BlochOut: Quantum Escape Game

A game-oriented introduction to quantum computing

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Abstract

This guide accompanies the BlochOut quantum escape game, providing detailed explanations of quantum computing concepts, solutions to puzzles, and theoretical background for students learning quantum mechanics. The game introduces fundamental quantum gates, the Bloch sphere representation, entanglement, and multi-qubit operations through an engaging narrative format.

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1 Introduction to Quantum Computing

1.1 What is a Qubit?

The Quantum Bit

A **qubit** (quantum bit) is the fundamental unit of quantum information. Unlike classical bits that can only be 0 or 1, a qubit can exist in a *superposition* of both states simultaneously.
Mathematically, a qubit state is written as:

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle \tag{1}$$

where α and β are complex numbers called *probability amplitudes*, and $|\alpha|^2 + |\beta|^2 = 1$.

1.2 The Bloch Sphere

Visualizing Quantum States

The **Bloch sphere** is a geometric representation of qubit states. Every point on the sphere's surface corresponds to a possible quantum state:

- North pole: $|0\rangle$ state
- South pole: $|1\rangle$ state
- Equator: Superposition states like $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$
- Interior points: Mixed states (not pure quantum states)

Any pure qubit state can be written as:

$$|\psi\rangle = \cos\frac{\theta}{2} |0\rangle + e^{i\phi} \sin\frac{\theta}{2} |1\rangle \tag{2}$$

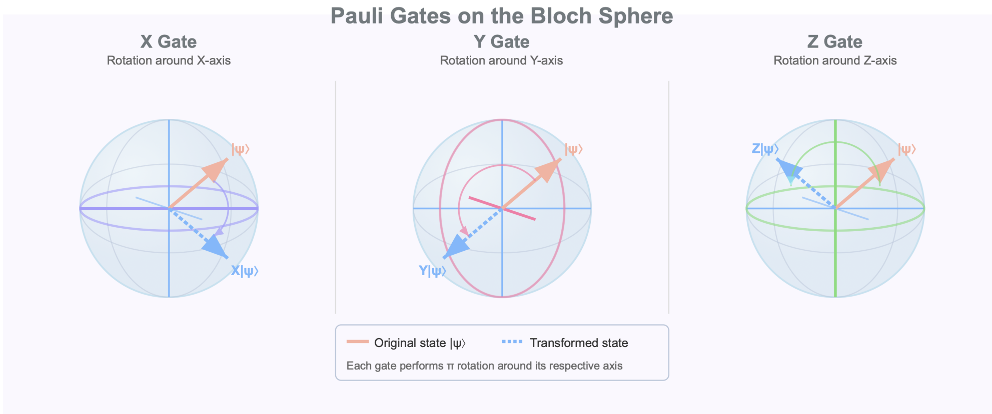
where θ is the polar angle and ϕ is the azimuthal angle on the Bloch sphere.

2 Level 1 & 2: Basic Quantum Gates

2.1 The Fundamental Gates

Pauli Gates

The Pauli gates are the most fundamental single-qubit operations:



$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{(Bit flip)} \tag{3}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \text{(Bit and phase flip)} \tag{4}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{(Phase flip)} \tag{5}$$

Gate Properties

Each Pauli gate is:

- **Hermitian**: $X^\dagger = X$, $Y^\dagger = Y$, $Z^\dagger = Z$
- **Unitary**: $XX^\dagger = I$, etc.
- **Involutory**: $X^2 = Y^2 = Z^2 = I$ (applying twice gives identity)

The game also introduces the player to some additional single qubit gates that are often used in quantum computing. These include:

Hadamard Gate

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{6}$$

The Hadamard gate is used to create an superposition of the basis states:

$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle \tag{7}$$

$$H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle \tag{8}$$

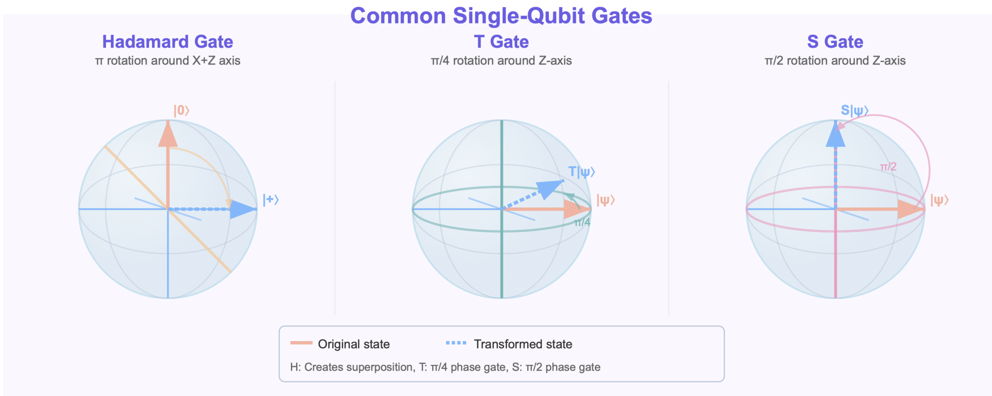


Figure 1: Caption

Phase Gates

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad \text{(Quarter phase, } S^4 = I\text{)} \tag{9}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \quad \text{(Eighth phase, } T^8 = I\text{)} \tag{10}$$

The S and T gates that can add a phase shift between the basis states $|0\rangle$ and $|1\rangle$. Equivalently, it rotates the Bloch vetor around the z-axis by $\theta = \frac{\pi}{2}$ for S and by $\theta = \frac{\pi}{4}$ for the T gate.

Did you know

- The Hadamard (H) gate and the S gate are part of a set called Clifford group gates [a]. These are a set of gates that can be classically simulable. This means that a classical computer can perform all algorithms and tasks that are based entirely of these computations. Effectively it means that if your computation or *algorithm* was entirely made up of these gates then they could be run on a classical computer with no need for quantum speedup!

- While Clifford gates are great in that they make some quantum algorithms classical simulable and therefore easy to study, they lack in that you can reach all possible quantum states using just those gates!

- T gate is an example of a gate whose transformation cannot be mimicked by S or H gate since $T = \sqrt{S}$ which is hard to perform!

[a] You will meet the final member of the Clifford group later in this odf!

2.2 Riddle Solutions

Level 1 Riddles and Solutions

1. *"Flip me over, flip me back, apply me twice and there's no lack."*
   Answer: X gate - The bit flip gate returns to identity when applied twice.

2. *"Around the Y-axis I'll rotate, apply me twice and seal your fate."*
   Answer: Y gate - Rotation around Y-axis, $Y^2 = I$.

3. *"I change your phase but not your chance."*
   Answer: Z gate - Changes phase of $|1\rangle$ but preserves probabilities.

4. *"I'm equal parts X, Y, and Z, apply me twice and you will see."*
   Answer: H gate - Creates equal superposition, $H^2 = I$.

5. *"A quarter turn around the Z, apply me four times, if you please."*
   Answer: S gate - $S^4 = I$.

6. *"One-eighth of a full rotation, apply me eight times for restoration."*
   Answer: T gate - $T^8 = I$.

Learning objective: Understanding that gates have specific orders (how many times you apply them to return to identity).

Single qubit gates are the simplest yet essential building blocks of quantum circuits. Since they act on a single qubit, at most they can only affect the state of a qubit locally and therefore cannot generate entanglement between qubits [1]. Still they are important as they allow for local manipulation of qubit states.

Level 2 Riddles and Solutions

1. X Gate - "Turn 1 into 0 and 0 into 1"

2. Y Gate - "Now lets rotate about the Y"

3. Z Gate - "Without changing your chance, lets do the phase dance"

4. S Gate - "Pass through the gate that takes half the amount of times as the longest gate" (T takes 8, so S takes 4, which is half)

Target State: The final state after applying X $\rightarrow$ Y $\rightarrow$ Z $\rightarrow$ S to the initial state.
Learning Objectives: Apply the gates learned in Level 1 in the correct sequence

[1] Which as we will see later is what really makes quantum stuff powerful

3 Level 3: Rotation Gates and Universality

3.1 Parameterized Rotation Gates

Continuous Rotations

Unlike the discrete Pauli gates, rotation gates allow continuous control:

$$R_x(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \tag{11}$$

$$R_y(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \tag{12}$$

$$R_z(\theta) = \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix} \tag{13}$$

Continuous rotations can be thought of as the equivalent of how we move around our when we walk. Rather than taking discrete steps, we can take any continuous value. Similarly, in the quantum world, this allows us to move the Bloch vector in any direction on the sphere.

Level 3 Riddles and Solutions

Use the three rotation gates Rx, Ry, and Rz gates with precise angle control. The target state requires specific rotation angles. So experiment with different angle combinations to reach the target.

1. $Rx(\theta)$: Rotation around X-axis by angle
2. $Ry(\theta)$: Rotation around Y-axis by angle
3. $Rz(\theta)$: Rotation around Z-axis by angle

Learning objective: Observe the target Bloch vector position. Use Ry to adjust latitude. Use Rz to adjust longitude and finally fine-tune with Rx if needed

3.2 Universal Gate Sets

Wouldn't it be great if instead of all these gates we could have just a few handful ones to do all the movement we want? Well, we're in luck because that's exactly what universal gate sets do! For a given qubit system size they correspond to the set of gates that can perform any movement! Here's the exact definition:

Quantum Universality

A set of gates is **universal** if any quantum computation can be approximated to arbitrary precision using only those gates.
One universal set consists of:

- $R_y(\theta)$ - Rotation around Y-axis
- $R_z(\phi)$ - Rotation around Z-axis
- Phase gate: $\begin{pmatrix} 1 & 0 \\ 0 & e^{i\delta} \end{pmatrix}$

Universal Decomposition

Any single-qubit unitary can be written as:

$$U = e^{i\alpha} R_z(\beta) R_y(\gamma) R_z(\delta) \tag{14}$$

Level 4 Solution Hint

The game asks you to recreate a target state using the universal gates. The key insight is that you need exactly the sequence $e^{i\phi} R_z(\gamma) R_y(\beta) R_z(\alpha)$ where the parameters are multiples of π.
Try values like:

- $\alpha = 0.4\pi$
- $\beta = 0.3\pi$
- $\gamma = 0.2\pi$
- $\phi = 0.1\pi$

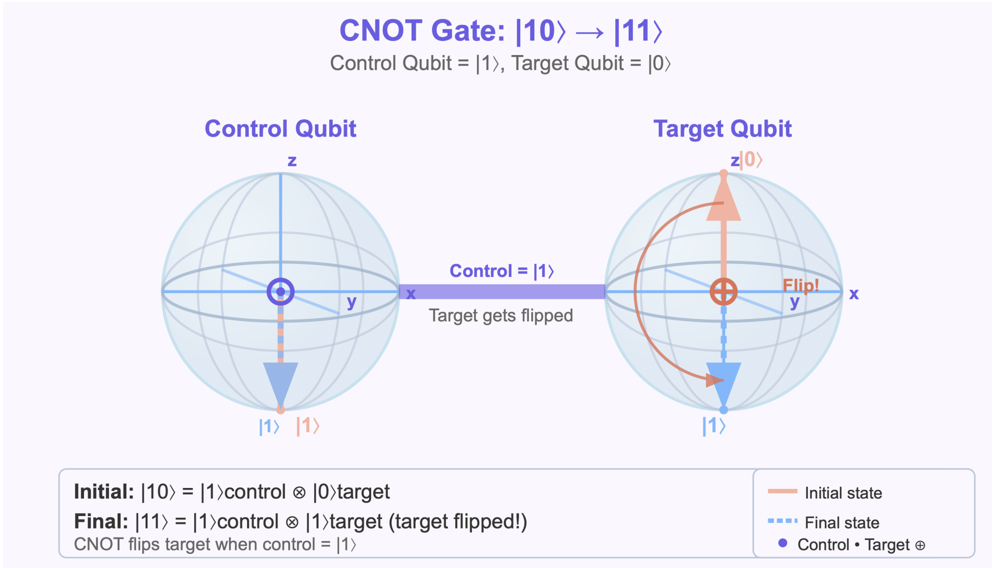
4 Levels 4 and 5: Two-Qubit Gates and Entanglement

4.1 Two-Qubit Systems

A two-qubit system has four basis states: $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$.

The general state is:

$$|\psi\rangle = \alpha_{00} |00\rangle + \alpha_{01} |01\rangle + \alpha_{10} |10\rangle + \alpha_{11} |11\rangle \tag{15}$$



4.2 Entangling Gates

CNOT Gate

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \tag{16}$$

The CNOT gate flips the target qubit if the control qubit is $|1\rangle$:

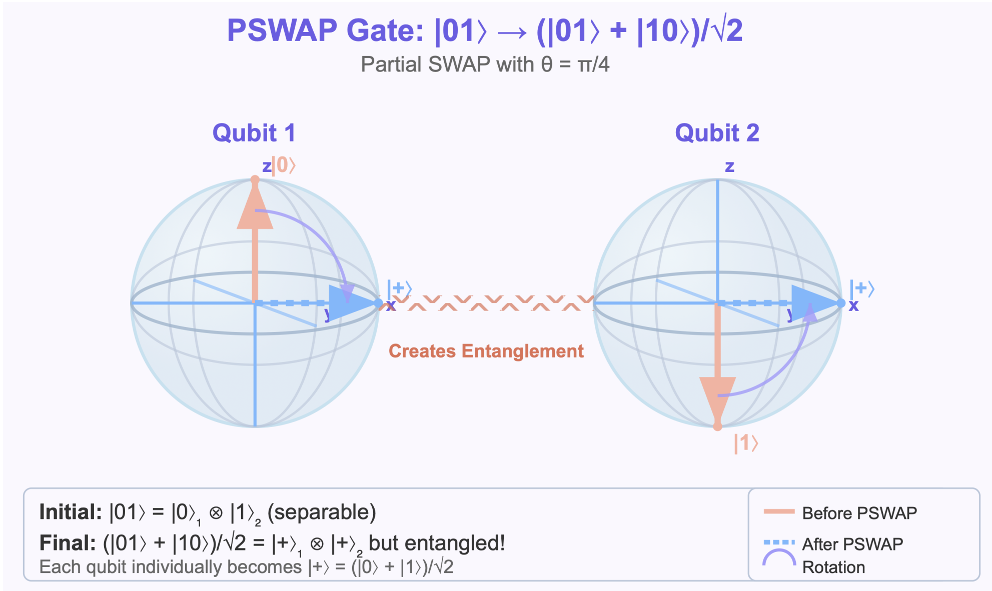
$$\mathrm{CNOT}\,|00\rangle = |00\rangle \tag{17}$$
$$\mathrm{CNOT}\,|01\rangle = |01\rangle \tag{18}$$
$$\mathrm{CNOT}\,|10\rangle = |11\rangle \tag{19}$$
$$\mathrm{CNOT}\,|11\rangle = |10\rangle \tag{20}$$

Did you know

The CNOT gate along with the H S and T single qubit gates forms a universal gate set implying that any two qubit rotation can be written as a combination of these gates.

Parameterized SWAP (PSWAP)

$$\mathrm{PSWAP}(\phi) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\phi & i\sin\phi & 0 \\ 0 & i\sin\phi & \cos\phi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{21}$$



This gate partially swaps two qubits with a phase factor. This gate can be used to generate or destroy entanglement. Starting with a pure state of two qubits where one qubit is in $|1\rangle$ state and the other in $|0\rangle$ state, the gate can generate bell state. Conversely, starting from entangled state, the qubits would have ended up en-entangled.

Controlled Rotation (CR)

$$\mathrm{CR}(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\theta} \end{pmatrix} \tag{22}$$

Applies rotation $e^{i\theta}$ to the target qubit only when the control is $|1\rangle$.

4.3 Bell States and Maximum Entanglement

Bell States

The four maximally entangled two-qubit states:

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{23}$$

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle) \tag{24}$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \tag{25}$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \tag{26}$$

These states cannot be written as a product of single-qubit states - they are *entangled*.

4.4 Creating Bell States

Bell State Circuit

To create the Bell state $|\Phi^+\rangle$ from $|00\rangle$:

1. Apply Hadamard to first qubit: $H \otimes I\,|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle)$
2. Apply CNOT: $\text{CNOT}\frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

5 Density Matrices and Mixed States

5.1 Pure vs Mixed States

Density Matrix Formalism

For a pure state $|\psi\rangle$, the density matrix is:

$$\rho = |\psi\rangle\langle\psi| \tag{27}$$

For mixed states (statistical mixtures), we have:

$$\rho = \sum_i p_i\,|\psi_i\rangle\langle\psi_i| \tag{28}$$

where p_i are classical probabilities.

5.2 Reduced Density Matrices

For entangled systems, we often look at subsystems using the **partial trace**:

If we have a two-qubit state ρ_{AB}, the reduced density matrix for qubit A is:

$$\rho_A = \mathrm{Tr}_B(\rho_{AB}) \tag{29}$$

Important Note

Even if the full two-qubit state is pure (represented by a point on the Bloch sphere), the individual qubits may be in mixed states when entangled. This is visualized in the game as the individual Bloch spheres showing mixed states when the qubits are entangled.

Levels 5+: Entanglement Challenges

- Coordinate actions between both qubits
- Use CNOT to create initial entanglement
- Apply CR gates for controlled rotations
- Use PSWAP for partial state exchange
- Remember: individual qubit states become mixed when entangled

5.3 Advanced Strategy Tips

Expert Tips

1. **Plan your sequence**: Write down the target state and work backwards
2. **Use symmetry**: Many quantum operations have elegant symmetries
3. **Check progress**: Monitor the distance to target state continuously
4. **Undo mistakes**: Remember which gates are self-inverse
5. **Coordinate timing**: In two-qubit levels, timing of operations matters

6 Mathematical Appendix

6.1 Matrix Representations

Single-Qubit Gates

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{30}$$

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{31}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{32}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{33}$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{34}$$

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \tag{35}$$

$$T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \tag{36}$$

Rotation Gates

$$R_x(\theta) = \cos\frac{\theta}{2} I - i \sin\frac{\theta}{2} X \tag{37}$$

$$R_y(\theta) = \cos\frac{\theta}{2} I - i \sin\frac{\theta}{2} Y \tag{38}$$

$$R_z(\theta) = \cos\frac{\theta}{2} I - i \sin\frac{\theta}{2} Z \tag{39}$$

6.2 Bloch Sphere Coordinates

For a qubit state $|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$, the Bloch vector components are:

$$r_x = 2\text{Re}(\alpha^* \beta) \tag{40}$$

$$r_y = 2\text{Im}(\alpha^* \beta) \tag{41}$$

$$r_z = |\alpha|^2 - |\beta|^2 \tag{42}$$

The length of the Bloch vector is $|\vec{r}| = 1$ for pure states.

6.3 Useful Identities

$$XYZ = iI \tag{43}$$
$$HXH = Z \tag{44}$$
$$HZH = X \tag{45}$$
$$SXS^\dagger = Y \tag{46}$$
$$R_z(\theta) X R_z^\dagger(\theta) = \cos\theta\, X + \sin\theta\, Y \tag{47}$$

A Quick Reference

A.1 Gate Cheat Sheet

Gate	**Matrix**	**Action**	**Order**
X	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	Bit flip	$X^2 = I$
Y	$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$	Bit+phase flip	$Y^2 = I$
Z	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	Phase flip	$Z^2 = I$
H	$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	Superposition	$H^2 = I$
S	$\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$	Quarter phase	$S^4 = I$
T	$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$	Eighth phase	$T^8 = I$

Table 1: Single-qubit gate reference

A.2 Common Quantum States

$$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = H\,|0\rangle \tag{48}$$
$$|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = H\,|1\rangle \tag{49}$$
$$|{+i}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle) \tag{50}$$
$$|{-i}\rangle = \frac{1}{\sqrt{2}}(|0\rangle - i\,|1\rangle) \tag{51}$$