DTU Compute
Department of Applied Mathematics and Computer Science

DTU

High-Performance Computing

Introduction to Parallel Computing

Overview

- Parallel Computing
  - The name of the game
  - Programming models
  - Caches revisited
  - Parallel architectures
  - Multi-core everywhere
  - Hardware examples
  - Memory and multi-core

Parallelism is everywhere

- In today's computer installations one has many levels of parallelism:
  - Instruction level (ILP)
  - Chip level (multi-core, multi-threading)
  - System level (SMP)
  - GP-GPUs
  - Clusters

What is Parallelization?

The Name of the Game

What is Parallelization?

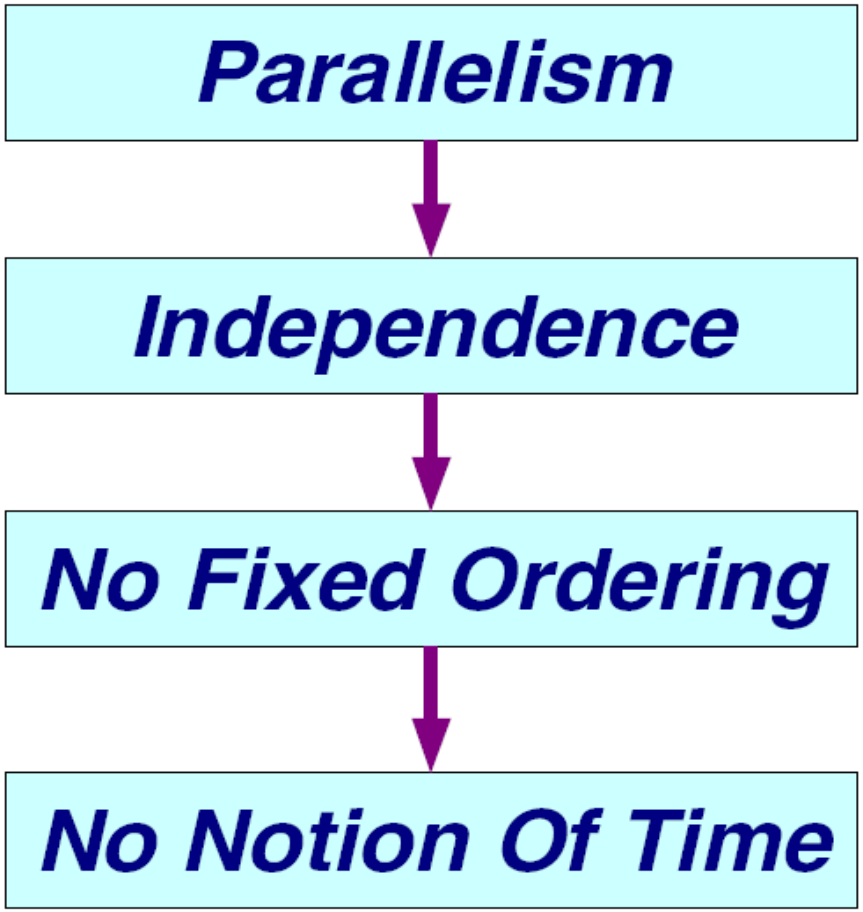
An attempt of a definition:

> "*Something*" is parallel, if there is a certain level of independence in the order of operations

"*Something*" can be:

- A collection of program statements
- An algorithm
- A part of your program
- The problem you are trying to solve

Parallelism – when?



Something that does not follow this rule is not parallel !!!

Parallelism – Example 1

```
for (i=0; i<n; i++]
   a[i] = a[i] + b[i];
```

Every iteration in this loop is independent of the other iterations

Thread	T=1	T=2
1	a[1]=a[1]+b[1]	a[5]=a[5]+b[5]
2	a[2]=a[2]+b[2]	a[8]=a[8]+b[8]
3	a[3]=a[3]+b[3]	a[12]=a[12]+b[12]
4	a[4]=a[4]+b[4]	a[7]=a[7]+b[7]

Parallelism – Example 2

```
for (i=0; i<n; i++]
   a[i] = a[i+1] + b[i];
```

This operation is not parallel !

Proc	T=1	T=2
1	a[1]=a[2]+b[1]	a[4]=a[5]+b[4]
2	a[2]=a[3]+b[2]	a[8]=a[9]+b[8]
3	a[3]=a[4]+b[3]	a[12]=a[13]+b[12]
4	a[5]=a[6]+b[5]	a[7]=a[8]+b[7]

Parallelism – Results example 2

```
Results for P=1
   12512501.0
   12512501.0
   12512501.0
   12512501.0
Results for P=8
   12512508.0
   12512508.0
   12512508.0
   12512508.0
Results for P=32
   12512526.0
   12512530.0
   12512528.0
   12512527.0
Results for P=64
   12512548.0
   12512545.0
   12512549.0
   12512547.0
```

- parallel version of example 2 was run 4 times each on 1, 8, 32 and 64 threads/processors
- Output: sum over all elements of vector a
- Except for P=1, the results are:
  - Wrong
  - Inconsistent
  - NOT reproducable
- This is called a 'Data Race'

Parallelism

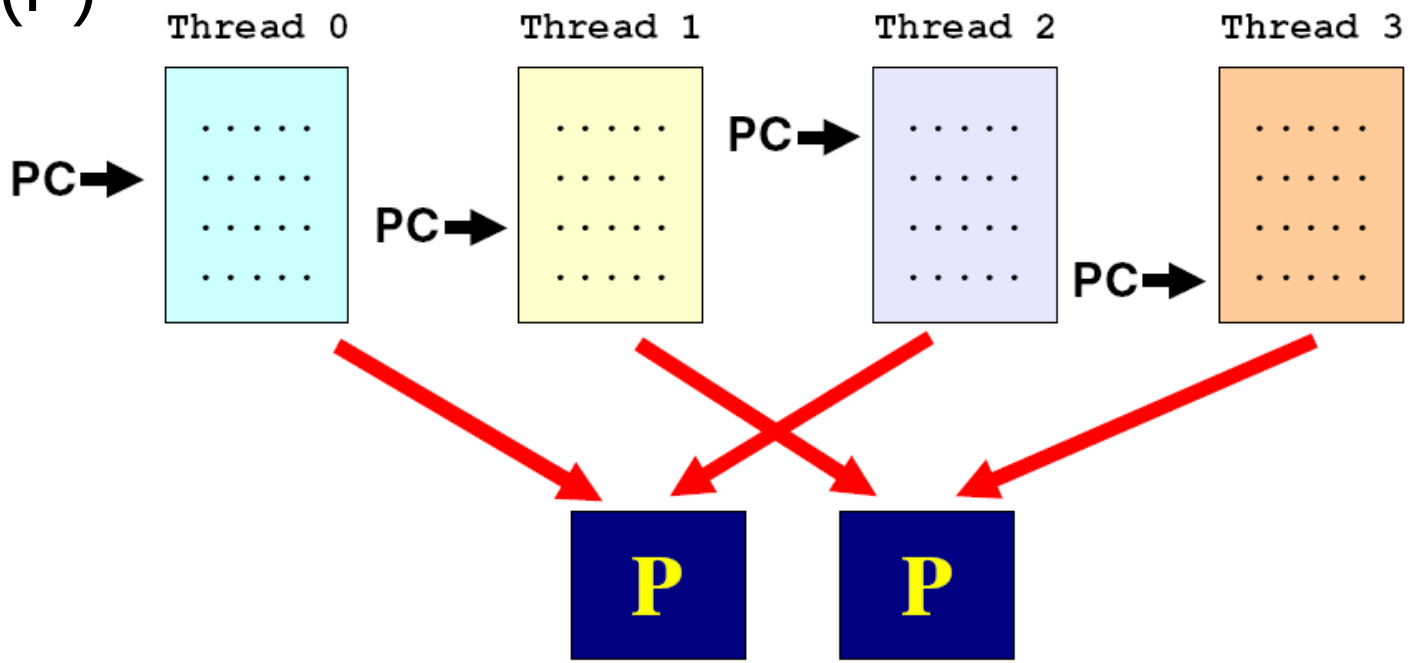
Fundamental problem:

```
for (i = 0; i < n; i++ )
    a[i]= a[i+M] + b[i];
```

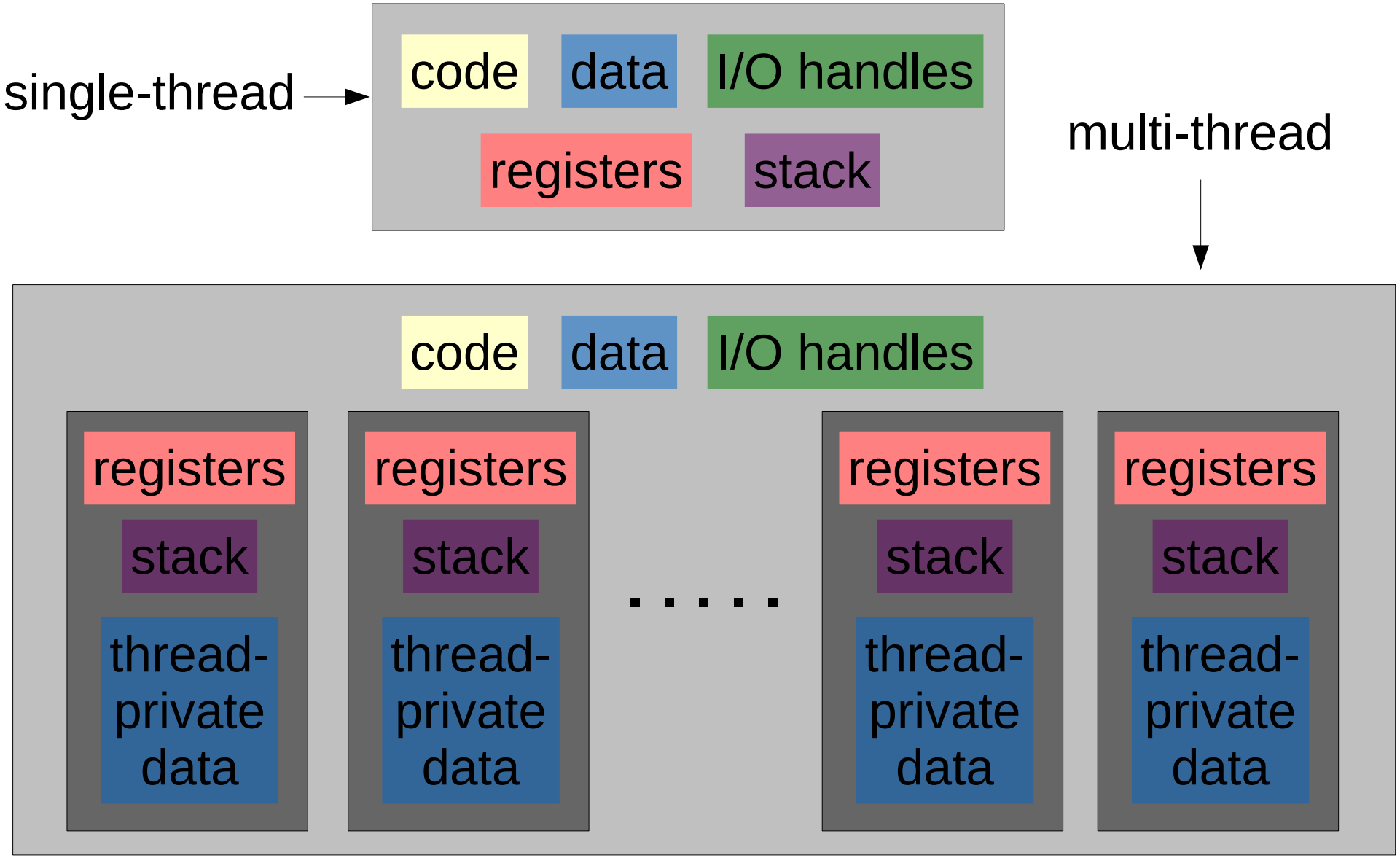
M = 0 : parallel
M >= 1 : not parallel

What is a Thread?

- Loosely said, a thread consists of a series of instructions with it's own program counter (“PC”) and state
- A parallel program will execute threads in parallel
- These threads are then scheduled onto processing units (P)

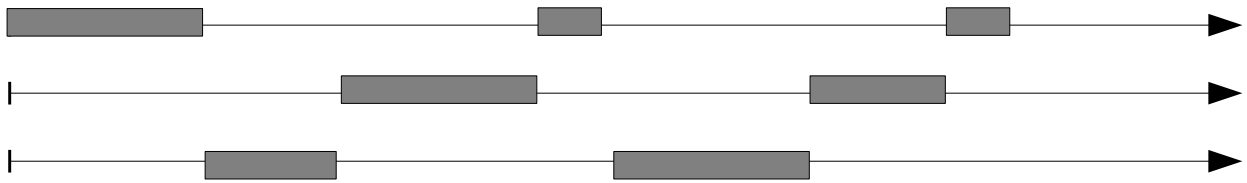


Single- vs. multi-threaded



Parallelism vs Concurrency

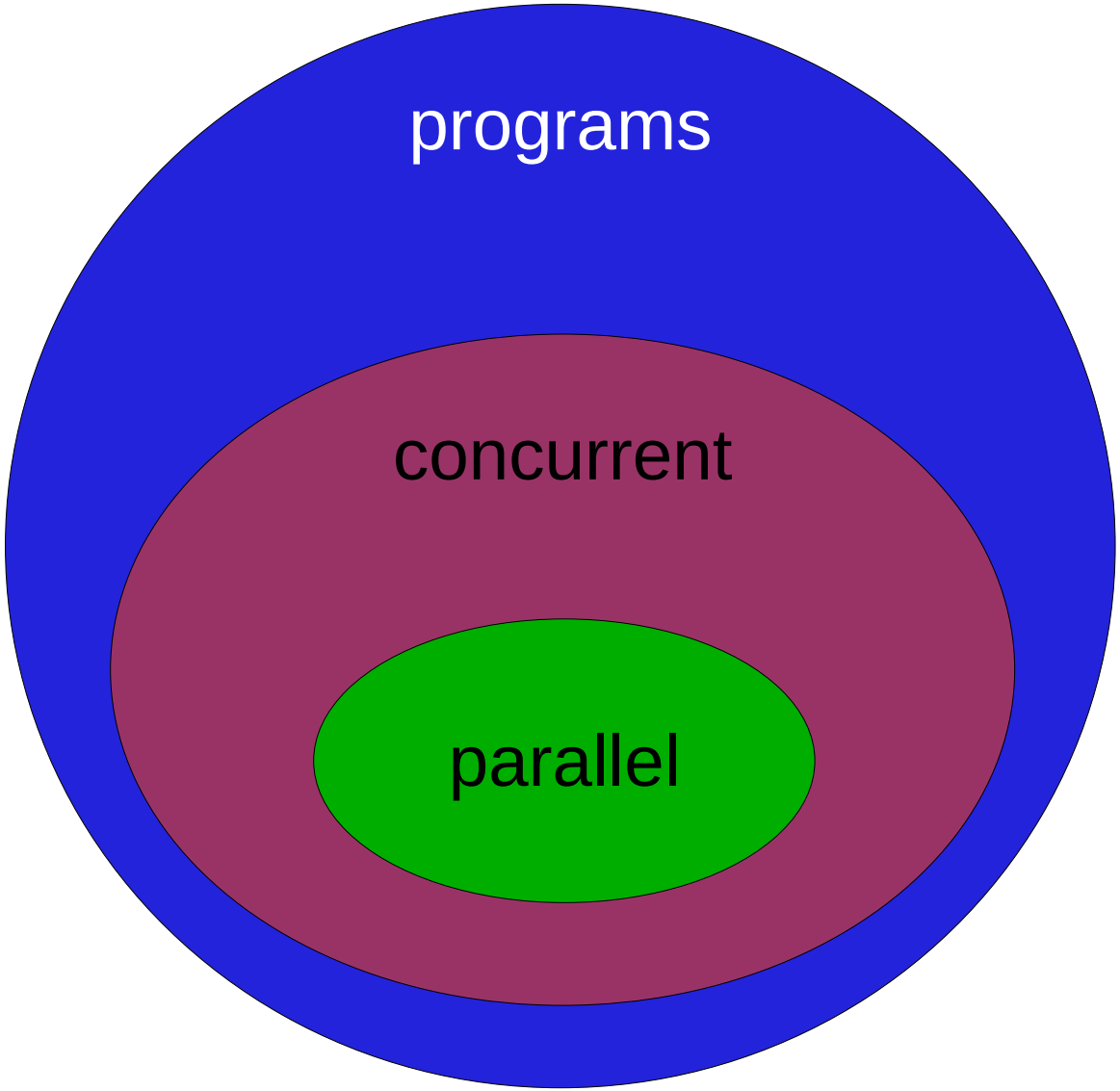
Concurrent, non-parallel execution:



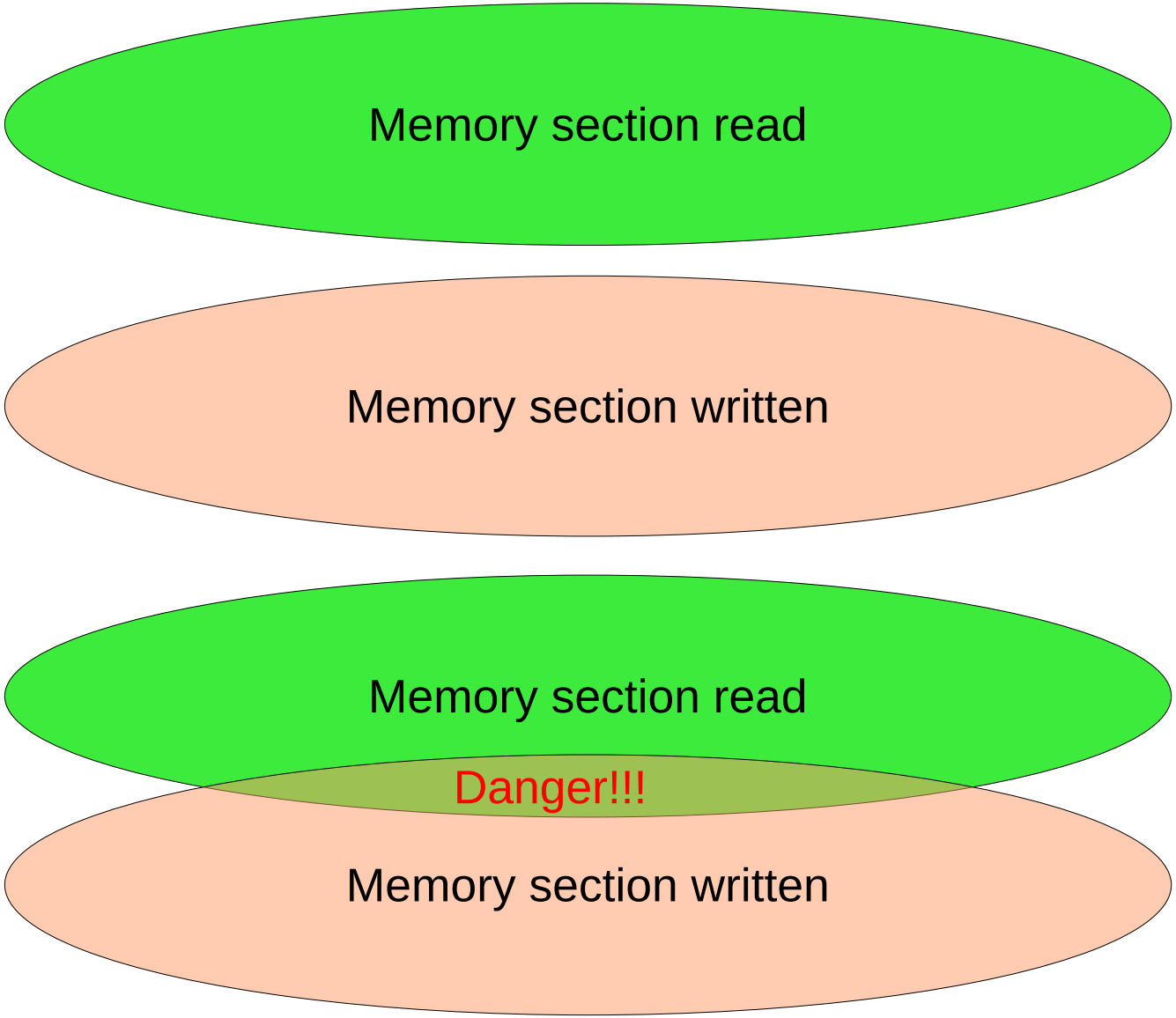
e.g. multiple threads on a single core CPU

Concurrent, and parallel execution

Parallelism vs Concurrency



Parallelism – Memory Access



parallel

not parallel! -
... unless one can protect the overlap area

Parallelism – Data Races

- Race conditions can be nasty – and difficult to detect:
  - Numerical results differ (slightly) from run to run
  - Difficult to distinguish from numerical side effects
  - Changing the number of threads can make the problem disappear – or appear again
  - Shows very often first when using many threads, i.e. late in development

Parallelism – Data Race Detection

- A few data race detection tools are available:
  - Intel: ThreadChecker (now: Inspector XE)
    - part of the Intel OneAPI HPC edition
    - not so easy to use
  - GCC/LLVM: come with a thread checker library
    - compiler option: -fsanitize-thread
    - problem: doesn’t work well with OpenMP ‘out-of-the-box’
    - solution: Archer (more in a later lecture)
- Those tools instrument your code, i.e. a detection run takes substantially longer

Numerical Results

Consider:

A = B + C + D + E

Serial Processing

```
A = B + C

A = A + D

A = A + E
```

Parallel Processing

Thread 0

```
T1 = B + C

T1 = T1 + T2
```

Thread 1

```
T2 = D + E
```

- ***The roundoff behaviour is different and so the numerical results may be different too***
- ***This is natural for parallel programs, but it may be hard to differentiate it from an ordinary bug ....***

Basic concepts

- Consider the following code with two loops

```
for (i = 0; i < n; i++)
    a[i] = b[i] + c[i];
```

```
for (i = 0; i < n; i++)
    d[i] = a[i] + e[i];
```

- Running this in parallel over i might give the wrong answer.

Basic concepts – the barrier

- The problem can be fixed:

```
for (i = 0; i < n; i++)
    a[i] = b[i] + c[i];
```

wait!

```
for (i = 0; i < n; i++)
    d[i] = a[i] + e[i];
```

- The barrier assures that no thread starts working on the second loop before the work on loop one is finished.

Basic concepts – the barrier

When to use barriers?

- To assure data integrity, e.g.
  - after one iteration in a solver
  - between parts of the code that read and write the same variables
- Barriers are expensive and don't scale to a large number of threads

Basic concepts – reduction

- A typical code fragment:

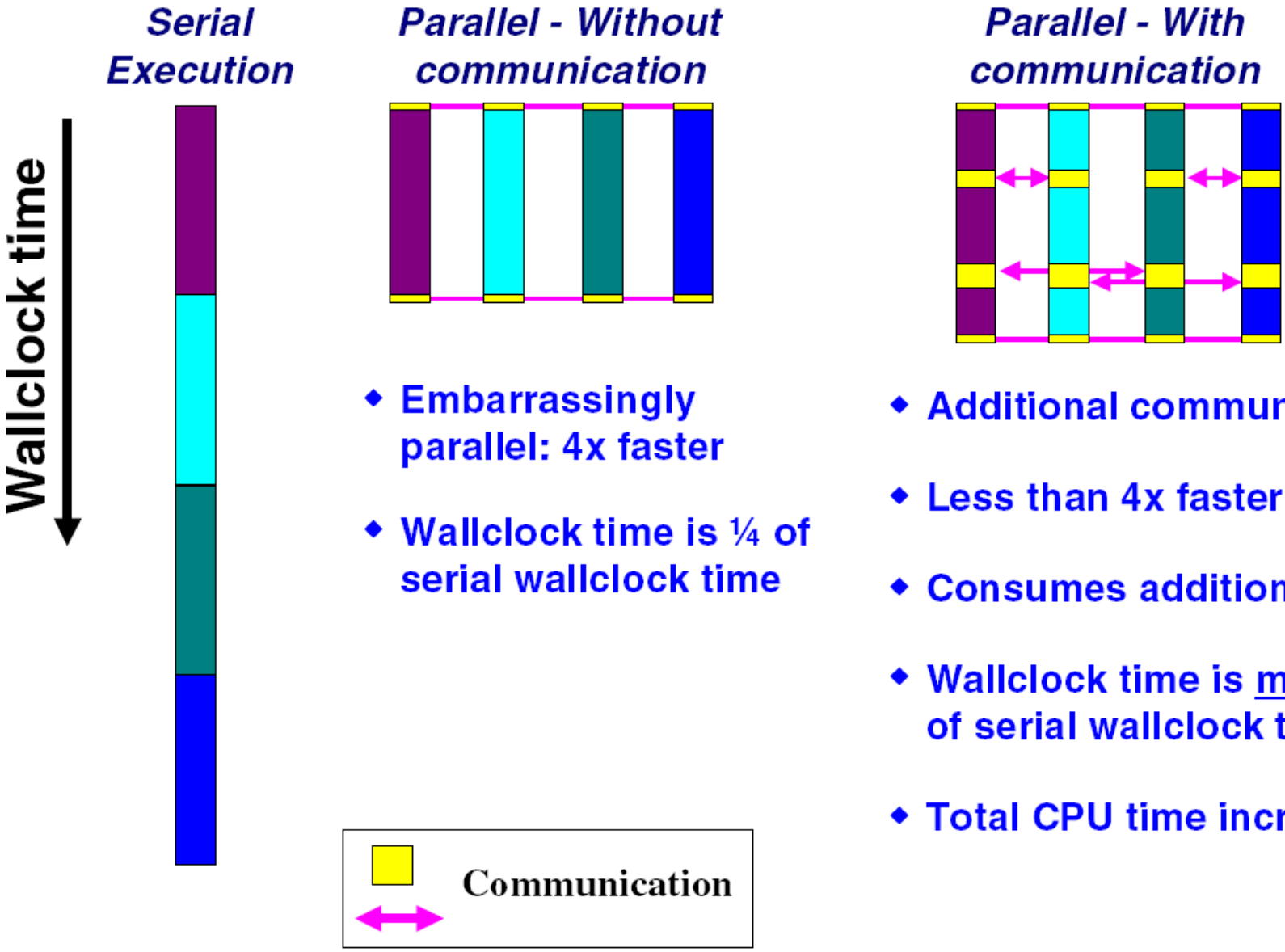
```
for( i = 0; i < n; i++ ) {
    ...
    sum += a[i];
    ...
}
```

- This loop can not run in parallel, unless the update of sum is protected. ☞ serial code
- An operation like the above is called a “reduction” operation, and there are ways to handle this issue (more later...).

Parallel Overhead

- The total CPU time may exceed the serial CPU time:
  - The newly introduced parallel portions in your program need to be executed
  - Threads need time for sending data to each other and for synchronizing (“communication”)
- Typically, things also get worse when increasing the number of threads
- Efficient parallelization is about minimizing the communication overhead

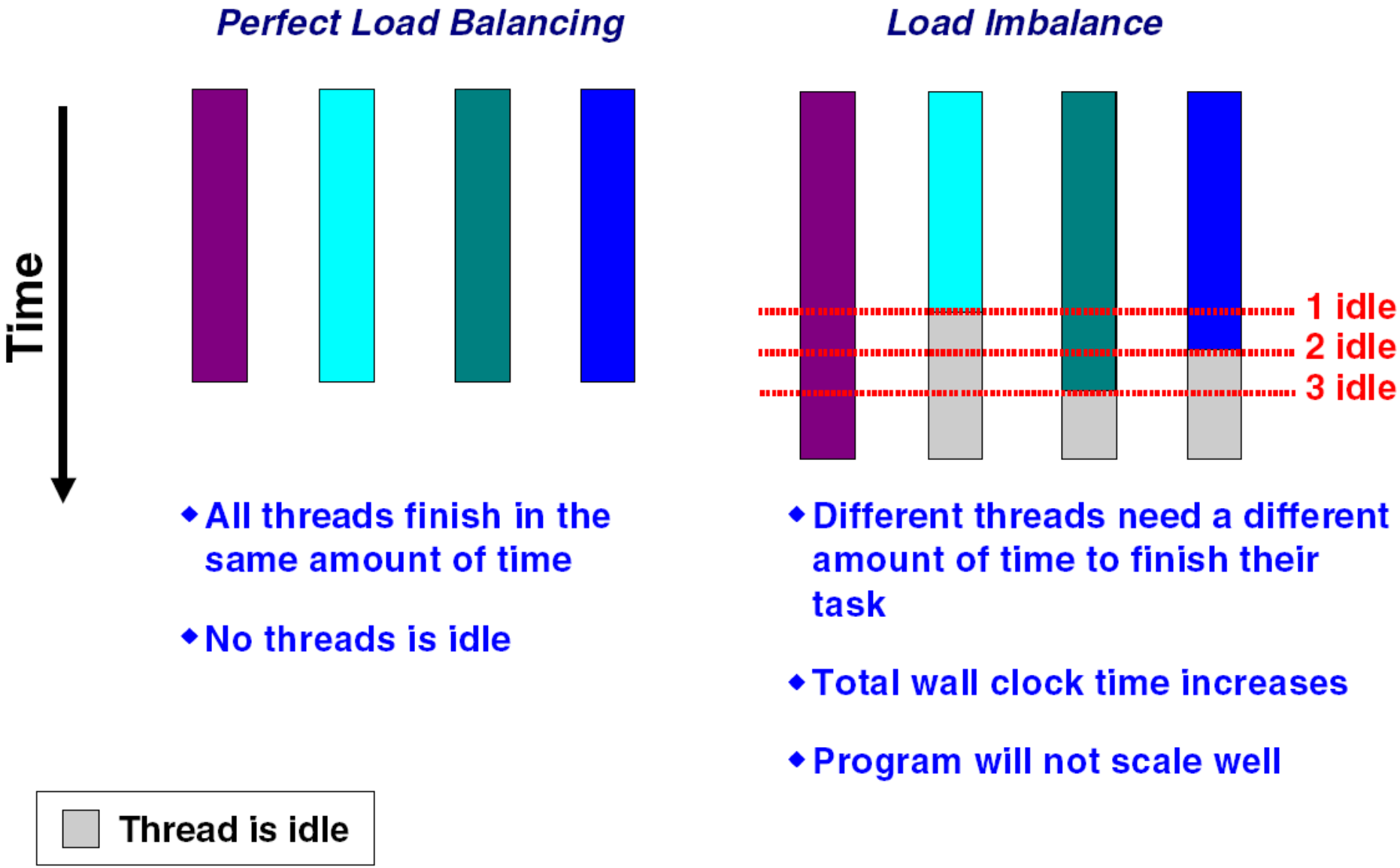
Communication



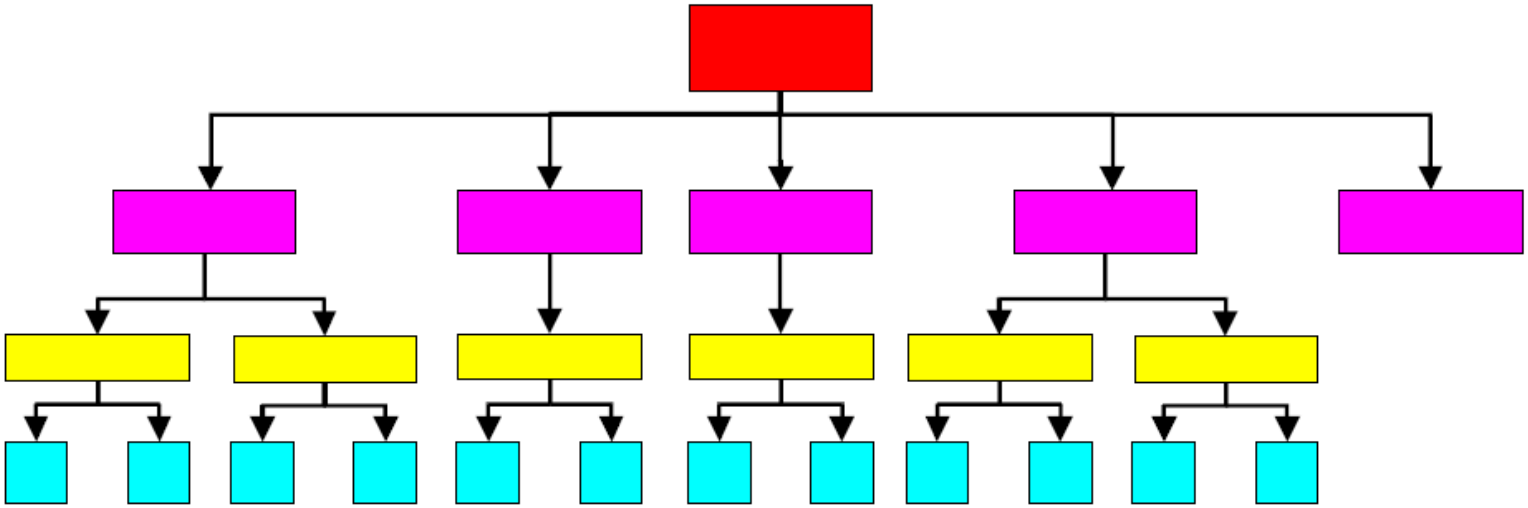
- Embarrassingly parallel: 4x faster
- Wallclock time is ¼ of serial wallclock time

- Additional communication
- Less than 4x faster
- Consumes additional resources
- Wallclock time is more than ¼ of serial wallclock time
- Total CPU time increases

Load Balancing



Dilemma – Where to parallelize?



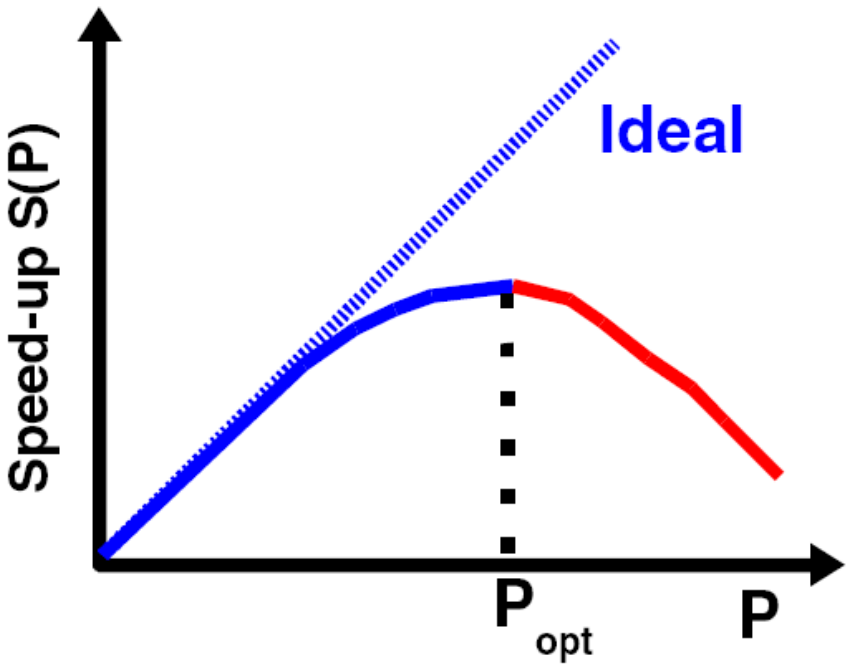
- Parallelization at the highest (■) level:
  - Low communication cost
  - Limited to 5 processors only
  - Potential load balancing issue
- Parallelization at the lowest (■) level:
  - Higher communication cost
  - Not limited to a certain number of processors
  - Load balancing probably less of an issue

Scalability

We distinguish ...

- ... how well a solution to some problem will work when the size of the problem increases.
  - typically associated with algorithmic complexity
- ... how well a parallel solution to some problem will work when the number of processing units (PUs) increases.
  - Strong scaling (speed-up) or weak scaling

Scalability – speed-up & efficiency



- *Define the speed-up S(P) as S(P) := T(1)/T(P)*
- *The efficiency E(P) is defined as E(P) := S(P)/P*
- *In the ideal case, S(P)=P and E(P)=P/P=1=100%*
- *Unless the application is embarrassingly parallel, S(P) will start to deviate from the ideal curve*
- *Past this point P_{opt}, the application will get less and less benefit from adding processors*
- *Note that both metrics give no information on the actual run-time*
- *As such, they can be dangerous to use*

Amdahl's Law

Assume our program has a parallel fraction "f"

This implies the execution time $T(1) := f*T(1) + (1-f)*T(1)$

On P processors: $T(P) = (f/P)*T(1) + (1-f)*T(1)$

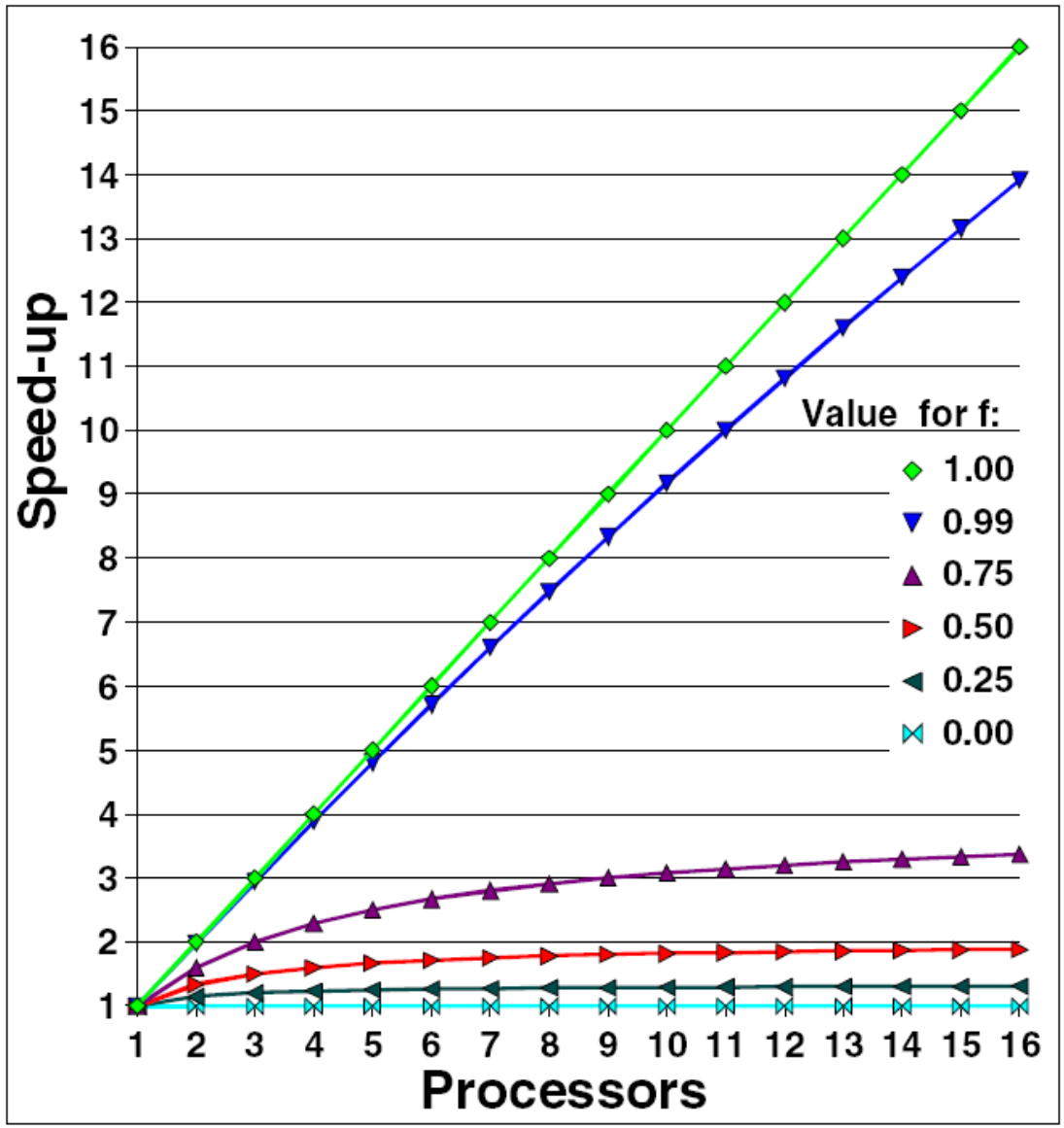
Amdahl's law:

$$S(P) := T(1) / T(P) = 1 / (f/P + 1-f)$$

Comments:

- ***This "law' describes the effect that the non-parallelizable part of a program has on scalability***
- ***Note that the additional overhead caused by parallelization and speed-up because of cache effects are not taken into account***

Amdahl's Law



- *It is easy to scale on a small number of processors*
- *Scalable performance however requires a high degree of parallelization i.e. f is very close to 1*
- *This implies that you need to parallelize that part of the code where the majority of the time is spent*

Amdahl's Law in Practice

We can estimate the parallel fraction "f"

Recall: $T(P) = (f/P)*T(1) + (1-f)*T(1)$

It is trivial to solve this equation for "f":

$$f = (1 - T(P)/T(1))/(1 - (1/P))$$

Example:

```
T(1) = 100 and T(4)=37 => S(4) = T(1)/T(4) = 2.70
f = (1-37/100)/(1-(1/4)) = 0.63/0.75 = 0.84 = 84%
```

Estimated performance on 8 processors is then:

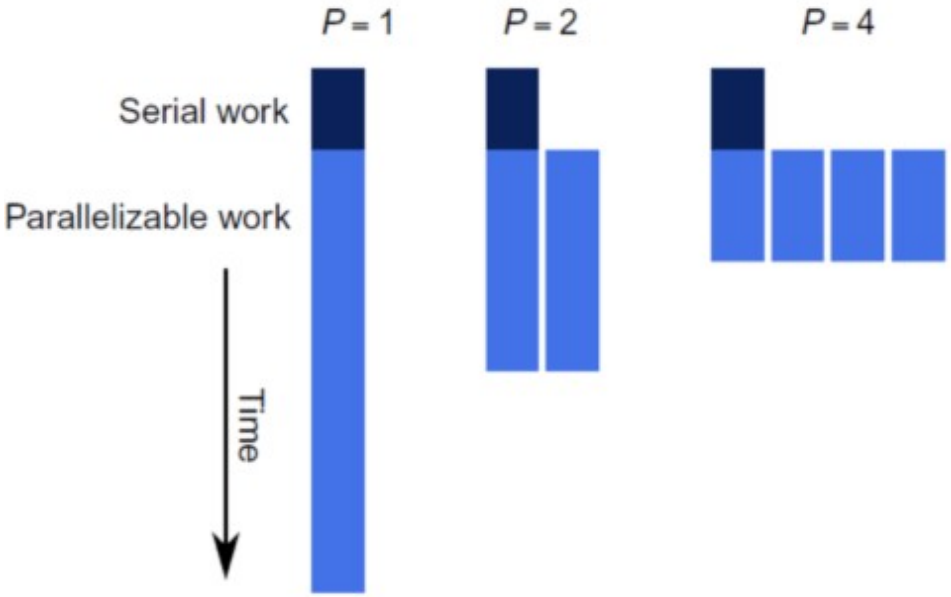
```
T(8) = (0.84/8)*100 + (1-0.84)*100 = 26.5
S(8) = T/T(8) = 3.78
```

Scaling: strong vs. weak

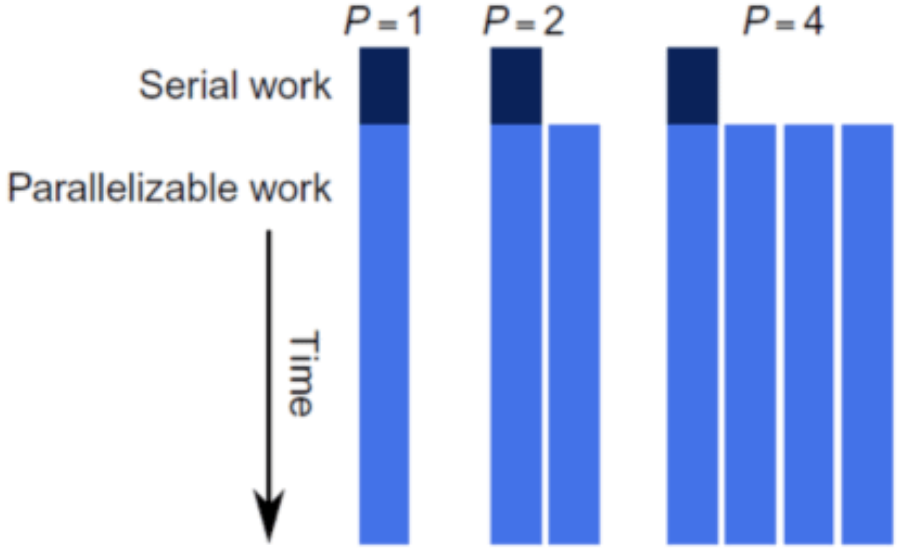
- How does the execution time go down for a fixed problem size by increasing the number of PUs?
  - Amdahl's law ⇒ speed-up, i.e. reduce time
  - also known as “strong scaling”
- How much can we increase the problem size by adding more PUs, keeping the execution time approx. constant?
  - Gustafson's law ⇒ scale-up, i.e. increase work
  - also known as “weak scaling”

Amdahl's vs Gustafson's law

Amdahl: fixed work



Gustafson: fixed work/PU



Amdahl's vs Gustafson's Law

- Amdahl's law
  - Theoretical performance of an application with a ***fixed amount of parallel work*** given a particular number of Processing Units (PUs)
- Gustafson's Law:
  - Theoretical performance of an application with a ***fixed amount of parallel work per PU*** given a particular number of PUs

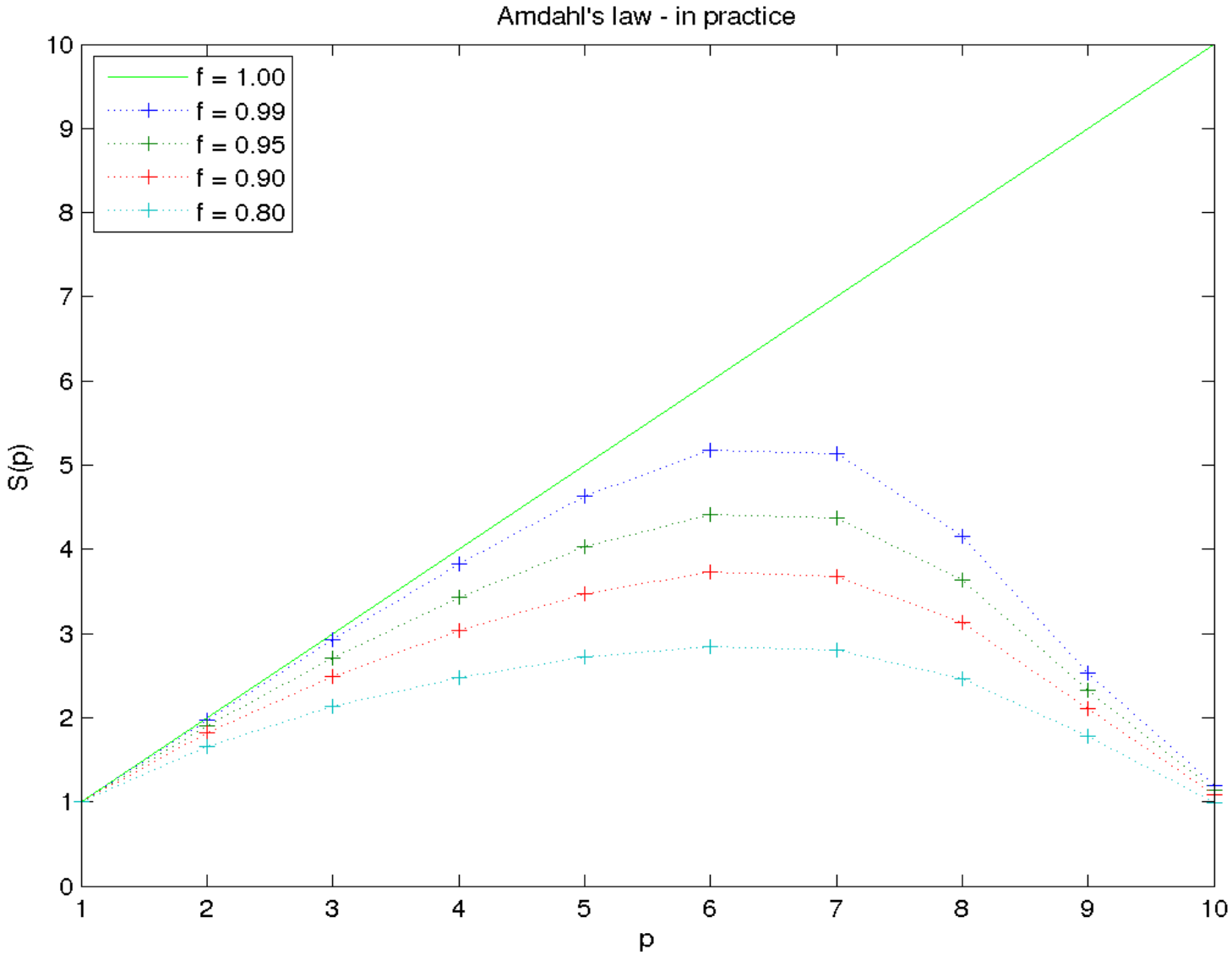
Code scalibility in practice – I

- Although Amdahl and Gustafson provide theoretical upper bounds, eventually real data are necessary for analysis
- Inconsistencies in performance – especially on shared systems – often appear in singular runs
- Best practice: Monitor codes several times and average the results to filter out periods of heavy usage due to other users

Code scalibility in practice – II

- Ideally, HPC codes would be able to scale to the theoretical limit, but ...
  - Never the case in reality
  - All codes eventually reach a real upper limit on speedup
  - At some point codes become “bound” to one or more limiting hardware factors (memory, network, I/O)

Code scalibility in practice – III



What is Parallelization? - Summary

- Parallelization is simply another optimization technique to get your results sooner
- To this end, more that one processor is used to solve the problem
- The “Elapsed Time” (also called wallclock time) will come down, but total CPU time will probably go up
- The latter is a difference with serial optimization, where one makes better use of existing resources, i.e. the cost will come down

Parallel Programming Models

Parallel Programming Models

Two “classic” parallel programming models:

- Distributed memory
  - PVM (standardized)
  - MPI (de-facto standard, widely used)
    - http://mpi-forum.org or http://open-mpi.org/
- Shared memory
  - Pthreads (standardized)
  - C++11 threads
  - OpenMP (de-facto standard) http://openmp.org/
  - Automatic parallelization (depends on compiler)

Parallel Programming Models

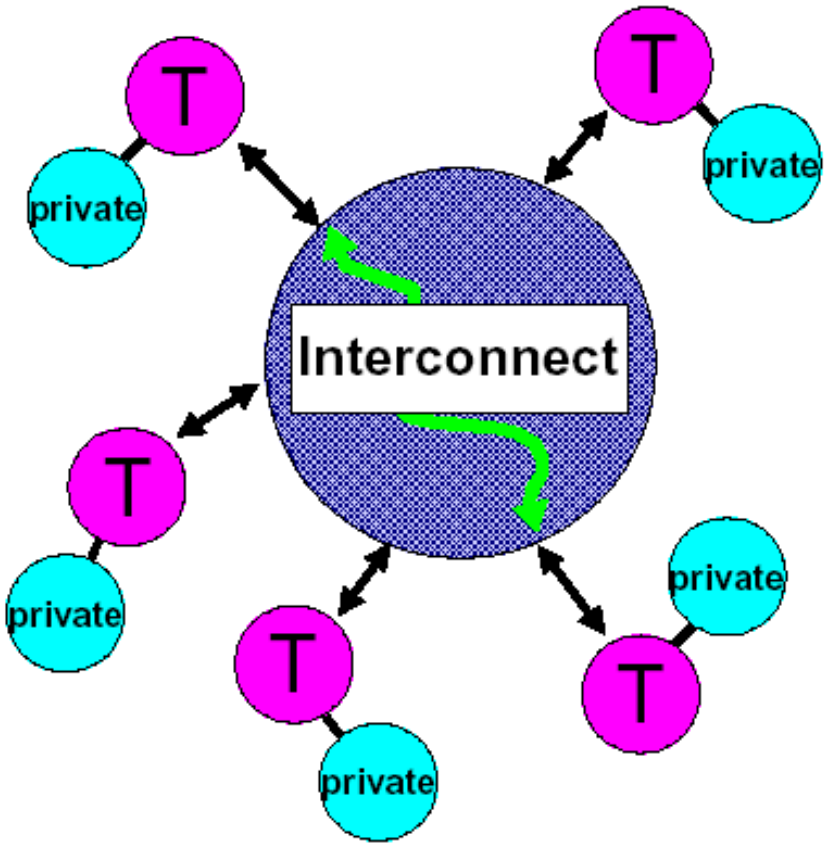
Other programming models

- PGAS (Partitioned Global Address Space):
  - UPC (Unified Parallel C)
  - Co-Array Fortran
- GPUs: massively parallel & shared memory
  - CUDA
  - OpenCL
  - Shader languages
  - OpenMP (target offloading)

Parallel Programming Models

Distributed memory programming model, e.g. MPI:

- all data is private to the threads
- data is shared by exchanging buffers
- explicit synchronization



Parallel Programming Models

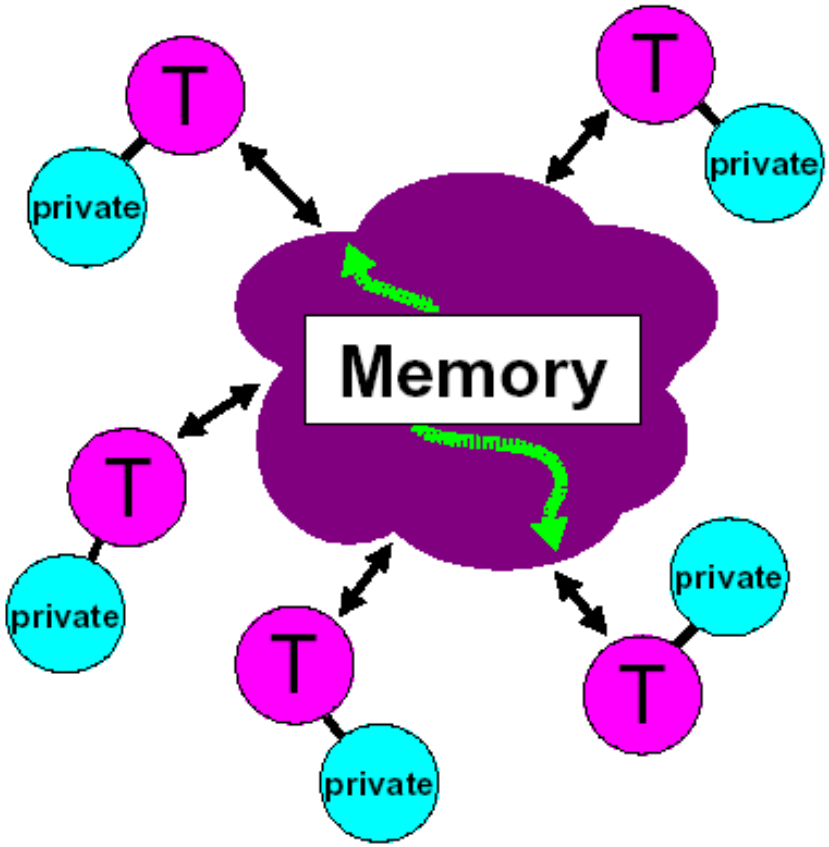
MPI:

- An MPI application is a set of independent processes (aka ranks)
  - on different machines
  - on the same machine
- communication over the interconnect
  - network (network of workstations, cluster, grid)
  - memory (SMP)
- communication is under control of the programmer

Parallel Programming Models

Shared memory model, e.g. OpenMP:

- all threads have access to the same global memory
- data transfer is transparent to the programmer
- synchronization is (mostly) implicit
- there is private data as well



Parallel Programming Models

OpenMP:

- needs an SMP
- but ... with current CPU designs, there is an SMP in every computer
  - multi-core CPUs (CMP)
  - chip multi-threading (CMT)
  - or a combination of both
  - or ... (whatever we'll see in the future)

MPI vs OpenMP

OpenMP version of “Hello world”:

```
#include <stdio.h>

int main(int argc, char *argv[]) {
    #pragma omp parallel
    {
    printf("Hello world!\n");
    } /* end parallel */
    return(0);
}
```

MPI vs OpenMP

```
% gcc -o hello -fopenmp hello.c
% ./hello
Hello world!

% OMP_NUM_THREADS=2 ./hello
Hello world!
Hello world!

% OMP_NUM_THREADS=8 ./hello
Hello world!
Hello world!
Hello world!
Hello world!
Hello world!
```

This behaviour is implementation dependent!

no. of threads: OMP_NUM_THREADS

MPI vs OpenMP

MPI version of “Hello world”:

```
#include <stdio.h>
#include <stdlib.h>
#include "mpi.h"

int main(int argc, char **argv) {
    int myrank, p;
    MPI_Init(&argc, &argv);
    MPI_Comm_rank(MPI_COMM_WORLD, &myrank);
    MPI_Comm_size(MPI_COMM_WORLD, &p);
    printf("Hello world from %d!\n", myrank);
    MPI_Finalize();
    return 0;
}
```

MPI vs OpenMP

MPI version: compile and run

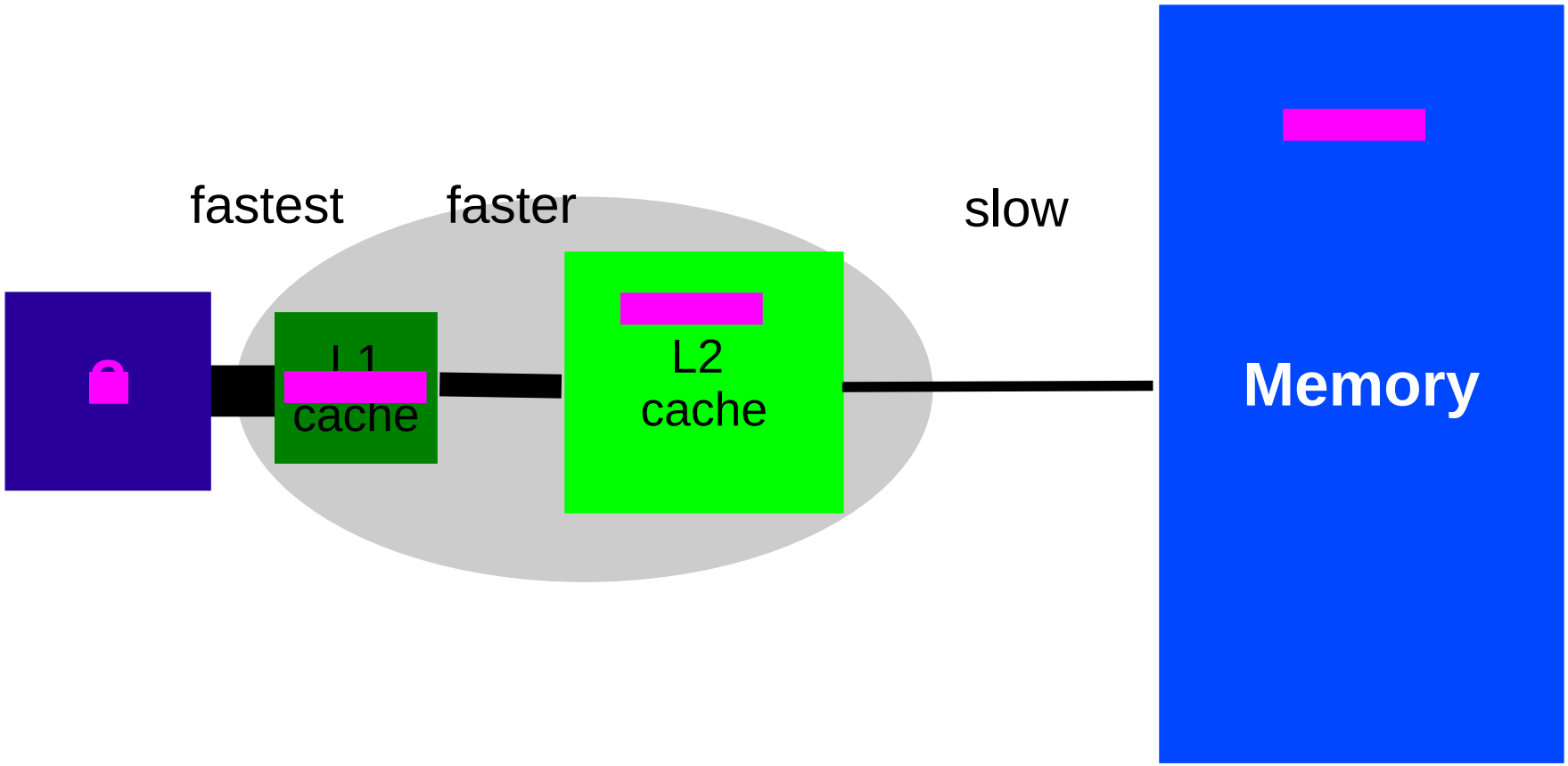
```
$ module load mpi
$ mpicc -o hello_mpi hello_mpi.c

$ ./hello_mpi
Hello world from 0!

$ mpirun -np 4 ./hello_mpi
Hello world from 1!
Hello world from 3!
Hello world from 0!
Hello world from 2!
```

Automatic Parallelization

- Some compilers are capable of generating parallel code for loops that can be safely executed in parallel.
- This is always loop based!
- Compilers:
  - Intel compilers: -parallel
  - Oracle Studio compilers: -xautopar
  - GCC: -floop-parallelize-all -ftree-parallelize-loops=N
    - limited, sets no. of threads (N) at compile time!
- For more information see the manual pages

Caches revisited

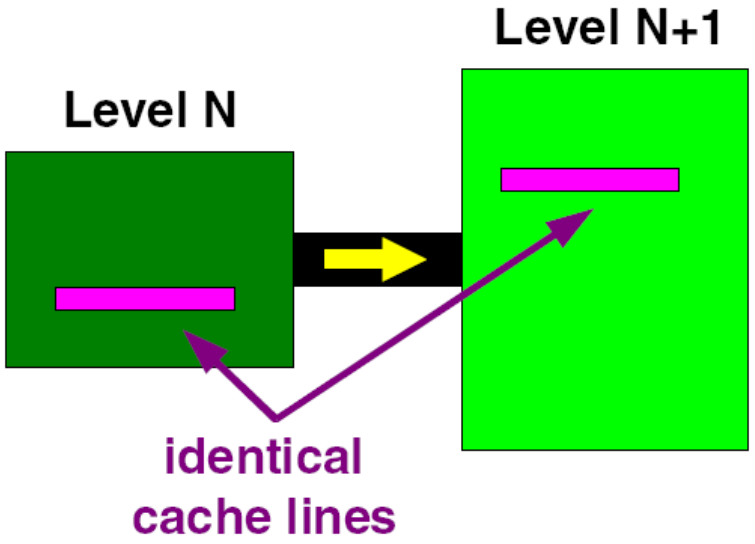
Typical cache based system



How are caches organized?

- Caches contain partial images of memory
- If data gets modified, the state of that data, i.e. the whole cache line, changes
- This has to be made known to the system
- There are two common approaches:
  - Write-through
  - Write-back

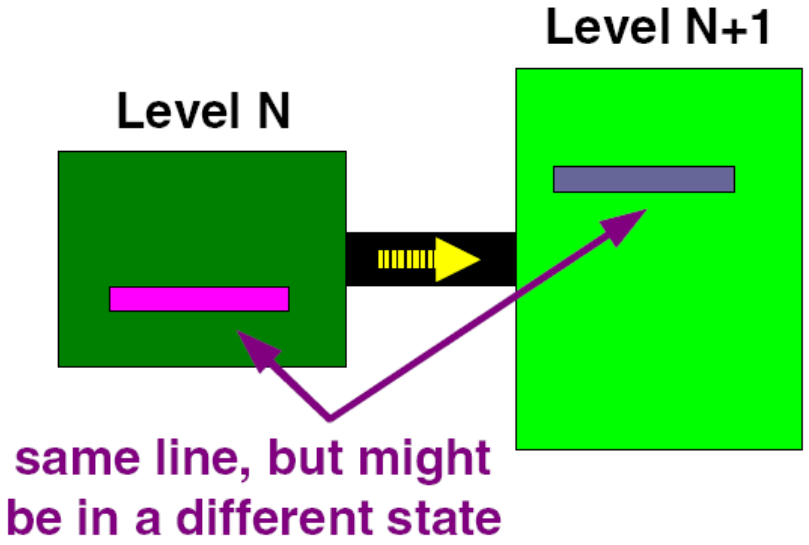
Write-through cache



Notes:
- simple to implement
- easy to find the right copy
- can result in waste of bandwidth

- Always flushes a modified cache line back to a higher level in the memory hierarchy
  - e.g. from L1-cache to main memory
- This assures, that the system always knows where to access the correct cache line

Write-back cache



Notes:
- minimizes cache traffic
- need to keep track of status
- this mechanism is called *'cache coherency'*

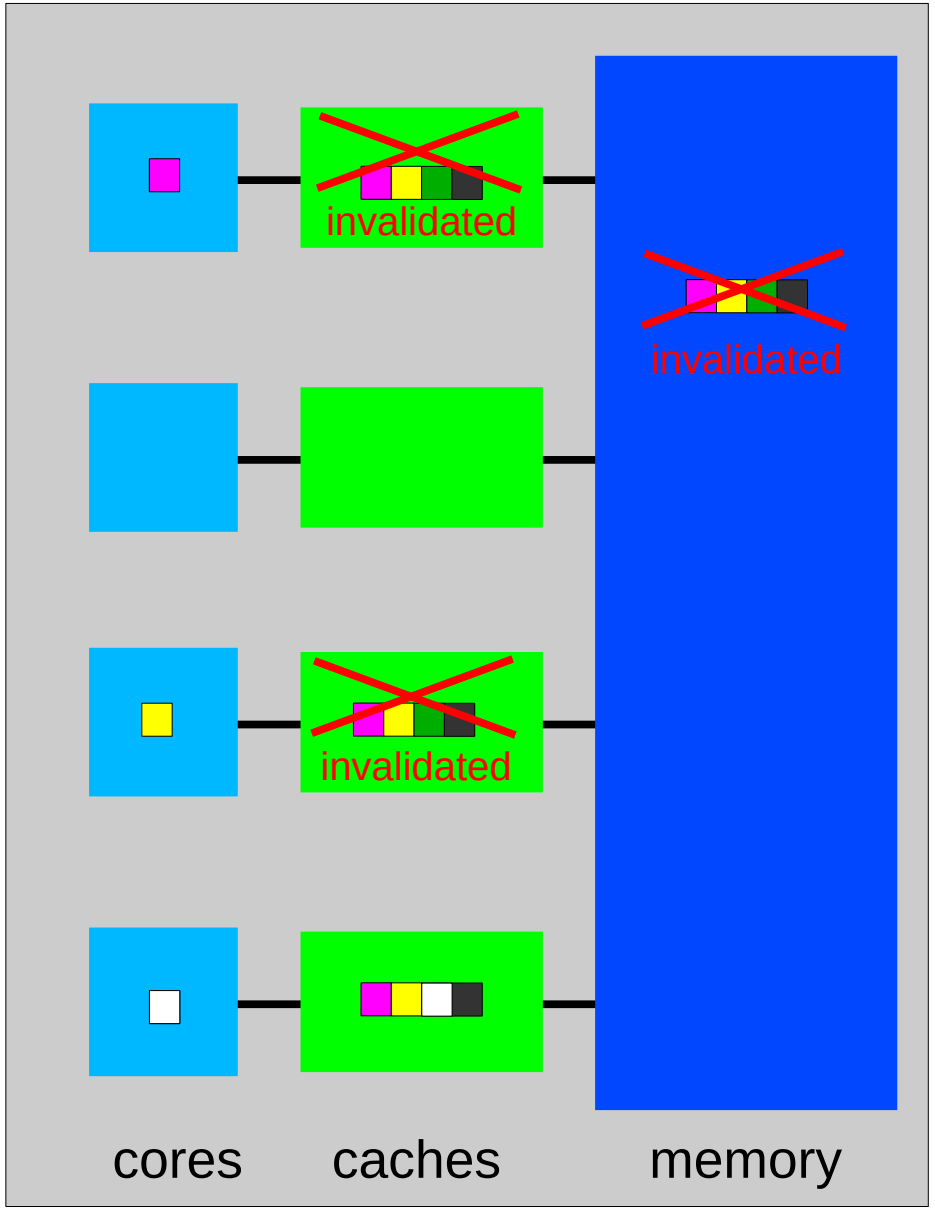
- Write a modified cache line back only if needed
  - capacity issues
  - another cache line maps onto this line
  - another CPU might need this cache line

Caches in MP/multi-core systems

- A cache line always starts in memory
- Over time multiple copies may exist

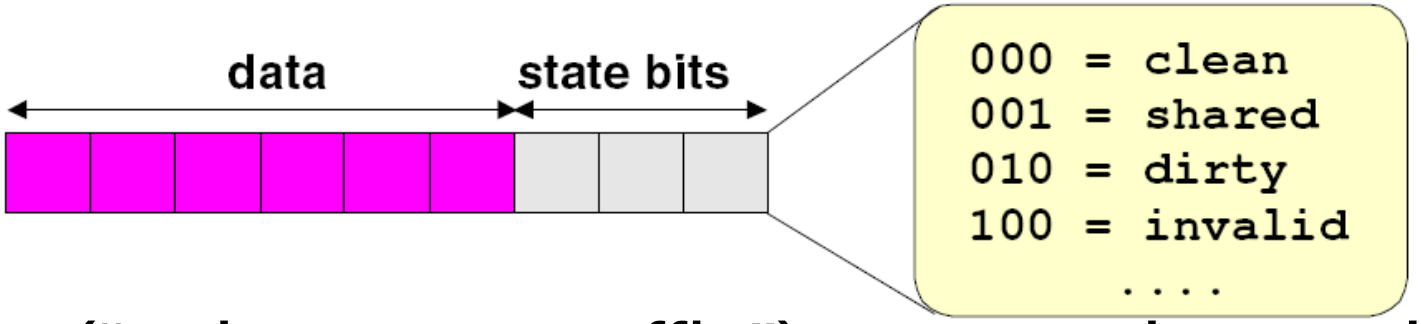
Cache coherency ('cc')

- tracks changes in copies
- assures correct cache line is used
- many implementations
- hardware support to be efficient



Cache coherency ('cc')

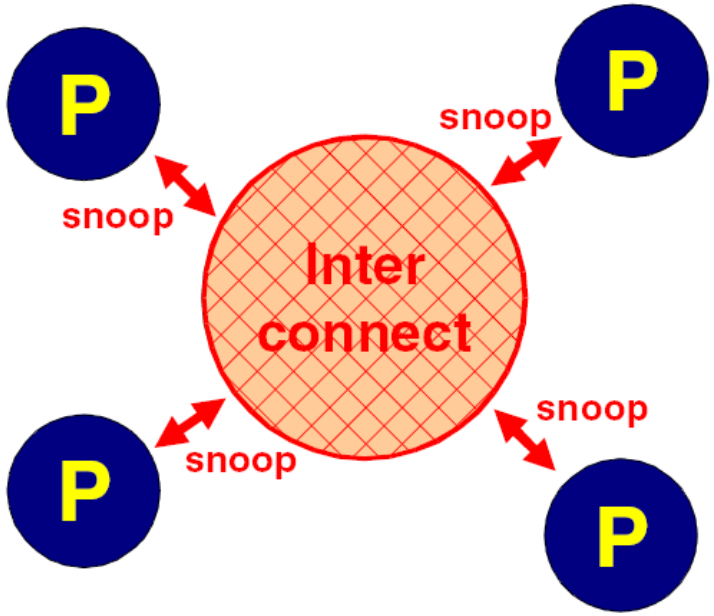
- Needed in write-back cache systems
- Keeps track of the status of all cache lines
- State information:



- Signals (“coherency traffic”) are used to update the state bits of the cache lines
- This allows to build efficient SMP systems

Snoopy based cache coherence

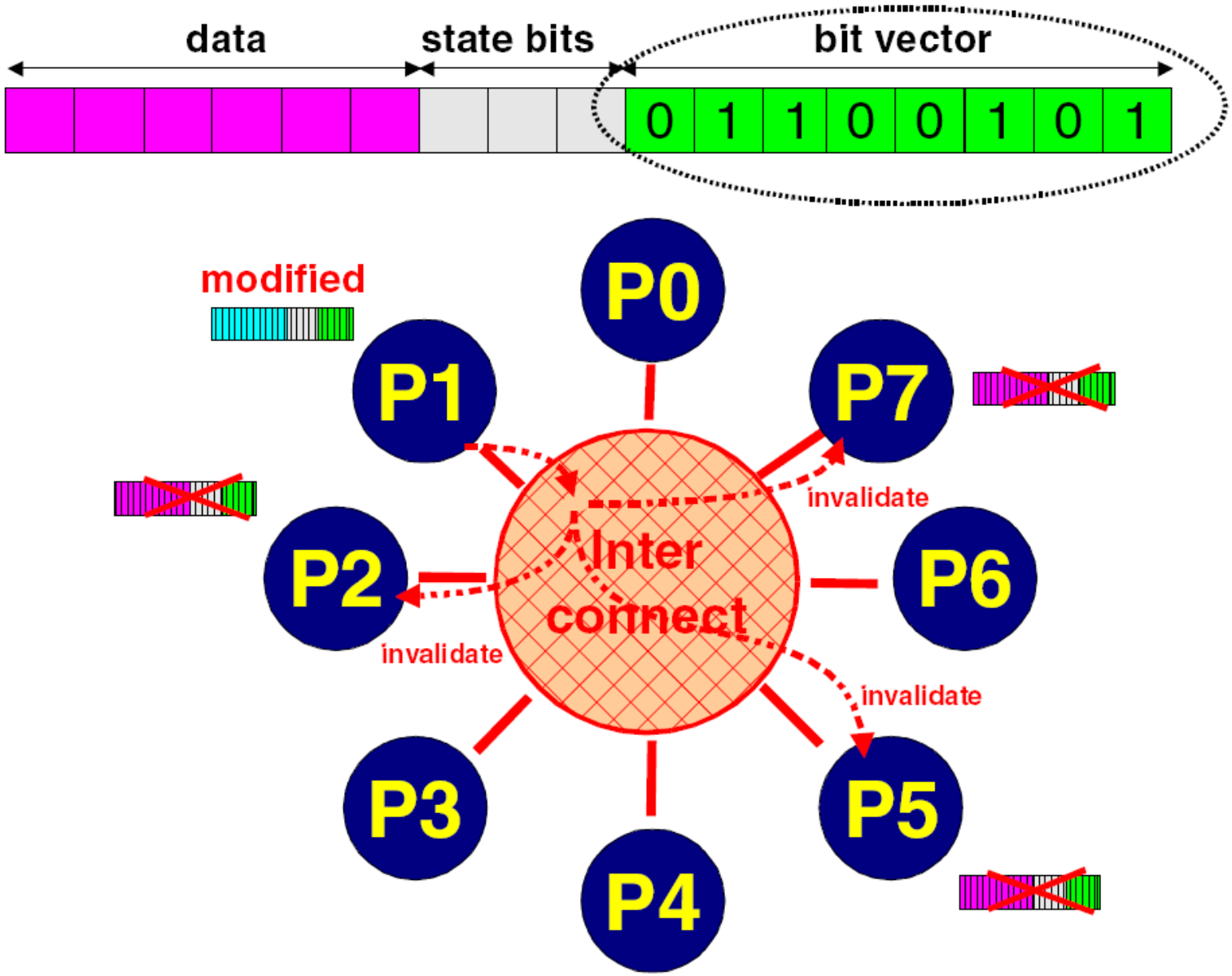
- Also known as “broadcast cache coherence”
  - all addresses are sent to all CPUs
  - result takes only a few cycles
- Advantages:
  - low latency
  - fast cache-to-cache transfer
- Disadvantages:
  - data bandwidth limited by snoop bandwidth
  - difficult to scale to many CPUs



Directory based cache coherence

- Also known as SSM (Scalable Shared Memory)
- point-to-point protocol, i.e. a directory keeps track which CPUs are involved with a particular cache line
- requests are sent to the involved CPUs only
- Advantages:
  - larger bandwidth & scalable to many CPUs
- Disadvantages:
  - longer & non-uniform latency
  - slower cache-to-cache transfer
  - need to store the directory entries

SSM example:

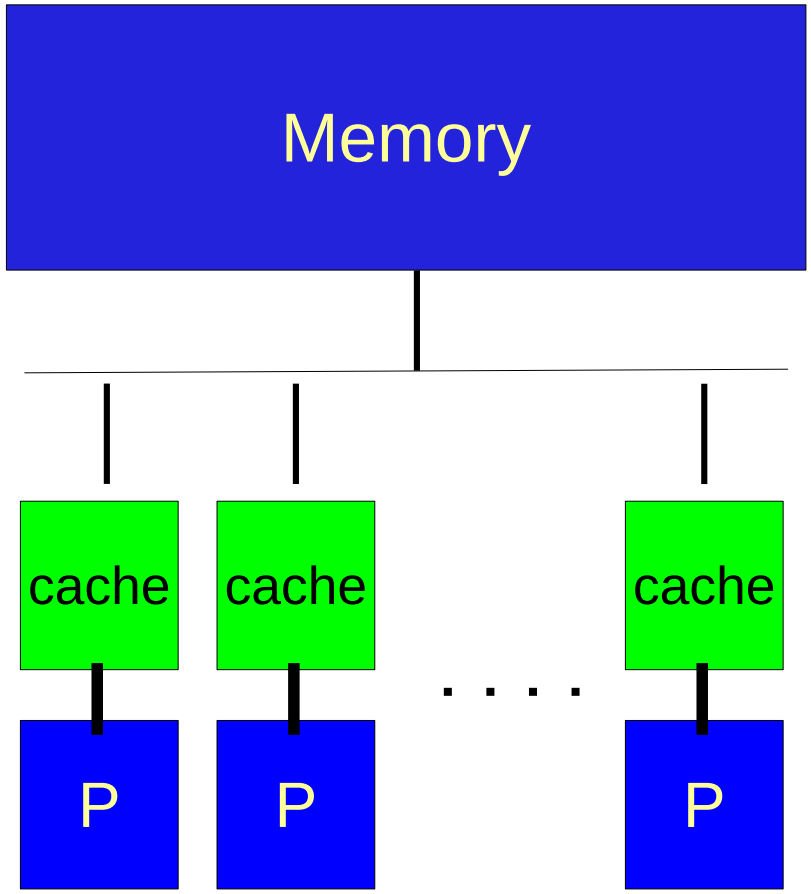

data
state bits
bit vector
0 1 1 0 0 1 0 1
modified
P0
P1
P2
P3
P4
P5
P6
P7
Inter connect
invalidate
invalidate
invalidate

Parallel Architectures

Parallel Architectures

- It is difficult to label systems:
  - most systems share some characteristics, but not all
  - the variety of systems is increasing
- In the (“*historical*”) overview presented here, systems are labelled based on main memory:
  - Shared or Distributed:
    - can all CPUs access all memory, or only a subset?
  - Memory access times
    - uniform vs non-uniform

Uniform Memory Access (UMA)



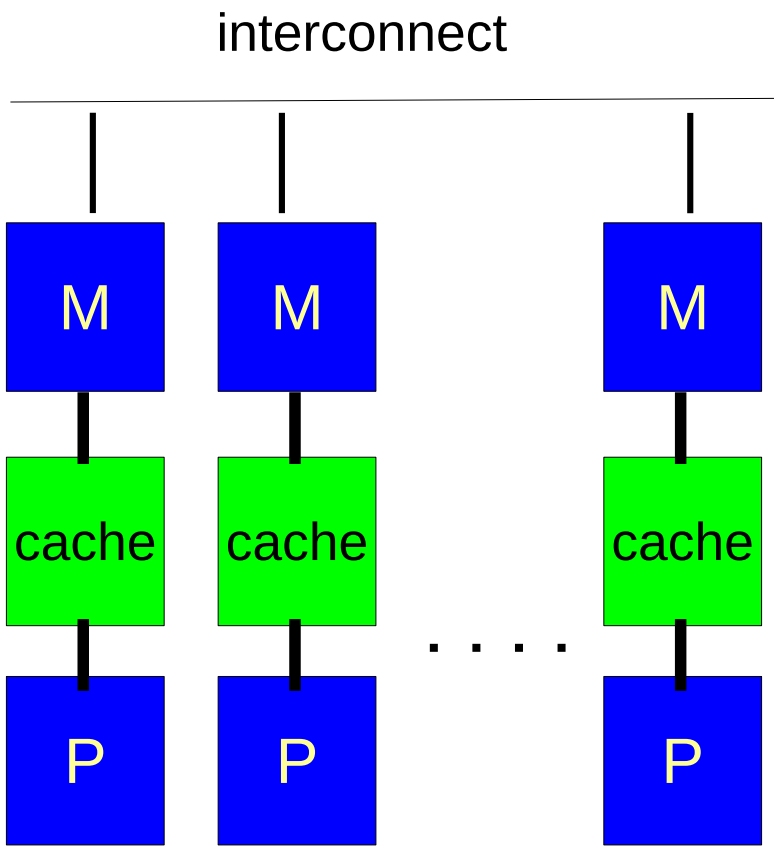
Memory access:

- uniform for all P

Example systems:

- single-socket multi-core CPU (e.g. your laptop)

Non-Uniform Memory Access (NUMA)



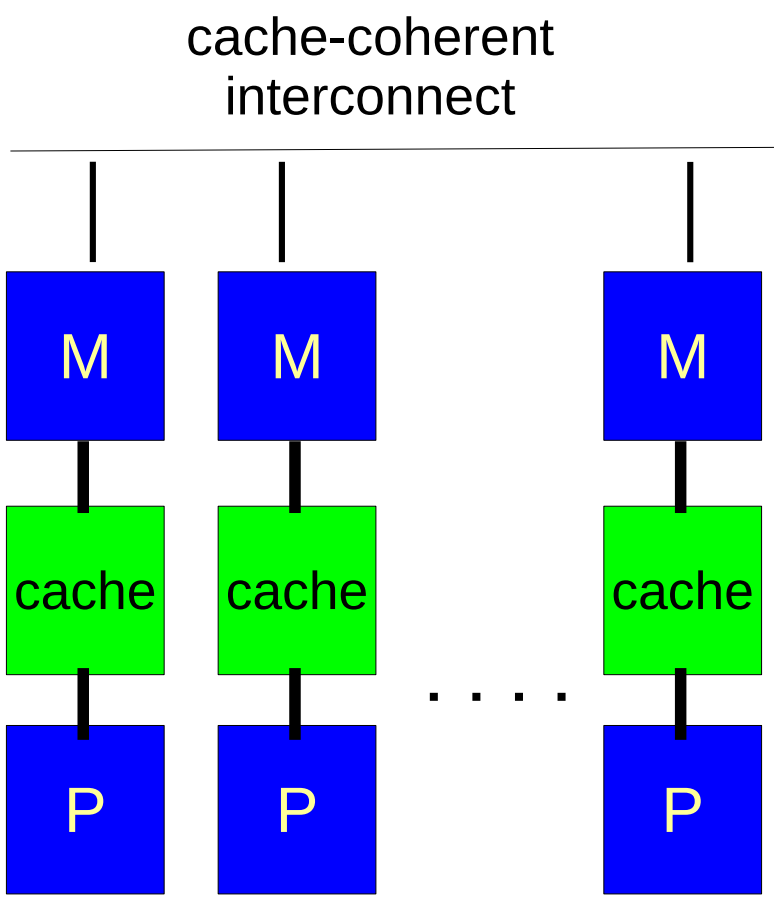
Memory access:

- non-uniform across P

Example systems:

- cluster of nodes, connected by a (fast) network

cc-NUMA



Memory access:

- non-uniform across P
- ... but cache-coherency on the interconnect

Example systems:

- (almost) all multi-socket x64 servers

Multi-core – everywhere!

Welcome to a “threaded” world

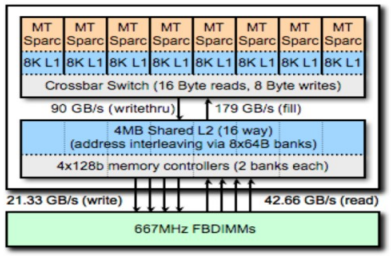
The first multi-core chips

- 2004 – multi-core arrives:
  - IBM POWER5
  - Sun UltraSPARC-IV
- 2005 is the year of the x86 dual-core CPUs:
  - AMD Opteron “Denmark” (August 2005)
  - Intel Xeon “Paxville DP” (October 2005)
- 2008/2009: quad-cores
  - AMD Opteron 'Barcelona'
  - Intel Xeon 'Nehalem'

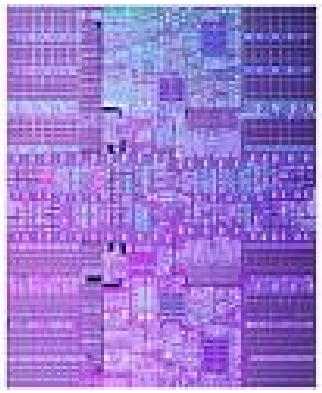
2009's Multi-cores

282 use Quad-Core
204 use Dual-Core
3 use Nona-core

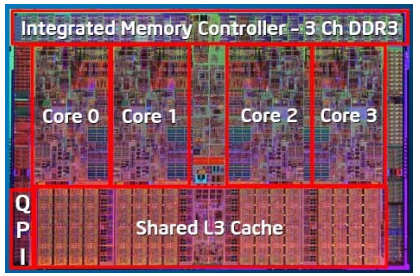
99% of Top500 Systems Are Based on Multi-core



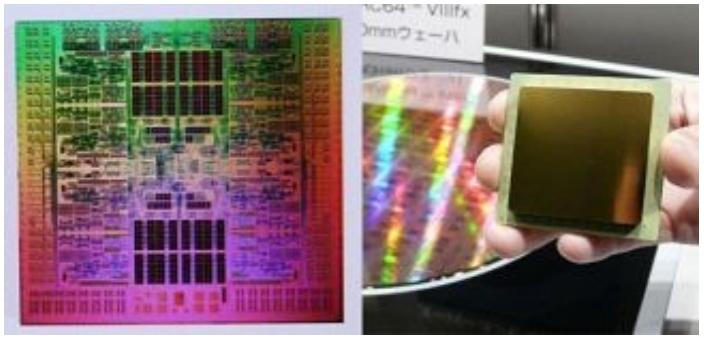
Sun Niagara2 (8 cores)



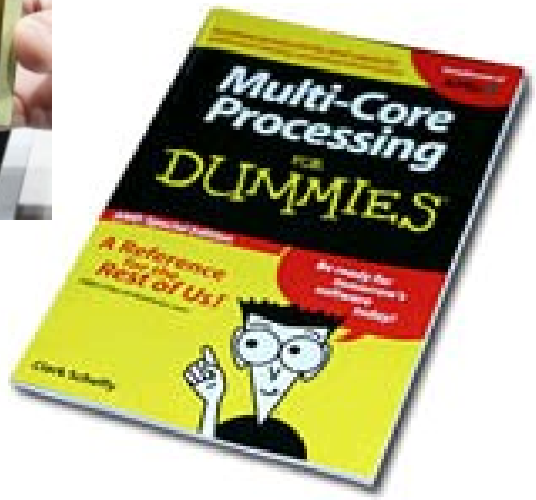
IBM Power 7 (8 cores)

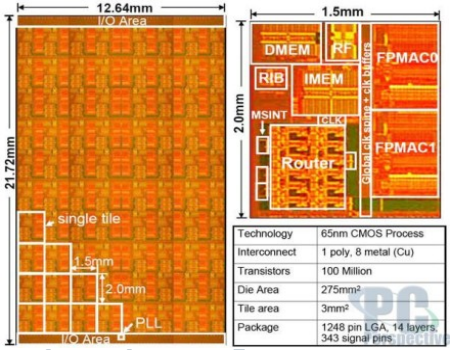


Intel Nehalem (4 cores)

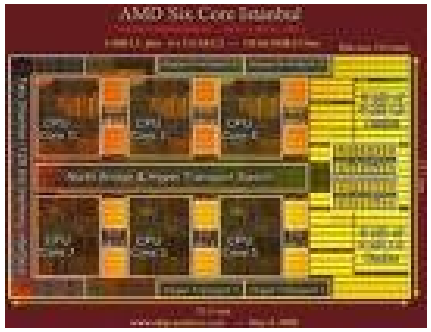


Fujitsu Venus (8 cores)

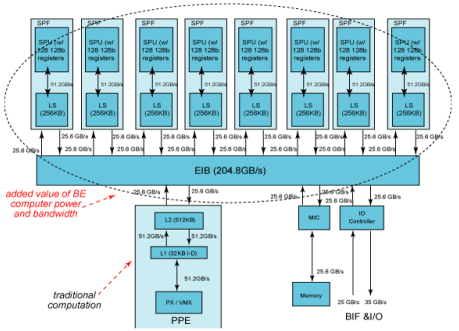




Intel Polaris [experimental] (80 cores)

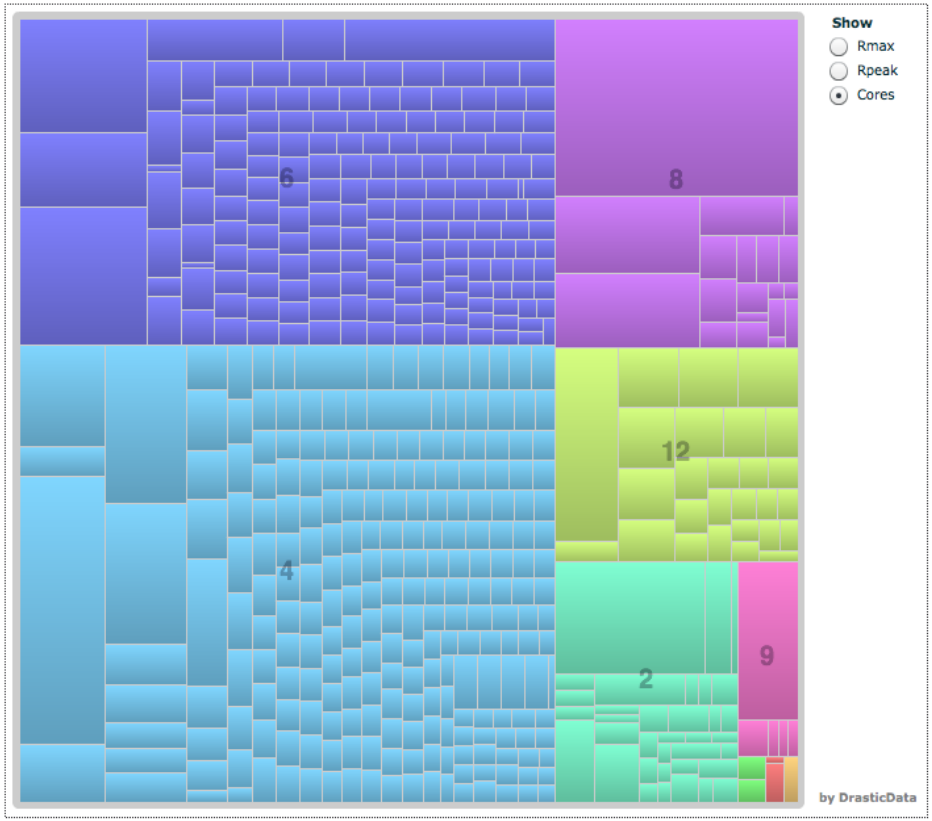


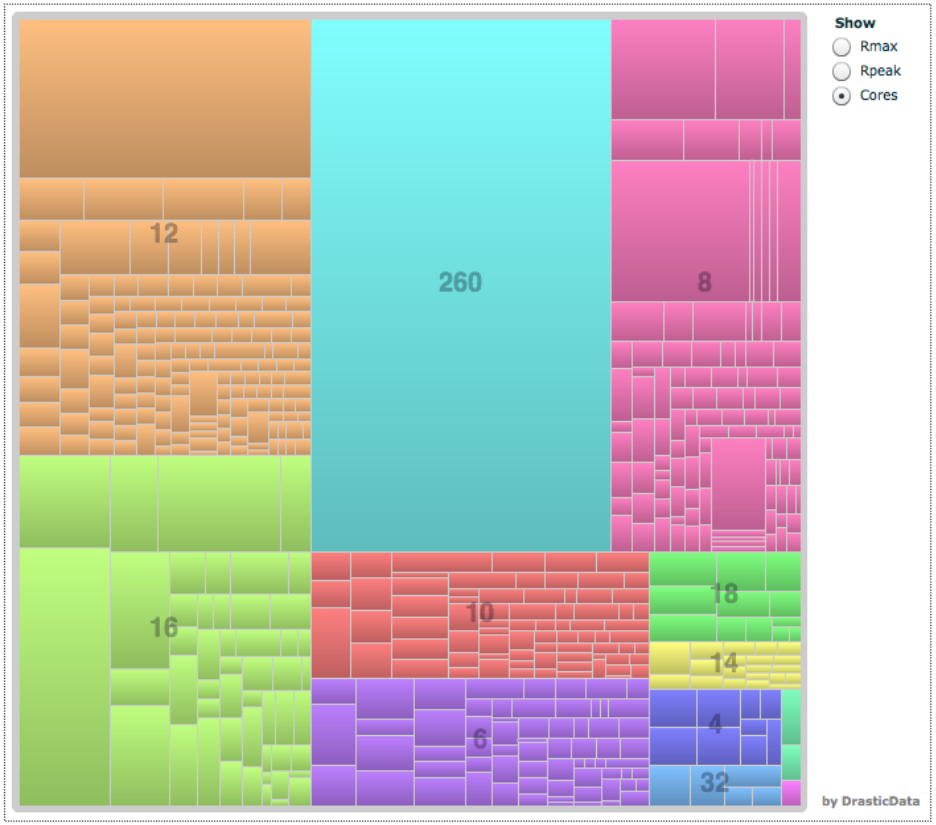
AMD Istanbul (6 cores)



IBM Cell (9 cores)

TOP500: multi-cores 2011 and 2016

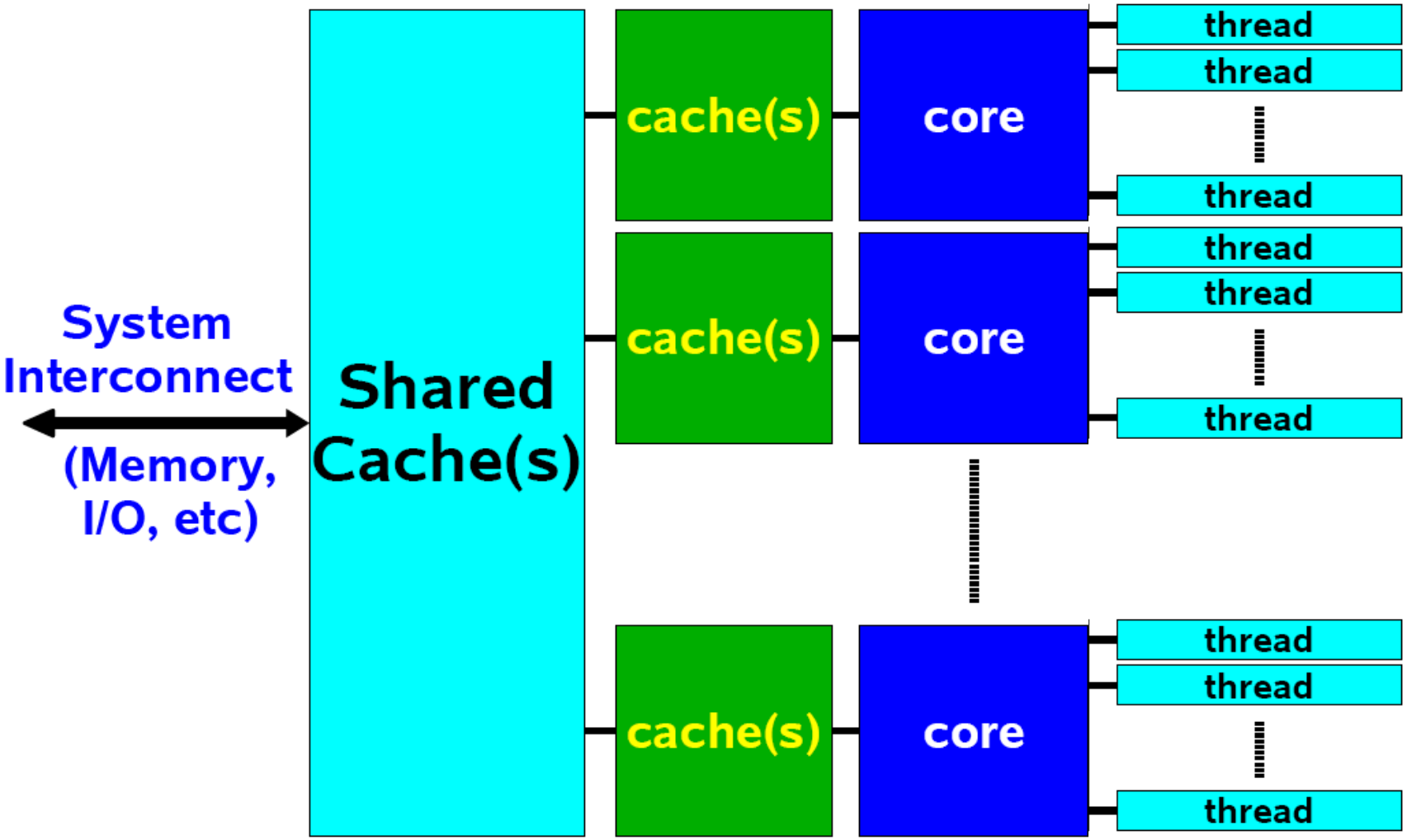




What is a multi-core chip?

- A “core” is not well-defined – let us assume it covers the processing units and the L1 caches (a very simplified CPU).
- Different implementations are possible – and available (examples follow), e.g. multi-threaded cores
- Cache hierarchy of private and shared caches
- For software developers it matters that there is parallellism in the hardware, they can take advantage of

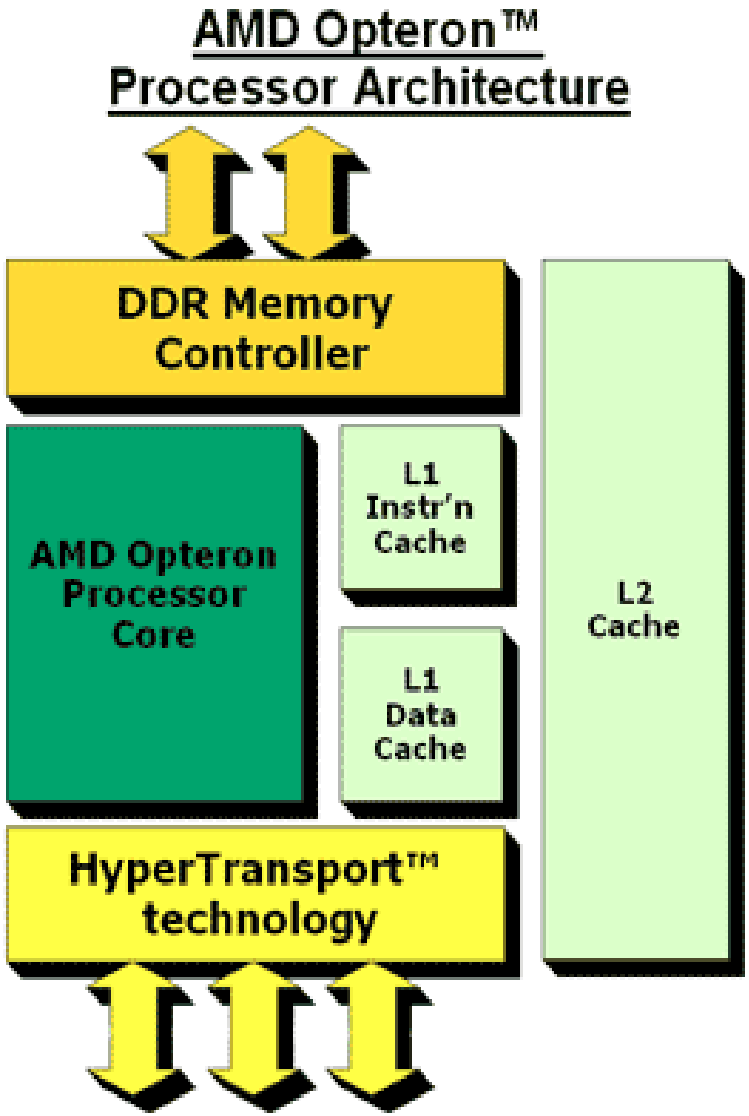
A generic multi-core design



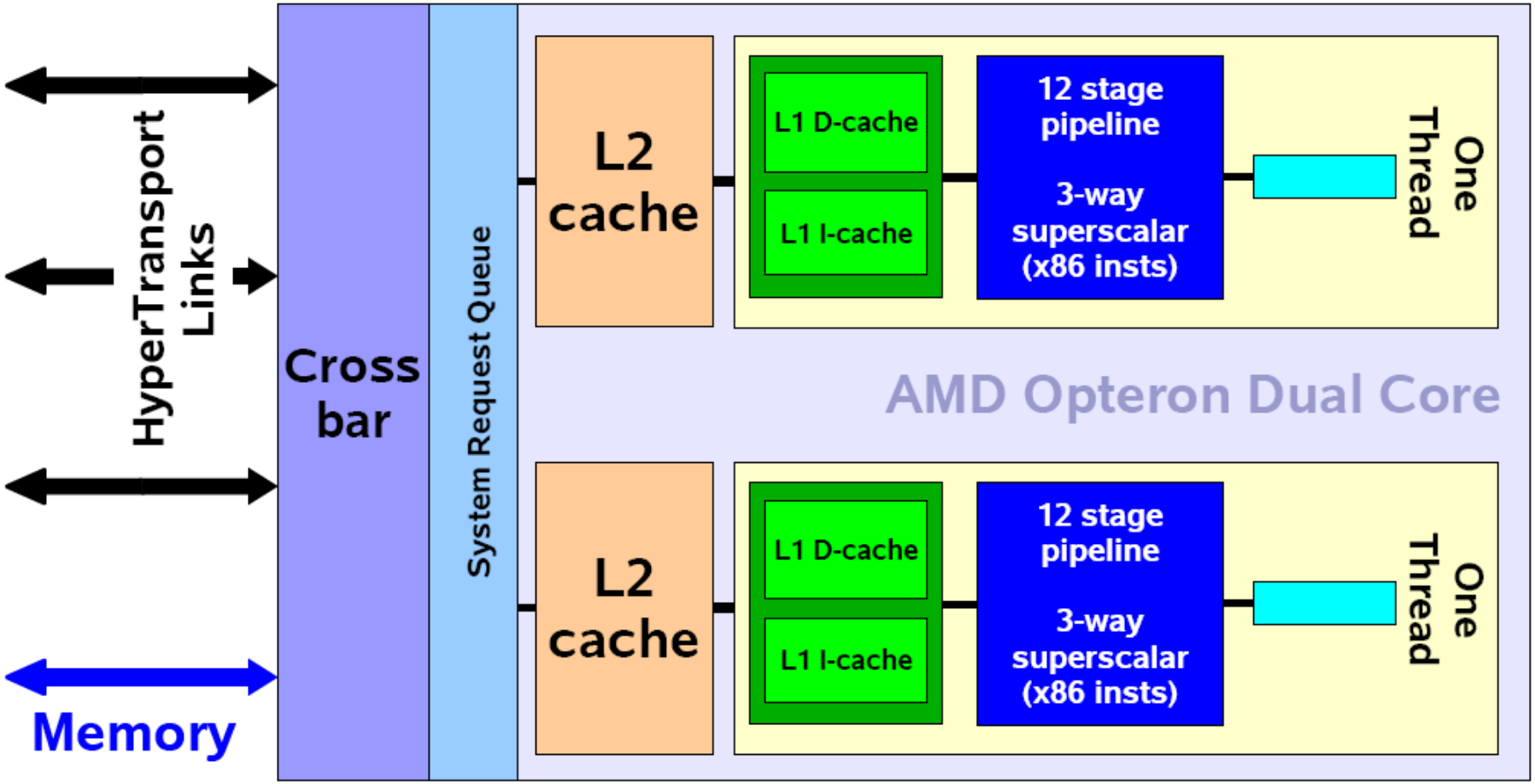
The AMD Opteron – single core

On-chip:

- Memory controller
- L2 cache
- 3 fast HyperTransport links: 6.4 GB/s per link

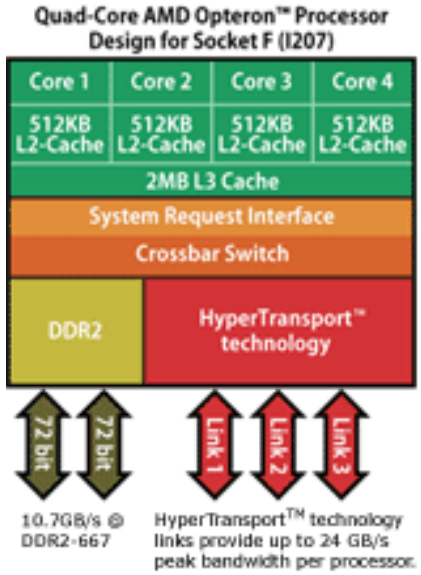


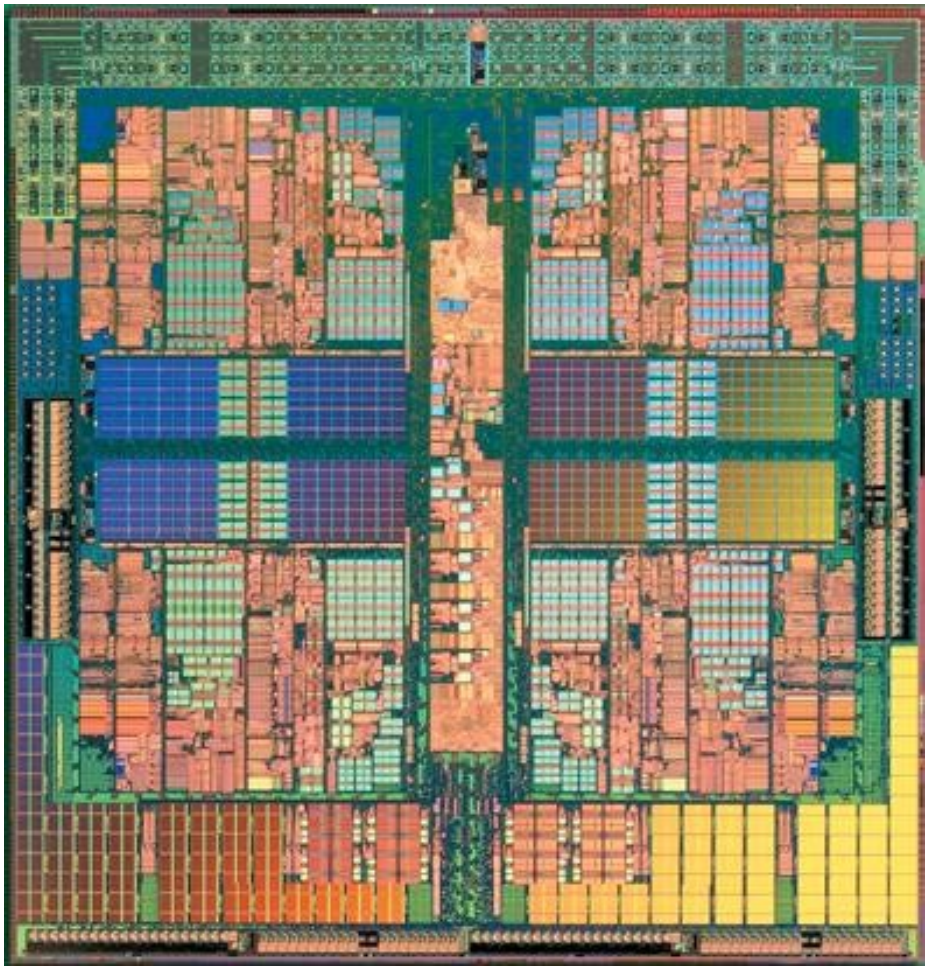
AMD Opteron - dual-core



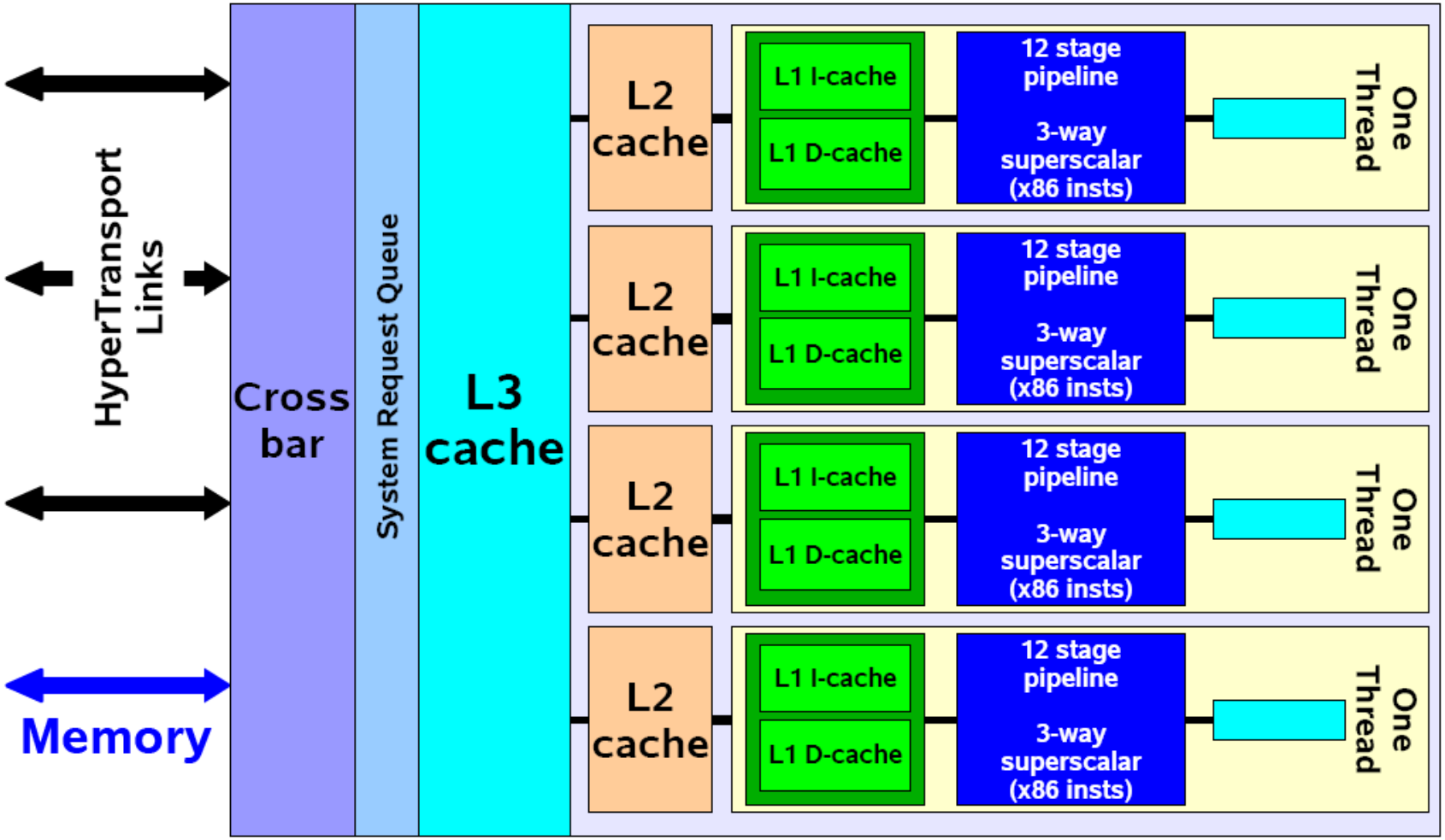
AMD Opteron – quad-core

- dedicated L2 caches
- shared L3 cache



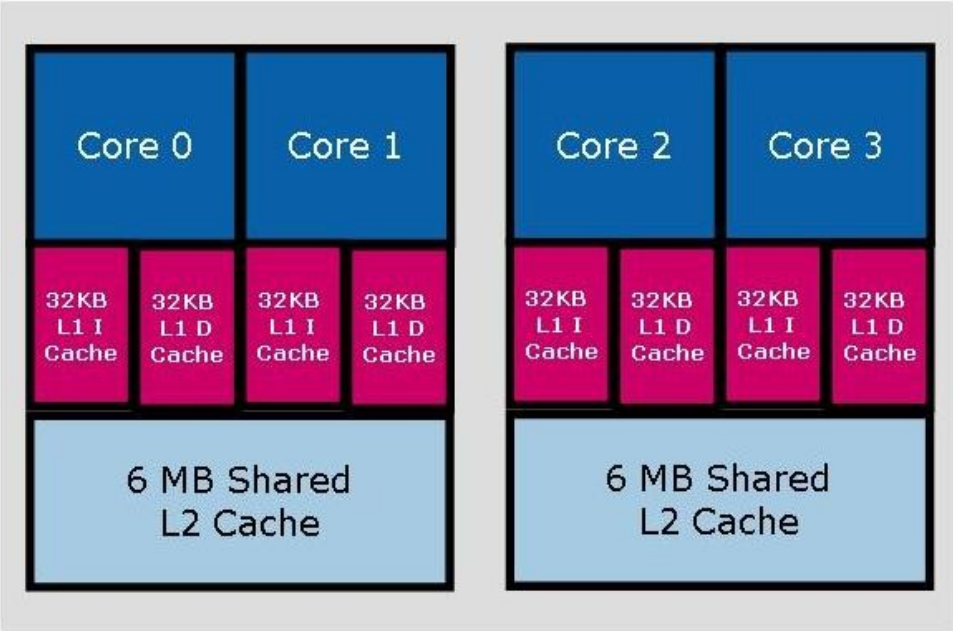


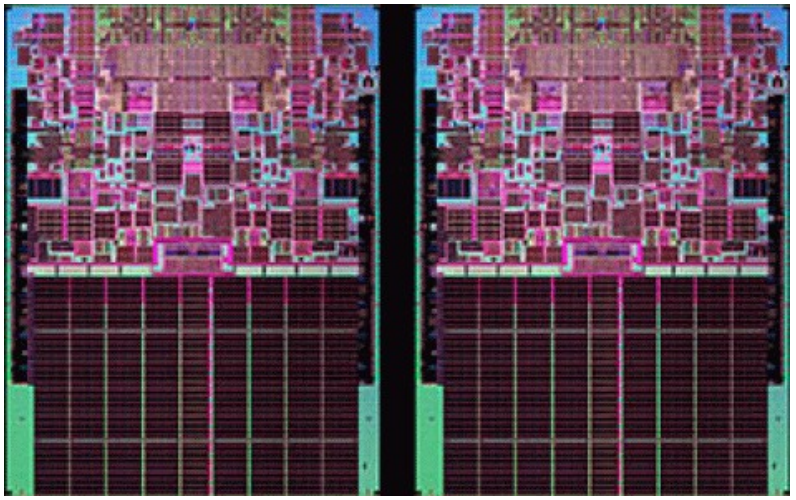
AMD Opteron – quad-core



Quad-core Intel Xeon

Intel® Xeon® processor 5400 series
(Codename 'Harpertown)

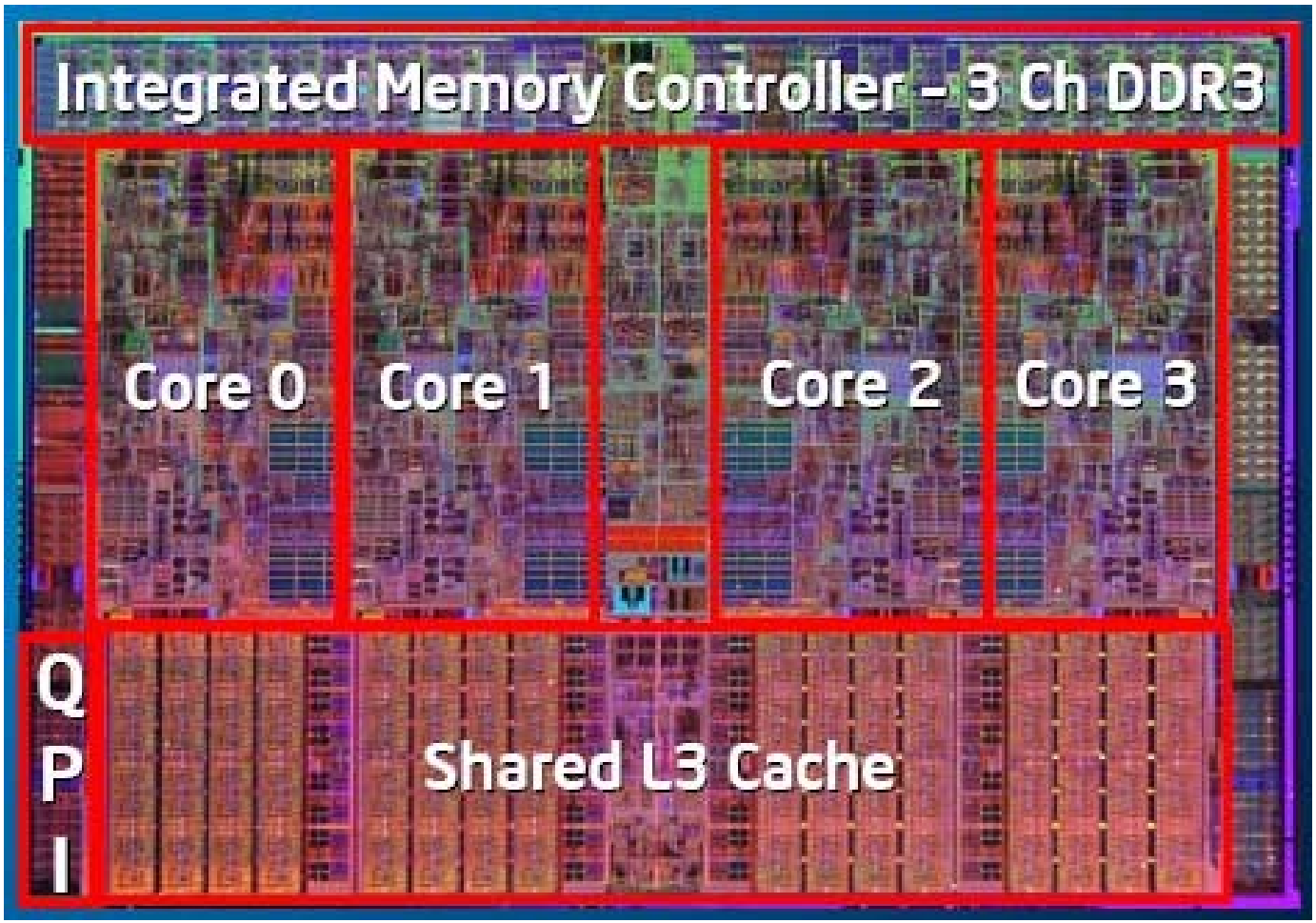




Two dual-core chips "glued" together

The Intel “Nehalem” CPU

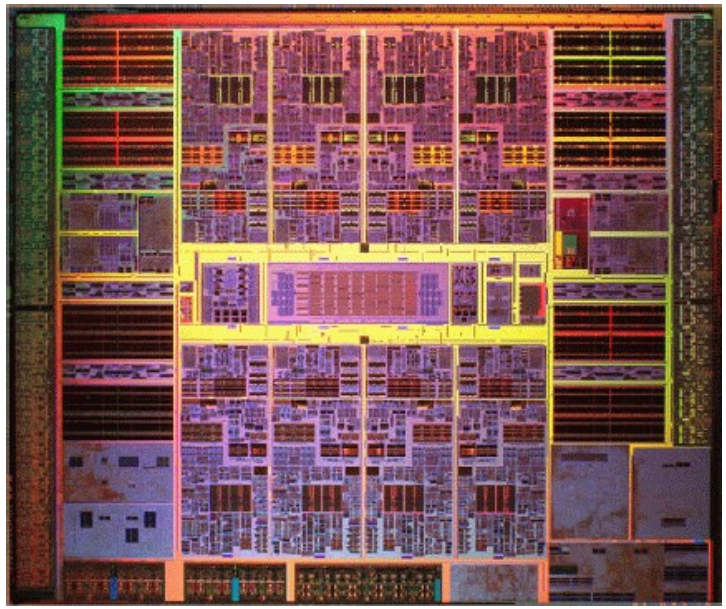
First quad-core CPU with QPI



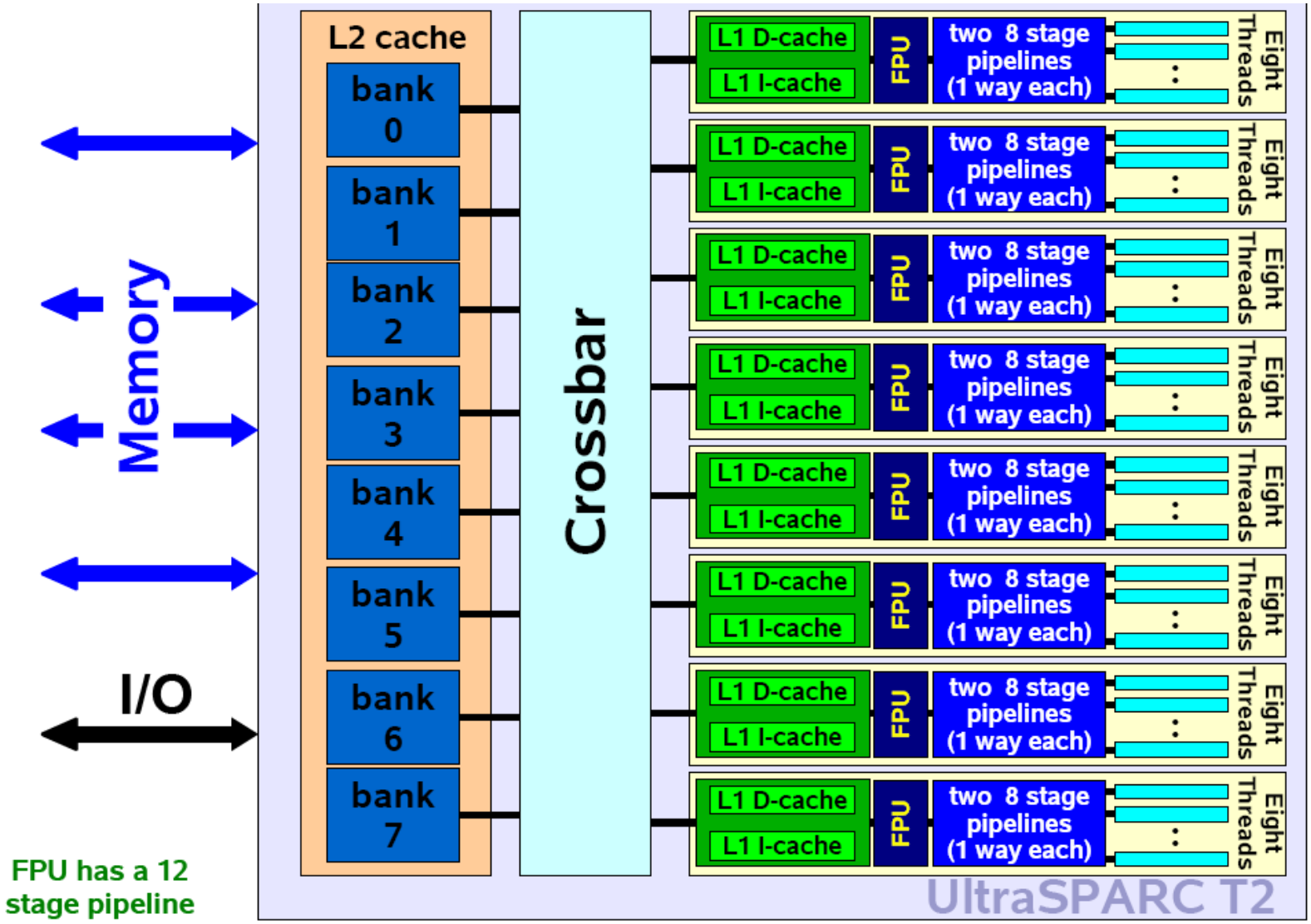
UltraSPARC-T2

System on a chip:

- 8 cores with 8 threads = 64 threads
- integrated multi-threaded 10 Gb/s Ethernet
- integrated crypto-unit per core
- low power (< 95W)
- < 1.5W/thread

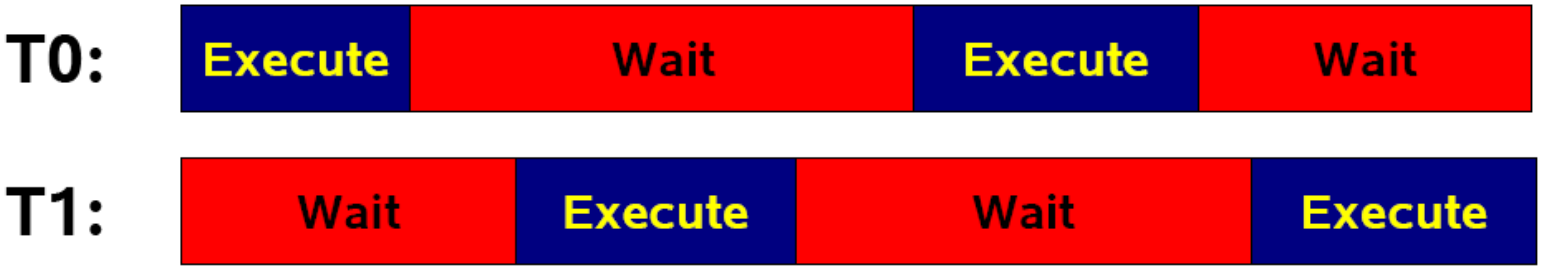


UltraSPARC-T2



Why adding threads to a core?

Execution of two threads:



Interleaving the work – better utilization:

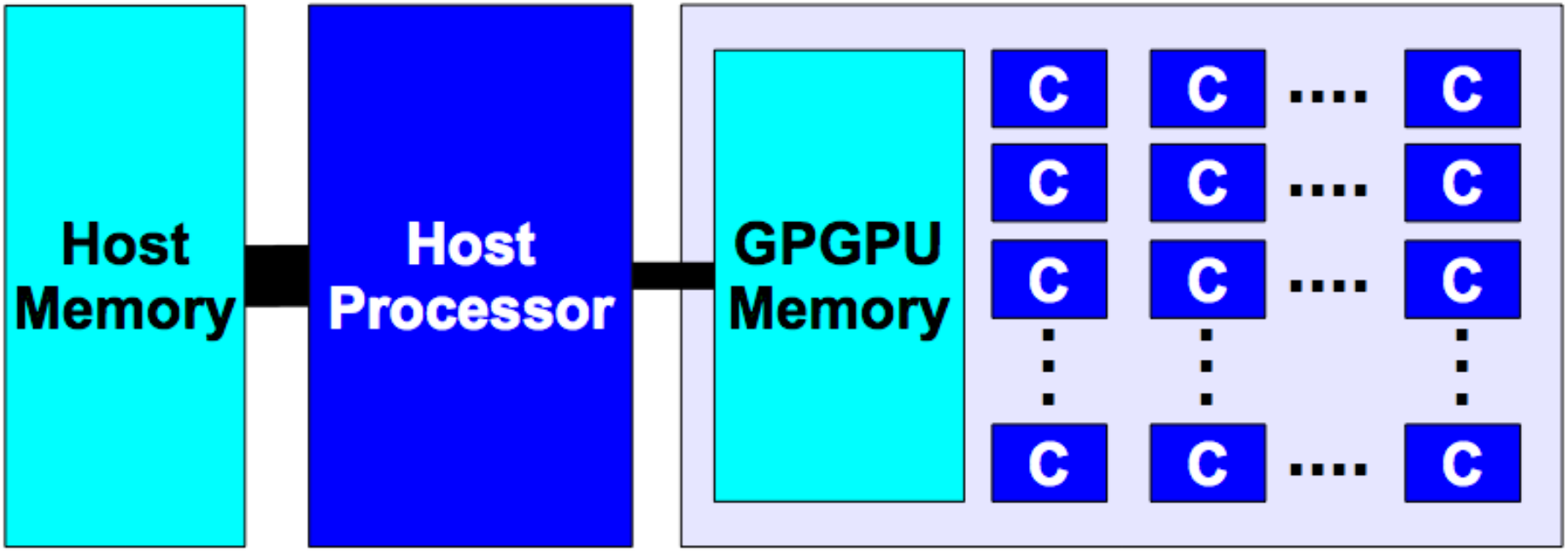
Keyword: “Throughput Computing”

GPU computing - Accelerators

Graphics Processors

- Specialized hardware for operations typical for graphics rendering
- lots of cores (SPs – scalar processors)
- very fast memory (expensive!) - limited in size (compared to main memory)
- more instructions have been added over the last years to do more general purpose computing
- programming environments (CUDA, OpenCL) to harness the power of the GPUs

A generic GPGPU



GPU “features”

- every “core” is a very simple processor
- “cores” cannot work independently
- no independent execution of threads, but SIMT (*Single Instruction, Multiple Threads*)
- no global address space, neither within the GPU, nor with the CPU
- no cache-coherency
- latency hiding by executing many threads
- ===> more next week

Multi-core and memory bandwidth

Remember from “Serial Tuning”:

- The TPP is easy to calculate:
  - 2 GHz CPU/core
  - 4 Flops per clock cycle
  - TPP: 8 GFlop/s (per core!)
- To obtain that peak performance, we need to be able to feed the core with the right amount of data!

Multi-core and memory bandwidth

How much data is that?

- 4 floating point operations (add, mult) need 8 floating numbers => 64 bytes (double prec.)
- That is 64 bytes / clock cycle or 128 GB/s
  - 128 GB is the content of more than 27 DVDs!!!
- But what is the memory bandwidth in modern machines?

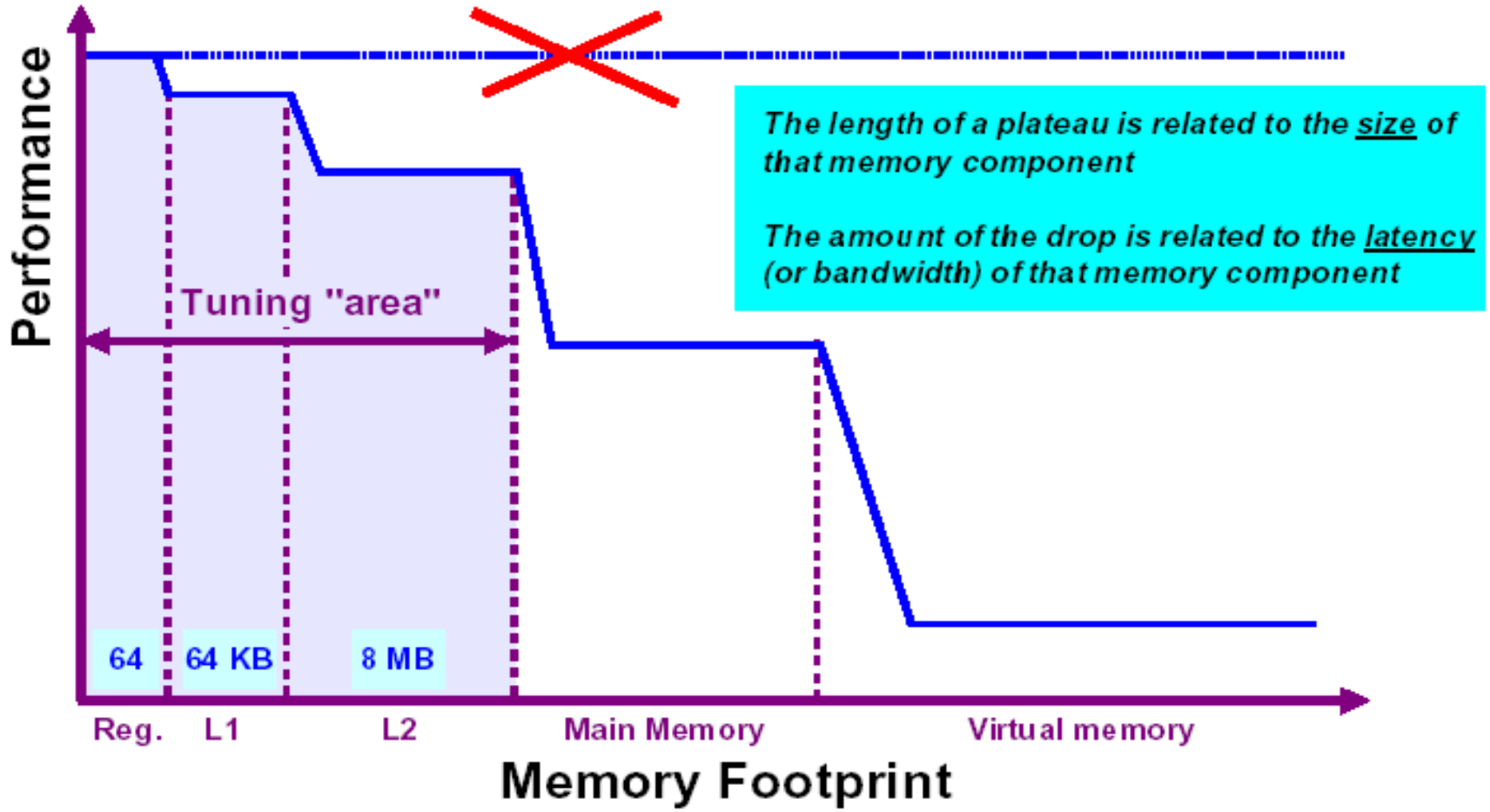
Multi-core and memory bandwidth

Memory bandwidth of the Xeon 5500 family

- equipped with DDR3-1333 DIMMS:
  - 1333 MT/s => 10.6 GB/s per memory channel
  - or 32 GB/s maximum (all channels equipped)
- with DDR3-1066 DIMMS:
  - max. 25.5 GB/s
- that's per socket – but each CPU has 4 cores!
- => the memory bandwidth per core is less!!!!

Multi-core and memory bandwidth

Flashback (single core):



Multi-core and memory bandwidth

Compute-bound vs memory-bound

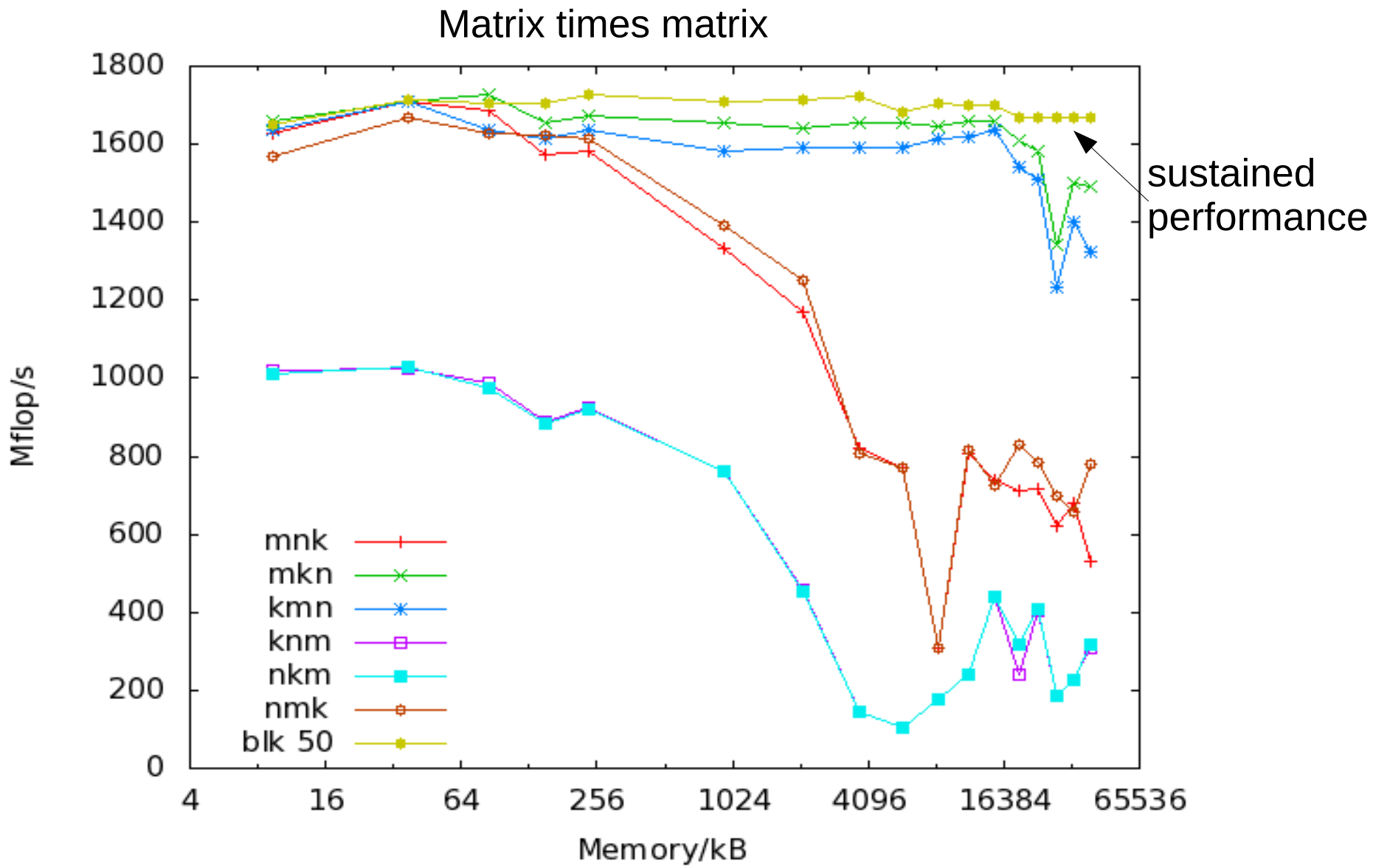
- Compute-bound:
  - the number of flops is higher than the number of mem-ops (load/store)
  - example: matrix times matrix
- Memory bound:
  - the number of mem-ops is dominating the flops
  - example: matrix times vector

Multi-core and memory bandwidth

Compute-bound vs memory-bound

- but ...
- for large problems, even matrix times matrix gets dominated by the memory operations, it turns into a memory bound problem
- we know how to solve that – e.g. by blocking algorithms, to keep the problem compute bound

Multi-core and memory bandwidth



Multi-core and memory bandwidth

Scaling of memory-bound applications

- What happens with memory bound applications on multi-core systems?
- N cores – one CPU, e.g. N = 4
- N threads execute a mem-bound kernel
  - => all N threads fight for the same mem-bw
  - => this is a performance bottleneck
  - => this has a negative effect on scaling (speed-up)

Multi-core and memory bandwidth

Scaling of memory-bound applications (cont'd)

- this effect is visible for all parallel applications, regardless of the programming model, i.e.
  - MPI applications
  - OpenMP applications
- adding more cores to the CPU makes the situation even worse!
- adding more sockets helps, but one still can't make use of all cores!

The Roofline Model

- Amdahl's law, i.e. the definition of speed-up, doesn't take bottlenecks into account!
- Where are those bottlenecks?
  - arithmetic operations: flop/s
  - memory access: bandwidth/latency
  - network access: communication or I/O (bw/latency)

The Roofline Model

Example: part of the STREAM benchmark (Triad)

```
for(i=0; i < N; ++i) {
  a[i] = b[i] + s * c[i];
}
```

- How ‘expensive’ is one iteration of this loop?
  - 2 floating point operations: add & mult
  - 3 memory operations (2 loads, 1 store)
  - s is kept in a register

The Roofline Model

Arithmetic operations:

- time for a Flop: about one nanosecond (1 ns)
- modern CPUs can do several Flops per cycle (vectorization, FMA)

Memory operations:

- latency: 90ns (DRAM)
- bandwidth: a few bytes per cycle

=> the Triad code is memory bound!

The Roofline Model

Operational (or arithmetic) intensity:

$$I = \frac{arithmetic\ operations\ [flops]}{data\ transfers\ (ld, st)\ [words]}$$

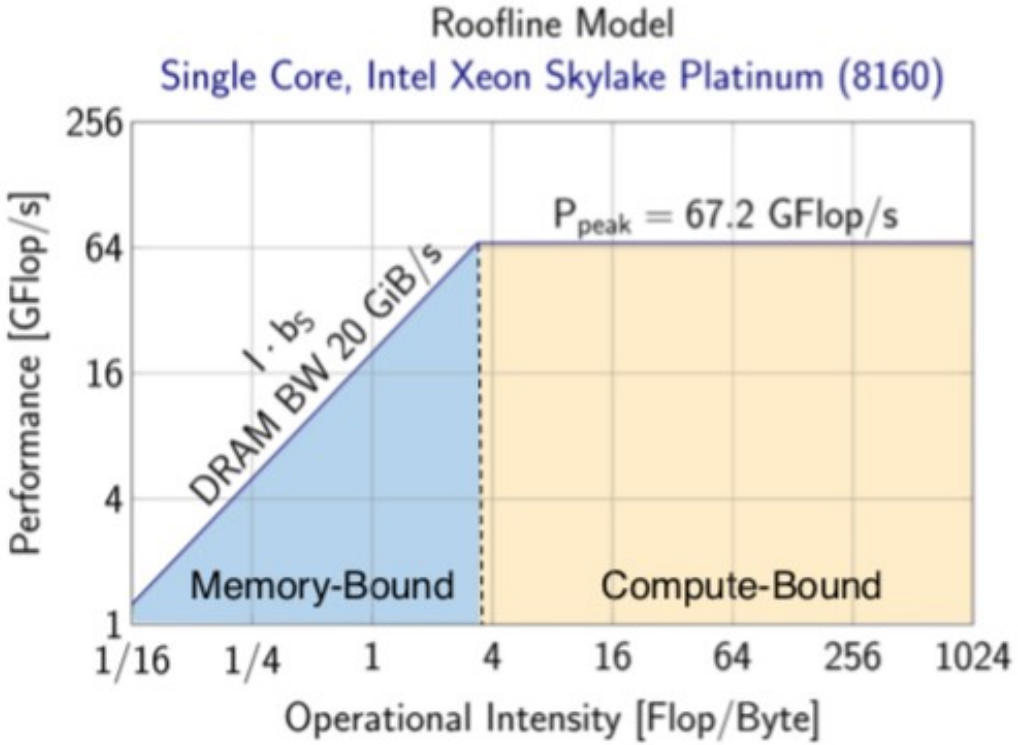
- e.g. for the triad problem:
  - I = 2 flops/3 words = 0.66 flops/word
  - with 8 bytes/word (double): I = 0.083 flops/byte
- now we also need to consider the bandwidth of the slowest data path, b_s (in bytes/second)

The Roofline Model

Achievable Performance P:

$$P := \min(I * b_s , P_{peak})$$

- This is called the 'Roofline (Performance) Model'



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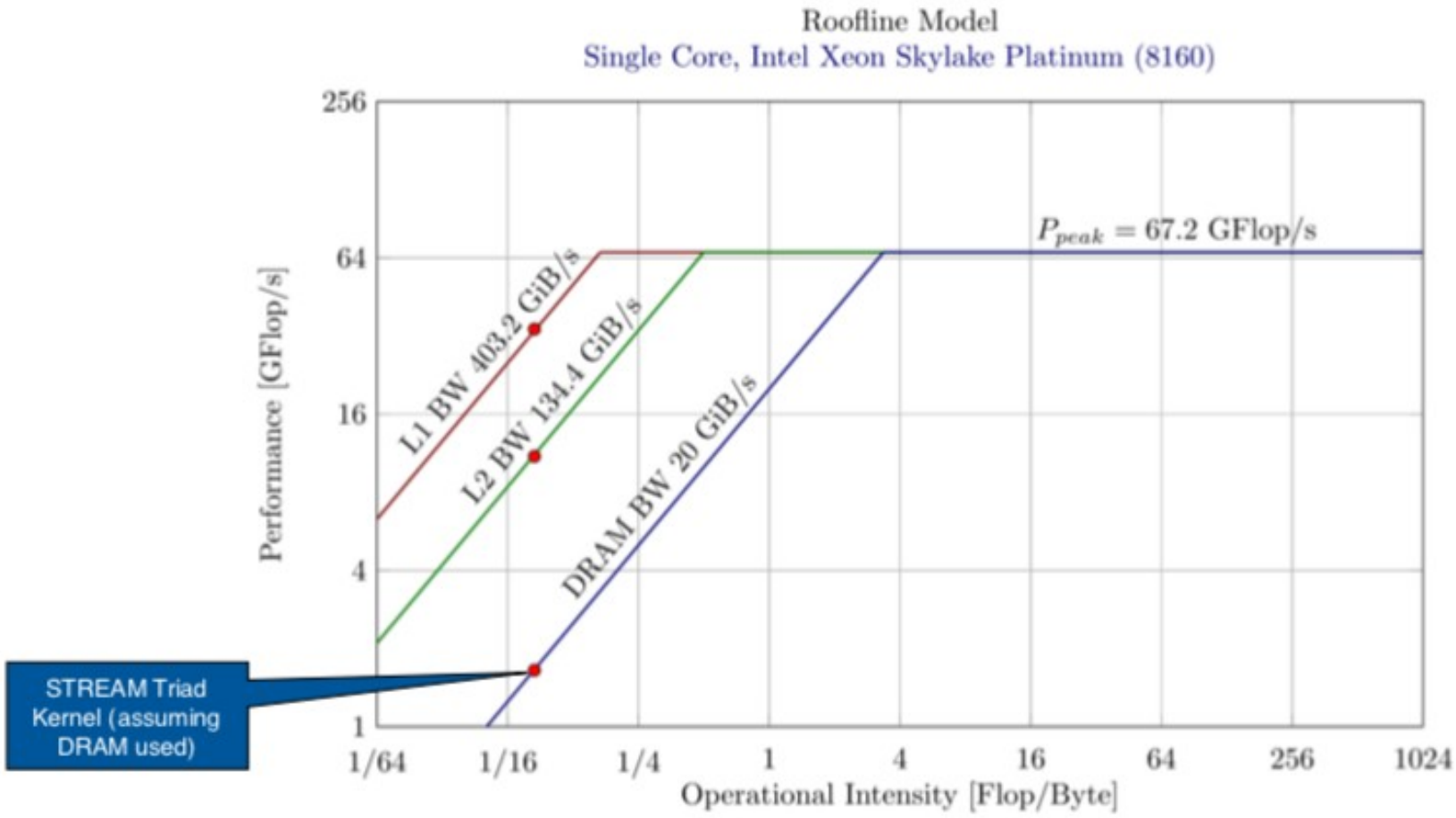
The Roofline Model



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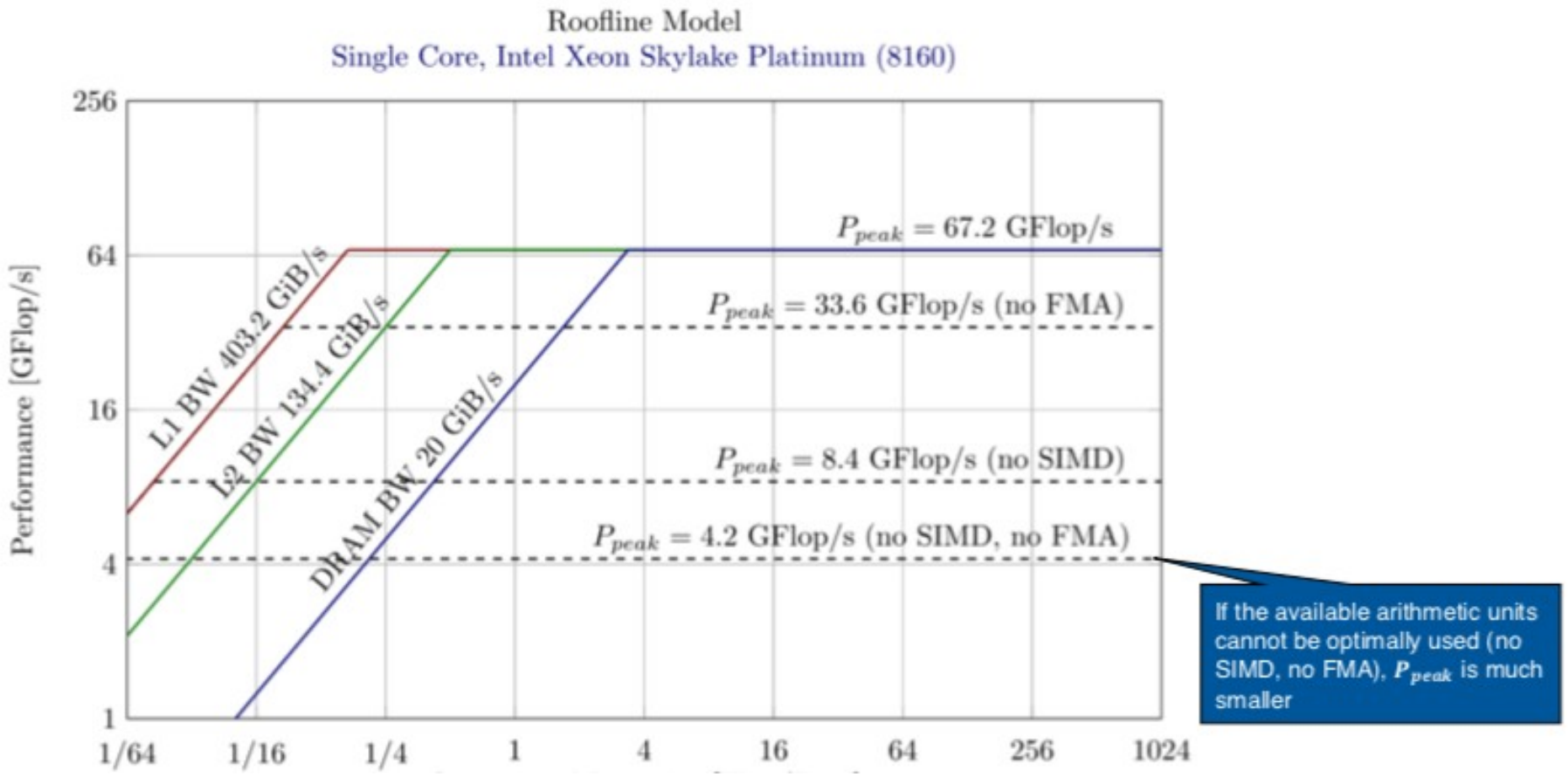
The Roofline Model



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The Roofline Model

Classification of problems:

Arithmetic oper.	Data transfers	Classification	Examples
O(N)	O(N)	memory bound	scalar product, vector ops, sparse matrix times vector
$O(N^2)$	$O(N^2)$	memory bound (but can benefit from cache)	dense matrix times vector, matrix addition
$O(N^x)$ (x>2)	$O(N^2)$	compute bound	dense matrix multiplication

Summary

- You have heard about:
  - Parallel programming models and basic concepts
  - Parallel architectures (shared / distributed memory)
  - Cache-coherency
  - Multi-core CPUs
  - GPUs/accelerators
  - Multi-core and memory bandwidth
- Next 3 lectures:

  Portable programming of shared memory systems with OpenMP