## Chapter 4

**Relational Algebra** 

&

**Relational Calculus** 

#### **Chapter 4 - Objectives**

- Meaning of the term relational completeness.
- How to form queries in relational algebra.
- How to form queries in tuple relational calculus.
- How to form queries in domain relational calculus.
- Categories of relational DML.

#### Introduction

- Relational algebra and relational calculus are formal languages associated with the relational model.
- Informally, relational algebra is a (high-level) procedural language and relational calculus a non-procedural language.
- However, formally both are equivalent to one another.
- A language that produces a relation that can be derived using relational calculus is <u>relationally complete</u>.

#### Database Query Languages

- Given a database, ask questions, get data as answers
  - Get all students with GPA > 3.7 who applied to USQ and QUT and nowhere else
  - Get all humanities departments at campuses in Queensland with < 200 applicants</li>
  - Get the campus with highest average accept rate over the last five years
- Some questions are easy to pose, some are not
- Some questions are easy for DBMS to answer, some are not.
- "Query language", but also used to update the database

## Relational Query Languages

- Formal:
  - relational algebra, relational calculus, Datalog
- Practical:
  - SQL,
  - Quel,
  - Query-by-Example (QBE)
- In **ALL** languages, a query is executed over a set of relations, <u>get</u> <u>single relation as the result</u>

#### Relational Algebra (RA)

- Relational algebra operations work on one or more relations to define another relation without changing the original relations.
- Both operands and results are relations, so output from one operation can become input to another operation.
- Allows expressions to be nested, just as in arithmetic. This property is called <u>closure</u>.

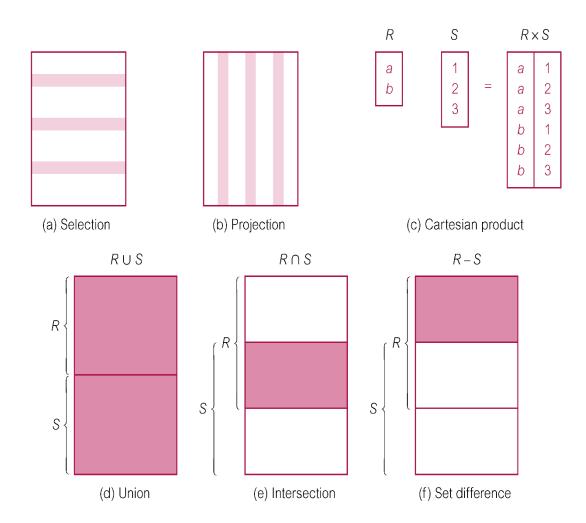
#### Relational Algebra

- 5 basic operations in relational algebra: Selection, Projection, Cartesian product, Union, and Set Difference.
- These perform most of the data retrieval operations needed.
- Also have Join, Intersection, and Division operations, which can be expressed in terms of 5 basic operations.

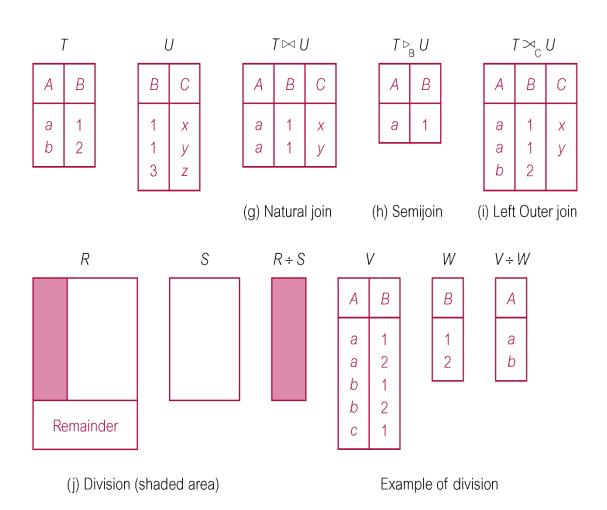
#### RA Operations

- Operations of traditional relational algebra fall into four broad classes:
  - 1. Operations that remove parts of a relation
  - 2. Renaming
  - 3. Set operations
  - Operations that combine tuples of two relations

## **Relational Algebra Operations**



#### **Relational Algebra Operations**



#### **Selection (or Restriction)**

- σ<sub>predicate</sub> (R)
  - Works on a single relation R (unary operation) and defines a relation that contains only those tuples (rows) of R that satisfy the specified condition (predicate).
  - Schema of  $\sigma_c(R)$  is the same as schema of R
  - Selection loses information
  - Condition C:
    - AND, OR, NOT, A  $\theta$  B, A  $\theta$  c, where  $\theta$  {<,  $\leq$ , >,  $\geq$ , =,  $\neq$ }

# **Example - Selection (or Restriction)**

• List all staff with a salary greater than £10,000.

 $\sigma_{\text{salary}}$  > 10000 (Staff)

staffNo	fName	IName	position	sex	DOB	salary	branchNo
SL21	John	White	Manager	M	1-Oct-45	30000	B005
SG37	Ann	Beech	Assistant	F	10-Nov-60	12000	B003
SG14	David	Ford	Supervisor	M	24- Mar-58	18000	B003
SG5	Susan	Brand	Manager	F	3-Jun-40	24000	B003

#### **Projection**

- $\Pi_{\text{col1},\ldots,\text{coln}}(R)$ 
  - Works on a single relation R (unary operation) and defines a relation that contains a vertical subset of R, extracting the values of specified attributes and eliminating duplicates.
  - Projection loses information
    - Possibly vertically, possibly horizontally
  - Schema of resulting relation: attributes subset of the attributes of R

#### **Example - Projection**

• Produce a list of salaries for all staff, showing only staffNo, fName, lName, and salary details.

 $\Pi_{\text{staffNo, fName, IName, salary}}$  (Staff)

staffNo	fName	IName	salary	
SL21	John	White	30000	
SG37	Ann	Beech	12000	
SG14	David	Ford	18000	
SA9	Mary	Howe	9000	
SG5	Susan	Brand	24000	
SL41	Julie	Lee	9000	

#### Rename

- $\rho_{v}(R)$  (unary operation)
  - ullet  $\gamma$  is a one-to-one function that maps a set of attributes to a new set of attributes
  - Schema is the same, up to renaming of attributes
  - Content, or instance, remains unchanged

#### Union

- $R \cup S$  (binary operation, set operation)
  - Union of two relations R and S defines a relation that contains all the tuples of R, or S, or both R and S, duplicate tuples being eliminated.
  - R and S must be union-compatible.
    - Same set of attributes + domains
- If R and S have I and J tuples, respectively, union is obtained by concatenating them into one relation with a maximum of (I + J) tuples.
- Lossless, but impossible to undo
- commutative, associative

#### **Example - Union**

• List all cities where there is either a branch office or a property for rent.

 $\Pi_{\text{city}}(Branch) \cup \Pi_{\text{city}}(PropertyForRent)$ 

#### city

London

Aberdeen

Glasgow

**Bristol** 

#### **Set Difference**

- **R S** (binary operation, set operation)
  - Defines a relation consisting of the tuples that are in relation R, but not in S.
  - R and S must be union-compatible.
  - Loses information
  - $R S \neq S R !!$

#### **Example - Set Difference**

• List all cities where there is a branch office but no properties for rent.

 $\Pi_{city}(Branch) - \Pi_{city}(PropertyForRent)$ 

city

**Bristol** 

#### Intersection

- $R \cap S$  (binary operation, set operation)
  - Defines a relation consisting of the set of all tuples that are in both R and S.
  - R and S must be union-compatible.
- Expressed using basic operations:

$$R \cap S = R - (R - S)$$

• commutative, associative

#### **Example - Intersection**

• List all cities where there is both a branch office and at least one property for rent.

 $\Pi_{\text{city}}(Branch) \cap \Pi_{\text{city}}(PropertyForRent)$ 

#### city

Aberdeen

London

Glasgow

#### **Cartesian product**

- R × S (binary operation, set operation)
  - Defines a relation that is the concatenation of every tuple of relation R with every tuple of relation S.
  - Lossless, possible to undo using projection
    - Unless one of R, S is empty!
  - $\#(R \times S) = \#R * \#S$
  - Schema: union of sets of attributes
  - commutative, associative

#### **Example - Cartesian Product**

 List the names and comments of all clients who have viewed a property for rent.

( $\Pi_{\text{clientNo, fName, IName}}$ (Client)) X ( $\Pi_{\text{clientNo, propertyNo, comment}}$ 

(Viewing))

Requires further restriction!

client.clientNo	fName	IName	Viewing.clientNo	propertyNo	comment
CR76	John	Kay	CR56	PA14	too small
CR76	John	Kay	CR76	PG4	too remote
CR76	John	Kay	CR56	PG4	
CR76	John	Kay	CR62	PA14	no dining room
CR76	John	Kay	CR56	PG36	
CR56	Aline	Stewart	CR56	PA14	too small
CR56	Aline	Stewart	CR76	PG4	too remote
CR56	Aline	Stewart	CR56	PG4	
CR56	Aline	Stewart	CR62	PA14	no dining room
CR56	Aline	Stewart	CR56	PG36	
CR74	Mike	Ritchie	CR56	PA14	too small
CR74	Mike	Ritchie	CR76	PG4	too remote
CR74	Mike	Ritchie	CR56	PG4	
CR74	Mike	Ritchie	CR62	PA14	no dining room
CR74	Mike	Ritchie	CR56	PG36	
CR62	Mary	Tregear	CR56	PA14	too small
CR62	Mary	Tregear	CR76	PG4	too remote
CR62	Mary	Tregear	CR56	PG4	
CR62	Mary	Tregear	CR62	PA14	no dining room
CR62	Mary	Tregear	CR56	PG36	

## **Example - Cartesian Product and Selection**

• Use selection operation to extract those tuples where Client.clientNo = Viewing.clientNo.

$$\sigma_{\substack{\text{Client.clientNo} = viewing.clientNo}}((\prod_{\substack{\text{clientNo,fName,lName}}}(Client)) \ X \\ (\prod_{\substack{\text{clientNo,propertyNo,comment}}}(Viewing)))$$

client.clientNo	fName	IName	Viewing.clientNo	propertyNo	comment
CR76	John	Kay	CR76	PG4	too remote
CR56	Aline	Stewart	CR56	PA14	too small
CR56	Aline	Stewart	CR56	PG4	
CR56	Aline	Stewart	CR56	PG36	
CR62	Mary	Tregear	CR62	PA14	no dining room

• Cartesian product and Selection can be reduced to a single operation called a *Join*.

## **Join Operations**

- Join is a derivative of Cartesian product.
- Equivalent to performing a Selection, using join predicate as selection formula, over Cartesian product of the two operand relations.
  - $\sigma_c(R \times S)$
- One of the most difficult operations to implement efficiently in an RDBMS and one reason why RDBMSs have intrinsic performance problems.
  - But can usually be optimized

#### **Join Operations**

- Various forms of join operation
  - Theta join
  - Equijoin (a particular type of Theta join)
  - Natural join
  - Outer join
  - Semijoin

## Theta join (θ-join)

- R
  - Defines a relation that contains tuples satisfying the predicate F from the Cartesian product of R and S.
  - The predicate F is of the form  $R.a_i \theta S.b_i$  where  $\theta$  may be one of the comparison operators  $(<, \le, >, \ge, =, \ne)$ .

## Theta join (θ-join)

 Can rewrite Theta join using basic Selection and Cartesian product operations.

$$R \qquad {}_{F}S = \sigma_{F}(R \times S)$$

- Degree of a Theta join is sum of degrees of the operand relations *R* and *S*.
- If predicate F contains only equality (=), the term *Equijoin* is used.

## **Example - Equijoin**

• List the names and comments of all clients who have viewed a property for rent.

$$(\Pi_{\text{clientNo},\text{fName},\text{IName}}(\text{Client})) \qquad \text{Client.clientNo} = \text{Viewing.clientNo} (\Pi_{\text{clientNo},\text{propertyNo},\text{comment}}(\text{Viewing}))$$



client.clientNo	fName	IName	Viewing.clientNo	propertyNo	comment
CR76	John	Kay	CR76	PG4	too remote
CR56	Aline	Stewart	CR56	PA14	too small
CR56	Aline	Stewart	CR56	PG4	
CR56	Aline	Stewart	CR56	PG36	
CR62	Mary	Tregear	CR62	PA14	no dining room

#### **Natural Join**

- R 🔊
  - An Equijoin of the two relations R and S over all common attributes x. One occurrence of each common attribute is eliminated from the result.
  - Usual simulation (selection and cartesian product), plus projection

#### **Example - Natural Join**

• List the names and comments of all clients who have viewed a property for rent.

$$(\Pi_{\text{clientNo,fName,IName}}(\text{Client}))$$
  $(\Pi_{\text{clientNo,propertyNo,comment}}(\text{Viewing}))$ 

clientNo	fName	IName	propertyNo	comment
CR76	John	Kay	PG4	too remote
CR56	Aline	Stewart	PA14	too small
CR56	Aline	Stewart	PG4	
CR56	Aline	Stewart	PG36	
CR62	Mary	Tregear	PA14	no dining room

#### **Outer join**

• To display rows in the result that do not have matching values in the join column, use Outer join.

#### • R S

- (Left) outer join is join in which tuples from R that do not have matching values in common columns of S are also included in result relation.
- Padded with NULLs

## **Example - Left Outer join**

• Produce a status report on property viewings.

 $\Pi_{\text{propertyNo,street,city}}$ (PropertyForRent) Viewing



propertyNo	street	city	clientNo	viewDate	comment
PA14	16 Holhead	Aberdeen	CR56	24-May-01	too small
PA14	16 Holhead	Aberdeen	CR62	14-May-01	no dining room
PL94	6 Argyll St	London	null	null	null
PG4	6 Lawrence St	Glasgow	CR76	20-Apr-01	too remote
PG4	6 Lawrence St	Glasgow	CR56	26-May-01	
PG36	2 Manor Rd	Glasgow	CR56	28-Apr-01	
PG21	18 Dale Rd	Glasgow	null	null	null
PG16	5 Novar Dr	Glasgow	null	null	null

## Semijoin

- R 🕸
  - Defines a relation that contains the tuples of R that participate in the join of R with S.

Can rewrite Semijoin using Projection and Join:

$$R \triangleright_F S = \Pi_A(R \bowtie S)$$

#### **Example - Semijoin**

List complete details of all staff who work at the branch in Glasgow.

Staff Staff.brancNo = Branch.branchNo and branch.city = 'Glasgow' Branch



staffNo	fName	IName	position	sex	DOB	salary	branchNo
SG37	Ann	Beech	Assistant	F	10-Nov-60	12000	B003
SG14	David	Ford	Supervisor	M	24- Mar-58	18000	B003
SG5	Susan	Brand	Manager	F	3-Jun-40	24000	B003

#### **Division**

- R ÷ S
  - Defines a relation over the attributes C that consists of set of tuples from R that match combination of every tuple in S.
- Expressed using basic operations:

$$T_1 \leftarrow \Pi_c(R)$$

$$T_2 \leftarrow \Pi_c((S \times T_1) - R)$$

$$T \leftarrow T_1 - T_2$$

# **Example - Division**

Identify all clients who have viewed all properties with three rooms.

$$(\Pi_{\text{clientNo,propertyNo}}(\text{Viewing})) \div (\Pi_{\text{propertyNo}}(\sigma_{\text{rooms}=3}(\text{PropertyForRent})))$$

clientNo	propertyNo
CR56	PA14
CR76	PG4
CR56	PG4
CR62	PA14
CR56	PG36
CK56	PG30

 $\Pi_{\text{clientNo,propertyNo}}(\text{Viewing}) \quad \Pi_{\text{propertyNo}}(\sigma_{\text{rooms}=3}(\text{PropertyForRent}))$ 

propertyNo	
PG4 PG36	

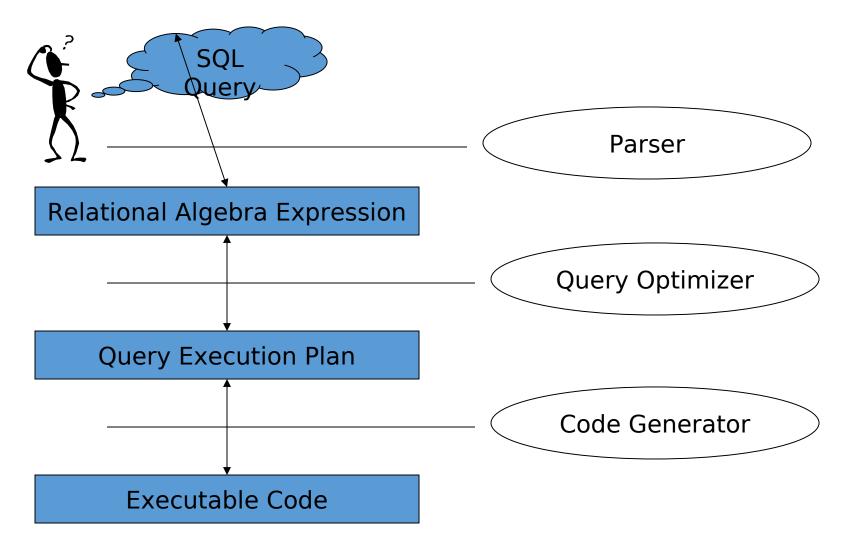
clientNo **CR56** 

**RESULT** 

# Why Relational Algebra?

- All DBMSs use relational algebra as intermediate language for specifying query evaluation algorithms
  - Parse SQL and translate it into expression in relational algebra
  - However, translated expression (or straight SQL) would be very inefficient
- Set of rules for manipulating algebraic expressions
  - Don't exist for SQL
  - Expressions can be converted into equivalent ones which take less time to execute
  - Done by query optimizer

# Overview of Query Processing



# Remarks about the Relational Algebra

- The Relational Algebra is not Turing Complete
  - No explicit loop
  - No recursion
- This is a feature, not a bug
  - Helps with query optimization and processing
  - Operations are linear in size of instance
- It is undecidable whether two algebra expressions are equivalent

No transitive closure!

- Restriction to Conjunctive Queries: decidable
  - CQ: Selection, projection, Cartesian product only

# Non-trivial example queries

- Consider the relation schema:
  - Visits(Drinker,Bar); Likes(Drinker,Beer); Serves(Bar,Beer);
- Give all the drinkers with the beers they do not like
  - $(\Pi_{Drinker}(Likes) \times \Pi_{Beer}(Likes))$  Likes
- Give the pairs of beers that are not served in a common bar
  - $(\Pi_{Beer}(Serves) \times \Pi_{Beer}(Serves))$  -
  - $\Pi_{\text{Beer1.Beer2}}$ . $\sigma_{\text{Bar1=Bar2}}$ (Serves x Serves)

## More hard RA expressions

- Give all the drinkers that like all beers that 'John' likes
  - Likes:  $\Pi_{\text{Beer}}$ .  $\sigma_{\text{Drinker='John'}}$  (Likes)
- Give all the drinkers that like exactly the same beers as 'John'
  - ( Likes :  $\Pi_{Beer}$  .  $\sigma_{Drinker='John'}$  (Likes) )  $\cap$
  - (((Π<sub>Drinker</sub>(Likes) X Π<sub>Beer</sub>(Likes)) Likes) :
  - $\Pi_{Beer}$ .  $\sigma_{Drinker='John'}$  (( $\Pi_{Drinker}$ (Likes) X  $\Pi_{Beer}$ (Likes)) Likes))

### Relational Calculus (RC)

- Relational calculus query specifies what is to be retrieved rather than how to retrieve it.
  - No description of how to evaluate a query.
- In first-order logic (or predicate calculus), *predicate* is a truth-valued function with arguments.
- When we substitute values for the arguments, function yields an expression, called a *proposition*, which can be either true or false.

#### **Relational Calculus**

- If predicate contains a variable (e.g. 'x is a member of staff'), there must be a range for x.
- When we substitute some values of this range for x, proposition may be true; for other values, it may be false.
- When applied to databases, relational calculus has two forms: tuple and domain.

#### **Tuple Relational Calculus (TRC)**

- Interested in finding tuples for which a predicate is true. Based on use of <u>tuple variables</u>.
- Tuple variable is a variable that 'ranges over' a named relation: i.e., variable whose only permitted values are tuples of the relation.
- Specify range of a tuple variable S as the Staff relation as: Staff(S)
- To find set of all tuples S such that P(S) is true:  $\{S \mid P(S)\}$

#### **Tuple Relational Calculus - Example**

- To find details of all staff earning more than £10,000: {S | Staff(S) ^ S.salary > 10000}
- To find a particular attribute, such as salary, write:

$$\{S.salary \mid Staff(S) \land S.salary > 10000\}$$

- Can use two *quantifiers* to tell how many instances the predicate applies to:
  - Existential quantifier ∃ ('there exists')
  - Universal quantifier ∀ ('for all')
- Tuple variables qualified by  $\forall$  or  $\exists$  are called *bound* variables, otherwise called *free* variables.

• Existential quantifier used in formulae that must be true for at least one instance, such as:

```
Staff(S) \wedge (\exists B)(Branch(B) \wedge (B.branchNo) = S.branchNo) \wedge B.city = 'London')
```

• Means 'There exists a Branch tuple that has the same branchNo as the branchNo of the current Staff tuple, S, and is located in London'.

• Universal quantifier is used in statements about every instance, such as:

```
\square(\forall B) (B.city \neq 'Paris')
```

- Means 'For all Branch tuples, the address is not in Paris'.
- Can also use  $\sim(\exists B)$  (B.city = 'Paris') which means 'There are no branches with an address in Paris'.

- Formulae should be unambiguous and make sense.
- A (well-formed) formula is made out of <u>atoms</u>:
  - $R(S_i)$ , where  $S_i$  is a tuple variable and R is a relation
  - $S_i a_1 \theta S_j a_2$
  - $S_{i}$ .  $a_{1} \theta c$
- Can recursively build up formulae from atoms:
  - An atom is a formula
  - If  $F_1$  and  $F_2$  are formulae, so are their conjunction,  $F_1 \,^{\wedge} F_2$ ; disjunction,  $F_1 \,^{\vee} F_2$ ; and negation,  $\sim F_1$
  - If F is a formula with free variable X, then  $(\exists X)(F)$  and  $(\forall X)(F)$  are also formulae.

#### **Example - Tuple Relational Calculus**

a) List the names of all managers who earn more than £25,000.

```
{S.fName, S.lName | Staff(S) ^
S.position = 'Manager' ^ S.salary > 25000}
```

b) List the staff who manage properties for rent in Glasgow.

```
{S | Staff(S) ^ (∃P) (PropertyForRent(P) ^ (P.staffNo = S.staffNo) ^ P.city = 'Glasgow')}
```

#### **Example - Tuple Relational Calculus**

c) List the names of staff who currently do not manage any properties.

```
{S.fName, S.lName | Staff(S) ^ (~(∃P) (PropertyForRent(P)^(S.staffNo = P.staffNo)))}
```

Or

```
{S.fName, S.lName | Staff(S) \land (\forallP) (\simPropertyForRent(P) \lor \sim(S.staffNo = P.staffNo)))}
```

#### **Example - Tuple Relational Calculus**

• List the names of clients who have viewed a property Glasgow.

property for rent in

```
{C.fName, C.lName | Client(C) ^ ((∃V)(∃P)
(Viewing(V) ^ PropertyForRent(P) ^ (
C.clientNo = V.clientNo) ^
(V.propertyNo=P.propertyNo)^P.city = 'Glasgow'))}
```

• Expressions can generate an infinite set. For example:

**{S | ~Staff(S)}** 

- To avoid this [an *unsafe* query], add restriction that all values in result must be values in the domain of the expression.
  - Basically, tie all tuple variables to a relation

# Unsafe queries in TRC

- The following TRC expressions are safe
  - $\{ t(A) \mid \exists u (R(u) AND u(A) = t(A)) \}$
  - { t(A) | NOT ∃u (R(u) AND u(A) ≠ t(A)) }
  - $\{ t(A) \mid \forall u (R(u) => u(A) = t(A)) \}$
- The following TRC expressions are unsafe
  - { t(A,B) | NOT R(t) }
  - $\{ t(A) \mid \exists u(u(A) = t(A)) \}$
  - $\{ t(A) \mid \forall u(R(u) \text{ AND } t(A) = 8) \}$

#### **Domain Relational Calculus (DRC)**

- Uses variables that take values from <u>domains</u> instead of tuples of relations.
- If  $F(d_1, d_2, ..., d_n)$  stands for a formula composed of atoms and  $d_1, d_2, ...$ .,  $d_n$  represent domain variables, then:

$$\{d_1, d_2, \ldots, d_n \mid F(d_1, d_2, \ldots, d_n)\}$$

is a general domain relational calculus expression.

a) Find the names of all managers who earn more than £25,000.

```
{fN, lN | (∃sN, posn, sex, DOB, sal, bN)
(Staff (sN, fN, lN, posn, sex, DOB, sal, bN) ^
posn = 'Manager' ^ sal > 25000)}
```

b) List the staff who manage properties for rent in Glasgow.

```
{sN, fN, lN, posn, sex, DOB, sal, bN | (∃sN1,cty)(Staff(sN,fN,lN,posn,sex,DOB,sal,bN) ^ (PropertyForRent(pN, st, cty, pc, typ, rms, rnt,oN, sN1, bN1) ^ (sN=sN1) ^ (cty='Glasgow')}
```

c) List the names of staff who currently do not manage any properties for rent.

```
{fN, IN | (∃sN)

(Staff(sN,fN,IN,posn,sex,DOB,sal,bN) ^

(~(∃sN1) (PropertyForRent(pN, st, cty, pc, typ,

rms, rnt,oN, sN1, bN1) ^ (sN = sN1))))}
```

Note: for brevity, some attributes here were not bound but should have been. See text book p. 106 (Third Edition), p. 108 (Fourth Edition).

You should always bind non-free variables in assignments and exams.

d) List the names of clients who have viewed a property for rent in Glasgow.

```
{fN, IN | (∃cN, cN1, pN, pN1, cty)

(Client(cN, fN, IN,tel, pT, mR) ^

Viewing(cN1, pN1, dt, cmt) ^

PropertyForRent(pN, st, cty, pc, typ,

rms, rnt,oN, sN, bN) ^

(cN = cN1) ^ (pN = pN1) ^ cty = 'Glasgow')}
```

#### **Domain Relational Calculus**

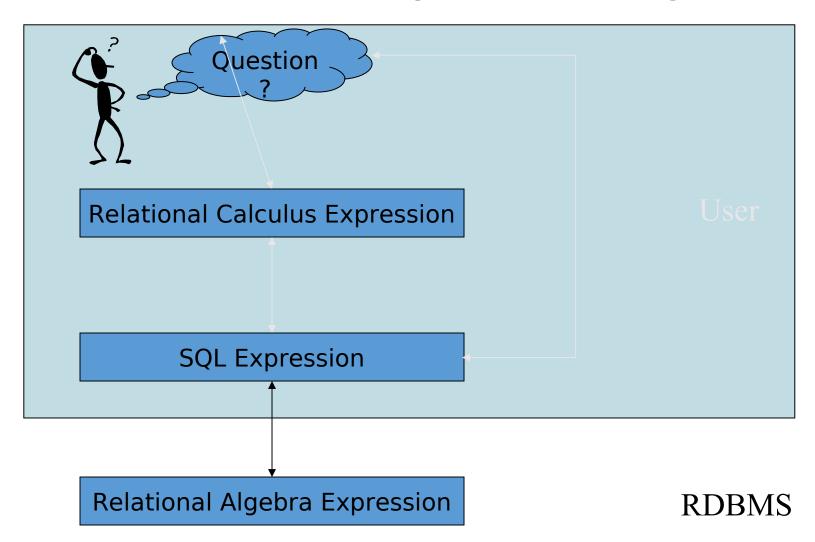
• When restricted to safe expressions, domain relational calculus is equivalent to tuple relational calculus restricted to safe expressions, which is equivalent to relational algebra.

• Means every relational algebra expression has an equivalent relational calculus expression, and vice versa.

# Why Relational Calculus?

- Easy queries can be written in SQL immediately
- Difficult queries require, either:
  - Very much experience; or
  - Trial-and-error iterative approach; or
  - Good understanding of Relational Calculus
- SQL, like RC, is a declarative language
  - With some procedural ingredients (e.g. union)
- Quantifiers in RC are directly translated in SQL (EXISTS)
- Following formal translation algorithms exist:
  - From Calculus to SQL
  - From SQL to Algebra
  - From Algebra to Calculus

# Overview of Query Processing



# Remarks about the Relational Calculus

- Corresponds to Predicate Logic
  - A.k.a First Order Logic
- Formally, a query is a function mapping a set of relations to a single relation
- Same expressive power as Relational Algebra
- Same theoretical results
- If a query language can express the same queries as the Relational Calculus, then it is relationally complete
- RC, like RA, does not have aggregate functions such as Count, and also not grouping
  - These are extra features provided by SQL
  - Instead, joining can be used for some types of counting

# **Other Languages**

- Transform-oriented languages are non-procedural languages that use relations to transform input data into required outputs (e.g. SQL).
- Graphical languages provide user with picture of the structure of the relation. User fills in example of what is wanted and system returns required data in that format (e.g. QBE).

# **Other Languages**

- 4GLs can create complete customized application using limited set of commands in a user-friendly, often menu-driven environment.
- Some systems accept a form of *natural language*, sometimes called a 5GL, although this development is still a an early stage.