3 Fluid and Solid Mechanics

3.6 Particle Drift in a Periodic Flow Field (4 units)

This project builds on material in the Part IB Fluid Dynamics course.

A one-dimensional periodic flow in a fluid has velocity u in the x-direction only, given by

$$u = \alpha \cos k(x - ct). \tag{1}$$

A material fluid element subject to this motion will have trajectory $X(t)$ satisfying

$$\frac{dX}{dt} = \alpha \cos k(X(t) - ct). \tag{2}$$

Question 1 Explain why, without loss of generality, distance and time units may be chosen so that $k = 2\pi$ and $c = 1$, giving

$$\frac{dX}{dt} = a \cos 2\pi(X(t) - t). \tag{3}$$

How is a related to α?

Question 2 Solve (3) numerically for a representative set of values of a, taking $X(0) = 0$. Describe your results qualitatively, and plot the solutions against time. You can use your own ODE integrator, or alternatively one such as the MATLAB function `ode45`. In either case you should justify the accuracy of your results (for example, by considering results produced with different step-sizes or tolerances). What if $X(0) \neq 0$?

Question 3 Verify from your numerical results that for $|a|$ sufficiently small, there is a time-averaged mean 'drift' velocity of $\frac{1}{2}a^2$. Include details of your method.

Question 4 Give a *physical* interpretation of the interaction between the flow and the material element. Do not confine your answer only to small $|a|$.

Hint: You may wish to consider a graph of $\frac{dX}{dt}$ against X.

Question 5 Analyse mathematically the above system, using any approach you see fit, e.g. in the case of question 3 you might seek an approximate solution for small $|a|$.

