Arithmetic

Classification of Numbers

Class	Symbol	Description
Natural Numbers	N	1, 2, 3, ...
Whole Numbers	W	0, 1, 2, 3, ...
Integers	Z	{...-3, -2, -1, 0, 1, 2, 3, ...}
Negative Integers		{..., -5, -4,-3, -2, -1}
Positive Integers		{1, 2, 3, 4, 5, 6, ...}
Non-Negative Integers		{0, 1, 2, 3, 4, 5, 6, ...}
Non-Positive Integers		{..., -5, -4,-3, -2, -1, 0}
Rational Numbers	Q	Numbers in p/q form; where *p, q* are integers and $q \neq$ zero.
Irrational numbers		It cannot be represented in p/q form; where *p, q* are integers. Example: $\sqrt{2}$, $\sqrt{3}$, π, e, etc.
Real Numbers	R	All the rational numbers and all the irrational numbers together form the real numbers. 0.45, 5/2, -0.726495 ..., 18, and -65, 1/4 are some example of real numbers.
Prime Number		It can be divided only by 1 and itself. Example: 2, 3, 5, 7, ...

Non-Prime Number		The positive integers which are not prime (except 1) are known as non prime numbers. It is also called composite numbers. Example: 4, 6, 8, ...
Co- Prime Number		A pair of numbers not having any common factors other than 1. Example: 15 and 28

Absolute Values:

The absolute value of A is denoted by |A|.
If $a < 0$, then $|a| = -(a)$
If $a > 0$, then $|a| = a$
If $a = 0$, then $|a| = 0$

For example: $|-3| = 3$, $|4| = 4$

Division:

Dividend = (Divisor X quotient) + Remainder
(Note: Remainder < Divisor)

Factors and Multiple:

If *b* divides A perfectly and the remainder is 0, then *b* is a factor of A
Factors can be positive and negative.

Positive factors of 15 are 1, 3, 5, and 15
15 is a **multiple** of 1, 3, 5, 15, -1, -3, -5, -15

GCD:

GCD (greatest common divisor) or HCF (highest common factor) of two non-zero integers, is the largest positive integer that perfectly divides both numbers.

LCM:

LCM (least common multiple) of two integers *a* and *b* is the smallest positive integer that is a multiple of both *a* and *b*.

Rules of Operation:

Addition and Subtraction:

- Positive + Positive = Positive
  5 + 4 = 9

- Negative + Negative = Negative
  (- 7) + (- 2) = - 9

- Negative and a positive number:
  Find the positive difference between the two numbers and use the sign of the larger number.
  (- 7) + 4 = - (7 – 4) = -3
  6 + (-9) = - (9 – 6) = - 3
  6 + (-4) = + (6 – 4) = 2

Multiplication:

- Positive x Positive = Positive
  3 x 2 = 6

- Negative x Negative = Positive
  (-2) x (-8) = 16

- Negative x Positive = Negative
  (-3) x 4 = -12

- Positive x Negative = Negative
  3 x (-4) = -12

Division:

- Positive ÷ Positive = Positive
  12 ÷ 3 = 4

- Negative ÷ Negative = Positive
  (-12) ÷ (-3) = 4

- Negative ÷ Positive = Negative
  (-12) ÷ 3 = -4

- Positive ÷ Negative = Negative
  12 ÷ (-3) = -4

Operations on Odd and Even Integers:

Odd + Odd = Even
Even + Even = Even
Odd + Even = Odd

Odd – Odd = Even
Even – Even = Even
Odd – Even = Odd

Odd × Odd = Odd
Even × Even = Even
Even × Odd = Even

Fractions, Decimals, and Percentages

Convert fractions to decimals:

Divide the denominator (the bottom part) by the numerator (the top part):
$\frac{1}{4}$ = 1 ÷ 4 = 0.25

Convert fractions to percentages:

Multiply the fraction by 100 and reduce the resulting fraction.
$\frac{1}{4}$ x 100 = $\frac{100}{4}$ = 25%

Convert decimals to fractions:

Divide the decimal by powers of 10.
If there is one digit after the decimal point, divide by 10^1

If there are two digits after the decimal point, divide by 10^2

If there are n digits after the decimal point, divide by 10^n

$0.25 = \frac{25}{100} = \frac{1}{4}$

Convert decimals to percentages:

Convert decimals to fractions and then covert the fraction to percentage.

$0.25 = \frac{25}{100} = \frac{25}{100} \times 100 = 25\%$

$0.235 = \frac{235}{1000} = \frac{23.5}{100} = \frac{23.5}{100} \times 100 = 23.5\%$

Convert percentages to decimals:

Move the decimal point two places to the left.

$25\% = \frac{25}{100} = 0.25$

$215\% = \frac{215}{100} = 2.15$

Convert percentages to fractions:

Divide the number by 100 and reduce the fraction.

$25\% = \frac{25}{100} = \frac{1}{4}$

Converting mixed fractions to improper fractions:

$$a\frac{c}{d} = \frac{ad + c}{d}$$

Rules of Operations in Fractions:

Addition:

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

Subtraction:

$$\frac{a}{b} - \frac{c}{d} = \frac{ad - bc}{bd}$$

Multiplication:
$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

Division:
$$\frac{a/b}{c/d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}$$

Ratio and Proportions

Ratios:

- If $a : b = c : d$, then $a : b = c : d = (a + c) : (b + d)$
- If $a < b$, then for a positive quantity x,

  $\frac{a+x}{b+x} > \frac{a}{b}$ and $\frac{a-x}{b-x} < \frac{a}{b}$

- If $a > b$, then for a positive quantity x,

  $\frac{a+x}{b+x} < \frac{a}{b}$ and $\frac{a-x}{b-x} > \frac{a}{b}$

Proportions:

If $a : b :: c : d$ (or) $\frac{a}{b} = \frac{c}{d}$, then

- $\frac{a}{c} = \frac{b}{d}$
- $\frac{b}{a} = \frac{d}{c}$
- $\frac{a+b}{b} = \frac{c+d}{d}$
- $\frac{a-b}{b} = \frac{c-d}{d}$
- $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \cdots = k$, then $\frac{a+c+e+\dots}{b+d+f+\dots} = k$

Sequence

Arithmetic Progression (*a, a + d, a + 2d, a + 3d ...)*

*n*th term of the sequence = $\mathrm{T_n} = \mathrm{a} + (\mathrm{n}-1)\mathrm{d}$
Sum of first *n* terms of the sequence = $S_n = \frac{n}{2}[2a + (n-1)d]$
Where *a* is the first term and *d* is the common difference.

Geometric Progression (a, ar, ar^2, ar^3...)

$\mathrm{T_n} = \mathrm{ar^{n-1}}$
If $r > 1$, $S_n = a\left(\frac{r^n - 1}{r - 1}\right)$

If $r < 1$, $S_n = a\left(\frac{1 - r^n}{1 - r}\right)$

Sum important series:

Sum of first *n* natural numbers
$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

Sum of the squares of the first *n* natural numbers
$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of the cubes of the first *n* natural numbers
$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$$

Algebra

Algebraic Expressions

Average (or) Mean/ Arithmetic Mean: Sum of elements/Number of elements

⇨ $\frac{x_1 + x_2 + \ldots + x_n}{n}$

Mathematical Formulae

- $(x + y)^2 = x^2 + 2xy + y^2$
- $(x - y)^2 = x^2 - 2xy + y^2$
- $(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$
- $(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$
- $x^2 - y^2 = (x + y)(x - y)$
- $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
- $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Linear and Quadratic Equations

Linear equation:

General form

A*x* + B*y* + C = 0

Quadratic equation:

A general quadratic equation can be written in the form $ax^2 + bx + c = 0$

And the roots are given by, x = $\left[\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$

Linear Inequalities

- Adding (or Subtracting) the same number to both sides of an inequality does not change the order of the inequality sign (i.e., '>', or '<').

If $a < b$ then $a + c < b + c$ and $a - c < b - c$.
Similarly, if $a > b$ then $a + c > b + c$ and $a - c > b - c$ for any three numbers *a*, *b*, and *c*.

- Multiplying (or Dividing) both sides of an inequality by the same positive number does not change the order of the inequality sign (i.e., '>', or '<').

  For any three numbers *a, b*, and *c* where $c > 0$,
  - if $a < b$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$
  - if $a > b$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$

- Multiplying (or Dividing) both sides of an Inequality by the same negative number reverses the order of the inequality sign (i.e., '>' to '<' and '<' to '>').

  For any three numbers *a, b*, and *c* where $c < 0$,
  - if $a < b$ then $ac > bc$ and $\frac{a}{c} > \frac{b}{c}$
  - if $a > b$ then $ac < bc$ and $\frac{a}{c} < \frac{b}{c}$

- If $a < b$ and $b < c$, then $a < c$, for any three numbers *a, b*, and *c*

- If *a* and *b* are of the same sign and $a < b$ ($a > b$), then $\frac{1}{a} > \frac{1}{b}$ ($\frac{1}{a} < \frac{1}{b}$).

Rules of Exponent

Law	**Example**
$a^m \times a^n = a^{m+n}$	$2^2 \times 2^3 = 2^{3+2} = 2^5$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{2^3}{2^2} = 2^{3-2} = 2^1$
$(a^m)^n = a^{mn}$	$(2^3)^2 = 2^{3 \times 2} = 2^6$

$(ab)^m = a^m \times b^m$	$(2 \times 3)^6 = 2^6 \times 3^6$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ where $b \neq 0$	$\left(\frac{2}{3}\right)^6 = \frac{2^6}{3^6}$
$a^{-m} = \frac{1}{a^m}$ where $a \neq 0$	$2^{-3} = \frac{1}{2^3}$
$a^0 = 1$	$5^0 = 1$
$a^1 = a$	$5^1 = 5$
$a^{m/n} = \sqrt[n]{a^m}$	$2^{6/3} = \sqrt[3]{2^6} = \sqrt[3]{64} = 4$

Laws for fractional exponents

Law	Example
$a^{m/n} = \sqrt[n]{a^m}$	$x^{2/3} = \sqrt[3]{x^2}$
$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}, b \neq 0$	$\frac{\sqrt[3]{16}}{\sqrt[3]{2}} = \sqrt[3]{8} = 2$
$a^{1/2} = \sqrt[2]{a^1} = \sqrt{a}, a \geq 0$	$\sqrt{25} = 5, (not \pm 5)$

Applied Mathematics

Profit and Loss

SP – Selling price
CP – Cost price
MP – Marked Price

Percentage Change:

Percentage Change = $\frac{Final\ value\ - Initial\ Value}{Initial\ Value} \times 100$

Interest:

Amount = Principal + Interest

Profit and Loss:

Profit = SP – CP

Loss = CP – SP

Percentage Profit = $\frac{Profit}{CP} \times 100 = \frac{SP-CP}{CP} \times 100$

Percentage Loss = $\frac{Loss}{CP} \times 100 = \frac{CP-SP}{CP} \times 100$

Discount:

Discount is usually expressed as a certain per cent of the MP

Discount = MP - SP

Rate of Discount = Discount% = $\frac{Discount}{MP} \times 100$

$$SP = MP \times \left(\frac{100 - Discount\%}{100}\right)$$

$$MP = \frac{100 \times SP}{100 - Discount\%}$$

Simple and Compound Interest:

Simple Interest = $\frac{Principal \times Rate \times Time}{100} = \frac{PNR}{100}$

Compound Interest = $P\left(1 + \frac{R}{100}\right)^n - P$

Total Amount = Principal + CI (Compound Interest)

a. Formula for Interest Compounded Annually

Total Amount = $\boldsymbol{P\left(1+\frac{R}{100}\right)^{n}}$

b. Formula for Interest Compounded Half Yearly

Total Amount = $\boldsymbol{P\left(1+\frac{R}{200}\right)^{2n}}$

c. Formulae for Interest Compounded Quarterly

Total Amount = $\boldsymbol{P\left(1+\frac{R}{400}\right)^{4n}}$

d. Formulae for Interest Compounded Annually with fractional years (example: 2.5 years)

Total Amount = $\boldsymbol{P\left(1+\frac{R}{100}\right)^{a} \times \left(1+\frac{bR}{100}\right)}$ Where if year is 2.5 then a =2 and b=0.5

e. With different interest rates for different years, say *x*% for year 1, *y*% for year2, *z*% for year3

Total Amount = $\boldsymbol{P\left(1+\frac{x}{100}\right) \times \left(1+\frac{y}{100}\right) \times \left(1+\frac{z}{100}\right)}$

CI = Compound Interest, P = Principal or Sum of amount, R = % Rate per annum, *n* = Time Span in years, here.

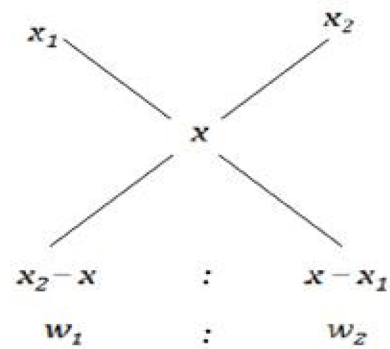
Mixture

Alligation Rule:

The ratio of the weights of the two items mixed will be inversely proportional to the deviation of attributes of these two items from the average attribute of the resultant mixture.

$$\frac{W_1}{W_2} = \frac{x_2 - x_1}{x - x_1}$$

Alligation Cross:



Speed

$$Speed = \frac{Distance}{Time\ taken\ to\ cover\ the\ distance}$$

Average Speed:

$$Average\ Speed = \frac{Total\ Distance\ covered}{Time\ taken\ taken\ to\ cover\ the\ distance}$$

$$= \frac{d_1 + d_2 + d_3 \ldots}{t_1 + t_2 + t_3 \ldots}$$

Some important formulas for Train Problem

Important Conversion:

- *a* km/h = ($a \times \frac{5}{18}$) m/s
- *a* m/s = ($a \times \frac{18}{5}$) km/h

Distance covered by a train to cross a pole/platform:

- Distance covered by a train of length L to pass a pole or a standing man or a signal post is equal to the length of the train L
- Distance covered by a train of length L to pass a stationary object of length b is equal to (Length of the train L + Length of the object b)

Relative Speed:

- Suppose two trains or two bodies are moving in opposite directions at speeds u and v, then their relative speed is given by $(u + v)$.
- Suppose two trains or two bodies are moving in the same direction at speeds u and v; where $u > v$, then their relative speed is given by $(u - v)$.

Time taken by two trains to cross each other:

If two trains of length a and b are moving in opposite directions at speeds u and v, respectively, then the time taken by the trains to cross each other is given by $\frac{a+b}{u+v}$

Time taken by faster train to cross a slower train:

If two trains of length a and b are moving in the same direction at speeds u and v, respectively, then the time taken by the faster train to cross the slower train is given by $\frac{a+b}{u-v}$

Two trains T_1 and T_2 start at the same time from points A and B, respectively toward each other. After crossing each other, if the time taken by the trains T_1 and T_2 to reach B and A is a and b respectively, then

$$Speed\ of\ train\ T_1 : Speed\ of\ train\ T_2 = \sqrt{b} : \sqrt{a}$$

Some important formula for Boat and stream

In water, the direction along the stream is called downstream and, the direction against the stream is called upstream.

If the speed of a boat in still water is *u* and the speed of the stream is *v*, then
Speed downstream = $(u + v)$
Speed upstream = $(u - v)$

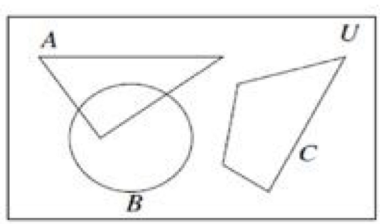
If the speed downstream is *a* and the speed upstream is *b*, then
Speed in still water = $\frac{a+b}{2}$
Rate of stream = $\frac{a-b}{2}$

Age Problem

1. If the current age of a person is *x*, then *n* times the age is *nx*.
2. If the current age of a person is *x*, then the person's age *n* years later/hence = *x* + *n*.
3. If the current age of a person is *x*, then the person's age *n* years ago = *x* - *n*.
4. The ages in a ratio *a: b* will be *ax* and *bx*.

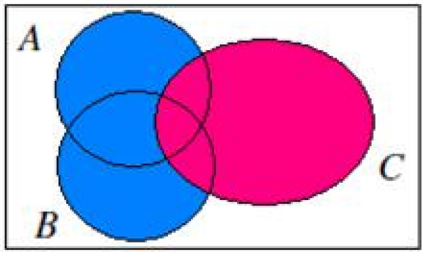
Venn diagram

The universal set U is usually represented by a rectangle.
Inside this rectangle, subsets of the universal set are represented by geometrical figures.

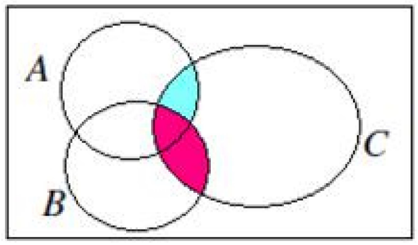


Venn diagrams help us identify some useful formulas in set operations.

To represent (AUB) ∩ C:



To represent (A∩C) U (B∩C):

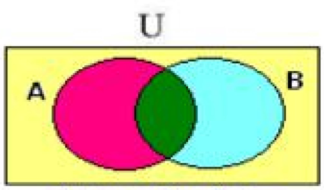


⇨ (AUB)∩ C = (A ∩ C) U(B ∩ C)

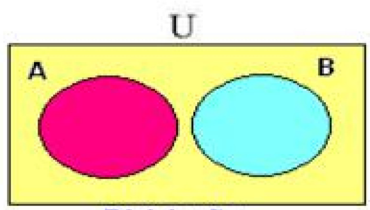
Important Formulas:

n (A U B) = n (A) + n (B) – n (A n B)

If A and B are independent sets, then n (A n B) = n (A) × n (B)



Intersecting Sets



Disjoint Sets

Work and Efficiency

Work done = Time Taken to complete the work x Efficiency

Time and Work:

If the time taken to complete a work is *d* units of time, then Efficiency = Work completed in one unit of time = $\frac{1}{d}$

Time taken to complete the work = $\frac{1}{Work\ completed\ in\ one\ unit\ of\ time}$

Data Analysis

Permutation and Combination

Factorial:

$$n! = 1 \times 2 \times 3 \times \ldots \times (n-1) \times n$$
$$n! = n \times (n-1)!$$

Permutations: If the order does matter, it is a Permutation.

$nP_r = \frac{n!}{(n-r)!}$

Combinations: If the order doesn't matter, it is a Combination.

$nC_r = \frac{n!}{r!\,(n-r)!}$

Where, *n*, *r* are non-negative integers and $r \le n$.

r is the size of each permutation.

n is the size of the set from which elements are permuted.

! is the factorial operator.

Permutation with and without repetition

Repetition Allowed:

Number of ways to choose *r* of them out of *n* number of things where repetition of things is allowed and the order in which the things are chosen matters = n^r

Repetition not allowed:

Number of ways to choose *r* of them out of *n* number of things where repetition of things is **not** allowed and the order in which the things are chosen matters = $\frac{n!}{(n-r)!}$

Combination with and without repetition:

Repetition not allowed:

Number of ways to choose *r* of them out of *n* number of things where repetition of things is **not** allowed and the order in which the things are chosen does not matter = $\frac{n!}{r!\,(n-r)!}$

Repetition allowed:

Number of ways to choose *r* of them out of *n* number of things where repetition of things is allowed and the order in which the things are chosen **does not** matters = $\frac{(n+r-1)!}{r!\,(n-1)!}$

Statistics

- **Mean** – Average value of a set

Mean:

$$\text{Sample Mean} = \bar{x} = \frac{\sum x}{n}$$

$$\text{Population Mean} = \mu = \frac{\sum x}{N}$$

Standard deviation:

$$\text{Sample standard deviation} = s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$$

$$\text{Population standard deviation} = \mu = \sqrt{\frac{\sum(x-\mu)^2}{N}}$$

Where *n* = Sample Size and *N* = Population Size

- **Mode** – Most frequently occurring value in a set
- **Median** – Midpoint between lowest and highest value of a set
- **Range** – Difference between largest and smallest value in a set

  Range = Largest data value - smallest data value

Sample mean for a frequency distribution:

$$\bar{x} = \frac{\sum fx}{n}$$

Sample standard deviation for a frequency distribution:

$$s = \sqrt{\frac{\sum f(x - \bar{x})^2}{n - 1}}$$

Sample coefficient of variation CV:

$$CV = \frac{s}{\bar{x}}$$

Probability

Probability of an Event = $\frac{\text{Number of Favorable Outcomes}}{\text{Number of Total Outcomes}}$

Odds in favor of an Event = $\frac{\text{Number of Favorable Outcomes}}{\text{Number of Unfavorable Outcomes}}$

Odds against an Event = $\frac{\text{Number of Unfavorable Outcomes}}{\text{Number of Favorable Outcomes}}$

Probability of the complement of event A (A^c)

$P(A^c) = 1 - P(A)$

General addition rule

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

General multiplication rules

P (A and B) = P (A) × P (B given A)

P (A and B) = P (B) × P (A given B)

Independent events

P (A and B) = P (A) × P (B)

- ⇨ *P*(*A* or *B*) = *P*(*A*) + *P*(*B*) - P(A) × P(B)
- ⇨ P(B given A) = P(B)
- ⇨ P(A given B) = P(A)

Mutually Exclusive Events:

P (A and B) = 0

- ⇨ *P* (*A* or *B*) = *P* (*A*) + *P* (*B*)
- ⇨ P (B given A) = 0
- ⇨ P (A given B) = 0

Probability Distribution

A **random variable** is a variable (typically represented by *x*) that has a single numerical value that is determined by chance.

A **probability distribution** is a graph, table, or formula that gives the probability for each value of the random variable.

If *x* is a random variable then denote by *P(x)* to be the probability that *x* occurs. It must be the case that $0 \leq P(x) \leq 1$ for each value of *x* and $\Sigma\ P(x) = 1$(the sum of all the probabilities is 1)

Normal Distribution:

The Normal Distribution is also called the Gaussian distribution. It is defined by two parameters mean ('average' m) and standard deviation (σ). A theoretical frequency distribution for a set of variable data, usually represented by a bell-shaped curve is symmetrical about the mean.

Formula:

X < mean = 0.5 - Z

X > mean = 0.5 + Z

X = mean = 0.5

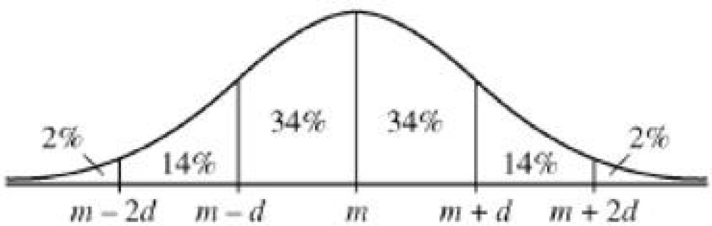
$$Z = \frac{X - m}{\sigma}$$

Where,

m = Mean

σ = Standard Deviation

X = Normal Random Variable



The figure shows a normal distribution with mean *m* and standard deviation *d*, including **approximate** percents of the distribution corresponding to the six regions shown.

Binomial Distribution:

Let X be a random variable. If X follows binomial probability distribution, then

$$P(X = r) = \frac{n!}{r!\,(n-r)!} \times p^{r} \times q^{n-r}$$

Where *r* = number of successes;

p = probability of success; *q* = 1 - *p*

Geometry

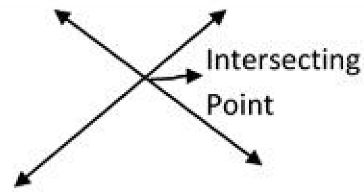
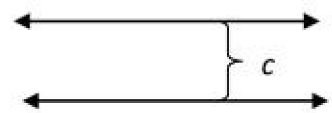
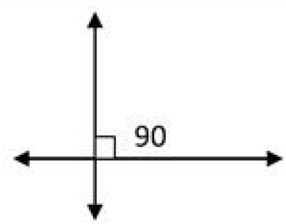
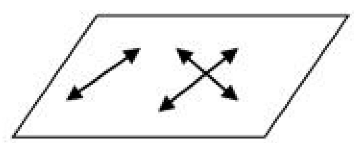
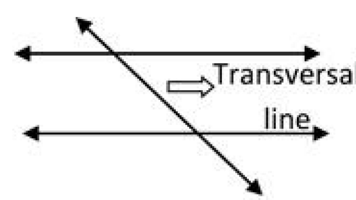
Basic Geometry

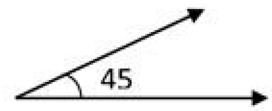
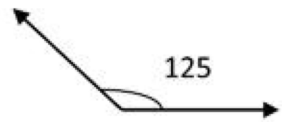
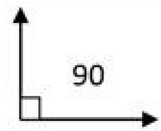
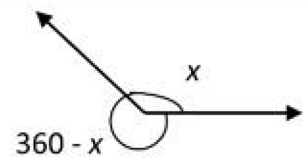
Line:

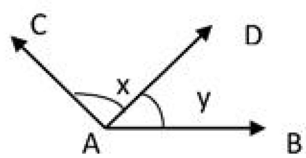
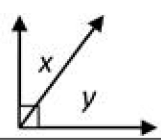
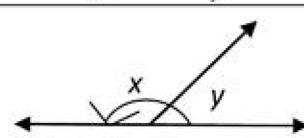
Has no starting or ending point. A line is often designated by a single variable. For example, given below is a line *m*

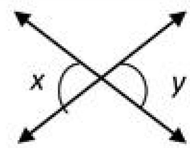
Line Segment:

Has a starting and ending point. A line segment is designated by its end points. For example, given below is a line segment AB

Types of Lines	Definition	Figure
Intersecting lines	Two or more line or line segments that meet at a point are called intersecting lines.	
Parallel lines	Parallel lines are lines that never meet. The distance between the lines at any point of time is a constant c.	
Perpendicular lines	Lines intersect to form right (90 degrees) angles.	
Coplanar Lines	Lines that lie on the same plane	
Transversal Line	A line that intersect two other coplanar lines	

Types of Angles	Definition	Figure
Acute angle	An acute angle is greater than 0^o and less than 90^o. See the figure for example.	
Obtuse angle	An obtuse angle is greater than 90^o and less than 180^o. See the figure for example.	
Right angle	A right angle equals 90^o.	
Straight angle	A straight angle is an angle of a straight line which equals 180^o	
Reflex angle	A reflex angle is greater than 180^o and less than 360^o. In the figure, reflex angle of x is 180 - x	

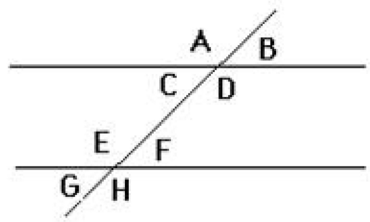
Angle Relationship	Definition	Figure
Adjacent Angles	Angles with a common vertex and a common side x and y share a common side AD and a common vertex A are adjacent angles	
Complementary Angles	Two angles whose measure sum up to 90 degree. $x + y = 90$	
Supplementary Angles	Two angles whose measure sum up to 180 degree $x + y = 180$	

Vertical Angles	Pair of opposite angles formed by two intersecting lines $x = y$	

When a transversal cuts two or more parallel lines, the following angles are formed.

Alternate interior angles

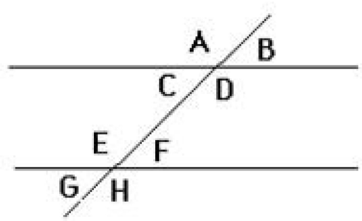
Pairs of interior angles on opposite sides of the transversal



C = F, E = D

Alternate exterior angles

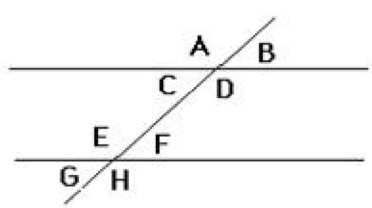
Pairs of exterior angles on opposite sides of the transversal



A = H B = G

Corresponding angles

When two lines are crossed by another line (which is called the Transversal), the angles in similar positions are called corresponding angles.



A = E, C = G, B = F, D = H

Triangle

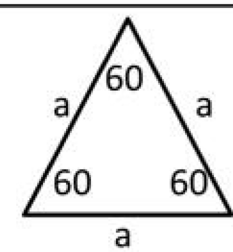
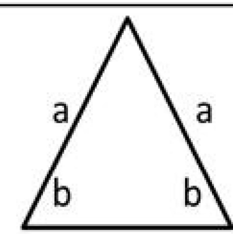
Properties of Triangles

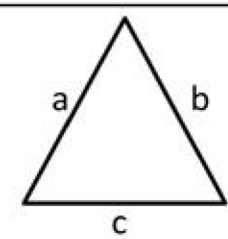
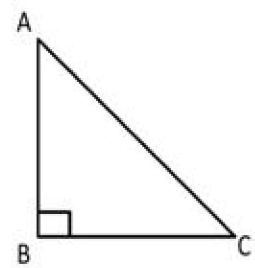
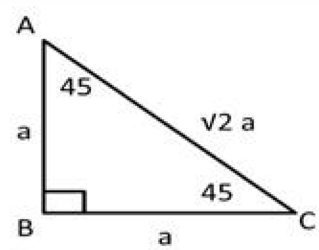
Triangles have the following properties:

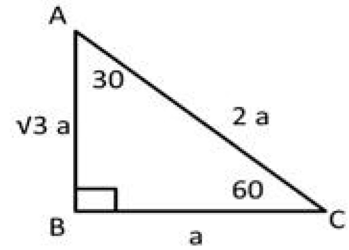
- All triangles have 3 straight sides, 3 corners (vertices), and 3 angles.
- All triangles have angles adding up to 180°.

Types of Triangle:

- Equilateral Triangles
- Isosceles Triangles
- Scalene Triangles
- Right angled Triangles

TRIANGLE NAME	DEFINITION	PROPERTIES	FIGURE
Equilateral	All the three angles and side lengths are equal	• All the angles are equal to 60 degree. • Median, Bisectors and Altitude are the same. See the figure for example. The triangle has three sides of equal length *a* and three angles are equal to 60 degree. It is equilateral.	
Isosceles	At least two angles and length of the sides opposite to these angles are equal.	• All equilateral triangles are isosceles but the converse is not true. • Median, Bisectors and Altitude drawn through the	

		vertex to the unequal side are the same. See the figure for example. The triangle has two angles of equal measure *b* and length of the sides opposite to these angles are equal (= *a*) . It is isosceles.	
Scalene	None of the angles and sides are equal	See the figure for example. The length of the sides of the triangle are of different measure, *a*, *b*, and *c*. The angles will be of different measure. It is scalene.	
Right Angled Triangle	One of the angles in the triangle is 90 degree. AC is the hypotenuse. AB and BC are the legs of the triangle.	• The other two angle measures less than 90 degree. • The other two angles are a pair of complementary angles. • Pythagorean Theorem: $AC^2 = AB^2 + BC^2$	
45-45-90 triangle	A right angled triangle in which the remaining two angles are 45 degree	• It is an isosceles triangle. • The ratio of length of the sides of this triangle is 1 : 1 : √2	

30-60-90 triangle	A right angled triangle in which the remaining two angles are 30 and 60 degrees	• It is a scalene triangle • The ratio of length of the sides of this triangle is 1 : √3 : 2	

Polygon

Types of Polygons:

Equiangular - all angles are equal.

Equilateral - all sides are of the same length.

Regular - all angles are equal and all sides are of the same length. Regular polygons are both equiangular and equilateral.

Convex - a straight line drawn through a convex polygon crosses at most two sides. Every interior angle is less than 180°.

Concave - you can draw at least one straight line through a concave polygon that crosses more than two sides. At least one interior angle is more than 180°.

Number of diagonals in an *n*-sided polygon = $\frac{n(n-3)}{2}$

Points to remember:

- Polygon is a closed figure. It starts with 3 sides and it can have *n* sides.
- There are two types of angles in a polygon. Internal or Interior angle; External or Exterior angle.
- Interior angle + Exterior angle = 180
- Sum of the Interior or internal angle of an n sided polygon = (n-2) × 180

- Sum of the exterior angles of an *n*-sided polygon is 360

Coordinate Geometry

- Equation of a straight line:
  $y = mx + c$ where m is the slope.
    - The slopes of two lines, m_1 and m_2 are equal if the lines are parallel. If the two lines are perpendicular, $\mathbf{m_1 \times m_2 = -1}$.
    - Finding the y-intercept: Put $x = 0$ in the above equation. c is the y-intercept.
    - Finding the x-intercept: Put y = 0 in the above equation. $\frac{-c}{m}$ is the x-intercept.

- The equation of a straight line which cuts off intercepts a and b on the x-axis and y-axis is

$$\frac{x}{a} + \frac{y}{b} = 1$$

- The equation of a straight line passing through the origin (0, 0) is $\boldsymbol{y = mx}$.
    - The equation of a straight line passing through the origin and making an angle 45 degree with the positive x-axis is $\mathbf{y = x}$
    - The equation of a straight line passing through the origin and making an angle 45 degree with the negative x-axis is $\mathbf{y = -x}$

- Equation of a straight line parallel to the y-axis at a distance 'a' from it is $x = a$.

- Equation of a straight line parallel to the x-axis at a distance 'b' from it is $y = b$.

- Equation of a line parallel to the x-axis and passing through the point (a, b) is $y = b$.

- Equation of a line perpendicular to x-axis and passing through (a, b) is $x = a$.

- Equation of a line parallel to the y-axis and passing through (a, b) is $x = a$.

- Equation of a line perpendicular to the y-axis and passing through (a, b) is $y = b$.

- Equation of x-axis is $y = 0$ and equation of y-axis is $x = 0$.

Slope intercept Form: Given y-intercept, b and the slope m of a line. The equation of the line is given by, $y = mx + b$

Point Slope form: Given a point (x, y) on a line and the slope m of the line. The equation of the line is given by, $y - y_1 = m(x - x_1)$

Two point form: Given any two points (x_1, y_1) and (x_2, y_2) on a line. The equation of the line is given by, $\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$

Intercept Form: Given the x-intercept, a and y-intercept, b of a line. The equation of the line is given by, $\frac{x}{a} + \frac{y}{b} = 1$

Quadratic equation:

It is an equation of the form, $y = ax^2 + bx + c$
Where a, b, and c are constant
If $a \neq 0$, then the graph of the equation will be a parabola.
If $a > 0$, then parabola opens upward
If $a < 0$, then parabola opens downward

Ways to find the vertex of the parabola:

x coordinate of vertex = $\frac{-b}{2a}$
y coordinate of vertex can obtained by substituting the above x value in the equation.
Hence the coordinate of the vertex is given by (x, y).

Standard form of equation of a Circle:

The graph of an equation of the form $(x-a)^2 + (y-b)^2 = r^2$ is a circle with its center at the point (*a*, *b*) and with radius *r*.

Solid Geometry and Shaded area

Two-dimension:

Rectangle:
Area of Rectangle = length × breadth
Perimeter of rectangle = 2 (length + breadth)

Square:
Area of square = $(side)^2$
Perimeter of square = 4 × side

Triangle:
Area of triangle = $\frac{1}{2} \times base \times height$
Area of equilateral triangle = $\frac{\sqrt{3}}{4} \times (side)^2$

Parallelogram:
Area of parallelogram = $base \times height$

Trapezium:
Area of trapezium = $\frac{1}{2} \times (a+b) \times h$
Where, *a* and *b* are length of the parallel sides, *h* is distance between them.

Rhombus:
Area of rhombus = $\frac{1}{2} \times d_1 \times d_2$
Where, d_1 and d_2 are the length of the diagonals of the rhombus

Circle:

Circumference of circle = $2\pi r$

Area of circle = πr^2

Where, r is the radius and d is the diameter of the circle.

Sector:

Length of the arc = $\frac{\theta}{360} \times 2\pi r$

Area of a sector of a circle = $\frac{\theta}{360} \times \pi \times r^2$

Where θ is the degree measure of sector's arc.

Three Dimension:

Cuboid:

Volume of cuboid = $l \times b \times h$

Surface area cuboid = $2(lb + bh + hl))$

Body Diagonal = $\sqrt{l^2 + b^2 + h^2}$

Where, $l, b, and\ h$ are the length, breadth, and height of the cuboid.

Cube:

Volume of cube = a^3

Surface area = $6a^2$

Body diagonal = $\sqrt{3}a$

Where, a is the length of the edge of the cube.

Sphere:

Volume of Sphere = $\frac{4}{3} \times \pi \times r^3$

Surface area of Sphere = $4\pi r^2$

Where, r is the radius.

Cylinder:

Volume of Cylinder = $\pi r^2 h$

Curved surface area of cylinder = $2\pi rh$

Total surface area of cylinder = $2\pi r$ ($h + r$)

Where, $h\ and\ r$ are the height and the radius of the cylinder.

Cone:

Volume of cone = $\frac{1}{3}\ \pi r^2 h$

Slant height of cone = $l = \sqrt{h^2 + r^2}$

Curved surface area of cone = $\pi r l$

Total surface area of cone = $\pi r\ (r + l)$

Where, $l, h, and\ r$ are the slant height, height, and radius of the cone.

Data Interpretation

Generally for any DI question, it is important to analyze the graph first and then check the question.

In the graph, you need to check,

- What is the graph talking about?
- What is given in the x-axis and what is given in the y-axis?
- Is the values given in numbers or percentages? If the values are percentages, then definitely a total value will be given somewhere. So, find the total value.
- In case, there is more than one graph, the additional thing that you should find is the relation between the different graphs given.

After finding all these, check the question and solve it.