National University of Singapore
School of Computing
CS1010X: Programming Methodology
Semester II, 2024/2025

Recitation 5
Working with Sequences

Python

1. Equality Testing
   - `==` returns `True` if two objects are equivalent.
   - `is` returns `True` if two objects are identical, i.e. they are the same object.
2. Membership Testing
   - x `in` y returns `True` if x is contained in the sequence type y. There are three basic sequence types in Python: tuples, lists, and range objects. We will discuss lists after the midterm break.
3. Range Type
   - The range type represents an immutable sequence of numbers and is commonly used for looping a specific number of times in for loops.
   - `range(start, stop[, step])`: Creates a sequence starting at `start`, at intervals of `step`, up to (but not including `stop`). If `step` is zero, `ValueError` will be raised.
   - `range(stop)` : Creates a sequence starting at 0, at intervals of 1, up to (but not including `stop`).
   - The advantage of the range type over a regular tuple is that a range object will always take the same (small) amount of memory, no matter the size of the range it represents (as it only stores the start, stop and step values, calculating individual items and subranges as needed), i.e. $O(1)$ space.

Problems

1. Evaluate the following expressions:

```
1 == 1

1 is 1

"foo" == 'foo'

0 == "0"

0 is "0"

False == False
False == 0

(1, 2) == (1, "2")

(1, 2) is (1, 2)

(1, 2, 3, 4, 5) == (1, 2, 3, 4, 5)

(1, 2, 3, 4, 5) == (1, 2, 3, 5, 4)

((1, 2), (2, 3)) == ((1, 2), (3, 2))

x = (1,2)
y = (1,2)
z = x

x is y

x == y

x is z

3 in (1, 2, 3, 4, 5)

(1, 2) in (1, 2, 3, 4, 5)

(2) in (1, 2, 3, 4, 5)

() == ()

(1) == 1

(1, ) == 1

a = ((1,2), (3,4))

b = (x, (3,4))

x in a

x in b
```

2. The function `in` will test if an object is inside a tuple by checking for equivalence. Write a function `contains` that will check if an object is inside a tuple by checking for identity. For example,

```
x = (1,2)
a = ((1,2), (3,4))
b = (x, (3,4))

contains(x,a) => False
contains(x,b) => True
```

Write a function `deep_contains` that will check if an object is nested arbitrarily deep within a tuple. For example,

```
x = (1,2)
a = ((1,2), ((3,4), x), (5,6))

contains(x,a) => False
deep_contains(x,a) => True
```

3. The `accumulate` procedure discussed in lecture is also known as `fold_right`, because it combines the first element of the sequence with the result of combining all the elements to the right. There is also a `fold_left`, which is similar to `fold_right`, except that it combines elements working in the opposite direction:

```
def accumulate(fn, initial, seq):
    if seq == ():
        return initial
    else:
        return fn(seq[0], accumulate(fn, initial, seq[1:]))

def fold_left(fn, initial, seq):
    if seq == ():
        return initial
    else:
        return fn(fold_left(fn, initial, seq[:-1]),seq[-1])
```

   (a) Suppose we also define the following functions:

```
def pair(a, b):
    return (a, b)

def divide(a, b):
    return a/b
```

   What are the values of

```
fold_right(divide, 1,(1, 2, 3))
fold_left(divide,1,(1, 2, 3))
fold_right(pair,(),(1, 2, 3))
fold_left(pair,(),(1, 2, 3))
```

(b) Give a property that `fn` should satisfy to guarantee that `fold_right` (or `accumulate`) and `fold_left` will produce the same values for any sequence.

4. A *queue* is a data structure that stores elements in order. Elements are enqueued onto the tail of the queue. Elements are dequeued from the head of the queue. Thus, the first element enqueued is also the first element dequeued (FIFO, first-in-first-out). The `qhead` operation is used to get the element at the head of the queue.

```
qhead(enqueue(5, empty_queue()))
# Value: 5

q = enqueue(4, enqueue(5, enqueue(6, empty_queue())))

qhead(q)
# Value: 6

qhead(dequeue(q))
# Value: 5
```

(a) Decide on an implementation for *queue*.

(b) Implement `empty_queue`

```
def empty_queue():
```

Order of growth in time? Space?

(c) Implement `enqueue`; a function that returns a new queue with the element `x` added to the tail of `q`.

```
def enqueue(x, q):
```

Order of growth in time? Space?

(d) Implement `dequeue`; a function that returns a new queue with the head element removed from `q`.

```
def dequeue(q):
```

Order of growth in time? Space?

(e) Implement `qhead`; a function that returns the value of the head element of `q`.

```
def qhead(q):
```

Order of growth in time? Space?

5. **Homework:** Suppose `x` is bound to the tuple `(1, 2, 3, 4, 5, 6, 7)`. Using `map`, `filter`, `accumulate` and/or lambdas (as discussed in Lecture), write an expression involving `x` that returns:

   (a) `(1, 4, 9, 16, 25, 36, 49)`

   (b) `(1, 3, 5, 7)`

   (c) `((1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6), (7, 7))`

   (d) `((2, 4), 6)`

   (e) `(((2,), 4), 6)`

   (f) The maximum element of `x`: `7`

   (g) The minimum element of `x`: `1`

   (h) The maximum squared even element of `x`: `36`

   (i) The sum of the square of each value in `x`: `140`

   You are encouraged to provide multiple solutions for the above questions.