CHAPTER 14

Wave Motion

General Equation of Wave Motion

$$\frac{d^2y}{dt^2} = v^2\frac{d^2y}{dx^2}$$

$$y(x,t) = f\left(t \pm \frac{x}{v}\right)$$

where, y(x, t) should be finite everywhere.

$f\left(t + \frac{x}{v}\right)$ represents wave travelling in –ve x-axis.

$f\left(t - \frac{x}{v}\right)$ represents wave travelling in + ve x-axis.

$y = A\sin(\omega t \pm kx + \phi)$

Terms Related to Wave Motion (For 1-D Progressive Sine Wave)

Wave Number (or Propagation Constant) (k)

$$k = 2\pi/\lambda = \frac{\omega}{v}\left(\text{rad m}^{-1}\right)$$

Phase of Wave

The argument of harmonic function ($\omega t \pm kx + \phi$) is called phase of the wave.

Phase difference ($\Delta\phi$): difference in phases of two particles at any time t.

$\Delta\phi = \frac{2\pi}{\lambda}\Delta x$ where Δx is path difference

Also $\Delta\phi = \frac{2\pi}{T}.\Delta t$

Speed of Transverse Wave Along the String

$v = \sqrt{\frac{T}{\mu}}$ where T = Tension

μ = mass per unit length

Velocity of Longitudinal Waves

- Velocity of longitudinal waves in solid, $v = \sqrt{\frac{Y}{\rho}}$
- Velocity of longitudinal waves in liquid and gas, $v = \sqrt{\frac{K}{\rho}}$

where, Y $\rightarrow$ Young's modulus, K $\rightarrow$ Bulk modulus.

Newton's Formula:

Velocity of sound in gas, $v = \sqrt{\frac{P}{\rho}}$

Laplace Formula:

$v = \sqrt{\frac{\gamma P}{\rho}}$, where $\gamma = \frac{C_P}{C_V}$ and P = adiabatic pressure.

Power Transmitted Along the String

Average Power $\langle P\rangle = 2\pi^2 f^2 A^2 \mu v$

Intensity $I = \frac{\langle P\rangle}{s} = 2\pi^2 f^2 A^2 \rho v$

Reflection of waves

If we have a wave

$y_i(x, t) = a\sin(wt - kx)$ then,

(*i*) Equation of wave reflected at a rigid boundary

$y_r(x, t) = a\sin(kx + wt + \pi)$

or $y_r(x, t) = -a\sin(kx + wt)$

i.e. the reflected wave is 180° out of phase.

(*ii*) Equation of wave reflected at an open boundary

$y_r(x, t) = a\sin(kx + wt)$

i.e. the reflected wave is in phase with the incident wave.

Standing/Stationary Waves

$y_1 = A\sin(\omega t - kx + \theta_i)$

$y_2 = A\sin(\omega t - kx + \theta_2)$

$$y_1 + y_2 = 2A\cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)\sin\left(\omega t + \frac{\theta_1 + \theta_2}{2}\right)$$

The quantity $2A\cos\left(kx + \frac{\theta_2 - \theta_1}{2}\right)$ represents resultant amplitude at x. At some position resultant amplitude is zero these are called **nodes**. At some positions resultant amplitude is 2A, these are called **antinodes**.

Distance between successive nodes or antinodes $= \frac{\lambda}{}$

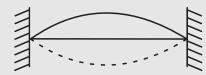
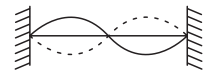
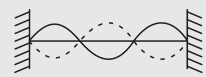
Distance between adjacent nodes and antinodes = $\lambda/4$.

All the particles in same segment (portion between two successive nodes) vibrate in same phase.

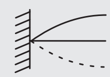
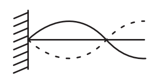
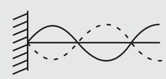
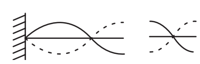
Since nodes are permanently at rest so energy can not be transmitted across these.

Vibrations of Strings (Standing Wave)

Fixed at Both Ends

First harmonics or Fundamental frequency	$L = \frac{\lambda}{2},\ f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$	
Second harmonics or First overtone	$L = \frac{2\lambda}{2},\ f_2 = \frac{2}{2L}\sqrt{\frac{T}{\mu}}$	
Third harmonics or Second overtone	$L = \frac{3\lambda}{2},\ f_3 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$	
n^{th} harmonics or $(n-1)^{th}$ overtone	$L = \frac{n\lambda}{2},\ f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$	

String Free at One End

First harmonics or Fundamental frequency	$L = \frac{\lambda}{4},\ f_1 = \frac{1}{4L}\sqrt{\frac{T}{\mu}}$	
Third harmonics or First overtone	$L = \frac{3\lambda}{4},\ f = \frac{3}{4L}\sqrt{\frac{T}{\mu}}$	
Fifth harmonics or Second overtone	$L = \frac{5\lambda}{4},\ f_5 = \frac{5}{4L}\sqrt{\frac{T}{\mu}}$	
$(2n+1)^{th}$ harmonic or n^{th} overtone	$L = \frac{(2n+1)\lambda}{4},$ $f_{2n+1} = \frac{(2n+1)}{4L}\sqrt{\frac{T}{\mu}}$	

Organ Pipes

1. In a closed organ pipe only odd harmonics are present.

$$\nu_1 = \frac{V}{4L} \quad \text{(fundamental)}$$

$$\nu_2 = 3\nu \quad \text{(third harmonic or first overtone)}$$

$$\nu_3 = 5\nu$$

$$\nu_n = (2n-1)\nu$$

2. In an open organ pipe both odd and even harmonics are present.

$$\nu'_1 = \frac{V}{2L} = \nu' \quad \text{(first harmonic)}$$

$$\nu'_2 = 2\nu' \quad \text{(second harmonic or first overtone)}$$

$$\nu'_3 = 3\nu'$$

$$\nu'_n = (2n-1)\nu'$$

3. Resonance tube: If l_1 and l_2 are the first and second resonance length with a tuning fork of frequency 'ν' then the speed of sound.

$v = 4v(L_2 + 0.3D)$

where, D = internal diameter of resonance tube

$v = 2v(l_2 + l_1)$

End correction = 0.3D = $\frac{l_2 - l_1}{2}$

Beats Frequency

- Beat frequency = Difference in frequency of two sources = No. of beats per second.

  beat frequency = $|v_1 - v_2|$
- $v_2 = v_1 \pm$ beat
- Beat frequency is always a positive value. This fact can be used to decide about + or – sign in the above equation.

Doppler Effect in Sound

1. If V, V_o, V_s and V_m are the velocity of sound, observer, source and medium respectively, then the apparent frequency

$$\nu = \frac{V + V_m \pm V_o}{V + V_m \mp V_s} \times \nu$$

2. If the medium is at rest ($v_m = 0$), then

$$\nu' = \frac{V \pm V_o}{V \mp V_s} \times \nu$$