Modeling Temporal Reproduction under Punishment Conditions

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Abstract

This study models human temporal reproduction behavior under two conditions: no-punishment and punishment, based on an experiment conducted with human participants (van der Mijn, Damsma, Taatgen, & van Rijn, 2021). The computational model employs a pulse-based internal clock, memory blending, and a feedback mechanism to simulate and reproduce the experimental results. By accurately capturing key patterns observed in the original experiment, such as adjustments in reproduced duration and variability across conditions, the model provides a framework for understanding the cognitive processes underlying temporal reproduction. While the model closely replicates the human data, discrepancies in variability and adjustment magnitude suggest areas for future refinement.

1 Introduction

How do humans perceive and reproduce time intervals? Temporal reproduction tasks provide a key tool to investigate this question, offering insights into how the human brain encodes, stores, and retrieves temporal information. Such tasks are fundamental to understanding cognitive processes underlying time perception, an ability critical for activities ranging from speech and motor control to decision-making (Gibbon, 1977; Block, 2014).

One prevailing theory in this domain is Scalar Expectancy Theory (SET), which posits that time perception relies on an internal clock composed of three stages: a pacemaker, an accumulator, and a reference memory (Gibbon, 1977). According to SET, variability in temporal reproductions arises from noise introduced during the accumulation and retrieval of time intervals. However, cognitive processes such as memory blending, where individuals integrate multiple past experiences, and decision-making strategies also play a role in shaping temporal behavior (Block, 2014; Wearden, 2004).

A key challenge in this domain lies in understanding how external feedback, such as punishment or reward, influences temporal reproduction. While previous research has demonstrated that feedback can reduce variability and lead to more cautious reproductions (van der Mijn et al., 2021), questions remain regarding the underlying mechanisms. For example, how do memory blending and feedback interact to produce the observed behavioral patterns? Are existing models sufficient to account for these dynamics?

This study addresses these questions by building a computational model to simulate and reproduce the results of a temporal reproduction experiment conducted under two conditions: no-punishment and punishment (van der Mijn et al., 2021). The model incorporates a pulse-based internal clock, memory blending, and a feedback mechanism to capture the adaptive strategies observed in human participants. Specifically, we hypothesize that:

1. The model will replicate the upward adjustment in reproduced durations and the reduction in variability observed under punishment conditions.

2. The model will capture the relationship between variability and adjustment magnitude, reflecting cautious behavior in response to feedback.

By testing these hypotheses, we aim to contribute to a deeper understanding of how feedback mechanisms interact with cognitive processes to shape temporal reproduction behavior.

2 The Model

In this section, we detail the rationale and implementation of the model. We use a discrete pulse representation to encode time intervals, integrate past experiences through memory blending, and incorporate a punishment mechanism that progressively shifts reproduced times if the reproduced duration is too short (Block, 2014).

2.1 Rationale and Implementation Details

Pulse Representation. The model assumes an internal "clock" where timing is represented by pulses whose durations lengthen slightly after each pulse. This approach is inspired by the scalar expectancy theory, which posits that time is represented proportionally (Gibbon, 1977).

Noise Functions. Empirical research shows that human timing is inherently variable. We capture this variability with a log-odds noise function (Wearden, 2004):

$$noise(s) = s * log((1 - rand)/rand) \qquad (1)$$

where `rand` is drawn from a uniform distribution, yielding fluctuations that simulate moment-to-moment variability.

Time-to-Pulses and Pulses-to-Time. Two helper functions, `time_to_pulses` and `pulses_to_time`, translate between continuous time and discrete pulse counts. This approach aligns with prior studies exploring pulse-based timing models (Gibbon, 1977). They follow the below equation:

$$\text{pulseduration}_{t1} = a \cdot \text{pulseduration}_{t0} + \text{noise} \cdot (b \cdot a \cdot \text{pulseduration}_{t0}) \tag{2}$$

where:

- $\text{pulseduration}_{t0}$ is the current pulse duration with $\text{pulseduration}_{t1}$ being the immediate next pulse duration (the starting pulse duration is 0.011 seconds).
- a is a constant with the value 1.1.
- noise represents a value obtained from a logistic noise function.
- b is a constant with the value 0.015.

Memory Blending. The model stores an internal "memory trace" of pulse counts for each reproduced time. It retrieves and "blends" pulses from previous trials to form a new estimate, simulating how humans integrate multiple past experiences for current judgments (Block, 2014).

Feedback Mechanism. Feedback is an essential component of temporal reproduction tasks. The feedback mechanism used in this model evaluates whether the reproduced duration deviates from the target time. This mechanism has been adapted from prior studies on human timing under feedback (van der Mijn et al., 2021; Wearden, 2004). The feedback mechanism in the non-punishment block evaluates whether the reproduced duration deviates by more than from the target time. If the reproduced duration is too slow, the model increases the retrieved pulse count (approximately 21 pulses) by drawing an additional number of pulses from a Gaussian distribution with mean of 3 and standard deviation of 3, ensuring a minimum increment of zero. Conversely, if the reproduction is too fast, the model reduces the retrieved pulse count by subtracting a number of pulses sampled from the same Gaussian distribution, also ensuring a minimum removal of zero. In the punishment block, if the reproduced duration is too fast, an pulse is added to the retrieved blended pulse count. This adjustment is drawn from a Gaussian distribution with a mean of 5 and a standard deviation of 1, ensuring a minimum increment of zero. Conversely, if the reproduced duration is too slow, the retrieved pulse count is reduced by a value sampled from a Gaussian distribution with a mean of 1 and a standard deviation of 1, also with a minimum reduction of zero. This mechanism simulates how participants exhibit increased caution and mindfulness when under punishment conditions, adapting their reproductions accordingly.

2.2 Algorithmic Flow

For each subject, the algorithm then enters a loop over trials that spans two blocks: the first without punishment and the second with punishment. But first the model begins with 5 training trials where the 750ms target duration is each time converted to pulse using the `time_to_pulses` function and saved to the model memory. This training trials begin with an initial random interval between 1000ms and 2000ms, then the target duration of 750ms and followed by a 1000ms time internval.

Continuing into the trainging trials, during each trial, the model retrieves a blended pulse count from memory, which represents an average of previous experiences with the target interval. This blended pulse count is then converted back into a time duration using the `pulses_to_time` function. The reproduced time is recorded, and based on whether the trial falls within the non-punishment or punishment block, as well as whether the reproduced duration is too fast or too slow relative to the target, pulses are either added or removed from the retrieved pulse count using the previously described feedback mechanism. This blended pulse, modified by punishment, is then stored in memory. Similar to the training trials, the algorithm waits for an interval of 1000 ms before proceeding to the next trial. This process is repeated for a total of 50 trials in the non-punishment block and 50 trials in the punishment block.

3 Results

3.1 Mean Produced Duration by Block

Figure 1 shows the mean reproduced duration by block for the baseline data, while Figure 2 depicts the corresponding data from the updated model. In the baseline data, participants slightly underestimated the target duration in Block 1 (no punishment) and adjusted their reproductions upward in Block 2 (punishment). The updated model accurately mirrors these trends, capturing both the initial underestimation and the subsequent increase in reproduced duration due to the punishment mechanism.

The generated model data exhibit trends consistent with the baseline data but with subtle differences in spread and central tendencies. For Block 1 (no punishment), the generated data show a similar spread to the baseline, with values ranging approximately from 700 to 850 ms. In Block 2 (punishment), the generated data accurately capture the upward adjustment in reproduced duration, centering around higher values, with a spread from 750 to 900 ms, which closely aligns with the baseline data. These results demonstrate that the model effectively replicates the cautious adjustments participants exhibit under punishment conditions.

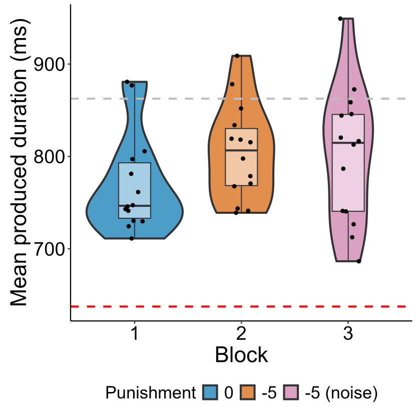

Figure 1: Mean reproduced duration by block (baseline data). Dashed red line: 750 ms target.

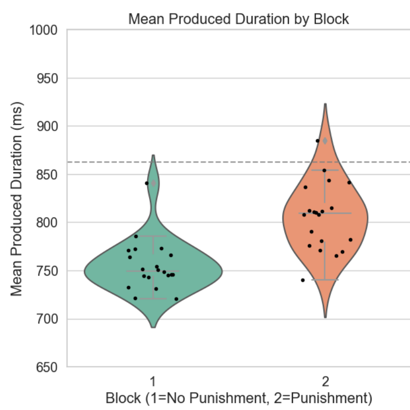

Figure 2: Mean reproduced duration by block (updated model data). Dashed red line: 750 ms target.

3.2 CV of Produced Duration by Block

Figure 3 shows the coefficient of variation (CV) for the baseline data across blocks, while Figure 4 presents the corresponding CV data generated by the updated model. In the baseline data, Block 1 represents the no-punishment condition, showing a wider spread in variability, while Block 2, under the punishment condition, exhibits reduced variability due to the effect of punishment.

The model captures these trends with remarkable accuracy. For Block 1, the generated CV data closely replicate the observed variability, demonstrating a similar range and central tendency. In Block 2, the model reflects a reduction in variability, mirroring the cautious behavior of participants under punishment conditions. Although the generated data align with the baseline, the CV spread in Block 2 for the model is slightly narrower than that of the baseline, suggesting a more uniform response to punishment.

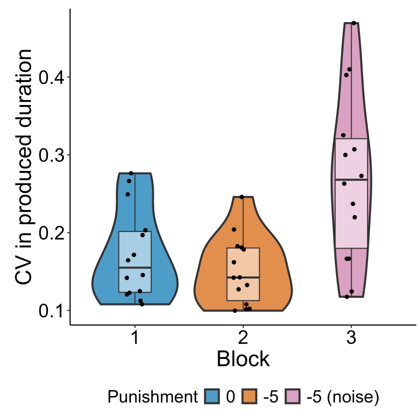

Figure 3: CV in produced duration by block (baseline data). Block 1 represents no-punishment, and Block 2 represents punishment.

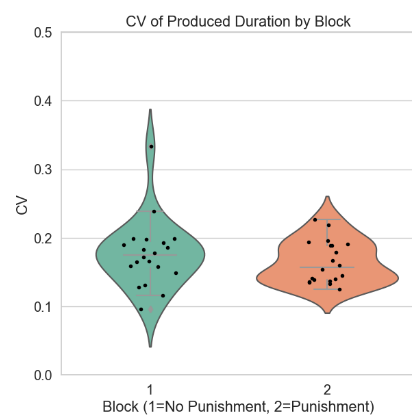

Figure 4: CV in produced duration by block (generated model data). Block 1 represents no-punishment, and Block 2 represents punishment.

3.3 Adjustment vs. Variability Relationship

Figures 5 and 6 illustrate the relationship between variability in the punishment block (Block 2) and the magnitude of adjustment in reproduced durations. In the baseline data, a positive correlation of 0.68 indicates that participants who were more variable also made more significant adjustments to their reproduced durations. The updated model data exhibit a similar positive trend, with a slightly lower correlation value of 0.6, reflecting less pronounced adjustments compared to the baseline.

While both the baseline and model-generated data highlight a clear positive trend, the baseline data display a wider range of adjustments, with some participants making extreme adjustments exceeding 150 ms. In contrast, the model data show a more constrained range of adjustments, with values generally staying below 120 ms. This difference suggests that while the model effectively captures the overall relationship, it underestimates the variability and extremity of adjustments seen in the baseline data.

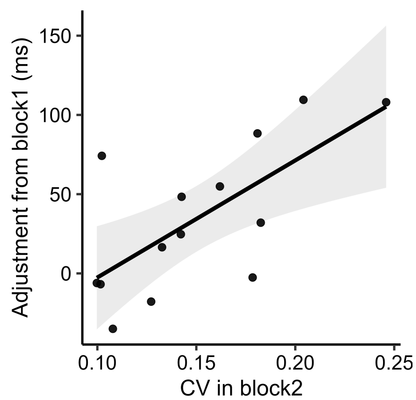

Figure 5: Relationship between CV and adjustment (baseline data).

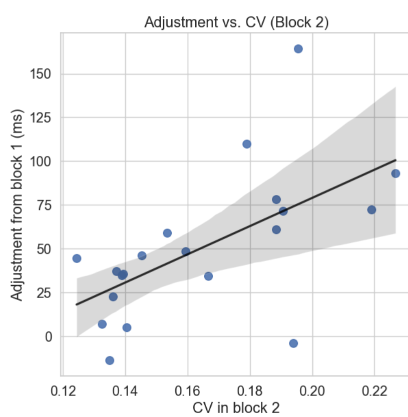

Figure 6: Relationship between CV and adjustment (updated model data).

4 Discussion

This study demonstrates that a computational model can effectively replicate key patterns observed in human temporal reproduction under no-punishment and punishment conditions. The model accurately reproduced the upward adjustment in durations and the reduction in variability under punishment, supporting the idea that feedback mechanisms play a critical role in shaping temporal reproduction behavior. Additionally, the model captured the positive relationship between variability and adjustment magnitude, highlighting its ability to simulate the cautious strategies employed by participants in response to feedback.

However, certain limitations were observed. While the model replicated overall trends, it underestimated the variability and extremity of adjustments seen in the experimental data, particularly in the punishment block. This suggests that additional factors, such as individual differences or more complex noise dynamics, may influence human responses to feedback. Future work could explore incorporating adaptive parameters or reinforcement learning mechanisms to address these discrepancies (Wearden, 2004).

Beyond the specific findings of this study, the results have broader implications for our understanding of time perception and cognitive control. By demonstrating how feedback influences temporal behavior, this study highlights the flexibility of the human cognitive system in adapting to external constraints. These findings also provide support for the broader applicability of Scalar Expectancy Theory (Gibbon, 1977), while pointing to areas where extensions or refinements may be necessary, such as the integration of feedback and decision-making processes.

Looking forward, this work lays the foundation for several potential directions. One avenue involves testing the model in more complex experimental paradigms, such as those involving variable or delayed feedback. Another direction involves applying the model to related domains, such as motor control or speech timing, where temporal accuracy is critical. Finally, integrating neural data into the model could provide insights into the biological underpinnings of time perception and feedback processing.

In conclusion, this study not only advances our understanding of temporal reproduction but also demonstrates the utility of computational modeling in bridging experimental findings and theoretical frameworks. While questions remain, the model provides a robust platform for investigating the interplay between time perception, memory, and feedback in dynamic environments.

References

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