x + c$

★ $\int \sin x = -\cos x + c$

★ $\int \cos x\, dx = \sin x + c$

★ $\int \frac{1}{ax+b} dx = \frac{1}{a}\ln(ax+b) + c$

★ $\int \sin(ax+b)dx = -\frac{1}{a}\cos(ax+b) + c$

★ $\int \cos(ax+b)dx = +\frac{1}{a}\sin(ax+b) + c$

★ $\int 2\cos(2x+3) = \sin(2x+3)$

★ $\int \cos(2x+3) = 1/2 \sin(2x+3) + c$

★ $\int \frac{1}{4x+5} dx = \frac{1}{4}\ln(4x+5) + c$

## Definite Integration

Upper limit of x

$$\int_{x_i = a}^{x_f = b} f(x)\,.\,dx = g(x)\Big|_{x_i = a}^{x_f = b} = g(b) - g(a)$$

Lower limit of x

Ex (1):

$$\int_0^3 x^2 dx = \frac{x^3}{3}\Big|_0^3 = \left(\frac{3^3}{3}\right) - \left(\frac{0^3}{3}\right) = 9 \text{ Ans}$$

Ex (2):

$$\int_2^3 4x^3 dx = 4\frac{x^4}{4}\Big|_2^3 = x^4\Big|_2^3 = 3^4 - 2^4 = 81 - 16 = 65 \text{ Ans}$$

Ex (3):

$$\int_0^{\pi/2} \cos x dx = \sin x\Big|_0^{\pi/2} = \left(\sin\frac{\pi}{2}\right) - \sin 0 = 1 - 0 = 1$$

Ex (4):

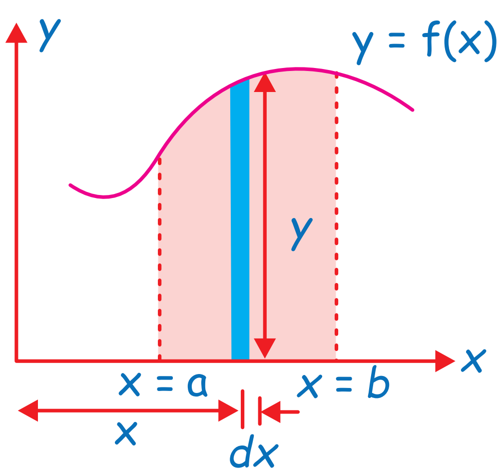
$$\int_2^3 e^x dx = e^x\Big|_2^3 = e^3 - e^2$$

**Q.** $\int_0^1 (3x^2 + 2x) = \cancel{3}\dfrac{x^3}{\cancel{3}} + 2\dfrac{x^2}{2}\Big|_0^1$

$= (x^3 + x^2)\Big|_0^1 = (1^3 + 1^2) - (0^3 + 0^2) = 2$ Ans

## APPLICATION OF INTEGRATION

★ Area under the curve :




Area of small element = ydx = f(x)dx

If we sum up all areas between x = a and x = b then

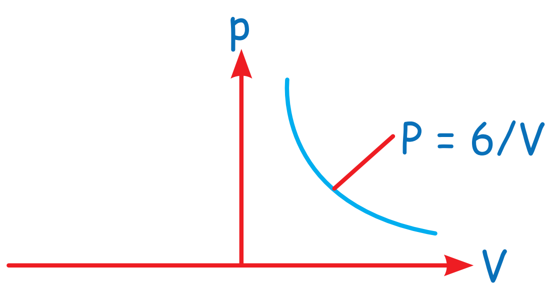
$\int_a^b f(x)dx$ = shaded area between curve and x-axis.

★ Average value

if y = f(x) then $\bar{y}$ or $y_{average} = \dfrac{\int_{x=a}^{x=b} f(x)dx}{b-a}$

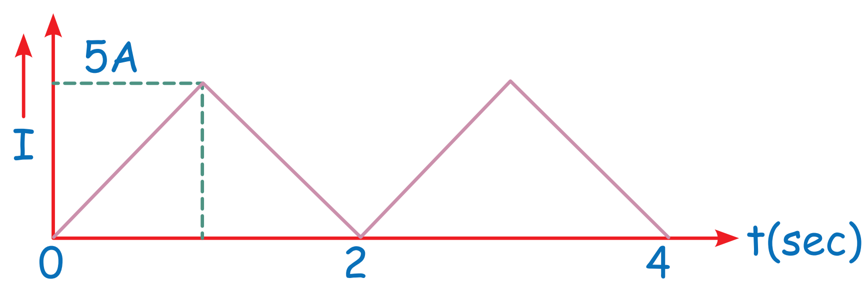
$= \dfrac{\text{Area under the curve}}{\text{interval}}$

**Q.** A gas expands its volume from V to 3V as shown in figure. Calculate the work in this process if $W = \int pdv$.



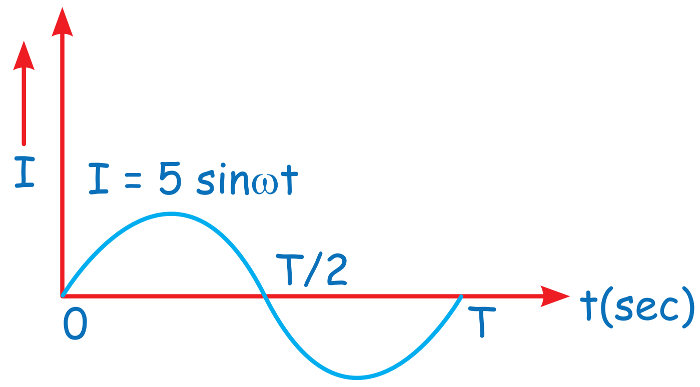

**Sol.** $W = \int_V^{3V} pdv \Rightarrow \int_V^{3V} \dfrac{6}{v}dv = \left[6\log_e v\right]_V^{3V} = 6\ln 3$

**Q.** Calculate average value of current from t = 0 to t = 4 seconds.




**Sol.** $I_{average} = \dfrac{\int_a^b Idt}{\int_a^b dt} = \dfrac{\text{total area}}{\text{total time}} = \dfrac{5 \times 2}{4} = 2.5$ amp

**Q.** Calculate average value of current from t = 0 to t = T seconds (T = 2π/ω).




**Sol.** $I_{average} = \dfrac{\int_0^T 5\sin\omega t}{\int_0^T dt} = \dfrac{5}{\omega T}\left[-\cos\omega t\right]_0^T$

$= -\dfrac{5}{\omega T}\left[\cos\omega T - \cos 0\right] = 0 \qquad \left[\omega = \dfrac{2\pi}{T}\right]$

★ For any moving object, the average speed can never be zero or negative, as total distance covered is always +ve.

★ If a particle travels distances $s_1$, $s_2$, $s_3$ , ..........., etc., at different speeds $v_1$, $v_2$, $v_3$, ........ etc., respectively, then

$v_{av} = \dfrac{\Delta s}{\Delta t} = \dfrac{\Sigma s_i}{\Sigma(s_i / v_i)}$

If $s_1 = s_2 = ..... = s_n = s$,

Then $\dfrac{1}{v_{av.}} = \dfrac{1}{n}\left[\dfrac{1}{v_1} + \dfrac{1}{v_2} + ......\right] = \dfrac{1}{n}\Sigma\dfrac{1}{v_i}$

**Special case:** If a particle moves a distance at speed $v_1$ and comes back to initial position with speed $v_2$, then

$v_{av} = \dfrac{2v_1v_2}{v_1 + v_2}$

★ If a particle travels at speeds $v_1$, $v_2$, ..., etc., for time intervals $t_1$, $t_2$, ..., then

$v_{av} = \dfrac{\Delta s}{\Delta t} = \dfrac{v_1t_1 + v_2t_2 + ...... + v_nt_n}{t_1 + t_2 + .....t_n} = \dfrac{\Sigma v_it_i}{\Sigma t_i}$

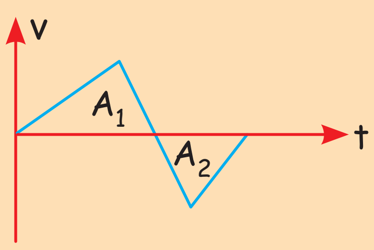
**Special case:** If a particle moves for two equal intervals of time at different speeds, then

$v_{av} = \dfrac{v_1 + v_2}{2}$.

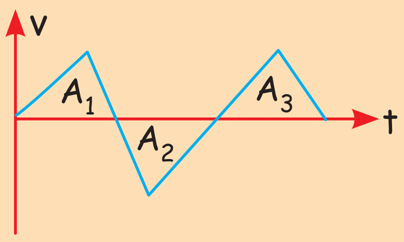
# काम का डब्बा

$$x \underset{\text{integration}}{\overset{\text{differentiation}}{\rightleftarrows}} v \underset{\text{integration}}{\overset{\text{differentiation}}{\rightleftarrows}} a$$

$\int y \, . \, dx$ = Area under curve
curve और x-axis के बीच का Area

- a-t graph का area change in velocity देता है।
- v-t graph का area change in position देता है बोले तो
  v-t graph का area displacement देता है
- 


  Displacement = $A_1 - A_2$
  Distance = $A_1 + A_2$


  Displacement = $A_1 - A_2 + A_3$
  Distance = $A_1 + A_2 + A_3$
- (x - t) graph ka slope $\Rightarrow$ v
- (v - t) graph ka slope $\Rightarrow$ $\vec{a}$
- $v = \dfrac{dx}{dt}$, $a = \dfrac{dv}{dt}$
- Displacement = $\int vdt$ = Area
- change in velocity = $\int adt$
- St line $\rightarrow$ slope $\rightarrow$ const
- (v - t) is st. line $\rightarrow$ a = constant
- If (x-t) is straight line $\rightarrow$ v = constant and a = 0.
- If a = 0 $\rightarrow$ v = constant $\rightarrow$ (x-t) straight line
- If a = const$\rightarrow$(v-t) straight line$\rightarrow$(x-t) parabola
- If a body moves with uniform acceleration and velocity changes from u to v in a time interval, then average velocity = $\dfrac{v+u}{2}$.
- If a body moving with uniform acceleration has velocities u and v at two points in its path, then the velocity at the midpoint of given two points = $\sqrt{\dfrac{u^2+v^2}{2}}$.

**Q.** At t = 0, particle is at x = 10 particle is moving such that its velocity vs time relation is given as $V = 3t^2 + 2t$. Find location of particle, t = 1 and x = f(t)

**Sol.** $x = \int vdt = \int (3t^2 + 2t)dt = t^3 + t^2 + c$

at t = 0, x = 10 (put the value)

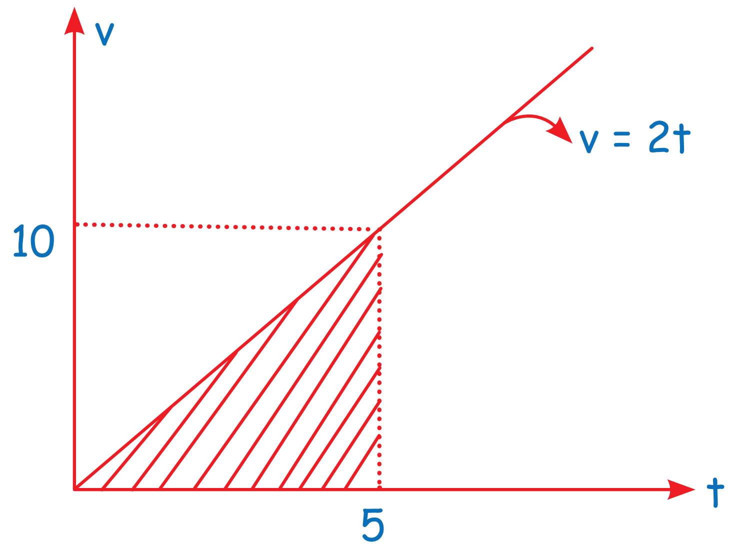
$x = t^3 + t^2 + c$

$10 = 0 + 0 + c \Rightarrow c = 10$

$x = t^3 + t^2 + 10$ (now you can find x for any t)

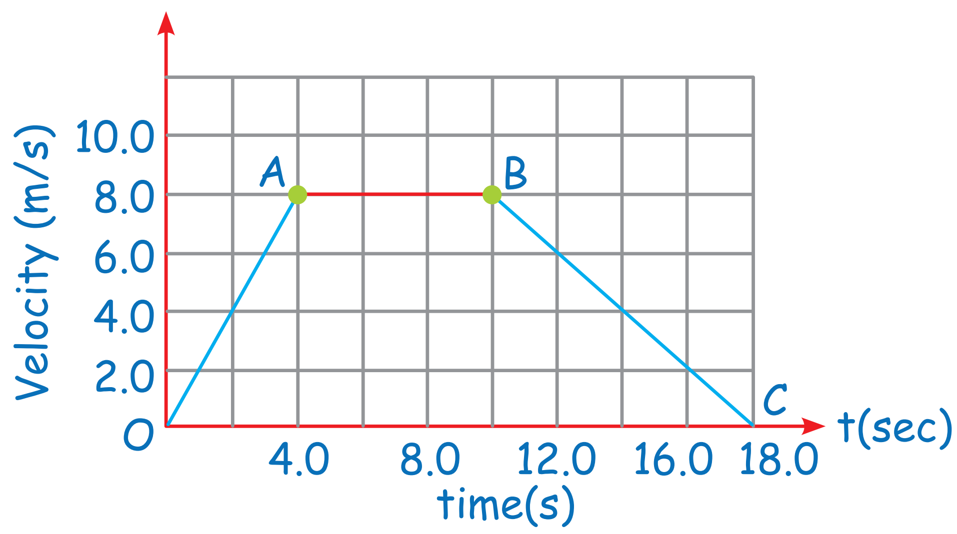
**Q.** A particle is moving along x-axis. Its velocity vs time relation is given as v = 2t. Find displacement from t = 0, t = 5

**Sol.**




Area = 1/2 × 5 × 10 = 25 = displacement

**Q.** What is the acceleration for each graph segment in figure? Describe the motion of the object over the total time interval. Also calculate displacement.




**Sol.** Segment OA; $a = \dfrac{8-0}{4-0} = 2 \text{ m/s}^2$

Segment AB; graph horizontal i.e., slope zero i.e., a = 0

Segment BC; $a = \dfrac{0-8}{18-10} = -1$ m/s2

The graph is trapezium. Its area between t = 0 to t = 18s is displacement.

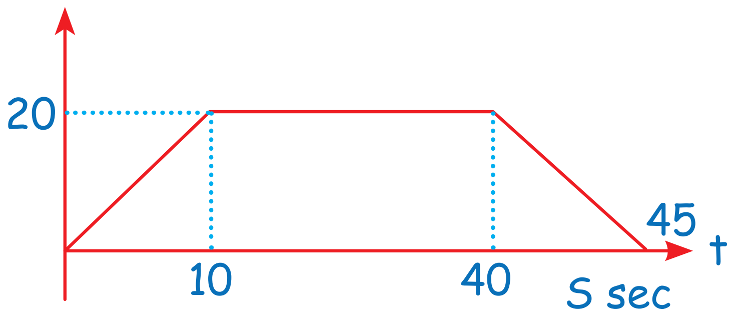
Area of v-t graph = displacement = $\dfrac{1}{2}(18 + 6) \times 8$

= 96 m

Particle accelerates uniformly for first 4 sec., then moves with uniform velocity for next 6 sec. and then retards uniformly to come to rest in next 8 sec.

**Q.** A particle starts from rest, accelerates at 2 $m/s^2$ for 10 s & then goes at constant velocity for 30s and then deaccelerate at 4 $m/s^2$ till it stops. What is the distance travelled by it.

**Sol.**




$$\text{Area} = \frac{1}{2}\times(30+45)\times 20 = 750$$

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

a = cons

u = intial velocity

v = final velocity

a = acc

s = displacement

भाई ये eqn तभी लगाना जब a constant हो

**Q.** A particle start motion having initial velocity 20 m/s and it move with const acc of 10 $m/s^2$

1.Find velocity at t = 4 sec

2.Find displacement of particle from t = 0 to t = 4 sec

3.Find displacement in $4^{th}$ sec

**Sol.** **1.** t = 4, v = u + at

= 20 + 10 × 4

v = 60

**2.** s = ut + 1/2 $at^2$

$= 20\times 4 + \frac{1}{2}\times 10\times 4^2 = 160$

**Q.** A truck travelling with uniform acceleration crosses two points A & B with velocities 60 m/s & 40 m/s respectively. The speed of the body at the midpoint of A & B is nearest to.

**Sol.** A • 60 →    C mid $V_c$ = ?    B • 40 →

← x →

$A \to B$   $40^2 = 60^2 + 2ax \Rightarrow 2ax = -2000$   (i)

$A \to C$   $V_C^2 = 60^2 + 2a\,x/2$   (ii)

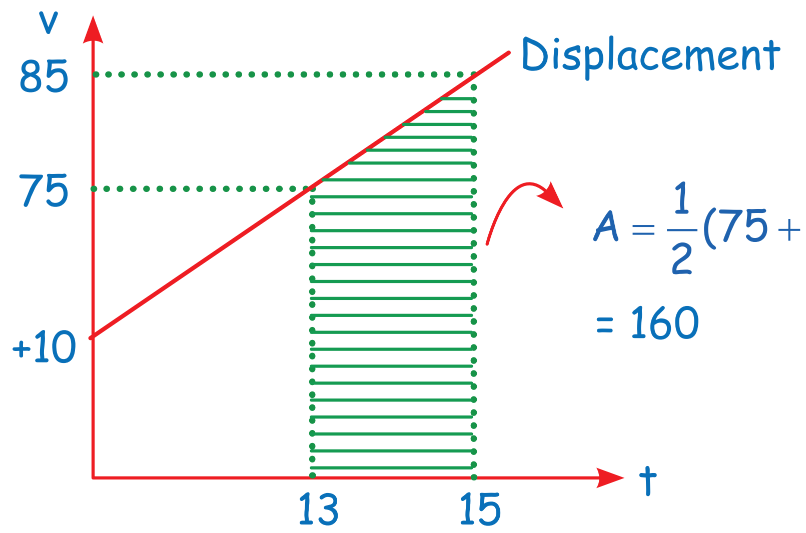
from (i) & (ii)

$V_C^2 = 3600 - 1000 = 2600$

$V_c = 10\sqrt{26}$

**Q.** A particle having initial velocity 10 m/s move with a constant acceleration 5 $ms^{-2}$, for a time 15 second along a straight line, what is the displacement of the particle in last 2 second?

**Sol.**




$A = \frac{1}{2}(75+85)2$

= 160

**Q.** A bullet is moving with a velocity of 200 cm/s. penetrates a wooden block & comes to rest after travelling 4cm insides it. What velocity is needed for travelling distance of 9cm in same block?

**Sol.** $0^2 = 2^2 + 2a\times\frac{4}{100}$

$-4 = \frac{8}{100}a \Rightarrow a = -50$

$0^2 = u^2 + 2as$

$0 = u^2 + 2x(-50)\frac{9}{100}$

$u^2 = 9$

u = 3 m/s

**Q.** A bullet going with speed 350 m/s enters a concrete wall and penetrates a distance of 5.0 cm before coming to rest. Find the deaceleration

**Sol.** $U_f = 0$, a = ?

$0^2 = (350)^2 + 2\times a\times\frac{5}{100}$

$a = -(350)^2 \times 10$

$a = -1225000\ m/s^2$

**Q.** A particle covered 100 m distance in first 10 s of its joumey and in next 10 s it travel 200 m. Find distance travelled in next 10s? (acc is const)

**Sol.** t = 0 → t = 10 $\quad 100 = u\times 10 + \frac{1}{2}a\times 10^2$ ...(i)

t = 0 → t = 20 $\quad 300 = u\times 20 + \frac{1}{2}a(20)^2$ ...(ii)

**Sol.** e (i) and (ii) and get a = 1, u = 5

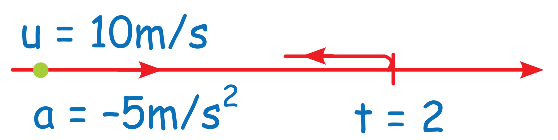
Let it travel $x_3$ in last 10s so in 30s a travel $300 + x_3$

$$300 + x_3 = 5\times 30 + \frac{1}{2}\times 1\times (30)^2$$

**Sol.** e and get $x_3 = 300$

**Q.** A particle moving with initial velocity of 10 m/s towards East has an acceleration of 5 $m/s^2$ towards west. Find the displacement and distance travelled by the particle in first 4 seconds?

**Sol.**




$v = u + at \Rightarrow 0 = 10 - 5t \Rightarrow t = 2s$

The direction of velocity changes after two seconds.

$S = 10\times 4 + \frac{1}{2}(-5)\times 4^2 = 0 =$ displacement

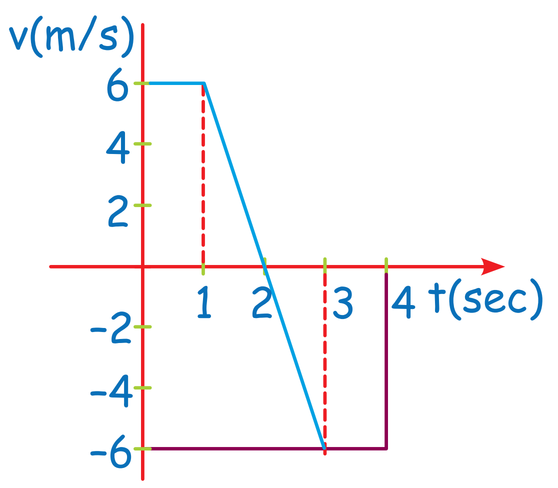
Distance travelled is not equal to displacement because during course of journey, velocity changes direction.

$D = S(\text{at } 2s) + |S(\text{at } 4s) - S(\text{at } 2s)|$

$= \left(10\times 2 - \frac{1}{2}\times 5\times 2^2\right) + \left|0 - (10\times 2) - \frac{1}{2}\times 5\times 2^2\right|$

= 10 + 10 = 20 m

**Q.** A particle moves along a straight line along $x$-axis. At time $t$ = 0, its position is at $x$ = 0. The velocity $v$ m/s of the object changes as a function of time $t$ seconds as shown in the figure.




(i) What is $x$ at $t$ = 1 sec?

(ii) What is the acceleration at $t$ = 2 sec?

(iii) What is $x$ at $t$ = 4 sec?

(iv) What is the average speed between $t$ = 0 and $t$ = 3 sec?

**Sol.** (i) $x$ is displacement at $t$ = 1 sec.

Area under the $v - t$ curve gives displacement

From $t$ = 0 to $t$ = 1 sec.

$x = 6 \times 1 = 6$ m

(ii) Slope of the $v$-$t$ curve gives acceleration from the given $v$-$t$ curve

Slope at $t$ = 2 sec. gives acceleration at $t$ = 2 sec.

$\tan\theta = a = -\frac{6}{1} = -6\ m/s^2$

(iii) $x$ (at $t$ = 4 sec):

Area under the curve from $t$ = 0 to $t$ = 4 sec

$= 6\times 1 + \frac{1}{2}\times 6\times 1 - \frac{1}{2}\times 6\times 1 - 6\times 1 = 0$

$\Rightarrow x\,(t = 4) = 0$ m

(iv) Average speed from $t$ = 0 to $t$ = 3 sec.

Displacement from $t$ = 0 to $t$ = 2 sec. = Area under the curve $= 6 + \frac{1}{2}\times 6\times 1 = 9$ m

Displacement form $t$ = 2 to $t$ = 3 sec.

$= -\frac{1}{2}\times 6\times 1 = -3$ m

Distance from $t$ = 0 to $t$ = 3 sec = |9| + |-3| = 12 m

Average speed $= \frac{\text{Distance}}{\text{Time}} = \frac{12}{3} = 4$ m/s

## Question Practice on Variable Acceleration and Utptang Integration

**Q.** The acceleration a of a particle moving in one dimension is given by $a = 6 - 2t$. If the particle is initially at $x = 0$ and its velocity is 2 m/s, find its position and velocity at time $t$.

**Sol.** $\frac{dv}{dt} = 6 - 2t$

$$\int_2^v dv = \int_0^t (6 - 2t)dt$$

$\Rightarrow v - 2 = (6t - t^2)|_0^t = 6t - t^2 \Rightarrow v(t) = 2 + 6t - t^2$

To find position, we integrate velocity.

$v = \frac{dx}{dt} = 2 + 6t - t^2 \Rightarrow dx = (2 + 6t - t^2)dt$

$$\int_0^x dx = \int_0^t (2 + 6t - t^2)dt = 2t + 3t^2 - \frac{t^3}{3}$$

or $x(t) = 2t + 3t^2 - \frac{t^3}{3}$

**Q.** The retardation of a car when its engine is shut off depends on its velocity as $a = -\alpha v$ where $\alpha$ is positive constant. Find the total distance travelled by the car if its initial velocity is 20 m/s and $\alpha = 0.5/s$.

**Sol.** $\frac{dv}{dt} = -\alpha v$

$$\frac{dv}{dx}\left(\frac{dx}{dt}\right) = -\alpha v \Rightarrow \frac{vdv}{dx} = -\alpha v$$

or $dv = -\alpha dx$

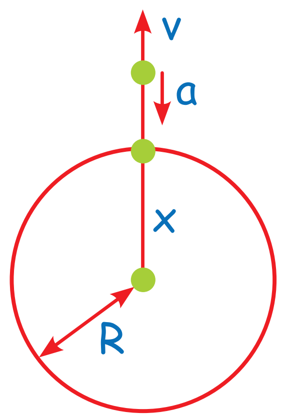
$$\int_{20}^{0} dv = -\alpha\int_{0}^{d} dx \Rightarrow v\Big|_{20}^{0} = -\alpha x\Big|_{0}^{d}$$

$-20 = -\alpha d$

$$d = \frac{20}{\alpha} = \frac{20}{0.5} = 40 \text{ m}$$

**Q.** With what velocity in vertical upward direction should a body be projected from the surface of earth so that it reaches a height equal to radius of earth? The acceleration of body is given by $a = -\frac{GM}{x^2}$ where x is the distance from centre of earth and $M$ is the mass of earth.

**Sol.** Note that acceleration due to gravity is nearly constant near the surface of earth. But if the height become too large its dependence on distance can not be ignored.




$$a = \frac{dv}{dt} = -\frac{GM}{x^2}$$

$$\text{or } \frac{dv}{dx}\cdot\frac{dx}{dt} = -\frac{GM}{x^2} \Rightarrow vdv - \frac{GM}{x^2}dx$$

At the highest point, velocity is zero. Also note $x_i = R$ and $x_f = 2R$.

$$\int_{u}^{0} vdv = -GM\int_{R}^{2R}\frac{dx}{x^2}$$

$$\frac{v^2}{2}\Big|_{u}^{0} = -GM\int_{R}^{2R} x^{-2}dx = \frac{GM}{x}\Big|_{R}^{2R}$$

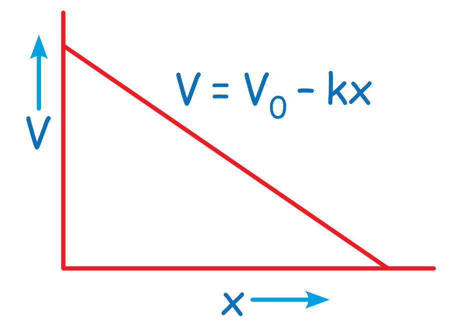
$$\Rightarrow -\frac{u^2}{2} = GM\left[\frac{1}{2R} - \frac{1}{R}\right]$$

$$\Rightarrow u^2 = \frac{GM}{R} \Rightarrow u = \sqrt{\frac{GM}{R}} = \sqrt{\frac{GM}{R^2}R}$$

$$\because g = \left(\frac{GM}{R^2}\right)$$

$$\therefore u = \sqrt{gR} = 8 \text{ km/s } [\because R = 6400 \text{ km}, g = 10 \text{ m/s}^2]$$

**Q.** A particle is moving along $x$-axis with velocity $V$ which varies according to the law $V = V_0 - Kx$ here $V_0$ and $K$ are constants. Choose the correct acceleration vs time plot for the time interval when particle moves from $x = 0$ to $x = \frac{V_0}{K}$.




**Sol.** $V = V_0 - Kx$

$$\frac{dx}{dt} = (V_0 - Kx) \Rightarrow \int_0^x \frac{dx}{(V_0 - Kx)} = \int_0^t dt$$

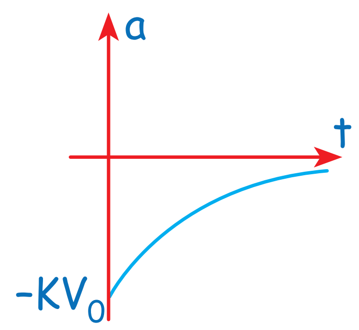
$$x = \frac{V_0}{K}(1 - e^{-Kt})$$

$$\therefore \frac{dx}{dt} = +V_0 e^{-Kt} \Rightarrow a = \frac{d^2x}{dt^2} = -KV_0 e^{-Kt}$$

At $t = 0$, $a = -KV_0$

At $t = \infty$, $a = 0$

Therefore, graph is as shown




**Q.** A block of mass m is fired horizontally along a level surface that is lubricated with oil. The oil provides a viscous resistance that varies as the 3/2 power of the speed. If the initial speed of the block is $v_0$ at $x = 0$, find the maximum distance reached by the block. Assume no resistance to motion other than that provided by the oil.

**Sol.** $F = -v^{3/2}$

$$a = -\frac{1}{m}v^{3/2} \qquad [\because F = ma]$$

$$v\frac{dv}{dx} = -\frac{1}{m}v^{3/2}$$

$$\int_{v_0}^{0} v^{-\frac{1}{2}}dv = -\frac{1}{m}\int_{0}^{d} dx$$

$2mv_0^{1/2} = d$

**Q.** Acceleration of particle moving rectilinearly is $a = 4 - 2x$ (where $x$ is position in metre and $a$ in $m/s^2$). It is at rest at $x = 0$. At what position $x$ (in metre) will the particle again come to instantaneous rest?

**Sol.** $\frac{vdv}{dx} = 4 - 2x$

$$\int_0^v vdv = \int_0^x (4-2x)dx \Rightarrow \frac{v^2}{2} = 4x - x^2$$

when $v = 0$, $4x - x^2 = 0$

$x = 0, 4$

$\therefore$ At $x = 4$ m, the particle will again come to rest.

## Question for Practice

**Q.** A particle start motion at $t = 0$ from $x = +10$, such that its 'v' vs t relation is given as $\vec{v} = 4t^3 + 3t^2 + 2t$. Find location of particle at $t = 1$ sec.

**Sol.** $x = \int vdt$

$x = t^4 + t^3 + t^2 + c$

$t = 0, x = 10$ (put $t = 0, x = 10$)

$10 = 0 + 0 + 0 + c$ $\boxed{c = 10}$

$x = t^4 + t^3 + t^2 + 10$

$t = 1, x = 13$

**Q.** A particle start motion from $x = 5$, at $t = 0$ such that $v = t^2 + t$. Find location of particle at $t = 6$ sec.

**Sol.** $dx = v\,dt$

$$\int_{x=5}^{x_f} dx = \int_{t=0}^{t=6} (t^2 + t)dt$$

$$x\Big|_5^{x_f} = (t^3/3 + t^2/2)\Big|_0^6$$

$$x_f - 5 = \left(\frac{6^3}{3} + \frac{6^2}{2}\right) - \left(\frac{0^3}{3} + \frac{0^2}{2}\right)$$

$x_f - 5 = 72 + 18 - 0$

$x_f = 90 + 5$

$x_f = 95$ Ans

**Q.** A particle start motion from rest from origin at $t = 0$ such that $a = 6t$ find v, x (location) at, $t = 2$ sec

**Sol.** $a = 6t$

$v = \int adt = \int 6tdt$

$v = 6t^2/2 + c$

$v = 3t^2 + c$

$t = 0, v = 0$

$0 = 0 + c$

$c = 0$

$v = 3t^2$ $\boxed{t = 2, v = 12}$

$x = \int 3t^2\,dt = 3\,t^3/3 + c'$

$x = t^3 + c'$

$t = 0, x = 0,\quad 0 = 0 + c \quad \rightarrow c' = 0$

$x = t^3$

$t = 2, x = 8, v = 12$

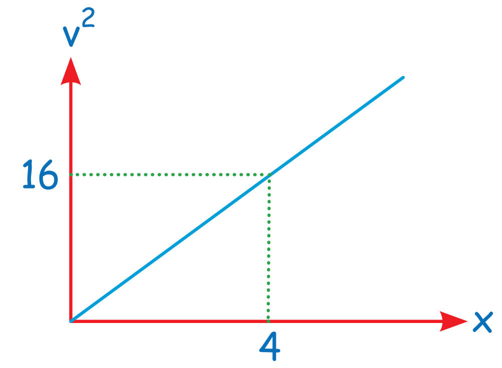
**Q.** If $v = x^2 + 3x$ Find acc at $x = 2$

**Sol.** $a = v\frac{dv}{dx}$ $\quad \frac{dv}{dx} = 2x + 3$

$a = (x^2 + 3x)(2x + 3)$

put $x = 2, a = 70$

**Q.** Find acc at $x = 4$

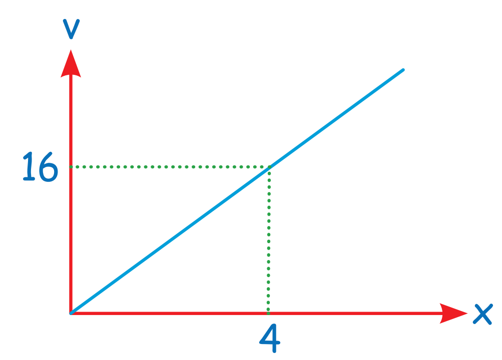



**Sol.** $\text{slope} = \frac{dy}{dx} = \frac{d(v^2)}{dx}$

$16/4 = 2v\frac{dv}{dx}$

$4 = 2a \rightarrow \boxed{a = 2}$

**Q.** Find acc at $x = 4$




**Sol.** $\text{slope} = \frac{dy}{dx} = \frac{dv}{dx} = 16/4$

$a = v\frac{dv}{dx}$

$$a = 16 \times \frac{16}{4} = \boxed{64}$$

**Q.** $x = \sqrt{v+1}$ find acc at x = 5

**Sol.** $x^2 = v + 1$

$v = x^2 - 1$

$$a = v\frac{dv}{dx} = (x^2 - 1)(2x)$$

$x = 5 \Rightarrow a = 240$

**Q.** Acceleration of a particle moving on x-axis having initial speed $v_0$ with distance from origin is given by $a = \sqrt{x}$ Distance covered by particle where its speed become thrice that of initial speed.

**Sol.** $a = \sqrt{x}$ $\qquad$ $a > 0$, speed ↑, $v > 0$

$$v\frac{dv}{dx} = \sqrt{x}$$

$$\int v\,dv = \sqrt{x}$$

$$\int_{V_0}^{3V_0} v\,dv = \int_0^{x_f} x^{1/2}dx$$

$$\left.\frac{v^2}{2}\right|_{V_0}^{3V_0} = \left.\frac{x^{1/2+1}}{1/2+1}\right|_0^{x_f}$$

$$\frac{9V_0^2}{2} - \frac{V_0^2}{2} = \left.\frac{2}{3}x^{3/2}\right|_0^{xf}$$

$$\frac{8V_0^2}{2} = 2/3\,x_f^{3/2}$$

**Q.** $x = a \sin \omega t$

$y = a(1 - \cos \omega t) = a - a\cos \omega t$

Find $\vec{v}$, speed of particle

**Sol.** $v_x = a\omega \cos \omega t$

$v_y = 0 - a\omega[-\sin \omega t]$

$v_y = a\omega \sin \omega t$

$$\vec{v} = (a\omega \cos \omega t)\,\hat{i} + (a\omega \sin \omega t)\,\hat{j}$$

speed = ? magnitude

$$|\vec{v}| = \sqrt{(a\omega\cos\omega t)^2 + (a\omega\sin\omega t)^2}$$

$$= a\omega\sqrt{\cos^2\omega t + \sin^2\omega t}$$

speed = $a\omega$

# MOTION UNDER GRAVITY

It's very important article जिसमे हम particle का motion under the effect of gravity पढ़ेंगे.....

**Assumption**

1. Air resistance force is neglected until mention.
2. Variation of gravity g is neglected until mention.

भईया kinematics के ques मे समझ नही आता कैसे करना है sign convention मे भी दिक्कत होती है sol. तो समझ आ जाते है पर सवाल खुद से नही बनते इसका कोई इलाज.....

हाँ भाई सबसे पहले तो जोर से घुटना लगाओ..... फिर जो given है वो लिख लो जो पूछा है वो भी लिखलो और देखो कौन-सी eqn of motion connect हो रही है।

+/- जहाँ मन करे ऊपर या नीचे मान सकते हो ans. same आएगा बस इस बात का ध्यान रखना u, s, a को with sign लिखना है।

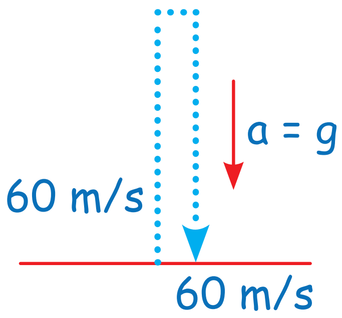
**Q.** A particle is projected from ground vertically upward with velocity 60 m/s. (1) Find velocity and location of particle at t = 10 (2). When will particle come to rest (3). Find $h_{max}$, time of flight.

**Sol.** Sign convention

Let upward direction (+ve)

u = 60 upar = +60

a = 10 neeche = -10




- t = 10, v = u + at = 60 + (-10)(10)

  v = − 40
  नीचे

- location = $y = 60 \times 10 + 1/2(-10)(10)^2 = +100$
  उपर

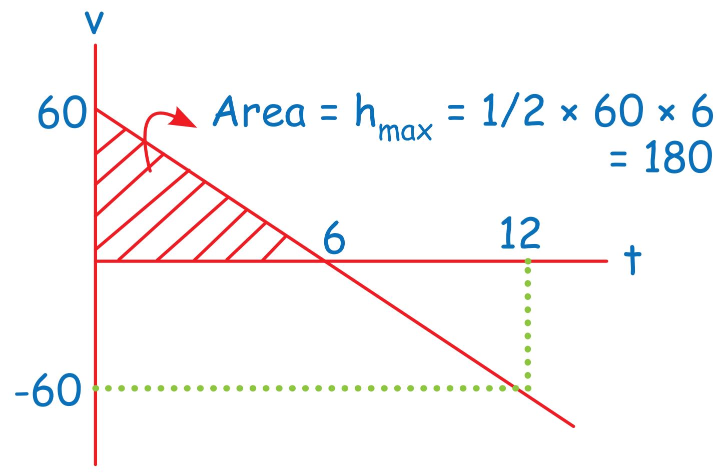
- When particle will come to rest

  $v = u + at \Rightarrow 0 = 60 - 10t \Rightarrow t = 6$

- $0^2 = 60^2 + 2 \times (-10)\,h_{max}$

  $h_{max} = 180$

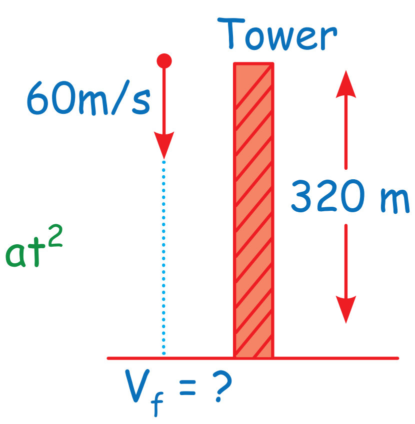
✦ Time of fight = 6 + 6 = 12 sec




भईया जब particle ऊपर जाता है तब उसका acceleration नीचे होता है और जब नीचे आता है तब भी उसका acceleration नीचे होता है।

हाँ भाई अभी के लिए बस तु ये याद रख अगर particle हवा मे है तो उसका acceleration g होगा चाहे ऊपर जाए या नीचे..... particle उधर जाता है जिस तरफ उसकी velocity होती है और उसका acceleration उधर होता है जिस तरफ उस पर net force होता है।

**Q.** A particle is thrown in vertically downward with velocity 60 m/s from top of a tower of height 320 m. Find when particle will hit the ground & with what velocity & $V_f$ = ?




**Sol.** Let (Downward +ve)

u = +60 a = +10

✦ displacement $s = ut + \frac{1}{2}at^2$

$+320 = 60t + \frac{1}{2} \times 10 \times t^2$

$t^2 + 12t - 64 = 0$

$(t + 16)(t - 4) = 0 \Rightarrow t = 4s$

✦ $V_f$ = ?

$V^2 = u^2 + 2as$

$V^2 = 60^2 + 2 \times 10 \times 320$

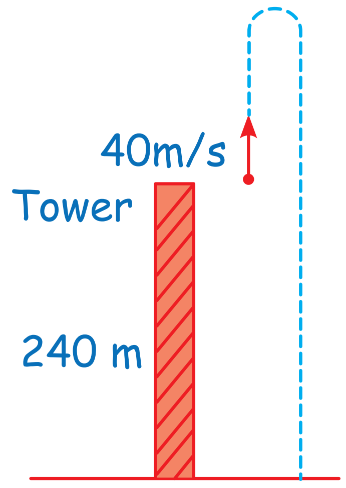
$V^2 = 3600 + 6400$

$V^2 = 10000$

$V_f$ = velocity V = 100 (नीचे)

**Q.** A particle is thrown in vertically upward with velocity 40 m/s from top of a tower of height 240 m. Find when particle will hit the ground & with what velocity & $V_f$ = ?

**Sol.** (1) let (up = +ve)




u = +40, a = -10, s = -240

$s = ut + \frac{1}{2}at^2$

$-240 = 40t + 1/2\,(-10)\,t^2$

$5t^2 - 40t - 240 = 0$

$(t - 12)(t + 4) = 0$

t = 12

(2) $V^2 = u^2 + 2as = (40)^2 + 2 \times (-10)(-240)$

V = 80

**Q.** A balloon is rising upward with constant velocity 40 m/s. When it reaches at a height of 240 m from ground a particle is drop from it. Find when will particle hit the ground with what velocity?

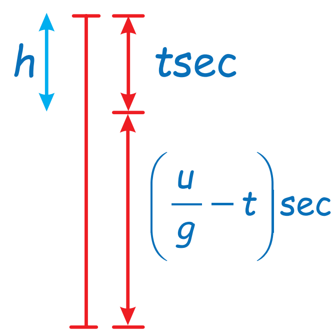
**Ans.** t = 12, v = 80 m/s

## SKC

अगर हम किसी चलती हुई गाडी लिफ्ट और उड़ते हुए गुब्बारे से किसी particle को drop करें तो drop के just बाद particle की velocity वही होती जो उस गाडी, लिफ्ट और गुब्बारे की हो. So, अबे ये सवाल तो वही टावर वाला सवाल है।

**Q.** If a ball is thrown vertically upwards with speed u, the distance covered during the last t seconds of its ascent is

**Sol.** If ball is thrown with velocity u, then time of ascent $= \frac{u}{g}$




Velocity after $\left(\frac{u}{g} - t\right)$sec, $v = u - g\left(\frac{u}{g} - t\right) = gt.$

So, distance in last 't' sec, $0^2 = (gt)^2 - 2(g)h.$

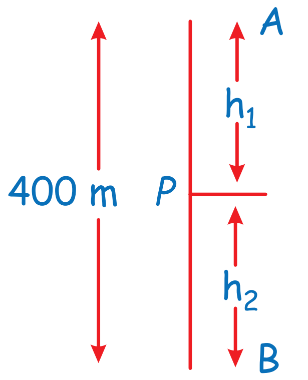
$h = \frac{1}{2}gt^2.$

**Q.** A man drops a ball downside from the roof of a tower of height 400 m. At the same time another ball is thrown upside with a velocity 50 m/s from the foot of tower. What is the height from the foot of the tower where the two balls would meet?

**Sol.** Let both balls meet at point P after time t.

The distance travelled by ball A

$$h_1 = \frac{1}{2}gt^2 \qquad \text{...(i)}$$




The distance travelled by ball B

$$h_2 = ut - \frac{1}{2}gt^2 \qquad \text{...(ii)}$$

By adding (i) and (ii) $h_1 + h_2 = ut = 400$

(Given $h = h_1 + h_2 = 400$)

$\therefore$ $t = 400/50 = 8$ s and $h_1 = 320$ m, $h_2 = 80$ m

**Q.** Water drops fall at regular intervals from a tap which is 5 m above the ground. The third drop is leaving the tap at the instant the first drop touches the ground. How far above the ground is the second drop at that instant?

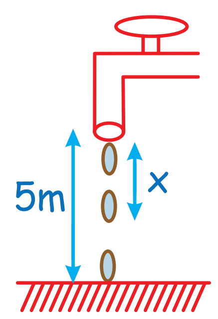
**Sol.** Let the interval between each drop be t then from question

For first drop $\frac{1}{2}g(2t)^2 = 5$ ...(i)

For second drop $x = \frac{1}{2}gt^2$ ...(ii)

By solving (i) and (ii) $x = \frac{5}{4}$ and

Hence required height $h = 5 - \frac{5}{4} = 3.75$m.




**Q.** A balloon is at a height of 81 m and is ascending vertically upward with a velocity of 12 m/sec. A body of 2 kg weight is dropped from it. If g = 10 m/s$^2$ the body will reach the surface of the earth in

**Sol.** As the balloon is going up so initial velocity of balloon

= + 12 m/s,

$\Delta y = -81$ m; $a = -g = -10$ m/s$^2$

By applying $h = ut + \frac{1}{2}gt^2$; $-81 = 12t - \frac{1}{2}(10)t^2$

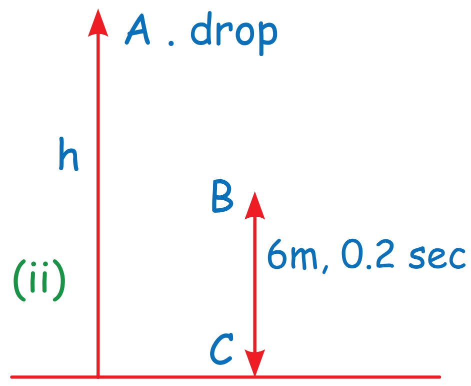
$\Rightarrow 5t^2 - 12t - 81 = 0$

$\Rightarrow t = \dfrac{12 \pm \sqrt{144 + 1620}}{10} = \dfrac{12 \pm \sqrt{1764}}{10} = 5.4$ s

**Q.** A ball dropped from a height h from ground, if it take 0.2 sec to cross the last 6m before hitting the ground. Find height from which it was dropped.

**Sol.** $t_{A\to C} = t = \sqrt{\dfrac{2h}{g}}$ ...(i)

$t_{A\to B} = t - \cdot 2 = \dfrac{\sqrt{2(h-6)}}{g}$ (ii)




## PROJECTILE MOTION

भाई ये बहुत मजेदार chapter है बस यहाँ x और y दोनो मे motion हो रहा है, बस ये याद रखना x और y दो independent axis है तो दोनो के साथ independently खेलना है जब x के साथ हो तो y को भुल जाओ और जब y के साथ हो तो x को भूल जाओ it's like your gf... अबे समझ गया ना।

In this chapter

★ Air Resistance neglected until mention

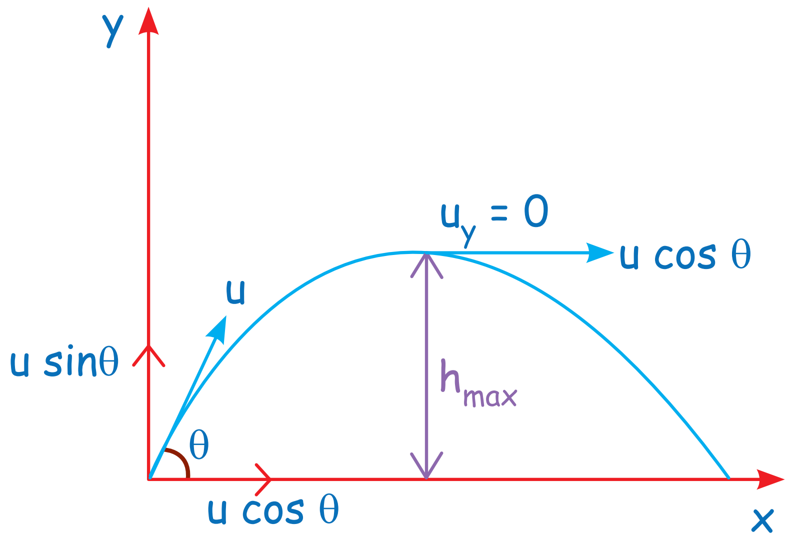
★ variation of g neglected g = 10m/s$^2$ (downward)

g = 9.8 m/s$^2$

★ Since, $a_x = 0 \Rightarrow v_x = \text{const} = u\cos\theta$

★ Particle चाहे ऊपर जा रहा हो या नीचे आ रहा हो उसका acc नीचे की तरफ g होगा

A particle is projected with speed u at angle $\theta$ with the horizontal as shown in fig.




**1.** Time of flight (T)

$u_y = u\sin\theta$,

$a_y = -g$

$v = u + at$

$0 = u\sin\theta - gt$

$t = u\sin\theta/g$ (time to reach highest point)

$$\boxed{T = 2t = \frac{2u\sin\theta}{g}}$$

Here, Kinematics का सबसे बड़ा हथियार $a_x = 0$ means horizontal मे velocity constant रहेगी।

**2.** Maximum height ($h_{max}$)

$v^2 = u^2 + 2as$ (y)

$0 = (u\sin\theta)^2 - 2g\,h_{max}$

$$\boxed{h_{max} = \frac{u^2\sin^2\theta}{2g}}$$

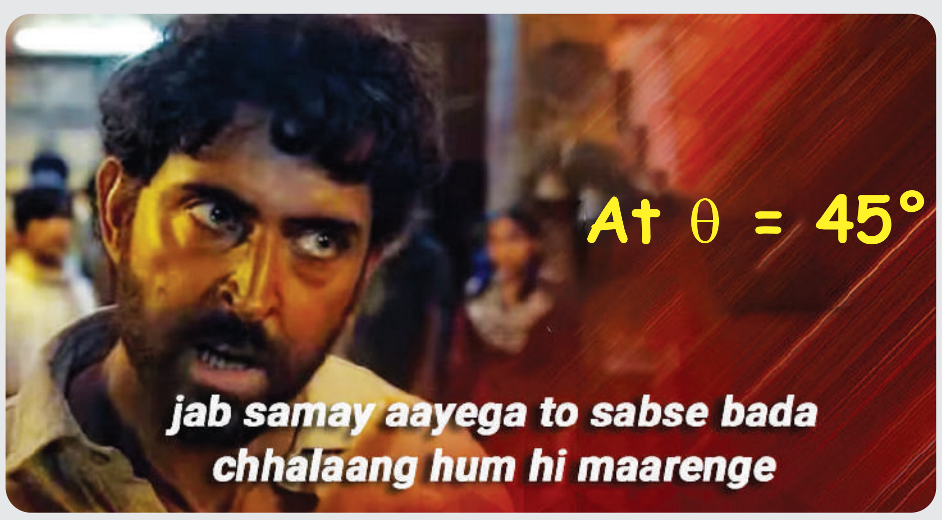
**3.** Range = $u\cos\theta \times T$

$$R = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{u^2 2\sin\theta\cos\theta}{2}$$

$$\boxed{R = \frac{u^2\sin 2\theta}{g}}$$

For R to be maximum by keeping u fix $\sin 2\theta$ should be maximum $\Rightarrow 2\theta = 90^\circ \Rightarrow \theta = 45^\circ$

If u is fixed then from ground to ground projectile range will be maximum at $\theta = 45^\circ$




# काम का डब्बा ($a_x = 0$, ground to ground)

$$T = \frac{2u\sin\theta}{g} = \frac{2U_y}{a_y}$$

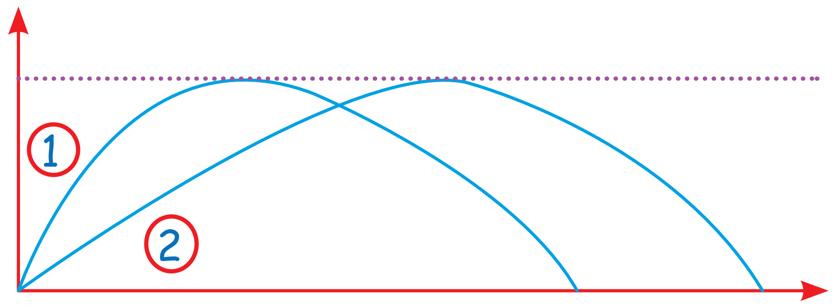
$$H_{max} = \frac{(u\sin\theta)^2}{2g} = \frac{U_y^2}{2a_y}$$

$$R = \frac{u^2\sin 2\theta}{g} = U_x T = \frac{2U_xU_y}{a_y}$$

★ If vertical velocity same $\Rightarrow$ T, $h_{max}$, same.

★ If U $\rightarrow$ fix, at $\theta$ and $90-\theta \Rightarrow$ Range Same

★

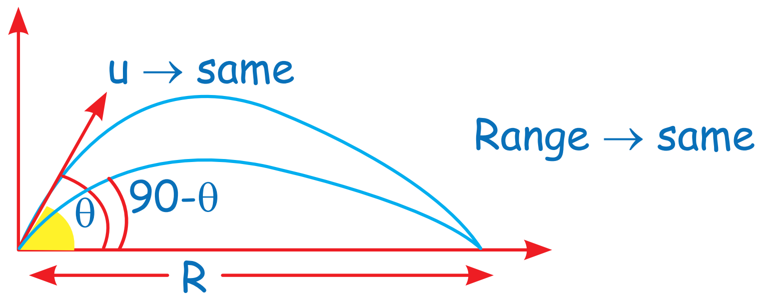



$H_{max} \rightarrow$ same

$U_y \rightarrow$ same

$u_1\sin\theta = u_2\sin\theta_2$

★

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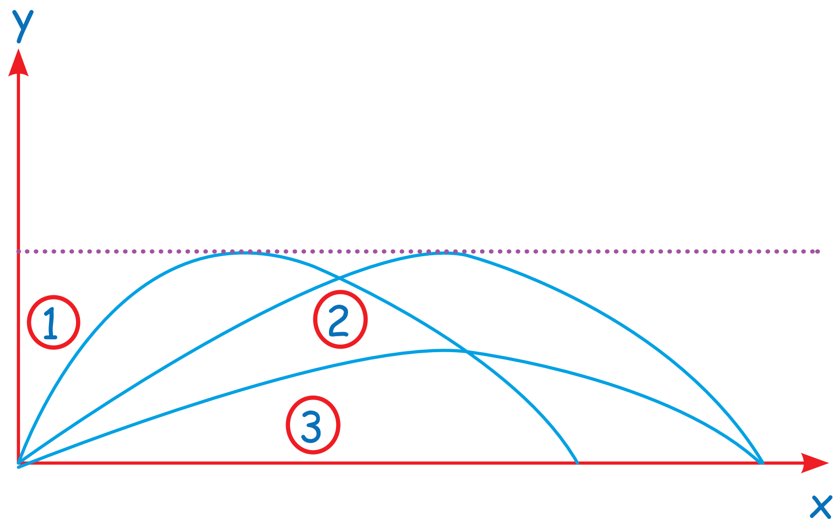



★ $(H_{max})_1 = (H_{max})_2 > (H_{max})_3$

★ $T_1 = T_2 > T_3$

$(U_y)_1 = (U_y)_2 > (U_y)_3$

$R_1 < R_2 = R_3$

**Q.** A particle is projected from ground with speed at an angle θ with horizontal such that its maximum height is half of range. Find θ

**Sol.** $h_{max} = 1/2\ R$

$$\frac{u^2\sin^2\theta}{2g} = \frac{1}{2}\frac{u^2\sin 2\theta}{g}$$

$\sin^2\theta = \sin 2\theta$

$\sin^2\theta = 2\sin\theta\cos\theta$

$\tan\theta = 2$

$\theta = \tan^{-1}(2)$

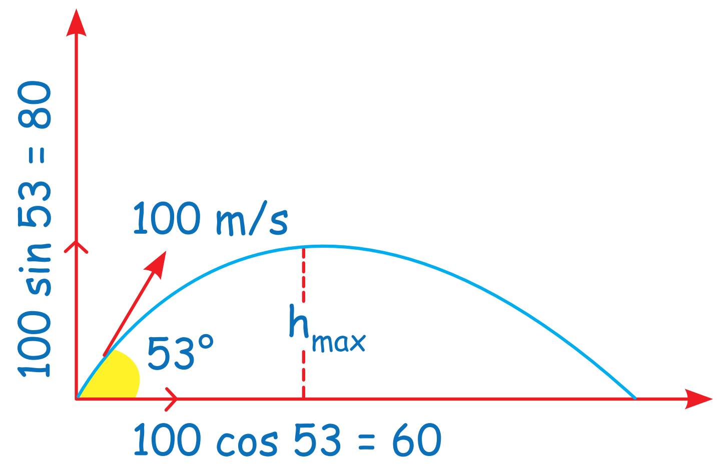
**Q.** Two particle are projected with same speed at angle 30 & 60 Find Ratio of their max height, range and time of flight respectively.

**Sol.** (1) $$\frac{(h_{max})_1}{(h_{max})_2} = \frac{\left(\frac{u^2\sin^2 30}{2g}\right)}{\left(\frac{u^2\sin^2 60}{2g}\right)} = \frac{(1/2)^2}{(\sqrt{3}/2)^2} = \frac{1}{3}$$

(2) $$\frac{R_1}{R_2} = \frac{\left(\frac{u^2\sin(2\times 30)}{g}\right)}{\left(\frac{u^2\sin(2\times 60)}{g}\right)} = \frac{\sin 60}{\sin 120} = \frac{\sqrt{3}/2}{\sqrt{3}/2} = 1$$

(3) $$T = \frac{2u\sin\theta}{g} \therefore \frac{T_1}{T_2} = \frac{\sin 30}{\sin 60} = \frac{1/2}{\sqrt{3}/2} = \frac{1}{\sqrt{3}}$$

**Q.** A particle is projected with velocity 100m/s at angle of 53° with the horizontal, answer following parts.




1. Find initial velocity, acc

**Sol.** $\vec{u} = 60\hat{i} + 80\hat{j}$

2. Acceleration.

$\vec{a} = g$ नीचे

$\vec{a} = 0\hat{i} - 10\hat{j}$

3. Find time of flight

**Sol.** T = 8 + 8 = 16 sec.

4. Range = ?

**Sol.** 60 × 16 = 960

5. Find velocity at

**Sol.**

| | | |
|---|---|---|
| t = 2 | $v_y = 60\hat{j}$ | $\vec{v} = 60\hat{i} + 60\hat{j}$ |
| t = 8 | $v_y = 0\hat{j}$ | $\vec{v} = 60\hat{i} + 0\hat{j}$ |
| t = 10 | $v_y = -20\hat{j}$ | $\vec{v} = 60\hat{i} - 20\hat{j}$ |
| t = 16 | $v_y = -80\hat{j}$ | $\vec{v} = 60\hat{i} - 80\hat{j}$ |

6. Find change in momentum for entire path (m = 1 kg) (momentum = mass × velocity)

**Sol.** $\Delta\vec{P} = \vec{P}_f - \vec{P}_i$

$\vec{P}_i = m\vec{v}_i = 1(60\hat{i} + 80\hat{j})$ $\quad \vec{P}_f = 60\hat{i} - 80\hat{j}$

$\Delta\vec{P} = \vec{P}_f - \vec{P}_i = 0\hat{i} - 160\hat{j}$

$\Delta\vec{P} = -160\hat{j}$

7. Find $h_{max}$

**Sol.** $v^2 = u^2 + 2as$

$0^2 = (80)^2 + 2(-10)\ h_{max}$

$h_{max} = 320$

8. Find $\vec{V}$ at any time

**Sol.** V = u +at

$V_y = 80 - 10t$

$\vec{V} = 60\hat{i} + (80 - 10t)\hat{j}$

9. Find time when $\vec{v}$ become perpendicular to $\vec{a}$

**Sol.** $\vec{V}_i = 60\hat{i} + (80 - 10t)\hat{j}$

$\vec{a} = \qquad -10\hat{j}$

$\vec{v}_i \cdot \vec{a} = -10(80 - 10t) = 0$

t = 8

10. Find the time when velocity of particles become perpendicular to initial velocity.

**Sol.** $\vec{V} = 60\hat{i} + 80\hat{j}$

$\vec{V}_f = 60\hat{i} + (80 - 10t)\hat{j}$

$\vec{V}_i \cdot \vec{V}_f = 0$

60 × 60 + 80(80 - 10t) = 0

3600 + 6400 - 800 t = 0

$$t = \frac{10000}{800} = \frac{100}{8} = 12.5$$

**11.** Find location of particle at t = 8

**Sol.** t = 8, $x = 60 \times 8 = 480$

$y = 80 \times 8 - \frac{1}{2} \times 10 \times 8^2$

$y = 640 - 320 = 320$

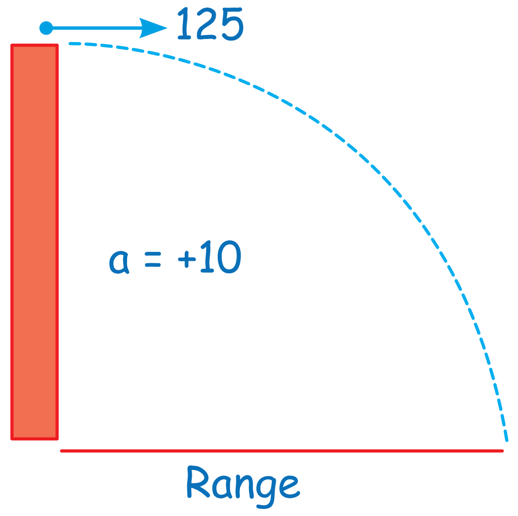
$(x, y) = (480, 320)$

**Q.** A particle is projected horizontally with velocity 40m/s from top of a tower of height 500 m above ground. Calculate range.

**Sol.** $u_y = 0$, $a_y = +10$, $s_y = +500$

$s = ut + 1/2\ at^2$

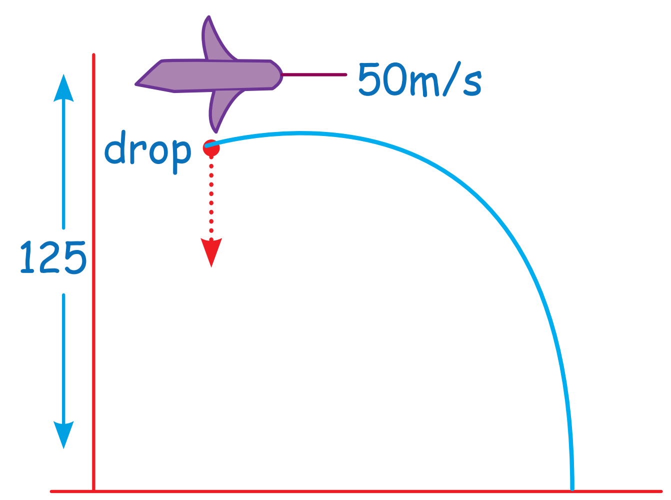
$500 = 0 + 1/2 \times 10 \times t^2$




t = 10 sec.

Range: $40 \times 10 = 400$ m

**Q.** A body is dropped from plane. Calculate range.

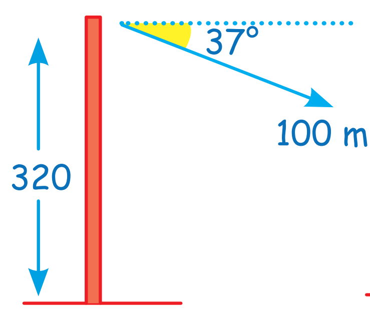


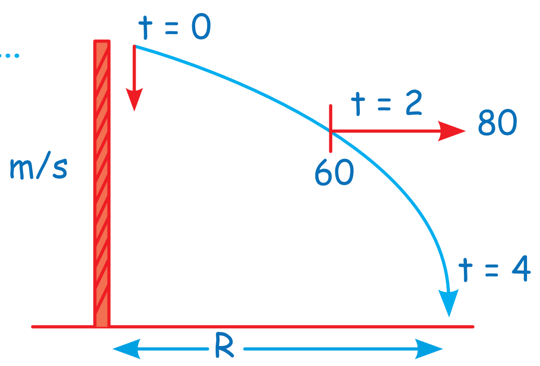

**Sol.** $t = \sqrt{\frac{2h}{a}} = \sqrt{2 \times \frac{125}{10}} = 5$

$R = 50 \times 5 = 250$m

**Q.** (1) When will particle strike the ground?

(2) Where will partice strike the ground




**Sol.** (1) y = ↓+ve $s = ut + 1/2\ at^2$

$320 = 60\,t + (1/2)10at^2$

$5t^2 + 60t - 320 = 0$

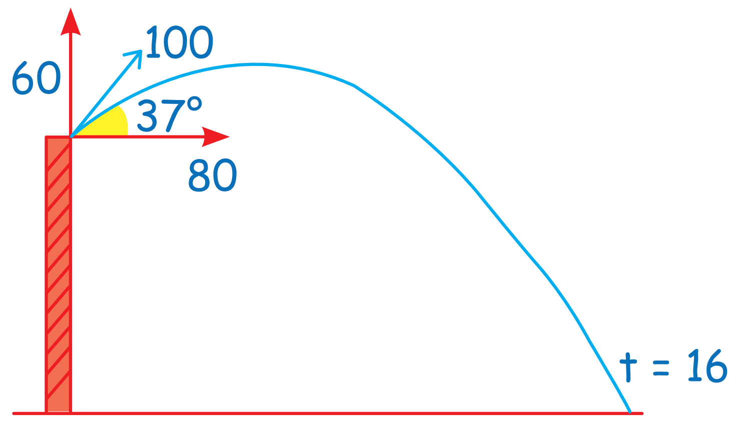
$\Rightarrow t = 4$ sec

(2) Range: $\rightarrow 80 \times 4 = 320$

**Q.** A particle is thrown with velocity 100 m/s at an angle of 37° with horizontal from a tower of height 320 m.

(A) Find when particle will hit the ground and where. (Take upward direction is positive)

**Sol.** (a) $u_y = +60$, $a_y = -10$, $s_y = -320$




$-320 = 60t - 1/2 \times 10 \times t^2$

$\boxed{t = 16 \text{ sec}}$

$R = 80 \times 16$

(B) Find speed of particle at t = 16 sec

$\vec{v} = 80\hat{i} - 100\hat{j}$

$|\vec{v}| = \sqrt{80^2 + 100^2}$

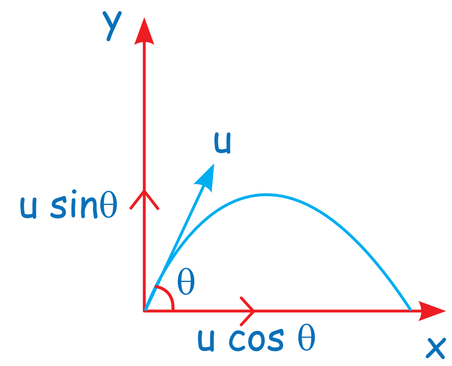
(C) Find angle made by horizontal at t = 16 sec

t = 16, $v = 80\hat{i} - 100\hat{j}$

$\tan\alpha = 100/80$

$\alpha = \tan^{-1} 5/4$

## Equation of Trajectory




$x = u\cos\theta \cdot t \Rightarrow t = \frac{x}{u\cos\theta}$

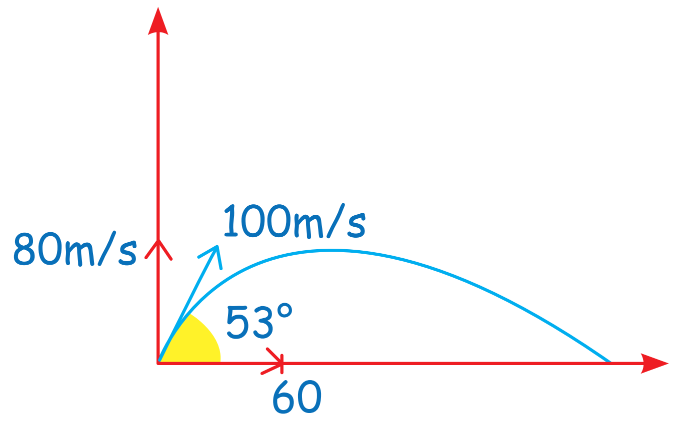
$y = u \sin\theta t - 1/2\ gt^2$

$y = u\sin\theta \frac{x}{u\cos\theta} - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$

$y = x\tan\theta - \frac{1}{2} g \frac{x^2}{u^2\cos^2\theta}$

$y = x\tan\theta(1 - x/R)$

**Q.** Find equation of trajectory.




$x = 60t \Rightarrow t = x/60$

$y = 80t - 1/2 \times 10 \times t^2$

$$y = 80\frac{x}{60} - \frac{1}{2} \times 10 \times \left(\frac{x}{60}\right)^2$$

$$\boxed{y = \frac{4x}{3} - \frac{x^2}{720}}$$

**Q.** If equation of trajectory is given by $y = x\sqrt{3} - \frac{x^2}{20}$. Find u, θ.

**Sol.** $y = x\tan\theta - 1/2\, g\frac{x^2}{u^2\cos^2\theta}$ (Formula)

$\Rightarrow \sqrt{3} = \tan\theta$

$\theta = 60°$

$$\frac{1}{20} = \frac{g}{2u^2\cos^2\theta}$$

$$\frac{1}{20} = \frac{10}{2u^2\cos^2 60}$$

Solve and get u = 20

> To find the range directly
>
> $$y = x\sqrt{3} - \frac{x^2}{20} = x\sqrt{3}\left(1 - \frac{x}{20\sqrt{3}}\right)$$
>
> Compare this with $y = x\tan\theta(1 - x/R)$
>
> $\tan\theta = \sqrt{3},\ R = 20\sqrt{3}$
>
> or Put y = 0 and get x = R

**Q.** If eqn of trajectory of a projectile from ground to ground is given as $y = 4x - \frac{x^2}{4}$. Find Range = ?

**Sol.** $y = 4x(1-x/16)$

$y = x\tan\theta(1-x/R)$

R = 16

**Q.** $y = ax - bx^2$ find θ and Range

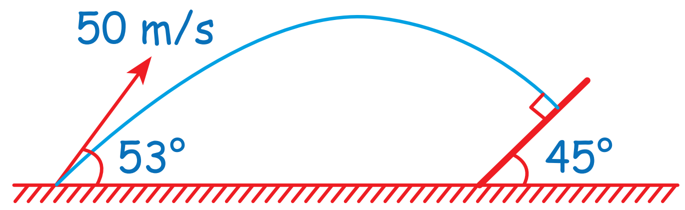
**Sol.** $y = ax(1 - bx^2/ax)$ $\qquad R = a/b$

$y = ax(1 - bx/a)$ $\qquad \tan\theta = a$

$y = 2\tan\theta(1 - x/R)$ $\qquad \theta = \tan^{-1} a$

**Q.** In following figure particle strike the inclined plane perpendicularly. Find

Time of collision and coordinate of point where particle collide if projection point is origin.

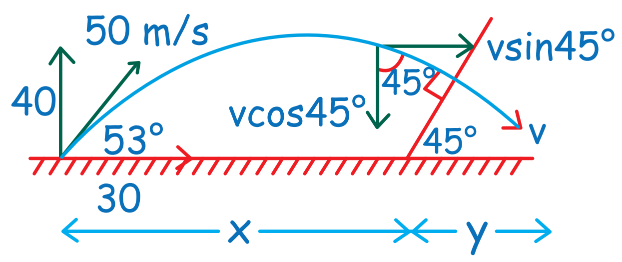



**Sol.** At the time of collision,

$v\sin45° = 50\cos53° \Rightarrow v = 30\sqrt{2}$

$v_y = 50\sin53° - 10T \Rightarrow -30\sqrt{2}\cos45° = 40 - 10T$

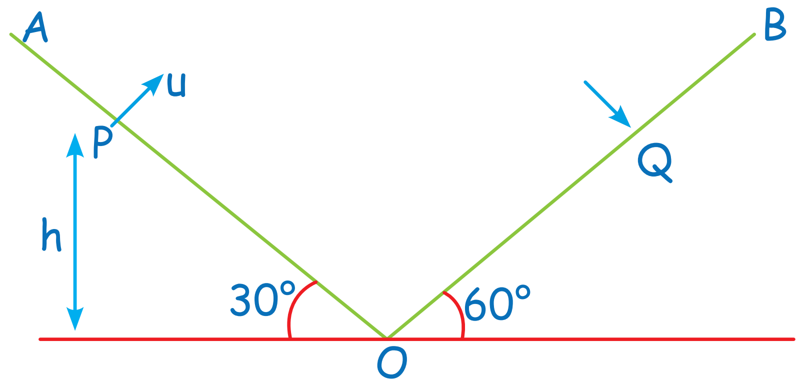
$\Rightarrow T = 7$ sec




$x = u\cos\theta \times T = 30 \times 7 = 2010$

$$y = 40 \times 7 - \frac{1}{2} \times 10 \times 7^2 = 35$$

**Q.** A particle thrown perpendicularly to an inclined plane and it's strike perpendicularly to another inclined plane with speed 100 m/s as shown in fig. Find with what velocity it will strike the inclined.




**Sol.** u cos 60° = 100 cos 30° (By keeping horizontal velocity)

> **SKC**
>
> Projectile motion का सबसे बड़ा हथियार horizontal मे velocity constant करना।

$$100 \times 1/2 = v\frac{\sqrt{3}}{2} \Rightarrow v = \frac{100}{\sqrt{3}}$$

**Q.** A particle is moving such that
$x = 4t^2$
$y = 2t^3$
$z = 4t^4$
Find velocity at t = 2 sec →

**Sol.** $u_x = \frac{dx}{dt} = 8t$

$u_y = \frac{dy}{dx} = 6t^2$

$u_z = \frac{dz}{dt} 16t^3$

Velocity $= 8t\hat{i} + 6t^2\hat{j} + 16t^3\hat{k}$
acc $= 8\hat{i} + 12t\hat{j} + 48t^2\hat{k}$

$\vec{V} = 8t\hat{i} + 6t^2\hat{j} + 16t^3\hat{k}$

**Q.** A particle is moving such that $\vec{r} = 3t^2 + 2t^3\hat{j} + 10t\hat{k}$
Find angle between $\vec{v}$ & $\vec{a}$ at t = 1 sec.

**Sol.** $\vec{v} = 6t\hat{i} + 6t^2\hat{j} + 10\hat{k}$

$\vec{a} = 6\hat{i} + 12t\hat{j} + 0\hat{k}$

At t = 1, $\vec{v} = 6\hat{i} + 6\hat{j} + 10\hat{k}$

$\vec{a} = 6\hat{i} + 12\hat{j}$

$\vec{a} \cdot \vec{v} = av \cos\theta$

$$\cos\theta = \frac{\vec{a}.\vec{v}}{av} = \frac{36 + 72 + 0}{\sqrt{6^2 + 12^2}\sqrt{6^2 + 6^2 + 10^2}}$$

## RELATIVE MOTION


Velocity of A wrt ground = 10 m/s i
Velocity of B wrt ground = 6 m/s i

$\boxed{\text{Velocity of A wrt B} = \vec{v}_{A/B} = \vec{v}_A - \vec{v}_B}$

Velocity of A wrt B = $\vec{V}_{A/B}$ ≡ B की खोपडी पर बैठकर A को देख

$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$

$\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

Similarly $\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B$

★ (A) → 6m/s  (B) → 8m/s

$\vec{v}_{B/A} = 8\hat{i} - 6\hat{i} = 2\hat{i}$

$\vec{v}_{A/B} = 6\hat{i} - 8\hat{i} = -2\hat{i}$

★ ← 10m/s (B)  → 6m/s (A)

$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = (-10\hat{i} - (6\hat{i})) = -16\hat{i}$

★ (A) → 10m/s  (B) → 10m/s

$\vec{V}_{B/A} = 10 - 10 = 0$

★ (A) → 10m/s  (B) ↑ 20m/s

$\vec{V}_{B/A} = 20\hat{j} - 10\hat{i}$

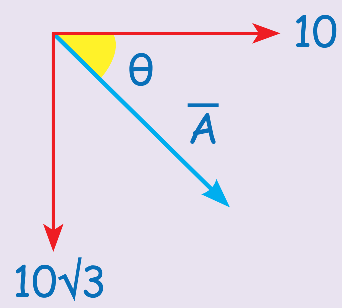
★
$\vec{V}_{A/B} = \vec{V}_A - \vec{V}_B$
$\vec{V}_{B/A} = \vec{V}_B - \vec{V}_A$
$\vec{V}_{A/B} = -\vec{V}_{B/A}$

अब आगे हमे relative motion, rain man, river boat problems पढ़ने है उसके लिए बहुत जरूरी है की हम vector को draw करना सीखें इसके लिए i am attaching few fundamental things grab it.

★ $\vec{A} = 10\hat{i} - 10\sqrt{3}\hat{j}$

$\tan\theta = \frac{10\sqrt{3}}{10}$

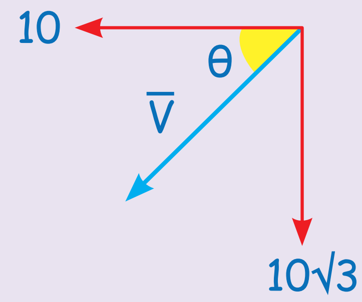
θ = 60°




★ $\vec{v} = -10\hat{i} - 10\sqrt{3}\hat{j}$

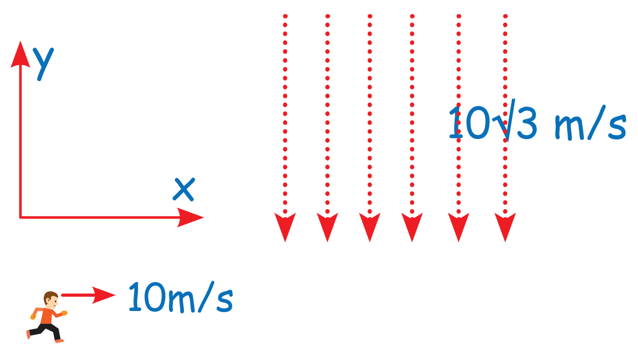
$\tan\theta = \frac{10\sqrt{3}}{10}$

θ = 60°




**Q.** A man is moving along +x-axis (east) with speed 10 m/s & rain is falling vertically downward with speed 10√3 m/s. In which direction man should hold umbrella to protect himself.

**Sol.**

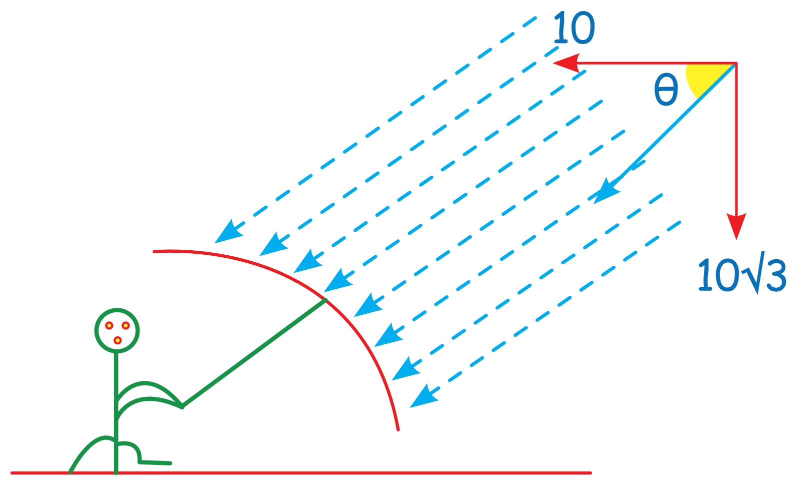



$\vec{v}_{man} = 10\hat{i}$

$\vec{v}_{rain} = -10\sqrt{3}\hat{j}$

$\vec{v}_{rain/man} = \vec{v}_r - \vec{v}_m$

$\vec{v}_{r/m} = -10\sqrt{3}\hat{j} - 10\hat{i} = -10\hat{i} - 10\sqrt{3}\hat{j}$




**Q.** A man is moving in east direction with speed 10 m/s in a car A bird is flying with speed 10√3 in south direction

(1) Find velocity of bird observed by man

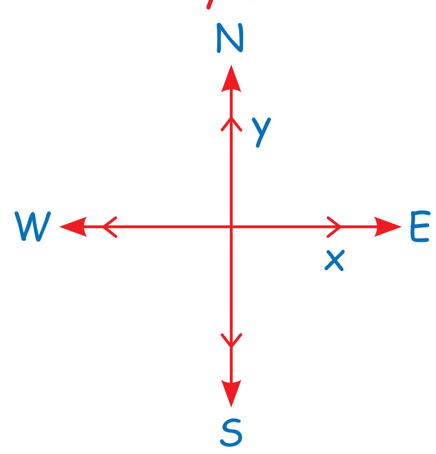
**Sol.** $\vec{v}_{man} = 10\hat{i}$

$\vec{v}_{bird} = -10\sqrt{3}\,\hat{j}$

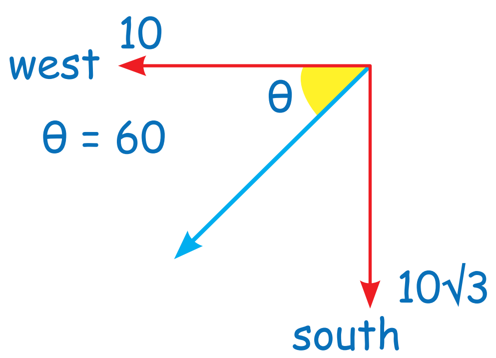
$\vec{v}_{b/m} = \vec{v}_b - \vec{v}_m$

$= -10\sqrt{3}\hat{j} - 10\hat{i}$

$|\vec{v}_{b/m}| = \sqrt{10^2 + (10\sqrt{3})^2}$




= 20 (60° south of west)

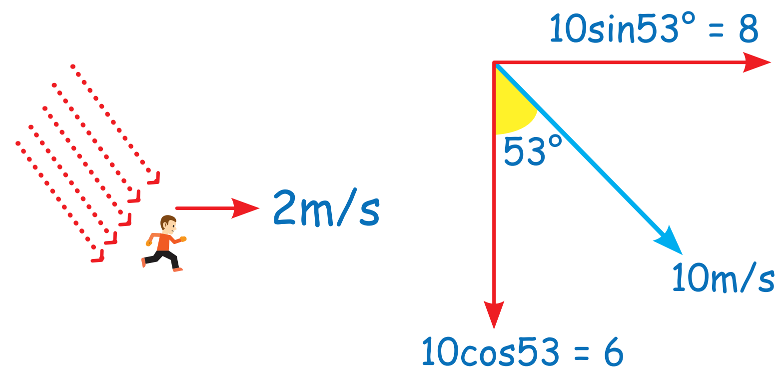



**SKC**

Rainman prob. जैसे ques के लिए ये 4 step follow करो

1. सबसे पहले velocity of man and rain निकालो
2. अब vectorly velocity of rain wrt man $\vec{V}_{r/m}$ निकालो
3. अब $\vec{V}_{r/m}$ को draw करके नया fig. बनाओ
4. अब हमे पता चल गया कि man को rain कहाँ आती दिख रही है so अब छाता लगा दो

**Q.** Rain is falling with speed 10 m/s at an angle 53° with vertical. A man is moving with speed 2 m/s along east as shown in diagram.



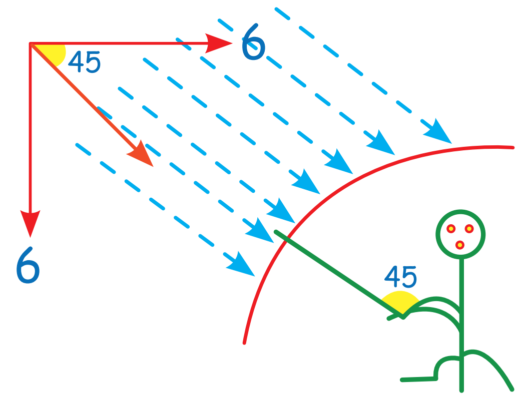

(1) In which direction man should hold umbrella to protect himself

**Sol.** $\vec{V}_r = 8\hat{i} - 6\hat{j}$

$\vec{V}_m = 2\hat{i}$

$\vec{v}_{r/m} = (8\hat{i} - 6\hat{j}) - (2\hat{i})$

$= 6\hat{i} - 6\hat{j}$



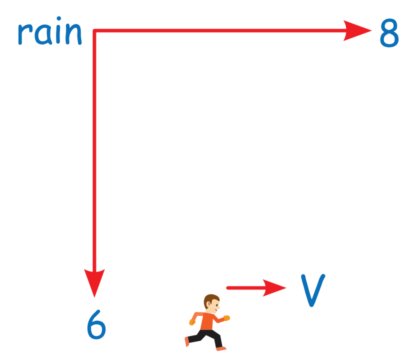

$\tan\theta = \frac{6}{6}$

$\tan\theta = 1$

$\tan 45 = 1$

$\theta = 45°$

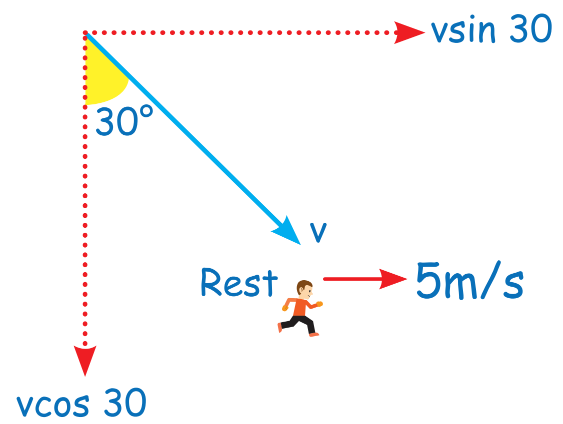
(2) What should be velocity of man so that rain appear falling vertically downward. to him?




V= 8 Ans.

**Q.** To a stationary man, rain appear to be falling at an angle 30° with the vertical As he start moving with speed of 5 m/s he feels that rain is falling vertically. Find speed of rain.

**Sol.**




v sin 30 = 5

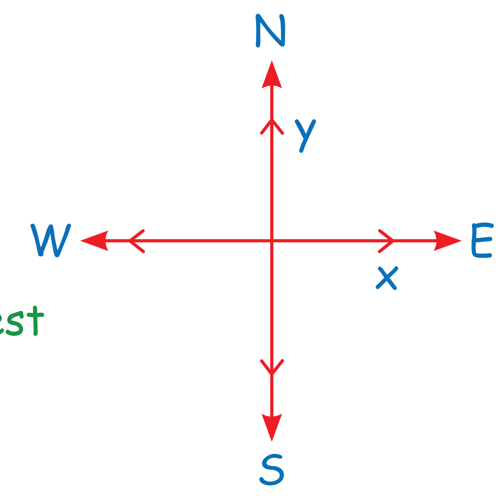
v = 10 speed of rain

**Q.** A man holding a flag is running with speed 10 m/s along east. If wind speed is 10√3 m/s along south then. In which direction flag will flutter?

**Sol.** $\vec{V}_{man} = 10\hat{i}$

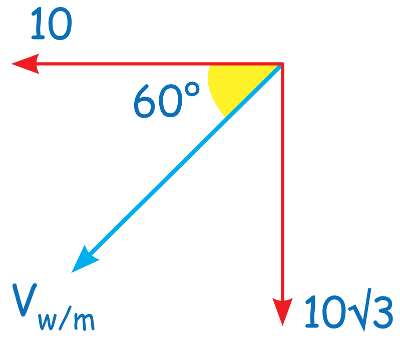
$\vec{V}_{wind} = -10\sqrt{3}\hat{j}$

$\vec{V}_{w/m} = -10\sqrt{3}\hat{j} - 10\hat{i}$

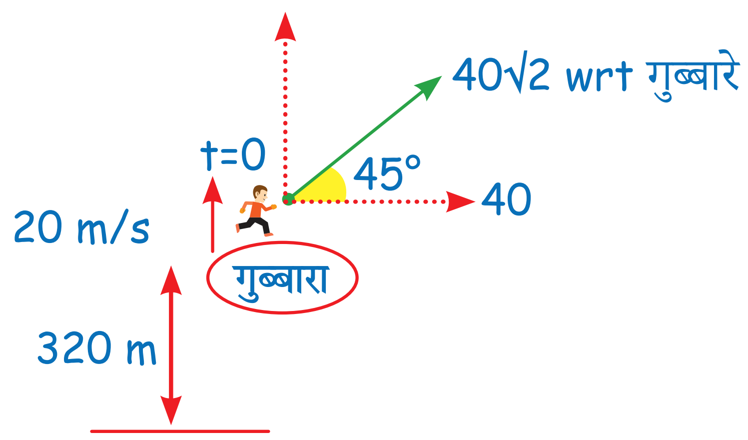



Ans = 60° south of west

30° west of south




**Q.** A balloon is rising upward with constant velocity 20 m/s. When it is at a height 320 m from ground a particle is projected from balloon with velocity $40\sqrt{2}$ m/s at an angle of 45° with horizontal at t = 0.
Find velocity of particle just after projection and time of flight of particle.




t = 0+ $\quad \vec{V}_p = ?$ (wrt ground)

**Sol.** $\vec{V}_{P/ballon} = 40\hat{i} + 40\hat{j}$

$\vec{V}_P - \vec{V}_{ballon} = 40\hat{i} + 40\hat{j}$

$\vec{V}_P - 20\hat{j} = 40\hat{i} + 40\hat{j}$

$\vec{V}_P = 40\hat{i} + 60\hat{j}$

$S = ut + \frac{1}{2}at^2$ (upward +ve)

$-320 = 60t - \frac{1}{2} \times 10 \times t^2$

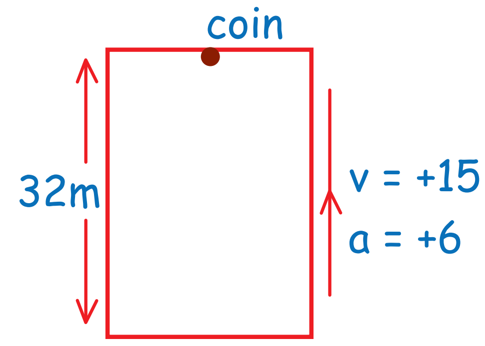
**Sol.**e and get t = 16 sec

अबे ये तो अब 320 m के tower से पत्थर फेकने वाला ques. बन गया।

## SKC

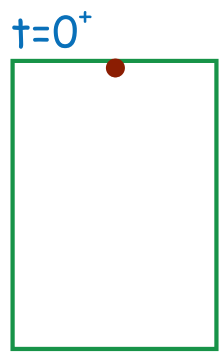
**Q.stion** में जब कभी भी **lift** का, गुब्बारे का **acc given** होता है, तो वो **g** को **consider** करने के बाद **given** होता है

**Q.** A lift of height 32 m is going up with constant acc a = 6 m/s$^2$. When velocity of lift is 15 m/s upward, a coin is drop from cealing of lift at t = 0. Find time when coin will hit the floor.




**Sol.** चुपचाप lift ke अंदर जाके बैठ जाओ.

wrt lift




(In lift frame or wrt lift)

At t = 0 $\vec{V}_{coin/lift} = 0$

$\vec{a}_{coin/lift} = -10 - (+6) = -16$

$\vec{s}_{coin/lift} = -32$

$S = ut + 1/2 \times 16 \times t^2$

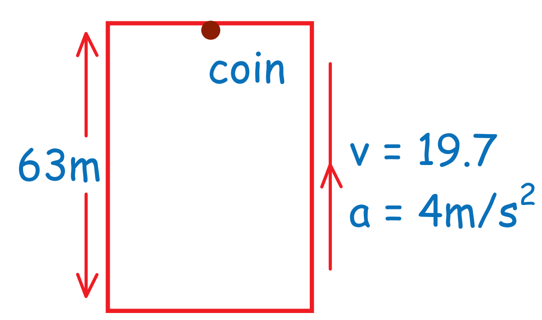
$-32 = 0 - 1/2 \times 16 \times t^2$

t = 2 sec

## SKC

ऐसे सवालो मे सबसे पहले चुप चाप lift के अंदर जाके बैठ जाओ और coin की velocity, coin का acc lift के respect मे लिखकर eqn of motion लगादो. बस ये याद रखना अगर coin हवा मे है तो ground के respect मे उसका acc नीचे g होगा

**Q.** At t = 0 coin drop. Find time when coin will hit the floor




**Sol.** $\vec{u}_{coin/lift} = 0$

$\vec{a}_{coin/lift} = -10 - (+4) = -14$

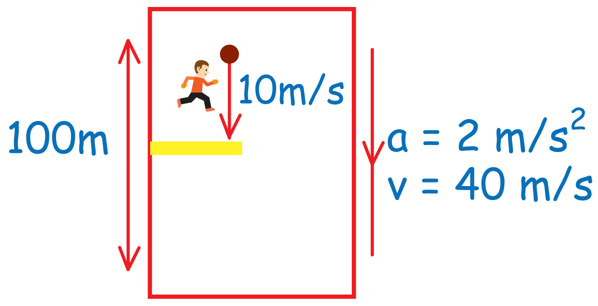
$\vec{s}_{coin/lift} = -63m$

$s = ut + 1/2\ at^2$

$-63 = 0 + 1/2 \times (-14)\ t^2$

$t = 3$ sec

**Q.** Ball thrown downward at 10m/s w.r.t lift.




When will coin strike the floor?

**Sol.** Lift ke अंदर आकर wrt lift.

$v_{coin/lift} = -10$

$a_{coin/lift} = -10 - (-2) = -8$

$S_{coin/lift} = -100$

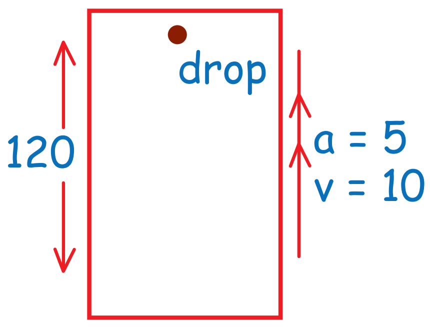
$-100 = -10t + 1/2\ (-8) \times t^2$

$100 = 10t + 4t^2$

$2t^2 + 5t - 50 = 0$

$t = 4$

**Q.** When will coin strike the floor of left.




**Sol.** $\vec{V}_{coin/lift} = 0$

$\vec{a}_{coin/lift} = -10 - (+5) = -15$

$\vec{S}_{coin/lift} = -120$

$-120 = 0 + 1/2\ (-15)t^2$

$t = 4$ sec

lift $t = 4,\quad S = ?,\quad u = 10,\quad a = 5$

$s = 10 \times 4 + 1/2 \times 5 \times 4^2 = 80$

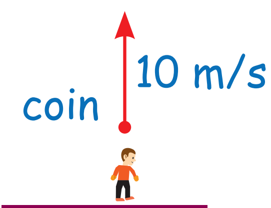
Find displacement of lift w.r.t ground before coin strike the floor?

Displacement of coin in t = 4 sec in ground frame

$S = ut + 1/2\ at^2$

$s = 10 \times 4 + 1/2(-10) \times 4^2$

$= 40 - 80 = -40$ Ans




**Q.** Car *A* and car *B* start moving simultaneously in the same direction along the line joining them. Car *A* moves with a constant acceleration $a = 4\ m/s^2$, while car *B* moves with a constant velocity $v = 1$ m/s. At time $t = 0$, car *A* is 10 m behind car *B*. Find the time when car *A* overtakes car *B*. How much time *A* take to overtake.

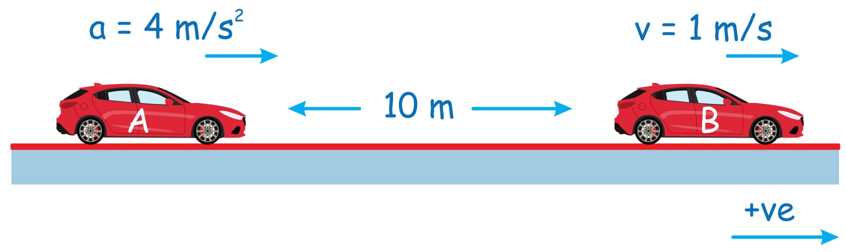
**Sol.** Given: $u_A = 0$, $u_B = 1$ m/s, $a_A = 4\ m/s^2$ and $a_B = 0$

Assuming car *B* to be at rest, we have

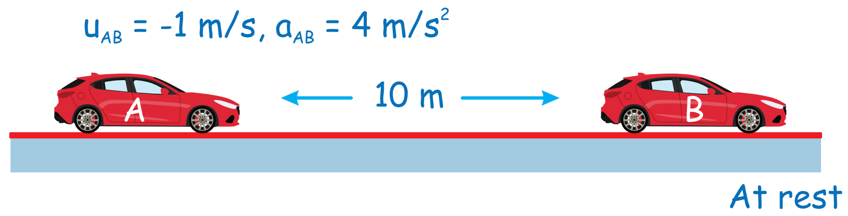
$u_{AB} = u_A - u_B = 0 - 1 = -1$ m/s

$a_{AB} = a_A - a_B = 4 - 0 = 4\ m/s^2$

Now, the problem can be assumed in simplified form as follow:




Substituting the proper values in equation




$S = ut + \frac{1}{2}at^2$

we get $10 = -t + \frac{1}{2}(4)(t^2)$ or $2t^2 - t - 10 = 0$

Ignoring the negative value, the desired time is 2.5s.

**Note:** The above problem can also be solved without using the concept of relative motion as under. At the time when *A* overtakes *B*,

$S_A = S_B + 10$

$\therefore\ \frac{1}{2} \times 4 \times t^2 = 1 \times t + 10$

or $2t^2 - t - 10 = 0$

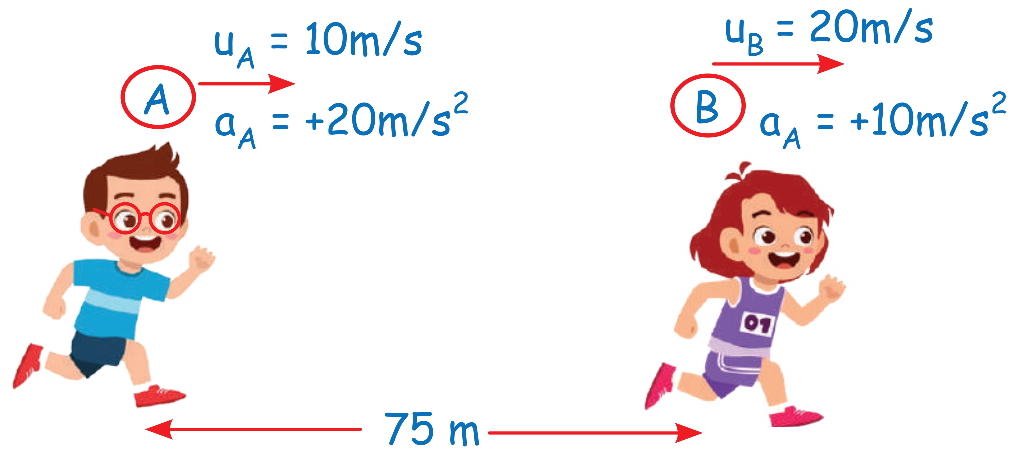
Which on solving gives $t = 2.5$ s and $-2$ s, the same as we found above.

## SKC

Catching वाले सवालो मे

- सबसे पहले आगे वाले बंदे पर बैठ जाओ
- बाद आगे वाले के respect मे intial v, a, s लिखो
- equt$^n$ of motion ठोक दो

**Q.** At $t = 0$ gap between saleemian boy A and girl B is 75 m. Find when boy will catch the girl.




**Sol.** M-1 (relative वाला method, आगे वाले के ऊपर जाके बैठ जाओ)

$\vec{u}_{A/B} = 10 - 20 = -10$

$\vec{a}_{A/B} = 20 - 10 = 10$

$s = ut + 1/2\ at^2$

$75 = -10t + (1/2)10\ t^2$

$5t^2 - 10t - 75 = 0$

**Sol**.e and get $t = 5$

M-2

$75 + x_b = x_a$

$75 + 20t + 1/2 \times 10 \times t^2 = 10t + 1/2 \times 20 \times t^2$

$75 + 20t + 5t^2 = 10t + 10t^2$

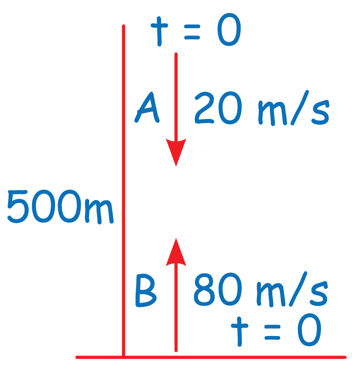
$5t^2 - 10t - 75 = 0$

$t^2 - 2t - 15 = 0$

$(t - 5)(t + 3) = 0$

$t = 5$

**Q.** A & B are thrown as shown simantaneously at $t = 0$ when will they meet?




**Sol.** $v_{A/B} = -20 - 80 = -100$

$\vec{a}_{A/B} = -10 - (-10) = 0$

$s_{A/B} = -500$

$-500 = -100t + 1/2 \times 0 \times t^2$

$t = 5$ sec

**Direct$^n$**

$\vec{a}_{rel} = 0$

$\vec{v}_{rel} = 100$

$500 = 100t$

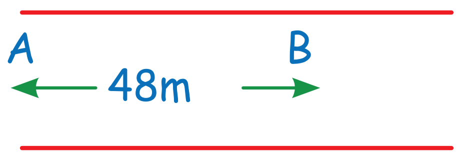
$t = 5$ sec

## RIVER-BOAT-MAN PROBLEM

भाई ये JEE-Mains और NEET का favourite article है बेशरमो की तरह बार-बार आता रहता है इसे अच्छे से कर लेना

**Q.** River is flowing with speed 6 m/s, and $\vec{v}_{man/river}$ is 10 m/s.

Man start swimming from A to B and return back to A. Find time taken




**Sol.** $(A \to B)t_{A \to B} = \dfrac{48}{10+6} = 3 sec$

$(B \to A)\ t_{B \to A} = \dfrac{48}{10-6} = 12 sec$

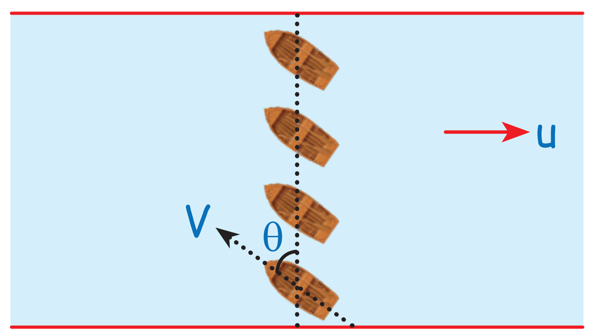
total time = 3 + 12 = 15 sec

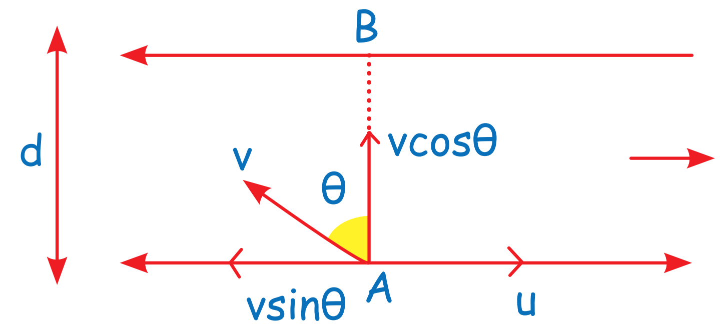
Average speed $= \dfrac{48+48}{15}$

Avrg velocity = 0

### Case-I

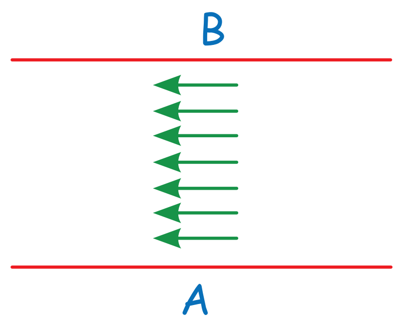
**Crossing of river in min distance.**



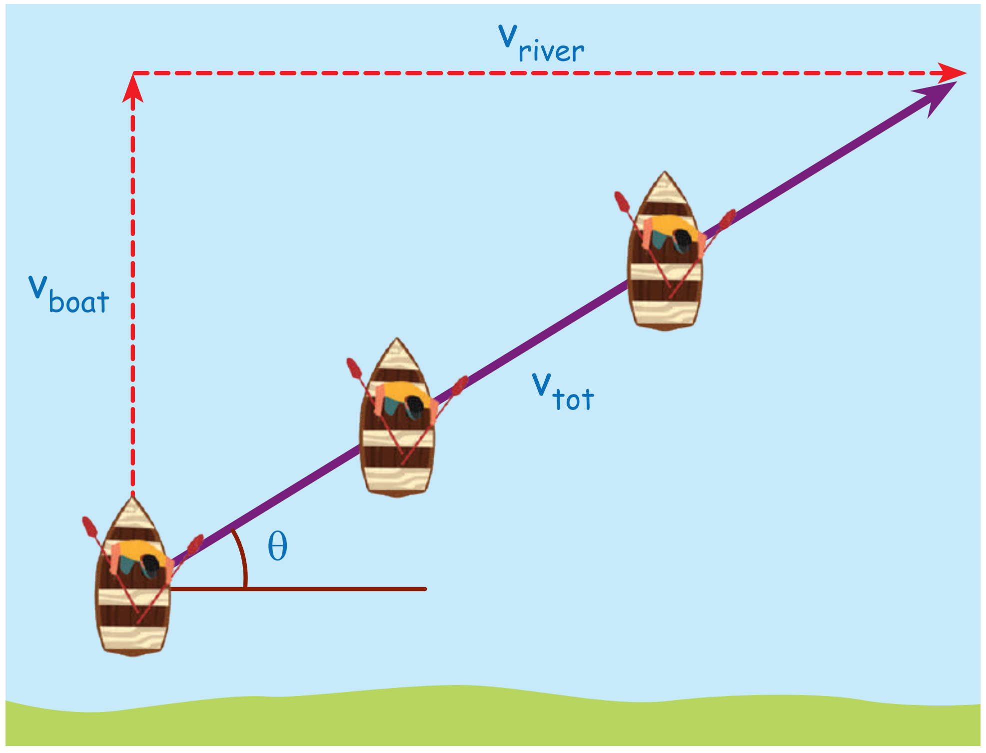

Drift = 0

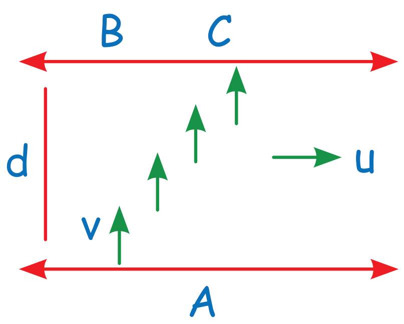
$$\boxed{\begin{aligned} V \sin\theta &= u \\ t &= \frac{d}{u\cos\theta} \end{aligned}}$$



## Case-II

### Crossing of river in min time

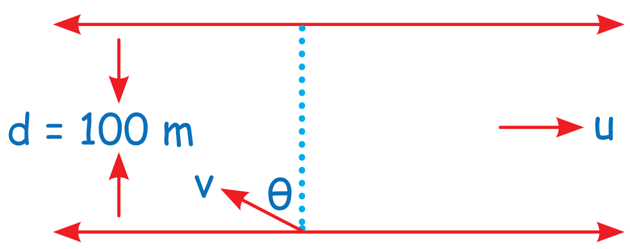



$$\boxed{\begin{array}{l}\text{Min time} = d/v \\ \text{Drift} = CB = ut\end{array}}$$

**Q.** A river is flowing with velocity 5 m/s wrt ground. A man can swim with velocity 10 m/s wrt river. The width of the river is 100 m.

(1) In which direction man should swim so that he crosses that river in min distance travel also find time taken to cross the river.




**Sol.** $\vec{v}_{m/r} = v = 10$ m/s

$u = 5 \qquad d = 100$

$v \sin\theta = u$

$10 \sin\theta = 5$

$\sin\theta = 1/2$

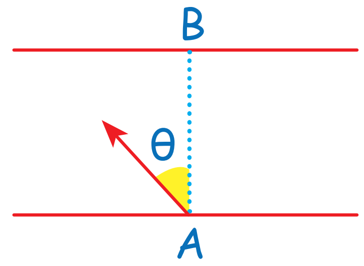
$\sin\theta = 30°$

$$\text{time} = \frac{d}{v\cos\theta} = \frac{100}{10\cos 30°}$$

(2) If he want to cross the river in min possible time find min time to cross the river & drift

**Sol.** $t = \frac{100}{10} = 10\,\text{sec}$

BC = drift = 5 × 10 = 50 m




$\sin\theta = \frac{\text{छोटी v}}{\text{बड़ी v}}$

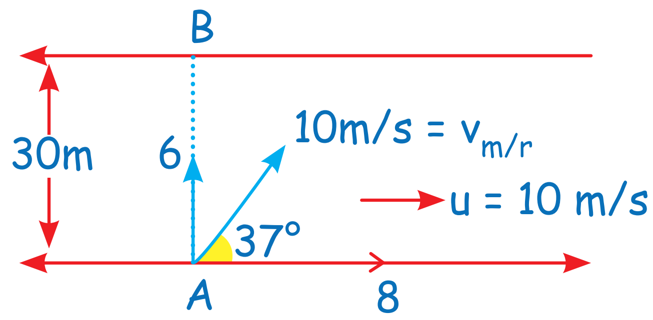
**Q.** $u = 10\sqrt{3}$, $\vec{V}_{m/r} = 20$

θ = ? for min distance

**Sol.** $\sin\theta = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$

θ = 60

**Q.** In following figure if man is swimming with velocity 10 m/s wrt river at an angle of 37° to the river velocity. Find time taken to cross the river and drift if velocity of river is 10 m/s and width of the river is 30 m/s

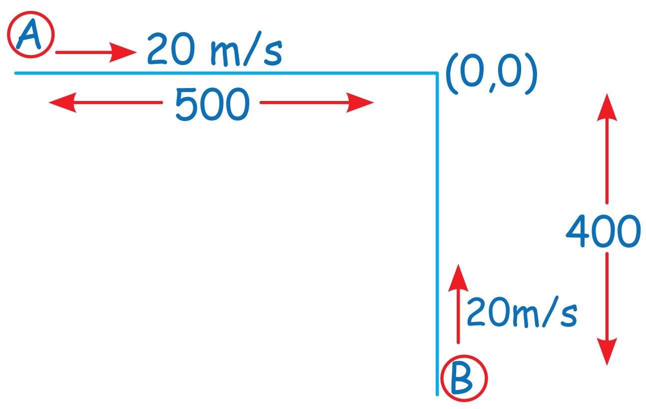



**Sol.** $t = \frac{30}{6} = 5\,\text{sec}$

Find drift

BC = 18 × 5 = 90 [$V_{net}$ = 10 + 8 = 18]

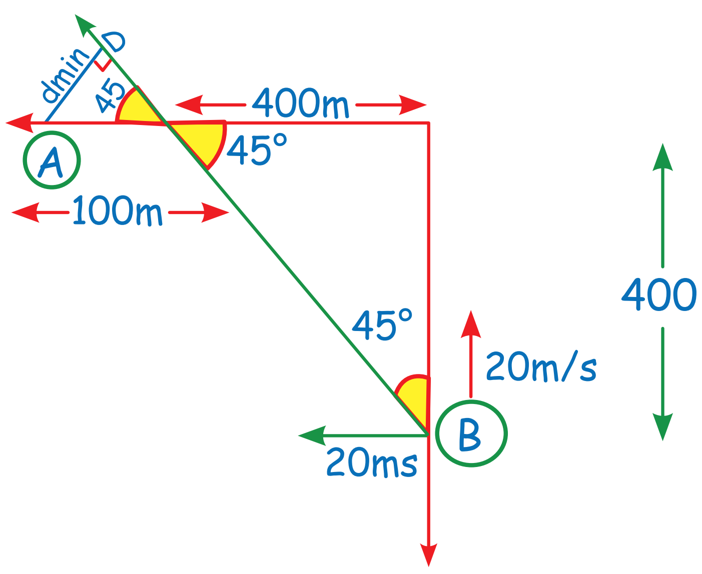
**Q.** Min distance between the moving particle.




**Sol.** $\vec{V}_A = -20\hat{i}$

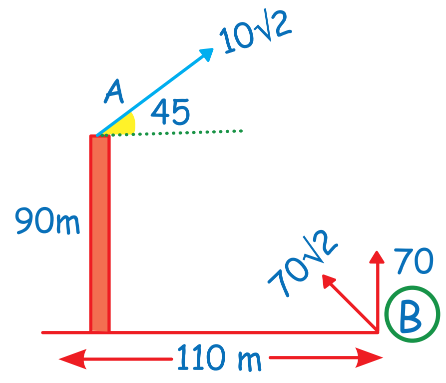
$\vec{V}_B = 20\hat{j}$

$\vec{V}_{B/A} = -20\hat{i} + 20\hat{j}$




AD = $d_{min}$ = 100 sin 45

**Q. Min Distance between them**

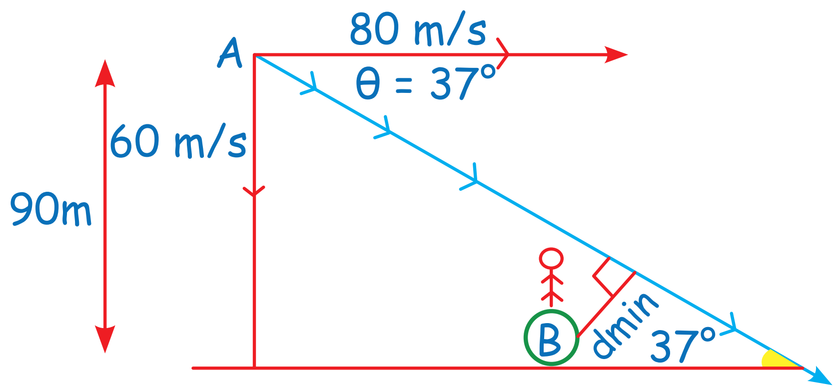



**Sol.** $\vec{V}_A = 10\hat{i} + 10\hat{j}$

$\vec{V}_B = -70\hat{i} + 70\hat{j}$

$\vec{V}_{A/B} = 80\hat{i} - 60\hat{j}$

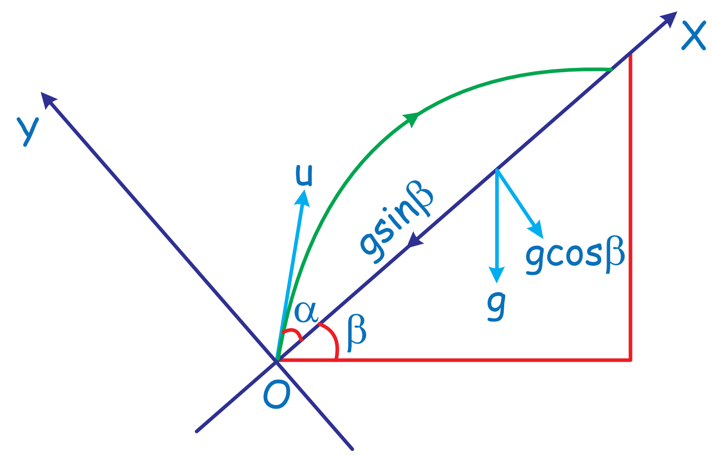
$\vec{a}_{A/B} = 0$




## PROJECTION ON AN INCLINED PLANE

**Case-I:** Particle is projected up the incline

Here $\alpha$ is angle of projection w.r.t. the inclined plane.




$a_x = -g\sin\beta$, $\quad u_x = u\cos\alpha$

$a_y = -g\cos\beta$, $\quad u_y = u\sin\alpha$

**Time of flight (T):** When the particle strikes the inclined plane, y becomes zero

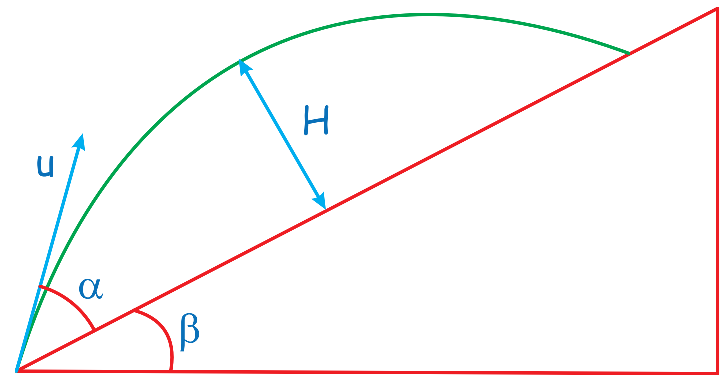
$$y = u_y t + \frac{1}{2} a_y t^2$$

$$\Rightarrow \quad 0 = u\sin\alpha T - \frac{1}{2} g\cos\beta T^2$$

$$\Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_y}{g_y}$$

where $u_\perp$ and $g_\perp$ are component of u and g perpendicular to the incline.

**Maximum Distance from Inclined Plane (H):**




When half of the time is elapsed y-coordinate is equal to maximum distance from the inclined plane of the projectile

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\cos\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u_y^2}{2g_y}$$

**Range Along the Inclined Plane (R):**

When the particle strikes the inclined plane, x coordinate is equal to range of the particle

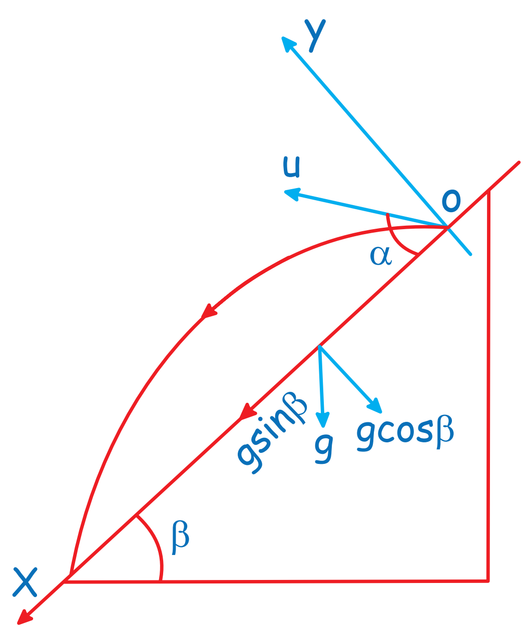
$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow \quad R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\Rightarrow \quad \boxed{R = \frac{2u^2\sin\alpha\cos(\alpha+\beta)}{g\cos^2\beta}}$$ (इसको रटना मत बस ये देखो eqn मे value कैसे put कर रहे है)

In this case after solving we got range will be maximum when $\alpha = \frac{\pi}{4} - \frac{\beta}{2}$ and Max. Range $= \frac{u^2}{g[1+\sin\beta]}$

**Case-II: Particle is projected down the incline**




$a_x = g\sin\beta \qquad u_x = u\cos\alpha$

$a_y = -g\cos\beta \qquad u_y = u\sin\alpha$

## Time of Flight (T):

When the particle strikes the inclined plane y coordinate becomes zero

$$y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow 0 = u \sin\alpha T - \frac{1}{2} g \cos\beta T^2$$

$$\Rightarrow \quad T = \frac{2u\sin\alpha}{g\cos\beta} = \frac{2u_y}{g_y}$$

## Maximum Distance (H):

When half of the time is elapsed y coordinate is equal to maximum distance of the projectile from the plane.

$$H = u\sin\alpha\left(\frac{u\sin\alpha}{g\cos\beta}\right) - \frac{1}{2}g\cos\beta\left(\frac{u\sin\alpha}{g\cos\beta}\right)^2$$

$$H = \frac{u^2\sin^2\alpha}{2g\cos\beta} = \frac{u_y^2}{2g_y}$$

## Range Along the Inclined Plane (R):

When the particle strikes the inclined plane x coordinate is equal to range of the particle

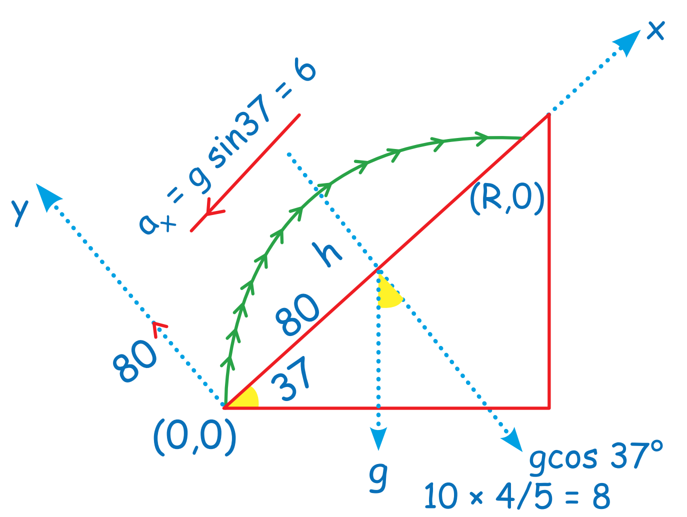
$$x = u_x t + \frac{1}{2} a_x t^2$$

$$\Rightarrow R = u\cos\alpha\left(\frac{2u\sin\alpha}{g\cos\beta}\right) + \frac{1}{2}g\sin\beta\left(\frac{2u\sin\alpha}{g\cos\beta}\right)^2$$

$$\boxed{R = \frac{2u^2\sin\alpha\cos(\alpha-\beta)}{g\cos^2\beta}}$$

In this case after solving we got range will be maximum when $\alpha = \frac{\pi}{4} + \frac{\beta}{2}$ and Max. Range $= \frac{u^2}{g[1-\sin\beta]}$

**Q.** A particle is projected with velocity $80\sqrt{2}$ at an angle 45° with inclined plane. If inclined plane makes angle 37° with horizontal. Time of flight and range of particle.




Time of flight = T = 10 + 10 = 20 sec

**Sol.** $T = \frac{2u_y}{a_y} = \frac{2\times 80}{8} = 20\,\text{sec}$

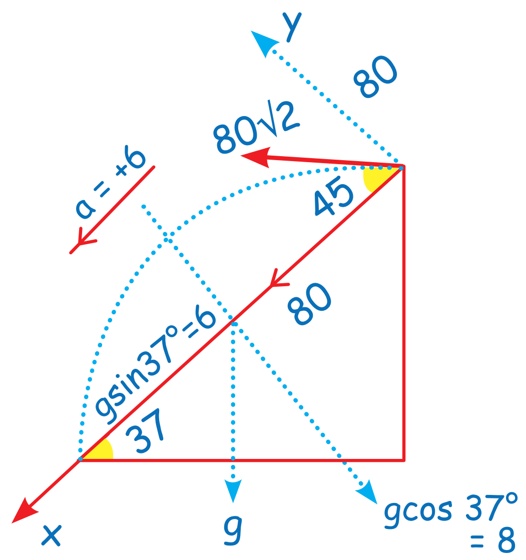
$R = ut + \frac{1}{2}at^2$ (x mai)

$R = 80\times 20 - \frac{1}{\cancel{2}}\cancel{6}^3\times(20)^2$

$= 1600 - (3 \times 400)$

$1600 - 1200 = 400$ m

**Q.** Repeat the above ques. for following fig.




**Sol.** $T = \frac{2\times 80}{8} = 20$

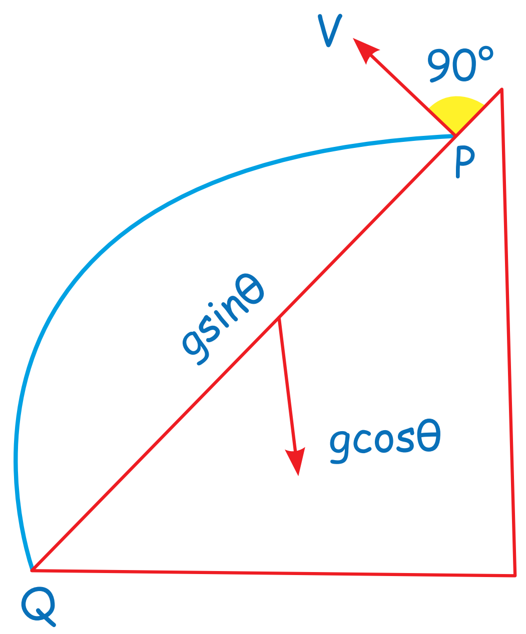
Range $= 80 \times 20 + 1/2 \times 6 \times 20^2$

$= 1600 + 3 \times 400$

$= 1600 + 1200$

$= 2800$

**Q.** Time of flight is T find PQ




**Sol.** $PQ = 1/2\, g\sin\theta\, T^2$

$= \frac{1}{\cancel{2}}\cancel{g}\sin\theta T \times \frac{\cancel{2}v}{\cancel{g}\cos\theta}$ $\quad \left[T = \frac{2V}{g\cos\theta}\right]$

$TV\tan\theta$

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